

Computer algebra independent integration tests

1-Algebraic-functions/1.2-Trinomial-products/1.2.2-Quartic/1.2.2.6-P-x-d-x-^m-a+b-x²+c-x⁴-^p

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September 20, 2021

Compiled on September 20, 2021 at 3:13pm

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Chapter 1

Introduction

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This report gives the result of running the computer algebra independent integration test. Elementary Algebraic integrals version.

The download section below contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [143]. This is test number [30].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.3.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.45 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)
9. IntegrateAlgebraic under Mathematica 12.3.1 on windows 10. https://github.com/stblake/algebraic_integration. September 15, 2021 version.

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (143)	0.00 (0)
Mathematica	100.00 (143)	0.00 (0)
Maple	100.00 (143)	0.00 (0)
Mupad	100.00 (143)	0.00 (0)
Giac	97.20 (139)	2.80 (4)
Fricas	80.42 (115)	19.58 (28)
Sympy	55.24 (79)	% 44.76 (64)
Maxima	51.05 (73)	48.95 (70)
IntegrateAlgebraic	9.79 (14)	90.21 (129)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

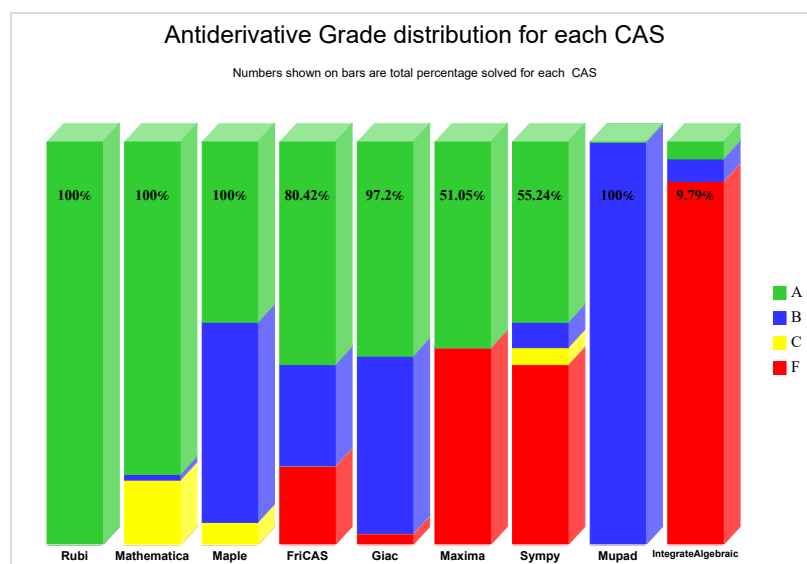
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

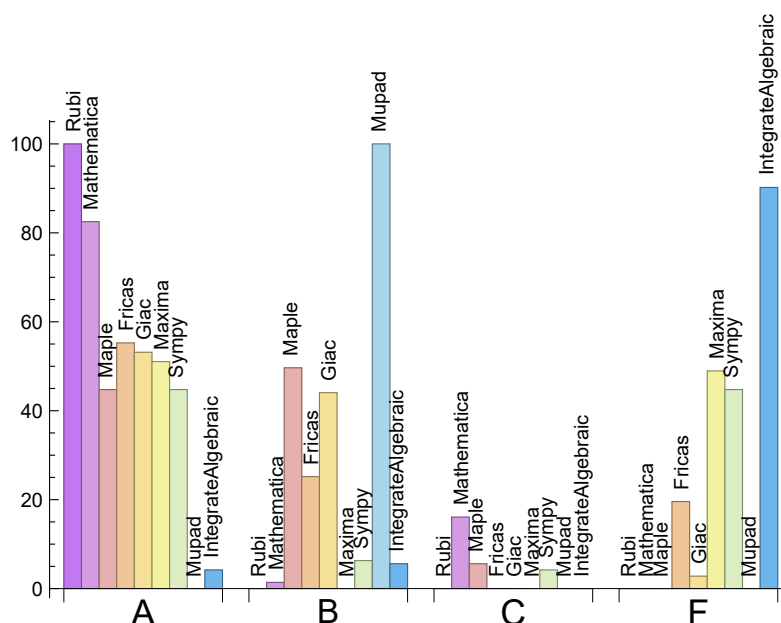
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	82.52	1.40	16.08	0.00
Fricas	55.24	25.17	0.00	19.58
Giac	53.15	44.06	0.00	2.80
Maxima	51.05	0.00	0.00	48.95
Maple	44.76	49.65	5.59	0.00
Sympy	44.76	6.29	4.20	44.76
IntegrateAlgebraic	4.20	5.59	0.00	90.21
Mupad	N/A	100.00	0.00	0.00

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This is the typical normal failure **F**.

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	0	0.00 %	0.00 %	0.00 %
Fricas	28	0.00 %	100.00 %	0.00 %
IntegrateAlgebraic	129	100.00 %	0.00 %	0.00 %
Giac	4	0.00 %	0.00 %	100.00 %
Maxima	70	77.14 %	0.00 %	22.86 %
Sympy	64	0.00 %	100.00 %	0.00 %
Mupad	0	0.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

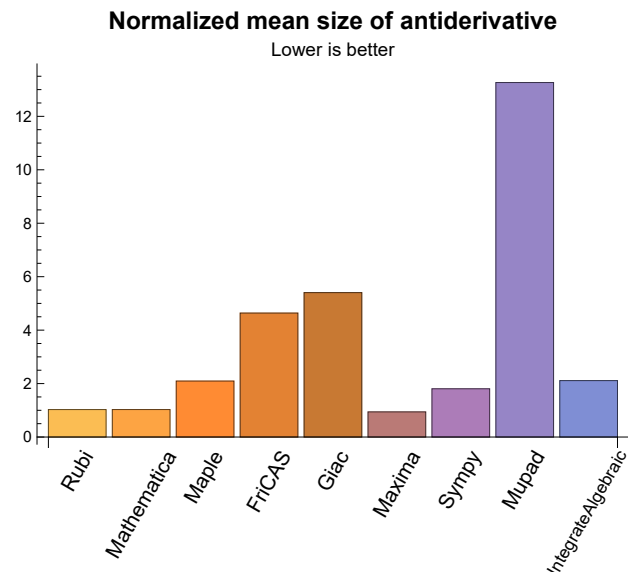
1.3 Performance

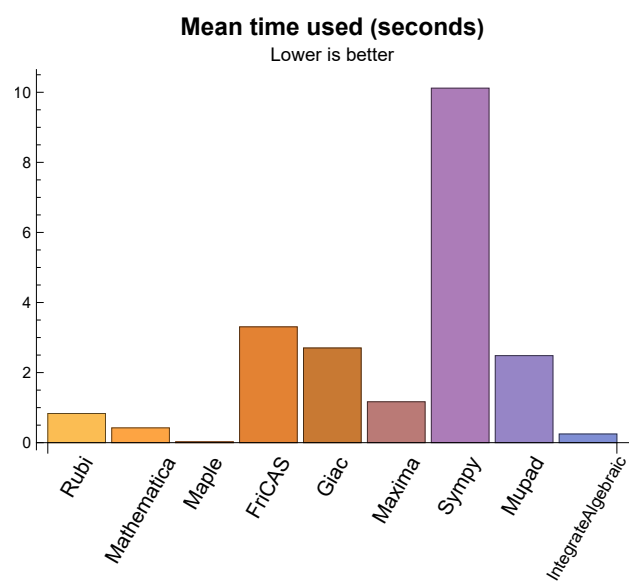
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.83	203.12	1.03	174.00	1.00
Mathematica	0.42	207.08	1.03	142.00	1.00
Maple	0.03	585.73	2.09	227.00	1.68
Maxima	1.17	104.12	0.94	65.00	0.88
Fricas	3.31	1240.70	4.64	137.00	1.37
Sympy	10.12	261.04	1.81	71.00	0.98
Giac	2.70	1805.94	5.40	227.00	1.22
Mupad	2.48	4593.26	13.26	176.00	0.96
IntegrateAlgebraic	0.25	345.43	2.11	329.00	2.08

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

IntegrateAlgebraic {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {}

IntegrateAlgebraic {}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error Exception raised: NotImplementedError

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the builtin system function LeafSize is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user slelievre at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

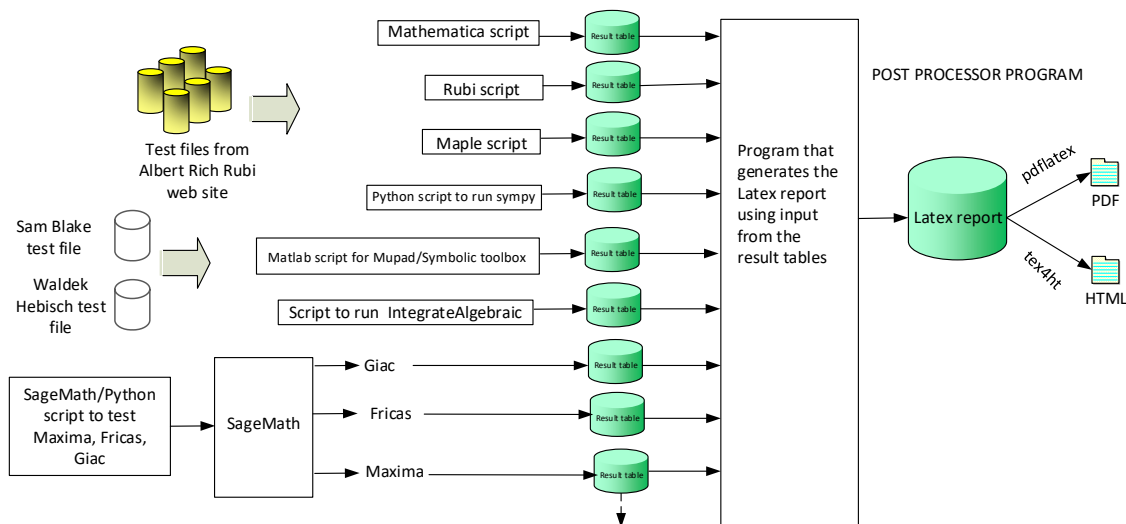
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine, 'cos(x)*sin(x)')
the_variable = evalin(symengine, 'x')
anti = int(integrand, the_variable)
```

Which gives $\sin(x) \sim 2/2$

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer. the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
The following field present only in Rubi and Mathematica Tables
13. integer. 1 if result was verified or 0 if not verified.
The following fields present only in Rubi Tables
14. integer. Number of rules used.
15. integer. Integrand leaf size.
16. real number. Ratio of field 14 over field 15
17. integer. 1 if result was verified or 0 if not verified.
18. String of form "{n,n,...}" which is list of the rules used by Rubi

High level overview of the CAS independent integration test build system

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May 11, 2021

Chapter 2

detailed summary tables of results

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2.1 List of integrals sorted by grade for each CAS

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2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 123, 124, 125, 126, 127, 128, 129, 135, 136, 137, 138, 139, 140, 141, 142, 143 }

B grade: { 133, 134 }

C grade: { 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 130, 131, 132 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 49, 62, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 129, 130, 131, 132, 141, 142, 143 }

B grade: { 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 63, 64, 65, 66, 67, 68, 69, 70, 71, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128 }

C grade: { 133, 134, 135, 136, 137, 138, 139, 140 }

F grade: { }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 37, 38, 39, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143 }

B grade: { }

C grade: { }

F grade: { 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 45, 46, 47, 48, 49, 50, 51, 52, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 123, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143 }

B grade: { 37, 38, 39, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 67, 68, 69, 70, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122 }

C grade: { }

F grade: { 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 40, 41, 42, 43, 44, 66, 71, 124, 125, 126, 127, 128 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 39, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 113, 116, 117, 121, 122 }

B grade: { 62, 108, 111, 112, 114, 115, 118, 119, 120 }

C grade: { 131, 132, 133, 134, 139, 140 }

F grade: { 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 63, 64, 65, 66, 67, 68, 69, 70, 71, 123, 124, 125, 126, 127, 128, 129, 130, 135, 136, 137, 138, 141, 142, 143 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 45, 46, 47, 48, 49, 50, 51, 52, 59, 60, 61, 62, 63, 64, 65, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 123, 130, 131, 132, 137, 138 }

B grade: { 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 53, 54, 55, 56, 57, 58, 66, 67, 68, 69, 70, 71, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 124, 125, 126, 127, 128, 129, 139, 140, 141, 142, 143 }

C grade: { }

F grade: { 133, 134, 135, 136 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143 }

C grade: { }

F grade: { }

2.1.9 Integrate Algebraic

A grade: { 135, 136, 137, 138, 139, 140 }

B grade: { 130, 131, 132, 133, 134, 141, 142, 143 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column N.S. in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, Mathematica was abbreviated to MMA and IntegrateAlgebraic to I.A.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	74	74	61	60	64	68	64	62	0
N.S.	1	1.00	1.00	0.82	0.81	0.86	0.92	0.86	0.84	0.00
time (sec)	N/A	0.082	0.015	0.000	0.452	1.225	0.071	0.286	0.036	0.000
Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	74	74	61	60	64	68	64	62	0
N.S.	1	1.00	1.00	0.82	0.81	0.86	0.92	0.86	0.84	0.00
time (sec)	N/A	0.057	0.011	0.003	0.465	1.171	0.072	0.227	0.030	0.000
Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	69	69	58	57	61	65	61	59	0
N.S.	1	1.00	1.00	0.84	0.83	0.88	0.94	0.88	0.86	0.00
time (sec)	N/A	0.036	0.012	0.000	0.455	1.062	0.069	0.377	0.029	0.000
Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	65	60	55	55	63	60	57	0
N.S.	1	1.00	1.00	0.92	0.85	0.85	0.97	0.92	0.88	0.00
time (sec)	N/A	0.040	0.015	0.002	0.604	1.287	0.157	0.372	0.036	0.001
Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	63	57	55	62	58	57	56	0
N.S.	1	1.00	1.00	0.90	0.87	0.98	0.92	0.90	0.89	0.00
time (sec)	N/A	0.051	0.022	0.006	0.680	1.234	0.175	0.299	0.037	0.001

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	58	58	55	62	61	58	56	0
N.S.	1	1.00	0.92	0.92	0.87	0.98	0.97	0.92	0.89	0.00
time (sec)	N/A	0.048	0.038	0.010	0.752	1.325	0.294	0.290	0.035	0.001
Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	60	57	56	62	63	56	55	0
N.S.	1	1.00	0.95	0.90	0.89	0.98	1.00	0.89	0.87	0.00
time (sec)	N/A	0.051	0.045	0.007	0.659	1.255	0.521	0.369	0.033	0.001
Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	62	58	56	62	63	57	56	0
N.S.	1	1.00	0.98	0.92	0.89	0.98	1.00	0.90	0.89	0.00
time (sec)	N/A	0.051	0.027	0.006	0.752	1.380	1.755	0.288	0.048	0.001
Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	63	60	56	62	66	57	56	0
N.S.	1	1.00	1.00	0.95	0.89	0.98	1.05	0.90	0.89	0.00
time (sec)	N/A	0.052	0.054	0.007	0.739	1.283	5.698	0.259	0.777	0.001
Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	68	63	59	62	70	60	60	0
N.S.	1	1.00	1.00	0.93	0.87	0.91	1.03	0.88	0.88	0.00
time (sec)	N/A	0.048	0.045	0.006	0.874	0.971	15.378	0.390	0.790	0.001
Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	159	159	159	142	143	154	168	154	141	0
N.S.	1	1.00	1.00	0.89	0.90	0.97	1.06	0.97	0.89	0.00
time (sec)	N/A	0.214	0.044	0.001	1.134	1.141	0.093	0.406	0.817	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	159	159	159	142	143	154	163	154	141	0
N.S.	1	1.00	1.00	0.89	0.90	0.97	1.03	0.97	0.89	0.00
time (sec)	N/A	0.143	0.036	0.001	1.182	1.093	0.094	0.306	0.068	0.000
Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	154	154	154	139	140	151	165	151	138	0
N.S.	1	1.00	1.00	0.90	0.91	0.98	1.07	0.98	0.90	0.00
time (sec)	N/A	0.111	0.029	0.002	0.629	0.978	0.093	0.300	0.070	0.000
Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	150	150	150	149	138	138	156	149	135	0
N.S.	1	1.00	1.00	0.99	0.92	0.92	1.04	0.99	0.90	0.00
time (sec)	N/A	0.107	0.038	0.004	0.843	1.361	0.306	0.364	0.799	0.001
Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	145	145	145	147	137	145	156	147	135	0
N.S.	1	1.00	1.00	1.01	0.94	1.00	1.08	1.01	0.93	0.00
time (sec)	N/A	0.121	0.091	0.007	0.734	1.527	0.322	0.282	0.797	0.001
Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	149	149	139	148	139	145	153	148	135	0
N.S.	1	1.00	0.93	0.99	0.93	0.97	1.03	0.99	0.91	0.00
time (sec)	N/A	0.123	0.094	0.007	0.622	1.260	0.460	0.395	0.792	0.001
Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	149	149	151	146	140	145	160	146	137	0
N.S.	1	1.00	1.01	0.98	0.94	0.97	1.07	0.98	0.92	0.00
time (sec)	N/A	0.137	0.077	0.009	0.681	1.381	0.719	0.284	0.059	0.001

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	148	148	130	144	139	145	153	142	134	0
N.S.	1	1.00	0.88	0.97	0.94	0.98	1.03	0.96	0.91	0.00
time (sec)	N/A	0.142	0.081	0.008	0.624	1.672	2.349	0.377	0.058	0.001
Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	143	143	142	144	138	145	155	140	136	0
N.S.	1	1.00	0.99	1.01	0.97	1.01	1.08	0.98	0.95	0.00
time (sec)	N/A	0.147	0.077	0.009	0.612	1.474	7.809	0.292	0.054	0.001
Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	149	149	144	148	140	145	158	141	136	0
N.S.	1	1.00	0.97	0.99	0.94	0.97	1.06	0.95	0.91	0.00
time (sec)	N/A	0.143	0.092	0.009	0.688	1.264	27.402	0.396	0.057	0.001
Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	339	339	460	1622	0	0	0	5304	2588	0
N.S.	1	1.00	1.36	4.78	0.00	0.00	0.00	15.65	7.63	0.00
time (sec)	N/A	1.856	0.572	0.065	0.000	0.000	0.000	5.749	0.958	0.001
Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	278	278	377	1171	0	0	0	3519	2696	0
N.S.	1	1.00	1.36	4.21	0.00	0.00	0.00	12.66	9.70	0.00
time (sec)	N/A	0.466	0.420	0.054	0.000	0.000	0.000	5.025	1.527	0.001
Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	270	270	360	1327	0	0	0	3843	1890	0
N.S.	1	1.00	1.33	4.91	0.00	0.00	0.00	14.23	7.00	0.00
time (sec)	N/A	0.835	0.366	0.050	0.000	0.000	0.000	5.566	2.001	0.001

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	223	223	240	728	0	0	0	2369	5594	0
N.S.	1	1.00	1.08	3.26	0.00	0.00	0.00	10.62	25.09	0.00
time (sec)	N/A	0.213	0.359	0.040	0.000	0.000	0.000	5.359	1.885	0.001
Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	211	211	234	616	0	0	0	1616	3942	0
N.S.	1	1.00	1.11	2.92	0.00	0.00	0.00	7.66	18.68	0.00
time (sec)	N/A	0.266	0.209	0.025	0.000	0.000	0.000	4.372	2.306	0.001
Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	229	229	285	488	0	0	0	2336	2258	0
N.S.	1	1.00	1.24	2.13	0.00	0.00	0.00	10.20	9.86	0.00
time (sec)	N/A	0.259	0.445	0.037	0.000	0.000	0.000	5.075	1.493	0.001
Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	260	260	315	811	0	0	0	3507	2588	0
N.S.	1	1.00	1.21	3.12	0.00	0.00	0.00	13.49	9.95	0.00
time (sec)	N/A	0.471	1.080	0.040	0.000	0.000	0.000	5.393	1.022	0.001
Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	288	288	377	1054	0	0	0	3353	3563	0
N.S.	1	1.00	1.31	3.66	0.00	0.00	0.00	11.64	12.37	0.00
time (sec)	N/A	0.474	0.889	0.056	0.000	0.000	0.000	5.872	1.172	0.001
Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	412	412	444	1429	0	0	0	5219	4754	0
N.S.	1	1.00	1.08	3.47	0.00	0.00	0.00	12.67	11.54	0.00
time (sec)	N/A	1.334	1.382	0.062	0.000	0.000	0.000	8.471	1.774	0.001

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	347	347	358	831	0	0	0	3228	3278	0
N.S.	1	1.00	1.03	2.39	0.00	0.00	0.00	9.30	9.45	0.00
time (sec)	N/A	0.618	0.894	0.044	0.000	0.000	0.000	5.369	1.613	0.001
Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	356	356	378	1119	0	0	0	4440	3835	0
N.S.	1	1.00	1.06	3.14	0.00	0.00	0.00	12.47	10.77	0.00
time (sec)	N/A	0.902	1.039	0.055	0.000	0.000	0.000	7.049	1.671	0.001
Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	317	317	335	1344	0	0	0	3013	3198	0
N.S.	1	1.00	1.06	4.24	0.00	0.00	0.00	9.50	10.09	0.00
time (sec)	N/A	0.415	1.249	0.177	0.000	0.000	0.000	5.173	1.595	0.001
Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	368	368	393	1813	0	0	0	5158	4707	0
N.S.	1	1.00	1.07	4.93	0.00	0.00	0.00	14.02	12.79	0.00
time (sec)	N/A	0.867	1.224	0.151	0.000	0.000	0.000	7.849	1.675	0.001
Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	403	403	458	1603	0	0	0	6022	8129	0
N.S.	1	1.00	1.14	3.98	0.00	0.00	0.00	14.94	20.17	0.00
time (sec)	N/A	0.932	1.473	0.063	0.000	0.000	0.000	6.553	1.838	0.001
Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	514	514	559	2398	0	0	0	9015	8684	0
N.S.	1	1.00	1.09	4.67	0.00	0.00	0.00	17.54	16.89	0.00
time (sec)	N/A	1.486	2.027	0.085	0.000	0.000	0.000	11.545	2.468	0.001

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	534	534	655	2512	0	0	0	6938	10595	0
N.S.	1	1.00	1.23	4.70	0.00	0.00	0.00	12.99	19.84	0.00
time (sec)	N/A	1.992	2.470	0.096	0.000	0.000	0.000	7.565	2.773	0.001
Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	399	399	296	5520	611	3898	0	7808	2443	0
N.S.	1	1.00	0.74	13.83	1.53	9.77	0.00	19.57	6.12	0.00
time (sec)	N/A	0.425	0.919	0.013	1.706	2.258	0.000	1.131	3.280	1.786
Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	260	260	185	2187	344	1603	0	3203	1314	0
N.S.	1	1.00	0.71	8.41	1.32	6.17	0.00	12.32	5.05	0.00
time (sec)	N/A	0.223	0.277	0.010	1.731	1.656	0.000	0.725	1.810	1.368
Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	137	137	90	585	155	444	3735	914	527	0
N.S.	1	1.00	0.66	4.27	1.13	3.24	27.26	6.67	3.85	0.00
time (sec)	N/A	0.088	0.104	0.005	0.815	1.291	2.575	0.533	1.075	0.644
Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	356	356	378	1119	0	0	0	4440	3835	0
N.S.	1	1.00	1.06	3.14	0.00	0.00	0.00	12.47	10.77	0.00
time (sec)	N/A	0.924	1.038	0.000	0.000	0.000	0.000	7.038	0.004	0.001
Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	356	356	378	1119	0	0	0	4440	3835	0
N.S.	1	1.00	1.06	3.14	0.00	0.00	0.00	12.47	10.77	0.00
time (sec)	N/A	0.371	0.425	0.036	0.000	0.000	0.000	7.135	1.551	0.001

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	356	356	378	1119	0	0	0	4439	3835	0
N.S.	1	1.00	1.06	3.14	0.00	0.00	0.00	12.47	10.77	0.00
time (sec)	N/A	0.370	0.154	0.039	0.000	0.000	0.000	6.896	1.474	0.001
Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	356	356	378	1119	0	0	0	4439	3835	0
N.S.	1	1.00	1.06	3.14	0.00	0.00	0.00	12.47	10.77	0.00
time (sec)	N/A	0.356	0.154	0.034	0.000	0.000	0.000	7.311	1.389	0.001
Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	356	356	378	1119	0	0	0	4439	3835	0
N.S.	1	1.00	1.06	3.14	0.00	0.00	0.00	12.47	10.77	0.00
time (sec)	N/A	0.359	0.153	0.033	0.000	0.000	0.000	6.836	1.414	0.001
Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	273	273	260	622	0	900	0	306	2972	0
N.S.	1	1.00	0.95	2.28	0.00	3.30	0.00	1.12	10.89	0.00
time (sec)	N/A	0.854	0.199	0.007	0.000	2.562	0.000	1.868	1.604	0.001
Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	203	203	193	474	0	677	0	214	2295	0
N.S.	1	1.00	0.95	2.33	0.00	3.33	0.00	1.05	11.31	0.00
time (sec)	N/A	0.424	0.143	0.006	0.000	2.388	0.000	2.005	1.626	0.001
Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	144	144	136	321	0	473	0	141	1689	0
N.S.	1	1.00	0.94	2.23	0.00	3.28	0.00	0.98	11.73	0.00
time (sec)	N/A	0.272	0.099	0.006	0.000	1.373	0.000	1.987	1.300	0.001

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	103	103	100	211	0	318	0	99	1081	0
N.S.	1	1.00	0.97	2.05	0.00	3.09	0.00	0.96	10.50	0.00
time (sec)	N/A	0.179	0.066	0.004	0.000	1.128	0.000	1.780	1.830	0.001
Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	178	165	0	309	0	97	3927	0
N.S.	1	1.00	1.84	1.70	0.00	3.19	0.00	1.00	40.48	0.00
time (sec)	N/A	0.200	0.137	0.009	0.000	1.750	0.000	1.903	8.881	0.001
Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	118	118	203	227	0	399	0	135	4437	0
N.S.	1	1.00	1.72	1.92	0.00	3.38	0.00	1.14	37.60	0.00
time (sec)	N/A	0.285	0.150	0.010	0.000	2.145	0.000	1.777	7.857	0.001
Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	174	174	314	356	0	609	0	212	6187	0
N.S.	1	1.00	1.80	2.05	0.00	3.50	0.00	1.22	35.56	0.00
time (sec)	N/A	0.407	0.345	0.012	0.000	3.384	0.000	1.719	9.917	0.001
Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	244	244	416	523	0	834	0	313	9141	0
N.S.	1	1.00	1.70	2.14	0.00	3.42	0.00	1.28	37.46	0.00
time (sec)	N/A	0.573	0.348	0.013	0.000	6.639	0.000	1.941	13.829	0.001
Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	369	369	456	1450	0	15467	0	7243	23332	0
N.S.	1	1.00	1.24	3.93	0.00	41.92	0.00	19.63	63.23	0.00
time (sec)	N/A	4.577	0.509	0.035	0.000	42.586	0.000	5.025	4.912	0.001

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	282	282	365	1035	0	9364	0	5461	15674	0
N.S.	1	1.00	1.29	3.67	0.00	33.21	0.00	19.37	55.58	0.00
time (sec)	N/A	3.590	0.492	0.030	0.000	7.852	0.000	4.755	3.359	0.001
Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	219	219	258	676	0	5788	0	4086	10209	0
N.S.	1	1.00	1.18	3.09	0.00	26.43	0.00	18.66	46.62	0.00
time (sec)	N/A	0.637	0.326	0.027	0.000	4.073	0.000	3.908	3.360	0.001
Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	213	213	253	563	0	5930	0	3988	10170	0
N.S.	1	1.00	1.19	2.64	0.00	27.84	0.00	18.72	47.75	0.00
time (sec)	N/A	0.839	0.302	0.025	0.000	2.209	0.000	5.936	3.515	0.001
Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	267	267	284	727	0	9850	0	3813	15505	0
N.S.	1	1.00	1.06	2.72	0.00	36.89	0.00	14.28	58.07	0.00
time (sec)	N/A	1.065	0.339	0.027	0.000	11.248	0.000	3.442	4.763	0.001
Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	329	329	394	1121	0	15830	0	6718	23019	0
N.S.	1	1.00	1.20	3.41	0.00	48.12	0.00	20.42	69.97	0.00
time (sec)	N/A	1.942	0.551	0.033	0.000	40.458	0.000	7.015	6.247	0.001
Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	320	320	309	1167	0	2111	0	424	3499	0
N.S.	1	1.00	0.97	3.65	0.00	6.60	0.00	1.32	10.93	0.00
time (sec)	N/A	1.233	0.497	0.024	0.000	1.867	0.000	1.952	1.333	0.001

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	236	236	236	832	0	1455	0	279	2450	0
N.S.	1	1.00	1.00	3.53	0.00	6.17	0.00	1.18	10.38	0.00
time (sec)	N/A	0.440	0.356	0.017	0.000	1.149	0.000	1.865	1.811	0.001
Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	165	165	175	336	0	970	0	195	1651	0
N.S.	1	1.00	1.06	2.04	0.00	5.88	0.00	1.18	10.01	0.00
time (sec)	N/A	0.287	0.249	0.015	0.000	1.004	0.000	1.838	2.717	0.001
Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	123	123	130	205	0	650	474	140	342	0
N.S.	1	1.00	1.06	1.67	0.00	5.28	3.85	1.14	2.78	0.00
time (sec)	N/A	0.184	0.102	0.012	0.000	0.885	38.035	2.167	0.378	0.001
Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	166	166	268	462	0	1103	0	227	8706	0
N.S.	1	1.00	1.61	2.78	0.00	6.64	0.00	1.37	52.45	0.00
time (sec)	N/A	0.394	0.445	0.017	0.000	3.328	0.000	2.001	11.849	0.001
Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	234	234	403	722	0	1764	0	287	11879	0
N.S.	1	1.00	1.72	3.09	0.00	7.54	0.00	1.23	50.76	0.00
time (sec)	N/A	0.725	0.658	0.023	0.000	7.267	0.000	1.846	12.979	0.001
Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	329	329	592	1078	0	2567	0	535	15905	0
N.S.	1	1.00	1.80	3.28	0.00	7.80	0.00	1.63	48.34	0.00
time (sec)	N/A	1.157	1.216	0.029	0.000	15.903	0.000	1.888	21.016	0.001

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	550	550	648	2558	0	0	0	8957	33799	0
N.S.	1	1.00	1.18	4.65	0.00	0.00	0.00	16.29	61.45	0.00
time (sec)	N/A	13.227	2.131	0.056	0.000	0.000	0.000	9.044	4.104	0.001
Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	436	436	511	1977	0	12597	0	7496	25862	0
N.S.	1	1.00	1.17	4.53	0.00	28.89	0.00	17.19	59.32	0.00
time (sec)	N/A	5.541	1.542	0.052	0.000	31.024	0.000	8.250	2.648	0.001
Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	362	362	414	1300	0	8951	0	6208	19494	0
N.S.	1	1.00	1.14	3.59	0.00	24.73	0.00	17.15	53.85	0.00
time (sec)	N/A	2.498	1.103	0.043	0.000	15.053	0.000	6.811	6.543	0.001
Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	346	346	382	1182	0	8991	0	6356	19589	0
N.S.	1	1.00	1.10	3.42	0.00	25.99	0.00	18.37	56.62	0.00
time (sec)	N/A	1.896	1.081	0.040	0.000	17.136	0.000	6.973	6.552	0.001
Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	399	399	444	1575	0	13111	0	7182	28164	0
N.S.	1	1.00	1.11	3.95	0.00	32.86	0.00	18.00	70.59	0.00
time (sec)	N/A	2.203	1.319	0.048	0.000	40.367	0.000	7.093	6.862	0.001
Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	575	575	548	2180	0	0	0	8660	36097	0
N.S.	1	1.00	0.95	3.79	0.00	0.00	0.00	15.06	62.78	0.00
time (sec)	N/A	9.906	1.797	0.065	0.000	0.000	0.000	8.591	7.370	0.001

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	62	56	58	82	61	63	57	0
N.S.	1	1.00	0.91	0.82	0.85	1.21	0.90	0.93	0.84	0.00
time (sec)	N/A	0.126	0.028	0.017	0.602	1.416	0.172	0.362	0.056	0.001
Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	61	51	53	77	56	58	53	0
N.S.	1	1.00	1.00	0.84	0.87	1.26	0.92	0.95	0.87	0.00
time (sec)	N/A	0.118	0.027	0.016	0.719	1.641	0.171	0.323	0.039	0.001
Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	54	46	48	72	48	53	47	0
N.S.	1	1.00	1.00	0.85	0.89	1.33	0.89	0.98	0.87	0.00
time (sec)	N/A	0.108	0.024	0.016	1.075	1.225	0.173	0.367	0.897	0.001
Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	49	41	43	67	44	45	43	0
N.S.	1	1.00	1.00	0.84	0.88	1.37	0.90	0.92	0.88	0.00
time (sec)	N/A	0.086	0.022	0.015	0.514	1.181	0.174	0.390	0.038	0.001
Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	42	42	36	38	57	36	40	37	0
N.S.	1	1.00	1.00	0.86	0.90	1.36	0.86	0.95	0.88	0.00
time (sec)	N/A	0.049	0.017	0.016	0.522	1.235	0.166	0.354	0.049	0.001
Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	44	38	44	71	41	47	40	0
N.S.	1	1.00	1.00	0.86	1.00	1.61	0.93	1.07	0.91	0.00
time (sec)	N/A	0.077	0.022	0.016	0.724	1.470	0.183	0.377	0.041	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	55	55	50	45	53	92	51	53	50	0
N.S.	1	1.00	0.91	0.82	0.96	1.67	0.93	0.96	0.91	0.00
time (sec)	N/A	0.104	0.025	0.017	0.792	1.345	0.205	0.374	0.044	0.001
Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	64	64	56	50	56	97	56	66	55	0
N.S.	1	1.00	0.88	0.78	0.88	1.52	0.88	1.03	0.86	0.00
time (sec)	N/A	0.111	0.027	0.019	0.679	1.216	0.213	0.341	0.919	0.001
Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	71	56	58	79	68	58	58	0
N.S.	1	1.00	1.01	0.80	0.83	1.13	0.97	0.83	0.83	0.00
time (sec)	N/A	0.085	0.044	0.012	1.639	1.340	0.210	0.331	0.952	0.001
Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	58	49	51	74	54	51	50	0
N.S.	1	1.00	1.02	0.86	0.89	1.30	0.95	0.89	0.88	0.00
time (sec)	N/A	0.082	0.048	0.012	1.621	1.309	0.207	0.314	0.054	0.001
Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	57	46	48	69	54	48	48	0
N.S.	1	1.00	1.02	0.82	0.86	1.23	0.96	0.86	0.86	0.00
time (sec)	N/A	0.073	0.042	0.013	1.635	1.103	0.210	0.306	0.918	0.001
Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	50	41	43	64	48	43	42	0
N.S.	1	1.00	1.02	0.84	0.88	1.31	0.98	0.88	0.86	0.00
time (sec)	N/A	0.066	0.038	0.011	1.633	1.312	0.207	0.340	0.068	0.001

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	46	38	40	59	46	40	40	0
N.S.	1	1.00	0.96	0.79	0.83	1.23	0.96	0.83	0.83	0.00
time (sec)	N/A	0.028	0.040	0.012	1.529	1.515	0.203	0.339	0.072	0.001
Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	51	43	45	68	49	45	45	0
N.S.	1	1.00	0.96	0.81	0.85	1.28	0.92	0.85	0.85	0.00
time (sec)	N/A	0.073	0.049	0.013	1.554	1.160	0.219	0.384	0.070	0.001
Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	62	62	56	48	52	79	56	52	51	0
N.S.	1	1.00	0.90	0.77	0.84	1.27	0.90	0.84	0.82	0.00
time (sec)	N/A	0.084	0.051	0.015	1.597	0.609	0.240	0.386	0.923	0.001
Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	69	61	53	57	84	61	57	57	0
N.S.	1	1.00	0.88	0.77	0.83	1.22	0.88	0.83	0.83	0.00
time (sec)	N/A	0.090	0.058	0.017	1.764	1.021	0.254	0.343	0.916	0.001
Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	76	76	77	58	62	89	66	62	61	0
N.S.	1	1.00	1.01	0.76	0.82	1.17	0.87	0.82	0.80	0.00
time (sec)	N/A	0.100	0.055	0.016	1.528	1.297	0.276	0.450	0.074	0.001
Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	81	71	64	71	114	75	61	70	0
N.S.	1	1.00	0.88	0.79	0.88	1.41	0.93	0.75	0.86	0.00
time (sec)	N/A	0.112	0.056	0.013	1.584	1.232	0.258	0.317	0.059	0.001

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	80	66	62	68	109	76	58	68	0
N.S.	1	1.00	0.82	0.78	0.85	1.36	0.95	0.72	0.85	0.00
time (sec)	N/A	0.100	0.054	0.013	1.914	1.322	0.256	0.414	0.052	0.001
Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	75	75	60	56	63	104	70	53	63	0
N.S.	1	1.00	0.80	0.75	0.84	1.39	0.93	0.71	0.84	0.00
time (sec)	N/A	0.091	0.061	0.013	1.850	1.187	0.255	0.324	0.928	0.001
Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	72	72	55	54	60	99	65	50	59	0
N.S.	1	1.00	0.76	0.75	0.83	1.38	0.90	0.69	0.82	0.00
time (sec)	N/A	0.068	0.058	0.011	1.527	0.690	0.262	0.345	0.928	0.001
Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	72	72	56	53	60	99	66	50	60	0
N.S.	1	1.00	0.78	0.74	0.83	1.38	0.92	0.69	0.83	0.00
time (sec)	N/A	0.066	0.061	0.013	1.744	1.397	0.251	0.379	0.071	0.001
Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	72	72	56	53	60	99	65	50	59	0
N.S.	1	1.00	0.78	0.74	0.83	1.38	0.90	0.69	0.82	0.00
time (sec)	N/A	0.037	0.059	0.013	1.671	1.270	0.248	0.328	0.070	0.001
Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	63	58	65	108	71	55	65	0
N.S.	1	1.00	0.80	0.73	0.82	1.37	0.90	0.70	0.82	0.00
time (sec)	N/A	0.103	0.065	0.014	1.797	1.269	0.276	0.338	0.920	0.001

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	86	78	64	72	119	76	62	71	0
N.S.	1	1.00	0.91	0.74	0.84	1.38	0.88	0.72	0.83	0.00
time (sec)	N/A	0.119	0.059	0.016	1.527	0.963	0.293	0.361	0.923	0.001
Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	93	93	73	68	77	124	82	67	77	0
N.S.	1	1.00	0.78	0.73	0.83	1.33	0.88	0.72	0.83	0.00
time (sec)	N/A	0.134	0.076	0.015	1.737	1.249	0.312	0.356	0.935	0.001
Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	86	78	74	71	95	87	76	75	0
N.S.	1	1.00	0.91	0.86	0.83	1.10	1.01	0.88	0.87	0.00
time (sec)	N/A	0.135	0.045	0.010	1.333	1.137	0.183	1.018	0.901	0.001
Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	81	73	69	66	90	80	71	69	0
N.S.	1	1.00	0.90	0.85	0.81	1.11	0.99	0.88	0.85	0.00
time (sec)	N/A	0.127	0.030	0.009	1.824	1.121	0.185	1.169	0.051	0.001
Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	74	66	64	59	85	73	66	65	0
N.S.	1	1.00	0.89	0.86	0.80	1.15	0.99	0.89	0.88	0.00
time (sec)	N/A	0.121	0.029	0.009	1.585	1.052	0.180	1.090	0.045	0.001
Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	61	59	54	80	68	54	60	0
N.S.	1	1.00	0.94	0.91	0.83	1.23	1.05	0.83	0.92	0.00
time (sec)	N/A	0.105	0.026	0.010	1.718	1.001	0.181	1.184	0.919	0.001

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	58	58	54	49	70	60	49	69	0
N.S.	1	1.00	1.00	0.93	0.84	1.21	1.03	0.84	1.19	0.00
time (sec)	N/A	0.067	0.022	0.010	1.406	1.330	0.179	1.133	0.048	0.001
Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	93	58	55	84	65	62	59	0
N.S.	1	1.00	1.41	0.88	0.83	1.27	0.98	0.94	0.89	0.00
time (sec)	N/A	0.108	0.062	0.012	1.685	0.840	0.198	1.103	0.907	0.001
Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	71	71	97	63	66	105	76	66	68	0
N.S.	1	1.00	1.37	0.89	0.93	1.48	1.07	0.93	0.96	0.00
time (sec)	N/A	0.134	0.050	0.013	1.427	1.190	0.212	1.072	0.061	0.001
Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	80	105	68	71	110	80	79	72	0
N.S.	1	1.00	1.31	0.85	0.89	1.38	1.00	0.99	0.90	0.00
time (sec)	N/A	0.137	0.060	0.014	2.381	1.067	0.226	1.165	0.060	0.001
Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	87	110	73	76	115	85	84	78	0
N.S.	1	1.00	1.26	0.84	0.87	1.32	0.98	0.97	0.90	0.00
time (sec)	N/A	0.149	0.066	0.014	2.474	1.215	0.241	1.165	0.065	0.001
Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	248	248	145	427	0	519	71	585	171	0
N.S.	1	1.00	0.58	1.72	0.00	2.09	0.29	2.36	0.69	0.00
time (sec)	N/A	0.345	0.172	0.119	0.000	1.306	0.611	1.887	0.106	0.001

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	237	237	132	419	0	476	1205	576	164	0
N.S.	1	1.00	0.56	1.77	0.00	2.01	5.08	2.43	0.69	0.00
time (sec)	N/A	0.293	0.158	0.032	0.000	1.304	1.360	1.851	0.943	0.001
Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	232	232	129	416	0	508	60	573	162	0
N.S.	1	1.00	0.56	1.79	0.00	2.19	0.26	2.47	0.70	0.00
time (sec)	N/A	0.292	0.159	0.030	0.000	1.376	0.615	1.854	0.095	0.001
Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	225	225	121	412	0	459	51	566	156	0
N.S.	1	1.00	0.54	1.83	0.00	2.04	0.23	2.52	0.69	0.00
time (sec)	N/A	0.297	0.164	0.034	0.000	1.222	0.602	1.825	0.958	0.001
Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	224	224	115	408	0	454	1185	565	153	0
N.S.	1	1.00	0.51	1.82	0.00	2.03	5.29	2.52	0.68	0.00
time (sec)	N/A	0.215	0.264	0.029	0.000	1.408	1.291	1.818	0.127	0.001
Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	229	229	126	414	0	471	1192	572	159	0
N.S.	1	1.00	0.55	1.81	0.00	2.06	5.21	2.50	0.69	0.00
time (sec)	N/A	0.310	0.176	0.033	0.000	1.347	1.323	1.937	0.136	0.001
Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	238	238	131	419	0	528	60	579	165	0
N.S.	1	1.00	0.55	1.76	0.00	2.22	0.25	2.43	0.69	0.00
time (sec)	N/A	0.335	0.291	0.035	0.000	1.362	0.653	1.851	0.137	0.001

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	245	245	140	424	0	496	1202	584	171	0
N.S.	1	1.00	0.57	1.73	0.00	2.02	4.91	2.38	0.70	0.00
time (sec)	N/A	0.329	0.290	0.034	0.000	1.445	1.332	1.785	0.143	0.001
Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	243	243	156	429	0	561	1204	588	184	0
N.S.	1	1.00	0.64	1.77	0.00	2.31	4.95	2.42	0.76	0.00
time (sec)	N/A	0.360	0.216	0.035	0.000	1.355	1.350	2.694	0.111	0.001
Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	242	242	155	426	0	557	82	585	182	0
N.S.	1	1.00	0.64	1.76	0.00	2.30	0.34	2.42	0.75	0.00
time (sec)	N/A	0.310	0.204	0.033	0.000	1.380	0.680	2.588	0.942	0.001
Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	235	235	138	422	0	551	71	580	176	0
N.S.	1	1.00	0.59	1.80	0.00	2.34	0.30	2.47	0.75	0.00
time (sec)	N/A	0.300	0.317	0.035	0.000	1.389	0.656	2.612	0.992	0.001
Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	238	238	129	418	0	546	1198	577	173	0
N.S.	1	1.00	0.54	1.76	0.00	2.29	5.03	2.42	0.73	0.00
time (sec)	N/A	0.290	0.297	0.032	0.000	1.402	1.306	2.600	0.146	0.001
Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	246	246	133	418	0	570	1200	577	174	0
N.S.	1	1.00	0.54	1.70	0.00	2.32	4.88	2.35	0.71	0.00
time (sec)	N/A	0.284	0.297	0.032	0.000	1.352	1.332	2.688	1.012	0.001

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	248	248	129	418	0	576	1195	577	173	0
N.S.	1	1.00	0.52	1.69	0.00	2.32	4.82	2.33	0.70	0.00
time (sec)	N/A	0.254	0.291	0.032	0.000	1.335	1.337	2.618	1.008	0.001
Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	253	253	140	424	0	630	75	582	179	0
N.S.	1	1.00	0.55	1.68	0.00	2.49	0.30	2.30	0.71	0.00
time (sec)	N/A	0.343	0.366	0.034	0.000	1.387	0.672	3.266	0.993	0.001
Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	262	262	139	429	0	652	80	589	185	0
N.S.	1	1.00	0.53	1.64	0.00	2.49	0.31	2.25	0.71	0.00
time (sec)	N/A	0.366	0.315	0.036	0.000	1.042	0.689	2.991	1.022	0.001
Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	149	149	142	357	0	486	0	146	1834	0
N.S.	1	1.00	0.95	2.40	0.00	3.26	0.00	0.98	12.31	0.00
time (sec)	N/A	0.295	0.116	0.006	0.000	1.593	0.000	1.898	1.685	0.001
Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	594	594	721	3028	0	0	0	10761	47339	0
N.S.	1	1.00	1.21	5.10	0.00	0.00	0.00	18.12	79.70	0.00
time (sec)	N/A	14.113	2.641	0.060	0.000	0.000	0.000	9.953	4.731	0.001
Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	471	471	575	2300	0	0	0	9170	36589	0
N.S.	1	1.00	1.22	4.88	0.00	0.00	0.00	19.47	77.68	0.00
time (sec)	N/A	6.662	1.946	0.047	0.000	0.000	0.000	8.921	4.218	0.001

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	449	449	512	1760	0	0	0	8913	32587	0
N.S.	1	1.00	1.14	3.92	0.00	0.00	0.00	19.85	72.58	0.00
time (sec)	N/A	2.867	1.651	0.053	0.000	0.000	0.000	8.514	5.821	0.001
Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	460	460	529	2045	0	0	0	9176	40860	0
N.S.	1	1.00	1.15	4.45	0.00	0.00	0.00	19.95	88.83	0.00
time (sec)	N/A	2.791	2.434	0.055	0.000	0.000	0.000	8.303	7.761	0.001
Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	542	542	612	2503	0	0	0	10422	51386	0
N.S.	1	1.00	1.13	4.62	0.00	0.00	0.00	19.23	94.81	0.00
time (sec)	N/A	7.265	2.153	0.067	0.000	0.000	0.000	9.055	8.467	0.001
Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	20	21	31	31	0	58	31	0
N.S.	1	1.00	1.00	1.05	1.55	1.55	0.00	2.90	1.55	0.00
time (sec)	N/A	0.036	0.141	0.010	1.028	1.057	0.000	0.609	1.098	0.362
Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	210	278	265	145	295	138	0	269	287	615
N.S.	1	1.32	1.26	0.69	1.40	0.66	0.00	1.28	1.37	2.93
time (sec)	N/A	0.315	1.379	0.009	1.026	1.360	0.000	0.894	1.645	0.371
Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	159	213	232	109	217	104	367	194	215	477
N.S.	1	1.34	1.46	0.69	1.36	0.65	2.31	1.22	1.35	3.00
time (sec)	N/A	0.189	1.069	0.007	1.062	1.246	135.140	0.749	1.494	0.286

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	109	149	194	73	139	71	350	121	143	329
N.S.	1	1.37	1.78	0.67	1.28	0.65	3.21	1.11	1.31	3.02
time (sec)	N/A	0.122	0.691	0.006	1.032	1.394	90.666	0.626	1.377	0.210
Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	C	A	A	C	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	93	151	217	143	105	80	304	0	161	199
N.S.	1	1.62	2.33	1.54	1.13	0.86	3.27	0.00	1.73	2.14
time (sec)	N/A	0.165	0.880	0.042	1.004	1.337	91.279	0.000	2.949	0.196
Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	C	A	A	C	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	99	155	233	163	123	98	270	0	422	223
N.S.	1	1.57	2.35	1.65	1.24	0.99	2.73	0.00	4.26	2.25
time (sec)	N/A	0.252	0.213	0.024	1.017	1.406	133.790	0.000	5.151	0.255
Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F(-1)	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	126	182	134	222	193	102	0	0	932	207
N.S.	1	1.44	1.06	1.76	1.53	0.81	0.00	0.00	7.40	1.64
time (sec)	N/A	0.277	0.161	0.030	1.034	1.235	0.000	0.000	10.816	0.236
Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F(-1)	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	212	248	173	306	271	137	0	0	1621	397
N.S.	1	1.17	0.82	1.44	1.28	0.65	0.00	0.00	7.65	1.87
time (sec)	N/A	0.373	0.192	0.036	1.033	1.300	0.000	0.000	20.051	0.444
Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	216	245	202	273	190	134	0	208	1132	401
N.S.	1	1.13	0.94	1.26	0.88	0.62	0.00	0.96	5.24	1.86
time (sec)	N/A	0.205	0.791	0.037	1.106	1.282	0.000	0.749	23.121	0.295

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	128	179	157	191	113	100	0	135	651	207
N.S.	1	1.40	1.23	1.49	0.88	0.78	0.00	1.05	5.09	1.62
time (sec)	N/A	0.091	0.562	0.020	1.018	1.247	0.000	0.595	12.861	0.187
Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	C	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	102	155	135	148	73	90	287	257	306	192
N.S.	1	1.52	1.32	1.45	0.72	0.88	2.81	2.52	3.00	1.88
time (sec)	N/A	0.122	0.555	0.023	1.052	1.177	104.024	1.046	7.002	0.161
Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	C	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	157	157	81	146	85	90	257	555	138	168
N.S.	1	1.00	0.52	0.93	0.54	0.57	1.64	3.54	0.88	1.07
time (sec)	N/A	0.125	0.124	0.026	1.005	1.223	116.433	1.467	2.268	0.151
Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	160	160	87	82	148	76	0	1103	146	329
N.S.	1	1.00	0.54	0.51	0.92	0.48	0.00	6.89	0.91	2.06
time (sec)	N/A	0.145	0.118	0.005	1.019	1.144	0.000	2.546	1.732	0.171
Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	226	226	124	118	226	110	0	1517	218	477
N.S.	1	1.00	0.55	0.52	1.00	0.49	0.00	6.71	0.96	2.11
time (sec)	N/A	0.178	0.142	0.007	1.010	1.521	0.000	4.731	1.817	0.224
Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	292	292	158	154	304	144	0	1931	290	615
N.S.	1	1.00	0.54	0.53	1.04	0.49	0.00	6.61	0.99	2.11
time (sec)	N/A	0.242	0.176	0.007	1.031	2.019	0.000	7.348	1.873	0.283

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [35] had the largest ratio of [.4643]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	1	1.00	26	0.038
2	A	2	1	1.00	24	0.042
3	A	2	1	1.00	23	0.043
4	A	2	1	1.00	26	0.038
5	A	2	1	1.00	26	0.038
6	A	2	1	1.00	26	0.038
7	A	2	1	1.00	26	0.038
8	A	2	1	1.00	26	0.038
9	A	2	1	1.00	26	0.038
10	A	2	1	1.00	26	0.038
11	A	2	1	1.00	28	0.036
12	A	2	1	1.00	26	0.038
13	A	2	1	1.00	25	0.040
14	A	2	1	1.00	28	0.036
15	A	2	1	1.00	28	0.036
16	A	2	1	1.00	28	0.036
17	A	2	1	1.00	28	0.036
18	A	2	1	1.00	28	0.036
19	A	2	1	1.00	28	0.036
20	A	2	1	1.00	28	0.036
21	A	13	11	1.00	28	0.393
22	A	12	11	1.00	28	0.393
23	A	11	10	1.00	28	0.357
24	A	10	9	1.00	26	0.346
25	A	8	7	1.00	25	0.280
26	A	12	10	1.00	28	0.357
27	A	13	12	1.00	28	0.429
28	A	13	11	1.00	28	0.393
29	A	11	10	1.00	28	0.357
30	A	10	9	1.00	28	0.321
31	A	10	9	1.00	28	0.321
32	A	10	9	1.00	26	0.346
33	A	10	9	1.00	25	0.360
34	A	14	12	1.00	28	0.429
35	A	15	13	1.00	28	0.464
36	A	15	13	1.00	28	0.464
37	A	2	1	1.00	30	0.033
38	A	2	1	1.00	30	0.033
39	A	2	1	1.00	28	0.036

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
40	A	10	9	1.00	28	0.321
41	A	11	10	1.00	30	0.333
42	A	11	10	1.00	31	0.323
43	A	11	10	1.00	34	0.294
44	A	11	10	1.00	34	0.294
45	A	7	6	1.00	30	0.200
46	A	7	6	1.00	30	0.200
47	A	7	6	1.00	30	0.200
48	A	7	6	1.00	28	0.214
49	A	7	6	1.00	30	0.200
50	A	7	6	1.00	30	0.200
51	A	7	6	1.00	30	0.200
52	A	7	6	1.00	30	0.200
53	A	5	3	1.00	30	0.100
54	A	5	3	1.00	30	0.100
55	A	5	3	1.00	27	0.111
56	A	5	3	1.00	30	0.100
57	A	5	3	1.00	30	0.100
58	A	5	3	1.00	30	0.100
59	A	8	7	1.00	30	0.233
60	A	7	7	1.00	30	0.233
61	A	6	6	1.00	30	0.200
62	A	5	5	1.00	28	0.179
63	A	8	7	1.00	30	0.233
64	A	8	7	1.00	30	0.233
65	A	8	7	1.00	30	0.233
66	A	6	4	1.00	30	0.133
67	A	6	4	1.00	30	0.133
68	A	4	3	1.00	30	0.100
69	A	4	3	1.00	27	0.111
70	A	6	4	1.00	30	0.133
71	A	6	4	1.00	30	0.133
72	A	7	5	1.00	31	0.161
73	A	7	5	1.00	31	0.161
74	A	7	5	1.00	31	0.161
75	A	7	5	1.00	31	0.161
76	A	5	4	1.00	29	0.138
77	A	4	3	1.00	31	0.097
78	A	4	3	1.00	31	0.097
79	A	4	3	1.00	31	0.097
80	A	6	4	1.00	31	0.129
81	A	6	4	1.00	31	0.129
82	A	6	4	1.00	31	0.129
83	A	6	4	1.00	31	0.129
84	A	4	3	1.00	28	0.107
85	A	5	3	1.00	31	0.097
86	A	5	3	1.00	31	0.097
87	A	5	3	1.00	31	0.097

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
88	A	5	3	1.00	31	0.097
89	A	7	5	1.00	31	0.161
90	A	7	5	1.00	31	0.161
91	A	7	5	1.00	31	0.161
92	A	5	4	1.00	31	0.129
93	A	5	4	1.00	31	0.129
94	A	5	4	1.00	28	0.143
95	A	6	3	1.00	31	0.097
96	A	6	3	1.00	31	0.097
97	A	6	3	1.00	31	0.097
98	A	8	7	1.00	31	0.226
99	A	8	7	1.00	31	0.226
100	A	8	7	1.00	31	0.226
101	A	8	7	1.00	31	0.226
102	A	6	6	1.00	29	0.207
103	A	8	7	1.00	31	0.226
104	A	8	7	1.00	31	0.226
105	A	8	7	1.00	31	0.226
106	A	8	7	1.00	31	0.226
107	A	12	7	1.00	31	0.226
108	A	12	7	1.00	31	0.226
109	A	12	7	1.00	31	0.226
110	A	12	7	1.00	31	0.226
111	A	10	6	1.00	28	0.214
112	A	12	7	1.00	31	0.226
113	A	12	7	1.00	31	0.226
114	A	12	7	1.00	31	0.226
115	A	13	8	1.00	31	0.258
116	A	13	8	1.00	31	0.258
117	A	13	8	1.00	31	0.258
118	A	11	7	1.00	31	0.226
119	A	11	7	1.00	31	0.226
120	A	11	7	1.00	28	0.250
121	A	13	7	1.00	31	0.226
122	A	13	7	1.00	31	0.226
123	A	7	6	1.00	33	0.182
124	A	6	4	1.00	35	0.114
125	A	6	4	1.00	35	0.114
126	A	4	3	1.00	32	0.094
127	A	6	4	1.00	35	0.114
128	A	6	4	1.00	35	0.114
129	A	1	1	1.00	42	0.024
130	A	5	4	1.32	35	0.114
131	A	4	3	1.34	35	0.086
132	A	4	3	1.37	33	0.091
133	A	6	5	1.62	35	0.143
134	A	6	6	1.57	35	0.171
135	A	6	6	1.44	35	0.171

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
136	A	7	7	1.17	35	0.200
137	A	6	6	1.13	35	0.171
138	A	5	5	1.40	32	0.156
139	A	5	5	1.52	35	0.143
140	A	5	5	1.00	35	0.143
141	A	4	4	1.00	35	0.114
142	A	5	5	1.00	35	0.143
143	A	6	5	1.00	35	0.143

Chapter 3

Listing of integrals

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3.19	$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^6} dx$	103
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3.21	$\int \frac{x^4(A+Bx+Cx^2)}{a+bx^2+cx^4} dx$	109
3.22	$\int \frac{x^3(A+Bx+Cx^2)}{a+bx^2+cx^4} dx$	117
3.23	$\int \frac{x^2(A+Bx+Cx^2)}{a+bx^2+cx^4} dx$	124
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3.36	$\int \frac{A+Bx+Cx^2}{x^3(a+bx^2+cx^4)^2} dx$	226
3.37	$\int (dx)^m (A+Bx+Cx^2)(a+bx^2+cx^4)^3 dx$	238
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3.45	$\int \frac{x^7(d+ex^2+fx^4)}{a+bx^2+cx^4} dx$	299
3.46	$\int \frac{x^5(d+ex^2+fx^4)}{a+bx^2+cx^4} dx$	304
3.47	$\int \frac{x^3(d+ex^2+fx^4)}{a+bx^2+cx^4} dx$	309
3.48	$\int \frac{x(d+ex^2+fx^4)}{a+bx^2+cx^4} dx$	313
3.49	$\int \frac{d+ex^2+fx^4}{x(a+bx^2+cx^4)} dx$	317
3.50	$\int \frac{d+ex^2+fx^4}{x^3(a+bx^2+cx^4)} dx$	322
3.51	$\int \frac{d+ex^2+fx^4}{x^5(a+bx^2+cx^4)} dx$	327
3.52	$\int \frac{d+ex^2+fx^4}{x^7(a+bx^2+cx^4)} dx$	333
3.53	$\int \frac{x^4(d+ex^2+fx^4)}{a+bx^2+cx^4} dx$	340
3.54	$\int \frac{x^2(d+ex^2+fx^4)}{a+bx^2+cx^4} dx$	360
3.55	$\int \frac{d+ex^2+fx^4}{a+bx^2+cx^4} dx$	374
3.56	$\int \frac{d+ex^2+fx^4}{x^2(a+bx^2+cx^4)} dx$	384

3.57	$\int \frac{d+ex^2+fx^4}{x^4(a+bx^2+cx^4)} dx$	394
3.58	$\int \frac{d+ex^2+fx^4}{x^6(a+bx^2+cx^4)} dx$	408
3.59	$\int \frac{x^7(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2} dx$	428
3.60	$\int \frac{x^5(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2} dx$	434
3.61	$\int \frac{x^3(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2} dx$	439
3.62	$\int \frac{x(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2} dx$	444
3.63	$\int \frac{d+ex^2+fx^4}{x(a+bx^2+cx^4)^2} dx$	448
3.64	$\int \frac{d+ex^2+fx^4}{x^3(a+bx^2+cx^4)^2} dx$	455
3.65	$\int \frac{d+ex^2+fx^4}{x^5(a+bx^2+cx^4)^2} dx$	464
3.66	$\int \frac{x^6(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2} dx$	475
3.67	$\int \frac{x^4(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2} dx$	496
3.68	$\int \frac{x^2(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2} dx$	517
3.69	$\int \frac{d+ex^2+fx^4}{(a+bx^2+cx^4)^2} dx$	533
3.70	$\int \frac{d+ex^2+fx^4}{x^2(a+bx^2+cx^4)^2} dx$	549
3.71	$\int \frac{d+ex^2+fx^4}{x^4(a+bx^2+cx^4)^2} dx$	571
3.72	$\int \frac{x^9(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} dx$	593
3.73	$\int \frac{x^7(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} dx$	596
3.74	$\int \frac{x^5(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} dx$	599
3.75	$\int \frac{x^3(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} dx$	602
3.76	$\int \frac{x(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} dx$	605
3.77	$\int \frac{4+x^2+3x^4+5x^6}{x(2+3x^2+x^4)^2} dx$	608
3.78	$\int \frac{4+x^2+3x^4+5x^6}{x^3(2+3x^2+x^4)^2} dx$	611
3.79	$\int \frac{4+x^2+3x^4+5x^6}{x^5(2+3x^2+x^4)^2} dx$	614
3.80	$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} dx$	617
3.81	$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} dx$	620
3.82	$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} dx$	623
3.83	$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} dx$	626

3.84	$\int \frac{4+x^2+3x^4+5x^6}{(2+3x^2+x^4)^2} dx$	629
3.85	$\int \frac{4+x^2+3x^4+5x^6}{x^2(2+3x^2+x^4)^2} dx$	632
3.86	$\int \frac{4+x^2+3x^4+5x^6}{x^4(2+3x^2+x^4)^2} dx$	635
3.87	$\int \frac{4+x^2+3x^4+5x^6}{x^6(2+3x^2+x^4)^2} dx$	638
3.88	$\int \frac{4+x^2+3x^4+5x^6}{x^8(2+3x^2+x^4)^2} dx$	641
3.89	$\int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3} dx$	644
3.90	$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3} dx$	648
3.91	$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3} dx$	652
3.92	$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3} dx$	656
3.93	$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3} dx$	659
3.94	$\int \frac{4+x^2+3x^4+5x^6}{(2+3x^2+x^4)^3} dx$	662
3.95	$\int \frac{4+x^2+3x^4+5x^6}{x^2(2+3x^2+x^4)^3} dx$	665
3.96	$\int \frac{4+x^2+3x^4+5x^6}{x^4(2+3x^2+x^4)^3} dx$	668
3.97	$\int \frac{4+x^2+3x^4+5x^6}{x^6(2+3x^2+x^4)^3} dx$	671
3.98	$\int \frac{x^9(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2} dx$	674
3.99	$\int \frac{x^7(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2} dx$	678
3.100	$\int \frac{x^5(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2} dx$	682
3.101	$\int \frac{x^3(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2} dx$	686
3.102	$\int \frac{x(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2} dx$	690
3.103	$\int \frac{4+x^2+3x^4+5x^6}{x(3+2x^2+x^4)^2} dx$	693
3.104	$\int \frac{4+x^2+3x^4+5x^6}{x^3(3+2x^2+x^4)^2} dx$	697
3.105	$\int \frac{4+x^2+3x^4+5x^6}{x^5(3+2x^2+x^4)^2} dx$	701
3.106	$\int \frac{4+x^2+3x^4+5x^6}{x^7(3+2x^2+x^4)^2} dx$	705
3.107	$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2} dx$	709
3.108	$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2} dx$	714
3.109	$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2} dx$	719
3.110	$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2} dx$	724

3.111	$\int \frac{4+x^2+3x^4+5x^6}{(3+2x^2+x^4)^2} dx$	729
3.112	$\int \frac{4+x^2+3x^4+5x^6}{x^2(3+2x^2+x^4)^2} dx$	734
3.113	$\int \frac{4+x^2+3x^4+5x^6}{x^4(3+2x^2+x^4)^2} dx$	739
3.114	$\int \frac{4+x^2+3x^4+5x^6}{x^6(3+2x^2+x^4)^2} dx$	744
3.115	$\int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3} dx$	750
3.116	$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3} dx$	756
3.117	$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3} dx$	761
3.118	$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3} dx$	767
3.119	$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3} dx$	773
3.120	$\int \frac{4+x^2+3x^4+5x^6}{(3+2x^2+x^4)^3} dx$	778
3.121	$\int \frac{4+x^2+3x^4+5x^6}{x^2(3+2x^2+x^4)^3} dx$	783
3.122	$\int \frac{4+x^2+3x^4+5x^6}{x^4(3+2x^2+x^4)^3} dx$	788
3.123	$\int \frac{x(d+ex^2+fx^4+gx^6)}{a+bx^2+cx^4} dx$	793
3.124	$\int \frac{x^4(d+ex^2+fx^4+gx^6)}{(a+bx^2+cx^4)^2} dx$	797
3.125	$\int \frac{x^2(d+ex^2+fx^4+gx^6)}{(a+bx^2+cx^4)^2} dx$	824
3.126	$\int \frac{d+ex^2+fx^4+gx^6}{(a+bx^2+cx^4)^2} dx$	846
3.127	$\int \frac{d+ex^2+fx^4+gx^6}{x^2(a+bx^2+cx^4)^2} dx$	865
3.128	$\int \frac{d+ex^2+fx^4+gx^6}{x^4(a+bx^2+cx^4)^2} dx$	889
3.129	$\int x^2 (a + bx^2 + cx^4)^p (3a + b(5 + 2p)x^2 + c(7 + 4p)x^4) dx$	918
3.130	$\int \frac{x^5(a+bx^2+cx^4)}{\sqrt{d-ex} \sqrt{d+ex}} dx$	920
3.131	$\int \frac{x^3(a+bx^2+cx^4)}{\sqrt{d-ex} \sqrt{d+ex}} dx$	924
3.132	$\int \frac{x(a+bx^2+cx^4)}{\sqrt{d-ex} \sqrt{d+ex}} dx$	928
3.133	$\int \frac{a+bx^2+cx^4}{x \sqrt{d-ex} \sqrt{d+ex}} dx$	932
3.134	$\int \frac{a+bx^2+cx^4}{x^3 \sqrt{d-ex} \sqrt{d+ex}} dx$	936
3.135	$\int \frac{a+bx^2+cx^4}{x^5 \sqrt{d-ex} \sqrt{d+ex}} dx$	940
3.136	$\int \frac{a+bx^2+cx^4}{x^7 \sqrt{d-ex} \sqrt{d+ex}} dx$	945
3.137	$\int \frac{x^2(a+bx^2+cx^4)}{\sqrt{d-ex} \sqrt{d+ex}} dx$	951
3.138	$\int \frac{a+bx^2+cx^4}{\sqrt{d-ex} \sqrt{d+ex}} dx$	955
3.139	$\int \frac{a+bx^2+cx^4}{x^2 \sqrt{d-ex} \sqrt{d+ex}} dx$	959
3.140	$\int \frac{a+bx^2+cx^4}{x^4 \sqrt{d-ex} \sqrt{d+ex}} dx$	963

3.141	$\int \frac{a+bx^2+cx^4}{x^6\sqrt{d-ex}\sqrt{d+ex}} dx$	967
3.142	$\int \frac{a+bx^2+cx^4}{x^8\sqrt{d-ex}\sqrt{d+ex}} dx$	971
3.143	$\int \frac{a+bx^2+cx^4}{x^{10}\sqrt{d-ex}\sqrt{d+ex}} dx$	975

3.1 $\int x^2 (A + Bx + Cx^2) (a + bx^2 + cx^4) dx$

Optimal. Leaf size=74

$$\frac{1}{5}x^5(aC + Ab) + \frac{1}{3}aAx^3 + \frac{1}{4}aBx^4 + \frac{1}{7}x^7(Ac + bC) + \frac{1}{6}bBx^6 + \frac{1}{8}Bcx^8 + \frac{1}{9}cCx^9$$

Rubi [A] time = 0.08, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {1628}

$$\frac{1}{5}x^5(aC + Ab) + \frac{1}{3}aAx^3 + \frac{1}{4}aBx^4 + \frac{1}{7}x^7(Ac + bC) + \frac{1}{6}bBx^6 + \frac{1}{8}Bcx^8 + \frac{1}{9}cCx^9$$

Antiderivative was successfully verified.

[In] Int[x^2*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4), x]

[Out] (a*A*x^3)/3 + (a*B*x^4)/4 + ((A*b + a*C)*x^5)/5 + (b*B*x^6)/6 + ((A*c + b*C)*x^7)/7 + (B*c*x^8)/8 + (c*C*x^9)/9

Rule 1628

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int x^2 (A + Bx + Cx^2) (a + bx^2 + cx^4) dx &= \int (aAx^2 + aBx^3 + (Ab + aC)x^4 + bBx^5 + (Ac + bC)x^6 + Bcx^7 + \\ &= \frac{1}{3}aAx^3 + \frac{1}{4}aBx^4 + \frac{1}{5}(Ab + aC)x^5 + \frac{1}{6}bBx^6 + \frac{1}{7}(Ac + bC)x^7 + \frac{1}{8}Bcx^8 + \frac{1}{9}cCx^9 \end{aligned}$$

Mathematica [A] time = 0.02, size = 74, normalized size = 1.00

$$\frac{1}{5}x^5(aC + Ab) + \frac{1}{3}aAx^3 + \frac{1}{4}aBx^4 + \frac{1}{7}x^7(Ac + bC) + \frac{1}{6}bBx^6 + \frac{1}{8}Bcx^8 + \frac{1}{9}cCx^9$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4), x]

[Out] (a*A*x^3)/3 + (a*B*x^4)/4 + ((A*b + a*C)*x^5)/5 + (b*B*x^6)/6 + ((A*c + b*C)*x^7)/7 + (B*c*x^8)/8 + (c*C*x^9)/9

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 (A + Bx + Cx^2) (a + bx^2 + cx^4) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4), x]

[Out] IntegrateAlgebraic[x^2*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4), x]

fricas [A] time = 1.23, size = 64, normalized size = 0.86

$$\frac{1}{9}x^9cC + \frac{1}{8}x^8cB + \frac{1}{7}x^7bC + \frac{1}{7}x^7cA + \frac{1}{6}x^6bB + \frac{1}{5}x^5aC + \frac{1}{5}x^5bA + \frac{1}{4}x^4aB + \frac{1}{3}x^3aA$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] 1/9*x^9*c*C + 1/8*x^8*c*B + 1/7*x^7*b*C + 1/7*x^7*c*A + 1/6*x^6*b*B + 1/5*x^5*a*C + 1/5*x^5*b*A + 1/4*x^4*a*B + 1/3*x^3*a*A

giac [A] time = 0.29, size = 64, normalized size = 0.86

$$\frac{1}{9} Ccx^9 + \frac{1}{8} Bcx^8 + \frac{1}{7} Cbx^7 + \frac{1}{7} Acx^7 + \frac{1}{6} Bbx^6 + \frac{1}{5} Cax^5 + \frac{1}{5} Abx^5 + \frac{1}{4} Bax^4 + \frac{1}{3} Aax^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] 1/9*C*c*x^9 + 1/8*B*c*x^8 + 1/7*C*b*x^7 + 1/7*A*c*x^7 + 1/6*B*b*x^6 + 1/5*C*a*x^5 + 1/5*A*b*x^5 + 1/4*B*a*x^4 + 1/3*A*a*x^3

maple [A] time = 0.00, size = 61, normalized size = 0.82

$$\frac{Ccx^9}{9} + \frac{Bcx^8}{8} + \frac{Bbx^6}{6} + \frac{(Ac + bC)x^7}{7} + \frac{Bax^4}{4} + \frac{Aax^3}{3} + \frac{(Ab + aC)x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x)

[Out] 1/3*a*A*x^3+1/4*a*B*x^4+1/5*(A*b+C*a)*x^5+1/6*b*B*x^6+1/7*(A*c+C*b)*x^7+1/8*B*c*x^8+1/9*c*C*x^9

maxima [A] time = 0.45, size = 60, normalized size = 0.81

$$\frac{1}{9} Ccx^9 + \frac{1}{8} Bcx^8 + \frac{1}{6} Bbx^6 + \frac{1}{7} (Cb + Ac)x^7 + \frac{1}{4} Bax^4 + \frac{1}{5} (Ca + Ab)x^5 + \frac{1}{3} Aax^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] 1/9*C*c*x^9 + 1/8*B*c*x^8 + 1/6*B*b*x^6 + 1/7*(C*b + A*c)*x^7 + 1/4*B*a*x^4 + 1/5*(C*a + A*b)*x^5 + 1/3*A*a*x^3

mupad [B] time = 0.04, size = 62, normalized size = 0.84

$$\frac{Ccx^9}{9} + \frac{Bcx^8}{8} + \left(\frac{Ac}{7} + \frac{Cb}{7}\right)x^7 + \frac{Bbx^6}{6} + \left(\frac{Ab}{5} + \frac{Ca}{5}\right)x^5 + \frac{Bax^4}{4} + \frac{Aax^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x)

[Out] x^5*((A*b)/5 + (C*a)/5) + x^7*((A*c)/7 + (C*b)/7) + (A*a*x^3)/3 + (B*a*x^4)/4 + (B*b*x^6)/6 + (B*c*x^8)/8 + (C*c*x^9)/9

sympy [A] time = 0.07, size = 68, normalized size = 0.92

$$\frac{Aax^3}{3} + \frac{Bax^4}{4} + \frac{Bbx^6}{6} + \frac{Bcx^8}{8} + \frac{Ccx^9}{9} + x^7\left(\frac{Ac}{7} + \frac{Cb}{7}\right) + x^5\left(\frac{Ab}{5} + \frac{Ca}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(C*x**2+B*x+A)*(c*x**4+b*x**2+a),x)

[Out] A*a*x**3/3 + B*a*x**4/4 + B*b*x**6/6 + B*c*x**8/8 + C*c*x**9/9 + x**7*(A*c/7 + C*b/7) + x**5*(A*b/5 + C*a/5)

3.2 $\int x(A + Bx + Cx^2)(a + bx^2 + cx^4) dx$

Optimal. Leaf size=74

$$\frac{1}{4}x^4(aC + Ab) + \frac{1}{2}aAx^2 + \frac{1}{3}aBx^3 + \frac{1}{6}x^6(Ac + bC) + \frac{1}{5}bBx^5 + \frac{1}{7}Bcx^7 + \frac{1}{8}cCx^8$$

Rubi [A] time = 0.06, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {1628}

$$\frac{1}{4}x^4(aC + Ab) + \frac{1}{2}aAx^2 + \frac{1}{3}aBx^3 + \frac{1}{6}x^6(Ac + bC) + \frac{1}{5}bBx^5 + \frac{1}{7}Bcx^7 + \frac{1}{8}cCx^8$$

Antiderivative was successfully verified.

[In] Int[x*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4), x]

[Out] (a*A*x^2)/2 + (a*B*x^3)/3 + ((A*b + a*C)*x^4)/4 + (b*B*x^5)/5 + ((A*c + b*C)*x^6)/6 + (B*c*x^7)/7 + (c*C*x^8)/8

Rule 1628

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int x(A + Bx + Cx^2)(a + bx^2 + cx^4) dx &= \int (aAx + aBx^2 + (Ab + aC)x^3 + bBx^4 + (Ac + bC)x^5 + Bcx^6 + cCx^7) dx \\ &= \frac{1}{2}aAx^2 + \frac{1}{3}aBx^3 + \frac{1}{4}(Ab + aC)x^4 + \frac{1}{5}bBx^5 + \frac{1}{6}(Ac + bC)x^6 + \frac{1}{7}Bcx^7 + \frac{1}{8}cCx^8 \end{aligned}$$

Mathematica [A] time = 0.01, size = 74, normalized size = 1.00

$$\frac{1}{4}x^4(aC + Ab) + \frac{1}{2}aAx^2 + \frac{1}{3}aBx^3 + \frac{1}{6}x^6(Ac + bC) + \frac{1}{5}bBx^5 + \frac{1}{7}Bcx^7 + \frac{1}{8}cCx^8$$

Antiderivative was successfully verified.

[In] Integrate[x*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4), x]

[Out] (a*A*x^2)/2 + (a*B*x^3)/3 + ((A*b + a*C)*x^4)/4 + (b*B*x^5)/5 + ((A*c + b*C)*x^6)/6 + (B*c*x^7)/7 + (c*C*x^8)/8

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x(A + Bx + Cx^2)(a + bx^2 + cx^4) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4), x]

[Out] IntegrateAlgebraic[x*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4), x]

fricas [A] time = 1.17, size = 64, normalized size = 0.86

$$\frac{1}{8}x^8cC + \frac{1}{7}x^7cB + \frac{1}{6}x^6bC + \frac{1}{6}x^6cA + \frac{1}{5}x^5bB + \frac{1}{4}x^4aC + \frac{1}{4}x^4bA + \frac{1}{3}x^3aB + \frac{1}{2}x^2aA$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] $\frac{1}{8}cx^8 + \frac{1}{7}bcx^7 + \frac{1}{6}cbx^6 + \frac{1}{6}cx^6 + \frac{1}{5}bx^5 + \frac{1}{4}cax^4 + \frac{1}{4}abx^4 + \frac{1}{3}Bax^3 + \frac{1}{2}Aax^2$

giac [A] time = 0.23, size = 64, normalized size = 0.86

$$\frac{1}{8} Ccx^8 + \frac{1}{7} Bcx^7 + \frac{1}{6} Cbx^6 + \frac{1}{6} Acx^6 + \frac{1}{5} Bbx^5 + \frac{1}{4} Cax^4 + \frac{1}{4} Abx^4 + \frac{1}{3} Bax^3 + \frac{1}{2} Aax^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] $\frac{1}{8}Ccx^8 + \frac{1}{7}Bbcx^7 + \frac{1}{6}Ccbx^6 + \frac{1}{6}Acx^6 + \frac{1}{5}Bbx^5 + \frac{1}{4}Cax^4 + \frac{1}{4}Abx^4 + \frac{1}{3}Bax^3 + \frac{1}{2}Aax^2$

maple [A] time = 0.00, size = 61, normalized size = 0.82

$$\frac{Ccx^8}{8} + \frac{Bcx^7}{7} + \frac{Bbx^5}{5} + \frac{(Ac + bC)x^6}{6} + \frac{Bax^3}{3} + \frac{Aax^2}{2} + \frac{(Ab + aC)x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x)

[Out] $\frac{1}{2}Aax^2 + \frac{1}{3}Bax^3 + \frac{1}{4}(Aab + Cca)x^4 + \frac{1}{5}Bbx^5 + \frac{1}{6}(Acb + Cba)x^6 + \frac{1}{7}Bbcx^7 + \frac{1}{8}Ccx^8$

maxima [A] time = 0.47, size = 60, normalized size = 0.81

$$\frac{1}{8} Ccx^8 + \frac{1}{7} Bcx^7 + \frac{1}{5} Bbx^5 + \frac{1}{6} (Cb + Ac)x^6 + \frac{1}{3} Bax^3 + \frac{1}{4} (Ca + Ab)x^4 + \frac{1}{2} Aax^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] $\frac{1}{8}Ccx^8 + \frac{1}{7}Bbcx^7 + \frac{1}{5}Bcbx^5 + \frac{1}{6}(Ccb + Aca)x^6 + \frac{1}{3}Bax^3 + \frac{1}{4}(Cca + Aab)x^4 + \frac{1}{2}Aax^2$

mupad [B] time = 0.03, size = 62, normalized size = 0.84

$$\frac{Ccx^8}{8} + \frac{Bcx^7}{7} + \left(\frac{Ac}{6} + \frac{Cb}{6}\right)x^6 + \frac{Bbx^5}{5} + \left(\frac{Ab}{4} + \frac{Ca}{4}\right)x^4 + \frac{Bax^3}{3} + \frac{Aax^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x)

[Out] $x^4*((A*b)/4 + (C*a)/4) + x^6*((A*c)/6 + (C*b)/6) + (A*a*x^2)/2 + (B*a*x^3)/3 + (B*b*x^5)/5 + (B*c*x^7)/7 + (C*c*x^8)/8$

sympy [A] time = 0.07, size = 68, normalized size = 0.92

$$\frac{Aax^2}{2} + \frac{Bax^3}{3} + \frac{Bbx^5}{5} + \frac{Bcx^7}{7} + \frac{Ccx^8}{8} + x^6\left(\frac{Ac}{6} + \frac{Cb}{6}\right) + x^4\left(\frac{Ab}{4} + \frac{Ca}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(C*x**2+B*x+A)*(c*x**4+b*x**2+a),x)

[Out] $Aax^2/2 + Bax^3/3 + Bbx^5/5 + Bcx^7/7 + Ccx^8/8 + x^6*(A*c/6 + C*b/6) + x^4*(A*b/4 + C*a/4)$

3.3 $\int (A + Bx + Cx^2)(a + bx^2 + cx^4) dx$

Optimal. Leaf size=69

$$\frac{1}{3}x^3(aC + Ab) + aAx + \frac{1}{2}aBx^2 + \frac{1}{5}x^5(Ac + bC) + \frac{1}{4}bBx^4 + \frac{1}{6}Bcx^6 + \frac{1}{7}cCx^7$$

Rubi [A] time = 0.04, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {1657}

$$\frac{1}{3}x^3(aC + Ab) + aAx + \frac{1}{2}aBx^2 + \frac{1}{5}x^5(Ac + bC) + \frac{1}{4}bBx^4 + \frac{1}{6}Bcx^6 + \frac{1}{7}cCx^7$$

Antiderivative was successfully verified.

[In] Int[(A + B*x + C*x^2)*(a + b*x^2 + c*x^4), x]

[Out] a*A*x + (a*B*x^2)/2 + ((A*b + a*C)*x^3)/3 + (b*B*x^4)/4 + ((A*c + b*C)*x^5)/5 + (B*c*x^6)/6 + (c*C*x^7)/7

Rule 1657

Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[Expand Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int (A + Bx + Cx^2)(a + bx^2 + cx^4) dx &= \int (aA + aBx + (Ab + aC)x^2 + bBx^3 + (Ac + bC)x^4 + Bcx^5 + cCx^6) dx \\ &= aAx + \frac{1}{2}aBx^2 + \frac{1}{3}(Ab + aC)x^3 + \frac{1}{4}bBx^4 + \frac{1}{5}(Ac + bC)x^5 + \frac{1}{6}Bcx^6 + \frac{1}{7}cCx^7 \end{aligned}$$

Mathematica [A] time = 0.01, size = 69, normalized size = 1.00

$$\frac{1}{3}x^3(aC + Ab) + aAx + \frac{1}{2}aBx^2 + \frac{1}{5}x^5(Ac + bC) + \frac{1}{4}bBx^4 + \frac{1}{6}Bcx^6 + \frac{1}{7}cCx^7$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x + C*x^2)*(a + b*x^2 + c*x^4), x]

[Out] a*A*x + (a*B*x^2)/2 + ((A*b + a*C)*x^3)/3 + (b*B*x^4)/4 + ((A*c + b*C)*x^5)/5 + (B*c*x^6)/6 + (c*C*x^7)/7

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + Bx + Cx^2)(a + bx^2 + cx^4) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B*x + C*x^2)*(a + b*x^2 + c*x^4), x]

[Out] IntegrateAlgebraic[(A + B*x + C*x^2)*(a + b*x^2 + c*x^4), x]

fricas [A] time = 1.06, size = 61, normalized size = 0.88

$$\frac{1}{7}x^7cC + \frac{1}{6}x^6cB + \frac{1}{5}x^5bC + \frac{1}{5}x^5cA + \frac{1}{4}x^4bB + \frac{1}{3}x^3aC + \frac{1}{3}x^3bA + \frac{1}{2}x^2aB + xaA$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] $\frac{1}{7}x^7*c*C + \frac{1}{6}x^6*c*B + \frac{1}{5}x^5*b*C + \frac{1}{5}x^5*c*A + \frac{1}{4}x^4*b*B + \frac{1}{3}x^3*a*C + \frac{1}{3}x^3*b*A + \frac{1}{2}x^2*a*B + x*a*A$

giac [A] time = 0.38, size = 61, normalized size = 0.88

$$\frac{1}{7}Ccx^7 + \frac{1}{6}Bcx^6 + \frac{1}{5}Cbx^5 + \frac{1}{5}Acx^5 + \frac{1}{4}Bbx^4 + \frac{1}{3}Cax^3 + \frac{1}{3}Abx^3 + \frac{1}{2}Bax^2 + Aax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] $\frac{1}{7}C*c*x^7 + \frac{1}{6}B*c*x^6 + \frac{1}{5}C*b*x^5 + \frac{1}{5}A*c*x^5 + \frac{1}{4}B*b*x^4 + \frac{1}{3}C*a*x^3 + \frac{1}{3}A*b*x^3 + \frac{1}{2}B*a*x^2 + A*a*x$

maple [A] time = 0.00, size = 58, normalized size = 0.84

$$\frac{Ccx^7}{7} + \frac{Bcx^6}{6} + \frac{Bbx^4}{4} + \frac{(Ac + bC)x^5}{5} + \frac{Bax^2}{2} + Aax + \frac{(Ab + aC)x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a),x)

[Out] $a*A*x + \frac{1}{2}a*B*x^2 + \frac{1}{3}(A*b + C*a)*x^3 + \frac{1}{4}b*B*x^4 + \frac{1}{5}(A*c + C*b)*x^5 + \frac{1}{6}B*c*x^6 + \frac{1}{7}c*C*x^7$

maxima [A] time = 0.46, size = 57, normalized size = 0.83

$$\frac{1}{7}Ccx^7 + \frac{1}{6}Bcx^6 + \frac{1}{4}Bbx^4 + \frac{1}{5}(Cb + Ac)x^5 + \frac{1}{2}Bax^2 + \frac{1}{3}(Ca + Ab)x^3 + Aax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] $\frac{1}{7}C*c*x^7 + \frac{1}{6}B*c*x^6 + \frac{1}{4}B*b*x^4 + \frac{1}{5}(C*b + A*c)*x^5 + \frac{1}{2}B*a*x^2 + \frac{1}{3}(C*a + A*b)*x^3 + A*a*x$

mupad [B] time = 0.03, size = 59, normalized size = 0.86

$$\frac{Ccx^7}{7} + \frac{Bcx^6}{6} + \left(\frac{Ac}{5} + \frac{Cb}{5}\right)x^5 + \frac{Bbx^4}{4} + \left(\frac{Ab}{3} + \frac{Ca}{3}\right)x^3 + \frac{Bax^2}{2} + Aax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x)

[Out] $x^3*((A*b)/3 + (C*a)/3) + x^5*((A*c)/5 + (C*b)/5) + A*a*x + (B*a*x^2)/2 + (B*b*x^4)/4 + (B*c*x^6)/6 + (C*c*x^7)/7$

sympy [A] time = 0.07, size = 65, normalized size = 0.94

$$Aax + \frac{Bax^2}{2} + \frac{Bbx^4}{4} + \frac{Bcx^6}{6} + \frac{Ccx^7}{7} + x^5\left(\frac{Ac}{5} + \frac{Cb}{5}\right) + x^3\left(\frac{Ab}{3} + \frac{Ca}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a),x)

[Out] $A*a*x + B*a*x**2/2 + B*b*x**4/4 + B*c*x**6/6 + C*c*x**7/7 + x**5*(A*c/5 + C*b/5) + x**3*(A*b/3 + C*a/3)$

$$3.4 \quad \int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x} dx$$

Optimal. Leaf size=65

$$\frac{1}{2}x^2(aC + Ab) + aA \log(x) + aBx + \frac{1}{4}x^4(Ac + bC) + \frac{1}{3}bBx^3 + \frac{1}{5}Bcx^5 + \frac{1}{6}cCx^6$$

Rubi [A] time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {1628}

$$\frac{1}{2}x^2(aC + Ab) + aA \log(x) + aBx + \frac{1}{4}x^4(Ac + bC) + \frac{1}{3}bBx^3 + \frac{1}{5}Bcx^5 + \frac{1}{6}cCx^6$$

Antiderivative was successfully verified.

[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x,x]

[Out] a*B*x + ((A*b + a*C)*x^2)/2 + (b*B*x^3)/3 + ((A*c + b*C)*x^4)/4 + (B*c*x^5)/5 + (c*C*x^6)/6 + a*A*Log[x]

Rule 1628

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x} dx &= \int \left(aB + \frac{aA}{x} + (Ab+aC)x + bBx^2 + (Ac+bC)x^3 + Bcx^4 + cCx^5 \right) dx \\ &= aBx + \frac{1}{2}(Ab+aC)x^2 + \frac{1}{3}bBx^3 + \frac{1}{4}(Ac+bC)x^4 + \frac{1}{5}Bcx^5 + \frac{1}{6}cCx^6 + \end{aligned}$$

Mathematica [A] time = 0.01, size = 65, normalized size = 1.00

$$\frac{1}{2}x^2(aC + Ab) + aA \log(x) + aBx + \frac{1}{4}x^4(Ac + bC) + \frac{1}{3}bBx^3 + \frac{1}{5}Bcx^5 + \frac{1}{6}cCx^6$$

Antiderivative was successfully verified.

[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x,x]

[Out] a*B*x + ((A*b + a*C)*x^2)/2 + (b*B*x^3)/3 + ((A*c + b*C)*x^4)/4 + (B*c*x^5)/5 + (c*C*x^6)/6 + a*A*Log[x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x,x]

[Out] IntegrateAlgebraic[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x, x]

fricas [A] time = 1.29, size = 55, normalized size = 0.85

$$\frac{1}{6} Ccx^6 + \frac{1}{5} Bcx^5 + \frac{1}{3} Bbx^3 + \frac{1}{4} (Cb + Ac)x^4 + Bax + \frac{1}{2} (Ca + Ab)x^2 + Aa \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x,x, algorithm="fricas")

[Out] 1/6*C*c*x^6 + 1/5*B*c*x^5 + 1/3*B*b*x^3 + 1/4*(C*b + A*c)*x^4 + B*a*x + 1/2*(C*a + A*b)*x^2 + A*a*log(x)

giac [A] time = 0.37, size = 60, normalized size = 0.92

$$\frac{1}{6} Ccx^6 + \frac{1}{5} Bcx^5 + \frac{1}{4} Cbx^4 + \frac{1}{4} Acx^4 + \frac{1}{3} Bbx^3 + \frac{1}{2} Cax^2 + \frac{1}{2} Abx^2 + Bax + Aa \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x,x, algorithm="giac")

[Out] 1/6*C*c*x^6 + 1/5*B*c*x^5 + 1/4*C*b*x^4 + 1/4*A*c*x^4 + 1/3*B*b*x^3 + 1/2*C*a*x^2 + 1/2*A*b*x^2 + B*a*x + A*a*log(abs(x))

maple [A] time = 0.00, size = 60, normalized size = 0.92

$$\frac{Ccx^6}{6} + \frac{Bcx^5}{5} + \frac{Acx^4}{4} + \frac{Cbx^4}{4} + \frac{Bbx^3}{3} + \frac{Abx^2}{2} + \frac{Cax^2}{2} + Aa \ln(x) + Bax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x,x)

[Out] 1/6*c*C*x^6+1/5*B*c*x^5+1/4*A*x^4*c+1/4*C*x^4*b+1/3*b*B*x^3+1/2*A*x^2*b+1/2*C*x^2*a+a*B*x+a*A*ln(x)

maxima [A] time = 0.60, size = 55, normalized size = 0.85

$$\frac{1}{6} Ccx^6 + \frac{1}{5} Bcx^5 + \frac{1}{3} Bbx^3 + \frac{1}{4} (Cb + Ac)x^4 + Bax + \frac{1}{2} (Ca + Ab)x^2 + Aa \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x,x, algorithm="maxima")

[Out] 1/6*C*c*x^6 + 1/5*B*c*x^5 + 1/3*B*b*x^3 + 1/4*(C*b + A*c)*x^4 + B*a*x + 1/2*(C*a + A*b)*x^2 + A*a*log(x)

mupad [B] time = 0.04, size = 57, normalized size = 0.88

$$x^2 \left(\frac{Ab}{2} + \frac{Ca}{2} \right) + x^4 \left(\frac{Ac}{4} + \frac{Cb}{4} \right) + Bax + \frac{Bbx^3}{3} + \frac{Bcx^5}{5} + \frac{Ccx^6}{6} + Aa \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x,x)

[Out] x^2*((A*b)/2 + (C*a)/2) + x^4*((A*c)/4 + (C*b)/4) + B*a*x + (B*b*x^3)/3 + (B*c*x^5)/5 + (C*c*x^6)/6 + A*a*log(x)

sympy [A] time = 0.16, size = 63, normalized size = 0.97

$$Aa \log(x) + Bax + \frac{Bbx^3}{3} + \frac{Bcx^5}{5} + \frac{Ccx^6}{6} + x^4 \left(\frac{Ac}{4} + \frac{Cb}{4} \right) + x^2 \left(\frac{Ab}{2} + \frac{Ca}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)/x,x)
```

```
[Out] A*a*log(x) + B*a*x + B*b*x**3/3 + B*c*x**5/5 + C*c*x**6/6 + x**4*(A*c/4 + C*b/4) + x**2*(A*b/2 + C*a/2)
```

$$3.5 \quad \int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^2} dx$$

Optimal. Leaf size=63

$$x(aC + Ab) - \frac{aA}{x} + aB \log(x) + \frac{1}{3}x^3(Ac + bC) + \frac{1}{2}bBx^2 + \frac{1}{4}Bcx^4 + \frac{1}{5}cCx^5$$

Rubi [A] time = 0.05, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {1628}

$$x(aC + Ab) - \frac{aA}{x} + aB \log(x) + \frac{1}{3}x^3(Ac + bC) + \frac{1}{2}bBx^2 + \frac{1}{4}Bcx^4 + \frac{1}{5}cCx^5$$

Antiderivative was successfully verified.

[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^2,x]

[Out] -((a*A)/x) + (A*b + a*C)*x + (b*B*x^2)/2 + ((A*c + b*C)*x^3)/3 + (B*c*x^4)/4 + (c*C*x^5)/5 + a*B*Log[x]

Rule 1628

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int \frac{(A + Bx + Cx^2)(a + bx^2 + cx^4)}{x^2} dx &= \int \left(Ab \left(1 + \frac{aC}{Ab} \right) + \frac{aA}{x^2} + \frac{aB}{x} + bBx + (Ac + bC)x^2 + Bcx^3 + cCx^4 \right) dx \\ &= -\frac{aA}{x} + (Ab + aC)x + \frac{1}{2}bBx^2 + \frac{1}{3}(Ac + bC)x^3 + \frac{1}{4}Bcx^4 + \frac{1}{5}cCx^5 + aB \log(x) \end{aligned}$$

Mathematica [A] time = 0.02, size = 63, normalized size = 1.00

$$x(aC + Ab) - \frac{aA}{x} + aB \log(x) + \frac{1}{3}x^3(Ac + bC) + \frac{1}{2}bBx^2 + \frac{1}{4}Bcx^4 + \frac{1}{5}cCx^5$$

Antiderivative was successfully verified.

[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^2,x]

[Out] -((a*A)/x) + (A*b + a*C)*x + (b*B*x^2)/2 + ((A*c + b*C)*x^3)/3 + (B*c*x^4)/4 + (c*C*x^5)/5 + a*B*Log[x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + Bx + Cx^2)(a + bx^2 + cx^4)}{x^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^2,x]

[Out] IntegrateAlgebraic[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^2, x]

fricas [A] time = 1.23, size = 62, normalized size = 0.98

$$\frac{12 Ccx^6 + 15 Bcx^5 + 30 Bbx^3 + 20 (Cb + Ac)x^4 + 60 Bax \log(x) + 60 (Ca + Ab)x^2 - 60 Aa}{60x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^2,x, algorithm="fricas")

[Out] 1/60*(12*C*c*x^6 + 15*B*c*x^5 + 30*B*b*x^3 + 20*(C*b + A*c)*x^4 + 60*B*a*x*log(x) + 60*(C*a + A*b)*x^2 - 60*A*a)/x

giac [A] time = 0.30, size = 57, normalized size = 0.90

$$\frac{1}{5} Ccx^5 + \frac{1}{4} Bcx^4 + \frac{1}{3} Cbx^3 + \frac{1}{3} Acx^3 + \frac{1}{2} Bbx^2 + Cax + Abx + Ba \log(|x|) - \frac{Aa}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^2,x, algorithm="giac")

[Out] 1/5*C*c*x^5 + 1/4*B*c*x^4 + 1/3*C*b*x^3 + 1/3*A*c*x^3 + 1/2*B*b*x^2 + C*a*x + A*b*x + B*a*log(abs(x)) - A*a/x

maple [A] time = 0.01, size = 57, normalized size = 0.90

$$\frac{Ccx^5}{5} + \frac{Bcx^4}{4} + \frac{Acx^3}{3} + \frac{Cbx^3}{3} + \frac{Bbx^2}{2} + Abx + Ba \ln(x) + Cax - \frac{Aa}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^2,x)

[Out] 1/5*c*C*x^5+1/4*B*c*x^4+1/3*A*x^3*c+1/3*C*x^3*b+1/2*b*B*x^2+A*b*x+a*C*x+a*B*ln(x)-a*A/x

maxima [A] time = 0.68, size = 55, normalized size = 0.87

$$\frac{1}{5} Ccx^5 + \frac{1}{4} Bcx^4 + \frac{1}{2} Bbx^2 + \frac{1}{3} (Cb + Ac)x^3 + Ba \log(x) + (Ca + Ab)x - \frac{Aa}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^2,x, algorithm="maxima")

[Out] 1/5*C*c*x^5 + 1/4*B*c*x^4 + 1/2*B*b*x^2 + 1/3*(C*b + A*c)*x^3 + B*a*log(x) + (C*a + A*b)*x - A*a/x

mupad [B] time = 0.04, size = 56, normalized size = 0.89

$$x (Ab + Ca) + x^3 \left(\frac{Ac}{3} + \frac{Cb}{3} \right) - \frac{Aa}{x} + \frac{Bbx^2}{2} + \frac{Bcx^4}{4} + \frac{Ccx^5}{5} + Ba \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^2,x)

[Out] x*(A*b + C*a) + x^3*((A*c)/3 + (C*b)/3) - (A*a)/x + (B*b*x^2)/2 + (B*c*x^4)/4 + (C*c*x^5)/5 + B*a*log(x)

sympy [A] time = 0.18, size = 58, normalized size = 0.92

$$-\frac{Aa}{x} + Ba \log(x) + \frac{Bbx^2}{2} + \frac{Bcx^4}{4} + \frac{Ccx^5}{5} + x^3 \left(\frac{Ac}{3} + \frac{Cb}{3} \right) + x (Ab + Ca)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)/x**2,x)
```

```
[Out] -A*a/x + B*a*log(x) + B*b*x**2/2 + B*c*x**4/4 + C*c*x**5/5 + x**3*(A*c/3 + C*b/3) + x*(A*b + C*a)
```

$$3.6 \quad \int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^3} dx$$

Optimal. Leaf size=63

$$\log(x)(aC + Ab) - \frac{aA}{2x^2} - \frac{aB}{x} + \frac{1}{2}x^2(Ac + bC) + bBx + \frac{1}{3}Bcx^3 + \frac{1}{4}cCx^4$$

Rubi [A] time = 0.05, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {1628}

$$\log(x)(aC + Ab) - \frac{aA}{2x^2} - \frac{aB}{x} + \frac{1}{2}x^2(Ac + bC) + bBx + \frac{1}{3}Bcx^3 + \frac{1}{4}cCx^4$$

Antiderivative was successfully verified.

[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^3, x]

[Out] -(a*A)/(2*x^2) - (a*B)/x + b*B*x + ((A*c + b*C)*x^2)/2 + (B*c*x^3)/3 + (c*C*x^4)/4 + (A*b + a*C)*Log[x]

Rule 1628

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int \frac{(A + Bx + Cx^2)(a + bx^2 + cx^4)}{x^3} dx &= \int \left(bB + \frac{aA}{x^3} + \frac{aB}{x^2} + \frac{Ab + aC}{x} + (Ac + bC)x + Bcx^2 + cCx^3 \right) dx \\ &= -\frac{aA}{2x^2} - \frac{aB}{x} + bBx + \frac{1}{2}(Ac + bC)x^2 + \frac{1}{3}Bcx^3 + \frac{1}{4}cCx^4 + (Ab + aC)\log(x) \end{aligned}$$

Mathematica [A] time = 0.04, size = 58, normalized size = 0.92

$$\log(x)(aC + Ab) - \frac{a(A + 2Bx)}{2x^2} + \frac{1}{12}x \left(cx(6A + 4Bx + 3Cx^2) + 6b(2B + Cx) \right)$$

Antiderivative was successfully verified.

[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^3, x]

[Out] -1/2*(a*(A + 2*B*x))/x^2 + (x*(6*b*(2*B + C*x) + c*x*(6*A + 4*B*x + 3*C*x^2)))/12 + (A*b + a*C)*Log[x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + Bx + Cx^2)(a + bx^2 + cx^4)}{x^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^3, x]

[Out] IntegrateAlgebraic[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^3, x]

fricas [A] time = 1.33, size = 62, normalized size = 0.98

$$\frac{3 C c x^6 + 4 B c x^5 + 12 B b x^3 + 6 (C b + A c) x^4 + 12 (C a + A b) x^2 \log(x) - 12 B a x - 6 A a}{12 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^3,x, algorithm="fricas")

[Out] 1/12*(3*C*c*x^6 + 4*B*c*x^5 + 12*B*b*x^3 + 6*(C*b + A*c)*x^4 + 12*(C*a + A*b)*x^2*log(x) - 12*B*a*x - 6*A*a)/x^2

giac [A] time = 0.29, size = 58, normalized size = 0.92

$$\frac{1}{4} C c x^4 + \frac{1}{3} B c x^3 + \frac{1}{2} C b x^2 + \frac{1}{2} A c x^2 + B b x + (C a + A b) \log(|x|) - \frac{2 B a x + A a}{2 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^3,x, algorithm="giac")

[Out] 1/4*C*c*x^4 + 1/3*B*c*x^3 + 1/2*C*b*x^2 + 1/2*A*c*x^2 + B*b*x + (C*a + A*b)*log(abs(x)) - 1/2*(2*B*a*x + A*a)/x^2

maple [A] time = 0.01, size = 58, normalized size = 0.92

$$\frac{C c x^4}{4} + \frac{B c x^3}{3} + \frac{A c x^2}{2} + \frac{C b x^2}{2} + A b \ln(x) + B b x + C a \ln(x) - \frac{B a}{x} - \frac{A a}{2 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^3,x)

[Out] 1/4*c*C*x^4+1/3*B*c*x^3+1/2*A*x^2*c+1/2*C*x^2*b+b*B*x+A*ln(x)*b+C*ln(x)*a-a*B/x-1/2*a*A/x^2

maxima [A] time = 0.75, size = 55, normalized size = 0.87

$$\frac{1}{4} C c x^4 + \frac{1}{3} B c x^3 + B b x + \frac{1}{2} (C b + A c) x^2 + (C a + A b) \log(x) - \frac{2 B a x + A a}{2 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^3,x, algorithm="maxima")

[Out] 1/4*C*c*x^4 + 1/3*B*c*x^3 + B*b*x + 1/2*(C*b + A*c)*x^2 + (C*a + A*b)*log(x) - 1/2*(2*B*a*x + A*a)/x^2

mupad [B] time = 0.03, size = 56, normalized size = 0.89

$$x^2 \left(\frac{A c}{2} + \frac{C b}{2} \right) - \frac{\frac{A a}{2} + B a x}{x^2} + \ln(x) (A b + C a) + B b x + \frac{B c x^3}{3} + \frac{C c x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^3,x)

[Out] x^2*((A*c)/2 + (C*b)/2) - ((A*a)/2 + B*a*x)/x^2 + log(x)*(A*b + C*a) + B*b*x + (B*c*x^3)/3 + (C*c*x^4)/4

sympy [A] time = 0.29, size = 61, normalized size = 0.97

$$B b x + \frac{B c x^3}{3} + \frac{C c x^4}{4} + x^2 \left(\frac{A c}{2} + \frac{C b}{2} \right) + (A b + C a) \log(x) + \frac{-A a - 2 B a x}{2 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)/x**3,x)
```

```
[Out] B*b*x + B*c*x**3/3 + C*c*x**4/4 + x**2*(A*c/2 + C*b/2) + (A*b + C*a)*log(x)
+ (-A*a - 2*B*a*x)/(2*x**2)
```

$$3.7 \quad \int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^4} dx$$

Optimal. Leaf size=63

$$-\frac{aC + Ab}{x} - \frac{aA}{3x^3} - \frac{aB}{2x^2} + x(Ac + bC) + bB \log(x) + \frac{1}{2}Bcx^2 + \frac{1}{3}cCx^3$$

Rubi [A] time = 0.05, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {1628}

$$-\frac{aC + Ab}{x} - \frac{aA}{3x^3} - \frac{aB}{2x^2} + x(Ac + bC) + bB \log(x) + \frac{1}{2}Bcx^2 + \frac{1}{3}cCx^3$$

Antiderivative was successfully verified.

[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^4,x]

[Out] -(a*A)/(3*x^3) - (a*B)/(2*x^2) - (A*b + a*C)/x + (A*c + b*C)*x + (B*c*x^2)/2 + (c*C*x^3)/3 + b*B*Log[x]

Rule 1628

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int \frac{(A + Bx + Cx^2)(a + bx^2 + cx^4)}{x^4} dx &= \int \left(Ac \left(1 + \frac{bC}{Ac} \right) + \frac{aA}{x^4} + \frac{aB}{x^3} + \frac{Ab + aC}{x^2} + \frac{bB}{x} + Bcx + cCx^2 \right) dx \\ &= -\frac{aA}{3x^3} - \frac{aB}{2x^2} - \frac{Ab + aC}{x} + (Ac + bC)x + \frac{1}{2}Bcx^2 + \frac{1}{3}cCx^3 + bB \log(x) \end{aligned}$$

Mathematica [A] time = 0.04, size = 60, normalized size = 0.95

$$-\frac{a(2A + 3x(B + 2Cx))}{6x^3} - \frac{Ab}{x} + Acx + bB \log(x) + bCx + \frac{1}{2}Bcx^2 + \frac{1}{3}cCx^3$$

Antiderivative was successfully verified.

[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^4,x]

[Out] -((A*b)/x) + A*c*x + b*C*x + (B*c*x^2)/2 + (c*C*x^3)/3 - (a*(2*A + 3*x*(B + 2*C*x)))/(6*x^3) + b*B*Log[x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + Bx + Cx^2)(a + bx^2 + cx^4)}{x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^4,x]

[Out] IntegrateAlgebraic[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^4, x]

fricas [A] time = 1.26, size = 62, normalized size = 0.98

$$\frac{2 C c x^6 + 3 B c x^5 + 6 B b x^3 \log(x) + 6 (C b + A c) x^4 - 3 B a x - 6 (C a + A b) x^2 - 2 A a}{6 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^4,x, algorithm="fricas")

[Out] 1/6*(2*C*c*x^6 + 3*B*c*x^5 + 6*B*b*x^3*log(x) + 6*(C*b + A*c)*x^4 - 3*B*a*x - 6*(C*a + A*b)*x^2 - 2*A*a)/x^3

giac [A] time = 0.37, size = 56, normalized size = 0.89

$$\frac{1}{3} C c x^3 + \frac{1}{2} B c x^2 + C b x + A c x + B b \log(|x|) - \frac{3 B a x + 6 (C a + A b) x^2 + 2 A a}{6 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^4,x, algorithm="giac")

[Out] 1/3*C*c*x^3 + 1/2*B*c*x^2 + C*b*x + A*c*x + B*b*log(abs(x)) - 1/6*(3*B*a*x + 6*(C*a + A*b)*x^2 + 2*A*a)/x^3

maple [A] time = 0.01, size = 57, normalized size = 0.90

$$\frac{C c x^3}{3} + \frac{B c x^2}{2} + A c x + B b \ln(x) + C b x - \frac{A b}{x} - \frac{C a}{x} - \frac{B a}{2 x^2} - \frac{A a}{3 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^4,x)

[Out] 1/3*c*C*x^3+1/2*B*c*x^2+A*c*x+b*C*x+b*B*ln(x)-1/x*A*b-1/x*a*C-1/3*a*A/x^3-1/2*a*B/x^2

maxima [A] time = 0.66, size = 56, normalized size = 0.89

$$\frac{1}{3} C c x^3 + \frac{1}{2} B c x^2 + B b \log(x) + (C b + A c) x - \frac{3 B a x + 6 (C a + A b) x^2 + 2 A a}{6 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^4,x, algorithm="maxima")

[Out] 1/3*C*c*x^3 + 1/2*B*c*x^2 + B*b*log(x) + (C*b + A*c)*x - 1/6*(3*B*a*x + 6*(C*a + A*b)*x^2 + 2*A*a)/x^3

mupad [B] time = 0.03, size = 55, normalized size = 0.87

$$x (A c + C b) - \frac{(A b + C a) x^2 + \frac{B a x}{2} + \frac{A a}{3}}{x^3} + \frac{B c x^2}{2} + \frac{C c x^3}{3} + B b \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^4,x)

[Out] x*(A*c + C*b) - ((A*a)/3 + x^2*(A*b + C*a) + (B*a*x)/2)/x^3 + (B*c*x^2)/2 + (C*c*x^3)/3 + B*b*log(x)

sympy [A] time = 0.52, size = 63, normalized size = 1.00

$$B b \log(x) + \frac{B c x^2}{2} + \frac{C c x^3}{3} + x (A c + C b) + \frac{-2 A a - 3 B a x + x^2 (-6 A b - 6 C a)}{6 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)/x**4,x)
```

```
[Out] B*b*log(x) + B*c*x**2/2 + C*c*x**3/3 + x*(A*c + C*b) + (-2*A*a - 3*B*a*x +  
x**2*(-6*A*b - 6*C*a))/(6*x**3)
```

$$3.8 \quad \int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^5} dx$$

Optimal. Leaf size=63

$$-\frac{aC + Ab}{2x^2} - \frac{aA}{4x^4} - \frac{aB}{3x^3} + \log(x)(Ac + bC) - \frac{bB}{x} + Bcx + \frac{1}{2}cCx^2$$

Rubi [A] time = 0.05, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {1628}

$$-\frac{aC + Ab}{2x^2} - \frac{aA}{4x^4} - \frac{aB}{3x^3} + \log(x)(Ac + bC) - \frac{bB}{x} + Bcx + \frac{1}{2}cCx^2$$

Antiderivative was successfully verified.

[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^5,x]

[Out] -(a*A)/(4*x^4) - (a*B)/(3*x^3) - (A*b + a*C)/(2*x^2) - (b*B)/x + B*c*x + (c*C*x^2)/2 + (A*c + b*C)*Log[x]

Rule 1628

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int \frac{(A + Bx + Cx^2)(a + bx^2 + cx^4)}{x^5} dx &= \int \left(Bc + \frac{aA}{x^5} + \frac{aB}{x^4} + \frac{Ab + aC}{x^3} + \frac{bB}{x^2} + \frac{Ac + bC}{x} + cCx \right) dx \\ &= -\frac{aA}{4x^4} - \frac{aB}{3x^3} - \frac{Ab + aC}{2x^2} - \frac{bB}{x} + Bcx + \frac{1}{2}cCx^2 + (Ac + bC) \log(x) \end{aligned}$$

Mathematica [A] time = 0.03, size = 62, normalized size = 0.98

$$-\frac{a(3A + 4Bx + 6Cx^2)}{12x^4} + \frac{-Ab - 2bBx + cx^3(2B + Cx)}{2x^2} + \log(x)(Ac + bC)$$

Antiderivative was successfully verified.

[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^5,x]

[Out] -1/12*(a*(3*A + 4*B*x + 6*C*x^2))/x^4 + ((-A*b) - 2*b*B*x + c*x^3*(2*B + C*x))/(2*x^2) + (A*c + b*C)*Log[x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + Bx + Cx^2)(a + bx^2 + cx^4)}{x^5} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^5,x]

[Out] IntegrateAlgebraic[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^5, x]

fricas [A] time = 1.38, size = 62, normalized size = 0.98

$$\frac{6 C c x^6 + 12 B c x^5 + 12 (C b + A c) x^4 \log(x) - 12 B b x^3 - 4 B a x - 6 (C a + A b) x^2 - 3 A a}{12 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^5,x, algorithm="fricas")

[Out] 1/12*(6*C*c*x^6 + 12*B*c*x^5 + 12*(C*b + A*c)*x^4*log(x) - 12*B*b*x^3 - 4*B*a*x - 6*(C*a + A*b)*x^2 - 3*A*a)/x^4

giac [A] time = 0.29, size = 57, normalized size = 0.90

$$\frac{1}{2} C c x^2 + B c x + (C b + A c) \log(|x|) - \frac{12 B b x^3 + 4 B a x + 6 (C a + A b) x^2 + 3 A a}{12 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^5,x, algorithm="giac")

[Out] 1/2*C*c*x^2 + B*c*x + (C*b + A*c)*log(abs(x)) - 1/12*(12*B*b*x^3 + 4*B*a*x + 6*(C*a + A*b)*x^2 + 3*A*a)/x^4

maple [A] time = 0.01, size = 58, normalized size = 0.92

$$\frac{C c x^2}{2} + A c \ln(x) + B c x + C b \ln(x) - \frac{B b}{x} - \frac{A b}{2 x^2} - \frac{C a}{2 x^2} - \frac{B a}{3 x^3} - \frac{A a}{4 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^5,x)

[Out] 1/2*c*C*x^2+B*c*x+A*ln(x)*c+C*ln(x)*b-b*B/x-1/3*a*B/x^3-1/4*a*A/x^4-1/2/x^2*A*b-1/2/x^2*a*C

maxima [A] time = 0.75, size = 56, normalized size = 0.89

$$\frac{1}{2} C c x^2 + B c x + (C b + A c) \log(x) - \frac{12 B b x^3 + 4 B a x + 6 (C a + A b) x^2 + 3 A a}{12 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^5,x, algorithm="maxima")

[Out] 1/2*C*c*x^2 + B*c*x + (C*b + A*c)*log(x) - 1/12*(12*B*b*x^3 + 4*B*a*x + 6*(C*a + A*b)*x^2 + 3*A*a)/x^4

mupad [B] time = 0.05, size = 56, normalized size = 0.89

$$\ln(x) (A c + C b) - \frac{B b x^3 + \left(\frac{A b}{2} + \frac{C a}{2}\right) x^2 + \frac{B a x}{3} + \frac{A a}{4}}{x^4} + B c x + \frac{C c x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^5,x)

[Out] log(x)*(A*c + C*b) - ((A*a)/4 + x^2*((A*b)/2 + (C*a)/2) + (B*a*x)/3 + B*b*x^3)/x^4 + B*c*x + (C*c*x^2)/2

sympy [A] time = 1.76, size = 63, normalized size = 1.00

$$B c x + \frac{C c x^2}{2} + (A c + C b) \log(x) + \frac{-3 A a - 4 B a x - 12 B b x^3 + x^2 (-6 A b - 6 C a)}{12 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)/x**5,x)
```

```
[Out] B*c*x + C*c*x**2/2 + (A*c + C*b)*log(x) + (-3*A*a - 4*B*a*x - 12*B*b*x**3 +  
x**2*(-6*A*b - 6*C*a))/(12*x**4)
```

$$3.9 \quad \int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^6} dx$$

Optimal. Leaf size=63

$$-\frac{aC + Ab}{3x^3} - \frac{aA}{5x^5} - \frac{aB}{4x^4} - \frac{Ac + bC}{x} - \frac{bB}{2x^2} + Bc \log(x) + cCx$$

Rubi [A] time = 0.05, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {1628}

$$-\frac{aC + Ab}{3x^3} - \frac{aA}{5x^5} - \frac{aB}{4x^4} - \frac{Ac + bC}{x} - \frac{bB}{2x^2} + Bc \log(x) + cCx$$

Antiderivative was successfully verified.

[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^6,x]

[Out] -(a*A)/(5*x^5) - (a*B)/(4*x^4) - (A*b + a*C)/(3*x^3) - (b*B)/(2*x^2) - (A*c + b*C)/x + c*C*x + B*c*Log[x]

Rule 1628

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int \frac{(A + Bx + Cx^2)(a + bx^2 + cx^4)}{x^6} dx &= \int \left(cC + \frac{aA}{x^6} + \frac{aB}{x^5} + \frac{Ab + aC}{x^4} + \frac{bB}{x^3} + \frac{Ac + bC}{x^2} + \frac{Bc}{x} \right) dx \\ &= -\frac{aA}{5x^5} - \frac{aB}{4x^4} - \frac{Ab + aC}{3x^3} - \frac{bB}{2x^2} - \frac{Ac + bC}{x} + cCx + Bc \log(x) \end{aligned}$$

Mathematica [A] time = 0.05, size = 63, normalized size = 1.00

$$Bc \log(x) - \frac{12aA + 5ax(3B + 4Cx) + 20Ax^2(b + 3cx^2) + 30bx^3(B + 2Cx) - 60cCx^6}{60x^5}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^6,x]

[Out] -1/60*(12*a*A - 60*c*C*x^6 + 30*b*x^3*(B + 2*C*x) + 5*a*x*(3*B + 4*C*x) + 20*A*x^2*(b + 3*c*x^2))/x^5 + B*c*Log[x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + Bx + Cx^2)(a + bx^2 + cx^4)}{x^6} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^6,x]

[Out] IntegrateAlgebraic[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^6, x]

fricas [A] time = 1.28, size = 62, normalized size = 0.98

$$\frac{60 Ccx^6 + 60 Bcx^5 \log(x) - 30 Bbx^3 - 60 (Cb + Ac)x^4 - 15 Bax - 20 (Ca + Ab)x^2 - 12 Aa}{60 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^6,x, algorithm="fricas")

[Out] 1/60*(60*C*c*x^6 + 60*B*c*x^5*log(x) - 30*B*b*x^3 - 60*(C*b + A*c)*x^4 - 15*B*a*x - 20*(C*a + A*b)*x^2 - 12*A*a)/x^5

giac [A] time = 0.26, size = 57, normalized size = 0.90

$$Ccx + Bc \log(|x|) - \frac{30 Bbx^3 + 60 (Cb + Ac)x^4 + 15 Bax + 20 (Ca + Ab)x^2 + 12 Aa}{60 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^6,x, algorithm="giac")

[Out] C*c*x + B*c*log(abs(x)) - 1/60*(30*B*b*x^3 + 60*(C*b + A*c)*x^4 + 15*B*a*x + 20*(C*a + A*b)*x^2 + 12*A*a)/x^5

maple [A] time = 0.01, size = 60, normalized size = 0.95

$$Bc \ln(x) + Ccx - \frac{Ac}{x} - \frac{Cb}{x} - \frac{Bb}{2x^2} - \frac{Ab}{3x^3} - \frac{Ca}{3x^3} - \frac{Ba}{4x^4} - \frac{Aa}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^6,x)

[Out] c*C*x+B*c*ln(x)-1/x*A*c-1/x*b*C-1/3/x^3*A*b-1/3/x^3*a*C-1/5*a*A/x^5-1/4*a*B/x^4-1/2*b*B/x^2

maxima [A] time = 0.74, size = 56, normalized size = 0.89

$$Ccx + Bc \log(x) - \frac{30 Bbx^3 + 60 (Cb + Ac)x^4 + 15 Bax + 20 (Ca + Ab)x^2 + 12 Aa}{60 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^6,x, algorithm="maxima")

[Out] C*c*x + B*c*log(x) - 1/60*(30*B*b*x^3 + 60*(C*b + A*c)*x^4 + 15*B*a*x + 20*(C*a + A*b)*x^2 + 12*A*a)/x^5

mupad [B] time = 0.78, size = 56, normalized size = 0.89

$$Ccx - \frac{(Ac + Cb)x^4 + \frac{Bbx^3}{2} + \left(\frac{Ab}{3} + \frac{Ca}{3}\right)x^2 + \frac{Bax}{4} + \frac{Aa}{5}}{x^5} + Bc \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^6,x)

[Out] C*c*x - ((A*a)/5 + x^2*((A*b)/3 + (C*a)/3) + x^4*(A*c + C*b) + (B*a*x)/4 + (B*b*x^3)/2)/x^5 + B*c*log(x)

sympy [A] time = 5.70, size = 66, normalized size = 1.05

$$Bc \log(x) + Ccx + \frac{-12Aa - 15Bax - 30Bbx^3 + x^4(-60Ac - 60Cb) + x^2(-20Ab - 20Ca)}{60x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)/x**6,x)
```

```
[Out] B*c*log(x) + C*c*x + (-12*A*a - 15*B*a*x - 30*B*b*x**3 + x**4*(-60*A*c - 60
*C*b) + x**2*(-20*A*b - 20*C*a))/(60*x**5)
```


$$3.10 \quad \int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^7} dx$$

Optimal. Leaf size=68

$$-\frac{aC + Ab}{4x^4} - \frac{aA}{6x^6} - \frac{aB}{5x^5} - \frac{Ac + bC}{2x^2} - \frac{bB}{3x^3} - \frac{Bc}{x} + cC \log(x)$$

Rubi [A] time = 0.05, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {1628}

$$-\frac{aC + Ab}{4x^4} - \frac{aA}{6x^6} - \frac{aB}{5x^5} - \frac{Ac + bC}{2x^2} - \frac{bB}{3x^3} - \frac{Bc}{x} + cC \log(x)$$

Antiderivative was successfully verified.

[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^7, x]

[Out] -(a*A)/(6*x^6) - (a*B)/(5*x^5) - (A*b + a*C)/(4*x^4) - (b*B)/(3*x^3) - (A*c + b*C)/(2*x^2) - (B*c)/x + c*C*Log[x]

Rule 1628

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int \frac{(A + Bx + Cx^2)(a + bx^2 + cx^4)}{x^7} dx &= \int \left(\frac{aA}{x^7} + \frac{aB}{x^6} + \frac{Ab + aC}{x^5} + \frac{bB}{x^4} + \frac{Ac + bC}{x^3} + \frac{Bc}{x^2} + \frac{cC}{x} \right) dx \\ &= -\frac{aA}{6x^6} - \frac{aB}{5x^5} - \frac{Ab + aC}{4x^4} - \frac{bB}{3x^3} - \frac{Ac + bC}{2x^2} - \frac{Bc}{x} + cC \log(x) \end{aligned}$$

Mathematica [A] time = 0.05, size = 68, normalized size = 1.00

$$cC \log(x) - \frac{a(10A + 3x(4B + 5Cx)) + 5x^2(3A(b + 2cx^2) + 2x(2bB + 3bCx + 6Bcx^2))}{60x^6}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^7, x]

[Out] -1/60*(a*(10*A + 3*x*(4*B + 5*C*x)) + 5*x^2*(3*A*(b + 2*c*x^2) + 2*x*(2*b*B + 3*b*C*x + 6*B*c*x^2)))/x^6 + c*C*Log[x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + Bx + Cx^2)(a + bx^2 + cx^4)}{x^7} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^7, x]

[Out] IntegrateAlgebraic[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^7, x]

fricas [A] time = 0.97, size = 62, normalized size = 0.91

$$\frac{60 Ccx^6 \log(x) - 60 Bcx^5 - 20 Bbx^3 - 30 (Cb + Ac)x^4 - 12 Bax - 15 (Ca + Ab)x^2 - 10 Aa}{60 x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^7,x, algorithm="fricas")

[Out] 1/60*(60*C*c*x^6*log(x) - 60*B*c*x^5 - 20*B*b*x^3 - 30*(C*b + A*c)*x^4 - 12*B*a*x - 15*(C*a + A*b)*x^2 - 10*A*a)/x^6

giac [A] time = 0.39, size = 60, normalized size = 0.88

$$Cc \log(|x|) - \frac{60 Bcx^5 + 20 Bbx^3 + 30 (Cb + Ac)x^4 + 12 Bax + 15 (Ca + Ab)x^2 + 10 Aa}{60 x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^7,x, algorithm="giac")

[Out] C*c*log(abs(x)) - 1/60*(60*B*c*x^5 + 20*B*b*x^3 + 30*(C*b + A*c)*x^4 + 12*B*a*x + 15*(C*a + A*b)*x^2 + 10*A*a)/x^6

maple [A] time = 0.01, size = 63, normalized size = 0.93

$$Cc \ln(x) - \frac{Bc}{x} - \frac{Ac}{2x^2} - \frac{Cb}{2x^2} - \frac{Bb}{3x^3} - \frac{Ab}{4x^4} - \frac{Ca}{4x^4} - \frac{Ba}{5x^5} - \frac{Aa}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^7,x)

[Out] c*C*ln(x)-B*c/x-1/3*b*B/x^3-1/5*a*B/x^5-1/4/x^4*A*b-1/4/x^4*a*C-1/2/x^2*A*c-1/2/x^2*b*C-1/6*a*A/x^6

maxima [A] time = 0.87, size = 59, normalized size = 0.87

$$Cc \log(x) - \frac{60 Bcx^5 + 20 Bbx^3 + 30 (Cb + Ac)x^4 + 12 Bax + 15 (Ca + Ab)x^2 + 10 Aa}{60 x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^7,x, algorithm="maxima")

[Out] C*c*log(x) - 1/60*(60*B*c*x^5 + 20*B*b*x^3 + 30*(C*b + A*c)*x^4 + 12*B*a*x + 15*(C*a + A*b)*x^2 + 10*A*a)/x^6

mupad [B] time = 0.79, size = 60, normalized size = 0.88

$$Cc \ln(x) - \frac{Bcx^5 + \left(\frac{Ac}{2} + \frac{Cb}{2}\right)x^4 + \frac{Bbx^3}{3} + \left(\frac{Ab}{4} + \frac{Ca}{4}\right)x^2 + \frac{Bax}{5} + \frac{Aa}{6}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^7,x)

[Out] C*c*log(x) - ((A*a)/6 + x^2*((A*b)/4 + (C*a)/4) + x^4*((A*c)/2 + (C*b)/2) + (B*a*x)/5 + (B*b*x^3)/3 + B*c*x^5)/x^6

sympy [A] time = 15.38, size = 70, normalized size = 1.03

$$Cc \log(x) + \frac{-10Aa - 12Bax - 20Bbx^3 - 60Bcx^5 + x^4(-30Ac - 30Cb) + x^2(-15Ab - 15Ca)}{60x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)/x**7,x)
```

```
[Out] C*c*log(x) + (-10*A*a - 12*B*a*x - 20*B*b*x**3 - 60*B*c*x**5 + x**4*(-30*A*c - 30*C*b) + x**2*(-15*A*b - 15*C*a))/(60*x**6)
```

$$3.11 \quad \int x^2 (A + Bx + Cx^2) (a + bx^2 + cx^4)^2 dx$$

Optimal. Leaf size=159

$$\frac{1}{3}a^2Ax^3 + \frac{1}{4}a^2Bx^4 + \frac{1}{9}x^9(C(2ac + b^2) + 2Abc) + \frac{1}{7}x^7(A(2ac + b^2) + 2abC) + \frac{1}{5}ax^5(aC + 2Ab) + \frac{1}{8}Bx^8(2ac + b^2) + \frac{1}{3}$$

Rubi [A] time = 0.21, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {1628}

$$\frac{1}{3}a^2Ax^3 + \frac{1}{4}a^2Bx^4 + \frac{1}{9}x^9(C(2ac + b^2) + 2Abc) + \frac{1}{7}x^7(A(2ac + b^2) + 2abC) + \frac{1}{5}ax^5(aC + 2Ab) + \frac{1}{8}Bx^8(2ac + b^2) + \frac{1}{3}abBx^6 + \frac{1}{11}cx^{11}(Ac + 2bC) + \frac{1}{5}bBcx^{10} + \frac{1}{12}Bc^2x^{12} + \frac{1}{13}c^2Cx^{13}$$

Antiderivative was successfully verified.

[In] Int[x^2*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]

[Out] (a^2*A*x^3)/3 + (a^2*B*x^4)/4 + (a*(2*A*b + a*C)*x^5)/5 + (a*b*B*x^6)/3 + ((A*(b^2 + 2*a*c) + 2*a*b*C)*x^7)/7 + (B*(b^2 + 2*a*c)*x^8)/8 + ((2*A*b*c + (b^2 + 2*a*c)*C)*x^9)/9 + (b*B*c*x^10)/5 + (c*(A*c + 2*b*C)*x^11)/11 + (B*c^2*x^12)/12 + (c^2*C*x^13)/13

Rule 1628

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int x^2 (A + Bx + Cx^2) (a + bx^2 + cx^4)^2 dx &= \int (a^2Ax^2 + a^2Bx^3 + a(2Ab + aC)x^4 + 2abBx^5 + (A(b^2 + 2ac) + 2abC)x^6 + (A(b^2 + 2ac) + 2abC)x^7 + (A(b^2 + 2ac) + 2abC)x^8 + (A(b^2 + 2ac) + 2abC)x^9 + (A(b^2 + 2ac) + 2abC)x^{10} + (A(b^2 + 2ac) + 2abC)x^{11} + (A(b^2 + 2ac) + 2abC)x^{12} + (A(b^2 + 2ac) + 2abC)x^{13}) dx \\ &= \frac{1}{3}a^2Ax^3 + \frac{1}{4}a^2Bx^4 + \frac{1}{5}a(2Ab + aC)x^5 + \frac{1}{3}abBx^6 + \frac{1}{7}(A(b^2 + 2ac) + 2abC)x^7 + \frac{1}{8}(A(b^2 + 2ac) + 2abC)x^8 + \frac{1}{9}(A(b^2 + 2ac) + 2abC)x^9 + \frac{1}{5}(A(b^2 + 2ac) + 2abC)x^{10} + \frac{1}{11}(A(b^2 + 2ac) + 2abC)x^{11} + \frac{1}{12}(A(b^2 + 2ac) + 2abC)x^{12} + \frac{1}{13}(A(b^2 + 2ac) + 2abC)x^{13} \end{aligned}$$

Mathematica [A] time = 0.04, size = 159, normalized size = 1.00

$$\frac{1}{3}a^2Ax^3 + \frac{1}{4}a^2Bx^4 + \frac{1}{9}x^9(2acC + 2Abc + b^2C) + \frac{1}{7}x^7(2aAc + 2abC + Ab^2) + \frac{1}{5}ax^5(aC + 2Ab) + \frac{1}{8}Bx^8(2ac + b^2) + \frac{1}{3}abBx^6 + \frac{1}{11}cx^{11}(Ac + 2bC) + \frac{1}{5}bBcx^{10} + \frac{1}{12}Bc^2x^{12} + \frac{1}{13}c^2Cx^{13}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]

[Out] (a^2*A*x^3)/3 + (a^2*B*x^4)/4 + (a*(2*A*b + a*C)*x^5)/5 + (a*b*B*x^6)/3 + ((A*b^2 + 2*a*A*c + 2*a*b*C)*x^7)/7 + (B*(b^2 + 2*a*c)*x^8)/8 + ((2*A*b*c + b^2*C + 2*a*c*C)*x^9)/9 + (b*B*c*x^10)/5 + (c*(A*c + 2*b*C)*x^11)/11 + (B*c^2*x^12)/12 + (c^2*C*x^13)/13

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 (A + Bx + Cx^2) (a + bx^2 + cx^4)^2 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]

[Out] IntegrateAlgebraic[x^2*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2, x]

fricas [A] time = 1.14, size = 154, normalized size = 0.97

$$\frac{1}{13}x^{13}c^2C + \frac{1}{12}x^{12}c^2B + \frac{2}{11}x^{11}cbC + \frac{1}{11}x^{11}c^2A + \frac{1}{5}x^{10}cbB + \frac{1}{9}x^9b^2C + \frac{2}{9}x^9caC + \frac{2}{9}x^9cbA + \frac{1}{8}x^8b^2B + \frac{1}{4}x^8caB + \frac{2}{7}x^7baC + \frac{1}{7}x^7b^2A + \frac{2}{7}x^7caA + \frac{1}{3}x^6baB + \frac{1}{5}x^5a^2C + \frac{2}{5}x^5baA + \frac{1}{4}x^4a^2B + \frac{1}{3}x^3a^2A$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")

[Out] 1/13*x^13*c^2*C + 1/12*x^12*c^2*B + 2/11*x^11*c*b*C + 1/11*x^11*c^2*A + 1/5*x^10*c*b*B + 1/9*x^9*b^2*C + 2/9*x^9*c*a*C + 2/9*x^9*c*b*A + 1/8*x^8*b^2*B + 1/4*x^8*c*a*B + 2/7*x^7*b*a*C + 1/7*x^7*b^2*A + 2/7*x^7*c*a*A + 1/3*x^6*b*a*B + 1/5*x^5*a^2*C + 2/5*x^5*b*a*A + 1/4*x^4*a^2*B + 1/3*x^3*a^2*A

giac [A] time = 0.41, size = 154, normalized size = 0.97

$$\frac{1}{13}C^2x^{13} + \frac{1}{12}Bc^2x^{12} + \frac{2}{11}Cbcbx^{11} + \frac{1}{11}Ac^2x^{11} + \frac{1}{5}Bbcx^{10} + \frac{1}{9}Cb^2x^9 + \frac{2}{9}Cacx^9 + \frac{2}{9}Abcx^9 + \frac{1}{8}Bb^2x^8 + \frac{1}{4}Bacx^8 + \frac{2}{7}Ccbx^7 + \frac{1}{7}Ab^2x^7 + \frac{2}{7}Aacx^7 + \frac{1}{3}Babx^6 + \frac{1}{5}Ca^2x^5 + \frac{2}{5}Aabx^5 + \frac{1}{4}Ba^2x^4 + \frac{1}{3}Aa^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="giac")

[Out] 1/13*C*c^2*x^13 + 1/12*B*c^2*x^12 + 2/11*C*b*c*x^11 + 1/11*A*c^2*x^11 + 1/5*B*b*c*x^10 + 1/9*C*b^2*x^9 + 2/9*C*a*c*x^9 + 2/9*A*b*c*x^9 + 1/8*B*b^2*x^8 + 1/4*B*a*c*x^8 + 2/7*C*a*b*x^7 + 1/7*A*b^2*x^7 + 2/7*A*a*c*x^7 + 1/3*B*a*b*x^6 + 1/5*C*a^2*x^5 + 2/5*A*a*b*x^5 + 1/4*B*a^2*x^4 + 1/3*A*a^2*x^3

maple [A] time = 0.00, size = 142, normalized size = 0.89

$$\frac{C^2x^{13}}{13} + \frac{Bc^2x^{12}}{12} + \frac{Bbcx^{10}}{5} + \frac{(Ac^2 + 2Cbc)x^{11}}{11} + \frac{Babx^6}{3} + \frac{(2ac + b^2)Bx^8}{8} + \frac{(2Abc + (2ac + b^2)C)x^9}{9} + \frac{Ba^2x^4}{4} + \frac{(2Cab + (2ac + b^2)A)x^7}{7} + \frac{Aa^2x^3}{3} + \frac{(2Aab + Ca^2)x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x)

[Out] 1/13*c^2*C*x^13+1/12*B*c^2*x^12+1/11*(A*c^2+2*C*b*c)*x^11+1/5*b*B*c*x^10+1/9*(2*A*b*c+(2*a*c+b^2)*C)*x^9+1/8*B*(2*a*c+b^2)*x^8+1/7*(A*(2*a*c+b^2)+2*a*b*C)*x^7+1/3*a*b*B*x^6+1/5*(2*A*a*b+C*a^2)*x^5+1/4*a^2*B*x^4+1/3*a^2*A*x^3

maxima [A] time = 1.13, size = 143, normalized size = 0.90

$$\frac{1}{13}C^2x^{13} + \frac{1}{12}Bc^2x^{12} + \frac{1}{5}Bbcx^{10} + \frac{1}{11}(2Cbc + Ac^2)x^{11} + \frac{1}{9}(Cb^2 + 2(Ca + Ab)c)x^9 + \frac{1}{3}Babx^6 + \frac{1}{8}(Bb^2 + 2Bac)x^8 + \frac{1}{7}(2Cab + Ab^2 + 2Aac)x^7 + \frac{1}{4}Ba^2x^4 + \frac{1}{3}Aa^2x^3 + \frac{1}{5}(Ca^2 + 2Aab)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")

[Out] 1/13*C*c^2*x^13 + 1/12*B*c^2*x^12 + 1/5*B*b*c*x^10 + 1/11*(2*C*b*c + A*c^2)*x^11 + 1/9*(C*b^2 + 2*(C*a + A*b)*c)*x^9 + 1/3*B*a*b*x^6 + 1/8*(B*b^2 + 2*B*a*c)*x^8 + 1/7*(2*C*a*b + A*b^2 + 2*A*a*c)*x^7 + 1/4*B*a^2*x^4 + 1/3*A*a^2*x^3 + 1/5*(C*a^2 + 2*A*a*b)*x^5

mupad [B] time = 0.82, size = 141, normalized size = 0.89

$$x^5 \left(\frac{Ca^2}{5} + \frac{2Aba}{5} \right) + x^{11} \left(\frac{Ac^2}{11} + \frac{2Cbc}{11} \right) + x^7 \left(\frac{Ab^2}{7} + \frac{2Cab}{7} + \frac{2Aac}{7} \right) + x^9 \left(\frac{Cb^2}{9} + \frac{2Acab}{9} + \frac{2Cac}{9} \right) + \frac{Aa^2x^3}{3} + \frac{Ba^2x^4}{4} + \frac{Bc^2x^{12}}{12} + \frac{C^2x^{13}}{13} + \frac{Bx^8(b^2 + 2ac)}{8} + \frac{Babx^6}{3} + \frac{Bbcx^{10}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x)

[Out] x^5*((C*a^2)/5 + (2*A*a*b)/5) + x^11*((A*c^2)/11 + (2*C*b*c)/11) + x^7*((A*b^2)/7 + (2*A*a*c)/7 + (2*C*a*b)/7) + x^9*((C*b^2)/9 + (2*A*b*c)/9 + (2*C*a*c)/9) + (A*a^2*x^3)/3 + (B*a^2*x^4)/4 + (B*c^2*x^12)/12 + (C*c^2*x^13)/13 + (B*x^8*(2*a*c + b^2))/8 + (B*a*b*x^6)/3 + (B*b*c*x^10)/5

sympy [A] time = 0.09, size = 168, normalized size = 1.06

$$\frac{Aa^2x^3}{3} + \frac{Ba^2x^4}{4} + \frac{Babx^6}{3} + \frac{Bbcx^{10}}{5} + \frac{Bc^2x^{12}}{12} + \frac{Cc^2x^{13}}{13} + x^{11}\left(\frac{Ac^2}{11} + \frac{2Cbc}{11}\right) + x^9\left(\frac{2Abc}{9} + \frac{2Cac}{9} + \frac{Cb^2}{9}\right) + x^8\left(\frac{Bac}{4} + \frac{Bb^2}{8}\right) + x^7\left(\frac{2Aac}{7} + \frac{Ab^2}{7} + \frac{2Cab}{7}\right) + x^5\left(\frac{2Aab}{5} + \frac{Ca^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2,x)

[Out] A*a**2*x**3/3 + B*a**2*x**4/4 + B*a*b*x**6/3 + B*b*c*x**10/5 + B*c**2*x**12/12 + C*c**2*x**13/13 + x**11*(A*c**2/11 + 2*C*b*c/11) + x**9*(2*A*b*c/9 + 2*C*a*c/9 + C*b**2/9) + x**8*(B*a*c/4 + B*b**2/8) + x**7*(2*A*a*c/7 + A*b**2/7 + 2*C*a*b/7) + x**5*(2*A*a*b/5 + C*a**2/5)

$$3.12 \quad \int x (A + Bx + Cx^2) (a + bx^2 + cx^4)^2 dx$$

Optimal. Leaf size=159

$$\frac{1}{2}a^2Ax^2 + \frac{1}{3}a^2Bx^3 + \frac{1}{8}x^8(C(2ac + b^2) + 2Abc) + \frac{1}{6}x^6(A(2ac + b^2) + 2abC) + \frac{1}{4}ax^4(aC + 2Ab) + \frac{1}{7}Bx^7(2ac + b^2)$$

Rubi [A] time = 0.14, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {1628}

$$\frac{1}{2}a^2Ax^2 + \frac{1}{3}a^2Bx^3 + \frac{1}{8}x^8(C(2ac + b^2) + 2Abc) + \frac{1}{6}x^6(A(2ac + b^2) + 2abC) + \frac{1}{4}ax^4(aC + 2Ab) + \frac{1}{7}Bx^7(2ac + b^2) + \frac{2}{5}abBx^5 + \frac{1}{10}cx^{10}(Ac + 2bC) + \frac{2}{9}bBcx^9 + \frac{1}{11}Bc^2x^{11} + \frac{1}{12}c^2Cx^{12}$$

Antiderivative was successfully verified.

[In] Int[x*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]

[Out] (a^2*A*x^2)/2 + (a^2*B*x^3)/3 + (a*(2*A*b + a*C)*x^4)/4 + (2*a*b*B*x^5)/5 + ((A*(b^2 + 2*a*c) + 2*a*b*C)*x^6)/6 + (B*(b^2 + 2*a*c)*x^7)/7 + ((2*A*b*c + (b^2 + 2*a*c)*C)*x^8)/8 + (2*b*B*c*x^9)/9 + (c*(A*c + 2*b*C)*x^10)/10 + (B*c^2*x^11)/11 + (c^2*C*x^12)/12

Rule 1628

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int x (A + Bx + Cx^2) (a + bx^2 + cx^4)^2 dx &= \int (a^2Ax + a^2Bx^2 + a(2Ab + aC)x^3 + 2abBx^4 + (A(b^2 + 2ac) + 2abC)x^5 + (A(b^2 + 2ac) + 2abC)x^6 + (2AbC + b^2C)x^7 + (2AbC + b^2C)x^8 + (2AbC + b^2C)x^9 + (2AbC + b^2C)x^{10} + (2AbC + b^2C)x^{11} + (2AbC + b^2C)x^{12}) dx \\ &= \frac{1}{2}a^2Ax^2 + \frac{1}{3}a^2Bx^3 + \frac{1}{4}a(2Ab + aC)x^4 + \frac{2}{5}abBx^5 + \frac{1}{6}(A(b^2 + 2ac) + 2abC)x^6 + \frac{1}{7}(2AbC + b^2C)x^7 + \frac{1}{8}(2AbC + b^2C)x^8 + \frac{1}{9}(2AbC + b^2C)x^9 + \frac{1}{10}(2AbC + b^2C)x^{10} + \frac{1}{11}(2AbC + b^2C)x^{11} + \frac{1}{12}(2AbC + b^2C)x^{12} \end{aligned}$$

Mathematica [A] time = 0.04, size = 159, normalized size = 1.00

$$\frac{1}{2}a^2Ax^2 + \frac{1}{3}a^2Bx^3 + \frac{1}{8}x^8(2acC + 2Abc + b^2C) + \frac{1}{6}x^6(2aAc + 2abC + Ab^2) + \frac{1}{4}ax^4(aC + 2Ab) + \frac{1}{7}Bx^7(2ac + b^2) + \frac{2}{5}abBx^5 + \frac{1}{10}cx^{10}(Ac + 2bC) + \frac{2}{9}bBcx^9 + \frac{1}{11}Bc^2x^{11} + \frac{1}{12}c^2Cx^{12}$$

Antiderivative was successfully verified.

[In] Integrate[x*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]

[Out] (a^2*A*x^2)/2 + (a^2*B*x^3)/3 + (a*(2*A*b + a*C)*x^4)/4 + (2*a*b*B*x^5)/5 + ((A*b^2 + 2*a*A*c + 2*a*b*C)*x^6)/6 + (B*(b^2 + 2*a*c)*x^7)/7 + ((2*A*b*c + b^2*C + 2*a*c*C)*x^8)/8 + (2*b*B*c*x^9)/9 + (c*(A*c + 2*b*C)*x^10)/10 + (B*c^2*x^11)/11 + (c^2*C*x^12)/12

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x (A + Bx + Cx^2) (a + bx^2 + cx^4)^2 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]

[Out] IntegrateAlgebraic[x*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2, x]

fricas [A] time = 1.09, size = 154, normalized size = 0.97

$$\frac{1}{12}x^{12}c^2C + \frac{1}{11}x^{11}c^2B + \frac{1}{5}x^{10}cbC + \frac{1}{10}x^{10}c^2A + \frac{2}{9}x^9cbB + \frac{1}{8}x^8b^2C + \frac{1}{4}x^8caC + \frac{1}{4}x^8cbA + \frac{1}{7}x^7b^2B + \frac{2}{7}x^7caB + \frac{1}{3}x^6baC + \frac{1}{6}x^6b^2A + \frac{1}{3}x^6caA + \frac{2}{5}x^5baB + \frac{1}{4}x^4a^2C + \frac{1}{2}x^4baA + \frac{1}{3}x^3a^2B + \frac{1}{2}x^2a^2A$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")

$$\begin{aligned} \text{[Out]} & \frac{1}{12}x^{12}c^2C + \frac{1}{11}x^{11}c^2B + \frac{1}{5}x^{10}c*b*C + \frac{1}{10}x^{10}c^2A + \frac{2}{9}x^9c*b*B \\ & + \frac{1}{8}x^8b^2C + \frac{1}{4}x^8c*a*C + \frac{1}{4}x^8c*b*A + \frac{1}{7}x^7b^2B + \frac{2}{7}x^7c*a*B \\ & + \frac{1}{3}x^6b*a*C + \frac{1}{6}x^6b^2A + \frac{1}{3}x^6c*a*A + \frac{2}{5}x^5b*a*B + \frac{1}{4}x^4a^2C \\ & + \frac{1}{2}x^4b*a*A + \frac{1}{3}x^3a^2B + \frac{1}{2}x^2a^2A \end{aligned}$$

giac [A] time = 0.31, size = 154, normalized size = 0.97

$$\frac{1}{12}C^2x^{12} + \frac{1}{11}Bc^2x^{11} + \frac{1}{5}Cbcbx^{10} + \frac{1}{10}Ac^2x^{10} + \frac{2}{9}Bbcx^9 + \frac{1}{8}Cb^2x^8 + \frac{1}{4}Cacx^8 + \frac{1}{4}Abcx^8 + \frac{1}{7}Bb^2x^7 + \frac{2}{7}Bacx^7 + \frac{1}{3}Cabx^6 + \frac{1}{6}Ab^2x^6 + \frac{1}{3}Aacx^6 + \frac{2}{5}Babx^5 + \frac{1}{4}Ca^2x^4 + \frac{1}{2}Aabx^4 + \frac{1}{3}Ba^2x^3 + \frac{1}{2}Aa^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="giac")

$$\begin{aligned} \text{[Out]} & \frac{1}{12}C^2x^{12} + \frac{1}{11}Bc^2x^{11} + \frac{1}{5}C*b*c*x^{10} + \frac{1}{10}A*c^2*x^{10} + \frac{2}{9}B*b*c*x^9 \\ & + \frac{1}{8}C*b^2*x^8 + \frac{1}{4}C*a*c*x^8 + \frac{1}{4}A*b*c*x^8 + \frac{1}{7}B*b^2*x^7 + \frac{2}{7}B*a*c*x^7 \\ & + \frac{1}{3}C*a*b*x^6 + \frac{1}{6}A*b^2*x^6 + \frac{1}{3}A*a*c*x^6 + \frac{2}{5}B*a*b*x^5 \\ & + \frac{1}{4}C*a^2*x^4 + \frac{1}{2}A*a*b*x^4 + \frac{1}{3}B*a^2*x^3 + \frac{1}{2}A*a^2*x^2 \end{aligned}$$

maple [A] time = 0.00, size = 142, normalized size = 0.89

$$\frac{C^2x^{12}}{12} + \frac{Bc^2x^{11}}{11} + \frac{2Bbcx^9}{9} + \frac{(A^2+2CbC)x^{10}}{10} + \frac{2Babx^5}{5} + \frac{(2ac+b^2)Bx^7}{7} + \frac{(2Abc+(2ac+b^2)C)x^8}{8} + \frac{Ba^2x^3}{3} + \frac{(2Cab+(2ac+b^2)A)x^6}{6} + \frac{Aa^2x^2}{2} + \frac{(2Aab+Ca^2)x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x)

$$\begin{aligned} \text{[Out]} & \frac{1}{12}c^2C*x^{12} + \frac{1}{11}B*c^2*x^{11} + \frac{1}{10}*(A*c^2+2*C*b*c)*x^{10} + \frac{2}{9}b*B*c*x^9 + \frac{1}{8} \\ & *(2*A*b*c+(2*a*c+b^2)*C)*x^8 + \frac{1}{7}B*(2*a*c+b^2)*x^7 + \frac{1}{6}*(2*C*a*b+(2*a*c+b^2) \\ & *A)*x^6 + \frac{2}{5}a*b*B*x^5 + \frac{1}{4}*(2*A*a*b+C*a^2)*x^4 + \frac{1}{3}a^2*B*x^3 + \frac{1}{2}a^2*A*x^2 \end{aligned}$$

maxima [A] time = 1.18, size = 143, normalized size = 0.90

$$\frac{1}{12}C^2x^{12} + \frac{1}{11}Bc^2x^{11} + \frac{2}{9}Bbcx^9 + \frac{1}{10}(2CbC+Ac^2)x^{10} + \frac{1}{8}(Cb^2+2(Ca+Ab)c)x^8 + \frac{2}{5}Babx^5 + \frac{1}{7}(Bb^2+2Bac)x^7 + \frac{1}{6}(2Cab+Ab^2+2Aac)x^6 + \frac{1}{3}Ba^2x^3 + \frac{1}{2}Aa^2x^2 + \frac{1}{4}(Ca^2+2Aab)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")

$$\begin{aligned} \text{[Out]} & \frac{1}{12}C^2x^{12} + \frac{1}{11}Bc^2x^{11} + \frac{2}{9}B*b*c*x^9 + \frac{1}{10}*(2*C*b*c + A*c^2)*x^{10} \\ & + \frac{1}{8}*(C*b^2 + 2*(C*a + A*b)*c)*x^8 + \frac{2}{5}B*a*b*x^5 + \frac{1}{7}*(B*b^2 + 2*B \\ & *a*c)*x^7 + \frac{1}{6}*(2*C*a*b + A*b^2 + 2*A*a*c)*x^6 + \frac{1}{3}B*a^2*x^3 + \frac{1}{2}A*a^2 \\ & *x^2 + \frac{1}{4}*(C*a^2 + 2*A*a*b)*x^4 \end{aligned}$$

mapad [B] time = 0.07, size = 141, normalized size = 0.89

$$x^4\left(\frac{Ca^2}{4} + \frac{Aba}{2}\right) + x^{10}\left(\frac{Ac^2}{10} + \frac{Cbc}{5}\right) + x^6\left(\frac{Ab^2}{6} + \frac{Cab}{3} + \frac{Aac}{3}\right) + x^8\left(\frac{Cb^2}{8} + \frac{Ac b}{4} + \frac{Cac}{4}\right) + \frac{Aa^2x^2}{2} + \frac{Ba^2x^3}{3} + \frac{Bc^2x^{11}}{11} + \frac{C^2x^{12}}{12} + \frac{Bx^7(b^2+2ac)}{7} + \frac{2Babx^5}{5} + \frac{2Bbcx^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x)

$$\begin{aligned} \text{[Out]} & x^4*((C*a^2)/4 + (A*a*b)/2) + x^{10}*((A*c^2)/10 + (C*b*c)/5) + x^6*((A*b^2)/ \\ & 6 + (A*a*c)/3 + (C*a*b)/3) + x^8*((C*b^2)/8 + (A*b*c)/4 + (C*a*c)/4) + (A*a \\ & ^2*x^2)/2 + (B*a^2*x^3)/3 + (B*c^2*x^{11})/11 + (C*c^2*x^{12})/12 + (B*x^7*(2*a \\ & *c + b^2))/7 + (2*B*a*b*x^5)/5 + (2*B*b*c*x^9)/9 \end{aligned}$$

sympy [A] time = 0.09, size = 163, normalized size = 1.03

$$\frac{Aa^2x^2}{2} + \frac{Ba^2x^3}{3} + \frac{2Babx^5}{5} + \frac{2Bbcx^9}{9} + \frac{Bc^2x^{11}}{11} + \frac{Cc^2x^{12}}{12} + x^{10} \left(\frac{Ac^2}{10} + \frac{Cbc}{5} \right) + x^8 \left(\frac{Abc}{4} + \frac{Cac}{4} + \frac{Cb^2}{8} \right) + x^7 \left(\frac{2Bac}{7} + \frac{Bb^2}{7} \right) + x^6 \left(\frac{Aac}{3} + \frac{Ab^2}{6} + \frac{Cab}{3} \right) + x^4 \left(\frac{Aab}{2} + \frac{Ca^2}{4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2,x)

[Out] A*a**2*x**2/2 + B*a**2*x**3/3 + 2*B*a*b*x**5/5 + 2*B*b*c*x**9/9 + B*c**2*x**11/11 + C*c**2*x**12/12 + x**10*(A*c**2/10 + C*b*c/5) + x**8*(A*b*c/4 + C*a*c/4 + C*b**2/8) + x**7*(2*B*a*c/7 + B*b**2/7) + x**6*(A*a*c/3 + A*b**2/6 + C*a*b/3) + x**4*(A*a*b/2 + C*a**2/4)

$$3.13 \quad \int (A + Bx + Cx^2) (a + bx^2 + cx^4)^2 dx$$

Optimal. Leaf size=154

$$a^2 Ax + \frac{1}{2} a^2 Bx^2 + \frac{1}{7} x^7 (C(2ac + b^2) + 2Abc) + \frac{1}{5} x^5 (A(2ac + b^2) + 2abC) + \frac{1}{3} ax^3 (aC + 2Ab) + \frac{1}{6} Bx^6 (2ac + b^2) + \frac{1}{2} abx^4$$

Rubi [A] time = 0.11, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {1657}

$$a^2 Ax + \frac{1}{2} a^2 Bx^2 + \frac{1}{7} x^7 (C(2ac + b^2) + 2Abc) + \frac{1}{5} x^5 (A(2ac + b^2) + 2abC) + \frac{1}{3} ax^3 (aC + 2Ab) + \frac{1}{6} Bx^6 (2ac + b^2) + \frac{1}{2} abBx^4 + \frac{1}{9} cx^9 (Ac + 2bC) + \frac{1}{4} bBcx^8 + \frac{1}{10} Bc^2 x^{10} + \frac{1}{11} c^2 Cx^{11}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]

[Out] a^2*A*x + (a^2*B*x^2)/2 + (a*(2*A*b + a*C)*x^3)/3 + (a*b*B*x^4)/2 + ((A*(b^2 + 2*a*c) + 2*a*b*C)*x^5)/5 + (B*(b^2 + 2*a*c)*x^6)/6 + ((2*A*b*c + (b^2 + 2*a*c)*C)*x^7)/7 + (b*B*c*x^8)/4 + (c*(A*c + 2*b*C)*x^9)/9 + (B*c^2*x^10)/10 + (c^2*C*x^11)/11

Rule 1657

Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[Expand Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int (A + Bx + Cx^2) (a + bx^2 + cx^4)^2 dx &= \int (a^2 A + a^2 Bx + a(2Ab + aC)x^2 + 2abBx^3 + (A(b^2 + 2ac) + 2abC)x^4 \\ &\quad + (2a^2 A^*c + 2a^*b*C)x^5 + (B*(b^2 + 2a*c)*x^6)/6 + ((2*A*b*c + (b^2 + 2*a*c)*C)*x^7)/7 + (b*B*c*x^8)/4 + (c*(A*c + 2*b*C)*x^9)/9 + (B*c^2*x^{10})/10 + (c^2*C*x^{11})/11 \end{aligned}$$

Mathematica [A] time = 0.03, size = 154, normalized size = 1.00

$$a^2 Ax + \frac{1}{2} a^2 Bx^2 + \frac{1}{7} x^7 (2acC + 2Abc + b^2 C) + \frac{1}{5} x^5 (2aAc + 2abC + Ab^2) + \frac{1}{3} ax^3 (aC + 2Ab) + \frac{1}{6} Bx^6 (2ac + b^2) + \frac{1}{2} abBx^4 + \frac{1}{9} cx^9 (Ac + 2bC) + \frac{1}{4} bBcx^8 + \frac{1}{10} Bc^2 x^{10} + \frac{1}{11} c^2 Cx^{11}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]

[Out] a^2*A*x + (a^2*B*x^2)/2 + (a*(2*A*b + a*C)*x^3)/3 + (a*b*B*x^4)/2 + ((A*(b^2 + 2*a*c) + 2*a*b*C)*x^5)/5 + (B*(b^2 + 2*a*c)*x^6)/6 + ((2*A*b*c + b^2*C + 2*a*c*C)*x^7)/7 + (b*B*c*x^8)/4 + (c*(A*c + 2*b*C)*x^9)/9 + (B*c^2*x^10)/10 + (c^2*C*x^11)/11

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + Bx + Cx^2) (a + bx^2 + cx^4)^2 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]

[Out] IntegrateAlgebraic[(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2, x]

fricas [A] time = 0.98, size = 151, normalized size = 0.98

$$\frac{1}{11}x^{11}c^2C + \frac{1}{10}x^{10}c^2B + \frac{2}{9}x^9cbC + \frac{1}{9}x^9c^2A + \frac{1}{4}x^8cbB + \frac{1}{7}x^7b^2C + \frac{2}{7}x^7caC + \frac{2}{7}x^7cbA + \frac{1}{6}x^6b^2B + \frac{1}{3}x^6caB + \frac{2}{5}x^5baC + \frac{1}{5}x^5b^2A + \frac{2}{5}x^5caA + \frac{1}{2}x^4baB + \frac{1}{3}x^3a^2C + \frac{2}{3}x^3baA + \frac{1}{2}x^2a^2B + xa^2A$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")

[Out] $\frac{1}{11}x^{11}c^2C + \frac{1}{10}x^{10}c^2B + \frac{2}{9}x^9c^2C + \frac{1}{9}x^9c^2A + \frac{1}{4}x^8cbB + \frac{1}{7}x^7b^2C + \frac{2}{7}x^7caC + \frac{2}{7}x^7cbA + \frac{1}{6}x^6b^2B + \frac{1}{3}x^6caB + \frac{2}{5}x^5baC + \frac{1}{5}x^5b^2A + \frac{2}{5}x^5caA + \frac{1}{2}x^4baB + \frac{1}{3}x^3a^2C + \frac{2}{3}x^3baA + \frac{1}{2}x^2a^2B + xa^2A$

giac [A] time = 0.30, size = 151, normalized size = 0.98

$$\frac{1}{11}C^2x^{11} + \frac{1}{10}Bc^2x^{10} + \frac{2}{9}Cbcx^9 + \frac{1}{9}Ac^2x^9 + \frac{1}{4}Bbcx^8 + \frac{1}{7}Cb^2x^7 + \frac{2}{7}Cacx^7 + \frac{2}{7}Abcx^7 + \frac{1}{6}Bb^2x^6 + \frac{1}{3}Bacx^6 + \frac{2}{5}Cabx^5 + \frac{1}{5}Ab^2x^5 + \frac{2}{5}Aacx^5 + \frac{1}{2}Babx^4 + \frac{1}{3}Ca^2x^3 + \frac{2}{3}Aabx^3 + \frac{1}{2}Ba^2x^2 + Aa^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="giac")

[Out] $\frac{1}{11}C^2x^{11} + \frac{1}{10}Bc^2x^{10} + \frac{2}{9}C^2x^9 + \frac{1}{9}A^2c^2x^9 + \frac{1}{4}B^2x^8 + \frac{1}{7}C^2x^7 + \frac{2}{7}C^2x^7 + \frac{2}{7}A^2x^7 + \frac{1}{6}B^2x^6 + \frac{1}{3}B^2x^6 + \frac{2}{5}C^2x^5 + \frac{1}{5}A^2x^5 + \frac{2}{5}A^2x^5 + \frac{1}{2}B^2x^4 + \frac{1}{3}C^2x^3 + \frac{2}{3}A^2x^3 + \frac{1}{2}B^2x^2 + A^2x$

maple [A] time = 0.00, size = 139, normalized size = 0.90

$$\frac{C^2x^{11}}{11} + \frac{Bc^2x^{10}}{10} + \frac{Bbcx^8}{4} + \frac{(A^2 + 2Cbc)x^9}{9} + \frac{Babx^4}{2} + \frac{(2ac + b^2)Bx^6}{6} + \frac{(2Abc + (2ac + b^2)C)x^7}{7} + \frac{Ba^2x^2}{2} + \frac{(2Cab + (2ac + b^2)A)x^5}{5} + Aa^2x + \frac{(2Aab + Ca^2)x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x)

[Out] $\frac{1}{11}C^2x^{11} + \frac{1}{10}Bc^2x^{10} + \frac{1}{9}(A^2c^2 + 2C^2bc)x^9 + \frac{1}{4}B^2x^8 + \frac{1}{7}(2A^2bc + (2ac + b^2)C)x^7 + \frac{1}{6}B^2x^6 + \frac{1}{5}(2C^2ab + (2ac + b^2)A)x^5 + \frac{1}{2}A^2x + \frac{1}{3}(2A^2ab + Ca^2)x^3$

maxima [A] time = 0.63, size = 140, normalized size = 0.91

$$\frac{1}{11}C^2x^{11} + \frac{1}{10}Bc^2x^{10} + \frac{1}{4}Bbcx^8 + \frac{1}{9}(2Cbc + Ac^2)x^9 + \frac{1}{7}(Cb^2 + 2(Ca + Ab)c)x^7 + \frac{1}{2}Babx^4 + \frac{1}{6}(Bb^2 + 2Bac)x^6 + \frac{1}{5}(2Cab + Ab^2 + 2Aac)x^5 + \frac{1}{2}Ba^2x^2 + Aa^2x + \frac{1}{3}(Ca^2 + 2Aab)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")

[Out] $\frac{1}{11}C^2x^{11} + \frac{1}{10}Bc^2x^{10} + \frac{1}{4}B^2x^8 + \frac{1}{9}(2C^2bc + A^2c^2)x^9 + \frac{1}{7}(C^2b^2 + 2(C^2a + A^2b)c)x^7 + \frac{1}{2}B^2x^4 + \frac{1}{6}(B^2b^2 + 2B^2a^2c)x^6 + \frac{1}{5}(2C^2ab + A^2b^2 + 2A^2a^2c)x^5 + \frac{1}{2}B^2x^2 + A^2x + \frac{1}{3}(C^2a^2 + 2A^2ab)x^3$

mupad [B] time = 0.07, size = 138, normalized size = 0.90

$$x^3 \left(\frac{Ca^2}{3} + \frac{2Aba}{3} \right) + x^9 \left(\frac{A^2}{9} + \frac{2Cbc}{9} \right) + x^5 \left(\frac{Ab^2}{5} + \frac{2Cab}{5} + \frac{2Aac}{5} \right) + x^7 \left(\frac{Cb^2}{7} + \frac{2Ac b}{7} + \frac{2Cac}{7} \right) + \frac{Ba^2x^2}{2} + \frac{Bc^2x^{10}}{10} + \frac{C^2x^{11}}{11} + \frac{Bx^6(b^2 + 2ac)}{6} + Aa^2x + \frac{Babx^4}{2} + \frac{Bbcx^8}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x)

[Out] $x^3 \left(\frac{(A^2)}{3} + \frac{(2A^2ab)}{3} \right) + x^9 \left(\frac{(A^2c^2)}{9} + \frac{(2C^2bc)}{9} \right) + x^5 \left(\frac{(A^2b^2)}{5} + \frac{(2A^2ac)}{5} + \frac{(2C^2ab)}{5} \right) + x^7 \left(\frac{(C^2b^2)}{7} + \frac{(2A^2bc)}{7} + \frac{(2C^2ac)}{7} \right) + \frac{(B^2x^2)}{2} + \frac{(B^2x^{10})}{10} + \frac{(C^2x^{11})}{11} + \frac{(B^2x^6(2ac + b^2))}{6} + Aa^2x + \frac{(B^2x^4)}{2} + \frac{(B^2x^8)}{4}$

sympy [A] time = 0.09, size = 165, normalized size = 1.07

$$Aa^2x + \frac{Ba^2x^2}{2} + \frac{Babx^4}{2} + \frac{Bbcx^8}{4} + \frac{Bc^2x^{10}}{10} + \frac{Cc^2x^{11}}{11} + x^9\left(\frac{Ac^2}{9} + \frac{2Cbc}{9}\right) + x^7\left(\frac{2Abc}{7} + \frac{2Cac}{7} + \frac{Cb^2}{7}\right) + x^6\left(\frac{Bac}{3} + \frac{Bb^2}{6}\right) + x^5\left(\frac{2Aac}{5} + \frac{Ab^2}{5} + \frac{2Cab}{5}\right) + x^3\left(\frac{2Aab}{3} + \frac{Ca^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2,x)

[Out] A*a**2*x + B*a**2*x**2/2 + B*a*b*x**4/2 + B*b*c*x**8/4 + B*c**2*x**10/10 + C*c**2*x**11/11 + x**9*(A*c**2/9 + 2*C*b*c/9) + x**7*(2*A*b*c/7 + 2*C*a*c/7 + C*b**2/7) + x**6*(B*a*c/3 + B*b**2/6) + x**5*(2*A*a*c/5 + A*b**2/5 + 2*C*a*b/5) + x**3*(2*A*a*b/3 + C*a**2/3)

$$3.14 \quad \int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x} dx$$

Optimal. Leaf size=150

$$a^2 A \log(x) + a^2 Bx + \frac{1}{6} x^6 (C(2ac + b^2) + 2Abc) + \frac{1}{4} x^4 (A(2ac + b^2) + 2abC) + \frac{1}{2} ax^2 (aC + 2Ab) + \frac{1}{5} Bx^5 (2ac + b^2)$$

Rubi [A] time = 0.11, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {1628}

$$a^2 A \log(x) + a^2 Bx + \frac{1}{6} x^6 (C(2ac + b^2) + 2Abc) + \frac{1}{4} x^4 (A(2ac + b^2) + 2abC) + \frac{1}{2} ax^2 (aC + 2Ab) + \frac{1}{5} Bx^5 (2ac + b^2) + \frac{2}{3} abBx^3 + \frac{1}{8} cx^8 (Ac + 2bC) + \frac{2}{7} bBcx^7 + \frac{1}{9} Bc^2 x^9 + \frac{1}{10} c^2 Cx^{10}$$

Antiderivative was successfully verified.

[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x, x]

[Out] a^2*B*x + (a*(2*A*b + a*C)*x^2)/2 + (2*a*b*B*x^3)/3 + ((A*(b^2 + 2*a*c) + 2*a*b*C)*x^4)/4 + (B*(b^2 + 2*a*c)*x^5)/5 + ((2*A*b*c + (b^2 + 2*a*c)*C)*x^6)/6 + (2*b*B*c*x^7)/7 + (c*(A*c + 2*b*C)*x^8)/8 + (B*c^2*x^9)/9 + (c^2*C*x^10)/10 + a^2*A*Log[x]

Rule 1628

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int \frac{(A + Bx + Cx^2)(a + bx^2 + cx^4)^2}{x} dx &= \int \left(a^2 B + \frac{a^2 A}{x} + a(2Ab + aC)x + 2abBx^2 + (A(b^2 + 2ac) + 2abC) \right. \\ &= a^2 Bx + \frac{1}{2} a(2Ab + aC)x^2 + \frac{2}{3} abBx^3 + \frac{1}{4} (A(b^2 + 2ac) + 2abC)x^4 + \end{aligned}$$

Mathematica [A] time = 0.04, size = 150, normalized size = 1.00

$$a^2 A \log(x) + a^2 Bx + \frac{1}{6} x^6 (2ac + 2Abc + b^2 C) + \frac{1}{4} x^4 (2aAc + 2abC + Ab^2) + \frac{1}{2} ax^2 (aC + 2Ab) + \frac{1}{5} Bx^5 (2ac + b^2) + \frac{2}{3} abBx^3 + \frac{1}{8} cx^8 (Ac + 2bC) + \frac{2}{7} bBcx^7 + \frac{1}{9} Bc^2 x^9 + \frac{1}{10} c^2 Cx^{10}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x, x]

[Out] a^2*B*x + (a*(2*A*b + a*C)*x^2)/2 + (2*a*b*B*x^3)/3 + ((A*b^2 + 2*a*A*c + 2*a*b*C)*x^4)/4 + (B*(b^2 + 2*a*c)*x^5)/5 + ((2*A*b*c + b^2*C + 2*a*c*C)*x^6)/6 + (2*b*B*c*x^7)/7 + (c*(A*c + 2*b*C)*x^8)/8 + (B*c^2*x^9)/9 + (c^2*C*x^10)/10 + a^2*A*Log[x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + Bx + Cx^2)(a + bx^2 + cx^4)^2}{x} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x, x]

[Out] IntegrateAlgebraic[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x, x]

fricas [A] time = 1.36, size = 138, normalized size = 0.92

$$\frac{1}{10}C^2x^{10} + \frac{1}{9}Bc^2x^9 + \frac{2}{7}Bbcx^7 + \frac{1}{8}(2Cbc + Ac^2)x^8 + \frac{1}{6}(Cb^2 + 2(Ca + Ab)c)x^6 + \frac{2}{3}Babx^3 + \frac{1}{5}(Bb^2 + 2Bac)x^5 + \frac{1}{4}(2Cab + Ab^2 + 2Aac)x^4 + Ba^2x + Aa^2\log(x) + \frac{1}{2}(Ca^2 + 2Aab)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x,x, algorithm="fricas")

[Out] 1/10*C*c^2*x^10 + 1/9*B*c^2*x^9 + 2/7*B*b*c*x^7 + 1/8*(2*C*b*c + A*c^2)*x^8 + 1/6*(C*b^2 + 2*(C*a + A*b)*c)*x^6 + 2/3*B*a*b*x^3 + 1/5*(B*b^2 + 2*B*a*c)*x^5 + 1/4*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 + B*a^2*x + A*a^2*log(x) + 1/2*(C*a^2 + 2*A*a*b)*x^2

giac [A] time = 0.36, size = 149, normalized size = 0.99

$$\frac{1}{10}C^2x^{10} + \frac{1}{9}Bc^2x^9 + \frac{1}{4}Cbcx^8 + \frac{1}{8}Ac^2x^8 + \frac{2}{7}Bbcx^7 + \frac{1}{6}Cb^2x^6 + \frac{1}{3}Cacx^6 + \frac{1}{3}Abcx^6 + \frac{1}{5}Bb^2x^5 + \frac{2}{3}Bacx^5 + \frac{1}{2}Cabx^4 + \frac{1}{4}Ab^2x^4 + \frac{1}{2}Aacx^4 + \frac{2}{3}Babx^3 + \frac{1}{2}Ca^2x^2 + Aabx^2 + Ba^2x + Aa^2\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x,x, algorithm="giac")

[Out] 1/10*C*c^2*x^10 + 1/9*B*c^2*x^9 + 1/4*C*b*c*x^8 + 1/8*A*c^2*x^8 + 2/7*B*b*c*x^7 + 1/6*C*b^2*x^6 + 1/3*C*a*c*x^6 + 1/3*A*b*c*x^6 + 1/5*B*b^2*x^5 + 2/5*B*a*c*x^5 + 1/2*C*a*b*x^4 + 1/4*A*b^2*x^4 + 1/2*A*a*c*x^4 + 2/3*B*a*b*x^3 + 1/2*C*a^2*x^2 + A*a*b*x^2 + B*a^2*x + A*a^2*log(abs(x))

maple [A] time = 0.00, size = 149, normalized size = 0.99

$$\frac{C^2x^{10}}{10} + \frac{Bc^2x^9}{9} + \frac{Ac^2x^8}{8} + \frac{Cbcx^8}{4} + \frac{2Bbcx^7}{7} + \frac{Abcx^6}{3} + \frac{Cacx^6}{3} + \frac{Cb^2x^6}{6} + \frac{2Bacx^5}{5} + \frac{Bb^2x^5}{5} + \frac{Aacx^4}{2} + \frac{Ab^2x^4}{4} + \frac{Cabx^4}{2} + \frac{2Babx^3}{3} + Aabx^2 + \frac{Ca^2x^2}{2} + Aa^2\ln(x) + Ba^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x,x)

[Out] 1/10*c^2*C*x^10+1/9*B*c^2*x^9+1/8*A*x^8*c^2+1/4*C*x^8*b*c+2/7*b*B*c*x^7+1/3*A*x^6*b*c+1/3*C*x^6*a*c+1/6*C*x^6*b^2+2/5*B*x^5*a*c+1/5*B*x^5*b^2+1/2*A*x^4*a*c+1/4*A*x^4*b^2+1/2*C*x^4*a*b+2/3*a*b*B*x^3+A*x^2*a*b+1/2*C*x^2*a^2+a^2*B*x+a^2*A*ln(x)

maxima [A] time = 0.84, size = 138, normalized size = 0.92

$$\frac{1}{10}C^2x^{10} + \frac{1}{9}Bc^2x^9 + \frac{2}{7}Bbcx^7 + \frac{1}{8}(2Cbc + Ac^2)x^8 + \frac{1}{6}(Cb^2 + 2(Ca + Ab)c)x^6 + \frac{2}{3}Babx^3 + \frac{1}{5}(Bb^2 + 2Bac)x^5 + \frac{1}{4}(2Cab + Ab^2 + 2Aac)x^4 + Ba^2x + Aa^2\log(x) + \frac{1}{2}(Ca^2 + 2Aab)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x,x, algorithm="maxima")

[Out] 1/10*C*c^2*x^10 + 1/9*B*c^2*x^9 + 2/7*B*b*c*x^7 + 1/8*(2*C*b*c + A*c^2)*x^8 + 1/6*(C*b^2 + 2*(C*a + A*b)*c)*x^6 + 2/3*B*a*b*x^3 + 1/5*(B*b^2 + 2*B*a*c)*x^5 + 1/4*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 + B*a^2*x + A*a^2*log(x) + 1/2*(C*a^2 + 2*A*a*b)*x^2

mupad [B] time = 0.80, size = 135, normalized size = 0.90

$$x^2\left(\frac{Ca^2}{2} + Aba\right) + x^8\left(\frac{Ac^2}{8} + \frac{Cbc}{4}\right) + x^4\left(\frac{Ab^2}{4} + \frac{Cab}{2} + \frac{Aac}{2}\right) + x^6\left(\frac{Cb^2}{6} + \frac{Ac b}{3} + \frac{Cac}{3}\right) + \frac{Bc^2x^9}{9} + \frac{C^2x^{10}}{10} + Aa^2\ln(x) + \frac{Bx^5(b^2 + 2ac)}{5} + Ba^2x + \frac{2Babx^3}{3} + \frac{2Bbcx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x,x)

[Out] x^2*((C*a^2)/2 + A*a*b) + x^8*((A*c^2)/8 + (C*b*c)/4) + x^4*((A*b^2)/4 + (A*a*c)/2 + (C*a*b)/2) + x^6*((C*b^2)/6 + (A*b*c)/3 + (C*a*c)/3) + (B*c^2*x^9

) / 9 + (C*c^2*x^10) / 10 + A*a^2*log(x) + (B*x^5*(2*a*c + b^2)) / 5 + B*a^2*x + (2*B*a*b*x^3) / 3 + (2*B*b*c*x^7) / 7

sympy [A] time = 0.31, size = 156, normalized size = 1.04

$$Aa^2 \log(x) + Ba^2x + \frac{2Babx^3}{3} + \frac{2Bbcx^7}{7} + \frac{Bc^2x^9}{9} + \frac{Cc^2x^{10}}{10} + x^8 \left(\frac{Ac^2}{8} + \frac{Cbc}{4} \right) + x^6 \left(\frac{Abc}{3} + \frac{Cac}{3} + \frac{Cb^2}{6} \right) + x^5 \left(\frac{2Bac}{5} + \frac{Bb^2}{5} \right) + x^4 \left(\frac{Aac}{2} + \frac{Ab^2}{4} + \frac{Cab}{2} \right) + x^2 \left(Aab + \frac{Ca^2}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2/x,x)

[Out] A*a**2*log(x) + B*a**2*x + 2*B*a*b*x**3/3 + 2*B*b*c*x**7/7 + B*c**2*x**9/9 + C*c**2*x**10/10 + x**8*(A*c**2/8 + C*b*c/4) + x**6*(A*b*c/3 + C*a*c/3 + C*b**2/6) + x**5*(2*B*a*c/5 + B*b**2/5) + x**4*(A*a*c/2 + A*b**2/4 + C*a*b/2) + x**2*(A*a*b + C*a**2/2)

$$3.15 \quad \int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^2} dx$$

Optimal. Leaf size=145

$$-\frac{a^2A}{x} + a^2B \log(x) + \frac{1}{5}x^5 (C(2ac + b^2) + 2Abc) + \frac{1}{3}x^3 (A(2ac + b^2) + 2abC) + ax(aC + 2Ab) + \frac{1}{4}Bx^4 (2ac + b^2) + abBx^2 + \frac{1}{7}cx^7(Ac + 2bC) + \frac{1}{3}bBcx^6 + \frac{1}{8}Bc^2x^8 + \frac{1}{9}c^2Cx^9$$

Rubi [A] time = 0.12, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {1628}

$$-\frac{a^2A}{x} + a^2B \log(x) + \frac{1}{5}x^5 (C(2ac + b^2) + 2Abc) + \frac{1}{3}x^3 (A(2ac + b^2) + 2abC) + ax(aC + 2Ab) + \frac{1}{4}Bx^4 (2ac + b^2) + abBx^2 + \frac{1}{7}cx^7(Ac + 2bC) + \frac{1}{3}bBcx^6 + \frac{1}{8}Bc^2x^8 + \frac{1}{9}c^2Cx^9$$

Antiderivative was successfully verified.

[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^2, x]

[Out] -((a^2*A)/x) + a*(2*A*b + a*C)*x + a*b*B*x^2 + ((A*(b^2 + 2*a*c) + 2*a*b*C)*x^3)/3 + (B*(b^2 + 2*a*c)*x^4)/4 + ((2*A*b*c + (b^2 + 2*a*c)*C)*x^5)/5 + (b*B*c*x^6)/3 + (c*(A*c + 2*b*C)*x^7)/7 + (B*c^2*x^8)/8 + (c^2*C*x^9)/9 + a^2*B*Log[x]

Rule 1628

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int \frac{(A + Bx + Cx^2)(a + bx^2 + cx^4)^2}{x^2} dx &= \int \left(a(2Ab + aC) + \frac{a^2A}{x^2} + \frac{a^2B}{x} + 2abBx + (A(b^2 + 2ac) + 2abC)x^2 + \right. \\ &\quad \left. - \frac{a^2A}{x} + a(2Ab + aC)x + abBx^2 + \frac{1}{3}(A(b^2 + 2ac) + 2abC)x^3 + \frac{1}{4}B(b^2 + 2ac) + abBx^2 + \frac{1}{7}cx^7(Ac + 2bC) + \frac{1}{3}bBcx^6 + \frac{1}{8}Bc^2x^8 + \frac{1}{9}c^2Cx^9 \right) dx \end{aligned}$$

Mathematica [A] time = 0.09, size = 145, normalized size = 1.00

$$-\frac{a^2A}{x} + a^2B \log(x) + \frac{1}{5}x^5 (2acC + 2Abc + b^2C) + \frac{1}{3}x^3 (2aAc + 2abC + Ab^2) + ax(aC + 2Ab) + \frac{1}{4}Bx^4 (2ac + b^2) + abBx^2 + \frac{1}{7}cx^7(Ac + 2bC) + \frac{1}{3}bBcx^6 + \frac{1}{8}Bc^2x^8 + \frac{1}{9}c^2Cx^9$$

Antiderivative was successfully verified.

[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^2, x]

[Out] -((a^2*A)/x) + a*(2*A*b + a*C)*x + a*b*B*x^2 + ((A*b^2 + 2*a*A*c + 2*a*b*C)*x^3)/3 + (B*(b^2 + 2*a*c)*x^4)/4 + ((2*A*b*c + b^2*C + 2*a*c*C)*x^5)/5 + (b*B*c*x^6)/3 + (c*(A*c + 2*b*C)*x^7)/7 + (B*c^2*x^8)/8 + (c^2*C*x^9)/9 + a^2*B*Log[x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + Bx + Cx^2)(a + bx^2 + cx^4)^2}{x^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^2, x]

[Out] IntegrateAlgebraic[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^2, x]
fricas [A] time = 1.53, size = 145, normalized size = 1.00

$$\frac{280 C^2 x^{10} + 315 B c^2 x^9 + 840 B b c x^7 + 360 (2 C b c + A c^2) x^8 + 504 (C b^2 + 2 (C a + A b) c) x^6 + 2520 B a b x^3 + 630 (B b^2 + 2 B a c) x^5 + 840 (2 C a b + A b^2 + 2 A a c) x^4 + 2520 B a^2 x \log(x) - 2520 A a^2 + 2520 (C a^2 + 2 A a b) x^2}{2520 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^2,x, algorithm="fricas")

[Out] 1/2520*(280*C*c^2*x^10 + 315*B*c^2*x^9 + 840*B*b*c*x^7 + 360*(2*C*b*c + A*c^2)*x^8 + 504*(C*b^2 + 2*(C*a + A*b)*c)*x^6 + 2520*B*a*b*x^3 + 630*(B*b^2 + 2*B*a*c)*x^5 + 840*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 + 2520*B*a^2*x*log(x) - 2520*A*a^2 + 2520*(C*a^2 + 2*A*a*b)*x^2)/x

giac [A] time = 0.28, size = 147, normalized size = 1.01

$$\frac{1}{9} C^2 x^9 + \frac{1}{8} B c^2 x^8 + \frac{2}{7} C b c x^7 + \frac{1}{7} A c^2 x^7 + \frac{1}{3} B b c x^6 + \frac{1}{5} C b^2 x^5 + \frac{2}{5} C a c x^5 + \frac{2}{5} A b c x^5 + \frac{1}{4} B b^2 x^4 + \frac{1}{2} B a c x^4 + \frac{2}{3} C a b x^3 + \frac{1}{3} A b^2 x^3 + \frac{2}{3} A a c x^3 + B a b x^2 + C a^2 x + 2 A a b x + B a^2 \log(x) - \frac{A a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^2,x, algorithm="giac")

[Out] 1/9*C*c^2*x^9 + 1/8*B*c^2*x^8 + 2/7*C*b*c*x^7 + 1/7*A*c^2*x^7 + 1/3*B*b*c*x^6 + 1/5*C*b^2*x^5 + 2/5*C*a*c*x^5 + 2/5*A*b*c*x^5 + 1/4*B*b^2*x^4 + 1/2*B*a*c*x^4 + 2/3*C*a*b*x^3 + 1/3*A*b^2*x^3 + 2/3*A*a*c*x^3 + B*a*b*x^2 + C*a^2*x + 2*A*a*b*x + B*a^2*log(abs(x)) - A*a^2/x

maple [A] time = 0.01, size = 147, normalized size = 1.01

$$\frac{C^2 x^9}{9} + \frac{B c^2 x^8}{8} + \frac{A c^2 x^7}{7} + \frac{2 C b c x^7}{7} + \frac{B b c x^6}{3} + \frac{2 A b c x^5}{5} + \frac{2 C a c x^5}{5} + \frac{C b^2 x^5}{5} + \frac{B a c x^4}{2} + \frac{B b^2 x^4}{4} + \frac{2 A a c x^3}{3} + \frac{A b^2 x^3}{3} + \frac{2 C a b x^3}{3} + B a b x^2 + 2 A a b x + B a^2 \ln(x) + C a^2 x - \frac{A a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^2,x)

[Out] 1/9*c^2*C*x^9+1/8*B*c^2*x^8+1/7*A*x^7*c^2+2/7*C*x^7*b*c+1/3*b*B*c*x^6+2/5*A*x^5*b*c+2/5*C*x^5*a*c+1/5*C*x^5*b^2+1/2*B*x^4*a*c+1/4*B*x^4*b^2+2/3*A*x^3*a*c+1/3*A*x^3*b^2+2/3*C*x^3*a*b+a*b*B*x^2+2*A*a*b*x+C*a^2*x+a^2*B*ln(x)-a^2*A/x

maxima [A] time = 0.73, size = 137, normalized size = 0.94

$$\frac{1}{9} C^2 x^9 + \frac{1}{8} B c^2 x^8 + \frac{1}{3} B b c x^6 + \frac{1}{7} (2 C b c + A c^2) x^7 + \frac{1}{5} (C b^2 + 2 (C a + A b) c) x^5 + B a b x^2 + \frac{1}{4} (B b^2 + 2 B a c) x^4 + \frac{1}{3} (2 C a b + A b^2 + 2 A a c) x^3 + B a^2 \log(x) - \frac{A a^2}{x} + (C a^2 + 2 A a b) x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^2,x, algorithm="maxima")

[Out] 1/9*C*c^2*x^9 + 1/8*B*c^2*x^8 + 1/3*B*b*c*x^6 + 1/7*(2*C*b*c + A*c^2)*x^7 + 1/5*(C*b^2 + 2*(C*a + A*b)*c)*x^5 + B*a*b*x^2 + 1/4*(B*b^2 + 2*B*a*c)*x^4 + 1/3*(2*C*a*b + A*b^2 + 2*A*a*c)*x^3 + B*a^2*log(x) - A*a^2/x + (C*a^2 + 2*A*a*b)*x

mupad [B] time = 0.80, size = 135, normalized size = 0.93

$$x^7 \left(\frac{A c^2}{7} + \frac{2 C b c}{7} \right) + x^3 \left(\frac{A b^2}{3} + \frac{2 C a b}{3} + \frac{2 A a c}{3} \right) + x^5 \left(\frac{C b^2}{5} + \frac{2 A c b}{5} + \frac{2 C a c}{5} \right) + x (C a^2 + 2 A a b) - \frac{A a^2}{x} + \frac{B c^2 x^8}{8} + \frac{C c^2 x^9}{9} + B a^2 \ln(x) + \frac{B x^4 (b^2 + 2 a c)}{4} + B a b x^2 + \frac{B b c x^6}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^2,x)

[Out] x^7*((A*c^2)/7 + (2*C*b*c)/7) + x^3*((A*b^2)/3 + (2*A*a*c)/3 + (2*C*a*b)/3) + x^5*((C*b^2)/5 + (2*A*b*c)/5 + (2*C*a*c)/5) + x*(C*a^2 + 2*A*a*b) - (A*a

$$\frac{a^2}{x} + (B*c^2*x^8)/8 + (C*c^2*x^9)/9 + B*a^2*\log(x) + (B*x^4*(2*a*c + b^2))/4 + B*a*b*x^2 + (B*b*c*x^6)/3$$

sympy [A] time = 0.32, size = 156, normalized size = 1.08

$$-\frac{Aa^2}{x} + Ba^2 \log(x) + Babx^2 + \frac{Bbcx^6}{3} + \frac{Bc^2x^8}{8} + \frac{Cc^2x^9}{9} + x^7\left(\frac{Ac^2}{7} + \frac{2Cbc}{7}\right) + x^5\left(\frac{2Abc}{5} + \frac{2Cac}{5} + \frac{Cb^2}{5}\right) + x^4\left(\frac{Bac}{2} + \frac{Bb^2}{4}\right) + x^3\left(\frac{2Aac}{3} + \frac{Ab^2}{3} + \frac{2Cab}{3}\right) + x(2Aab + Ca^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2/x**2,x)

[Out] -A*a**2/x + B*a**2*log(x) + B*a*b*x**2 + B*b*c*x**6/3 + B*c**2*x**8/8 + C*c**2*x**9/9 + x**7*(A*c**2/7 + 2*C*b*c/7) + x**5*(2*A*b*c/5 + 2*C*a*c/5 + C*b**2/5) + x**4*(B*a*c/2 + B*b**2/4) + x**3*(2*A*a*c/3 + A*b**2/3 + 2*C*a*b/3) + x*(2*A*a*b + C*a**2)

$$3.16 \quad \int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^3} dx$$

Optimal. Leaf size=149

$$-\frac{a^2 A}{2x^2} - \frac{a^2 B}{x} + \frac{1}{4}x^4 (C(2ac + b^2) + 2Abc) + \frac{1}{2}x^2 (A(2ac + b^2) + 2abC) + a \log(x)(aC + 2Ab) + \frac{1}{3}Bx^3 (2ac + b^2) + 2$$

Rubi [A] time = 0.12, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {1628}

$$-\frac{a^2 A}{2x^2} - \frac{a^2 B}{x} + \frac{1}{4}x^4 (C(2ac + b^2) + 2Abc) + \frac{1}{2}x^2 (A(2ac + b^2) + 2abC) + a \log(x)(aC + 2Ab) + \frac{1}{3}Bx^3 (2ac + b^2) + 2abBx + \frac{1}{6}cx^6 (Ac + 2bC) + \frac{2}{5}bBcx^5 + \frac{1}{7}Bc^2x^7 + \frac{1}{8}c^2Cx^8$$

Antiderivative was successfully verified.

[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^3, x]

[Out] -(a^2*A)/(2*x^2) - (a^2*B)/x + 2*a*b*B*x + ((A*(b^2 + 2*a*c) + 2*a*b*C)*x^2)/2 + (B*(b^2 + 2*a*c)*x^3)/3 + ((2*A*b*c + (b^2 + 2*a*c)*C)*x^4)/4 + (2*b*B*c*x^5)/5 + (c*(A*c + 2*b*C)*x^6)/6 + (B*c^2*x^7)/7 + (c^2*C*x^8)/8 + a*(2*A*b + a*C)*Log[x]

Rule 1628

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int \frac{(A + Bx + Cx^2)(a + bx^2 + cx^4)^2}{x^3} dx &= \int \left(2abB + \frac{a^2 A}{x^3} + \frac{a^2 B}{x^2} + \frac{a(2Ab + aC)}{x} + (A(b^2 + 2ac) + 2abC)x \right. \\ &= -\frac{a^2 A}{2x^2} - \frac{a^2 B}{x} + 2abBx + \frac{1}{2} (A(b^2 + 2ac) + 2abC)x^2 + \frac{1}{3} B(b^2 + 2ac)x^3 \end{aligned}$$

Mathematica [A] time = 0.09, size = 139, normalized size = 0.93

$$-\frac{a^2(A + 2Bx)}{2x^2} + \frac{1}{6}ax(cx(6A + 4Bx + 3Cx^2) + 6b(2B + Cx) + a \log(x)(aC + 2Ab) + \frac{1}{840}x^2(140A(3b^2 + 3bcx^2 + c^2x^4) + 70b^2x(4B + 3Cx) + 56bcx^3(6B + 5Cx) + 15c^2x^5(8B + 7Cx)))$$

Antiderivative was successfully verified.

[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^3, x]

[Out] -1/2*(a^2*(A + 2*B*x))/x^2 + (a*x*(6*b*(2*B + C*x) + c*x*(6*A + 4*B*x + 3*C*x^2)))/6 + (x^2*(70*b^2*x*(4*B + 3*C*x) + 56*b*c*x^3*(6*B + 5*C*x) + 15*c^2*x^5*(8*B + 7*C*x) + 140*A*(3*b^2 + 3*b*c*x^2 + c^2*x^4)))/840 + a*(2*A*b + a*C)*Log[x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + Bx + Cx^2)(a + bx^2 + cx^4)^2}{x^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^3, x]

[Out] IntegrateAlgebraic[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^3, x]

fricas [A] time = 1.26, size = 145, normalized size = 0.97

$$\frac{105 C c^2 x^{10} + 120 B c^2 x^9 + 336 B b c x^7 + 140 (2 C b c + A c^2) x^8 + 210 (C b^2 + 2 (C a + A b) c) x^6 + 1680 B a b x^3 + 280 (B b^2 + 2 B a c) x^5 + 420 (2 C a b + A b^2 + 2 A a c) x^4 - 840 B a^2 x + 840 (C a^2 + 2 A a b) x^2 \log(x) - 420 A a^2}{840 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^3,x, algorithm="fricas")

[Out] 1/840*(105*C*c^2*x^10 + 120*B*c^2*x^9 + 336*B*b*c*x^7 + 140*(2*C*b*c + A*c^2)*x^8 + 210*(C*b^2 + 2*(C*a + A*b)*c)*x^6 + 1680*B*a*b*x^3 + 280*(B*b^2 + 2*B*a*c)*x^5 + 420*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 - 840*B*a^2*x + 840*(C*a^2 + 2*A*a*b)*x^2*log(x) - 420*A*a^2)/x^2

giac [A] time = 0.40, size = 148, normalized size = 0.99

$$\frac{1}{8} C c^2 x^8 + \frac{1}{7} B c^2 x^7 + \frac{1}{3} C b c x^6 + \frac{1}{6} A c^2 x^6 + \frac{2}{5} B b c x^5 + \frac{1}{4} C b^2 x^4 + \frac{1}{2} C a c x^4 + \frac{1}{2} A b c x^4 + \frac{1}{3} B b^2 x^3 + \frac{2}{3} B a c x^3 + C a b x^2 + \frac{1}{2} A b^2 x^2 + A a c x^2 + 2 B a b x + (C a^2 + 2 A a b) \log(|x|) - \frac{2 B a^2 x + A a^2}{2 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^3,x, algorithm="giac")

[Out] 1/8*C*c^2*x^8 + 1/7*B*c^2*x^7 + 1/3*C*b*c*x^6 + 1/6*A*c^2*x^6 + 2/5*B*b*c*x^5 + 1/4*C*b^2*x^4 + 1/2*C*a*c*x^4 + 1/2*A*b*c*x^4 + 1/3*B*b^2*x^3 + 2/3*B*a*c*x^3 + C*a*b*x^2 + 1/2*A*b^2*x^2 + A*a*c*x^2 + 2*B*a*b*x + (C*a^2 + 2*A*a*b)*log(abs(x)) - 1/2*(2*B*a^2*x + A*a^2)/x^2

maple [A] time = 0.01, size = 148, normalized size = 0.99

$$\frac{C c^2 x^8}{8} + \frac{B c^2 x^7}{7} + \frac{A c^2 x^6}{6} + \frac{C b c x^6}{3} + \frac{2 B b c x^5}{5} + \frac{A b c x^4}{2} + \frac{C a c x^4}{2} + \frac{C b^2 x^4}{4} + \frac{2 B a c x^3}{3} + \frac{B b^2 x^3}{3} + A a c x^2 + \frac{A b^2 x^2}{2} + C a b x^2 + 2 A a b \ln(x) + 2 B a b x + C a^2 \ln(x) - \frac{B a^2}{x} - \frac{A a^2}{2 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^3,x)

[Out] 1/8*c^2*C*x^8+1/7*B*c^2*x^7+1/6*A*x^6*c^2+1/3*C*x^6*b*c+2/5*b*B*c*x^5+1/2*A*x^4*b*c+1/2*C*x^4*a*c+1/4*C*x^4*b^2+2/3*B*x^3*a*c+1/3*B*x^3*b^2+A*x^2*a*c+1/2*A*x^2*b^2+C*x^2*a*b+2*a*b*B*x+2*A*ln(x)*a*b+C*ln(x)*a^2-a^2*B/x-1/2*a^2*A/x^2

maxima [A] time = 0.62, size = 139, normalized size = 0.93

$$\frac{1}{8} C c^2 x^8 + \frac{1}{7} B c^2 x^7 + \frac{2}{5} B b c x^5 + \frac{1}{6} (2 C b c + A c^2) x^6 + \frac{1}{4} (C b^2 + 2 (C a + A b) c) x^4 + 2 B a b x + \frac{1}{3} (B b^2 + 2 B a c) x^3 + \frac{1}{2} (2 C a b + A b^2 + 2 A a c) x^2 + (C a^2 + 2 A a b) \log(x) - \frac{2 B a^2 x + A a^2}{2 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^3,x, algorithm="maxima")

[Out] 1/8*C*c^2*x^8 + 1/7*B*c^2*x^7 + 2/5*B*b*c*x^5 + 1/6*(2*C*b*c + A*c^2)*x^6 + 1/4*(C*b^2 + 2*(C*a + A*b)*c)*x^4 + 2*B*a*b*x + 1/3*(B*b^2 + 2*B*a*c)*x^3 + 1/2*(2*C*a*b + A*b^2 + 2*A*a*c)*x^2 + (C*a^2 + 2*A*a*b)*log(x) - 1/2*(2*B*a^2*x + A*a^2)/x^2

mupad [B] time = 0.79, size = 135, normalized size = 0.91

$$x^6 \left(\frac{A c^2}{6} + \frac{C b c}{3} \right) + \ln(x) (C a^2 + 2 A a b) + x^2 \left(\frac{A b^2}{2} + C a b + A a c \right) + x^4 \left(\frac{C b^2}{4} + \frac{A c b}{2} + \frac{C a c}{2} \right) - \frac{A a^2 + B a^2 x}{x^2} + \frac{B c^2 x^7}{7} + \frac{C c^2 x^8}{8} + \frac{B x^3 (b^2 + 2 a c)}{3} + \frac{2 B b c x^5}{5} + 2 B a b x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^3,x)

[Out] x^6*((A*c^2)/6 + (C*b*c)/3) + log(x)*(C*a^2 + 2*A*a*b) + x^2*((A*b^2)/2 + A*a*c + C*a*b) + x^4*((C*b^2)/4 + (A*b*c)/2 + (C*a*c)/2) - ((A*a^2)/2 + B*a^2

$$2*x)/x^2 + (B*c^2*x^7)/7 + (C*c^2*x^8)/8 + (B*x^3*(2*a*c + b^2))/3 + (2*B*b*c*x^5)/5 + 2*B*a*b*x$$

sympy [A] time = 0.46, size = 153, normalized size = 1.03

$$2Babx + \frac{2Bbcx^5}{5} + \frac{Bc^2x^7}{7} + \frac{Cc^2x^8}{8} + a(2Ab + Ca)\log(x) + x^6\left(\frac{Ac^2}{6} + \frac{Cbc}{3}\right) + x^4\left(\frac{Abc}{2} + \frac{Cac}{2} + \frac{Cb^2}{4}\right) + x^3\left(\frac{2Bac}{3} + \frac{Bb^2}{3}\right) + x^2\left(Aac + \frac{Ab^2}{2} + Cab\right) + \frac{-Aa^2 - 2Ba^2x}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2/x**3,x)

[Out] 2*B*a*b*x + 2*B*b*c*x**5/5 + B*c**2*x**7/7 + C*c**2*x**8/8 + a*(2*A*b + C*a)*log(x) + x**6*(A*c**2/6 + C*b*c/3) + x**4*(A*b*c/2 + C*a*c/2 + C*b**2/4) + x**3*(2*B*a*c/3 + B*b**2/3) + x**2*(A*a*c + A*b**2/2 + C*a*b) + (-A*a**2 - 2*B*a**2*x)/(2*x**2)

$$3.17 \quad \int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^4} dx$$

Optimal. Leaf size=149

$$-\frac{a^2A}{3x^3} - \frac{a^2B}{2x^2} + \frac{1}{3}x^3(C(2ac+b^2) + 2Abc) + x(A(2ac+b^2) + 2abC) - \frac{a(aC+2Ab)}{x} + \frac{1}{2}Bx^2(2ac+b^2) + 2abB \log(x)$$

Rubi [A] time = 0.14, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {1628}

$$-\frac{a^2A}{3x^3} - \frac{a^2B}{2x^2} + \frac{1}{3}x^3(C(2ac+b^2) + 2Abc) + x(A(2ac+b^2) + 2abC) - \frac{a(aC+2Ab)}{x} + \frac{1}{2}Bx^2(2ac+b^2) + 2abB \log(x) + \frac{1}{5}cx^5(Ac+2bC) + \frac{1}{2}bBcx^4 + \frac{1}{6}Bc^2x^6 + \frac{1}{7}c^2Cx^7$$

Antiderivative was successfully verified.

[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^4, x]

[Out] $-(a^2A)/(3x^3) - (a^2B)/(2x^2) - (a*(2A*b + a*C))/x + (A*(b^2 + 2*a*c) + 2*a*b*C)*x + (B*(b^2 + 2*a*c)*x^2)/2 + ((2*A*b*c + (b^2 + 2*a*c)*C)*x^3)/3 + (b*B*c*x^4)/2 + (c*(A*c + 2*b*C)*x^5)/5 + (B*c^2*x^6)/6 + (c^2*C*x^7)/7 + 2*a*b*B*Log[x]$

Rule 1628

Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^4} dx &= \int \left(Ab^2 \left(1 + \frac{2a(Ac+bc)}{Ab^2} \right) + \frac{a^2A}{x^4} + \frac{a^2B}{x^3} + \frac{a(2Ab+aC)}{x^2} + \frac{2abB}{x} + B(b^2+2ac) \right) dx \\ &= -\frac{a^2A}{3x^3} - \frac{a^2B}{2x^2} - \frac{a(2Ab+aC)}{x} + (A(b^2+2ac) + 2abC)x + \frac{1}{2}B(b^2+2ac)x^2 + B(b^2+2ac)x^3 + \frac{1}{4}B^2x^4 + \frac{1}{5}Bc^2x^5 + \frac{1}{6}Bc^2x^6 + \frac{1}{7}c^2Cx^7 + 2abB \log(x) \end{aligned}$$

Mathematica [A] time = 0.08, size = 151, normalized size = 1.01

$$\frac{a^2(-C) - 2aAb}{x} - \frac{a^2A}{3x^3} - \frac{a^2B}{2x^2} + \frac{1}{3}x^3(2acC + 2Abc + b^2C) + x(2aAc + 2abC + Ab^2) + \frac{1}{2}Bx^2(2ac+b^2) + 2abB \log(x) + \frac{1}{5}cx^5(Ac+2bC) + \frac{1}{2}bBcx^4 + \frac{1}{6}Bc^2x^6 + \frac{1}{7}c^2Cx^7$$

Antiderivative was successfully verified.

[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^4, x]

[Out] $-1/3*(a^2A)/x^3 - (a^2B)/(2x^2) + (-2*a*A*b - a^2*C)/x + (A*b^2 + 2*a*A*c + 2*a*b*C)*x + (B*(b^2 + 2*a*c)*x^2)/2 + ((2*A*b*c + b^2*C + 2*a*c*C)*x^3)/3 + (b*B*c*x^4)/2 + (c*(A*c + 2*b*C)*x^5)/5 + (B*c^2*x^6)/6 + (c^2*C*x^7)/7 + 2*a*b*B*Log[x]$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^4, x]

[Out] IntegrateAlgebraic[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^4, x]

fricas [A] time = 1.38, size = 145, normalized size = 0.97

$$\frac{30 C^2 x^{10} + 35 B^2 x^9 + 105 B b c x^7 + 42 (2 C b c + A c^2) x^6 + 70 (C b^2 + 2 (C a + A b) c) x^5 + 420 B a b x^3 \log(x) + 105 (B b^2 + 2 B a c) x^2 + 210 (2 C a b + A b^2 + 2 A a c) x - 105 B a^2 x - 70 A a^2 - 210 (C a^2 + 2 A a b) x^2}{210 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^4,x, algorithm="fricas")

[Out] 1/210*(30*C*c^2*x^10 + 35*B*c^2*x^9 + 105*B*b*c*x^7 + 42*(2*C*b*c + A*c^2)*x^8 + 70*(C*b^2 + 2*(C*a + A*b)*c)*x^6 + 420*B*a*b*x^3*log(x) + 105*(B*b^2 + 2*B*a*c)*x^5 + 210*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 - 105*B*a^2*x - 70*A*a^2 - 210*(C*a^2 + 2*A*a*b)*x^2)/x^3

giac [A] time = 0.28, size = 146, normalized size = 0.98

$$\frac{1}{7} C^2 x^7 + \frac{1}{6} B^2 x^6 + \frac{2}{5} C b c x^5 + \frac{1}{5} A c^2 x^5 + \frac{1}{2} B b c x^4 + \frac{1}{3} C b^2 x^3 + \frac{2}{3} C a c x^3 + \frac{2}{3} A b c x^3 + \frac{1}{2} B b^2 x^2 + B a c x^2 + 2 C a b x + A b^2 x + 2 A a c x + 2 B a b \log(|x|) - \frac{3 B a^2 x + 2 A a^2 + 6 (C a^2 + 2 A a b) x^2}{6 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^4,x, algorithm="giac")

[Out] 1/7*C*c^2*x^7 + 1/6*B*c^2*x^6 + 2/5*C*b*c*x^5 + 1/5*A*c^2*x^5 + 1/2*B*b*c*x^4 + 1/3*C*b^2*x^3 + 2/3*C*a*c*x^3 + 2/3*A*b*c*x^3 + 1/2*B*b^2*x^2 + B*a*c*x^2 + 2*C*a*b*x + A*b^2*x + 2*A*a*c*x + 2*B*a*b*log(abs(x)) - 1/6*(3*B*a^2*x + 2*A*a^2 + 6*(C*a^2 + 2*A*a*b)*x^2)/x^3

maple [A] time = 0.01, size = 146, normalized size = 0.98

$$\frac{C^2 x^7}{7} + \frac{B^2 x^6}{6} + \frac{A c^2 x^5}{5} + \frac{2 C b c x^5}{5} + \frac{B b c x^4}{2} + \frac{2 A b c x^3}{3} + \frac{2 C a c x^3}{3} + \frac{C b^2 x^3}{3} + B a c x^2 + \frac{B b^2 x^2}{2} + 2 A a c x + A b^2 x + 2 B a b \ln(x) + 2 C a b x - \frac{2 A a b}{x} - \frac{C a^2}{x} - \frac{B a^2}{2 x^2} - \frac{A a^2}{3 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^4,x)

[Out] 1/7*c^2*C*x^7+1/6*B*c^2*x^6+1/5*A*x^5*c^2+2/5*C*x^5*b*c+1/2*b*B*c*x^4+2/3*A*x^3*b*c+2/3*C*x^3*a*c+1/3*C*x^3*b^2+B*x^2*a*c+1/2*B*x^2*b^2+2*a*A*c*x+A*b^2*x+2*C*a*b*x+2*a*b*B*ln(x)-2*a/x*A*b-a^2/x*C-1/3*a^2*A/x^3-1/2*a^2*B/x^2

maxima [A] time = 0.68, size = 140, normalized size = 0.94

$$\frac{1}{7} C^2 x^7 + \frac{1}{6} B^2 x^6 + \frac{1}{2} B b c x^4 + \frac{1}{5} (2 C b c + A c^2) x^5 + \frac{1}{3} (C b^2 + 2 (C a + A b) c) x^3 + 2 B a b \log(x) + \frac{1}{2} (B b^2 + 2 B a c) x^2 + (2 C a b + A b^2 + 2 A a c) x - \frac{3 B a^2 x + 2 A a^2 + 6 (C a^2 + 2 A a b) x^2}{6 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^4,x, algorithm="maxima")

[Out] 1/7*C*c^2*x^7 + 1/6*B*c^2*x^6 + 1/2*B*b*c*x^4 + 1/5*(2*C*b*c + A*c^2)*x^5 + 1/3*(C*b^2 + 2*(C*a + A*b)*c)*x^3 + 2*B*a*b*log(x) + 1/2*(B*b^2 + 2*B*a*c)*x^2 + (2*C*a*b + A*b^2 + 2*A*a*c)*x - 1/6*(3*B*a^2*x + 2*A*a^2 + 6*(C*a^2 + 2*A*a*b)*x^2)/x^3

mupad [B] time = 0.06, size = 137, normalized size = 0.92

$$x^5 \left(\frac{A c^2}{5} + \frac{2 C b c}{5} \right) - \frac{x^2 (C a^2 + 2 A b a) + \frac{A a^2}{3} + \frac{B a^2 x}{2}}{x^3} + x (A b^2 + 2 C a b + 2 A a c) + x^3 \left(\frac{C b^2}{3} + \frac{2 A c b}{3} + \frac{2 C a c}{3} \right) + \frac{B c^2 x^6}{6} + \frac{C c^2 x^7}{7} + \frac{B x^2 (b^2 + 2 a c)}{2} + \frac{B b c x^4}{2} + 2 B a b \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^4,x)

[Out] x^5*((A*c^2)/5 + (2*C*b*c)/5) - (x^2*(C*a^2 + 2*A*a*b) + (A*a^2)/3 + (B*a^2*x)/2)/x^3 + x*(A*b^2 + 2*A*a*c + 2*C*a*b) + x^3*((C*b^2)/3 + (2*A*b*c)/3 +

$$(2Cac)/3 + (Bc^2x^6)/6 + (C^2x^7)/7 + (Bx^2(2ac + b^2))/2 + (Bbcx^4)/2 + 2Bab \log(x)$$

sympy [A] time = 0.72, size = 160, normalized size = 1.07

$$2Bab \log(x) + \frac{Bbcx^4}{2} + \frac{Bc^2x^6}{6} + \frac{C^2x^7}{7} + x^5 \left(\frac{Ac^2}{5} + \frac{2Cbc}{5} \right) + x^3 \left(\frac{2Abc}{3} + \frac{2Cac}{3} + \frac{Cb^2}{3} \right) + x^2 \left(Bac + \frac{Bb^2}{2} \right) + x(2Aac + Ab^2 + 2Cab) + \frac{-2Aa^2 - 3Ba^2x + x^2(-12Aab - 6Ca^2)}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2/x**4,x)

[Out] 2*B*a*b*log(x) + B*b*c*x**4/2 + B*c**2*x**6/6 + C*c**2*x**7/7 + x**5*(A*c**2/5 + 2*C*b*c/5) + x**3*(2*A*b*c/3 + 2*C*a*c/3 + C*b**2/3) + x**2*(B*a*c + B*b**2/2) + x*(2*A*a*c + A*b**2 + 2*C*a*b) + (-2*A*a**2 - 3*B*a**2*x + x**2*(-12*A*a*b - 6*C*a**2))/(6*x**3)

$$3.18 \quad \int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^5} dx$$

Optimal. Leaf size=148

$$-\frac{a^2A}{4x^4} - \frac{a^2B}{3x^3} + \frac{1}{2}x^2(C(2ac+b^2) + 2Abc) + \log(x)(A(2ac+b^2) + 2abC) - \frac{a(aC+2Ab)}{2x^2} + Bx(2ac+b^2) - \frac{2abB}{x}$$

Rubi [A] time = 0.14, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {1628}

$$-\frac{a^2A}{4x^4} - \frac{a^2B}{3x^3} + \frac{1}{2}x^2(C(2ac+b^2) + 2Abc) + \log(x)(A(2ac+b^2) + 2abC) - \frac{a(aC+2Ab)}{2x^2} + Bx(2ac+b^2) - \frac{2abB}{x} + \frac{1}{4}cx^4(Ac+2bC) + \frac{2}{3}bBcx^3 + \frac{1}{5}Bc^2x^5 + \frac{1}{6}c^2Cx^6$$

Antiderivative was successfully verified.

[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^5, x]

[Out] $-(a^2A)/(4x^4) - (a^2B)/(3x^3) - (a*(2A*b + a*C))/(2x^2) - (2*a*b*B)/x + B*(b^2 + 2*a*c)*x + ((2*A*b*c + (b^2 + 2*a*c)*C)*x^2)/2 + (2*b*B*c*x^3)/3 + (c*(A*c + 2*b*C)*x^4)/4 + (B*c^2*x^5)/5 + (c^2*C*x^6)/6 + (A*(b^2 + 2*a*c) + 2*a*b*C)*\text{Log}[x]$

Rule 1628

Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^5} dx &= \int \left(B(b^2+2ac) + \frac{a^2A}{x^5} + \frac{a^2B}{x^4} + \frac{a(2Ab+aC)}{x^3} + \frac{2abB}{x^2} + \frac{A(b^2+2ac)}{x} \right) dx \\ &= -\frac{a^2A}{4x^4} - \frac{a^2B}{3x^3} - \frac{a(2Ab+aC)}{2x^2} - \frac{2abB}{x} + B(b^2+2ac)x + \frac{1}{2}(2Abc + \dots) \end{aligned}$$

Mathematica [A] time = 0.08, size = 130, normalized size = 0.88

$$-\frac{a^2(3A+4Bx+6Cx^2)}{12x^4} + \log(x)(A(2ac+b^2) + 2abC) + \frac{a(-Ab-2bBx+cx^3(2B+Cx))}{x^2} + \frac{1}{60}x(10bcx(6A+x(4B+3Cx)) + c^2x^3(15A+2x(6B+5Cx)) + 30b^2(2B+Cx))$$

Antiderivative was successfully verified.

[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^5, x]

[Out] $-1/12*(a^2*(3*A + 4*B*x + 6*C*x^2))/x^4 + (a*(-(A*b) - 2*b*B*x + c*x^3*(2*B + C*x)))/x^2 + (x*(30*b^2*(2*B + C*x) + 10*b*c*x*(6*A + x*(4*B + 3*C*x)) + c^2*x^3*(15*A + 2*x*(6*B + 5*C*x))))/60 + (A*(b^2 + 2*a*c) + 2*a*b*C)*\text{Log}[x]$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^5} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^5, x]

[Out] IntegrateAlgebraic[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^5, x]

fricas [A] time = 1.67, size = 145, normalized size = 0.98

$$\frac{10C^2x^{10} + 12Bc^2x^9 + 40Bbcx^7 + 15(2Cbc + Ac^2)x^8 + 30(Cb^2 + 2(Ca + Ab)c)x^6 - 120Babx^3 + 60(Bb^2 + 2Bac)x^5 + 60(2Cab + Ab^2 + 2Aac)x^4 \log(x) - 20Ba^2x - 15Aa^2 - 30(Ca^2 + 2Aab)x^2}{60x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^5,x, algorithm="fricas")

[Out] 1/60*(10*C*c^2*x^10 + 12*B*c^2*x^9 + 40*B*b*c*x^7 + 15*(2*C*b*c + A*c^2)*x^8 + 30*(C*b^2 + 2*(C*a + A*b)*c)*x^6 - 120*B*a*b*x^3 + 60*(B*b^2 + 2*B*a*c)*x^5 + 60*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4*log(x) - 20*B*a^2*x - 15*A*a^2 - 30*(C*a^2 + 2*A*a*b)*x^2)/x^4

giac [A] time = 0.38, size = 142, normalized size = 0.96

$$\frac{1}{6}C^2x^6 + \frac{1}{5}Bc^2x^5 + \frac{1}{2}Cbcx^4 + \frac{1}{4}Ac^2x^4 + \frac{2}{3}Bbcx^3 + \frac{1}{2}Cb^2x^2 + Cax^2 + Abcx^2 + Bb^2x + 2Bacx + (2Cab + Ab^2 + 2Aac)\log(|x|) - \frac{24Babx^3 + 4Ba^2x + 3Aa^2 + 6(Ca^2 + 2Aab)x^2}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^5,x, algorithm="giac")

[Out] 1/6*C*c^2*x^6 + 1/5*B*c^2*x^5 + 1/2*C*b*c*x^4 + 1/4*A*c^2*x^4 + 2/3*B*b*c*x^3 + 1/2*C*b^2*x^2 + C*a*c*x^2 + A*b*c*x^2 + B*b^2*x + 2*B*a*c*x + (2*C*a*b + A*b^2 + 2*A*a*c)*log(abs(x)) - 1/12*(24*B*a*b*x^3 + 4*B*a^2*x + 3*A*a^2 + 6*(C*a^2 + 2*A*a*b)*x^2)/x^4

maple [A] time = 0.01, size = 144, normalized size = 0.97

$$\frac{C^2x^6}{6} + \frac{Bc^2x^5}{5} + \frac{Ac^2x^4}{4} + \frac{Cbcx^4}{2} + \frac{2Bbcx^3}{3} + Abcx^2 + Cax^2 + \frac{Cb^2x^2}{2} + 2Aac\ln(x) + Ab^2\ln(x) + 2Bacx + Bb^2x + 2Cab\ln(x) - \frac{2Bab}{x} - \frac{Aab}{x^2} - \frac{Ca^2}{2x^2} - \frac{Ba^2}{3x^3} - \frac{Aa^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^5,x)

[Out] 1/6*c^2*C*x^6+1/5*B*c^2*x^5+1/4*A*x^4*c^2+1/2*C*x^4*b*c+2/3*b*B*c*x^3+A*x^2*b*c+C*x^2*a*c+1/2*C*x^2*b^2+2*a*B*c*x+b^2*B*x+2*A*ln(x)*a*c+A*ln(x)*b^2+2*C*ln(x)*a*b-2*a*b*B/x-1/3*a^2*B/x^3-1/4*a^2*A/x^4-a/x^2*A*b-1/2*a^2/x^2*C

maxima [A] time = 0.62, size = 139, normalized size = 0.94

$$\frac{1}{6}C^2x^6 + \frac{1}{5}Bc^2x^5 + \frac{2}{3}Bbcx^3 + \frac{1}{4}(2Cbc + Ac^2)x^4 + \frac{1}{2}(Cb^2 + 2(Ca + Ab)c)x^2 + (Bb^2 + 2Bac)x + (2Cab + Ab^2 + 2Aac)\log(x) - \frac{24Babx^3 + 4Ba^2x + 3Aa^2 + 6(Ca^2 + 2Aab)x^2}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^5,x, algorithm="maxima")

[Out] 1/6*C*c^2*x^6 + 1/5*B*c^2*x^5 + 2/3*B*b*c*x^3 + 1/4*(2*C*b*c + A*c^2)*x^4 + 1/2*(C*b^2 + 2*(C*a + A*b)*c)*x^2 + (B*b^2 + 2*B*a*c)*x + (2*C*a*b + A*b^2 + 2*A*a*c)*log(x) - 1/12*(24*B*a*b*x^3 + 4*B*a^2*x + 3*A*a^2 + 6*(C*a^2 + 2*A*a*b)*x^2)/x^4

mupad [B] time = 0.06, size = 134, normalized size = 0.91

$$x^4 \left(\frac{Ac^2}{4} + \frac{Cbc}{2} \right) - \frac{x^2 \left(\frac{Ca^2}{2} + Aba \right) + \frac{Aa^2}{4} + \frac{Ba^2x}{3} + 2Babx^3}{x^4} + x^2 \left(\frac{Cb^2}{2} + Acb + Cac \right) + \ln(x) (Ab^2 + 2Cab + 2Aac) + \frac{Bc^2x^5}{5} + \frac{Cc^2x^6}{6} + Bx (b^2 + 2ac) + \frac{2Bbcx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^5,x)

[Out] x^4*((A*c^2)/4 + (C*b*c)/2) - (x^2*((C*a^2)/2 + A*a*b) + (A*a^2)/4 + (B*a^2*x)/3 + 2*B*a*b*x^3)/x^4 + x^2*((C*b^2)/2 + A*b*c + C*a*c) + log(x)*(A*b^2

$$+ 2*A*a*c + 2*C*a*b) + (B*c^2*x^5)/5 + (C*c^2*x^6)/6 + B*x*(2*a*c + b^2) + (2*B*b*c*x^3)/3$$

sympy [A] time = 2.35, size = 153, normalized size = 1.03

$$\frac{2Bbcx^3}{3} + \frac{Bc^2x^5}{5} + \frac{Cc^2x^6}{6} + x^4\left(\frac{Ac^2}{4} + \frac{Cbc}{2}\right) + x^2\left(ABC + Cac + \frac{Cb^2}{2}\right) + x(2Bac + Bb^2) + (2Aac + Ab^2 + 2Cab)\log(x) + \frac{-3Aa^2 - 4Ba^2x - 24Babx^3 + x^2(-12Aab - 6Ca^2)}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2/x**5,x)

[Out] 2*B*b*c*x**3/3 + B*c**2*x**5/5 + C*c**2*x**6/6 + x**4*(A*c**2/4 + C*b*c/2) + x**2*(A*b*c + C*a*c + C*b**2/2) + x*(2*B*a*c + B*b**2) + (2*A*a*c + A*b**2 + 2*C*a*b)*log(x) + (-3*A*a**2 - 4*B*a**2*x - 24*B*a*b*x**3 + x**2*(-12*A*a*b - 6*C*a**2))/(12*x**4)

$$3.19 \quad \int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^6} dx$$

Optimal. Leaf size=143

$$-\frac{a^2A}{5x^5} - \frac{a^2B}{4x^4} + x(C(2ac+b^2) + 2Abc) - \frac{A(2ac+b^2) + 2abC}{x} - \frac{a(aC+2Ab)}{3x^3} + B \log(x)(2ac+b^2) - \frac{abB}{x^2} + \frac{1}{3}cx^3(Ac + 2bC)$$

Rubi [A] time = 0.15, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {1628}

$$-\frac{a^2A}{5x^5} - \frac{a^2B}{4x^4} + x(C(2ac+b^2) + 2Abc) - \frac{A(2ac+b^2) + 2abC}{x} - \frac{a(aC+2Ab)}{3x^3} + B \log(x)(2ac+b^2) - \frac{abB}{x^2} + \frac{1}{3}cx^3(Ac + 2bC) + bBcx^2 + \frac{1}{4}Bc^2x^4 + \frac{1}{5}c^2Cx^5$$

Antiderivative was successfully verified.

[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^6, x]

[Out] $-(a^2A)/(5x^5) - (a^2B)/(4x^4) - (a(2Ab + aC))/(3x^3) - (abB)/x^2 - (A(b^2 + 2ac) + 2abC)/x + (2Abc + (b^2 + 2ac)Cx + bBcx^2 + (c(Ac + 2bC)x^3)/3 + (Bc^2x^4)/4 + (c^2Cx^5)/5 + B(b^2 + 2ac) \log(x)$

Rule 1628

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^6} dx &= \int \left(2Abc \left(1 + \frac{b \left(1 + \frac{2ac}{b^2} \right) C}{2Ac} \right) + \frac{a^2A}{x^6} + \frac{a^2B}{x^5} + \frac{a(2Ab+aC)}{x^4} + \frac{2abB}{x^3} + \frac{A(b^2+2ac) + 2abC}{x} + (2Abc + \dots) \right) dx \\ &= -\frac{a^2A}{5x^5} - \frac{a^2B}{4x^4} - \frac{a(2Ab+aC)}{3x^3} - \frac{abB}{x^2} - \frac{A(b^2+2ac) + 2abC}{x} + (2Abc + \dots) \end{aligned}$$

Mathematica [A] time = 0.08, size = 142, normalized size = 0.99

$$-\frac{a^2A}{5x^5} - \frac{a^2B}{4x^4} - \frac{2aAc + 2abC + Ab^2}{x} - \frac{a(aC+2Ab)}{3x^3} + B \log(x)(2ac+b^2) + Cx(2ac+b^2) - \frac{abB}{x^2} + \frac{1}{3}cx^3(Ac + 2bC) + 2Abcx + bBcx^2 + \frac{1}{4}Bc^2x^4 + \frac{1}{5}c^2Cx^5$$

Antiderivative was successfully verified.

[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^6, x]

[Out] $-1/5*(a^2A)/x^5 - (a^2B)/(4x^4) - (a(2Ab + aC))/(3x^3) - (abB)/x^2 - (A(b^2 + 2ac) + 2abC)/x + 2Abc + (b^2 + 2ac)Cx + bBcx^2 + (c(Ac + 2bC)x^3)/3 + (Bc^2x^4)/4 + (c^2Cx^5)/5 + B(b^2 + 2ac) \log(x)$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^6} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^6,x]

[Out] IntegrateAlgebraic[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^6, x]

fricas [A] time = 1.47, size = 145, normalized size = 1.01

$$\frac{12 C^2 x^{10} + 15 B c^2 x^9 + 60 B b c x^7 + 20 (2 C b c + A c^2) x^8 + 60 (C b^2 + 2 (C a + A b) c) x^6 + 60 (B b^2 + 2 B a c) x^5 \log(x) - 60 B a b x^3 - 60 (2 C a b + A b^2 + 2 A a c) x^4 - 15 B a^2 x - 12 A a^2 - 20 (C a^2 + 2 A a b) x^2}{60 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^6,x, algorithm="fricas")

[Out] 1/60*(12*C*c^2*x^10 + 15*B*c^2*x^9 + 60*B*b*c*x^7 + 20*(2*C*b*c + A*c^2)*x^8 + 60*(C*b^2 + 2*(C*a + A*b)*c)*x^6 + 60*(B*b^2 + 2*B*a*c)*x^5*log(x) - 60*B*a*b*x^3 - 60*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 - 15*B*a^2*x - 12*A*a^2 - 20*(C*a^2 + 2*A*a*b)*x^2)/x^5

giac [A] time = 0.29, size = 140, normalized size = 0.98

$$\frac{1}{5} C^2 x^5 + \frac{1}{4} B c^2 x^4 + \frac{2}{3} C b c x^3 + \frac{1}{3} A c^2 x^3 + B b c x^2 + C b^2 x + 2 C a c x + 2 A b c x + (B b^2 + 2 B a c) \log(x) - \frac{60 B a b x^3 + 60 (2 C a b + A b^2 + 2 A a c) x^4 + 15 B a^2 x + 12 A a^2 + 20 (C a^2 + 2 A a b) x^2}{60 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^6,x, algorithm="giac")

[Out] 1/5*C*c^2*x^5 + 1/4*B*c^2*x^4 + 2/3*C*b*c*x^3 + 1/3*A*c^2*x^3 + B*b*c*x^2 + C*b^2*x + 2*C*a*c*x + 2*A*b*c*x + (B*b^2 + 2*B*a*c)*log(abs(x)) - 1/60*(60*B*a*b*x^3 + 60*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 + 15*B*a^2*x + 12*A*a^2 + 20*(C*a^2 + 2*A*a*b)*x^2)/x^5

maple [A] time = 0.01, size = 144, normalized size = 1.01

$$\frac{C c^2 x^5}{5} + \frac{B c^2 x^4}{4} + \frac{A c^2 x^3}{3} + \frac{2 C b c x^3}{3} + B b c x^2 + 2 A b c x + 2 B a c \ln(x) + B b^2 \ln(x) + 2 C a c x + C b^2 x - \frac{2 A a c}{x} - \frac{A b^2}{x} - \frac{2 C a b}{x} - \frac{B a b}{x^2} - \frac{2 A a b}{3 x^3} - \frac{C a^2}{3 x^3} - \frac{B a^2}{4 x^4} - \frac{A a^2}{5 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^6,x)

[Out] 1/5*c^2*C*x^5+1/4*B*c^2*x^4+1/3*A*x^3*c^2+2/3*C*x^3*b*c+b*B*c*x^2+2*A*b*c*x+2*a*c*C*x+b^2*C*x+2*B*ln(x)*a*c+B*ln(x)*b^2-2/x*a*A*c-1/x*A*b^2-2/x*C*a*b-2/3*a/x^3*A*b-1/3*a^2/x^3*C-1/5*a^2*A/x^5-1/4*a^2*B/x^4-a*b*B/x^2

maxima [A] time = 0.61, size = 138, normalized size = 0.97

$$\frac{1}{5} C^2 x^5 + \frac{1}{4} B c^2 x^4 + B b c x^2 + \frac{1}{3} (2 C b c + A c^2) x^3 + (C b^2 + 2 (C a + A b) c) x + (B b^2 + 2 B a c) \log(x) - \frac{60 B a b x^3 + 60 (2 C a b + A b^2 + 2 A a c) x^4 + 15 B a^2 x + 12 A a^2 + 20 (C a^2 + 2 A a b) x^2}{60 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^6,x, algorithm="maxima")

[Out] 1/5*C*c^2*x^5 + 1/4*B*c^2*x^4 + B*b*c*x^2 + 1/3*(2*C*b*c + A*c^2)*x^3 + (C*b^2 + 2*(C*a + A*b)*c)*x + (B*b^2 + 2*B*a*c)*log(x) - 1/60*(60*B*a*b*x^3 + 60*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 + 15*B*a^2*x + 12*A*a^2 + 20*(C*a^2 + 2*A*a*b)*x^2)/x^5

mupad [B] time = 0.05, size = 136, normalized size = 0.95

$$x^3 \left(\frac{A c^2}{3} + \frac{2 C b c}{3} \right) - \frac{x^2 \left(\frac{C a^2}{3} + \frac{2 A b a}{3} \right) + \frac{A a^2}{5} + x^4 (A b^2 + 2 C a b + 2 A a c) + \frac{B a^2 x}{4} + B a b x^3}{x^5} + x (C b^2 + 2 A c b + 2 C a c) + \ln(x) (B b^2 + 2 B a c) + \frac{B c^2 x^4}{4} + \frac{C c^2 x^5}{5} + B b c x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^6,x)

```
[Out] x^3*((A*c^2)/3 + (2*C*b*c)/3) - (x^2*((C*a^2)/3 + (2*A*a*b)/3) + (A*a^2)/5
+ x^4*(A*b^2 + 2*A*a*c + 2*C*a*b) + (B*a^2*x)/4 + B*a*b*x^3/x^5 + x*(C*b^2
+ 2*A*b*c + 2*C*a*c) + log(x)*(B*b^2 + 2*B*a*c) + (B*c^2*x^4)/4 + (C*c^2*x
^5)/5 + B*b*c*x^2
```

sympy [A] time = 7.81, size = 155, normalized size = 1.08

$$Bbcx^2 + \frac{Bc^2x^4}{4} + B(2ac + b^2)\log(x) + \frac{Cc^2x^5}{5} + x^3\left(\frac{Ac^2}{3} + \frac{2Cbc}{3}\right) + x(2Abc + 2Cac + Cb^2) + \frac{-12Aa^2 - 15Ba^2x - 60Babx^3 + x^4(-120Aac - 60Ab^2 - 120Cab) + x^2(-40Aab - 20Ca^2)}{60x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2/x**6,x)
```

```
[Out] B*b*c*x**2 + B*c**2*x**4/4 + B*(2*a*c + b**2)*log(x) + C*c**2*x**5/5 + x**3
*(A*c**2/3 + 2*C*b*c/3) + x*(2*A*b*c + 2*C*a*c + C*b**2) + (-12*A*a**2 - 15
*B*a**2*x - 60*B*a*b*x**3 + x**4*(-120*A*a*c - 60*A*b**2 - 120*C*a*b) + x**
2*(-40*A*a*b - 20*C*a**2))/(60*x**5)
```

$$3.20 \quad \int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^7} dx$$

Optimal. Leaf size=149

$$\frac{a^2 A}{6x^6} - \frac{a^2 B}{5x^5} - \frac{A(2ac + b^2) + 2abC}{2x^2} + \log(x) (C(2ac + b^2) + 2Abc) - \frac{a(aC + 2Ab)}{4x^4} - \frac{B(2ac + b^2)}{x} - \frac{2abB}{3x^3} + \frac{1}{2}cx^2$$

Rubi [A] time = 0.14, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {1628}

$$\frac{a^2 A}{6x^6} - \frac{a^2 B}{5x^5} - \frac{A(2ac + b^2) + 2abC}{2x^2} + \log(x) (C(2ac + b^2) + 2Abc) - \frac{a(aC + 2Ab)}{4x^4} - \frac{B(2ac + b^2)}{x} - \frac{2abB}{3x^3} + \frac{1}{2}cx^2(Ac + 2bC) + 2bBcx + \frac{1}{3}Bc^2x^3 + \frac{1}{4}c^2Cx^4$$

Antiderivative was successfully verified.

[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^7, x]

[Out] $-(a^2 A)/(6x^6) - (a^2 B)/(5x^5) - (a(2Ab + aC))/(4x^4) - (2a*b*B)/(3x^3) - (A*(b^2 + 2*a*c) + 2*a*b*C)/(2x^2) - (B*(b^2 + 2*a*c))/x + 2*b*B*c*x + (c*(A*c + 2*b*C)*x^2)/2 + (B*c^2*x^3)/3 + (c^2*C*x^4)/4 + (2*A*b*c + (b^2 + 2*a*c)*C)*\text{Log}[x]$

Rule 1628

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int \frac{(A + Bx + Cx^2)(a + bx^2 + cx^4)^2}{x^7} dx &= \int \left(2bBc + \frac{a^2 A}{x^7} + \frac{a^2 B}{x^6} + \frac{a(2Ab + aC)}{x^5} + \frac{2abB}{x^4} + \frac{A(b^2 + 2ac) + 2abC}{x^3} \right. \\ &\quad \left. - \frac{a^2 A}{6x^6} - \frac{a^2 B}{5x^5} - \frac{a(2Ab + aC)}{4x^4} - \frac{2abB}{3x^3} - \frac{A(b^2 + 2ac) + 2abC}{2x^2} - \frac{B(b^2 + 2ac)}{x} \right) dx \end{aligned}$$

Mathematica [A] time = 0.09, size = 144, normalized size = 0.97

$$-\frac{a^2(10A + 3x(4B + 5Cx))}{60x^6} + \log(x) (C(2ac + b^2) + 2Abc) - \frac{a(3A(b + 2cx^2) + 2x(2bB + 3bCx + 6Bcx^2))}{6x^4} + \frac{A(c^2x^4 - b^2)}{2x^2} - \frac{b^2B}{x} + bcx(2B + Cx) + \frac{1}{12}c^2x^3(4B + 3Cx)$$

Antiderivative was successfully verified.

[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^7, x]

[Out] $-(b^2*B)/x + b*c*x*(2*B + C*x) + (c^2*x^3*(4*B + 3*C*x))/12 + (A*(-b^2 + c^2*x^4))/(2*x^2) - (a^2*(10*A + 3*x*(4*B + 5*C*x)))/(60*x^6) - (a*(3*A*(b + 2*c*x^2) + 2*x*(2*b*B + 3*b*C*x + 6*B*c*x^2)))/(6*x^4) + (2*A*b*c + (b^2 + 2*a*c)*C)*\text{Log}[x]$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + Bx + Cx^2)(a + bx^2 + cx^4)^2}{x^7} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^7, x]

[Out] IntegrateAlgebraic[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^7, x]

fricas [A] time = 1.26, size = 145, normalized size = 0.97

$$\frac{15 C^2 x^{10} + 20 B c^2 x^9 + 120 B b c x^7 + 30 (2 C b c + A c^2) x^8 + 60 (C b^2 + 2 (C a + A b) c) x^6 \log(x) - 40 B a b x^3 - 60 (B b^2 + 2 B a c) x^5 - 30 (2 C a b + A b^2 + 2 A a c) x^4 - 12 B a^2 x - 10 A a^2 - 15 (C a^2 + 2 A a b) x^2}{60 x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^7, x, algorithm="fricas")

[Out] 1/60*(15*C*c^2*x^10 + 20*B*c^2*x^9 + 120*B*b*c*x^7 + 30*(2*C*b*c + A*c^2)*x^8 + 60*(C*b^2 + 2*(C*a + A*b)*c)*x^6*log(x) - 40*B*a*b*x^3 - 60*(B*b^2 + 2*B*a*c)*x^5 - 30*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 - 12*B*a^2*x - 10*A*a^2 - 15*(C*a^2 + 2*A*a*b)*x^2)/x^6

giac [A] time = 0.40, size = 141, normalized size = 0.95

$$\frac{1}{4} C^2 x^4 + \frac{1}{3} B c^2 x^3 + C b c x^2 + \frac{1}{2} A c^2 x^2 + 2 B b c x + (C b^2 + 2 C a c + 2 A b c) \log(x) - \frac{40 B a b x^3 + 60 (B b^2 + 2 B a c) x^5 + 30 (2 C a b + A b^2 + 2 A a c) x^4 + 12 B a^2 x + 10 A a^2 + 15 (C a^2 + 2 A a b) x^2}{60 x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^7, x, algorithm="giac")

[Out] 1/4*C*c^2*x^4 + 1/3*B*c^2*x^3 + C*b*c*x^2 + 1/2*A*c^2*x^2 + 2*B*b*c*x + (C*b^2 + 2*C*a*c + 2*A*b*c)*log(abs(x)) - 1/60*(40*B*a*b*x^3 + 60*(B*b^2 + 2*B*a*c)*x^5 + 30*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 + 12*B*a^2*x + 10*A*a^2 + 15*(C*a^2 + 2*A*a*b)*x^2)/x^6

maple [A] time = 0.01, size = 148, normalized size = 0.99

$$\frac{C^2 x^4}{4} + \frac{B c^2 x^3}{3} + \frac{A c^2 x^2}{2} + C b c x^2 + 2 A b c \ln(x) + 2 B b c x + 2 C a c \ln(x) + C b^2 \ln(x) - \frac{2 B a c}{x} - \frac{B b^2}{x} - \frac{A a c}{x^2} - \frac{A b^2}{2 x^2} - \frac{C a b}{x^2} - \frac{2 B a b}{3 x^3} - \frac{A a b}{2 x^4} - \frac{C a^2}{4 x^4} - \frac{B a^2}{5 x^5} - \frac{A a^2}{6 x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^7, x)

[Out] 1/4*c^2*C*x^4+1/3*B*c^2*x^3+1/2*A*x^2*c^2+C*x^2*b*c+2*b*B*c*x+2*A*ln(x)*b*c+2*C*ln(x)*a*c+C*ln(x)*b^2-2*B/x*a*c-B/x*b^2-2/3*a*b*B/x^3-1/5*a^2*B/x^5-1/2*a/x^4*A*b-1/4*a^2/x^4*C-1/x^2*a*A*c-1/2/x^2*A*b^2-1/x^2*C*a*b-1/6*a^2*A/x^6

maxima [A] time = 0.69, size = 140, normalized size = 0.94

$$\frac{1}{4} C^2 x^4 + \frac{1}{3} B c^2 x^3 + 2 B b c x + \frac{1}{2} (2 C b c + A c^2) x^2 + (C b^2 + 2 (C a + A b) c) \log(x) - \frac{40 B a b x^3 + 60 (B b^2 + 2 B a c) x^5 + 30 (2 C a b + A b^2 + 2 A a c) x^4 + 12 B a^2 x + 10 A a^2 + 15 (C a^2 + 2 A a b) x^2}{60 x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^7, x, algorithm="maxima")

[Out] 1/4*C*c^2*x^4 + 1/3*B*c^2*x^3 + 2*B*b*c*x + 1/2*(2*C*b*c + A*c^2)*x^2 + (C*b^2 + 2*(C*a + A*b)*c)*log(x) - 1/60*(40*B*a*b*x^3 + 60*(B*b^2 + 2*B*a*c)*x^5 + 30*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 + 12*B*a^2*x + 10*A*a^2 + 15*(C*a^2 + 2*A*a*b)*x^2)/x^6

mupad [B] time = 0.06, size = 136, normalized size = 0.91

$$x^2 \left(\frac{A c^2}{2} + C b c \right) - \frac{x^2 \left(\frac{C a^2}{4} + \frac{A b a}{2} \right) + x^5 (B b^2 + 2 B a c) + \frac{A a^2}{6} + x^4 \left(\frac{A b^2}{2} + C a b + A a c \right) + \frac{B a^2 x}{5} + \frac{2 B a b x^3}{3} + \ln(x) (C b^2 + 2 A c b + 2 C a c) + \frac{B c^2 x^3}{3} + \frac{C c^2 x^4}{4} + 2 B b c x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^7, x)


```
[Out] x^2*((A*c^2)/2 + C*b*c) - (x^2*((C*a^2)/4 + (A*a*b)/2) + x^5*(B*b^2 + 2*B*a*c) + (A*a^2)/6 + x^4*((A*b^2)/2 + A*a*c + C*a*b) + (B*a^2*x)/5 + (2*B*a*b*x^3)/3)/x^6 + log(x)*(C*b^2 + 2*A*b*c + 2*C*a*c) + (B*c^2*x^3)/3 + (C*c^2*x^4)/4 + 2*B*b*c*x
```

sympy [A] time = 27.40, size = 158, normalized size = 1.06

$$2Bbcx + \frac{Bc^2x^3}{3} + \frac{C^2x^4}{4} + x^2\left(\frac{Ac^2}{2} + Cbc\right) + (2Abc + 2Cac + Cb^2)\log(x) + \frac{-10Aa^2 - 12Ba^2x - 40Babx^3 + x^5(-120Bac - 60Bb^2) + x^4(-60Aac - 30Ab^2 - 60Cab) + x^2(-30Aab - 15Ca^2)}{60x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2/x**7,x)
```

```
[Out] 2*B*b*c*x + B*c**2*x**3/3 + C*c**2*x**4/4 + x**2*(A*c**2/2 + C*b*c) + (2*A*b*c + 2*C*a*c + C*b**2)*log(x) + (-10*A*a**2 - 12*B*a**2*x - 40*B*a*b*x**3 + x**5*(-120*B*a*c - 60*B*b**2) + x**4*(-60*A*a*c - 30*A*b**2 - 60*C*a*b) + x**2*(-30*A*a*b - 15*C*a**2))/(60*x**6)
```

$$3.21 \quad \int \frac{x^4(A+Bx+Cx^2)}{a+bx^2+cx^4} dx$$

Optimal. Leaf size=339

$$\frac{\left(-\frac{Ac(b^2-2ac)-bC(b^2-3ac)}{\sqrt{b^2-4ac}} + acC + Abc + b^2(-C)\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right) \left(\frac{Ac(b^2-2ac)-bC(b^2-3ac)}{\sqrt{b^2-4ac}} + acC + Abc + b^2(-C)\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(\frac{Ac(b^2-2ac)-bC(b^2-3ac)}{\sqrt{b^2-4ac}} + acC + Abc + b^2(-C)\right)}{\sqrt{2}c^{5/2}\sqrt{b^2-4ac} + b}$$

Rubi [A] time = 1.86, antiderivative size = 339, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 28, number of rules / integrand size = 0.393, Rules used = {1662, 1279, 1166, 205, 12, 1114, 703, 634, 618, 206, 628}

$$\frac{\left(-\frac{Ac(b^2-2ac)-bC(b^2-3ac)}{\sqrt{b^2-4ac}} + acC + Abc + b^2(-C)\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right) \left(\frac{Ac(b^2-2ac)-bC(b^2-3ac)}{\sqrt{b^2-4ac}} + acC + Abc + b^2(-C)\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2-4ac}+b}}\right) - \frac{B(b^2-2ac) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right) - bB \log(a+bx^2+cx^4) + \frac{x(Ac-bC)}{c^2} + \frac{Bx^2}{2c} + \frac{Cx^3}{3c}}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}} \sqrt{2}c^{5/2}\sqrt{\sqrt{b^2-4ac}+b} - \frac{B(b^2-2ac) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right) - bB \log(a+bx^2+cx^4) + \frac{x(Ac-bC)}{c^2} + \frac{Bx^2}{2c} + \frac{Cx^3}{3c}}{2c^2\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

[In] Int[(x^4*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4), x]

[Out] ((A*c - b*C)*x)/c^2 + (B*x^2)/(2*c) + (C*x^3)/(3*c) - ((A*b*c - b^2*C + a*c*C - (A*c*(b^2 - 2*a*c) - b*(b^2 - 3*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/(Sqrt[2]*c^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((A*b*c - b^2*C + a*c*C + (A*c*(b^2 - 2*a*c) - b*(b^2 - 3*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]/(Sqrt[2]*c^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (B*(b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^2*Sqrt[b^2 - 4*a*c]) - (b*B*Log[a + b*x^2 + c*x^4])/(4*c^2)

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_)^m)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1114

```
Int[(x_)^m*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1279

```
Int[((f_.)*(x_)^m)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p + 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1662

```
Int[(Pq_)*((d_.)*(x_)^m)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2 + 1})*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1})*(a + b*x^2 + c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] && !PolyQ[Pq, x^2]
```

Rubi steps

$$\int \frac{x^4 (A + Bx + Cx^2)}{a + bx^2 + cx^4} dx = \int \frac{Bx^5}{a + bx^2 + cx^4} dx + \int \frac{x^4 (A + Cx^2)}{a + bx^2 + cx^4} dx$$

$$= \frac{Cx^3}{3c} + B \int \frac{x^5}{a + bx^2 + cx^4} dx - \frac{\int \frac{x^2(3aC-3(Ac-bC)x^2)}{a+bx^2+cx^4} dx}{3c}$$

$$= \frac{(Ac - bC)x}{c^2} + \frac{Cx^3}{3c} + \frac{1}{2}B \text{Subst} \left(\int \frac{x^2}{a + bx + cx^2} dx, x, x^2 \right) + \frac{\int \frac{-3a(Ac-bC)-3(Abc-b^2C+acC)}{a+bx^2+cx^4} dx}{3c^2}$$

$$= \frac{(Ac - bC)x}{c^2} + \frac{Bx^2}{2c} + \frac{Cx^3}{3c} + \frac{B \text{Subst} \left(\int \frac{-a-bx}{a+bx+cx^2} dx, x, x^2 \right)}{2c} - \frac{\left(Abc - b^2C + acC - \frac{Ac(b^2-2ac)-b(b^2-3ac)C}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{bx - \sqrt{b^2-4ac}}}{\sqrt{b - \sqrt{b^2-4ac}}} \right)}{\sqrt{2} c^{5/2} \sqrt{b - \sqrt{b^2-4ac}}}$$

$$= \frac{(Ac - bC)x}{c^2} + \frac{Bx^2}{2c} + \frac{Cx^3}{3c} - \frac{\left(Abc - b^2C + acC - \frac{Ac(b^2-2ac)-b(b^2-3ac)C}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{bx - \sqrt{b^2-4ac}}}{\sqrt{b - \sqrt{b^2-4ac}}} \right)}{\sqrt{2} c^{5/2} \sqrt{b - \sqrt{b^2-4ac}}}$$

$$= \frac{(Ac - bC)x}{c^2} + \frac{Bx^2}{2c} + \frac{Cx^3}{3c} - \frac{\left(Abc - b^2C + acC - \frac{Ac(b^2-2ac)-b(b^2-3ac)C}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{bx - \sqrt{b^2-4ac}}}{\sqrt{b - \sqrt{b^2-4ac}}} \right)}{\sqrt{2} c^{5/2} \sqrt{b - \sqrt{b^2-4ac}}}$$

Mathematica [A] time = 0.57, size = 460, normalized size = 1.36

$$\frac{6\sqrt{2}(Ac + \sqrt{b^2-4ac} - 2ac + b^2) + c \left(b^2 \sqrt{b^2-4ac} - ac \sqrt{b^2-4ac} + 3abc - b^3 \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{bx - \sqrt{b^2-4ac}}}{\sqrt{b - \sqrt{b^2-4ac}}} \right) + 6\sqrt{2} \left(c \left(b^2 \sqrt{b^2-4ac} - ac \sqrt{b^2-4ac} - 3abc + b^3 \right) - Ac \left(\sqrt{b^2-4ac} - 2ac + b^2 \right) \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{bx - \sqrt{b^2-4ac}}}{\sqrt{b - \sqrt{b^2-4ac}}} \right) - 3B \sqrt{c} \left(b \sqrt{b^2-4ac} + 2ac - b^2 \right) \log \left(\frac{\sqrt{b^2-4ac} - b + 2cx^2}{\sqrt{b^2-4ac}} \right) - 3B \sqrt{c} \left(b \sqrt{b^2-4ac} - 2ac + b^2 \right) \log \left(\frac{\sqrt{b^2-4ac} + b + 2cx^2}{\sqrt{b^2-4ac}} \right) + 12\sqrt{c} x (Ac - bC) + 6Bc^3 x^2 + 4c^3 C x^3}{12c^{5/2} \sqrt{b - \sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4), x]
[Out] (12*Sqrt[c]*(A*c - b*C)*x + 6*B*c^(3/2)*x^2 + 4*c^(3/2)*C*x^3 + (6*Sqrt[2]*(A*c*(b^2 - 2*a*c - b*Sqrt[b^2 - 4*a*c]) + (-b^3 + 3*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 4*a*c) - a*c*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (6*Sqrt[2]*(-A*c*(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])) + (b^3 - 3*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - a*c*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (3*B*Sqrt[c]*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/Sqrt[b^2 - 4*a*c] - (3*B*Sqrt[c]*(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c]/(12*c^(5/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 (A + Bx + Cx^2)}{a + bx^2 + cx^4} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^4*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4), x]
[Out] IntegrateAlgebraic[(x^4*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] Timed out

giac [B] time = 5.75, size = 5304, normalized size = 15.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out]
$$-1/4*B*b*\log(\text{abs}(c*x^4 + b*x^2 + a))/c^2 - 1/8*((2*b^5*c^3 - 16*a*b^3*c^4 + 32*a^2*b*c^5 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^5*c + 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^3*c^2 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^4*c^2 - 16*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b*c^3 - 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c^3 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^3*c^3 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^3 + 8*(b^2 - 4*a*c)*a*b*c^4)*A*c^2 - (2*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2*b^2*c^4 - 32*a^3*c^5 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^6 + 9*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^4*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^5*c - 24*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b^2*c^2 - 10*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^3*c^2 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^4*c^2 + 16*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*c^3 + 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b*c^3 + 5*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c^3 - 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2 + 10*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b^2 - 4*a*c)*a^2*c^4)*C*c^2 + 2*(\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^4*c^3 - 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b^2*c^4 - 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^3*c^4 + 2*a*b^4*c^4 + 16*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*c^5 + 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b*c^5 + \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c^5 - 16*a^2*b^2*c^5 - 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*c^6 + 32*a^3*c^6 - 2*(b^2 - 4*a*c)*a*b^2*c^4 + 8*(b^2 - 4*a*c)*a^2*c^5)*A*\text{abs}(c) - 2*(\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^5*c^2 - 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b^3*c^3 - 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^4*c^3 + 2*a*b^5*c^3 + 16*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*b*c^4 + 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b^2*c^4 + \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^3*c^4 - 16*a^2*b^3*c^4 - 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b*c^5 + 32*a^3*b*c^5 - 2*(b^2 - 4*a*c)*a*b^3*c^3 + 8*(b^2 - 4*a*c)*a^2*b*c^4)*C*\text{abs}(c) - (2*b^5*c^5 - 12*a*b^3*c^6 + 16*a^2*b*c^7 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^5*c^3 + 6*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^3*c^4 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^4*c^4 - 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b*c^5 - 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c^5 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^3*c^5 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b*c^6 - 2*(b^2 - 4*a*c)*b^3*c^5 + 4*(b^2 - 4*a*c)*a*b*c^6)*A + (2*b^6*c^4 - 14*a*b^4*c^5 + 24*a^2*b^2*c^6 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^6*c^2 + 7*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^4*c^3 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)$$

$$\begin{aligned}
& c) * b^5 * c^3 - 12 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c - \sqrt{b^2 - 4 * a * c} * c} * a \\
& ^2 * b^2 * c^4 - 6 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c - \sqrt{b^2 - 4 * a * c} * c} * a * \\
& b^3 * c^4 - \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c - \sqrt{b^2 - 4 * a * c} * c} * b^4 * c^4 \\
& + 3 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c - \sqrt{b^2 - 4 * a * c} * c} * a * b^2 * c^5 - \\
& 2 * (b^2 - 4 * a * c) * b^4 * c^4 + 6 * (b^2 - 4 * a * c) * a * b^2 * c^5 * C) * \arctan(2 * \sqrt{1/2} * \\
& x / \sqrt{(b * c^7 + \sqrt{b^2 * c^{14} - 4 * a * c^{15}}) / c^8}) / ((a * b^4 * c^4 - 8 * a^2 * b^2 * c^5 \\
& - 2 * a * b^3 * c^5 + 16 * a^3 * c^6 + 8 * a^2 * b * c^6 + a * b^2 * c^6 - 4 * a^2 * c^7) * c^2) + \\
& 1/8 * ((2 * b^5 * c^3 - 16 * a * b^3 * c^4 + 32 * a^2 * b * c^5 - \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c} * c} * b^5 * c \\
& + 8 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c} * c} * a * b^3 * c^2 + 2 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c} * c} * b^4 * c^2 - 16 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c} * c} * a^2 * b * c^3 - 8 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c} * c} * a * b^2 * c^3 - \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c} * c} * b^3 * c^3 + 4 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c} * c} * a * b * c^4 - 2 * (b^2 - 4 * a * c) * b^3 * c^3 + 8 * (b^2 - 4 * a * c) * a * b * c^4) * A * c^2 - (2 * b^6 * c^2 - 18 * a * b^4 * c^3 + 48 * a^2 * b^2 * c^4 - 32 * a^3 * c^5 - \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c} * c} * b^6 + 9 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c} * c} * a * b^4 * c + 2 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c} * c} * b^5 * c - 24 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c} * c} * a^2 * b^2 * c^2 - 10 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c} * c} * a * b^3 * c^2 - \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c} * c} * b^4 * c^2 + 16 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c} * c} * a^3 * c^3 + 8 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c} * c} * a^2 * b * c^3 + 5 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c} * c} * a * b^2 * c^3 - 4 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c} * c} * a^2 * c^4 - 2 * (b^2 - 4 * a * c) * b^4 * c^2 + 10 * (b^2 - 4 * a * c) * a * b^2 * c^3 - 8 * (b^2 - 4 * a * c) * a^2 * c^4) * C * c^2 - 2 * (\sqrt{2} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c} * c} * a * b^4 * c^3 - 8 * \sqrt{2} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c} * c} * a^2 * b^2 * c^4 - 2 * \sqrt{2} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c} * c} * a * b^3 * c^4 - 2 * a * b^5 * c^3 + 16 * \sqrt{2} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c} * c} * a^3 * b * c^4 + 8 * \sqrt{2} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c} * c} * a^2 * b^2 * c^4 + \sqrt{2} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c} * c} * a * b^3 * c^4 + 16 * a^2 * b^3 * c^4 - 4 * \sqrt{2} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c} * c} * a^2 * b * c^5 - 32 * a^3 * b * c^5 + 2 * (b^2 - 4 * a * c) * a * b^3 * c^3 - 8 * (b^2 - 4 * a * c) * a^2 * b * c^4) * C * \text{abs}(c) - (2 * b^5 * c^5 - 12 * a * b^3 * c^6 + 16 * a^2 * b * c^7 - \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c} * c} * b^5 * c^3 + 6 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c} * c} * a * b^3 * c^4 + 2 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c} * c} * b^4 * c^4 - 8 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c} * c} * a^2 * b * c^5 - 4 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c} * c} * a * b^2 * c^5 - \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c} * c} * b^3 * c^5 + 2 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c} * c} * a * b * c^6 - 2 * (b^2 - 4 * a * c) * b^3 * c^5 + 4 * (b^2 - 4 * a * c) * a * b * c^6) * A + (2 * b^6 * c^4 - 14 * a * b^4 * c^5 + 24 * a^2 * b^2 * c^6 - \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c} * c} * b^6 * c^2 + 7 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c} * c} * a * b^4 * c^3 + 2 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c} * c} * b^5 * c^3 - 12 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c} * c} * a^2 * b^2 * c^4 - 6 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c} * c} * a * b^3 * c^4 - \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c} * c} * b^4 * c^4 + 3 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c} * c} * a * b^2 * c^5 - 2 * (b^2 - 4 * a * c) * b^4 * c^4 + 6 * (b^2 - 4 * a * c) * a * b^2 * c^5) * C) * \arctan(2 * \sqrt{1/2} * x / \sqrt{(b * c^7 - \sqrt{b^2 * c^{14} - 4 * a * c^{15}}) / c^8}) / ((a * b^4 * c^4 - 8 * a^2 * b^2 * c^5 - 2 * a * b^3 * c^5 + 16 * a^3 * c^6 + 8 * a^2 * b * c^6 + a * b^2 * c^6 - 4 * a^2 * c^7) * c^2) + 1/6 * (2 * C * c^2 * x^3 + 3 * B * c^2 * x^2 - 6 * C * b * c * x + 6 * A * c^2 * x) / c^3 + 1/16 * ((b^7 - 10 * a * b^5 * c - 2 * b^6 * c + 32 * a^2 * b^3 * c^2 + 12 *
\end{aligned}$$

$$\left(\frac{1}{2}\right) * c^{\left(\frac{1}{2}\right)} * c * x * C * a^{-2} - 2 / (4 * a * c - b^2) * 2^{\left(\frac{1}{2}\right)} / ((b + (-4 * a * c + b^2)^{\left(\frac{1}{2}\right)}) * c)^{\left(\frac{1}{2}\right)} * \arctan(2^{\left(\frac{1}{2}\right)} / ((b + (-4 * a * c + b^2)^{\left(\frac{1}{2}\right)}) * c)^{\left(\frac{1}{2}\right)} * c * x) * C * a^{-2} - 1 / c / (4 * a * c - b^2) * B * \ln(2 * c * x^2 + b + (-4 * a * c + b^2)^{\left(\frac{1}{2}\right)}) * a * b + 1 / 4 / c^2 / (4 * a * c - b^2) * B * \ln(-2 * c * x^2 - b + (-4 * a * c + b^2)^{\left(\frac{1}{2}\right)}) * b^3 + 1 / 4 / c^2 / (4 * a * c - b^2) * B * \ln(2 * c * x^2 + b + (-4 * a * c + b^2)^{\left(\frac{1}{2}\right)}) * b^3$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] 1/6*(2*C*c*x^3 + 3*B*c*x^2 - 6*(C*b - A*c)*x)/c^2 - integrate((B*b*c*x^3 + B*a*c*x - C*a*b + A*a*c - (C*b^2 - (C*a + A*b)*c)*x^2)/(c*x^4 + b*x^2 + a), x)/c^2

mupad [B] time = 0.96, size = 2588, normalized size = 7.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x)

[Out] x*(A/c - (C*b)/c^2) + symsum(log((C^3*a^4*c - C^3*a^3*b^2 - A*B^2*a^3*c^2 + A*C^2*a^2*b^3 + A^2*C*a^3*c^2 + A^3*a^2*b*c^2 + A*B^2*a^2*b^2*c - 2*A^2*C*a^2*b^2*c - B^2*C*a^3*b*c)/c^3 - root(128*a*b^2*c^6*z^4 - 16*b^4*c^5*z^4 - 256*a^2*c^7*z^4 - 256*B*a^2*b*c^5*z^3 + 128*B*a*b^3*c^4*z^3 - 16*B*b^5*c^3*z^3 - 64*A*C*a*b^4*c^2*z^2 + 144*A*C*a^2*b^2*c^3*z^2 + 8*A*C*b^6*c*z^2 + 80*C^2*a^3*b*c^3*z^2 + 32*B^2*a*b^4*c^2*z^2 - 48*A^2*a^2*b*c^4*z^2 + 28*A^2*a*b^3*c^3*z^2 + 36*C^2*a*b^5*c*z^2 - 64*A*C*a^3*c^4*z^2 - 100*C^2*a^2*b^3*c^2*z^2 - 56*B^2*a^2*b^2*c^3*z^2 - 4*B^2*b^6*c*z^2 - 32*B^2*a^3*c^4*z^2 - 4*A^2*b^5*c^2*z^2 - 4*C^2*b^7*z^2 + 32*A*B*C*a^3*b*c^2*z - 8*A*B*C*a^2*b^3*c*z - 20*B*C^2*a^3*b^2*c*z + 4*A^2*B*a^2*b^2*c^2*z - 16*B^3*a^3*b*c^2*z + 4*B^3*a^2*b^3*c*z + 16*B*C^2*a^4*c^2*z + 4*B*C^2*a^2*b^4*z - 16*A^2*B*a^3*c^3*z + 2*A^3*C*a^3*b*c + 4*A*B^2*C*a^4*c - 2*A^2*C^2*a^4*c + 2*A*C^3*a^4*b - A^2*B^2*a^3*b*c - B^2*C^2*a^4*b - A^2*C^2*a^3*b^2 - A^4*a^3*c^2 - B^4*a^4*c - C^4*a^5, z, k)*(root(128*a*b^2*c^6*z^4 - 16*b^4*c^5*z^4 - 256*a^2*c^7*z^4 - 256*B*a^2*b*c^5*z^3 + 128*B*a*b^3*c^4*z^3 - 16*B*b^5*c^3*z^3 - 64*A*C*a*b^4*c^2*z^2 + 144*A*C*a^2*b^2*c^3*z^2 + 8*A*C*b^6*c*z^2 + 80*C^2*a^3*b*c^3*z^2 + 32*B^2*a*b^4*c^2*z^2 - 48*A^2*a^2*b*c^4*z^2 + 28*A^2*a*b^3*c^3*z^2 + 36*C^2*a*b^5*c*z^2 - 64*A*C*a^3*c^4*z^2 - 100*C^2*a^2*b^3*c^2*z^2 - 56*B^2*a^2*b^2*c^3*z^2 - 4*B^2*b^6*c*z^2 - 32*B^2*a^3*c^4*z^2 - 4*A^2*b^5*c^2*z^2 - 4*C^2*b^7*z^2 + 32*A*B*C*a^3*b*c^2*z - 8*A*B*C*a^2*b^3*c*z - 20*B*C^2*a^3*b^2*c*z + 4*A^2*B*a^2*b^2*c^2*z - 16*B^3*a^3*b*c^2*z + 4*B^3*a^2*b^3*c*z + 16*B*C^2*a^4*c^2*z + 4*B*C^2*a^2*b^4*z - 16*A^2*B*a^3*c^3*z + 2*A^3*C*a^3*b*c + 4*A*B^2*C*a^4*c - 2*A^2*C^2*a^4*c + 2*A*C^3*a^4*b - A^2*B^2*a^3*b*c - B^2*C^2*a^4*b - A^2*C^2*a^3*b^2 - A^4*a^3*c^2 - B^4*a^4*c - C^4*a^5, z, k))*((x*(16*B*a^2*c^5 + 8*B*b^4*c^3 - 36*B*a*b^2*c^4))/c^3 - (16*A*a^2*c^5 - 4*A*a*b^2*c^4 + 4*C*a*b^3*c^3 - 16*C*a^2*b*c^4)/c^3 + (root(128*a*b^2*c^6*z^4 - 16*b^4*c^5*z^4 - 256*a^2*c^7*z^4 - 256*B*a^2*b*c^5*z^3 + 128*B*a*b^3*c^4*z^3 - 16*B*b^5*c^3*z^3 - 64*A*C*a*b^4*c^2*z^2 + 144*A*C*a^2*b^2*c^3*z^2 + 8*A*C*b^6*c*z^2 + 80*C^2*a^3*b*c^3*z^2 + 32*B^2*a*b^4*c^2*z^2 - 48*A^2*a^2*b*c^4*z^2 + 28*A^2*a*b^3*c^3*z^2 + 36*C^2*a*b^5*c*z^2 - 64*A*C*a^3*c^4*z^2 - 100*C^2*a^2*b^3*c^2*z^2 - 56*B^2*a^2*b^2*c^3*z^2 - 4*B^2*b^6*c*z^2 - 32*B^2*a^3*c^4*z^2 - 4*A^2*b^5*c^2*z^2 - 4*C^2*b^7*z^2 + 32*A*B*C*a^3*b*c^2*z - 8*A*B*C*a^2*b^3*c*z - 20*B*C^2*a^3*b^2*c*z + 4*A^2*B*a^2*b^2*c^2*z - 16*B^3*a^3*b*c^2*z + 4*B^3*a^2*b^3*c*z + 16*B*C^2*a^4*c^2*z + 4*B*C^2*a^2*b^4*z - 16*A^2*B*a^3*c^3*z + 2*A^3*C*a^3*b*c + 4*A*B^2*C*a^4*c - 2*A^2*C^2*a^4*c + 2*A*C^3*a^4*b - A^2*B^2*a^3*b*c - B^2*C^2*a^4*b - A^2*C^2*a^3*b^2 - A^4*a^3*c^2 - B^4*a^4*c - C^4*a^5, z, k))*x*(8*b^3*c^5 - 32*a*b*c^6))/c^3) + (8*B

$$\begin{aligned} & *C*a^3*c^3 - 4*A*B*a^2*b*c^3)/c^3 + (x*(2*C^2*b^6 + 2*B^2*b^5*c + 4*A^2*a^2 \\ & *c^4 + 2*A^2*b^4*c^2 - 4*C^2*a^3*c^3 - 4*A*C*b^5*c + 18*C^2*a^2*b^2*c^2 - 1 \\ & 2*C^2*a*b^4*c - 8*A^2*a*b^2*c^3 - 10*B^2*a*b^3*c^2 + 6*B^2*a^2*b*c^3 + 20*A \\ & *C*a*b^3*c^2 - 20*A*C*a^2*b*c^3))/c^3) + (x*(B*C^2*a^2*b^3 - B^3*a^3*c^2 + \\ & B^3*a^2*b^2*c + A^2*B*a^2*b*c^2 + 2*A*B*C*a^3*c^2 - 2*B*C^2*a^3*b*c - 2*A*B \\ & *C*a^2*b^2*c))/c^3)*\text{root}(128*a*b^2*c^6*z^4 - 16*b^4*c^5*z^4 - 256*a^2*c^7*z \\ & ^4 - 256*B*a^2*b*c^5*z^3 + 128*B*a*b^3*c^4*z^3 - 16*B*b^5*c^3*z^3 - 64*A*C* \\ & a*b^4*c^2*z^2 + 144*A*C*a^2*b^2*c^3*z^2 + 8*A*C*b^6*c*z^2 + 80*C^2*a^3*b*c^ \\ & 3*z^2 + 32*B^2*a*b^4*c^2*z^2 - 48*A^2*a^2*b*c^4*z^2 + 28*A^2*a*b^3*c^3*z^2 \\ & + 36*C^2*a*b^5*c*z^2 - 64*A*C*a^3*c^4*z^2 - 100*C^2*a^2*b^3*c^2*z^2 - 56*B^ \\ & 2*a^2*b^2*c^3*z^2 - 4*B^2*b^6*c*z^2 - 32*B^2*a^3*c^4*z^2 - 4*A^2*b^5*c^2*z^ \\ & 2 - 4*C^2*b^7*z^2 + 32*A*B*C*a^3*b*c^2*z - 8*A*B*C*a^2*b^3*c*z - 20*B*C^2*a \\ & ^3*b^2*c*z + 4*A^2*B*a^2*b^2*c^2*z - 16*B^3*a^3*b*c^2*z + 4*B^3*a^2*b^3*c*z \\ & + 16*B*C^2*a^4*c^2*z + 4*B*C^2*a^2*b^4*z - 16*A^2*B*a^3*c^3*z + 2*A^3*C*a^ \\ & 3*b*c + 4*A*B^2*C*a^4*c - 2*A^2*C^2*a^4*c + 2*A*C^3*a^4*b - A^2*B^2*a^3*b*c \\ & - B^2*C^2*a^4*b - A^2*C^2*a^3*b^2 - A^4*a^3*c^2 - B^4*a^4*c - C^4*a^5, z, \\ & k), k, 1, 4) + (B*x^2)/(2*c) + (C*x^3)/(3*c) \end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4*(C*x**2+B*x+A)/(c*x**4+b*x**2+a), x)

[Out] Timed out

$$3.22 \quad \int \frac{x^3(A+Bx+Cx^2)}{a+bx^2+cx^4} dx$$

Optimal. Leaf size=278

$$\frac{(2acC + Abc + b^2(-C)) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right) + (Ac - bC) \log(a + bx^2 + cx^4)}{2c^2\sqrt{b^2 - 4ac}} - \frac{B\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b - \sqrt{b^2 - 4ac}}}$$

Rubi [A] time = 0.47, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$, Rules used = {1662, 1251, 773, 634, 618, 206, 628, 12, 1122, 1166, 205}

$$\frac{(2acC + Abc + b^2(-C)) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right) + (Ac - bC) \log(a + bx^2 + cx^4)}{2c^2\sqrt{b^2 - 4ac}} - \frac{B\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{B\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}} + b\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2 - 4ac} + b}} + \frac{Bx}{c} + \frac{Cx^2}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4), x]
```

```
[Out] (B*x)/c + (C*x^2)/(2*c) - (B*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (B*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + ((A*b*c - b^2*C + 2*a*c*C)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^2*Sqrt[b^2 - 4*a*c]) + ((A*c - b*C)*Log[a + b*x^2 + c*x^4])/(4*c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 773

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1122

```
Int[((d_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(d^3*(d*x)^(m-3)*(a + b*x^2 + c*x^4)^(p+1))/(c*(m+4*p+1)), x] - Dist[d^4/(c*(m+4*p+1)), Int[(d*x)^(m-4)*Simp[a*(m-3) + b*(m+2*p-1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m+4*p+1, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m-1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m-1)/2]
```

Rule 1662

```
Int[(Pq_)*((d_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m+1)*Sum[Coeff[Pq, x, 2*k+1]*x^(2*k), {k, 0, (q-1)/2 + 1}]*(a + b*x^2 + c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] && !PolyQ[Pq, x^2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3 (A + Bx + Cx^2)}{a + bx^2 + cx^4} dx &= \int \frac{Bx^4}{a + bx^2 + cx^4} dx + \int \frac{x^3 (A + Cx^2)}{a + bx^2 + cx^4} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{x(A + Cx)}{a + bx + cx^2} dx, x, x^2 \right) + B \int \frac{x^4}{a + bx^2 + cx^4} dx \\
&= \frac{Bx}{c} + \frac{Cx^2}{2c} + \frac{\text{Subst} \left(\int \frac{-aC + (Ac - bC)x}{a + bx + cx^2} dx, x, x^2 \right)}{2c} - \frac{B \int \frac{a + bx^2}{a + bx^2 + cx^4} dx}{c} \\
&= \frac{Bx}{c} + \frac{Cx^2}{2c} - \frac{\left(B \left(b - \frac{b^2 - 2ac}{\sqrt{b^2 - 4ac}} \right) \right) \int \frac{1}{\frac{b}{2} - \frac{1}{2} \sqrt{b^2 - 4ac} + cx^2} dx}{2c} - \frac{\left(B \left(b + \frac{b^2 - 2ac}{\sqrt{b^2 - 4ac}} \right) \right) \int \frac{1}{\frac{b}{2} + \frac{1}{2} \sqrt{b^2 - 4ac} + cx^2} dx}{2c} \\
&= \frac{Bx}{c} + \frac{Cx^2}{2c} - \frac{B \left(b - \frac{b^2 - 2ac}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{\sqrt{2} c^{3/2} \sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{B \left(b + \frac{b^2 - 2ac}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b + \sqrt{b^2 - 4ac}}} \right)}{\sqrt{2} c^{3/2} \sqrt{b + \sqrt{b^2 - 4ac}}} \\
&= \frac{Bx}{c} + \frac{Cx^2}{2c} - \frac{B \left(b - \frac{b^2 - 2ac}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{\sqrt{2} c^{3/2} \sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{B \left(b + \frac{b^2 - 2ac}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b + \sqrt{b^2 - 4ac}}} \right)}{\sqrt{2} c^{3/2} \sqrt{b + \sqrt{b^2 - 4ac}}}
\end{aligned}$$

Mathematica [A] time = 0.42, size = 377, normalized size = 1.36

$$\frac{\left(A \sqrt{b^2 - 4ac} - b \right) + C \left(-b \sqrt{b^2 - 4ac} - 2ac + b^2 \right) \log \left(\sqrt{b^2 - 4ac} - b - 2cx^2 \right) - \left(C \left(b \sqrt{b^2 - 4ac} - 2ac + b^2 \right) - A \sqrt{b^2 - 4ac} \right) \log \left(\sqrt{b^2 - 4ac} + b + 2cx^2 \right)}{\sqrt{b^2 - 4ac}} - \frac{2\sqrt{2}B\sqrt{c} \left(b \sqrt{b^2 - 4ac} + 2ac - b^2 \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{\sqrt{b^2 - 4ac} \sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{2\sqrt{2}B\sqrt{c} \left(b \sqrt{b^2 - 4ac} - 2ac + b^2 \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b + \sqrt{b^2 - 4ac}}} \right)}{\sqrt{b^2 - 4ac} \sqrt{b + \sqrt{b^2 - 4ac}}} + 4Bcx + 2cCx^2$$

Antiderivative was successfully verified.

[In] Integrate[(x^3*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4), x]

[Out] (4*B*c*x + 2*c*C*x^2 - (2*Sqrt[2]*B*Sqrt[c]*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (2*Sqrt[2]*B*Sqrt[c]*(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + ((A*c*(-b + Sqrt[b^2 - 4*a*c]) + (b^2 - 2*a*c - b*Sqrt[b^2 - 4*a*c])*C)*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/Sqrt[b^2 - 4*a*c] - (((-A*c*(b + Sqrt[b^2 - 4*a*c])) + (b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*C)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c])/(4*c^2)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 (A + Bx + Cx^2)}{a + bx^2 + cx^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^3*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4), x]

[Out] IntegrateAlgebraic[(x^3*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(C*x^2+B*x+A)/(c*x^4+b*x^2+a), x, algorithm="fricas")

[Out] Timed out

giac [B] time = 5.03, size = 3519, normalized size = 12.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x³*(C*x²+B*x+A)/(c*x⁴+b*x²+a),x, algorithm="giac")

[Out]
$$-1/4*(C*b - A*c)*\log(\text{abs}(c*x^4 + b*x^2 + a))/c^2 + 1/2*(C*c*x^2 + 2*B*c*x)/c^2 - 1/8*((2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^5 + 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^3*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^4*c - 16*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b*c^2 - 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c^2 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^3*c^2 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*B*c^2 + 2*(\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^4*c^2 - 8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b^2*c^3 - 2*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^3*c^3 + 2*a*b^4*c^3 + 16*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*c^4 + 8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b*c^4 + \sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c^4 - 16*a^2*b^2*c^4 - 4*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*c^5 + 32*a^3*c^5 - 2*(b^2 - 4*a*c)*a*b^2*c^3 + 8*(b^2 - 4*a*c)*a^2*c^4)*B*\text{abs}(c) - (2*b^5*c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^5*c^2 + 6*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^3*c^3 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^4*c^3 - 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b*c^4 - 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c^4 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^3*c^4 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^5)*B)*\arctan(2*\sqrt{1/2}*x/\sqrt{(b*c^5 + \sqrt{b^2*c^{10} - 4*a*c^{11}})/c^6}))/((a*b^4*c^3 - 8*a^2*b^2*c^4 - 2*a*b^3*c^4 + 16*a^3*c^5 + 8*a^2*b*c^5 + a*b^2*c^5 - 4*a^2*c^6)*c^2) + 1/8*((2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^5 + 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^3*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^4*c - 16*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b*c^2 - 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c^2 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^3*c^2 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*B*c^2 - 2*(\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^4*c^2 - 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b^2*c^3 - 2*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^3*c^3 - 2*a*b^4*c^3 + 16*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^3*c^4 + 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b*c^4 + \sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c^4 + 16*a^2*b^2*c^4 - 4*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*c^5 - 32*a^3*c^5 + 2*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b^2 - 4*a*c)*a^2*c^4)*B*\text{abs}(c) - (2*b^5*c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^5*c^2 + 6*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^3*c^3 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^4*c^3 - 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b*c^4 - 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c^4 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^3*c^4 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^5)*B)*\arctan(2*\sqrt{1/2}*x/\sqrt{(b*c^5 - \sqrt{b^2*c^{10} - 4*a*c^{11}})/c^6}))/((a*b^4*c^3 - 8*a^2*b^2*c^4 - 2*a*b^3*c^4 + 16*a^3*c^5 + 8*a^2*b*c^5 + a*b^2*c^5 - 4*a^2*c^6)*c^2) - 1/16*((b^6*c - 8*a*b^4*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a*b^3*c^3 + b^4*c$$

$$\begin{aligned}
& c^3 - 4ab^2c^4 - (b^5c - 8a^2b^3c^2 - 2b^4c^2 + 16a^2b^2c^3 + 8a^2b^2c^3 + b^3c^3 - 4ab^2c^4) \sqrt{b^2 - 4ac} * A * \text{abs}(c) - (b^7 - 10a^2b^5c - 2b^6c + 32a^2b^3c^2 + 12a^2b^4c^2 + b^5c^2 - 32a^3b^2c^3 - 16a^2b^2c^3 - 6a^2b^3c^3 + 8a^2b^2c^4 - (b^6 - 10a^2b^4c - 2b^5c + 32a^2b^2c^2 + 12a^2b^3c^2 + b^4c^2 - 32a^3c^3 - 16a^2b^2c^3 - 6a^2b^2c^3 + 8a^2c^4) \sqrt{b^2 - 4ac}) * C * \text{abs}(c) + (b^6c^2 - 8a^2b^4c^3 - 2b^5c^3 + 16a^2b^2c^4 + 8a^2b^3c^4 + b^4c^4 - 4a^2b^2c^5 + (b^5c^2 - 4a^2b^3c^3 - 2b^4c^3 + b^3c^4) \sqrt{b^2 - 4ac}) * A - (b^7c - 10a^2b^5c^2 - 2b^6c^2 + 32a^2b^3c^3 + 12a^2b^4c^3 + b^5c^3 - 32a^3b^2c^4 - 16a^2b^2c^4 - 6a^2b^3c^4 + 8a^2b^2c^5 - (b^6c - 6a^2b^4c^2 - 2b^5c^2 + 8a^2b^2c^3 + 4a^2b^3c^3 + b^4c^3 - 2a^2b^2c^4) \sqrt{b^2 - 4ac}) * C * \log(x^2 + 1/2(b^2c^5 + \sqrt{b^2c^{10} - 4a^2c^{11}})/c^6) / ((a^2b^4c - 8a^2b^2c^2 - 2a^2b^3c^2 + 16a^3c^3 + 8a^2b^2c^3 + a^2b^2c^3 - 4a^2c^4) * c^2 * \text{abs}(c)) - 1/16 * ((b^6c - 8a^2b^4c^2 - 2b^5c^2 + 16a^2b^2c^3 + 8a^2b^3c^3 + b^4c^3 - 4a^2b^2c^4 + (b^5c - 8a^2b^3c^2 - 2b^4c^2 + 16a^2b^2c^3 + 8a^2b^2c^3 + b^3c^3 - 4a^2b^2c^4) \sqrt{b^2 - 4ac}) * A * \text{abs}(c) - (b^7 - 10a^2b^5c - 2b^6c + 32a^2b^3c^2 + 12a^2b^4c^2 + b^5c^2 - 32a^3b^2c^3 - 16a^2b^2c^3 - 6a^2b^3c^3 + 8a^2b^2c^4 + (b^6 - 10a^2b^4c - 2b^5c + 32a^2b^2c^2 + 12a^2b^3c^2 + b^4c^2 - 32a^3c^3 - 16a^2b^2c^3 - 6a^2b^2c^3 + 8a^2c^4) \sqrt{b^2 - 4ac}) * C * \text{abs}(c) + (b^6c^2 - 8a^2b^4c^3 - 2b^5c^3 + 16a^2b^2c^4 + 8a^2b^3c^4 + b^4c^4 - 4a^2b^2c^5 + (b^5c^2 - 4a^2b^3c^3 - 2b^4c^3 + b^3c^4) \sqrt{b^2 - 4ac}) * A - (b^7c - 10a^2b^5c^2 - 2b^6c^2 + 32a^2b^3c^3 + 12a^2b^4c^3 + b^5c^3 - 32a^3b^2c^4 - 16a^2b^2c^4 - 6a^2b^3c^4 + 8a^2b^2c^5 + (b^6c - 6a^2b^4c^2 - 2b^5c^2 + 8a^2b^2c^3 + 4a^2b^3c^3 + b^4c^3 - 2a^2b^2c^4) \sqrt{b^2 - 4ac}) * C * \log(x^2 + 1/2(b^2c^5 - \sqrt{b^2c^{10} - 4a^2c^{11}})/c^6) / ((a^2b^4c - 8a^2b^2c^2 - 2a^2b^3c^2 + 16a^3c^3 + 8a^2b^2c^3 + a^2b^2c^3 - 4a^2c^4) * c^2 * \text{abs}(c))
\end{aligned}$$

maple [B] time = 0.05, size = 1171, normalized size = 4.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^3 * (C * x^2 + B * x + A) / (c * x^4 + b * x^2 + a), x)$

[Out] $\begin{aligned}
& 1/2 * C * x^2 / c + B / c * x + 1/4 / c / (4 * a * c - b^2) * \ln(-2 * c * x^2 - b + (-4 * a * c + b^2)^{(1/2)}) * A * (-4 * a * c + b^2)^{(1/2)} * b + 1 / (4 * a * c - b^2) * \ln(-2 * c * x^2 - b + (-4 * a * c + b^2)^{(1/2)}) * a * A - 1/4 / c / (4 * a * c - b^2) * \ln(-2 * c * x^2 - b + (-4 * a * c + b^2)^{(1/2)}) * A * b^2 + 1/2 / c / (4 * a * c - b^2) * \ln(-2 * c * x^2 - b + (-4 * a * c + b^2)^{(1/2)}) * C * (-4 * a * c + b^2)^{(1/2)} * a - 1/4 / c^2 / (4 * a * c - b^2) * \ln(-2 * c * x^2 - b + (-4 * a * c + b^2)^{(1/2)}) * C * (-4 * a * c + b^2)^{(1/2)} * b^2 - 1 / c / (4 * a * c - b^2) * \ln(-2 * c * x^2 - b + (-4 * a * c + b^2)^{(1/2)}) * C * a * b + 1/4 / c^2 / (4 * a * c - b^2) * \ln(-2 * c * x^2 - b + (-4 * a * c + b^2)^{(1/2)}) * b^3 * C - 1 / (4 * a * c - b^2) * 2^{(1/2)} / ((-b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * \arctanh(2^{(1/2)} / ((-b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * B * (-4 * a * c + b^2)^{(1/2)} * a + 1/2 / c / (4 * a * c - b^2) * 2^{(1/2)} / ((-b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * \arctanh(2^{(1/2)} / ((-b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * B * (-4 * a * c + b^2)^{(1/2)} * b^2 + 2 / (4 * a * c - b^2) * 2^{(1/2)} / ((-b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * \arctanh(2^{(1/2)} / ((-b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * a * b * B - 1/2 / c / (4 * a * c - b^2) * 2^{(1/2)} / ((-b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * \arctanh(2^{(1/2)} / ((-b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * b^3 * B - 1/4 / c / (4 * a * c - b^2) * \ln(2 * c * x^2 + b + (-4 * a * c + b^2)^{(1/2)}) * A * (-4 * a * c + b^2)^{(1/2)} * b + 1 / (4 * a * c - b^2) * \ln(2 * c * x^2 + b + (-4 * a * c + b^2)^{(1/2)}) * a * A - 1/4 / c / (4 * a * c - b^2) * \ln(2 * c * x^2 + b + (-4 * a * c + b^2)^{(1/2)}) * A * b^2 - 1/2 / c / (4 * a * c - b^2) * \ln(2 * c * x^2 + b + (-4 * a * c + b^2)^{(1/2)}) * C * (-4 * a * c + b^2)^{(1/2)} * a + 1/4 / c^2 / (4 * a * c - b^2) * \ln(2 * c * x^2 + b + (-4 * a * c + b^2)^{(1/2)}) * C * (-4 * a * c + b^2)^{(1/2)} * b^2 - 1 / c / (4 * a * c - b^2) * \ln(2 * c * x^2 + b + (-4 * a * c + b^2)^{(1/2)}) * C * a * b + 1/4 / c^2 / (4 * a * c - b^2) * \ln(2 * c * x^2 + b + (-4 * a * c + b^2)^{(1/2)}) * b^3 * C - 1 / (4 * a * c - b^2) * 2^{(1/2)} / ((b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * \arctan(2^{(1/2)} / ((b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * B * (-4 * a * c + b^2)^{(1/2)} * a + 1/2 / c / (4 * a * c - b^2) * 2^{(1/2)} / ((b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * \arctan(2^{(1/2)} / ((b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * B * (-4 * a * c + b^2)^{(1/2)} * b^2 - 2 / (4 * a * c - b^2) * 2^{(1/2)} / ((b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * \arctan(2^{(1/2)} / ((b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * c)
\end{aligned}$

$\frac{1}{2} * c * x) * a * b * B + \frac{1}{2} / c / (4 * a * c - b^2) * 2^{(1/2)} / ((b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * a$
 $\text{rctan}(2^{(1/2)} / ((b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * b^3 * B$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{Cx^2 + 2Bx}{2c} + \frac{-\int \frac{Bbx^2 + (Cb - Ac)x^3 + Cax + Ba}{cx^4 + bx^2 + a} dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] $\frac{1}{2} * (C * x^2 + 2 * B * x) / c + \text{integrate}(- (B * b * x^2 + (C * b - A * c) * x^3 + C * a * x + B * a) / (c * x^4 + b * x^2 + a), x) / c$

mupad [B] time = 1.53, size = 2696, normalized size = 9.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x)

[Out] $\text{symsum}(\log((B^3 * a^2 * b * c - B * C^2 * a^3 * c + A^2 * B * a^2 * c^2 + B * C^2 * a^2 * b^2 - 2 * A * B * C * a^2 * b * c) / c^2 - \text{root}(128 * a * b^2 * c^5 * z^4 - 16 * b^4 * c^4 * z^4 - 256 * a^2 * c^6 * z^4 - 256 * C * a^2 * b * c^4 * z^3 + 128 * C * a * b^3 * c^3 * z^3 - 128 * A * a * b^2 * c^4 * z^3 - 16 * C * b^5 * c^2 * z^3 + 16 * A * b^4 * c^3 * z^3 + 256 * A * a^2 * c^5 * z^3 + 160 * A * C * a^2 * b * c^3 * z^2 - 72 * A * C * a * b^3 * c^2 * z^2 + 8 * A * C * b^5 * c * z^2 - 48 * B^2 * a^2 * b * c^3 * z^2 + 28 * B^2 * a * b^3 * c^2 * z^2 + 40 * A^2 * a * b^2 * c^3 * z^2 + 32 * C^2 * a * b^4 * c * z^2 - 56 * C^2 * a^2 * b^2 * c^2 * z^2 - 4 * B^2 * b^5 * c * z^2 - 32 * C^2 * a^3 * c^3 * z^2 - 4 * A^2 * b^4 * c^2 * z^2 - 96 * A^2 * a^2 * c^4 * z^2 - 4 * C^2 * b^6 * z^2 + 4 * B^2 * C * a^2 * b^2 * c * z - 32 * A^2 * C * a^2 * b * c^2 * z + 12 * A * C^2 * a^2 * b^2 * c * z + 16 * A * B^2 * a^2 * b * c^2 * z + 8 * A^2 * C * a * b^3 * c * z - 4 * A * B^2 * a * b^3 * c * z - 4 * A * C^2 * a * b^4 * z - 4 * A^3 * a * b^2 * c^2 * z - 16 * B^2 * C * a^3 * c^2 * z + 16 * A * C^2 * a^3 * c^2 * z - 16 * C^3 * a^3 * b * c * z + 4 * C^3 * a^2 * b^3 * z + 16 * A^3 * a^2 * c^3 * z + 2 * A^3 * C * a^2 * b * c + 4 * A * B^2 * C * a^3 * c - 2 * A^2 * C^2 * a^3 * c + 2 * A * C^3 * a^3 * b - A^2 * B^2 * a^2 * b * c - B^2 * C^2 * a^3 * b - A^2 * C^2 * a^2 * b^2 - A^4 * a^2 * c^2 - B^4 * a^3 * c - C^4 * a^4, z, k) * (\text{root}(128 * a * b^2 * c^5 * z^4 - 16 * b^4 * c^4 * z^4 - 256 * a^2 * c^6 * z^4 - 256 * C * a^2 * b * c^4 * z^3 + 128 * C * a * b^3 * c^3 * z^3 - 128 * A * a * b^2 * c^4 * z^3 - 16 * C * b^5 * c^2 * z^3 + 16 * A * b^4 * c^3 * z^3 + 256 * A * a^2 * c^5 * z^3 + 160 * A * C * a^2 * b * c^3 * z^2 - 72 * A * C * a * b^3 * c^2 * z^2 + 8 * A * C * b^5 * c * z^2 - 48 * B^2 * a^2 * b * c^3 * z^2 + 28 * B^2 * a * b^3 * c^2 * z^2 + 40 * A^2 * a * b^2 * c^3 * z^2 + 32 * C^2 * a * b^4 * c * z^2 - 56 * C^2 * a^2 * b^2 * c^2 * z^2 - 4 * B^2 * b^5 * c * z^2 - 32 * C^2 * a^3 * c^3 * z^2 - 4 * A^2 * b^4 * c^2 * z^2 - 96 * A^2 * a^2 * c^4 * z^2 - 4 * C^2 * b^6 * z^2 + 4 * B^2 * C * a^2 * b^2 * c * z - 32 * A^2 * C * a^2 * b * c^2 * z + 12 * A * C^2 * a^2 * b^2 * c * z + 16 * A * B^2 * a^2 * b * c^2 * z + 8 * A^2 * C * a * b^3 * c * z - 4 * A * B^2 * a * b^3 * c * z - 4 * A * C^2 * a * b^4 * z - 4 * A^3 * a * b^2 * c^2 * z - 16 * B^2 * C * a^3 * c^2 * z + 16 * A * C^2 * a^3 * c^2 * z - 16 * C^3 * a^3 * b * c * z + 4 * C^3 * a^2 * b^3 * z + 16 * A^3 * a^2 * c^3 * z + 2 * A^3 * C * a^2 * b * c + 4 * A * B^2 * C * a^3 * c - 2 * A^2 * C^2 * a^3 * c + 2 * A * C^3 * a^3 * b - A^2 * B^2 * a^2 * b * c - B^2 * C^2 * a^3 * b - A^2 * C^2 * a^2 * b^2 - A^4 * a^2 * c^2 - B^4 * a^3 * c - C^4 * a^4, z, k) * ((x * (16 * C * a^2 * c^4 - 8 * A * b^3 * c^3 + 8 * C * b^4 * c^2 + 32 * A * a * b * c^4 - 36 * C * a * b^2 * c^3)) / c^2 - (16 * B * a^2 * c^4 - 4 * B * a * b^2 * c^3) / c^2 + (\text{root}(128 * a * b^2 * c^5 * z^4 - 16 * b^4 * c^4 * z^4 - 256 * a^2 * c^6 * z^4 - 256 * C * a^2 * b * c^4 * z^3 + 128 * C * a * b^3 * c^3 * z^3 - 128 * A * a * b^2 * c^4 * z^3 - 16 * C * b^5 * c^2 * z^3 + 16 * A * b^4 * c^3 * z^3 + 256 * A * a^2 * c^5 * z^3 + 160 * A * C * a^2 * b * c^3 * z^2 - 72 * A * C * a * b^3 * c^2 * z^2 + 8 * A * C * b^5 * c * z^2 - 48 * B^2 * a^2 * b * c^3 * z^2 + 28 * B^2 * a * b^3 * c^2 * z^2 + 40 * A^2 * a * b^2 * c^3 * z^2 + 32 * C^2 * a * b^4 * c * z^2 - 56 * C^2 * a^2 * b^2 * c^2 * z^2 - 4 * B^2 * b^5 * c * z^2 - 32 * C^2 * a^3 * c^3 * z^2 - 4 * A^2 * b^4 * c^2 * z^2 - 96 * A^2 * a^2 * c^4 * z^2 - 4 * C^2 * b^6 * z^2 + 4 * B^2 * C * a^2 * b^2 * c * z - 32 * A^2 * C * a^2 * b * c^2 * z + 12 * A * C^2 * a^2 * b^2 * c * z + 16 * A * B^2 * a^2 * b * c^2 * z + 8 * A^2 * C * a * b^3 * c * z - 4 * A * B^2 * a * b^3 * c * z - 4 * A * C^2 * a * b^4 * z - 4 * A^3 * a * b^2 * c^2 * z - 16 * B^2 * C * a^3 * c^2 * z + 16 * A * C^2 * a^3 * c^2 * z - 16 * C^3 * a^3 * b * c * z + 4 * C^3 * a^2 * b^3 * z + 16 * A^3 * a^2 * c^3 * z + 2 * A^3 * C * a^2 * b * c + 4 * A * B^2 * C * a^3 * c - 2 * A^2 * C^2 * a^3 * c + 2 * A * C^3 * a^3 * b - A^2 * B^2 * a^2 * b * c - B^2 * C^2 * a^3 * b - A^2 * C^2 * a^2 * b^2 - A^4 * a^2 * c^2 - B^4 * a^3 * c - C^4 * a^4, z, k) * x * (8 * b^3 * c^4 - 32 * a * b * c^5)) / c^2) +$

$$\begin{aligned} & (8*A*B*a^2*c^3 - 4*B*C*a^2*b*c^2)/c^2 + (x*(2*C^2*b^5 + 2*B^2*b^4*c + 2*A^2 \\ & *b^3*c^2 + 4*B^2*a^2*c^3 - 4*A*C*b^4*c - 8*A*C*a^2*c^3 - 10*A^2*a*b*c^3 - 1 \\ & 0*C^2*a*b^3*c - 8*B^2*a*b^2*c^2 + 6*C^2*a^2*b*c^2 + 20*A*C*a*b^2*c^2))/c^2) \\ & - (x*(C^3*a^3*c - C^3*a^2*b^2 + A*C^2*a*b^3 + A^3*a*b*c^2 - A*B^2*a^2*c^2 \\ & + A^2*C*a^2*c^2 + A*B^2*a*b^2*c - 2*A^2*C*a*b^2*c - B^2*C*a^2*b*c))/c^2)*ro \\ & ot(128*a*b^2*c^5*z^4 - 16*b^4*c^4*z^4 - 256*a^2*c^6*z^4 - 256*C*a^2*b*c^4*z \\ & ^3 + 128*C*a*b^3*c^3*z^3 - 128*A*a*b^2*c^4*z^3 - 16*C*b^5*c^2*z^3 + 16*A*b^ \\ & 4*c^3*z^3 + 256*A*a^2*c^5*z^3 + 160*A*C*a^2*b*c^3*z^2 - 72*A*C*a*b^3*c^2*z^ \\ & 2 + 8*A*C*b^5*c*z^2 - 48*B^2*a^2*b*c^3*z^2 + 28*B^2*a*b^3*c^2*z^2 + 40*A^2* \\ & a*b^2*c^3*z^2 + 32*C^2*a*b^4*c*z^2 - 56*C^2*a^2*b^2*c^2*z^2 - 4*B^2*b^5*c*z \\ & ^2 - 32*C^2*a^3*c^3*z^2 - 4*A^2*b^4*c^2*z^2 - 96*A^2*a^2*c^4*z^2 - 4*C^2*b^ \\ & 6*z^2 + 4*B^2*C*a^2*b^2*c*z - 32*A^2*C*a^2*b*c^2*z + 12*A*C^2*a^2*b^2*c*z + \\ & 16*A*B^2*a^2*b*c^2*z + 8*A^2*C*a*b^3*c*z - 4*A*B^2*a*b^3*c*z - 4*A*C^2*a*b \\ & ^4*z - 4*A^3*a*b^2*c^2*z - 16*B^2*C*a^3*c^2*z + 16*A*C^2*a^3*c^2*z - 16*C^3 \\ & *a^3*b*c*z + 4*C^3*a^2*b^3*z + 16*A^3*a^2*c^3*z + 2*A^3*C*a^2*b*c + 4*A*B^2 \\ & *C*a^3*c - 2*A^2*C^2*a^3*c + 2*A*C^3*a^3*b - A^2*B^2*a^2*b*c - B^2*C^2*a^3* \\ & b - A^2*C^2*a^2*b^2 - A^4*a^2*c^2 - B^4*a^3*c - C^4*a^4, z, k), k, 1, 4) + \\ & (C*x^2)/(2*c) + (B*x)/c \end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(C*x**2+B*x+A)/(c*x**4+b*x**2+a), x)

[Out] Timed out

$$3.23 \quad \int \frac{x^2(A+Bx+Cx^2)}{a+bx^2+cx^4} dx$$

Optimal. Leaf size=270

$$\frac{\left(-\frac{Abc-C(b^2-2ac)}{\sqrt{b^2-4ac}} + Ac - bC\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right) + \left(\frac{2acC+Abc+b^2(-C)}{\sqrt{b^2-4ac}} + Ac - bC\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2-4ac}+b}}\right) + \frac{bB \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}} + \sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Rubi [A] time = 0.83, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {1662, 1279, 1166, 205, 12, 1114, 634, 618, 206, 628}

$$\frac{\left(-\frac{Abc-C(b^2-2ac)}{\sqrt{b^2-4ac}} + Ac - bC\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right) + \left(\frac{2acC+Abc+b^2(-C)}{\sqrt{b^2-4ac}} + Ac - bC\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2-4ac}+b}}\right) + \frac{bB \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}} + \frac{B \log(a+bx^2+cx^4)}{4c} + \frac{Cx}{c}}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}} + \sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

[In] Int[(x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4), x]

[Out] (C*x)/c + ((A*c - b*C - (A*b*c - (b^2 - 2*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((A*c - b*C + (A*b*c - b^2*C + 2*a*c*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (b*B*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c*Sqrt[b^2 - 4*a*c]) + (B*Log[a + b*x^2 + c*x^4])/(4*c)

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1114

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1166

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1279

```
Int[((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)*((a_) + (b_)*(x_)^2 + (c_)*(
x_)^4)^(p_), x_Symbol] := Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
IntegerQ[m])
```

Rule 1662

```
Int[(Pq)*((d_)*(x_))^(m_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_S
ymbol] := Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1})*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
+ 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1})*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] && !Po
lyQ[Pq, x^2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2(A+Bx+Cx^2)}{a+bx^2+cx^4} dx &= \int \frac{Bx^3}{a+bx^2+cx^4} dx + \int \frac{x^2(A+Cx^2)}{a+bx^2+cx^4} dx \\
&= \frac{Cx}{c} + B \int \frac{x^3}{a+bx^2+cx^4} dx - \frac{\int \frac{aC+(-Ac+bC)x^2}{a+bx^2+cx^4} dx}{c} \\
&= \frac{Cx}{c} + \frac{1}{2}B \operatorname{Subst} \left(\int \frac{x}{a+bx+cx^2} dx, x, x^2 \right) - \frac{\left(-Ac+bC + \frac{Abc-b^2C+2acC}{\sqrt{b^2-4ac}}\right) \int \frac{\frac{b}{2}-\frac{1}{2}\sqrt{b}}{\frac{b}{2}-\frac{1}{2}\sqrt{b}}}{2c} \\
&= \frac{Cx}{c} + \frac{\left(Ac-bC - \frac{Abc-b^2C+2acC}{\sqrt{b^2-4ac}}\right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}} \right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(Ac-bC + \frac{Abc-(b^2-2ac)C}{\sqrt{b^2-4ac}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}} \\
&= \frac{Cx}{c} + \frac{\left(Ac-bC - \frac{Abc-b^2C+2acC}{\sqrt{b^2-4ac}}\right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}} \right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(Ac-bC + \frac{Abc-(b^2-2ac)C}{\sqrt{b^2-4ac}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}} \\
&= \frac{Cx}{c} + \frac{\left(Ac-bC - \frac{Abc-b^2C+2acC}{\sqrt{b^2-4ac}}\right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}} \right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(Ac-bC + \frac{Abc-(b^2-2ac)C}{\sqrt{b^2-4ac}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}
\end{aligned}$$

Mathematica [A] time = 0.37, size = 360, normalized size = 1.33

$$\frac{2\sqrt{2}\left(Ac(b-\sqrt{b^2-4ac})+C(b\sqrt{b^2-4ac}+2ac-b^2)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{2\sqrt{2}\left(C(b\sqrt{b^2-4ac}-2ac+b^2)-Ac(\sqrt{b^2-4ac}+b)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b^2-4ac}+b}\right)}{\sqrt{b^2-4ac}\sqrt{b^2-4ac}+b} + \frac{B\sqrt{c}\left(\sqrt{b^2-4ac}-b\right)\log\left(\sqrt{b^2-4ac}-b-2cx^2\right)}{\sqrt{b^2-4ac}} + \frac{B\sqrt{c}\left(\sqrt{b^2-4ac}+b\right)\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)}{\sqrt{b^2-4ac}} + 4\sqrt{c}Cx$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4), x]

[Out] (4*sqrt[c]*C*x - (2*sqrt[2]*(A*c*(b - sqrt[b^2 - 4*a*c]) + (-b^2 + 2*a*c + b*sqrt[b^2 - 4*a*c])*C)*ArcTan[(sqrt[2]*sqrt[c]*x)/sqrt[b - sqrt[b^2 - 4*a*c]]])/(sqrt[b^2 - 4*a*c]*sqrt[b - sqrt[b^2 - 4*a*c]]) - (2*sqrt[2]*(-A*c*(b + sqrt[b^2 - 4*a*c])) + (b^2 - 2*a*c + b*sqrt[b^2 - 4*a*c])*C)*ArcTan[(sqrt[2]*sqrt[c]*x)/sqrt[b + sqrt[b^2 - 4*a*c]]])/(sqrt[b^2 - 4*a*c]*sqrt[b + sqrt[b^2 - 4*a*c]]) + (B*sqrt[c]*(-b + sqrt[b^2 - 4*a*c])*Log[-b + sqrt[b^2 - 4*a*c]] - 2*c*x^2)/sqrt[b^2 - 4*a*c] + (B*sqrt[c]*(b + sqrt[b^2 - 4*a*c])*Log[b + sqrt[b^2 - 4*a*c]] + 2*c*x^2)/sqrt[b^2 - 4*a*c]/(4*c^(3/2))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(A+Bx+Cx^2)}{a+bx^2+cx^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4), x]

[Out] IntegrateAlgebraic[(x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] Timed out

giac [B] time = 5.57, size = 3843, normalized size = 14.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] $C*x/c + 1/4*B*\log(\text{abs}(c*x^4 + b*x^2 + a))/c + 1/8*((2*b^4*c^3 - 16*a*b^2*c^4 + 32*a^2*c^5 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c})*$
 $b^4*c + 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c}*a*b^2*c^2 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c}*b^3*c^2 -$
 $16*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c}*a^2*c^3 - 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c}*a*b*c^3 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c}*b^2*c^3 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c}*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3$
 $+ 8*(b^2 - 4*a*c)*a*c^4)*A*c^2 - (2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c}*b^5 + 8*\sqrt{2}*$
 $*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c}*a*b^3*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c}*b^4*c - 16*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c}*a^2*b*c^2 - 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c}*a*b^2*c^2 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c}*b^3*c^2 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c}*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*C*c^2 - 2*(\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c}*a*b^4*c^2 - 8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c}*a^2*b^2*c^3 - 2*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c}*a*b^3*c^3 + 2*a*b^4*c^3 + 16*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c}*a^3*c^4 + 8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c}*a^2*b*c^4 + \sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c}*a*b^2*c^4 - 16*a^2*b^2*c^4 - 4*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c}*a^2*c^5 + 32*a^3*c^5 - 2*(b^2 - 4*a*c)*a*b^2*c^3 + 8*(b^2 - 4*a*c)*a^2*c^4)*C*abs(c) - (2*b^4*c^5 - 8*a*b^2*c^6 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c}*b^4*c^3 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c}*a*b^2*c^4 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c}*b^3*c^4 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c}*b^2*c^5 - 2*(b^2 - 4*a*c)*b^2*c^5)*A + (2*b^5*c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c}*b^5*c^2 + 6*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c}*a*b^3*c^3 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c}*b^4*c^3 - 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c}*a^2*b*c^4 - 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c}*a*b^2*c^4 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c}*b^3*c^4 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c}*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^5)*C*\arctan(2*\sqrt{1/2}*x/\sqrt{(b*c^3 + \sqrt{b^2*c^6 - 4*a*c^7}))/c^4))/(a*b^4*c^3 - 8*a^2*b^2*c^4 - 2*a*b^3*c^4 + 16*a^3*c^5 + 8*a^2*b*c^5 + a*b^2*c^5 - 4*a^2*c^6)*c^2) - 1/8*((2*b^4*c^3 - 16*a*b^2*c^4 + 32*a^2*c^5 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c}*b^4*c + 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c}*a*b^2*c^2 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c}*b^3*c^2 - 16*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c}*a^2*c^3 - 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c}*a*b*c^3 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c}*b^2*c^3 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c}*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 8*(b^2 - 4*a*c)*a*c^4)*A*c^2 - (2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c}*b^5 + 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c}*a*b^3*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c}*b^4*c - 16*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c}*a^2*b*c^2 - 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c}$

$$\begin{aligned}
& 4*a*c)*c)*a*b^2*c^2 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c) \\
& c)*c)*b^3*c^2 + 4*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c)*c) \\
& *a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*C*c^2 + 2*(\text{sqrt} \\
& \text{rt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c)*c)*a*b^4*c^2 - 8*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt} \\
& (b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c)*c)*a \\
& b^3*c^3 - 2*a*b^4*c^3 + 16*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c)*c)*a^3*c^4 \\
& + 8*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c)*c)*a^2*b*c^4 + \text{sqrt}(2)*\text{sqrt}(b*c + \\
& \text{sqrt}(b^2 - 4*a*c)*c)*a*b^2*c^4 + 16*a^2*b^2*c^4 - 4*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt} \\
& (b^2 - 4*a*c)*c)*a^2*c^5 - 32*a^3*c^5 + 2*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b^2 \\
& - 4*a*c)*a^2*c^4)*C*abs(c) - (2*b^4*c^5 - 8*a*b^2*c^6 - \text{sqrt}(2)*\text{sqrt}(b^2 - \\
& 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c)*c)*b^4*c^3 + 4*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c) \\
&)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c)*c)*a*b^2*c^4 + 2*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{s} \\
& \text{qrt}(b*c + \text{sqrt}(b^2 - 4*a*c)*c)*b^3*c^4 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c \\
& + \text{sqrt}(b^2 - 4*a*c)*c)*b^2*c^5 - 2*(b^2 - 4*a*c)*b^2*c^5)*A + (2*b^5*c^4 - \\
& 12*a*b^3*c^5 + 16*a^2*b*c^6 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^ \\
& 2 - 4*a*c)*c)*b^5*c^2 + 6*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4 \\
& *a*c)*c)*a*b^3*c^3 + 2*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a* \\
& c)*c)*b^4*c^3 - 8*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c)*c) \\
& *a^2*b*c^4 - 4*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c)*c)*a* \\
& b^2*c^4 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c)*c)*b^3*c^4 \\
& + 2*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c)*c)*a*b*c^5 - 2* \\
& (b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^5)*C)*\text{arctan}(2*\text{sqrt}(1/2)*x/\text{sq} \\
& \text{rt}((b*c^3 - \text{sqrt}(b^2*c^6 - 4*a*c^7))/c^4))/((a*b^4*c^3 - 8*a^2*b^2*c^4 - 2* \\
& a*b^3*c^4 + 16*a^3*c^5 + 8*a^2*b*c^5 + a*b^2*c^5 - 4*a^2*c^6)*c^2) - 1/16*(\\
& (b^6 - 8*a*b^4*c - 2*b^5*c + 16*a^2*b^2*c^2 + 8*a*b^3*c^2 + b^4*c^2 - 4*a*b \\
& ^2*c^3 - (b^5 - 8*a*b^3*c - 2*b^4*c + 16*a^2*b*c^2 + 8*a*b^2*c^2 + b^3*c^2 \\
& - 4*a*b*c^3)*\text{sqrt}(b^2 - 4*a*c))*B*abs(c) + (b^6*c - 8*a*b^4*c^2 - 2*b^5*c^2 \\
& + 16*a^2*b^2*c^3 + 8*a*b^3*c^3 + b^4*c^3 - 4*a*b^2*c^4 - (b^5*c - 4*a*b^3* \\
& c^2 - 2*b^4*c^2 + b^3*c^3)*\text{sqrt}(b^2 - 4*a*c))*B)*\log(x^2 + 1/2*(b*c^3 + \text{sq} \\
& \text{rt}(b^2*c^6 - 4*a*c^7))/c^4)/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + \\
& 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2*abs(c)) - 1/16*((b^6 - 8*a*b^4*c \\
& - 2*b^5*c + 16*a^2*b^2*c^2 + 8*a*b^3*c^2 + b^4*c^2 - 4*a*b^2*c^3 + (b^5 - 8 \\
& *a*b^3*c - 2*b^4*c + 16*a^2*b*c^2 + 8*a*b^2*c^2 + b^3*c^2 - 4*a*b*c^3)*\text{sqrt} \\
& (b^2 - 4*a*c))*B*abs(c) + (b^6*c - 8*a*b^4*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3 \\
& + 8*a*b^3*c^3 + b^4*c^3 - 4*a*b^2*c^4 + (b^5*c - 4*a*b^3*c^2 - 2*b^4*c^2 + \\
& b^3*c^3)*\text{sqrt}(b^2 - 4*a*c))*B)*\log(x^2 + 1/2*(b*c^3 - \text{sqrt}(b^2*c^6 - 4*a*c \\
& ^7))/c^4)/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a \\
& b^2*c^2 - 4*a^2*c^3)*c^2*abs(c))
\end{aligned}$$

maple [B] time = 0.05, size = 1327, normalized size = 4.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a), x)$

[Out] $C*x/c+1/4/c/(4*a*c-b^2)*B*\ln(-2*c*x^2-b+(-4*a*c+b^2)^{(1/2)})*(-4*a*c+b^2)^{(1/2)}$
 $*b+1/(4*a*c-b^2)*B*\ln(-2*c*x^2-b+(-4*a*c+b^2)^{(1/2)})*a-1/4/c/(4*a*c-b^2)$
 $*B*\ln(-2*c*x^2-b+(-4*a*c+b^2)^{(1/2)})*b^2-1/2/(4*a*c-b^2)*2^{(1/2)}/((-b+(-4*a$
 $*c+b^2)^{(1/2))*c)^{(1/2)}*\text{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2))*c)^{(1/2)}*c$
 $*x)*A*(-4*a*c+b^2)^{(1/2)}*b-2*c/(4*a*c-b^2)*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2))$
 $*c)^{(1/2)}*\text{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2))*c)^{(1/2)}*c*x)*A*a+1/2/(4$
 $*a*c-b^2)*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2))*c)^{(1/2)}*\text{arctanh}(2^{(1/2)}/((-b+(-$
 $4*a*c+b^2)^{(1/2))*c)^{(1/2)}*c*x)*A*b^2-1/4/c/(4*a*c-b^2)*2^{(1/2)}/((-b+(-4*a$
 $*c+b^2)^{(1/2))*c)^{(1/2)}*\text{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2))*c)^{(1/2)}*c$
 $*x)*C*(-4*a*c+b^2)*b-1/(4*a*c-b^2)*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2))*c)^{(1/2)}$
 $*\text{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2))*c)^{(1/2)}*c*x)*C*(-4*a*c+b^2)^{(1/2)}$
 $*a+1/2/c/(4*a*c-b^2)*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2))*c)^{(1/2)}*\text{arctanh}(2^{(1/2)}/$
 $(-b+(-4*a*c+b^2)^{(1/2))*c)^{(1/2)}*c*x)*C*(-4*a*c+b^2)^{(1/2)}*b^2+1/(4*a$
 $*c-b^2)*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2))*c)^{(1/2)}*\text{arctanh}(2^{(1/2)}/((-b+(-4*$

$$\begin{aligned}
& a*c+b^2)^{(1/2))*c)^{(1/2)*c*x)*b*C*a-1/4/c/(4*a*c-b^2)*2^{(1/2)/((-b+(-4*a*c+ \\
& b^2)^{(1/2))*c)^{(1/2)*\operatorname{arctanh}(2^{(1/2)/((-b+(-4*a*c+b^2)^{(1/2))*c)^{(1/2)*c*x} \\
& *b^3*C-1/4/c/(4*a*c-b^2)*B*\ln(2*c*x^2+b+(-4*a*c+b^2)^{(1/2))*(-4*a*c+b^2)^{(1/2) \\
& /2)*b+1/(4*a*c-b^2)*B*\ln(2*c*x^2+b+(-4*a*c+b^2)^{(1/2))*a-1/4/c/(4*a*c-b^2)* \\
& B*\ln(2*c*x^2+b+(-4*a*c+b^2)^{(1/2))*b^2-1/2/(4*a*c-b^2)*2^{(1/2)/((b+(-4*a*c+ \\
& b^2)^{(1/2))*c)^{(1/2)*\operatorname{arctan}(2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2))*c)^{(1/2)*c*x}*A \\
& *(-4*a*c+b^2)^{(1/2)*b+2*c/(4*a*c-b^2)*2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2))*c)^{(1/2) \\
& /2)*\operatorname{arctan}(2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2))*c)^{(1/2)*c*x}*A*a-1/2/(4*a*c-b^2) \\
&)*2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2))*c)^{(1/2)*\operatorname{arctan}(2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2) \\
& /2))*c)^{(1/2)*c*x}*A*b^2+1/4/c/(4*a*c-b^2)*2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2) \\
& /2))*c)^{(1/2)*\operatorname{arctan}(2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2))*c)^{(1/2)*c*x}*C*(-4*a*c+b \\
& ^2)*b-1/(4*a*c-b^2)*2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2))*c)^{(1/2)*\operatorname{arctan}(2^{(1/2) \\
& /2)/((b+(-4*a*c+b^2)^{(1/2))*c)^{(1/2)*c*x}*C*(-4*a*c+b^2)^{(1/2)*a+1/2/c/(4*a*c- \\
& b^2)*2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2))*c)^{(1/2)*\operatorname{arctan}(2^{(1/2)/((b+(-4*a*c+b^2) \\
& ^2)^{(1/2))*c)^{(1/2)*c*x}*C*(-4*a*c+b^2)^{(1/2)*b^2-1/(4*a*c-b^2)*2^{(1/2)/((b+ \\
& (-4*a*c+b^2)^{(1/2))*c)^{(1/2)*\operatorname{arctan}(2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2))*c)^{(1/2) \\
& /2))*c*x)*b*C*a+1/4/c/(4*a*c-b^2)*2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2))*c)^{(1/2)*\operatorname{arc} \\
& \tan(2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2))*c)^{(1/2)*c*x)*b^3*C}
\end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] C*x/c + integrate((B*c*x^3 - (C*b - A*c)*x^2 - C*a)/(c*x^4 + b*x^2 + a), x) /c

mupad [B] time = 2.00, size = 1890, normalized size = 7.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x)

[Out] symsum(log(- root(128*a*b^2*c^4*z^4 - 16*b^4*c^3*z^4 - 256*a^2*c^5*z^4 - 128*B*a*b^2*c^3*z^3 + 16*B*b^4*c^2*z^3 + 256*B*a^2*c^4*z^3 - 48*A*C*a*b^2*c^2*z^2 + 8*A*C*b^4*c*z^2 - 48*C^2*a^2*b*c^2*z^2 + 40*B^2*a*b^2*c^2*z^2 + 28*C^2*a*b^3*c*z^2 + 16*A^2*a*b*c^3*z^2 + 64*A*C*a^2*c^3*z^2 - 4*B^2*b^4*c*z^2 - 96*B^2*a^2*c^3*z^2 - 4*A^2*b^3*c^2*z^2 - 4*C^2*b^5*z^2 + 8*A*B*C*a*b^2*c*z + 16*B*C^2*a^2*b*c*z - 32*A*B*C*a^2*c^2*z - 4*B*C^2*a*b^3*z - 4*B^3*a*b^2*c*z + 16*B^3*a^2*c^2*z + 4*A*B^2*C*a^2*c + 2*A^3*C*a*b*c - A^2*B^2*a*b*c - 2*A^2*C^2*a^2*c + 2*A*C^3*a^2*b - B^2*C^2*a^2*b - A^2*C^2*a*b^2 - B^4*a^2*c - A^4*a*c^2 - C^4*a^3, z, k)*((8*B*C*a^2*c^2 - 4*A*B*a*b*c^2)/c - root(128*a*b^2*c^4*z^4 - 16*b^4*c^3*z^4 - 256*a^2*c^5*z^4 - 128*B*a*b^2*c^3*z^3 + 16*B*b^4*c^2*z^3 + 256*B*a^2*c^4*z^3 - 48*A*C*a*b^2*c^2*z^2 + 8*A*C*b^4*c*z^2 - 48*C^2*a^2*b*c^2*z^2 + 40*B^2*a*b^2*c^2*z^2 + 28*C^2*a*b^3*c*z^2 + 16*A^2*a*b*c^3*z^2 + 64*A*C*a^2*c^3*z^2 - 4*B^2*b^4*c*z^2 - 96*B^2*a^2*c^3*z^2 - 4*A^2*b^3*c^2*z^2 - 4*C^2*b^5*z^2 + 8*A*B*C*a*b^2*c*z + 16*B*C^2*a^2*b*c*z - 32*A*B*C*a^2*c^2*z - 4*B*C^2*a*b^3*z - 4*B^3*a*b^2*c*z + 16*B^3*a^2*c^2*z + 4*A*B^2*C*a^2*c + 2*A^3*C*a*b*c - A^2*B^2*a*b*c - 2*A^2*C^2*a^2*c + 2*A*C^3*a^2*b - B^2*C^2*a^2*b - A^2*C^2*a*b^2 - B^4*a^2*c - A^4*a*c^2 - C^4*a^3, z, k)*((16*C*a^2*c^3 - 4*C*a*b^2*c^2)/c + (x*(8*B*b^3*c^2 - 32*B*a*b*c^3))/c - (root(128*a*b^2*c^4*z^4 - 16*b^4*c^3*z^4 - 256*a^2*c^5*z^4 - 128*B*a*b^2*c^3*z^3 + 16*B*b^4*c^2*z^3 + 256*B*a^2*c^4*z^3 - 48*A*C*a*b^2*c^2*z^2 + 8*A*C*b^4*c*z^2 - 48*C^2*a^2*b*c^2*z^2 + 40*B^2*a*b^2*c^2*z^2 + 28*C^2*a*b^3*c*z^2 + 16*A^2*a*b*c^3*z^2 + 64*A*C*a^2*c^3*z^2 - 4*B^2*b^4*c*z^2 - 96*B^2*a^2*c^3*z^2 - 4*A^2*b^3*c^2*z^2 - 4*C^2*b^5*z^2 + 8*A*B*C*a*b^2*c*z + 16*B*C^2*a^2*b*c*z - 32*A*B*C*a^2*c^2*z - 4*B*C^2*a*b^3*z - 4*B^3*a*b^2*c*z + 16*B^3*a^2*c^2*z + 4*A*B^2*C*a^2*c + 2*A^3*C*a*b*c - A^2*B^2*a*b*c - 2

$$\begin{aligned}
& A^2 C^2 a^2 c + 2 A C^3 a^2 b - B^2 C^2 a^2 b - A^2 C^2 a b^2 - B^4 a^2 c - \\
& A^4 a c^2 - C^4 a^3, z, k) * x * (8 b^3 c^3 - 32 a b c^4) / c) + (x * (2 C^2 b^4 - \\
& 4 A^2 a c^3 + 2 B^2 b^3 c + 2 A^2 b^2 c^2 + 4 C^2 a^2 c^2 - 4 A C b^3 c - \\
& 10 B^2 a b c^2 - 8 C^2 a b^2 c + 12 A C a b c^2)) / c) - (A^3 a c^2 - C^3 a^2 \\
& 2 b + A C^2 a b^2 + A C^2 a^2 c - B^2 C a^2 c + A B^2 a b c - 2 A^2 C a b c \\
&) / c - (x * (B^3 a b c + A^2 B a c^2 + B C^2 a b^2 - B C^2 a^2 c - 2 A B C a b \\
& c)) / c) * \text{root}(128 a b^2 c^4 z^4 - 16 b^4 c^3 z^4 - 256 a^2 c^5 z^4 - 128 B a \\
& b^2 c^3 z^3 + 16 B b^4 c^2 z^3 + 256 B a^2 c^4 z^3 - 48 A C a b^2 c^2 z^2 \\
& + 8 A C b^4 c z^2 - 48 C^2 a^2 b c^2 z^2 + 40 B^2 a b^2 c^2 z^2 + 28 C^2 a \\
& b^3 c z^2 + 16 A^2 a b c^3 z^2 + 64 A C a^2 c^3 z^2 - 4 B^2 b^4 c z^2 - 96 B \\
& B^2 a^2 c^3 z^2 - 4 A^2 b^3 c^2 z^2 - 4 C^2 b^5 z^2 + 8 A B C a b^2 c z + 1 \\
& 6 B C^2 a^2 b c z - 32 A B C a^2 c^2 z - 4 B C^2 a b^3 z - 4 B^3 a b^2 c z \\
& + 16 B^3 a^2 c^2 z + 4 A B^2 C a^2 c + 2 A^3 C a b c - A^2 B^2 a b c - 2 A^2 \\
& C^2 a^2 c + 2 A C^3 a^2 b - B^2 C^2 a^2 b - A^2 C^2 a b^2 - B^4 a^2 c - A \\
& ^4 a c^2 - C^4 a^3, z, k), k, 1, 4) + (C x) / c
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(C*x**2+B*x+A)/(c*x**4+b*x**2+a),x)

[Out] Timed out

$$3.24 \quad \int \frac{x(A+Bx+Cx^2)}{a+bx^2+cx^4} dx$$

Optimal. Leaf size=223

$$\frac{(2Ac - bC) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}} - \frac{B\sqrt{b-\sqrt{b^2-4ac}} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}} + \frac{B\sqrt{\sqrt{b^2-4ac}+b} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Rubi [A] time = 0.21, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {1662, 1247, 634, 618, 206, 628, 12, 1130, 205}

$$\frac{(2Ac - bC) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}} - \frac{B\sqrt{b-\sqrt{b^2-4ac}} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}} + \frac{B\sqrt{\sqrt{b^2-4ac}+b} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}} + \frac{C \log(a + bx^2 + cx^4)}{4c}$$

Antiderivative was successfully verified.

[In] Int[(x*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4), x]

[Out] -((B*Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])) + (B*Sqrt[b + Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]) - ((2*A*c - b*C)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c*Sqrt[b^2 - 4*a*c]) + (C*Log[a + b*x^2 + c*x^4])/(4*c)

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ

$[2*c*d - b*e, 0] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ !\text{NiceSqrtQ}[b^2 - 4*a*c]$

Rule 1130

$\text{Int}[(d_*)(x_)^{(m_)} / ((a_) + (b_*)(x_)^2 + (c_*)(x_)^4), x_Symbol] \ :> \ \text{With}\{q = \text{Rt}[b^2 - 4*a*c, 2]\}, \ \text{Dist}[(d^2*(b/q + 1))/2, \ \text{Int}[(d*x)^{(m-2)} / (b/2 + q/2 + c*x^2), x], x] - \text{Dist}[(d^2*(b/q - 1))/2, \ \text{Int}[(d*x)^{(m-2)} / (b/2 - q/2 + c*x^2), x], x] \ /; \ \text{FreeQ}\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{eQ}[m, 2]$

Rule 1247

$\text{Int}[(x_)*((d_) + (e_*)(x_)^2)^{(q_)}*((a_) + (b_*)(x_)^2 + (c_*)(x_)^4)^{(p_)}, x_Symbol] \ :> \ \text{Dist}[1/2, \ \text{Subst}[\text{Int}[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] \ /; \ \text{FreeQ}\{a, b, c, d, e, p, q\}, x]$

Rule 1662

$\text{Int}[(Pq_)*((d_*)(x_)^{(m_)}*((a_) + (b_*)(x_)^2 + (c_*)(x_)^4)^{(p_)}, x_Symbol] \ :> \ \text{Module}\{q = \text{Expon}[Pq, x], k\}, \ \text{Int}[(d*x)^m*\text{Sum}[\text{Coeff}[Pq, x, 2*k]*x^{(2*k)}, \{k, 0, q/2 + 1\}]* (a + b*x^2 + c*x^4)^p, x] + \text{Dist}[1/d, \ \text{Int}[(d*x)^{(m+1)}*\text{Sum}[\text{Coeff}[Pq, x, 2*k + 1]*x^{(2*k)}, \{k, 0, (q-1)/2 + 1\}]* (a + b*x^2 + c*x^4)^p, x], x] \ /; \ \text{FreeQ}\{a, b, c, d, m, p\}, x] \ \&\& \ \text{PolyQ}[Pq, x] \ \&\& \ !\text{PolyQ}[Pq, x^2]$

Rubi steps

$$\begin{aligned} \int \frac{x(A + Bx + Cx^2)}{a + bx^2 + cx^4} dx &= \int \frac{Bx^2}{a + bx^2 + cx^4} dx + \int \frac{x(A + Cx^2)}{a + bx^2 + cx^4} dx \\ &= \frac{1}{2} \text{Subst}\left(\int \frac{A + Cx}{a + bx + cx^2} dx, x, x^2\right) + B \int \frac{x^2}{a + bx^2 + cx^4} dx \\ &= \frac{1}{2} \left(B \left(1 - \frac{b}{\sqrt{b^2 - 4ac}}\right)\right) \int \frac{1}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2 - 4ac} + cx^2} dx + \frac{1}{2} \left(B \left(1 + \frac{b}{\sqrt{b^2 - 4ac}}\right)\right) \int \frac{1}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2 - 4ac} + cx^2} dx \\ &= -\frac{B\sqrt{b - \sqrt{b^2 - 4ac}} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2 - 4ac}} + \frac{B\sqrt{b + \sqrt{b^2 - 4ac}} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b + \sqrt{b^2 - 4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2 - 4ac}} \\ &= -\frac{B\sqrt{b - \sqrt{b^2 - 4ac}} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2 - 4ac}} + \frac{B\sqrt{b + \sqrt{b^2 - 4ac}} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b + \sqrt{b^2 - 4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2 - 4ac}} \end{aligned}$$

Mathematica [A] time = 0.36, size = 240, normalized size = 1.08

$$\frac{(c(\sqrt{b^2 - 4ac} - b) + 2Ac) \log(\sqrt{b^2 - 4ac} - b - 2cx^2) - (2Ac - c(\sqrt{b^2 - 4ac} + b)) \log(\sqrt{b^2 - 4ac} + b + 2cx^2) - 2\sqrt{2}B\sqrt{c}\sqrt{b - \sqrt{b^2 - 4ac}} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right) + 2\sqrt{2}B\sqrt{c}\sqrt{b + \sqrt{b^2 - 4ac}} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b + \sqrt{b^2 - 4ac}}}\right)}{4c\sqrt{b^2 - 4ac}}$$

Antiderivative was successfully verified.

[In] Integrate[(x*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4), x]

[Out] $(-2*\text{Sqrt}[2]*B*\text{Sqrt}[c]*\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]]*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]]] + 2*\text{Sqrt}[2]*B*\text{Sqrt}[c]*\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]]*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]]] + (2*A*c + (-b + \text{Sqrt}[b^2 - 4*a*c])*C)*\text{Log}[-b + \text{Sqrt}[b^2 - 4*a*c] - 2*c*x^2] - (2*A*c -$

$(b + \sqrt{b^2 - 4ac}) \cdot C \cdot \log[b + \sqrt{b^2 - 4ac} + 2cx^2] / (4c\sqrt{b^2 - 4ac})$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(A + Bx + Cx^2)}{a + bx^2 + cx^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]

[Out] IntegrateAlgebraic[(x*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] Timed out

giac [B] time = 5.36, size = 2369, normalized size = 10.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] $\frac{1}{4}C \cdot \log(\text{abs}(c x^4 + b x^2 + a)) / c + \frac{1}{8} \cdot ((2 b^4 c^2 - 16 a b^2 c^3 + 32 a^2 c^4 - \sqrt{2}) \sqrt{b^2 - 4 a c}) \sqrt{b c - \sqrt{b^2 - 4 a c}} \sqrt{b^4 + 8 \sqrt{2}) \sqrt{b^2 - 4 a c}) \sqrt{b c - \sqrt{b^2 - 4 a c}} \sqrt{b^3 c - 16 \sqrt{2}) \sqrt{b^2 - 4 a c}) \sqrt{b c - \sqrt{b^2 - 4 a c}} \sqrt{a^2 c^2 - 8 \sqrt{2}) \sqrt{b^2 - 4 a c}) \sqrt{b c - \sqrt{b^2 - 4 a c}} \sqrt{a b c^2 - \sqrt{2}) \sqrt{b^2 - 4 a c}) \sqrt{b c - \sqrt{b^2 - 4 a c}} \sqrt{b^2 c^2 + 4 \sqrt{2}) \sqrt{b^2 - 4 a c}) \sqrt{b c - \sqrt{b^2 - 4 a c}} \sqrt{a c^3 - 2 (b^2 - 4 a c) b^2 c^2 + 8 (b^2 - 4 a c) a c^3} B c^2 - (2 b^4 c^4 - 8 a b^2 c^5 - \sqrt{2}) \sqrt{b^2 - 4 a c}) \sqrt{b c - \sqrt{b^2 - 4 a c}} \sqrt{b^4 c^2 + 4 \sqrt{2}) \sqrt{b^2 - 4 a c}) \sqrt{b c - \sqrt{b^2 - 4 a c}} \sqrt{a b^2 c^3 + 2 \sqrt{2}) \sqrt{b^2 - 4 a c}) \sqrt{b c - \sqrt{b^2 - 4 a c}} \sqrt{b^3 c^3 - \sqrt{2}) \sqrt{b^2 - 4 a c}) \sqrt{b c - \sqrt{b^2 - 4 a c}} \sqrt{b^2 c^4 - 2 (b^2 - 4 a c) b^2 c^4} B) \arctan(2 \sqrt{1/2}) x / \sqrt{(b c + \sqrt{b^2 c^2 - 4 a c^3}) / c^2}) / ((a b^4 c^2 - 8 a^2 b^2 c^3 - 2 a b^3 c^3 + 16 a^3 c^4 + 8 a^2 b c^4 + a b^2 c^4 - 4 a^2 c^5) c^2) - \frac{1}{8} \cdot ((2 b^4 c^2 - 16 a b^2 c^3 + 32 a^2 c^4 - \sqrt{2}) \sqrt{b^2 - 4 a c}) \sqrt{b c + \sqrt{b^2 - 4 a c}} \sqrt{b^4 + 8 \sqrt{2}) \sqrt{b^2 - 4 a c}) \sqrt{b c + \sqrt{b^2 - 4 a c}} \sqrt{a b^2 c^3 + 2 \sqrt{2}) \sqrt{b^2 - 4 a c}) \sqrt{b c + \sqrt{b^2 - 4 a c}} \sqrt{b^3 c^3 - 16 \sqrt{2}) \sqrt{b^2 - 4 a c}) \sqrt{b c + \sqrt{b^2 - 4 a c}} \sqrt{a^2 c^2 - 8 \sqrt{2}) \sqrt{b^2 - 4 a c}) \sqrt{b c + \sqrt{b^2 - 4 a c}} \sqrt{a b c^2 - \sqrt{2}) \sqrt{b^2 - 4 a c}) \sqrt{b c + \sqrt{b^2 - 4 a c}} \sqrt{b^2 c^2 + 4 \sqrt{2}) \sqrt{b^2 - 4 a c}) \sqrt{b c + \sqrt{b^2 - 4 a c}} \sqrt{a c^3 - 2 (b^2 - 4 a c) b^2 c^2 + 8 (b^2 - 4 a c) a c^3} B c^2 - (2 b^4 c^4 - 8 a b^2 c^5 - \sqrt{2}) \sqrt{b^2 - 4 a c}) \sqrt{b c + \sqrt{b^2 - 4 a c}} \sqrt{b^4 c^2 + 4 \sqrt{2}) \sqrt{b^2 - 4 a c}) \sqrt{b c + \sqrt{b^2 - 4 a c}} \sqrt{a b^2 c^3 + 2 \sqrt{2}) \sqrt{b^2 - 4 a c}) \sqrt{b c + \sqrt{b^2 - 4 a c}} \sqrt{b^3 c^3 - \sqrt{2}) \sqrt{b^2 - 4 a c}) \sqrt{b c + \sqrt{b^2 - 4 a c}} \sqrt{b^2 c^4 - 2 (b^2 - 4 a c) b^2 c^4} B) \arctan(2 \sqrt{1/2}) x / \sqrt{(b c - \sqrt{b^2 c^2 - 4 a c^3}) / c^2}) / ((a b^4 c^2 - 8 a^2 b^2 c^3 - 2 a b^3 c^3 + 16 a^3 c^4 + 8 a^2 b c^4 + a b^2 c^4 - 4 a^2 c^5) c^2) + \frac{1}{16} \cdot (2 (b^5 c - 8 a b^3 c^2 - 2 b^4 c^2 + 16 a^2 b c^3 + 8 a b^2 c^3 + b^3 c^3 - 4 a b c^4 - (b^4 c - 8 a b^2 c^2 - 2 b^3 c^2$

+ 16*a^2*c^3 + 8*a*b*c^3 + b^2*c^3 - 4*a*c^4)*sqrt(b^2 - 4*a*c))*A*abs(c) - (b^6 - 8*a*b^4*c - 2*b^5*c + 16*a^2*b^2*c^2 + 8*a*b^3*c^2 + b^4*c^2 - 4*a*b^2*c^3 - (b^5 - 8*a*b^3*c - 2*b^4*c + 16*a^2*b*c^2 + 8*a*b^2*c^2 + b^3*c^2 - 4*a*b*c^3)*sqrt(b^2 - 4*a*c))*C*abs(c) + 2*(b^5*c^2 - 8*a*b^3*c^3 - 2*b^4*c^3 + 16*a^2*b*c^4 + 8*a*b^2*c^4 + b^3*c^4 - 4*a*b*c^5 + (b^4*c^2 - 4*a*b^2*c^3 - 2*b^3*c^3 + b^2*c^4)*sqrt(b^2 - 4*a*c))*A - (b^6*c - 8*a*b^4*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a*b^3*c^3 + b^4*c^3 - 4*a*b^2*c^4 - (b^5*c - 4*a*b^3*c^2 - 2*b^4*c^2 + b^3*c^3)*sqrt(b^2 - 4*a*c))*C)*log(x^2 + 1/2*(b*c + sqrt(b^2*c^2 - 4*a*c^3))/c^2)/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2*abs(c)) + 1/16*(2*(b^5*c - 8*a*b^3*c^2 - 2*b^4*c^2 + 16*a^2*b*c^3 + 8*a*b^2*c^3 + b^3*c^3 - 4*a*b*c^4 + (b^4*c - 8*a*b^2*c^2 - 2*b^3*c^2 + 16*a^2*c^3 + 8*a*b*c^3 + b^2*c^3 - 4*a*c^4)*sqrt(b^2 - 4*a*c))*A*abs(c) - (b^6 - 8*a*b^4*c - 2*b^5*c + 16*a^2*b^2*c^2 + 8*a*b^3*c^2 + b^4*c^2 - 4*a*b^2*c^3 + (b^5 - 8*a*b^3*c - 2*b^4*c + 16*a^2*b*c^2 + 8*a*b^2*c^2 + b^3*c^2 - 4*a*b*c^3)*sqrt(b^2 - 4*a*c))*C*abs(c) + 2*(b^5*c^2 - 8*a*b^3*c^3 - 2*b^4*c^3 + 16*a^2*b*c^4 + 8*a*b^2*c^4 + b^3*c^4 - 4*a*b*c^5 + (b^4*c^2 - 4*a*b^2*c^3 - 2*b^3*c^3 + b^2*c^4)*sqrt(b^2 - 4*a*c))*A - (b^6*c - 8*a*b^4*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a*b^3*c^3 + b^4*c^3 - 4*a*b^2*c^4 + (b^5*c - 4*a*b^3*c^2 - 2*b^4*c^2 + b^3*c^3)*sqrt(b^2 - 4*a*c))*C)*log(x^2 + 1/2*(b*c - sqrt(b^2*c^2 - 4*a*c^3))/c^2)/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2*abs(c))

maple [B] time = 0.04, size = 728, normalized size = 3.26



Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(C*x^2+B*x+A)/(c*x^4+b*x^2+a), x)

[Out] -1/2/(4*a*c-b^2)*ln(-2*c*x^2-b+(-4*a*c+b^2)^(1/2))*A*(-4*a*c+b^2)^(1/2)+1/4/c/(4*a*c-b^2)*ln(-2*c*x^2-b+(-4*a*c+b^2)^(1/2))*C*(-4*a*c+b^2)^(1/2)*b+1/(4*a*c-b^2)*ln(-2*c*x^2-b+(-4*a*c+b^2)^(1/2))*a*C-1/4/c/(4*a*c-b^2)*ln(-2*c*x^2-b+(-4*a*c+b^2)^(1/2))*b^2*C-1/2/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B*(-4*a*c+b^2)^(1/2)*b-2*c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*B+1/2/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*B+1/2/(4*a*c-b^2)*ln(2*c*x^2+b+(-4*a*c+b^2)^(1/2))*A*(-4*a*c+b^2)^(1/2)-1/4/c/(4*a*c-b^2)*ln(2*c*x^2+b+(-4*a*c+b^2)^(1/2))*C*(-4*a*c+b^2)^(1/2)*b+1/(4*a*c-b^2)*ln(2*c*x^2+b+(-4*a*c+b^2)^(1/2))*a*C-1/4/c/(4*a*c-b^2)*ln(2*c*x^2+b+(-4*a*c+b^2)^(1/2))*b^2*C-1/2/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B*(-4*a*c+b^2)^(1/2)*b+2*c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*B-1/2/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*B

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Cx^2 + Bx + A)x}{cx^4 + bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(C*x^2+B*x+A)/(c*x^4+b*x^2+a), x, algorithm="maxima")

[Out] integrate((C*x^2 + B*x + A)*x/(c*x^4 + b*x^2 + a), x)

mupad [B] time = 1.89, size = 5594, normalized size = 25.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((x*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4), x)$

[Out] $\text{symsum}(\log(A^3*c^2*x - B^3*a*c - B*C^2*a*b - 8*\text{root}(128*a*b^2*c^3*z^4 - 16*b^4*c^2*z^4 - 256*a^2*c^4*z^4 - 128*C*a*b^2*c^2*z^3 + 256*C*a^2*c^3*z^3 + 16*C*b^4*c*z^3 + 32*A*C*a*b*c^2*z^2 - 8*A*C*b^3*c*z^2 + 40*C^2*a*b^2*c*z^2 + 16*B^2*a*b*c^2*z^2 - 4*B^2*b^3*c*z^2 - 32*A^2*a*c^3*z^2 - 96*C^2*a^2*c^2*z^2 + 8*A^2*b^2*c^2*z^2 - 4*C^2*b^4*z^2 - 16*A*C^2*a*b*c*z - 4*A^2*C*b^2*c*z + 16*A^2*C*a*c^2*z + 4*A*B^2*b^2*c*z - 16*A*B^2*a*c^2*z + 16*C^3*a^2*c*z - 4*C^3*a*b^2*z + 4*A*C^2*b^3*z + 4*A*B^2*C*a*c - 2*A^2*C^2*a*c + 2*A^3*C*b*c + 2*A*C^3*a*b - B^2*C^2*a*b - A^2*B^2*b*c - B^4*a*c - A^2*C^2*b^2 - C^4*a^2 - A^4*c^2, z, k)^3*b^3*c^2*x - C^3*a*b*x + A*C^2*b^2*x - 2*C^2*\text{root}(128*a*b^2*c^3*z^4 - 16*b^4*c^2*z^4 - 256*a^2*c^4*z^4 - 128*C*a*b^2*c^2*z^3 + 256*C*a^2*c^3*z^3 + 16*C*b^4*c*z^3 + 32*A*C*a*b*c^2*z^2 - 8*A*C*b^3*c*z^2 + 40*C^2*a*b^2*c*z^2 + 16*B^2*a*b*c^2*z^2 - 4*B^2*b^3*c*z^2 - 32*A^2*a*c^3*z^2 - 96*C^2*a^2*c^2*z^2 + 8*A^2*b^2*c^2*z^2 - 4*C^2*b^4*z^2 - 16*A*C^2*a*b*c*z - 4*A^2*C*b^2*c*z + 16*A^2*C*a*c^2*z + 4*A*B^2*b^2*c*z - 16*A*B^2*a*c^2*z + 16*C^3*a^2*c*z - 4*C^3*a*b^2*z + 4*A*C^2*b^3*z + 4*A*B^2*C*a*c - 2*A^2*C^2*a*c + 2*A^3*C*b*c + 2*A*C^3*a*b - B^2*C^2*a*b - A^2*B^2*b*c - B^4*a*c - A^2*C^2*b^2 - C^4*a^2 - A^4*c^2, z, k)*b^3*x + 32*\text{root}(128*a*b^2*c^3*z^4 - 16*b^4*c^2*z^4 - 256*a^2*c^4*z^4 - 128*C*a*b^2*c^2*z^3 + 256*C*a^2*c^3*z^3 + 16*C*b^4*c*z^3 + 32*A*C*a*b*c^2*z^2 - 8*A*C*b^3*c*z^2 + 40*C^2*a*b^2*c*z^2 + 16*B^2*a*b*c^2*z^2 - 4*B^2*b^3*c*z^2 - 32*A^2*a*c^3*z^2 - 96*C^2*a^2*c^2*z^2 + 8*A^2*b^2*c^2*z^2 - 4*C^2*b^4*z^2 - 16*A*C^2*a*b*c*z - 4*A^2*C*b^2*c*z + 16*A^2*C*a*c^2*z + 4*A*B^2*b^2*c*z - 16*A*B^2*a*c^2*z + 16*C^3*a^2*c*z - 4*C^3*a*b^2*z + 4*A*C^2*b^3*z + 4*A*B^2*C*a*c - 2*A^2*C^2*a*c + 2*A^3*C*b*c + 2*A*C^3*a*b - B^2*C^2*a*b - A^2*B^2*b*c - B^4*a*c - A^2*C^2*b^2 - C^4*a^2 - A^4*c^2, z, k)^2*b^2*c^2*x - 8*A*B*\text{root}(128*a*b^2*c^3*z^4 - 16*b^4*c^2*z^4 - 256*a^2*c^4*z^4 - 128*C*a*b^2*c^2*z^3 + 256*C*a^2*c^3*z^3 + 16*C*b^4*c*z^3 + 32*A*C*a*b*c^2*z^2 - 8*A*C*b^3*c*z^2 + 40*C^2*a*b^2*c*z^2 + 16*B^2*a*b*c^2*z^2 - 4*B^2*b^3*c*z^2 - 32*A^2*a*c^3*z^2 - 96*C^2*a^2*c^2*z^2 + 8*A^2*b^2*c^2*z^2 - 4*C^2*b^4*z^2 - 16*A*C^2*a*b*c*z - 4*A^2*C*b^2*c*z + 16*A^2*C*a*c^2*z + 4*A*B^2*b^2*c*z - 16*A*B^2*a*c^2*z + 16*C^3*a^2*c*z - 4*C^3*a*b^2*z + 4*A*C^2*b^3*z + 4*A*B^2*C*a*c - 2*A^2*C^2*a*c + 2*A^3*C*b*c + 2*A*C^3*a*b - B^2*C^2*a*b - A^2*B^2*b*c - B^4*a*c - A^2*C^2*b^2 - C^4*a^2 - A^4*c^2, z, k)*a*c^2 + A*B^2*b*c*x + A*C^2*a*c*x - 2*A^2*C*b*c*x - B^2*C*a*c*x + 2*A*B*C*a*c + 16*A*\text{root}(128*a*b^2*c^3*z^4 - 16*b^4*c^2*z^4 - 256*a^2*c^4*z^4 - 128*C*a*b^2*c^2*z^3 + 256*C*a^2*c^3*z^3 + 16*C*b^4*c*z^3 + 32*A*C*a*b*c^2*z^2 - 8*A*C*b^3*c*z^2 + 40*C^2*a*b^2*c*z^2 + 16*B^2*a*b*c^2*z^2 - 4*B^2*b^3*c*z^2 - 32*A^2*a*c^3*z^2 - 96*C^2*a^2*c^2*z^2 + 8*A^2*b^2*c^2*z^2 - 4*C^2*b^4*z^2 - 16*A*C^2*a*b*c*z - 4*A^2*C*b^2*c*z + 16*A^2*C*a*c^2*z + 4*A*B^2*b^2*c*z - 16*A*B^2*a*c^2*z + 16*C^3*a^2*c*z - 4*C^3*a*b^2*z + 4*A*C^2*b^3*z + 4*A*B^2*C*a*c - 2*A^2*C^2*a*c + 2*A^3*C*b*c + 2*A*C^3*a*b - B^2*C^2*a*b - A^2*B^2*b*c - B^4*a*c - A^2*C^2*b^2 - C^4*a^2 - A^4*c^2, z, k)^2*a*c^3*x + 2*A^2*\text{root}(128*a*b^2*c^3*z^4 - 16*b^4*c^2*z^4 - 256*a^2*c^4*z^4 - 128*C*a*b^2*c^2*z^3 + 256*C*a^2*c^3*z^3 + 16*C*b^4*c*z^3 + 32*A*C*a*b*c^2*z^2 - 8*A*C*b^3*c*z^2 + 40*C^2*a*b^2*c*z^2 + 16*B^2*a*b*c^2*z^2 - 4*B^2*b^3*c*z^2 - 32*A^2*a*c^3*z^2 - 96*C^2*a^2*c^2*z^2 + 8*A^2*b^2*c^2*z^2 - 4*C^2*b^4*z^2 - 16*A*C^2*a*b*c*z - 4*A^2*C*b^2*c*z + 16*A^2*C*a*c^2*z + 4*A*B^2*b^2*c*z - 16*A*B^2*a*c^2*z + 16*C^3*a^2*c*z - 4*C^3*a*b^2*z + 4*A*C^2*b^3*z + 4*A*B^2*C*a*c - 2*A^2*C^2*a*c + 2*A^3*C*b*c + 2*A*C^3*a*b - B^2*C^2*a*b - A^2*B^2*b*c - B^4*a*c - A^2*C^2*b^2 - C^4*a^2 - A^4*c^2, z, k)^2*a*c^3*x + 2*A^2*\text{root}(128*a*b^2*c^3*z^4 - 16*b^4*c^2*z^4 - 256*a^2*c^4*z^4 - 128*C*a*b^2*c^2*z^3 + 256*C*a^2*c^3*z^3 + 16*C*b^4*c*z^3 + 32*A*C*a*b*c^2*z^2 - 8*A*C*b^3*c*z^2 + 40*C^2*a*b^2*c*z^2 + 16*B^2*a*b*c^2*z^2 - 4*B^2*b^3*c*z^2 - 32*A^2*a*c^3*z^2 - 96*C^2*a^2*c^2*z^2 + 8*A^2*b^2*c^2*z^2 - 4*C^2*b^4*z^2 - 16*A*C^2*a*b*c*z - 4*A^2*C*b^2*c*z + 16*A^2*C*a*c^2*z + 4*A*B^2*b^2*c*z - 16*A*B^2*a*c^2*z + 16*C^3*a^2*c*z - 4*C^3*a*b^2*z + 4*A*C^2*b^3*z + 4*A*B^2*C*a*c - 2*A^2*C^2*a*c + 2*A^3*C*b*c + 2*A*C^3*a*b - B^2*C^2*a*b - A^2*B^2*b*c - B^4*a*c - A^2*C^2*b^2 - C^4*a^2 - A^4*c^2, z, k)^2*a*c^3*x + 2*A^2*\text{root}(128*a*b^2*c^3*z^4 - 16*b^4*c^2*z^4 - 256*a^2*c^4*z^4 - 128*C*a*b^2*c^2*z^3 + 256*C*a^2*c^3*z^3 + 16*C*b^4*c*z^3 + 32*A*C*a*b*c^2*z^2 - 8*A*C*b^3*c*z^2 + 40*C^2*a*b^2*c*z^2 + 16*B^2*a*b*c^2*z^2 - 4*B^2*b^3*c*z^2 - 32*A^2*a*c^3*z^2 - 96*C^2*a^2*c^2*z^2 + 8*A^2*b^2*c^2*z^2 - 4*C^2*b^4*z^2 - 16*A*C^2*a*b*c*z - 4*A^2*C*b^2*c*z + 16*A^2*C*a*c^2*z + 4*A*B^2*b^2*c*z - 16*A*B^2*a*c^2*z + 16*C^3*a^2*c*z - 4*C^3*a*b^2*z + 4*A*C^2*b^3*z + 4*A*B^2*C*a*c - 2*A^2*C^2*a*c + 2*A^3*C*b*c + 2*A*C^3*a*b - B^2*C^2*a*b - A^2*B^2*b*c - B^4*a*c - A^2*C^2*b^2 - C^4*a^2 - A^4*c^2, z, k)^2*a*c^3*x$

$$\begin{aligned}
& 16*A*B^2*a*c^2*z + 16*C^3*a^2*c*z - 4*C^3*a*b^2*z + 4*A*C^2*b^3*z + 4*A*B^2 \\
& 2*C*a*c - 2*A^2*C^2*a*c + 2*A^3*C*b*c + 2*A*C^3*a*b - B^2*C^2*a*b - A^2*B^2 \\
& *b*c - B^4*a*c - A^2*C^2*b^2 - C^4*a^2 - A^4*c^2, z, k)*b*c^2*x + 4*B^2*roo \\
& t(128*a*b^2*c^3*z^4 - 16*b^4*c^2*z^4 - 256*a^2*c^4*z^4 - 128*C*a*b^2*c^2*z^3 \\
& + 256*C*a^2*c^3*z^3 + 16*C*b^4*c*z^3 + 32*A*C*a*b*c^2*z^2 - 8*A*C*b^3*c*z \\
& ^2 + 40*C^2*a*b^2*c*z^2 + 16*B^2*a*b*c^2*z^2 - 4*B^2*b^3*c*z^2 - 32*A^2*a*c \\
& ^3*z^2 - 96*C^2*a^2*c^2*z^2 + 8*A^2*b^2*c^2*z^2 - 4*C^2*b^4*z^2 - 16*A*C^2* \\
& a*b*c*z - 4*A^2*C*b^2*c*z + 16*A^2*C*a*c^2*z + 4*A*B^2*b^2*c*z - 16*A*B^2*a \\
& *c^2*z + 16*C^3*a^2*c*z - 4*C^3*a*b^2*z + 4*A*C^2*b^3*z + 4*A*B^2*C*a*c - 2 \\
& *A^2*C^2*a*c + 2*A^3*C*b*c + 2*A*C^3*a*b - B^2*C^2*a*b - A^2*B^2*b*c - B^4* \\
& a*c - A^2*C^2*b^2 - C^4*a^2 - A^4*c^2, z, k)*a*c^2*x - 2*B^2*root(128*a*b^2 \\
& *c^3*z^4 - 16*b^4*c^2*z^4 - 256*a^2*c^4*z^4 - 128*C*a*b^2*c^2*z^3 + 256*C*a \\
& ^2*c^3*z^3 + 16*C*b^4*c*z^3 + 32*A*C*a*b*c^2*z^2 - 8*A*C*b^3*c*z^2 + 40*C^2 \\
& *a*b^2*c*z^2 + 16*B^2*a*b*c^2*z^2 - 4*B^2*b^3*c*z^2 - 32*A^2*a*c^3*z^2 - 96 \\
& *C^2*a^2*c^2*z^2 + 8*A^2*b^2*c^2*z^2 - 4*C^2*b^4*z^2 - 16*A*C^2*a*b*c*z - 4 \\
& *A^2*C*b^2*c*z + 16*A^2*C*a*c^2*z + 4*A*B^2*b^2*c*z - 16*A*B^2*a*c^2*z + 16 \\
& *C^3*a^2*c*z - 4*C^3*a*b^2*z + 4*A*C^2*b^3*z + 4*A*B^2*C*a*c - 2*A^2*C^2*a* \\
& c + 2*A^3*C*b*c + 2*A*C^3*a*b - B^2*C^2*a*b - A^2*B^2*b*c - B^4*a*c - A^2*C \\
& ^2*b^2 - C^4*a^2 - A^4*c^2, z, k)*b^2*c*x + 8*C*root(128*a*b^2*c^3*z^4 - 16 \\
& *b^4*c^2*z^4 - 256*a^2*c^4*z^4 - 128*C*a*b^2*c^2*z^3 + 256*C*a^2*c^3*z^3 + \\
& 16*C*b^4*c*z^3 + 32*A*C*a*b*c^2*z^2 - 8*A*C*b^3*c*z^2 + 40*C^2*a*b^2*c*z^2 \\
& + 16*B^2*a*b*c^2*z^2 - 4*B^2*b^3*c*z^2 - 32*A^2*a*c^3*z^2 - 96*C^2*a^2*c^2* \\
& z^2 + 8*A^2*b^2*c^2*z^2 - 4*C^2*b^4*z^2 - 16*A*C^2*a*b*c*z - 4*A^2*C*b^2*c* \\
& z + 16*A^2*C*a*c^2*z + 4*A*B^2*b^2*c*z - 16*A*B^2*a*c^2*z + 16*C^3*a^2*c*z \\
& - 4*C^3*a*b^2*z + 4*A*C^2*b^3*z + 4*A*B^2*C*a*c - 2*A^2*C^2*a*c + 2*A^3*C*b* \\
& *c + 2*A*C^3*a*b - B^2*C^2*a*b - A^2*B^2*b*c - B^4*a*c - A^2*C^2*b^2 - C^4* \\
& a^2 - A^4*c^2, z, k)^2*b^3*c*x - 32*C*root(128*a*b^2*c^3*z^4 - 16*b^4*c^2*z \\
& ^4 - 256*a^2*c^4*z^4 - 128*C*a*b^2*c^2*z^3 + 256*C*a^2*c^3*z^3 + 16*C*b^4*c \\
& *z^3 + 32*A*C*a*b*c^2*z^2 - 8*A*C*b^3*c*z^2 + 40*C^2*a*b^2*c*z^2 + 16*B^2*a \\
& *b*c^2*z^2 - 4*B^2*b^3*c*z^2 - 32*A^2*a*c^3*z^2 - 96*C^2*a^2*c^2*z^2 + 8*A^ \\
& 2*b^2*c^2*z^2 - 4*C^2*b^4*z^2 - 16*A*C^2*a*b*c*z - 4*A^2*C*b^2*c*z + 16*A^2 \\
& *C*a*c^2*z + 4*A*B^2*b^2*c*z - 16*A*B^2*a*c^2*z + 16*C^3*a^2*c*z - 4*C^3*a* \\
& b^2*z + 4*A*C^2*b^3*z + 4*A*B^2*C*a*c - 2*A^2*C^2*a*c + 2*A^3*C*b*c + 2*A*C \\
& ^3*a*b - B^2*C^2*a*b - A^2*B^2*b*c - B^4*a*c - A^2*C^2*b^2 - C^4*a^2 - A^4* \\
& c^2, z, k)^2*a*b*c^2*x + 4*B*C*root(128*a*b^2*c^3*z^4 - 16*b^4*c^2*z^4 - 25 \\
& 6*a^2*c^4*z^4 - 128*C*a*b^2*c^2*z^3 + 256*C*a^2*c^3*z^3 + 16*C*b^4*c*z^3 + \\
& 32*A*C*a*b*c^2*z^2 - 8*A*C*b^3*c*z^2 + 40*C^2*a*b^2*c*z^2 + 16*B^2*a*b*c^2* \\
& z^2 - 4*B^2*b^3*c*z^2 - 32*A^2*a*c^3*z^2 - 96*C^2*a^2*c^2*z^2 + 8*A^2*b^2*c \\
& ^2*z^2 - 4*C^2*b^4*z^2 - 16*A*C^2*a*b*c*z - 4*A^2*C*b^2*c*z + 16*A^2*C*a*c^ \\
& 2*z + 4*A*B^2*b^2*c*z - 16*A*B^2*a*c^2*z + 16*C^3*a^2*c*z - 4*C^3*a*b^2*z \\
& + 4*A*C^2*b^3*z + 4*A*B^2*C*a*c - 2*A^2*C^2*a*c + 2*A^3*C*b*c + 2*A*C^3*a*b \\
& - B^2*C^2*a*b - A^2*B^2*b*c - B^4*a*c - A^2*C^2*b^2 - C^4*a^2 - A^4*c^2, z, \\
& k)*a*b*c - 8*A*C*root(128*a*b^2*c^3*z^4 - 16*b^4*c^2*z^4 - 256*a^2*c^4*z^4 \\
& - 128*C*a*b^2*c^2*z^3 + 256*C*a^2*c^3*z^3 + 16*C*b^4*c*z^3 + 32*A*C*a*b*c^ \\
& 2*z^2 - 8*A*C*b^3*c*z^2 + 40*C^2*a*b^2*c*z^2 + 16*B^2*a*b*c^2*z^2 - 4*B^2*b \\
& ^3*c*z^2 - 32*A^2*a*c^3*z^2 - 96*C^2*a^2*c^2*z^2 + 8*A^2*b^2*c^2*z^2 - 4*C^ \\
& 2*b^4*z^2 - 16*A*C^2*a*b*c*z - 4*A^2*C*b^2*c*z + 16*A^2*C*a*c^2*z + 4*A*B^2 \\
& *b^2*c*z - 16*A*B^2*a*c^2*z + 16*C^3*a^2*c*z - 4*C^3*a*b^2*z + 4*A*C^2*b^3* \\
& z + 4*A*B^2*C*a*c - 2*A^2*C^2*a*c + 2*A^3*C*b*c + 2*A*C^3*a*b - B^2*C^2*a*b \\
& - A^2*B^2*b*c - B^4*a*c - A^2*C^2*b^2 - C^4*a^2 - A^4*c^2, z, k)*a*c^2*x + \\
& 10*C^2*root(128*a*b^2*c^3*z^4 - 16*b^4*c^2*z^4 - 256*a^2*c^4*z^4 - 128*C*a \\
& *b^2*c^2*z^3 + 256*C*a^2*c^3*z^3 + 16*C*b^4*c*z^3 + 32*A*C*a*b*c^2*z^2 - 8* \\
& A*C*b^3*c*z^2 + 40*C^2*a*b^2*c*z^2 + 16*B^2*a*b*c^2*z^2 - 4*B^2*b^3*c*z^2 - \\
& 32*A^2*a*c^3*z^2 - 96*C^2*a^2*c^2*z^2 + 8*A^2*b^2*c^2*z^2 - 4*C^2*b^4*z^2 \\
& - 16*A*C^2*a*b*c*z - 4*A^2*C*b^2*c*z + 16*A^2*C*a*c^2*z + 4*A*B^2*b^2*c*z - \\
& 16*A*B^2*a*c^2*z + 16*C^3*a^2*c*z - 4*C^3*a*b^2*z + 4*A*C^2*b^3*z + 4*A*B^ \\
& 2*C*a*c - 2*A^2*C^2*a*c + 2*A^3*C*b*c + 2*A*C^3*a*b - B^2*C^2*a*b - A^2*B^2 \\
& *b*c - B^4*a*c - A^2*C^2*b^2 - C^4*a^2 - A^4*c^2, z, k)*a*b*c*x)*root(128*a \\
& *b^2*c^3*z^4 - 16*b^4*c^2*z^4 - 256*a^2*c^4*z^4 - 128*C*a*b^2*c^2*z^3 + 256
\end{aligned}$$

```

*C*a^2*c^3*z^3 + 16*C*b^4*c*z^3 + 32*A*C*a*b*c^2*z^2 - 8*A*C*b^3*c*z^2 + 40
*C^2*a*b^2*c*z^2 + 16*B^2*a*b*c^2*z^2 - 4*B^2*b^3*c*z^2 - 32*A^2*a*c^3*z^2
- 96*C^2*a^2*c^2*z^2 + 8*A^2*b^2*c^2*z^2 - 4*C^2*b^4*z^2 - 16*A*C^2*a*b*c*z
- 4*A^2*C*b^2*c*z + 16*A^2*C*a*c^2*z + 4*A*B^2*b^2*c*z - 16*A*B^2*a*c^2*z
+ 16*C^3*a^2*c*z - 4*C^3*a*b^2*z + 4*A*C^2*b^3*z + 4*A*B^2*C*a*c - 2*A^2*C^
2*a*c + 2*A^3*C*b*c + 2*A*C^3*a*b - B^2*C^2*a*b - A^2*B^2*b*c - B^4*a*c - A
^2*C^2*b^2 - C^4*a^2 - A^4*c^2, z, k), k, 1, 4)

```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(C*x**2+B*x+A)/(c*x**4+b*x**2+a),x)

[Out] Timed out

$$3.25 \quad \int \frac{A+Bx+Cx^2}{a+bx^2+cx^4} dx$$

Optimal. Leaf size=211

$$\frac{\left(\frac{2Ac-bC}{\sqrt{b^2-4ac}} + C\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(C - \frac{2Ac-bC}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}} - \frac{B \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Rubi [A] time = 0.27, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1673, 1166, 205, 12, 1107, 618, 206}

$$\frac{\left(\frac{2Ac-bC}{\sqrt{b^2-4ac}} + C\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(C - \frac{2Ac-bC}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}} - \frac{B \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x + C*x^2)/(a + b*x^2 + c*x^4), x]

[Out] ((C + (2*A*c - b*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((C - (2*A*c - b*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (B*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1107

Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2

- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1673

Int[(Pq_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :> Module[{q = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && !PolyQ[Pq, x^2]

Rubi steps

$$\int \frac{A + Bx + Cx^2}{a + bx^2 + cx^4} dx = \int \frac{Bx}{a + bx^2 + cx^4} dx + \int \frac{A + Cx^2}{a + bx^2 + cx^4} dx$$

$$= B \int \frac{x}{a + bx^2 + cx^4} dx + \frac{1}{2} \left(C - \frac{2Ac - bC}{\sqrt{b^2 - 4ac}} \right) \int \frac{1}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2 - 4ac} + cx^2} dx + \frac{1}{2} \left(C + \frac{2Ac - bC}{\sqrt{b^2 - 4ac}} \right) \int \frac{1}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2 - 4ac} + cx^2} dx$$

$$= \frac{\left(C + \frac{2Ac - bC}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{\sqrt{2} \sqrt{c} \sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{\left(C - \frac{2Ac - bC}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b + \sqrt{b^2 - 4ac}}} \right)}{\sqrt{2} \sqrt{c} \sqrt{b + \sqrt{b^2 - 4ac}}} + \frac{1}{2} B \text{Subst} \left(\int \frac{1}{b^2 + cx^2} dx \right)$$

$$= \frac{\left(C + \frac{2Ac - bC}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{\sqrt{2} \sqrt{c} \sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{\left(C - \frac{2Ac - bC}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b + \sqrt{b^2 - 4ac}}} \right)}{\sqrt{2} \sqrt{c} \sqrt{b + \sqrt{b^2 - 4ac}}} - B \text{Subst} \left(\int \frac{1}{b^2 + cx^2} dx \right)$$

$$= \frac{\left(C + \frac{2Ac - bC}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{\sqrt{2} \sqrt{c} \sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{\left(C - \frac{2Ac - bC}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b + \sqrt{b^2 - 4ac}}} \right)}{\sqrt{2} \sqrt{c} \sqrt{b + \sqrt{b^2 - 4ac}}} - \frac{B \tanh^{-1} \left(\frac{b + 2cx^2}{\sqrt{b^2 - 4ac}} \right)}{\sqrt{b^2 - 4ac}}$$

Mathematica [A] time = 0.21, size = 234, normalized size = 1.11

$$\frac{\sqrt{2} \left(C \left(\sqrt{b^2 - 4ac} - b \right) + 2Ac \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{\sqrt{c} \sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{\sqrt{2} \left(C \left(\sqrt{b^2 - 4ac} + b \right) - 2Ac \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b + \sqrt{b^2 - 4ac}}} \right)}{\sqrt{c} \sqrt{b + \sqrt{b^2 - 4ac}}} + B \log \left(\sqrt{b^2 - 4ac} - b - 2cx^2 \right) - B \log \left(\sqrt{b^2 - 4ac} + b + 2cx^2 \right)}{2\sqrt{b^2 - 4ac}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x + C*x^2)/(a + b*x^2 + c*x^4), x]

[Out] ((Sqrt[2]*(2*A*c + (-b + Sqrt[b^2 - 4*a*c]))*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/(Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(-2*A*c + (b + Sqrt[b^2 - 4*a*c]))*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]/(Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + B*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2] - B*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(2*Sqrt[b^2 - 4*a*c])

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx + Cx^2}{a + bx^2 + cx^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B*x + C*x^2)/(a + b*x^2 + c*x^4),x]

[Out] IntegrateAlgebraic[(A + B*x + C*x^2)/(a + b*x^2 + c*x^4), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] Timed out

giac [B] time = 4.37, size = 1616, normalized size = 7.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out]
$$-1/2*(b^2*c^2 - 4*a*c^3 - 2*b*c^3 + c^4)*\sqrt{b^2 - 4*a*c}*B*\log(x^2 + 1/2*(b + \sqrt{b^2 - 4*a*c}))/c)/((b^4 - 8*a*b^2*c - 2*b^3*c + 16*a^2*c^2 + 8*a*b*c^2 + b^2*c^2 - 4*a*c^3)*c^2) + 1/2*(b^2*c^2 - 4*a*c^3 - 2*b*c^3 + c^4)*\sqrt{b^2 - 4*a*c}*B*\log(x^2 + 1/2*(b - \sqrt{b^2 - 4*a*c}))/c)/((b^4 - 8*a*b^2*c - 2*b^3*c + 16*a^2*c^2 + 8*a*b*c^2 + b^2*c^2 - 4*a*c^3)*c^2) + 1/4*((\sqrt{2})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^4 - 8*\sqrt{2})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c - 2*\sqrt{2})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^3*c - 2*b^4*c + 16*\sqrt{2})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*c^2 + 8*\sqrt{2})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b*c^2 + \sqrt{2})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^2*c^2 + 16*a*b^2*c^2 - 2*b^3*c^2 - 4*\sqrt{2})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*c^3 - 32*a^2*c^3 + 8*a*b*c^3 + \sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^3 - 4*\sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b*c - 2*\sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^2*c + \sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 + 2*(b^2 - 4*a*c)*b*c^2)*A + 2*(2*a*b^2*c^2 - 8*a^2*c^3 - \sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^2 + 4*\sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*c + 2*\sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b*c - \sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*C)*\arctan(2*\sqrt{1/2}*x/\sqrt{(b + \sqrt{b^2 - 4*a*c}))/c}))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*\text{abs}(c)) + 1/4*((\sqrt{2})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^4 - 8*\sqrt{2})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c - 2*\sqrt{2})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^3*c + 2*b^4*c + 16*\sqrt{2})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*c^2 + 8*\sqrt{2})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b*c^2 + \sqrt{2})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^2*c^2 - 16*a*b^2*c^2 - 2*b^3*c^2 - 4*\sqrt{2})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*c^3 + 32*a^2*c^3 + 8*a*b*c^3 + \sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^3 - 4*\sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b*c - 2*\sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^2*c + \sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b*c^2 - 2*(b^2 - 4*a*c)*b^2*c + 8*(b^2 - 4*a*c)*a*c^2 + 2*(b^2 - 4*a*c)*b*c^2)*A + 2*(2*a*b^2*c^2 - 8*a^2*c^3 - \sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^2 + 4*\sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*c + 2*\sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b*c - \sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*C)*\arctan(2*\sqrt{1/2}*x/\sqrt{(b - \sqrt{b^2 - 4*a*c}))/c}))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*\text{abs}(c))$$

maple [B] time = 0.02, size = 616, normalized size = 2.92

$$\frac{2\sqrt{c}\operatorname{Arctanh}\left(\frac{\sqrt{a-cx}}{\sqrt{b+\sqrt{4ac+c^2}}}\right)}{(4a-c)\sqrt{b+\sqrt{4ac+c^2}}}-\frac{2\sqrt{c}\operatorname{Arctanh}\left(\frac{\sqrt{a-cx}}{\sqrt{b-\sqrt{4ac+c^2}}}\right)}{(4a-c)\sqrt{b-\sqrt{4ac+c^2}}}-\frac{\sqrt{c}\operatorname{P}\operatorname{arctanh}\left(\frac{\sqrt{a-cx}}{\sqrt{b+\sqrt{4ac+c^2}}}\right)}{2(4a-c)\sqrt{b+\sqrt{4ac+c^2}}}-\frac{\sqrt{c}\operatorname{P}\operatorname{arctanh}\left(\frac{\sqrt{a-cx}}{\sqrt{b-\sqrt{4ac+c^2}}}\right)}{2(4a-c)\sqrt{b-\sqrt{4ac+c^2}}}-\frac{\sqrt{4ac+c^2}\sqrt{c}\operatorname{Arctanh}\left(\frac{\sqrt{a-cx}}{\sqrt{b+\sqrt{4ac+c^2}}}\right)}{(4a-c)\sqrt{b+\sqrt{4ac+c^2}}}-\frac{\sqrt{4ac+c^2}\sqrt{c}\operatorname{Arctanh}\left(\frac{\sqrt{a-cx}}{\sqrt{b-\sqrt{4ac+c^2}}}\right)}{(4a-c)\sqrt{b-\sqrt{4ac+c^2}}}-\frac{\sqrt{4ac+c^2}\sqrt{c}\operatorname{Arctanh}\left(\frac{\sqrt{a-cx}}{\sqrt{b+\sqrt{4ac+c^2}}}\right)}{2(4a-c)\sqrt{b+\sqrt{4ac+c^2}}}-\frac{\sqrt{4ac+c^2}\sqrt{c}\operatorname{Arctanh}\left(\frac{\sqrt{a-cx}}{\sqrt{b-\sqrt{4ac+c^2}}}\right)}{2(4a-c)\sqrt{b-\sqrt{4ac+c^2}}}-\frac{\sqrt{4ac+c^2}\sqrt{c}\operatorname{Arctanh}\left(\frac{\sqrt{a-cx}}{\sqrt{b+\sqrt{4ac+c^2}}}\right)}{2(4a-c)\sqrt{b+\sqrt{4ac+c^2}}}-\frac{\sqrt{4ac+c^2}\sqrt{c}\operatorname{Arctanh}\left(\frac{\sqrt{a-cx}}{\sqrt{b-\sqrt{4ac+c^2}}}\right)}{2(4a-c)\sqrt{b-\sqrt{4ac+c^2}}}-\frac{\sqrt{4ac+c^2}\sqrt{c}\operatorname{Arctanh}\left(\frac{\sqrt{a-cx}}{\sqrt{b+\sqrt{4ac+c^2}}}\right)}{2(4a-c)\sqrt{b+\sqrt{4ac+c^2}}}-\frac{\sqrt{4ac+c^2}\sqrt{c}\operatorname{Arctanh}\left(\frac{\sqrt{a-cx}}{\sqrt{b-\sqrt{4ac+c^2}}}\right)}{2(4a-c)\sqrt{b-\sqrt{4ac+c^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((C*x^2+B*x+A)/(c*x^4+b*x^2+a),x)`

[Out]
$$-1/2*(-4*a*c+b^2)^{(1/2)}/(4*a*c-b^2)*B*\ln(-2*c*x^2-b+(-4*a*c+b^2)^{(1/2)})+c*(-4*a*c+b^2)^{(1/2)}/(4*a*c-b^2)*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*A-2*c/(4*a*c-b^2)*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*C*a+1/2/(4*a*c-b^2)*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*C*b^2-1/2*(-4*a*c+b^2)^{(1/2)}/(4*a*c-b^2)*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b*C+1/2*(-4*a*c+b^2)^{(1/2)}/(4*a*c-b^2)*B*\ln(2*c*x^2+b+(-4*a*c+b^2)^{(1/2)})+c*(-4*a*c+b^2)^{(1/2)}/(4*a*c-b^2)*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*A+2*c/(4*a*c-b^2)*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*C*a-1/2/(4*a*c-b^2)*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*C*b^2-1/2*(-4*a*c+b^2)^{(1/2)}/(4*a*c-b^2)*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b*C$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Cx^2 + Bx + A}{cx^4 + bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")`

[Out] `integrate((C*x^2 + B*x + A)/(c*x^4 + b*x^2 + a), x)`

mupad [B] time = 2.31, size = 3942, normalized size = 18.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B*x + C*x^2)/(a + b*x^2 + c*x^4),x)`

[Out] `symsum(log(A*B^2*c^2 - A^2*C*c^2 + B^3*c^2*x - C^3*a*c + A*C^2*b*c - 8*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 16*A*C*a*b^2*c*z^2 - 16*C^2*a^2*b*c*z^2 - 8*B^2*a*b^2*c*z^2 - 16*A^2*a*b*c^2*z^2 + 64*A*C*a^2*c^2*z^2 + 4*C^2*a*b^3*z^2 + 4*A^2*b^3*c*z^2 + 32*B^2*a^2*c^2*z^2 + 16*B*C^2*a^2*c*z + 4*A^2*B*b^2*c*z - 4*B*C^2*a*b^2*z - 16*A^2*B*a*c^2*z - 4*A*B^2*C*a*c + 2*A^2*C^2*a*c - 2*A^3*C*b*c - 2*A*C^3*a*b + B^2*C^2*a*b + A^2*B^2*b*c + B^4*a*c + A^2*C^2*b^2 + C^4*a^2 + A^4*c^2, z, k)^3*b^3*c^2*x - 16*A*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 16*A*C*a*b^2*c*z^2 - 16*C^2*a^2*b*c*z^2 - 8*B^2*a*b^2*c*z^2 - 16*A^2*a*b*c^2*z^2 + 64*A*C*a^2*c^2*z^2 + 4*C^2*a*b^3*z^2 + 4*A^2*b^3*c*z^2 + 32*B^2*a^2*c^2*z^2 + 16*B*C^2*a^2*c*z + 4*A^2*B*b^2*c*z - 4*B*C^2*a*b^2*z - 16*A^2*B*a*c^2*z - 4*A*B^2*C*a*c + 2*A^2*C^2*a*c - 2*A^3*C*b*c - 2*A*C^3*a*b + B^2*C^2*a*b + A^2*B^2*b*c + B^4*a*c + A^2*C^2*b^2 + C^4*a^2 + A^4*c^2, z, k)^2*a*c^3 - 4*A^2*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 16*A*C*a*b^2*c*z^2 - 16*C^2*a^2*b*c*z^2 - 8*B^2*a*b^2*c*z^2 - 16*A^2*a*b*c^2*z^2 + 64*A*C*a^2*c^2*z^2 + 4*C^2*a*b^3*z^2 + 4*A^2*b^3*c*z^2 + 32*B^2*a^2*c^2*z^2 + 16*B*C^2*a^2*c*z + 4*A^2*B*b^2*c*z - 4*B*C^2*a*b^2*z - 16*A^2*B*a*c^2*z - 4*A*B^2*C*a*c + 2*A^2*C^2*a*c - 2*A^3*C*b*c - 2*A*C^3*a*b + B^2*C^2*a*b + A^2*B^2*b*c + B^4*a*c + A^2*C^2*b^2 + C^4*a^2 + A^4*c^2, z, k)*c^3*x + 4*A*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 16*A*C*a*b^2*c*z^2 - 16*C^2*a^2*b*c*z^2 - 8*B^2*a*b^2*c*z^2 - 16*A^2*a*b*c^2*z^2 + 64*A*C*a^2*c^2*z^2 + 4*C^2*a*b^3*z^2 + 4*A^2*b^3*c*z^2 + 32*B^2*a^2*c^2*z^2 + 16*B*C^2*a^2*c*z + 4*A^2*B*b^2*c*z - 4*B*C^2*a*b^2*z - 16*A^2*B*a*c^2*z - 4*A*B^2*`

$$B^2*a^2*c^2*z^2 + 16*B*C^2*a^2*c*z + 4*A^2*B*b^2*c*z - 4*B*C^2*a*b^2*z - 16$$

$$*A^2*B*a*c^2*z - 4*A*B^2*C*a*c + 2*A^2*C^2*a*c - 2*A^3*C*b*c - 2*A*C^3*a*b$$

$$+ B^2*C^2*a*b + A^2*B^2*b*c + B^4*a*c + A^2*C^2*b^2 + C^4*a^2 + A^4*c^2, z,$$

$$k), k, 1, 4)$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x**2+B*x+A)/(c*x**4+b*x**2+a),x)

[Out] Timed out

$$3.26 \quad \int \frac{A+Bx+Cx^2}{x(a+bx^2+cx^4)} dx$$

Optimal. Leaf size=229

$$\frac{(Ab - 2aC) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right) - A \log(a + bx^2 + cx^4) + A \log(x) + \frac{\sqrt{2} B \sqrt{c} \tan^{-1}\left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b - \sqrt{b^2-4ac}}}\right) - \sqrt{2} B \sqrt{c} \tan^{-1}\left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b + \sqrt{b^2-4ac}}}\right)}{\sqrt{b^2 - 4ac} \sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{\sqrt{2} B \sqrt{c} \tan^{-1}\left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b + \sqrt{b^2-4ac}}}\right) - \sqrt{2} B \sqrt{c} \tan^{-1}\left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b - \sqrt{b^2-4ac}}}\right)}{\sqrt{b^2 - 4ac} \sqrt{b + \sqrt{b^2 - 4ac}}}}{2a\sqrt{b^2 - 4ac} - \frac{A \log(a + bx^2 + cx^4)}{4a} + \frac{A \log(x)}{a} + \frac{\sqrt{2} B \sqrt{c} \tan^{-1}\left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b - \sqrt{b^2-4ac}}}\right) - \sqrt{2} B \sqrt{c} \tan^{-1}\left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b + \sqrt{b^2-4ac}}}\right)}{\sqrt{b^2 - 4ac} \sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{\sqrt{2} B \sqrt{c} \tan^{-1}\left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b + \sqrt{b^2-4ac}}}\right) - \sqrt{2} B \sqrt{c} \tan^{-1}\left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b - \sqrt{b^2-4ac}}}\right)}{\sqrt{b^2 - 4ac} \sqrt{b + \sqrt{b^2 - 4ac}}}}$$

Rubi [A] time = 0.26, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {1662, 1251, 800, 634, 618, 206, 628, 12, 1093, 205}

$$\frac{(Ab - 2aC) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right) - A \log(a + bx^2 + cx^4) + A \log(x) + \frac{\sqrt{2} B \sqrt{c} \tan^{-1}\left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b - \sqrt{b^2-4ac}}}\right) - \sqrt{2} B \sqrt{c} \tan^{-1}\left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b + \sqrt{b^2-4ac}}}\right)}{\sqrt{b^2 - 4ac} \sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{\sqrt{2} B \sqrt{c} \tan^{-1}\left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b + \sqrt{b^2-4ac}}}\right) - \sqrt{2} B \sqrt{c} \tan^{-1}\left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b - \sqrt{b^2-4ac}}}\right)}{\sqrt{b^2 - 4ac} \sqrt{b + \sqrt{b^2 - 4ac}}}}{2a\sqrt{b^2 - 4ac} - \frac{A \log(a + bx^2 + cx^4)}{4a} + \frac{A \log(x)}{a} + \frac{\sqrt{2} B \sqrt{c} \tan^{-1}\left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b - \sqrt{b^2-4ac}}}\right) - \sqrt{2} B \sqrt{c} \tan^{-1}\left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b + \sqrt{b^2-4ac}}}\right)}{\sqrt{b^2 - 4ac} \sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{\sqrt{2} B \sqrt{c} \tan^{-1}\left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b + \sqrt{b^2-4ac}}}\right) - \sqrt{2} B \sqrt{c} \tan^{-1}\left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b - \sqrt{b^2-4ac}}}\right)}{\sqrt{b^2 - 4ac} \sqrt{b + \sqrt{b^2 - 4ac}}}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x + C*x^2)/(x*(a + b*x^2 + c*x^4)), x]

[Out] (Sqrt[2]*B*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]) / (Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*B*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]) / (Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + ((A*b - 2*a*C)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]]) / (2*a*Sqrt[b^2 - 4*a*c]) + (A*Log[x])/a - (A*Log[a + b*x^2 + c*x^4]) / (4*a)

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1093

```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]
```

Rule 1662

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2 + 1}]*((a + b*x^2 + c*x^4)^p), x] + Dist[1/d, Int[(d*x)^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*((a + b*x^2 + c*x^4)^p), x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] && !PolyQ[Pq, x^2]
```

Rubi steps

$$\begin{aligned}
\int \frac{A + Bx + Cx^2}{x(a + bx^2 + cx^4)} dx &= \int \frac{B}{a + bx^2 + cx^4} dx + \int \frac{A + Cx^2}{x(a + bx^2 + cx^4)} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{A + Cx}{x(a + bx + cx^2)} dx, x, x^2 \right) + B \int \frac{1}{a + bx^2 + cx^4} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \left(\frac{A}{ax} + \frac{-Ab + aC - Acx}{a(a + bx + cx^2)} \right) dx, x, x^2 \right) + \frac{(Bc) \int \frac{1}{\frac{b}{2} - \frac{1}{2} \sqrt{b^2 - 4ac} + cx^2} dx}{\sqrt{b^2 - 4ac}} - \frac{(Bc)}{\sqrt{b^2 - 4ac}} \\
&= \frac{\sqrt{2} B \sqrt{c} \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{\sqrt{b^2 - 4ac} \sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{\sqrt{2} B \sqrt{c} \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b + \sqrt{b^2 - 4ac}}} \right)}{\sqrt{b^2 - 4ac} \sqrt{b + \sqrt{b^2 - 4ac}}} + \frac{A \log(x)}{a} + \frac{\text{Subst} \left(\int \frac{1}{a + bx + cx^2} dx, x, x^2 \right)}{2} \\
&= \frac{\sqrt{2} B \sqrt{c} \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{\sqrt{b^2 - 4ac} \sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{\sqrt{2} B \sqrt{c} \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b + \sqrt{b^2 - 4ac}}} \right)}{\sqrt{b^2 - 4ac} \sqrt{b + \sqrt{b^2 - 4ac}}} + \frac{A \log(x)}{a} - \frac{A \text{Subst} \left(\int \frac{1}{a + bx + cx^2} dx, x, x^2 \right)}{2} \\
&= \frac{\sqrt{2} B \sqrt{c} \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{\sqrt{b^2 - 4ac} \sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{\sqrt{2} B \sqrt{c} \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b + \sqrt{b^2 - 4ac}}} \right)}{\sqrt{b^2 - 4ac} \sqrt{b + \sqrt{b^2 - 4ac}}} + \frac{A \log(x)}{a} - \frac{A \log(a)}{a} \\
&= \frac{\sqrt{2} B \sqrt{c} \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{\sqrt{b^2 - 4ac} \sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{\sqrt{2} B \sqrt{c} \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b + \sqrt{b^2 - 4ac}}} \right)}{\sqrt{b^2 - 4ac} \sqrt{b + \sqrt{b^2 - 4ac}}} + \frac{(Ab - 2aC) \tanh^{-1} \left(\frac{\sqrt{b^2 - 4ac} x}{a + bx^2 + cx^4} \right)}{2a\sqrt{b^2 - 4ac}}
\end{aligned}$$

Mathematica [A] time = 0.44, size = 285, normalized size = 1.24

$$\frac{(A(\sqrt{b^2 - 4ac} + b) - 2aC) \log(\sqrt{b^2 - 4ac} - b - 2cx^2)}{4a\sqrt{b^2 - 4ac}} - \frac{(A(\sqrt{b^2 - 4ac} - b) + 2aC) \log(\sqrt{b^2 - 4ac} + b + 2cx^2)}{4a\sqrt{b^2 - 4ac}} + \frac{A \log(x)}{a} + \frac{\sqrt{2} B \sqrt{c} \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{\sqrt{b^2 - 4ac} \sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{\sqrt{2} B \sqrt{c} \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b + \sqrt{b^2 - 4ac}}} \right)}{\sqrt{b^2 - 4ac} \sqrt{b + \sqrt{b^2 - 4ac}}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x + C*x^2)/(x*(a + b*x^2 + c*x^4)), x]

[Out] (Sqrt[2]*B*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*B*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (A*Log[x])/a - ((A*(b + Sqrt[b^2 - 4*a*c]) - 2*a*C)*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(4*a*Sqrt[b^2 - 4*a*c]) - ((A*(-b + Sqrt[b^2 - 4*a*c]) + 2*a*C)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(4*a*Sqrt[b^2 - 4*a*c])

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx + Cx^2}{x(a + bx^2 + cx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B*x + C*x^2)/(x*(a + b*x^2 + c*x^4)), x]

[Out] IntegrateAlgebraic[(A + B*x + C*x^2)/(x*(a + b*x^2 + c*x^4)), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)/x/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] Timed out

giac [B] time = 5.08, size = 2336, normalized size = 10.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)/x/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out]
$$-1/4*A*\log(\text{abs}(c*x^4 + b*x^2 + a))/a + A*\log(\text{abs}(x))/a + 1/4*((\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*b^4 - 8*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a*b^2*c - 2*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*b^3*c + 2*b^4*c + 16*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a^2*c^2 + 8*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a*b*c^2 + \text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*b^2*c^2 - 16*a*b^2*c^2 - 4*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a*c^3 + 32*a^2*c^3 - 2*(b^2 - 4*a*c)*b^2*c + 8*(b^2 - 4*a*c)*a*c^2)*B*\text{abs}(c) + (2*b^3*c^3 - 8*a*b*c^4 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*b^3*c + 4*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a*b*c^2 + 2*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*b^2*c^2 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*b*c^3 - 2*(b^2 - 4*a*c)*b*c^3)*B)*\arctan(2*\text{sqrt}(1/2)*x/\text{sqrt}((a^2*b*c + \text{sqrt}(a^4*b^2*c^2 - 4*a^5*c^3)))/(a^2*c^2)))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2) + 1/4*((\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*b^4 - 8*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a*b^2*c - 2*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*b^3*c - 2*b^4*c + 16*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^2*c^2 + 8*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a*b*c^2 + \text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*b^2*c^2 + 16*a*b^2*c^2 - 4*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a*c^3 - 32*a^2*c^3 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2)*B*\text{abs}(c) - (2*b^3*c^3 - 8*a*b*c^4 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*b^3*c + 4*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a*b*c^2 + 2*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*b^2*c^2 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*b*c^3 - 2*(b^2 - 4*a*c)*b*c^3)*B)*\arctan(2*\text{sqrt}(1/2)*x/\text{sqrt}((a^2*b*c - \text{sqrt}(a^4*b^2*c^2 - 4*a^5*c^3)))/(a^2*c^2)))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2) - 1/16*((b^6*c - 8*a*b^4*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a*b^3*c^3 + b^4*c^3 - 4*a*b^2*c^4 - (b^5*c - 8*a*b^3*c^2 - 2*b^4*c^2 + 16*a^2*b*c^3 + 8*a*b^2*c^3 + b^3*c^3 - 4*a*b*c^4))*\text{sqrt}(b^2 - 4*a*c))*A*\text{abs}(c) - 2*(a*b^5*c - 8*a^2*b^3*c^2 - 2*a*b^4*c^2 + 16*a^3*b*c^3 + 8*a^2*b^2*c^3 + a*b^3*c^3 - 4*a^2*b*c^4 - (a*b^4*c - 8*a^2*b^2*c^2 - 2*a*b^3*c^2 + 16*a^3*c^3 + 8*a^2*b*c^3 + a*b^2*c^3 - 4*a^2*c^4))*\text{sqrt}(b^2 - 4*a*c))*C*\text{abs}(c) + (b^6*c^2 - 8*a*b^4*c^3 - 2*b^5*c^3 + 16*a^2*b^2*c^4 + 8*a*b^3*c^4 + b^4*c^4 - 4*a*b^2*c^5 + (b^5*c^2 - 4*a*b^3*c^3 - 2*b^4*c^3 + b^3*c^4))*\text{sqrt}(b^2 - 4*a*c))*A - 2*(a*b^5*c^2 - 8*a^2*b^3*c^3 - 2*a*b^4*c^3 + 16*a^3*b*c^4 + 8*a^2*b^2*c^4 + a*b^3*c^4 - 4*a^2*b*c^5 - (a*b^4*c^2 - 4*a^2*b^2*c^3 - 2*a*b^3*c^3 + a*b^2*c^4))*\text{sqrt}(b^2 - 4*a*c))*C)*\log(x^2 + 1/2*(a^2*b*c + \text{sqrt}(a^4*b^2*c^2 - 4*a^5*c^3)))/(a^2*c^2)))/((a^2*b^4 - 8*a^3*b^2*c - 2*a^2*b^3*c + 16*a^4*c^2 + 8*a^3*b*c^2 + a^2*b^2*c^2 - 4*a^3*c^3)*c^2*\text{abs}(c)) - 1/16*((b^6*c - 8*a*b^4*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a*b^3*c^3 + b^4*c^3 - 4*a*b^2*c^4 + (b^5*c - 8*a*b^3*c^2 - 2*b^4*c^2 + 16*a^2*b*c^3 + 8*a*b^2*c^3 + b^3*c^3 - 4*a*b*c^4))*\text{sqrt}(b^2 - 4*a*c))*A*\text{abs}(c) - 2*(a*b^5*c - 8*a^2*b^3*c^2 - 2*a*b^4*c^2 + 16*a^3*b*c^3 + 8*a^2*b^2*c^3 + a*b^3*c^3 - 4*a^2*b*c^4 + (a*b^4*c - 8*a^2*b^2*c^2 - 2*a*b^3*c^2 + 16*a^3*c^3 + 8*a^2*b*c^3 + a*b^2*c^3 - 4*a^2*c^3$$

$$c^4) \sqrt{b^2 - 4ac}) C \operatorname{abs}(c) + (b^6 c^2 - 8 a b^4 c^3 - 2 b^5 c^3 + 16 a^2 b^2 c^4 + 8 a b^3 c^4 + b^4 c^4 - 4 a b^2 c^5 + (b^5 c^2 - 4 a b^3 c^3 - 2 b^4 c^3 + b^3 c^4) \sqrt{b^2 - 4ac}) A - 2 (a b^5 c^2 - 8 a^2 b^3 c^3 - 2 a b^4 c^3 + 16 a^3 b c^4 + 8 a^2 b^2 c^4 + a b^3 c^4 - 4 a^2 b c^5 + (a b^4 c^2 - 4 a^2 b^2 c^3 - 2 a b^3 c^3 + a b^2 c^4) \sqrt{b^2 - 4ac}) C \log(x^2 + \frac{1}{2}(a^2 b c - \sqrt{a^4 b^2 c^2 - 4 a^5 c^3})) / (a^2 c^2) / ((a^2 b^4 - 8 a^3 b^2 c - 2 a^2 b^3 c + 16 a^4 c^2 + 8 a^3 b c^2 + a^2 b^2 c^2 - 4 a^3 c^3) c^2 \operatorname{abs}(c))$$

maple [B] time = 0.04, size = 488, normalized size = 2.13

$$\frac{A^2 \ln\left(\frac{2a^2 - b + \sqrt{4ac + b^2}}{(4ac - b^2)^2}\right)}{(4ac - b^2)^2} + \frac{A^2 \ln\left(\frac{2a^2 + b + \sqrt{4ac + b^2}}{(4ac - b^2)^2}\right)}{(4ac - b^2)^2} + \frac{A \operatorname{arctan}\left(\frac{2a^2 - b + \sqrt{4ac + b^2}}{(4ac - b^2)}\right)}{(4ac - b^2)} + \frac{A \operatorname{arctan}\left(\frac{2a^2 + b + \sqrt{4ac + b^2}}{(4ac - b^2)}\right)}{(4ac - b^2)} + \frac{\sqrt{4ac + b^2} \sqrt{B} \operatorname{arctan}\left(\frac{a^2}{\sqrt{(4ac + b^2)z}}\right)}{(4ac - b^2) \sqrt{(4ac + b^2)z}} + \frac{\sqrt{4ac + b^2} \sqrt{B} \operatorname{arctan}\left(\frac{a^2}{\sqrt{(4ac + b^2)z}}\right)}{(4ac - b^2) \sqrt{(4ac + b^2)z}} + \frac{\sqrt{4ac + b^2} \operatorname{arctan}\left(\frac{2a^2 - b + \sqrt{4ac + b^2}}{(4ac - b^2)}\right)}{(4ac - b^2)^2} + \frac{\sqrt{4ac + b^2} \operatorname{arctan}\left(\frac{2a^2 + b + \sqrt{4ac + b^2}}{(4ac - b^2)}\right)}{(4ac - b^2)^2} + \frac{2\sqrt{4ac + b^2} C \ln\left(\frac{2a^2 - b + \sqrt{4ac + b^2}}{(4ac - b^2)}\right)}{(4ac - b^2)^2} + \frac{2\sqrt{4ac + b^2} C \ln\left(\frac{2a^2 + b + \sqrt{4ac + b^2}}{(4ac - b^2)}\right)}{(4ac - b^2)^2} + \frac{A \ln(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((C*x^2+B*x+A)/x/(c*x^4+b*x^2+a), x)`

[Out] $A/a \ln(x) - 4c/(16ac - 4b^2) \ln(-2cx^2 - b + (-4ac + b^2)^{1/2}) A + 1/a/(16ac - 4b^2) \ln(-2cx^2 - b + (-4ac + b^2)^{1/2}) A b^2 + 1/a(-4ac + b^2)^{1/2}/(16ac - 4b^2) \ln(-2cx^2 - b + (-4ac + b^2)^{1/2}) A b - 2(-4ac + b^2)^{1/2}/(16ac - 4b^2) \ln(-2cx^2 - b + (-4ac + b^2)^{1/2}) C + 4c(-4ac + b^2)^{1/2}/(16ac - 4b^2) B^2^{1/2}/((-b + (-4ac + b^2)^{1/2})c)^{1/2} \operatorname{arctanh}(2^{1/2}/((-b + (-4ac + b^2)^{1/2})c)^{1/2} c x) - 4c/(16ac - 4b^2) \ln(2cx^2 + b + (-4ac + b^2)^{1/2}) A + 1/a/(16ac - 4b^2) \ln(2cx^2 + b + (-4ac + b^2)^{1/2}) A b^2 - 1/a(-4ac + b^2)^{1/2}/(16ac - 4b^2) \ln(2cx^2 + b + (-4ac + b^2)^{1/2}) A b + 2(-4ac + b^2)^{1/2}/(16ac - 4b^2) \ln(2cx^2 + b + (-4ac + b^2)^{1/2}) C + 4c(-4ac + b^2)^{1/2}/(16ac - 4b^2) B^2^{1/2}/((b + (-4ac + b^2)^{1/2})c)^{1/2} \operatorname{arctan}(2^{1/2}/((b + (-4ac + b^2)^{1/2})c)^{1/2} c x)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((C*x^2+B*x+A)/x/(c*x^4+b*x^2+a), x, algorithm="maxima")`

[Out] $A \log(x)/a - \operatorname{integrate}((A c x^3 - B a - (C a - A b) x)/(c x^4 + b x^2 + a), x)/a$

mupad [B] time = 1.49, size = 2258, normalized size = 9.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B*x + C*x^2)/(x*(a + b*x^2 + c*x^4)), x)`

[Out] $\operatorname{symsum}(\log(x(B^4 c^3 + C^4 a c^2 + A^2 C^2 c^3 - 3 A B^2 C c^3 - A C^3 b c^2 + B^2 C^2 b c^2) - \operatorname{root}(128 a^3 b^2 c z^4 - 256 a^4 c^2 z^4 - 16 a^2 b^4 z^4 + 128 A a^2 b^2 c z^3 - 256 A a^3 c^2 z^3 - 16 A a b^4 z^3 + 32 A C a^2 b c z^2 - 8 A C a b^3 z^2 + 16 B^2 a^2 b c z^2 + 40 A^2 a b^2 c z^2 - 32 C^2 a^3 c z^2 - 4 B^2 a b^3 z^2 + 8 C^2 a^2 b^2 z^2 - 96 A^2 a^2 c^2 z^2 - 4 A^2 b^4 z^2 + 16 A^2 C a b c z + 16 B^2 C a^2 c z - 16 A C^2 a^2 c z - 4 B^2 C a b^2 z + 4 A C^2 a b^2 z + 4 A^3 b^2 c z - 16 A^3 a c^2 z - 4 A^2 C b^3 z + 4 A B^2 C a c - 2 A^2 C^2 a c + 2 A^3 C b c + 2 A C^3 a b - B^2 C^2 a b - A^2 B^2 b c - B^4 a c - A^2 C^2 b^2 - C^4 a^2 - A^4 c^2, z, k))(x(A B^2 b c^3 - 5 A^3 c^4 - 13 A C^2 a c^3 + 6 A^2 C b c^3 + 17 B^2 C a c^3 + C^3 a b c^2 + A C^2 b^2 c^2 - 4 B^2 C b^2 c^2) - \operatorname{root}(128 a^3 b^2 c z^4 - 256 a^4 c^2 z^4 - 16 a^2 b^4 z^4 + 128 A a^2 b^2 c z^3 - 256 A a^3 c^2 z^3 - 16 A a b^4 z^3 + 32 A C a^2 b c z^2 - 8 A C a b^3 z^2 + 16 B^2 a^2 b c z^2 + 40 A^2 a b^2 c z^2 - 32 C^2 a^3 c z^2 - 4 B^2 a b^3 z^2 + 8 C^2 a^2 b^2 z^2 - 96 A^2 a^2 c^2 z^2 - 4 A^2 b^4 z^2 + 16 A^2 C a b c z + 16 B^2 C a^2 c z - 16 A C^2 a^2 c z - 4 B^2 C a b^2 z + 4 A C^2 a b^2 z + 4 A^3 b^2 c z$

```

- 16*A^3*a*c^2*z - 4*A^2*C*b^3*z + 4*A*B^2*C*a*c - 2*A^2*C^2*a*c + 2*A^3*C*
b*c + 2*A*C^3*a*b - B^2*C^2*a*b - A^2*B^2*b*c - B^4*a*c - A^2*C^2*b^2 - C^4
*a^2 - A^4*c^2, z, k)*(x*(60*A^2*a*c^4 - 16*A^2*b^2*c^3 + 4*B^2*b^3*c^2 + 3
6*C^2*a^2*c^3 + 8*A*C*b^3*c^2 - 14*B^2*a*b*c^3 - 10*C^2*a*b^2*c^2 - 28*A*C*
a*b*c^3) + root(128*a^3*b^2*c*z^4 - 256*a^4*c^2*z^4 - 16*a^2*b^4*z^4 + 128*
A*a^2*b^2*c*z^3 - 256*A*a^3*c^2*z^3 - 16*A*a*b^4*z^3 + 32*A*C*a^2*b*c*z^2 -
8*A*C*a*b^3*z^2 + 16*B^2*a^2*b*c*z^2 + 40*A^2*a*b^2*c*z^2 - 32*C^2*a^3*c*z
^2 - 4*B^2*a*b^3*z^2 + 8*C^2*a^2*b^2*z^2 - 96*A^2*a^2*c^2*z^2 - 4*A^2*b^4*z
^2 + 16*A^2*C*a*b*c*z + 16*B^2*C*a^2*c*z - 16*A*C^2*a^2*c*z - 4*B^2*C*a*b^2
*z + 4*A*C^2*a*b^2*z + 4*A^3*b^2*c*z - 16*A^3*a*c^2*z - 4*A^2*C*b^3*z + 4*A
*B^2*C*a*c - 2*A^2*C^2*a*c + 2*A^3*C*b*c + 2*A*C^3*a*b - B^2*C^2*a*b - A^2*
B^2*b*c - B^4*a*c - A^2*C^2*b^2 - C^4*a^2 - A^4*c^2, z, k)*(x*(240*A*a^2*c^
4 + 12*A*b^4*c^2 - 108*A*a*b^2*c^3 + 4*C*a*b^3*c^2 - 16*C*a^2*b*c^3) + root
(128*a^3*b^2*c*z^4 - 256*a^4*c^2*z^4 - 16*a^2*b^4*z^4 + 128*A*a^2*b^2*c*z^3
- 256*A*a^3*c^2*z^3 - 16*A*a*b^4*z^3 + 32*A*C*a^2*b*c*z^2 - 8*A*C*a*b^3*z^
2 + 16*B^2*a^2*b*c*z^2 + 40*A^2*a*b^2*c*z^2 - 32*C^2*a^3*c*z^2 - 4*B^2*a*b^
3*z^2 + 8*C^2*a^2*b^2*z^2 - 96*A^2*a^2*c^2*z^2 - 4*A^2*b^4*z^2 + 16*A^2*C*a
*b*c*z + 16*B^2*C*a^2*c*z - 16*A*C^2*a^2*c*z - 4*B^2*C*a*b^2*z + 4*A*C^2*a*
b^2*z + 4*A^3*b^2*c*z - 16*A^3*a*c^2*z - 4*A^2*C*b^3*z + 4*A*B^2*C*a*c - 2*
A^2*C^2*a*c + 2*A^3*C*b*c + 2*A*C^3*a*b - B^2*C^2*a*b - A^2*B^2*b*c - B^4*a
*c - A^2*C^2*b^2 - C^4*a^2 - A^4*c^2, z, k))*x*(320*a^3*c^4 + 24*a*b^4*c^2 -
176*a^2*b^2*c^3) - 4*B*a*b^3*c^2 + 16*B*a^2*b*c^3) + 4*A*B*b^3*c^2 + 8*B*C
*a^2*c^3 - 12*A*B*a*b*c^3 - 4*B*C*a*b^2*c^2) + B^3*a*c^3 + 4*A^2*B*b*c^3 +
6*A*B*C*a*c^3 - 4*A*B*C*b^2*c^2 + B*C^2*a*b*c^2) + A*B^3*c^3 - 2*A^2*B*C*c^
3 + A*B*C^2*b*c^2)*root(128*a^3*b^2*c*z^4 - 256*a^4*c^2*z^4 - 16*a^2*b^4*z^
4 + 128*A*a^2*b^2*c*z^3 - 256*A*a^3*c^2*z^3 - 16*A*a*b^4*z^3 + 32*A*C*a^2*b
*c*z^2 - 8*A*C*a*b^3*z^2 + 16*B^2*a^2*b*c*z^2 + 40*A^2*a*b^2*c*z^2 - 32*C^2
*a^3*c*z^2 - 4*B^2*a*b^3*z^2 + 8*C^2*a^2*b^2*z^2 - 96*A^2*a^2*c^2*z^2 - 4*A
^2*b^4*z^2 + 16*A^2*C*a*b*c*z + 16*B^2*C*a^2*c*z - 16*A*C^2*a^2*c*z - 4*B^2
*C*a*b^2*z + 4*A*C^2*a*b^2*z + 4*A^3*b^2*c*z - 16*A^3*a*c^2*z - 4*A^2*C*b^3
*z + 4*A*B^2*C*a*c - 2*A^2*C^2*a*c + 2*A^3*C*b*c + 2*A*C^3*a*b - B^2*C^2*a*
b - A^2*B^2*b*c - B^4*a*c - A^2*C^2*b^2 - C^4*a^2 - A^4*c^2, z, k), k, 1, 4
) + (A*log(x))/a

```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x**2+B*x+A)/x/(c*x**4+b*x**2+a),x)

[Out] Timed out

$$3.27 \quad \int \frac{A+Bx+Cx^2}{x^2(a+bx^2+cx^4)} dx$$

Optimal. Leaf size=260

$$\frac{\sqrt{c} \left(\frac{Ab-2aC}{\sqrt{b^2-4ac}} + A \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b-\sqrt{b^2-4ac}}} \right) - \sqrt{c} \left(A - \frac{Ab-2aC}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{\sqrt{b^2-4ac}+b}} \right) - \frac{A}{ax} + \frac{bB \tanh^{-1} \left(\frac{b+2cx^2}{\sqrt{b^2-4ac}} \right)}{2a\sqrt{b^2-4ac}}}{\sqrt{2} a \sqrt{b-\sqrt{b^2-4ac}} - \sqrt{2} a \sqrt{\sqrt{b^2-4ac}+b}}$$

Rubi [A] time = 0.47, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1662, 1281, 1166, 205, 12, 1114, 705, 29, 634, 618, 206, 628}

$$\frac{\sqrt{c} \left(\frac{Ab-2aC}{\sqrt{b^2-4ac}} + A \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b-\sqrt{b^2-4ac}}} \right) - \sqrt{c} \left(A - \frac{Ab-2aC}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{\sqrt{b^2-4ac}+b}} \right) - \frac{A}{ax} + \frac{bB \tanh^{-1} \left(\frac{b+2cx^2}{\sqrt{b^2-4ac}} \right)}{2a\sqrt{b^2-4ac}} - \frac{B \log(a+bx^2+cx^4)}{4a} + \frac{B \log(x)}{a}}{\sqrt{2} a \sqrt{b-\sqrt{b^2-4ac}} - \sqrt{2} a \sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x + C*x^2)/(x^2*(a + b*x^2 + c*x^4)),x]

[Out] -(A/(a*x)) - (Sqrt[c]*(A + (A*b - 2*a*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*a*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(A - (A*b - 2*a*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (b*B*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a*Sqrt[b^2 - 4*a*c]) + (B*Log[x])/a - (B*Log[a + b*x^2 + c*x^4])/(4*a)

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 29

Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,

e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1281

```
Int[((f_.)*(x_)^(m_.))*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1662

```
Int[(Pq_)*((d_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2 + 1}]*((a + b*x^2 + c*x^4)^p), x] + Dist[1/d, Int[(d*x)^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*((a + b*x^2 + c*x^4)^p), x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] && !PolyQ[Pq, x^2]
```

Rubi steps

$$\begin{aligned}
\int \frac{A + Bx + Cx^2}{x^2(a + bx^2 + cx^4)} dx &= \int \frac{B}{x(a + bx^2 + cx^4)} dx + \int \frac{A + Cx^2}{x^2(a + bx^2 + cx^4)} dx \\
&= -\frac{A}{ax} - \frac{\int \frac{Ab - aC + Acx^2}{a + bx^2 + cx^4} dx}{a} + B \int \frac{1}{x(a + bx^2 + cx^4)} dx \\
&= -\frac{A}{ax} + \frac{1}{2} B \operatorname{Subst} \left(\int \frac{1}{x(a + bx + cx^2)} dx, x, x^2 \right) - \frac{\left(c \left(A - \frac{Ab - 2aC}{\sqrt{b^2 - 4ac}} \right) \right) \int \frac{1}{\frac{b}{2} + \frac{1}{2} \sqrt{b^2 - 4ac} + cx}}{2a} \\
&= -\frac{A}{ax} - \frac{\sqrt{c} \left(A + \frac{Ab - 2aC}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{\sqrt{2} a \sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{\sqrt{c} \left(A - \frac{Ab - 2aC}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{cx}}{\sqrt{b + \sqrt{b^2 - 4ac}}} \right)}{\sqrt{2} a \sqrt{b + \sqrt{b^2 - 4ac}}} \\
&= -\frac{A}{ax} - \frac{\sqrt{c} \left(A + \frac{Ab - 2aC}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{\sqrt{2} a \sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{\sqrt{c} \left(A - \frac{Ab - 2aC}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{cx}}{\sqrt{b + \sqrt{b^2 - 4ac}}} \right)}{\sqrt{2} a \sqrt{b + \sqrt{b^2 - 4ac}}} \\
&= -\frac{A}{ax} - \frac{\sqrt{c} \left(A + \frac{Ab - 2aC}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{\sqrt{2} a \sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{\sqrt{c} \left(A - \frac{Ab - 2aC}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{cx}}{\sqrt{b + \sqrt{b^2 - 4ac}}} \right)}{\sqrt{2} a \sqrt{b + \sqrt{b^2 - 4ac}}} \\
&= -\frac{A}{ax} - \frac{\sqrt{c} \left(A + \frac{Ab - 2aC}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{\sqrt{2} a \sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{\sqrt{c} \left(A - \frac{Ab - 2aC}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{cx}}{\sqrt{b + \sqrt{b^2 - 4ac}}} \right)}{\sqrt{2} a \sqrt{b + \sqrt{b^2 - 4ac}}} \\
&= -\frac{A}{ax} - \frac{\sqrt{c} \left(A + \frac{Ab - 2aC}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{\sqrt{2} a \sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{\sqrt{c} \left(A - \frac{Ab - 2aC}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{cx}}{\sqrt{b + \sqrt{b^2 - 4ac}}} \right)}{\sqrt{2} a \sqrt{b + \sqrt{b^2 - 4ac}}}
\end{aligned}$$

Mathematica [A] time = 1.08, size = 315, normalized size = 1.21

$$\frac{2\sqrt{2}\sqrt{c}\left(A\left(\sqrt{b^2-4ac}+b\right)-2aC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{2\sqrt{2}\sqrt{c}\left(A\left(\sqrt{b^2-4ac}-b\right)+2aC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}} + \frac{B\left(\sqrt{b^2-4ac}+b\right)\log\left(\sqrt{b^2-4ac}-b-2cx^2\right)}{\sqrt{b^2-4ac}} + \frac{B\left(\sqrt{b^2-4ac}-b\right)\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)}{\sqrt{b^2-4ac}} + \frac{4A}{x} - 4B\log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x + C*x^2)/(x^2*(a + b*x^2 + c*x^4)), x]

[Out]
$$-\frac{1}{4} \left(\frac{4A}{x} + \frac{2\sqrt{2}\sqrt{c}\left(A\left(\sqrt{b^2-4ac}+b\right)-2aC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{2\sqrt{2}\sqrt{c}\left(A\left(\sqrt{b^2-4ac}-b\right)+2aC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}} + \frac{B\left(\sqrt{b^2-4ac}+b\right)\log\left(\sqrt{b^2-4ac}-b-2cx^2\right)}{\sqrt{b^2-4ac}} + \frac{B\left(\sqrt{b^2-4ac}-b\right)\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)}{\sqrt{b^2-4ac}} + \frac{4A}{x} - 4B\log(x) \right)$$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx + Cx^2}{x^2(a + bx^2 + cx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B*x + C*x^2)/(x^2*(a + b*x^2 + c*x^4)), x]

[Out] IntegrateAlgebraic[(A + B*x + C*x^2)/(x^2*(a + b*x^2 + c*x^4)), x]

$$\begin{aligned}
& 4*a*c)*c)*a*b^2*c^3 + \text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*b^3*c^3 + 16 \\
& *a*b^3*c^3 - 4*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a*b*c^4 - 32*a^2*b*c \\
& ^4 + 2*(b^2 - 4*a*c)*b^3*c^2 - 8*(b^2 - 4*a*c)*a*b*c^3)*A*\text{abs}(c) + 2*(\text{sqrt}(\\
& 2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a*b^4*c - 8*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 \\
& - 4*a*c))*c)*a^2*b^2*c^2 - 2*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a*b^3*c \\
& ^2 - 2*a*b^4*c^2 + 16*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^3*c^3 + 8*s \\
& \text{qrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^2*b*c^3 + \text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(\\
& b^2 - 4*a*c))*c)*a*b^2*c^3 + 16*a^2*b^2*c^3 - 4*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 \\
& - 4*a*c))*c)*a^2*c^4 - 32*a^3*c^4 + 2*(b^2 - 4*a*c)*a*b^2*c^2 - 8*(b^2 - 4*a \\
& *c)*a^2*c^3)*C*\text{abs}(c) + (2*b^4*c^4 - 8*a*b^2*c^5 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c \\
&)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*b^4*c^2 + 4*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sq} \\
& \text{rt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a*b^2*c^3 + 2*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sq} \\
& \text{rt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*b^3*c^3 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sq} \\
& \text{rt}(b^2 - 4*a*c))*c)*b^2*c^4 - 2*(b^2 - 4*a*c)*b^2*c^4)*A - 2*(2*a*b^3*c^4 - \\
& 8*a^2*b*c^5 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a*b \\
& ^3*c^2 + 4*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^2*b* \\
& c^3 + 2*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a*b^2*c^3 \\
& - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a*b*c^4 - 2*(b \\
& ^2 - 4*a*c)*a*b*c^4)*C)*\arctan(2*\text{sqrt}(1/2)*x/\text{sqrt}((a^2*b*c - \text{sqrt}(a^4*b^2*c \\
& ^2 - 4*a^5*c^3))/(a^2*c^2)))/((a^2*b^4*c - 8*a^3*b^2*c^2 - 2*a^2*b^3*c^2 + \\
& 16*a^4*c^3 + 8*a^3*b*c^3 + a^2*b^2*c^3 - 4*a^3*c^4)*c^2) - 1/16*((b^6*c - 8 \\
& *a*b^4*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a*b^3*c^3 + b^4*c^3 - 4*a*b^2*c \\
& ^4 - (b^5*c - 8*a*b^3*c^2 - 2*b^4*c^2 + 16*a^2*b*c^3 + 8*a*b^2*c^3 + b^3*c^ \\
& 3 - 4*a*b*c^4)*\text{sqrt}(b^2 - 4*a*c))*B*\text{abs}(c) + (b^6*c^2 - 8*a*b^4*c^3 - 2*b^5 \\
& *c^3 + 16*a^2*b^2*c^4 + 8*a*b^3*c^4 + b^4*c^4 - 4*a*b^2*c^5 - (b^5*c^2 - 4* \\
& a*b^3*c^3 - 2*b^4*c^3 + b^3*c^4)*\text{sqrt}(b^2 - 4*a*c))*B)*\log(x^2 + 1/2*(a^2*b \\
& *c + \text{sqrt}(a^4*b^2*c^2 - 4*a^5*c^3))/(a^2*c^2)))/((a^2*b^4 - 8*a^3*b^2*c - 2* \\
& a^2*b^3*c + 16*a^4*c^2 + 8*a^3*b*c^2 + a^2*b^2*c^2 - 4*a^3*c^3)*c^2*\text{abs}(c)) \\
& - 1/16*((b^6*c - 8*a*b^4*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a*b^3*c^3 + \\
& b^4*c^3 - 4*a*b^2*c^4 + (b^5*c - 8*a*b^3*c^2 - 2*b^4*c^2 + 16*a^2*b*c^3 + 8 \\
& *a*b^2*c^3 + b^3*c^3 - 4*a*b*c^4)*\text{sqrt}(b^2 - 4*a*c))*B*\text{abs}(c) + (b^6*c^2 - \\
& 8*a*b^4*c^3 - 2*b^5*c^3 + 16*a^2*b^2*c^4 + 8*a*b^3*c^4 + b^4*c^4 - 4*a*b^2* \\
& c^5 + (b^5*c^2 - 4*a*b^3*c^3 - 2*b^4*c^3 + b^3*c^4)*\text{sqrt}(b^2 - 4*a*c))*B)*\log(x^2 + 1/2*(a^2*b*c - \text{sqrt}(a^4*b^2*c^2 - 4*a^5*c^3))/(a^2*c^2)))/((a^2*b^4 \\
& - 8*a^3*b^2*c - 2*a^2*b^3*c + 16*a^4*c^2 + 8*a^3*b*c^2 + a^2*b^2*c^2 - 4*a \\
& ^3*c^3)*c^2*\text{abs}(c))
\end{aligned}$$

maple [B] time = 0.04, size = 811, normalized size = 3.12

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((C*x^2+B*x+A)/x^2/(c*x^4+b*x^2+a), x)$

[Out]
$$\begin{aligned}
& -A/a/x+B/a*\ln(x)+1/a/(16*a*c-4*b^2)*\ln(-2*c*x^2-b+(-4*a*c+b^2)^{(1/2)})*B*(-4 \\
& *a*c+b^2)^{(1/2)}*b-4*c/(16*a*c-4*b^2)*\ln(-2*c*x^2-b+(-4*a*c+b^2)^{(1/2)})*B+1/ \\
& a/(16*a*c-4*b^2)*\ln(-2*c*x^2-b+(-4*a*c+b^2)^{(1/2)})*b^2*B-2/a*c/(16*a*c-4*b^ \\
& 2)*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\text{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b \\
& ^2)^{(1/2)})*c)^{(1/2)}*c*x)*A*(-4*a*c+b^2)^{(1/2)}*b+8*c^2/(16*a*c-4*b^2)*2^{(1/2 \\
&)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\text{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2 \\
&)}*c)^{(1/2)}*c*x)*A-2/a*c/(16*a*c-4*b^2)*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^ \\
& (1/2)*\text{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*A*b^2+4*c/(16* \\
& a*c-4*b^2)*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\text{arctanh}(2^{(1/2)}/((-b+(\\
& -4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*C*(-4*a*c+b^2)^{(1/2)}-1/a/(16*a*c-4*b^2)*\ln \\
& (2*c*x^2+b+(-4*a*c+b^2)^{(1/2)})*B*(-4*a*c+b^2)^{(1/2)}*b-4*c/(16*a*c-4*b^2)*\ln \\
& (2*c*x^2+b+(-4*a*c+b^2)^{(1/2)})*B+1/a/(16*a*c-4*b^2)*\ln(2*c*x^2+b+(-4*a*c+b^ \\
& 2)^{(1/2)})*b^2*B-2/a*c/(16*a*c-4*b^2)*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/ \\
& 2)}*\text{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*A*(-4*a*c+b^2)^{(1/2 \\
&)}*b-8*c^2/(16*a*c-4*b^2)*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\text{arctan}(2^ \\
& (1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*A+2/a*c/(16*a*c-4*b^2)*2^{(1/2)}/
\end{aligned}$$

$$\left((b + (-4ac + b^2)^{1/2})^c \right)^{1/2} \arctan\left(2^{1/2} / \left((b + (-4ac + b^2)^{1/2})^c \right)^{1/2} \right) \cdot (c \cdot x)^{A \cdot b^2 + 4c} / \left(16ac - 4b^2 \right)^{1/2} / \left((b + (-4ac + b^2)^{1/2})^c \right)^{1/2} \cdot \arctan\left(2^{1/2} / \left((b + (-4ac + b^2)^{1/2})^c \right)^{1/2} \right) \cdot C \cdot (-4ac + b^2)^{1/2}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)/x^2/(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] B*log(x)/a - integrate((B*c*x^3 + A*c*x^2 + B*b*x - C*a + A*b)/(c*x^4 + b*x^2 + a), x)/a - A/(a*x)

mupad [B] time = 1.02, size = 2588, normalized size = 9.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x + C*x^2)/(x^2*(a + b*x^2 + c*x^4)),x)

[Out] symsum(log(root(128*a^4*b^2*c*z^4 - 256*a^5*c^2*z^4 - 16*a^3*b^4*z^4 + 128*B*a^3*b^2*c*z^3 - 256*B*a^4*c^2*z^3 - 16*B*a^2*b^4*z^3 - 48*A*C*a^2*b^2*c*z^2 + 8*A*C*a*b^4*z^2 + 40*B^2*a^2*b^2*c*z^2 - 48*A^2*a^2*b*c^2*z^2 + 16*C^2*a^3*b*c*z^2 + 28*A^2*a*b^3*c*z^2 + 64*A*C*a^3*c^2*z^2 - 4*B^2*a*b^4*z^2 - 96*B^2*a^3*c^2*z^2 - 4*C^2*a^2*b^3*z^2 - 4*A^2*b^5*z^2 - 8*A*B*C*a*b^2*c*z - 16*A^2*B*a*b*c^2*z + 32*A*B*C*a^2*c^2*z + 4*A^2*B*b^3*c*z + 4*B^3*a*b^2*c*z - 16*B^3*a^2*c^2*z + 4*A*B^2*C*a*c^2 + 2*A*C^3*a*b*c - B^2*C^2*a*b*c - 2*A^2*C^2*a*c^2 + 2*A^3*C*b*c^2 - A^2*C^2*b^2*c - A^2*B^2*b*c^2 - C^4*a^2*c - B^4*a*c^2 - A^4*c^3, z, k)*(root(128*a^4*b^2*c*z^4 - 256*a^5*c^2*z^4 - 16*a^3*b^4*z^4 + 128*B*a^3*b^2*c*z^3 - 256*B*a^4*c^2*z^3 - 16*B*a^2*b^4*z^3 - 48*A*C*a^2*b^2*c*z^2 + 8*A*C*a*b^4*z^2 + 40*B^2*a^2*b^2*c*z^2 - 48*A^2*a^2*b*c^2*z^2 + 16*C^2*a^3*b*c*z^2 + 28*A^2*a*b^3*c*z^2 + 64*A*C*a^3*c^2*z^2 - 4*B^2*a*b^4*z^2 - 96*B^2*a^3*c^2*z^2 - 4*C^2*a^2*b^3*z^2 - 4*A^2*b^5*z^2 - 8*A*B*C*a*b^2*c*z - 16*A^2*B*a*b*c^2*z + 32*A*B*C*a^2*c^2*z + 4*A^2*B*b^3*c*z + 4*B^3*a*b^2*c*z - 16*B^3*a^2*c^2*z + 4*A*B^2*C*a*c^2 + 2*A*C^3*a*b*c - B^2*C^2*a*b*c - 2*A^2*C^2*a*c^2 + 2*A^3*C*b*c^2 - A^2*C^2*b^2*c - A^2*B^2*b*c^2 - C^4*a^2*c - B^4*a*c^2 - A^4*c^3, z, k)*(root(128*a^4*b^2*c*z^4 - 256*a^5*c^2*z^4 - 16*a^3*b^4*z^4 + 128*B*a^3*b^2*c*z^3 - 256*B*a^4*c^2*z^3 - 16*B*a^2*b^4*z^3 - 48*A*C*a^2*b^2*c*z^2 + 8*A*C*a*b^4*z^2 + 40*B^2*a^2*b^2*c*z^2 - 48*A^2*a^2*b*c^2*z^2 + 16*C^2*a^3*b*c*z^2 + 28*A^2*a*b^3*c*z^2 + 64*A*C*a^3*c^2*z^2 - 4*B^2*a*b^4*z^2 - 96*B^2*a^3*c^2*z^2 - 4*C^2*a^2*b^3*z^2 - 4*A^2*b^5*z^2 - 8*A*B*C*a*b^2*c*z - 16*A^2*B*a*b*c^2*z + 32*A*B*C*a^2*c^2*z + 4*A^2*B*b^3*c*z + 4*B^3*a*b^2*c*z - 16*B^3*a^2*c^2*z + 4*A*B^2*C*a*c^2 + 2*A*C^3*a*b*c - B^2*C^2*a*b*c - 2*A^2*C^2*a*c^2 + 2*A^3*C*b*c^2 - A^2*C^2*b^2*c - A^2*B^2*b*c^2 - C^4*a^2*c - B^4*a*c^2 - A^4*c^3, z, k)*x*(320*a^5*c^4 + 24*a^3*b^4*c^2 - 176*a^4*b^2*c^3))/a^2) - (8*A*B*a^2*c^4 + 4*A*B*b^4*c^2 - 16*A*B*a*b^2*c^3 - 4*B*C*a*b^3*c^2 + 12*B*C*a^2*b*c^3)/a + (x*(4*A^2*b^5*c^2 + 60*B^2*a^3*c^4 - 16*B^2*a^2*b^2*c^3 + 4*C^2*a^2*b^3*c^2 - 72*A*C*a^3*c^4 - 28*A^2*a*b^3*c^3 + 50*A^2*a^2*b*c^4 - 14*C^2*a^3*b*c^3 + 48*A*C*a^2*b^2*c^3 - 8*A*C*a*b^4*c^2))/a^2) - (C^3*a^2*c

$$\begin{aligned} &^3 + 7AB^2ac^4 + A^2C^2ac^4 - 4AB^2b^2c^3 - AC^2ab^2c^3 + 4B^2C^2ab^2c^3)/a + (x(5B^3a^2c^4 - 4A^2B^2b^3c^3 - B^2C^2a^2b^2c^3 - 26A \\ &B^2C^2a^2c^4 + 14A^2B^2ab^2c^4 + 8AB^2C^2ab^2c^3))/a^2) - (AB^3c^4 - A \\ &^2B^2C^2c^4 - B^2C^3ac^3 + AB^2C^2b^2c^3)/a + (x(A^4c^5 + C^4a^2c^3 + A \\ &^2C^2b^2c^3 - 2A^3C^2b^2c^4 + A^2B^2b^2c^4 + 2A^2C^2a^2c^4 - 2AB^2C^2 \\ &C^2ac^4 - 2AC^3ab^2c^3))/a^2)*\text{root}(128a^4b^2c^2z^4 - 256a^5c^2z^4 - \\ &16a^3b^4z^4 + 128B^2a^3b^2c^2z^3 - 256B^2a^4c^2z^3 - 16B^2a^2b^4z^3 \\ &3 - 48AC^2a^2b^2c^2z^2 + 8AC^2a^2b^4z^2 + 40B^2a^2b^2c^2z^2 - 48A^2 \\ &a^2b^2c^2z^2 + 16C^2a^3b^2c^2z^2 + 28A^2a^2b^3c^2z^2 + 64AC^2a^3c^2z^2 \\ &2 - 4B^2a^2b^4z^2 - 96B^2a^3c^2z^2 - 4C^2a^2b^3z^2 - 4A^2b^5z^2 \\ &2 - 8AB^2C^2ab^2c^2z - 16A^2B^2ab^2c^2z + 32AB^2C^2a^2c^2z + 4A^2B^2b \\ &^3c^2z + 4B^3a^2b^2c^2z - 16B^3a^2c^2z + 4AB^2C^2a^2c^2 + 2AC^3ab \\ &^2c - B^2C^2ab^2c - 2A^2C^2a^2c^2 + 2A^3C^2b^2c^2 - A^2C^2b^2c - A^2 \\ &B^2b^2c^2 - C^4a^2c - B^4a^2c^2 - A^4c^3, z, k), k, 1, 4) - A/(ax) + (B \\ &*\log(x))/a \end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x**2+B*x+A)/x**2/(c*x**4+b*x**2+a),x)

[Out] Timed out

$$3.28 \quad \int \frac{A+Bx+Cx^2}{x^3(a+bx^2+cx^4)} dx$$

Optimal. Leaf size=288

$$\frac{(A(b^2 - 2ac) - abC) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right) + (Ab - aC) \log(a + bx^2 + cx^4) - \frac{\log(x)(Ab - aC)}{a^2} - \frac{A}{2ax^2} - \frac{B\sqrt{c} \left(\frac{b}{\sqrt{b^2-4ac}} + 1\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right) - B\sqrt{c} \left(1 - \frac{b}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2-4ac}+b}}\right) - \frac{B}{ax}}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}}{2a^2\sqrt{b^2-4ac}} + \frac{(Ab - aC) \log(a + bx^2 + cx^4)}{4a^2} - \frac{\log(x)(Ab - aC)}{a^2} - \frac{A}{2ax^2} - \frac{B\sqrt{c} \left(\frac{b}{\sqrt{b^2-4ac}} + 1\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right) - B\sqrt{c} \left(1 - \frac{b}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2-4ac}+b}}\right) - \frac{B}{ax}}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}$$

Rubi [A] time = 0.47, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$, Rules used = {1662, 1251, 800, 634, 618, 206, 628, 12, 1123, 1166, 205}

$$\frac{(A(b^2 - 2ac) - abC) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right) + (Ab - aC) \log(a + bx^2 + cx^4) - \frac{\log(x)(Ab - aC)}{a^2} - \frac{A}{2ax^2} - \frac{B\sqrt{c} \left(\frac{b}{\sqrt{b^2-4ac}} + 1\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right) - B\sqrt{c} \left(1 - \frac{b}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2-4ac}+b}}\right) - \frac{B}{ax}}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}}{2a^2\sqrt{b^2-4ac}} + \frac{(Ab - aC) \log(a + bx^2 + cx^4)}{4a^2} - \frac{\log(x)(Ab - aC)}{a^2} - \frac{A}{2ax^2} - \frac{B\sqrt{c} \left(\frac{b}{\sqrt{b^2-4ac}} + 1\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right) - B\sqrt{c} \left(1 - \frac{b}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2-4ac}+b}}\right) - \frac{B}{ax}}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x + C*x^2)/(x^3*(a + b*x^2 + c*x^4)), x]

[Out] $-\frac{A}{2ax^2} - \frac{B}{ax} - \frac{(B\sqrt{c} \left(1 + \frac{b}{\sqrt{b^2 - 4ac}}\right) \text{ArcTan}\left[\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right])}{(\sqrt{2}a\sqrt{b - \sqrt{b^2 - 4ac}})} - \frac{(B\sqrt{c} \left(1 - \frac{b}{\sqrt{b^2 - 4ac}}\right) \text{ArcTan}\left[\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b + \sqrt{b^2 - 4ac}}}\right])}{(\sqrt{2}a\sqrt{b + \sqrt{b^2 - 4ac}})} - \frac{((A(b^2 - 2ac) - abC) \text{ArcTanh}\left[\frac{b + 2cx^2}{\sqrt{b^2 - 4ac}}\right])}{(2a^2\sqrt{b^2 - 4ac})} - \frac{((Ab - aC) \log[a + bx^2 + cx^4])}{(4a^2)}$

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dist[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1251

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]
```

Rule 1662

```
Int[(Pq_)*((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2 + c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] && !PolyQ[Pq, x^2]
```

Rubi steps

$$\begin{aligned}
 \int \frac{A + Bx + Cx^2}{x^3(a + bx^2 + cx^4)} dx &= \int \frac{B}{x^2(a + bx^2 + cx^4)} dx + \int \frac{A + Cx^2}{x^3(a + bx^2 + cx^4)} dx \\
 &= \frac{1}{2} \text{Subst} \left(\int \frac{A + Cx}{x^2(a + bx + cx^2)} dx, x, x^2 \right) + B \int \frac{1}{x^2(a + bx^2 + cx^4)} dx \\
 &= -\frac{B}{ax} + \frac{1}{2} \text{Subst} \left(\int \left(\frac{A}{ax^2} + \frac{-Ab + aC}{a^2x} + \frac{A(b^2 - ac) - abC + c(Ab - aC)x}{a^2(a + bx + cx^2)} \right) dx, x, x^2 \right) \\
 &= -\frac{A}{2ax^2} - \frac{B}{ax} - \frac{(Ab - aC) \log(x)}{a^2} + \frac{\text{Subst} \left(\int \frac{A(b^2 - ac) - abC + c(Ab - aC)x}{a + bx + cx^2} dx, x, x^2 \right)}{2a^2} - \frac{Bc}{2a^2} \left(1 - \frac{b}{\sqrt{b^2 - 4ac}} \right) \\
 &= -\frac{A}{2ax^2} - \frac{B}{ax} - \frac{B\sqrt{c} \left(1 + \frac{b}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{\sqrt{2}a\sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{B\sqrt{c} \left(1 - \frac{b}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b + \sqrt{b^2 - 4ac}}} \right)}{\sqrt{2}a\sqrt{b + \sqrt{b^2 - 4ac}}} \\
 &= -\frac{A}{2ax^2} - \frac{B}{ax} - \frac{B\sqrt{c} \left(1 + \frac{b}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{\sqrt{2}a\sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{B\sqrt{c} \left(1 - \frac{b}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b + \sqrt{b^2 - 4ac}}} \right)}{\sqrt{2}a\sqrt{b + \sqrt{b^2 - 4ac}}} \\
 &= -\frac{A}{2ax^2} - \frac{B}{ax} - \frac{B\sqrt{c} \left(1 + \frac{b}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{\sqrt{2}a\sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{B\sqrt{c} \left(1 - \frac{b}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b + \sqrt{b^2 - 4ac}}} \right)}{\sqrt{2}a\sqrt{b + \sqrt{b^2 - 4ac}}}
 \end{aligned}$$

Mathematica [A] time = 0.89, size = 377, normalized size = 1.31

$$\frac{\left(\frac{A(b\sqrt{b^2-4ac}-2ac+b^2)-ac(\sqrt{b^2-4ac}+b)}{\sqrt{b^2-4ac}} \right) \log(\sqrt{b^2-4ac}-b-2cx^2)}{4a^2} + \frac{\left(\frac{A(b\sqrt{b^2-4ac}+2ac-b^2)+ac(b-\sqrt{b^2-4ac})}{\sqrt{b^2-4ac}} \right) \log(\sqrt{b^2-4ac}+b+2cx^2)}{4a^2} + 4 \log(x)(aC - Ab) - \frac{2aA}{x^2} - \frac{2\sqrt{2}aB\sqrt{c}(\sqrt{b^2-4ac}+b) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{2\sqrt{2}aB\sqrt{c}(\sqrt{b^2-4ac}-b) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}} - \frac{4aB}{x}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x + C*x^2)/(x^3*(a + b*x^2 + c*x^4)), x]

[Out] ((-2*a*A)/x^2 - (4*a*B)/x - (2*Sqrt[2]*a*B*Sqrt[c]*(b + Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (2*Sqrt[2]*a*B*Sqrt[c]*(-b + Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + 4*(-(A*b) + a*C)*Log[x] + ((A*(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c]) - a*(b + Sqrt[b^2 - 4*a*c])*C)*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/Sqrt[b^2 - 4*a*c] + ((A*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c]) + a*(b - Sqrt[b^2 - 4*a*c])*C)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c])/(4*a^2)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx + Cx^2}{x^3(a + bx^2 + cx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B*x + C*x^2)/(x^3*(a + b*x^2 + c*x^4)), x]

[Out] IntegrateAlgebraic[(A + B*x + C*x^2)/(x^3*(a + b*x^2 + c*x^4)), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)/x^3/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] Timed out

giac [B] time = 5.87, size = 3353, normalized size = 11.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)/x^3/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out]
$$-1/4*(C*a - A*b)*\log(\text{abs}(c*x^4 + b*x^2 + a))/a^2 + (C*a - A*b)*\log(\text{abs}(x))/a^2 - 1/8*((2*b^4*c^2 - 16*a*b^2*c^3 + 32*a^2*c^4 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*b^4 + 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a*b^2*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*b^3*c - 16*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a^2*c^2 - 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a*b*c^2 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^2*c^2 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*B*c^2 + 2*(\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*b^5*c - 8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a*b^3*c^2 - 2*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*b^4*c^2 + 2*b^5*c^2 + 16*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a^2*b*c^3 + 8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a*b^2*c^3 + \sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*b^3*c^3 - 16*a*b^3*c^3 - 4*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a*b*c^4 + 32*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*B*abs(c) + (2*b^4*c^4 - 8*a*b^2*c^5 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*b^4*c^2 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a*b^2*c^3 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*b^3*c^3 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^2*c^4 - 2*(b^2 - 4*a*c)*b^2*c^4)*B)*\arctan(2*\sqrt{1/2}*x/\sqrt{(a^4*b*c + \sqrt{a^8*b^2*c^2 - 4*a^9*c^3})/(a^4*c^2)))/((a^2*b^4*c - 8*a^3*b^2*c^2 - 2*a^2*b^3*c^2 + 16*a^4*c^3 + 8*a^3*b*c^3 + a^2*b^2*c^3 - 4*a^3*c^4)*c^2) + 1/8*((2*b^4*c^2 - 16*a*b^2*c^3 + 32*a^2*c^4 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*b^4 + 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*a*b^2*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*b^3*c - 16*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*a^2*c^2 - 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*a*b*c^2 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^2*c^2 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*B*c^2 - 2*(\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*b^5*c - 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*a*b^3*c^2 - 2*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*b^4*c^2 - 2*b^5*c^2 + 16*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*a^2*b*c^3 + 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*a*b^2*c^3 + \sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*b^3*c^3 + 16*a*b^3*c^3 - 4*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*a*b*c^4 - 32*a^2*b*c^4 + 2*(b^2 - 4*a*c)*b^3*c^2 - 8*(b^2 - 4*a*c)*a*b*c^3)*B*abs(c) + (2*b^4*c^4 - 8*a*b^2*c^5 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*b^4*c^2 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*a*b^2*c^3 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*b^3*c^3 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^2*c^4 - 2*(b^2 - 4*a*c)*b^2*c^4)*B)*\arctan(2*\sqrt{1/2}*x/\sqrt{(a^4*b*c - \sqrt{a^8*b^2*c^2 - 4*a^9*c^3})/(a^4*c^2)))/((a^2*b^4*c - 8*a^3*b^2*c^2 - 2*a^2*b^3*c^2 + 16*a^4*c^3 + 8*a^3*b*c^3 + a^2*b^2*c^3 - 4*a^3*c^4)*c^2) + 1/16*((b^7*c - 10*a*b^5*c^2 - 2*b^6*c^2 + 32*a^2*b^3*c^3 + 12*a*b^4*c^3 + b$$

```

^5*c^3 - 32*a^3*b*c^4 - 16*a^2*b^2*c^4 - 6*a*b^3*c^4 + 8*a^2*b*c^5 - (b^6*c
- 10*a*b^4*c^2 - 2*b^5*c^2 + 32*a^2*b^2*c^3 + 12*a*b^3*c^3 + b^4*c^3 - 32*
a^3*c^4 - 16*a^2*b*c^4 - 6*a*b^2*c^4 + 8*a^2*c^5)*sqrt(b^2 - 4*a*c))*A*abs(
c) - (a*b^6*c - 8*a^2*b^4*c^2 - 2*a*b^5*c^2 + 16*a^3*b^2*c^3 + 8*a^2*b^3*c^
3 + a*b^4*c^3 - 4*a^2*b^2*c^4 - (a*b^5*c - 8*a^2*b^3*c^2 - 2*a*b^4*c^2 + 16
*a^3*b*c^3 + 8*a^2*b^2*c^3 + a*b^3*c^3 - 4*a^2*b*c^4)*sqrt(b^2 - 4*a*c))*C*
abs(c) + (b^7*c^2 - 10*a*b^5*c^3 - 2*b^6*c^3 + 32*a^2*b^3*c^4 + 12*a*b^4*c^
4 + b^5*c^4 - 32*a^3*b*c^5 - 16*a^2*b^2*c^5 - 6*a*b^3*c^5 + 8*a^2*b*c^6 + (
b^6*c^2 - 6*a*b^4*c^3 - 2*b^5*c^3 + 8*a^2*b^2*c^4 + 4*a*b^3*c^4 + b^4*c^4 -
2*a*b^2*c^5)*sqrt(b^2 - 4*a*c))*A - (a*b^6*c^2 - 8*a^2*b^4*c^3 - 2*a*b^5*c
^3 + 16*a^3*b^2*c^4 + 8*a^2*b^3*c^4 + a*b^4*c^4 - 4*a^2*b^2*c^5 - (a*b^5*c^
2 - 4*a^2*b^3*c^3 - 2*a*b^4*c^3 + a*b^3*c^4)*sqrt(b^2 - 4*a*c))*C)*log(x^2
+ 1/2*(a^4*b*c + sqrt(a^8*b^2*c^2 - 4*a^9*c^3))/(a^4*c^2))/((a^3*b^4 - 8*a^
4*b^2*c - 2*a^3*b^3*c + 16*a^5*c^2 + 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a^4*c^3)
*c^2*abs(c)) + 1/16*((b^7*c - 10*a*b^5*c^2 - 2*b^6*c^2 + 32*a^2*b^3*c^3 + 1
2*a*b^4*c^3 + b^5*c^3 - 32*a^3*b*c^4 - 16*a^2*b^2*c^4 - 6*a*b^3*c^4 + 8*a^2
*b*c^5 + (b^6*c - 10*a*b^4*c^2 - 2*b^5*c^2 + 32*a^2*b^2*c^3 + 12*a*b^3*c^3
+ b^4*c^3 - 32*a^3*c^4 - 16*a^2*b*c^4 - 6*a*b^2*c^4 + 8*a^2*c^5)*sqrt(b^2 -
4*a*c))*A*abs(c) - (a*b^6*c - 8*a^2*b^4*c^2 - 2*a*b^5*c^2 + 16*a^3*b^2*c^3
+ 8*a^2*b^3*c^3 + a*b^4*c^3 - 4*a^2*b^2*c^4 + (a*b^5*c - 8*a^2*b^3*c^2 - 2
*a*b^4*c^2 + 16*a^3*b*c^3 + 8*a^2*b^2*c^3 + a*b^3*c^3 - 4*a^2*b*c^4)*sqrt(b
^2 - 4*a*c))*C*abs(c) - (b^7*c^2 - 10*a*b^5*c^3 - 2*b^6*c^3 + 32*a^2*b^3*c^
4 + 12*a*b^4*c^4 + b^5*c^4 - 32*a^3*b*c^5 - 16*a^2*b^2*c^5 - 6*a*b^3*c^5 +
8*a^2*b*c^6 + (b^6*c^2 - 6*a*b^4*c^3 - 2*b^5*c^3 + 8*a^2*b^2*c^4 + 4*a*b^3*
c^4 + b^4*c^4 - 2*a*b^2*c^5)*sqrt(b^2 - 4*a*c))*A + (a*b^6*c^2 - 8*a^2*b^4*
c^3 - 2*a*b^5*c^3 + 16*a^3*b^2*c^4 + 8*a^2*b^3*c^4 + a*b^4*c^4 - 4*a^2*b^2*
c^5 + (a*b^5*c^2 - 4*a^2*b^3*c^3 - 2*a*b^4*c^3 + a*b^3*c^4)*sqrt(b^2 - 4*a*
c))*C)*log(x^2 + 1/2*(a^4*b*c - sqrt(a^8*b^2*c^2 - 4*a^9*c^3))/(a^4*c^2))/
((a^3*b^4 - 8*a^4*b^2*c - 2*a^3*b^3*c + 16*a^5*c^2 + 8*a^4*b*c^2 + a^3*b^2*c
^2 - 4*a^4*c^3)*c^2*abs(c)) - 1/2*(2*B*a*x + A*a)/(a^2*x^2)

```

maple [B] time = 0.06, size = 1054, normalized size = 3.66

Verification of antiderivative is not currently implemented for this CAS.

[In] int((C*x^2+B*x+A)/x^3/(c*x^4+b*x^2+a), x)

[Out]
$$\begin{aligned}
 & -1/2*A/a/x^2 - B/a/x - A/a^2*b*\ln(x) + 1/a*\ln(x)*C + 8/a*c/(32*a*c - 8*b^2)*\ln(-2*c*x \\
 & ^2 - b + (-4*a*c + b^2)^{(1/2)}) * A*b - 2/a^2/(32*a*c - 8*b^2)*\ln(-2*c*x^2 - b + (-4*a*c + b^2 \\
 &)^{(1/2)}) * A*b^3 + 4/a*c/(32*a*c - 8*b^2)*\ln(-2*c*x^2 - b + (-4*a*c + b^2)^{(1/2)}) * A*(-4 \\
 & *a*c + b^2)^{(1/2)} - 2/a^2/(32*a*c - 8*b^2)*\ln(-2*c*x^2 - b + (-4*a*c + b^2)^{(1/2)}) * A*(-4 \\
 & *a*c + b^2)^{(1/2)} * b^2 + 2/a/(32*a*c - 8*b^2)*\ln(-2*c*x^2 - b + (-4*a*c + b^2)^{(1/2)}) * C \\
 & * (-4*a*c + b^2)^{(1/2)} * b - 8*c/(32*a*c - 8*b^2)*\ln(-2*c*x^2 - b + (-4*a*c + b^2)^{(1/2)}) * \\
 & C + 2/a/(32*a*c - 8*b^2)*\ln(-2*c*x^2 - b + (-4*a*c + b^2)^{(1/2)}) * C*b^2 - 4/a*c/(32*a*c - \\
 & 8*b^2)*B*2^{(1/2)}/((-b + (-4*a*c + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctanh}(2^{(1/2)}/((-b + (-4 \\
 & *a*c + b^2)^{(1/2)}) * c)^{(1/2)} * c*x) * (-4*a*c + b^2)^{(1/2)} * b + 16*c^2/(32*a*c - 8*b^2)*B \\
 & *2^{(1/2)}/((-b + (-4*a*c + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctanh}(2^{(1/2)}/((-b + (-4*a*c + b^2 \\
 &)^{(1/2)}) * c)^{(1/2)} * c*x) - 4/a*c/(32*a*c - 8*b^2)*B*2^{(1/2)}/((-b + (-4*a*c + b^2)^{(1/ \\
 & 2)) * c)^{(1/2)} * \operatorname{arctanh}(2^{(1/2)}/((-b + (-4*a*c + b^2)^{(1/2)}) * c)^{(1/2)} * c*x) * b^2 + 8/a \\
 & *c/(32*a*c - 8*b^2)*\ln(2*c*x^2 + b + (-4*a*c + b^2)^{(1/2)}) * A*b - 2/a^2/(32*a*c - 8*b^2) \\
 & *\ln(2*c*x^2 + b + (-4*a*c + b^2)^{(1/2)}) * A*b^3 - 4/a*c/(32*a*c - 8*b^2)*\ln(2*c*x^2 + b + (\\
 & -4*a*c + b^2)^{(1/2)}) * A*(-4*a*c + b^2)^{(1/2)} + 2/a^2/(32*a*c - 8*b^2)*\ln(2*c*x^2 + b + (\\
 & -4*a*c + b^2)^{(1/2)}) * A*(-4*a*c + b^2)^{(1/2)} * b^2 - 2/a/(32*a*c - 8*b^2)*\ln(2*c*x^2 + b \\
 & + (-4*a*c + b^2)^{(1/2)}) * C * (-4*a*c + b^2)^{(1/2)} * b - 8*c/(32*a*c - 8*b^2)*\ln(2*c*x^2 + b \\
 & + (-4*a*c + b^2)^{(1/2)}) * C + 2/a/(32*a*c - 8*b^2)*\ln(2*c*x^2 + b + (-4*a*c + b^2)^{(1/2)}) * \\
 & C*b^2 - 4/a*c/(32*a*c - 8*b^2)*B*2^{(1/2)}/((b + (-4*a*c + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctan} \\
 & (2^{(1/2)}/((b + (-4*a*c + b^2)^{(1/2)}) * c)^{(1/2)} * c*x) * (-4*a*c + b^2)^{(1/2)} * b - 16*c^2 \\
 & / (32*a*c - 8*b^2)*B*2^{(1/2)}/((b + (-4*a*c + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctan}(2^{(1/2)}/(\\
 & (b + (-4*a*c + b^2)^{(1/2)}) * c)^{(1/2)} * c*x) + 4/a*c/(32*a*c - 8*b^2)*B*2^{(1/2)}/((b + (-4
 \end{aligned}$$

$(a^2c + b^2)^{1/2} \arctan\left(\frac{2^{1/2}}{(b + (-4ac + b^2)^{1/2})} \sqrt{cx + b^2}\right)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(Ca - Ab) \log(x)}{a^2} + \frac{-\int \frac{Bacx^2 + (Ca - Ab)cx^3 + Bab + (Cab - Ab^2 + Aac)x}{cx^4 + bx^2 + a} dx}{a^2} - \frac{2Bx + A}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)/x^3/(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] (C*a - A*b)*log(x)/a^2 + integrate(-(B*a*c*x^2 + (C*a - A*b)*c*x^3 + B*a*b + (C*a*b - A*b^2 + A*a*c)*x)/(c*x^4 + b*x^2 + a), x)/a^2 - 1/2*(2*B*x + A)/(a*x^2)

mupad [B] time = 1.17, size = 3563, normalized size = 12.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x + C*x^2)/(x^3*(a + b*x^2 + c*x^4)),x)

[Out] symsum(log(root(128*a^5*b^2*c*z^4 - 256*a^6*c^2*z^4 - 16*a^4*b^4*z^4 + 128*C*a^4*b^2*c*z^3 + 256*A*a^4*b*c^2*z^3 - 128*A*a^3*b^3*c*z^3 - 256*C*a^5*c^2*z^3 - 16*C*a^3*b^4*z^3 + 16*A*a^2*b^5*z^3 + 160*A*C*a^3*b*c^2*z^2 - 72*A*C*a^2*b^3*c*z^2 + 8*A*C*a*b^5*z^2 + 40*C^2*a^3*b^2*c*z^2 - 48*B^2*a^3*b*c^2*z^2 + 28*B^2*a^2*b^3*c*z^2 + 32*A^2*a*b^4*c*z^2 - 56*A^2*a^2*b^2*c^2*z^2 - 4*B^2*a*b^5*z^2 - 96*C^2*a^4*c^2*z^2 - 4*C^2*a^2*b^4*z^2 - 32*A^2*a^3*c^3*z^2 - 4*A^2*b^6*z^2 - 16*B^2*C*a^2*b*c^2*z + 32*A*C^2*a^2*b*c^2*z - 12*A^2*C*a*b^2*c^2*z - 4*A*B^2*a*b^2*c^2*z + 4*B^2*C*a*b^3*c*z - 8*A*C^2*a*b^3*c*z + 16*A^3*a*b*c^3*z + 4*A^2*C*b^4*c*z + 4*C^3*a^2*b^2*c*z - 16*A^2*C*a^2*c^3*z + 16*A*B^2*a^2*c^3*z - 16*C^3*a^3*c^2*z - 4*A^3*b^3*c^2*z + 2*A*C^3*a*b*c^2 + 4*A*B^2*C*a*c^3 - 2*A^2*C^2*a*c^3 + 2*A^3*C*b*c^3 - B^2*C^2*a*b*c^2 - A^2*B^2*b*c^3 - A^2*C^2*b^2*c^2 - C^4*a^2*c^2 - B^4*a*c^3 - A^4*c^4, z, k)*(root(128*a^5*b^2*c*z^4 - 256*a^6*c^2*z^4 - 16*a^4*b^4*z^4 + 128*C*a^4*b^2*c*z^3 + 256*A*a^4*b*c^2*z^3 - 128*A*a^3*b^3*c*z^3 - 256*C*a^5*c^2*z^3 - 16*C*a^3*b^4*z^3 + 16*A*a^2*b^5*z^3 + 160*A*C*a^3*b*c^2*z^2 - 72*A*C*a^2*b^3*c*z^2 + 8*A*C*a*b^5*z^2 + 40*C^2*a^3*b^2*c*z^2 - 48*B^2*a^3*b*c^2*z^2 + 28*B^2*a^2*b^3*c*z^2 + 32*A^2*a*b^4*c*z^2 - 56*A^2*a^2*b^2*c^2*z^2 - 4*B^2*a*b^5*z^2 - 96*C^2*a^4*c^2*z^2 - 4*C^2*a^2*b^4*z^2 - 32*A^2*a^3*c^3*z^2 - 4*A^2*b^6*z^2 - 16*B^2*C*a^2*b*c^2*z + 32*A*C^2*a^2*b*c^2*z - 12*A^2*C*a*b^2*c^2*z - 4*A*B^2*a*b^2*c^2*z + 4*B^2*C*a*b^3*c*z - 8*A*C^2*a*b^3*c*z + 16*A^3*a*b*c^3*z + 4*A^2*C*b^4*c*z + 4*C^3*a^2*b^2*c*z - 16*A^2*C*a^2*c^3*z + 16*A*B^2*a^2*c^3*z - 16*C^3*a^3*c^2*z - 4*A^3*b^3*c^2*z + 2*A*C^3*a*b*c^2 + 4*A*B^2*C*a*c^3 - 2*A^2*C^2*a*c^3 + 2*A^3*C*b*c^3 - B^2*C^2*a*b*c^2 - A^2*B^2*b*c^3 - A^2*C^2*b^2*c^2 - C^4*a^2*c^2 - B^4*a*c^3 - A^4*c^4, z, k))*((16*B*a^5*c^4 + 4*B*a^3*b^4*c^2 - 20*B*a^4*b^2*c^3)/a^3 + (x*(240*C*a^5*c^4 - 224*A*a^4*b*c^4 - 12*A*a^2*b^5*c^2 + 104*A*a^3*b^3*c^3 + 12*C*a^3*b^4*c^2 - 108*C*a^4*b^

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2*c^3))/a^3 + (root(128*a^5*b^2*c*z^4 - 256*a^6*c^2*z^4 - 16*a^4*b^4*z^4 +
128*C*a^4*b^2*c*z^3 + 256*A*a^4*b*c^2*z^3 - 128*A*a^3*b^3*c*z^3 - 256*C*a^5
*c^2*z^3 - 16*C*a^3*b^4*z^3 + 16*A*a^2*b^5*z^3 + 160*A*C*a^3*b*c^2*z^2 - 72
*A*C*a^2*b^3*c*z^2 + 8*A*C*a*b^5*z^2 + 40*C^2*a^3*b^2*c*z^2 - 48*B^2*a^3*b*
c^2*z^2 + 28*B^2*a^2*b^3*c*z^2 + 32*A^2*a*b^4*c*z^2 - 56*A^2*a^2*b^2*c^2*z^
2 - 4*B^2*a*b^5*z^2 - 96*C^2*a^4*c^2*z^2 - 4*C^2*a^2*b^4*z^2 - 32*A^2*a^3*c
^3*z^2 - 4*A^2*b^6*z^2 - 16*B^2*C*a^2*b*c^2*z + 32*A*C^2*a^2*b*c^2*z - 12*A
^2*C*a*b^2*c^2*z - 4*A*B^2*a*b^2*c^2*z + 4*B^2*C*a*b^3*c*z - 8*A*C^2*a*b^3*
c*z + 16*A^3*a*b*c^3*z + 4*A^2*C*b^4*c*z + 4*C^3*a^2*b^2*c*z - 16*A^2*C*a^2
*c^3*z + 16*A*B^2*a^2*c^3*z - 16*C^3*a^3*c^2*z - 4*A^3*b^3*c^2*z + 2*A*C^3*
a*b*c^2 + 4*A*B^2*C*a*c^3 - 2*A^2*C^2*a*c^3 + 2*A^3*C*b*c^3 - B^2*C^2*a*b*c
^2 - A^2*B^2*b*c^3 - A^2*C^2*b^2*c^2 - C^4*a^2*c^2 - B^4*a*c^3 - A^4*c^4, z
, k)*x*(320*a^6*c^4 + 24*a^4*b^4*c^2 - 176*a^5*b^2*c^3))/a^3) - (8*B*C*a^4*
c^4 + 20*A*B*a^2*b^3*c^3 + 4*B*C*a^2*b^4*c^2 - 16*B*C*a^3*b^2*c^3 - 4*A*B*a
*b^5*c^2 - 20*A*B*a^3*b*c^4)/a^3 + (x*(36*A^2*a^3*c^5 + 60*C^2*a^4*c^4 + 22
*A^2*a^2*b^2*c^4 - 28*B^2*a^2*b^3*c^3 - 16*C^2*a^3*b^2*c^3 - 8*A^2*a*b^4*c^
3 + 4*B^2*a*b^5*c^2 + 50*B^2*a^3*b*c^4 + 24*A*C*a^2*b^3*c^3 - 92*A*C*a^3*b*
c^4))/a^3) - (A^2*B*a^2*c^5 + 7*B*C^2*a^3*c^4 - 4*A^2*B*a*b^2*c^4 - 4*B*C^2
*a^2*b^2*c^3 + 4*A*B*C*a*b^3*c^3 - 4*A*B*C*a^2*b*c^4)/a^3 + (x*(2*A^3*b^3*c
^4 + 5*C^3*a^3*c^4 - 12*A^3*a*b*c^5 - 17*A*B^2*a^2*c^5 + 13*A^2*C*a^2*c^5 +
6*A*B^2*a*b^2*c^4 - 9*A*C^2*a^2*b*c^4 + 2*A^2*C*a*b^2*c^4 - 4*B^2*C*a*b^3*
c^3 + 14*B^2*C*a^2*b*c^4))/a^3) - (A^3*B*b*c^5 + B*C^3*a^2*c^4 - A^2*B*C*a*
c^5 - A*B*C^2*a*b*c^4)/a^3 + (x*(A^4*c^6 + B^4*a*c^5 - A^3*C*b*c^5 + A^2*C^
2*a*c^5 + B^2*C^2*a*b*c^4 - 3*A*B^2*C*a*c^5))/a^3)*root(128*a^5*b^2*c*z^4 -
256*a^6*c^2*z^4 - 16*a^4*b^4*z^4 + 128*C*a^4*b^2*c*z^3 + 256*A*a^4*b*c^2*z
^3 - 128*A*a^3*b^3*c*z^3 - 256*C*a^5*c^2*z^3 - 16*C*a^3*b^4*z^3 + 16*A*a^2*
b^5*z^3 + 160*A*C*a^3*b*c^2*z^2 - 72*A*C*a^2*b^3*c*z^2 + 8*A*C*a*b^5*z^2 +
40*C^2*a^3*b^2*c*z^2 - 48*B^2*a^3*b*c^2*z^2 + 28*B^2*a^2*b^3*c*z^2 + 32*A^2
*a*b^4*c*z^2 - 56*A^2*a^2*b^2*c^2*z^2 - 4*B^2*a*b^5*z^2 - 96*C^2*a^4*c^2*z^
2 - 4*C^2*a^2*b^4*z^2 - 32*A^2*a^3*c^3*z^2 - 4*A^2*b^6*z^2 - 16*B^2*C*a^2*b
*c^2*z + 32*A*C^2*a^2*b*c^2*z - 12*A^2*C*a*b^2*c^2*z - 4*A*B^2*a*b^2*c^2*z
+ 4*B^2*C*a*b^3*c*z - 8*A*C^2*a*b^3*c*z + 16*A^3*a*b*c^3*z + 4*A^2*C*b^4*c*
z + 4*C^3*a^2*b^2*c*z - 16*A^2*C*a^2*c^3*z + 16*A*B^2*a^2*c^3*z - 16*C^3*a^
3*c^2*z - 4*A^3*b^3*c^2*z + 2*A*C^3*a*b*c^2 + 4*A*B^2*C*a*c^3 - 2*A^2*C^2*a
*c^3 + 2*A^3*C*b*c^3 - B^2*C^2*a*b*c^2 - A^2*B^2*b*c^3 - A^2*C^2*b^2*c^2 -
C^4*a^2*c^2 - B^4*a*c^3 - A^4*c^4, z, k), k, 1, 4) - (A/(2*a) + (B*x)/a)/x^
2 - (log(x)*(A*b - C*a))/a^2

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sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x**2+B*x+A)/x**3/(c*x**4+b*x**2+a), x)

[Out] Timed out

$$3.29 \quad \int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2} dx$$

Optimal. Leaf size=412

$$\frac{\left(-\frac{Ac(4ac+b^2)+bC(b^2-8ac)}{\sqrt{b^2-4ac}} + C(b^2-6ac) + Abc\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right) + \left(\frac{Ac(4ac+b^2)+bC(b^2-8ac)}{\sqrt{b^2-4ac}} + C(b^2-6ac) + Abc\right)}{2\sqrt{2}c^{3/2}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(\frac{Ac(4ac+b^2)+bC(b^2-8ac)}{\sqrt{b^2-4ac}} + C(b^2-6ac) + Abc\right)}{2\sqrt{2}c^{3/2}(b^2-4ac)\sqrt{\sqrt{b^2-4ac}}}$$

Rubi [A] time = 1.33, antiderivative size = 412, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 28, number of rules / integrand size = 0.357, Rules used = {1662, 1275, 1279, 1166, 205, 12, 1114, 722, 618, 206}

$$\frac{\left(-\frac{Ac(4ac+b^2)+bC(b^2-8ac)}{\sqrt{b^2-4ac}} + C(b^2-6ac) + Abc\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right) + \left(\frac{Ac(4ac+b^2)+bC(b^2-8ac)}{\sqrt{b^2-4ac}} + C(b^2-6ac) + Abc\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right) - \frac{x^3(-2aC+x^2(2Ac-bC)+Ab)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{x(2Ac-bC)}{2c(b^2-4ac)} + \frac{Bx^2(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{2aB \tanh^{-1}\left(\frac{b+2x^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}}{2\sqrt{2}c^{3/2}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}} + 2\sqrt{2}c^{3/2}(b^2-4ac)\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

[In] Int[(x^4*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2, x]

[Out] ((2*A*c - b*C)*x)/(2*c*(b^2 - 4*a*c)) + (B*x^2*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (x^3*(A*b - 2*a*C + (2*A*c - b*C)*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((A*b*c + (b^2 - 6*a*c)*C - (A*c*(b^2 + 4*a*c) + b*(b^2 - 8*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/(2*Sqrt[2]*c^(3/2)*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((A*b*c + (b^2 - 6*a*c)*C + (A*c*(b^2 + 4*a*c) + b*(b^2 - 8*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]/(2*Sqrt[2]*c^(3/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (2*a*B*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 722

Int[((d_.) + (e_.)*(x_)^(m_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m-1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p+1))/((p+1)*(b^2 - 4*a*c)), x] - Dist[(2*(2*p+3)*(c*d^2 - b*d*e + a*e^2))/((p+1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m-2)*(a + b*x + c*x^2)^(p+1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &

& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0] && LtQ[p, -1]

Rule 1114

Int[(x_)^(m_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]

Rule 1166

Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1275

Int[((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1279

Int[((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p + 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1662

Int[(Pq_)*((d_)*(x_))^(m_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2 + 1}](a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}](a + b*x^2 + c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] && !PolyQ[Pq, x^2]

Rubi steps

$$\begin{aligned}
\int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2} dx &= \int \frac{Bx^5}{(a+bx^2+cx^4)^2} dx + \int \frac{x^4(A+Cx^2)}{(a+bx^2+cx^4)^2} dx \\
&= -\frac{x^3(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + B \int \frac{x^5}{(a+bx^2+cx^4)^2} dx + \frac{\int \frac{x^2(3(Ab-2aC)+(2Ac-bC)x^2)}{a+bx^2+cx^4}}{2(b^2-4ac)} dx \\
&= \frac{(2Ac-bC)x}{2c(b^2-4ac)} - \frac{x^3(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{1}{2}B \text{Subst} \left(\int \frac{x^2}{(a+bx+cx^2)^2} dx \right) \\
&= \frac{(2Ac-bC)x}{2c(b^2-4ac)} + \frac{Bx^2(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{x^3(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} \\
&= \frac{(2Ac-bC)x}{2c(b^2-4ac)} + \frac{Bx^2(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{x^3(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \dots \\
&= \frac{(2Ac-bC)x}{2c(b^2-4ac)} + \frac{Bx^2(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{x^3(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \dots
\end{aligned}$$

Mathematica [A] time = 1.38, size = 444, normalized size = 1.08

$$\frac{1}{4} \left(\frac{2(a(B+C)-2c(A+B+C)) + b^2(B+C)-Ac^2}{c(4ac-b^2)(a+bx^2+cx^4)}, \frac{\sqrt{2} \left(c \left(b^2 \sqrt{b^2-4ac} - 6ac \sqrt{b^2-4ac} + 8abc - b^3 \right) - Ac \left(-4 \sqrt{b^2-4ac} + 4ac + b^2 \right) \right) \tan^{-1} \left(\frac{\sqrt{2} cx}{\sqrt{b-\sqrt{b^2-4ac}}} \right)}{c^{3/2} (b^2-4ac)^{3/2} \sqrt{b-\sqrt{b^2-4ac}}}, \frac{\sqrt{2} \left(Ac \left(b \sqrt{b^2-4ac} + 4ac + b^2 \right) + c \left(b^2 \sqrt{b^2-4ac} - 6ac \sqrt{b^2-4ac} - 8abc + b^3 \right) \right) \tan^{-1} \left(\frac{\sqrt{2} cx}{\sqrt{b+\sqrt{b^2-4ac}}} \right)}{c^{3/2} (b^2-4ac)^{3/2} \sqrt{b+\sqrt{b^2-4ac}}}, \frac{4ab \log(\sqrt{b^2-4ac}-b-2cx)}{(b^2-4ac)^{3/2}}, \frac{4ab \log(\sqrt{b^2-4ac}+b+2cx)}{(b^2-4ac)^{3/2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x^4*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]

[Out] ((2*(b*x^2*(-(A*c*x) + b*(B + C*x)) + a*(b*(B + C*x) - 2*c*x*(A + x*(B + C*x))))/(c*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(-(A*c*(b^2 + 4*a*c - b*Sqrt[b^2 - 4*a*c])) + (-b^3 + 8*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 6*a*c*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(c^(3/2)*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(A*c*(b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c]) + (b^3 - 8*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 6*a*c*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(c^(3/2)*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (4*a*B*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + (4*a*B*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^4*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]

[Out] IntegrateAlgebraic[(x^4*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2, x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

```
giac [B] time = 8.47, size = 5219, normalized size = 12.67
```

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(C*b^2*x^3 - 2*C*a*c*x^3 - A*b*c*x^3 + B*b^2*x^2 - 2*B*a*c*x^2 + C*a*b
*x - 2*A*a*c*x + B*a*b)/((c*x^4 + b*x^2 + a)*(b^2*c - 4*a*c^2)) + 1/16*((2*
b^3*c^3 - 8*a*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*
b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2
- sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^3 - 2*(b^2
- 4*a*c)*b*c^3)*(b^2*c - 4*a*c^2)^2*A + (2*b^4*c^2 - 20*a*b^2*c^3 + 48*a^2
*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 + 10*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 24*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 12*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 12*(b^2 - 4
*a*c)*a*c^3)*(b^2*c - 4*a*c^2)^2*C - 4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a*b^4*c^3 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 2*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - 2*a*b^4*c^4 + 16*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^5 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^2*b*c^5 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 +
16*a^2*b^2*c^5 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^6 - 32*a^3
*c^6 + 2*(b^2 - 4*a*c)*a*b^2*c^4 - 8*(b^2 - 4*a*c)*a^2*c^5)*A*abs(b^2*c - 4
*a*c^2) + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 - 8*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b^4*c^3 - 2*a*b^5*c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^3*b*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 +
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 + 16*a^2*b^3*c^4 - 4*sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 - 32*a^3*b*c^5 + 2*(b^2 - 4*a
*c)*a*b^3*c^3 - 8*(b^2 - 4*a*c)*a^2*b*c^4)*C*abs(b^2*c - 4*a*c^2) - (2*b^7*
c^5 - 8*a*b^5*c^6 - 32*a^2*b^3*c^7 + 128*a^3*b*c^8 - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^4 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b^5*c^5 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^3*b*c^6 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^2*b^2*c^6 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^2*b*c^7 - 2*(b^2 - 4*a*c)*b^5*c^5 + 32*(b^2 - 4*a*c)*a^2*b
*c^7)*A - (2*b^8*c^4 - 32*a*b^6*c^5 + 160*a^2*b^4*c^6 - 256*a^3*b^2*c^7 - s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^8*c^2 + 16*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3 + 2*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c^3 - 80*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 - 24*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^4 + 128*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^5 + 64*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 + 12*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 - 32*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 - 2*(b^2 - 4*a*c)*b^6*c^4 +
```

$$\begin{aligned}
& 24*(b^2 - 4*a*c)*a*b^4*c^5 - 64*(b^2 - 4*a*c)*a^2*b^2*c^6)*C)*\arctan(2*\sqrt{x}/\sqrt{((b^3*c - 4*a*b*c^2 + \sqrt{(b^3*c - 4*a*b*c^2)^2 - 4*(a*b^2*c - 4*a^2*c^2)*(b^2*c^2 - 4*a*c^3)))/(b^2*c^2 - 4*a*c^3))})/((a*b^6*c^3 - 12*a^2*b^4*c^4 - 2*a*b^5*c^4 + 48*a^3*b^2*c^5 + 16*a^2*b^3*c^5 + a*b^4*c^5 - 64*a^4*c^6 - 32*a^3*b*c^6 - 8*a^2*b^2*c^6 + 16*a^3*c^7)*\text{abs}(b^2*c - 4*a*c^2)*\text{abs}(c)) - 1/16*((2*b^3*c^3 - 8*a*b*c^4 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*b^3*c + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a*b*c^2 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*b^2*c^2 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*b*c^3 - 2*(b^2 - 4*a*c)*b*c^3*(b^2*c - 4*a*c^2)^2*A + (2*b^4*c^2 - 20*a*b^2*c^3 + 48*a^2*c^4 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*b^4 + 10*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a*b^2*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*b^3*c - 24*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^2*c^2 - 12*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a*b*c^2 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*b^2*c^2 + 6*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 12*(b^2 - 4*a*c)*a*c^3*(b^2*c - 4*a*c^2)^2*C + 4*(\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a*b^4*c^3 - 8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^2*b^2*c^4 - 2*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a*b^3*c^4 + 2*a*b^4*c^4 + 16*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^3*c^5 + 8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^2*b*c^5 + \sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a*b^2*c^5 - 16*a^2*b^2*c^5 - 4*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^2*c^6 + 32*a^3*c^6 - 2*(b^2 - 4*a*c)*a*b^2*c^4 + 8*(b^2 - 4*a*c)*a^2*c^5)*A*\text{abs}(b^2*c - 4*a*c^2) - 2*(\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a*b^5*c^2 - 8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^2*b^3*c^3 - 2*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a*b^4*c^3 + 2*a*b^5*c^3 + 16*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^3*b*c^4 + 8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^2*b^2*c^4 + \sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a*b^3*c^4 - 16*a^2*b^3*c^4 - 4*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^2*b*c^5 + 32*a^3*b*c^5 - 2*(b^2 - 4*a*c)*a*b^3*c^3 + 8*(b^2 - 4*a*c)*a^2*b*c^4)*C*\text{abs}(b^2*c - 4*a*c^2) - (2*b^7*c^5 - 8*a*b^5*c^6 - 32*a^2*b^3*c^7 + 128*a^3*b*c^8 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*b^7*c^3 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a*b^5*c^4 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*b^6*c^4 + 16*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^2*b^3*c^5 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*b^5*c^5 - 64*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^3*b*c^6 - 32*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^2*b^2*c^6 + 16*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^2*b*c^7 - 2*(b^2 - 4*a*c)*b^5*c^5 + 32*(b^2 - 4*a*c)*a^2*b*c^7)*A - (2*b^8*c^4 - 32*a*b^6*c^5 + 160*a^2*b^4*c^6 - 256*a^3*b^2*c^7 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*b^8*c^2 + 16*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a*b^6*c^3 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*b^7*c^3 - 80*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^2*b^4*c^4 - 24*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a*b^5*c^4 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*b^6*c^4 + 128*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^3*b^2*c^5 + 64*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^2*b^3*c^5 + 12*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a*b^4*c^5 - 32*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^2*b^2*c^6 - 2*(b^2 - 4*a*c)*b^6*c^4 + 24*(b^2 - 4*a*c)*a*b^4*c^5 - 64*(b^2 - 4*a*c)*a^2*b^2*c^6)*C)*\arctan(2*\sqrt{1/2}*x/\sqrt{((b^3*c - 4*a*b*c^2 - \sqrt{(b^3*c - 4*a*b*c^2)^2 - 4*(a*b^2*c - 4*a^2*c^2)*(b^2*c^2 - 4*a*c^3)))/(b^2*c^2 - 4*a*c^3))})/((a*b^6*c^3 - 12*a^2*b^4*c^4 - 2*a*b^5*c^4 + 48*a^3*b^2*c^5 + 16*a^2*b^3*c^5 + a*b^4*c^5 - 64*a^4*c^6 - 32*a^3*b*c^6 - 8*a^2*b^2*c^6 + 16*a^3*c^7)*\text{abs}(b^2*c - 4*a*c^2)*\text{abs}(c)) - 1/4*((b^3*c - 4*a*b*c^2 - 2*b^2*c^2 + b*c^3 + (b^2*c - 4*a*c^2 - 2*b*c^2 + c^3)*\sqrt{b^2 - 4*a*c}))*B*\text{abs}(b^2*c - 4*a*c^2) + (b^5*c^2 - 8*a*b^3*c^3 - 2*b^4*c^3 + 16*a^2*b*c^4 + 8*a*b^2*c^4 + b^3*c^4
\end{aligned}$$

4 - 4*a*b*c^5 + (b^4*c^2 - 4*a*b^2*c^3 - 2*b^3*c^3 + b^2*c^4)*sqrt(b^2 - 4*a*c))*B)*log(x^2 + 1/2*(b^3*c - 4*a*b*c^2 + sqrt((b^3*c - 4*a*b*c^2)^2 - 4*(a*b^2*c - 4*a^2*c^2)*(b^2*c^2 - 4*a*c^3)))/(b^2*c^2 - 4*a*c^3))/((b^4 - 8*a*b^2*c - 2*b^3*c + 16*a^2*c^2 + 8*a*b*c^2 + b^2*c^2 - 4*a*c^3)*c^2*abs(b^2*c - 4*a*c^2)) - 1/4*((b^3*c - 4*a*b*c^2 - 2*b^2*c^2 + b*c^3 - (b^2*c - 4*a*c^2 - 2*b*c^2 + c^3)*sqrt(b^2 - 4*a*c))*B*abs(b^2*c - 4*a*c^2) - (b^5*c^2 - 8*a*b^3*c^3 - 2*b^4*c^3 + 16*a^2*b*c^4 + 8*a*b^2*c^4 + b^3*c^4 - 4*a*b*c^5 - (b^4*c^2 - 4*a*b^2*c^3 - 2*b^3*c^3 + b^2*c^4)*sqrt(b^2 - 4*a*c))*B)*log(x^2 + 1/2*(b^3*c - 4*a*b*c^2 - sqrt((b^3*c - 4*a*b*c^2)^2 - 4*(a*b^2*c - 4*a^2*c^2)*(b^2*c^2 - 4*a*c^3)))/(b^2*c^2 - 4*a*c^3))/((b^4 - 8*a*b^2*c - 2*b^3*c + 16*a^2*c^2 + 8*a*b*c^2 + b^2*c^2 - 4*a*c^3)*c^2*abs(b^2*c - 4*a*c^2))

maple [B] time = 0.06, size = 1429, normalized size = 3.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x)

[Out] (-1/2*(A*b*c+2*C*a*c-C*b^2)/(4*a*c-b^2)/c*x^3-1/2*B*(2*a*c-b^2)/(4*a*c-b^2)/c*x^2-1/2*a*(2*A*c-C*b)/(4*a*c-b^2)/c*x+1/2*a*b*B/c/(4*a*c-b^2))/(c*x^4+b*x^2+a)-1/(4*a*c-b^2)^2*B*(-4*a*c+b^2)^(1/2)*a*ln(-2*c*x^2-b+(-4*a*c+b^2)^(1/2))+1/(4*a*c-b^2)^2*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*(-4*a*c+b^2)^(1/2)*a+1/4/(4*a*c-b^2)^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*(-4*a*c+b^2)^(1/2)*b^2+1/(4*a*c-b^2)^2*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b-1/4/(4*a*c-b^2)^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b^3-2/(4*a*c-b^2)^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*C*(-4*a*c+b^2)^(1/2)*a*b+1/4/(4*a*c-b^2)^2/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*C*(-4*a*c+b^2)^(1/2)*b^3-6/(4*a*c-b^2)^2*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*C*a^2+5/2/(4*a*c-b^2)^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*C*a*b^2-1/4/(4*a*c-b^2)^2/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*C*b^4+1/(4*a*c-b^2)^2*B*(-4*a*c+b^2)^(1/2)*a*ln(2*c*x^2+b+(-4*a*c+b^2)^(1/2))+1/(4*a*c-b^2)^2*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*(-4*a*c+b^2)^(1/2)*a+1/4/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*(-4*a*c+b^2)^(1/2)*b^2-1/(4*a*c-b^2)^2*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b+1/4/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b^3-2/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*C*(-4*a*c+b^2)^(1/2)*a*b+1/4/(4*a*c-b^2)^2/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*C*(-4*a*c+b^2)^(1/2)*b^3+6/(4*a*c-b^2)^2*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*C*a^2-5/2/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*C*a*b^2+1/4/(4*a*c-b^2)^2/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*C*b^4

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(Cb^2 - (2Ca + Ab)c)x^3 + Bab + (Bb^2 - 2Bac)x^2 + (Cab - 2Aac)x}{2((b^2c^2 - 4ac^3)x^4 + ab^2c - 4a^2c^2 + (b^3c - 4abc^2)x^2)} + \frac{-\int \frac{4Bacx - Cab + 2Aac - (Cb^2 - (6Ca - Ab)c)x^2}{cx^4 + bx^2 + a} dx}{2(b^2c - 4ac^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")

[Out]
$$-1/2*((C*b^2 - (2*C*a + A*b)*c)*x^3 + B*a*b + (B*b^2 - 2*B*a*c)*x^2 + (C*a*b - 2*A*a*c)*x)/(b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2) + 1/2*integrate(-(4*B*a*c*x - C*a*b + 2*A*a*c - (C*b^2 - (6*C*a - A*b)*c)*x^2)/(c*x^4 + b*x^2 + a), x)/(b^2*c - 4*a*c^2)$$

mupad [B] time = 1.77, size = 4754, normalized size = 11.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x)

[Out]
$$\text{symsum}(\log(-\text{root}(1572864*a^5*b^2*c^8*z^4 - 983040*a^4*b^4*c^7*z^4 + 327680*a^3*b^6*c^6*z^4 - 61440*a^2*b^8*c^5*z^4 + 6144*a*b^{10}*c^4*z^4 - 256*b^{12}*c^3*z^4 - 1048576*a^6*c^9*z^4 + 576*A*C*a*b^8*c^2*z^2 + 24576*A*C*a^4*b^2*c^5*z^2 - 3072*A*C*a^2*b^6*c^3*z^2 + 2048*A*C*a^3*b^4*c^4*z^2 - 32*A*C*b^{10}*c*z^2 + 61440*C^2*a^5*b*c^5*z^2 + 12288*A^2*a^4*b*c^6*z^2 + 432*C^2*a*b^9*c*z^2 - 49152*A*C*a^5*c^6*z^2 - 61440*C^2*a^4*b^3*c^4*z^2 + 24064*C^2*a^3*b^5*c^3*z^2 - 4608*C^2*a^2*b^7*c^2*z^2 + 24576*B^2*a^4*b^2*c^5*z^2 - 6144*B^2*a^3*b^4*c^4*z^2 + 512*B^2*a^2*b^6*c^3*z^2 - 8192*A^2*a^3*b^3*c^5*z^2 + 1536*A^2*a^2*b^5*c^4*z^2 - 32768*B^2*a^5*c^6*z^2 - 16*A^2*b^9*c^2*z^2 - 16*C^2*b^{11}*z^2 - 3072*A*B*C*a^3*b^3*c^3*z + 768*A*B*C*a^2*b^5*c^2*z + 4096*A*B*C*a^4*b*c^4*z - 64*A*B*C*a*b^7*c*z + 672*B*C^2*a^2*b^6*c*z - 32*A^2*B*a*b^6*c^2*z + 15872*B*C^2*a^4*b^2*c^3*z - 4992*B*C^2*a^3*b^4*c^2*z - 1536*A^2*B*a^3*b^2*c^4*z + 384*A^2*B*a^2*b^4*c^3*z - 32*B*C^2*a*b^8*z - 18432*B*C^2*a^5*c^4*z + 2048*A^2*B*a^4*c^5*z + 192*A*B^2*C*a^3*b^2*c^2 - 32*A*B^2*C*a^2*b^4*c - 960*A^2*C^2*a^3*b^2*c^2 - 16*A^2*B^2*a^2*b^3*c^2 - 18*A^3*C*a*b^5*c - 960*B^2*C^2*a^4*b*c^2 + 240*B^2*C^2*a^3*b^3*c + 198*A^2*C^2*a^2*b^4*c + 144*A^3*C*a^2*b^3*c^2 - 192*A^2*B^2*a^3*b*c^3 + 2016*A*C^3*a^4*b*c^2 - 496*A*C^3*a^3*b^3*c + 224*A^3*C*a^3*b*c^3 + 768*A*B^2*C*a^4*c^3 + 360*C^4*a^4*b^2*c - 9*A^4*a*b^4*c^2 + 30*A*C^3*a^2*b^5 - 9*A^2*C^2*a*b^6 - 24*A^4*a^2*b^2*c^3 - 288*A^2*C^2*a^4*c^3 - 16*B^2*C^2*a^2*b^5 - 1296*C^4*a^5*c^2 - 256*B^4*a^4*c^3 - 25*C^4*a^3*b^4 - 16*A^4*a^3*c^4, z, k)*(\text{root}(1572864*a^5*b^2*c^8*z^4 - 983040*a^4*b^4*c^7*z^4 + 327680*a^3*b^6*c^6*z^4 - 61440*a^2*b^8*c^5*z^4 + 6144*a*b^{10}*c^4*z^4 - 256*b^{12}*c^3*z^4 - 1048576*a^6*c^9*z^4 + 576*A*C*a*b^8*c^2*z^2 + 24576*A*C*a^4*b^2*c^5*z^2 - 3072*A*C*a^2*b^6*c^3*z^2 + 2048*A*C*a^3*b^4*c^4*z^2 - 32*A*C*b^{10}*c*z^2 + 61440*C^2*a^5*b*c^5*z^2 + 12288*A^2*a^4*b*c^6*z^2 + 432*C^2*a*b^9*c*z^2 - 49152*A*C*a^5*c^6*z^2 - 61440*C^2*a^4*b^3*c^4*z^2 + 24064*C^2*a^3*b^5*c^3*z^2 - 4608*C^2*a^2*b^7*c^2*z^2 + 24576*B^2*a^4*b^2*c^5*z^2 - 6144*B^2*a^3*b^4*c^4*z^2 + 512*B^2*a^2*b^6*c^3*z^2 - 8192*A^2*a^3*b^3*c^5*z^2 + 1536*A^2*a^2*b^5*c^4*z^2 - 32768*B^2*a^5*c^6*z^2 - 16*A^2*b^9*c^2*z^2 - 16*C^2*b^{11}*z^2 - 3072*A*B*C*a^3*b^3*c^3*z + 768*A*B*C*a^2*b^5*c^2*z + 4096*A*B*C*a^4*b*c^4*z - 64*A*B*C*a*b^7*c*z + 672*B*C^2*a^2*b^6*c*z - 32*A^2*B*a*b^6*c^2*z + 15872*B*C^2*a^4*b^2*c^3*z - 4992*B*C^2*a^3*b^4*c^2*z - 1536*A^2*B*a^3*b^2*c^4*z + 384*A^2*B*a^2*b^4*c^3*z - 32*B*C^2*a*b^8*z - 18432*B*C^2*a^5*c^4*z + 2048*A^2*B*a^4*c^5*z + 192*A*B^2*C*a^3*b^2*c^2 - 32*A*B^2*C*a^2*b^4*c - 960*A^2*C^2*a^3*b^2*c^2 - 16*A^2*B^2*a^2*b^3*c^2 - 18*A^3*C*a*b^5*c - 960*B^2*C^2*a^4*b*c^2 + 240*B^2*C^2*a^3*b^3*c + 198*A^2*C^2*a^2*b^4*c + 144*A^3*C*a^2*b^3*c^2 - 192*A^2*B^2*a^3*b*c^3 + 2016*A*C^3*a^4*b*c^2 - 496*A*C^3*a^3*b^3*c + 224*A^3*C*a^3*b*c^3 + 768*A*B^2*C*a^4*c^3 + 360*C^4*a^4*b^2*c - 9*A^4*a*b^4*c^2 + 30*A*C^3*a^2*b^5 - 9*A^2*C^2*a*b^6 - 24*A^4*a^2*b^2*c^3 - 288*A^2*C^2*a^4*c^3 - 16*B^2*C^2*a^2*b^5 - 1296*C^4*a^5*c^2 - 256*B^4*a^4*c^3 - 25*C^4*a^3*b^4 - 16*A^4*a^3*c^4, z, k))*((x*(1024*B*a^4*c^6 - 16*B*a*b^6*c^3 + 192*B*a^2*b^4*c^4 - 768*B*a^3*b^2*c^5))/(2*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)) - (2048*A*a^4*c^6 - 32*A*a*b^6*c^3 + 16*C*a*b^7*c^2 - 1024*C*a^4*b*c^5 + 384*A*a^2*b^4*c^4 - 1536*A*a^3*b^2*c^5 - 192*C*a^2*b^5*c^3 + 768*C*a^3*b^3*c^4)/$$

$$\begin{aligned}
& (8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)) + (\text{root}(1572864*a^5*b^2*c^8*z^4 - 983040*a^4*b^4*c^7*z^4 + 327680*a^3*b^6*c^6*z^4 - 61440*a^2*b^8*c^5*z^4 + 6144*a*b^{10}*c^4*z^4 - 256*b^{12}*c^3*z^4 - 1048576*a^6*c^9*z^4 + 576*A*C*a*b^8*c^2*z^2 + 24576*A*C*a^4*b^2*c^5*z^2 - 3072*A*C*a^2*b^6*c^3*z^2 + 2048*A*C*a^3*b^4*c^4*z^2 - 32*A*C*b^{10}*c*z^2 + 61440*C^2*a^5*b*c^5*z^2 + 12288*A^2*a^4*b*c^6*z^2 + 432*C^2*a*b^9*c*z^2 - 49152*A*C*a^5*c^6*z^2 - 61440*C^2*a^4*b^3*c^4*z^2 + 24064*C^2*a^3*b^5*c^3*z^2 - 4608*C^2*a^2*b^7*c^2*z^2 + 24576*B^2*a^4*b^2*c^5*z^2 - 6144*B^2*a^3*b^4*c^4*z^2 + 512*B^2*a^2*b^6*c^3*z^2 - 8192*A^2*a^3*b^3*c^5*z^2 + 1536*A^2*a^2*b^5*c^4*z^2 - 32768*B^2*a^5*c^6*z^2 - 16*A^2*b^9*c^2*z^2 - 16*C^2*b^{11}*z^2 - 3072*A*B*C*a^3*b^3*c^3*z + 768*A*B*C*a^2*b^5*c^2*z + 4096*A*B*C*a^4*b*c^4*z - 64*A*B*C*a*b^7*c*z + 672*B*C^2*a^2*b^6*c*z - 32*A^2*B*a*b^6*c^2*z + 15872*B*C^2*a^4*b^2*c^3*z - 4992*B*C^2*a^3*b^4*c^2*z - 1536*A^2*B*a^3*b^2*c^4*z + 384*A^2*B*a^2*b^4*c^3*z - 32*B*C^2*a*b^8*z - 18432*B*C^2*a^5*c^4*z + 2048*A^2*B*a^4*c^5*z + 192*A*B^2*C*a^3*b^2*c^2 - 32*A*B^2*C*a^2*b^4*c - 960*A^2*C^2*a^3*b^2*c^2 - 16*A^2*B^2*a^2*b^3*c^2 - 18*A^3*C*a*b^5*c - 960*B^2*C^2*a^4*b*c^2 + 240*B^2*C^2*a^3*b^3*c + 198*A^2*C^2*a^2*b^4*c + 144*A^3*C*a^2*b^3*c^2 - 192*A^2*B^2*a^3*b*c^3 + 2016*A*C^3*a^4*b*c^2 - 496*A*C^3*a^3*b^3*c + 224*A^3*C*a^3*b*c^3 + 768*A*B^2*C*a^4*c^3 + 360*C^4*a^4*b^2*c - 9*A^4*a*b^4*c^2 + 30*A*C^3*a^2*b^5 - 9*A^2*C^2*a*b^6 - 24*A^4*a^2*b^2*c^3 - 288*A^2*C^2*a^4*c^3 - 16*B^2*C^2*a^2*b^5 - 1296*C^4*a^5*c^2 - 256*B^4*a^4*c^3 - 25*C^4*a^3*b^4 - 16*A^4*a^3*c^4, z, k)*x*(16*b^9*c^3 - 256*a*b^7*c^4 + 4096*a^4*b*c^7 + 1536*a^2*b^5*c^5 - 4096*a^3*b^3*c^6))/(2*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3))) - (1536*B*C*a^4*c^4 + 128*A*B*a^2*b^3*c^3 + 32*B*C*a^2*b^4*c^2 - 512*B*C*a^3*b^2*c^3 - 512*A*B*a^3*b*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)) + (x*(C^2*b^8 - 32*A^2*a^3*c^5 + A^2*b^6*c^2 + 288*C^2*a^4*c^4 + 2*A*C*b^7*c - 16*B^2*a^2*b^3*c^3 + 138*C^2*a^2*b^4*c^2 - 368*C^2*a^3*b^2*c^3 - 20*C^2*a*b^6*c - 2*A^2*a*b^4*c^3 + 64*B^2*a^3*b*c^4 + 48*A*C*a^2*b^3*c^3 - 22*A*C*a*b^5*c^2 + 32*A*C*a^3*b*c^4))/(2*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3))) - (3*A*C^2*a*b^5 - 216*C^3*a^4*c^2 - 5*C^3*a^2*b^4 + 32*A*B^2*a^3*c^3 - 24*A^2*C*a^3*c^3 + 3*A^3*a*b^3*c^2 + 4*A^3*a^2*b*c^3 + 66*C^3*a^3*b^2*c - 51*A*C^2*a^2*b^3*c + 204*A*C^2*a^3*b*c^2 - 16*B^2*C*a^3*b*c^2 - 42*A^2*C*a^2*b^2*c^2 + 6*A^2*C*a*b^4*c)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)) - (x*(16*B^3*a^3*c^3 + B*C^2*a*b^5 + A^2*B*a*b^3*c^2 + 4*A^2*B*a^2*b*c^3 - 14*B*C^2*a^2*b^3*c + 48*B*C^2*a^3*b*c^2 - 24*A*B*C*a^3*c^3 - 10*A*B*C*a^2*b^2*c^2 + 2*A*B*C*a*b^4*c))/(2*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)))*\text{root}(1572864*a^5*b^2*c^8*z^4 - 983040*a^4*b^4*c^7*z^4 + 327680*a^3*b^6*c^6*z^4 - 61440*a^2*b^8*c^5*z^4 + 6144*a*b^{10}*c^4*z^4 - 256*b^{12}*c^3*z^4 - 1048576*a^6*c^9*z^4 + 576*A*C*a*b^8*c^2*z^2 + 24576*A*C*a^4*b^2*c^5*z^2 - 3072*A*C*a^2*b^6*c^3*z^2 + 2048*A*C*a^3*b^4*c^4*z^2 - 32*A*C*b^{10}*c*z^2 + 61440*C^2*a^5*b*c^5*z^2 + 12288*A^2*a^4*b*c^6*z^2 + 432*C^2*a*b^9*c*z^2 - 49152*A*C*a^5*c^6*z^2 - 61440*C^2*a^4*b^3*c^4*z^2 + 24064*C^2*a^3*b^5*c^3*z^2 - 4608*C^2*a^2*b^7*c^2*z^2 + 24576*B^2*a^4*b^2*c^5*z^2 - 6144*B^2*a^3*b^4*c^4*z^2 + 512*B^2*a^2*b^6*c^3*z^2 - 8192*A^2*a^3*b^3*c^5*z^2 + 1536*A^2*a^2*b^5*c^4*z^2 - 32768*B^2*a^5*c^6*z^2 - 16*A^2*b^9*c^2*z^2 - 16*C^2*b^{11}*z^2 - 3072*A*B*C*a^3*b^3*c^3*z + 768*A*B*C*a^2*b^5*c^2*z + 4096*A*B*C*a^4*b*c^4*z - 64*A*B*C*a*b^7*c*z + 672*B*C^2*a^2*b^6*c*z - 32*A^2*B*a*b^6*c^2*z + 15872*B*C^2*a^4*b^2*c^3*z - 4992*B*C^2*a^3*b^4*c^2*z - 1536*A^2*B*a^3*b^2*c^4*z + 384*A^2*B*a^2*b^4*c^3*z - 32*B*C^2*a*b^8*z - 18432*B*C^2*a^5*c^4*z + 2048*A^2*B*a^4*c^5*z + 192*A*B^2*C*a^3*b^2*c^2 - 32*A*B^2*C*a^2*b^4*c - 960*A^2*C^2*a^3*b^2*c^2 - 16*A^2*B^2*a^2*b^3*c^2 - 18*A^3*C*a*b^5*c - 960*B^2*C^2*a^4*b*c^2 + 240*B^2*C^2*a^3*b^3*c + 198*A^2*C^2*a^2*b^4*c + 144*A^3*C*a^2*b^3*c^2 - 192*A^2*B^2*a^3*b*c^3 + 2016*A*C^3*a^4*b*c^2 - 496*A*C^3*a^3*b^3*c + 224*A^3*C*a^3*b*c^3 + 768*A*B^2*C*a^4*c^3 + 360*C^4*a^4*b^2*c - 9*A^4*a*b^4*c^2 + 30*A*C^3*a^2*b^5 - 9*A^2*C^2*a*b^6 - 24*A^4*a^2*b^2*c^3 - 288*A^2*C^2*a^4*c^3 - 16*B^2*C^2*a^2*b^5 - 1296*C^4*a^5*c^2 - 256*B^4*a^4*c^3 - 25*C^4*a^3*b^4 - 16*A^4*a^3*c^4, z, k), k, 1, 4) - ((x^3*(A*b*c - C*b^2 + 2*C*a*c))/(2*c*(4*a*c - b^2)) + (x*(2*A*a*c - C*a*b))/(2*c*(4*a*c - b^2)) - (B*a*b)/(2*c*(4*a
\end{aligned}$$

$*c - b^2)) + (B*x^2*(2*a*c - b^2))/(2*c*(4*a*c - b^2)))/(a + b*x^2 + c*x^4)$
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4*(C*x**2+B*x+A)/(c*x**4+b*x**2+a)**2,x)

[Out] Timed out

$$3.30 \quad \int \frac{x^3(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2} dx$$

Optimal. Leaf size=347

$$-\frac{(Ab-2aC)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}} + \frac{x^2(2acC+Abc+b^2(-C))+a(2Ac-bC)}{2c(b^2-4ac)(a+bx^2+cx^4)} + \frac{Bx(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{B(b^2-4ac)}{2\sqrt{2}\sqrt{b^2-4ac}}$$

Rubi [A] time = 0.62, antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {1662, 1251, 777, 618, 206, 12, 1120, 1166, 205}

$$\frac{x^2(2acC+Abc+b^2(-C))+a(2Ac-bC)}{2c(b^2-4ac)(a+bx^2+cx^4)} - \frac{(Ab-2aC)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}} + \frac{Bx(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{B\left(b-\frac{4ac+b^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} + \frac{B(b\sqrt{b^2-4ac}+4ac+b^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b^2-4ac+b}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)^{3/2}\sqrt{b^2-4ac+b}}$$

Antiderivative was successfully verified.

[In] Int[(x^3*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]

[Out] (B*x*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (a*(2*A*c - b*C) + (A*b*c - b^2*C + 2*a*c*C)*x^2)/(2*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (B*(b - (b^2 + 4*a*c)/Sqrt[b^2 - 4*a*c])/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (B*(b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - ((A*b - 2*a*C)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 777

Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := -Simp[((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (b^2*e*g - b*c*(e*f + d*g) + 2*c*(c*d*f - a*e*g))*x)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(c*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x

+ c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]

Rule 1120

Int[((d_)*(x_))^(m_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :> -Simp[(d^3*(d*x)^(m - 3)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1166

Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1251

Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]

Rule 1662

Int[(Pq_)*((d_)*(x_))^(m_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2 + 1})*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1})*(a + b*x^2 + c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] && !PolyQ[Pq, x^2]

Rubi steps

$$\begin{aligned}
 \int \frac{x^3(A + Bx + Cx^2)}{(a + bx^2 + cx^4)^2} dx &= \int \frac{Bx^4}{(a + bx^2 + cx^4)^2} dx + \int \frac{x^3(A + Cx^2)}{(a + bx^2 + cx^4)^2} dx \\
 &= \frac{1}{2} \text{Subst} \left(\int \frac{x(A + Cx)}{(a + bx + cx^2)^2} dx, x, x^2 \right) + B \int \frac{x^4}{(a + bx^2 + cx^4)^2} dx \\
 &= \frac{Bx(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{a(2Ac - bC) + (Abc - b^2C + 2acC)x^2}{2c(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{B \int \frac{2a -}{a + bx^2}}{2(b^2 - 4ac)} \\
 &= \frac{Bx(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{a(2Ac - bC) + (Abc - b^2C + 2acC)x^2}{2c(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{B(b^2 + 4ac)}{2(b^2 - 4ac)} \\
 &= \frac{Bx(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{a(2Ac - bC) + (Abc - b^2C + 2acC)x^2}{2c(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{B(b^2 + 4ac)}{2\sqrt{2}}
 \end{aligned}$$

Mathematica [A] time = 0.89, size = 358, normalized size = 1.03

$$\frac{1}{4} \left(\frac{2(a(2Ac - bC + 2cx(B + Cx)) + bx^2(Ac - bC + Bcx))}{c(4ac - b^2)(a + bx^2 + cx^2)} + \frac{2(Ab - 2aC) \log(\sqrt{b^2 - 4ac} - b - 2cx^2)}{(b^2 - 4ac)^{3/2}} - \frac{2(Ab - 2aC) \log(\sqrt{b^2 - 4ac} + b + 2cx^2)}{(b^2 - 4ac)^{3/2}} + \frac{\sqrt{2} B (b\sqrt{b^2 - 4ac} - 4ac - b^2) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{\sqrt{c}(b^2 - 4ac)^{3/2}\sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{\sqrt{2} B (b\sqrt{b^2 - 4ac} + 4ac + b^2) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b + \sqrt{b^2 - 4ac}}}\right)}{\sqrt{c}(b^2 - 4ac)^{3/2}\sqrt{b + \sqrt{b^2 - 4ac}}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x^3*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]

[Out] ((-2*(b*x^2*(A*c - b*C + B*c*x) + a*(2*A*c - b*C + 2*c*x*(B + C*x)))/(c*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*B*(-b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*B*(b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (2*(A*b - 2*a*C)*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) - (2*(A*b - 2*a*C)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 (A + Bx + Cx^2)}{(a + bx^2 + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^3*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]

[Out] IntegrateAlgebraic[(x^3*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2, x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")

[Out] Timed out

giac [B] time = 5.37, size = 3228, normalized size = 9.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")

[Out] 1/2*(B*b*c*x^3 - C*b^2*x^2 + 2*C*a*c*x^2 + A*b*c*x^2 + 2*B*a*c*x - C*a*b + 2*A*a*c)/((c*x^4 + b*x^2 + a)*(b^2*c - 4*a*c^2)) + 1/16*((2*b^3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*(b^2 - 4*a*c)^2*B - 4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a*b^3*c^2 - 2*a*b^4*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^3*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^2*b*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a*b^2*c^3 + 16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^2*c^4 - 32*a^3*c^4 + 2*(b^2 - 4*a*c)*a*b^2*c^2 - 8*(b^2 - 4*a*c)*a^2*c^3)*B*abs(b^2 - 4*a*c) - (2*b^7*c^2 - 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*b^7 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2

$$\begin{aligned}
& - 4ac)c)b^6c + 16\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}} \\
& c)a^2b^3c^2 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}} \\
& c)b^5c^2 - 64\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}} \\
& c)a^3b^3c^3 - 32\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}} \\
& c)a^2b^2c^3 + 16\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}} \\
& c)a^2b^3c^4 - 2(b^2 - 4ac)b^5c^2 + 32(b^2 - 4ac)a^2b^3c^4) \\
& B)\arctan(2\sqrt{1/2}x/\sqrt{(b^3 - 4ab^2c + \sqrt{(b^3 - 4ab^2c)^2 - 4(a^2b^2 - 4a^2c^2)} \\
& c)(b^2c - 4ac^2)))/(b^2c - 4ac^2)))/((ab^6c - 12a^2b^4c^2 - 2 \\
& ab^5c^2 + 48a^3b^2c^3 + 16a^2b^3c^3 + ab^4c^3 - 64a^4c^4 - 32a \\
& ^3b^3c^4 - 8a^2b^2c^4 + 16a^3c^5)\text{abs}(b^2 - 4ac)\text{abs}(c)) - 1/16*((2 \\
& b^3c^2 - 8ab^3c^3 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}} \\
& c)b^3 + 4\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}} \\
& c)ab^2c + 2\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}} \\
& c)b^2c - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}} \\
& c)b^2c - 2(b^2 - 4ac)b^2c^2) \\
& (b^2 - 4ac)^2B + 4(\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}} \\
& c)ab^4c - 8\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}} \\
& c)a^2b^2c^2 - 2\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}} \\
& c)ab^3c^2 + 2ab^4c^2 + 16\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}} \\
& c)a^3c^3 + 8\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}} \\
& c)a^2b^3c^3 + \sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}} \\
& c)ab^2c^3 - 16a^2b^2c^3 - 4\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}} \\
& c)a^2c^4 + 32a^3c^4 - 2(b^2 - 4ac)ab^2c^2 + 8(b^2 - 4ac) \\
& a^2c^3)B\text{abs}(b^2 - 4ac) - (2b^7c^2 - 8ab^5c^3 - 32a^2b^3c^4 + 128a^3b^3c^5 - \sqrt{2}\sqrt{b^2 - 4ac} \\
& \sqrt{bc - \sqrt{b^2 - 4ac}} \\
& c)b^7 + 4\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}} \\
& c)ab^5c + 2\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}} \\
& c)b^6c + 16\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}} \\
& c)a^2b^3c^2 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}} \\
& c)b^5c^2 - 64\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}} \\
& c)a^3b^3c^3 - 32\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}} \\
& c)a^2b^2c^3 + 16\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}} \\
& c)a^2b^3c^4 - 2(b^2 - 4ac)b^5c^2 + 32(b^2 - 4ac)a^2b^3c^4) \\
& B)\arctan(2\sqrt{1/2}x/\sqrt{(b^3 - 4ab^2c - \sqrt{(b^3 - 4ab^2c)^2 - 4(a^2b^2 - 4a^2c^2)} \\
& c)(b^2c - 4ac^2)))/(b^2c - 4ac^2)))/((ab^6c - 12a^2b^4c^2 - 2a^2b^5c^2 + 48a^3b^2c^3 + 16a^2b^3c^3 + ab^4c^3 - 64 \\
& a^4c^4 - 32a^3b^3c^4 - 8a^2b^2c^4 + 16a^3c^5)\text{abs}(b^2 - 4ac)\text{abs}(c)) + 1/8*((b^4c - 4ab^2c^2 - 2b^3c^2 + b^2c^3 + (b^3c - 4ab^3c^2 \\
& - 2b^2c^2 + bc^3)\sqrt{b^2 - 4ac})A\text{abs}(b^2 - 4ac) - 2(ab^3c - 4a^2b^3c^2 - 2ab^2c^2 + ab^3c^3 + (ab^2c - 4a^2c^2 - 2ab^3c^2 + ac^3) \\
& \sqrt{b^2 - 4ac})C\text{abs}(b^2 - 4ac) - (b^6c - 8ab^4c^2 - 2b^5c^2 + 16a^2b^2c^3 + 8ab^3c^3 + b^4c^3 - 4ab^2c^4 + (b^5c - 4ab^3 \\
& c^2 - 2b^4c^2 + b^3c^3)\sqrt{b^2 - 4ac})A + 2(ab^5c - 8a^2b^3c^2 - 2ab^4c^2 + 16a^3b^3c^3 + 8a^2b^2c^3 + ab^3c^3 - 4a^2b^3c^4 + \\
& (ab^4c - 4a^2b^2c^2 - 2ab^3c^2 + ab^2c^3)\sqrt{b^2 - 4ac})C) \\
& \log(x^2 + 1/2(b^3 - 4ab^2c + \sqrt{(b^3 - 4ab^2c)^2 - 4(a^2b^2 - 4a^2c^2)} \\
& c)(b^2c - 4ac^2)))/(b^2c - 4ac^2)))/((ab^4 - 8a^2b^2c - 2ab^3c + 16a^3c^2 + 8a^2b^2c^2 + ab^2c^2 - 4a^2c^3)c^2\text{abs}(b^2 - 4ac)) + \\
& 1/8*((b^4c - 4ab^2c^2 - 2b^3c^2 + b^2c^3 - (b^3c - 4ab^3c^2 - 2b^2c^2 + bc^3)\sqrt{b^2 - 4ac})A\text{abs}(b^2 - 4ac) - 2(ab^3c - 4a^2b^3 \\
& c^2 - 2ab^2c^2 + ab^3c^3 - (ab^2c - 4a^2c^2 - 2ab^3c^2 + ac^3)\sqrt{b^2 - 4ac})C\text{abs}(b^2 - 4ac) - (b^6c - 8ab^4c^2 - 2b^5c^2 + 16 \\
& a^2b^2c^3 + 8ab^3c^3 + b^4c^3 - 4ab^2c^4 - (b^5c - 4ab^3c^2 - 2b^4c^2 + b^3c^3)\sqrt{b^2 - 4ac})A + 2(ab^5c - 8a^2b^3c^2 - 2 \\
& ab^4c^2 + 16a^3b^3c^3 + 8a^2b^2c^3 + ab^3c^3 - 4a^2b^3c^4 - (ab^4c - 4a^2b^2c^2 - 2ab^3c^2 + ab^2c^3)\sqrt{b^2 - 4ac})C) \\
& \log(x^2 + 1/2(b^3 - 4ab^2c - \sqrt{(b^3 - 4ab^2c)^2 - 4(a^2b^2 - 4a^2c^2)} \\
& c)(b^2c - 4ac^2)))/(b^2c - 4ac^2)))/((ab^4 - 8a^2b^2c - 2ab^3c + 16a^3c^2 + 8a^2b^2c^2 + ab^2c^2 - 4a^2c^3)c^2\text{abs}(b^2 - 4ac))
\end{aligned}$$

maple [B] time = 0.04, size = 831, normalized size = 2.39

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^3*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x)$

[Out]
$$\begin{aligned} & (-1/2*b*B/(4*a*c-b^2)*x^3-1/2*(A*b*c+2*C*a*c-C*b^2)/(4*a*c-b^2)/c*x^2-a*B/(4*a*c-b^2)*x-1/2*a*(2*A*c-C*b)/(4*a*c-b^2)/c)/(c*x^4+b*x^2+a)+1/2/(4*a*c-b^2)^2*\ln(-2*c*x^2-b+(-4*a*c+b^2)^(1/2))*A*(-4*a*c+b^2)^(1/2)*b-1/(4*a*c-b^2)^2*\ln(-2*c*x^2-b+(-4*a*c+b^2)^(1/2))*C*(-4*a*c+b^2)^(1/2)*a+c/(4*a*c-b^2)^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*\text{arctanh}(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*c*x)*B*(-4*a*c+b^2)^(1/2)*a+1/4/(4*a*c-b^2)^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*\text{arctanh}(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*c*x)*B*(-4*a*c+b^2)^(1/2)*b^2+c/(4*a*c-b^2)^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*\text{arctanh}(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*c*x)*a*b*B-1/4/(4*a*c-b^2)^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*\text{arctanh}(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*c*x)*b^3*B-1/2/(4*a*c-b^2)^2*\ln(2*c*x^2+b+(-4*a*c+b^2)^(1/2))*A*(-4*a*c+b^2)^(1/2)*b+1/(4*a*c-b^2)^2*\ln(2*c*x^2+b+(-4*a*c+b^2)^(1/2))*C*(-4*a*c+b^2)^(1/2)*a+c/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*\text{arctan}(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*c*x)*B*(-4*a*c+b^2)^(1/2)*a+1/4/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*\text{arctan}(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*c*x)*B*(-4*a*c+b^2)^(1/2)*b^2-c/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*\text{arctan}(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*c*x)*a*b*B+1/4/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*\text{arctan}(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*c*x)*b^3*B \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^3*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, \text{algorithm}="maxima")$

[Out]
$$\begin{aligned} & 1/2*(B*b*c*x^3 + 2*B*a*c*x - C*a*b + 2*A*a*c - (C*b^2 - (2*C*a + A*b)*c)*x^2)/((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2) \\ & + 1/2*\text{integrate}((B*b*x^2 - 2*B*a - 2*(2*C*a - A*b)*x)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c) \end{aligned}$$

mupad [B] time = 1.61, size = 3278, normalized size = 9.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((x^3*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x)$

[Out]
$$\begin{aligned} & \text{symsum}(\log(\text{root}(1572864*a^5*b^2*c^6*z^4 - 983040*a^4*b^4*c^5*z^4 + 327680*a^3*b^6*c^4*z^4 - 61440*a^2*b^8*c^3*z^4 + 6144*a*b^10*c^2*z^4 - 1048576*a^6*c^7*z^4 - 256*b^12*c*z^4 + 32768*A*C*a^4*b*c^4*z^2 - 512*A*C*a*b^7*c*z^2 - 24576*A*C*a^3*b^3*c^3*z^2 + 6144*A*C*a^2*b^5*c^2*z^2 + 512*C^2*a^2*b^6*c*z^2 + 12288*B^2*a^4*b*c^4*z^2 - 1536*A^2*a*b^6*c^2*z^2 + 24576*C^2*a^4*b^2*c^3*z^2 - 6144*C^2*a^3*b^4*c^2*z^2 - 8192*B^2*a^3*b^3*c^3*z^2 + 1536*B^2*a^2*b^5*c^2*z^2 - 8192*A^2*a^3*b^2*c^4*z^2 + 6144*A^2*a^2*b^4*c^3*z^2 + 128*A^2*b^8*c*z^2 - 32768*C^2*a^5*c^4*z^2 - 16*B^2*b^9*z^2 + 384*B^2*C*a^2*b^4*c*z - 1024*A*B^2*a^3*b*c^3*z - 192*A*B^2*a*b^5*c*z - 1536*B^2*C*a^3*b^2*c^2*z + 768*A*B^2*a^2*b^3*c^2*z - 32*B^2*C*a*b^6*z + 2048*B^2*C*a^4*c^3*z + 16*A*B^2*b^7*z + 192*A*B^2*C*a^2*b^2*c + 512*A*C^3*a^3*b*c + 128*A^3*C*a*b^3*c + 16*A*B^2*C*a*b^4 - 384*A^2*C^2*a^2*b^2*c - 192*B^2*C^2*a^3*b*c - 48*A^2*B^2*a*b^3*c - 24*B^4*a^2*b^2*c - 16*B^2*C^2*a^2*b^3 - 16*B^4*a^3*c^2 - 4*A^2*B^2*b^5 - 256*C^4*a^4*c - 16*A^4*b^4*c - 9*B^4*a*b^4, z, k))*((256*A*B*a^2*b^2*c^3 + 128*B*C*a^2*b^3*c^2 - 64*A*B*a*b^4*c^2 - 512*B*C*a^3*b*c^3)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - \text{root}(1572864*a^5*b^2*c^6*z^4 - 983040*a^4*b^4*c^5*z^4 + 327680*a^3*b^6*c^4*z^4 - 61440*a^2*b^8*c^3*z^4 + 6144*a*b^10*c^2*z^4 - 1048576*a^6*c^7*z^4 - 256*b^12*c*z^4 + 32768*A*C \end{aligned}$$

$$\begin{aligned}
& a^4 b^5 c^4 z^2 - 512 A C a^2 b^7 c^2 z^2 - 24576 A C a^3 b^3 c^3 z^2 + 6144 A C a^2 b^5 c^2 z^2 + 512 C^2 a^2 b^6 c^2 z^2 + 12288 B^2 a^4 b^4 c^4 z^2 - 1536 A^2 a^2 b^6 c^2 z^2 + 24576 C^2 a^4 b^2 c^3 z^2 - 6144 C^2 a^3 b^4 c^2 z^2 - 8192 B^2 a^3 b^3 c^3 z^2 + 1536 B^2 a^2 b^5 c^2 z^2 - 8192 A^2 a^3 b^2 c^4 z^2 + 6144 A^2 a^2 b^4 c^3 z^2 + 128 A^2 b^8 c^2 z^2 - 32768 C^2 a^5 c^4 z^2 - 16 B^2 b^9 z^2 + 384 B^2 C a^2 b^4 c z - 1024 A B^2 a^3 b^3 c^3 z - 192 A B^2 a^2 b^5 c z - 1536 B^2 C a^3 b^2 c^2 z + 768 A B^2 a^2 b^3 c^2 z - 32 B^2 C a^2 b^6 z + 2048 B^2 C a^4 c^3 z + 16 A B^2 b^7 z + 192 A B^2 C a^2 b^2 c + 512 A C^3 a^3 b^3 c + 128 A^3 C a^2 b^3 c + 16 A B^2 C a^2 b^4 - 384 A^2 C^2 a^2 b^2 c - 192 B^2 C^2 a^3 b^3 c - 48 A^2 B^2 a^2 b^3 c - 24 B^4 a^2 b^2 c - 16 B^2 C^2 a^2 b^3 - 16 B^4 a^3 c^2 - 4 A^2 B^2 b^5 - 256 C^4 a^4 c - 16 A^4 b^4 c - 9 B^4 a^2 b^4, z, k) * ((x * (16 A b^7 c^2 + 2048 C a^4 c^5 - 192 A a^2 b^5 c^3 - 1024 A a^3 b^3 c^5 - 32 C a^2 b^6 c^2 + 768 A a^2 b^3 c^4 + 384 C a^2 b^4 c^3 - 1536 C a^3 b^2 c^4)) / (4 * (b^6 - 64 a^3 c^3 + 48 a^2 b^2 c^2 - 12 a b^4 c))) - (2048 B a^4 c^5 - 32 B a^2 b^6 c^2 + 384 B a^2 b^4 c^3 - 1536 B a^3 b^2 c^4) / (8 * (b^6 - 64 a^3 c^3 + 48 a^2 b^2 c^2 - 12 a b^4 c))) + (root(1572864 a^5 b^2 c^6 z^4 - 983040 a^4 b^4 c^5 z^4 + 327680 a^3 b^6 c^4 z^4 - 61440 a^2 b^8 c^3 z^4 + 6144 a b^10 c^2 z^4 - 1048576 a^6 c^7 z^4 - 256 b^12 c z^4 + 32768 A C a^4 b^4 c^4 z^2 - 512 A C a^2 b^7 c^2 z^2 - 24576 A C a^3 b^3 c^3 z^2 + 6144 A C a^2 b^5 c^2 z^2 + 512 C^2 a^2 b^6 c^2 z^2 + 12288 B^2 a^4 b^4 c^4 z^2 - 1536 A^2 a^2 b^6 c^2 z^2 + 24576 C^2 a^4 b^2 c^3 z^2 - 6144 C^2 a^3 b^4 c^2 z^2 - 8192 B^2 a^3 b^3 c^3 z^2 + 1536 B^2 a^2 b^5 c^2 z^2 - 8192 A^2 a^3 b^2 c^4 z^2 + 6144 A^2 a^2 b^4 c^3 z^2 + 128 A^2 b^8 c^2 z^2 - 32768 C^2 a^5 c^4 z^2 - 16 B^2 b^9 z^2 + 384 B^2 C a^2 b^4 c z - 1024 A B^2 a^3 b^3 c^3 z - 192 A B^2 a^2 b^5 c z - 1536 B^2 C a^3 b^2 c^2 z + 768 A B^2 a^2 b^3 c^2 z - 32 B^2 C a^2 b^6 z + 2048 B^2 C a^4 c^3 z + 16 A B^2 b^7 z + 192 A B^2 C a^2 b^2 c + 512 A C^3 a^3 b^3 c + 128 A^3 C a^2 b^3 c + 16 A B^2 C a^2 b^4 - 384 A^2 C^2 a^2 b^2 c - 192 B^2 C^2 a^3 b^3 c - 48 A^2 B^2 a^2 b^3 c - 24 B^4 a^2 b^2 c - 16 B^2 C^2 a^2 b^3 - 16 B^4 a^3 c^2 - 4 A^2 B^2 b^5 - 256 C^4 a^4 c - 16 A^4 b^4 c - 9 B^4 a^2 b^4, z, k) * x * (32 b^9 c^2 - 512 a b^7 c^3 + 8192 a^4 b^3 c^6 + 3072 a^2 b^5 c^4 - 8192 a^3 b^3 c^5)) / (4 * (b^6 - 64 a^3 c^3 + 48 a^2 b^2 c^2 - 12 a b^4 c))) + (x * (8 A^2 b^5 c^2 - 2 B^2 b^6 c + 64 B^2 a^3 c^4 + 32 C^2 a^2 b^3 c^2 - 32 A^2 a^2 b^3 c^3 + 4 B^2 a^2 b^4 c^2 - 128 C^2 a^3 b^3 c^3 + 128 A C a^2 b^2 c^3 - 32 A C a^2 b^4 c^2)) / (4 * (b^6 - 64 a^3 c^3 + 48 a^2 b^2 c^2 - 12 a b^4 c))) - (3 B^3 a^2 b^3 c + 32 B C^2 a^3 c^2 + 4 B^3 a^2 b^3 c^2 + 8 A^2 B a^2 b^2 c^2 - 32 A B C a^2 b^3 c^2) / (8 * (b^6 - 64 a^3 c^3 + 48 a^2 b^2 c^2 - 12 a b^4 c))) + (x * (4 A^3 b^3 c^2 - 32 C^3 a^3 c^2 + A B^2 b^4 c + 4 A B^2 a^2 b^2 c^2 + 48 A C^2 a^2 b^3 c^2 - 24 A^2 C a^2 b^2 c^2 - 8 B^2 C a^2 b^3 c^2 - 2 B^2 C a^2 b^3 c)) / (4 * (b^6 - 64 a^3 c^3 + 48 a^2 b^2 c^2 - 12 a b^4 c))) * root(1572864 a^5 b^2 c^6 z^4 - 983040 a^4 b^4 c^5 z^4 + 327680 a^3 b^6 c^4 z^4 - 61440 a^2 b^8 c^3 z^4 + 6144 a b^10 c^2 z^4 - 1048576 a^6 c^7 z^4 - 256 b^12 c z^4 + 32768 A C a^4 b^4 c^4 z^2 - 512 A C a^2 b^7 c^2 z^2 - 24576 A C a^3 b^3 c^3 z^2 + 6144 A C a^2 b^5 c^2 z^2 + 512 C^2 a^2 b^6 c^2 z^2 + 12288 B^2 a^4 b^4 c^4 z^2 - 1536 A^2 a^2 b^6 c^2 z^2 + 24576 C^2 a^4 b^2 c^3 z^2 - 6144 C^2 a^3 b^4 c^2 z^2 - 8192 B^2 a^3 b^3 c^3 z^2 + 1536 B^2 a^2 b^5 c^2 z^2 - 8192 A^2 a^3 b^2 c^4 z^2 + 6144 A^2 a^2 b^4 c^3 z^2 + 128 A^2 b^8 c^2 z^2 - 32768 C^2 a^5 c^4 z^2 - 16 B^2 b^9 z^2 + 384 B^2 C a^2 b^4 c z - 1024 A B^2 a^3 b^3 c^3 z - 192 A B^2 a^2 b^5 c z - 1536 B^2 C a^3 b^2 c^2 z + 768 A B^2 a^2 b^3 c^2 z - 32 B^2 C a^2 b^6 z + 2048 B^2 C a^4 c^3 z + 16 A B^2 b^7 z + 192 A B^2 C a^2 b^2 c + 512 A C^3 a^3 b^3 c + 128 A^3 C a^2 b^3 c + 16 A B^2 C a^2 b^4 - 384 A^2 C^2 a^2 b^2 c - 192 B^2 C^2 a^3 b^3 c - 48 A^2 B^2 a^2 b^3 c - 24 B^4 a^2 b^2 c - 16 B^2 C^2 a^2 b^3 - 16 B^4 a^3 c^2 - 4 A^2 B^2 b^5 - 256 C^4 a^4 c - 16 A^4 b^4 c - 9 B^4 a^2 b^4, z, k), k, 1, 4) - ((B a x) / (4 a c - b^2) + (x^2 (A b c - C b^2 + 2 C a c)) / (2 c (4 a c - b^2)) + (B b x^3) / (2 (4 a c - b^2)) + (a (2 A c - C b)) / (2 c (4 a c - b^2))) / (a + b x^2 + c x^4)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(C*x**2+B*x+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```


$$3.31 \quad \int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2} dx$$

Optimal. Leaf size=356

$$\frac{x(-2aC + x^2(2Ac - bC) + Ab)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} \frac{\left(-\frac{4Abc - C(4ac + b^2)}{\sqrt{b^2 - 4ac}} + 2Ac - bC\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2 - 4ac)\sqrt{b - \sqrt{b^2 - 4ac}}} \frac{\left(\frac{4Abc - C(4ac + b^2)}{\sqrt{b^2 - 4ac}} + 2Ac\right)}{2\sqrt{2}\sqrt{c}(b^2 - 4ac)}$$

Rubi [A] time = 0.90, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {1662, 1275, 1166, 205, 12, 1114, 638, 618, 206}

$$\frac{x(-2aC + x^2(2Ac - bC) + Ab)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{\left(-\frac{4Abc - C(4ac + b^2)}{\sqrt{b^2 - 4ac}} + 2Ac - bC\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2 - 4ac)\sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{\left(\frac{4Abc - C(4ac + b^2)}{\sqrt{b^2 - 4ac}} + 2Ac - bC\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b^2 - 4ac + b}}\right)}{2\sqrt{2}\sqrt{c}(b^2 - 4ac)\sqrt{b^2 - 4ac + b}} + \frac{B(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{bB \tanh^{-1}\left(\frac{b + 2cx^2}{\sqrt{b^2 - 4ac}}\right)}{(b^2 - 4ac)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]

[Out] (B*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (x*(A*b - 2*a*C + (2*A*c - b*C)*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((2*A*c - b*C - (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((2*A*c - b*C + (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (b*B*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 638

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1275

```
Int[((f_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1662

```
Int[(Pq_)*((d_.)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2 + 1}]*((a + b*x^2 + c*x^4)^p), x] + Dist[1/d, Int[(d*x)^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k + 1), {k, 0, (q - 1)/2 + 1}]*((a + b*x^2 + c*x^4)^p), x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] && !PolyQ[Pq, x^2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2 (A + Bx + Cx^2)}{(a + bx^2 + cx^4)^2} dx &= \int \frac{Bx^3}{(a + bx^2 + cx^4)^2} dx + \int \frac{x^2 (A + Cx^2)}{(a + bx^2 + cx^4)^2} dx \\
&= -\frac{x (Ab - 2aC + (2Ac - bC)x^2)}{2 (b^2 - 4ac) (a + bx^2 + cx^4)} + B \int \frac{x^3}{(a + bx^2 + cx^4)^2} dx + \frac{\int \frac{Ab - 2aC + (-2Ac + bC)x}{a + bx^2 + cx^4}}{2 (b^2 - 4ac)} \\
&= -\frac{x (Ab - 2aC + (2Ac - bC)x^2)}{2 (b^2 - 4ac) (a + bx^2 + cx^4)} + \frac{1}{2} B \text{Subst} \left(\int \frac{x}{(a + bx + cx^2)^2} dx, x, x^2 \right) - \frac{(2Ac - bC)}{2\sqrt{2}\sqrt{c}} \frac{1}{\sqrt{b^2 - 4ac}} \\
&= \frac{B (2a + bx^2)}{2 (b^2 - 4ac) (a + bx^2 + cx^4)} - \frac{x (Ab - 2aC + (2Ac - bC)x^2)}{2 (b^2 - 4ac) (a + bx^2 + cx^4)} - \frac{(2Ac - bC - \frac{4Abc}{\sqrt{b^2 - 4ac}})}{2\sqrt{2}\sqrt{c}} \frac{1}{\sqrt{b^2 - 4ac}} \\
&= \frac{B (2a + bx^2)}{2 (b^2 - 4ac) (a + bx^2 + cx^4)} - \frac{x (Ab - 2aC + (2Ac - bC)x^2)}{2 (b^2 - 4ac) (a + bx^2 + cx^4)} - \frac{(2Ac - bC - \frac{4Abc}{\sqrt{b^2 - 4ac}})}{2\sqrt{2}\sqrt{c}} \frac{1}{\sqrt{b^2 - 4ac}} \\
&= \frac{B (2a + bx^2)}{2 (b^2 - 4ac) (a + bx^2 + cx^4)} - \frac{x (Ab - 2aC + (2Ac - bC)x^2)}{2 (b^2 - 4ac) (a + bx^2 + cx^4)} - \frac{(2Ac - bC - \frac{4Abc}{\sqrt{b^2 - 4ac}})}{2\sqrt{2}\sqrt{c}} \frac{1}{\sqrt{b^2 - 4ac}}
\end{aligned}$$

Mathematica [A] time = 1.04, size = 378, normalized size = 1.06

$$\frac{1}{4} \left(\frac{4a(B + Cx) + 2x(B + Cx) - A(b + 2cx^2)}{(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\sqrt{2} (c(b\sqrt{b^2 - 4ac} - 4ac - b^2) - 2Ac(\sqrt{b^2 - 4ac} - 2b)) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b^2 - 4ac}} \right)}{\sqrt{c} (b^2 - 4ac)^{3/2} \sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{\sqrt{2} (c(b\sqrt{b^2 - 4ac} + 4ac + b^2) - 2Ac(\sqrt{b^2 - 4ac} + 2b)) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b^2 - 4ac}} \right)}{\sqrt{c} (b^2 - 4ac)^{3/2} \sqrt{b^2 - 4ac} + b} + \frac{2bB \log(\sqrt{b^2 - 4ac} - b - 2cx^2)}{(b^2 - 4ac)^{3/2}} - \frac{2bB \log(\sqrt{b^2 - 4ac} + b + 2cx^2)}{(b^2 - 4ac)^{3/2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]

[Out] ((4*a*(B + C*x) + 2*x*(b*x*(B + C*x) - A*(b + 2*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(-2*A*c*(-2*b + Sqrt[b^2 - 4*a*c]) + (-b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(-2*A*c*(2*b + Sqrt[b^2 - 4*a*c]) + (b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (2*b*B*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) - (2*b*B*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 (A + Bx + Cx^2)}{(a + bx^2 + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]

[Out] IntegrateAlgebraic[(x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2, x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

```
giac [B] time = 7.05, size = 4440, normalized size = 12.47
```

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(C*b*x^3 - 2*A*c*x^3 + B*b*x^2 + 2*C*a*x - A*b*x + 2*B*a)/((c*x^4 + b*x^2 + a)*(b^2 - 4*a*c)) - 1/16*(2*(2*b^2*c^3 - 8*a*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c))*sqrt(b*c + sqrt(b^2 - 4*a*c))*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*c^3 - 2*(b^2 - 4*a*c)*c^3*(b^2 - 4*a*c)^2*A - (2*b^3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*(b^2 - 4*a*c)^2*C - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*b^5*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*b^4*c^2 - 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a*b^2*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*b^3*c^3 + 16*a*b^3*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a*b*c^4 - 32*a^2*b*c^4 + 2*(b^2 - 4*a*c)*b^3*c^2 - 8*(b^2 - 4*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) + 4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a*b^3*c^2 - 2*a*b^4*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^3*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^2*b*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*c)*a*b^2*c^3 + 16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^2*c^4 - 32*a^3*c^4 + 2*(b^2 - 4*a*c)*a*b^2*c^2 - 8*(b^2 - 4*a*c)*a^2*c^3)*C*abs(b^2 - 4*a*c) - 4*(2*b^6*c^3 - 16*a*b^4*c^4 + 32*a^2*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*b^6*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a*b^4*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*b^5*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^2*b^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a*b^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^3 + 8*(b^2 - 4*a*c)*a*b^2*c^4)*A + (2*b^7*c^2 - 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*b^7 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*b^6*c + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^2*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^3*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^2*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 + 32*(b^2 - 4*a*c)*a^2*b*c^4)*C)*arctan(2*sqrt(1/2)*x/sqrt((b^3 - 4*a*b*c + sqrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2)))/((a*b^6*c - 12*a^2*b^4*c^2 - 2*a*b^5*c^2 + 48*a^3*b^2*c^3 + 16*a^2*b^3*c^3 + a*b^4*c^3 - 64*a^4*c^4 - 32*a^3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*abs(b^2 - 4*a*c)*abs(c)) + 1/16*(2*(2*b^2*c^3 - 8*a*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c))*sqrt(b*c - sqrt(b^2 - 4*a*c))*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
```

```

rt(b^2 - 4*a*c)*c)^3 - 2*(b^2 - 4*a*c)*c^3)*(b^2 - 4*a*c)^2*A - (2*b^3*c^
2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*b
^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a*b*c + 2*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*b^2*c - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*b*c^2 - 2*(b^2 - 4*a*c)*b*
c^2)*(b^2 - 4*a*c)^2*C + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*b^5*c -
8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c))*b^4*c^2 + 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c))*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a*b^2*c^3
+ sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*b^3*c^3 - 16*a*b^3*c^3 - 4*sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a*b*c^4 + 32*a^2*b*c^4 - 2*(b^2 - 4*a*c)
*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) - 4*(sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c))*a*b^4*c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*
a^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a*b^3*c^2 + 2*a*b
^4*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^3*c^3 + 8*sqrt(2)*sq
rt(b*c - sqrt(b^2 - 4*a*c))*a^2*b*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*
c))*a*b^2*c^3 - 16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*
a^2*c^4 + 32*a^3*c^4 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + 8*(b^2 - 4*a*c)*a^2*c
^3)*C*abs(b^2 - 4*a*c) - 4*(2*b^6*c^3 - 16*a*b^4*c^4 + 32*a^2*b^2*c^5 - s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*b^6*c + 8*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a*b^4*c^2 + 2*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*b^5*c^2 - 16*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^2*b^2*c^3 - 8*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)
)*sqrt(b*c - sqrt(b^2 - 4*a*c))*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c))*a*b^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^3 + 8*(b^2 -
4*a*c)*a*b^2*c^4)*A + (2*b^7*c^2 - 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*
b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*b^7 + 4*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a*b^5*c + 2*sqrt(2)
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*b^6*c + 16*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^2*b^3*c^2 - sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^3*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^2*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 +
32*(b^2 - 4*a*c)*a^2*b*c^4)*C)*arctan(2*sqrt(1/2)*x/sqrt((b^3 - 4*a*b*c - s
qrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*
a*c^2)))/((a*b^6*c - 12*a^2*b^4*c^2 - 2*a*b^5*c^2 + 48*a^3*b^2*c^3 + 16*a^2
*b^3*c^3 + a*b^4*c^3 - 64*a^4*c^4 - 32*a^3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c
^5)*abs(b^2 - 4*a*c)*abs(c)) + 1/8*((b^4*c - 4*a*b^2*c^2 - 2*b^3*c^2 + b^2*
c^3 + (b^3*c - 4*a*b*c^2 - 2*b^2*c^2 + b*c^3)*sqrt(b^2 - 4*a*c))*B*abs(b^2
- 4*a*c) - (b^6*c - 8*a*b^4*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a*b^3*c^3
+ b^4*c^3 - 4*a*b^2*c^4 + (b^5*c - 4*a*b^3*c^2 - 2*b^4*c^2 + b^3*c^3)*sqrt(
b^2 - 4*a*c))*B)*log(x^2 + 1/2*(b^3 - 4*a*b*c + sqrt((b^3 - 4*a*b*c)^2 - 4*
(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2)))/((a*b^4 - 8*a^2*b
^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2*abs
(b^2 - 4*a*c)) + 1/8*((b^4*c - 4*a*b^2*c^2 - 2*b^3*c^2 + b^2*c^3 - (b^3*c -
4*a*b*c^2 - 2*b^2*c^2 + b*c^3)*sqrt(b^2 - 4*a*c))*B*abs(b^2 - 4*a*c) - (b^
6*c - 8*a*b^4*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a*b^3*c^3 + b^4*c^3 - 4*
a*b^2*c^4 - (b^5*c - 4*a*b^3*c^2 - 2*b^4*c^2 + b^3*c^3)*sqrt(b^2 - 4*a*c))*
B)*log(x^2 + 1/2*(b^3 - 4*a*b*c - sqrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2
*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2)))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*
c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2*abs(b^2 - 4*a*c))

```

maple [B] time = 0.06, size = 1119, normalized size = 3.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x)

[Out] $(1/2*(2*A*c-C*b)/(4*a*c-b^2)*x^3-1/2*b*B/(4*a*c-b^2)*x^2+1/2*(A*b-2*C*a)/(4*a*c-b^2)*x-a*B/(4*a*c-b^2))/(c*x^4+b*x^2+a)+1/2/(4*a*c-b^2)^2*B*(-4*a*c+b^2)^{(1/2)}*b*\ln(-2*c*x^2-b+(-4*a*c+b^2)^{(1/2)})-c/(4*a*c-b^2)^2*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*A*(-4*a*c+b^2)^{(1/2)}*b-2*c^2/(4*a*c-b^2)^2*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*A+1/2*c/(4*a*c-b^2)^2*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*A*b^2+c/(4*a*c-b^2)^2*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*C*(-4*a*c+b^2)^{(1/2)}*a+1/4/(4*a*c-b^2)^2*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*C*a*b-1/4/(4*a*c-b^2)^2*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^3*C-1/2/(4*a*c-b^2)^2*B*(-4*a*c+b^2)^{(1/2)}*b*\ln(2*c*x^2+b+(-4*a*c+b^2)^{(1/2)})-c/(4*a*c-b^2)^2*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*A*(-4*a*c+b^2)^{(1/2)}*b+2*c^2/(4*a*c-b^2)^2*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*A-1/2*c/(4*a*c-b^2)^2*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*A*b^2+c/(4*a*c-b^2)^2*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*C*(-4*a*c+b^2)^{(1/2)}*a+1/4/(4*a*c-b^2)^2*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*C*(-4*a*c+b^2)^{(1/2)}*b^2-c/(4*a*c-b^2)^2*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*C*a*b+1/4/(4*a*c-b^2)^2*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^3*C$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{Bbx^2 + (Cb - 2Ac)x^3 + 2Ba + (2Ca - Ab)x}{2((b^2c - 4ac^2)x^4 + ab^2 - 4a^2c + (b^3 - 4abc)x^2)} - \frac{-\int \frac{2Bbx+(Cb-2Ac)x^2-2Ca+Ab}{cx^4+bx^2+a} dx}{2(b^2 - 4ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")`

[Out] $1/2*(B*b*x^2 + (C*b - 2*A*c)*x^3 + 2*B*a + (2*C*a - A*b)*x)/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2) - 1/2*\operatorname{integrate}(-2*B*b*x + (C*b - 2*A*c)*x^2 - 2*C*a + A*b)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c)$

mupad [B] time = 1.67, size = 3835, normalized size = 10.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x)`

[Out] `symsum(log((8*A^3*a*c^4 + 6*A^3*b^2*c^3 + A*C^2*b^4*c - 3*C^3*a*b^3*c + 4*A*B^2*b^3*c^2 + 8*A*C^2*a^2*c^3 - 5*A^2*C*b^3*c^2 - 4*C^3*a^2*b*c^2 + 18*A*C^2*a*b^2*c^2 - 8*B^2*C*a*b^2*c^2 - 28*A^2*C*a*b*c^3)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - root(256*a*b^12*c*z^4 - 1572864*a^6*b^2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^3*b^8*c^3*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8*c*z^2 - 6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^5*b*c^4*z^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5*c^5*z^2 + 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a^4*b^2*c^4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8192*A^2*a^3*b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A^2*b^9*c*z^2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B*a^3*b*c^4*z`

$$\begin{aligned}
& - 192A^2B^*ab^5c^2z - 768B^*C^2a^3b^3c^2z + 768A^2B^*a^2b^3c^3z \\
& + 16A^2B^*b^7c^z - 16B^*C^2a^*b^7z - 64A^*B^2C^*a^2b^2c^2 - 48A^*B^2 \\
& *C^*a^*b^4c + 192A^2C^2a^2b^2c^2 + 48B^2C^2a^2b^3c + 48A^2B^2a^* \\
& b^3c^2 - 96A^3C^*a^2b^*c^3 - 96A^*C^3a^3b^*c^2 - 80A^3C^*a^*b^3c^2 - 80 \\
& *A^*C^3a^2b^3c + 42A^2C^2a^*b^4c + 24C^4a^3b^2c + 24A^4a^*b^2c^3 \\
& + 4B^2C^2a^*b^5 + 4A^2B^2b^5c + 16B^4a^*b^4c - 6A^3C^*b^5c - 6A^* \\
& C^3a^*b^5 + 32A^2C^2a^3c^3 + 16C^4a^4c^2 + 9C^4a^2b^4 + 9A^4b^4 \\
& c^2 + 16A^4a^2c^4 + A^2C^2b^6, z, k) * (\text{root}(256a^*b^12c^*z^4 - 157286 \\
& 4a^6b^2c^6z^4 + 983040a^5b^4c^5z^4 - 327680a^4b^6c^4z^4 + 61440 \\
& *a^3b^8c^3z^4 - 6144a^2b^10c^2z^4 + 1048576a^7c^7z^4 - 192A^*C^*a^* \\
& b^8c^*z^2 - 6144A^*C^*a^3b^4c^3z^2 + 2048A^*C^*a^2b^6c^2z^2 - 12288C^2 \\
& *a^5b^*c^4z^2 - 12288A^2a^4b^*c^5z^2 - 128B^2a^*b^8c^*z^2 + 16384A^*C^* \\
& a^5c^5z^2 + 8192C^2a^4b^3c^3z^2 - 1536C^2a^3b^5c^2z^2 + 8192B^2 \\
& *a^4b^2c^4z^2 - 6144B^2a^3b^4c^3z^2 + 1536B^2a^2b^6c^2z^2 + 8 \\
& 192A^2a^3b^3c^4z^2 - 1536A^2a^2b^5c^3z^2 + 16C^2a^*b^9z^2 + 16A^2 \\
& *b^9c^*z^2 + 1024B^*C^2a^4b^*c^3z + 192B^*C^2a^2b^5c^*z - 1024A^2B^* \\
& a^3b^*c^4z - 192A^2B^*a^*b^5c^2z - 768B^*C^2a^3b^3c^2z + 768A^2B^* \\
& a^2b^3c^3z + 16A^2B^*b^7c^z - 16B^*C^2a^*b^7z - 64A^*B^2C^*a^2b^2c^2 \\
& - 48A^*B^2C^*a^*b^4c + 192A^2C^2a^2b^2c^2 + 48B^2C^2a^2b^3c + 4 \\
& 8A^2B^2a^*b^3c^2 - 96A^3C^*a^2b^*c^3 - 96A^*C^3a^3b^*c^2 - 80A^3C^*a^* \\
& b^3c^2 - 80A^*C^3a^2b^3c + 42A^2C^2a^*b^4c + 24C^4a^3b^2c + 24A^4 \\
& a^*b^2c^3 + 4B^2C^2a^*b^5 + 4A^2B^2b^5c + 16B^4a^*b^4c - 6A^3C^* \\
& b^5c - 6A^*C^3a^*b^5 + 32A^2C^2a^3c^3 + 16C^4a^4c^2 + 9C^4a^2b^4 \\
& + 9A^4b^4c^2 + 16A^4a^2c^4 + A^2C^2b^6, z, k) * ((x*(16B^*b^7c^2 - \\
& 192B^*a^*b^5c^3 - 1024B^*a^3b^*c^5 + 768B^*a^2b^3c^4))/(4*(b^6 - 64a^3c^3 \\
& + 48a^2b^2c^2 - 12a^*b^4c)) - (16A^*b^7c^2 + 2048C^*a^4c^5 - 192A^* \\
& A^*a^*b^5c^3 - 1024A^*a^3b^*c^5 - 32C^*a^*b^6c^2 + 768A^*a^2b^3c^4 + 384C^* \\
& a^2b^4c^3 - 1536C^*a^3b^2c^4))/(8*(b^6 - 64a^3c^3 + 48a^2b^2c^2 - \\
& 12a^*b^4c)) + (\text{root}(256a^*b^12c^*z^4 - 1572864a^6b^2c^6z^4 + 983040a^5 \\
& b^4c^5z^4 - 327680a^4b^6c^4z^4 + 61440a^3b^8c^3z^4 - 6144a^2b^ \\
& ^10c^2z^4 + 1048576a^7c^7z^4 - 192A^*C^*a^*b^8c^*z^2 - 6144A^*C^*a^3b^4c^ \\
& c^3z^2 + 2048A^*C^*a^2b^6c^2z^2 - 12288C^2a^5b^*c^4z^2 - 12288A^2a^4 \\
& b^*c^5z^2 - 128B^2a^*b^8c^*z^2 + 16384A^*C^*a^5c^5z^2 + 8192C^2a^4b^ \\
& 3c^3z^2 - 1536C^2a^3b^5c^2z^2 + 8192B^2a^4b^2c^4z^2 - 6144B^2a^3 \\
& b^4c^3z^2 + 1536B^2a^2b^6c^2z^2 + 8192A^2a^3b^3c^4z^2 - 153 \\
& 6A^2a^2b^5c^3z^2 + 16C^2a^*b^9z^2 + 16A^2b^9c^*z^2 + 1024B^*C^2a^4 \\
& b^*c^3z + 192B^*C^2a^2b^5c^*z - 1024A^2B^*a^3b^*c^4z - 192A^2B^*a^*b^ \\
& 5c^2z - 768B^*C^2a^3b^3c^2z + 768A^2B^*a^2b^3c^3z + 16A^2B^*b^7c^ \\
& c^z - 16B^*C^2a^*b^7z - 64A^*B^2C^*a^2b^2c^2 - 48A^*B^2C^*a^*b^4c + 192A^2 \\
& C^2a^2b^2c^2 + 48B^2C^2a^2b^3c + 48A^2B^2a^*b^3c^2 - 96A^3C^* \\
& C^*a^2b^*c^3 - 96A^*C^3a^3b^*c^2 - 80A^3C^*a^*b^3c^2 - 80A^*C^3a^2b^3c \\
& + 42A^2C^2a^*b^4c + 24C^4a^3b^2c + 24A^4a^*b^2c^3 + 4B^2C^2a^*b^ \\
& 5 + 4A^2B^2b^5c + 16B^4a^*b^4c - 6A^3C^*b^5c - 6A^*C^3a^*b^5 + 32A^2 \\
& C^2a^3c^3 + 16C^4a^4c^2 + 9C^4a^2b^4 + 9A^4b^4c^2 + 16A^4a^2 \\
& c^4 + A^2C^2b^6, z, k) * x * (32b^9c^2 - 512a^*b^7c^3 + 8192a^4b^*c^6 + \\
& 3072a^2b^5c^4 - 8192a^3b^3c^5))/(4*(b^6 - 64a^3c^3 + 48a^2b^2c^2 - \\
& 12a^*b^4c)) - (16A^*B^*b^5c^2 + 256B^*C^*a^2b^2c^3 - 256A^*B^*a^2b^*c^ \\
& ^4 - 64B^*C^*a^*b^4c^2))/(8*(b^6 - 64a^3c^3 + 48a^2b^2c^2 - 12a^*b^4c)) \\
& + (x*(2C^2b^6c + 64A^2a^2c^5 + 20A^2b^4c^3 - 8B^2b^5c^2 - 64C^2 \\
& a^3c^4 - 12A^*C^*b^5c^2 - 96A^2a^*b^2c^4 + 32B^2a^*b^3c^3 - 4C^2a^* \\
& b^4c^2 + 32A^*C^*a^*b^3c^3 + 64A^*C^*a^2b^*c^4))/(4*(b^6 - 64a^3c^3 + 48a^2 \\
& b^2c^2 - 12a^*b^4c)) + (x*(4B^3b^3c^2 + B^*C^2b^4c + 8A^2B^*b^2 \\
& c^3 + 4B^*C^2a^*b^2c^2 - 6A^*B^*C^*b^3c^2 - 8A^*B^*C^*a^*b^*c^3))/(4*(b^6 - 64 \\
& a^3c^3 + 48a^2b^2c^2 - 12a^*b^4c)) * \text{root}(256a^*b^12c^*z^4 - 1572864a^6 \\
& b^2c^6z^4 + 983040a^5b^4c^5z^4 - 327680a^4b^6c^4z^4 + 61440a^3 \\
& b^8c^3z^4 - 6144a^2b^10c^2z^4 + 1048576a^7c^7z^4 - 192A^*C^*a^*b^8 \\
& c^*z^2 - 6144A^*C^*a^3b^4c^3z^2 + 2048A^*C^*a^2b^6c^2z^2 - 12288C^2a^5 \\
& b^*c^4z^2 - 12288A^2a^4b^*c^5z^2 - 128B^2a^*b^8c^*z^2 + 16384A^*C^*a^5 \\
& c^5z^2 + 8192C^2a^4b^3c^3z^2 - 1536C^2a^3b^5c^2z^2 + 8192B^2a^
\end{aligned}$$

$$\begin{aligned}
& ^4*b^2*c^4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8192 \\
& *A^2*a^3*b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A^2 \\
& *b^9*c*z^2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B*a^ \\
& 3*b*c^4*z - 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*a^2 \\
& *b^3*c^3*z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^2 - \\
& 48*A*B^2*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 48*A \\
& ^2*B^2*a*b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*b^3 \\
& *c^2 - 80*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A^4* \\
& a*b^2*c^3 + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C*b^ \\
& 5*c - 6*A*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^4 + \\
& 9*A^4*b^4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k), k, 1, 4) - ((B*a)/(4* \\
& a*c - b^2) - (x*(A*b - 2*C*a))/(2*(4*a*c - b^2)) - (x^3*(2*A*c - C*b))/(2*(\\
& 4*a*c - b^2)) + (B*b*x^2)/(2*(4*a*c - b^2)))/(a + b*x^2 + c*x^4)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(C*x**2+B*x+A)/(c*x**4+b*x**2+a)**2,x)

[Out] Timed out

$$3.32 \quad \int \frac{x(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2} dx$$

Optimal. Leaf size=317

$$\frac{(2Ac - bC) \tanh^{-1} \left(\frac{b+2cx^2}{\sqrt{b^2-4ac}} \right)}{(b^2 - 4ac)^{3/2}} - \frac{-2aC + x^2(2Ac - bC) + Ab}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{Bx(b + 2cx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{B\sqrt{c} \left(2b - \sqrt{b^2 - 4ac} \right)}{\sqrt{2} (b^2 - 4ac)}$$

Rubi [A] time = 0.42, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {1662, 1247, 638, 618, 206, 12, 1119, 1166, 205}

$$\frac{-2aC + x^2(2Ac - bC) + Ab}{2(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{(2Ac - bC) \tanh^{-1} \left(\frac{b+2cx^2}{\sqrt{b^2-4ac}} \right)}{(b^2 - 4ac)^{3/2}} - \frac{Bx(b + 2cx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{B\sqrt{c} \left(2b - \sqrt{b^2 - 4ac} \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{\sqrt{2} (b^2 - 4ac)^{3/2} \sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{B\sqrt{c} \left(\sqrt{b^2 - 4ac} + 2b \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2 - 4ac} + b}} \right)}{\sqrt{2} (b^2 - 4ac)^{3/2} \sqrt{\sqrt{b^2 - 4ac} + b}}$$

Antiderivative was successfully verified.

[In] Int[(x*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2, x]

[Out] $-(B*x*(b + 2*c*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (A*b - 2*a*C + (2*A*c - b*C)*x^2)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (B*\text{Sqrt}[c]*(2*b - \text{Sqrt}[b^2 - 4*a*c])* \text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]])]/(\text{Sqrt}[2]*(b^2 - 4*a*c)^{(3/2)}*\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]]) - (B*\text{Sqrt}[c]*(2*b + \text{Sqrt}[b^2 - 4*a*c])* \text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]])]/(\text{Sqrt}[2]*(b^2 - 4*a*c)^{(3/2)}*\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]]) + ((2*A*c - b*C)* \text{ArcTanh}[(b + 2*c*x^2)/\text{Sqrt}[b^2 - 4*a*c]])/(b^2 - 4*a*c)^{(3/2)}$

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 638

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]

Rule 1119

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*(d*x)^(m - 1)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*(p
+ 1)*(b^2 - 4*a*c)), x] - Dist[d^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^(m
- 2)*(b*(m - 1) + 2*c*(m + 4*p + 5)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x
] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m,
1] && LeQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1662

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1})*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
+ 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1})*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] && !Po
lyQ[Pq, x^2]
```

Rubi steps

$$\int \frac{x(A + Bx + Cx^2)}{(a + bx^2 + cx^4)^2} dx = \int \frac{Bx^2}{(a + bx^2 + cx^4)^2} dx + \int \frac{x(A + Cx^2)}{(a + bx^2 + cx^4)^2} dx$$

$$= \frac{1}{2} \text{Subst} \left(\int \frac{A + Cx}{(a + bx + cx^2)^2} dx, x, x^2 \right) + B \int \frac{x^2}{(a + bx^2 + cx^4)^2} dx$$

$$= -\frac{Bx(b + 2cx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{Ab - 2aC + (2Ac - bC)x^2}{2(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{B \int \frac{b-2cx^2}{a+bx^2+cx^4} dx}{2(b^2 - 4ac)} - \frac{(2Ac - bC)x^2}{2(b^2 - 4ac)(a + bx^2 + cx^4)}$$

$$= -\frac{Bx(b + 2cx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{Ab - 2aC + (2Ac - bC)x^2}{2(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{(Bc(2b - \sqrt{b^2 - 4ac}))}{2(b^2 - 4ac)}$$

$$= -\frac{Bx(b + 2cx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{Ab - 2aC + (2Ac - bC)x^2}{2(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{B\sqrt{c}(2b - \sqrt{b^2 - 4ac})}{\sqrt{2}(b^2 - 4ac)^{3/2}}$$

Mathematica [A] time = 1.25, size = 335, normalized size = 1.06

$$\frac{1}{2} \left(\frac{2aC - A(b + 2cx^2) + x(-bB + bCx - 2Bcx^2)}{(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{(bC - 2Ac) \log(\sqrt{b^2 - 4ac} - b - 2cx^2)}{(b^2 - 4ac)^{3/2}} + \frac{(2Ac - bC) \log(\sqrt{b^2 - 4ac} + b + 2cx^2)}{(b^2 - 4ac)^{3/2}} - \frac{\sqrt{2}B\sqrt{c}(\sqrt{b^2 - 4ac} - 2b) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{c} - \sqrt{b^2 - 4ac}}\right)}{(b^2 - 4ac)^{3/2}\sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{\sqrt{2}B\sqrt{c}(\sqrt{b^2 - 4ac} + 2b) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{c} + \sqrt{b^2 - 4ac}}\right)}{(b^2 - 4ac)^{3/2}\sqrt{\sqrt{b^2 - 4ac} + b}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]

[Out]
$$\frac{((2*a*C - A*(b + 2*c*x^2) + x*(-(b*B) + b*C*x - 2*B*c*x^2))/(b^2 - 4*a*c)*(a + b*x^2 + c*x^4) - (\text{Sqrt}[2]*B*\text{Sqrt}[c]*(-2*b + \text{Sqrt}[b^2 - 4*a*c])*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]]])/(b^2 - 4*a*c)^{(3/2)}*\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]]) - (\text{Sqrt}[2]*B*\text{Sqrt}[c]*(2*b + \text{Sqrt}[b^2 - 4*a*c])*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]]])/(b^2 - 4*a*c)^{(3/2)}*\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]]) + ((-2*A*c + b*C)*\text{Log}[-b + \text{Sqrt}[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^{(3/2)} + ((2*A*c - b*C)*\text{Log}[b + \text{Sqrt}[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^{(3/2)})/2$$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(A + Bx + Cx^2)}{(a + bx^2 + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]

[Out] IntegrateAlgebraic[(x*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2, x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")

[Out] Timed out

giac [B] time = 5.17, size = 3013, normalized size = 9.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")

[Out]
$$\begin{aligned} & -1/2*(2*B*c*x^3 - C*b*x^2 + 2*A*c*x^2 + B*b*x - 2*C*a + A*b)/((c*x^4 + b*x^2 + a)*(b^2 - 4*a*c)) - 1/8*((2*b^2*c^2 - 8*a*c^3 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*b^2 + 4*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*a*c + 2*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*b*c - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c^2 - 2*(b^2 - 4*a*c)*c^2)*(b^2 - 4*a*c)^2*B - (\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*b^5 - 8*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*a*b^3*c - 2*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*b^4*c - 2*b^5*c + 16*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*a^2*b*c^2 + 8*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*a*b^2*c^2 + \text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*b^3*c^2 + 16*a*b^3*c^2 - 4*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*a*b*c^3 - 32*a^2*b*c^3 + 2*(b^2 - 4*a*c)*b^3*c - 8*(b^2 - 4*a*c)*a*b*c^2)*B*\text{abs}(b^2 - 4*a*c) - 2*(2*b^6*c^2 - 16*a*b^4*c^3 + 32*a^2*b^2*c^4 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*b^6 + 8*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*a*b^4*c + 2*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*b^5*c - 16*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*a^2*b^2*c^2 - 8*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*a*b^3*c^2 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*b^4*c^2 + 4*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*a*b^2*c^3 - 2*(b^2 - 4*a*c)*b^4*c^2 + 8*(b^2 - 4*a*c)*a*b^2*c^3)*B)*\text{arctan}(2*\text{sqrt}(1/2)*x/\text{sqrt}((b^3 - 4*a*b*c + \text{sqrt}((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*c)))) \end{aligned}$$

$$\begin{aligned}
& c) \cdot (b^2c - 4ac^2)) / (b^2c - 4ac^2)) / ((a^6b^6 - 12a^2b^4c - 2ab^5c + 48a^3b^2c^2 + 16a^2b^3c^2 + a^4b^4c^2 - 64a^4c^3 - 32a^3b^3c^3 - 8a^2b^2c^3 + 16a^3c^4) \cdot \text{abs}(b^2 - 4ac) \cdot \text{abs}(c)) + 1/8 \cdot ((2b^2c^2 - 8ac^3 - \sqrt{2}) \cdot \sqrt{b^2 - 4ac}) \cdot \sqrt{b^2c - \sqrt{b^2 - 4ac}} \cdot c) \cdot b^2 + \\
& 4 \cdot \sqrt{2} \cdot \sqrt{b^2 - 4ac}) \cdot \sqrt{b^2c - \sqrt{b^2 - 4ac}} \cdot c) \cdot ac + 2 \cdot \sqrt{2} \cdot \sqrt{b^2 - 4ac}) \cdot \sqrt{b^2c - \sqrt{b^2 - 4ac}} \cdot c) \cdot b^2c - \sqrt{2} \cdot \sqrt{b^2 - 4ac}) \cdot \sqrt{b^2c - \sqrt{b^2 - 4ac}} \cdot c) \cdot c^2 - 2 \cdot (b^2 - 4ac) \cdot c^2) \cdot (b^2 - 4ac)^2 \cdot B + \\
& (\sqrt{2} \cdot \sqrt{b^2c - \sqrt{b^2 - 4ac}} \cdot c) \cdot b^5 - 8 \cdot \sqrt{2} \cdot \sqrt{b^2c - \sqrt{b^2 - 4ac}} \cdot c) \cdot a \cdot b^3c - 2 \cdot \sqrt{2} \cdot \sqrt{b^2c - \sqrt{b^2 - 4ac}} \cdot c) \cdot b^4c + 2 \cdot b^5c + 16 \cdot \sqrt{2} \cdot \sqrt{b^2c - \sqrt{b^2 - 4ac}} \cdot c) \cdot a^2 \cdot b^2c^2 + \\
& 8 \cdot \sqrt{2} \cdot \sqrt{b^2c - \sqrt{b^2 - 4ac}} \cdot c) \cdot a \cdot b^2c^2 + \sqrt{2} \cdot \sqrt{b^2c - \sqrt{b^2 - 4ac}} \cdot c) \cdot b^3c^2 - 16 \cdot a \cdot b^3c^2 - 4 \cdot \sqrt{2} \cdot \sqrt{b^2c - \sqrt{b^2 - 4ac}} \cdot c) \cdot a \cdot b^3c^3 + 32 \cdot a^2 \cdot b^2c^3 - 2 \cdot (b^2 - 4ac) \cdot b^3c + 8 \cdot (b^2 - 4ac) \cdot c) \cdot a \cdot b^2c^2) \cdot B \cdot \text{abs}(b^2 - 4ac) - \\
& 2 \cdot (2b^6c^2 - 16a \cdot b^4c^3 + 32a^2 \cdot b^2c^4 - \sqrt{2} \cdot \sqrt{b^2 - 4ac}) \cdot \sqrt{b^2c - \sqrt{b^2 - 4ac}} \cdot c) \cdot b^6 + 8 \cdot \sqrt{2} \cdot \sqrt{b^2 - 4ac}) \cdot \sqrt{b^2c - \sqrt{b^2 - 4ac}} \cdot c) \cdot a \cdot b^4c + 2 \cdot \sqrt{2} \cdot \sqrt{b^2 - 4ac}) \cdot \sqrt{b^2c - \sqrt{b^2 - 4ac}} \cdot c) \cdot b^5c - 16 \cdot \sqrt{2} \cdot \sqrt{b^2 - 4ac}) \cdot \sqrt{b^2c - \sqrt{b^2 - 4ac}} \cdot c) \cdot a^2 \cdot b^2c^2 - 8 \cdot \sqrt{2} \cdot \sqrt{b^2 - 4ac}) \cdot \sqrt{b^2c - \sqrt{b^2 - 4ac}} \cdot c) \cdot a \cdot b^3c^2 - \sqrt{2} \cdot \sqrt{b^2 - 4ac}) \cdot \sqrt{b^2c - \sqrt{b^2 - 4ac}} \cdot c) \cdot b^4c^2 + 4 \cdot \sqrt{2} \cdot \sqrt{b^2 - 4ac}) \cdot \sqrt{b^2c - \sqrt{b^2 - 4ac}} \cdot c) \cdot a \cdot b^2c^3 - 2 \cdot (b^2 - 4ac) \cdot b^4c^2 + 8 \cdot (b^2 - 4ac) \cdot a \cdot b^2c^3) \cdot B) \cdot \arctan(2 \cdot \sqrt{1/2} \cdot x / \sqrt{(b^3 - 4a \cdot b^2c - \sqrt{(b^3 - 4a \cdot b^2c)^2 - 4 \cdot (a \cdot b^2 - 4a^2c) \cdot (b^2c - 4ac^2))}) / (b^2c - 4ac^2)}) / ((a^6b^6 - 12a^2b^4c - 2ab^5c + 48a^3b^2c^2 + 16a^2b^3c^2 + a^4b^4c^2 - 64a^4c^3 - 32a^3b^3c^3 - 8a^2b^2c^3 + 16a^3c^4) \cdot \text{abs}(b^2 - 4ac) \cdot \text{abs}(c)) - 1/8 \cdot (2 \cdot (b^3c^2 - 4a \cdot b^2c^3 - 2b^2c^3 + b^4c + (b^2c^2 - 4ac^3 - 2b^2c^3 + c^4) \cdot \sqrt{b^2 - 4ac})) \cdot A \cdot \text{abs}(b^2 - 4ac) - (b^4c - 4a \cdot b^2c^2 - 2b^3c^2 + b^2c^3 + (b^3c - 4a \cdot b^2c^2 - 2b^2c^2 + b^2c^3) \cdot \sqrt{b^2 - 4ac})) \cdot C \cdot \text{abs}(b^2 - 4ac) - 2 \cdot (b^5c^2 - 8a \cdot b^3c^3 - 2b^4c^3 + 16a^2 \cdot b^2c^4 + 8a \cdot b^2c^4 + b^3c^4 - 4a \cdot b^2c^5 + (b^4c^2 - 4a \cdot b^2c^3 - 2b^3c^3 + b^2c^4) \cdot \sqrt{b^2 - 4ac})) \cdot A + (b^6c - 8a \cdot b^4c^2 - 2b^5c^2 + 16a^2 \cdot b^2c^3 + 8a \cdot b^3c^3 + b^4c^3 - 4a \cdot b^2c^4 + (b^5c - 4a \cdot b^3c^2 - 2b^4c^2 + b^3c^3) \cdot \sqrt{b^2 - 4ac})) \cdot C) \cdot \log(x^2 + 1/2 \cdot (b^3 - 4a \cdot b^2c + \sqrt{(b^3 - 4a \cdot b^2c)^2 - 4 \cdot (a \cdot b^2 - 4a^2c) \cdot (b^2c - 4ac^2)})) / (b^2c - 4ac^2)) / ((a^4b^4 - 8a^2 \cdot b^2c - 2a \cdot b^3c + 16a^3c^2 + 8a^2 \cdot b^2c^2 + a \cdot b^2c^2 - 4a^2c^3) \cdot c^2 \cdot \text{abs}(b^2 - 4ac)) - 1/8 \cdot (2 \cdot (b^3c^2 - 4a \cdot b^2c^3 - 2b^2c^3 + b^4c - (b^2c^2 - 4ac^3 - 2b^2c^3 + c^4) \cdot \sqrt{b^2 - 4ac})) \cdot A \cdot \text{abs}(b^2 - 4ac) - (b^4c - 4a \cdot b^2c^2 - 2b^3c^2 + b^2c^3 - (b^3c - 4a \cdot b^2c^2 - 2b^2c^2 + b^2c^3) \cdot \sqrt{b^2 - 4ac})) \cdot C \cdot \text{abs}(b^2 - 4ac) - 2 \cdot (b^5c^2 - 8a \cdot b^3c^3 - 2b^4c^3 + 16a^2 \cdot b^2c^4 + 8a \cdot b^2c^4 + b^3c^4 - 4a \cdot b^2c^5 - (b^4c^2 - 4a \cdot b^2c^3 - 2b^3c^3 + b^2c^4) \cdot \sqrt{b^2 - 4ac})) \cdot A + (b^6c - 8a \cdot b^4c^2 - 2b^5c^2 + 16a^2 \cdot b^2c^3 + 8a \cdot b^3c^3 + b^4c^3 - 4a \cdot b^2c^4 - (b^5c - 4a \cdot b^3c^2 - 2b^4c^2 + b^3c^3) \cdot \sqrt{b^2 - 4ac})) \cdot C) \cdot \log(x^2 + 1/2 \cdot (b^3 - 4a \cdot b^2c - \sqrt{(b^3 - 4a \cdot b^2c)^2 - 4 \cdot (a \cdot b^2 - 4a^2c) \cdot (b^2c - 4ac^2)})) / (b^2c - 4ac^2)) / ((a^4b^4 - 8a^2 \cdot b^2c - 2a \cdot b^3c + 16a^3c^2 + 8a^2 \cdot b^2c^2 + a \cdot b^2c^2 - 4a^2c^3) \cdot c^2 \cdot \text{abs}(b^2 - 4ac))
\end{aligned}$$

maple [B] time = 0.18, size = 1344, normalized size = 4.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x \cdot (C \cdot x^2 + B \cdot x + A) / (c \cdot x^4 + b \cdot x^2 + a)^2, x)$

[Out] $-2c^2 / (4ac - b^2)^2 \cdot 2^{1/2} / ((-b + (-4ac + b^2)^{1/2}) \cdot c)^{1/2} \cdot \text{arctanh}(2^{1/2} / 2) / ((-b + (-4ac + b^2)^{1/2}) \cdot c)^{1/2} \cdot c \cdot x) \cdot a \cdot B + 1/2 \cdot c / (4ac - b^2)^2 \cdot 2^{1/2} / ((-b + (-4ac + b^2)^{1/2}) \cdot c)^{1/2} \cdot \text{arctanh}(2^{1/2} / 2) / ((-b + (-4ac + b^2)^{1/2}) \cdot c)^{1/2} \cdot c \cdot x) \cdot b^2 \cdot B + 2c^2 / (4ac - b^2)^2 \cdot 2^{1/2} / ((b + (-4ac + b^2)^{1/2}) \cdot c)^{1/2} \cdot \text{arctan}(2^{1/2} / 2) / ((b + (-4ac + b^2)^{1/2}) \cdot c)^{1/2} \cdot c \cdot x) \cdot a \cdot B - 1/2 \cdot c / (4ac$

$$\begin{aligned}
& -b^2)^2 \cdot 2^{1/2} / ((b + (-4ac + b^2)^{1/2}) \cdot c)^{1/2} \cdot \arctan(2^{1/2} / ((b + (-4ac + b^2)^{1/2}) \cdot c)^{1/2} \cdot cx) \cdot b^2 \cdot B - 1/2 / (4ac - b^2)^2 / (x^2 + 1/2b/c + 1/2(-4ac + b^2)^{1/2} / c) \cdot A \cdot b^2 - 1/2 / (4ac - b^2)^2 / (x^2 + 1/2b/c - 1/2(-4ac + b^2)^{1/2} / c) \cdot A \cdot b^2 - c / (4ac - b^2)^2 \cdot 2^{1/2} / ((b + (-4ac + b^2)^{1/2}) \cdot c)^{1/2} \cdot \arctan(2^{1/2} / ((b + (-4ac + b^2)^{1/2}) \cdot c)^{1/2} \cdot cx) \cdot (-4ac + b^2)^{1/2} \cdot b \cdot B - c / (4ac - b^2)^2 \cdot 2^{1/2} / ((-b + (-4ac + b^2)^{1/2}) \cdot c)^{1/2} \cdot \operatorname{arctanh}(2^{1/2} / ((-b + (-4ac + b^2)^{1/2}) \cdot c)^{1/2} \cdot cx) \cdot (-4ac + b^2)^{1/2} \cdot b \cdot B + 1/4c / (4ac - b^2)^2 / (x^2 + 1/2b/c + 1/2(-4ac + b^2)^{1/2} / c) \cdot b^3 \cdot C + c / (4ac - b^2)^2 \cdot \ln(2 \cdot cx^2 + b + (-4ac + b^2)^{1/2}) \cdot A \cdot (-4ac + b^2)^{1/2} + 2c / (4ac - b^2)^2 / (x^2 + 1/2b/c - 1/2(-4ac + b^2)^{1/2} / c) \cdot A \cdot a + 1/4c / (4ac - b^2)^2 / (x^2 + 1/2b/c - 1/2(-4ac + b^2)^{1/2} / c) \cdot b^3 \cdot C - c / (4ac - b^2)^2 \cdot \ln(-2 \cdot cx^2 - b + (-4ac + b^2)^{1/2}) \cdot A \cdot (-4ac + b^2)^{1/2} + 2c / (4ac - b^2)^2 / (x^2 + 1/2b/c + 1/2(-4ac + b^2)^{1/2} / c) \cdot A \cdot a - 1/2 / (4ac - b^2)^2 / (x^2 + 1/2b/c - 1/2(-4ac + b^2)^{1/2} / c) \cdot B \cdot x \cdot b^2 + 1 / (4ac - b^2)^2 / (x^2 + 1/2b/c - 1/2(-4ac + b^2)^{1/2} / c) \cdot C \cdot (-4ac + b^2)^{1/2} \cdot a - 1 / (4ac - b^2)^2 / (x^2 + 1/2b/c - 1/2(-4ac + b^2)^{1/2} / c) \cdot C \cdot a \cdot b + 1/2 / (4ac - b^2)^2 \cdot \ln(-2 \cdot cx^2 - b + (-4ac + b^2)^{1/2}) \cdot C \cdot (-4ac + b^2)^{1/2} \cdot b - 1/2 / (4ac - b^2)^2 / (x^2 + 1/2b/c + 1/2(-4ac + b^2)^{1/2} / c) \cdot B \cdot x \cdot b^2 - 1 / (4ac - b^2)^2 / (x^2 + 1/2b/c + 1/2(-4ac + b^2)^{1/2} / c) \cdot C \cdot (-4ac + b^2)^{1/2} \cdot a - 1 / (4ac - b^2)^2 / (x^2 + 1/2b/c + 1/2(-4ac + b^2)^{1/2} / c) \cdot C \cdot a \cdot b - 1/2 / (4ac - b^2)^2 \cdot \ln(2 \cdot cx^2 + b + (-4ac + b^2)^{1/2}) \cdot C \cdot (-4ac + b^2)^{1/2} \cdot b + 2c / (4ac - b^2)^2 / (x^2 + 1/2b/c - 1/2(-4ac + b^2)^{1/2} / c) \cdot B \cdot a \cdot x - 1/4c / (4ac - b^2)^2 / (x^2 + 1/2b/c - 1/2(-4ac + b^2)^{1/2} / c) \cdot C \cdot (-4ac + b^2)^{1/2} \cdot b^2 + 2c / (4ac - b^2)^2 / (x^2 + 1/2b/c + 1/2(-4ac + b^2)^{1/2} / c) \cdot B \cdot a \cdot x + 1/4c / (4ac - b^2)^2 / (x^2 + 1/2b/c + 1/2(-4ac + b^2)^{1/2} / c) \cdot C \cdot (-4ac + b^2)^{1/2} \cdot b^2
\end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")

[Out] $-1/2 \cdot (2Bcx^3 + Bbx - (Cb - 2Ac)x^2 - 2Ca + Ab) / ((b^2c - 4ac^2)x^4 + ab^2 - 4a^2c + (b^3 - 4ab^2c)x^2) - 1/2 \cdot \int (2Bcx^2 - Bb - 2(Cb - 2Ac)x) / (cx^4 + bx^2 + a), x) / (b^2 - 4ac)$

mupad [B] time = 1.60, size = 3198, normalized size = 10.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x)

[Out] $\operatorname{symsum}(\log((4B^3ac^4 + 3B^3b^2c^3 + 8A^2B^2bc^4 + 2B^2C^2b^3c^2 - 8AB^2Cb^2c^3) / (4(b^6 - 64a^3c^3 + 48a^2b^2c^2 - 12ab^4c)) - \operatorname{root}(1572864a^6b^2c^5z^4 - 983040a^5b^4c^4z^4 + 327680a^4b^6c^3z^4 - 61440a^3b^8c^2z^4 + 6144a^2b^{10}cz^4 - 1048576a^7c^6z^4 - 256ab^{12}z^4 + 32768A^2Ca^4b^2c^4z^2 - 512A^2Ca^3b^7c^2z^2 - 24576A^2Ca^3b^3c^3z^2 + 6144A^2Ca^2b^5c^2z^2 - 1536C^2a^2b^6c^2z^2 + 12288B^2a^4b^2c^4z^2 + 512A^2a^2b^6c^2z^2 - 8192C^2a^4b^2c^3z^2 + 6144C^2a^3b^4c^2z^2 - 8192B^2a^3b^3c^3z^2 + 1536B^2a^2b^5c^2z^2 + 24576A^2a^3b^2c^4z^2 - 6144A^2a^2b^4c^3z^2 + 128C^2a^2b^8z^2 - 32768A^2a^4c^5z^2 - 16B^2b^9z^2 + 1024B^2Ca^3b^3c^3z - 384AB^2ab^4c^2z + 192B^2Ca^2b^5cz - 768B^2Ca^2b^3c^2z + 1536AB^2a^2b^2c^3z + 32AB^2b^6cz - 2048AB^2a^3c^4z - 16B^2Cb^7z + 192AB^2Ca^2b^2c^2 + 512A^3Cab^3c^3 + 128A^3Cab^3c + 16AB^2Cb^4c - 384A^2C^2ab^2c^2 - 48B^2C^2ab^3c - 192A^2B^2ab^3c^3 - 24B^4ab^2c^2 - 16A^2B^2b^3c^2 - 16B^4a^2c^3 - 4B^2C^2b^5 - 9B^4b^4c - 16C^4ab^4 - 256A^4ac^4, z, k) \cdot (\operatorname{root}(1572864a^6b^2c^5z^4 - 983040a^5b^4c^4z^4 + 327680a^4b^6c^3z^4 - 61440a^3b^8c^2z^4 + 6144a^2b^{10}cz^4 - 1048576a^7c^6z^4 - 256ab^{12}z^4 + 32768A^2Ca^4b^2c^4z^2 - 512A^2Ca^3b^7c^2z^2 - 24576A^2Ca^3b^3c^3z^2 + 6144A^2Ca^2b^5c^2z^2 - 1536C^2a^2b^6c^2z^2 + 12288B^2a^4b^2c^4z^2 + 512A^2a^2b^6c^2z^2 - 8192C^2a^4b^2c^3z^2 + 6144C^2a^3b^4c^2z^2 - 8192B^2a^3b^3c^3z^2 + 1536B^2a^2b^5c^2z^2 + 24576A^2a^3b^2c^4z^2 - 6144A^2a^2b^4c^3z^2 + 128C^2a^2b^8z^2 - 32768A^2a^4c^5z^2 - 16B^2b^9z^2 + 1024B^2Ca^3b^3c^3z - 384AB^2ab^4c^2z + 192B^2Ca^2b^5cz - 768B^2Ca^2b^3c^2z + 1536AB^2a^2b^2c^3z + 32AB^2b^6cz - 2048AB^2a^3c^4z - 16B^2Cb^7z + 192AB^2Ca^2b^2c^2 + 512A^3Cab^3c^3 + 128A^3Cab^3c + 16AB^2Cb^4c - 384A^2C^2ab^2c^2 - 48B^2C^2ab^3c - 192A^2B^2ab^3c^3 - 24B^4ab^2c^2 - 16A^2B^2b^3c^2 - 16B^4a^2c^3 - 4B^2C^2b^5 - 9B^4b^4c - 16C^4ab^4 - 256A^4ac^4, z, k))$

$$\begin{aligned}
& ^4*b*c^4*z^2 - 512*A*C*a*b^7*c*z^2 - 24576*A*C*a^3*b^3*c^3*z^2 + 6144*A*C*a^2*b^5*c^2*z^2 - 1536*C^2*a^2*b^6*c*z^2 + 12288*B^2*a^4*b*c^4*z^2 + 512*A^2*a*b^6*c^2*z^2 - 8192*C^2*a^4*b^2*c^3*z^2 + 6144*C^2*a^3*b^4*c^2*z^2 - 8192*B^2*a^3*b^3*c^3*z^2 + 1536*B^2*a^2*b^5*c^2*z^2 + 24576*A^2*a^3*b^2*c^4*z^2 - 6144*A^2*a^2*b^4*c^3*z^2 + 128*C^2*a*b^8*z^2 - 32768*A^2*a^4*c^5*z^2 - 16*B^2*b^9*z^2 + 1024*B^2*C*a^3*b*c^3*z - 384*A*B^2*a*b^4*c^2*z + 192*B^2*C*a*b^5*c*z - 768*B^2*C*a^2*b^3*c^2*z + 1536*A*B^2*a^2*b^2*c^3*z + 32*A*B^2*b^6*c*z - 2048*A*B^2*a^3*c^4*z - 16*B^2*C*b^7*z + 192*A*B^2*C*a*b^2*c^2 + 512*A^3*C*a*b*c^3 + 128*A*C^3*a*b^3*c + 16*A*B^2*C*b^4*c - 384*A^2*C^2*a*b^2*c^2 - 48*B^2*C^2*a*b^3*c - 192*A^2*B^2*a*b*c^3 - 24*B^4*a*b^2*c^2 - 16*A^2*B^2*b^3*c^2 - 16*B^4*a^2*c^3 - 4*B^2*C^2*b^5 - 9*B^4*b^4*c - 16*C^4*a*b^4 - 256*A^4*a*c^4, z, k)*(x*(512*A*a^3*c^6 - 8*A*b^6*c^3 + 4*C*b^7*c^2 + 96*A*a*b^4*c^4 - 48*C*a*b^5*c^3 - 256*C*a^3*b*c^5 - 384*A*a^2*b^2*c^5 + 192*C*a^2*b^3*c^4))/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) - (8*B*b^7*c^2 - 96*B*a*b^5*c^3 - 512*B*a^3*b*c^5 + 384*B*a^2*b^3*c^4)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (root(1572864*a^6*b^2*c^5*z^4 - 983040*a^5*b^4*c^4*z^4 + 327680*a^4*b^6*c^3*z^4 - 61440*a^3*b^8*c^2*z^4 + 6144*a^2*b^10*c*z^4 - 1048576*a^7*c^6*z^4 - 256*a*b^12*z^4 + 32768*A*C*a^4*b*c^4*z^2 - 512*A*C*a*b^7*c*z^2 - 24576*A*C*a^3*b^3*c^3*z^2 + 6144*A*C*a^2*b^5*c^2*z^2 - 1536*C^2*a^2*b^6*c*z^2 + 12288*B^2*a^4*b*c^4*z^2 + 512*A^2*a*b^6*c^2*z^2 - 8192*C^2*a^4*b^2*c^3*z^2 + 6144*C^2*a^3*b^4*c^2*z^2 - 8192*B^2*a^3*b^3*c^3*z^2 + 1536*B^2*a^2*b^5*c^2*z^2 + 24576*A^2*a^3*b^2*c^4*z^2 - 6144*A^2*a^2*b^4*c^3*z^2 + 128*C^2*a*b^8*z^2 - 32768*A^2*a^4*c^5*z^2 - 16*B^2*b^9*z^2 + 1024*B^2*C*a^3*b*c^3*z - 384*A*B^2*a*b^4*c^2*z + 192*B^2*C*a*b^5*c*z - 768*B^2*C*a^2*b^3*c^2*z + 1536*A*B^2*a^2*b^2*c^3*z + 32*A*B^2*b^6*c*z - 2048*A*B^2*a^3*c^4*z - 16*B^2*C*b^7*z + 192*A*B^2*C*a*b^2*c^2 + 512*A^3*C*a*b*c^3 + 128*A*C^3*a*b^3*c + 16*A*B^2*C*b^4*c - 384*A^2*C^2*a*b^2*c^2 - 48*B^2*C^2*a*b^3*c - 192*A^2*B^2*a*b*c^3 - 24*B^4*a*b^2*c^2 - 16*A^2*B^2*b^3*c^2 - 16*B^4*a^2*c^3 - 4*B^2*C^2*b^5 - 9*B^4*b^4*c - 16*C^4*a*b^4 - 256*A^4*a*c^4, z, k)*x*(8*b^9*c^2 - 128*a*b^7*c^3 + 2048*a^4*b*c^6 + 768*a^2*b^5*c^4 - 2048*a^3*b^3*c^5))/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) - (256*A*B*a^2*c^5 - 16*A*B*b^4*c^3 + 8*B*C*b^5*c^2 - 128*B*C*a^2*b*c^4)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(16*B^2*a^2*c^5 - 8*A^2*b^3*c^4 + 5*B^2*b^4*c^3 - 2*C^2*b^5*c^2 + 8*A*C*b^4*c^3 + 32*A^2*a*b*c^5 - 24*B^2*a*b^2*c^4 + 8*C^2*a*b^3*c^3 - 32*A*C*a*b^2*c^4))/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) - (x*(8*A^3*c^5 - C^3*b^3*c^2 + 4*A*B^2*b*c^4 - 12*A^2*C*b*c^4 + 6*A*C^2*b^2*c^3 - 2*B^2*C*b^2*c^3))/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))*root(1572864*a^6*b^2*c^5*z^4 - 983040*a^5*b^4*c^4*z^4 + 327680*a^4*b^6*c^3*z^4 - 61440*a^3*b^8*c^2*z^4 + 6144*a^2*b^10*c*z^4 - 1048576*a^7*c^6*z^4 - 256*a*b^12*z^4 + 32768*A*C*a^4*b*c^4*z^2 - 512*A*C*a*b^7*c*z^2 - 24576*A*C*a^3*b^3*c^3*z^2 + 6144*A*C*a^2*b^5*c^2*z^2 - 1536*C^2*a^2*b^6*c*z^2 + 12288*B^2*a^4*b*c^4*z^2 + 512*A^2*a*b^6*c^2*z^2 - 8192*C^2*a^4*b^2*c^3*z^2 + 6144*C^2*a^3*b^4*c^2*z^2 - 8192*B^2*a^3*b^3*c^3*z^2 + 1536*B^2*a^2*b^5*c^2*z^2 + 24576*A^2*a^3*b^2*c^4*z^2 - 6144*A^2*a^2*b^4*c^3*z^2 + 128*C^2*a*b^8*z^2 - 32768*A^2*a^4*c^5*z^2 - 16*B^2*b^9*z^2 + 1024*B^2*C*a^3*b*c^3*z - 384*A*B^2*a*b^4*c^2*z + 192*B^2*C*a*b^5*c*z - 768*B^2*C*a^2*b^3*c^2*z + 1536*A*B^2*a^2*b^2*c^3*z + 32*A*B^2*b^6*c*z - 2048*A*B^2*a^3*c^4*z - 16*B^2*C*b^7*z + 192*A*B^2*C*a*b^2*c^2 + 512*A^3*C*a*b*c^3 + 128*A*C^3*a*b^3*c + 16*A*B^2*C*b^4*c - 384*A^2*C^2*a*b^2*c^2 - 48*B^2*C^2*a*b^3*c - 192*A^2*B^2*a*b*c^3 - 24*B^4*a*b^2*c^2 - 16*A^2*B^2*b^3*c^2 - 16*B^4*a^2*c^3 - 4*B^2*C^2*b^5 - 9*B^4*b^4*c - 16*C^4*a*b^4 - 256*A^4*a*c^4, z, k), k, 1, 4) + ((A*b - 2*C*a)/(2*(4*a*c - b^2)) + (x^2*(2*A*c - C*b))/(2*(4*a*c - b^2)) + (B*b*x)/(2*(4*a*c - b^2)) + (B*c*x^3)/(4*a*c - b^2))/(a + b*x^2 + c*x^4)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x**2+B*x+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

$$3.33 \quad \int \frac{A+Bx+Cx^2}{(a+bx^2+cx^4)^2} dx$$

Optimal. Leaf size=368

$$\frac{x(cx^2(Ab-2aC)-2aAc-abC+Ab^2)}{2a(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{c}\left(\frac{A(b^2-12ac)+4abC}{\sqrt{b^2-4ac}} - 2aC + Ab\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{c}\left(-\frac{12aAc+4abC}{\sqrt{b^2-4ac}} - 2aC + Ab\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}}$$

Rubi [A] time = 0.87, antiderivative size = 368, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 25, number of rules / integrand size = 0.360, Rules used = {1673, 1178, 1166, 205, 12, 1107, 614, 618, 206}

$$\frac{x(cx^2(Ab-2aC)-2aAc-abC+Ab^2)}{2a(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{c}\left(\frac{A(b^2-12ac)+4abC}{\sqrt{b^2-4ac}} - 2aC + Ab\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{c}\left(-\frac{12aAc+4abC}{\sqrt{b^2-4ac}} - 2aC + Ab\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}} - \frac{B(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{2Bc\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x + C*x^2)/(a + b*x^2 + c*x^4)^2,x]

[Out] $-(B*(b + 2*c*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (x*(A*b^2 - 2*a*A*c - a*b*C + c*(A*b - 2*a*C)*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (\text{Sqrt}[c]*(A*b - 2*a*C + (A*(b^2 - 12*a*c) + 4*a*b*C)/\text{Sqrt}[b^2 - 4*a*c]))*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]])]/(2*\text{Sqrt}[2]*a*(b^2 - 4*a*c)*\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]]) + (\text{Sqrt}[c]*(A*b - 2*a*C - (A*b^2 - 12*a*A*c + 4*a*b*C)/\text{Sqrt}[b^2 - 4*a*c]))*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]])]/(2*\text{Sqrt}[2]*a*(b^2 - 4*a*c)*\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]]) + (2*B*c*\text{ArcTanh}[(b + 2*c*x^2)/\text{Sqrt}[b^2 - 4*a*c]])/(b^2 - 4*a*c)^{(3/2)}$

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 614

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p + 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && IntegerQ[4*p]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1107

Int[(x_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]

Rule 1166

Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1178

Int[((d_) + (e_)*(x_)^2)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2*p]

Rule 1673

Int[(Pq_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Module[{q = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && !PolyQ[Pq, x^2]

Rubi steps

$$\begin{aligned} \int \frac{A + Bx + Cx^2}{(a + bx^2 + cx^4)^2} dx &= \int \frac{Bx}{(a + bx^2 + cx^4)^2} dx + \int \frac{A + Cx^2}{(a + bx^2 + cx^4)^2} dx \\ &= \frac{x(Ab^2 - 2aAc - abC + c(Ab - 2aC)x^2)}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + B \int \frac{x}{(a + bx^2 + cx^4)^2} dx - \frac{\int \frac{-Ab^2 + 6aAc - a^2}{a + bx^2 + cx^4} dx}{2a(b^2 - 4ac)} \\ &= \frac{x(Ab^2 - 2aAc - abC + c(Ab - 2aC)x^2)}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{1}{2}B \text{Subst} \left(\int \frac{1}{(a + bx + cx^2)^2} dx, x, x^2 \right) \\ &= -\frac{B(b + 2cx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{x(Ab^2 - 2aAc - abC + c(Ab - 2aC)x^2)}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\sqrt{c} \left(A \left(\frac{b + 2cx^2}{2(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{x(Ab^2 - 2aAc - abC + c(Ab - 2aC)x^2)}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} \right) \right)}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} \\ &= -\frac{B(b + 2cx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{x(Ab^2 - 2aAc - abC + c(Ab - 2aC)x^2)}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\sqrt{c} \left(A \left(\frac{b + 2cx^2}{2(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{x(Ab^2 - 2aAc - abC + c(Ab - 2aC)x^2)}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} \right) \right)}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} \\ &= -\frac{B(b + 2cx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{x(Ab^2 - 2aAc - abC + c(Ab - 2aC)x^2)}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\sqrt{c} \left(A \left(\frac{b + 2cx^2}{2(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{x(Ab^2 - 2aAc - abC + c(Ab - 2aC)x^2)}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} \right) \right)}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} \end{aligned}$$

Mathematica [A] time = 1.22, size = 393, normalized size = 1.07

$$\frac{1}{4} \left(\frac{4acx(A+x(B+Cx)) + 2ab(B+Cx) - 2Abx(b+cx^2)}{a(4ac-b^2)(a+bx^2+cx^4)}, \frac{\sqrt{2}\sqrt{c}\left(A(b\sqrt{b^2-4ac}-12ac+b^2)-2ac(\sqrt{b^2-4ac}-2b)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}, \frac{\sqrt{2}\sqrt{c}\left(A(-b\sqrt{b^2-4ac}-12ac+b^2)+2ac(\sqrt{b^2-4ac}+2b)\right)\tan^{-1}\left(\frac{-\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{a(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}, \frac{4Bc\log(\sqrt{b^2-4ac}-b-2cx^2)}{(b^2-4ac)^{3/2}}, \frac{4Bc\log(\sqrt{b^2-4ac}+b+2cx^2)}{(b^2-4ac)^{3/2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x + C*x^2)/(a + b*x^2 + c*x^4)^2, x]

[Out]
$$\frac{((2ab(B+Cx) - 2Abx(b+cx^2) + 4acx(A+x(B+Cx)))/(a(-b^2 + 4ac)(a+bx^2+cx^4)) + (\sqrt{2}\sqrt{c}(A(b\sqrt{b^2-4ac}-12ac+b^2)-2ac(\sqrt{b^2-4ac}-2b))\tan^{-1}(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}) - 2a(-2b+\sqrt{b^2-4ac})C)\text{ArcTan}[(\sqrt{2}\sqrt{c}x)/\sqrt{b-\sqrt{b^2-4ac}}]) - (\sqrt{2}\sqrt{c}(A(-b\sqrt{b^2-4ac}-12ac+b^2)+2ac(\sqrt{b^2-4ac}+2b))\tan^{-1}(\frac{-\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}) + 2a(2b+\sqrt{b^2-4ac})C)\text{ArcTan}[(\sqrt{2}\sqrt{c}x)/\sqrt{b+\sqrt{b^2-4ac}}]) - (4Bc\text{Log}[-b+\sqrt{b^2-4ac}-2cx^2])/(b^2-4ac)^{3/2} + (4Bc\text{Log}[b+\sqrt{b^2-4ac}+2cx^2])/(b^2-4ac)^{3/2})/4$$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx + Cx^2}{(a + bx^2 + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B*x + C*x^2)/(a + b*x^2 + c*x^4)^2, x]

[Out] IntegrateAlgebraic[(A + B*x + C*x^2)/(a + b*x^2 + c*x^4)^2, x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")

[Out] Timed out

giac [B] time = 7.85, size = 5158, normalized size = 14.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")

[Out]
$$-1/2*(2Cacx^3 - Abcx^3 + 2Bacx^2 + Cabx - Ab^2x + 2Aacx + B^2a)/(c^2x^4 + b^2x^2 + a^2)(ab^2 - 4a^2c) + 1/16*((2b^3c^2 - 8ab^2c^3 - \sqrt{2}\sqrt{b^2-4ac})\sqrt{bc + \sqrt{b^2-4ac}}b^3 + 4\sqrt{2}\sqrt{b^2-4ac})\sqrt{bc + \sqrt{b^2-4ac}}ab^2c + 2\sqrt{2}\sqrt{b^2-4ac})\sqrt{bc + \sqrt{b^2-4ac}}b^2c - \sqrt{2}\sqrt{b^2-4ac})\sqrt{bc + \sqrt{b^2-4ac}}b^2c^2 - 2(b^2-4ac)b^2c^2)(ab^2 - 4a^2c)^2A - 2(2ab^2c^2 - 8a^2c^3 - \sqrt{2}\sqrt{b^2-4ac})\sqrt{bc + \sqrt{b^2-4ac}}ab^2 + 4\sqrt{2}\sqrt{b^2-4ac})\sqrt{bc + \sqrt{b^2-4ac}}a^2c + 2\sqrt{2}\sqrt{b^2-4ac})\sqrt{bc + \sqrt{b^2-4ac}}ab^2c - \sqrt{2}\sqrt{b^2-4ac})\sqrt{bc + \sqrt{b^2-4ac}}a^2c^2 - 2(b^2-4ac)a^2c^2)(ab^2 - 4a^2c)^2C + 2(\sqrt{2}\sqrt{b^2-4ac})\sqrt{bc + \sqrt{b^2-4ac}}ab^6 - 14\sqrt{2}\sqrt{b^2-4ac})\sqrt{bc + \sqrt{b^2-4ac}}a^2b^4c - 2\sqrt{2}\sqrt{b^2-4ac})\sqrt{bc + \sqrt{b^2-4ac}}ab^5c - 2ab^6c + 64\sqrt{2}\sqrt{b^2-4ac})\sqrt{bc + \sqrt{b^2-4ac}}a^3b^2c^2 + 20\sqrt{2}\sqrt{b^2-4ac})\sqrt{bc + \sqrt{b^2-4ac}}a^2b^3c^2 + \sqrt{2}\sqrt{b^2-4ac})\sqrt{bc + \sqrt{b^2-4ac}}$$

$$\begin{aligned}
& 2 - 4ac)c)ab^4c^2 + 28a^2b^4c^2 - 96\sqrt{2}\sqrt{b^2 - 4ac)c)a^4c^3 - 48\sqrt{2}\sqrt{b^2 - 4ac)c)a^3b^2c^3 - 10\sqrt{2}\sqrt{b^2 - 4ac)c)a^2b^2c^3 - 128a^3b^2c^3 + 24\sqrt{2}\sqrt{b^2 - 4ac)c)a^3c^4 + 192a^4c^4 + 2(b^2 - 4ac)ab^4c - 20(b^2 - 4ac)a^2b^2c^2 + 48(b^2 - 4ac)a^3c^3) \\
& A\text{abs}(ab^2 - 4a^2c) + 2(\sqrt{2}\sqrt{b^2 - 4ac)c)a^2b^5 - 8\sqrt{2}\sqrt{b^2 - 4ac)c)a^3b^3c - 2\sqrt{2}\sqrt{b^2 - 4ac)c)a^2b^4c - 2a^2b^5c + 16\sqrt{2}\sqrt{b^2 - 4ac)c)a^4b^2c^2 + 8\sqrt{2}\sqrt{b^2 - 4ac)c)a^3b^2c^2 + \sqrt{2}\sqrt{b^2 - 4ac)c)a^2b^3c^2 + 16a^3b^3c^2 - 4\sqrt{2}\sqrt{b^2 - 4ac)c)a^3b^2c^3 - 32a^4b^2c^3 + 2(b^2 - 4ac)a^2b^3c - 8(b^2 - 4ac)a^3b^2c^2)C\text{abs}(ab^2 - 4a^2c) + (2a^2b^7c^2 - 40a^3b^5c^3 + 224a^4b^3c^4 - 384a^5b^2c^5 - \sqrt{2}\sqrt{b^2 - 4ac)c)a^2b^7 + 20\sqrt{2}\sqrt{b^2 - 4ac)c)a^3b^5c + 2\sqrt{2}\sqrt{b^2 - 4ac)c)a^2b^6c - 112\sqrt{2}\sqrt{b^2 - 4ac)c)a^4b^3c^2 - 32\sqrt{2}\sqrt{b^2 - 4ac)c)a^3b^4c^2 - \sqrt{2}\sqrt{b^2 - 4ac)c)a^2b^5c^2 + 192\sqrt{2}\sqrt{b^2 - 4ac)c)a^5b^2c^3 + 96\sqrt{2}\sqrt{b^2 - 4ac)c)a^4b^2c^3 + 16\sqrt{2}\sqrt{b^2 - 4ac)c)a^3b^3c^3 - 48\sqrt{2}\sqrt{b^2 - 4ac)c)a^4b^2c^4 - 2(b^2 - 4ac)a^2b^5c^2 + 32(b^2 - 4ac)a^3b^3c^3 - 96(b^2 - 4ac)a^4b^2c^4)A + 4(2a^3b^6c^2 - 16a^4b^4c^3 + 32a^5b^2c^4 - \sqrt{2}\sqrt{b^2 - 4ac)c)a^3b^6 + 8\sqrt{2}\sqrt{b^2 - 4ac)c)a^4b^4c + 2\sqrt{2}\sqrt{b^2 - 4ac)c)a^3b^5c - 16\sqrt{2}\sqrt{b^2 - 4ac)c)a^5b^2c^2 - 8\sqrt{2}\sqrt{b^2 - 4ac)c)a^4b^3c^2 - \sqrt{2}\sqrt{b^2 - 4ac)c)a^3b^4c^2 + 4\sqrt{2}\sqrt{b^2 - 4ac)c)a^4b^2c^3 - 2(b^2 - 4ac)a^3b^4c^2 + 8(b^2 - 4ac)a^4b^2c^3)C)\arctan(2\sqrt{1/2}x/\sqrt{(ab^3 - 4a^2bc + \sqrt{(ab^3 - 4a^2bc)^2 - 4(a^2b^2 - 4a^3c)(ab^2c - 4a^2c^2))})/(ab^2c - 4a^2c^2)))/((a^3b^6 - 12a^4b^4c - 2a^3b^5c + 48a^5b^2c^2 + 16a^4b^3c^2 + a^3b^4c^2 - 64a^6c^3 - 32a^5b^2c^3 - 8a^4b^2c^3 + 16a^5c^4)\text{abs}(ab^2 - 4a^2c)\text{abs}(c)) - 1/16((2b^3c^2 - 8ab^2c^3 - \sqrt{2}\sqrt{b^2 - 4ac)c)b^3 + 4\sqrt{2}\sqrt{b^2 - 4ac)c)ab^2c - 2\sqrt{2}\sqrt{b^2 - 4ac)c)b^2c - \sqrt{2}\sqrt{b^2 - 4ac)c)b^2c - \sqrt{2}\sqrt{b^2 - 4ac)c)b^2c - 2(b^2 - 4ac)b^2c^2)(ab^2 - 4a^2c)^2A - 2(2ab^2c^2 - 8a^2c^3 - \sqrt{2}\sqrt{b^2 - 4ac)c)ab^2 + 4\sqrt{2}\sqrt{b^2 - 4ac)c)ab^2c - \sqrt{2}\sqrt{b^2 - 4ac)c)a^2c + 2\sqrt{2}\sqrt{b^2 - 4ac)c)ab^2c - \sqrt{2}\sqrt{b^2 - 4ac)c)ab^2c - \sqrt{2}\sqrt{b^2 - 4ac)c)ab^2c - 2(b^2 - 4ac)a^2c^2)(ab^2 - 4a^2c)^2C - 2(\sqrt{2}\sqrt{b^2 - 4ac)c)ab^6 - 14\sqrt{2}\sqrt{b^2 - 4ac)c)a^2b^4c - 2\sqrt{2}\sqrt{b^2 - 4ac)c)ab^5c + 2ab^6c + 64\sqrt{2}\sqrt{b^2 - 4ac)c)a^3b^2c^2 + 20\sqrt{2}\sqrt{b^2 - 4ac)c)a^2b^3c^2 + \sqrt{2}\sqrt{b^2 - 4ac)c)ab^4c^2 - 28a^2b^4c^2 - 96\sqrt{2}\sqrt{b^2 - 4ac)c)a^4c^3 - 48\sqrt{2}\sqrt{b^2 - 4ac)c)a^3b^2c^3 - 10\sqrt{2}\sqrt{b^2 - 4ac)c)a^2b^2c^3 + 128a^3b^2c^3 + 24\sqrt{2}\sqrt{b^2 - 4ac)c)a^3c^4 - 192a^4c^4 - 2(b^2 - 4ac)ab^4c + 20(b^2 - 4ac)a^2b^2c^2 - 48(b^2 - 4ac)a^3c^3)A\text{abs}(ab^2 - 4a^2c) - 2(\sqrt{2}\sqrt{b^2 - 4ac)c)a^2b^5 - 8\sqrt{2}\sqrt{b^2 - 4ac)c)a^3b^3c - 2\sqrt{2}\sqrt{b^2 - 4ac)c)a^2b^4c + 2a^2b^5c + 16\sqrt{2}\sqrt{b^2 - 4ac)c)a^4b^2c^2 + 8\sqrt{2}\sqrt{b^2 - 4ac)c)a^3b^2c^2}
\end{aligned}$$

$$\begin{aligned}
&^2 + \sqrt{2} \sqrt{b*c - \sqrt{b^2 - 4*a*c}} * c) * a^2 * b^3 * c^2 - 16 * a^3 * b^3 * c^2 - \\
&4 * \sqrt{2} \sqrt{b*c - \sqrt{b^2 - 4*a*c}} * c) * a^3 * b * c^3 + 32 * a^4 * b * c^3 - 2 * (b^2 - 4*a*c) * \\
&a^2 * b^3 * c + 8 * (b^2 - 4*a*c) * a^3 * b * c^2) * C * \text{abs}(a * b^2 - 4 * a^2 * c) + \\
&(2 * a^2 * b^7 * c^2 - 40 * a^3 * b^5 * c^3 + 224 * a^4 * b^3 * c^4 - 384 * a^5 * b * c^5 - \sqrt{2}) \\
&* \sqrt{b^2 - 4*a*c} * \sqrt{b*c - \sqrt{b^2 - 4*a*c}} * c) * a^2 * b^7 + 20 * \sqrt{2} * \sqrt{b^2 - 4*a*c} * \\
&\sqrt{b*c - \sqrt{b^2 - 4*a*c}} * c) * a^3 * b^5 * c + 2 * \sqrt{2} * \sqrt{b^2 - 4*a*c} * \sqrt{b^2 - 4*a*c} * \\
&\sqrt{b*c - \sqrt{b^2 - 4*a*c}} * c) * a^2 * b^6 * c - 112 * \sqrt{2} * \sqrt{b^2 - 4*a*c} * \sqrt{b^2 - 4*a*c} * \\
&\sqrt{b*c - \sqrt{b^2 - 4*a*c}} * c) * a^4 * b^3 * c^2 - 32 * \sqrt{2} * \sqrt{b^2 - 4*a*c} * \sqrt{b^2 - 4*a*c} * \\
&\sqrt{b*c - \sqrt{b^2 - 4*a*c}} * c) * a^3 * b^4 * c^2 - \sqrt{2} * \sqrt{b^2 - 4*a*c} * \sqrt{b^2 - 4*a*c} * \\
&\sqrt{b*c - \sqrt{b^2 - 4*a*c}} * c) * a^2 * b^5 * c^2 + 192 * \sqrt{2} * \sqrt{b^2 - 4*a*c} * \sqrt{b^2 - 4*a*c} * \\
&\sqrt{b*c - \sqrt{b^2 - 4*a*c}} * c) * a^5 * b * c^3 + 96 * \sqrt{2} * \sqrt{b^2 - 4*a*c} * \sqrt{b^2 - 4*a*c} * \\
&\sqrt{b*c - \sqrt{b^2 - 4*a*c}} * c) * a^4 * b^2 * c^3 + 16 * \sqrt{2} * \sqrt{b^2 - 4*a*c} * \sqrt{b^2 - 4*a*c} * \\
&\sqrt{b*c - \sqrt{b^2 - 4*a*c}} * c) * a^3 * b^3 * c^3 - 48 * \sqrt{2} * \sqrt{b^2 - 4*a*c} * \sqrt{b^2 - 4*a*c} * \\
&\sqrt{b*c - \sqrt{b^2 - 4*a*c}} * c) * a^4 * b * c^4 - 2 * (b^2 - 4*a*c) * a^2 * b^5 * c^2 + 32 * (b^2 - 4*a*c) * \\
&a^3 * b^3 * c^3 - 96 * (b^2 - 4*a*c) * a^4 * b * c^4) * A + 4 * (2 * a^3 * b^6 * c^2 - 16 * a^4 * b^4 * c^3 + \\
&32 * a^5 * b^2 * c^4 - \sqrt{2}) * \sqrt{b^2 - 4*a*c} * \sqrt{b*c - \sqrt{b^2 - 4*a*c}} * c) * a^3 * b^6 + 8 * \sqrt{2} * \\
&\sqrt{b^2 - 4*a*c} * \sqrt{b*c - \sqrt{b^2 - 4*a*c}} * c) * a^4 * b^4 * c + 2 * \sqrt{2} * \sqrt{b^2 - 4*a*c} * \sqrt{b*c - \\
&\sqrt{b^2 - 4*a*c}} * c) * a^3 * b^5 * c - 16 * \sqrt{2} * \sqrt{b^2 - 4*a*c} * \sqrt{b*c - \sqrt{b^2 - 4*a*c}} * c) * \\
&a^5 * b^2 * c^2 - 8 * \sqrt{2} * \sqrt{b^2 - 4*a*c} * \sqrt{b*c - \sqrt{b^2 - 4*a*c}} * c) * a^4 * b^3 * c^2 - \\
&\sqrt{2} * \sqrt{b^2 - 4*a*c} * \sqrt{b*c - \sqrt{b^2 - 4*a*c}} * c) * a^3 * b^4 * c^2 + 4 * \sqrt{2} * \sqrt{b^2 - 4*a*c} * \\
&\sqrt{b*c - \sqrt{b^2 - 4*a*c}} * c) * a^4 * b^2 * c^3 - 2 * (b^2 - 4*a*c) * a^3 * b^4 * c^2 + 8 * (b^2 - 4*a*c) * \\
&a^4 * b^2 * c^3) * C * \arctan(2 * \sqrt{1/2} * x / \sqrt{(a * b^3 - 4 * a^2 * b * c - \sqrt{(a * b^3 - 4 * a^2 * b * c)^2 - 4 * (a^2 * b^2 - 4 * a^3 * c) * (a * b^2 * c - 4 * a^2 * c^2))}) / (a * b^2 * c - 4 * a^2 * c^2)) / ((a^3 * b^6 - 12 * a^4 * b^4 * c - 2 * a^3 * b^5 * c + 48 * a^5 * b^2 * c^2 + 16 * a^4 * b^3 * c^2 + a^3 * b^4 * c^2 - 64 * a^6 * c^3 - 32 * a^5 * b * c^3 - 8 * a^4 * b^2 * c^3 + 16 * a^5 * c^4) * \text{abs}(a * b^2 - 4 * a^2 * c) * \text{abs}(c)) - 1/4 * ((b^3 * c^2 - 4 * a * b * c^3 - 2 * b^2 * c^3 + b * c^4 + (b^2 * c^2 - 4 * a * c^3 - 2 * b * c^3 + c^4) * \sqrt{b^2 - 4 * a * c})) * B * \text{abs}(a * b^2 - 4 * a^2 * c) - (a * b^5 * c^2 - 8 * a^2 * b^3 * c^3 - 2 * a * b^4 * c^3 + 16 * a^3 * b * c^4 + 8 * a^2 * b^2 * c^4 + a * b^3 * c^4 - 4 * a^2 * b * c^5 + (a * b^4 * c^2 - 4 * a^2 * b^2 * c^3 - 2 * a * b^3 * c^3 + a * b^2 * c^4) * \sqrt{b^2 - 4 * a * c})) * B) * \log(x^2 + 1/2 * (a * b^3 - 4 * a^2 * b * c + \sqrt{(a * b^3 - 4 * a^2 * b * c)^2 - 4 * (a^2 * b^2 - 4 * a^3 * c) * (a * b^2 * c - 4 * a^2 * c^2))}) / (a * b^2 * c - 4 * a^2 * c^2)) / ((a * b^4 - 8 * a^2 * b^2 * c - 2 * a * b^3 * c + 16 * a^3 * c^2 + 8 * a^2 * b * c^2 + a * b^2 * c^2 - 4 * a^2 * c^3) * c^2 * \text{abs}(a * b^2 - 4 * a^2 * c)) - 1/4 * ((b^3 * c^2 - 4 * a * b * c^3 - 2 * b^2 * c^3 + b * c^4 - (b^2 * c^2 - 4 * a * c^3 - 2 * b * c^3 + c^4) * \sqrt{b^2 - 4 * a * c})) * B * \text{abs}(a * b^2 - 4 * a^2 * c) - (a * b^5 * c^2 - 8 * a^2 * b^3 * c^3 - 2 * a * b^4 * c^3 + 16 * a^3 * b * c^4 + 8 * a^2 * b^2 * c^4 + a * b^3 * c^4 - 4 * a^2 * b * c^5 - (a * b^4 * c^2 - 4 * a^2 * b^2 * c^3 - 2 * a * b^3 * c^3 + a * b^2 * c^4) * \sqrt{b^2 - 4 * a * c})) * B) * \log(x^2 + 1/2 * (a * b^3 - 4 * a^2 * b * c - \sqrt{(a * b^3 - 4 * a^2 * b * c)^2 - 4 * (a^2 * b^2 - 4 * a^3 * c) * (a * b^2 * c - 4 * a^2 * c^2))}) / (a * b^2 * c - 4 * a^2 * c^2)) / ((a * b^4 - 8 * a^2 * b^2 * c - 2 * a * b^3 * c + 16 * a^3 * c^2 + 8 * a^2 * b * c^2 + a * b^2 * c^2 - 4 * a^2 * c^3) * c^2 * \text{abs}(a * b^2 - 4 * a^2 * c))
\end{aligned}$$

maple [B] time = 0.15, size = 1813, normalized size = 4.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x)$

[Out] $-1/2/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^{(1/2)}/c)*B*b^2-1/2/(4*a*c-b^2)^2/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^{(1/2)}/c)*B*b^2-1/2/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^{(1/2)}/c)*B*a-c/(4*a*c-b^2)^2*B*(-4*a*c+b^2)^{(1/2)}*\ln(-2*c*x^2-b+(-4*a*c+b^2)^{(1/2)})+2*c/(4*a*c-b^2)^2/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^{(1/2)}/c)*B*a+c/(4*a*c-b^2)^2*B*(-4*a*c+b^2)^{(1/2)}*\ln(2*c*x^2+b+(-4*a*c+b^2)^{(1/2)})+3*c^2/(4*a*c-b^2)^2*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*A*(-4*a*c+b^2)^{(1/2)}-c^2/(4*a*c-b^2)^2*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctan(2^{(1/2)}/((b+(-4*a*c$

$$\begin{aligned}
& +b^2)^{(1/2)} * c)^{(1/2)} * c * x) * A * b + 2 * c^2 / (4 * a * c - b^2)^2 * a * 2^{(1/2)} / ((b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * \arctan(2^{(1/2)} / ((b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * C - 1 / 4 * c / (4 * a * c - b^2)^2 / a * 2^{(1/2)} / ((-b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctanh}(2^{(1/2)} / ((-b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * A * b^3 - c / (4 * a * c - b^2)^2 * 2^{(1/2)} / ((-b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctanh}(2^{(1/2)} / ((-b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * C * (-4 * a * c + b^2)^{(1/2)} * b + 1 / 4 * c / (4 * a * c - b^2)^2 / a * 2^{(1/2)} / ((b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * \arctan(2^{(1/2)} / ((b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * A * b^3 - c / (4 * a * c - b^2)^2 * 2^{(1/2)} / ((b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * \arctan(2^{(1/2)} / ((b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * C * (-4 * a * c + b^2)^{(1/2)} * b + 1 / 4 * c / (4 * a * c - b^2)^2 / (x^2 + 1 / 2 * b / c + 1 / 2 * (-4 * a * c + b^2)^{(1/2)} / c) / a * x * A * b^3 + 1 / 4 * c / (4 * a * c - b^2)^2 / (x^2 + 1 / 2 * b / c - 1 / 2 * (-4 * a * c + b^2)^{(1/2)} / c) / a * x * A * b^3 - c / (4 * a * c - b^2)^2 / (x^2 + 1 / 2 * b / c - 1 / 2 * (-4 * a * c + b^2)^{(1/2)} / c) * x * A * (-4 * a * c + b^2)^{(1/2)} - c / (4 * a * c - b^2)^2 / (x^2 + 1 / 2 * b / c - 1 / 2 * (-4 * a * c + b^2)^{(1/2)} / c) * A * b * x + 2 * c / (4 * a * c - b^2)^2 / (x^2 + 1 / 2 * b / c - 1 / 2 * (-4 * a * c + b^2)^{(1/2)} / c) * a * C * x + c / (4 * a * c - b^2)^2 / (x^2 + 1 / 2 * b / c + 1 / 2 * (-4 * a * c + b^2)^{(1/2)} / c) * x * A * (-4 * a * c + b^2)^{(1/2)} - c / (4 * a * c - b^2)^2 / (x^2 + 1 / 2 * b / c + 1 / 2 * (-4 * a * c + b^2)^{(1/2)} / c) * A * b * x + 2 * c / (4 * a * c - b^2)^2 / (x^2 + 1 / 2 * b / c + 1 / 2 * (-4 * a * c + b^2)^{(1/2)} / c) * a * C * x - 1 / 2 / (4 * a * c - b^2)^2 / (x^2 + 1 / 2 * b / c - 1 / 2 * (-4 * a * c + b^2)^{(1/2)} / c) * x * C * b^2 - 1 / 4 * c / (4 * a * c - b^2)^2 / a * 2^{(1/2)} / ((-b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctanh}(2^{(1/2)} / ((-b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * A * (-4 * a * c + b^2)^{(1/2)} * b^2 - 1 / 4 * c / (4 * a * c - b^2)^2 / a * 2^{(1/2)} / ((b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * \arctan(2^{(1/2)} / ((b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * A * (-4 * a * c + b^2)^{(1/2)} * b^2 - 1 / 2 * c / (4 * a * c - b^2)^2 * 2^{(1/2)} / ((b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * \arctan(2^{(1/2)} / ((b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * C * b^2 + 3 * c^2 / (4 * a * c - b^2)^2 * 2^{(1/2)} / ((-b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctanh}(2^{(1/2)} / ((-b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * A * (-4 * a * c + b^2)^{(1/2)} * c^2 / (4 * a * c - b^2)^2 * a * 2^{(1/2)} / ((-b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctanh}(2^{(1/2)} / ((-b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * C + 1 / 2 * c / (4 * a * c - b^2)^2 * 2^{(1/2)} / ((-b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctanh}(2^{(1/2)} / ((-b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * C * b^2 + 1 / 4 * c / (4 * a * c - b^2)^2 / (x^2 + 1 / 2 * b / c - 1 / 2 * (-4 * a * c + b^2)^{(1/2)} / c) / a * x * A * (-4 * a * c + b^2)^{(1/2)} * b^2 - 1 / 4 * c / (4 * a * c - b^2)^2 / (x^2 + 1 / 2 * b / c + 1 / 2 * (-4 * a * c + b^2)^{(1/2)} / c) / a * x * A * (-4 * a * c + b^2)^{(1/2)} * b^2
\end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2 B a c x^2 + (2 C a - A b) c x^3 + B a b + (C a b - A b^2 + 2 A a c) x}{2 \left((a b^2 c - 4 a^2 c^2) x^4 + a^2 b^2 - 4 a^3 c + (a b^3 - 4 a^2 b c) x^2 \right)} + \frac{- \int \frac{4 B a c x + (2 C a - A b) c x^2 - C a b - A b^2 + 6 A a c}{c x^4 + b x^2 + a} dx}{2 (a b^2 - 4 a^2 c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")

[Out] -1/2*(2*B*a*c*x^2 + (2*C*a - A*b)*c*x^3 + B*a*b + (C*a*b - A*b^2 + 2*A*a*c)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2) + 1/2*integrate(-(4*B*a*c*x + (2*C*a - A*b)*c*x^2 - C*a*b - A*b^2 + 6*A*a*c)/(c*x^4 + b*x^2 + a), x)/(a*b^2 - 4*a^2*c)

mupad [B] time = 1.67, size = 4707, normalized size = 12.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x + C*x^2)/(a + b*x^2 + c*x^4)^2,x)

[Out] ((B*b)/(2*(4*a*c - b^2)) + (x*(2*A*a*c - A*b^2 + C*a*b))/(2*a*(4*a*c - b^2)) + (B*c*x^2)/(4*a*c - b^2) - (c*x^3*(A*b - 2*C*a))/(2*a*(4*a*c - b^2)))/(a + b*x^2 + c*x^4) + symsum(log((5*A^3*b^3*c^4 + 8*C^3*a^3*c^4 + 6*C^3*a^2*b^2*c^3 - 36*A^3*a*b*c^5 - 96*A*B^2*a^2*c^5 + 72*A^2*C*a^2*c^5 - 3*A^2*C*b^4*c^3 + 16*A*B^2*a*b^2*c^4 + 3*A*C^2*a*b^3*c^3 - 60*A*C^2*a^2*b*c^4 + 18*A^2*C*a*b^2*c^4 + 16*B^2*C*a^2*b*c^4)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - root(1572864*a^8*b^2*c^5*z^4 - 983040*a^7*b^4*c^4*z^4

$$\begin{aligned}
& + 327680*a^6*b^6*c^3*z^4 - 61440*a^5*b^8*c^2*z^4 + 6144*a^4*b^{10}*c*z^4 - 1048576*a^9*c^6*z^4 - 256*a^3*b^{12}*z^4 + 576*A*C*a^2*b^8*c*z^2 + 24576*A*C*a^5*b^2*c^4*z^2 - 3072*A*C*a^3*b^6*c^2*z^2 + 2048*A*C*a^4*b^4*c^3*z^2 - 32*A*C*a*b^{10}*z^2 + 12288*C^2*a^6*b*c^4*z^2 + 61440*A^2*a^5*b*c^5*z^2 + 432*A^2*a*b^9*c*z^2 - 49152*A*C*a^6*c^5*z^2 - 8192*C^2*a^5*b^3*c^3*z^2 + 1536*C^2*a^4*b^5*c^2*z^2 + 24576*B^2*a^5*b^2*c^4*z^2 - 6144*B^2*a^4*b^4*c^3*z^2 + 512*B^2*a^3*b^6*c^2*z^2 - 61440*A^2*a^4*b^3*c^4*z^2 + 24064*A^2*a^3*b^5*c^3*z^2 - 4608*A^2*a^2*b^7*c^2*z^2 - 32768*B^2*a^6*c^5*z^2 - 16*C^2*a^2*b^9*z^2 - 16*A^2*b^{11}*z^2 + 3072*A*B*C*a^3*b^3*c^3*z - 768*A*B*C*a^2*b^5*c^2*z - 4096*A*B*C*a^4*b*c^4*z + 64*A*B*C*a*b^7*c*z + 32*B*C^2*a^2*b^6*c*z - 672*A^2*B*a*b^6*c^2*z + 1536*B*C^2*a^4*b^2*c^3*z - 384*B*C^2*a^3*b^4*c^2*z - 15872*A^2*B*a^3*b^2*c^4*z + 4992*A^2*B*a^2*b^4*c^3*z + 32*A^2*B*b^8*c*z - 2048*B*C^2*a^5*c^4*z + 18432*A^2*B*a^4*c^5*z + 192*A*B^2*C*a^2*b^2*c^3 - 32*A*B^2*C*a*b^4*c^2 - 16*B^2*C^2*a^2*b^3*c^2 - 960*A^2*C^2*a^2*b^2*c^3 - 18*A*C^3*a*b^5*c - 192*B^2*C^2*a^3*b*c^3 + 198*A^2*C^2*a*b^4*c^2 + 144*A*C^3*a^2*b^3*c^2 - 960*A^2*B^2*a^2*b*c^4 + 240*A^2*B^2*a*b^3*c^3 + 2016*A^3*C*a^2*b*c^4 - 496*A^3*C*a*b^3*c^3 + 224*A*C^3*a^3*b*c^3 + 768*A*B^2*C*a^3*c^4 - 9*C^4*a^2*b^4*c + 360*A^4*a*b^2*c^4 + 30*A^3*C*b^5*c^2 - 9*A^2*C^2*b^6*c - 24*C^4*a^3*b^2*c^2 - 288*A^2*C^2*a^3*c^4 - 16*A^2*B^2*b^5*c^2 - 16*C^4*a^4*c^3 - 256*B^4*a^3*c^4 - 25*A^4*b^4*c^3 - 1296*A^4*a^2*c^5, z, k)*(root(1572864*a^8*b^2*c^5*z^4 - 983040*a^7*b^4*c^4*z^4 + 327680*a^6*b^6*c^3*z^4 - 61440*a^5*b^8*c^2*z^4 + 6144*a^4*b^{10}*c*z^4 - 1048576*a^9*c^6*z^4 - 256*a^3*b^{12}*z^4 + 576*A*C*a^2*b^8*c*z^2 + 24576*A*C*a^5*b^2*c^4*z^2 - 3072*A*C*a^3*b^6*c^2*z^2 + 2048*A*C*a^4*b^4*c^3*z^2 - 32*A*C*a*b^{10}*z^2 + 12288*C^2*a^6*b*c^4*z^2 + 61440*A^2*a^5*b*c^5*z^2 + 432*A^2*a*b^9*c*z^2 - 49152*A*C*a^6*c^5*z^2 - 8192*C^2*a^5*b^3*c^3*z^2 + 1536*C^2*a^4*b^5*c^2*z^2 + 24576*B^2*a^5*b^2*c^4*z^2 - 6144*B^2*a^4*b^4*c^3*z^2 + 512*B^2*a^3*b^6*c^2*z^2 - 61440*A^2*a^4*b^3*c^4*z^2 + 24064*A^2*a^3*b^5*c^3*z^2 - 4608*A^2*a^2*b^7*c^2*z^2 - 32768*B^2*a^6*c^5*z^2 - 16*C^2*a^2*b^9*z^2 - 16*A^2*b^{11}*z^2 + 3072*A*B*C*a^3*b^3*c^3*z - 768*A*B*C*a^2*b^5*c^2*z - 4096*A*B*C*a^4*b*c^4*z + 64*A*B*C*a*b^7*c*z + 32*B*C^2*a^2*b^6*c*z - 672*A^2*B*a*b^6*c^2*z + 1536*B*C^2*a^4*b^2*c^3*z - 384*B*C^2*a^3*b^4*c^2*z - 15872*A^2*B*a^3*b^2*c^4*z + 4992*A^2*B*a^2*b^4*c^3*z + 32*A^2*B*b^8*c*z - 2048*B*C^2*a^5*c^4*z + 18432*A^2*B*a^4*c^5*z + 192*A*B^2*C*a^2*b^2*c^3 - 32*A*B^2*C*a*b^4*c^2 - 16*B^2*C^2*a^2*b^3*c^2 - 960*A^2*C^2*a^2*b^2*c^3 - 18*A*C^3*a*b^5*c - 192*B^2*C^2*a^3*b*c^3 + 198*A^2*C^2*a*b^4*c^2 + 144*A*C^3*a^2*b^3*c^2 - 960*A^2*B^2*a^2*b*c^4 + 240*A^2*B^2*a*b^3*c^3 + 2016*A^3*C*a^2*b*c^4 - 496*A^3*C*a*b^3*c^3 + 224*A*C^3*a^3*b*c^3 + 768*A*B^2*C*a^3*c^4 - 9*C^4*a^2*b^4*c + 360*A^4*a*b^2*c^4 + 30*A^3*C*b^5*c^2 - 9*A^2*C^2*b^6*c - 24*C^4*a^3*b^2*c^2 - 288*A^2*C^2*a^3*c^4 - 16*A^2*B^2*b^5*c^2 - 16*C^4*a^4*c^3 - 256*B^4*a^3*c^4 - 25*A^4*b^4*c^3 - 1296*A^4*a^2*c^5, z, k)*((x*(1024*B*a^5*c^6 - 16*B*a^2*b^6*c^3 + 192*B*a^3*b^4*c^4 - 768*B*a^4*b^2*c^5))/(2*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (6144*A*a^5*c^6 + 16*A*a*b^8*c^2 - 1024*C*a^5*b*c^5 - 288*A*a^2*b^6*c^3 + 1920*A*a^3*b^4*c^4 - 5632*A*a^4*b^2*c^5 + 16*C*a^2*b^7*c^2 - 192*C*a^3*b^5*c^3 + 768*C*a^4*b^3*c^4)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + (root(1572864*a^8*b^2*c^5*z^4 - 983040*a^7*b^4*c^4*z^4 + 327680*a^6*b^6*c^3*z^4 - 61440*a^5*b^8*c^2*z^4 + 6144*a^4*b^{10}*c*z^4 - 1048576*a^9*c^6*z^4 - 256*a^3*b^{12}*z^4 + 576*A*C*a^2*b^8*c*z^2 + 24576*A*C*a^5*b^2*c^4*z^2 - 3072*A*C*a^3*b^6*c^2*z^2 + 2048*A*C*a^4*b^4*c^3*z^2 - 32*A*C*a*b^{10}*z^2 + 12288*C^2*a^6*b*c^4*z^2 + 61440*A^2*a^5*b*c^5*z^2 + 432*A^2*a*b^9*c*z^2 - 49152*A*C*a^6*c^5*z^2 - 8192*C^2*a^5*b^3*c^3*z^2 + 1536*C^2*a^4*b^5*c^2*z^2 + 24576*B^2*a^5*b^2*c^4*z^2 - 6144*B^2*a^4*b^4*c^3*z^2 + 512*B^2*a^3*b^6*c^2*z^2 - 61440*A^2*a^4*b^3*c^4*z^2 + 24064*A^2*a^3*b^5*c^3*z^2 - 4608*A^2*a^2*b^7*c^2*z^2 - 32768*B^2*a^6*c^5*z^2 - 16*C^2*a^2*b^9*z^2 - 16*A^2*b^{11}*z^2 + 3072*A*B*C*a^3*b^3*c^3*z - 768*A*B*C*a^2*b^5*c^2*z - 4096*A*B*C*a^4*b*c^4*z + 64*A*B*C*a*b^7*c*z + 32*B*C^2*a^2*b^6*c*z - 672*A^2*B*a*b^6*c^2*z + 1536*B*C^2*a^4*b^2*c^3*z - 384*B*C^2*a^3*b^4*c^2*z - 15872*A^2*B*a^3*b^2*c^4*z + 4992*A^2*B*a^2*b^4*c^3*z + 32*A^2*B*b^8*c*z - 2048*B*C^2*a^5*c^4*z + 18432*A^2*B*a^4*c^5*z + 192*A*B^2*C*a^2*b^2*c^3 - 32*A*B^2*
\end{aligned}$$

$$\begin{aligned}
& C^2 a^2 b^4 c^2 - 16 B^2 C^2 a^2 b^3 c^2 - 960 A^2 C^2 a^2 b^2 c^3 - 18 A^3 C^3 a^2 b^5 c - 192 B^2 C^2 a^3 b^3 c^3 + 198 A^2 C^2 a^2 b^4 c^2 + 144 A^3 C^3 a^2 b^3 c^2 \\
& - 960 A^2 B^2 a^2 b^3 c^4 + 240 A^2 B^2 a^2 b^3 c^3 + 2016 A^3 C^3 a^2 b^3 c^4 - 496 A^3 C^3 a^2 b^3 c^3 + 224 A^3 C^3 a^3 b^3 c^3 + 768 A^2 B^2 C^3 a^3 c^4 - 9 C^4 a^2 b^4 c \\
& + 360 A^4 a^2 b^2 c^4 + 30 A^3 C^3 b^5 c^2 - 9 A^2 C^2 b^6 c - 24 C^4 a^3 b^2 c^2 - 288 A^2 C^2 a^3 c^4 - 16 A^2 B^2 b^5 c^2 - 16 C^4 a^4 c^3 - 256 B^4 a^3 c^4 \\
& - 25 A^4 b^4 c^3 - 1296 A^4 a^2 c^5, z, k) \cdot x \cdot (4096 a^6 b^6 c^6 + 16 a^2 b^9 c^2 - 256 a^3 b^7 c^3 + 1536 a^4 b^5 c^4 - 4096 a^5 b^3 c^5) \\
& / (2 (a^2 b^6 - 64 a^5 c^3 - 12 a^3 b^4 c + 48 a^4 b^2 c^2)) + (32 B^2 C^3 a^2 b^4 c^3 - 384 A^2 B^2 a^2 b^3 c^4 - 512 B^2 C^3 a^4 c^5 + 32 A^2 B^2 a^2 b^5 c^3 + 1024 A^2 B^2 a^3 b^3 c^5) \\
& / (8 (a^2 b^6 - 64 a^5 c^3 - 12 a^3 b^4 c + 48 a^4 b^2 c^2)) + (x (A^2 b^6 c^3 - 288 A^2 a^3 c^6 + 32 C^2 a^4 c^5 + 128 A^2 a^2 b^2 c^5 - 16 B^2 a^2 b^3 c^4 \\
& + 10 C^2 a^2 b^4 c^3 - 48 C^2 a^3 b^2 c^4 - 18 A^2 a^2 b^4 c^4 + 64 B^2 a^3 b^3 c^5 - 48 A^2 C^2 a^2 b^3 c^4 + 2 A^2 C^2 a^2 b^5 c^3 + 160 A^2 C^2 a^3 b^3 c^5)) \\
& / (2 (a^2 b^6 - 64 a^5 c^3 - 12 a^3 b^4 c + 48 a^4 b^2 c^2)) - (x (16 B^3 a^2 c^5 - A^2 B^2 b^3 c^4 + 8 B^2 C^2 a^2 b^3 c^4 - 24 A^2 B^2 C^2 a^2 c^5 + 12 A^2 B^2 a^2 b^3 c^5 \\
& - 2 A^2 B^2 C^2 a^2 b^2 c^4)) / (2 (a^2 b^6 - 64 a^5 c^3 - 12 a^3 b^4 c + 48 a^4 b^2 c^2)) \cdot \text{root}(1572864 a^8 b^2 c^5 z^4 - 983040 a^7 b^4 c^4 z^4 \\
& + 327680 a^6 b^6 c^3 z^4 - 61440 a^5 b^8 c^2 z^4 + 6144 a^4 b^{10} c^2 z^4 - 1048576 a^9 c^6 z^4 - 256 a^3 b^{12} z^4 + 576 A^2 C^2 a^2 b^8 c^2 z^2 + 24576 A^2 C^2 a^5 b^2 c^4 z^2 \\
& - 3072 A^2 C^2 a^3 b^6 c^2 z^2 + 2048 A^2 C^2 a^4 b^4 c^3 z^2 - 32 A^2 C^2 a^2 b^10 z^2 + 12288 C^2 a^6 b^3 c^4 z^2 + 61440 A^2 a^5 b^3 c^5 z^2 + 432 A^2 a^2 b^9 c^3 z^2 \\
& - 49152 A^2 C^2 a^6 c^5 z^2 - 8192 C^2 a^5 b^3 c^3 z^2 + 1536 C^2 a^4 b^5 c^2 z^2 + 24576 B^2 a^5 b^2 c^4 z^2 - 6144 B^2 a^4 b^4 c^3 z^2 + 512 B^2 a^3 b^6 c^2 z^2 \\
& - 61440 A^2 a^4 b^3 c^4 z^2 + 24064 A^2 a^3 b^5 c^3 z^2 - 4608 A^2 a^2 b^7 c^2 z^2 - 32768 B^2 a^6 c^5 z^2 - 16 C^2 a^2 b^9 z^2 - 16 A^2 b^{11} z^2 \\
& + 3072 A^2 B^2 C^2 a^3 b^3 c^3 z - 768 A^2 B^2 C^2 a^2 b^5 c^2 z - 4096 A^2 B^2 C^2 a^4 b^3 c^4 z + 64 A^2 B^2 C^2 a^2 b^7 c^2 z + 32 B^2 C^2 a^2 b^6 c^2 z - 672 A^2 B^2 a^2 b^6 c^2 z \\
& + 1536 B^2 C^2 a^4 b^2 c^3 z - 384 B^2 C^2 a^3 b^4 c^2 z - 15872 A^2 B^2 a^3 b^2 c^4 z + 4992 A^2 B^2 a^2 b^4 c^3 z + 32 A^2 B^2 b^8 c^2 z - 2048 B^2 C^2 a^5 c^4 z \\
& + 18432 A^2 B^2 a^4 c^5 z + 192 A^2 B^2 C^2 a^2 b^2 c^3 - 32 A^2 B^2 C^2 a^2 b^4 c^2 - 16 B^2 C^2 a^2 b^3 c^2 - 960 A^2 C^2 a^2 b^2 c^3 - 18 A^3 C^3 a^2 b^5 c \\
& - 192 B^2 C^2 a^3 b^3 c^3 + 198 A^2 C^2 a^2 b^4 c^2 + 144 A^3 C^3 a^2 b^3 c^2 - 960 A^2 B^2 a^2 b^3 c^4 + 240 A^2 B^2 a^2 b^3 c^3 + 2016 A^3 C^3 a^2 b^3 c^4 \\
& - 496 A^3 C^3 a^2 b^3 c^3 + 224 A^3 C^3 a^3 b^3 c^3 + 768 A^2 B^2 C^3 a^3 c^4 - 9 C^4 a^2 b^4 c + 360 A^4 a^2 b^2 c^4 + 30 A^3 C^3 b^5 c^2 - 9 A^2 C^2 b^6 c \\
& - 24 C^4 a^3 b^2 c^2 - 288 A^2 C^2 a^3 c^4 - 16 A^2 B^2 b^5 c^2 - 16 C^4 a^4 c^3 - 256 B^4 a^3 c^4 - 25 A^4 b^4 c^3 - 1296 A^4 a^2 c^5, z, k), k, 1, 4)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x**2+B*x+A)/(c*x**4+b*x**2+a)**2,x)

[Out] Timed out

$$3.34 \quad \int \frac{A+Bx+Cx^2}{x(a+bx^2+cx^4)^2} dx$$

Optimal. Leaf size=403

$$\frac{(4a^2cC + A(b^3 - 6abc)) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right) - A \log(a + bx^2 + cx^4) + \frac{A \log(x)}{a^2} + \frac{A(b^2 - 2ac) + cx^2(Ab - 2aC) - a}{2a(b^2 - 4ac)(a + bx^2 + cx^4)}}{2a^2(b^2 - 4ac)^{3/2}}$$

Rubi [A] time = 0.93, antiderivative size = 403, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1662, 1251, 822, 800, 634, 618, 206, 628, 12, 1092, 1166, 205}

$$\frac{(4a^2cC + A(b^3 - 6abc)) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right) - A \log(a + bx^2 + cx^4) + \frac{A \log(x)}{a^2} + \frac{A(b^2 - 2ac) + cx^2(Ab - 2aC) - a}{2a(b^2 - 4ac)(a + bx^2 + cx^4)}}{2a^2(b^2 - 4ac)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x + C*x^2)/(x*(a + b*x^2 + c*x^4)^2), x]

[Out] (B*x*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (A*(b^2 - 2*a*c) - a*b*C + c*(A*b - 2*a*C)*x^2)/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (B*sqrt[c]*(b^2 - 12*a*c + b*sqrt[b^2 - 4*a*c])*ArcTan[(sqrt[2]*sqrt[c]*x)/sqrt[b - sqrt[b^2 - 4*a*c]])]/(2*sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*sqrt[b - sqrt[b^2 - 4*a*c]]) - (B*sqrt[c]*(b^2 - 12*a*c - b*sqrt[b^2 - 4*a*c])*ArcTan[(sqrt[2]*sqrt[c]*x)/sqrt[b + sqrt[b^2 - 4*a*c]])]/(2*sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*sqrt[b + sqrt[b^2 - 4*a*c]]) + ((A*(b^3 - 6*a*b*c) + 4*a^2*c*C)*ArcTanh[(b + 2*c*x^2)/sqrt[b^2 - 4*a*c]])/(2*a^2*(b^2 - 4*a*c)^(3/2)) + (A*Log[x])/a^2 - (A*Log[a + b*x^2 + c*x^4])/(4*a^2)

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^(m)*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m + 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1092

```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := -Simp[(x*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_.) + (e_.)*(x_)^2)/((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1251

```
Int[(x_)^(m_)*((d_.) + (e_.)*(x_)^2)^(q_)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]
```

Rule 1662

```
Int[(Pq_)*((d_.)*(x_))^(m_)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2 + 1})*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1})*(a + b*x^2 + c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] && !PolyQ[Pq, x^2]
```

Rubi steps

$$\begin{aligned}
 \int \frac{A + Bx + Cx^2}{x(a + bx^2 + cx^4)^2} dx &= \int \frac{B}{(a + bx^2 + cx^4)^2} dx + \int \frac{A + Cx^2}{x(a + bx^2 + cx^4)^2} dx \\
 &= \frac{1}{2} \text{Subst} \left(\int \frac{A + Cx}{x(a + bx + cx^2)^2} dx, x, x^2 \right) + B \int \frac{1}{(a + bx^2 + cx^4)^2} dx \\
 &= \frac{Bx(b^2 - 2ac + bcx^2)}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{A(b^2 - 2ac) - abC + c(Ab - 2aC)x^2}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{\text{Subst} \left(\int \frac{-A}{x(a + bx + cx^2)} dx, x, x^2 \right)}{2a(b^2 - 4ac)} \\
 &= \frac{Bx(b^2 - 2ac + bcx^2)}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{A(b^2 - 2ac) - abC + c(Ab - 2aC)x^2}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{\text{Subst} \left(\int \frac{A}{x(a + bx + cx^2)} dx, x, x^2 \right)}{2a(b^2 - 4ac)} \\
 &= \frac{Bx(b^2 - 2ac + bcx^2)}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{A(b^2 - 2ac) - abC + c(Ab - 2aC)x^2}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{B\sqrt{c}(b^2 - 12ac)}{2\sqrt{2}a} \\
 &= \frac{Bx(b^2 - 2ac + bcx^2)}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{A(b^2 - 2ac) - abC + c(Ab - 2aC)x^2}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{B\sqrt{c}(b^2 - 12ac)}{2\sqrt{2}a} \\
 &= \frac{Bx(b^2 - 2ac + bcx^2)}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{A(b^2 - 2ac) - abC + c(Ab - 2aC)x^2}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{B\sqrt{c}(b^2 - 12ac)}{2\sqrt{2}a} \\
 &= \frac{Bx(b^2 - 2ac + bcx^2)}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{A(b^2 - 2ac) - abC + c(Ab - 2aC)x^2}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{B\sqrt{c}(b^2 - 12ac)}{2\sqrt{2}a} \\
 &= \frac{Bx(b^2 - 2ac + bcx^2)}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{A(b^2 - 2ac) - abC + c(Ab - 2aC)x^2}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{B\sqrt{c}(b^2 - 12ac)}{2\sqrt{2}a}
 \end{aligned}$$

Mathematica [A] time = 1.47, size = 458, normalized size = 1.14

$$\frac{(4a^2C + A(\sqrt{b^2 - 4ac} - 4ac\sqrt{b^2 - 4ac} - 6abc + b^3)) \log(\sqrt{b^2 - 4ac} - b - 2cx^2)}{(b^2 - 4ac)^{3/2}} - \frac{A(\sqrt{b^2 - 4ac} - 4ac\sqrt{b^2 - 4ac} + 6abc - b^3) - 4a^2C}{(b^2 - 4ac)^{3/2}} \log(\sqrt{b^2 - 4ac} + b + 2cx^2)}{4a^2} - \frac{2a(-A(-2ac + b^2 + bcx^2) + abC + 2acx(B + C) - bB)(b + cx^2)}{(b^2 - 4ac)(b + bx^2 + cx^4)} + \frac{\sqrt{2}aB\sqrt{c}(b\sqrt{b^2 - 4ac} - 12ac + b^2) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{(b^2 - 4ac)^{3/2}\sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{\sqrt{2}aB\sqrt{c}(b\sqrt{b^2 - 4ac} + 12ac - b^2) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b + \sqrt{b^2 - 4ac}}}\right)}{(b^2 - 4ac)^{3/2}\sqrt{b + \sqrt{b^2 - 4ac}}} + 4A \log(x)$$

Antiderivative was successfully verified.

```

[In] Integrate[(A + B*x + C*x^2)/(x*(a + b*x^2 + c*x^4)^2), x]
[Out] ((-2*a*(a*b*C + 2*a*c*x*(B + C*x) - b*B*x*(b + c*x^2) - A*(b^2 - 2*a*c + b*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*a*B*Sqrt[c]*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*a*B*Sqrt[c]*(-b^2 + 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + 4*A*Log[x] - ((A*(b^3 - 6*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 4*a*c*Sqrt[b^2 - 4*a*c]) + 4*a^2*c*C)*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) - ((A*(-b^3 + 6*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 4*a*c*Sqrt[b^2 - 4*a*c]) - 4*a^2*c*C)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/(4*a^2)
    
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx + Cx^2}{x(a + bx^2 + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B*x + C*x^2)/(x*(a + b*x^2 + c*x^4)^2), x]

[Out] IntegrateAlgebraic[(A + B*x + C*x^2)/(x*(a + b*x^2 + c*x^4)^2), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)/x/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")

[Out] Timed out

giac [B] time = 6.55, size = 6022, normalized size = 14.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)/x/(c*x^4+b*x^2+a)^2,x, algorithm="giac")

[Out]
$$-1/4*A*\log(\text{abs}(c*x^4 + b*x^2 + a))/a^2 + A*\log(\text{abs}(x))/a^2 + 1/16*((a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)^2*(2*b^3*c^2 - 8*a*b*c^3 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c}*b^3 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c}*a*b*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c}*b^2*c - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c}*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*B + 2*(\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c}*a^4*b^8*c - 18*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c}*a^5*b^6*c^2 - 2*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c}*a^4*b^7*c^2 - 2*a^4*b^8*c^2 + 120*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c}*a^6*b^4*c^3 + 28*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c}*a^5*b^5*c^3 + \sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c}*a^4*b^6*c^3 + 36*a^5*b^6*c^3 - 352*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c}*a^7*b^2*c^4 - 128*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c}*a^6*b^3*c^4 - 14*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c}*a^5*b^4*c^4 - 240*a^6*b^4*c^4 + 384*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c}*a^8*c^5 + 192*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c}*a^7*b*c^5 + 64*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c}*a^6*b^2*c^5 + 704*a^7*b^2*c^5 - 96*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c}*a^7*c^6 - 768*a^8*c^6 + 2*(b^2 - 4*a*c)*a^4*b^6*c^2 - 28*(b^2 - 4*a*c)*a^5*b^4*c^3 + 128*(b^2 - 4*a*c)*a^6*b^2*c^4 - 192*(b^2 - 4*a*c)*a^7*c^5)*B*\text{abs}(a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3) + (2*a^8*b^11*c^4 - 56*a^9*b^9*c^5 + 576*a^10*b^7*c^6 - 2816*a^11*b^5*c^7 + 6656*a^12*b^3*c^8 - 6144*a^13*b*c^9 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c}*a^8*b^11*c^2 + 28*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c}*a^9*b^9*c^3 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c}*a^8*b^10*c^3 - 288*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c}*a^10*b^7*c^4 - 48*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c}*a^9*b^8*c^4 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c}*a^8*b^9*c^4 + 1408*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c}*a^11*b^5*c^5 + 384*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c}*a^10*b^6*c^5 + 24*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c}*a^9*b^7*c^5 - 3328*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c}*a^12*b^3*c^6 - 1280*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c}*a^11*b^4*c^6 - 192*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c}*a^10*b^5*c^6 + 3072*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c}$$

$$\begin{aligned}
& \text{rt}(b^2 - 4ac)c \cdot a^{13}b^7c^7 + 1536\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c \cdot a^{12}b^2c^7 + 640\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c \cdot a^{11}b^3c^7 - 768\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c \cdot a^{12}b^8c^8 - 2(b^2 - 4ac)a^8b^9c^4 + 48(b^2 - 4ac)a^9b^7c^5 - 384(b^2 - 4ac)a^{10}b^5c^6 + 1280(b^2 - 4ac)a^{11}b^3c^7 - 1536(b^2 - 4ac)a^{12}b^8c^8) \cdot B \cdot \arctan(2\sqrt{1/2}x/\sqrt{(a^4b^5c - 8a^5b^3c^2 + 16a^6b^2c^3 + \sqrt{(a^4b^5c - 8a^5b^3c^2 + 16a^6b^2c^3)^2 - 4(a^5b^4c - 8a^6b^2c^2 + 16a^7c^3)}(a^4b^4c^2 - 8a^5b^2c^3 + 16a^6c^4)))/((a^6b^8c - 16a^7b^6c^2 - 2a^6b^7c^2 + 96a^8b^4c^3 + 24a^7b^5c^3 + a^6b^6c^3 - 256a^9b^2c^4 - 96a^8b^3c^4 - 12a^7b^4c^4 + 256a^{10}c^5 + 128a^9b^2c^5 + 48a^8b^2c^5 - 64a^9c^6) \cdot \text{abs}(a^4b^4c - 8a^5b^2c^2 + 16a^6c^3) \cdot \text{abs}(c)) - 1/16((a^4b^4c - 8a^5b^2c^2 + 16a^6c^3)^2 \cdot (2b^3c^2 - 8ab^2c^3 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c) \cdot b^3 + 4\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c) \cdot a \cdot b^2c - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c) \cdot b^2c^2 - 2(b^2 - 4ac) \cdot b^2c^2) \cdot B - 2(\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}c) \cdot a^4b^8c - 18\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}c) \cdot a^5b^6c^2 - 2\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}c) \cdot a^4b^7c^2 + 2a^4b^8c^2 + 120\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}c) \cdot a^6b^4c^3 + 28\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}c) \cdot a^5b^5c^3 + \sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}c) \cdot a^4b^6c^3 - 36a^5b^6c^3 - 352\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}c) \cdot a^7b^2c^4 - 128\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}c) \cdot a^6b^3c^4 - 14\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}c) \cdot a^5b^4c^4 + 240a^6b^4c^4 + 384\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}c) \cdot a^8c^5 + 192\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}c) \cdot a^7b^2c^5 + 64\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}c) \cdot a^6b^2c^5 - 704a^7b^2c^5 - 96\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}c) \cdot a^7c^6 + 768a^8c^6 - 2(b^2 - 4ac) \cdot a^4b^6c^2 + 28(b^2 - 4ac) \cdot a^5b^4c^3 - 128(b^2 - 4ac) \cdot a^6b^2c^4 + 192(b^2 - 4ac) \cdot a^7c^5) \cdot B \cdot \text{abs}(a^4b^4c - 8a^5b^2c^2 + 16a^6c^3) + (2a^8b^{11}c^4 - 56a^9b^9c^5 + 576a^{10}b^7c^6 - 2816a^{11}b^5c^7 + 6656a^{12}b^3c^8 - 6144a^{13}b^2c^9 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c) \cdot a^8b^{11}c^2 + 28\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c) \cdot a^9b^9c^3 + 2\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c) \cdot a^8b^{10}c^3 - 288\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c) \cdot a^{10}b^7c^4 - 48\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c) \cdot a^9b^8c^4 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c) \cdot a^8b^9c^4 + 1408\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c) \cdot a^{11}b^5c^5 + 384\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c) \cdot a^{10}b^6c^5 + 24\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c) \cdot a^9b^7c^5 - 3328\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c) \cdot a^{12}b^3c^6 - 1280\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c) \cdot a^{11}b^4c^6 - 192\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c) \cdot a^{10}b^5c^6 + 3072\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c) \cdot a^{13}b^2c^7 + 1536\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c) \cdot a^{12}b^2c^7 + 640\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c) \cdot a^{11}b^3c^7 - 768\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c) \cdot a^{12}b^8c^8 - 2(b^2 - 4ac) \cdot a^8b^9c^4 + 48(b^2 - 4ac) \cdot a^9b^7c^5 - 384(b^2 - 4ac) \cdot a^{10}b^5c^6 + 1280(b^2 - 4ac) \cdot a^{11}b^3c^7 - 1536(b^2 - 4ac) \cdot a^{12}b^8c^8) \cdot B \cdot \arctan(2\sqrt{1/2}x/\sqrt{(a^4b^5c - 8a^5b^3c^2 + 16a^6b^2c^3 - \sqrt{(a^4b^5c - 8a^5b^3c^2 + 16a^6b^2c^3)^2 - 4(a^5b^4c - 8a^6b^2c^2 + 16a^7c^3)}(a^4b^4c^2 - 8a^5b^2c^3 + 16a^6c^4)))/((a^6b^8c - 16a^7b^6c^2 - 2a^6b^7c^2 + 96a^8b^4c^3 + 24a^7b^5c^3 + a^6b^6c^3 - 256a^9b^2c^4 - 96a^8b^3c^4 - 12a^7b^4c^4 + 256a^{10}c^5 + 128a^9b^2c^5 + 48a^8b^2c^5 - 64a^9c^6) \cdot \text{abs}(a^4b^4c - 8a^5b^2c^2 + 16a^6c^3) \cdot \text{abs}(c)) - 1/16((b^6c - 10ab^4c^2 - 2b^5c^2 + 24a^2b^2c^3 + 12ab^3c^3 + b^4c^3 - 6ab^2c^4 + (b^5c - 10ab^3c^2 - 2b^4c^2
\end{aligned}$$

$$\begin{aligned}
& + 24a^2b^2c^3 + 12ab^2c^3 + b^3c^3 - 6ab^2c^4) \sqrt{b^2 - 4ac}) * A * \\
& \text{abs}(a^4b^4c - 8a^5b^2c^2 + 16a^6c^3) + 4(a^2b^3c^2 - 4a^3b^2c^3 - 2a^2b^2c^3 + a^2b^2c^4) * \sqrt{b^2 - 4ac}) * C * \text{abs}(a^4b^4c - 8a^5b^2c^2 + 16a^6c^3) + (a^4b^4c^2 - 18a^5b^8c^3 - 2a^4b^9c^3 + 120a^6b^6c^4 + 28a^5b^7c^4 + a^4b^8c^4 - 352a^7b^4c^5 - 128a^6b^5c^5 - 14a^5b^6c^5 + 384a^8b^2c^6 + 192a^7b^3c^6 + 64a^6b^4c^6 - 96a^7b^2c^7 + (a^4b^9c^2 - 14a^5b^7c^3 - 2a^4b^8c^3 + 64a^6b^5c^4 + 20a^5b^6c^4 + a^4b^7c^4 - 96a^7b^3c^5 - 48a^6b^4c^5 - 10a^5b^5c^5 + 24a^6b^3c^6) * \sqrt{b^2 - 4ac}) * A + 4(a^6b^7c^3 - 12a^7b^5c^4 - 2a^6b^6c^4 + 48a^8b^3c^5 + 16a^7b^4c^5 + a^6b^5c^5 - 64a^9b^2c^6 - 32a^8b^2c^6 - 8a^7b^3c^6 + 16a^8b^2c^7 + (a^6b^6c^3 - 8a^7b^4c^4 - 2a^6b^5c^4 + 16a^8b^2c^5 + 8a^7b^3c^5 + a^6b^4c^5 - 4a^7b^2c^6) * \sqrt{b^2 - 4ac}) * C * \log(x^2 + 1/2(a^4b^5c - 8a^5b^3c^2 + 16a^6b^2c^3 + \sqrt{(a^4b^5c - 8a^5b^3c^2 + 16a^6b^2c^3)^2 - 4(a^5b^4c - 8a^6b^2c^2 + 16a^7c^3)(a^4b^4c^2 - 8a^5b^2c^3 + 16a^6c^4)})) / (a^4b^4c^2 - 8a^5b^2c^3 + 16a^6c^4) / ((a^3b^4 - 8a^4b^2c - 2a^3b^3c + 16a^5c^2 + 8a^4b^2c^2 + a^3b^2c^2 - 4a^4c^3) * c^2 * \text{abs}(a^4b^4c - 8a^5b^2c^2 + 16a^6c^3)) - 1/16 * ((b^6c - 10ab^4c^2 - 2b^5c^2 + 24a^2b^2c^3 + 12ab^3c^3 + b^4c^3 - 6ab^2c^4 - (b^5c - 10ab^3c^2 - 2b^4c^2 + 24a^2b^2c^3 + 12ab^3c^3 + b^4c^3 - 6ab^2c^4) * \sqrt{b^2 - 4ac})) * A * \text{abs}(a^4b^4c - 8a^5b^2c^2 + 16a^6c^3) + 4(a^2b^3c^2 - 4a^3b^2c^3 - 2a^2b^2c^3 + a^2b^2c^4 - (a^2b^2c^2 - 4a^3c^3 - 2a^2b^2c^3 + a^2c^4) * \sqrt{b^2 - 4ac})) * C * \text{abs}(a^4b^4c - 8a^5b^2c^2 + 16a^6c^3) - (a^4b^10c^2 - 18a^5b^8c^3 - 2a^4b^9c^3 + 120a^6b^6c^4 + 28a^5b^7c^4 + a^4b^8c^4 - 352a^7b^4c^5 - 128a^6b^5c^5 - 14a^5b^6c^5 + 384a^8b^2c^6 + 192a^7b^3c^6 + 64a^6b^4c^6 - 96a^7b^2c^7 - (a^4b^9c^2 - 14a^5b^7c^3 - 2a^4b^8c^3 + 64a^6b^5c^4 + 20a^5b^6c^4 + a^4b^7c^4 - 96a^7b^3c^5 - 48a^6b^4c^5 - 10a^5b^5c^5 + 24a^6b^3c^6) * \sqrt{b^2 - 4ac})) * A - 4(a^6b^7c^3 - 12a^7b^5c^4 - 2a^6b^6c^4 + 48a^8b^3c^5 + 16a^7b^4c^5 + a^6b^5c^5 - 64a^9b^2c^6 - 32a^8b^2c^6 - 8a^7b^3c^6 + 16a^8b^2c^7 - (a^6b^6c^3 - 8a^7b^4c^4 - 2a^6b^5c^4 + 16a^8b^2c^5 + 8a^7b^3c^5 + a^6b^4c^5 - 4a^7b^2c^6) * \sqrt{b^2 - 4ac})) * C * \log(x^2 + 1/2(a^4b^5c - 8a^5b^3c^2 + 16a^6b^2c^3 - \sqrt{(a^4b^5c - 8a^5b^3c^2 + 16a^6b^2c^3)^2 - 4(a^5b^4c - 8a^6b^2c^2 + 16a^7c^3)(a^4b^4c^2 - 8a^5b^2c^3 + 16a^6c^4)})) / (a^4b^4c^2 - 8a^5b^2c^3 + 16a^6c^4) / ((a^3b^4 - 8a^4b^2c - 2a^3b^3c + 16a^5c^2 + 8a^4b^2c^2 + a^3b^2c^2 - 4a^4c^3) * c^2 * \text{abs}(a^4b^4c - 8a^5b^2c^2 + 16a^6c^3)) + 1/2 * (B * a * b * c * x^3 - C * a^2 * b + A * a * b^2 - 2 * A * a^2 * c - (2 * C * a^2 * c - A * a * b * c) * x^2 + (B * a * b^2 - 2 * B * a^2 * c) * x) / ((c * x^4 + b * x^2 + a) * (b^2 - 4 * a * c) * a^2)
\end{aligned}$$

maple [B] time = 0.06, size = 1603, normalized size = 3.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((C*x^2+B*x+A)/x/(c*x^4+b*x^2+a)^2, x)$

[Out]
$$\begin{aligned}
& -1/a*c/(4*a*c-b^2)/(16*a*c-4*b^2)*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} * \\
& \arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} * c * x) * B * (-4*a*c+b^2)^{(1/2)} * b \\
& ^2 - 1/a*c/(4*a*c-b^2)/(16*a*c-4*b^2)*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} * \\
& \operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} * c * x) * B * (-4*a*c+b^2)^{(1/2)} * b \\
& ^2 - 1/2/(c*x^4+b*x^2+a)/(4*a*c-b^2) * B/a*b^2*x - 1/2/(c*x^4+b*x^2+a)/(4*a*c-b^2) * \\
& B/a*b*c*x^3 + A/a^2*\ln(x) + 1/2/(c*x^4+b*x^2+a)/(4*a*c-b^2) * C*b + 1/(c*x^4+b*x^2+a)/(4*a*c-b^2) * \\
& A*c - 1/2/(c*x^4+b*x^2+a)/(4*a*c-b^2) * A/a*b*c*x^2 - 1/a^2/(4*a*c-b^2)/(16*a*c-4*b^2) * \\
& \ln(-2*c*x^2-b+(-4*a*c+b^2)^{(1/2)}) * A*b^4 - 1/a^2/(4*a*c-b^2)/(16*a*c-4*b^2) * \ln(2*c*x^2+b+(-4*a*c+b^2)^{(1/2)}) * A*b^4 - 4*c/(4*a*c-b^2)/(16*a*c-4*b^2) * \ln(-2*c*x^2-b+(-4*a*c+b^2)^{(1/2)}) * C * (-4*a*c+b^2)^{(1/2)} + 4*c/(4*a*c-b^2)/(16*a*c-4*b^2) * \ln(2*c*x^2+b+(-4*a*c+b^2)^{(1/2)}) * C * (-4*a*c+b^2)^{(1/2)}
\end{aligned}$$

$$\begin{aligned}
& b^2)^{(1/2)} - 6/a*c/(4*a*c - b^2)/(16*a*c - 4*b^2) * \ln(2*c*x^2 + b + (-4*a*c + b^2)^{(1/2)}) \\
&) * A * (-4*a*c + b^2)^{(1/2)} * b + 12*c^2/(4*a*c - b^2)/(16*a*c - 4*b^2) * 2^{(1/2)} / ((-b + (-4 \\
& *a*c + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctanh}(2^{(1/2)} / ((-b + (-4*a*c + b^2)^{(1/2)}) * c)^{(1/2)} \\
& * c * x) * B * (-4*a*c + b^2)^{(1/2)} + 4*c^2/(4*a*c - b^2)/(16*a*c - 4*b^2) * 2^{(1/2)} / ((-b + (- \\
& 4*a*c + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctanh}(2^{(1/2)} / ((-b + (-4*a*c + b^2)^{(1/2)}) * c)^{(1/2)} \\
&) * c * x) * B * B - 16*c^2/(4*a*c - b^2)/(16*a*c - 4*b^2) * \ln(-2*c*x^2 - b + (-4*a*c + b^2)^{(1/2)}) \\
&) * A - 16*c^2/(4*a*c - b^2)/(16*a*c - 4*b^2) * \ln(2*c*x^2 + b + (-4*a*c + b^2)^{(1/2)}) * A - \\
& 1/a^2/(4*a*c - b^2)/(16*a*c - 4*b^2) * \ln(-2*c*x^2 - b + (-4*a*c + b^2)^{(1/2)}) * A * (-4*a*c \\
& + b^2)^{(1/2)} * b^3 + 8/a*c/(4*a*c - b^2)/(16*a*c - 4*b^2) * \ln(-2*c*x^2 - b + (-4*a*c + b^2)^{(1/2)}) \\
&) * A * b^2 + 1/a^2/(4*a*c - b^2)/(16*a*c - 4*b^2) * \ln(2*c*x^2 + b + (-4*a*c + b^2)^{(1/2)}) * A * (-4*a*c + b^2)^{(1/2)} * b^3 + 8/a*c/(4*a*c - b^2)/(16*a*c - 4*b^2) * \ln(2*c*x^2 + \\
& b + (-4*a*c + b^2)^{(1/2)}) * A * b^2 - 1/a*c/(4*a*c - b^2)/(16*a*c - 4*b^2) * 2^{(1/2)} / ((-b + (-4 \\
& *a*c + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctanh}(2^{(1/2)} / ((-b + (-4*a*c + b^2)^{(1/2)}) * c)^{(1/2)} \\
&) * c * x) * B * b^3 + 1/a*c/(4*a*c - b^2)/(16*a*c - 4*b^2) * 2^{(1/2)} / ((b + (-4*a*c + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctan}(2^{(1/2)} / ((b + (-4*a*c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * B * b^3 - 1/2 \\
& / (c*x^4 + b*x^2 + a) / (4*a*c - b^2) * A / a * b^2 + 1 / (c*x^4 + b*x^2 + a) * c / (4*a*c - b^2) * x^2 * C + \\
& 12*c^2/(4*a*c - b^2)/(16*a*c - 4*b^2) * 2^{(1/2)} / ((b + (-4*a*c + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctan}(2^{(1/2)} / ((b + (-4*a*c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * B * (-4*a*c + b^2)^{(1/2)} - 4 \\
& * c^2/(4*a*c - b^2)/(16*a*c - 4*b^2) * 2^{(1/2)} / ((b + (-4*a*c + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctan}(2^{(1/2)} / ((b + (-4*a*c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * b * B + 6/a*c/(4*a*c - b^2)/(16 \\
& *a*c - 4*b^2) * \ln(-2*c*x^2 - b + (-4*a*c + b^2)^{(1/2)}) * A * (-4*a*c + b^2)^{(1/2)} * b + 1 / (c * \\
& x^4 + b*x^2 + a) / (4*a*c - b^2) * B * c * x
\end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)/x/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")

[Out] $\frac{1}{2} * (B * b * c * x^3 - (2 * C * a - A * b) * c * x^2 - C * a * b + A * b^2 - 2 * A * a * c + (B * b^2 - 2 * B * a * c) * x) / ((a * b^2 * c - 4 * a^2 * c^2) * x^4 + a^2 * b^2 - 4 * a^3 * c + (a * b^3 - 4 * a^2 * b * c) * x^2) + \frac{1}{2} * \operatorname{integrate}((B * a * b * c * x^2 + B * a * b^2 - 6 * B * a^2 * c - 2 * (A * b^2 * c - 4 * A * a * c^2) * x^3 - 2 * (A * b^3 + (2 * C * a^2 - 5 * A * a * b) * c) * x) / (c * x^4 + b * x^2 + a), x) / (a^2 * b^2 - 4 * a^3 * c) + A * \log(x) / a^2$

mupad [B] time = 1.84, size = 8129, normalized size = 20.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x + C*x^2)/(x*(a + b*x^2 + c*x^4)^2),x)

[Out] $\frac{(2 * A * a * c - A * b^2 + C * a * b) / (2 * a * (4 * a * c - b^2)) + (B * x * (2 * a * c - b^2)) / (2 * a * (4 * a * c - b^2)) - (c * x^2 * (A * b - 2 * C * a)) / (2 * a * (4 * a * c - b^2)) - (B * b * c * x^3) / (2 * a * (4 * a * c - b^2))}{(a + b * x^2 + c * x^4)} + \operatorname{symsum}(\log(\operatorname{root}(1572864 * a^9 * b^2 * c^5 * z^4 - 983040 * a^8 * b^4 * c^4 * z^4 + 327680 * a^7 * b^6 * c^3 * z^4 - 61440 * a^6 * b^8 * c^2 * z^4 + 6144 * a^5 * b^10 * c * z^4 - 1048576 * a^10 * c^6 * z^4 - 256 * a^4 * b^12 * z^4 + 1572864 * A * a^7 * b^2 * c^5 * z^3 - 983040 * A * a^6 * b^4 * c^4 * z^3 + 327680 * A * a^5 * b^6 * c^3 * z^3 - 61440 * A * a^4 * b^8 * c^2 * z^3 + 6144 * A * a^3 * b^10 * c * z^3 - 1048576 * A * a^8 * c^6 * z^3 - 256 * A * a^2 * b^12 * z^3 + 98304 * A * C * a^6 * b * c^5 * z^2 + 256 * A * C * a^2 * b^9 * c * z^2 - 90112 * A * C * a^5 * b^3 * c^4 * z^2 + 30720 * A * C * a^4 * b^5 * c^3 * z^2 - 4608 * A * C * a^3 * b^7 * c^2 * z^2 + 61440 * B^2 * a^6 * b * c^5 * z^2 + 432 * B^2 * a^2 * b^9 * c * z^2 + 1536 * A^2 * a * b^10 * c * z^2 + 24576 * C^2 * a^6 * b^2 * c^4 * z^2 - 6144 * C^2 * a^5 * b^4 * c^3 * z^2 + 512 * C^2 * a^4 * b^6 * c^2 * z^2 - 61440 * B^2 * a^5 * b^3 * c^4 * z^2 + 24064 * B^2 * a^4 * b^5 * c^3 * z^2 - 4608 * B^2 * a^3 * b^7 * c^2 * z^2 + 516096 * A^2 * a^5 * b^2 * c^5 * z^2 - 288768 * A^2 * a^4 * b^4 * c^4 * z^2 + 88576 * A^2 * a^3 * b^6 * c^3 * z^2 - 15744 * A^2 * a^2 * b^8 * c^2 * z^2 - 16 * B^2 * a * b^11 * z^2 - 32768 * C^2 * a^7 * c^5 * z^2 - 393216 * A^2 * a^6 * c^6 * z^2 - 64 * A^2 * b^12 * z^2 + 49152 * A^2 * C * a^4 * b * c^5 * z - 2304 * A^2 * C * a * b^7 * c^2 * z + 3072 * A * B^2 * a^4 * b * c^5 * z - 48 * A * B^2 * a * b^7 * c^2 * z + 32 * B^2 * C * a * b^8 * c * z - 15872 * B^2 * C * a^4 * b^2 * c^4 * z + 4992 * B^2 * C * a^3 * b^4 * c^3 * z - 672 * B^2 * C * a^2 * b^6 * c^2 * z - 45056 * A^2 * C * a^3 * b^3 * c^4 * z + 15$

$$\begin{aligned}
& 360A^2C^2a^2b^5c^3z + 12288A^2C^2a^4b^2c^4z - 3072A^2C^2a^3b^4c^3z + 256A^2C^2a^2b^6c^2z - 2304A^2B^2a^3b^3c^4z + 576A^2B^2a^2b^5c^3z + 128A^2C^2b^9c^2z + 61440A^3a^3b^2c^5z - 21504A^3a^2b^4c^4z + 3328A^3a^2b^6c^3z + 18432B^2C^2a^5c^5z - 16384A^2C^2a^5c^5z \\
& - 192A^3b^8c^2z - 65536A^3a^4c^6z - 1088A^2B^2C^2a^2b^2c^4 + 48A^2B^2C^2a^2b^4c^3 + 240B^2C^2a^2b^3c^3 - 1920A^2C^2a^2b^2c^4 - 960B^2C^2a^3b^2c^4 - 16B^2C^2a^2b^5c^2 + 768A^2C^2a^2b^4c^3 - 256A^2C^3a^2b^3c^3 - 3072A^2B^2a^2b^2c^5 + 1104A^2B^2a^2b^3c^4 + 6144A^3C^2a^2b^2c^5 - 2176A^3C^2a^2b^3c^4 + 1536A^2C^3a^3b^2c^4 + 4608A^2B^2C^2a^3c^5 - 25B^4a^2b^4c^3 + 1536A^4a^2b^2c^5 + 192A^3C^2b^5c^3 + 360B^4a^2b^2c^4 - 64A^2C^2b^6c^2 - 2048A^2C^2a^3c^5 - 100A^2B^2b^5c^3 - 256C^4a^4c^4 - 1296B^4a^3c^5 - 144A^4b^4c^4 - 4096A^4a^2c^6, z, k) \cdot (\text{root}(1572864a^9b^2c^5z^4 - 983040a^8b^4c^4z^4 + 327680a^7b^6c^3z^4 - 61440a^6b^8c^2z^4 + 6144a^5b^10c^2z^4 - 1048576a^10c^6z^4 - 256a^4b^12z^4 + 1572864A^2a^7b^2c^5z^3 - 983040A^2a^6b^4c^4z^3 + 327680A^2a^5b^6c^3z^3 - 61440A^2a^4b^8c^2z^3 + 6144A^2a^3b^10c^2z^3 - 1048576A^2a^8c^6z^3 - 256A^2a^2b^12z^3 + 98304A^2C^2a^6b^2c^5z^2 + 256A^2C^2a^2b^9c^2z^2 - 90112A^2C^2a^5b^3c^4z^2 + 30720A^2C^2a^4b^5c^3z^2 - 4608A^2C^2a^3b^7c^2z^2 + 61440B^2a^6b^2c^5z^2 + 432B^2a^2b^9c^2z^2 + 1536A^2a^2b^10c^2z^2 + 24576C^2a^6b^2c^4z^2 - 6144C^2a^5b^4c^3z^2 + 512C^2a^4b^6c^2z^2 - 61440B^2a^5b^3c^4z^2 + 24064B^2a^4b^5c^3z^2 - 4608B^2a^3b^7c^2z^2 + 516096A^2a^5b^2c^5z^2 - 288768A^2a^4b^4c^4z^2 + 88576A^2a^3b^6c^3z^2 - 15744A^2a^2b^8c^2z^2 - 16B^2a^2b^11z^2 - 32768C^2a^7c^5z^2 - 393216A^2a^6c^6z^2 - 64A^2b^12z^2 + 49152A^2C^2a^4b^2c^5z - 2304A^2C^2a^2b^7c^2z + 3072A^2B^2a^4b^2c^5z - 48A^2B^2a^2b^7c^2z + 32B^2C^2a^2b^8c^2z - 15872B^2C^2a^4b^2c^4z + 4992B^2C^2a^3b^4c^3z - 672B^2C^2a^2b^6c^2z - 45056A^2C^2a^3b^3c^4z + 15360A^2C^2a^2b^5c^3z + 12288A^2C^2a^4b^2c^4z - 3072A^2C^2a^3b^4c^3z + 256A^2C^2a^2b^6c^2z - 2304A^2B^2a^3b^3c^4z + 576A^2B^2a^2b^5c^3z + 128A^2C^2b^9c^2z + 61440A^3a^3b^2c^5z - 21504A^3a^2b^4c^4z + 3328A^3a^2b^6c^3z + 18432B^2C^2a^5c^5z - 16384A^2C^2a^5c^5z - 192A^3b^8c^2z - 65536A^3a^4c^6z - 1088A^2B^2C^2a^2b^2c^4 + 48A^2B^2C^2a^2b^4c^3 + 240B^2C^2a^2b^3c^3 - 1920A^2C^2a^2b^2c^4 - 960B^2C^2a^3b^2c^4 - 16B^2C^2a^2b^5c^2 + 768A^2C^2a^2b^4c^3 - 256A^2C^3a^2b^3c^3 - 3072A^2B^2a^2b^2c^5 + 1104A^2B^2a^2b^3c^4 + 6144A^3C^2a^2b^2c^5 - 2176A^3C^2a^2b^3c^4 + 1536A^2C^3a^3b^2c^4 + 4608A^2B^2C^2a^3c^5 - 25B^4a^2b^4c^3 + 1536A^4a^2b^2c^5 + 192A^3C^2b^5c^3 + 360B^4a^2b^2c^4 - 64A^2C^2b^6c^2 - 2048A^2C^2a^3c^5 - 100A^2B^2b^5c^3 - 256C^4a^4c^4 - 1296B^4a^3c^5 - 144A^4b^4c^4 - 4096A^4a^2c^6, z, k) \cdot ((1032A^2B^2a^3b^5c^4 - 152A^2B^2a^2b^7c^3 - 768B^2C^2a^6c^6 - 2944A^2B^2a^4b^3c^5 + 16B^2C^2a^3b^6c^3 - 208B^2C^2a^4b^4c^4 + 768B^2C^2a^5b^2c^5 + 8A^2B^2a^2b^9c^2 + 2944A^2B^2a^5b^2c^6) / (4(a^3b^6 - 64a^6c^3 - 12a^4b^4c + 48a^5b^2c^2))) + \text{root}(1572864a^9b^2c^5z^4 - 983040a^8b^4c^4z^4 + 327680a^7b^6c^3z^4 - 61440a^6b^8c^2z^4 + 6144a^5b^10c^2z^4 - 1048576a^10c^6z^4 - 256a^4b^12z^4 + 1572864A^2a^7b^2c^5z^3 - 983040A^2a^6b^4c^4z^3 + 327680A^2a^5b^6c^3z^3 - 61440A^2a^4b^8c^2z^3 + 6144A^2a^3b^10c^2z^3 - 1048576A^2a^8c^6z^3 - 256A^2a^2b^12z^3 + 98304A^2C^2a^6b^2c^5z^2 + 256A^2C^2a^2b^9c^2z^2 - 90112A^2C^2a^5b^3c^4z^2 + 30720A^2C^2a^4b^5c^3z^2 - 4608A^2C^2a^3b^7c^2z^2 + 61440B^2a^6b^2c^5z^2 + 432B^2a^2b^9c^2z^2 + 1536A^2a^2b^10c^2z^2 + 24576C^2a^6b^2c^4z^2 - 6144C^2a^5b^4c^3z^2 + 512C^2a^4b^6c^2z^2 - 61440B^2a^5b^3c^4z^2 + 24064B^2a^4b^5c^3z^2 - 4608B^2a^3b^7c^2z^2 + 516096A^2a^5b^2c^5z^2 - 288768A^2a^4b^4c^4z^2 + 88576A^2a^3b^6c^3z^2 - 15744A^2a^2b^8c^2z^2 - 16B^2a^2b^11z^2 - 32768C^2a^7c^5z^2 - 393216A^2a^6c^6z^2 - 64A^2b^12z^2 + 49152A^2C^2a^4b^2c^5z - 2304A^2C^2a^2b^7c^2z + 3072A^2B^2a^4b^2c^5z - 48A^2B^2a^2b^7c^2z + 32B^2C^2a^2b^8c^2z - 15872B^2C^2a^4b^2c^4z + 4992B^2C^2a^3b^4c^3z - 672B^2C^2a^2b^6c^2z - 45056A^2C^2a^3b^3c^4z + 15360A^2C^2a^2b^5c^3z + 12288A^2C^2a^4b^2c^4z
\end{aligned}$$

$$\begin{aligned}
& z - 3072*A*C^2*a^3*b^4*c^3*z + 256*A*C^2*a^2*b^6*c^2*z - 2304*A*B^2*a^3*b^3 \\
& *c^4*z + 576*A*B^2*a^2*b^5*c^3*z + 128*A^2*C*b^9*c*z + 61440*A^3*a^3*b^2*c^ \\
& 5*z - 21504*A^3*a^2*b^4*c^4*z + 3328*A^3*a*b^6*c^3*z + 18432*B^2*C*a^5*c^5* \\
& z - 16384*A*C^2*a^5*c^5*z - 192*A^3*b^8*c^2*z - 65536*A^3*a^4*c^6*z - 1088* \\
& A*B^2*C*a^2*b^2*c^4 + 48*A*B^2*C*a*b^4*c^3 + 240*B^2*C^2*a^2*b^3*c^3 - 1920 \\
& *A^2*C^2*a^2*b^2*c^4 - 960*B^2*C^2*a^3*b*c^4 - 16*B^2*C^2*a*b^5*c^2 + 768*A \\
& ^2*C^2*a*b^4*c^3 - 256*A*C^3*a^2*b^3*c^3 - 3072*A^2*B^2*a^2*b*c^5 + 1104*A^ \\
& 2*B^2*a*b^3*c^4 + 6144*A^3*C*a^2*b*c^5 - 2176*A^3*C*a*b^3*c^4 + 1536*A*C^3* \\
& a^3*b*c^4 + 4608*A*B^2*C*a^3*c^5 - 25*B^4*a*b^4*c^3 + 1536*A^4*a*b^2*c^5 + \\
& 192*A^3*C*b^5*c^3 + 360*B^4*a^2*b^2*c^4 - 64*A^2*C^2*b^6*c^2 - 2048*A^2*C^2 \\
& *a^3*c^5 - 100*A^2*B^2*b^5*c^3 - 256*C^4*a^4*c^4 - 1296*B^4*a^3*c^5 - 144*A \\
& ^4*b^4*c^4 - 4096*A^4*a^2*c^6, z, k)*((x*(983040*A*a^8*c^8 - 32768*C*a^8*b* \\
& c^7 + 192*A*a^2*b^12*c^2 - 4736*A*a^3*b^10*c^3 + 48896*A*a^4*b^8*c^4 - 2703 \\
& 36*A*a^5*b^6*c^5 + 843776*A*a^6*b^4*c^6 - 1409024*A*a^7*b^2*c^7 - 128*C*a^4 \\
& *b^9*c^3 + 2048*C*a^5*b^7*c^4 - 12288*C*a^6*b^5*c^5 + 32768*C*a^7*b^3*c^6)) \\
& /((16*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c \\
& ^3)) - (3584*B*a^7*b*c^6 + 8*B*a^3*b^9*c^2 - 152*B*a^4*b^7*c^3 + 1056*B*a^5 \\
& *b^5*c^4 - 3200*B*a^6*b^3*c^5)/(4*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48 \\
& *a^5*b^2*c^2)) + (root(1572864*a^9*b^2*c^5*z^4 - 983040*a^8*b^4*c^4*z^4 + 3 \\
& 27680*a^7*b^6*c^3*z^4 - 61440*a^6*b^8*c^2*z^4 + 6144*a^5*b^10*c*z^4 - 10485 \\
& 76*a^10*c^6*z^4 - 256*a^4*b^12*z^4 + 1572864*A*a^7*b^2*c^5*z^3 - 983040*A*a \\
& ^6*b^4*c^4*z^3 + 327680*A*a^5*b^6*c^3*z^3 - 61440*A*a^4*b^8*c^2*z^3 + 6144* \\
& A*a^3*b^10*c*z^3 - 1048576*A*a^8*c^6*z^3 - 256*A*a^2*b^12*z^3 + 98304*A*C*a \\
& ^6*b*c^5*z^2 + 256*A*C*a^2*b^9*c*z^2 - 90112*A*C*a^5*b^3*c^4*z^2 + 30720*A* \\
& C*a^4*b^5*c^3*z^2 - 4608*A*C*a^3*b^7*c^2*z^2 + 61440*B^2*a^6*b*c^5*z^2 + 43 \\
& 2*B^2*a^2*b^9*c*z^2 + 1536*A^2*a*b^10*c*z^2 + 24576*C^2*a^6*b^2*c^4*z^2 - 6 \\
& 144*C^2*a^5*b^4*c^3*z^2 + 512*C^2*a^4*b^6*c^2*z^2 - 61440*B^2*a^5*b^3*c^4*z \\
& ^2 + 24064*B^2*a^4*b^5*c^3*z^2 - 4608*B^2*a^3*b^7*c^2*z^2 + 516096*A^2*a^5* \\
& b^2*c^5*z^2 - 288768*A^2*a^4*b^4*c^4*z^2 + 88576*A^2*a^3*b^6*c^3*z^2 - 1574 \\
& 4*A^2*a^2*b^8*c^2*z^2 - 16*B^2*a*b^11*z^2 - 32768*C^2*a^7*c^5*z^2 - 393216* \\
& A^2*a^6*c^6*z^2 - 64*A^2*b^12*z^2 + 49152*A^2*C*a^4*b*c^5*z - 2304*A^2*C*a* \\
& b^7*c^2*z + 3072*A*B^2*a^4*b*c^5*z - 48*A*B^2*a*b^7*c^2*z + 32*B^2*C*a*b^8* \\
& c*z - 15872*B^2*C*a^4*b^2*c^4*z + 4992*B^2*C*a^3*b^4*c^3*z - 672*B^2*C*a^2* \\
& b^6*c^2*z - 45056*A^2*C*a^3*b^3*c^4*z + 15360*A^2*C*a^2*b^5*c^3*z + 12288*A \\
& *C^2*a^4*b^2*c^4*z - 3072*A*C^2*a^3*b^4*c^3*z + 256*A*C^2*a^2*b^6*c^2*z - 2 \\
& 304*A*B^2*a^3*b^3*c^4*z + 576*A*B^2*a^2*b^5*c^3*z + 128*A^2*C*b^9*c*z + 614 \\
& 40*A^3*a^3*b^2*c^5*z - 21504*A^3*a^2*b^4*c^4*z + 3328*A^3*a*b^6*c^3*z + 184 \\
& 32*B^2*C*a^5*c^5*z - 16384*A*C^2*a^5*c^5*z - 192*A^3*b^8*c^2*z - 65536*A^3* \\
& a^4*c^6*z - 1088*A*B^2*C*a^2*b^2*c^4 + 48*A*B^2*C*a*b^4*c^3 + 240*B^2*C^2*a \\
& ^2*b^3*c^3 - 1920*A^2*C^2*a^2*b^2*c^4 - 960*B^2*C^2*a^3*b*c^4 - 16*B^2*C^2* \\
& a*b^5*c^2 + 768*A^2*C^2*a*b^4*c^3 - 256*A*C^3*a^2*b^3*c^3 - 3072*A^2*B^2*a^ \\
& 2*b*c^5 + 1104*A^2*B^2*a*b^3*c^4 + 6144*A^3*C*a^2*b*c^5 - 2176*A^3*C*a*b^3* \\
& c^4 + 1536*A*C^3*a^3*b*c^4 + 4608*A*B^2*C*a^3*c^5 - 25*B^4*a*b^4*c^3 + 1536 \\
& *A^4*a*b^2*c^5 + 192*A^3*C*b^5*c^3 + 360*B^4*a^2*b^2*c^4 - 64*A^2*C^2*b^6*c \\
& ^2 - 2048*A^2*C^2*a^3*c^5 - 100*A^2*B^2*b^5*c^3 - 256*C^4*a^4*c^4 - 1296*B^ \\
& 4*a^3*c^5 - 144*A^4*b^4*c^4 - 4096*A^4*a^2*c^6, z, k)*x*(1310720*a^10*c^8 + \\
& 384*a^4*b^12*c^2 - 8960*a^5*b^10*c^3 + 87040*a^6*b^8*c^4 - 450560*a^7*b^6* \\
& c^5 + 1310720*a^8*b^4*c^6 - 2031616*a^9*b^2*c^7))/(16*(a^3*b^8 + 256*a^7*c^ \\
& 4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3))) - (x*(26560*A^2*a^3* \\
& b^6*c^5 - 36864*C^2*a^7*c^7 - 2912*A^2*a^2*b^8*c^4 - 245760*A^2*a^6*c^8 - 1 \\
& 20832*A^2*a^4*b^4*c^6 + 273408*A^2*a^5*b^2*c^7 + 432*B^2*a^2*b^9*c^3 - 4616 \\
& *B^2*a^3*b^7*c^4 + 24032*B^2*a^4*b^5*c^5 - 60800*B^2*a^5*b^3*c^6 + 640*C^2* \\
& a^4*b^6*c^4 - 7424*C^2*a^5*b^4*c^5 + 28672*C^2*a^6*b^2*c^6 + 128*A^2*a*b^10 \\
& *c^3 - 16*B^2*a*b^11*c^2 + 59904*B^2*a^6*b*c^7 + 256*A*C*a^2*b^9*c^3 - 4608 \\
& *A*C*a^3*b^7*c^4 + 30464*A*C*a^4*b^5*c^5 - 88064*A*C*a^5*b^3*c^6 + 94208*A* \\
& C*a^6*b*c^7))/(16*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - \\
& 256*a^6*b^2*c^3))) + (108*B^3*a^4*c^6 - 15*B^3*a^3*b^2*c^5 + 24*A^2*B*a*b^5 \\
& *c^4 + 704*A^2*B*a^3*b*c^6 + 56*B*C^2*a^4*b*c^5 - 266*A^2*B*a^2*b^3*c^5 - 8 \\
& *B*C^2*a^3*b^3*c^4 + 576*A*B*C*a^4*c^6 - 16*A*B*C*a*b^6*c^3 + 208*A*B*C*a^2
\end{aligned}$$

$$\begin{aligned} & *b^4*c^4 - 744*A*B*C*a^3*b^2*c^5)/(4*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + \\ & 48*a^5*b^2*c^2)) + (x*(20480*A^3*a^4*c^8 - 32*A^3*b^8*c^4 + 1216*A^3*a^2*b \\ & ^4*c^6 - 11008*A^3*a^3*b^2*c^7 + 128*C^3*a^4*b^3*c^5 + 13312*A*C^2*a^5*c^7 \\ & - 19584*B^2*C*a^5*c^7 + 192*A^3*a*b^6*c^5 - 512*C^3*a^5*b*c^6 + 40*A*B^2*a* \\ & b^7*c^4 - 2496*A*B^2*a^4*b*c^7 + 256*A^2*C*a*b^7*c^4 - 25600*A^2*C*a^4*b*c^ \\ & 7 - 32*B^2*C*a*b^8*c^3 - 508*A*B^2*a^2*b^5*c^5 + 2016*A*B^2*a^3*b^3*c^6 - 6 \\ & 4*A*C^2*a^2*b^6*c^4 + 1152*A*C^2*a^3*b^4*c^5 - 6912*A*C^2*a^4*b^2*c^6 - 355 \\ & 2*A^2*C*a^2*b^5*c^5 + 16512*A^2*C*a^3*b^3*c^6 + 672*B^2*C*a^2*b^6*c^4 - 500 \\ & 0*B^2*C*a^3*b^4*c^5 + 16192*B^2*C*a^4*b^2*c^6))/(16*(a^3*b^8 + 256*a^7*c^4 \\ & - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3))) - (108*A*B^3*a^2*c^6 - \\ & 10*A^3*B*b^3*c^5 - 192*A^2*B*C*a^2*c^6 - 15*A*B^3*a*b^2*c^5 + 64*A^3*B*a*b \\ & *c^6 - 8*A*B*C^2*a*b^3*c^4 + 56*A*B*C^2*a^2*b*c^5 + 24*A^2*B*C*a*b^2*c^5)/(\\ & 4*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)) + (x*(1296*B^4*a^ \\ & 3*c^7 - 48*A^4*b^4*c^6 + 256*C^4*a^4*c^6 + 1024*A^2*C^2*a^3*c^7 - 360*B^4*a \\ & ^2*b^2*c^6 + 32*A^3*C*b^5*c^5 + 256*A^4*a*b^2*c^7 + 25*B^4*a*b^4*c^5 - 3456 \\ & *A*B^2*C*a^3*c^7 - 1024*A*C^3*a^3*b*c^6 - 1024*A^3*C*a^2*b*c^7 - 176*A^2*B^ \\ & 2*a*b^3*c^6 + 960*A^2*B^2*a^2*b*c^7 + 128*A*C^3*a^2*b^3*c^5 - 128*A^2*C^2*a \\ & *b^4*c^5 + 16*B^2*C^2*a*b^5*c^4 + 960*B^2*C^2*a^3*b*c^6 + 640*A^2*C^2*a^2*b \\ & ^2*c^6 - 240*B^2*C^2*a^2*b^3*c^5 - 40*A*B^2*C*a*b^4*c^5 + 768*A*B^2*C*a^2*b \\ & ^2*c^6))/(16*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a \\ & ^6*b^2*c^3))) *root(1572864*a^9*b^2*c^5*z^4 - 983040*a^8*b^4*c^4*z^4 + 32768 \\ & 0*a^7*b^6*c^3*z^4 - 61440*a^6*b^8*c^2*z^4 + 6144*a^5*b^10*c*z^4 - 1048576*a \\ & ^10*c^6*z^4 - 256*a^4*b^12*z^4 + 1572864*A*a^7*b^2*c^5*z^3 - 983040*A*a^6*b \\ & ^4*c^4*z^3 + 327680*A*a^5*b^6*c^3*z^3 - 61440*A*a^4*b^8*c^2*z^3 + 6144*A*a^ \\ & 3*b^10*c*z^3 - 1048576*A*a^8*c^6*z^3 - 256*A*a^2*b^12*z^3 + 98304*A*C*a^6*b \\ & *c^5*z^2 + 256*A*C*a^2*b^9*c*z^2 - 90112*A*C*a^5*b^3*c^4*z^2 + 30720*A*C*a^ \\ & 4*b^5*c^3*z^2 - 4608*A*C*a^3*b^7*c^2*z^2 + 61440*B^2*a^6*b*c^5*z^2 + 432*B^ \\ & 2*a^2*b^9*c*z^2 + 1536*A^2*a*b^10*c*z^2 + 24576*C^2*a^6*b^2*c^4*z^2 - 6144* \\ & C^2*a^5*b^4*c^3*z^2 + 512*C^2*a^4*b^6*c^2*z^2 - 61440*B^2*a^5*b^3*c^4*z^2 + \\ & 24064*B^2*a^4*b^5*c^3*z^2 - 4608*B^2*a^3*b^7*c^2*z^2 + 516096*A^2*a^5*b^2* \\ & c^5*z^2 - 288768*A^2*a^4*b^4*c^4*z^2 + 88576*A^2*a^3*b^6*c^3*z^2 - 15744*A^ \\ & 2*a^2*b^8*c^2*z^2 - 16*B^2*a*b^11*z^2 - 32768*C^2*a^7*c^5*z^2 - 393216*A^2* \\ & a^6*c^6*z^2 - 64*A^2*b^12*z^2 + 49152*A^2*C*a^4*b*c^5*z - 2304*A^2*C*a*b^7* \\ & c^2*z + 3072*A*B^2*a^4*b*c^5*z - 48*A*B^2*a*b^7*c^2*z + 32*B^2*C*a*b^8*c*z \\ & - 15872*B^2*C*a^4*b^2*c^4*z + 4992*B^2*C*a^3*b^4*c^3*z - 672*B^2*C*a^2*b^6* \\ & c^2*z - 45056*A^2*C*a^3*b^3*c^4*z + 15360*A^2*C*a^2*b^5*c^3*z + 12288*A*C^2 \\ & *a^4*b^2*c^4*z - 3072*A*C^2*a^3*b^4*c^3*z + 256*A*C^2*a^2*b^6*c^2*z - 2304* \\ & A*B^2*a^3*b^3*c^4*z + 576*A*B^2*a^2*b^5*c^3*z + 128*A^2*C*b^9*c*z + 61440*A \\ & ^3*a^3*b^2*c^5*z - 21504*A^3*a^2*b^4*c^4*z + 3328*A^3*a*b^6*c^3*z + 18432*B \\ & ^2*C*a^5*c^5*z - 16384*A*C^2*a^5*c^5*z - 192*A^3*b^8*c^2*z - 65536*A^3*a^4* \\ & c^6*z - 1088*A*B^2*C*a^2*b^2*c^4 + 48*A*B^2*C*a*b^4*c^3 + 240*B^2*C^2*a^2*b \\ & ^3*c^3 - 1920*A^2*C^2*a^2*b^2*c^4 - 960*B^2*C^2*a^3*b*c^4 - 16*B^2*C^2*a*b^ \\ & 5*c^2 + 768*A^2*C^2*a*b^4*c^3 - 256*A*C^3*a^2*b^3*c^3 - 3072*A^2*B^2*a^2*b* \\ & c^5 + 1104*A^2*B^2*a*b^3*c^4 + 6144*A^3*C*a^2*b*c^5 - 2176*A^3*C*a*b^3*c^4 \\ & + 1536*A*C^3*a^3*b*c^4 + 4608*A*B^2*C*a^3*c^5 - 25*B^4*a*b^4*c^3 + 1536*A^4 \\ & *a*b^2*c^5 + 192*A^3*C*b^5*c^3 + 360*B^4*a^2*b^2*c^4 - 64*A^2*C^2*b^6*c^2 - \\ & 2048*A^2*C^2*a^3*c^5 - 100*A^2*B^2*b^5*c^3 - 256*C^4*a^4*c^4 - 1296*B^4*a^ \\ & 3*c^5 - 144*A^4*b^4*c^4 - 4096*A^4*a^2*c^6, z, k), k, 1, 4) + (A*log(x))/a^ \\ & 2 \end{aligned}$$

`sympy [F(-1)]` time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((C*x**2+B*x+A)/x/(c*x**4+b*x**2+a)**2,x)`

[Out] Timed out

3.35 $\int \frac{A+Bx+Cx^2}{x^2(a+bx^2+cx^4)^2} dx$

Optimal. Leaf size=514

$$\frac{-10aAc - abC + 3Ab^2}{2a^2x(b^2 - 4ac)} \sqrt{c} \left(A(3b^2\sqrt{b^2 - 4ac} - 10ac\sqrt{b^2 - 4ac} - 16abc + 3b^3) - aC(b\sqrt{b^2 - 4ac} - 12ac + b^2) \right) - \frac{2\sqrt{2} a^2 (b^2 - 4ac)^{3/2} \sqrt{b - \sqrt{b^2 - 4ac}}}{2\sqrt{2} a^2 (b^2 - 4ac)^{3/2} \sqrt{b - \sqrt{b^2 - 4ac}}}$$

Rubi [A] time = 1.49, antiderivative size = 514, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 28, number of rules / integrand size = 0.464, Rules used = {1662, 1277, 1281, 1166, 205, 12, 1114, 740, 800, 634, 618, 206, 628}

$$\frac{-10aAc - abC + 3Ab^2}{2a^2x(b^2 - 4ac)} \sqrt{c} \left(A(3b^2\sqrt{b^2 - 4ac} - 10ac\sqrt{b^2 - 4ac} - 16abc + 3b^3) - aC(b\sqrt{b^2 - 4ac} - 12ac + b^2) \right) - \frac{2\sqrt{2} a^2 (b^2 - 4ac)^{3/2} \sqrt{b - \sqrt{b^2 - 4ac}}}{2\sqrt{2} a^2 (b^2 - 4ac)^{3/2} \sqrt{b - \sqrt{b^2 - 4ac}}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(x^2*(a + b*x^2 + c*x^4)^2), x]
[Out] -(3*A*b^2 - 10*A*A*c - a*b*C)/(2*a^2*(b^2 - 4*a*c)*x) + (B*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (A*(b^2 - 2*a*c) - a*b*C + c*(A*b - 2*a*C)*x^2)/(2*a*(b^2 - 4*a*c)*x*(a + b*x^2 + c*x^4)) - (Sqrt[c]*(A*(3*b^3 - 16*a*b*c + 3*b^2*Sqrt[b^2 - 4*a*c] - 10*a*c*Sqrt[b^2 - 4*a*c]) - a*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(3*A*b^2 - 10*A*A*c - a*b*C - (A*(3*b^3 - 16*a*b*c) - a*(b^2 - 12*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (b*B*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^2*(b^2 - 4*a*c)^(3/2)) + (B*Log[x])/a^2 - (B*Log[a + b*x^2 + c*x^4])/(4*a^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d}, x]
```

e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 740

Int[((d_.) + (e_.)*(x_)^m)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^p, x_Symbol] := Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]

Rule 800

Int[((d_.) + (e_.)*(x_)^m)*((f_.) + (g_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]

Rule 1114

Int[(x_)^m*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^p, x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]

Rule 1166

Int[((d_.) + (e_.)*(x_)^2)/((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1277

Int[((f_.)*(x_)^m)*((d_.) + (e_.)*(x_)^2)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^p, x_Symbol] := -Simp[((f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)*(d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^2))/(2*a*f*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[d*(b^2*(m + 2*(p + 1) + 1) - 2*a*c*(m + 4*(p + 1) + 1)) - a*b*e*(m + 1) + c*(m + 2*(2*p + 3) + 1)*(b*d - 2*a*e)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1281

Int[((f_.)*(x_)^m)*((d_.) + (e_.)*(x_)^2)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^p, x_Symbol] := Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]

, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1662

Int[(Pq_)*((d_)*(x_))^(m_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2 + 1}]*a + b*x^2 + c*x^4]^p, x] + Dist[1/d, Int[(d*x)^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*a + b*x^2 + c*x^4]^p, x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] && !PolyQ[Pq, x^2]

Rubi steps

$$\begin{aligned} \int \frac{A + Bx + Cx^2}{x^2(a + bx^2 + cx^4)^2} dx &= \int \frac{B}{x(a + bx^2 + cx^4)^2} dx + \int \frac{A + Cx^2}{x^2(a + bx^2 + cx^4)^2} dx \\ &= \frac{A(b^2 - 2ac) - abC + c(Ab - 2aC)x^2}{2a(b^2 - 4ac)x(a + bx^2 + cx^4)} + B \int \frac{1}{x(a + bx^2 + cx^4)^2} dx - \frac{\int \frac{-3Ab^2 + 10aAc + abC}{x^2(a + bx^2 + cx^4)} dx}{2a(b^2 - 4ac)} \\ &= -\frac{3Ab^2 - 10aAc - abC}{2a^2(b^2 - 4ac)x} + \frac{A(b^2 - 2ac) - abC + c(Ab - 2aC)x^2}{2a(b^2 - 4ac)x(a + bx^2 + cx^4)} + \frac{1}{2}B \operatorname{Subst}\left(\int \frac{1}{x(a + bx^2 + cx^4)} dx, x, \sqrt{x}\right) \\ &= -\frac{3Ab^2 - 10aAc - abC}{2a^2(b^2 - 4ac)x} + \frac{B(b^2 - 2ac + bcx^2)}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{A(b^2 - 2ac) - abC + c(Ab - 2aC)x^2}{2a(b^2 - 4ac)x(a + bx^2 + cx^4)} \\ &= -\frac{3Ab^2 - 10aAc - abC}{2a^2(b^2 - 4ac)x} + \frac{B(b^2 - 2ac + bcx^2)}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{A(b^2 - 2ac) - abC + c(Ab - 2aC)x^2}{2a(b^2 - 4ac)x(a + bx^2 + cx^4)} \\ &= -\frac{3Ab^2 - 10aAc - abC}{2a^2(b^2 - 4ac)x} + \frac{B(b^2 - 2ac + bcx^2)}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{A(b^2 - 2ac) - abC + c(Ab - 2aC)x^2}{2a(b^2 - 4ac)x(a + bx^2 + cx^4)} \\ &= -\frac{3Ab^2 - 10aAc - abC}{2a^2(b^2 - 4ac)x} + \frac{B(b^2 - 2ac + bcx^2)}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{A(b^2 - 2ac) - abC + c(Ab - 2aC)x^2}{2a(b^2 - 4ac)x(a + bx^2 + cx^4)} \\ &= -\frac{3Ab^2 - 10aAc - abC}{2a^2(b^2 - 4ac)x} + \frac{B(b^2 - 2ac + bcx^2)}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{A(b^2 - 2ac) - abC + c(Ab - 2aC)x^2}{2a(b^2 - 4ac)x(a + bx^2 + cx^4)} \\ &= -\frac{3Ab^2 - 10aAc - abC}{2a^2(b^2 - 4ac)x} + \frac{B(b^2 - 2ac + bcx^2)}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{A(b^2 - 2ac) - abC + c(Ab - 2aC)x^2}{2a(b^2 - 4ac)x(a + bx^2 + cx^4)} \\ &= -\frac{3Ab^2 - 10aAc - abC}{2a^2(b^2 - 4ac)x} + \frac{B(b^2 - 2ac + bcx^2)}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{A(b^2 - 2ac) - abC + c(Ab - 2aC)x^2}{2a(b^2 - 4ac)x(a + bx^2 + cx^4)} \end{aligned}$$

Mathematica [A] time = 2.03, size = 559, normalized size = 1.09

$$\frac{-4A^2(B+C)+2A(B+C)(a+cb^2+4c^2)+2A^2(a^2+2a^2b^2+2a^2c^2)-2A^2(b^2+c^2)}{(b^2-4ac)^2\sqrt{a+bx^2+cx^4}}, \frac{\sqrt{2}\sqrt{a^2(-3b^2\sqrt{b^2-4ac}+10a\sqrt{b^2-4ac}+10ab^2-10ab^2-3b^2)+c(\sqrt{b^2-4ac}-12a^2)}}{\sqrt{a+bx^2+cx^4}}, \frac{\sqrt{2}\sqrt{a^2(-3b^2\sqrt{b^2-4ac}+10a\sqrt{b^2-4ac}+10ab^2-10ab^2-3b^2)+c(\sqrt{b^2-4ac}+12a^2)}}{\sqrt{a+bx^2+cx^4}} - \frac{\sqrt{2}\sqrt{a^2}}{\sqrt{a+bx^2+cx^4}} \frac{B(\sqrt{b^2-4ac}-4a\sqrt{b^2-4ac}-6ab^2+3b^2)}{(b^2-4ac)^2} \log(\sqrt{b^2-4ac}-2ax) - \frac{B(\sqrt{b^2-4ac}-4a\sqrt{b^2-4ac}-6ab^2+3b^2)}{(b^2-4ac)^2} \log(\sqrt{b^2-4ac}+2ax) - \frac{4A}{7} + 4B \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(x^2*(a + b*x^2 + c*x^4)^2), x]
[Out] ((-4*A)/x + (-4*a^2*c*(B + C*x) - 2*A*b^2*x*(b + c*x^2) + 2*a*(2*A*c^2*x^3 + b^2*(B + C*x) + b*c*x*(3*A + x*(B + C*x))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(A*(-3*b^3 + 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c]) + a*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(A*(3*b^3 - 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c]) + a*(-b^2 + 12*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + 4*B*Log[x] - (B*(b^3 - 6*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 4*a*c*Sqrt[b^2 - 4*a*c])*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) - (B*(-b^3 + 6*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 4*a*c*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/(4*a^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx + Cx^2}{x^2 (a + bx^2 + cx^4)^2} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x + C*x^2)/(x^2*(a + b*x^2 + c*x^4)^2), x]
[Out] IntegrateAlgebraic[(A + B*x + C*x^2)/(x^2*(a + b*x^2 + c*x^4)^2), x]
fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
[Out] Timed out
giac [B] time = 11.55, size = 9015, normalized size = 17.54
```

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
[Out] -1/4*B*log(abs(c*x^4 + b*x^2 + a))/a^2 + B*log(abs(x))/a^2 + 1/2*(C*a*b*c*x^4 - 3*A*b^2*c*x^4 + 10*A*a*c^2*x^4 + B*a*b*c*x^3 + C*a*b^2*x^2 - 3*A*b^3*x^2 - 2*C*a^2*c*x^2 + 11*A*a*b*c*x^2 + B*a*b^2*x - 2*B*a^2*c*x - 2*A*a*b^2 + 8*A*a^2*c)/(c*x^5 + b*x^3 + a*x)*(a^2*b^2 - 4*a^3*c) - 1/16*((a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)^2*(6*b^4*c^2 - 44*a*b^2*c^3 + 80*a^2*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*b^4 + 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a*b^2*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*b^3*c - 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^2*c^2 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a*b*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*b^2*c^2 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a*c^3 - 6*(b^2 - 4*a*c)*b^2*c^2 + 20*(b^2 - 4*a*c)*a*c^3)*A - (a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)^2*(2*a*b^3*c^2 - 8*a^2*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^2*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a*b^2*c - sqrt(2)*s
```

$$\begin{aligned}
& \text{qrt}(b^2 - 4ac) * \text{sqrt}(bc + \text{sqrt}(b^2 - 4ac) * c) * a * b * c^2 - 2 * (b^2 - 4ac) * \\
& a * b * c^2) * C + 2 * (3 * \text{sqrt}(2) * \text{sqrt}(bc + \text{sqrt}(b^2 - 4ac) * c) * a^4 * b^9 * c - 49 * \text{sq} \\
& \text{rt}(2) * \text{sqrt}(bc + \text{sqrt}(b^2 - 4ac) * c) * a^5 * b^7 * c^2 - 6 * \text{sqrt}(2) * \text{sqrt}(bc + \text{sq} \\
& \text{rt}(b^2 - 4ac) * c) * a^4 * b^8 * c^2 - 6 * a^4 * b^9 * c^2 + 300 * \text{sqrt}(2) * \text{sqrt}(bc + \text{sq} \\
& \text{rt}(b^2 - 4ac) * c) * a^6 * b^5 * c^3 + 74 * \text{sqrt}(2) * \text{sqrt}(bc + \text{sqrt}(b^2 - 4ac) * c) * \\
& a^5 * b^6 * c^3 + 3 * \text{sqrt}(2) * \text{sqrt}(bc + \text{sqrt}(b^2 - 4ac) * c) * a^4 * b^7 * c^3 + 98 * a^ \\
& 5 * b^7 * c^3 - 816 * \text{sqrt}(2) * \text{sqrt}(bc + \text{sqrt}(b^2 - 4ac) * c) * a^7 * b^3 * c^4 - 304 * \text{sq} \\
& \text{qrt}(2) * \text{sqrt}(bc + \text{sqrt}(b^2 - 4ac) * c) * a^6 * b^4 * c^4 - 37 * \text{sqrt}(2) * \text{sqrt}(bc + \\
& \text{sqrt}(b^2 - 4ac) * c) * a^5 * b^5 * c^4 - 600 * a^6 * b^5 * c^4 + 832 * \text{sqrt}(2) * \text{sqrt}(bc + \\
& \text{sqrt}(b^2 - 4ac) * c) * a^8 * b * c^5 + 416 * \text{sqrt}(2) * \text{sqrt}(bc + \text{sqrt}(b^2 - 4ac) * \\
& c) * a^7 * b^2 * c^5 + 152 * \text{sqrt}(2) * \text{sqrt}(bc + \text{sqrt}(b^2 - 4ac) * c) * a^6 * b^3 * c^5 + \\
& 1632 * a^7 * b^3 * c^5 - 208 * \text{sqrt}(2) * \text{sqrt}(bc + \text{sqrt}(b^2 - 4ac) * c) * a^7 * b * c^6 - \\
& 1664 * a^8 * b * c^6 + 6 * (b^2 - 4ac) * a^4 * b^7 * c^2 - 74 * (b^2 - 4ac) * a^5 * b^5 * c^3 \\
& + 304 * (b^2 - 4ac) * a^6 * b^3 * c^4 - 416 * (b^2 - 4ac) * a^7 * b * c^5) * A * \text{abs}(a^4 * b \\
& ^4 * c - 8 * a^5 * b^2 * c^2 + 16 * a^6 * c^3) - 2 * (\text{sqrt}(2) * \text{sqrt}(bc + \text{sqrt}(b^2 - 4ac) \\
&) * c) * a^5 * b^8 * c - 18 * \text{sqrt}(2) * \text{sqrt}(bc + \text{sqrt}(b^2 - 4ac) * c) * a^6 * b^6 * c^2 - 2 \\
& * \text{sqrt}(2) * \text{sqrt}(bc + \text{sqrt}(b^2 - 4ac) * c) * a^5 * b^7 * c^2 - 2 * a^5 * b^8 * c^2 + 120 * \\
& \text{sqrt}(2) * \text{sqrt}(bc + \text{sqrt}(b^2 - 4ac) * c) * a^7 * b^4 * c^3 + 28 * \text{sqrt}(2) * \text{sqrt}(bc + \\
& \text{sqrt}(b^2 - 4ac) * c) * a^6 * b^5 * c^3 + \text{sqrt}(2) * \text{sqrt}(bc + \text{sqrt}(b^2 - 4ac) * c) \\
& * a^5 * b^6 * c^3 + 36 * a^6 * b^6 * c^3 - 352 * \text{sqrt}(2) * \text{sqrt}(bc + \text{sqrt}(b^2 - 4ac) * c) \\
& * a^8 * b^2 * c^4 - 128 * \text{sqrt}(2) * \text{sqrt}(bc + \text{sqrt}(b^2 - 4ac) * c) * a^7 * b^3 * c^4 - 14 \\
& * \text{sqrt}(2) * \text{sqrt}(bc + \text{sqrt}(b^2 - 4ac) * c) * a^6 * b^4 * c^4 - 240 * a^7 * b^4 * c^4 + 38 \\
& 4 * \text{sqrt}(2) * \text{sqrt}(bc + \text{sqrt}(b^2 - 4ac) * c) * a^9 * c^5 + 192 * \text{sqrt}(2) * \text{sqrt}(bc + \\
& \text{sqrt}(b^2 - 4ac) * c) * a^8 * b * c^5 + 64 * \text{sqrt}(2) * \text{sqrt}(bc + \text{sqrt}(b^2 - 4ac) * c) \\
& * a^7 * b^2 * c^5 + 704 * a^8 * b^2 * c^5 - 96 * \text{sqrt}(2) * \text{sqrt}(bc + \text{sqrt}(b^2 - 4ac) * c) \\
& * a^8 * c^6 - 768 * a^9 * c^6 + 2 * (b^2 - 4ac) * a^5 * b^6 * c^2 - 28 * (b^2 - 4ac) * a^6 \\
& * b^4 * c^3 + 128 * (b^2 - 4ac) * a^7 * b^2 * c^4 - 192 * (b^2 - 4ac) * a^8 * c^5) * C * \text{abs} \\
& (a^4 * b^4 * c - 8 * a^5 * b^2 * c^2 + 16 * a^6 * c^3) + (6 * a^8 * b^{12} * c^4 - 128 * a^9 * b^{10} * c \\
& ^5 + 1088 * a^{10} * b^8 * c^6 - 4608 * a^{11} * b^6 * c^7 + 9728 * a^{12} * b^4 * c^8 - 8192 * a^{13} * \\
& b^2 * c^9 - 3 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(bc + \text{sqrt}(b^2 - 4ac) * c) * a^8 * b \\
& ^{12} * c^2 + 64 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(bc + \text{sqrt}(b^2 - 4ac) * c) * a^9 * \\
& b^{10} * c^3 + 6 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(bc + \text{sqrt}(b^2 - 4ac) * c) * a^8 * \\
& b^{11} * c^3 - 544 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(bc + \text{sqrt}(b^2 - 4ac) * c) * a^ \\
& 10 * b^8 * c^4 - 104 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(bc + \text{sqrt}(b^2 - 4ac) * c) * \\
& a^9 * b^9 * c^4 - 3 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(bc + \text{sqrt}(b^2 - 4ac) * c) * a \\
& ^8 * b^{10} * c^4 + 2304 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(bc + \text{sqrt}(b^2 - 4ac) * c) \\
&) * a^{11} * b^6 * c^5 + 672 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(bc + \text{sqrt}(b^2 - 4ac) \\
&) * c) * a^{10} * b^7 * c^5 + 52 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(bc + \text{sqrt}(b^2 - 4ac) \\
&) * c) * a^9 * b^8 * c^5 - 4864 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(bc + \text{sqrt}(b^2 - 4ac) \\
&) * c) * a^{12} * b^4 * c^6 - 1920 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(bc + \text{sqrt}(b^2 - \\
& 4ac) * c) * a^{11} * b^5 * c^6 - 336 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(bc + \text{sqrt}(b^2 \\
& - 4ac) * c) * a^{10} * b^6 * c^6 + 4096 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(bc + \text{sqrt}(b^2 \\
& - 4ac) * c) * a^{13} * b^2 * c^7 + 2048 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(bc + \text{sq} \\
& \text{rt}(b^2 - 4ac) * c) * a^{12} * b^3 * c^7 + 960 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(bc + \text{s} \\
& \text{qrt}(b^2 - 4ac) * c) * a^{11} * b^4 * c^7 - 1024 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(bc \\
& + \text{sqrt}(b^2 - 4ac) * c) * a^{12} * b^2 * c^8 - 6 * (b^2 - 4ac) * a^8 * b^{10} * c^4 + 104 * (b \\
& ^2 - 4ac) * a^9 * b^8 * c^5 - 672 * (b^2 - 4ac) * a^{10} * b^6 * c^6 + 1920 * (b^2 - 4ac) \\
&) * a^{11} * b^4 * c^7 - 2048 * (b^2 - 4ac) * a^{12} * b^2 * c^8) * A - (2 * a^9 * b^{11} * c^4 - 56 \\
& * a^{10} * b^9 * c^5 + 576 * a^{11} * b^7 * c^6 - 2816 * a^{12} * b^5 * c^7 + 6656 * a^{13} * b^3 * c^8 - \\
& 6144 * a^{14} * b * c^9 - \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(bc + \text{sqrt}(b^2 - 4ac) * c) \\
& * a^9 * b^{11} * c^2 + 28 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(bc + \text{sqrt}(b^2 - 4ac) * c) \\
&) * a^{10} * b^9 * c^3 + 2 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(bc + \text{sqrt}(b^2 - 4ac) * c) \\
&) * a^9 * b^{10} * c^3 - 288 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(bc + \text{sqrt}(b^2 - 4ac) \\
&) * c) * a^{11} * b^7 * c^4 - 48 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(bc + \text{sqrt}(b^2 - 4ac) \\
&) * c) * a^{10} * b^8 * c^4 - \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(bc + \text{sqrt}(b^2 - 4ac) * \\
& c) * a^9 * b^9 * c^4 + 1408 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(bc + \text{sqrt}(b^2 - 4ac) \\
&) * c) * a^{12} * b^5 * c^5 + 384 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(bc + \text{sqrt}(b^2 - 4ac) \\
&) * c) * a^{11} * b^6 * c^5 + 24 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(bc + \text{sqrt}(b^2 - 4ac) \\
&) * c) * a^{10} * b^7 * c^5 - 3328 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(bc + \text{sqrt}(b^2 -
\end{aligned}$$

$$\begin{aligned}
& 4*a*c)*c)*a^{13}*b^3*c^6 - 1280*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a^{12}*b^4*c^6 - 192*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a^{11}*b^5*c^6 + 3072*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a^{14}*b*c^7 + 1536*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a^{13}*b^2*c^7 + 640*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a^{12}*b^3*c^7 - 768*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a^{13}*b*c^8 - 2*(b^2 - 4*a*c)*a^9*b^9*c^4 + 48*(b^2 - 4*a*c)*a^{10}*b^7*c^5 - 384*(b^2 - 4*a*c)*a^{11}*b^5*c^6 + 1280*(b^2 - 4*a*c)*a^{12}*b^3*c^7 - 1536*(b^2 - 4*a*c)*a^{13}*b*c^8)*C)*\arctan(2*\sqrt{1/2}*x/\sqrt{((a^4*b^5*c - 8*a^5*b^3*c^2 + 16*a^6*b*c^3 + \sqrt{(a^4*b^5*c - 8*a^5*b^3*c^2 + 16*a^6*b*c^3)^2 - 4*(a^5*b^4*c - 8*a^6*b^2*c^2 + 16*a^7*c^3)*(a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^4)))/(a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^4)))/((a^7*b^8*c - 16*a^8*b^6*c^2 - 2*a^7*b^7*c^2 + 96*a^9*b^4*c^3 + 24*a^8*b^5*c^3 + a^7*b^6*c^3 - 256*a^{10}*b^2*c^4 - 96*a^9*b^3*c^4 - 12*a^8*b^4*c^4 + 256*a^{11}*c^5 + 128*a^{10}*b*c^5 + 48*a^9*b^2*c^5 - 64*a^{10}*c^6)*\text{abs}(a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*\text{abs}(c)) + 1/16*((a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)^2*(6*b^4*c^2 - 44*a*b^2*c^3 + 80*a^2*c^4 - 3*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*b^4 + 22*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a*b^2*c + 6*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*b^3*c - 40*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a^2*c^2 - 20*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a*b*c^2 - 3*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*b^2*c^2 + 10*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a*c^3 - 6*(b^2 - 4*a*c)*b^2*c^2 + 20*(b^2 - 4*a*c)*a*c^3)*A - (a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)^2*(2*a*b^3*c^2 - 8*a^2*b*c^3 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a*b^3 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a^2*b*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a*b^2*c - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a*b*c^2 - 2*(b^2 - 4*a*c)*a*b*c^2)*C - 2*(3*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a^4*b^9*c - 49*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a^5*b^7*c^2 - 6*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a^4*b^8*c^2 + 6*a^4*b^9*c^2 + 300*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a^6*b^5*c^3 + 74*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a^5*b^6*c^3 + 3*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a^4*b^7*c^3 - 98*a^5*b^7*c^3 - 816*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a^7*b^3*c^4 - 304*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a^6*b^4*c^4 - 37*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a^5*b^5*c^4 + 600*a^6*b^5*c^4 + 832*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a^8*b*c^5 + 416*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a^7*b^2*c^5 + 152*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a^6*b^3*c^5 - 1632*a^7*b^3*c^5 - 208*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a^7*b*c^6 + 1664*a^8*b*c^6 - 6*(b^2 - 4*a*c)*a^4*b^7*c^2 + 74*(b^2 - 4*a*c)*a^5*b^5*c^3 - 304*(b^2 - 4*a*c)*a^6*b^3*c^4 + 416*(b^2 - 4*a*c)*a^7*b*c^5)*A*\text{abs}(a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3) + 2*(\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a^5*b^8*c - 18*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a^6*b^6*c^2 - 2*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a^5*b^7*c^2 + 2*a^5*b^8*c^2 + 120*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a^7*b^4*c^3 + 28*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a^6*b^5*c^3 + \sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a^5*b^6*c^3 - 36*a^6*b^6*c^3 - 352*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a^8*b^2*c^4 - 128*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a^7*b^3*c^4 - 14*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a^6*b^4*c^4 + 240*a^7*b^4*c^4 + 384*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a^9*c^5 + 192*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a^8*b*c^5 + 64*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a^7*b^2*c^5 - 704*a^8*b^2*c^5 - 96*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a^8*c^6 + 768*a^9*c^6 - 2*(b^2 - 4*a*c)*a^5*b^6*c^2 + 28*(b^2 - 4*a*c)*a^6*b^4*c^3 - 128*(b^2 - 4*a*c)*a^7*b^2*c^4 + 192*(b^2 - 4*a*c)*a^8*c^5)*C*\text{abs}(a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3) + (6*a^8*b^12*c^4 - 128*a^9*b^10*c^5 + 1088*a^{10}*b^8*c^6 - 4608*a^{11}*b^6*c^7 + 9728*a^{12}*b^4*c^8 - 8192*a^{13}*b^2*c^9 - 3*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a^8*b^12*c^2 + 64*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a^9*b^10*c^3 + 6*s
\end{aligned}$$

$$\begin{aligned}
& \text{qrt}(2) * \text{sqrt}(b^2 - 4*a*c) * \text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c) * a^8 * b^{11} * c^3 - 544 \\
& * \text{sqrt}(2) * \text{sqrt}(b^2 - 4*a*c) * \text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c) * a^{10} * b^8 * c^4 - 1 \\
& 04 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4*a*c) * \text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c) * a^9 * b^9 * c^4 - \\
& 3 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4*a*c) * \text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c) * a^8 * b^{10} * c^4 + \\
& 2304 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4*a*c) * \text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c) * a^{11} * b^6 * c^5 \\
& + 672 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4*a*c) * \text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c) * a^{10} * b^7 * c \\
& ^5 + 52 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4*a*c) * \text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c) * a^9 * b^8 * c \\
& ^5 - 4864 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4*a*c) * \text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c) * a^{12} * b^4 \\
& * c^6 - 1920 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4*a*c) * \text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c) * a^{11} \\
& * b^5 * c^6 - 336 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4*a*c) * \text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c) * a^{10} \\
& * b^6 * c^6 + 4096 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4*a*c) * \text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c) \\
& * a^{13} * b^2 * c^7 + 2048 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4*a*c) * \text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c) \\
& * c) * a^{12} * b^3 * c^7 + 960 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4*a*c) * \text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a* \\
& c) * c) * a^{11} * b^4 * c^7 - 1024 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4*a*c) * \text{sqrt}(b*c - \text{sqrt}(b^2 - 4 \\
& *a*c) * c) * a^{12} * b^2 * c^8 - 6 * (b^2 - 4*a*c) * a^8 * b^{10} * c^4 + 104 * (b^2 - 4*a*c) * a^ \\
& 9 * b^8 * c^5 - 672 * (b^2 - 4*a*c) * a^{10} * b^6 * c^6 + 1920 * (b^2 - 4*a*c) * a^{11} * b^4 * c^ \\
& 7 - 2048 * (b^2 - 4*a*c) * a^{12} * b^2 * c^8) * A - (2*a^9*b^{11}*c^4 - 56*a^{10}*b^9*c^5 \\
& + 576*a^{11}*b^7*c^6 - 2816*a^{12}*b^5*c^7 + 6656*a^{13}*b^3*c^8 - 6144*a^{14}*b*c^ \\
& 9 - \text{sqrt}(2) * \text{sqrt}(b^2 - 4*a*c) * \text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c) * a^9 * b^{11} * c^2 \\
& + 28 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4*a*c) * \text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c) * a^{10} * b^9 * c^3 \\
& + 2 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4*a*c) * \text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c) * a^9 * b^{10} * c^3 \\
& - 288 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4*a*c) * \text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c) * a^{11} * b^7 * c \\
& ^4 - 48 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4*a*c) * \text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c) * a^{10} * b^8 * \\
& c^4 - \text{sqrt}(2) * \text{sqrt}(b^2 - 4*a*c) * \text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c) * a^9 * b^9 * c^4 \\
& + 1408 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4*a*c) * \text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c) * a^{12} * b^5 * \\
& c^5 + 384 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4*a*c) * \text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c) * a^{11} * b^6 \\
& * c^5 + 24 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4*a*c) * \text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c) * a^{10} * b^7 \\
& * c^5 - 3328 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4*a*c) * \text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c) * a^{13} \\
& * b^3 * c^6 - 1280 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4*a*c) * \text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c) * \\
& a^{12} * b^4 * c^6 - 192 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4*a*c) * \text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c) \\
&) * a^{11} * b^5 * c^6 + 3072 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4*a*c) * \text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c) \\
& * c) * a^{14} * b * c^7 + 1536 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4*a*c) * \text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a* \\
& c) * c) * a^{13} * b^2 * c^7 + 640 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4*a*c) * \text{sqrt}(b*c - \text{sqrt}(b^2 - 4* \\
& a*c) * c) * a^{12} * b^3 * c^7 - 768 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4*a*c) * \text{sqrt}(b*c - \text{sqrt}(b^2 - \\
& 4*a*c) * c) * a^{13} * b * c^8 - 2 * (b^2 - 4*a*c) * a^9 * b^9 * c^4 + 48 * (b^2 - 4*a*c) * a^{10} \\
& * b^7 * c^5 - 384 * (b^2 - 4*a*c) * a^{11} * b^5 * c^6 + 1280 * (b^2 - 4*a*c) * a^{12} * b^3 * c^7 \\
& - 1536 * (b^2 - 4*a*c) * a^{13} * b * c^8) * C) * \arctan(2 * \text{sqrt}(1/2) * x / \text{sqrt}((a^4 * b^5 * c - \\
& 8 * a^5 * b^3 * c^2 + 16 * a^6 * b * c^3 - \text{sqrt}((a^4 * b^5 * c - 8 * a^5 * b^3 * c^2 + 16 * a^6 * b * c \\
& ^3)^2 - 4 * (a^5 * b^4 * c - 8 * a^6 * b^2 * c^2 + 16 * a^7 * c^3) * (a^4 * b^4 * c^2 - 8 * a^5 * b^2 \\
& * c^3 + 16 * a^6 * c^4))) / (a^4 * b^4 * c^2 - 8 * a^5 * b^2 * c^3 + 16 * a^6 * c^4))) / ((a^7 * b^8 \\
& * c - 16 * a^8 * b^6 * c^2 - 2 * a^7 * b^7 * c^2 + 96 * a^9 * b^4 * c^3 + 24 * a^8 * b^5 * c^3 + a^7 \\
& * b^6 * c^3 - 256 * a^{10} * b^2 * c^4 - 96 * a^9 * b^3 * c^4 - 12 * a^8 * b^4 * c^4 + 256 * a^{11} * c^ \\
& 5 + 128 * a^{10} * b * c^5 + 48 * a^9 * b^2 * c^5 - 64 * a^{10} * c^6) * \text{abs}(a^4 * b^4 * c - 8 * a^5 * b^ \\
& 2 * c^2 + 16 * a^6 * c^3) * \text{abs}(c) - 1/16 * ((b^6 * c - 10 * a * b^4 * c^2 - 2 * b^5 * c^2 + 24 * \\
& a^2 * b^2 * c^3 + 12 * a * b^3 * c^3 + b^4 * c^3 - 6 * a * b^2 * c^4 + (b^5 * c - 10 * a * b^3 * c^2 \\
& - 2 * b^4 * c^2 + 24 * a^2 * b * c^3 + 12 * a * b^2 * c^3 + b^3 * c^3 - 6 * a * b * c^4) * \text{sqrt}(b^2 - \\
& 4 * a * c)) * B * \text{abs}(a^4 * b^4 * c - 8 * a^5 * b^2 * c^2 + 16 * a^6 * c^3) - (a^4 * b^{10} * c^2 - 18 \\
& * a^5 * b^8 * c^3 - 2 * a^4 * b^9 * c^3 + 120 * a^6 * b^6 * c^4 + 28 * a^5 * b^7 * c^4 + a^4 * b^8 * c \\
& ^4 - 352 * a^7 * b^4 * c^5 - 128 * a^6 * b^5 * c^5 - 14 * a^5 * b^6 * c^5 + 384 * a^8 * b^2 * c^6 + \\
& 192 * a^7 * b^3 * c^6 + 64 * a^6 * b^4 * c^6 - 96 * a^7 * b^2 * c^7 + (a^4 * b^9 * c^2 - 14 * a^5 * \\
& b^7 * c^3 - 2 * a^4 * b^8 * c^3 + 64 * a^6 * b^5 * c^4 + 20 * a^5 * b^6 * c^4 + a^4 * b^7 * c^4 - 9 \\
& 6 * a^7 * b^3 * c^5 - 48 * a^6 * b^4 * c^5 - 10 * a^5 * b^5 * c^5 + 24 * a^6 * b^3 * c^6) * \text{sqrt}(b^2 \\
& - 4 * a * c)) * B) * \log(x^2 + 1/2 * (a^4 * b^5 * c - 8 * a^5 * b^3 * c^2 + 16 * a^6 * b * c^3 + \text{sqrt} \\
& ((a^4 * b^5 * c - 8 * a^5 * b^3 * c^2 + 16 * a^6 * b * c^3)^2 - 4 * (a^5 * b^4 * c - 8 * a^6 * b^2 * c^ \\
& 2 + 16 * a^7 * c^3) * (a^4 * b^4 * c^2 - 8 * a^5 * b^2 * c^3 + 16 * a^6 * c^4))) / (a^4 * b^4 * c^2 - \\
& 8 * a^5 * b^2 * c^3 + 16 * a^6 * c^4)) / ((a^3 * b^4 - 8 * a^4 * b^2 * c - 2 * a^3 * b^3 * c + 16 * a^ \\
& 5 * c^2 + 8 * a^4 * b * c^2 + a^3 * b^2 * c^2 - 4 * a^4 * c^3) * c^2 * \text{abs}(a^4 * b^4 * c - 8 * a^5 * b^ \\
& 2 * c^2 + 16 * a^6 * c^3) - 1/16 * ((b^6 * c - 10 * a * b^4 * c^2 - 2 * b^5 * c^2 + 24 * a^2 * b^2 \\
& * c^3 + 12 * a * b^3 * c^3 + b^4 * c^3 - 6 * a * b^2 * c^4 - (b^5 * c - 10 * a * b^3 * c^2 - 2 * b^4
\end{aligned}$$

$$\begin{aligned}
& *c^2 + 24*a^2*b*c^3 + 12*a*b^2*c^3 + b^3*c^3 - 6*a*b*c^4)*\text{sqrt}(b^2 - 4*a*c) \\
&) * B * \text{abs}(a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3) - (a^4*b^10*c^2 - 18*a^5*b^8*c^3 - 2*a^4*b^9*c^3 + 120*a^6*b^6*c^4 + 28*a^5*b^7*c^4 + a^4*b^8*c^4 - 35 \\
& 2*a^7*b^4*c^5 - 128*a^6*b^5*c^5 - 14*a^5*b^6*c^5 + 384*a^8*b^2*c^6 + 192*a^7*b^3*c^6 + 64*a^6*b^4*c^6 - 96*a^7*b^2*c^7 - (a^4*b^9*c^2 - 14*a^5*b^7*c^3 \\
& - 2*a^4*b^8*c^3 + 64*a^6*b^5*c^4 + 20*a^5*b^6*c^4 + a^4*b^7*c^4 - 96*a^7*b^3*c^5 - 48*a^6*b^4*c^5 - 10*a^5*b^5*c^5 + 24*a^6*b^3*c^6)*\text{sqrt}(b^2 - 4*a*c) \\
&)) * B * \log(x^2 + 1/2*(a^4*b^5*c - 8*a^5*b^3*c^2 + 16*a^6*b*c^3 - \text{sqrt}((a^4*b^5*c - 8*a^5*b^3*c^2 + 16*a^6*b*c^3)^2 - 4*(a^5*b^4*c - 8*a^6*b^2*c^2 + 16*a^7*c^3) * (a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^4))) / (a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^4)) / ((a^3*b^4 - 8*a^4*b^2*c - 2*a^3*b^3*c + 16*a^5*c^2 + 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a^4*c^3) * c^2 * \text{abs}(a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3))
\end{aligned}$$

maple [B] time = 0.08, size = 2398, normalized size = 4.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((C*x^2+B*x+A)/x^2/(c*x^4+b*x^2+a)^2, x)$

[Out]
$$\begin{aligned}
& -1/2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*B/a*b^2-1/a*c/(4*a*c-b^2)/(16*a*c-4*b^2)*2 \\
& ^{(1/2)/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\text{arctanh}(2^{(1/2)/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)})*c*x)*C*(-4*a*c+b^2)^{(1/2)}*b^2-1/a*c/(4*a*c-b^2)/(16*a*c-4*b \\
& ^2)*2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\text{arctan}(2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)})*c*x)*C*(-4*a*c+b^2)^{(1/2)}*b^2-16/a*c^2/(4*a*c-b^2)/(16*a* \\
& c-4*b^2)*2^{(1/2)/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\text{arctanh}(2^{(1/2)/((-b+(-4 \\
& *a*c+b^2)^{(1/2)})*c)^{(1/2)})*c*x)*A*(-4*a*c+b^2)^{(1/2)}*b+3/a^2*c/(4*a*c-b^2)/(\\
& 16*a*c-4*b^2)*2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\text{arctan}(2^{(1/2)/((b+ \\
& -4*a*c+b^2)^{(1/2)})*c)^{(1/2)})*c*x)*A*(-4*a*c+b^2)^{(1/2)}*b^3+3/a^2*c/(4*a*c-b^ \\
& 2)/(16*a*c-4*b^2)*2^{(1/2)/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\text{arctanh}(2^{(1/2) \\
& /((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)})*c*x)*A*(-4*a*c+b^2)^{(1/2)}*b^3-16/a*c^2/(\\
& 4*a*c-b^2)/(16*a*c-4*b^2)*2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\text{arctan}(2 \\
& ^{(1/2)/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)})*c*x)*A*(-4*a*c+b^2)^{(1/2)}*b+B/a^2*1 \\
& n(x)-1/(c*x^4+b*x^2+a)/(4*a*c-b^2)*A/a*c^2*x^3+1/2/(c*x^4+b*x^2+a)/(4*a*c-b \\
& ^2)*A/a^2*b^3*x+1/2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*A/a^2*b^2*c*x^3-3/2/(c*x^4+ \\
& b*x^2+a)/(4*a*c-b^2)*A/a*b*c*x+1/(c*x^4+b*x^2+a)/(4*a*c-b^2)*B*c-1/2/(c*x^4 \\
& +b*x^2+a)/(4*a*c-b^2)*B/a*b*c*x^2-1/a*c/(4*a*c-b^2)/(16*a*c-4*b^2)*2^{(1/2)/ \\
& ((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\text{arctanh}(2^{(1/2)/((-b+(-4*a*c+b^2)^{(1/2)}) \\
& *c)^{(1/2)})*c*x)*C*b^3+22/a*c^2/(4*a*c-b^2)/(16*a*c-4*b^2)*2^{(1/2)/((b+(-4*a*c \\
& +b^2)^{(1/2)})*c)^{(1/2)}*\text{arctan}(2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)})*c*x)* \\
& A*b^2-3/a^2*c/(4*a*c-b^2)/(16*a*c-4*b^2)*2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2)}) \\
& *c)^{(1/2)}*\text{arctan}(2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)})*c*x)*A*b^4+3/a^2*c/(\\
& 4*a*c-b^2)/(16*a*c-4*b^2)*2^{(1/2)/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\text{arctanh} \\
& (2^{(1/2)/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)})*c*x)*A*b^4-22/a*c^2/(4*a*c-b^2)/ \\
& (16*a*c-4*b^2)*2^{(1/2)/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\text{arctanh}(2^{(1/2)/((\\
& -b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)})*c*x)*A*b^2+1/a*c/(4*a*c-b^2)/(16*a*c-4*b^2) \\
& *2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\text{arctan}(2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2) \\
& /2)})*c)^{(1/2)})*c*x)*C*b^3+6/a*c/(4*a*c-b^2)/(16*a*c-4*b^2)*\ln(-2*c*x^2-b+(- \\
& 4*a*c+b^2)^{(1/2)})*B*(-4*a*c+b^2)^{(1/2)}*b-6/a*c/(4*a*c-b^2)/(16*a*c-4*b^2)*1 \\
& n(2*c*x^2+b+(-4*a*c+b^2)^{(1/2)})*B*(-4*a*c+b^2)^{(1/2)}*b+12*c^2/(4*a*c-b^2)/(\\
& 16*a*c-4*b^2)*2^{(1/2)/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\text{arctanh}(2^{(1/2)/((- \\
& b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)})*c*x)*C*(-4*a*c+b^2)^{(1/2)}+12*c^2/(4*a*c-b^2) \\
& / (16*a*c-4*b^2)*2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\text{arctan}(2^{(1/2)/((b \\
& +(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)})*c*x)*C*(-4*a*c+b^2)^{(1/2)}-4*c^2/(4*a*c-b^2)/(\\
& 16*a*c-4*b^2)*2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\text{arctan}(2^{(1/2)/((b+ \\
& -4*a*c+b^2)^{(1/2)})*c)^{(1/2)})*c*x)*C*b+4*c^2/(4*a*c-b^2)/(16*a*c-4*b^2)*2^{(1/ \\
& 2)/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\text{arctanh}(2^{(1/2)/((-b+(-4*a*c+b^2)^{(1/2) \\
&))*c)^{(1/2)})*c*x)*C*b+1/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*c*C-1/2/a/(c*x^4+b*x^2 \\
& +a)*c/(4*a*c-b^2)*x^3*C*b-1/a^2/(4*a*c-b^2)/(16*a*c-4*b^2)*\ln(-2*c*x^2-b+(-
\end{aligned}$$

$$4*a*c+b^2)^{(1/2)}*B*(-4*a*c+b^2)^{(1/2)}*b^3+1/a^2/(4*a*c-b^2)/(16*a*c-4*b^2) * \ln(2*c*x^2+b+(-4*a*c+b^2)^{(1/2)}) * B*(-4*a*c+b^2)^{(1/2)}*b^3+8/a*c/(4*a*c-b^2) / (16*a*c-4*b^2) * \ln(2*c*x^2+b+(-4*a*c+b^2)^{(1/2)}) * b^2*B+8/a*c/(4*a*c-b^2) / (16*a*c-4*b^2) * \ln(-2*c*x^2-b+(-4*a*c+b^2)^{(1/2)}) * b^2*B+40*c^3/(4*a*c-b^2) / (16*a*c-4*b^2) * 2^{(1/2)} / ((-b+(-4*a*c+b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctanh}(2^{(1/2)} / ((-b+(-4*a*c+b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * A - 40*c^3 / (4*a*c-b^2) / (16*a*c-4*b^2) * 2^{(1/2)} / ((b+(-4*a*c+b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctan}(2^{(1/2)} / ((b+(-4*a*c+b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * A - 16*c^2 / (4*a*c-b^2) / (16*a*c-4*b^2) * \ln(2*c*x^2+b+(-4*a*c+b^2)^{(1/2)}) * B - 16*c^2 / (4*a*c-b^2) / (16*a*c-4*b^2) * \ln(-2*c*x^2-b+(-4*a*c+b^2)^{(1/2)}) * B - 1/2/a / (c*x^4+b*x^2+a) / (4*a*c-b^2) * x * C * b^2 - 1/a^2 / (4*a*c-b^2) / (16*a*c-4*b^2) * \ln(2*c*x^2+b+(-4*a*c+b^2)^{(1/2)}) * b^4 * B - 1/a^2 / (4*a*c-b^2) / (16*a*c-4*b^2) * \ln(-2*c*x^2-b+(-4*a*c+b^2)^{(1/2)}) * b^4 * B - A/a^2/x$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")

[Out] $\frac{1}{2} * (B * a * b * c * x^3 + (10 * A * a * c^2 + (C * a * b - 3 * A * b^2) * c) * x^4 - 2 * A * a * b^2 + 8 * A * a^2 * c + (C * a * b^2 - 3 * A * b^3 - (2 * C * a^2 - 11 * A * a * b) * c) * x^2 + (B * a * b^2 - 2 * B * a^2 * c) * x) / ((a^2 * b^2 * c - 4 * a^3 * c^2) * x^5 + (a^2 * b^3 - 4 * a^3 * b * c) * x^3 + (a^3 * b^2 - 4 * a^4 * c) * x) + \frac{1}{2} * \operatorname{integrate}((C * a * b^2 - 3 * A * b^3 - 2 * (B * b^2 * c - 4 * B * a * c^2) * x^3 + (10 * A * a * c^2 + (C * a * b - 3 * A * b^2) * c) * x^2 - (6 * C * a^2 - 13 * A * a * b) * c - 2 * (B * b^3 - 5 * B * a * b * c) * x) / (c * x^4 + b * x^2 + a), x) / (a^2 * b^2 - 4 * a^3 * c) + B * \log(x) / a^2$

mupad [B] time = 2.47, size = 8684, normalized size = 16.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x + C*x^2)/(x^2*(a + b*x^2 + c*x^4)^2),x)

[Out] $\operatorname{symsum}(\log(\operatorname{root}(1572864 * a^{10} * b^2 * c^5 * z^4 - 983040 * a^9 * b^4 * c^4 * z^4 + 327680 * a^8 * b^6 * c^3 * z^4 - 61440 * a^7 * b^8 * c^2 * z^4 + 6144 * a^6 * b^{10} * c * z^4 - 1048576 * a^1 * c^6 * z^4 - 256 * a^5 * b^{12} * z^4 + 1572864 * B * a^8 * b^2 * c^5 * z^3 - 983040 * B * a^7 * b^4 * c^4 * z^3 + 327680 * B * a^6 * b^6 * c^3 * z^3 - 61440 * B * a^5 * b^8 * c^2 * z^3 + 6144 * B * a^4 * b^{10} * c * z^3 - 1048576 * B * a^9 * c^6 * z^3 - 256 * B * a^3 * b^{12} * z^3 - 2432 * A * C * a^2 * b^{10} * c * z^2 - 491520 * A * C * a^6 * b^2 * c^5 * z^2 + 358400 * A * C * a^5 * b^4 * c^4 * z^2 - 129024 * A * C * a^4 * b^6 * c^3 * z^2 + 24768 * A * C * a^3 * b^8 * c^2 * z^2 + 96 * A * C * a * b^{12} * z^2 + 61440 * C^2 * a^7 * b * c^5 * z^2 + 432 * C^2 * a^3 * b^9 * c * z^2 + 1536 * B^2 * a^2 * b^{10} * c * z^2 - 43008 * 0 * A^2 * a^6 * b * c^6 * z^2 + 3408 * A^2 * a * b^{11} * c * z^2 + 245760 * A * C * a^7 * c^6 * z^2 - 6144 * 0 * C^2 * a^6 * b^3 * c^4 * z^2 + 24064 * C^2 * a^5 * b^5 * c^3 * z^2 - 4608 * C^2 * a^4 * b^7 * c^2 * z^2 + 516096 * B^2 * a^6 * b^2 * c^5 * z^2 - 288768 * B^2 * a^5 * b^4 * c^4 * z^2 + 88576 * B^2 * a^4 * b^6 * c^3 * z^2 - 15744 * B^2 * a^3 * b^8 * c^2 * z^2 + 716800 * A^2 * a^5 * b^3 * c^5 * z^2 - 483 * 840 * A^2 * a^4 * b^5 * c^4 * z^2 + 170496 * A^2 * a^3 * b^7 * c^3 * z^2 - 33232 * A^2 * a^2 * b^9 * c^2 * z^2 - 64 * B^2 * a * b^{12} * z^2 - 393216 * B^2 * a^7 * c^6 * z^2 - 16 * C^2 * a^2 * b^{11} * z^2 - 144 * A^2 * b^{13} * z^2 - 110592 * A * B * C * a^4 * b^2 * c^5 * z + 36864 * A * B * C * a^3 * b^4 * c^4 * z - 5376 * A * B * C * a^2 * b^6 * c^3 * z + 288 * A * B * C * a * b^8 * c^2 * z + 3072 * B * C^2 * a^5 * b * c^5 * z - 138240 * A^2 * B * a^4 * b * c^6 * z + 7344 * A^2 * B * a * b^7 * c^3 * z + 122880 * A * B * C * a^5 * c^6 * z - 2304 * B * C^2 * a^4 * b^3 * c^4 * z + 576 * B * C^2 * a^3 * b^5 * c^3 * z - 48 * B * C^2 * a^2 * b^7 * c^2 * z + 131328 * A^2 * B * a^3 * b^3 * c^5 * z - 46656 * A^2 * B * a^2 * b^5 * c^4 * z + 61440 * B^3 * a^4 * b^2 * c^5 * z - 21504 * B^3 * a^3 * b^4 * c^4 * z + 3328 * B^3 * a^2 * b^6 * c^3 * z - 192 * B^3 * a * b^8 * c^2 * z - 432 * A^2 * B * b^9 * c^2 * z - 65536 * B^3 * a^5 * c^6 * z - 5568 * A * B^2 * C * a^2 * b^2 * c^5 + 496 * A * B^2 * C * a * b^4 * c^4 + 1104 * B^2 * C^2 * a^2 * b^3 * c^4 - 3264 * A^2 * C^2 * a^2 * b^2 * c^5 - 3072 * B^2 * C^2 * a^3 * b * c^5 - 100 * B^2 * C^2 * a * b^5 * c^3 + 2070 * A^2 * C^2 * a * b^4 * c^4 - 1840 * A * C^3 * a^2 * b^3 * c^4 - 7680 * A^2 * B^2 * a^2 * b * c^6 + 3152 * A^2 * B^2 * a * b^3 * c^5 + 15200 * A^3 * C * a^2 * b * c^6 - 6192 * A^3 * C * a * b^3 * c^5 + 5472 * A * C^3 * a^3 * b * c^5 + 150 * A * C^3 * a * b^5 * c^3 + 15360 * A * B^2 * C * a^3 * c^6 - 144 * B^4 * a * b^4 * c^4 + 420$

$$\begin{aligned}
& 0*A^4*a*b^2*c^6 + 630*A^3*C*b^5*c^4 + 360*C^4*a^3*b^2*c^4 - 25*C^4*a^2*b^4*c^3 + 1536*B^4*a^2*b^2*c^5 - 225*A^2*C^2*b^6*c^3 - 7200*A^2*C^2*a^3*c^6 - 324*A^2*B^2*b^5*c^4 - 1296*C^4*a^4*c^5 - 4096*B^4*a^3*c^6 - 441*A^4*b^4*c^5 \\
& - 10000*A^4*a^2*c^7, z, k)*(root(1572864*a^10*b^2*c^5*z^4 - 983040*a^9*b^4*c^4*z^4 + 327680*a^8*b^6*c^3*z^4 - 61440*a^7*b^8*c^2*z^4 + 6144*a^6*b^10*c^1*z^4 - 1048576*a^11*c^6*z^4 - 256*a^5*b^12*z^4 + 1572864*B*a^8*b^2*c^5*z^3 - \\
& 983040*B*a^7*b^4*c^4*z^3 + 327680*B*a^6*b^6*c^3*z^3 - 61440*B*a^5*b^8*c^2*z^3 + 6144*B*a^4*b^10*c*z^3 - 1048576*B*a^9*c^6*z^3 - 256*B*a^3*b^12*z^3 - \\
& 2432*A*C*a^2*b^10*c*z^2 - 491520*A*C*a^6*b^2*c^5*z^2 + 358400*A*C*a^5*b^4*c^4*z^2 - 129024*A*C*a^4*b^6*c^3*z^2 + 24768*A*C*a^3*b^8*c^2*z^2 + 96*A*C*a*b^12*z^2 + 61440*C^2*a^7*b*c^5*z^2 + 432*C^2*a^3*b^9*c*z^2 + 1536*B^2*a^2*b^10*c*z^2 - 430080*A^2*a^6*b*c^6*z^2 + 3408*A^2*a*b^11*c*z^2 + 245760*A*C*a^7*c^6*z^2 - 61440*C^2*a^6*b^3*c^4*z^2 + 24064*C^2*a^5*b^5*c^3*z^2 - 4608*C^2*a^4*b^7*c^2*z^2 + 516096*B^2*a^6*b^2*c^5*z^2 - 288768*B^2*a^5*b^4*c^4*z^2 + 88576*B^2*a^4*b^6*c^3*z^2 - 15744*B^2*a^3*b^8*c^2*z^2 + 716800*A^2*a^5*b^3*c^5*z^2 - 483840*A^2*a^4*b^5*c^4*z^2 + 170496*A^2*a^3*b^7*c^3*z^2 - 33232*A^2*a^2*b^9*c^2*z^2 - 64*B^2*a*b^12*z^2 - 393216*B^2*a^7*c^6*z^2 - 16*C^2*a^2*b^11*z^2 - 144*A^2*b^13*z^2 - 110592*A*B*C*a^4*b^2*c^5*z + 36864*A*B*C*a^3*b^4*c^4*z - 5376*A*B*C*a^2*b^6*c^3*z + 288*A*B*C*a*b^8*c^2*z + 3072*B*C^2*a^5*b*c^5*z - 138240*A^2*B*a^4*b*c^6*z + 7344*A^2*B*a*b^7*c^3*z + 122880*A*B*C*a^5*c^6*z - 2304*B*C^2*a^4*b^3*c^4*z + 576*B*C^2*a^3*b^5*c^3*z - 48*B*C^2*a^2*b^7*c^2*z + 131328*A^2*B*a^3*b^3*c^5*z - 46656*A^2*B*a^2*b^5*c^4*z + 61440*B^3*a^4*b^2*c^5*z - 21504*B^3*a^3*b^4*c^4*z + 3328*B^3*a^2*b^6*c^3*z - 192*B^3*a*b^8*c^2*z - 432*A^2*B*b^9*c^2*z - 65536*B^3*a^5*c^6*z - 5568*A*B^2*C*a^2*b^2*c^5 + 496*A*B^2*C*a*b^4*c^4 + 1104*B^2*C^2*a^2*b^3*c^4 - 3264*A^2*C^2*a^2*b^2*c^5 - 3072*B^2*C^2*a^3*b*c^5 - 100*B^2*C^2*a*b^5*c^3 + 2070*A^2*C^2*a*b^4*c^4 - 1840*A*C^3*a^2*b^3*c^4 - 7680*A^2*B^2*a^2*b*c^6 + 3152*A^2*B^2*a*b^3*c^5 + 15200*A^3*C*a^2*b*c^6 - 6192*A^3*C*a*b^3*c^5 + 5472*A*C^3*a^3*b*c^5 + 150*A*C^3*a*b^5*c^3 + 15360*A*B^2*C*a^3*c^6 - 144*B^4*a*b^4*c^4 + 4200*A^4*a*b^2*c^6 + 630*A^3*C*b^5*c^4 + 360*C^4*a^3*b^2*c^4 - 25*C^4*a^2*b^4*c^3 + 1536*B^4*a^2*b^2*c^5 - 225*A^2*C^2*b^6*c^3 - 7200*A^2*C^2*a^3*c^6 - 324*A^2*B^2*b^5*c^4 - 1296*C^4*a^4*c^5 - 4096*B^4*a^3*c^6 - 441*A^4*b^4*c^5 - 10000*A^4*a^2*c^7, z, k)*(root(1572864*a^10*b^2*c^5*z^4 - 983040*a^9*b^4*c^4*z^4 + 327680*a^8*b^6*c^3*z^4 - 61440*a^7*b^8*c^2*z^4 + 6144*a^6*b^10*c^1*z^4 - 1048576*a^11*c^6*z^4 - 256*a^5*b^12*z^4 + 1572864*B*a^8*b^2*c^5*z^3 - 983040*B*a^7*b^4*c^4*z^3 + 327680*B*a^6*b^6*c^3*z^3 - 61440*B*a^5*b^8*c^2*z^3 + 6144*B*a^4*b^10*c*z^3 - 1048576*B*a^9*c^6*z^3 - 256*B*a^3*b^12*z^3 - 2432*A*C*a^2*b^10*c*z^2 - 491520*A*C*a^6*b^2*c^5*z^2 + 358400*A*C*a^5*b^4*c^4*z^2 - 129024*A*C*a^4*b^6*c^3*z^2 + 24768*A*C*a^3*b^8*c^2*z^2 + 96*A*C*a*b^12*z^2 + 61440*C^2*a^7*b*c^5*z^2 + 432*C^2*a^3*b^9*c*z^2 + 1536*B^2*a^2*b^10*c*z^2 - 430080*A^2*a^6*b*c^6*z^2 + 3408*A^2*a*b^11*c*z^2 + 245760*A*C*a^7*c^6*z^2 - 61440*C^2*a^6*b^3*c^4*z^2 + 24064*C^2*a^5*b^5*c^3*z^2 - 4608*C^2*a^4*b^7*c^2*z^2 + 516096*B^2*a^6*b^2*c^5*z^2 - 288768*B^2*a^5*b^4*c^4*z^2 + 88576*B^2*a^4*b^6*c^3*z^2 - 15744*B^2*a^3*b^8*c^2*z^2 + 716800*A^2*a^5*b^3*c^5*z^2 - 483840*A^2*a^4*b^5*c^4*z^2 + 170496*A^2*a^3*b^7*c^3*z^2 - 33232*A^2*a^2*b^9*c^2*z^2 - 64*B^2*a*b^12*z^2 - 393216*B^2*a^7*c^6*z^2 - 16*C^2*a^2*b^11*z^2 - 144*A^2*b^13*z^2 - 110592*A*B*C*a^4*b^2*c^5*z + 36864*A*B*C*a^3*b^4*c^4*z - 5376*A*B*C*a^2*b^6*c^3*z + 288*A*B*C*a*b^8*c^2*z + 3072*B*C^2*a^5*b*c^5*z - 138240*A^2*B*a^4*b*c^6*z + 7344*A^2*B*a*b^7*c^3*z + 122880*A*B*C*a^5*c^6*z - 2304*B*C^2*a^4*b^3*c^4*z + 576*B*C^2*a^3*b^5*c^3*z - 48*B*C^2*a^2*b^7*c^2*z + 131328*A^2*B*a^3*b^3*c^5*z - 46656*A^2*B*a^2*b^5*c^4*z + 61440*B^3*a^4*b^2*c^5*z - 21504*B^3*a^3*b^4*c^4*z + 3328*B^3*a^2*b^6*c^3*z - 192*B^3*a*b^8*c^2*z - 432*A^2*B*b^9*c^2*z - 65536*B^3*a^5*c^6*z - 5568*A*B^2*C*a^2*b^2*c^5 + 496*A*B^2*C*a*b^4*c^4 + 1104*B^2*C^2*a^2*b^3*c^4 - 3264*A^2*C^2*a^2*b^2*c^5 - 3072*B^2*C^2*a^3*b*c^5 - 100*B^2*C^2*a*b^5*c^3 + 2070*A^2*C^2*a*b^4*c^4 - 1840*A*C^3*a^2*b^3*c^4 - 7680*A^2*B^2*a^2*b*c^6 + 3152*A^2*B^2*a*b^3*c^5 + 15200*A^3*C*a^2*b*c^6 - 6192*A^3*C*a*b^3*c^5 + 5472*A*C^3*a^3*b*c^5 + 150*A*C^3*a*b^5*c^3 + 15360*A*B^2*C*a^3*c^6 - 144*B^4*a*b^4*c^4 + 4200*A^4*a*b^2*c^6 + 630*A^3*C*b^5*c^4 + 360
\end{aligned}$$

$$\begin{aligned}
& *C^4*a^3*b^2*c^4 - 25*C^4*a^2*b^4*c^3 + 1536*B^4*a^2*b^2*c^5 - 225*A^2*C^2* \\
& b^6*c^3 - 7200*A^2*C^2*a^3*c^6 - 324*A^2*B^2*b^5*c^4 - 1296*C^4*a^4*c^5 - 4 \\
& 096*B^4*a^3*c^6 - 441*A^4*b^4*c^5 - 10000*A^4*a^2*c^7, z, k) * ((x*(983040*B* \\
& a^9*c^8 + 192*B*a^3*b^12*c^2 - 4736*B*a^4*b^10*c^3 + 48896*B*a^5*b^8*c^4 - \\
& 270336*B*a^6*b^6*c^5 + 843776*B*a^7*b^4*c^6 - 1409024*B*a^8*b^2*c^7)) / (16*(\\
& a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)) - \\
& (10240*A*a^8*c^7 + 7168*C*a^8*b*c^6 - 48*A*a^3*b^10*c^2 + 832*A*a^4*b^8*c^ \\
& 3 - 5536*A*a^5*b^6*c^4 + 17280*A*a^6*b^4*c^5 - 24064*A*a^7*b^2*c^6 + 16*C*a \\
& ^4*b^9*c^2 - 304*C*a^5*b^7*c^3 + 2112*C*a^6*b^5*c^4 - 6400*C*a^7*b^3*c^5) / (\\
& 8*(a^4*b^6 - 64*a^7*c^3 - 12*a^5*b^4*c + 48*a^6*b^2*c^2)) + (root(1572864*a \\
& ^10*b^2*c^5*z^4 - 983040*a^9*b^4*c^4*z^4 + 327680*a^8*b^6*c^3*z^4 - 61440*a \\
& ^7*b^8*c^2*z^4 + 6144*a^6*b^10*c*z^4 - 1048576*a^11*c^6*z^4 - 256*a^5*b^12* \\
& z^4 + 1572864*B*a^8*b^2*c^5*z^3 - 983040*B*a^7*b^4*c^4*z^3 + 327680*B*a^6*b \\
& ^6*c^3*z^3 - 61440*B*a^5*b^8*c^2*z^3 + 6144*B*a^4*b^10*c*z^3 - 1048576*B*a^ \\
& 9*c^6*z^3 - 256*B*a^3*b^12*z^3 - 2432*A*C*a^2*b^10*c*z^2 - 491520*A*C*a^6*b \\
& ^2*c^5*z^2 + 358400*A*C*a^5*b^4*c^4*z^2 - 129024*A*C*a^4*b^6*c^3*z^2 + 2476 \\
& 8*A*C*a^3*b^8*c^2*z^2 + 96*A*C*a*b^12*z^2 + 61440*C^2*a^7*b*c^5*z^2 + 432*C \\
& ^2*a^3*b^9*c*z^2 + 1536*B^2*a^2*b^10*c*z^2 - 430080*A^2*a^6*b*c^6*z^2 + 340 \\
& 8*A^2*a*b^11*c*z^2 + 245760*A*C*a^7*c^6*z^2 - 61440*C^2*a^6*b^3*c^4*z^2 + 2 \\
& 4064*C^2*a^5*b^5*c^3*z^2 - 4608*C^2*a^4*b^7*c^2*z^2 + 516096*B^2*a^6*b^2*c^ \\
& 5*z^2 - 288768*B^2*a^5*b^4*c^4*z^2 + 88576*B^2*a^4*b^6*c^3*z^2 - 15744*B^2* \\
& a^3*b^8*c^2*z^2 + 716800*A^2*a^5*b^3*c^5*z^2 - 483840*A^2*a^4*b^5*c^4*z^2 + \\
& 170496*A^2*a^3*b^7*c^3*z^2 - 33232*A^2*a^2*b^9*c^2*z^2 - 64*B^2*a*b^12*z^2 \\
& - 393216*B^2*a^7*c^6*z^2 - 16*C^2*a^2*b^11*z^2 - 144*A^2*b^13*z^2 - 110592 \\
& *A*B*C*a^4*b^2*c^5*z + 36864*A*B*C*a^3*b^4*c^4*z - 5376*A*B*C*a^2*b^6*c^3*z \\
& + 288*A*B*C*a*b^8*c^2*z + 3072*B*C^2*a^5*b*c^5*z - 138240*A^2*B*a^4*b*c^6* \\
& z + 7344*A^2*B*a*b^7*c^3*z + 122880*A*B*C*a^5*c^6*z - 2304*B*C^2*a^4*b^3*c^ \\
& 4*z + 576*B*C^2*a^3*b^5*c^3*z - 48*B*C^2*a^2*b^7*c^2*z + 131328*A^2*B*a^3*b \\
& ^3*c^5*z - 46656*A^2*B*a^2*b^5*c^4*z + 61440*B^3*a^4*b^2*c^5*z - 21504*B^3* \\
& a^3*b^4*c^4*z + 3328*B^3*a^2*b^6*c^3*z - 192*B^3*a*b^8*c^2*z - 432*A^2*B*b^ \\
& 9*c^2*z - 65536*B^3*a^5*c^6*z - 5568*A*B^2*C*a^2*b^2*c^5 + 496*A*B^2*C*a*b^ \\
& 4*c^4 + 1104*B^2*C^2*a^2*b^3*c^4 - 3264*A^2*C^2*a^2*b^2*c^5 - 3072*B^2*C^2* \\
& a^3*b*c^5 - 100*B^2*C^2*a*b^5*c^3 + 2070*A^2*C^2*a*b^4*c^4 - 1840*A*C^3*a^2 \\
& *b^3*c^4 - 7680*A^2*B^2*a^2*b*c^6 + 3152*A^2*B^2*a*b^3*c^5 + 15200*A^3*C*a^ \\
& 2*b*c^6 - 6192*A^3*C*a*b^3*c^5 + 5472*A*C^3*a^3*b*c^5 + 150*A*C^3*a*b^5*c^3 \\
& + 15360*A*B^2*C*a^3*c^6 - 144*B^4*a*b^4*c^4 + 4200*A^4*a*b^2*c^6 + 630*A^3 \\
& *C*b^5*c^4 + 360*C^4*a^3*b^2*c^4 - 25*C^4*a^2*b^4*c^3 + 1536*B^4*a^2*b^2*c^ \\
& 5 - 225*A^2*C^2*b^6*c^3 - 7200*A^2*C^2*a^3*c^6 - 324*A^2*B^2*b^5*c^4 - 1296 \\
& *C^4*a^4*c^5 - 4096*B^4*a^3*c^6 - 441*A^4*b^4*c^5 - 10000*A^4*a^2*c^7, z, k \\
&) * x * ((1310720*a^11*c^8 + 384*a^5*b^12*c^2 - 8960*a^6*b^10*c^3 + 87040*a^7*b^ \\
& 8*c^4 - 450560*a^8*b^6*c^5 + 1310720*a^9*b^4*c^6 - 2031616*a^10*b^2*c^7)) / (\\
& 16*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3 \\
&)) + (5120*A*B*a^6*c^7 + 832*A*B*a^2*b^8*c^3 - 5392*A*B*a^3*b^6*c^4 + 1574 \\
& 4*A*B*a^4*b^4*c^5 - 18944*A*B*a^5*b^2*c^6 + 16*B*C*a^2*b^9*c^2 - 304*B*C*a^ \\
& 3*b^7*c^3 + 2064*B*C*a^4*b^5*c^4 - 5888*B*C*a^5*b^3*c^5 - 48*A*B*a*b^10*c^2 \\
& + 5888*B*C*a^6*b*c^6) / (8*(a^4*b^6 - 64*a^7*c^3 - 12*a^5*b^4*c + 48*a^6*b^2 \\
& *c^2)) + (x*(144*A^2*b^13*c^2 + 245760*B^2*a^7*c^8 + 33304*A^2*a^2*b^9*c^4 \\
& - 171768*A^2*a^3*b^7*c^5 + 492320*A^2*a^4*b^5*c^6 - 742016*A^2*a^5*b^3*c^7 \\
& - 128*B^2*a^2*b^10*c^3 + 2912*B^2*a^3*b^8*c^4 - 26560*B^2*a^4*b^6*c^5 + 120 \\
& 832*B^2*a^5*b^4*c^6 - 273408*B^2*a^6*b^2*c^7 + 16*C^2*a^2*b^11*c^2 - 432*C^ \\
& 2*a^3*b^9*c^3 + 4616*C^2*a^4*b^7*c^4 - 24032*C^2*a^5*b^5*c^5 + 60800*C^2*a^ \\
& 6*b^3*c^6 - 276480*A*C*a^7*c^8 - 3408*A^2*a*b^11*c^3 + 458240*A^2*a^6*b*c^8 \\
& - 59904*C^2*a^7*b*c^7 + 2432*A*C*a^2*b^10*c^3 - 24816*A*C*a^3*b^8*c^4 + 12 \\
& 9952*A*C*a^4*b^6*c^5 - 365440*A*C*a^5*b^4*c^6 + 515584*A*C*a^6*b^2*c^7 - 96 \\
& *A*C*a*b^12*c^2)) / (16*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^ \\
& 2 - 256*a^7*b^2*c^3)) + (216*C^3*a^5*c^6 + 63*A^3*a^2*b^3*c^6 - 30*C^3*a^4 \\
& *b^2*c^5 + 4480*A*B^2*a^4*c^7 + 600*A^2*C*a^4*c^7 - 300*A^3*a^3*b*c^7 - 144 \\
& *A*B^2*a*b^6*c^4 - 564*A*C^2*a^4*b*c^6 + 1408*B^2*C*a^4*b*c^6 + 1536*A*B^2* \\
& a^2*b^4*c^5 - 4984*A*B^2*a^3*b^2*c^6 + 105*A*C^2*a^3*b^3*c^5 - 45*A^2*C*a^2
\end{aligned}$$

$$\begin{aligned}
& *b^4*c^5 + 102*A^2*C*a^3*b^2*c^6 + 48*B^2*C*a^2*b^5*c^4 - 532*B^2*C*a^3*b^3 \\
& *c^5)/(8*(a^4*b^6 - 64*a^7*c^3 - 12*a^5*b^4*c + 48*a^6*b^2*c^2)) + (x*(2048 \\
& 0*B^3*a^5*c^8 + 192*B^3*a^2*b^6*c^5 + 1216*B^3*a^3*b^4*c^6 - 11008*B^3*a^4* \\
& b^2*c^7 + 360*A^2*B*b^9*c^4 - 32*B^3*a*b^8*c^4 - 6072*A^2*B*a*b^7*c^5 + 112 \\
& 320*A^2*B*a^4*b*c^8 - 2496*B*C^2*a^5*b*c^7 + 38284*A^2*B*a^2*b^5*c^6 - 1071 \\
& 04*A^2*B*a^3*b^3*c^7 + 40*B*C^2*a^2*b^7*c^4 - 508*B*C^2*a^3*b^5*c^5 + 2016* \\
& B*C^2*a^4*b^3*c^6 - 99840*A*B*C*a^5*c^8 - 240*A*B*C*a*b^8*c^4 + 4448*A*B*C* \\
& a^2*b^6*c^5 - 30176*A*B*C*a^3*b^4*c^6 + 89856*A*B*C*a^4*b^2*c^7))/(16*(a^4* \\
& b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3))) - (6 \\
& 3*A^3*B*b^3*c^6 - 640*A*B^3*a^2*c^7 + 216*B*C^3*a^3*c^6 + 600*A^2*B*C*a^2*c \\
& ^7 - 45*A^2*B*C*b^4*c^5 + 136*A*B^3*a*b^2*c^6 - 20*B^3*C*a*b^3*c^5 + 128*B^ \\
& 3*C*a^2*b*c^6 - 30*B*C^3*a^2*b^2*c^5 - 300*A^3*B*a*b*c^7 + 105*A*B*C^2*a*b^ \\
& 3*c^5 - 564*A*B*C^2*a^2*b*c^6 + 102*A^2*B*C*a*b^2*c^6)/(8*(a^4*b^6 - 64*a^7 \\
& *c^3 - 12*a^5*b^4*c + 48*a^6*b^2*c^2)) + (x*(10000*A^4*a^2*c^9 + 441*A^4*b^ \\
& 4*c^7 + 1296*C^4*a^4*c^7 + 216*A^2*B^2*b^5*c^6 + 7200*A^2*C^2*a^3*c^8 + 225 \\
& *A^2*C^2*b^6*c^5 + 256*B^4*a^2*b^2*c^7 + 25*C^4*a^2*b^4*c^5 - 360*C^4*a^3*b \\
& ^2*c^6 - 630*A^3*C*b^5*c^6 - 4200*A^4*a*b^2*c^8 - 48*B^4*a*b^4*c^6 - 7680*A \\
& *B^2*C*a^3*c^8 - 150*A*C^3*a*b^5*c^5 - 5472*A*C^3*a^3*b*c^7 + 6192*A^3*C*a* \\
& b^3*c^7 - 15200*A^3*C*a^2*b*c^8 - 2160*A^2*B^2*a*b^3*c^7 + 5440*A^2*B^2*a^2 \\
& *b*c^8 + 1840*A*C^3*a^2*b^3*c^6 - 2070*A^2*C^2*a*b^4*c^6 + 960*B^2*C^2*a^3* \\
& b*c^7 + 3264*A^2*C^2*a^2*b^2*c^7 - 176*B^2*C^2*a^2*b^3*c^6 - 144*A*B^2*C*a* \\
& b^4*c^6 + 2240*A*B^2*C*a^2*b^2*c^7))/(16*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^ \\
& 6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*root(1572864*a^10*b^2*c^5*z^4 - 9 \\
& 83040*a^9*b^4*c^4*z^4 + 327680*a^8*b^6*c^3*z^4 - 61440*a^7*b^8*c^2*z^4 + 61 \\
& 44*a^6*b^10*c*z^4 - 1048576*a^11*c^6*z^4 - 256*a^5*b^12*z^4 + 1572864*B*a^8 \\
& *b^2*c^5*z^3 - 983040*B*a^7*b^4*c^4*z^3 + 327680*B*a^6*b^6*c^3*z^3 - 61440* \\
& B*a^5*b^8*c^2*z^3 + 6144*B*a^4*b^10*c*z^3 - 1048576*B*a^9*c^6*z^3 - 256*B*a \\
& ^3*b^12*z^3 - 2432*A*C*a^2*b^10*c*z^2 - 491520*A*C*a^6*b^2*c^5*z^2 + 358400 \\
& *A*C*a^5*b^4*c^4*z^2 - 129024*A*C*a^4*b^6*c^3*z^2 + 24768*A*C*a^3*b^8*c^2*z \\
& ^2 + 96*A*C*a*b^12*z^2 + 61440*C^2*a^7*b*c^5*z^2 + 432*C^2*a^3*b^9*c*z^2 + \\
& 1536*B^2*a^2*b^10*c*z^2 - 430080*A^2*a^6*b*c^6*z^2 + 3408*A^2*a*b^11*c*z^2 \\
& + 245760*A*C*a^7*c^6*z^2 - 61440*C^2*a^6*b^3*c^4*z^2 + 24064*C^2*a^5*b^5*c^ \\
& 3*z^2 - 4608*C^2*a^4*b^7*c^2*z^2 + 516096*B^2*a^6*b^2*c^5*z^2 - 288768*B^2* \\
& a^5*b^4*c^4*z^2 + 88576*B^2*a^4*b^6*c^3*z^2 - 15744*B^2*a^3*b^8*c^2*z^2 + 7 \\
& 16800*A^2*a^5*b^3*c^5*z^2 - 483840*A^2*a^4*b^5*c^4*z^2 + 170496*A^2*a^3*b^7 \\
& *c^3*z^2 - 33232*A^2*a^2*b^9*c^2*z^2 - 64*B^2*a*b^12*z^2 - 393216*B^2*a^7*c \\
& ^6*z^2 - 16*C^2*a^2*b^11*z^2 - 144*A^2*b^13*z^2 - 110592*A*B*C*a^4*b^2*c^5* \\
& z + 36864*A*B*C*a^3*b^4*c^4*z - 5376*A*B*C*a^2*b^6*c^3*z + 288*A*B*C*a*b^8* \\
& c^2*z + 3072*B*C^2*a^5*b*c^5*z - 138240*A^2*B*a^4*b*c^6*z + 7344*A^2*B*a*b^ \\
& 7*c^3*z + 122880*A*B*C*a^5*c^6*z - 2304*B*C^2*a^4*b^3*c^4*z + 576*B*C^2*a^3 \\
& *b^5*c^3*z - 48*B*C^2*a^2*b^7*c^2*z + 131328*A^2*B*a^3*b^3*c^5*z - 46656*A^ \\
& 2*B*a^2*b^5*c^4*z + 61440*B^3*a^4*b^2*c^5*z - 21504*B^3*a^3*b^4*c^4*z + 332 \\
& 8*B^3*a^2*b^6*c^3*z - 192*B^3*a*b^8*c^2*z - 432*A^2*B*b^9*c^2*z - 65536*B^3 \\
& *a^5*c^6*z - 5568*A*B^2*C*a^2*b^2*c^5 + 496*A*B^2*C*a*b^4*c^4 + 1104*B^2*C^ \\
& 2*a^2*b^3*c^4 - 3264*A^2*C^2*a^2*b^2*c^5 - 3072*B^2*C^2*a^3*b*c^5 - 100*B^2 \\
& *C^2*a*b^5*c^3 + 2070*A^2*C^2*a*b^4*c^4 - 1840*A*C^3*a^2*b^3*c^4 - 7680*A^2 \\
& *B^2*a^2*b*c^6 + 3152*A^2*B^2*a*b^3*c^5 + 15200*A^3*C*a^2*b*c^6 - 6192*A^3* \\
& C*a*b^3*c^5 + 5472*A*C^3*a^3*b*c^5 + 150*A*C^3*a*b^5*c^3 + 15360*A*B^2*C*a^ \\
& 3*c^6 - 144*B^4*a*b^4*c^4 + 4200*A^4*a*b^2*c^6 + 630*A^3*C*b^5*c^4 + 360*C^ \\
& 4*a^3*b^2*c^4 - 25*C^4*a^2*b^4*c^3 + 1536*B^4*a^2*b^2*c^5 - 225*A^2*C^2*b^6 \\
& *c^3 - 7200*A^2*C^2*a^3*c^6 - 324*A^2*B^2*b^5*c^4 - 1296*C^4*a^4*c^5 - 4096 \\
& *B^4*a^3*c^6 - 441*A^4*b^4*c^5 - 10000*A^4*a^2*c^7, z, k), k, 1, 4) - (A/a \\
& - (x^2*(3*A*b^3 - C*a*b^2 + 2*C*a^2*c - 11*A*a*b*c))/(2*a^2*(4*a*c - b^2)) \\
& + (x^4*(10*A*a*c^2 - 3*A*b^2*c + C*a*b*c))/(2*a^2*(4*a*c - b^2)) - (B*x*(2* \\
& a*c - b^2))/(2*a*(4*a*c - b^2)) + (B*b*c*x^3)/(2*a*(4*a*c - b^2)))/(a*x + b \\
& *x^3 + c*x^5) + (B*log(x))/a^2
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/x**2/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

$$3.36 \quad \int \frac{A+Bx+Cx^2}{x^3(a+bx^2+cx^4)^2} dx$$

Optimal. Leaf size=534

$$\frac{(2Ab - aC) \log(a + bx^2 + cx^4)}{4a^3} - \frac{\log(x)(2Ab - aC)}{a^3} - \frac{-6aAc - abC + 2AB^2}{2a^2x^2(b^2 - 4ac)} - \frac{B(3b^2 - 10ac)}{2a^2x(b^2 - 4ac)} - \frac{B\sqrt{c} \left((3b^2 - 10ac) \right)}{2a^2x(b^2 - 4ac)}$$

Rubi [A] time = 1.99, antiderivative size = 534, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.464$, Rules used = {1662, 1251, 822, 800, 634, 618, 206, 628, 12, 1121, 1281, 1166, 205}

$$\frac{(2A(b^2c^2 - 6a^2c + b^4) - abC(b^2 - 4ac)) \operatorname{tanh}^{-1}\left(\frac{bx^2 + c}{\sqrt{b^2 - 4ac}}\right) - \frac{6aAc - abC + 2AB^2}{2a^2(b^2 - 4ac)} + \frac{(2Ab - aC) \log(a + bx^2 + cx^4)}{4a^3} - \frac{\log(x)(2Ab - aC)}{a^3} - \frac{B(3b^2 - 10ac)}{2a^2x^2(b^2 - 4ac)} - \frac{B\sqrt{c} \left((3b^2 - 10ac) \sqrt{b^2 - 4ac} - 16abc + 3b^3 \right) \operatorname{tan}^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b^2 - 4ac}}\right) + \frac{B\sqrt{c} \left((3b^2 - 10ac) \sqrt{b^2 - 4ac} - 16abc + 3b^3 \right) \operatorname{tan}^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b^2 - 4ac}}\right)}{2\sqrt{c}x^2(b^2 - 4ac)^{3/2} \sqrt{b^2 - 4ac}} + \frac{B\sqrt{c} \left((3b^2 - 10ac) \sqrt{b^2 - 4ac} - 16abc + 3b^3 \right) \operatorname{tan}^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b^2 - 4ac}}\right)}{2\sqrt{c}x^2(b^2 - 4ac)^{3/2} \sqrt{b^2 - 4ac}} + \frac{A(b^2 - 2ac) + c^2(Ab - 2aC) - abC}{2a^2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{B(-2ac + b^2 + bc^2)}{2ax(b^2 - 4ac)(a + bx^2 + cx^4)}}{2a^2(b^2 - 4ac)^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x + C*x^2)/(x^3*(a + b*x^2 + c*x^4)^2), x]

[Out] $-(2A*b^2 - 6*a*A*c - a*b*C)/(2*a^2*(b^2 - 4*a*c)*x^2) - (B*(3*b^2 - 10*a*c))/(2*a^2*(b^2 - 4*a*c)*x) + (B*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*x*(a + b*x^2 + c*x^4)) + (A*(b^2 - 2*a*c) - a*b*C + c*(A*b - 2*a*C)*x^2)/(2*a*(b^2 - 4*a*c)*x^2*(a + b*x^2 + c*x^4)) - (B*sqrt[c]*(3*b^3 - 16*a*b*c + (3*b^2 - 10*a*c)*sqrt[b^2 - 4*a*c])*ArcTan[(sqrt[2]*sqrt[c]*x)/sqrt[b - sqrt[b^2 - 4*a*c]])/(2*sqrt[2]*a^2*(b^2 - 4*a*c)^(3/2)*sqrt[b - sqrt[b^2 - 4*a*c]]) + (B*sqrt[c]*(3*b^3 - 16*a*b*c - (3*b^2 - 10*a*c)*sqrt[b^2 - 4*a*c])*ArcTan[(sqrt[2]*sqrt[c]*x)/sqrt[b + sqrt[b^2 - 4*a*c]])/(2*sqrt[2]*a^2*(b^2 - 4*a*c)^(3/2)*sqrt[b + sqrt[b^2 - 4*a*c]]) - ((2*A*(b^4 - 6*a*b^2*c + 6*a^2*c^2) - a*b*(b^2 - 6*a*c)*C)*ArcTanh[(b + 2*c*x^2)/sqrt[b^2 - 4*a*c]])/(2*a^3*(b^2 - 4*a*c)^(3/2)) - ((2*A*b - a*C)*Log[x])/a^3 + ((2*A*b - a*C)*Log[a + b*x^2 + c*x^4])/(4*a^3)$

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,

e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 800

Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]

Rule 822

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m + 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])

Rule 1121

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := -Simp[((d*x)^(m + 1)*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/((2*a*d*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m + 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1251

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]

Rule 1281

Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)


```
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1662

```
Int[(Pq_)*((d_)*(x_)^(m_))*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Module[{q = Expon[Pq, x], k}, Int[(d*x)^(m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2 + 1}])*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}])*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] && !PolyQ[Pq, x^2]
```

Rubi steps

$$\begin{aligned}
 \int \frac{A + Bx + Cx^2}{x^3(a + bx^2 + cx^4)^2} dx &= \int \frac{B}{x^2(a + bx^2 + cx^4)^2} dx + \int \frac{A + Cx^2}{x^3(a + bx^2 + cx^4)^2} dx \\
 &= \frac{1}{2} \text{Subst} \left(\int \frac{A + Cx}{x^2(a + bx + cx^2)^2} dx, x, x^2 \right) + B \int \frac{1}{x^2(a + bx^2 + cx^4)^2} dx \\
 &= \frac{B(b^2 - 2ac + bcx^2)}{2a(b^2 - 4ac)x(a + bx^2 + cx^4)} + \frac{A(b^2 - 2ac) - abC + c(Ab - 2aC)x^2}{2a(b^2 - 4ac)x^2(a + bx^2 + cx^4)} - \frac{\text{Subst} \left(\int \frac{A + Cx}{x^2(a + bx + cx^2)^2} dx, x, x^2 \right)}{2a(b^2 - 4ac)x^2(a + bx^2 + cx^4)} \\
 &= -\frac{B(3b^2 - 10ac)}{2a^2(b^2 - 4ac)x} + \frac{B(b^2 - 2ac + bcx^2)}{2a(b^2 - 4ac)x(a + bx^2 + cx^4)} + \frac{A(b^2 - 2ac) - abC + c(Ab - 2aC)x^2}{2a(b^2 - 4ac)x^2(a + bx^2 + cx^4)} - \frac{\text{Subst} \left(\int \frac{A + Cx}{x^2(a + bx + cx^2)^2} dx, x, x^2 \right)}{2a(b^2 - 4ac)x^2(a + bx^2 + cx^4)} \\
 &= -\frac{2Ab^2 - 6aAc - abC}{2a^2(b^2 - 4ac)x^2} - \frac{B(3b^2 - 10ac)}{2a^2(b^2 - 4ac)x} + \frac{B(b^2 - 2ac + bcx^2)}{2a(b^2 - 4ac)x(a + bx^2 + cx^4)} + \frac{A(b^2 - 2ac) - abC + c(Ab - 2aC)x^2}{2a(b^2 - 4ac)x^2(a + bx^2 + cx^4)} - \frac{\text{Subst} \left(\int \frac{A + Cx}{x^2(a + bx + cx^2)^2} dx, x, x^2 \right)}{2a(b^2 - 4ac)x^2(a + bx^2 + cx^4)} \\
 &= -\frac{2Ab^2 - 6aAc - abC}{2a^2(b^2 - 4ac)x^2} - \frac{B(3b^2 - 10ac)}{2a^2(b^2 - 4ac)x} + \frac{B(b^2 - 2ac + bcx^2)}{2a(b^2 - 4ac)x(a + bx^2 + cx^4)} + \frac{A(b^2 - 2ac) - abC + c(Ab - 2aC)x^2}{2a(b^2 - 4ac)x^2(a + bx^2 + cx^4)} - \frac{\text{Subst} \left(\int \frac{A + Cx}{x^2(a + bx + cx^2)^2} dx, x, x^2 \right)}{2a(b^2 - 4ac)x^2(a + bx^2 + cx^4)} \\
 &= -\frac{2Ab^2 - 6aAc - abC}{2a^2(b^2 - 4ac)x^2} - \frac{B(3b^2 - 10ac)}{2a^2(b^2 - 4ac)x} + \frac{B(b^2 - 2ac + bcx^2)}{2a(b^2 - 4ac)x(a + bx^2 + cx^4)} + \frac{A(b^2 - 2ac) - abC + c(Ab - 2aC)x^2}{2a(b^2 - 4ac)x^2(a + bx^2 + cx^4)} - \frac{\text{Subst} \left(\int \frac{A + Cx}{x^2(a + bx + cx^2)^2} dx, x, x^2 \right)}{2a(b^2 - 4ac)x^2(a + bx^2 + cx^4)}
 \end{aligned}$$

Mathematica [A] time = 2.47, size = 655, normalized size = 1.23

$$\frac{B(b^2 - 2ac + bcx^2)}{2a(b^2 - 4ac)x(a + bx^2 + cx^4)} + \frac{A(b^2 - 2ac) - abC + c(Ab - 2aC)x^2}{2a(b^2 - 4ac)x^2(a + bx^2 + cx^4)} - \frac{B(3b^2 - 10ac)}{2a^2(b^2 - 4ac)x} - \frac{2Ab^2 - 6aAc - abC}{2a^2(b^2 - 4ac)x^2} - \frac{\text{Subst} \left(\int \frac{A + Cx}{x^2(a + bx + cx^2)^2} dx, x, x^2 \right)}{2a(b^2 - 4ac)x^2(a + bx^2 + cx^4)}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x + C*x^2)/(x^3*(a + b*x^2 + c*x^4)^2), x]

```
[Out] ((-2*a*A)/x^2 - (4*a*B)/x - (2*a*(2*a^2*c*C + b^2*B*x*(b + c*x^2) + A*(b^3 - 3*a*b*c + b^2*c*x^2 - 2*a*c^2*x^2) - a*(b^2*C + 2*B*c^2*x^3 + b*c*x*(3*B + C*x))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*a*B*Sqrt[c]*(-3*b^3 + 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*a*B*Sqrt[c]*(3*b^3 - 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + 4*(-2*A*b + a*C)*Log[x] + ((2*A*(b^4 - 6*a*b^2*c + 6*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 4*a*b*c*Sqrt[b^2 - 4*a*c]) + a*(-b^3 + 6*a*b*c - b^2*Sqrt[b^2 - 4*a*c] + 4*a*c*Sqrt[b^2 - 4*a*c])*C)*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/((b^2 - 4*a*c)^(3/2)) + ((2*A*(-b^4 + 6*a*b^2*c - 6*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 4*a*b*c*Sqrt[b^2 - 4*a*c]) + a*(b^3 - 6*a*b*c - b^2*Sqrt[b^2 - 4*a*c] + 4*a*c*Sqrt[b^2 - 4*a*c])*C)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/((b^2 - 4*a*c)^(3/2))/(4*a^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx + Cx^2}{x^3(a + bx^2 + cx^4)^2} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x + C*x^2)/(x^3*(a + b*x^2 + c*x^4)^2), x]
```

```
[Out] IntegrateAlgebraic[(A + B*x + C*x^2)/(x^3*(a + b*x^2 + c*x^4)^2), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 7.56, size = 6938, normalized size = 12.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/16*((a^6*b^4*c - 8*a^7*b^2*c^2 + 16*a^8*c^3)^2*(6*b^4*c^2 - 44*a*b^2*c^3 + 80*a^2*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*b^4 + 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a*b^2*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*b^3*c - 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^2*c^2 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a*b*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*b^2*c^2 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a*c^3 - 6*(b^2 - 4*a*c)*b^2*c^2 + 20*(b^2 - 4*a*c)*a*c^3)*B + 2*(3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^6*b^9*c - 49*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^7*b^7*c^2 - 6*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^6*b^8*c^2 - 6*a^6*b^9*c^2 + 300*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^8*b^5*c^3 + 74*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^7*b^6*c^3 + 3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^6*b^7*c^3 + 98*a^7*b^7*c^3 - 816*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^9*b^3*c^4 - 304*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^8*b^4*c^4 - 37*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^7*b^5*c^4 - 600*a^8*b^5*c^4 + 832*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^10*b*c^5 + 416*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^9*b^2*c^5 + 152*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
```


$$\begin{aligned}
& \sqrt{b^2 - 4ac} \cdot c \cdot a^{13} b^9 c^4 - 3\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c \cdot a^{12} b^{10} c^4 + 2304\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c \cdot a^{15} b^6 c^5 + 672\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c \cdot a^{14} b^7 c^5 + 52\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c \cdot a^{13} b^8 c^5 - 4864\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c \cdot a^{16} b^4 c^6 - 1920\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c \cdot a^{15} b^5 c^6 - 336\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c \cdot a^{14} b^6 c^6 + 4096\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c \cdot a^{17} b^2 c^7 + 2048\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c \cdot a^{16} b^3 c^7 + 960\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c \cdot a^{15} b^4 c^7 - 1024\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c \cdot a^{16} b^2 c^8 - 6(b^2 - 4ac) \cdot a^{12} b^{10} c^4 + 104(b^2 - 4ac) \cdot a^{13} b^8 c^5 - 672(b^2 - 4ac) \cdot a^{14} b^6 c^6 + 1920(b^2 - 4ac) \cdot a^{15} b^4 c^7 - 2048(b^2 - 4ac) \cdot a^{16} b^2 c^8 \\
&) \cdot B \cdot \arctan\left(\frac{2\sqrt{1/2} \cdot x / \sqrt{(a^6 b^5 c - 8a^7 b^3 c^2 + 16a^8 b c^3 - \sqrt{(a^6 b^5 c - 8a^7 b^3 c^2 + 16a^8 b c^3)^2 - 4(a^7 b^4 c - 8a^8 b^2 c^2 + 16a^9 c^3)(a^6 b^4 c^2 - 8a^7 b^2 c^3 + 16a^8 c^4)})}}{(a^6 b^4 c^2 - 8a^7 b^2 c^3 + 16a^8 c^4)}\right) / \left(\frac{a^9 b^8 c - 16a^{10} b^6 c^2 - 2a^9 b^7 c^2 + 96a^{11} b^4 c^3 + 24a^{10} b^5 c^3 + a^9 b^6 c^3 - 256a^{12} b^2 c^4 - 96a^{11} b^3 c^4 - 12a^{10} b^4 c^4 + 256a^{13} c^5 + 128a^{12} b c^5 + 48a^{11} b^2 c^5 - 64a^{12} c^6}{(a^6 b^4 c - 8a^7 b^2 c^2 + 16a^8 c^3)} \cdot \text{abs}(c)\right) - \frac{1}{4}(Ca - 2Ab) \cdot \log(\text{abs}(c \cdot x^4 + b \cdot x^2 + a)) / a^3 + (Ca - 2Ab) \cdot \log(\text{abs}(x)) / a^3 + \frac{1}{16}(2(b^7 c - 10a \cdot b^5 c^2 - 2b^6 c^2 + 30a^2 b^3 c^3 + 12a \cdot b^4 c^3 + b^5 c^3 - 24a^3 b c^4 - 12a^2 b^2 c^4 - 6a \cdot b^3 c^4 + 6a^2 b c^5 + (b^6 c - 10a \cdot b^4 c^2 - 2b^5 c^2 + 30a^2 b^2 c^3 + 12a \cdot b^3 c^3 + b^4 c^3 - 24a^3 c^4 - 12a^2 b c^4 - 6a \cdot b^2 c^4 + 6a^2 c^5) \cdot \sqrt{b^2 - 4ac})) \cdot A \cdot \text{abs}(a^6 b^4 c - 8a^7 b^2 c^2 + 16a^8 c^3) - (a \cdot b^6 c - 10a^2 b^4 c^2 - 2a \cdot b^5 c^2 + 24a^3 b^2 c^3 + 12a^2 b^3 c^3 + a \cdot b^4 c^3 - 6a^2 b^2 c^4 + (a \cdot b^5 c - 10a^2 b^3 c^2 - 2a \cdot b^4 c^2 + 24a^3 b c^3 + 12a^2 b^2 c^3 + a \cdot b^3 c^3 - 6a^2 b c^4) \cdot \sqrt{b^2 - 4ac})) \cdot C \cdot \text{abs}(a^6 b^4 c - 8a^7 b^2 c^2 + 16a^8 c^3) - 2(a^6 b^{11} c^2 - 18a^7 b^9 c^3 - 2a^6 b^{10} c^3 + 126a^8 b^7 c^4 + 28a^7 b^8 c^4 + a^6 b^9 c^4 - 424a^9 b^5 c^5 - 140a^8 b^6 c^5 - 14a^7 b^7 c^5 + 672a^{10} b^3 c^6 + 288a^9 b^4 c^6 + 70a^8 b^5 c^6 - 384a^{11} b c^7 - 192a^{10} b^2 c^7 - 144a^9 b^3 c^7 + 96a^{10} b c^8 + (a^6 b^{10} c^2 - 14a^7 b^8 c^3 - 2a^6 b^9 c^3 + 70a^8 b^6 c^4 + 20a^7 b^7 c^4 + a^6 b^8 c^4 - 144a^9 b^4 c^5 - 60a^8 b^5 c^5 - 10a^7 b^6 c^5 + 96a^{10} b^2 c^6 + 48a^9 b^3 c^6 + 30a^8 b^4 c^6 - 24a^9 b^2 c^7) \cdot \sqrt{b^2 - 4ac})) \cdot A + (a^7 b^{10} c^2 - 18a^8 b^8 c^3 - 2a^7 b^9 c^3 + 120a^9 b^6 c^4 + 28a^8 b^7 c^4 + a^7 b^8 c^4 - 352a^{10} b^4 c^5 - 128a^9 b^5 c^5 - 14a^8 b^6 c^5 + 384a^{11} b^2 c^6 + 192a^{10} b^3 c^6 + 64a^9 b^4 c^6 - 96a^{10} b^2 c^7 + (a^7 b^9 c^2 - 14a^8 b^7 c^3 - 2a^7 b^8 c^3 + 64a^9 b^5 c^4 + 20a^8 b^6 c^4 + a^7 b^7 c^4 - 96a^{10} b^3 c^5 - 48a^9 b^4 c^5 - 10a^8 b^5 c^5 + 24a^9 b^3 c^6) \cdot \sqrt{b^2 - 4ac})) \cdot C \cdot \log(x^2 + 1/2(a^6 b^5 c - 8a^7 b^3 c^2 + 16a^8 b c^3 + \sqrt{(a^6 b^5 c - 8a^7 b^3 c^2 + 16a^8 b c^3)^2 - 4(a^7 b^4 c - 8a^8 b^2 c^2 + 16a^9 c^3)(a^6 b^4 c^2 - 8a^7 b^2 c^3 + 16a^8 c^4)})) / (a^4 b^4 - 8a^5 b^2 c - 2a^4 b^3 c + 16a^6 c^2 + 8a^5 b c^2 + a^4 b^2 c^2 - 4a^5 c^3) \cdot c^2 \cdot \text{abs}(a^6 b^4 c - 8a^7 b^2 c^2 + 16a^8 c^3)) + \frac{1}{16}(2(b^7 c - 10a \cdot b^5 c^2 - 2b^6 c^2 + 30a^2 b^3 c^3 + 12a \cdot b^4 c^3 + b^5 c^3 - 24a^3 b c^4 - 12a^2 b^2 c^4 - 6a \cdot b^3 c^4 + 6a^2 b c^5 - (b^6 c - 10a \cdot b^4 c^2 - 2b^5 c^2 + 30a^2 b^2 c^3 + 12a \cdot b^3 c^3 + b^4 c^3 - 24a^3 c^4 - 12a^2 b c^4 - 6a \cdot b^2 c^4 + 6a^2 c^5) \cdot \sqrt{b^2 - 4ac})) \cdot A \cdot \text{abs}(a^6 b^4 c - 8a^7 b^2 c^2 + 16a^8 c^3) - (a \cdot b^6 c - 10a^2 b^4 c^2 - 2a \cdot b^5 c^2 + 24a^3 b^2 c^3 + 12a^2 b^3 c^3 + a \cdot b^4 c^3 - 6a^2 b^2 c^4 - (a \cdot b^5 c - 10a^2 b^3 c^2 - 2a \cdot b^4 c^2 + 24a^3 b c^3 + 12a^2 b^2 c^3 + a \cdot b^3 c^3 - 6a^2 b c^4) \cdot \sqrt{b^2 - 4ac})) \cdot C \cdot \text{abs}(a^6 b^4 c - 8a^7 b^2 c^2 + 16a^8 c^3) - 2(a^6 b^{11} c^2 - 18a^7 b^9 c^3 - 2a^6 b^{10} c^3 + 126a^8 b^7 c^4 + 28a^7 b^8 c^4 + a^6 b^9 c^4 - 424a^9 b^5 c^5 - 140a^8 b^6 c^5 - 14a^7 b^7 c^5 + 672a^{10} b^3 c^6 + 288a^9 b^4 c^6 + 70a^8 b^5 c^6 - 384a^{11} b
\end{aligned}$$

$$\begin{aligned}
& c^7 - 192a^{10}b^2c^7 - 144a^9b^3c^7 + 96a^{10}b^4c^8 - (a^6b^{10}c^2 - 14a^7b^8c^3 - 2a^6b^9c^3 + 70a^8b^6c^4 + 20a^7b^7c^4 + a^6b^8c^4 - 144a^9b^4c^5 - 60a^8b^5c^5 - 10a^7b^6c^5 + 96a^{10}b^2c^6 + 48a^9b^3c^6 + 30a^8b^4c^6 - 24a^9b^2c^7) \sqrt{b^2 - 4ac} * A + (\\
& a^7b^{10}c^2 - 18a^8b^8c^3 - 2a^7b^9c^3 + 120a^9b^6c^4 + 28a^8b^7c^4 + a^7b^8c^4 - 352a^{10}b^4c^5 - 128a^9b^5c^5 - 14a^8b^6c^5 + 384a^{11}b^2c^6 + 192a^{10}b^3c^6 + 64a^9b^4c^6 - 96a^{10}b^2c^7 - (\\
& a^7b^9c^2 - 14a^8b^7c^3 - 2a^7b^8c^3 + 64a^9b^5c^4 + 20a^8b^6c^4 + a^7b^7c^4 - 96a^{10}b^3c^5 - 48a^9b^4c^5 - 10a^8b^5c^5 + 24a^9b^3c^6) \sqrt{b^2 - 4ac} * C) * \log(x^2 + 1/2(a^6b^5c - 8a^7b^3c^2 + 16a^8b^2c^3 - \sqrt{(a^6b^5c - 8a^7b^3c^2 + 16a^8b^2c^3)^2 - 4(a^7b^4c - 8a^8b^2c^2 + 16a^9c^3)} * (a^6b^4c^2 - 8a^7b^2c^3 + 16a^8c^4))) / (a^6b^4c^2 - 8a^7b^2c^3 + 16a^8c^4) / ((a^4b^4 - 8a^5b^2c - 2a^4b^3c + 16a^6c^2 + 8a^5b^2c^2 + a^4b^2c^2 - 4a^5c^3) * c^2 * \text{abs}(a^6b^4c - 8a^7b^2c^2 + 16a^8c^3)) - 1/2((3B*a*b^2*c - 10B*a^2*c^2)*x^5 + A*a^2*b^2 - 4A*a^3*c - (C*a^2*b*c - 2A*a*b^2*c + 6A*a^2*c^2)*x^4 + (3B*a*b^3 - 11B*a^2*b*c)*x^3 - (C*a^2*b^2 - 2A*a*b^3 - 2C*a^3*c + 7A*a^2*b*c)*x^2 + 2*(B*a^2*b^2 - 4B*a^3*c)*x) / ((c*x^4 + b*x^2 + a)*(b^2 - 4*a*c)*a^3*x^2)
\end{aligned}$$

maple [B] time = 0.10, size = 2512, normalized size = 4.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((C*x^2+B*x+A)/x^3/(c*x^4+b*x^2+a)^2,x)

[Out] $1/2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*A/a^2*b^3-1/2*A/a^2/x^2-1/(c*x^4+b*x^2+a)/(4*a*c-b^2)*A/a*c^2*x^2-3/2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*A/a*b*c+1/2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*B/a^2*b^2*c*x^3-3/2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*B/a*b*c*x+1/(c*x^4+b*x^2+a)/(4*a*c-b^2)*C-2*A/a^3*b*\ln(x)+1/2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*A/a^2*b^2*c*x^2+6/a*c/(4*a*c-b^2)/(16*a*c-4*b^2)*\ln(-2*c*x^2-b+(-4*a*c+b^2)^(1/2))*C*(-4*a*c+b^2)^(1/2)*b-12/a^2*c/(4*a*c-b^2)/(16*a*c-4*b^2)*\ln(-2*c*x^2-b+(-4*a*c+b^2)^(1/2))*A*(-4*a*c+b^2)^(1/2)*b^2-6/a*c/(4*a*c-b^2)/(16*a*c-4*b^2)*\ln(2*c*x^2+b+(-4*a*c+b^2)^(1/2))*C*(-4*a*c+b^2)^(1/2)*b+12/a^2*c/(4*a*c-b^2)/(16*a*c-4*b^2)*\ln(2*c*x^2+b+(-4*a*c+b^2)^(1/2))*A*(-4*a*c+b^2)^(1/2)*b^2+2/a^3/(4*a*c-b^2)/(16*a*c-4*b^2)*\ln(2*c*x^2+b+(-4*a*c+b^2)^(1/2))*A*b^5+2/a^3/(4*a*c-b^2)/(16*a*c-4*b^2)*\ln(-2*c*x^2-b+(-4*a*c+b^2)^(1/2))*A*b^5-1/a^2/(4*a*c-b^2)/(16*a*c-4*b^2)*\ln(-2*c*x^2-b+(-4*a*c+b^2)^(1/2))*C*b^4-1/a^2/(4*a*c-b^2)/(16*a*c-4*b^2)*\ln(2*c*x^2+b+(-4*a*c+b^2)^(1/2))*C*b^4+1/a^2*\ln(x)*C-16*c^2/(4*a*c-b^2)/(16*a*c-4*b^2)*\ln(2*c*x^2+b+(-4*a*c+b^2)^(1/2))*C-B/a^2/x-16/a*c^2/(4*a*c-b^2)/(16*a*c-4*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*\arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B*(-4*a*c+b^2)^(1/2)*b+3/a^2*c/(4*a*c-b^2)/(16*a*c-4*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*\arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B*(-4*a*c+b^2)^(1/2)*b^3+3/a^2*c/(4*a*c-b^2)/(16*a*c-4*b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*\arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B*(-4*a*c+b^2)^(1/2)*b^3-16/a*c^2/(4*a*c-b^2)/(16*a*c-4*b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*\arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B*(-4*a*c+b^2)^(1/2)*b-22/a*c^2/(4*a*c-b^2)/(16*a*c-4*b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*\arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B*b^2-3/a^2*c/(4*a*c-b^2)/(16*a*c-4*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*\arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B*b^4+3/a^2*c/(4*a*c-b^2)/(16*a*c-4*b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*\arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B*b^4+22/a*c^2/(4*a*c-b^2)/(16*a*c-4*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*\arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B*b^2-1/(c*x^4+b*x^2+a)/(4*a*c-b^2)*B/a*c^2*x^3+1/2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*B/a^2*b^3*x-1/2/a/(c*x^4+b*x^2+a)*c/(4*a*c-b^2)*x^2*C*b-40*c^3/(4*a*c-b^2)/(16*a*c-4*b^2$

$$\begin{aligned} &) * 2^{(1/2)} / ((b + (-4*a*c + b^2)^{(1/2)}) * c)^{(1/2)} * \arctan(2^{(1/2)} / ((b + (-4*a*c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * B + 40 * c^3 / (4*a*c - b^2) / (16*a*c - 4*b^2) * 2^{(1/2)} / ((-b + (-4*a*c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * B + 12/a * c^2 / (4*a*c - b^2) / (16*a*c - 4*b^2) * \ln(-2*c*x^2 - b + (-4*a*c + b^2)^{(1/2)}) \\ & * A * (-4*a*c + b^2)^{(1/2)} + 2/a^3 / (4*a*c - b^2) / (16*a*c - 4*b^2) * \ln(-2*c*x^2 - b + (-4*a*c + b^2)^{(1/2)}) * A * (-4*a*c + b^2)^{(1/2)} * b^4 + 32/a * c^2 / (4*a*c - b^2) / (16*a*c - 4*b^2) * \\ & \ln(-2*c*x^2 - b + (-4*a*c + b^2)^{(1/2)}) * A * b - 16/a^2 * c / (4*a*c - b^2) / (16*a*c - 4*b^2) * \ln(2*c*x^2 + b + (-4*a*c + b^2)^{(1/2)}) * A * b^3 + 8/a * c / (4*a*c - b^2) / (16*a*c - 4*b^2) * \ln(2 \\ & * c*x^2 + b + (-4*a*c + b^2)^{(1/2)}) * C * b^2 - 16/a^2 * c / (4*a*c - b^2) / (16*a*c - 4*b^2) * \ln(-2*c*x^2 - b + (-4*a*c + b^2)^{(1/2)}) * A * b^3 + 8/a * c / (4*a*c - b^2) / (16*a*c - 4*b^2) * \ln(-2 * \\ & c*x^2 - b + (-4*a*c + b^2)^{(1/2)}) * C * b^2 - 12/a * c^2 / (4*a*c - b^2) / (16*a*c - 4*b^2) * \ln(2 * \\ & c*x^2 + b + (-4*a*c + b^2)^{(1/2)}) * A * (-4*a*c + b^2)^{(1/2)} - 2/a^3 / (4*a*c - b^2) / (16*a*c - \\ & 4*b^2) * \ln(2*c*x^2 + b + (-4*a*c + b^2)^{(1/2)}) * A * (-4*a*c + b^2)^{(1/2)} * b^4 + 32/a * c^2 / (\\ & 4*a*c - b^2) / (16*a*c - 4*b^2) * \ln(2*c*x^2 + b + (-4*a*c + b^2)^{(1/2)}) * A * b + 1/a^2 / (4*a*c \\ & - b^2) / (16*a*c - 4*b^2) * \ln(2*c*x^2 + b + (-4*a*c + b^2)^{(1/2)}) * C * (-4*a*c + b^2)^{(1/2)} * \\ & b^3 - 1/a^2 / (4*a*c - b^2) / (16*a*c - 4*b^2) * \ln(-2*c*x^2 - b + (-4*a*c + b^2)^{(1/2)}) * C * (- \\ & 4*a*c + b^2)^{(1/2)} * b^3 - 1/2/a / (c*x^4 + b*x^2 + a) / (4*a*c - b^2) * C * b^2 \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")

[Out]
$$\begin{aligned} & -1/2 * ((3*B*b^2*c - 10*B*a*c^2) * x^5 - (6*A*a*c^2 + (C*a*b - 2*A*b^2) * c) * x^4 \\ & + A*a*b^2 - 4*A*a^2*c + (3*B*b^3 - 11*B*a*b*c) * x^3 - (C*a*b^2 - 2*A*b^3 - (\\ & 2*C*a^2 - 7*A*a*b) * c) * x^2 + 2 * (B*a*b^2 - 4*B*a^2*c) * x) / ((a^2*b^2*c - 4*a^3 * \\ & c^2) * x^6 + (a^2*b^3 - 4*a^3*b*c) * x^4 + (a^3*b^2 - 4*a^4*c) * x^2) - 1/2 * \text{integ} \\ & \text{rate}((3*B*a*b^3 - 13*B*a^2*b*c - 2*(4*(C*a^2 - 2*A*a*b) * c^2 - (C*a*b^2 - 2 * \\ & A*b^3) * c) * x^3 + (3*B*a*b^2*c - 10*B*a^2*c^2) * x^2 + 2*(C*a*b^3 - 2*A*b^4 - 6 \\ & *A*a^2*c^2 - 5*(C*a^2*b - 2*A*a*b^2) * c) * x) / (c*x^4 + b*x^2 + a), x) / (a^3*b^2 \\ & - 4*a^4*c) + (C*a - 2*A*b) * \log(x) / a^3 \end{aligned}$$

mupad [B] time = 2.77, size = 10595, normalized size = 19.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x + C*x^2)/(x^3*(a + b*x^2 + c*x^4)^2),x)

[Out]
$$\begin{aligned} & \text{symsum}(\log(\text{root}(1572864*a^{11}*b^2*c^5*z^4 - 983040*a^{10}*b^4*c^4*z^4 + 327680 \\ & *a^9*b^6*c^3*z^4 - 61440*a^8*b^8*c^2*z^4 + 6144*a^7*b^{10}*c*z^4 - 1048576*a^ \\ & 12*c^6*z^4 - 256*a^6*b^{12}*z^4 + 1572864*C*a^9*b^2*c^5*z^3 - 983040*C*a^8*b^ \\ & 4*c^4*z^3 + 327680*C*a^7*b^6*c^3*z^3 - 61440*C*a^6*b^8*c^2*z^3 - 3145728*A* \\ & a^8*b^3*c^5*z^3 + 1966080*A*a^7*b^5*c^4*z^3 - 655360*A*a^6*b^7*c^3*z^3 + 12 \\ & 2880*A*a^5*b^9*c^2*z^3 + 6144*C*a^5*b^{10}*c*z^3 + 2097152*A*a^9*b*c^6*z^3 - \\ & 12288*A*a^4*b^{11}*c*z^3 - 1048576*C*a^{10}*c^6*z^3 - 256*C*a^4*b^{12}*z^3 + 512* \\ & A*a^3*b^{13}*z^3 + 1277952*A*C*a^7*b*c^6*z^2 - 6144*A*C*a^2*b^{11}*c*z^2 - 1794 \\ & 048*A*C*a^6*b^3*c^5*z^2 + 1062912*A*C*a^5*b^5*c^4*z^2 - 340480*A*C*a^4*b^7* \\ & c^3*z^2 + 62208*A*C*a^3*b^9*c^2*z^2 + 256*A*C*a*b^{13}*z^2 + 1536*C^2*a^3*b^1 \\ & 0*c*z^2 - 430080*B^2*a^7*b*c^6*z^2 + 3408*B^2*a^2*b^{11}*c*z^2 + 6144*A^2*a*b \\ & ^{12}*c*z^2 + 516096*C^2*a^7*b^2*c^5*z^2 - 288768*C^2*a^6*b^4*c^4*z^2 + 88576 \\ & *C^2*a^5*b^6*c^3*z^2 - 15744*C^2*a^4*b^8*c^2*z^2 + 716800*B^2*a^6*b^3*c^5*z \\ & ^2 - 483840*B^2*a^5*b^5*c^4*z^2 + 170496*B^2*a^4*b^7*c^3*z^2 - 33232*B^2*a^ \\ & 3*b^9*c^2*z^2 + 1468416*A^2*a^5*b^4*c^5*z^2 - 966144*A^2*a^4*b^6*c^4*z^2 - \\ & 761856*A^2*a^6*b^2*c^6*z^2 + 326656*A^2*a^3*b^8*c^3*z^2 - 61440*A^2*a^2*b^1 \\ & 0*c^2*z^2 - 144*B^2*a*b^{13}*z^2 - 393216*C^2*a^8*c^6*z^2 - 64*C^2*a^2*b^{12}* \\ & ^2 - 294912*A^2*a^7*c^7*z^2 - 256*A^2*b^{14}*z^2 - 138240*B^2*C*a^5*b*c^6*z - \\ & 432*B^2*C*a*b^9*c^2*z + 245760*A*C^2*a^5*b*c^6*z + 12288*A^2*C*a*b^8*c^3*z \\ & + 768*A*C^2*a*b^9*c^2*z + 576*A*B^2*a*b^8*c^3*z + 131328*B^2*C*a^4*b^3*c^5 \end{aligned}$$

$$\begin{aligned}
& *z - 46656*B^2*C*a^3*b^5*c^4*z + 7344*B^2*C*a^2*b^7*c^3*z - 233472*A*C^2*a^4 \\
& 4*b^3*c^5*z + 168960*A^2*C*a^3*b^4*c^5*z - 86016*A^2*C*a^4*b^2*c^6*z + 8294 \\
& 4*A*C^2*a^3*b^5*c^4*z - 71424*A^2*C*a^2*b^6*c^4*z - 13056*A*C^2*a^2*b^7*c^3 \\
& *z - 152064*A*B^2*a^4*b^2*c^6*z + 56448*A*B^2*a^3*b^4*c^5*z - 9312*A*B^2*a^2 \\
& 2*b^6*c^4*z + 61440*C^3*a^5*b^2*c^5*z - 21504*C^3*a^4*b^4*c^4*z + 3328*C^3*a^3 \\
& a^3*b^6*c^3*z - 192*C^3*a^2*b^8*c^2*z - 286720*A^3*a^3*b^3*c^6*z + 104448*A^3 \\
& a^2*b^5*c^5*z + 294912*A^3*a^4*b*c^7*z - 16896*A^3*a*b^7*c^4*z - 768*A^2 \\
& *C*b^10*c^2*z - 147456*A^2*C*a^5*c^7*z + 153600*A*B^2*a^5*c^7*z - 65536*C^3 \\
& *a^6*c^6*z + 1024*A^3*b^9*c^3*z - 15936*A*B^2*C*a^2*b^2*c^6 + 1648*A*B^2*C* \\
& a*b^4*c^5 + 3152*B^2*C^2*a^2*b^3*c^5 - 4992*A^2*C^2*a^2*b^2*c^6 - 7680*B^2* \\
& C^2*a^3*b*c^6 - 324*B^2*C^2*a*b^5*c^4 - 5760*A*C^3*a^2*b^3*c^5 + 4608*A^2*C^2 \\
& a*b^4*c^5 - 16320*A^2*B^2*a^2*b*c^7 + 7152*A^2*B^2*a*b^3*c^6 + 32256*A^3 \\
& *C*a^2*b*c^7 + 14336*A*C^3*a^3*b*c^6 - 14080*A^3*C*a*b^3*c^6 + 576*A*C^3*a* \\
& b^5*c^4 + 38400*A*B^2*C*a^3*c^7 - 441*B^4*a*b^4*c^5 + 9216*A^4*a*b^2*c^7 + \\
& 1536*A^3*C*b^5*c^5 + 1536*C^4*a^3*b^2*c^5 - 144*C^4*a^2*b^4*c^4 + 4200*B^4*a^2 \\
& a^2*b^2*c^6 - 576*A^2*C^2*b^6*c^4 - 18432*A^2*C^2*a^3*c^7 - 784*A^2*B^2*b^5 \\
& *c^5 - 4096*C^4*a^4*c^6 - 10000*B^4*a^3*c^7 - 1024*A^4*b^4*c^6 - 20736*A^4* \\
& a^2*c^8, z, k)*(root(1572864*a^11*b^2*c^5*z^4 - 983040*a^10*b^4*c^4*z^4 + 3 \\
& 27680*a^9*b^6*c^3*z^4 - 61440*a^8*b^8*c^2*z^4 + 6144*a^7*b^10*c*z^4 - 10485 \\
& 76*a^12*c^6*z^4 - 256*a^6*b^12*z^4 + 1572864*C*a^9*b^2*c^5*z^3 - 983040*C*a^8 \\
& *b^4*c^4*z^3 + 327680*C*a^7*b^6*c^3*z^3 - 61440*C*a^6*b^8*c^2*z^3 - 31457 \\
& 28*A*a^8*b^3*c^5*z^3 + 1966080*A*a^7*b^5*c^4*z^3 - 655360*A*a^6*b^7*c^3*z^3 \\
& + 122880*A*a^5*b^9*c^2*z^3 + 6144*C*a^5*b^10*c*z^3 + 2097152*A*a^9*b*c^6*z^3 \\
& - 12288*A*a^4*b^11*c*z^3 - 1048576*C*a^10*c^6*z^3 - 256*C*a^4*b^12*z^3 + \\
& 512*A*a^3*b^13*z^3 + 1277952*A*C*a^7*b*c^6*z^2 - 6144*A*C*a^2*b^11*c*z^2 - \\
& 1794048*A*C*a^6*b^3*c^5*z^2 + 1062912*A*C*a^5*b^5*c^4*z^2 - 340480*A*C*a^4 \\
& *b^7*c^3*z^2 + 62208*A*C*a^3*b^9*c^2*z^2 + 256*A*C*a*b^13*z^2 + 1536*C^2*a^3 \\
& 3*b^10*c*z^2 - 430080*B^2*a^7*b*c^6*z^2 + 3408*B^2*a^2*b^11*c*z^2 + 6144*A^2 \\
& 2*a*b^12*c*z^2 + 516096*C^2*a^7*b^2*c^5*z^2 - 288768*C^2*a^6*b^4*c^4*z^2 + \\
& 88576*C^2*a^5*b^6*c^3*z^2 - 15744*C^2*a^4*b^8*c^2*z^2 + 716800*B^2*a^6*b^3* \\
& c^5*z^2 - 483840*B^2*a^5*b^5*c^4*z^2 + 170496*B^2*a^4*b^7*c^3*z^2 - 33232*B^2 \\
& a^3*b^9*c^2*z^2 + 1468416*A^2*a^5*b^4*c^5*z^2 - 966144*A^2*a^4*b^6*c^4*z^2 \\
& - 761856*A^2*a^6*b^2*c^6*z^2 + 326656*A^2*a^3*b^8*c^3*z^2 - 61440*A^2*a^2 \\
& 2*b^10*c^2*z^2 - 144*B^2*a*b^13*z^2 - 393216*C^2*a^8*c^6*z^2 - 64*C^2*a^2*b^12 \\
& z^2 - 294912*A^2*a^7*c^7*z^2 - 256*A^2*b^14*z^2 - 138240*B^2*C*a^5*b*c^6 \\
& *z - 432*B^2*C*a*b^9*c^2*z + 245760*A*C^2*a^5*b*c^6*z + 12288*A^2*C*a*b^8* \\
& c^3*z + 768*A*C^2*a*b^9*c^2*z + 576*A*B^2*a*b^8*c^3*z + 131328*B^2*C*a^4*b^3 \\
& c^5*z - 46656*B^2*C*a^3*b^5*c^4*z + 7344*B^2*C*a^2*b^7*c^3*z - 233472*A*C^2 \\
& a^4*b^3*c^5*z + 168960*A^2*C*a^3*b^4*c^5*z - 86016*A^2*C*a^4*b^2*c^6*z + \\
& 82944*A*C^2*a^3*b^5*c^4*z - 71424*A^2*C*a^2*b^6*c^4*z - 13056*A*C^2*a^2*b^7 \\
& c^3*z - 152064*A*B^2*a^4*b^2*c^6*z + 56448*A*B^2*a^3*b^4*c^5*z - 9312*A*B^2 \\
& a^2*b^6*c^4*z + 61440*C^3*a^5*b^2*c^5*z - 21504*C^3*a^4*b^4*c^4*z + 3328 \\
& *C^3*a^3*b^6*c^3*z - 192*C^3*a^2*b^8*c^2*z - 286720*A^3*a^3*b^3*c^6*z + 104 \\
& 448*A^3*a^2*b^5*c^5*z + 294912*A^3*a^4*b*c^7*z - 16896*A^3*a*b^7*c^4*z - 76 \\
& 8*A^2*C*b^10*c^2*z - 147456*A^2*C*a^5*c^7*z + 153600*A*B^2*a^5*c^7*z - 6553 \\
& 6*C^3*a^6*c^6*z + 1024*A^3*b^9*c^3*z - 15936*A*B^2*C*a^2*b^2*c^6 + 1648*A*B^2 \\
& C*a*b^4*c^5 + 3152*B^2*C^2*a^2*b^3*c^5 - 4992*A^2*C^2*a^2*b^2*c^6 - 7680 \\
& *B^2*C^2*a^3*b*c^6 - 324*B^2*C^2*a*b^5*c^4 - 5760*A*C^3*a^2*b^3*c^5 + 4608* \\
& A^2*C^2*a*b^4*c^5 - 16320*A^2*B^2*a^2*b*c^7 + 7152*A^2*B^2*a*b^3*c^6 + 3225 \\
& 6*A^3*C*a^2*b*c^7 + 14336*A*C^3*a^3*b*c^6 - 14080*A^3*C*a*b^3*c^6 + 576*A*C^3 \\
& a*b^5*c^4 + 38400*A*B^2*C*a^3*c^7 - 441*B^4*a*b^4*c^5 + 9216*A^4*a*b^2*c^7 + \\
& 1536*A^3*C*b^5*c^5 + 1536*C^4*a^3*b^2*c^5 - 144*C^4*a^2*b^4*c^4 + 4200 \\
& *B^4*a^2*b^2*c^6 - 576*A^2*C^2*b^6*c^4 - 18432*A^2*C^2*a^3*c^7 - 784*A^2*B^2 \\
& b^5*c^5 - 4096*C^4*a^4*c^6 - 10000*B^4*a^3*c^7 - 1024*A^4*b^4*c^6 - 20736 \\
& *A^4*a^2*c^8, z, k)*(root(1572864*a^11*b^2*c^5*z^4 - 983040*a^10*b^4*c^4*z^4 \\
& + 327680*a^9*b^6*c^3*z^4 - 61440*a^8*b^8*c^2*z^4 + 6144*a^7*b^10*c*z^4 - 10485 \\
& 76*a^12*c^6*z^4 - 256*a^6*b^12*z^4 + 1572864*C*a^9*b^2*c^5*z^3 - 98304 \\
& 0*C*a^8*b^4*c^4*z^3 + 327680*C*a^7*b^6*c^3*z^3 - 61440*C*a^6*b^8*c^2*z^3 - \\
& 3145728*A*a^8*b^3*c^5*z^3 + 1966080*A*a^7*b^5*c^4*z^3 - 655360*A*a^6*b^7*c^
\end{aligned}$$

$$\begin{aligned}
& 3z^3 + 122880Aa^5b^9c^2z^3 + 6144Ca^5b^{10}cz^3 + 2097152Aa^9b^8c^6z^3 - 12288Aa^4b^{11}cz^3 - 1048576Ca^{10}c^6z^3 - 256Ca^4b^{12}z^3 \\
& + 512Aa^3b^{13}z^3 + 1277952ACa^7b^6c^6z^2 - 6144ACa^2b^{11}cz^2 - 1794048ACa^6b^3c^5z^2 + 1062912ACa^5b^5c^4z^2 - 340480ACa^4b^7c^3z^2 \\
& + 62208ACa^3b^9c^2z^2 + 256ACa^2b^{13}z^2 + 1536C^2a^3b^{10}cz^2 - 430080B^2a^7b^6c^6z^2 + 3408B^2a^2b^{11}cz^2 + 6144A^2a^2b^{12}cz^2 \\
& + 516096C^2a^7b^2c^5z^2 - 288768C^2a^6b^4c^4z^2 + 88576C^2a^5b^6c^3z^2 - 15744C^2a^4b^8c^2z^2 + 716800B^2a^6b^3c^5z^2 \\
& - 483840B^2a^5b^5c^4z^2 + 170496B^2a^4b^7c^3z^2 - 33232B^2a^3b^9c^2z^2 + 1468416A^2a^5b^4c^5z^2 - 966144A^2a^4b^6c^4z^2 - 761856A^2a^6b^2c^6z^2 \\
& + 326656A^2a^3b^8c^3z^2 - 61440A^2a^2b^{10}c^2z^2 - 144B^2a^2b^{13}z^2 - 393216C^2a^8c^6z^2 - 64C^2a^2b^{12}z^2 - 294912A^2a^7c^7z^2 \\
& - 256A^2b^{14}z^2 - 138240B^2Ca^5b^6c^6z - 432B^2Ca^2b^9c^2z + 245760AC^2a^5b^6c^6z + 12288A^2Ca^2b^8c^3z + 768AC^2a^2b^9c^2z \\
& + 576AB^2a^2b^8c^3z + 131328B^2Ca^4b^3c^5z - 46656B^2Ca^3b^5c^4z + 7344B^2Ca^2b^7c^3z - 233472AC^2a^4b^3c^5z \\
& + 168960A^2Ca^3b^4c^5z - 86016A^2Ca^4b^2c^6z + 82944AC^2a^3b^5c^4z - 71424A^2Ca^2b^6c^4z - 13056AC^2a^2b^7c^3z \\
& - 152064AB^2a^4b^2c^6z + 56448AB^2a^3b^4c^5z - 9312AB^2a^2b^6c^4z + 61440C^3a^5b^2c^5z - 21504C^3a^4b^4c^4z + 3328C^3a^3b^6c^3z \\
& - 192C^3a^2b^8c^2z - 286720A^3a^3b^3c^6z + 104448A^3a^2b^5c^5z + 294912A^3a^4b^7c^4z - 16896A^3a^2b^7c^4z - 768A^2Cb^{10}c^2z \\
& - 147456A^2Ca^5c^7z + 153600AB^2a^5c^7z - 65536C^3a^6c^6z + 1024A^3b^9c^3z - 15936AB^2Ca^2b^2c^6 + 1648AB^2Ca^2b^4c^5 \\
& + 3152B^2C^2a^2b^3c^5 - 4992A^2C^2a^2b^2c^6 - 7680B^2C^2a^3b^6c^6 - 324B^2C^2a^2b^5c^4 - 5760AC^3a^2b^3c^5 + 4608A^2C^2a^2b^4c^5 \\
& - 16320A^2B^2a^2b^2c^7 + 7152A^2B^2a^2b^3c^6 + 32256A^3Ca^2b^2c^7 + 14336AC^3a^3b^6c^6 - 14080A^3Ca^2b^3c^6 + 576AC^3a^2b^5c^4 \\
& + 38400AB^2Ca^3c^7 - 441B^4a^2b^4c^5 + 9216A^4a^2b^2c^7 + 1536A^3Cb^5c^5 + 1536C^4a^3b^2c^5 - 144C^4a^2b^4c^4 + 4200B^4a^2b^2c^6 \\
& - 576A^2C^2b^6c^4 - 18432A^2C^2a^3c^7 - 784A^2B^2b^5c^5 - 4096C^4a^4c^6 - 10000B^4a^3c^7 - 1024A^4b^4c^6 - 20736A^4a^2c^8, z, k) \\
& \cdot ((x(983040Ca^{11}c^8 - 1867776Aa^{10}b^6c^8 - 384Aa^4b^{13}c^2 + 9472Aa^5b^{11}c^3 - 97408Aa^6b^9c^4 + 534528Aa^7b^7c^5 - 1650688Aa^8b^5c^6 \\
& + 2719744Aa^9b^3c^7 + 192Ca^5b^{12}c^2 - 4736Ca^6b^{10}c^3 + 48896Ca^7b^8c^4 - 270336Ca^8b^6c^5 + 843776Ca^9b^4c^6 - 1409024Ca^{10}b^2c^7)) / (16(a^6b^8 + 256a^{10}c^4 - 16a^7b^6c + 96a^8b^4c^2 - 256a^9b^2c^3)) - (10240Ba^{10}c^7 - 48Ba^5b^{10}c^2 \\
& + 832B^2a^6b^8c^3 - 5536B^2a^7b^6c^4 + 17280B^2a^8b^4c^5 - 24064B^2a^9b^2c^6) / (8(a^6b^6 - 64a^9c^3 - 12a^7b^4c + 48a^8b^2c^2)) + (\text{root}(1572864a^{11}b^2c^5z^4 - 983040a^{10}b^4c^4z^4 + 327680a^9b^6c^3z^4 \\
& - 61440a^8b^8c^2z^4 + 6144a^7b^{10}cz^4 - 1048576a^{12}c^6z^4 - 256a^6b^{12}z^4 + 1572864Ca^9b^2c^5z^3 - 983040Ca^8b^4c^4z^3 + 327680Ca^7b^6c^3z^3 - 61440Ca^6b^8c^2z^3 - 3145728Aa^8b^3c^5z^3 \\
& + 1966080Aa^7b^5c^4z^3 - 655360Aa^6b^7c^3z^3 + 122880Aa^5b^9c^2z^3 + 6144Ca^5b^{10}cz^3 + 2097152Aa^9b^8c^6z^3 - 12288Aa^4b^{11}cz^3 - 1048576Ca^{10}c^6z^3 - 256Ca^4b^{12}z^3 + 512Aa^3b^{13}z^3 \\
& + 1277952ACa^7b^6c^6z^2 - 6144ACa^2b^{11}cz^2 - 1794048ACa^6b^3c^5z^2 + 1062912ACa^5b^5c^4z^2 - 340480ACa^4b^7c^3z^2 + 62208ACa^3b^9c^2z^2 + 256ACa^2b^{13}z^2 + 1536C^2a^3b^{10}cz^2 \\
& - 430080B^2a^7b^6c^6z^2 + 3408B^2a^2b^{11}cz^2 + 6144A^2a^2b^{12}cz^2 + 516096C^2a^7b^2c^5z^2 - 288768C^2a^6b^4c^4z^2 + 88576C^2a^5b^6c^3z^2 - 15744C^2a^4b^8c^2z^2 + 716800B^2a^6b^3c^5z^2 \\
& - 483840B^2a^5b^5c^4z^2 + 170496B^2a^4b^7c^3z^2 - 33232B^2a^3b^9c^2z^2 + 1468416A^2a^5b^4c^5z^2 - 966144A^2a^4b^6c^4z^2 - 761856A^2a^6b^2c^6z^2 + 326656A^2a^3b^8c^3z^2 - 61440A^2a^2b^{10}c^2z^2 \\
& - 144B^2a^2b^{13}z^2 - 393216C^2a^8c^6z^2 - 64C^2a^2b^{12}z^2 - 294912A^2a^7c^7z^2 - 256A^2b^{14}z^2 - 138240B^2Ca^5b^6c^6z - 432B^2Ca^2b^9c^2z + 245760AC^2a^5b^6c^6z + 12288A^2Ca^2b^8c^3z
\end{aligned}$$

$$\begin{aligned}
& *z + 768*A*C^2*a*b^9*c^2*z + 576*A*B^2*a*b^8*c^3*z + 131328*B^2*C*a^4*b^3*c^5*z - 46656*B^2*C*a^3*b^5*c^4*z + 7344*B^2*C*a^2*b^7*c^3*z - 233472*A*C^2*a^4*b^3*c^5*z + 168960*A^2*C*a^3*b^4*c^5*z - 86016*A^2*C*a^4*b^2*c^6*z + 82944*A*C^2*a^3*b^5*c^4*z - 71424*A^2*C*a^2*b^6*c^4*z - 13056*A*C^2*a^2*b^7*c^3*z - 152064*A*B^2*a^4*b^2*c^6*z + 56448*A*B^2*a^3*b^4*c^5*z - 9312*A*B^2*a^2*b^6*c^4*z + 61440*C^3*a^5*b^2*c^5*z - 21504*C^3*a^4*b^4*c^4*z + 3328*C^3*a^3*b^6*c^3*z - 192*C^3*a^2*b^8*c^2*z - 286720*A^3*a^3*b^3*c^6*z + 104448*A^3*a^2*b^5*c^5*z + 294912*A^3*a^4*b*c^7*z - 16896*A^3*a*b^7*c^4*z - 768*A^2*C*b^10*c^2*z - 147456*A^2*C*a^5*c^7*z + 153600*A*B^2*a^5*c^7*z - 65536*C^3*a^6*c^6*z + 1024*A^3*b^9*c^3*z - 15936*A*B^2*C*a^2*b^2*c^6 + 1648*A*B^2*C*a*b^4*c^5 + 3152*B^2*C^2*a^2*b^3*c^5 - 4992*A^2*C^2*a^2*b^2*c^6 - 7680*B^2*C^2*a^3*b*c^6 - 324*B^2*C^2*a*b^5*c^4 - 5760*A*C^3*a^2*b^3*c^5 + 4608*A^2*C^2*a*b^4*c^5 - 16320*A^2*B^2*a^2*b*c^7 + 7152*A^2*B^2*a*b^3*c^6 + 32256*A^3*C*a^2*b*c^7 + 14336*A*C^3*a^3*b*c^6 - 14080*A^3*C*a*b^3*c^6 + 576*A*C^3*a*b^5*c^4 + 38400*A*B^2*C*a^3*c^7 - 441*B^4*a*b^4*c^5 + 9216*A^4*a*b^2*c^7 + 1536*A^3*C*b^5*c^5 + 1536*C^4*a^3*b^2*c^5 - 144*C^4*a^2*b^4*c^4 + 4200*B^4*a^2*b^2*c^6 - 576*A^2*C^2*b^6*c^4 - 18432*A^2*C^2*a^3*c^7 - 784*A^2*B^2*b^5*c^5 - 4096*C^4*a^4*c^6 - 10000*B^4*a^3*c^7 - 1024*A^4*b^4*c^6 - 20736*A^4*a^2*c^8, z, k)*x*(1310720*a^13*c^8 + 384*a^7*b^12*c^2 - 8960*a^8*b^10*c^3 + 87040*a^9*b^8*c^4 - 450560*a^10*b^6*c^5 + 1310720*a^11*b^4*c^6 - 2031616*a^12*b^2*c^7))/(16*(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*a^9*b^2*c^3))) + (5120*B*C*a^8*c^7 + 96*A*B*a^2*b^11*c^2 - 1664*A*B*a^3*b^9*c^3 + 11072*A*B*a^4*b^7*c^4 - 34752*A*B*a^5*b^5*c^5 + 49792*A*B*a^6*b^3*c^6 - 48*B*C*a^3*b^10*c^2 + 832*B*C*a^4*b^8*c^3 - 5392*B*C*a^5*b^6*c^4 + 15744*B*C*a^6*b^4*c^5 - 18944*B*C*a^7*b^2*c^6 - 24064*A*B*a^7*b*c^7)/(8*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)) + (x*(331776*A^2*a^8*c^9 + 245760*C^2*a^9*c^8 - 512*A^2*a^2*b^12*c^3 + 10112*A^2*a^3*b^10*c^4 - 78592*A^2*a^4*b^8*c^5 + 294784*A^2*a^5*b^6*c^6 - 498432*A^2*a^6*b^4*c^7 + 159744*A^2*a^7*b^2*c^8 + 144*B^2*a^2*b^13*c^2 - 3408*B^2*a^3*b^11*c^3 + 33304*B^2*a^4*b^9*c^4 - 171768*B^2*a^5*b^7*c^5 + 492320*B^2*a^6*b^5*c^6 - 742016*B^2*a^7*b^3*c^7 - 128*C^2*a^4*b^10*c^3 + 2912*C^2*a^5*b^8*c^4 - 26560*C^2*a^6*b^6*c^5 + 120832*C^2*a^7*b^4*c^6 - 273408*C^2*a^8*b^2*c^7 + 458240*B^2*a^8*b*c^8 + 512*A*C*a^3*b^11*c^3 - 10880*A*C*a^4*b^9*c^4 + 92416*A*C*a^5*b^7*c^5 - 391936*A*C*a^6*b^5*c^6 + 829440*A*C*a^7*b^3*c^7 - 700416*A*C*a^8*b*c^8))/(16*(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*a^9*b^2*c^3))) + (63*B^3*a^4*b^3*c^6 + 1440*A^2*B*a^5*c^8 + 4480*B*C^2*a^6*c^7 - 300*B^3*a^5*b*c^7 - 384*A^2*B*a^2*b^6*c^5 + 3440*A^2*B*a^3*b^4*c^6 - 8000*A^2*B*a^4*b^2*c^7 - 144*B*C^2*a^3*b^6*c^4 + 1536*B*C^2*a^4*b^4*c^5 - 4984*B*C^2*a^5*b^2*c^6 - 6112*A*B*C*a^5*b*c^7 + 288*A*B*C*a^2*b^7*c^4 - 2880*A*B*C*a^3*b^5*c^5 + 8464*A*B*C*a^4*b^3*c^6)/(8*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)) + (x*(256*A^3*b^11*c^4 + 20480*C^3*a^7*c^8 + 34048*A^3*a^2*b^7*c^6 - 130816*A^3*a^3*b^5*c^7 + 264320*A^3*a^4*b^3*c^8 - 32*C^3*a^3*b^8*c^4 + 192*C^3*a^4*b^6*c^5 + 1216*C^3*a^5*b^4*c^6 - 11008*C^3*a^6*b^2*c^7 - 163200*A*B^2*a^6*c^9 + 119808*A^2*C*a^6*c^9 - 4608*A^3*a*b^9*c^5 - 225792*A^3*a^5*b*c^9 + 144*A*B^2*a*b^10*c^4 - 46080*A*C^2*a^6*b*c^8 - 384*A^2*C*a*b^10*c^4 + 112320*B^2*C*a^6*b*c^8 - 3120*A*B^2*a^2*b^8*c^5 + 26272*A*B^2*a^3*b^6*c^6 - 107416*A*B^2*a^4*b^4*c^7 + 212928*A*B^2*a^5*b^2*c^8 + 192*A*C^2*a^2*b^9*c^4 - 1920*A*C^2*a^3*b^7*c^5 + 3360*A*C^2*a^4*b^5*c^6 + 16512*A*C^2*a^5*b^3*c^7 + 5376*A^2*C*a^2*b^8*c^5 - 28608*A^2*C*a^3*b^6*c^6 + 76416*A^2*C*a^4*b^4*c^7 - 123648*A^2*C*a^5*b^2*c^8 + 360*B^2*C*a^2*b^9*c^4 - 6072*B^2*C*a^3*b^7*c^5 + 38284*B^2*C*a^4*b^5*c^6 - 107104*B^2*C*a^5*b^3*c^7))/(16*(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*a^9*b^2*c^3))) + (224*A^3*B*b^5*c^6 + 640*B*C^3*a^4*c^7 - 1440*A^2*B*C*a^3*c^8 + 126*A*B^3*a*b^4*c^6 - 1664*A^3*B*a*b^3*c^7 + 2880*A^3*B*a^2*b*c^8 + 300*B^3*C*a^3*b*c^7 - 600*A*B^3*a^2*b^2*c^7 - 136*B*C^3*a^3*b^2*c^6 - 63*B^3*C*a^2*b^3*c^6 - 1824*A*B*C^2*a^3*b*c^7 - 336*A^2*B*C*a*b^4*c^6 + 384*A*B*C^2*a^2*b^3*c^6 + 1920*A^2*B*C*a^2*b^2*c^7)/(8*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)) + (x*(20736*A^4*a^3*c^10 - 512*A^4*b^6*c^7 + 10000*B^4*a^4*c^9 + 9216*A^2*C^2*a^4*c^9 - 18432*A^4*a^2*b^2*c^9 + 441*B^4*a^2
\end{aligned}$$

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*b^4*c^7 - 4200*B^4*a^3*b^2*c^8 - 48*C^4*a^3*b^4*c^6 + 256*C^4*a^4*b^2*c^7
+ 384*A^3*C*b^7*c^6 + 5376*A^4*a*b^4*c^8 - 28800*A*B^2*C*a^4*c^9 + 3072*A*C
^3*a^4*b*c^8 - 3584*A^3*C*a*b^5*c^7 - 9216*A^3*C*a^3*b*c^9 - 288*A^2*B^2*a*
b^5*c^7 - 2880*A^2*B^2*a^3*b*c^9 + 288*A*C^3*a^2*b^5*c^6 - 2048*A*C^3*a^3*b
^3*c^7 - 576*A^2*C^2*a*b^6*c^6 + 10368*A^3*C*a^2*b^3*c^8 + 5440*B^2*C^2*a^4
*b*c^8 + 1936*A^2*B^2*a^2*b^3*c^8 + 4992*A^2*C^2*a^2*b^4*c^7 - 12672*A^2*C^
2*a^3*b^2*c^8 + 216*B^2*C^2*a^2*b^5*c^6 - 2160*B^2*C^2*a^3*b^3*c^7 + 216*A*
B^2*C*a*b^6*c^6 - 3096*A*B^2*C*a^2*b^4*c^7 + 15872*A*B^2*C*a^3*b^2*c^8))/(1
6*(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*a^9*b^2*c^3
)))*root(1572864*a^11*b^2*c^5*z^4 - 983040*a^10*b^4*c^4*z^4 + 327680*a^9*b^
6*c^3*z^4 - 61440*a^8*b^8*c^2*z^4 + 6144*a^7*b^10*c*z^4 - 1048576*a^12*c^6*
z^4 - 256*a^6*b^12*z^4 + 1572864*C*a^9*b^2*c^5*z^3 - 983040*C*a^8*b^4*c^4*z
^3 + 327680*C*a^7*b^6*c^3*z^3 - 61440*C*a^6*b^8*c^2*z^3 - 3145728*A*a^8*b^3
*c^5*z^3 + 1966080*A*a^7*b^5*c^4*z^3 - 655360*A*a^6*b^7*c^3*z^3 + 122880*A*
a^5*b^9*c^2*z^3 + 6144*C*a^5*b^10*c*z^3 + 2097152*A*a^9*b*c^6*z^3 - 12288*A
*a^4*b^11*c*z^3 - 1048576*C*a^10*c^6*z^3 - 256*C*a^4*b^12*z^3 + 512*A*a^3*b
^13*z^3 + 1277952*A*C*a^7*b*c^6*z^2 - 6144*A*C*a^2*b^11*c*z^2 - 1794048*A*C
*a^6*b^3*c^5*z^2 + 1062912*A*C*a^5*b^5*c^4*z^2 - 340480*A*C*a^4*b^7*c^3*z^2
+ 62208*A*C*a^3*b^9*c^2*z^2 + 256*A*C*a*b^13*z^2 + 1536*C^2*a^3*b^10*c*z^2
- 430080*B^2*a^7*b*c^6*z^2 + 3408*B^2*a^2*b^11*c*z^2 + 6144*A^2*a*b^12*c*z
^2 + 516096*C^2*a^7*b^2*c^5*z^2 - 288768*C^2*a^6*b^4*c^4*z^2 + 88576*C^2*a^
5*b^6*c^3*z^2 - 15744*C^2*a^4*b^8*c^2*z^2 + 716800*B^2*a^6*b^3*c^5*z^2 - 48
3840*B^2*a^5*b^5*c^4*z^2 + 170496*B^2*a^4*b^7*c^3*z^2 - 33232*B^2*a^3*b^9*c
^2*z^2 + 1468416*A^2*a^5*b^4*c^5*z^2 - 966144*A^2*a^4*b^6*c^4*z^2 - 761856*
A^2*a^6*b^2*c^6*z^2 + 326656*A^2*a^3*b^8*c^3*z^2 - 61440*A^2*a^2*b^10*c^2*z
^2 - 144*B^2*a*b^13*z^2 - 393216*C^2*a^8*c^6*z^2 - 64*C^2*a^2*b^12*z^2 - 29
4912*A^2*a^7*c^7*z^2 - 256*A^2*b^14*z^2 - 138240*B^2*C*a^5*b*c^6*z - 432*B^
2*C*a*b^9*c^2*z + 245760*A*C^2*a^5*b*c^6*z + 12288*A^2*C*a*b^8*c^3*z + 768*
A*C^2*a*b^9*c^2*z + 576*A*B^2*a*b^8*c^3*z + 131328*B^2*C*a^4*b^3*c^5*z - 46
656*B^2*C*a^3*b^5*c^4*z + 7344*B^2*C*a^2*b^7*c^3*z - 233472*A*C^2*a^4*b^3*c
^5*z + 168960*A^2*C*a^3*b^4*c^5*z - 86016*A^2*C*a^4*b^2*c^6*z + 82944*A*C^2
*a^3*b^5*c^4*z - 71424*A^2*C*a^2*b^6*c^4*z - 13056*A*C^2*a^2*b^7*c^3*z - 15
2064*A*B^2*a^4*b^2*c^6*z + 56448*A*B^2*a^3*b^4*c^5*z - 9312*A*B^2*a^2*b^6*c
^4*z + 61440*C^3*a^5*b^2*c^5*z - 21504*C^3*a^4*b^4*c^4*z + 3328*C^3*a^3*b^6
*c^3*z - 192*C^3*a^2*b^8*c^2*z - 286720*A^3*a^3*b^3*c^6*z + 104448*A^3*a^2*
b^5*c^5*z + 294912*A^3*a^4*b*c^7*z - 16896*A^3*a*b^7*c^4*z - 768*A^2*C*b^10
*c^2*z - 147456*A^2*C*a^5*c^7*z + 153600*A*B^2*a^5*c^7*z - 65536*C^3*a^6*c^
6*z + 1024*A^3*b^9*c^3*z - 15936*A*B^2*C*a^2*b^2*c^6 + 1648*A*B^2*C*a*b^4*c
^5 + 3152*B^2*C^2*a^2*b^3*c^5 - 4992*A^2*C^2*a^2*b^2*c^6 - 7680*B^2*C^2*a^3
*b*c^6 - 324*B^2*C^2*a*b^5*c^4 - 5760*A*C^3*a^2*b^3*c^5 + 4608*A^2*C^2*a*b^
4*c^5 - 16320*A^2*B^2*a^2*b*c^7 + 7152*A^2*B^2*a*b^3*c^6 + 32256*A^3*C*a^2*
b*c^7 + 14336*A*C^3*a^3*b*c^6 - 14080*A^3*C*a*b^3*c^6 + 576*A*C^3*a*b^5*c^4
+ 38400*A*B^2*C*a^3*c^7 - 441*B^4*a*b^4*c^5 + 9216*A^4*a*b^2*c^7 + 1536*A^
3*C*b^5*c^5 + 1536*C^4*a^3*b^2*c^5 - 144*C^4*a^2*b^4*c^4 + 4200*B^4*a^2*b^2
*c^6 - 576*A^2*C^2*b^6*c^4 - 18432*A^2*C^2*a^3*c^7 - 784*A^2*B^2*b^5*c^5 -
4096*C^4*a^4*c^6 - 10000*B^4*a^3*c^7 - 1024*A^4*b^4*c^6 - 20736*A^4*a^2*c^8
, z, k), k, 1, 4) - (A/(2*a) + (B*x)/a - (x^2*(2*A*b^3 - C*a*b^2 + 2*C*a^2*
c - 7*A*a*b*c))/(2*a^2*(4*a*c - b^2)) + (B*x^5*(10*a*c^2 - 3*b^2*c))/(2*a^2
*(4*a*c - b^2)) + (c*x^4*(6*A*a*c - 2*A*b^2 + C*a*b))/(2*a^2*(4*a*c - b^2))
+ (B*b*x^3*(11*a*c - 3*b^2))/(2*a^2*(4*a*c - b^2)))/(a*x^2 + b*x^4 + c*x^6
) - (log(x)*(2*A*b - C*a))/a^3

```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x**2+B*x+A)/x**3/(c*x**4+b*x**2+a)**2,x)

[Out] Timed out

$$3.37 \quad \int (dx)^m (A + Bx + Cx^2) (a + bx^2 + cx^4)^3 dx$$

Optimal. Leaf size=399

$$\frac{a^3 A(dx)^{m+1}}{d(m+1)} + \frac{a^3 B(dx)^{m+2}}{d^2(m+2)} + \frac{a^2(dx)^{m+3}(aC + 3Ab)}{d^3(m+3)} + \frac{3a^2 bB(dx)^{m+4}}{d^4(m+4)} + \frac{3c(dx)^{m+11}(C(ac + b^2) + Abc)}{d^{11}(m+11)} + \frac{(dx)^{m+9}}{d^9(m+9)}$$

Rubi [A] time = 0.42, antiderivative size = 399, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {1628}

$$\frac{a^3(dx)^{m+1}}{d^{m+1}} + \frac{a^3B(dx)^{m+2}}{d^{m+2}} + \frac{3a^2B(dx)^{m+3}}{d^{m+3}} + \frac{3a^2(dx)^{m+4}(aC + 3Ab)}{d^{m+4}} + \frac{3a^2bB(dx)^{m+5}}{d^{m+5}} + \frac{3c(dx)^{m+6}(C(ac + b^2) + Abc)}{d^{m+6}} + \frac{3c(dx)^{m+7}(3Ac + b^2C + 3bC(ac + b^2))}{d^{m+7}} + \frac{3c(dx)^{m+8}(3Ac(ac + b^2) + bC(ac + b^2))}{d^{m+8}} + \frac{3c(dx)^{m+9}(C(ac + b^2) + Abc)}{d^{m+9}} + \frac{3ab(ac + b^2)(dx)^{m+10}}{d^{m+10}} + \frac{3b^2(ac + b^2)(dx)^{m+11}}{d^{m+11}} + \frac{3b^2(dx)^{m+12}(Ac + 3bC)}{d^{m+12}} + \frac{3b^2(dx)^{m+13}(Ac + 3bC)}{d^{m+13}} + \frac{3b^2(dx)^{m+14}(Ac + 3bC)}{d^{m+14}} + \frac{3c(dx)^{m+15}}{d^{m+15}}$$

Antiderivative was successfully verified.

[In] Int[(d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^3,x]

[Out] (a^3*A*(d*x)^(1 + m))/(d*(1 + m)) + (a^3*B*(d*x)^(2 + m))/(d^2*(2 + m)) + (a^2*(3*A*b + a*C)*(d*x)^(3 + m))/(d^3*(3 + m)) + (3*a^2*b*B*(d*x)^(4 + m))/(d^4*(4 + m)) + (3*a*(A*(b^2 + a*c) + a*b*C)*(d*x)^(5 + m))/(d^5*(5 + m)) + (3*a*B*(b^2 + a*c)*(d*x)^(6 + m))/(d^6*(6 + m)) + ((A*(b^3 + 6*a*b*c) + 3*a*(b^2 + a*c)*C)*(d*x)^(7 + m))/(d^7*(7 + m)) + (b*B*(b^2 + 6*a*c)*(d*x)^(8 + m))/(d^8*(8 + m)) + ((3*A*c*(b^2 + a*c) + b*(b^2 + 6*a*c)*C)*(d*x)^(9 + m))/(d^9*(9 + m)) + (3*B*c*(b^2 + a*c)*(d*x)^(10 + m))/(d^10*(10 + m)) + (3*c*(A*b*c + (b^2 + a*c)*C)*(d*x)^(11 + m))/(d^11*(11 + m)) + (3*b*B*c^2*(d*x)^(12 + m))/(d^12*(12 + m)) + (c^2*(A*c + 3*b*C)*(d*x)^(13 + m))/(d^13*(13 + m)) + (B*c^3*(d*x)^(14 + m))/(d^14*(14 + m)) + (c^3*C*(d*x)^(15 + m))/(d^15*(15 + m))

Rule 1628

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int (dx)^m (A + Bx + Cx^2) (a + bx^2 + cx^4)^3 dx &= \int \left(a^3 A(dx)^m + \frac{a^3 B(dx)^{1+m}}{d} + \frac{a^2(3Ab + aC)(dx)^{2+m}}{d^2} + \frac{3a^2 bB(dx)^{3+m}}{d^3} \right. \\ &= \frac{a^3 A(dx)^{1+m}}{d(1+m)} + \frac{a^3 B(dx)^{2+m}}{d^2(2+m)} + \frac{a^2(3Ab + aC)(dx)^{3+m}}{d^3(3+m)} + \frac{3a^2 bB(dx)^{4+m}}{d^4(4+m)} \end{aligned}$$

Mathematica [A] time = 0.92, size = 296, normalized size = 0.74

$$x(dx)^m \left(\frac{a^3 A}{m+1} + \frac{a^3 Bx}{m+2} + \frac{a^2(3Ac + 3Ab)}{m+3} + \frac{3a^2 bBx^3}{m+4} + \frac{3a^2(C(ac + b^2) + Abc)}{m+11} + \frac{3a^2(3Ac(ac + b^2) + bC(ac + b^2))}{m+9} + \frac{3ax^4(A(ac + b^2) + abC)}{m+5} + \frac{x^6(A(6abc + b^2) + 3aC(ac + b^2))}{m+7} + \frac{3Bcx^8(ac + b^2)}{m+10} + \frac{bBx^7(6ac + b^2)}{m+8} + \frac{3aBx^6(ac + b^2)}{m+6} + \frac{c^2x^{12}(Ac + 3bC)}{m+13} + \frac{3bBx^{11}}{m+12} + \frac{Bc^2x^{13}}{m+14} + \frac{c^2Cx^{14}}{m+15} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^3,x]

[Out] x*(d*x)^m*((a^3*A)/(1 + m) + (a^3*B*x)/(2 + m) + (a^2*(3*A*b + a*C)*x^2)/(3 + m) + (3*a^2*b*B*x^3)/(4 + m) + (3*a*(A*(b^2 + a*c) + a*b*C)*x^4)/(5 + m) + (3*a*B*(b^2 + a*c)*x^5)/(6 + m) + ((A*(b^3 + 6*a*b*c) + 3*a*(b^2 + a*c)*C)*x^6)/(7 + m) + (b*B*(b^2 + 6*a*c)*x^7)/(8 + m) + ((3*A*c*(b^2 + a*c) + b*(b^2 + 6*a*c)*C)*x^8)/(9 + m) + (3*B*c*(b^2 + a*c)*x^9)/(10 + m) + (3*c*(A

$*b*c + (b^2 + a*c)*C)*x^{10})/(11 + m) + (3*b*B*c^2*x^{11})/(12 + m) + (c^2*(A*c + 3*b*C)*x^{12})/(13 + m) + (B*c^3*x^{13})/(14 + m) + (c^3*C*x^{14})/(15 + m)$

IntegrateAlgebraic [F] time = 1.79, size = 0, normalized size = 0.00

$$\int (dx)^m (A + Bx + Cx^2) (a + bx^2 + cx^4)^3 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^3,x]

[Out] Defer[IntegrateAlgebraic] [(d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^3, x]

fricas [B] time = 2.26, size = 3898, normalized size = 9.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")

[Out] ((C*c^3*m^14 + 105*C*c^3*m^13 + 5005*C*c^3*m^12 + 143325*C*c^3*m^11 + 2749747*C*c^3*m^10 + 37312275*C*c^3*m^9 + 368411615*C*c^3*m^8 + 2681453775*C*c^3*m^7 + 14409322928*C*c^3*m^6 + 56663366760*C*c^3*m^5 + 159721605680*C*c^3*m^4 + 310989260400*C*c^3*m^3 + 392156797824*C*c^3*m^2 + 283465647360*C*c^3*m + 87178291200*C*c^3)*x^15 + (B*c^3*m^14 + 106*B*c^3*m^13 + 5096*B*c^3*m^12 + 147056*B*c^3*m^11 + 2840838*B*c^3*m^10 + 38786748*B*c^3*m^9 + 385081268*B*c^3*m^8 + 2816490248*B*c^3*m^7 + 15200266081*B*c^3*m^6 + 59999485546*B*c^3*m^5 + 169679309436*B*c^3*m^4 + 331303013496*B*c^3*m^3 + 418753514880*B*c^3*m^2 + 303268406400*B*c^3*m + 93405312000*B*c^3)*x^14 + ((3*C*b*c^2 + A*c^3)*m^14 + 107*(3*C*b*c^2 + A*c^3)*m^13 + 5189*(3*C*b*c^2 + A*c^3)*m^12 + 150943*(3*C*b*c^2 + A*c^3)*m^11 + 2937363*(3*C*b*c^2 + A*c^3)*m^10 + 40372761*(3*C*b*c^2 + A*c^3)*m^9 + 403249847*(3*C*b*c^2 + A*c^3)*m^8 + 2965379989*(3*C*b*c^2 + A*c^3)*m^7 + 16081189696*(3*C*b*c^2 + A*c^3)*m^6 + 63747744632*(3*C*b*c^2 + A*c^3)*m^5 + 180951426864*(3*C*b*c^2 + A*c^3)*m^4 + 30177100800*C*b*c^2 + 100590336000*A*c^3 + 354444796368*(3*C*b*c^2 + A*c^3)*m^3 + 449213351040*(3*C*b*c^2 + A*c^3)*m^2 + 326044051200*(3*C*b*c^2 + A*c^3)*m)*x^13 + 3*(B*b*c^2*m^14 + 108*B*b*c^2*m^13 + 5284*B*b*c^2*m^12 + 154992*B*b*c^2*m^11 + 3039718*B*b*c^2*m^10 + 42081864*B*b*c^2*m^9 + 423113372*B*b*c^2*m^8 + 3130267536*B*b*c^2*m^7 + 17067919121*B*b*c^2*m^6 + 67988181228*B*b*c^2*m^5 + 193813932344*B*b*c^2*m^4 + 381046157472*B*b*c^2*m^3 + 484441814160*B*b*c^2*m^2 + 352515844800*B*b*c^2*m + 108972864000*B*b*c^2)*x^12 + 3*((C*b^2*c + (C*a + A*b)*c^2)*m^14 + 109*(C*b^2*c + (C*a + A*b)*c^2)*m^13 + 5381*(C*b^2*c + (C*a + A*b)*c^2)*m^12 + 159209*(C*b^2*c + (C*a + A*b)*c^2)*m^11 + 3148323*(C*b^2*c + (C*a + A*b)*c^2)*m^10 + 43926927*(C*b^2*c + (C*a + A*b)*c^2)*m^9 + 444899543*(C*b^2*c + (C*a + A*b)*c^2)*m^8 + 3313733027*(C*b^2*c + (C*a + A*b)*c^2)*m^7 + 18180066256*(C*b^2*c + (C*a + A*b)*c^2)*m^6 + 72822481864*(C*b^2*c + (C*a + A*b)*c^2)*m^5 + 208624806576*(C*b^2*c + (C*a + A*b)*c^2)*m^4 + 118879488000*C*b^2*c + 411940473264*(C*b^2*c + (C*a + A*b)*c^2)*m^3 + 118879488000*(C*a + A*b)*c^2 + 525650497920*(C*b^2*c + (C*a + A*b)*c^2)*m^2 + 383662137600*(C*b^2*c + (C*a + A*b)*c^2)*m)*x^11 + 3*((B*b^2*c + B*a*c^2)*m^14 + 110*(B*b^2*c + B*a*c^2)*m^13 + 5480*(B*b^2*c + B*a*c^2)*m^12 + 163600*(B*b^2*c + B*a*c^2)*m^11 + 3263622*(B*b^2*c + B*a*c^2)*m^10 + 45922260*(B*b^2*c + B*a*c^2)*m^9 + 468873140*(B*b^2*c + B*a*c^2)*m^8 + 3518896600*(B*b^2*c + B*a*c^2)*m^7 + 19442163553*(B*b^2*c + B*a*c^2)*m^6 + 78381575150*(B*b^2*c + B*a*c^2)*m^5 + 225856355580*(B*b^2*c + B*a*c^2)*m^4 + 130767436800*B*b^2*c + 130767436800*B*a*c^2 + 448249789800*(B*b^2*c + B*a*c^2)*m^3 + 574497805824*(B*b^2*c + B*a*c^2)*m^2 + 420839556480*(B*b^2*c + B*a*c^2)*m)*x^10 + ((C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^2)*c)*m^14 + 111*(C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^2)*c)*m^13 + 5581*(C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^2)*c)*m^12 + 168171*(C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^2)*c)*m^11 + 168171*(C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^2)*c)*m^10 + 168171*(C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^2)*c)*m^9 + 168171*(C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^2)*c)*m^8 + 168171*(C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^2)*c)*m^7 + 168171*(C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^2)*c)*m^6 + 168171*(C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^2)*c)*m^5 + 168171*(C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^2)*c)*m^4 + 168171*(C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^2)*c)*m^3 + 168171*(C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^2)*c)*m^2 + 168171*(C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^2)*c)*m + 168171*(C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^2)*c)

$*c) * m^{11} + 3386083 * (C * b^3 + 3 * A * a * c^2 + 3 * (2 * C * a * b + A * b^2) * c) * m^{10} + 48083$
 $733 * (C * b^3 + 3 * A * a * c^2 + 3 * (2 * C * a * b + A * b^2) * c) * m^9 + 495342143 * (C * b^3 + 3 * A * a * c^2 + 3 * (2 * C * a * b + A * b^2) * c) * m^8 + 3749548713 * (C * b^3 + 3 * A * a * c^2 + 3 * (2 * C * a * b + A * b^2) * c) * m^7 + 20885191136 * (C * b^3 + 3 * A * a * c^2 + 3 * (2 * C * a * b + A * b^2) * c) * m^6 + 84836490456 * (C * b^3 + 3 * A * a * c^2 + 3 * (2 * C * a * b + A * b^2) * c) * m^5 + 246143692976 * (C * b^3 + 3 * A * a * c^2 + 3 * (2 * C * a * b + A * b^2) * c) * m^4 + 145297152000 * C * b^3 + 435891456000 * A * a * c^2 + 491520108816 * (C * b^3 + 3 * A * a * c^2 + 3 * (2 * C * a * b + A * b^2) * c) * m^3 + 633314724480 * (C * b^3 + 3 * A * a * c^2 + 3 * (2 * C * a * b + A * b^2) * c) * m^2 + 435891456000 * (2 * C * a * b + A * b^2) * c + 465985094400 * (C * b^3 + 3 * A * a * c^2 + 3 * (2 * C * a * b + A * b^2) * c) * m * x^9 + ((B * b^3 + 6 * B * a * b * c) * m^{14} + 112 * (B * b^3 + 6 * B * a * b * c) * m^{13} + 5684 * (B * b^3 + 6 * B * a * b * c) * m^{12} + 172928 * (B * b^3 + 6 * B * a * b * c) * m^{11} + 3516198 * (B * b^3 + 6 * B * a * b * c) * m^{10} + 50428896 * (B * b^3 + 6 * B * a * b * c) * m^9 + 524664572 * (B * b^3 + 6 * B * a * b * c) * m^8 + 4010311424 * (B * b^3 + 6 * B * a * b * c) * m^7 + 22548638161 * (B * b^3 + 6 * B * a * b * c) * m^6 + 92414105392 * (B * b^3 + 6 * B * a * b * c) * m^5 + 270359263944 * (B * b^3 + 6 * B * a * b * c) * m^4 + 163459296000 * B * b^3 + 980755776000 * B * a * b * c + 543939234048 * (B * b^3 + 6 * B * a * b * c) * m^3 + 705481831440 * (B * b^3 + 6 * B * a * b * c) * m^2 + 521962963200 * (B * b^3 + 6 * B * a * b * c) * m * x^8 + ((3 * C * a * b^2 + A * b^3 + 3 * (C * a^2 + 2 * A * a * b) * c) * m^{14} + 113 * (3 * C * a * b^2 + A * b^3 + 3 * (C * a^2 + 2 * A * a * b) * c) * m^{13} + 5789 * (3 * C * a * b^2 + A * b^3 + 3 * (C * a^2 + 2 * A * a * b) * c) * m^{12} + 177877 * (3 * C * a * b^2 + A * b^3 + 3 * (C * a^2 + 2 * A * a * b) * c) * m^{11} + 3654483 * (3 * C * a * b^2 + A * b^3 + 3 * (C * a^2 + 2 * A * a * b) * c) * m^{10} + 52977099 * (3 * C * a * b^2 + A * b^3 + 3 * (C * a^2 + 2 * A * a * b) * c) * m^9 + 557256047 * (3 * C * a * b^2 + A * b^3 + 3 * (C * a^2 + 2 * A * a * b) * c) * m^8 + 4306835671 * (3 * C * a * b^2 + A * b^3 + 3 * (C * a^2 + 2 * A * a * b) * c) * m^7 + 2448327985 * (3 * C * a * b^2 + A * b^3 + 3 * (C * a^2 + 2 * A * a * b) * c) * m^6 + 101420251688 * (3 * C * a * b^2 + A * b^3 + 3 * (C * a^2 + 2 * A * a * b) * c) * m^5 + 299730345264 * (3 * C * a * b^2 + A * b^3 + 3 * (C * a^2 + 2 * A * a * b) * c) * m^4 + 560431872000 * C * a * b^2 + 186810624000 * A * b^3 + 608700928752 * (3 * C * a * b^2 + A * b^3 + 3 * (C * a^2 + 2 * A * a * b) * c) * m^3 + 796089202560 * (3 * C * a * b^2 + A * b^3 + 3 * (C * a^2 + 2 * A * a * b) * c) * m^2 + 560431872000 * (C * a^2 + 2 * A * a * b) * c + 593193196800 * (3 * C * a * b^2 + A * b^3 + 3 * (C * a^2 + 2 * A * a * b) * c) * m * x^7 + 3 * ((B * a * b^2 + B * a^2 * c) * m^{14} + 114 * (B * a * b^2 + B * a^2 * c) * m^{13} + 5896 * (B * a * b^2 + B * a^2 * c) * m^{12} + 183024 * (B * a * b^2 + B * a^2 * c) * m^{11} + 3801478 * (B * a * b^2 + B * a^2 * c) * m^{10} + 55749612 * (B * a * b^2 + B * a^2 * c) * m^9 + 593598068 * (B * a * b^2 + B * a^2 * c) * m^8 + 4646039592 * (B * a * b^2 + B * a^2 * c) * m^7 + 26754892001 * (B * a * b^2 + B * a^2 * c) * m^6 + 112273858674 * (B * a * b^2 + B * a^2 * c) * m^5 + 336028955036 * (B * a * b^2 + B * a^2 * c) * m^4 + 217945728000 * B * a * b^2 + 217945728000 * B * a^2 * c + 690639615384 * (B * a * b^2 + B * a^2 * c) * m^3 + 913158011520 * (B * a * b^2 + B * a^2 * c) * m^2 + 686869545600 * (B * a * b^2 + B * a^2 * c) * m * x^6 + 3 * ((C * a^2 * b + A * a * b^2 + A * a^2 * c) * m^{14} + 115 * (C * a^2 * b + A * a * b^2 + A * a^2 * c) * m^{13} + 6005 * (C * a^2 * b + A * a * b^2 + A * a^2 * c) * m^{12} + 188375 * (C * a^2 * b + A * a * b^2 + A * a^2 * c) * m^{11} + 3957747 * (C * a^2 * b + A * a * b^2 + A * a^2 * c) * m^{10} + 58769745 * (C * a^2 * b + A * a * b^2 + A * a^2 * c) * m^9 + 634247015 * (C * a^2 * b + A * a * b^2 + A * a^2 * c) * m^8 + 5036392925 * (C * a^2 * b + A * a * b^2 + A * a^2 * c) * m^7 + 29449164928 * (C * a^2 * b + A * a * b^2 + A * a^2 * c) * m^6 + 125557386040 * (C * a^2 * b + A * a * b^2 + A * a^2 * c) * m^5 + 381885176880 * (C * a^2 * b + A * a * b^2 + A * a^2 * c) * m^4 + 261534873600 * C * a^2 * b + 261534873600 * A * a * b^2 + 261534873600 * A * a^2 * c + 797387461200 * (C * a^2 * b + A * a * b^2 + A * a^2 * c) * m^3 + 1070058397824 * (C * a^2 * b + A * a * b^2 + A * a^2 * c) * m^2 + 815525625600 * (C * a^2 * b + A * a * b^2 + A * a^2 * c) * m * x^5 + 3 * (B * a^2 * b * m^{14} + 116 * B * a^2 * b * m^{13} + 6116 * B * a^2 * b * m^{12} + 193936 * B * a^2 * b * m^{11} + 4123878 * B * a^2 * b * m^{10} + 62062968 * B * a^2 * b * m^9 + 679843868 * B * a^2 * b * m^8 + 5488252528 * B * a^2 * b * m^7 + 32678119441 * B * a^2 * b * m^6 + 142090732916 * B * a^2 * b * m^5 + 441309175416 * B * a^2 * b * m^4 + 941576643936 * B * a^2 * b * m^3 + 1290689128080 * B * a^2 * b * m^2 + 1003061102400 * B * a^2 * b * m + 326918592000 * B * a^2 * b) * x^4 + ((C * a^3 + 3 * A * a^2 * b) * m^{14} + 117 * (C * a^3 + 3 * A * a^2 * b) * m^{13} + 6229 * (C * a^3 + 3 * A * a^2 * b) * m^{12} + 199713 * (C * a^3 + 3 * A * a^2 * b) * m^{11} + 4300483 * (C * a^3 + 3 * A * a^2 * b) * m^{10} + 65657031 * (C * a^3 + 3 * A * a^2 * b) * m^9 + 731124647 * (C * a^3 + 3 * A * a^2 * b) * m^8 + 6014254059 * (C * a^3 + 3 * A * a^2 * b) * m^7 + 36588367376 * (C * a^3 + 3 * A * a^2 * b) * m^6 + 163038108552 * (C * a^3 + 3 * A * a^2 * b) * m^5 + 520557781424 * (C * a^3 + 3 * A * a^2 * b) * m^4 + 43589145600 * C * a^3 + 1307674368000 * A * a^2 * b + 1145140001328 * (C * a^3 + 3 * A * a^2 * b) * m^3 + 1621575699840 * (C * a^3 + 3 * A * a^2 * b) * m^2 + 1301090515200 * (C * a^3 + 3 * A * a^2 * b) * m * x^3 + (B * a^3 * m^{14} + 118 * B * a^3 * m^{13} + 6344 * B * a^3 * m^{12} + 205712 * B * a^3 * m^{11} +$

$$4488198*B*a^3*m^{10} + 69582084*B*a^3*m^9 + 788931572*B*a^3*m^8 + 6629764856*B*a^3*m^7 + 41371599841*B*a^3*m^6 + 190060010998*B*a^3*m^5 + 629552085084*B*a^3*m^4 + 1447709175432*B*a^3*m^3 + 2161577352960*B*a^3*m^2 + 1842662908800*B*a^3*m + 653837184000*B*a^3)*x^2 + (A*a^3*m^{14} + 119*A*a^3*m^{13} + 6461*A*a^3*m^{12} + 211939*A*a^3*m^{11} + 4687683*A*a^3*m^{10} + 73870797*A*a^3*m^9 + 854224943*A*a^3*m^8 + 7353403057*A*a^3*m^7 + 47277726496*A*a^3*m^6 + 225525484184*A*a^3*m^5 + 784146622896*A*a^3*m^4 + 1922666722704*A*a^3*m^3 + 3134328981120*A*a^3*m^2 + 3031488633600*A*a^3*m + 1307674368000*A*a^3)*x)*(d*x)^m/(m^{15} + 120*m^{14} + 6580*m^{13} + 218400*m^{12} + 4899622*m^{11} + 78558480*m^{10} + 928095740*m^9 + 8207628000*m^8 + 54631129553*m^7 + 272803210680*m^6 + 1009672107080*m^5 + 2706813345600*m^4 + 5056995703824*m^3 + 6165817614720*m^2 + 4339163001600*m + 1307674368000)$$

giac [B] time = 1.13, size = 7808, normalized size = 19.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^3,x, algorithm="giac")

[Out] ((d*x)^m*C*c^3*m^{14}*x^{15} + (d*x)^m*B*c^3*m^{14}*x^{14} + 105*(d*x)^m*C*c^3*m^{13}*x^{15} + 3*(d*x)^m*C*b*c^2*m^{14}*x^{13} + (d*x)^m*A*c^3*m^{14}*x^{13} + 106*(d*x)^m*B*c^3*m^{13}*x^{14} + 5005*(d*x)^m*C*c^3*m^{12}*x^{15} + 3*(d*x)^m*B*b*c^2*m^{14}*x^{12} + 321*(d*x)^m*C*b*c^2*m^{13}*x^{13} + 107*(d*x)^m*A*c^3*m^{13}*x^{13} + 5096*(d*x)^m*B*c^3*m^{12}*x^{14} + 143325*(d*x)^m*C*c^3*m^{11}*x^{15} + 3*(d*x)^m*C*b^2*c*m^{14}*x^{11} + 3*(d*x)^m*C*a*c^2*m^{14}*x^{11} + 3*(d*x)^m*A*b*c^2*m^{14}*x^{11} + 324*(d*x)^m*B*b*c^2*m^{13}*x^{12} + 15567*(d*x)^m*C*b*c^2*m^{12}*x^{13} + 5189*(d*x)^m*A*c^3*m^{12}*x^{13} + 147056*(d*x)^m*B*c^3*m^{11}*x^{14} + 2749747*(d*x)^m*C*c^3*m^{10}*x^{15} + 3*(d*x)^m*B*b^2*c*m^{14}*x^{10} + 3*(d*x)^m*B*a*c^2*m^{14}*x^{10} + 327*(d*x)^m*C*b^2*c*m^{13}*x^{11} + 327*(d*x)^m*C*a*c^2*m^{13}*x^{11} + 327*(d*x)^m*A*b*c^2*m^{13}*x^{11} + 15852*(d*x)^m*B*b*c^2*m^{12}*x^{12} + 452829*(d*x)^m*C*b*c^2*m^{11}*x^{13} + 150943*(d*x)^m*A*c^3*m^{11}*x^{13} + 2840838*(d*x)^m*B*c^3*m^{10}*x^{14} + 37312275*(d*x)^m*C*c^3*m^9*x^{15} + (d*x)^m*C*b^3*m^{14}*x^9 + 6*(d*x)^m*C*a*b*c*m^{14}*x^9 + 3*(d*x)^m*A*b^2*c*m^{14}*x^9 + 3*(d*x)^m*A*a*c^2*m^{14}*x^9 + 330*(d*x)^m*B*b^2*c*m^{13}*x^{10} + 330*(d*x)^m*B*a*c^2*m^{13}*x^{10} + 16143*(d*x)^m*C*b^2*c*m^{12}*x^{11} + 16143*(d*x)^m*C*a*c^2*m^{12}*x^{11} + 16143*(d*x)^m*A*b*c^2*m^{12}*x^{11} + 464976*(d*x)^m*B*b*c^2*m^{11}*x^{12} + 8812089*(d*x)^m*C*b*c^2*m^{10}*x^{13} + 2937363*(d*x)^m*A*c^3*m^{10}*x^{13} + 38786748*(d*x)^m*B*c^3*m^9*x^{14} + 368411615*(d*x)^m*C*c^3*m^8*x^{15} + (d*x)^m*B*b^3*m^{14}*x^8 + 6*(d*x)^m*B*a*b*c*m^{14}*x^8 + 111*(d*x)^m*C*b^3*m^{13}*x^9 + 666*(d*x)^m*C*a*b*c*m^{13}*x^9 + 333*(d*x)^m*A*b^2*c*m^{13}*x^9 + 333*(d*x)^m*A*a*c^2*m^{13}*x^9 + 16440*(d*x)^m*B*b^2*c*m^{12}*x^{10} + 16440*(d*x)^m*B*a*c^2*m^{12}*x^{10} + 477627*(d*x)^m*C*b^2*c*m^{11}*x^{11} + 477627*(d*x)^m*C*a*c^2*m^{11}*x^{11} + 477627*(d*x)^m*A*b*c^2*m^{11}*x^{11} + 9119154*(d*x)^m*B*b*c^2*m^{10}*x^{12} + 121118283*(d*x)^m*C*b*c^2*m^9*x^{13} + 40372761*(d*x)^m*A*c^3*m^9*x^{13} + 385081268*(d*x)^m*B*c^3*m^8*x^{14} + 2681453775*(d*x)^m*C*c^3*m^7*x^{15} + 3*(d*x)^m*C*a*b^2*m^{14}*x^7 + (d*x)^m*A*b^3*m^{14}*x^7 + 3*(d*x)^m*C*a^2*c*m^{14}*x^7 + 6*(d*x)^m*A*a*b*c*m^{14}*x^7 + 112*(d*x)^m*B*b^3*m^{13}*x^8 + 672*(d*x)^m*B*a*b*c*m^{13}*x^8 + 5581*(d*x)^m*C*b^3*m^{12}*x^9 + 33486*(d*x)^m*C*a*b*c*m^{12}*x^9 + 16743*(d*x)^m*A*b^2*c*m^{12}*x^9 + 16743*(d*x)^m*A*a*c^2*m^{12}*x^9 + 490800*(d*x)^m*B*b^2*c*m^{11}*x^{10} + 490800*(d*x)^m*B*a*c^2*m^{11}*x^{10} + 9444969*(d*x)^m*C*b^2*c*m^{10}*x^{11} + 9444969*(d*x)^m*C*a*c^2*m^{10}*x^{11} + 9444969*(d*x)^m*A*b*c^2*m^{10}*x^{11} + 126245592*(d*x)^m*B*b*c^2*m^9*x^{12} + 1209749541*(d*x)^m*C*b*c^2*m^8*x^{13} + 403249847*(d*x)^m*A*c^3*m^8*x^{13} + 2816490248*(d*x)^m*B*c^3*m^7*x^{14} + 14409322928*(d*x)^m*C*c^3*m^6*x^{15} + 3*(d*x)^m*B*a*b^2*m^{14}*x^6 + 3*(d*x)^m*B*a^2*c*m^{14}*x^6 + 339*(d*x)^m*C*a*b^2*m^{13}*x^7 + 113*(d*x)^m*A*b^3*m^{13}*x^7 + 339*(d*x)^m*C*a^2*c*m^{13}*x^7 + 678*(d*x)^m*A*a*b*c*m^{13}*x^7 + 5684*(d*x)^m*B*b^3*m^{12}*x^8 + 34104*(d*x)^m*B*a*b*c*m^{12}*x^8 + 168171*(d*x)^m*C*b^3*m^{11}*x^9 + 1009026*(d*x)^m*C*a*b*c*m^{11}*x^9 + 504513*(d*x)^m*A*b^2*c*m^{11}*x^9 + 504513*(d*x)^m*A*a*c^2*m^{11}*x^9 + 9790866*(d*x)^m*B*b^2*c*m^{10}*x^{10} + 9790866*(

$$\begin{aligned}
& (d*x)^m*B*a*c^2*m^10*x^10 + 131780781*(d*x)^m*C*b^2*c*m^9*x^11 + 131780781*(d*x)^m*C*a*c^2*m^9*x^11 + 131780781*(d*x)^m*A*b*c^2*m^9*x^11 + 1269340116*(d*x)^m*B*b*c^2*m^8*x^12 + 8896139967*(d*x)^m*C*b*c^2*m^7*x^13 + 2965379989*(d*x)^m*A*c^3*m^7*x^13 + 15200266081*(d*x)^m*B*c^3*m^6*x^14 + 56663366760*(d*x)^m*C*c^3*m^5*x^15 + 3*(d*x)^m*C*a^2*b*m^14*x^5 + 3*(d*x)^m*A*a*b^2*m^14*x^5 + 3*(d*x)^m*A*a^2*c*m^14*x^5 + 342*(d*x)^m*B*a*b^2*m^13*x^6 + 342*(d*x)^m*B*a^2*c*m^13*x^6 + 17367*(d*x)^m*C*a*b^2*m^12*x^7 + 5789*(d*x)^m*A*b^3*m^12*x^7 + 17367*(d*x)^m*C*a^2*c*m^12*x^7 + 34734*(d*x)^m*A*a*b*c*m^12*x^7 + 172928*(d*x)^m*B*b^3*m^11*x^8 + 1037568*(d*x)^m*B*a*b*c*m^11*x^8 + 3386083*(d*x)^m*C*b^3*m^10*x^9 + 20316498*(d*x)^m*C*a*b*c*m^10*x^9 + 10158249*(d*x)^m*A*b^2*c*m^10*x^9 + 10158249*(d*x)^m*A*a*c^2*m^10*x^9 + 137766780*(d*x)^m*B*b^2*c*m^9*x^10 + 137766780*(d*x)^m*B*a*c^2*m^9*x^10 + 1334698629*(d*x)^m*C*b^2*c*m^8*x^11 + 1334698629*(d*x)^m*C*a*c^2*m^8*x^11 + 1334698629*(d*x)^m*A*b*c^2*m^8*x^11 + 9390802608*(d*x)^m*B*b*c^2*m^7*x^12 + 48243569088*(d*x)^m*C*b*c^2*m^6*x^13 + 16081189696*(d*x)^m*A*c^3*m^6*x^13 + 59999485546*(d*x)^m*B*c^3*m^5*x^14 + 159721605680*(d*x)^m*C*c^3*m^4*x^15 + 3*(d*x)^m*B*a^2*b*m^14*x^4 + 345*(d*x)^m*C*a^2*b*m^13*x^5 + 345*(d*x)^m*A*a*b^2*m^13*x^5 + 345*(d*x)^m*A*a^2*c*m^13*x^5 + 17688*(d*x)^m*B*a*b^2*m^12*x^6 + 17688*(d*x)^m*B*a^2*c*m^12*x^6 + 533631*(d*x)^m*C*a*b^2*m^11*x^7 + 177877*(d*x)^m*A*b^3*m^11*x^7 + 533631*(d*x)^m*C*a^2*c*m^11*x^7 + 1067262*(d*x)^m*A*a*b*c*m^11*x^7 + 3516198*(d*x)^m*B*b^3*m^10*x^8 + 21097188*(d*x)^m*B*a*b*c*m^10*x^8 + 48083733*(d*x)^m*C*b^3*m^9*x^9 + 288502398*(d*x)^m*C*a*b*c*m^9*x^9 + 144251199*(d*x)^m*A*b^2*c*m^9*x^9 + 144251199*(d*x)^m*A*a*c^2*m^9*x^9 + 1406619420*(d*x)^m*B*b^2*c*m^8*x^10 + 1406619420*(d*x)^m*B*a*c^2*m^8*x^10 + 9941199081*(d*x)^m*C*b^2*c*m^7*x^11 + 9941199081*(d*x)^m*C*a*c^2*m^7*x^11 + 9941199081*(d*x)^m*A*b*c^2*m^7*x^11 + 51203757363*(d*x)^m*B*b*c^2*m^6*x^12 + 191243233896*(d*x)^m*C*b*c^2*m^5*x^13 + 63747744632*(d*x)^m*A*c^3*m^5*x^13 + 169679309436*(d*x)^m*B*c^3*m^4*x^14 + 310989260400*(d*x)^m*C*c^3*m^3*x^15 + (d*x)^m*C*a^3*m^14*x^3 + 3*(d*x)^m*A*a^2*b*m^14*x^3 + 348*(d*x)^m*B*a^2*b*m^13*x^4 + 18015*(d*x)^m*C*a^2*b*m^12*x^5 + 18015*(d*x)^m*A*a*b^2*m^12*x^5 + 18015*(d*x)^m*A*a^2*c*m^12*x^5 + 549072*(d*x)^m*B*a*b^2*m^11*x^6 + 549072*(d*x)^m*B*a^2*c*m^11*x^6 + 10963449*(d*x)^m*C*a*b^2*m^10*x^7 + 3654483*(d*x)^m*A*b^3*m^10*x^7 + 10963449*(d*x)^m*C*a^2*c*m^10*x^7 + 21926898*(d*x)^m*A*a*b*c*m^10*x^7 + 50428896*(d*x)^m*B*b^3*m^9*x^8 + 302573376*(d*x)^m*B*a*b*c*m^9*x^8 + 495342143*(d*x)^m*C*b^3*m^8*x^9 + 2972052858*(d*x)^m*C*a*b*c*m^8*x^9 + 1486026429*(d*x)^m*A*b^2*c*m^8*x^9 + 1486026429*(d*x)^m*A*a*c^2*m^8*x^9 + 10556689800*(d*x)^m*B*b^2*c*m^7*x^10 + 10556689800*(d*x)^m*B*a*c^2*m^7*x^10 + 54540198768*(d*x)^m*C*b^2*c*m^6*x^11 + 54540198768*(d*x)^m*C*a*c^2*m^6*x^11 + 54540198768*(d*x)^m*A*b*c^2*m^6*x^11 + 203964543684*(d*x)^m*B*b*c^2*m^5*x^12 + 542854280592*(d*x)^m*C*b*c^2*m^4*x^13 + 180951426864*(d*x)^m*A*c^3*m^4*x^13 + 331303013496*(d*x)^m*B*c^3*m^3*x^14 + 392156797824*(d*x)^m*C*c^3*m^2*x^15 + (d*x)^m*B*a^3*m^14*x^2 + 117*(d*x)^m*C*a^3*m^13*x^3 + 351*(d*x)^m*A*a^2*b*m^13*x^3 + 18348*(d*x)^m*B*a^2*b*m^12*x^4 + 565125*(d*x)^m*C*a^2*b*m^11*x^5 + 565125*(d*x)^m*A*a*b^2*m^11*x^5 + 565125*(d*x)^m*A*a^2*c*m^11*x^5 + 11404434*(d*x)^m*B*a*b^2*m^10*x^6 + 11404434*(d*x)^m*B*a^2*c*m^10*x^6 + 158931297*(d*x)^m*C*a*b^2*m^9*x^7 + 52977099*(d*x)^m*A*b^3*m^9*x^7 + 158931297*(d*x)^m*C*a^2*c*m^9*x^7 + 317862594*(d*x)^m*A*a*b*c*m^9*x^7 + 524664572*(d*x)^m*B*b^3*m^8*x^8 + 3147987432*(d*x)^m*B*a*b*c*m^8*x^8 + 3749548713*(d*x)^m*C*b^3*m^7*x^9 + 22497292278*(d*x)^m*C*a*b*c*m^7*x^9 + 11248646139*(d*x)^m*A*b^2*c*m^7*x^9 + 11248646139*(d*x)^m*A*a*c^2*m^7*x^9 + 58326490659*(d*x)^m*B*b^2*c*m^6*x^10 + 58326490659*(d*x)^m*B*a*c^2*m^6*x^10 + 218467445592*(d*x)^m*C*b^2*c*m^5*x^11 + 218467445592*(d*x)^m*C*a*c^2*m^5*x^11 + 218467445592*(d*x)^m*A*b*c^2*m^5*x^11 + 581441797032*(d*x)^m*B*b*c^2*m^4*x^12 + 1063334389104*(d*x)^m*C*b*c^2*m^3*x^13 + 354444796368*(d*x)^m*A*c^3*m^3*x^13 + 418753514880*(d*x)^m*B*c^3*m^2*x^14 + 283465647360*(d*x)^m*C*c^3*m*x^15 + (d*x)^m*A*a^3*m^14*x + 118*(d*x)^m*B*a^3*m^13*x^2 + 6229*(d*x)^m*C*a^3*m^12*x^3 + 18687*(d*x)^m*A*a^2*b*m^12*x^3 + 581808*(d*x)^m*B*a^2*b*m^11*x^4 + 11873241*(d*x)^m*C*a^2*b*m^10*x^5 + 11873241*(d*x)^m*A*a*b^2*m^10*x^5 + 11873241*(d*x)^m*A*a^2*c*m^10*x^5 + 167248836*(d*x)^m*B*a*b^2*
\end{aligned}$$

$m^9x^6 + 167248836(d*x)^mB*a^2*c*m^9x^6 + 1671768141(d*x)^mC*a*b^2*m^8x^7 + 557256047(d*x)^m*A*b^3*m^8x^7 + 1671768141(d*x)^mC*a^2*c*m^8x^7 + 3343536282(d*x)^m*A*a*b*c*m^8x^7 + 4010311424(d*x)^m*B*b^3*m^7x^8 + 24061868544(d*x)^m*B*a*b*c*m^7x^8 + 20885191136(d*x)^mC*b^3*m^6x^9 + 125311146816(d*x)^mC*a*b*c*m^6x^9 + 62655573408(d*x)^m*A*b^2*c*m^6x^9 + 62655573408(d*x)^m*A*a*c^2*m^6x^9 + 235144725450(d*x)^m*B*b^2*c*m^5x^10 + 235144725450(d*x)^m*B*a*c^2*m^5x^10 + 625874419728(d*x)^mC*b^2*c*m^4x^11 + 625874419728(d*x)^mC*a*c^2*m^4x^11 + 625874419728(d*x)^m*A*b*c^2*m^4x^11 + 1143138472416(d*x)^m*B*b*c^2*m^3x^12 + 1347640053120(d*x)^mC*b*c^2*m^2x^13 + 449213351040(d*x)^m*A*c^3*m^2x^13 + 303268406400(d*x)^m*B*c^3*m*x^14 + 87178291200(d*x)^mC*c^3x^15 + 119(d*x)^m*A*a^3*m^13x + 6344(d*x)^m*B*a^3*m^12x^2 + 199713(d*x)^mC*a^3*m^11x^3 + 599139(d*x)^m*A*a^2*b*m^11x^3 + 12371634(d*x)^m*B*a^2*b*m^10x^4 + 176309235(d*x)^mC*a^2*b*m^9x^5 + 176309235(d*x)^m*A*a*b^2*m^9x^5 + 176309235(d*x)^m*A*a^2*c*m^9x^5 + 1780794204(d*x)^m*B*a*b^2*m^8x^6 + 1780794204(d*x)^m*B*a^2*c*m^8x^6 + 12920507013(d*x)^mC*a*b^2*m^7x^7 + 4306835671(d*x)^m*A*b^3*m^7x^7 + 12920507013(d*x)^mC*a^2*c*m^7x^7 + 25841014026(d*x)^m*A*a*b*c*m^7x^7 + 22548638161(d*x)^m*B*b^3*m^6x^8 + 135291828966(d*x)^m*B*a*b*c*m^6x^8 + 84836490456(d*x)^mC*b^3*m^5x^9 + 509018942736(d*x)^mC*a*b*c*m^5x^9 + 254509471368(d*x)^m*A*b^2*c*m^5x^9 + 254509471368(d*x)^m*A*a*c^2*m^5x^9 + 677569066740(d*x)^m*B*b^2*c*m^4x^10 + 677569066740(d*x)^m*B*a*c^2*m^4x^10 + 1235821419792(d*x)^mC*b^2*c*m^3x^11 + 1235821419792(d*x)^mC*a*c^2*m^3x^11 + 1235821419792(d*x)^m*A*b*c^2*m^3x^11 + 1453325442480(d*x)^m*B*b*c^2*m^2x^12 + 978132153600(d*x)^mC*b*c^2*m*x^13 + 326044051200(d*x)^m*A*c^3*m*x^13 + 93405312000(d*x)^m*B*c^3x^14 + 6461(d*x)^m*A*a^3*m^12x + 205712(d*x)^m*B*a^3*m^11x^2 + 4300483(d*x)^mC*a^3*m^10x^3 + 12901449(d*x)^m*A*a^2*b*m^10x^3 + 186188904(d*x)^m*B*a^2*b*m^9x^4 + 1902741045(d*x)^mC*a^2*b*m^8x^5 + 1902741045(d*x)^m*A*a*b^2*m^8x^5 + 1902741045(d*x)^m*A*a^2*c*m^8x^5 + 13938118776(d*x)^m*B*a*b^2*m^7x^6 + 13938118776(d*x)^m*B*a^2*c*m^7x^6 + 73449839568(d*x)^mC*a*b^2*m^6x^7 + 24483279856(d*x)^m*A*b^3*m^6x^7 + 73449839568(d*x)^mC*a^2*c*m^6x^7 + 146899679136(d*x)^m*A*a*b*c*m^6x^7 + 92414105392(d*x)^m*B*b^3*m^5x^8 + 554484632352(d*x)^m*B*a*b*c*m^5x^8 + 246143692976(d*x)^mC*b^3*m^4x^9 + 1476862157856(d*x)^mC*a*b*c*m^4x^9 + 738431078928(d*x)^m*A*b^2*c*m^4x^9 + 738431078928(d*x)^m*A*a*c^2*m^4x^9 + 1344749369400(d*x)^m*B*b^2*c*m^3x^10 + 1344749369400(d*x)^m*B*a*c^2*m^3x^10 + 1576951493760(d*x)^mC*b^2*c*m^2x^11 + 1576951493760(d*x)^mC*a*c^2*m^2x^11 + 1576951493760(d*x)^m*A*b*c^2*m^2x^11 + 1057547534400(d*x)^m*B*b*c^2*m*x^12 + 301771008000(d*x)^mC*b*c^2x^13 + 100590336000(d*x)^m*A*c^3x^13 + 211939(d*x)^m*A*a^3*m^11x + 4488198(d*x)^m*B*a^3*m^10x^2 + 65657031(d*x)^mC*a^3*m^9x^3 + 196971093(d*x)^m*A*a^2*b*m^9x^3 + 2039531604(d*x)^m*B*a^2*b*m^8x^4 + 15109178775(d*x)^mC*a^2*b*m^7x^5 + 15109178775(d*x)^m*A*a*b^2*m^7x^5 + 15109178775(d*x)^m*A*a^2*c*m^7x^5 + 80264676003(d*x)^m*B*a*b^2*m^6x^6 + 80264676003(d*x)^m*B*a^2*c*m^6x^6 + 304260755064(d*x)^mC*a*b^2*m^5x^7 + 101420251688(d*x)^m*A*b^3*m^5x^7 + 304260755064(d*x)^mC*a^2*c*m^5x^7 + 608521510128(d*x)^m*A*a*b*c*m^5x^7 + 270359263944(d*x)^m*B*b^3*m^4x^8 + 1622155583664(d*x)^m*B*a*b*c*m^4x^8 + 491520108816(d*x)^mC*b^3*m^3x^9 + 2949120652896(d*x)^mC*a*b*c*m^3x^9 + 1474560326448(d*x)^m*A*b^2*c*m^3x^9 + 1474560326448(d*x)^m*A*a*c^2*m^3x^9 + 1723493417472(d*x)^m*B*b^2*c*m^2x^10 + 1723493417472(d*x)^m*B*a*c^2*m^2x^10 + 1150986412800(d*x)^mC*b^2*c*m*x^11 + 1150986412800(d*x)^mC*a*c^2*m*x^11 + 1150986412800(d*x)^m*A*b*c^2*m*x^11 + 326918592000(d*x)^m*B*b*c^2x^12 + 4687683(d*x)^m*A*a^3*m^10x + 69582084(d*x)^m*B*a^3*m^9x^2 + 731124647(d*x)^mC*a^3*m^8x^3 + 2193373941(d*x)^m*A*a^2*b*m^8x^3 + 16464757584(d*x)^m*B*a^2*b*m^7x^4 + 88347494784(d*x)^mC*a^2*b*m^6x^5 + 88347494784(d*x)^m*A*a*b^2*m^6x^5 + 88347494784(d*x)^m*A*a^2*c*m^6x^5 + 336821576022(d*x)^m*B*a*b^2*m^5x^6 + 336821576022(d*x)^m*B*a^2*c*m^5x^6 + 899191035792(d*x)^mC*a*b^2*m^4x^7 + 299730345264(d*x)^m*A*b^3*m^4x^7 + 899191035792(d*x)^mC*a^2*c*m^4x^7 + 1798382071584(d*x)^m*A*a*b*c*m^4x^7 + 543$

$939234048*(d*x)^m*B*b^3*m^3*x^8 + 3263635404288*(d*x)^m*B*a*b*c*m^3*x^8 + 6$
 $33314724480*(d*x)^m*C*b^3*m^2*x^9 + 3799888346880*(d*x)^m*C*a*b*c*m^2*x^9 +$
 $1899944173440*(d*x)^m*A*b^2*c*m^2*x^9 + 1899944173440*(d*x)^m*A*a*c^2*m^2*$
 $x^9 + 1262518669440*(d*x)^m*B*b^2*c*m*x^10 + 1262518669440*(d*x)^m*B*a*c^2*$
 $m*x^10 + 356638464000*(d*x)^m*C*b^2*c*x^11 + 356638464000*(d*x)^m*C*a*c^2*x$
 $^11 + 356638464000*(d*x)^m*A*b*c^2*x^11 + 73870797*(d*x)^m*A*a^3*m^9*x + 78$
 $8931572*(d*x)^m*B*a^3*m^8*x^2 + 6014254059*(d*x)^m*C*a^3*m^7*x^3 + 18042762$
 $177*(d*x)^m*A*a^2*b*m^7*x^3 + 98034358323*(d*x)^m*B*a^2*b*m^6*x^4 + 3766721$
 $58120*(d*x)^m*C*a^2*b*m^5*x^5 + 376672158120*(d*x)^m*A*a*b^2*m^5*x^5 + 3766$
 $72158120*(d*x)^m*A*a^2*c*m^5*x^5 + 1008086865108*(d*x)^m*B*a*b^2*m^4*x^6 +$
 $1008086865108*(d*x)^m*B*a^2*c*m^4*x^6 + 1826102786256*(d*x)^m*C*a*b^2*m^3*x$
 $^7 + 608700928752*(d*x)^m*A*b^3*m^3*x^7 + 1826102786256*(d*x)^m*C*a^2*c*m^3$
 $*x^7 + 3652205572512*(d*x)^m*A*a*b*c*m^3*x^7 + 705481831440*(d*x)^m*B*b^3*m$
 $^2*x^8 + 4232890988640*(d*x)^m*B*a*b*c*m^2*x^8 + 465985094400*(d*x)^m*C*b^3$
 $*m*x^9 + 2795910566400*(d*x)^m*C*a*b*c*m*x^9 + 1397955283200*(d*x)^m*A*b^2*$
 $c*m*x^9 + 1397955283200*(d*x)^m*A*a*c^2*m*x^9 + 392302310400*(d*x)^m*B*b^2*$
 $c*x^10 + 392302310400*(d*x)^m*B*a*c^2*x^10 + 854224943*(d*x)^m*A*a^3*m^8*x$
 $+ 6629764856*(d*x)^m*B*a^3*m^7*x^2 + 36588367376*(d*x)^m*C*a^3*m^6*x^3 + 10$
 $9765102128*(d*x)^m*A*a^2*b*m^6*x^3 + 426272198748*(d*x)^m*B*a^2*b*m^5*x^4 +$
 $1145655530640*(d*x)^m*C*a^2*b*m^4*x^5 + 1145655530640*(d*x)^m*A*a*b^2*m^4*$
 $x^5 + 1145655530640*(d*x)^m*A*a^2*c*m^4*x^5 + 2071918846152*(d*x)^m*B*a*b^2$
 $*m^3*x^6 + 2071918846152*(d*x)^m*B*a^2*c*m^3*x^6 + 2388267607680*(d*x)^m*C*$
 $a*b^2*m^2*x^7 + 796089202560*(d*x)^m*A*b^3*m^2*x^7 + 2388267607680*(d*x)^m*$
 $C*a^2*c*m^2*x^7 + 4776535215360*(d*x)^m*A*a*b*c*m^2*x^7 + 521962963200*(d*x$
 $)^m*B*b^3*m*x^8 + 313177779200*(d*x)^m*B*a*b*c*m*x^8 + 145297152000*(d*x)^$
 $m*C*b^3*x^9 + 871782912000*(d*x)^m*C*a*b*c*x^9 + 435891456000*(d*x)^m*A*b^2$
 $*c*x^9 + 435891456000*(d*x)^m*A*a*c^2*x^9 + 7353403057*(d*x)^m*A*a^3*m^7*x$
 $+ 41371599841*(d*x)^m*B*a^3*m^6*x^2 + 163038108552*(d*x)^m*C*a^3*m^5*x^3 +$
 $489114325656*(d*x)^m*A*a^2*b*m^5*x^3 + 1323927526248*(d*x)^m*B*a^2*b*m^4*x^4$
 $+ 2392162383600*(d*x)^m*C*a^2*b*m^3*x^5 + 2392162383600*(d*x)^m*A*a*b^2*m$
 $^3*x^5 + 2392162383600*(d*x)^m*A*a^2*c*m^3*x^5 + 2739474034560*(d*x)^m*B*a*$
 $b^2*m^2*x^6 + 2739474034560*(d*x)^m*B*a^2*c*m^2*x^6 + 1779579590400*(d*x)^m$
 $*C*a*b^2*m*x^7 + 593193196800*(d*x)^m*A*b^3*m*x^7 + 1779579590400*(d*x)^m*$
 $C*a^2*c*m*x^7 + 3559159180800*(d*x)^m*A*a*b*c*m*x^7 + 163459296000*(d*x)^m*$
 $B*b^3*x^8 + 980755776000*(d*x)^m*B*a*b*c*x^8 + 47277726496*(d*x)^m*A*a^3*m^6$
 $*x + 190060010998*(d*x)^m*B*a^3*m^5*x^2 + 520557781424*(d*x)^m*C*a^3*m^4*x^3$
 $+ 1561673344272*(d*x)^m*A*a^2*b*m^4*x^3 + 2824729931808*(d*x)^m*B*a^2*b*m$
 $^3*x^4 + 3210175193472*(d*x)^m*C*a^2*b*m^2*x^5 + 3210175193472*(d*x)^m*A*a*$
 $b^2*m^2*x^5 + 3210175193472*(d*x)^m*A*a^2*c*m^2*x^5 + 2060608636800*(d*x)^m$
 $*B*a*b^2*m*x^6 + 2060608636800*(d*x)^m*B*a^2*c*m*x^6 + 560431872000*(d*x)^m$
 $*C*a*b^2*x^7 + 186810624000*(d*x)^m*A*b^3*x^7 + 560431872000*(d*x)^m*C*a^2*$
 $c*x^7 + 1120863744000*(d*x)^m*A*a*b*c*x^7 + 225525484184*(d*x)^m*A*a^3*m^5*$
 $x + 629552085084*(d*x)^m*B*a^3*m^4*x^2 + 1145140001328*(d*x)^m*C*a^3*m^3*x^3$
 $+ 3435420003984*(d*x)^m*A*a^2*b*m^3*x^3 + 3872067384240*(d*x)^m*B*a^2*b*m$
 $^2*x^4 + 2446576876800*(d*x)^m*C*a^2*b*m*x^5 + 2446576876800*(d*x)^m*A*a*b^$
 $2*m*x^5 + 2446576876800*(d*x)^m*A*a^2*c*m*x^5 + 653837184000*(d*x)^m*B*a*b^$
 $2*x^6 + 653837184000*(d*x)^m*B*a^2*c*x^6 + 784146622896*(d*x)^m*A*a^3*m^4*x$
 $+ 1447709175432*(d*x)^m*B*a^3*m^3*x^2 + 1621575699840*(d*x)^m*C*a^3*m^2*x^3$
 $+ 4864727099520*(d*x)^m*A*a^2*b*m^2*x^3 + 3009183307200*(d*x)^m*B*a^2*b*m$
 $*x^4 + 784604620800*(d*x)^m*C*a^2*b*x^5 + 784604620800*(d*x)^m*A*a*b^2*x^5$
 $+ 784604620800*(d*x)^m*A*a^2*c*x^5 + 1922666722704*(d*x)^m*A*a^3*m^3*x + 21$
 $61577352960*(d*x)^m*B*a^3*m^2*x^2 + 1301090515200*(d*x)^m*C*a^3*m*x^3 + 390$
 $3271545600*(d*x)^m*A*a^2*b*m*x^3 + 980755776000*(d*x)^m*B*a^2*b*x^4 + 31343$
 $28981120*(d*x)^m*A*a^3*m^2*x + 1842662908800*(d*x)^m*B*a^3*m*x^2 + 43589145$
 $6000*(d*x)^m*C*a^3*x^3 + 1307674368000*(d*x)^m*A*a^2*b*x^3 + 3031488633600*$
 $(d*x)^m*A*a^3*m*x + 653837184000*(d*x)^m*B*a^3*x^2 + 1307674368000*(d*x)^m*$
 $A*a^3*x)/(m^15 + 120*m^14 + 6580*m^13 + 218400*m^12 + 4899622*m^11 + 785584$
 $80*m^10 + 928095740*m^9 + 8207628000*m^8 + 54631129553*m^7 + 272803210680*m$
 $^6 + 1009672107080*m^5 + 2706813345600*m^4 + 5056995703824*m^3 + 6165817614$

$720*m^2 + 4339163001600*m + 1307674368000$)

maple [B] time = 0.01, size = 5520, normalized size = 13.83

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^3,x)$

[Out] result too large to display

maxima [A] time = 1.71, size = 611, normalized size = 1.53

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^3,x, \text{algorithm}="maxima")$

[Out] $C*c^3*d^m*x^{15}*x^m/(m + 15) + B*c^3*d^m*x^{14}*x^m/(m + 14) + 3*C*b*c^2*d^m*x^{13}*x^m/(m + 13) + A*c^3*d^m*x^{13}*x^m/(m + 13) + 3*B*b*c^2*d^m*x^{12}*x^m/(m + 12) + 3*C*b^2*c*d^m*x^{11}*x^m/(m + 11) + 3*C*a*c^2*d^m*x^{11}*x^m/(m + 11) + 3*A*b*c^2*d^m*x^{11}*x^m/(m + 11) + 3*B*b^2*c*d^m*x^{10}*x^m/(m + 10) + 3*B*a*c^2*d^m*x^{10}*x^m/(m + 10) + C*b^3*d^m*x^9*x^m/(m + 9) + 6*C*a*b*c*d^m*x^9*x^m/(m + 9) + 3*A*b^2*c*d^m*x^9*x^m/(m + 9) + 3*A*a*c^2*d^m*x^9*x^m/(m + 9) + B*b^3*d^m*x^8*x^m/(m + 8) + 6*B*a*b*c*d^m*x^8*x^m/(m + 8) + 3*C*a*b^2*d^m*x^7*x^m/(m + 7) + A*b^3*d^m*x^7*x^m/(m + 7) + 3*C*a^2*c*d^m*x^7*x^m/(m + 7) + 6*A*a*b*c*d^m*x^7*x^m/(m + 7) + 3*B*a*b^2*d^m*x^6*x^m/(m + 6) + 3*B*a^2*c*d^m*x^6*x^m/(m + 6) + 3*C*a^2*b*d^m*x^5*x^m/(m + 5) + 3*A*a*b^2*d^m*x^5*x^m/(m + 5) + 3*A*a^2*c*d^m*x^5*x^m/(m + 5) + 3*B*a^2*b*d^m*x^4*x^m/(m + 4) + C*a^3*d^m*x^3*x^m/(m + 3) + 3*A*a^2*b*d^m*x^3*x^m/(m + 3) + B*a^3*d^m*x^2*x^m/(m + 2) + (d*x)^{(m + 1)}*A*a^3/(d*(m + 1))$

mapad [B] time = 3.28, size = 2443, normalized size = 6.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^3,x)$

[Out] $(x^7*(d*x)^m*(A*b^3 + 3*C*a*b^2 + 3*C*a^2*c + 6*A*a*b*c)*(593193196800*m + 796089202560*m^2 + 608700928752*m^3 + 299730345264*m^4 + 101420251688*m^5 + 24483279856*m^6 + 4306835671*m^7 + 557256047*m^8 + 52977099*m^9 + 3654483*m^10 + 177877*m^11 + 5789*m^12 + 113*m^13 + m^14 + 186810624000))/(4339163001600*m + 6165817614720*m^2 + 5056995703824*m^3 + 2706813345600*m^4 + 1009672107080*m^5 + 272803210680*m^6 + 54631129553*m^7 + 8207628000*m^8 + 928095740*m^9 + 78558480*m^10 + 4899622*m^11 + 218400*m^12 + 6580*m^13 + 120*m^14 + m^15 + 1307674368000) + (x^9*(d*x)^m*(C*b^3 + 3*A*a*c^2 + 3*A*b^2*c + 6*C*a*b*c)*(465985094400*m + 633314724480*m^2 + 491520108816*m^3 + 246143692976*m^4 + 84836490456*m^5 + 20885191136*m^6 + 3749548713*m^7 + 495342143*m^8 + 48083733*m^9 + 3386083*m^10 + 168171*m^11 + 5581*m^12 + 111*m^13 + m^14 + 145297152000))/(4339163001600*m + 6165817614720*m^2 + 5056995703824*m^3 + 2706813345600*m^4 + 1009672107080*m^5 + 272803210680*m^6 + 54631129553*m^7 + 8207628000*m^8 + 928095740*m^9 + 78558480*m^10 + 4899622*m^11 + 218400*m^12 + 6580*m^13 + 120*m^14 + m^15 + 1307674368000) + (B*c^3*x^14*(d*x)^m*(303268406400*m + 418753514880*m^2 + 331303013496*m^3 + 169679309436*m^4 + 59999485546*m^5 + 15200266081*m^6 + 2816490248*m^7 + 385081268*m^8 + 38786748*m^9 + 2840838*m^10 + 147056*m^11 + 5096*m^12 + 106*m^13 + m^14 + 93405312000))/(4339163001600*m + 6165817614720*m^2 + 5056995703824*m^3 + 2706813345600*m^4 + 1009672107080*m^5 + 272803210680*m^6 + 54631129553*m^7 + 8207628000*m^8 + 928095740*m^9 + 78558480*m^10 + 4899622*m^11 + 218400*m^12 + 6580*m^13 + 120*m^14 + m^15 + 1307674368000)$

$$\begin{aligned}
& ^{13} + 120m^{14} + m^{15} + 1307674368000) + (B*a^3*x^2*(d*x)^m*(1842662908800*m \\
& + 2161577352960*m^2 + 1447709175432*m^3 + 629552085084*m^4 + 190060010998 \\
& *m^5 + 41371599841*m^6 + 6629764856*m^7 + 788931572*m^8 + 69582084*m^9 + 44 \\
& 88198*m^{10} + 205712*m^{11} + 6344*m^{12} + 118*m^{13} + m^{14} + 653837184000))/(43 \\
& 39163001600*m + 6165817614720*m^2 + 5056995703824*m^3 + 2706813345600*m^4 + \\
& 1009672107080*m^5 + 272803210680*m^6 + 54631129553*m^7 + 8207628000*m^8 + \\
& 928095740*m^9 + 78558480*m^{10} + 4899622*m^{11} + 218400*m^{12} + 6580*m^{13} + 12 \\
& 0*m^{14} + m^{15} + 1307674368000) + (3*a*x^5*(d*x)^m*(A*b^2 + A*a*c + C*a*b)*(\\
& 815525625600*m + 1070058397824*m^2 + 797387461200*m^3 + 381885176880*m^4 + \\
& 125557386040*m^5 + 29449164928*m^6 + 5036392925*m^7 + 634247015*m^8 + 58769 \\
& 745*m^9 + 3957747*m^{10} + 188375*m^{11} + 6005*m^{12} + 115*m^{13} + m^{14} + 261534 \\
& 873600))/(4339163001600*m + 6165817614720*m^2 + 5056995703824*m^3 + 2706813 \\
& 345600*m^4 + 1009672107080*m^5 + 272803210680*m^6 + 54631129553*m^7 + 82076 \\
& 28000*m^8 + 928095740*m^9 + 78558480*m^{10} + 4899622*m^{11} + 218400*m^{12} + 65 \\
& 80*m^{13} + 120*m^{14} + m^{15} + 1307674368000) + (3*c*x^{11}*(d*x)^m*(C*b^2 + A*b \\
& *c + C*a*c)*(383662137600*m + 525650497920*m^2 + 411940473264*m^3 + 2086248 \\
& 06576*m^4 + 72822481864*m^5 + 18180066256*m^6 + 3313733027*m^7 + 444899543* \\
& m^8 + 43926927*m^9 + 3148323*m^{10} + 159209*m^{11} + 5381*m^{12} + 109*m^{13} + m^{ \\
& 14} + 118879488000))/(4339163001600*m + 6165817614720*m^2 + 5056995703824*m^ \\
& 3 + 2706813345600*m^4 + 1009672107080*m^5 + 272803210680*m^6 + 54631129553* \\
& m^7 + 8207628000*m^8 + 928095740*m^9 + 78558480*m^{10} + 4899622*m^{11} + 21840 \\
& 0*m^{12} + 6580*m^{13} + 120*m^{14} + m^{15} + 1307674368000) + (a^2*x^3*(d*x)^m*(3 \\
& *A*b + C*a)*(1301090515200*m + 1621575699840*m^2 + 1145140001328*m^3 + 5205 \\
& 57781424*m^4 + 163038108552*m^5 + 36588367376*m^6 + 6014254059*m^7 + 731124 \\
& 647*m^8 + 65657031*m^9 + 4300483*m^{10} + 199713*m^{11} + 6229*m^{12} + 117*m^{13} \\
& + m^{14} + 435891456000))/(4339163001600*m + 6165817614720*m^2 + 505699570382 \\
& 4*m^3 + 2706813345600*m^4 + 1009672107080*m^5 + 272803210680*m^6 + 54631129 \\
& 553*m^7 + 8207628000*m^8 + 928095740*m^9 + 78558480*m^{10} + 4899622*m^{11} + 2 \\
& 18400*m^{12} + 6580*m^{13} + 120*m^{14} + m^{15} + 1307674368000) + (c^2*x^{13}*(d*x) \\
& ^m*(A*c + 3*C*b)*(326044051200*m + 449213351040*m^2 + 354444796368*m^3 + 18 \\
& 0951426864*m^4 + 63747744632*m^5 + 16081189696*m^6 + 2965379989*m^7 + 40324 \\
& 9847*m^8 + 40372761*m^9 + 2937363*m^{10} + 150943*m^{11} + 5189*m^{12} + 107*m^{13} \\
& + m^{14} + 100590336000))/(4339163001600*m + 6165817614720*m^2 + 50569957038 \\
& 24*m^3 + 2706813345600*m^4 + 1009672107080*m^5 + 272803210680*m^6 + 54631129 \\
& 9553*m^7 + 8207628000*m^8 + 928095740*m^9 + 78558480*m^{10} + 4899622*m^{11} + \\
& 218400*m^{12} + 6580*m^{13} + 120*m^{14} + m^{15} + 1307674368000) + (A*a^3*x*(d*x) \\
& ^m*(3031488633600*m + 3134328981120*m^2 + 1922666722704*m^3 + 784146622896* \\
& m^4 + 225525484184*m^5 + 47277726496*m^6 + 7353403057*m^7 + 854224943*m^8 + \\
& 73870797*m^9 + 4687683*m^{10} + 211939*m^{11} + 6461*m^{12} + 119*m^{13} + m^{14} + \\
& 1307674368000))/(4339163001600*m + 6165817614720*m^2 + 5056995703824*m^3 + \\
& 2706813345600*m^4 + 1009672107080*m^5 + 272803210680*m^6 + 54631129553*m^7 \\
& + 8207628000*m^8 + 928095740*m^9 + 78558480*m^{10} + 4899622*m^{11} + 218400*m^ \\
& 12 + 6580*m^{13} + 120*m^{14} + m^{15} + 1307674368000) + (C*c^3*x^{15}*(d*x)^m*(28 \\
& 3465647360*m + 392156797824*m^2 + 310989260400*m^3 + 159721605680*m^4 + 566 \\
& 63366760*m^5 + 14409322928*m^6 + 2681453775*m^7 + 368411615*m^8 + 37312275* \\
& m^9 + 2749747*m^{10} + 143325*m^{11} + 5005*m^{12} + 105*m^{13} + m^{14} + 8717829120 \\
& 0))/(4339163001600*m + 6165817614720*m^2 + 5056995703824*m^3 + 270681334560 \\
& 0*m^4 + 1009672107080*m^5 + 272803210680*m^6 + 54631129553*m^7 + 8207628000 \\
& *m^8 + 928095740*m^9 + 78558480*m^{10} + 4899622*m^{11} + 218400*m^{12} + 6580*m^ \\
& 13 + 120*m^{14} + m^{15} + 1307674368000) + (3*B*c*x^{10}*(d*x)^m*(a*c + b^2)*(42 \\
& 0839556480*m + 574497805824*m^2 + 448249789800*m^3 + 225856355580*m^4 + 783 \\
& 81575150*m^5 + 19442163553*m^6 + 3518896600*m^7 + 468873140*m^8 + 45922260* \\
& m^9 + 3263622*m^{10} + 163600*m^{11} + 5480*m^{12} + 110*m^{13} + m^{14} + 1307674368 \\
& 00))/(4339163001600*m + 6165817614720*m^2 + 5056995703824*m^3 + 27068133456 \\
& 00*m^4 + 1009672107080*m^5 + 272803210680*m^6 + 54631129553*m^7 + 820762800 \\
& 0*m^8 + 928095740*m^9 + 78558480*m^{10} + 4899622*m^{11} + 218400*m^{12} + 6580*m \\
& ^{13} + 120*m^{14} + m^{15} + 1307674368000) + (3*B*a*x^6*(d*x)^m*(a*c + b^2)*(68 \\
& 6869545600*m + 913158011520*m^2 + 690639615384*m^3 + 336028955036*m^4 + 112 \\
& 273858674*m^5 + 26754892001*m^6 + 4646039592*m^7 + 593598068*m^8 + 55749612
\end{aligned}$$

```

*m^9 + 3801478*m^10 + 183024*m^11 + 5896*m^12 + 114*m^13 + m^14 + 217945728
000))/(4339163001600*m + 6165817614720*m^2 + 5056995703824*m^3 + 2706813345
600*m^4 + 1009672107080*m^5 + 272803210680*m^6 + 54631129553*m^7 + 82076280
00*m^8 + 928095740*m^9 + 78558480*m^10 + 4899622*m^11 + 218400*m^12 + 6580*
m^13 + 120*m^14 + m^15 + 1307674368000) + (3*B*b*c^2*x^12*(d*x)^m*(35251584
4800*m + 484441814160*m^2 + 381046157472*m^3 + 193813932344*m^4 + 679881812
28*m^5 + 17067919121*m^6 + 3130267536*m^7 + 423113372*m^8 + 42081864*m^9 +
3039718*m^10 + 154992*m^11 + 5284*m^12 + 108*m^13 + m^14 + 108972864000))/(
4339163001600*m + 6165817614720*m^2 + 5056995703824*m^3 + 2706813345600*m^4
+ 1009672107080*m^5 + 272803210680*m^6 + 54631129553*m^7 + 8207628000*m^8
+ 928095740*m^9 + 78558480*m^10 + 4899622*m^11 + 218400*m^12 + 6580*m^13 +
120*m^14 + m^15 + 1307674368000) + (B*b*x^8*(d*x)^m*(6*a*c + b^2)*(52196296
3200*m + 705481831440*m^2 + 543939234048*m^3 + 270359263944*m^4 + 924141053
92*m^5 + 22548638161*m^6 + 4010311424*m^7 + 524664572*m^8 + 50428896*m^9 +
3516198*m^10 + 172928*m^11 + 5684*m^12 + 112*m^13 + m^14 + 163459296000))/(
4339163001600*m + 6165817614720*m^2 + 5056995703824*m^3 + 2706813345600*m^4
+ 1009672107080*m^5 + 272803210680*m^6 + 54631129553*m^7 + 8207628000*m^8
+ 928095740*m^9 + 78558480*m^10 + 4899622*m^11 + 218400*m^12 + 6580*m^13 +
120*m^14 + m^15 + 1307674368000) + (3*B*a^2*b*x^4*(d*x)^m*(1003061102400*m
+ 1290689128080*m^2 + 941576643936*m^3 + 441309175416*m^4 + 142090732916*m^
5 + 32678119441*m^6 + 5488252528*m^7 + 679843868*m^8 + 62062968*m^9 + 41238
78*m^10 + 193936*m^11 + 6116*m^12 + 116*m^13 + m^14 + 326918592000))/(43391
63001600*m + 6165817614720*m^2 + 5056995703824*m^3 + 2706813345600*m^4 + 10
09672107080*m^5 + 272803210680*m^6 + 54631129553*m^7 + 8207628000*m^8 + 928
095740*m^9 + 78558480*m^10 + 4899622*m^11 + 218400*m^12 + 6580*m^13 + 120*m
^14 + m^15 + 1307674368000)

```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)**m*(C*x**2+B*x+A)*(c*x**4+b*x**2+a)**3,x)

[Out] Timed out

$$3.38 \quad \int (dx)^m (A + Bx + Cx^2) (a + bx^2 + cx^4)^2 dx$$

Optimal. Leaf size=260

$$\frac{a^2 A (dx)^{m+1}}{d(m+1)} + \frac{a^2 B (dx)^{m+2}}{d^2(m+2)} + \frac{(dx)^{m+7} (C(2ac + b^2) + 2Abc)}{d^7(m+7)} + \frac{(dx)^{m+5} (A(2ac + b^2) + 2abC)}{d^5(m+5)} + \frac{a(dx)^{m+3} (aC + b^2)}{d^3(m+3)}$$

Rubi [A] time = 0.22, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {1628}

$$\frac{a^2 A (dx)^{m+1}}{d(m+1)} + \frac{a^2 B (dx)^{m+2}}{d^2(m+2)} + \frac{(dx)^{m+5} (A(2ac + b^2) + 2abC)}{d^5(m+5)} + \frac{(dx)^{m+7} (C(2ac + b^2) + 2Abc)}{d^7(m+7)} + \frac{a(dx)^{m+3} (aC + 2Ab)}{d^3(m+3)} + \frac{B(2ac + b^2)(dx)^{m+6}}{d^6(m+6)} + \frac{2abB(dx)^{m+4}}{d^4(m+4)} + \frac{c(dx)^{m+9} (Ac + 2bC)}{d^9(m+9)} + \frac{2bBc(dx)^{m+8}}{d^8(m+8)} + \frac{Bc^2(dx)^{m+10}}{d^{10}(m+10)} + \frac{c^2C(dx)^{m+11}}{d^{11}(m+11)}$$

Antiderivative was successfully verified.

[In] Int[(d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]

[Out] (a^2*A*(d*x)^(1 + m))/(d*(1 + m)) + (a^2*B*(d*x)^(2 + m))/(d^2*(2 + m)) + (a*(2*A*b + a*C)*(d*x)^(3 + m))/(d^3*(3 + m)) + (2*a*b*B*(d*x)^(4 + m))/(d^4*(4 + m)) + ((A*(b^2 + 2*a*c) + 2*a*b*C)*(d*x)^(5 + m))/(d^5*(5 + m)) + (B*(b^2 + 2*a*c)*(d*x)^(6 + m))/(d^6*(6 + m)) + ((2*A*b*c + (b^2 + 2*a*c)*C)*(d*x)^(7 + m))/(d^7*(7 + m)) + (2*b*B*c*(d*x)^(8 + m))/(d^8*(8 + m)) + (c*(A*c + 2*b*C)*(d*x)^(9 + m))/(d^9*(9 + m)) + (B*c^2*(d*x)^(10 + m))/(d^10*(10 + m)) + (c^2*C*(d*x)^(11 + m))/(d^11*(11 + m))

Rule 1628

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int (dx)^m (A + Bx + Cx^2) (a + bx^2 + cx^4)^2 dx &= \int \left(a^2 A (dx)^m + \frac{a^2 B (dx)^{1+m}}{d} + \frac{a(2Ab + aC)(dx)^{2+m}}{d^2} + \frac{2abB(dx)^{3+m}}{d^3} \right. \\ &\quad \left. + \frac{a^2 A (dx)^{1+m}}{d(1+m)} + \frac{a^2 B (dx)^{2+m}}{d^2(2+m)} + \frac{a(2Ab + aC)(dx)^{3+m}}{d^3(3+m)} + \frac{2abB(dx)^{4+m}}{d^4(4+m)} \right) dx \end{aligned}$$

Mathematica [A] time = 0.28, size = 185, normalized size = 0.71

$$x(dx)^m \left(\frac{a^2 A}{m+1} + \frac{a^2 Bx}{m+2} + \frac{x^6 (C(2ac + b^2) + 2Abc)}{m+7} + \frac{x^4 (A(2ac + b^2) + 2abC)}{m+5} + \frac{ax^2 (aC + 2Ab)}{m+3} + \frac{Bx^5 (2ac + b^2)}{m+6} + \frac{2abBx^3}{m+4} + \frac{cx^8 (Ac + 2bC)}{m+9} + \frac{2bBcx^7}{m+8} + \frac{Bc^2x^9}{m+10} + \frac{c^2Cx^{10}}{m+11} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]

[Out] x*(d*x)^m*((a^2*A)/(1 + m) + (a^2*B*x)/(2 + m) + (a*(2*A*b + a*C)*x^2)/(3 + m) + (2*a*b*B*x^3)/(4 + m) + ((A*(b^2 + 2*a*c) + 2*a*b*C)*x^4)/(5 + m) + (B*(b^2 + 2*a*c)*x^5)/(6 + m) + ((2*A*b*c + (b^2 + 2*a*c)*C)*x^6)/(7 + m) + (2*b*B*c*x^7)/(8 + m) + (c*(A*c + 2*b*C)*x^8)/(9 + m) + (B*c^2*x^9)/(10 + m) + (c^2*C*x^10)/(11 + m))

IntegrateAlgebraic [F] time = 1.37, size = 0, normalized size = 0.00

$$\int (dx)^m (A + Bx + Cx^2) (a + bx^2 + cx^4)^2 dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] Defer[IntegrateAlgebraic] [(d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,
x]
```

fricas [B] time = 1.66, size = 1603, normalized size = 6.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] ((C*c^2*m^10 + 55*C*c^2*m^9 + 1320*C*c^2*m^8 + 18150*C*c^2*m^7 + 157773*C*c^
^2*m^6 + 902055*C*c^2*m^5 + 3416930*C*c^2*m^4 + 8409500*C*c^2*m^3 + 1275357
6*C*c^2*m^2 + 10628640*C*c^2*m + 3628800*C*c^2)*x^11 + (B*c^2*m^10 + 56*B*c
^2*m^9 + 1365*B*c^2*m^8 + 19020*B*c^2*m^7 + 167223*B*c^2*m^6 + 965328*B*c^2
*m^5 + 3686255*B*c^2*m^4 + 9133180*B*c^2*m^3 + 13926276*B*c^2*m^2 + 1165521
6*B*c^2*m + 3991680*B*c^2)*x^10 + ((2*C*b*c + A*c^2)*m^10 + 57*(2*C*b*c + A
*c^2)*m^9 + 1412*(2*C*b*c + A*c^2)*m^8 + 19962*(2*C*b*c + A*c^2)*m^7 + 1777
65*(2*C*b*c + A*c^2)*m^6 + 1037673*(2*C*b*c + A*c^2)*m^5 + 4000478*(2*C*b*c
+ A*c^2)*m^4 + 9991428*(2*C*b*c + A*c^2)*m^3 + 8870400*C*b*c + 4435200*A*c
^2 + 15335224*(2*C*b*c + A*c^2)*m^2 + 12900960*(2*C*b*c + A*c^2)*m)*x^9 + 2
*(B*b*c*m^10 + 58*B*b*c*m^9 + 1461*B*b*c*m^8 + 20982*B*b*c*m^7 + 189567*B*b
*c*m^6 + 1121022*B*b*c*m^5 + 4371359*B*b*c*m^4 + 11024858*B*b*c*m^3 + 17059
212*B*b*c*m^2 + 14444280*B*b*c*m + 4989600*B*b*c)*x^8 + ((C*b^2 + 2*(C*a +
A*b)*c)*m^10 + 59*(C*b^2 + 2*(C*a + A*b)*c)*m^9 + 1512*(C*b^2 + 2*(C*a + A
b)*c)*m^8 + 22086*(C*b^2 + 2*(C*a + A*b)*c)*m^7 + 202821*(C*b^2 + 2*(C*a +
A*b)*c)*m^6 + 1217811*(C*b^2 + 2*(C*a + A*b)*c)*m^5 + 4814858*(C*b^2 + 2*(C
*a + A*b)*c)*m^4 + 12291724*(C*b^2 + 2*(C*a + A*b)*c)*m^3 + 5702400*C*b^2 +
19216008*(C*b^2 + 2*(C*a + A*b)*c)*m^2 + 11404800*(C*a + A*b)*c + 16405920
*(C*b^2 + 2*(C*a + A*b)*c)*m)*x^7 + ((B*b^2 + 2*B*a*c)*m^10 + 60*(B*b^2 + 2
*B*a*c)*m^9 + 1565*(B*b^2 + 2*B*a*c)*m^8 + 23280*(B*b^2 + 2*B*a*c)*m^7 + 21
7743*(B*b^2 + 2*B*a*c)*m^6 + 1331100*(B*b^2 + 2*B*a*c)*m^5 + 5352935*(B*b^2
+ 2*B*a*c)*m^4 + 13878120*(B*b^2 + 2*B*a*c)*m^3 + 6652800*B*b^2 + 13305600
*B*a*c + 21989356*(B*b^2 + 2*B*a*c)*m^2 + 18981840*(B*b^2 + 2*B*a*c)*m)*x^6
+ ((2*C*a*b + A*b^2 + 2*A*a*c)*m^10 + 61*(2*C*a*b + A*b^2 + 2*A*a*c)*m^9 +
1620*(2*C*a*b + A*b^2 + 2*A*a*c)*m^8 + 24570*(2*C*a*b + A*b^2 + 2*A*a*c)*m
^7 + 234573*(2*C*a*b + A*b^2 + 2*A*a*c)*m^6 + 1464693*(2*C*a*b + A*b^2 + 2*
A*a*c)*m^5 + 6016070*(2*C*a*b + A*b^2 + 2*A*a*c)*m^4 + 15915380*(2*C*a*b +
A*b^2 + 2*A*a*c)*m^3 + 15966720*C*a*b + 7983360*A*b^2 + 15966720*A*a*c + 25
681176*(2*C*a*b + A*b^2 + 2*A*a*c)*m^2 + 22512096*(2*C*a*b + A*b^2 + 2*A*a*
c)*m)*x^5 + 2*(B*a*b*m^10 + 62*B*a*b*m^9 + 1677*B*a*b*m^8 + 25962*B*a*b*m^7
+ 253575*B*a*b*m^6 + 1623258*B*a*b*m^5 + 6846503*B*a*b*m^4 + 18609718*B*a*
b*m^3 + 30819204*B*a*b*m^2 + 27641160*B*a*b*m + 9979200*B*a*b)*x^4 + ((C*a^
2 + 2*A*a*b)*m^10 + 63*(C*a^2 + 2*A*a*b)*m^9 + 1736*(C*a^2 + 2*A*a*b)*m^8 +
27462*(C*a^2 + 2*A*a*b)*m^7 + 275037*(C*a^2 + 2*A*a*b)*m^6 + 1812447*(C*a^
2 + 2*A*a*b)*m^5 + 7902194*(C*a^2 + 2*A*a*b)*m^4 + 22289148*(C*a^2 + 2*A*a*
b)*m^3 + 13305600*C*a^2 + 26611200*A*a*b + 38390632*(C*a^2 + 2*A*a*b)*m^2 +
35746080*(C*a^2 + 2*A*a*b)*m)*x^3 + (B*a^2*m^10 + 64*B*a^2*m^9 + 1797*B*a^
2*m^8 + 29076*B*a^2*m^7 + 299271*B*a^2*m^6 + 2039016*B*a^2*m^5 + 9261503*B*
a^2*m^4 + 27472724*B*a^2*m^3 + 50312628*B*a^2*m^2 + 50292720*B*a^2*m + 1995
8400*B*a^2)*x^2 + (A*a^2*m^10 + 65*A*a^2*m^9 + 1860*A*a^2*m^8 + 30810*A*a^2
*m^7 + 326613*A*a^2*m^6 + 2310945*A*a^2*m^5 + 11028590*A*a^2*m^4 + 34967140
*A*a^2*m^3 + 70290936*A*a^2*m^2 + 80627040*A*a^2*m + 39916800*A*a^2)*x)*(d*
x)^m/(m^11 + 66*m^10 + 1925*m^9 + 32670*m^8 + 357423*m^7 + 2637558*m^6 + 13
339535*m^5 + 45995730*m^4 + 105258076*m^3 + 150917976*m^2 + 120543840*m + 3
9916800)
```

giac [B] time = 0.73, size = 3203, normalized size = 12.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
[Out] ((d*x)^m*C*c^2*m^10*x^11 + (d*x)^m*B*c^2*m^10*x^10 + 55*(d*x)^m*C*c^2*m^9*x^11 + 2*(d*x)^m*C*b*c*m^10*x^9 + (d*x)^m*A*c^2*m^10*x^9 + 56*(d*x)^m*B*c^2*m^9*x^10 + 1320*(d*x)^m*C*c^2*m^8*x^11 + 2*(d*x)^m*B*b*c*m^10*x^8 + 114*(d*x)^m*C*b*c*m^9*x^9 + 57*(d*x)^m*A*c^2*m^9*x^9 + 1365*(d*x)^m*B*c^2*m^8*x^10 + 18150*(d*x)^m*C*c^2*m^7*x^11 + (d*x)^m*C*b^2*m^10*x^7 + 2*(d*x)^m*C*a*c*m^10*x^7 + 2*(d*x)^m*A*b*c*m^10*x^7 + 116*(d*x)^m*B*b*c*m^9*x^8 + 2824*(d*x)^m*C*b*c*m^8*x^9 + 1412*(d*x)^m*A*c^2*m^8*x^9 + 19020*(d*x)^m*B*c^2*m^7*x^10 + 157773*(d*x)^m*C*c^2*m^6*x^11 + (d*x)^m*B*b^2*m^10*x^6 + 2*(d*x)^m*B*a*c*m^10*x^6 + 59*(d*x)^m*C*b^2*m^9*x^7 + 118*(d*x)^m*C*a*c*m^9*x^7 + 118*(d*x)^m*A*b*c*m^9*x^7 + 2922*(d*x)^m*B*b*c*m^8*x^8 + 39924*(d*x)^m*C*b*c*m^7*x^9 + 19962*(d*x)^m*A*c^2*m^7*x^9 + 167223*(d*x)^m*B*c^2*m^6*x^10 + 902055*(d*x)^m*C*c^2*m^5*x^11 + 2*(d*x)^m*C*a*b*m^10*x^5 + (d*x)^m*A*b^2*m^10*x^5 + 2*(d*x)^m*A*a*c*m^10*x^5 + 60*(d*x)^m*B*b^2*m^9*x^6 + 120*(d*x)^m*B*a*c*m^9*x^6 + 1512*(d*x)^m*C*b^2*m^8*x^7 + 3024*(d*x)^m*C*a*c*m^8*x^7 + 3024*(d*x)^m*A*b*c*m^8*x^7 + 41964*(d*x)^m*B*b*c*m^7*x^8 + 355530*(d*x)^m*C*b*c*m^6*x^9 + 177765*(d*x)^m*A*c^2*m^6*x^9 + 965328*(d*x)^m*B*c^2*m^5*x^10 + 3416930*(d*x)^m*C*c^2*m^4*x^11 + 2*(d*x)^m*B*a*b*m^10*x^4 + 122*(d*x)^m*C*a*b*m^9*x^5 + 61*(d*x)^m*A*b^2*m^9*x^5 + 122*(d*x)^m*A*a*c*m^9*x^5 + 1565*(d*x)^m*B*b^2*m^8*x^6 + 3130*(d*x)^m*B*a*c*m^8*x^6 + 22086*(d*x)^m*C*b^2*m^7*x^7 + 44172*(d*x)^m*C*a*c*m^7*x^7 + 44172*(d*x)^m*A*b*c*m^7*x^7 + 379134*(d*x)^m*B*b*c*m^6*x^8 + 2075346*(d*x)^m*C*b*c*m^5*x^9 + 1037673*(d*x)^m*A*c^2*m^5*x^9 + 3686255*(d*x)^m*B*c^2*m^4*x^10 + 8409500*(d*x)^m*C*c^2*m^3*x^11 + (d*x)^m*C*a^2*m^10*x^3 + 2*(d*x)^m*A*a*b*m^10*x^3 + 124*(d*x)^m*B*a*b*m^9*x^4 + 3240*(d*x)^m*C*a*b*m^8*x^5 + 1620*(d*x)^m*A*b^2*m^8*x^5 + 3240*(d*x)^m*A*a*c*m^8*x^5 + 23280*(d*x)^m*B*b^2*m^7*x^6 + 46560*(d*x)^m*B*a*c*m^7*x^6 + 202821*(d*x)^m*C*b^2*m^6*x^7 + 405642*(d*x)^m*C*a*c*m^6*x^7 + 405642*(d*x)^m*A*b*c*m^6*x^7 + 2242044*(d*x)^m*B*b*c*m^5*x^8 + 8000956*(d*x)^m*C*b*c*m^4*x^9 + 4000478*(d*x)^m*A*c^2*m^4*x^9 + 9133180*(d*x)^m*B*c^2*m^3*x^10 + 12753576*(d*x)^m*C*c^2*m^2*x^11 + (d*x)^m*B*a^2*m^10*x^2 + 63*(d*x)^m*C*a^2*m^9*x^3 + 126*(d*x)^m*A*a*b*m^9*x^3 + 3354*(d*x)^m*B*a*b*m^8*x^4 + 49140*(d*x)^m*C*a*b*m^7*x^5 + 24570*(d*x)^m*A*b^2*m^7*x^5 + 49140*(d*x)^m*A*a*c*m^7*x^5 + 217743*(d*x)^m*B*b^2*m^6*x^6 + 435486*(d*x)^m*B*a*c*m^6*x^6 + 1217811*(d*x)^m*C*b^2*m^5*x^7 + 2435622*(d*x)^m*C*a*c*m^5*x^7 + 2435622*(d*x)^m*A*b*c*m^5*x^7 + 8742718*(d*x)^m*B*b*c*m^4*x^8 + 19982856*(d*x)^m*C*b*c*m^3*x^9 + 9991428*(d*x)^m*A*c^2*m^3*x^9 + 13926276*(d*x)^m*B*c^2*m^2*x^10 + 10628640*(d*x)^m*C*c^2*m*x^11 + (d*x)^m*A*a^2*m^10*x + 64*(d*x)^m*B*a^2*m^9*x^2 + 1736*(d*x)^m*C*a^2*m^8*x^3 + 3472*(d*x)^m*A*a*b*m^8*x^3 + 51924*(d*x)^m*B*a*b*m^7*x^4 + 469146*(d*x)^m*C*a*b*m^6*x^5 + 234573*(d*x)^m*A*b^2*m^6*x^5 + 469146*(d*x)^m*A*a*c*m^6*x^5 + 1331100*(d*x)^m*B*b^2*m^5*x^6 + 2662200*(d*x)^m*B*a*c*m^5*x^6 + 4814858*(d*x)^m*C*b^2*m^4*x^7 + 9629716*(d*x)^m*C*a*c*m^4*x^7 + 9629716*(d*x)^m*A*b*c*m^4*x^7 + 22049716*(d*x)^m*B*b*c*m^3*x^8 + 30670448*(d*x)^m*C*b*c*m^2*x^9 + 15335224*(d*x)^m*A*c^2*m^2*x^9 + 11655216*(d*x)^m*B*c^2*m*x^10 + 3628800*(d*x)^m*C*c^2*x^11 + 65*(d*x)^m*A*a^2*m^9*x + 1797*(d*x)^m*B*a^2*m^8*x^2 + 27462*(d*x)^m*C*a^2*m^7*x^3 + 54924*(d*x)^m*A*a*b*m^7*x^3 + 507150*(d*x)^m*B*a*b*m^6*x^4 + 2929386*(d*x)^m*C*a*b*m^5*x^5 + 1464693*(d*x)^m*A*b^2*m^5*x^5 + 2929386*(d*x)^m*A*a*c*m^5*x^5 + 5352935*(d*x)^m*B*b^2*m^4*x^6 + 10705870*(d*x)^m*B*a*c*m^4*x^6 + 12291724*(d*x)^m*C*b^2*m^3*x^7 + 24583448*(d*x)^m*C*a*c*m^3*x^7 + 24583448*(d*x)^m*A*b*c*m^3*x^7 + 34118424*(d*x)^m*B*b*c*m^2*x^8 + 25801920*(d*x)^m*C*b*c*m*x^9 + 12900960*(d*x)^m*A*c^2*m*x^9 + 3991680*(d*x)^m*B*c^2*x^10 + 1860*(d*x)^m*A*a^2*m^8*x + 29076*(d*x)^m*B*a^2*m^7*x^2 + 275037*(d*x)^m*C*a^2*m^6*x^3 + 550074*(d*x)^m*A*a*b*m^6*x^3 + 3246516*(d*x)^m*B*a*b*m^5*x^4 + 12032140*(d*x)^m*C*a*b*m^4*x^5 + 6016070*(d*x)^m*A*b^2*m^4*x^5 + 12032140*(d*x)^m*A*a*c*m^4*x^5 + 13878120*(d*x)^m*B*b^2*m^3*x^6 + 27756240*(d*x)^m*B*a*c*m^3*x^6 + 19216
```

```

008*(d*x)^m*C*b^2*m^2*x^7 + 38432016*(d*x)^m*C*a*c*m^2*x^7 + 38432016*(d*x)
^m*A*b*c*m^2*x^7 + 28888560*(d*x)^m*B*b*c*m*x^8 + 8870400*(d*x)^m*C*b*c*x^9
+ 4435200*(d*x)^m*A*c^2*x^9 + 30810*(d*x)^m*A*a^2*m^7*x + 299271*(d*x)^m*B
*a^2*m^6*x^2 + 1812447*(d*x)^m*C*a^2*m^5*x^3 + 3624894*(d*x)^m*A*a*b*m^5*x^
3 + 13693006*(d*x)^m*B*a*b*m^4*x^4 + 31830760*(d*x)^m*C*a*b*m^3*x^5 + 15915
380*(d*x)^m*A*b^2*m^3*x^5 + 31830760*(d*x)^m*A*a*c*m^3*x^5 + 21989356*(d*x)
^m*B*b^2*m^2*x^6 + 43978712*(d*x)^m*B*a*c*m^2*x^6 + 16405920*(d*x)^m*C*b^2*
m*x^7 + 32811840*(d*x)^m*C*a*c*m*x^7 + 32811840*(d*x)^m*A*b*c*m*x^7 + 99792
00*(d*x)^m*B*b*c*x^8 + 326613*(d*x)^m*A*a^2*m^6*x + 2039016*(d*x)^m*B*a^2*m
^5*x^2 + 7902194*(d*x)^m*C*a^2*m^4*x^3 + 15804388*(d*x)^m*A*a*b*m^4*x^3 + 3
7219436*(d*x)^m*B*a*b*m^3*x^4 + 51362352*(d*x)^m*C*a*b*m^2*x^5 + 25681176*(
d*x)^m*A*b^2*m^2*x^5 + 51362352*(d*x)^m*A*a*c*m^2*x^5 + 18981840*(d*x)^m*B*
b^2*m*x^6 + 37963680*(d*x)^m*B*a*c*m*x^6 + 5702400*(d*x)^m*C*b^2*x^7 + 1140
4800*(d*x)^m*C*a*c*x^7 + 11404800*(d*x)^m*A*b*c*x^7 + 2310945*(d*x)^m*A*a^2
*m^5*x + 9261503*(d*x)^m*B*a^2*m^4*x^2 + 22289148*(d*x)^m*C*a^2*m^3*x^3 + 4
4578296*(d*x)^m*A*a*b*m^3*x^3 + 61638408*(d*x)^m*B*a*b*m^2*x^4 + 45024192*(
d*x)^m*C*a*b*m*x^5 + 22512096*(d*x)^m*A*b^2*m*x^5 + 45024192*(d*x)^m*A*a*c*
m*x^5 + 6652800*(d*x)^m*B*b^2*x^6 + 13305600*(d*x)^m*B*a*c*x^6 + 11028590*(
d*x)^m*A*a^2*m^4*x + 27472724*(d*x)^m*B*a^2*m^3*x^2 + 38390632*(d*x)^m*C*a^
2*m^2*x^3 + 76781264*(d*x)^m*A*a*b*m^2*x^3 + 55282320*(d*x)^m*B*a*b*m*x^4 +
15966720*(d*x)^m*C*a*b*x^5 + 7983360*(d*x)^m*A*b^2*x^5 + 15966720*(d*x)^m*
A*a*c*x^5 + 34967140*(d*x)^m*A*a^2*m^3*x + 50312628*(d*x)^m*B*a^2*m^2*x^2 +
35746080*(d*x)^m*C*a^2*m*x^3 + 71492160*(d*x)^m*A*a*b*m*x^3 + 19958400*(d
x)^m*B*a*b*x^4 + 70290936*(d*x)^m*A*a^2*m^2*x + 50292720*(d*x)^m*B*a^2*m*x^
2 + 13305600*(d*x)^m*C*a^2*x^3 + 26611200*(d*x)^m*A*a*b*x^3 + 80627040*(d*x
)^m*A*a^2*m*x + 19958400*(d*x)^m*B*a^2*x^2 + 39916800*(d*x)^m*A*a^2*x)/(m^1
1 + 66*m^10 + 1925*m^9 + 32670*m^8 + 357423*m^7 + 2637558*m^6 + 13339535*m^
5 + 45995730*m^4 + 105258076*m^3 + 150917976*m^2 + 120543840*m + 39916800)

```

maple [B] time = 0.01, size = 2187, normalized size = 8.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2, x)$

```

[Out] x*(C*c^2*m^10*x^10+B*c^2*m^10*x^9+55*C*c^2*m^9*x^10+A*c^2*m^10*x^8+56*B*c^2
*m^9*x^9+2*C*b*c*m^10*x^8+1320*C*c^2*m^8*x^10+57*A*c^2*m^9*x^8+2*B*b*c*m^10
*x^7+1365*B*c^2*m^8*x^9+114*C*b*c*m^9*x^8+18150*C*c^2*m^7*x^10+2*A*b*c*m^10
*x^6+1412*A*c^2*m^8*x^8+116*B*b*c*m^9*x^7+19020*B*c^2*m^7*x^9+2*C*a*c*m^10*
x^6+C*b^2*m^10*x^6+2824*C*b*c*m^8*x^8+157773*C*c^2*m^6*x^10+118*A*b*c*m^9*x
^6+19962*A*c^2*m^7*x^8+2*B*a*c*m^10*x^5+B*b^2*m^10*x^5+2922*B*b*c*m^8*x^7+1
67223*B*c^2*m^6*x^9+118*C*a*c*m^9*x^6+59*C*b^2*m^9*x^6+39924*C*b*c*m^7*x^8+
902055*C*c^2*m^5*x^10+2*A*a*c*m^10*x^4+A*b^2*m^10*x^4+3024*A*b*c*m^8*x^6+17
7765*A*c^2*m^6*x^8+120*B*a*c*m^9*x^5+60*B*b^2*m^9*x^5+41964*B*b*c*m^7*x^7+9
65328*B*c^2*m^5*x^9+2*C*a*b*m^10*x^4+3024*C*a*c*m^8*x^6+1512*C*b^2*m^8*x^6+
355530*C*b*c*m^6*x^8+3416930*C*c^2*m^4*x^10+122*A*a*c*m^9*x^4+61*A*b^2*m^9*
x^4+44172*A*b*c*m^7*x^6+1037673*A*c^2*m^5*x^8+2*B*a*b*m^10*x^3+3130*B*a*c*m
^8*x^5+1565*B*b^2*m^8*x^5+379134*B*b*c*m^6*x^7+3686255*B*c^2*m^4*x^9+122*C*
a*b*m^9*x^4+44172*C*a*c*m^7*x^6+22086*C*b^2*m^7*x^6+2075346*C*b*c*m^5*x^8+8
409500*C*c^2*m^3*x^10+2*A*a*b*m^10*x^2+3240*A*a*c*m^8*x^4+1620*A*b^2*m^8*x^
4+405642*A*b*c*m^6*x^6+4000478*A*c^2*m^4*x^8+124*B*a*b*m^9*x^3+46560*B*a*c*
m^7*x^5+23280*B*b^2*m^7*x^5+2242044*B*b*c*m^5*x^7+9133180*B*c^2*m^3*x^9+C*a
^2*m^10*x^2+3240*C*a*b*m^8*x^4+405642*C*a*c*m^6*x^6+202821*C*b^2*m^6*x^6+80
00956*C*b*c*m^4*x^8+12753576*C*c^2*m^2*x^10+126*A*a*b*m^9*x^2+49140*A*a*c*m
^7*x^4+24570*A*b^2*m^7*x^4+2435622*A*b*c*m^5*x^6+9991428*A*c^2*m^3*x^8+B*a^
2*m^10*x+3354*B*a*b*m^8*x^3+435486*B*a*c*m^6*x^5+217743*B*b^2*m^6*x^5+87427
18*B*b*c*m^4*x^7+13926276*B*c^2*m^2*x^9+63*C*a^2*m^9*x^2+49140*C*a*b*m^7*x^
4+2435622*C*a*c*m^5*x^6+1217811*C*b^2*m^5*x^6+19982856*C*b*c*m^3*x^8+106286
40*C*c^2*m*x^10+A*a^2*m^10+3472*A*a*b*m^8*x^2+469146*A*a*c*m^6*x^4+234573*A

```



```
*b^2*m^6*x^4+9629716*A*b*c*m^4*x^6+15335224*A*c^2*m^2*x^8+64*B*a^2*m^9*x+51
924*B*a*b*m^7*x^3+2662200*B*a*c*m^5*x^5+1331100*B*b^2*m^5*x^5+22049716*B*b*
c*m^3*x^7+11655216*B*c^2*m*x^9+1736*C*a^2*m^8*x^2+469146*C*a*b*m^6*x^4+9629
716*C*a*c*m^4*x^6+4814858*C*b^2*m^4*x^6+30670448*C*b*c*m^2*x^8+3628800*C*c^
2*x^10+65*A*a^2*m^9+54924*A*a*b*m^7*x^2+2929386*A*a*c*m^5*x^4+1464693*A*b^2
*m^5*x^4+24583448*A*b*c*m^3*x^6+12900960*A*c^2*m*x^8+1797*B*a^2*m^8*x+50715
0*B*a*b*m^6*x^3+10705870*B*a*c*m^4*x^5+5352935*B*b^2*m^4*x^5+34118424*B*b*c
*m^2*x^7+3991680*B*c^2*x^9+27462*C*a^2*m^7*x^2+2929386*C*a*b*m^5*x^4+245834
48*C*a*c*m^3*x^6+12291724*C*b^2*m^3*x^6+25801920*C*b*c*m*x^8+1860*A*a^2*m^8
+550074*A*a*b*m^6*x^2+12032140*A*a*c*m^4*x^4+6016070*A*b^2*m^4*x^4+38432016
*A*b*c*m^2*x^6+4435200*A*c^2*x^8+29076*B*a^2*m^7*x+3246516*B*a*b*m^5*x^3+27
756240*B*a*c*m^3*x^5+13878120*B*b^2*m^3*x^5+28888560*B*b*c*m*x^7+275037*C*a
^2*m^6*x^2+12032140*C*a*b*m^4*x^4+38432016*C*a*c*m^2*x^6+19216008*C*b^2*m^2
*x^6+8870400*C*b*c*x^8+30810*A*a^2*m^7+3624894*A*a*b*m^5*x^2+31830760*A*a*c
*m^3*x^4+15915380*A*b^2*m^3*x^4+32811840*A*b*c*m*x^6+299271*B*a^2*m^6*x+136
93006*B*a*b*m^4*x^3+43978712*B*a*c*m^2*x^5+21989356*B*b^2*m^2*x^5+9979200*B
*b*c*x^7+1812447*C*a^2*m^5*x^2+31830760*C*a*b*m^3*x^4+32811840*C*a*c*m*x^6+
16405920*C*b^2*m*x^6+326613*A*a^2*m^6+15804388*A*a*b*m^4*x^2+51362352*A*a*c
*m^2*x^4+25681176*A*b^2*m^2*x^4+11404800*A*b*c*x^6+2039016*B*a^2*m^5*x+3721
9436*B*a*b*m^3*x^3+37963680*B*a*c*m*x^5+18981840*B*b^2*m*x^5+7902194*C*a^2*
m^4*x^2+51362352*C*a*b*m^2*x^4+11404800*C*a*c*x^6+5702400*C*b^2*x^6+2310945
*A*a^2*m^5+44578296*A*a*b*m^3*x^2+45024192*A*a*c*m*x^4+22512096*A*b^2*m*x^4
+9261503*B*a^2*m^4*x+61638408*B*a*b*m^2*x^3+13305600*B*a*c*x^5+6652800*B*b^
2*x^5+22289148*C*a^2*m^3*x^2+45024192*C*a*b*m*x^4+11028590*A*a^2*m^4+767812
64*A*a*b*m^2*x^2+15966720*A*a*c*x^4+7983360*A*b^2*x^4+27472724*B*a^2*m^3*x+
55282320*B*a*b*m*x^3+38390632*C*a^2*m^2*x^2+15966720*C*a*b*x^4+34967140*A*a
^2*m^3+71492160*A*a*b*m*x^2+50312628*B*a^2*m^2*x+19958400*B*a*b*x^3+3574608
0*C*a^2*m*x^2+70290936*A*a^2*m^2+26611200*A*a*b*x^2+50292720*B*a^2*m*x+1330
5600*C*a^2*x^2+80627040*A*a^2*m+19958400*B*a^2*x+39916800*A*a^2)*(d*x)^m/(m
+11)/(10+m)/(m+9)/(8+m)/(m+7)/(6+m)/(m+5)/(4+m)/(m+3)/(2+m)/(m+1)
```

maxima [A] time = 1.73, size = 344, normalized size = 1.32

$$\frac{C^2 d^{m+11} x^m}{m+11} + \frac{B c^2 d^m x^{10} x^m}{m+10} + \frac{2 C b c d^m x^9 x^m}{m+9} + \frac{A c^2 d^m x^9 x^m}{m+9} + \frac{2 B b c d^m x^8 x^m}{m+8} + \frac{C b^2 d^m x^7 x^m}{m+7} + \frac{2 C a c d^m x^7 x^m}{m+7} + \frac{2 A b c d^m x^6 x^m}{m+6} + \frac{2 B a c d^m x^6 x^m}{m+6} + \frac{2 C a b d^m x^5 x^m}{m+5} + \frac{A b^2 d^m x^5 x^m}{m+5} + \frac{2 A a c d^m x^5 x^m}{m+5} + \frac{2 B a b d^m x^4 x^m}{m+4} + \frac{C a^2 d^m x^4 x^m}{m+4} + \frac{2 A a b d^m x^3 x^m}{m+3} + \frac{B a^2 d^m x^3 x^m}{m+3} + \frac{C a^2 d^m x^2 x^m}{m+2} + \frac{B a^2 d^m x^2 x^m}{m+2} + \frac{(d x)^{m+1} A a^2}{d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
[Out] C*c^2*d^m*x^11*x^m/(m + 11) + B*c^2*d^m*x^10*x^m/(m + 10) + 2*C*b*c*d^m*x^9
*x^m/(m + 9) + A*c^2*d^m*x^9*x^m/(m + 9) + 2*B*b*c*d^m*x^8*x^m/(m + 8) + C*
b^2*d^m*x^7*x^m/(m + 7) + 2*C*a*c*d^m*x^7*x^m/(m + 7) + 2*A*b*c*d^m*x^7*x^m
/(m + 7) + B*b^2*d^m*x^6*x^m/(m + 6) + 2*B*a*c*d^m*x^6*x^m/(m + 6) + 2*C*a*
b*d^m*x^5*x^m/(m + 5) + A*b^2*d^m*x^5*x^m/(m + 5) + 2*A*a*c*d^m*x^5*x^m/(m
+ 5) + 2*B*a*b*d^m*x^4*x^m/(m + 4) + C*a^2*d^m*x^3*x^m/(m + 3) + 2*A*a*b*d^
m*x^3*x^m/(m + 3) + B*a^2*d^m*x^2*x^m/(m + 2) + (d*x)^(m + 1)*A*a^2/(d*(m +
1))
```

mupad [B] time = 1.81, size = 1314, normalized size = 5.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x)
[Out] (x^5*(d*x)^m*(A*b^2 + 2*A*a*c + 2*C*a*b)*(22512096*m + 25681176*m^2 + 15915
380*m^3 + 6016070*m^4 + 1464693*m^5 + 234573*m^6 + 24570*m^7 + 1620*m^8 + 6
1*m^9 + m^10 + 7983360))/(120543840*m + 150917976*m^2 + 105258076*m^3 + 459
95730*m^4 + 13339535*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9
+ 66*m^10 + m^11 + 39916800) + (x^7*(d*x)^m*(C*b^2 + 2*A*b*c + 2*C*a*c)*(16
405920*m + 19216008*m^2 + 12291724*m^3 + 4814858*m^4 + 1217811*m^5 + 202821
*m^6 + 22086*m^7 + 1512*m^8 + 59*m^9 + m^10 + 5702400))/(120543840*m + 1509
```

$$\begin{aligned}
& 17976m^2 + 105258076m^3 + 45995730m^4 + 13339535m^5 + 2637558m^6 + 357423m^7 + 32670m^8 + 1925m^9 + 66m^{10} + m^{11} + 39916800) + (Bx^6(dx)^m \\
& m(2ac + b^2)(18981840m + 21989356m^2 + 13878120m^3 + 5352935m^4 + 1331100m^5 + 217743m^6 + 23280m^7 + 1565m^8 + 60m^9 + m^{10} + 6652800))/ \\
& (120543840m + 150917976m^2 + 105258076m^3 + 45995730m^4 + 13339535m^5 + 2637558m^6 + 357423m^7 + 32670m^8 + 1925m^9 + 66m^{10} + m^{11} + 39916800) \\
& + (Aa^2x(dx)^m(80627040m + 70290936m^2 + 34967140m^3 + 11028590m^4 + 2310945m^5 + 326613m^6 + 30810m^7 + 1860m^8 + 65m^9 + m^{10} + 39916800))/ \\
& (120543840m + 150917976m^2 + 105258076m^3 + 45995730m^4 + 13339535m^5 + 2637558m^6 + 357423m^7 + 32670m^8 + 1925m^9 + 66m^{10} + m^{11} + 39916800) \\
& + (cx^9(dx)^m(Ac + 2Cb)(12900960m + 15335224m^2 + 9991428m^3 + 4000478m^4 + 1037673m^5 + 177765m^6 + 19962m^7 + 1412m^8 + 57m^9 + m^{10} + 4435200))/ \\
& (120543840m + 150917976m^2 + 105258076m^3 + 45995730m^4 + 13339535m^5 + 2637558m^6 + 357423m^7 + 32670m^8 + 1925m^9 + 66m^{10} + m^{11} + 39916800) \\
& + (ax^3(dx)^m(2Ab + Ca)(35746080m + 38390632m^2 + 22289148m^3 + 7902194m^4 + 1812447m^5 + 275037m^6 + 27462m^7 + 1736m^8 + 63m^9 + m^{10} + 13305600))/ \\
& (120543840m + 150917976m^2 + 105258076m^3 + 45995730m^4 + 13339535m^5 + 2637558m^6 + 357423m^7 + 32670m^8 + 1925m^9 + 66m^{10} + m^{11} + 39916800) \\
& + (Bc^2x^{10}(dx)^m(11655216m + 13926276m^2 + 9133180m^3 + 3686255m^4 + 965328m^5 + 167223m^6 + 19020m^7 + 1365m^8 + 56m^9 + m^{10} + 3991680))/ \\
& (120543840m + 150917976m^2 + 105258076m^3 + 45995730m^4 + 13339535m^5 + 2637558m^6 + 357423m^7 + 32670m^8 + 1925m^9 + 66m^{10} + m^{11} + 39916800) \\
& + (Ccx^{11}(dx)^m(10628640m + 12753576m^2 + 8409500m^3 + 3416930m^4 + 902055m^5 + 157773m^6 + 18150m^7 + 1320m^8 + 55m^9 + m^{10} + 3628800))/ \\
& (120543840m + 150917976m^2 + 105258076m^3 + 45995730m^4 + 13339535m^5 + 2637558m^6 + 357423m^7 + 32670m^8 + 1925m^9 + 66m^{10} + m^{11} + 39916800) \\
& + (Ba^2x^2(dx)^m(50292720m + 50312628m^2 + 27472724m^3 + 9261503m^4 + 2039016m^5 + 299271m^6 + 29076m^7 + 1797m^8 + 64m^9 + m^{10} + 19958400))/ \\
& (120543840m + 150917976m^2 + 105258076m^3 + 45995730m^4 + 13339535m^5 + 2637558m^6 + 357423m^7 + 32670m^8 + 1925m^9 + 66m^{10} + m^{11} + 39916800) \\
& + (2Bb^2cx^8(dx)^m(14444280m + 17059212m^2 + 11024858m^3 + 4371359m^4 + 1121022m^5 + 189567m^6 + 20982m^7 + 1461m^8 + 58m^9 + m^{10} + 4989600))/ \\
& (120543840m + 150917976m^2 + 105258076m^3 + 45995730m^4 + 13339535m^5 + 2637558m^6 + 357423m^7 + 32670m^8 + 1925m^9 + 66m^{10} + m^{11} + 39916800) \\
& + (2Babx^4(dx)^m(27641160m + 30819204m^2 + 18609718m^3 + 6846503m^4 + 1623258m^5 + 253575m^6 + 25962m^7 + 1677m^8 + 62m^9 + m^{10} + 9979200))/ \\
& (120543840m + 150917976m^2 + 105258076m^3 + 45995730m^4 + 13339535m^5 + 2637558m^6 + 357423m^7 + 32670m^8 + 1925m^9 + 66m^{10} + m^{11} + 39916800)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((dx)**m*(Cx**2+Bx+A)*(c*x**4+b*x**2+a)**2,x)

[Out] Timed out

3.39 $\int (dx)^m (A + Bx + Cx^2) (a + bx^2 + cx^4) dx$

Optimal. Leaf size=137

$$\frac{(dx)^{m+3}(aC + Ab)}{d^3(m+3)} + \frac{aA(dx)^{m+1}}{d(m+1)} + \frac{aB(dx)^{m+2}}{d^2(m+2)} + \frac{(dx)^{m+5}(Ac + bC)}{d^5(m+5)} + \frac{bB(dx)^{m+4}}{d^4(m+4)} + \frac{Bc(dx)^{m+6}}{d^6(m+6)} + \frac{cC(dx)^{m+7}}{d^7(m+7)}$$

Rubi [A] time = 0.09, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {1628}

$$\frac{(dx)^{m+3}(aC + Ab)}{d^3(m+3)} + \frac{aA(dx)^{m+1}}{d(m+1)} + \frac{aB(dx)^{m+2}}{d^2(m+2)} + \frac{(dx)^{m+5}(Ac + bC)}{d^5(m+5)} + \frac{bB(dx)^{m+4}}{d^4(m+4)} + \frac{Bc(dx)^{m+6}}{d^6(m+6)} + \frac{cC(dx)^{m+7}}{d^7(m+7)}$$

Antiderivative was successfully verified.

[In] Int[(d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4), x]

[Out] (a*A*(d*x)^(1 + m))/(d*(1 + m)) + (a*B*(d*x)^(2 + m))/(d^2*(2 + m)) + ((A*b + a*C)*(d*x)^(3 + m))/(d^3*(3 + m)) + (b*B*(d*x)^(4 + m))/(d^4*(4 + m)) + ((A*c + b*C)*(d*x)^(5 + m))/(d^5*(5 + m)) + (B*c*(d*x)^(6 + m))/(d^6*(6 + m)) + (c*C*(d*x)^(7 + m))/(d^7*(7 + m))

Rule 1628

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int (dx)^m (A + Bx + Cx^2) (a + bx^2 + cx^4) dx &= \int \left(aA(dx)^m + \frac{aB(dx)^{1+m}}{d} + \frac{(Ab + aC)(dx)^{2+m}}{d^2} + \frac{bB(dx)^{3+m}}{d^3} \right. \\ &= \frac{aA(dx)^{1+m}}{d(1+m)} + \frac{aB(dx)^{2+m}}{d^2(2+m)} + \frac{(Ab + aC)(dx)^{3+m}}{d^3(3+m)} + \frac{bB(dx)^{4+m}}{d^4(4+m)} \end{aligned}$$

Mathematica [A] time = 0.10, size = 90, normalized size = 0.66

$$x(dx)^m \left(\frac{x^2(aC + Ab)}{m+3} + \frac{aA}{m+1} + \frac{aBx}{m+2} + \frac{x^4(Ac + bC)}{m+5} + \frac{bBx^3}{m+4} + \frac{Bcx^5}{m+6} + \frac{cCx^6}{m+7} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4), x]

[Out] x*(d*x)^m*((a*A)/(1 + m) + (a*B*x)/(2 + m) + ((A*b + a*C)*x^2)/(3 + m) + (b*B*x^3)/(4 + m) + ((A*c + b*C)*x^4)/(5 + m) + (B*c*x^5)/(6 + m) + (c*C*x^6)/(7 + m))

IntegrateAlgebraic [F] time = 0.64, size = 0, normalized size = 0.00

$$\int (dx)^m (A + Bx + Cx^2) (a + bx^2 + cx^4) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4), x]

[Out] Defer[IntegrateAlgebraic] [(d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4), x]
fricas [B] time = 1.29, size = 444, normalized size = 3.24

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] ((C*c*m^6 + 21*C*c*m^5 + 175*C*c*m^4 + 735*C*c*m^3 + 1624*C*c*m^2 + 1764*C*c*m + 720*C*c)*x^7 + (B*c*m^6 + 22*B*c*m^5 + 190*B*c*m^4 + 820*B*c*m^3 + 1849*B*c*m^2 + 2038*B*c*m + 840*B*c)*x^6 + ((C*b + A*c)*m^6 + 23*(C*b + A*c)*m^5 + 207*(C*b + A*c)*m^4 + 925*(C*b + A*c)*m^3 + 2144*(C*b + A*c)*m^2 + 1008*C*b + 1008*A*c + 2412*(C*b + A*c)*m)*x^5 + (B*b*m^6 + 24*B*b*m^5 + 226*B*b*m^4 + 1056*B*b*m^3 + 2545*B*b*m^2 + 2952*B*b*m + 1260*B*b)*x^4 + ((C*a + A*b)*m^6 + 25*(C*a + A*b)*m^5 + 247*(C*a + A*b)*m^4 + 1219*(C*a + A*b)*m^3 + 3112*(C*a + A*b)*m^2 + 1680*C*a + 1680*A*b + 3796*(C*a + A*b)*m)*x^3 + (B*a*m^6 + 26*B*a*m^5 + 270*B*a*m^4 + 1420*B*a*m^3 + 3929*B*a*m^2 + 5274*B*a*m + 2520*B*a)*x^2 + (A*a*m^6 + 27*A*a*m^5 + 295*A*a*m^4 + 1665*A*a*m^3 + 5104*A*a*m^2 + 8028*A*a*m + 5040*A*a)*x)*(d*x)^m/(m^7 + 28*m^6 + 322*m^5 + 1960*m^4 + 6769*m^3 + 13132*m^2 + 13068*m + 5040)

giac [B] time = 0.53, size = 914, normalized size = 6.67

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] ((d*x)^m*C*c*m^6*x^7 + (d*x)^m*B*c*m^6*x^6 + 21*(d*x)^m*C*c*m^5*x^7 + (d*x)^m*C*b*m^6*x^5 + (d*x)^m*A*c*m^6*x^5 + 22*(d*x)^m*B*c*m^5*x^6 + 175*(d*x)^m*C*c*m^4*x^7 + (d*x)^m*B*b*m^6*x^4 + 23*(d*x)^m*C*b*m^5*x^5 + 23*(d*x)^m*A*c*m^5*x^5 + 190*(d*x)^m*B*c*m^4*x^6 + 735*(d*x)^m*C*c*m^3*x^7 + (d*x)^m*C*a*m^6*x^3 + (d*x)^m*A*b*m^6*x^3 + 24*(d*x)^m*B*b*m^5*x^4 + 207*(d*x)^m*C*b*m^4*x^5 + 207*(d*x)^m*A*c*m^4*x^5 + 820*(d*x)^m*B*c*m^3*x^6 + 1624*(d*x)^m*C*c*m^2*x^7 + (d*x)^m*B*a*m^6*x^2 + 25*(d*x)^m*C*a*m^5*x^3 + 25*(d*x)^m*A*b*m^5*x^3 + 226*(d*x)^m*B*b*m^4*x^4 + 925*(d*x)^m*C*b*m^3*x^5 + 925*(d*x)^m*A*c*m^3*x^5 + 1849*(d*x)^m*B*c*m^2*x^6 + 1764*(d*x)^m*C*c*m*x^7 + (d*x)^m*A*a*m^6*x + 26*(d*x)^m*B*a*m^5*x^2 + 247*(d*x)^m*C*a*m^4*x^3 + 247*(d*x)^m*A*b*m^4*x^3 + 1056*(d*x)^m*B*b*m^3*x^4 + 2144*(d*x)^m*C*b*m^2*x^5 + 2144*(d*x)^m*A*c*m^2*x^5 + 2038*(d*x)^m*B*c*m*x^6 + 720*(d*x)^m*C*c*x^7 + 27*(d*x)^m*A*a*m^5*x + 270*(d*x)^m*B*a*m^4*x^2 + 1219*(d*x)^m*C*a*m^3*x^3 + 1219*(d*x)^m*A*b*m^3*x^3 + 2545*(d*x)^m*B*b*m^2*x^4 + 2412*(d*x)^m*C*b*m*x^5 + 2412*(d*x)^m*A*c*m*x^5 + 840*(d*x)^m*B*c*x^6 + 295*(d*x)^m*A*a*m^4*x + 1420*(d*x)^m*B*a*m^3*x^2 + 3112*(d*x)^m*C*a*m^2*x^3 + 3112*(d*x)^m*A*b*m^2*x^3 + 2952*(d*x)^m*B*b*m*x^4 + 1008*(d*x)^m*C*b*x^5 + 1008*(d*x)^m*A*c*x^5 + 1665*(d*x)^m*A*a*m^3*x + 3929*(d*x)^m*B*a*m^2*x^2 + 3796*(d*x)^m*C*a*m*x^3 + 3796*(d*x)^m*A*b*m*x^3 + 1260*(d*x)^m*B*b*x^4 + 5104*(d*x)^m*A*a*m^2*x + 5274*(d*x)^m*B*a*m*x^2 + 1680*(d*x)^m*C*a*x^3 + 1680*(d*x)^m*A*b*x^3 + 8028*(d*x)^m*A*a*m*x + 2520*(d*x)^m*B*a*x^2 + 5040*(d*x)^m*A*a*x)/(m^7 + 28*m^6 + 322*m^5 + 1960*m^4 + 6769*m^3 + 13132*m^2 + 13068*m + 5040)

maple [B] time = 0.00, size = 585, normalized size = 4.27

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x)

```
[Out] x*(C*c*m^6*x^6+B*c*m^6*x^5+21*C*c*m^5*x^6+A*c*m^6*x^4+22*B*c*m^5*x^5+C*b*m^6*x^4+175*C*c*m^4*x^6+23*A*c*m^5*x^4+B*b*m^6*x^3+190*B*c*m^4*x^5+23*C*b*m^5*x^4+735*C*c*m^3*x^6+A*b*m^6*x^2+207*A*c*m^4*x^4+24*B*b*m^5*x^3+820*B*c*m^3*x^5+C*a*m^6*x^2+207*C*b*m^4*x^4+1624*C*c*m^2*x^6+25*A*b*m^5*x^2+925*A*c*m^3*x^4+B*a*m^6*x+226*B*b*m^4*x^3+1849*B*c*m^2*x^5+25*C*a*m^5*x^2+925*C*b*m^3*x^4+1764*C*c*m*x^6+A*a*m^6+247*A*b*m^4*x^2+2144*A*c*m^2*x^4+26*B*a*m^5*x+1056*B*b*m^3*x^3+2038*B*c*m*x^5+247*C*a*m^4*x^2+2144*C*b*m^2*x^4+720*C*c*x^6+27*A*a*m^5+1219*A*b*m^3*x^2+2412*A*c*m*x^4+270*B*a*m^4*x+2545*B*b*m^2*x^3+840*B*c*x^5+1219*C*a*m^3*x^2+2412*C*b*m*x^4+295*A*a*m^4+3112*A*b*m^2*x^2+1008*A*c*x^4+1420*B*a*m^3*x+2952*B*b*m*x^3+3112*C*a*m^2*x^2+1008*C*b*x^4+1665*A*a*m^3+3796*A*b*m*x^2+3929*B*a*m^2*x+1260*B*b*x^3+3796*C*a*m*x^2+5104*A*a*m^2+1680*A*b*x^2+5274*B*a*m*x+1680*C*a*x^2+8028*A*a*m+2520*B*a*x+5040*A*a)*(d*x)^m/(m+7)/(m+6)/(m+5)/(m+4)/(m+3)/(m+2)/(m+1)
```

maxima [A] time = 0.82, size = 155, normalized size = 1.13

$$\frac{Ccd^m x^7 x^m}{m+7} + \frac{Bcd^m x^6 x^m}{m+6} + \frac{Cbd^m x^5 x^m}{m+5} + \frac{Acd^m x^5 x^m}{m+5} + \frac{Bbd^m x^4 x^m}{m+4} + \frac{Cad^m x^3 x^m}{m+3} + \frac{Abd^m x^3 x^m}{m+3} + \frac{Bad^m x^2 x^m}{m+2} + \frac{(dx)^{m+1} Aa}{d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] C*c*d^m*x^7*x^m/(m + 7) + B*c*d^m*x^6*x^m/(m + 6) + C*b*d^m*x^5*x^m/(m + 5) + A*c*d^m*x^5*x^m/(m + 5) + B*b*d^m*x^4*x^m/(m + 4) + C*a*d^m*x^3*x^m/(m + 3) + A*b*d^m*x^3*x^m/(m + 3) + B*a*d^m*x^2*x^m/(m + 2) + (d*x)^(m + 1)*A*a/(d*(m + 1))
```

mupad [B] time = 1.07, size = 527, normalized size = 3.85

^1/10^10 (13 + C3) (d^m x^7 + 20d^m x^6 + 120d^m x^5 + 280d^m x^4 + 280d^m x^3 + 168d^m x^2 + 504d^m x + 360d^m) (c x^4 + b x^2 + a) x^m / (m + 7) + ...

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x)
[Out] (x^3*(d*x)^m*(A*b + C*a)*(3796*m + 3112*m^2 + 1219*m^3 + 247*m^4 + 25*m^5 + m^6 + 1680))/(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040) + (x^5*(d*x)^m*(A*c + C*b)*(2412*m + 2144*m^2 + 925*m^3 + 207*m^4 + 23*m^5 + m^6 + 1008))/(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040) + (A*a*x*(d*x)^m*(8028*m + 5104*m^2 + 1665*m^3 + 295*m^4 + 27*m^5 + m^6 + 5040))/(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040) + (B*a*x^2*(d*x)^m*(5274*m + 3929*m^2 + 1420*m^3 + 270*m^4 + 26*m^5 + m^6 + 2520))/(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040) + (B*b*x^4*(d*x)^m*(2952*m + 2545*m^2 + 1056*m^3 + 226*m^4 + 24*m^5 + m^6 + 1260))/(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040) + (B*c*x^6*(d*x)^m*(2038*m + 1849*m^2 + 820*m^3 + 190*m^4 + 22*m^5 + m^6 + 840))/(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040) + (C*c*x^7*(d*x)^m*(1764*m + 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6 + 720))/(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040)
```

sympy [A] time = 2.58, size = 3735, normalized size = 27.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(C*x**2+B*x+A)*(c*x**4+b*x**2+a),x)
[Out] Piecewise((( -A*a/(6*x**6) - A*b/(4*x**4) - A*c/(2*x**2) - B*a/(5*x**5) - B*b/(3*x**3) - B*c/x - C*a/(4*x**4) - C*b/(2*x**2) + C*c*log(x))/d**7, Eq(m, -7)), (( -A*a/(5*x**5) - A*b/(3*x**3) - A*c/x - B*a/(4*x**4) - B*b/(2*x**2) + B*c*log(x) - C*a/(3*x**3) - C*b/x + C*c*x)/d**6, Eq(m, -6)), (( -A*a/(4*x**
```

$*4) - A*b/(2*x**2) + A*c*log(x) - B*a/(3*x**3) - B*b/x + B*c*x - C*a/(2*x**2) + C*b*log(x) + C*c*x**2/2/d**5, Eq(m, -5)), ((-A*a/(3*x**3) - A*b/x + A*c*x - B*a/(2*x**2) + B*b*log(x) + B*c*x**2/2 - C*a/x + C*b*x + C*c*x**3/3)/d**4, Eq(m, -4)), ((-A*a/(2*x**2) + A*b*log(x) + A*c*x**2/2 - B*a/x + B*b*x + B*c*x**3/3 + C*a*log(x) + C*b*x**2/2 + C*c*x**4/4)/d**3, Eq(m, -3)), ((-A*a/x + A*b*x + A*c*x**3/3 + B*a*log(x) + B*b*x**2/2 + B*c*x**4/4 + C*a*x + C*b*x**3/3 + C*c*x**5/5)/d**2, Eq(m, -2)), ((A*a*log(x) + A*b*x**2/2 + A*c*x**4/4 + B*a*x + B*b*x**3/3 + B*c*x**5/5 + C*a*x**2/2 + C*b*x**4/4 + C*c*x**6/6)/d, Eq(m, -1)), (A*a*d**m**6*x*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 27*A*a*d**m**5*x*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 295*A*a*d**m**4*x*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 1665*A*a*d**m**3*x*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 5104*A*a*d**m**2*x*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 8028*A*a*d**m*x*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 5040*A*a*d**m*x*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + A*b*d**m**6*x**3*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 25*A*b*d**m**5*x**3*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 247*A*b*d**m**4*x**3*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 1219*A*b*d**m**3*x**3*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 3112*A*b*d**m**2*x**3*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 3796*A*b*d**m*x**3*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 1680*A*b*d**m*x**3*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + A*c*d**m**6*x**5*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 23*A*c*d**m**5*x**5*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 207*A*c*d**m**4*x**5*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 925*A*c*d**m**3*x**5*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 2144*A*c*d**m**2*x**5*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 2412*A*c*d**m*x**5*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 1008*A*c*d**m*x**5*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + B*a*d**m**6*x**2*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 26*B*a*d**m**5*x**2*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 270*B*a*d**m**4*x**2*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 1420*B*a*d**m**3*x**2*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 3929*B*a*d**m**2*x**2*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 5274*B*a*d**m*x**2*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 2520*B*a*d**m*x**2*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + B*b*d**m**6*x**4*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 24*B*b*d**m**5*x**4*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 226*B*b*d**m**4*x**4*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 1056*B*b*d**m**3*x**4*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 2545*B*b*d**m**2*x**4*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 2952*B*b*d**m*x**4*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 1260*B*b*d**m*x**4*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 1260*B*b*d**m*x**4*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040)$

```

0*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + B*c*d**m**6*x**6*x**m
/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m
+ 5040) + 22*B*c*d**m**5*x**6*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4
+ 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 190*B*c*d**m**4*x**6*x**m/(
m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m +
5040) + 820*B*c*d**m**3*x**6*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4
+ 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 1849*B*c*d**m**2*x**6*x**m/(
m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m +
5040) + 2038*B*c*d**m*x**6*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 +
6769*m**3 + 13132*m**2 + 13068*m + 5040) + 840*B*c*d**m*x**6*x**m/(m**7 + 2
8*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) +
C*a*d**m**6*x**3*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3
+ 13132*m**2 + 13068*m + 5040) + 25*C*a*d**m**5*x**3*x**m/(m**7 + 28*m**6
+ 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 247*C*
a*d**m**4*x**3*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 +
13132*m**2 + 13068*m + 5040) + 1219*C*a*d**m**3*x**3*x**m/(m**7 + 28*m**6
+ 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 3112*C
*a*d**m**2*x**3*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 +
13132*m**2 + 13068*m + 5040) + 3796*C*a*d**m*x**3*x**m/(m**7 + 28*m**6 +
322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 1680*C*a
*d**m*x**3*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*
m**2 + 13068*m + 5040) + C*b*d**m**6*x**5*x**m/(m**7 + 28*m**6 + 322*m**5
+ 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 23*C*b*d**m**5*
x**5*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 +
13068*m + 5040) + 207*C*b*d**m**4*x**5*x**m/(m**7 + 28*m**6 + 322*m**5 +
1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 925*C*b*d**m**3*x
**5*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 +
13068*m + 5040) + 2144*C*b*d**m**2*x**5*x**m/(m**7 + 28*m**6 + 322*m**5 +
1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 2412*C*b*d**m*x**
5*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13
068*m + 5040) + 1008*C*b*d**m*x**5*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m
**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + C*c*d**m**6*x**7*x**m/(m
**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5
040) + 21*C*c*d**m**5*x**7*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 +
6769*m**3 + 13132*m**2 + 13068*m + 5040) + 175*C*c*d**m**4*x**7*x**m/(m**
7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 504
0) + 735*C*c*d**m**3*x**7*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6
769*m**3 + 13132*m**2 + 13068*m + 5040) + 1624*C*c*d**m**2*x**7*x**m/(m**
7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 504
0) + 1764*C*c*d**m*x**7*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 676
9*m**3 + 13132*m**2 + 13068*m + 5040) + 720*C*c*d**m*x**7*x**m/(m**7 + 28*m
**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040), True
))

```

$$3.40 \quad \int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2} dx$$

Optimal. Leaf size=356

$$\frac{x(-2aC + x^2(2Ac - bC) + Ab)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{\left(-\frac{4Abc - C(4ac + b^2)}{\sqrt{b^2 - 4ac}} + 2Ac - bC\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2 - 4ac)\sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{\left(\frac{4Abc - C(4ac + b^2)}{\sqrt{b^2 - 4ac}} + 2Ac - bC\right)}{2\sqrt{2}\sqrt{c}(b^2 - 4ac)}$$

Rubi [A] time = 0.92, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {1662, 1275, 1166, 205, 12, 1114, 638, 618, 206}

$$\frac{x(-2aC + x^2(2Ac - bC) + Ab)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{\left(-\frac{4Abc - C(4ac + b^2)}{\sqrt{b^2 - 4ac}} + 2Ac - bC\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2 - 4ac)\sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{\left(\frac{4Abc - C(4ac + b^2)}{\sqrt{b^2 - 4ac}} + 2Ac - bC\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b + \sqrt{b^2 - 4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2 - 4ac)\sqrt{b + \sqrt{b^2 - 4ac}}} + \frac{B(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{bB \tanh^{-1}\left(\frac{b + 2cx^2}{\sqrt{b^2 - 4ac}}\right)}{(b^2 - 4ac)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2, x]
```

```
[Out] (B*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (x*(A*b - 2*a*C + (2*A*c - b*C)*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((2*A*c - b*C - (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((2*A*c - b*C + (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (b*B*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```


Rule 1114

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1166

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1275

```
Int[((f_)*(x_)^(m_)*((d_) + (e_)*(x_)^2)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1662

```
Int[(Pq_)*((d_)*(x_)^(m_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2 + c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] && !PolyQ[Pq, x^2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2 (A + Bx + Cx^2)}{(a + bx^2 + cx^4)^2} dx &= \int \frac{Bx^3}{(a + bx^2 + cx^4)^2} dx + \int \frac{x^2 (A + Cx^2)}{(a + bx^2 + cx^4)^2} dx \\
&= -\frac{x (Ab - 2aC + (2Ac - bC)x^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} + B \int \frac{x^3}{(a + bx^2 + cx^4)^2} dx + \frac{\int \frac{Ab - 2aC + (-2Ac + bC)x^2}{a + bx^2 + cx^4} dx}{2(b^2 - 4ac)} \\
&= -\frac{x (Ab - 2aC + (2Ac - bC)x^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{1}{2} B \operatorname{Subst} \left(\int \frac{x}{(a + bx + cx^2)^2} dx, x, x^2 \right) - \frac{(2Ac - bC)}{2(b^2 - 4ac)} \\
&= \frac{B(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{x (Ab - 2aC + (2Ac - bC)x^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{(2Ac - bC - \frac{4Abc - (b^2 - 4ac)C}{\sqrt{b^2 - 4ac}})}{2\sqrt{2} \sqrt{c} (b^2 - 4ac)} \\
&= \frac{B(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{x (Ab - 2aC + (2Ac - bC)x^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{(2Ac - bC - \frac{4Abc - (b^2 - 4ac)C}{\sqrt{b^2 - 4ac}})}{2\sqrt{2} \sqrt{c} (b^2 - 4ac)} \\
&= \frac{B(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{x (Ab - 2aC + (2Ac - bC)x^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{(2Ac - bC - \frac{4Abc - (b^2 - 4ac)C}{\sqrt{b^2 - 4ac}})}{2\sqrt{2} \sqrt{c} (b^2 - 4ac)}
\end{aligned}$$

Mathematica [A] time = 1.04, size = 378, normalized size = 1.06

$$\frac{1}{4} \left(\frac{4a(B + Cx) + 2x(B + Cx) - A(b + 2cx^2)}{(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\sqrt{2} \left(C(b\sqrt{b^2 - 4ac} - 4ac - b^2) - 2Ac(\sqrt{b^2 - 4ac} - 2b) \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right) + \sqrt{2} \left(C(b\sqrt{b^2 - 4ac} + 4ac + b^2) - 2Ac(\sqrt{b^2 - 4ac} + 2b) \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b + \sqrt{b^2 - 4ac}}} \right) + \frac{2bB \log(\sqrt{b^2 - 4ac} - b - 2cx^2)}{(b^2 - 4ac)^{3/2}} - \frac{2bB \log(\sqrt{b^2 - 4ac} + b + 2cx^2)}{(b^2 - 4ac)^{3/2}}}{\sqrt{c} (b^2 - 4ac)^{3/2} \sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{\sqrt{2} \left(C(b\sqrt{b^2 - 4ac} + 4ac + b^2) - 2Ac(\sqrt{b^2 - 4ac} + 2b) \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right) + \sqrt{2} \left(C(b\sqrt{b^2 - 4ac} - 4ac - b^2) - 2Ac(\sqrt{b^2 - 4ac} - 2b) \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b + \sqrt{b^2 - 4ac}}} \right) + \frac{2bB \log(\sqrt{b^2 - 4ac} - b - 2cx^2)}{(b^2 - 4ac)^{3/2}} - \frac{2bB \log(\sqrt{b^2 - 4ac} + b + 2cx^2)}{(b^2 - 4ac)^{3/2}}}{\sqrt{c} (b^2 - 4ac)^{3/2} \sqrt{b + \sqrt{b^2 - 4ac}}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]

[Out] ((4*a*(B + C*x) + 2*x*(b*x*(B + C*x) - A*(b + 2*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(-2*A*c*(-2*b + Sqrt[b^2 - 4*a*c]) + (-b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(-2*A*c*(2*b + Sqrt[b^2 - 4*a*c]) + (b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (2*b*B*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) - (2*b*B*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 (A + Bx + Cx^2)}{(a + bx^2 + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]

[Out] IntegrateAlgebraic[(x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2, x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

```
giac [B] time = 7.04, size = 4440, normalized size = 12.47
```

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(C*b*x^3 - 2*A*c*x^3 + B*b*x^2 + 2*C*a*x - A*b*x + 2*B*a)/((c*x^4 + b*x^2 + a)*(b^2 - 4*a*c)) - 1/16*(2*(2*b^2*c^3 - 8*a*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c))*sqrt(b*c + sqrt(b^2 - 4*a*c))*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*c^3 - 2*(b^2 - 4*a*c)*c^3)*(b^2 - 4*a*c)^2*A - (2*b^3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*(b^2 - 4*a*c)^2*C - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*b^5*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*b^4*c^2 - 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a*b^2*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*b^3*c^3 + 16*a*b^3*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a*b*c^4 - 32*a^2*b*c^4 + 2*(b^2 - 4*a*c)*b^3*c^2 - 8*(b^2 - 4*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) + 4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a*b^3*c^2 - 2*a*b^4*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^3*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^2*b*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*c)*a*b^2*c^3 + 16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^2*c^4 - 32*a^3*c^4 + 2*(b^2 - 4*a*c)*a*b^2*c^2 - 8*(b^2 - 4*a*c)*a^2*c^3)*C*abs(b^2 - 4*a*c) - 4*(2*b^6*c^3 - 16*a*b^4*c^4 + 32*a^2*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*b^6*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a*b^4*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*b^5*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^2*b^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a*b^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^3 + 8*(b^2 - 4*a*c)*a*b^2*c^4)*A + (2*b^7*c^2 - 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*b^7 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*b^6*c + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^2*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^3*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^2*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 + 32*(b^2 - 4*a*c)*a^2*b*c^4)*C)*arctan(2*sqrt(1/2)*x/sqrt((b^3 - 4*a*b*c + sqrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2)))/((a*b^6*c - 12*a^2*b^4*c^2 - 2*a*b^5*c^2 + 48*a^3*b^2*c^3 + 16*a^2*b^3*c^3 + a*b^4*c^3 - 64*a^4*c^4 - 32*a^3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*abs(b^2 - 4*a*c)*abs(c)) + 1/16*(2*(2*b^2*c^3 - 8*a*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c))*sqrt(b*c - sqrt(b^2 - 4*a*c))*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
```

```

rt(b^2 - 4*a*c)*c)*c^3 - 2*(b^2 - 4*a*c)*c^3)*(b^2 - 4*a*c)^2*A - (2*b^3*c^
2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*b
^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a*b*c + 2*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*b^2*c - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*b*c^2 - 2*(b^2 - 4*a*c)*b*
c^2)*(b^2 - 4*a*c)^2*C + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*b^5*c -
8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c))*b^4*c^2 + 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c))*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a*b^2*c^3
+ sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*b^3*c^3 - 16*a*b^3*c^3 - 4*sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a*b*c^4 + 32*a^2*b*c^4 - 2*(b^2 - 4*a*c)
*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) - 4*(sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c))*a*b^4*c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a*b^3*c^2 + 2*a*b
^4*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^3*c^3 + 8*sqrt(2)*sqr
t(b*c - sqrt(b^2 - 4*a*c))*a^2*b*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*
c))*a*b^2*c^3 - 16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^2*c^4 + 32*a^3*c^4 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + 8*(b^2 - 4*a*c)*a^2*c
^3)*C*abs(b^2 - 4*a*c) - 4*(2*b^6*c^3 - 16*a*b^4*c^4 + 32*a^2*b^2*c^5 - sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*b^6*c + 8*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a*b^4*c^2 + 2*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*b^5*c^2 - 16*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^2*b^2*c^3 - 8*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)
)*sqrt(b*c - sqrt(b^2 - 4*a*c))*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c))*a*b^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^3 + 8*(b^2 -
4*a*c)*a*b^2*c^4)*A + (2*b^7*c^2 - 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*
b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*b^7 + 4*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a*b^5*c + 2*sqrt(2)
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*b^6*c + 16*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^2*b^3*c^2 - sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^3*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^2*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 +
32*(b^2 - 4*a*c)*a^2*b*c^4)*C)*arctan(2*sqrt(1/2)*x/sqrt((b^3 - 4*a*b*c - s
qrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*
a*c^2)))/((a*b^6*c - 12*a^2*b^4*c^2 - 2*a*b^5*c^2 + 48*a^3*b^2*c^3 + 16*a^2
*b^3*c^3 + a*b^4*c^3 - 64*a^4*c^4 - 32*a^3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c
^5)*abs(b^2 - 4*a*c)*abs(c)) + 1/8*((b^4*c - 4*a*b^2*c^2 - 2*b^3*c^2 + b^2*
c^3 + (b^3*c - 4*a*b*c^2 - 2*b^2*c^2 + b*c^3)*sqrt(b^2 - 4*a*c))*B*abs(b^2
- 4*a*c) - (b^6*c - 8*a*b^4*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a*b^3*c^3
+ b^4*c^3 - 4*a*b^2*c^4 + (b^5*c - 4*a*b^3*c^2 - 2*b^4*c^2 + b^3*c^3)*sqrt(
b^2 - 4*a*c))*B)*log(x^2 + 1/2*(b^3 - 4*a*b*c + sqrt((b^3 - 4*a*b*c)^2 - 4*
(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2)))/((a*b^4 - 8*a^2*b
^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2*abs
(b^2 - 4*a*c)) + 1/8*((b^4*c - 4*a*b^2*c^2 - 2*b^3*c^2 + b^2*c^3 - (b^3*c -
4*a*b*c^2 - 2*b^2*c^2 + b*c^3)*sqrt(b^2 - 4*a*c))*B*abs(b^2 - 4*a*c) - (b^
6*c - 8*a*b^4*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a*b^3*c^3 + b^4*c^3 - 4*
a*b^2*c^4 - (b^5*c - 4*a*b^3*c^2 - 2*b^4*c^2 + b^3*c^3)*sqrt(b^2 - 4*a*c))*
B)*log(x^2 + 1/2*(b^3 - 4*a*b*c - sqrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2
*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2)))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*
c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2*abs(b^2 - 4*a*c))

```

maple [B] time = 0.00, size = 1119, normalized size = 3.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x)

```
[Out] (-1/2/(4*a*c-b^2)*B*b*x^2+1/2*(2*A*c-C*b)/(4*a*c-b^2)*x^3-1/(4*a*c-b^2)*B*a
+1/2*(A*b-2*C*a)/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+1/2/(4*a*c-b^2)^2*(-4*a*c+b
^2)^(1/2)*B*b*ln(-2*c*x^2-b+(-4*a*c+b^2)^(1/2))-c/(4*a*c-b^2)^2*2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*c*x)*A*(-4*a*c+b^2)^(1/2)*b-2*c^2/(4*a*c-b^2)^2*2^(1/2)/((-b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)
*a*A+1/2*c/(4*a*c-b^2)^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b^2+c/(4*a*c-b^2)^2*2^(1/2)
)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*c*x)*C*(-4*a*c+b^2)^(1/2)*a+1/4/(4*a*c-b^2)^2*2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c
*x)*C*(-4*a*c+b^2)^(1/2)*b^2+c/(4*a*c-b^2)^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*C*a*b-1/
4/(4*a*c-b^2)^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/(-
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3*C-1/2/(4*a*c-b^2)^2*(-4*a*c+b^2)^(
1/2)*B*b*ln(2*c*x^2+b+(-4*a*c+b^2)^(1/2))-1/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*(-4*a*c+b^2)^(1/2)*A*b*c*arctan(2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*c*x)+2/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
)*c)^(1/2)*A*a*c^2*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)-1/2/
(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*A*b^2*c*arctan(2^(1/
2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)+1/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*(-4*a*c+b^2)^(1/2)*C*a*c*arctan(2^(1/2)/((b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*c*x)+1/4/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
)*c)^(1/2)*(-4*a*c+b^2)^(1/2)*C*b^2*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*c*x)-1/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*C*a*b
*c*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)+1/4/(4*a*c-b^2)^2*2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*C*b^3*arctan(2^(1/2)/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*c*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{Bbx^2 + (Cb - 2Ac)x^3 + 2Ba + (2Ca - Ab)x}{2((b^2c - 4ac^2)x^4 + ab^2 - 4a^2c + (b^3 - 4abc)x^2)} - \frac{\int \frac{2Bbx+(Cb-2Ac)x^2-2Ca+Ab}{cx^4+bx^2+a} dx}{2(b^2 - 4ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*b*x^2 + (C*b - 2*A*c)*x^3 + 2*B*a + (2*C*a - A*b)*x)/((b^2*c - 4*a*c
^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2) - 1/2*integrate(-(2*B*b*x
+ (C*b - 2*A*c)*x^2 - 2*C*a + A*b)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c)
```

mupad [B] time = 0.00, size = 3835, normalized size = 10.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] symsum(log((8*A^3*a*c^4 + 6*A^3*b^2*c^3 + A*C^2*b^4*c - 3*C^3*a*b^3*c + 4*A
*B^2*b^3*c^2 + 8*A*C^2*a^2*c^3 - 5*A^2*C*b^3*c^2 - 4*C^3*a^2*b*c^2 + 18*A*C
^2*a*b^2*c^2 - 8*B^2*C*a*b^2*c^2 - 28*A^2*C*a*b*c^3)/(8*(b^6 - 64*a^3*c^3 +
48*a^2*b^2*c^2 - 12*a*b^4*c)) - root(256*a*b^12*c*z^4 - 1572864*a^6*b^2*c^
6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^3*b^8*c^3
*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8*c*z^2 -
6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^5*b*c^4*z
^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5*c^5*z^2
+ 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a^4*b^2*c^
4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8192*A^2*a^3*
b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A^2*b^9*c*z^
2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B*a^3*b*c^4*z
```

$$\begin{aligned}
& - 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*a^2*b^3*c^3* \\
& z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^2 - 48*A*B^2 \\
& *C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 48*A^2*B^2*a* \\
& b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*b^3*c^2 - 80 \\
& *A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A^4*a*b^2*c^3 \\
& + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C*b^5*c - 6*A \\
& *C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^4 + 9*A^4*b^ \\
& 4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k)*(root(256*a*b^12*c*z^4 - 157286 \\
& 4*a^6*b^2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440 \\
& *a^3*b^8*c^3*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a* \\
& b^8*c*z^2 - 6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2 \\
& *a^5*b*c^4*z^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C* \\
& a^5*c^5*z^2 + 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^ \\
& 2*a^4*b^2*c^4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8 \\
& 192*A^2*a^3*b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16* \\
& A^2*b^9*c*z^2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B \\
& *a^3*b*c^4*z - 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B* \\
& a^2*b^3*c^3*z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^ \\
& 2 - 48*A*B^2*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 4 \\
& 8*A^2*B^2*a*b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a* \\
& b^3*c^2 - 80*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A \\
& ^4*a*b^2*c^3 + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C \\
& *b^5*c - 6*A*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^ \\
& 4 + 9*A^4*b^4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k)*((x*(16*B*b^7*c^2 - \\
& 192*B*a*b^5*c^3 - 1024*B*a^3*b*c^5 + 768*B*a^2*b^3*c^4))/(4*(b^6 - 64*a^3* \\
& c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (16*A*b^7*c^2 + 2048*C*a^4*c^5 - 192* \\
& A*a*b^5*c^3 - 1024*A*a^3*b*c^5 - 32*C*a*b^6*c^2 + 768*A*a^2*b^3*c^4 + 384*C \\
& *a^2*b^4*c^3 - 1536*C*a^3*b^2*c^4))/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - \\
& 12*a*b^4*c)) + (root(256*a*b^12*c*z^4 - 1572864*a^6*b^2*c^6*z^4 + 983040*a^ \\
& 5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^3*b^8*c^3*z^4 - 6144*a^2*b \\
& ^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8*c*z^2 - 6144*A*C*a^3*b^4* \\
& c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^5*b*c^4*z^2 - 12288*A^2*a^ \\
& 4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5*c^5*z^2 + 8192*C^2*a^4*b^ \\
& 3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a^4*b^2*c^4*z^2 - 6144*B^2* \\
& a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8192*A^2*a^3*b^3*c^4*z^2 - 153 \\
& 6*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A^2*b^9*c*z^2 + 1024*B*C^2*a^ \\
& 4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B*a^3*b*c^4*z - 192*A^2*B*a*b^ \\
& 5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*a^2*b^3*c^3*z + 16*A^2*B*b^7* \\
& c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^2 - 48*A*B^2*C*a*b^4*c + 192* \\
& A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 48*A^2*B^2*a*b^3*c^2 - 96*A^3* \\
& C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*b^3*c^2 - 80*A*C^3*a^2*b^3*c \\
& + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A^4*a*b^2*c^3 + 4*B^2*C^2*a*b^ \\
& 5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C*b^5*c - 6*A*C^3*a*b^5 + 32*A \\
& ^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^4 + 9*A^4*b^4*c^2 + 16*A^4*a^ \\
& 2*c^4 + A^2*C^2*b^6, z, k)*x*(32*b^9*c^2 - 512*a*b^7*c^3 + 8192*a^4*b*c^6 + \\
& 3072*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^ \\
& 2 - 12*a*b^4*c))) - (16*A*B*b^5*c^2 + 256*B*C*a^2*b^2*c^3 - 256*A*B*a^2*b*c \\
& ^4 - 64*B*C*a*b^4*c^2))/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) \\
& + (x*(2*C^2*b^6*c + 64*A^2*a^2*c^5 + 20*A^2*b^4*c^3 - 8*B^2*b^5*c^2 - 64*C \\
& ^2*a^3*c^4 - 12*A*C*b^5*c^2 - 96*A^2*a*b^2*c^4 + 32*B^2*a*b^3*c^3 - 4*C^2*a \\
& *b^4*c^2 + 32*A*C*a*b^3*c^3 + 64*A*C*a^2*b*c^4))/(4*(b^6 - 64*a^3*c^3 + 48* \\
& a^2*b^2*c^2 - 12*a*b^4*c))) + (x*(4*B^3*b^3*c^2 + B*C^2*b^4*c + 8*A^2*B*b^2 \\
& *c^3 + 4*B*C^2*a*b^2*c^2 - 6*A*B*C*b^3*c^2 - 8*A*B*C*a*b*c^3))/(4*(b^6 - 64 \\
& *a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)))*root(256*a*b^12*c*z^4 - 1572864*a \\
& ^6*b^2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^ \\
& 3*b^8*c^3*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8 \\
& *c*z^2 - 6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^ \\
& 5*b*c^4*z^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5 \\
& *c^5*z^2 + 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a
\end{aligned}$$

$$\begin{aligned}
&^4b^2c^4z^2 - 6144B^2a^3b^4c^3z^2 + 1536B^2a^2b^6c^2z^2 + 8192 \\
&A^2a^3b^3c^4z^2 - 1536A^2a^2b^5c^3z^2 + 16C^2a^2b^9z^2 + 16A^2 \\
&b^9c^3z^2 + 1024B^2C^2a^4b^3c^3z + 192B^2C^2a^2b^5c^3z - 1024A^2B^2a^3 \\
&b^3c^4z - 192A^2B^2a^2b^5c^2z - 768B^2C^2a^3b^3c^2z + 768A^2B^2a^2 \\
&b^3c^3z + 16A^2B^2b^7c^3z - 16B^2C^2a^2b^7z - 64A^2B^2C^2a^2b^2c^2 - \\
&48A^2B^2C^2a^2b^4c + 192A^2C^2a^2b^2c^2 + 48B^2C^2a^2b^3c + 48A^2 \\
&B^2a^2b^3c^2 - 96A^3C^2a^2b^3c - 96A^3C^3a^3b^3c^2 - 80A^3C^3a^2b^3 \\
&c^2 - 80A^3C^3a^2b^3c + 42A^2C^2a^2b^4c + 24C^4a^3b^2c + 24A^4a^2 \\
&b^2c^3 + 4B^2C^2a^2b^5 + 4A^2B^2b^5c + 16B^4a^2b^4c - 6A^3C^3b^5 \\
&c - 6A^3C^3a^2b^5 + 32A^2C^2a^3c^3 + 16C^4a^4c^2 + 9C^4a^2b^4 + \\
&9A^4b^4c^2 + 16A^4a^2c^4 + A^2C^2b^6, z, k), k, 1, 4) - ((B*a)/(4*a*c - b^2) - (x*(A*b - 2*C*a))/(2*(4*a*c - b^2)) - (x^3*(2*A*c - C*b))/(2*(4*a*c - b^2)) + (B*b*x^2)/(2*(4*a*c - b^2)))/(a + b*x^2 + c*x^4)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(C*x**2+B*x+A)/(c*x**4+b*x**2+a)**2,x)

[Out] Timed out

$$3.41 \quad \int \frac{x(Ax+Bx^2+Cx^3)}{(a+bx^2+cx^4)^2} dx$$

Optimal. Leaf size=356

$$\frac{x(-2aC + x^2(2Ac - bC) + Ab)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{\left(-\frac{4Abc - C(4ac + b^2)}{\sqrt{b^2 - 4ac}} + 2Ac - bC\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2 - 4ac)\sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{\left(\frac{4Abc - C(4ac + b^2)}{\sqrt{b^2 - 4ac}} + 2Ac - bC\right)}{2\sqrt{2}\sqrt{c}(b^2 - 4ac)}$$

Rubi [A] time = 0.37, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1585, 1662, 1275, 1166, 205, 12, 1114, 638, 618, 206}

$$\frac{x(-2aC + x^2(2Ac - bC) + Ab)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{\left(-\frac{4Abc - C(4ac + b^2)}{\sqrt{b^2 - 4ac}} + 2Ac - bC\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2 - 4ac)\sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{\left(\frac{4Abc - C(4ac + b^2)}{\sqrt{b^2 - 4ac}} + 2Ac - bC\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b + \sqrt{b^2 - 4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2 - 4ac)\sqrt{b + \sqrt{b^2 - 4ac}}} + \frac{B(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{bB \tanh^{-1}\left(\frac{b + 2cx^2}{\sqrt{b^2 - 4ac}}\right)}{(b^2 - 4ac)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(x*(A*x + B*x^2 + C*x^3))/(a + b*x^2 + c*x^4)^2,x]

[Out] (B*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (x*(A*b - 2*a*C + (2*A*c - b*C)*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((2*A*c - b*C - (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((2*A*c - b*C + (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (b*B*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 638

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]

Rule 1114

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1166

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1275

```
Int[((f_)*(x_)^(m_)*((d_) + (e_)*(x_)^2)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1585

```
Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_) + (c_)*(x_)^(r_))^(n_), x_Symbol] := Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1662

```
Int[(Pq)*((d_)*(x_)^(m_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2 + 1})*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1})*(a + b*x^2 + c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] && !PolyQ[Pq, x^2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x(Ax + Bx^2 + Cx^3)}{(a + bx^2 + cx^4)^2} dx &= \int \frac{x^2(A + Bx + Cx^2)}{(a + bx^2 + cx^4)^2} dx \\
&= \int \frac{Bx^3}{(a + bx^2 + cx^4)^2} dx + \int \frac{x^2(A + Cx^2)}{(a + bx^2 + cx^4)^2} dx \\
&= -\frac{x(Ab - 2aC + (2Ac - bC)x^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} + B \int \frac{x^3}{(a + bx^2 + cx^4)^2} dx + \frac{\int \frac{Ab - 2aC + (-2Ac + bC)x^2}{a + bx^2 + cx^4}}{2(b^2 - 4ac)} \\
&= -\frac{x(Ab - 2aC + (2Ac - bC)x^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{1}{2}B \text{Subst} \left(\int \frac{x}{(a + bx + cx^2)^2} dx, x, x^2 \right) - \frac{(2Ac - bC)}{2(b^2 - 4ac)} \\
&= \frac{B(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{x(Ab - 2aC + (2Ac - bC)x^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{(2Ac - bC - \frac{4Abc - (b^2 - 4ac)^2}{\sqrt{b^2 - 4ac}})}{2\sqrt{2}\sqrt{c}(b^2 - 4ac)} \\
&= \frac{B(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{x(Ab - 2aC + (2Ac - bC)x^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{(2Ac - bC - \frac{4Abc - (b^2 - 4ac)^2}{\sqrt{b^2 - 4ac}})}{2\sqrt{2}\sqrt{c}(b^2 - 4ac)} \\
&= \frac{B(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{x(Ab - 2aC + (2Ac - bC)x^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{(2Ac - bC - \frac{4Abc - (b^2 - 4ac)^2}{\sqrt{b^2 - 4ac}})}{2\sqrt{2}\sqrt{c}(b^2 - 4ac)}
\end{aligned}$$

Mathematica [A] time = 0.43, size = 378, normalized size = 1.06

$$\frac{1}{4} \left(\frac{4a(B + Cx) + 2x(B + Cx) - A(b + 2cx^2)}{(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\sqrt{2} \left(C(b\sqrt{b^2 - 4ac} - 4ac - b^2) - 2Ac(\sqrt{b^2 - 4ac} - 2b) \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c-x}}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right) + \sqrt{2} \left(C(b\sqrt{b^2 - 4ac} + 4ac + b^2) - 2Ac(\sqrt{b^2 - 4ac} + 2b) \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c+x}}{\sqrt{b + \sqrt{b^2 - 4ac}}} \right) + \frac{2bB \log(\sqrt{b^2 - 4ac} - b - 2cx^2)}{(b^2 - 4ac)^{3/2}} - \frac{2bB \log(\sqrt{b^2 - 4ac} + b + 2cx^2)}{(b^2 - 4ac)^{3/2}}}{\sqrt{c}(b^2 - 4ac)^{3/2} \sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{\sqrt{2} \left(C(b\sqrt{b^2 - 4ac} + 4ac + b^2) - 2Ac(\sqrt{b^2 - 4ac} + 2b) \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c-x}}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right) + \sqrt{2} \left(C(b\sqrt{b^2 - 4ac} - 4ac - b^2) - 2Ac(\sqrt{b^2 - 4ac} - 2b) \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c+x}}{\sqrt{b + \sqrt{b^2 - 4ac}}} \right) + \frac{2bB \log(\sqrt{b^2 - 4ac} - b - 2cx^2)}{(b^2 - 4ac)^{3/2}} - \frac{2bB \log(\sqrt{b^2 - 4ac} + b + 2cx^2)}{(b^2 - 4ac)^{3/2}}}{\sqrt{c}(b^2 - 4ac)^{3/2} \sqrt{b + \sqrt{b^2 - 4ac}}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x*(A*x + B*x^2 + C*x^3))/(a + b*x^2 + c*x^4)^2,x]

[Out] ((4*a*(B + C*x) + 2*x*(b*x*(B + C*x) - A*(b + 2*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(-2*A*c*(-2*b + Sqrt[b^2 - 4*a*c]) + (-b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(-2*A*c*(2*b + Sqrt[b^2 - 4*a*c]) + (b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (2*b*B*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) - (2*b*B*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(Ax + Bx^2 + Cx^3)}{(a + bx^2 + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x*(A*x + B*x^2 + C*x^3))/(a + b*x^2 + c*x^4)^2,x]

[Out] IntegrateAlgebraic[(x*(A*x + B*x^2 + C*x^3))/(a + b*x^2 + c*x^4)^2, x]
fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(C*x^3+B*x^2+A*x)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")

[Out] Timed out

giac [B] time = 7.13, size = 4440, normalized size = 12.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(C*x^3+B*x^2+A*x)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")

[Out]
$$\frac{1}{2}(Cbx^3 - 2Acx^3 + Bbx^2 + 2Cax - Abx + 2Ba) / ((cx^4 + bx^2 + a)(b^2 - 4ac)) - \frac{1}{16}(2(2b^2c^3 - 8a^2c^4 - \sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}}c) * b^2c + 4\sqrt{2}\sqrt{b^2 - 4ac} * \sqrt{bc + \sqrt{b^2 - 4ac}}c) * ac^2 + 2\sqrt{2}\sqrt{b^2 - 4ac} * \sqrt{bc + \sqrt{b^2 - 4ac}}c) * b^2c^2 - \sqrt{2}\sqrt{b^2 - 4ac} * \sqrt{bc + \sqrt{b^2 - 4ac}}c) * c^3 - 2(b^2 - 4ac) * c^3) * (b^2 - 4ac)^2A - (2b^3c^2 - 8a^2bc^3 - \sqrt{2}\sqrt{b^2 - 4ac}) * \sqrt{bc + \sqrt{b^2 - 4ac}}c) * b^3 + 4\sqrt{2}\sqrt{b^2 - 4ac} * \sqrt{bc + \sqrt{b^2 - 4ac}}c) * ab^2c + 2\sqrt{2}\sqrt{b^2 - 4ac} * \sqrt{bc + \sqrt{b^2 - 4ac}}c) * b^2c - \sqrt{2}\sqrt{b^2 - 4ac} * \sqrt{bc + \sqrt{b^2 - 4ac}}c) * b^2c^2 - 2(b^2 - 4ac) * b^2c^2) * (b^2 - 4ac)^2C - 2(\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * b^5c - 8\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * ab^3c^2 - 2\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * b^4c^2 - 2b^5c^2 + 16\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * a^2bc^3 + 8\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * ab^2c^3 + \sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * b^3c^3 + 16ab^3c^3 - 4\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * ab^2c^4 - 32a^2bc^4 + 2(b^2 - 4ac) * b^3c^2 - 8(b^2 - 4ac) * ab^2c^3) * A * \text{abs}(b^2 - 4ac) + 4(\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * ab^4c - 8\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * a^2b^2c^2 - 2\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * ab^3c^2 - 2ab^4c^2 + 16\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * a^3c^3 + 8\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * a^2bc^3 + \sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * a^2b^2c^3 + 16a^2b^2c^3 - 4\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * a^2c^4 - 32a^3c^4 + 2(b^2 - 4ac) * ab^2c^2 - 8(b^2 - 4ac) * a^2c^3) * C * \text{abs}(b^2 - 4ac) - 4(2b^6c^3 - 16ab^4c^4 + 32a^2b^2c^5 - \sqrt{2}\sqrt{b^2 - 4ac}) * \sqrt{bc + \sqrt{b^2 - 4ac}}c) * b^6c + 8\sqrt{2}\sqrt{b^2 - 4ac}) * \sqrt{bc + \sqrt{b^2 - 4ac}}c) * ab^4c^2 + 2\sqrt{2}\sqrt{b^2 - 4ac}) * \sqrt{bc + \sqrt{b^2 - 4ac}}c) * b^5c^2 - 16\sqrt{2}\sqrt{b^2 - 4ac}) * \sqrt{bc + \sqrt{b^2 - 4ac}}c) * a^2b^2c^3 - 8\sqrt{2}\sqrt{b^2 - 4ac}) * \sqrt{bc + \sqrt{b^2 - 4ac}}c) * ab^3c^3 - \sqrt{2}\sqrt{b^2 - 4ac}) * \sqrt{bc + \sqrt{b^2 - 4ac}}c) * b^4c^3 + 4\sqrt{2}\sqrt{b^2 - 4ac}) * \sqrt{bc + \sqrt{b^2 - 4ac}}c) * ab^2c^4 - 2(b^2 - 4ac) * b^4c^3 + 8(b^2 - 4ac) * ab^2c^4) * A + (2b^7c^2 - 8a^2b^5c^3 - 32a^2b^3c^4 + 128a^3bc^5 - \sqrt{2}\sqrt{b^2 - 4ac}) * \sqrt{bc + \sqrt{b^2 - 4ac}}c) * b^7 + 4\sqrt{2}\sqrt{b^2 - 4ac}) * \sqrt{bc + \sqrt{b^2 - 4ac}}c) * ab^5c + 2\sqrt{2}\sqrt{b^2 - 4ac}) * \sqrt{bc + \sqrt{b^2 - 4ac}}c) * b^6c + 16\sqrt{2}\sqrt{b^2 - 4ac}) * \sqrt{bc + \sqrt{b^2 - 4ac}}c) * a^2b^3c^2 - \sqrt{2}\sqrt{b^2 - 4ac}) * \sqrt{bc + \sqrt{b^2 - 4ac}}c) * b^5c^2 - 64\sqrt{2}\sqrt{b^2 - 4ac}) * \sqrt{bc + \sqrt{b^2 - 4ac}}c) * a^3bc^3 - 32\sqrt{2}\sqrt{b^2 - 4ac}) * \sqrt{bc + \sqrt{b^2 - 4ac}}c) * a^2b^2c^3 + 16\sqrt{2}\sqrt{b^2 - 4ac}) * \sqrt{bc + \sqrt{b^2 - 4ac}}c) * a^2bc^4 - 2(b^2 - 4ac) * b^5c^2 + 32(b^2 - 4ac) * a^2bc^4) * C) * \arctan(2\sqrt{1/2} * x / \sqrt{(b^3 - 4a^2bc + \sqrt{2}\sqrt{(b^3 - 4a^2bc)^2 - 4(a^2b^2 - 4a^2c)(b^2c - 4ac^2))}) / (b^2c - 4ac^2))} / ((ab^6c - 12a^2b^4c^2 - 2ab^5c^2 + 48a^3b^2c^3 + 16a^2b^$$

$$\begin{aligned}
& 3*c^3 + a*b^4*c^3 - 64*a^4*c^4 - 32*a^3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c^5) \\
& *abs(b^2 - 4*a*c)*abs(c)) + 1/16*(2*(2*b^2*c^3 - 8*a*c^4 - sqrt(2)*sqrt(b^2 \\
& - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a* \\
& c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt \\
& (b*c - sqrt(b^2 - 4*a*c))*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq \\
& rt(b^2 - 4*a*c))*c^3 - 2*(b^2 - 4*a*c)*c^3)*(b^2 - 4*a*c)^2*A - (2*b^3*c^ \\
& 2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*b \\
& ^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a*b*c + 2* \\
& sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*b^2*c - sqrt(2)*s \\
& qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*b*c^2 - 2*(b^2 - 4*a*c)*b* \\
& c^2)*(b^2 - 4*a*c)^2*C + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*b^5*c - \\
& 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c - \\
& sqrt(b^2 - 4*a*c))*b^4*c^2 + 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 \\
& - 4*a*c))*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a*b^2*c^3 \\
& + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*b^3*c^3 - 16*a*b^3*c^3 - 4*sqrt(\\
& 2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a*b*c^4 + 32*a^2*b*c^4 - 2*(b^2 - 4*a*c) \\
& *b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) - 4*(sqrt(2)*sqrt(b* \\
& c - sqrt(b^2 - 4*a*c))*a*b^4*c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*c \\
&)*a^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a*b^3*c^2 + 2*a*b \\
& ^4*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^3*c^3 + 8*sqrt(2)*sq \\
& rt(b*c - sqrt(b^2 - 4*a*c))*a^2*b*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a* \\
& c))*a*b^2*c^3 - 16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*c \\
&)*a^2*c^4 + 32*a^3*c^4 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + 8*(b^2 - 4*a*c)*a^2*c^ \\
& 3)*C*abs(b^2 - 4*a*c) - 4*(2*b^6*c^3 - 16*a*b^4*c^4 + 32*a^2*b^2*c^5 - sqrt \\
& (2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*b^6*c + 8*sqrt(2)*sq \\
& rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a*b^4*c^2 + 2*sqrt(2)*sqrt(b \\
& ^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*b^5*c^2 - 16*sqrt(2)*sqrt(b^2 - \\
& 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^2*b^2*c^3 - 8*sqrt(2)*sqrt(b^2 - \\
& 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c) \\
&)*sqrt(b*c - sqrt(b^2 - 4*a*c))*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sq \\
& rt(b*c - sqrt(b^2 - 4*a*c))*a*b^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^3 + 8*(b^2 - \\
& 4*a*c)*a*b^2*c^4)*A + (2*b^7*c^2 - 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3* \\
& b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*b^7 + 4*s \\
& qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a*b^5*c + 2*sqrt(2) \\
&)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*b^6*c + 16*sqrt(2)*sqrt \\
& (b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^2*b^3*c^2 - sqrt(2)*sqrt(b^ \\
& 2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 - \\
& 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^3*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4* \\
& a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^2*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4* \\
& a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 + \\
& 32*(b^2 - 4*a*c)*a^2*b*c^4)*C)*arctan(2*sqrt(1/2)*x/sqrt((b^3 - 4*a*b*c - s \\
& qrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4* \\
& a*c^2)))/((a*b^6*c - 12*a^2*b^4*c^2 - 2*a*b^5*c^2 + 48*a^3*b^2*c^3 + 16*a^2 \\
& *b^3*c^3 + a*b^4*c^3 - 64*a^4*c^4 - 32*a^3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c \\
& ^5)*abs(b^2 - 4*a*c)*abs(c)) + 1/8*((b^4*c - 4*a*b^2*c^2 - 2*b^3*c^2 + b^2* \\
& c^3 + (b^3*c - 4*a*b*c^2 - 2*b^2*c^2 + b*c^3)*sqrt(b^2 - 4*a*c))*B*abs(b^2 \\
& - 4*a*c) - (b^6*c - 8*a*b^4*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a*b^3*c^3 \\
& + b^4*c^3 - 4*a*b^2*c^4 + (b^5*c - 4*a*b^3*c^2 - 2*b^4*c^2 + b^3*c^3)*sqrt(\\
& b^2 - 4*a*c))*B)*log(x^2 + 1/2*(b^3 - 4*a*b*c + sqrt((b^3 - 4*a*b*c)^2 - 4* \\
& (a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2))/((a*b^4 - 8*a^2*b^ \\
& 2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2*abs \\
& (b^2 - 4*a*c)) + 1/8*((b^4*c - 4*a*b^2*c^2 - 2*b^3*c^2 + b^2*c^3 - (b^3*c - \\
& 4*a*b*c^2 - 2*b^2*c^2 + b*c^3)*sqrt(b^2 - 4*a*c))*B*abs(b^2 - 4*a*c) - (b^ \\
& 6*c - 8*a*b^4*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a*b^3*c^3 + b^4*c^3 - 4* \\
& a*b^2*c^4 - (b^5*c - 4*a*b^3*c^2 - 2*b^4*c^2 + b^3*c^3)*sqrt(b^2 - 4*a*c))* \\
& B)*log(x^2 + 1/2*(b^3 - 4*a*b*c - sqrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2 \\
& *c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3* \\
& c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2*abs(b^2 - 4*a*c))
\end{aligned}$$

maple [B] time = 0.04, size = 1119, normalized size = 3.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(C*x^3+B*x^2+A*x)/(c*x^4+b*x^2+a)^2,x)`

[Out]
$$\begin{aligned} & (-1/2/(4*a*c-b^2)*B*b*x^2+1/2*(2*A*c-C*b)/(4*a*c-b^2)*x^3-1/(4*a*c-b^2)*B*a \\ & +1/2*(A*b-2*C*a)/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+1/2/(4*a*c-b^2)^2*(-4*a*c+b \\ & ^2)^{(1/2)}*B*b*\ln(-2*c*x^2-b+(-4*a*c+b^2)^{(1/2)})-c/(4*a*c-b^2)^2*2^{(1/2)}/((- \\ & b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} \\ & *c*x)*A*(-4*a*c+b^2)^{(1/2)}*b-2*c^2/(4*a*c-b^2)^2*2^{(1/2)}/((-b+(-4*a*c+ \\ & b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x) \\ & *a*A+1/2*c/(4*a*c-b^2)^2*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(\\ & 2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*A*b^2+c/(4*a*c-b^2)^2*2^{(1/2)}/ \\ & ((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)}) \\ &)*c)^{(1/2)}*c*x)*C*(-4*a*c+b^2)^{(1/2)}*a+1/4/(4*a*c-b^2)^2*2^{(1/2)}/((-b+(-4*a \\ & *c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c \\ & *x)*C*(-4*a*c+b^2)^{(1/2)}*b^2+c/(4*a*c-b^2)^2*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2) \\ &))*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*C*a*b-1/ \\ & 4/(4*a*c-b^2)^2*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/(\\ & (-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^3*C-1/2/(4*a*c-b^2)^2*(-4*a*c+b^2)^{(1/2)} \\ & *B*b*\ln(2*c*x^2+b+(-4*a*c+b^2)^{(1/2)})-1/(4*a*c-b^2)^2*2^{(1/2)}/((b+(-4* \\ & a*c+b^2)^{(1/2)})*c)^{(1/2)}*(-4*a*c+b^2)^{(1/2)}*A*b*c*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a* \\ & c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)+2/(4*a*c-b^2)^2*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2) \\ &))*c)^{(1/2)}*A*a*c^2*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)-1/2/ \\ & (4*a*c-b^2)^2*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*A*b^2*c*\operatorname{arctan}(2^{(1/2)}/ \\ & ((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)+1/(4*a*c-b^2)^2*2^{(1/2)}/((b+(-4*a* \\ & c+b^2)^{(1/2)})*c)^{(1/2)}*(-4*a*c+b^2)^{(1/2)}*C*a*c*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+ \\ & b^2)^{(1/2)})*c)^{(1/2)}*c*x)+1/4/(4*a*c-b^2)^2*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2) \\ &))*c)^{(1/2)}*(-4*a*c+b^2)^{(1/2)}*C*b^2*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c \\ &)^{(1/2)}*c*x)-1/(4*a*c-b^2)^2*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*C*a*b \\ & *c*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)+1/4/(4*a*c-b^2)^2*2 \\ & ^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*C*b^3*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2) \\ &)^{(1/2)})*c)^{(1/2)}*c*x) \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{Bbx^2 + (Cb - 2Ac)x^3 + 2Ba + (2Ca - Ab)x}{2((b^2c - 4ac^2)x^4 + ab^2 - 4a^2c + (b^3 - 4abc)x^2)} - \frac{\int \frac{2Bbx + (Cb - 2Ac)x^2 - 2Ca + Ab}{cx^4 + bx^2 + a} dx}{2(b^2 - 4ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(C*x^3+B*x^2+A*x)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")`

[Out]
$$\frac{1/2*(B*b*x^2 + (C*b - 2*A*c)*x^3 + 2*B*a + (2*C*a - A*b)*x)/(b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2 - 1/2*\operatorname{integrate}(- (2*B*b*x + (C*b - 2*A*c)*x^2 - 2*C*a + A*b)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c)}$$

mupad [B] time = 1.55, size = 3835, normalized size = 10.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x*(A*x + B*x^2 + C*x^3))/(a + b*x^2 + c*x^4)^2,x)`

[Out]
$$\operatorname{symsum}(\log((8*A^3*a*c^4 + 6*A^3*b^2*c^3 + A*C^2*b^4*c - 3*C^3*a*b^3*c + 4*A*B^2*b^3*c^2 + 8*A*C^2*a^2*c^3 - 5*A^2*C*b^3*c^2 - 4*C^3*a^2*b*c^2 + 18*A*C^2*a*b^2*c^2 - 8*B^2*C*a*b^2*c^2 - 28*A^2*C*a*b*c^3)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - \operatorname{root}(256*a*b^12*c*z^4 - 1572864*a^6*b^2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^3*b^8*c^3*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8*c*z^2 - 6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^5*b*c^4*z$$

$$\begin{aligned}
& ^6b^2c^6z^4 + 983040a^5b^4c^5z^4 - 327680a^4b^6c^4z^4 + 61440a^3b^8c^3z^4 - 6144a^2b^{10}c^2z^4 + 1048576a^7c^7z^4 - 192A^2C^2a^8 \\
& *c^2z^2 - 6144A^2C^2a^3b^4c^3z^2 + 2048A^2C^2a^2b^6c^2z^2 - 12288C^2a^5b^2c^4z^2 - 12288A^2a^4b^2c^5z^2 - 128B^2a^2b^8c^2z^2 + 16384A^2C^2a^5 \\
& *c^5z^2 + 8192C^2a^4b^3c^3z^2 - 1536C^2a^3b^5c^2z^2 + 8192B^2a^4b^2c^4z^2 - 6144B^2a^3b^4c^3z^2 + 1536B^2a^2b^6c^2z^2 + 8192 \\
& *A^2a^3b^3c^4z^2 - 1536A^2a^2b^5c^3z^2 + 16C^2a^2b^9z^2 + 16A^2 \\
& *b^9c^2z^2 + 1024B^2C^2a^4b^2c^3z + 192B^2C^2a^2b^5c^2z - 1024A^2B^2a^3b^2c^4z - 192A^2B^2a^2b^5c^2z - 768B^2C^2a^3b^3c^2z + 768A^2B^2a^2 \\
& *b^3c^3z + 16A^2B^2b^7c^2z - 16B^2C^2a^2b^7z - 64A^2B^2C^2a^2b^2c^2 - 48A^2B^2C^2a^2b^4c + 192A^2C^2a^2b^2c^2 + 48B^2C^2a^2b^3c + 48A^2 \\
& *B^2a^2b^3c^2 - 96A^3C^2a^2b^2c^3 - 96A^2C^3a^3b^2c^2 - 80A^3C^2a^2b^3c^2 - 80A^2C^3a^2b^3c + 42A^2C^2a^2b^4c + 24C^4a^3b^2c + 24A^4a \\
& *b^2c^3 + 4B^2C^2a^2b^5 + 4A^2B^2b^5c + 16B^4a^2b^4c - 6A^3C^2b^5c - 6A^2C^3a^2b^5 + 32A^2C^2a^3c^3 + 16C^4a^4c^2 + 9C^4a^2b^4 + \\
& 9A^4b^4c^2 + 16A^4a^2c^4 + A^2C^2b^6, z, k), k, 1, 4) - ((B*a)/(4*a*c - b^2) - (x*(A*b - 2*C*a))/(2*(4*a*c - b^2)) - (x^3*(2*A*c - C*b))/(2*(4*a*c - b^2)) + (B*b*x^2)/(2*(4*a*c - b^2)))/(a + b*x^2 + c*x^4)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(C*x**3+B*x**2+A*x)/(c*x**4+b*x**2+a)**2,x)

[Out] Timed out

$$3.42 \quad \int \frac{Ax^2+Bx^3+Cx^4}{(a+bx^2+cx^4)^2} dx$$

Optimal. Leaf size=356

$$\frac{x(-2aC + x^2(2Ac - bC) + Ab) \left(-\frac{4Abc - C(4ac + b^2)}{\sqrt{b^2 - 4ac}} + 2Ac - bC \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right) \left(\frac{4Abc - C(4ac + b^2)}{\sqrt{b^2 - 4ac}} + 2Ac - bC \right)}{2(b^2 - 4ac)(a + bx^2 + cx^4) \cdot 2\sqrt{2} \sqrt{c} (b^2 - 4ac) \sqrt{b - \sqrt{b^2 - 4ac}} \cdot 2\sqrt{2} \sqrt{c} (b^2 - 4ac)}$$

Rubi [A] time = 0.37, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {1594, 1662, 1275, 1166, 205, 12, 1114, 638, 618, 206}

$$\frac{x(-2aC + x^2(2Ac - bC) + Ab) \left(-\frac{4Abc - C(4ac + b^2)}{\sqrt{b^2 - 4ac}} + 2Ac - bC \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right) \left(\frac{4Abc - C(4ac + b^2)}{\sqrt{b^2 - 4ac}} + 2Ac - bC \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b^2 - 4ac + b}} \right) + \frac{B(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{bB \tanh^{-1} \left(\frac{b + 2cx^2}{\sqrt{b^2 - 4ac}} \right)}{(b^2 - 4ac)^{3/2}}}{2\sqrt{2} \sqrt{c} (b^2 - 4ac) \sqrt{b - \sqrt{b^2 - 4ac}} \cdot 2\sqrt{2} \sqrt{c} (b^2 - 4ac) \sqrt{\sqrt{b^2 - 4ac} + b}}$$

Antiderivative was successfully verified.

[In] Int[(A*x^2 + B*x^3 + C*x^4)/(a + b*x^2 + c*x^4)^2, x]

[Out] (B*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (x*(A*b - 2*a*C + (2*A*c - b*C)*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((2*A*c - b*C - (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((2*A*c - b*C + (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (b*B*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 638

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]

Rule 1114

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1166

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1275

```
Int[((f_)*(x_)^(m_)*((d_) + (e_)*(x_)^2)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1594

```
Int[(u_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_) + (c_)*(x_)^(r_))^(n_), x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1662

```
Int[(Pq_)*((d_)*(x_)^(m_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2 + 1})*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1})*(a + b*x^2 + c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] && !PolyQ[Pq, x^2]
```

Rubi steps

$$\begin{aligned}
\int \frac{Ax^2 + Bx^3 + Cx^4}{(a + bx^2 + cx^4)^2} dx &= \int \frac{x^2(A + Bx + Cx^2)}{(a + bx^2 + cx^4)^2} dx \\
&= \int \frac{Bx^3}{(a + bx^2 + cx^4)^2} dx + \int \frac{x^2(A + Cx^2)}{(a + bx^2 + cx^4)^2} dx \\
&= -\frac{x(Ab - 2aC + (2Ac - bC)x^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} + B \int \frac{x^3}{(a + bx^2 + cx^4)^2} dx + \frac{\int \frac{Ab - 2aC + (-2Ac + bC)x^2}{a + bx^2 + cx^4} dx}{2(b^2 - 4ac)} \\
&= -\frac{x(Ab - 2aC + (2Ac - bC)x^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{1}{2}B \operatorname{Subst} \left(\int \frac{x}{(a + bx + cx^2)^2} dx, x, x^2 \right) - \frac{(2Ac - bC)}{2(b^2 - 4ac)} \\
&= \frac{B(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{x(Ab - 2aC + (2Ac - bC)x^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{(2Ac - bC - \frac{4Abc - (b^2 + 4ac)^2}{\sqrt{b^2 - 4ac}})}{2\sqrt{2}\sqrt{c}(b^2 - 4ac)} \\
&= \frac{B(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{x(Ab - 2aC + (2Ac - bC)x^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{(2Ac - bC - \frac{4Abc - (b^2 + 4ac)^2}{\sqrt{b^2 - 4ac}})}{2\sqrt{2}\sqrt{c}(b^2 - 4ac)} \\
&= \frac{B(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{x(Ab - 2aC + (2Ac - bC)x^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{(2Ac - bC - \frac{4Abc - (b^2 + 4ac)^2}{\sqrt{b^2 - 4ac}})}{2\sqrt{2}\sqrt{c}(b^2 - 4ac)}
\end{aligned}$$

Mathematica [A] time = 0.15, size = 378, normalized size = 1.06

$$\frac{1}{4} \left(\frac{4a(B + Cx) + 2x(Bx + Cx) - A(b + 2cx^2)}{(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\sqrt{2} \left(C(b\sqrt{b^2 - 4ac} - 4ac - b^2) - 2Ac(\sqrt{b^2 - 4ac} - 2b) \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right) + \sqrt{2} \left(C(b\sqrt{b^2 - 4ac} + 4ac + b^2) - 2Ac(\sqrt{b^2 - 4ac} + 2b) \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{b + \sqrt{b^2 - 4ac}}} \right) + \frac{2bB \log(\sqrt{b^2 - 4ac} - b - 2cx^2)}{(b^2 - 4ac)^{3/2}} - \frac{2bB \log(\sqrt{b^2 - 4ac} + b + 2cx^2)}{(b^2 - 4ac)^{3/2}}}{\sqrt{c}(b^2 - 4ac)^{3/2} \sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{\sqrt{2} \left(C(b\sqrt{b^2 - 4ac} + 4ac + b^2) - 2Ac(\sqrt{b^2 - 4ac} + 2b) \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right) + \sqrt{2} \left(C(b\sqrt{b^2 - 4ac} - 4ac - b^2) - 2Ac(\sqrt{b^2 - 4ac} - 2b) \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{b + \sqrt{b^2 - 4ac}}} \right) + \frac{2bB \log(\sqrt{b^2 - 4ac} - b - 2cx^2)}{(b^2 - 4ac)^{3/2}} - \frac{2bB \log(\sqrt{b^2 - 4ac} + b + 2cx^2)}{(b^2 - 4ac)^{3/2}}}{\sqrt{c}(b^2 - 4ac)^{3/2} \sqrt{b + \sqrt{b^2 - 4ac}}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(A*x^2 + B*x^3 + C*x^4)/(a + b*x^2 + c*x^4)^2,x]

[Out] ((4*a*(B + C*x) + 2*x*(b*x*(B + C*x) - A*(b + 2*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(-2*A*c*(-2*b + Sqrt[b^2 - 4*a*c]) + (-b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(-2*A*c*(2*b + Sqrt[b^2 - 4*a*c]) + (b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (2*b*B*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) - (2*b*B*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Ax^2 + Bx^3 + Cx^4}{(a + bx^2 + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A*x^2 + B*x^3 + C*x^4)/(a + b*x^2 + c*x^4)^2,x]

[Out] IntegrateAlgebraic[(A*x^2 + B*x^3 + C*x^4)/(a + b*x^2 + c*x^4)^2, x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^4+B*x^3+A*x^2)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")

[Out] Timed out

giac [B] time = 6.90, size = 4439, normalized size = 12.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^4+B*x^3+A*x^2)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")

[Out]
$$\frac{1}{2}*(C*b*x^3 - 2*A*c*x^3 + B*b*x^2 + 2*C*a*x - A*b*x + 2*B*a)/((c*x^4 + b*x^2 + a)*(b^2 - 4*a*c)) - \frac{1}{16}*(2*(2*b^2*c^3 - 8*a*c^4 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^2*c + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*c^2 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b*c^2 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*c^3 - 2*(b^2 - 4*a*c)*c^3*(b^2 - 4*a*c)^2*A - (2*b^3*c^2 - 8*a*b*c^3 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^3 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^2*c - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2*(b^2 - 4*a*c)^2*C - 2*(\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^5*c - 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^3*c^2 - 2*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^4*c^2 - 2*b^5*c^2 + 16*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b*c^3 + 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c^3 + \sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^3*c^3 + 16*a*b^3*c^3 - 4*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b*c^4 - 32*a^2*b*c^4 + 2*(b^2 - 4*a*c)*b^3*c^2 - 8*(b^2 - 4*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) + 4*(\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^4*c - 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b^2*c^2 - 2*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^3*c^2 - 2*a*b^4*c^2 + 16*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^3*c^3 + 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b*c^3 + \sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c^3 + 16*a^2*b^2*c^3 - 4*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*c^4 - 32*a^3*c^4 + 2*(b^2 - 4*a*c)*a*b^2*c^2 - 8*(b^2 - 4*a*c)*a^2*c^3)*C*abs(b^2 - 4*a*c) - 4*(2*b^6*c^3 - 16*a*b^4*c^4 + 32*a^2*b^2*c^5 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^6*c + 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^4*c^2 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^5*c^2 - 16*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b^2*c^3 - 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^3*c^3 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^4*c^3 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^3 + 8*(b^2 - 4*a*c)*a*b^2*c^4)*A + (2*b^7*c^2 - 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*b*c^5 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^7 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^5*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^6*c + 16*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b^3*c^2 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^5*c^2 - 64*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^3*b*c^3 - 32*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b^2*c^3 + 16*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 + 32*(b^2 - 4*a*c)*a^2*b*c^4)*C)*arctan(2*\sqrt{1/2}*x/\sqrt{(b^3 - 4*a*b*c + \sqrt{(b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)})))/(b^2*c - 4*a*c^2)))/((a*b^6*c - 12*a^2*b^4*c^2 - 2*a*b^5*c^2 + 48*a^3*b^2*c^3 + 16*a^2*b^3*c^3 + a*b^4*c^3 - 64*a^4*c^4 - 32*a^3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c^5)$$

```

*abs(b^2 - 4*a*c)*abs(c)) + 1/16*(2*(2*b^2*c^3 - 8*a*c^4 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*c^3 - 2*(b^2 - 4*a*c)*c^3)*(b^2 - 4*a*c)^2*A - (2*b^3*c^
2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b
^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*
c^2)*(b^2 - 4*a*c)^2*C + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c -
8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3
+ sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 16*a*b^3*c^3 - 4*sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 + 32*a^2*b*c^4 - 2*(b^2 - 4*a*c)
*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) - 4*(sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*a*b
^4*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a*b^2*c^3 - 16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^2*c^4 + 32*a^3*c^4 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + 8*(b^2 - 4*a*c)*a^2*c^
3)*C*abs(b^2 - 4*a*c) - 4*(2*b^6*c^3 - 16*a*b^4*c^4 + 32*a^2*b^2*c^5 - sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c + 8*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 2*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 16*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 8*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^3 + 8*(b^2 -
4*a*c)*a*b^2*c^4)*A + (2*b^7*c^2 - 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*
b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7 + 4*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c + 16*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 +
32*(b^2 - 4*a*c)*a^2*b*c^4)*C)*arctan(2*sqrt(1/2)*x/sqrt((b^3 - 4*a*b*c - s
qrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*
a*c^2)))/((a*b^6*c - 12*a^2*b^4*c^2 - 2*a*b^5*c^2 + 48*a^3*b^2*c^3 + 16*a^2
*b^3*c^3 + a*b^4*c^3 - 64*a^4*c^4 - 32*a^3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c
^5)*abs(b^2 - 4*a*c)*abs(c)) + 1/8*((b^4*c - 4*a*b^2*c^2 - 2*b^3*c^2 + b^2*
c^3 + (b^3*c - 4*a*b*c^2 - 2*b^2*c^2 + b*c^3)*sqrt(b^2 - 4*a*c))*B*abs(b^2
- 4*a*c) - (b^6*c - 8*a*b^4*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a*b^3*c^3
+ b^4*c^3 - 4*a*b^2*c^4 + (b^5*c - 4*a*b^3*c^2 - 2*b^4*c^2 + b^3*c^3)*sqrt(
b^2 - 4*a*c))*B)*log(x^2 + 1/2*(b^3 - 4*a*b*c + sqrt((b^3 - 4*a*b*c)^2 - 4*
(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2))/((a*b^4 - 8*a^2*b^
2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2*abs
(b^2 - 4*a*c)) + 1/8*((b^4*c - 4*a*b^2*c^2 - 2*b^3*c^2 + b^2*c^3 - (b^3*c -
4*a*b*c^2 - 2*b^2*c^2 + b*c^3)*sqrt(b^2 - 4*a*c))*B*abs(b^2 - 4*a*c) + (b^
6*c - 8*a*b^4*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a*b^3*c^3 + b^4*c^3 - 4*
a*b^2*c^4 - (b^5*c - 4*a*b^3*c^2 - 2*b^4*c^2 + b^3*c^3)*sqrt(b^2 - 4*a*c))*
B)*log(x^2 + 1/2*(b^3 - 4*a*b*c - sqrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2
*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*
c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2*abs(b^2 - 4*a*c))

```

maple [B] time = 0.04, size = 1119, normalized size = 3.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((C*x^4+B*x^3+A*x^2)/(c*x^4+b*x^2+a)^2,x)`

[Out]
$$\begin{aligned} & (-1/2/(4*a*c-b^2)*B*b*x^2+1/2*(2*A*c-C*b)/(4*a*c-b^2)*x^3-1/(4*a*c-b^2)*B*a \\ & +1/2*(A*b-2*C*a)/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+1/2/(4*a*c-b^2)^2*(-4*a*c+b \\ & ^2)^{(1/2)}*B*b*\ln(-2*c*x^2-b+(-4*a*c+b^2)^{(1/2)})-c/(4*a*c-b^2)^2*2^{(1/2)}/((- \\ & b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} \\ & *c*x)*A*(-4*a*c+b^2)^{(1/2)}*b-2*c^2/(4*a*c-b^2)^2*2^{(1/2)}/((-b+(-4*a*c+ \\ & b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x) \\ & *a*A+1/2*c/(4*a*c-b^2)^2*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(\\ & 2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*A*b^2+c/(4*a*c-b^2)^2*2^{(1/2)}/ \\ & ((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)}) \\ &)*c)^{(1/2)}*c*x)*C*(-4*a*c+b^2)^{(1/2)}*a+1/4/(4*a*c-b^2)^2*2^{(1/2)}/((-b+(-4*a \\ & *c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c \\ & *x)*C*(-4*a*c+b^2)^{(1/2)}*b^2+c/(4*a*c-b^2)^2*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2) \\ &))*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*C*a*b-1/ \\ & 4/(4*a*c-b^2)^2*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/(\\ & (-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^3*C-1/2/(4*a*c-b^2)^2*(-4*a*c+b^2)^{(1/2)} \\ & *B*b*\ln(2*c*x^2+b+(-4*a*c+b^2)^{(1/2)})-1/(4*a*c-b^2)^2*2^{(1/2)}/((b+(-4* \\ & a*c+b^2)^{(1/2)})*c)^{(1/2)}*(-4*a*c+b^2)^{(1/2)}*A*b*c*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a* \\ & c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)+2/(4*a*c-b^2)^2*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2) \\ &))*c)^{(1/2)}*A*a*c^2*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)-1/2/ \\ & (4*a*c-b^2)^2*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*A*b^2*c*\operatorname{arctan}(2^{(1/ \\ & 2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)+1/(4*a*c-b^2)^2*2^{(1/2)}/((b+(-4*a* \\ & c+b^2)^{(1/2)})*c)^{(1/2)}*(-4*a*c+b^2)^{(1/2)}*C*a*c*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+ \\ & b^2)^{(1/2)})*c)^{(1/2)}*c*x)+1/4/(4*a*c-b^2)^2*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2) \\ &))*c)^{(1/2)}*(-4*a*c+b^2)^{(1/2)}*C*b^2*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c \\ &)^{(1/2)}*c*x)-1/(4*a*c-b^2)^2*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*C*a*b \\ & *c*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)+1/4/(4*a*c-b^2)^2*2 \\ & ^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*C*b^3*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^ \\ & 2)^{(1/2)})*c)^{(1/2)}*c*x) \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{Bbx^2 + (Cb - 2Ac)x^3 + 2Ba + (2Ca - Ab)x}{2((b^2c - 4ac^2)x^4 + ab^2 - 4a^2c + (b^3 - 4abc)x^2)} - \frac{\int \frac{2Bbx + (Cb - 2Ac)x^2 - 2Ca + Ab}{cx^4 + bx^2 + a} dx}{2(b^2 - 4ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((C*x^4+B*x^3+A*x^2)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")`

[Out]
$$\frac{1/2*(B*b*x^2 + (C*b - 2*A*c)*x^3 + 2*B*a + (2*C*a - A*b)*x)/(b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2 - 1/2*\integrate(-(2*B*b*x + (C*b - 2*A*c)*x^2 - 2*C*a + A*b)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c)}$$

mupad [B] time = 1.47, size = 3835, normalized size = 10.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A*x^2 + B*x^3 + C*x^4)/(a + b*x^2 + c*x^4)^2,x)`

[Out]
$$\operatorname{symsum}(\log((8*A^3*a*c^4 + 6*A^3*b^2*c^3 + A*C^2*b^4*c - 3*C^3*a*b^3*c + 4*A*B^2*b^3*c^2 + 8*A*C^2*a^2*c^3 - 5*A^2*C*b^3*c^2 - 4*C^3*a^2*b*c^2 + 18*A*C^2*a*b^2*c^2 - 8*B^2*C*a*b^2*c^2 - 28*A^2*C*a*b*c^3)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - \operatorname{root}(256*a*b^12*c*z^4 - 1572864*a^6*b^2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^3*b^8*c^3*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8*c*z^2 - 6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^5*b*c^4*z$$

$$\begin{aligned}
& ^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5*c^5*z^2 \\
& + 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a^4*b^2*c^4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8192*A^2*a^3*b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A^2*b^9*c*z^2 \\
& + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B*a^3*b*c^4*z - 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*a^2*b^3*c^3*z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^2 - 48*A*B^2*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 48*A^2*B^2*a*b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*b^3*c^2 - 80*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A^4*a*b^2*c^3 + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C*b^5*c - 6*A*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^4 + 9*A^4*b^4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k)*(root(256*a*b^12*c*z^4 - 1572864*a^6*b^2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^3*b^8*c^3*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8*c*z^2 - 6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^5*b*c^4*z^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5*c^5*z^2 + 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a^4*b^2*c^4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8192*A^2*a^3*b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A^2*b^9*c*z^2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B*a^3*b*c^4*z - 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*a^2*b^3*c^3*z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^2 - 48*A*B^2*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 48*A^2*B^2*a*b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*b^3*c^2 - 80*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A^4*a*b^2*c^3 + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C*b^5*c - 6*A*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^4 + 9*A^4*b^4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k)*((x*(16*B*b^7*c^2 - 192*B*a*b^5*c^3 - 1024*B*a^3*b*c^5 + 768*B*a^2*b^3*c^4))/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (16*A*b^7*c^2 + 2048*C*a^4*c^5 - 192*A*a*b^5*c^3 - 1024*A*a^3*b*c^5 - 32*C*a*b^6*c^2 + 768*A*a^2*b^3*c^4 + 384*C*a^2*b^4*c^3 - 1536*C*a^3*b^2*c^4))/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (root(256*a*b^12*c*z^4 - 1572864*a^6*b^2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^3*b^8*c^3*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8*c*z^2 - 6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^5*b*c^4*z^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5*c^5*z^2 + 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a^4*b^2*c^4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8192*A^2*a^3*b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A^2*b^9*c*z^2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B*a^3*b*c^4*z - 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*a^2*b^3*c^3*z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^2 - 48*A*B^2*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 48*A^2*B^2*a*b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*b^3*c^2 - 80*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A^4*a*b^2*c^3 + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C*b^5*c - 6*A*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^4 + 9*A^4*b^4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k)*x*(32*b^9*c^2 - 512*a*b^7*c^3 + 8192*a^4*b*c^6 + 3072*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))) - (16*A*B*b^5*c^2 + 256*B*C*a^2*b^2*c^3 - 256*A*B*a^2*b*c^4 - 64*B*C*a*b^4*c^2))/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(2*C^2*b^6*c + 64*A^2*a^2*c^5 + 20*A^2*b^4*c^3 - 8*B^2*b^5*c^2 - 64*C^2*a^3*c^4 - 12*A*C*b^5*c^2 - 96*A^2*a*b^2*c^4 + 32*B^2*a*b^3*c^3 - 4*C^2*a*b^4*c^2 + 32*A*C*a*b^3*c^3 + 64*A*C*a^2*b*c^4))/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))) + (x*(4*B^3*b^3*c^2 + B*C^2*b^4*c + 8*A^2*B*b^2*c^3 + 4*B*C^2*a*b^2*c^2 - 6*A*B*C*b^3*c^2 - 8*A*B*C*a*b*c^3))/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)))*root(256*a*b^12*c*z^4 - 1572864*a
\end{aligned}$$

$$\begin{aligned}
& ^6b^2c^6z^4 + 983040a^5b^4c^5z^4 - 327680a^4b^6c^4z^4 + 61440a^3b^8c^3z^4 - 6144a^2b^{10}c^2z^4 + 1048576a^7c^7z^4 - 192A^2C^2a^8b^8c^2z^2 - 6144A^2C^2a^3b^4c^3z^2 + 2048A^2C^2a^2b^6c^2z^2 - 12288C^2a^5b^2c^4z^2 - 12288A^2a^4b^2c^5z^2 - 128B^2a^2b^8c^2z^2 + 16384A^2C^2a^5c^5z^2 + 8192C^2a^4b^3c^3z^2 - 1536C^2a^3b^5c^2z^2 + 8192B^2a^4b^2c^4z^2 - 6144B^2a^3b^4c^3z^2 + 1536B^2a^2b^6c^2z^2 + 8192A^2a^3b^3c^4z^2 - 1536A^2a^2b^5c^3z^2 + 16C^2a^2b^9z^2 + 16A^2b^9c^2z^2 + 1024B^2C^2a^4b^2c^3z + 192B^2C^2a^2b^5c^2z - 1024A^2B^2a^3b^2c^4z - 192A^2B^2a^2b^5c^2z - 768B^2C^2a^3b^3c^2z + 768A^2B^2a^2b^3c^3z + 16A^2B^2b^7c^2z - 16B^2C^2a^2b^7z - 64A^2B^2C^2a^2b^2c^2 - 48A^2B^2C^2a^2b^4c + 192A^2C^2a^2b^2c^2 + 48B^2C^2a^2b^3c + 48A^2B^2a^2b^3c^2 - 96A^3C^2a^2b^2c^3 - 96A^3C^3a^3b^2c^2 - 80A^3C^2a^2b^3c^2 - 80A^3C^3a^2b^3c + 42A^2C^2a^2b^4c + 24C^4a^3b^2c + 24A^4a^2b^2c^3 + 4B^2C^2a^2b^5 + 4A^2B^2b^5c + 16B^4a^2b^4c - 6A^3C^2b^5c - 6A^3C^3a^2b^5 + 32A^2C^2a^3c^3 + 16C^4a^4c^2 + 9C^4a^2b^4 + 9A^4b^4c^2 + 16A^4a^2c^4 + A^2C^2b^6, z, k), k, 1, 4) - ((B*a)/(4*a*c - b^2) - (x*(A*b - 2*C*a))/(2*(4*a*c - b^2)) - (x^3*(2*A*c - C*b))/(2*(4*a*c - b^2)) + (B*b*x^2)/(2*(4*a*c - b^2)))/(a + b*x^2 + c*x^4)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x**4+B*x**3+A*x**2)/(c*x**4+b*x**2+a)**2,x)

[Out] Timed out

$$3.43 \quad \int \frac{Ax^3+Bx^4+Cx^5}{x(a+bx^2+cx^4)^2} dx$$

Optimal. Leaf size=356

$$\frac{x(-2aC + x^2(2Ac - bC) + Ab) \left(-\frac{4Abc - C(4ac + b^2)}{\sqrt{b^2 - 4ac}} + 2Ac - bC \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right) \left(\frac{4Abc - C(4ac + b^2)}{\sqrt{b^2 - 4ac}} + 2Ac - bC \right)}{2(b^2 - 4ac)(a + bx^2 + cx^4) \cdot 2\sqrt{2} \sqrt{c} (b^2 - 4ac) \sqrt{b - \sqrt{b^2 - 4ac}} \cdot 2\sqrt{2} \sqrt{c} (b^2 - 4ac)}$$

Rubi [A] time = 0.36, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {1585, 1662, 1275, 1166, 205, 12, 1114, 638, 618, 206}

$$\frac{x(-2aC + x^2(2Ac - bC) + Ab)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{\left(-\frac{4Abc - C(4ac + b^2)}{\sqrt{b^2 - 4ac}} + 2Ac - bC \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{2\sqrt{2} \sqrt{c} (b^2 - 4ac) \sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{\left(\frac{4Abc - C(4ac + b^2)}{\sqrt{b^2 - 4ac}} + 2Ac - bC \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b + \sqrt{b^2 - 4ac}}} \right)}{2\sqrt{2} \sqrt{c} (b^2 - 4ac) \sqrt{b + \sqrt{b^2 - 4ac}}} + \frac{B(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{bB \tanh^{-1} \left(\frac{b + 2cx^2}{\sqrt{b^2 - 4ac}} \right)}{(b^2 - 4ac)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(A*x^3 + B*x^4 + C*x^5)/(x*(a + b*x^2 + c*x^4)^2), x]

[Out] (B*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (x*(A*b - 2*a*C + (2*A*c - b*C)*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((2*A*c - b*C - (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((2*A*c - b*C + (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (b*B*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 638

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]

Rule 1114

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1166

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1275

```
Int[((f_)*(x_)^(m_)*((d_) + (e_)*(x_)^2)*((a_) + (b_)*(x_)^2 + (c_)*(
x_)^4)^(p_), x_Symbol] := Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1
)*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1585

```
Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_) + (c_)*(x_)^(r_
))^n, x_Symbol] := Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1662

```
Int[(Pq)*((d_)*(x_)^(m_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_S
ymbol] := Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1})*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
+ 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1})*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] && !Po
lyQ[Pq, x^2]
```

Rubi steps

$$\begin{aligned}
\int \frac{Ax^3 + Bx^4 + Cx^5}{x(a + bx^2 + cx^4)^2} dx &= \int \frac{x^2(A + Bx + Cx^2)}{(a + bx^2 + cx^4)^2} dx \\
&= \int \frac{Bx^3}{(a + bx^2 + cx^4)^2} dx + \int \frac{x^2(A + Cx^2)}{(a + bx^2 + cx^4)^2} dx \\
&= -\frac{x(Ab - 2aC + (2Ac - bC)x^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} + B \int \frac{x^3}{(a + bx^2 + cx^4)^2} dx + \frac{\int \frac{Ab - 2aC + (-2Ac + bC)x^2}{a + bx^2 + cx^4} dx}{2(b^2 - 4ac)} \\
&= -\frac{x(Ab - 2aC + (2Ac - bC)x^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{1}{2} B \operatorname{Subst} \left(\int \frac{x}{(a + bx + cx^2)^2} dx, x, x^2 \right) - \frac{(2Ac - bC)}{2(b^2 - 4ac)} \\
&= \frac{B(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{x(Ab - 2aC + (2Ac - bC)x^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{(2Ac - bC - \frac{4Abc - (b^2 - 4ac)^2}{\sqrt{b^2 - 4ac}})}{2\sqrt{2}\sqrt{c}(b^2 - 4ac)} \\
&= \frac{B(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{x(Ab - 2aC + (2Ac - bC)x^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{(2Ac - bC - \frac{4Abc - (b^2 - 4ac)^2}{\sqrt{b^2 - 4ac}})}{2\sqrt{2}\sqrt{c}(b^2 - 4ac)} \\
&= \frac{B(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{x(Ab - 2aC + (2Ac - bC)x^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{(2Ac - bC - \frac{4Abc - (b^2 - 4ac)^2}{\sqrt{b^2 - 4ac}})}{2\sqrt{2}\sqrt{c}(b^2 - 4ac)}
\end{aligned}$$

Mathematica [A] time = 0.15, size = 378, normalized size = 1.06

$$\frac{1}{4} \left(\frac{4a(B + Cx) + 2x(Bx + Cx) - A(b + 2cx^2)}{(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\sqrt{2} \left(C(b\sqrt{b^2 - 4ac} - 4ac - b^2) - 2Ac(\sqrt{b^2 - 4ac} - 2b) \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right) + \sqrt{2} \left(C(b\sqrt{b^2 - 4ac} + 4ac + b^2) - 2Ac(\sqrt{b^2 - 4ac} + 2b) \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{b + \sqrt{b^2 - 4ac}}} \right) + \frac{2bB \log(\sqrt{b^2 - 4ac} - b - 2cx^2)}{(b^2 - 4ac)^{3/2}} - \frac{2bB \log(\sqrt{b^2 - 4ac} + b + 2cx^2)}{(b^2 - 4ac)^{3/2}}}{\sqrt{c}(b^2 - 4ac)^{3/2} \sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{\sqrt{2} \left(C(b\sqrt{b^2 - 4ac} + 4ac + b^2) - 2Ac(\sqrt{b^2 - 4ac} + 2b) \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{b + \sqrt{b^2 - 4ac}}} \right) + \frac{2bB \log(\sqrt{b^2 - 4ac} - b - 2cx^2)}{(b^2 - 4ac)^{3/2}} - \frac{2bB \log(\sqrt{b^2 - 4ac} + b + 2cx^2)}{(b^2 - 4ac)^{3/2}}}{\sqrt{c}(b^2 - 4ac)^{3/2} \sqrt{b + \sqrt{b^2 - 4ac}}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(A*x^3 + B*x^4 + C*x^5)/(x*(a + b*x^2 + c*x^4)^2), x]

[Out] ((4*a*(B + C*x) + 2*x*(b*x*(B + C*x) - A*(b + 2*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(-2*A*c*(-2*b + Sqrt[b^2 - 4*a*c]) + (-b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(-2*A*c*(2*b + Sqrt[b^2 - 4*a*c]) + (b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (2*b*B*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) - (2*b*B*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Ax^3 + Bx^4 + Cx^5}{x(a + bx^2 + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A*x^3 + B*x^4 + C*x^5)/(x*(a + b*x^2 + c*x^4)^2), x]

[Out] IntegrateAlgebraic[(A*x^3 + B*x^4 + C*x^5)/(x*(a + b*x^2 + c*x^4)^2), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^5+B*x^4+A*x^3)/x/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")

[Out] Timed out

giac [B] time = 7.31, size = 4439, normalized size = 12.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^5+B*x^4+A*x^3)/x/(c*x^4+b*x^2+a)^2,x, algorithm="giac")

[Out]
$$\frac{1}{2}(Cbx^3 - 2A^2cx^3 + B^2bx^2 + 2C^2ax - Ab^2x + 2B^2a) / ((cx^4 + bx^2 + a)(b^2 - 4ac)) - \frac{1}{16}(2(2b^2c^3 - 8a^2c^4 - \sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}}c) * b^2c + 4\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * a^2c^2 + 2\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * b^2c^2 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * c^3 - 2(b^2 - 4ac)c^3)(b^2 - 4ac)^2A - (2b^3c^2 - 8a^2bc^3 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * b^3 + 4\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * a^2bc + 2\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * b^2c - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * b^2c^2 - 2(b^2 - 4ac) * b^2c^2) * (b^2 - 4ac)^2C - 2(\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * b^5c - 8\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * a^2b^3c^2 - 2\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * b^4c^2 - 2b^5c^2 + 16\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * a^2b^2c^3 + 8\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * a^2b^2c^3 + \sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * b^3c^3 + 16a^2b^3c^3 - 4\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * a^2b^2c^4 - 32a^2b^2c^4 + 2(b^2 - 4ac) * b^3c^2 - 8(b^2 - 4ac) * a^2b^2c^3) * A * \text{abs}(b^2 - 4ac) + 4(\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * a^2b^4c - 8\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * a^2b^2c^2 - 2\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * a^2b^3c^2 - 2a^2b^4c^2 + 16\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * a^3c^3 + 8\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * a^2b^2c^3 + \sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * c) * a^2b^2c^3 + 16a^2b^2c^3 - 4\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * a^2c^4 - 32a^3c^4 + 2(b^2 - 4ac) * a^2b^2c^2 - 8(b^2 - 4ac) * a^2c^3) * C * \text{abs}(b^2 - 4ac) - 4(2b^6c^3 - 16a^2b^4c^4 + 32a^2b^2c^5 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * b^6c + 8\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * a^2b^4c^2 + 2\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * b^5c^2 - 16\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * a^2b^2c^3 - 8\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * a^2b^3c^3 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * b^4c^3 + 4\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * a^2b^2c^4 - 2(b^2 - 4ac) * b^4c^3 + 8(b^2 - 4ac) * a^2b^2c^4) * A + (2b^7c^2 - 8a^2b^5c^3 - 32a^2b^3c^4 + 128a^3b^2c^5 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * b^7 + 4\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * a^2b^5c + 2\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * b^6c + 16\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * a^2b^3c^2 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * b^5c^2 - 64\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * a^3b^2c^3 - 32\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * a^2b^2c^3 + 16\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * a^2b^2c^4 - 2(b^2 - 4ac) * b^5c^2 + 32(b^2 - 4ac) * a^2b^2c^4) * C) * \arctan(2\sqrt{1/2} * x / \sqrt{(b^3 - 4a^2bc + \sqrt{2}\sqrt{(b^3 - 4a^2bc)^2 - 4(a^2b^2 - 4a^2c)(b^2c - 4a^2c^2))}) / (b^2c - 4a^2c^2)) / ((a^2b^6c - 12a^2b^4c^2 - 2a^2b^5c^2 + 48a^3b^2c^3 + 16a^2b^3c^3 + a^2b^4c^3 - 64a^4c^4 - 32a^3b^2c^4 - 8a^2b^2c^4 + 16a^3c^5)$$

```

*abs(b^2 - 4*a*c)*abs(c)) + 1/16*(2*(2*b^2*c^3 - 8*a*c^4 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*c^3 - 2*(b^2 - 4*a*c)*c^3)*(b^2 - 4*a*c)^2*A - (2*b^3*c^
2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b
^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*
c^2)*(b^2 - 4*a*c)^2*C + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c -
8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3
+ sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 16*a*b^3*c^3 - 4*sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 + 32*a^2*b*c^4 - 2*(b^2 - 4*a*c)
*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) - 4*(sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*a*b
^4*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a*b^2*c^3 - 16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^2*c^4 + 32*a^3*c^4 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + 8*(b^2 - 4*a*c)*a^2*c^
3)*C*abs(b^2 - 4*a*c) - 4*(2*b^6*c^3 - 16*a*b^4*c^4 + 32*a^2*b^2*c^5 - sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c + 8*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 2*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 16*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 8*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^3 + 8*(b^2 -
4*a*c)*a*b^2*c^4)*A + (2*b^7*c^2 - 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*
b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7 + 4*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c + 16*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 +
32*(b^2 - 4*a*c)*a^2*b*c^4)*C)*arctan(2*sqrt(1/2)*x/sqrt((b^3 - 4*a*b*c - s
qrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*
a*c^2)))/((a*b^6*c - 12*a^2*b^4*c^2 - 2*a*b^5*c^2 + 48*a^3*b^2*c^3 + 16*a^2
*b^3*c^3 + a*b^4*c^3 - 64*a^4*c^4 - 32*a^3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c
^5)*abs(b^2 - 4*a*c)*abs(c)) + 1/8*((b^4*c - 4*a*b^2*c^2 - 2*b^3*c^2 + b^2*
c^3 + (b^3*c - 4*a*b*c^2 - 2*b^2*c^2 + b*c^3)*sqrt(b^2 - 4*a*c))*B*abs(b^2
- 4*a*c) - (b^6*c - 8*a*b^4*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a*b^3*c^3
+ b^4*c^3 - 4*a*b^2*c^4 + (b^5*c - 4*a*b^3*c^2 - 2*b^4*c^2 + b^3*c^3)*sqrt(
b^2 - 4*a*c))*B)*log(x^2 + 1/2*(b^3 - 4*a*b*c + sqrt((b^3 - 4*a*b*c)^2 - 4*
(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2))/((a*b^4 - 8*a^2*b^
2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2*abs
(b^2 - 4*a*c)) + 1/8*((b^4*c - 4*a*b^2*c^2 - 2*b^3*c^2 + b^2*c^3 - (b^3*c -
4*a*b*c^2 - 2*b^2*c^2 + b*c^3)*sqrt(b^2 - 4*a*c))*B*abs(b^2 - 4*a*c) + (b^
6*c - 8*a*b^4*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a*b^3*c^3 + b^4*c^3 - 4*
a*b^2*c^4 - (b^5*c - 4*a*b^3*c^2 - 2*b^4*c^2 + b^3*c^3)*sqrt(b^2 - 4*a*c))*
B)*log(x^2 + 1/2*(b^3 - 4*a*b*c - sqrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2
*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*
c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2*abs(b^2 - 4*a*c))

```

maple [B] time = 0.03, size = 1119, normalized size = 3.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((C*x^5+B*x^4+A*x^3)/x/(c*x^4+b*x^2+a)^2,x)`

[Out]
$$\begin{aligned} & (-1/2/(4*a*c-b^2)*B*b*x^2+1/2*(2*A*c-C*b)/(4*a*c-b^2)*x^3-1/(4*a*c-b^2)*B*a \\ & +1/2*(A*b-2*C*a)/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+1/2/(4*a*c-b^2)^2*(-4*a*c+b \\ & ^2)^{(1/2)}*B*b*\ln(-2*c*x^2-b+(-4*a*c+b^2)^{(1/2)})-c/(4*a*c-b^2)^2*2^{(1/2)}/((- \\ & b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} \\ & *c*x)*A*(-4*a*c+b^2)^{(1/2)}*b-2*c^2/(4*a*c-b^2)^2*2^{(1/2)}/((-b+(-4*a*c+ \\ & b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x) \\ & *a*A+1/2*c/(4*a*c-b^2)^2*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(\\ & 2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*A*b^2+c/(4*a*c-b^2)^2*2^{(1/2)}/ \\ & ((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)}) \\ &)*c)^{(1/2)}*c*x)*C*(-4*a*c+b^2)^{(1/2)}*a+1/4/(4*a*c-b^2)^2*2^{(1/2)}/((-b+(-4*a \\ & *c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c \\ & *x)*C*(-4*a*c+b^2)^{(1/2)}*b^2+c/(4*a*c-b^2)^2*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2) \\ &))*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*C*a*b-1/ \\ & 4/(4*a*c-b^2)^2*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/(\\ & (-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^3*C-1/2/(4*a*c-b^2)^2*(-4*a*c+b^2)^{(1/2)} \\ & *B*b*\ln(2*c*x^2+b+(-4*a*c+b^2)^{(1/2)})-1/(4*a*c-b^2)^2*2^{(1/2)}/((b+(-4* \\ & a*c+b^2)^{(1/2)})*c)^{(1/2)}*(-4*a*c+b^2)^{(1/2)}*A*b*c*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a* \\ & c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)+2/(4*a*c-b^2)^2*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2) \\ &)*c)^{(1/2)}*A*a*c^2*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)-1/2/ \\ & (4*a*c-b^2)^2*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*A*b^2*c*\operatorname{arctan}(2^{(1/2)}/ \\ & (b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)+1/(4*a*c-b^2)^2*2^{(1/2)}/((b+(-4*a* \\ & c+b^2)^{(1/2)})*c)^{(1/2)}*(-4*a*c+b^2)^{(1/2)}*C*a*c*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+ \\ & b^2)^{(1/2)})*c)^{(1/2)}*c*x)+1/4/(4*a*c-b^2)^2*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2) \\ &)*c)^{(1/2)}*(-4*a*c+b^2)^{(1/2)}*C*b^2*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c \\ &)^{(1/2)}*c*x)-1/(4*a*c-b^2)^2*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*C*a*b \\ & *c*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)+1/4/(4*a*c-b^2)^2*2 \\ & ^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*C*b^3*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2) \\ &)^{(1/2)})*c)^{(1/2)}*c*x) \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{Bbx^2 + (Cb - 2Ac)x^3 + 2Ba + (2Ca - Ab)x}{2((b^2c - 4ac^2)x^4 + ab^2 - 4a^2c + (b^3 - 4abc)x^2)} - \frac{\int \frac{2Bbx + (Cb - 2Ac)x^2 - 2Ca + Ab}{cx^4 + bx^2 + a} dx}{2(b^2 - 4ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((C*x^5+B*x^4+A*x^3)/x/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")`

[Out]
$$\frac{1/2*(B*b*x^2 + (C*b - 2*A*c)*x^3 + 2*B*a + (2*C*a - A*b)*x)/(b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2 - 1/2*\operatorname{integrate}(- (2*B*b*x + (C*b - 2*A*c)*x^2 - 2*C*a + A*b)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c)}$$

mupad [B] time = 1.39, size = 3835, normalized size = 10.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A*x^3 + B*x^4 + C*x^5)/(x*(a + b*x^2 + c*x^4)^2),x)`

[Out]
$$\operatorname{symsum}(\log((8*A^3*a*c^4 + 6*A^3*b^2*c^3 + A*C^2*b^4*c - 3*C^3*a*b^3*c + 4*A*B^2*b^3*c^2 + 8*A*C^2*a^2*c^3 - 5*A^2*C*b^3*c^2 - 4*C^3*a^2*b*c^2 + 18*A*C^2*a*b^2*c^2 - 8*B^2*C*a*b^2*c^2 - 28*A^2*C*a*b*c^3)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - \operatorname{root}(256*a*b^12*c*z^4 - 1572864*a^6*b^2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^3*b^8*c^3*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8*c*z^2 - 6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^5*b*c^4*z$$

$$\begin{aligned} & ^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5*c^5*z^2 \\ & + 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a^4*b^2*c^4 \\ & *z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8192*A^2*a^3*b \\ & b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A^2*b^9*c*z^2 \\ & + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B*a^3*b*c^4*z \\ & - 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*a^2*b^3*c^3* \\ & z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^2 - 48*A*B^2 \\ & *C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 48*A^2*B^2*a* \\ & b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*b^3*c^2 - 80 \\ & *A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A^4*a*b^2*c^3 \\ & + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C*b^5*c - 6*A \\ & *C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^4 + 9*A^4*b^ \\ & 4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k)*(root(256*a*b^12*c*z^4 - 157286 \\ & 4*a^6*b^2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440 \\ & *a^3*b^8*c^3*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a* \\ & b^8*c*z^2 - 6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2 \\ & *a^5*b*c^4*z^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C* \\ & a^5*c^5*z^2 + 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^ \\ & 2*a^4*b^2*c^4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8 \\ & 192*A^2*a^3*b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16* \\ & A^2*b^9*c*z^2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B \\ & *a^3*b*c^4*z - 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B* \\ & a^2*b^3*c^3*z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^ \\ & ^2 - 48*A*B^2*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 4 \\ & 8*A^2*B^2*a*b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a* \\ & b^3*c^2 - 80*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A \\ & ^4*a*b^2*c^3 + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C \\ & *b^5*c - 6*A*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^ \\ & ^4 + 9*A^4*b^4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k)*((x*(16*B*b^7*c^2 - \\ & 192*B*a*b^5*c^3 - 1024*B*a^3*b*c^5 + 768*B*a^2*b^3*c^4))/(4*(b^6 - 64*a^3*c^ \\ & ^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (16*A*b^7*c^2 + 2048*C*a^4*c^5 - 192* \\ & A*a*b^5*c^3 - 1024*A*a^3*b*c^5 - 32*C*a*b^6*c^2 + 768*A*a^2*b^3*c^4 + 384*C \\ & *a^2*b^4*c^3 - 1536*C*a^3*b^2*c^4))/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - \\ & 12*a*b^4*c)) + (root(256*a*b^12*c*z^4 - 1572864*a^6*b^2*c^6*z^4 + 983040*a^ \\ & 5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^3*b^8*c^3*z^4 - 6144*a^2*b^ \\ & ^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8*c*z^2 - 6144*A*C*a^3*b^4* \\ & c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^5*b*c^4*z^2 - 12288*A^2*a^ \\ & 4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5*c^5*z^2 + 8192*C^2*a^4*b^ \\ & 3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a^4*b^2*c^4*z^2 - 6144*B^2* \\ & a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8192*A^2*a^3*b^3*c^4*z^2 - 153 \\ & 6*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A^2*b^9*c*z^2 + 1024*B*C^2*a^ \\ & 4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B*a^3*b*c^4*z - 192*A^2*B*a*b^ \\ & 5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*a^2*b^3*c^3*z + 16*A^2*B*b^7* \\ & c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^2 - 48*A*B^2*C*a*b^4*c + 192* \\ & A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 48*A^2*B^2*a*b^3*c^2 - 96*A^3* \\ & C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*b^3*c^2 - 80*A*C^3*a^2*b^3*c \\ & + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A^4*a*b^2*c^3 + 4*B^2*C^2*a*b^ \\ & ^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C*b^5*c - 6*A*C^3*a*b^5 + 32*A \\ & ^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^4 + 9*A^4*b^4*c^2 + 16*A^4*a^ \\ & ^2*c^4 + A^2*C^2*b^6, z, k)*x*(32*b^9*c^2 - 512*a*b^7*c^3 + 8192*a^4*b*c^6 + \\ & 3072*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^ \\ & ^2 - 12*a*b^4*c)) - (16*A*B*b^5*c^2 + 256*B*C*a^2*b^2*c^3 - 256*A*B*a^2*b*c \\ & ^4 - 64*B*C*a*b^4*c^2))/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) \\ & + (x*(2*C^2*b^6*c + 64*A^2*a^2*c^5 + 20*A^2*b^4*c^3 - 8*B^2*b^5*c^2 - 64*C \\ & ^2*a^3*c^4 - 12*A*C*b^5*c^2 - 96*A^2*a*b^2*c^4 + 32*B^2*a*b^3*c^3 - 4*C^2*a \\ & *b^4*c^2 + 32*A*C*a*b^3*c^3 + 64*A*C*a^2*b*c^4))/(4*(b^6 - 64*a^3*c^3 + 48* \\ & a^2*b^2*c^2 - 12*a*b^4*c))) + (x*(4*B^3*b^3*c^2 + B*C^2*b^4*c + 8*A^2*B*b^2 \\ & *c^3 + 4*B*C^2*a*b^2*c^2 - 6*A*B*C*b^3*c^2 - 8*A*B*C*a*b*c^3))/(4*(b^6 - 64 \\ & *a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)))*root(256*a*b^12*c*z^4 - 1572864*a$$

$$\begin{aligned}
& ^6b^2c^6z^4 + 983040a^5b^4c^5z^4 - 327680a^4b^6c^4z^4 + 61440a^3b^8c^3z^4 - 6144a^2b^{10}c^2z^4 + 1048576a^7c^7z^4 - 192A^2C^2a^8 \\
& *c^2z^2 - 6144A^2C^2a^3b^4c^3z^2 + 2048A^2C^2a^2b^6c^2z^2 - 12288C^2a^5b^2c^4z^2 - 12288A^2a^4b^2c^5z^2 - 128B^2a^2b^8c^2z^2 + 16384A^2C^2a^5 \\
& *c^5z^2 + 8192C^2a^4b^3c^3z^2 - 1536C^2a^3b^5c^2z^2 + 8192B^2a^4b^2c^4z^2 - 6144B^2a^3b^4c^3z^2 + 1536B^2a^2b^6c^2z^2 + 8192 \\
& *A^2a^3b^3c^4z^2 - 1536A^2a^2b^5c^3z^2 + 16C^2a^2b^9z^2 + 16A^2 \\
& *b^9c^2z^2 + 1024B^2C^2a^4b^2c^3z + 192B^2C^2a^2b^5c^2z - 1024A^2B^2a^3b^2c^4z - 192A^2B^2a^2b^5c^2z - 768B^2C^2a^3b^3c^2z + 768A^2B^2a^2 \\
& *b^3c^3z + 16A^2B^2b^7c^2z - 16B^2C^2a^2b^7z - 64A^2B^2C^2a^2b^2c^2 - 48A^2B^2C^2a^2b^4c + 192A^2C^2a^2b^2c^2 + 48B^2C^2a^2b^3c + 48A^2 \\
& *B^2a^2b^3c^2 - 96A^3C^2a^2b^2c^3 - 96A^3C^3a^3b^2c^2 - 80A^3C^2a^2b^3c^2 - 80A^3C^3a^2b^3c + 42A^2C^2a^2b^4c + 24C^4a^3b^2c + 24A^4a \\
& *b^2c^3 + 4B^2C^2a^2b^5 + 4A^2B^2b^5c + 16B^4a^2b^4c - 6A^3C^2b^5c - 6A^3C^3a^2b^5 + 32A^2C^2a^3c^3 + 16C^4a^4c^2 + 9C^4a^2b^4 + \\
& 9A^4b^4c^2 + 16A^4a^2c^4 + A^2C^2b^6, z, k), k, 1, 4) - ((B*a)/(4*a*c - b^2) - (x*(A*b - 2*C*a))/(2*(4*a*c - b^2)) - (x^3*(2*A*c - C*b))/(2*(4*a*c - b^2)) + (B*b*x^2)/(2*(4*a*c - b^2)))/(a + b*x^2 + c*x^4)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x**5+B*x**4+A*x**3)/x/(c*x**4+b*x**2+a)**2,x)

[Out] Timed out

$$3.44 \quad \int \frac{Ax^4+Bx^5+Cx^6}{x^2(a+bx^2+cx^4)^2} dx$$

Optimal. Leaf size=356

$$\frac{x(-2aC + x^2(2Ac - bC) + Ab) \left(-\frac{4Abc - C(4ac + b^2)}{\sqrt{b^2 - 4ac}} + 2Ac - bC \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right) \left(\frac{4Abc - C(4ac + b^2)}{\sqrt{b^2 - 4ac}} + 2Ac - bC \right)}{2(b^2 - 4ac)(a + bx^2 + cx^4) \cdot 2\sqrt{2} \sqrt{c} (b^2 - 4ac) \sqrt{b - \sqrt{b^2 - 4ac}} \cdot 2\sqrt{2} \sqrt{c} (b^2 - 4ac)}$$

Rubi [A] time = 0.36, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {1585, 1662, 1275, 1166, 205, 12, 1114, 638, 618, 206}

$$\frac{x(-2aC + x^2(2Ac - bC) + Ab) \left(-\frac{4Abc - C(4ac + b^2)}{\sqrt{b^2 - 4ac}} + 2Ac - bC \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right) \left(\frac{4Abc - C(4ac + b^2)}{\sqrt{b^2 - 4ac}} + 2Ac - bC \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b^2 - 4ac + b}} \right) + \frac{B(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{bB \tanh^{-1} \left(\frac{b + 2cx^2}{\sqrt{b^2 - 4ac}} \right)}{(b^2 - 4ac)^{3/2}}}{2\sqrt{2} \sqrt{c} (b^2 - 4ac) \sqrt{b - \sqrt{b^2 - 4ac}} \cdot 2\sqrt{2} \sqrt{c} (b^2 - 4ac) \sqrt{\sqrt{b^2 - 4ac} + b}}$$

Antiderivative was successfully verified.

[In] Int[(A*x^4 + B*x^5 + C*x^6)/(x^2*(a + b*x^2 + c*x^4)^2), x]

[Out] (B*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (x*(A*b - 2*a*C + (2*A*c - b*C)*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((2*A*c - b*C - (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((2*A*c - b*C + (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (b*B*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 638

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]

Rule 1114

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1166

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1275

```
Int[((f_)*(x_)^(m_)*((d_) + (e_)*(x_)^2)*((a_) + (b_)*(x_)^2 + (c_)*(
x_)^4)^(p_), x_Symbol] := Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1
)*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1585

```
Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_) + (c_)*(x_)^(r_
))^n, x_Symbol] := Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1662

```
Int[(Pq)*((d_)*(x_)^(m_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_S
ymbol] := Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1})*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
+ 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1})*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] && !Po
lyQ[Pq, x^2]
```

Rubi steps

$$\begin{aligned}
\int \frac{Ax^4 + Bx^5 + Cx^6}{x^2(a + bx^2 + cx^4)^2} dx &= \int \frac{x^2(A + Bx + Cx^2)}{(a + bx^2 + cx^4)^2} dx \\
&= \int \frac{Bx^3}{(a + bx^2 + cx^4)^2} dx + \int \frac{x^2(A + Cx^2)}{(a + bx^2 + cx^4)^2} dx \\
&= -\frac{x(Ab - 2aC + (2Ac - bC)x^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} + B \int \frac{x^3}{(a + bx^2 + cx^4)^2} dx + \frac{\int \frac{Ab - 2aC + (-2Ac + bC)x^2}{a + bx^2 + cx^4} dx}{2(b^2 - 4ac)} \\
&= -\frac{x(Ab - 2aC + (2Ac - bC)x^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{1}{2} B \operatorname{Subst} \left(\int \frac{x}{(a + bx + cx^2)^2} dx, x, x^2 \right) - \frac{(2Ac - bC)}{2(b^2 - 4ac)} \\
&= \frac{B(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{x(Ab - 2aC + (2Ac - bC)x^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{(2Ac - bC - \frac{4Abc - b^2}{\sqrt{b^2 - 4ac}})}{2\sqrt{2}\sqrt{c}(b^2 - 4ac)} \\
&= \frac{B(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{x(Ab - 2aC + (2Ac - bC)x^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{(2Ac - bC - \frac{4Abc - b^2}{\sqrt{b^2 - 4ac}})}{2\sqrt{2}\sqrt{c}(b^2 - 4ac)} \\
&= \frac{B(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{x(Ab - 2aC + (2Ac - bC)x^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{(2Ac - bC - \frac{4Abc - b^2}{\sqrt{b^2 - 4ac}})}{2\sqrt{2}\sqrt{c}(b^2 - 4ac)}
\end{aligned}$$

Mathematica [A] time = 0.15, size = 378, normalized size = 1.06

$$\frac{1}{4} \left(\frac{4a(B + Cx) + 2x(Bx + Cx) - A(b + 2cx^2)}{(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\sqrt{2} \left(C(b\sqrt{b^2 - 4ac} - 4ac - b^2) - 2Ac(\sqrt{b^2 - 4ac} - 2b) \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right) + \sqrt{2} \left(C(b\sqrt{b^2 - 4ac} + 4ac + b^2) - 2Ac(\sqrt{b^2 - 4ac} + 2b) \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{b + \sqrt{b^2 - 4ac}}} \right) + \frac{2bB \log(\sqrt{b^2 - 4ac} - b - 2cx^2)}{(b^2 - 4ac)^{3/2}} - \frac{2bB \log(\sqrt{b^2 - 4ac} + b + 2cx^2)}{(b^2 - 4ac)^{3/2}}}{\sqrt{c}(b^2 - 4ac)^{3/2} \sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{\sqrt{2} \left(C(b\sqrt{b^2 - 4ac} + 4ac + b^2) - 2Ac(\sqrt{b^2 - 4ac} + 2b) \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right) + \sqrt{2} \left(C(b\sqrt{b^2 - 4ac} - 4ac - b^2) - 2Ac(\sqrt{b^2 - 4ac} - 2b) \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{b + \sqrt{b^2 - 4ac}}} \right) + \frac{2bB \log(\sqrt{b^2 - 4ac} - b - 2cx^2)}{(b^2 - 4ac)^{3/2}} - \frac{2bB \log(\sqrt{b^2 - 4ac} + b + 2cx^2)}{(b^2 - 4ac)^{3/2}}}{\sqrt{c}(b^2 - 4ac)^{3/2} \sqrt{b + \sqrt{b^2 - 4ac}}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(A*x^4 + B*x^5 + C*x^6)/(x^2*(a + b*x^2 + c*x^4)^2), x]

[Out] ((4*a*(B + C*x) + 2*x*(b*x*(B + C*x) - A*(b + 2*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(-2*A*c*(-2*b + Sqrt[b^2 - 4*a*c]) + (-b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(-2*A*c*(2*b + Sqrt[b^2 - 4*a*c]) + (b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (2*b*B*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) - (2*b*B*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Ax^4 + Bx^5 + Cx^6}{x^2(a + bx^2 + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A*x^4 + B*x^5 + C*x^6)/(x^2*(a + b*x^2 + c*x^4)^2), x]

[Out] IntegrateAlgebraic[(A*x^4 + B*x^5 + C*x^6)/(x^2*(a + b*x^2 + c*x^4)^2), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^6+B*x^5+A*x^4)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")

[Out] Timed out

giac [B] time = 6.84, size = 4439, normalized size = 12.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^6+B*x^5+A*x^4)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="giac")

[Out]
$$\frac{1}{2}(Cbx^3 - 2A^2cx^3 + B^2bx^2 + 2C^2ax - Ab^2x + 2B^2a) / ((cx^4 + bx^2 + a)(b^2 - 4ac)) - \frac{1}{16}(2(2b^2c^3 - 8a^2c^4 - \sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}}c) * b^2c + 4\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * ac^2 + 2\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * b^2c - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * b^2c^2 - 2(b^2 - 4ac) * b^2c^2) * (b^2 - 4ac)^2 * A - (2b^3c^2 - 8a^2bc^3 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * b^3 + 4\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * ab^2c + 2\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * b^2c - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * b^2c^2 - 2(b^2 - 4ac) * b^2c^2) * (b^2 - 4ac)^2 * C - 2(\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * b^5c - 8\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * ab^3c^2 - 2\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * b^4c^2 - 2b^5c^2 + 16\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * a^2 * b^2c^3 + 8\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * ab^2c^3 + \sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * b^3c^3 + 16a^2 * b^3c^3 - 4\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * ab^2c^4 - 32a^2 * b^2c^4 + 2(b^2 - 4ac) * b^3c^2 - 8(b^2 - 4ac) * ab^2c^3) * A * \text{abs}(b^2 - 4ac) + 4(\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * ab^4c - 8\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * a^2 * b^2c^2 - 2\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * ab^3c^2 - 2a * b^4c^2 + 16\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * a^3c^3 + 8\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * a^2 * b^2c^3 + \sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * a * b^2c^3 + 16a^2 * b^2c^3 - 4\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * a^2 * c^4 - 32a^3 * c^4 + 2(b^2 - 4ac) * a * b^2c^2 - 8(b^2 - 4ac) * a^2 * c^3) * C * \text{abs}(b^2 - 4ac) - 4(2b^6c^3 - 16a^2 * b^4c^4 + 32a^2 * b^2c^5 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * b^6c + 8\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * ab^4c^2 + 2\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * b^5c^2 - 16\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * a^2 * b^2c^3 - 8\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * ab^3c^3 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * b^4c^3 + 4\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * ab^2c^4 - 2(b^2 - 4ac) * b^4c^3 + 8(b^2 - 4ac) * a * b^2c^4) * A + (2b^7c^2 - 8a^2 * b^5c^3 - 32a^2 * b^3c^4 + 128a^3 * b^2c^5 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * b^7 + 4\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * ab^5c + 2\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * b^6c + 16\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * a^2 * b^3c^2 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * b^5c^2 - 64\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * a^3 * b^2c^3 - 32\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * a^2 * b^2c^3 + 16\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c) * a^2 * b^2c^4 - 2(b^2 - 4ac) * b^5c^2 + 32(b^2 - 4ac) * a^2 * b^2c^4) * C) * \arctan(2\sqrt{1/2} * x / \sqrt{(b^3 - 4a^2 * b^2c + \sqrt{2}\sqrt{(b^3 - 4a^2 * b^2c)^2 - 4(a^2 * b^2c - 4a^2 * c^2)}) / (b^2 * c - 4a^2 * c^2)}) / ((a * b^6c - 12a^2 * b^4c^2 - 2a * b^5c^2 + 48a^3 * b^2c^3 + 16a^2 * b^3c^3 + a * b^4c^3 - 64a^4c^4 - 32a^3 * b^2c^4 - 8a^2 * b^2c^4 + 16a^3 * c^5)$$

$$\begin{aligned}
& *abs(b^2 - 4*a*c)*abs(c)) + 1/16*(2*(2*b^2*c^3 - 8*a*c^4 - \sqrt{2}*\sqrt{b^2 - 4*a*c} \\
& - 4*a*c)*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c}*b^2*c + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c} \\
& c)*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c}*a*c^2 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{ \\
& (b*c - \sqrt{b^2 - 4*a*c}*c)*b*c^2 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{ \\
& (b^2 - 4*a*c)*c}*c^3 - 2*(b^2 - 4*a*c)*c^3)*(b^2 - 4*a*c)^2*A - (2*b^3*c^ \\
& 2 - 8*a*b*c^3 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c}*b \\
& ^3 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c}*a*b*c + 2* \\
& \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c}*b^2*c - \sqrt{2}*\sqrt{ \\
& (b^2 - 4*a*c)*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c}*b*c^2 - 2*(b^2 - 4*a*c)*b* \\
& c^2)*(b^2 - 4*a*c)^2*C + 2*(\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c}*b^5*c - \\
& 8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c}*a*b^3*c^2 - 2*\sqrt{2}*\sqrt{b*c - \\
& \sqrt{b^2 - 4*a*c}*c}*b^4*c^2 + 2*b^5*c^2 + 16*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - \\
& 4*a*c}*c}*a^2*b*c^3 + 8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c}*a*b^2*c^3 \\
& + \sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c}*b^3*c^3 - 16*a*b^3*c^3 - 4*\sqrt{2} \\
& (\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c}*a*b*c^4 + 32*a^2*b*c^4 - 2*(b^2 - 4*a*c) \\
& *b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) - 4*(\sqrt{2}*\sqrt{b* \\
& c - \sqrt{b^2 - 4*a*c}*c}*a*b^4*c - 8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c} \\
&)*a^2*b^2*c^2 - 2*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c}*a*b^3*c^2 + 2*a*b \\
& ^4*c^2 + 16*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c}*a^3*c^3 + 8*\sqrt{2}*\sqrt{ \\
& (b*c - \sqrt{b^2 - 4*a*c}*c)*a^2*b*c^3 + \sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a* \\
& c}*c}*a*b^2*c^3 - 16*a^2*b^2*c^3 - 4*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c} \\
&)*a^2*c^4 + 32*a^3*c^4 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + 8*(b^2 - 4*a*c)*a^2*c^ \\
& 3)*C*abs(b^2 - 4*a*c) - 4*(2*b^6*c^3 - 16*a*b^4*c^4 + 32*a^2*b^2*c^5 - \sqrt{2} \\
& (\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c}*b^6*c + 8*\sqrt{2}*\sqrt{ \\
& (b^2 - 4*a*c)*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c}*a*b^4*c^2 + 2*\sqrt{2}*\sqrt{b \\
& ^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c}*b^5*c^2 - 16*\sqrt{2}*\sqrt{b^2 - \\
& 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c}*a^2*b^2*c^3 - 8*\sqrt{2}*\sqrt{b^2 - \\
& 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c}*a*b^3*c^3 - \sqrt{2}*\sqrt{b^2 - 4*a*c} \\
&)*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c}*b^4*c^3 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{ \\
& (b*c - \sqrt{b^2 - 4*a*c}*c)*a*b^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^3 + 8*(b^2 - \\
& 4*a*c)*a*b^2*c^4)*A + (2*b^7*c^2 - 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3* \\
& b*c^5 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c}*b^7 + 4*\sqrt{2} \\
& (\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c}*a*b^5*c + 2*\sqrt{2} \\
&)*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c}*b^6*c + 16*\sqrt{2}*\sqrt{ \\
& (b^2 - 4*a*c)*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c}*a^2*b^3*c^2 - \sqrt{2}*\sqrt{b^2 - \\
& 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c}*b^5*c^2 - 64*\sqrt{2}*\sqrt{b^2 - \\
& 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c}*a^3*b*c^3 - 32*\sqrt{2}*\sqrt{b^2 - 4* \\
& a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c}*a^2*b^2*c^3 + 16*\sqrt{2}*\sqrt{b^2 - 4* \\
& a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c}*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 + \\
& 32*(b^2 - 4*a*c)*a^2*b*c^4)*C)*arctan(2*\sqrt{1/2}*x/\sqrt{((b^3 - 4*a*b*c - \sqrt{ \\
& (b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4* \\
& a*c^2)))/((a*b^6*c - 12*a^2*b^4*c^2 - 2*a*b^5*c^2 + 48*a^3*b^2*c^3 + 16*a^2 \\
& *b^3*c^3 + a*b^4*c^3 - 64*a^4*c^4 - 32*a^3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c \\
& ^5)*abs(b^2 - 4*a*c)*abs(c)) + 1/8*((b^4*c - 4*a*b^2*c^2 - 2*b^3*c^2 + b^2* \\
& c^3 + (b^3*c - 4*a*b*c^2 - 2*b^2*c^2 + b*c^3)*\sqrt{b^2 - 4*a*c}))*B*abs(b^2 - \\
& 4*a*c) - (b^6*c - 8*a*b^4*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a*b^3*c^3 \\
& + b^4*c^3 - 4*a*b^2*c^4 + (b^5*c - 4*a*b^3*c^2 - 2*b^4*c^2 + b^3*c^3)*\sqrt{ \\
& (b^2 - 4*a*c)})*B)*log(x^2 + 1/2*(b^3 - 4*a*b*c + \sqrt{((b^3 - 4*a*b*c)^2 - 4* \\
& (a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2)))/((a*b^4 - 8*a^2*b^ \\
& 2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2*abs \\
& (b^2 - 4*a*c)) + 1/8*((b^4*c - 4*a*b^2*c^2 - 2*b^3*c^2 + b^2*c^3 - (b^3*c - \\
& 4*a*b*c^2 - 2*b^2*c^2 + b*c^3)*\sqrt{b^2 - 4*a*c}))*B*abs(b^2 - 4*a*c) + (b^ \\
& 6*c - 8*a*b^4*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a*b^3*c^3 + b^4*c^3 - 4* \\
& a*b^2*c^4 - (b^5*c - 4*a*b^3*c^2 - 2*b^4*c^2 + b^3*c^3)*\sqrt{b^2 - 4*a*c})* \\
& B)*log(x^2 + 1/2*(b^3 - 4*a*b*c - \sqrt{((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2 \\
& *c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2)))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3* \\
& c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2*abs(b^2 - 4*a*c))
\end{aligned}$$

maple [B] time = 0.03, size = 1119, normalized size = 3.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((C*x^6+B*x^5+A*x^4)/x^2/(c*x^4+b*x^2+a)^2,x)`

[Out]
$$\begin{aligned} & (-1/2/(4*a*c-b^2)*B*b*x^2+1/2*(2*A*c-C*b)/(4*a*c-b^2)*x^3-1/(4*a*c-b^2)*B*a \\ & +1/2*(A*b-2*C*a)/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+1/2/(4*a*c-b^2)^2*(-4*a*c+b \\ & ^2)^{(1/2)}*B*b*\ln(-2*c*x^2-b+(-4*a*c+b^2)^{(1/2)})-c/(4*a*c-b^2)^2*2^{(1/2)}/((- \\ & b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} \\ & *c*x)*A*(-4*a*c+b^2)^{(1/2)}*b-2*c^2/(4*a*c-b^2)^2*2^{(1/2)}/((-b+(-4*a*c+ \\ & b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x) \\ & *a*A+1/2*c/(4*a*c-b^2)^2*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(\\ & 2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*A*b^2+c/(4*a*c-b^2)^2*2^{(1/2)}/ \\ & ((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)}) \\ &)*c)^{(1/2)}*c*x)*C*(-4*a*c+b^2)^{(1/2)}*a+1/4/(4*a*c-b^2)^2*2^{(1/2)}/((-b+(-4*a \\ & *c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c \\ & *x)*C*(-4*a*c+b^2)^{(1/2)}*b^2+c/(4*a*c-b^2)^2*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2) \\ &))*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*C*a*b-1/ \\ & 4/(4*a*c-b^2)^2*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/(\\ & (-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^3*C-1/2/(4*a*c-b^2)^2*(-4*a*c+b^2)^{(1/2)} \\ & *B*b*\ln(2*c*x^2+b+(-4*a*c+b^2)^{(1/2)})-1/(4*a*c-b^2)^2*2^{(1/2)}/((b+(-4* \\ & a*c+b^2)^{(1/2)})*c)^{(1/2)}*(-4*a*c+b^2)^{(1/2)}*A*b*c*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a* \\ & c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)+2/(4*a*c-b^2)^2*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2) \\ &))*c)^{(1/2)}*A*a*c^2*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)-1/2/ \\ & (4*a*c-b^2)^2*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*A*b^2*c*\operatorname{arctan}(2^{(1/2)}/ \\ & ((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)+1/(4*a*c-b^2)^2*2^{(1/2)}/((b+(-4*a* \\ & c+b^2)^{(1/2)})*c)^{(1/2)}*(-4*a*c+b^2)^{(1/2)}*C*a*c*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+ \\ & b^2)^{(1/2)})*c)^{(1/2)}*c*x)+1/4/(4*a*c-b^2)^2*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2) \\ &))*c)^{(1/2)}*(-4*a*c+b^2)^{(1/2)}*C*b^2*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c \\ &)^{(1/2)}*c*x)-1/(4*a*c-b^2)^2*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*C*a*b \\ & *c*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)+1/4/(4*a*c-b^2)^2*2 \\ & ^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*C*b^3*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2) \\ &)^{(1/2)})*c)^{(1/2)}*c*x) \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{Bbx^2 + (Cb - 2Ac)x^3 + 2Ba + (2Ca - Ab)x}{2((b^2c - 4ac^2)x^4 + ab^2 - 4a^2c + (b^3 - 4abc)x^2)} - \frac{\int \frac{2Bbx + (Cb - 2Ac)x^2 - 2Ca + Ab}{cx^4 + bx^2 + a} dx}{2(b^2 - 4ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((C*x^6+B*x^5+A*x^4)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")`

[Out]
$$\begin{aligned} & 1/2*(B*b*x^2 + (C*b - 2*A*c)*x^3 + 2*B*a + (2*C*a - A*b)*x)/((b^2*c - 4*a*c \\ & ^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2) - 1/2*\operatorname{integrate}(- (2*B*b*x \\ & + (C*b - 2*A*c)*x^2 - 2*C*a + A*b)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c) \end{aligned}$$

mupad [B] time = 1.41, size = 3835, normalized size = 10.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A*x^4 + B*x^5 + C*x^6)/(x^2*(a + b*x^2 + c*x^4)^2),x)`

[Out]
$$\begin{aligned} & \operatorname{symsum}(\log((8*A^3*a*c^4 + 6*A^3*b^2*c^3 + A*C^2*b^4*c - 3*C^3*a*b^3*c + 4*A \\ & *B^2*b^3*c^2 + 8*A*C^2*a^2*c^3 - 5*A^2*C*b^3*c^2 - 4*C^3*a^2*b*c^2 + 18*A*C \\ & ^2*a*b^2*c^2 - 8*B^2*C*a*b^2*c^2 - 28*A^2*C*a*b*c^3)/(8*(b^6 - 64*a^3*c^3 + \\ & 48*a^2*b^2*c^2 - 12*a*b^4*c)) - \operatorname{root}(256*a*b^12*c*z^4 - 1572864*a^6*b^2*c^ \\ & 6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^3*b^8*c^3 \\ & *z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8*c*z^2 - \\ & 6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^5*b*c^4*z \end{aligned}$$

$$\begin{aligned}
 &^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5*c^5*z^2 \\
 &+ 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a^4*b^2*c^4 \\
 &+ z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8192*A^2*a^3* \\
 &b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A^2*b^9*c*z^2 \\
 &+ 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B*a^3*b*c^4*z \\
 &- 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*a^2*b^3*c^3* \\
 &z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^2 - 48*A*B^2 \\
 &*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 48*A^2*B^2*a* \\
 &b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*b^3*c^2 - 80 \\
 &*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A^4*a*b^2*c^3 \\
 &+ 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C*b^5*c - 6*A \\
 &*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^4 + 9*A^4*b^4 \\
 &+ c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k)*(root(256*a*b^12*c*z^4 - 157286 \\
 &4*a^6*b^2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440 \\
 &*a^3*b^8*c^3*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a* \\
 &b^8*c*z^2 - 6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2 \\
 &*a^5*b*c^4*z^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C* \\
 &a^5*c^5*z^2 + 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2 \\
 &a^4*b^2*c^4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8 \\
 &192*A^2*a^3*b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16* \\
 &A^2*b^9*c*z^2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B \\
 &*a^3*b*c^4*z - 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B* \\
 &a^2*b^3*c^3*z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^2 \\
 &- 48*A*B^2*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 4 \\
 &8*A^2*B^2*a*b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a* \\
 &b^3*c^2 - 80*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A \\
 &^4*a*b^2*c^3 + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C \\
 &*b^5*c - 6*A*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^4 \\
 &+ 9*A^4*b^4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k)*((x*(16*B*b^7*c^2 - \\
 &192*B*a*b^5*c^3 - 1024*B*a^3*b*c^5 + 768*B*a^2*b^3*c^4))/(4*(b^6 - 64*a^3* \\
 &c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (16*A*b^7*c^2 + 2048*C*a^4*c^5 - 192* \\
 &A*a*b^5*c^3 - 1024*A*a^3*b*c^5 - 32*C*a*b^6*c^2 + 768*A*a^2*b^3*c^4 + 384*C \\
 &*a^2*b^4*c^3 - 1536*C*a^3*b^2*c^4))/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - \\
 &12*a*b^4*c)) + (root(256*a*b^12*c*z^4 - 1572864*a^6*b^2*c^6*z^4 + 983040*a^ \\
 &5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^3*b^8*c^3*z^4 - 6144*a^2*b \\
 &^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8*c*z^2 - 6144*A*C*a^3*b^4* \\
 &c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^5*b*c^4*z^2 - 12288*A^2*a^ \\
 &4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5*c^5*z^2 + 8192*C^2*a^4*b^ \\
 &3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a^4*b^2*c^4*z^2 - 6144*B^2* \\
 &a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8192*A^2*a^3*b^3*c^4*z^2 - 153 \\
 &6*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A^2*b^9*c*z^2 + 1024*B*C^2*a^ \\
 &4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B*a^3*b*c^4*z - 192*A^2*B*a*b^ \\
 &5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*a^2*b^3*c^3*z + 16*A^2*B*b^7* \\
 &c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^2 - 48*A*B^2*C*a*b^4*c + 192* \\
 &A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 48*A^2*B^2*a*b^3*c^2 - 96*A^3* \\
 &C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*b^3*c^2 - 80*A*C^3*a^2*b^3*c \\
 &+ 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A^4*a*b^2*c^3 + 4*B^2*C^2*a*b^ \\
 &5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C*b^5*c - 6*A*C^3*a*b^5 + 32*A \\
 &^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^4 + 9*A^4*b^4*c^2 + 16*A^4*a^ \\
 &2*c^4 + A^2*C^2*b^6, z, k)*x*(32*b^9*c^2 - 512*a*b^7*c^3 + 8192*a^4*b*c^6 + \\
 &3072*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 \\
 &- 12*a*b^4*c))) - (16*A*B*b^5*c^2 + 256*B*C*a^2*b^2*c^3 - 256*A*B*a^2*b*c \\
 &^4 - 64*B*C*a*b^4*c^2))/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) \\
 &+ (x*(2*C^2*b^6*c + 64*A^2*a^2*c^5 + 20*A^2*b^4*c^3 - 8*B^2*b^5*c^2 - 64*C \\
 &^2*a^3*c^4 - 12*A*C*b^5*c^2 - 96*A^2*a*b^2*c^4 + 32*B^2*a*b^3*c^3 - 4*C^2*a \\
 &*b^4*c^2 + 32*A*C*a*b^3*c^3 + 64*A*C*a^2*b*c^4))/(4*(b^6 - 64*a^3*c^3 + 48* \\
 &a^2*b^2*c^2 - 12*a*b^4*c))) + (x*(4*B^3*b^3*c^2 + B*C^2*b^4*c + 8*A^2*B*b^2 \\
 &*c^3 + 4*B*C^2*a*b^2*c^2 - 6*A*B*C*b^3*c^2 - 8*A*B*C*a*b*c^3))/(4*(b^6 - 64 \\
 &*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)))*root(256*a*b^12*c*z^4 - 1572864*a
 \end{aligned}$$

$$\begin{aligned}
& ^6b^2c^6z^4 + 983040a^5b^4c^5z^4 - 327680a^4b^6c^4z^4 + 61440a^3b^8c^3z^4 - 6144a^2b^{10}c^2z^4 + 1048576a^7c^7z^4 - 192A^2C^2a^8b^8c^2z^2 - 6144A^2C^2a^3b^4c^3z^2 + 2048A^2C^2a^2b^6c^2z^2 - 12288C^2a^5b^2c^4z^2 - 12288A^2a^4b^2c^5z^2 - 128B^2a^2b^8c^2z^2 + 16384A^2C^2a^5c^5z^2 + 8192C^2a^4b^3c^3z^2 - 1536C^2a^3b^5c^2z^2 + 8192B^2a^4b^2c^4z^2 - 6144B^2a^3b^4c^3z^2 + 1536B^2a^2b^6c^2z^2 + 8192A^2a^3b^3c^4z^2 - 1536A^2a^2b^5c^3z^2 + 16C^2a^2b^9z^2 + 16A^2b^9c^2z^2 + 1024B^2C^2a^4b^2c^3z + 192B^2C^2a^2b^5c^2z - 1024A^2B^2a^3b^2c^4z - 192A^2B^2a^2b^5c^2z - 768B^2C^2a^3b^3c^2z + 768A^2B^2a^2b^3c^3z + 16A^2B^2b^7c^2z - 16B^2C^2a^2b^7z - 64A^2B^2C^2a^2b^2c^2 - 48A^2B^2C^2a^2b^4c + 192A^2C^2a^2b^2c^2 + 48B^2C^2a^2b^3c + 48A^2B^2a^2b^3c^2 - 96A^3C^2a^2b^2c^3 - 96A^3C^3a^3b^2c^2 - 80A^3C^2a^2b^3c^2 - 80A^3C^3a^2b^3c + 42A^2C^2a^2b^4c + 24C^4a^3b^2c + 24A^4a^2b^2c^3 + 4B^2C^2a^2b^5 + 4A^2B^2b^5c + 16B^4a^2b^4c - 6A^3C^2b^5c - 6A^3C^3a^2b^5 + 32A^2C^2a^3c^3 + 16C^4a^4c^2 + 9C^4a^2b^4 + 9A^4b^4c^2 + 16A^4a^2c^4 + A^2C^2b^6, z, k), k, 1, 4) - ((B^2a)/(4a^2c - b^2) - (x*(A^2b - 2C^2a))/(2*(4a^2c - b^2)) - (x^3*(2A^2c - C^2b))/(2*(4a^2c - b^2)) + (B^2bx^2)/(2*(4a^2c - b^2)))/(a + b*x^2 + c*x^4)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x**6+B*x**5+A*x**4)/x**2/(c*x**4+b*x**2+a)**2,x)

[Out] Timed out

$$3.45 \quad \int \frac{x^7(d+ex^2+fx^4)}{a+bx^2+cx^4} dx$$

Optimal. Leaf size=273

$$\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(2a^2c^3e - b^3c(cd - 5af) - 4ab^2c^2e + abc^2(3cd - 5af) + b^5(-f) + b^4ce\right)}{2c^5\sqrt{b^2-4ac}} + \frac{x^4(-c(af + be) + b^2f)}{4c^3}$$

Rubi [A] time = 0.85, antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1663, 1628, 634, 618, 206, 628}

$$\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(2a^2c^3e - 4ab^2c^2e - b^3c(cd - 5af) + abc^2(3cd - 5af) + b^4ce + b^5(-f)\right)}{2c^5\sqrt{b^2-4ac}} + \frac{x^4(-c(af + be) + b^2f + c^2d)}{4c^3} + \frac{x^2(-bc(cd - 2af) - ac^2e + b^2ce + b^3(-f))}{2c^4} - \frac{\log(a + bx^2 + cx^4)\left(-b^2c(cd - 3af) - 2abc^2e + ac^2(cd - af) + b^3ce + b^4(-f)\right)}{4c^5} + \frac{x^6(cx - bf)}{6c^2} + \frac{fx^8}{8c}$$

Antiderivative was successfully verified.

[In] Int[(x^7*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]

[Out] ((b^2*c*e - a*c^2*e - b^3*f - b*c*(c*d - 2*a*f))*x^2)/(2*c^4) + ((c^2*d + b^2*f - c*(b*e + a*f))*x^4)/(4*c^3) + ((c*e - b*f)*x^6)/(6*c^2) + (f*x^8)/(8*c) - ((b^4*c*e - 4*a*b^2*c^2*e + 2*a^2*c^3*e - b^5*f - b^3*c*(c*d - 5*a*f) + a*b*c^2*(3*c*d - 5*a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^5*Sqrt[b^2 - 4*a*c]) - ((b^3*c*e - 2*a*b*c^2*e - b^4*f - b^2*c*(c*d - 3*a*f) + a*c^2*(c*d - a*f))*Log[a + b*x^2 + c*x^4])/(4*c^5)

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1628

Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 1663

Int[(Pq_)*(x_)^(m_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^p, x], x], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned} \int \frac{x^7 (d + ex^2 + fx^4)}{a + bx^2 + cx^4} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x^3 (d + ex + fx^2)}{a + bx + cx^2} dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left(\int \left(\frac{b^2ce - ac^2e - b^3f - bc(cd - 2af)}{c^4} + \frac{(c^2d + b^2f - c(be + af))x}{c^3} + \frac{(ce - bf)x^6}{6c^2} \right) dx, x, x^2 \right) \\ &= \frac{(b^2ce - ac^2e - b^3f - bc(cd - 2af))x^2}{2c^4} + \frac{(c^2d + b^2f - c(be + af))x^4}{4c^3} + \frac{(ce - bf)x^6}{6c^2} \\ &= \frac{(b^2ce - ac^2e - b^3f - bc(cd - 2af))x^2}{2c^4} + \frac{(c^2d + b^2f - c(be + af))x^4}{4c^3} + \frac{(ce - bf)x^6}{6c^2} \\ &= \frac{(b^2ce - ac^2e - b^3f - bc(cd - 2af))x^2}{2c^4} + \frac{(c^2d + b^2f - c(be + af))x^4}{4c^3} + \frac{(ce - bf)x^6}{6c^2} \\ &= \frac{(b^2ce - ac^2e - b^3f - bc(cd - 2af))x^2}{2c^4} + \frac{(c^2d + b^2f - c(be + af))x^4}{4c^3} + \frac{(ce - bf)x^6}{6c^2} \end{aligned}$$

Mathematica [A] time = 0.20, size = 260, normalized size = 0.95

$$\frac{-12 \tan^{-1} \left(\frac{bx^2}{\sqrt{4ac-b^2}} \right) \left(2b^2c^2 + b^3c(cd-5af) + 4ab^2c^2 + ab^2(5af-3cd) + b^5f - b^4ce \right) + 6c^2x^4(-c(af+bc) + b^2f + c^2d) - 12cx^2(bc(cd-2af) + ac^2e + b^3f - b^2ce) + 6 \log(a + bx^2 + cx^4) (b^2c(cd-3af) + 2abc^2e + ac^2(af-cd) + b^4f - b^3ce) + 4c^3x^6(ce-bf) + 3c^4fx^8}{24c^5}$$

Antiderivative was successfully verified.

[In] Integrate[(x^7*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4), x]

[Out] (-12*c*(-(b^2*c*e) + a*c^2*e + b^3*f + b*c*(c*d - 2*a*f))*x^2 + 6*c^2*(c^2*d + b^2*f - c*(b*e + a*f))*x^4 + 4*c^3*(c*e - b*f)*x^6 + 3*c^4*f*x^8 - (12*(-(b^4*c*e) + 4*a*b^2*c^2*e - 2*a^2*c^3*e + b^5*f + b^3*c*(c*d - 5*a*f) + a*b*c^2*(-3*c*d + 5*a*f))*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + 6*(-(b^3*c*e) + 2*a*b*c^2*e + b^4*f + b^2*c*(c*d - 3*a*f) + a*c^2*(-(c*d) + a*f))*Log[a + b*x^2 + c*x^4]/(24*c^5)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7 (d + ex^2 + fx^4)}{a + bx^2 + cx^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^7*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4), x]

[Out] IntegrateAlgebraic[(x^7*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4), x]

fricas [A] time = 2.56, size = 900, normalized size = 3.30

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a), x, algorithm="fricas")

```
[Out] [1/24*(3*(b^2*c^4 - 4*a*c^5)*f*x^8 + 4*((b^2*c^4 - 4*a*c^5)*e - (b^3*c^3 - 4*a*b*c^4)*f)*x^6 + 6*((b^2*c^4 - 4*a*c^5)*d - (b^3*c^3 - 4*a*b*c^4)*e + (b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*f)*x^4 - 12*((b^3*c^3 - 4*a*b*c^4)*d - (b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*e + (b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*f)*x^2 + 6*sqrt(b^2 - 4*a*c)*((b^3*c^2 - 3*a*b*c^3)*d - (b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*e + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*f)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + 6*((b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*d - (b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*e + (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*f)*log(c*x^4 + b*x^2 + a))/(b^2*c^5 - 4*a*c^6), 1/24*(3*(b^2*c^4 - 4*a*c^5)*f*x^8 + 4*((b^2*c^4 - 4*a*c^5)*e - (b^3*c^3 - 4*a*b*c^4)*f)*x^6 + 6*((b^2*c^4 - 4*a*c^5)*d - (b^3*c^3 - 4*a*b*c^4)*e + (b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*f)*x^4 - 12*((b^3*c^3 - 4*a*b*c^4)*d - (b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*e + (b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*f)*x^2 + 12*sqrt(-b^2 + 4*a*c)*((b^3*c^2 - 3*a*b*c^3)*d - (b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*e + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*f)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + 6*((b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*d - (b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*e + (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*f)*log(c*x^4 + b*x^2 + a))/(b^2*c^5 - 4*a*c^6)]
```

giac [A] time = 1.87, size = 306, normalized size = 1.12

$$\frac{3c^2f^8 - 4b^2f^8 + 4c^3e^8 + 6c^3d^8 + 6b^2f^8 - 6ac^2f^8 - 6bc^2e^8 - 12b^2d^8 - 12b^3f^8 + 24abcf^8 + 12b^2ce^8 - 12a^2e^8}{24c^8} \cdot \frac{(b^2d - ac^2d + b^2f - 3ab^2f + a^2c^2f - b^3ce + 2abc^2e)\log(cx^4 + bx^2 + a)}{4c^8} \cdot \frac{(b^2d - 3abc^2d + b^2f - 5ab^2f + 5a^2bc^2f - b^4ce + 4ab^2c^2e - 2a^2c^3e)\arctan\left(\frac{2cx^2 + b}{\sqrt{-b^2 + 4ac}}\right)}{2\sqrt{-b^2 + 4ac}c^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/24*(3*c^3*f*x^8 - 4*b*c^2*f*x^6 + 4*c^3*x^6*e + 6*c^3*d*x^4 + 6*b^2*c*f*x^4 - 6*a*c^2*f*x^4 - 6*b*c^2*x^4*e - 12*b*c^2*d*x^2 - 12*b^3*f*x^2 + 24*a*b*c*f*x^2 + 12*b^2*c*x^2*e - 12*a*c^2*x^2*e)/c^4 + 1/4*(b^2*c^2*d - a*c^3*d + b^4*f - 3*a*b^2*c*f + a^2*c^2*f - b^3*c*e + 2*a*b*c^2*e)*log(c*x^4 + b*x^2 + a)/c^5 - 1/2*(b^3*c^2*d - 3*a*b*c^3*d + b^5*f - 5*a*b^3*c*f + 5*a^2*b*c^2*f - b^4*c*e + 4*a*b^2*c^2*e - 2*a^2*c^3*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^5)
```

maple [B] time = 0.01, size = 622, normalized size = 2.28

$$\frac{c^3 f^8 - 4 b^2 f^8 + 4 c^3 e^8 + 6 c^3 d^8 + 6 b^2 f^8 - 6 a c^2 f^8 - 6 b c^2 e^8 - 12 b^2 d^8 - 12 b^3 f^8 + 24 a b c f^8 + 12 b^2 c e^8 - 12 a^2 e^8}{24 c^8} \cdot \frac{(b^2 d - a c^2 d + b^2 f - 3 a b^2 f + a^2 c^2 f - b^3 c e + 2 a b c^2 e) \log(c x^4 + b x^2 + a)}{4 c^8} \cdot \frac{(b^2 d - 3 a b c^2 d + b^2 f - 5 a b^2 f + 5 a^2 b c^2 f - b^4 c e + 4 a b^2 c^2 e - 2 a^2 c^3 e) \arctan\left(\frac{2 c x^2 + b}{\sqrt{-b^2 + 4 a c}}\right)}{2 \sqrt{-b^2 + 4 a c} c^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/c^3*x^2*a*b*f+1/2/c^4/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^4*e+1/8*f*x^8/c-1/2/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^3*d+1/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a^2*e-1/2/c^5/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^5*f-3/4/c^4*ln(c*x^4+b*x^2+a)*a*b^2*f+1/2/c^3*ln(c*x^4+b*x^2+a)*a*b*e+1/6/c*x^6*e+1/4/c*x^4*d+3/2/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*b*d+5/2/c^4/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*b^3*f-2/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*b^2*e-5/2/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a^2*b*f-1/2/c^2*x^2*a*e-1/6/c^2*x^6*b*f-1/4/c^2*x^4*a*f-1/2/c^4*x^2*b^3*f+1/2/c^3*x^2*b^2*e-1/2/c^2*x^2*b*d+1/4/c^3*x^4*b^2*f-1/4/c^2*x^4*b*e+1/4/c^3*ln(c*x^4+b*x^2+a)*b^2*d+1/4/c^3*ln(c*x^4+b*x^2+a)*a^2*f-1/4/c^2*ln(c*x^4+b*x^2+a)*a*d+1/4/c^5*ln(c*x^4+b*x^2+a)*b^4*f-1/4/c^4*ln(c*x^4+b*x^2+a)*b^3*e
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 1.60, size = 2972, normalized size = 10.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x)
```

```
[Out] x^6*(e/(6*c) - (b*f)/(6*c^2)) - x^4*((b*(e/c - (b*f)/c^2))/(4*c) - d/(4*c)
+ (a*f)/(4*c^2)) - x^2*((a*(e/c - (b*f)/c^2))/(2*c) - (b*((b*(e/c - (b*f)/c
^2))/c - d/c + (a*f)/c^2))/(2*c)) + (f*x^8)/(8*c) - (log(a + b*x^2 + c*x^4)
*(2*b^6*f + 8*a^2*c^4*d + 2*b^4*c^2*d - 8*a^3*c^3*f - 2*b^5*c*e + 26*a^2*b^
2*c^2*f - 14*a*b^4*c*f - 10*a*b^2*c^3*d + 12*a*b^3*c^2*e - 16*a^2*b*c^3*e))
/(2*(16*a*c^6 - 4*b^2*c^5)) + (atan((2*c^8*(4*a*c - b^2)*(x^2*(((4*a^2*c^
8*e - 6*b^3*c^7*d + 6*b^4*c^6*e - 6*b^5*c^5*f + 10*a*b*c^8*d - 16*a*b^2*c^7
*e + 22*a*b^3*c^6*f - 14*a^2*b*c^7*f)/c^8 - (4*b*c^2*(2*b^6*f + 8*a^2*c^4*d
+ 2*b^4*c^2*d - 8*a^3*c^3*f - 2*b^5*c*e + 26*a^2*b^2*c^2*f - 14*a*b^4*c*f
- 10*a*b^2*c^3*d + 12*a*b^3*c^2*e - 16*a^2*b*c^3*e))/(16*a*c^6 - 4*b^2*c^5)
)*(b^5*f - 2*a^2*c^3*e + b^3*c^2*d - b^4*c*e - 3*a*b*c^3*d - 5*a*b^3*c*f +
4*a*b^2*c^2*e + 5*a^2*b*c^2*f))/(8*c^5*(4*a*c - b^2)^(1/2)) - (b*(b^5*f - 2
*a^2*c^3*e + b^3*c^2*d - b^4*c*e - 3*a*b*c^3*d - 5*a*b^3*c*f + 4*a*b^2*c^2*
e + 5*a^2*b*c^2*f)*(2*b^6*f + 8*a^2*c^4*d + 2*b^4*c^2*d - 8*a^3*c^3*f - 2*b
^5*c*e + 26*a^2*b^2*c^2*f - 14*a*b^4*c*f - 10*a*b^2*c^3*d + 12*a*b^3*c^2*e
- 16*a^2*b*c^3*e))/(2*c^3*(4*a*c - b^2)^(1/2)*(16*a*c^6 - 4*b^2*c^5)))/a -
(b*(((4*a^2*c^8*e - 6*b^3*c^7*d + 6*b^4*c^6*e - 6*b^5*c^5*f + 10*a*b*c^8*d
- 16*a*b^2*c^7*e + 22*a*b^3*c^6*f - 14*a^2*b*c^7*f)/c^8 - (4*b*c^2*(2*b^6*
f + 8*a^2*c^4*d + 2*b^4*c^2*d - 8*a^3*c^3*f - 2*b^5*c*e + 26*a^2*b^2*c^2*f
- 14*a*b^4*c*f - 10*a*b^2*c^3*d + 12*a*b^3*c^2*e - 16*a^2*b*c^3*e))/(16*a*c
^6 - 4*b^2*c^5))*(2*b^6*f + 8*a^2*c^4*d + 2*b^4*c^2*d - 8*a^3*c^3*f - 2*b^5
*c*e + 26*a^2*b^2*c^2*f - 14*a*b^4*c*f - 10*a*b^2*c^3*d + 12*a*b^3*c^2*e -
16*a^2*b*c^3*e))/(2*(16*a*c^6 - 4*b^2*c^5)) - (b^9*f^2 + b^5*c^4*d^2 + b^7*
c^2*e^2 - 3*a*b^3*c^5*d^2 + 2*a^2*b*c^6*d^2 - 5*a*b^5*c^3*e^2 - 2*a^3*b*c^5
*e^2 + 3*a^4*b*c^4*f^2 - 2*b^8*c*e*f + 7*a^2*b^3*c^4*e^2 + 16*a^2*b^5*c^2*f
^2 - 13*a^3*b^3*c^3*f^2 - 7*a*b^7*c*f^2 + a^3*c^6*d*e - 2*b^6*c^3*d*e - a^4
*c^5*e*f + 2*b^7*c^2*d*f + 8*a*b^4*c^4*d*e - 10*a*b^5*c^3*d*f - 5*a^3*b*c^5
*d*f + 12*a*b^6*c^2*e*f - 8*a^2*b^2*c^5*d*e + 14*a^2*b^3*c^4*d*f - 22*a^2*b
^4*c^3*e*f + 12*a^3*b^2*c^4*e*f)/c^8 + (b*(b^5*f - 2*a^2*c^3*e + b^3*c^2*d
- b^4*c*e - 3*a*b*c^3*d - 5*a*b^3*c*f + 4*a*b^2*c^2*e + 5*a^2*b*c^2*f))^2)/
(2*c^8*(4*a*c - b^2)))/(2*a*(4*a*c - b^2)^(1/2)) - (((8*a^3*c^7*f - 8*a^2
*c^8*d - 24*a^2*b^2*c^6*f + 8*a*b^2*c^7*d - 8*a*b^3*c^6*e + 16*a^2*b*c^7*e
+ 8*a*b^4*c^5*f)/c^8 + (8*a*c^2*(2*b^6*f + 8*a^2*c^4*d + 2*b^4*c^2*d - 8*a^
3*c^3*f - 2*b^5*c*e + 26*a^2*b^2*c^2*f - 14*a*b^4*c*f - 10*a*b^2*c^3*d + 12
*a*b^3*c^2*e - 16*a^2*b*c^3*e))/(16*a*c^6 - 4*b^2*c^5))*(b^5*f - 2*a^2*c^3*
e + b^3*c^2*d - b^4*c*e - 3*a*b*c^3*d - 5*a*b^3*c*f + 4*a*b^2*c^2*e + 5*a^2
*b*c^2*f))/(8*c^5*(4*a*c - b^2)^(1/2)) + (a*(b^5*f - 2*a^2*c^3*e + b^3*c^2*
d - b^4*c*e - 3*a*b*c^3*d - 5*a*b^3*c*f + 4*a*b^2*c^2*e + 5*a^2*b*c^2*f)*(2
*b^6*f + 8*a^2*c^4*d + 2*b^4*c^2*d - 8*a^3*c^3*f - 2*b^5*c*e + 26*a^2*b^2*c
^2*f - 14*a*b^4*c*f - 10*a*b^2*c^3*d + 12*a*b^3*c^2*e - 16*a^2*b*c^3*e))/(c
^3*(4*a*c - b^2)^(1/2)*(16*a*c^6 - 4*b^2*c^5)))/a + (b*((a*b^8*f^2 + a^3*c^
6*d^2 + a^5*c^4*f^2 + a*b^4*c^4*d^2 + a*b^6*c^2*e^2 - 6*a^2*b^6*c*f^2 - 2*a
^2*b^2*c^5*d^2 - 4*a^2*b^4*c^3*e^2 + 4*a^3*b^2*c^4*e^2 + 11*a^3*b^4*c^2*f^2
- 6*a^4*b^2*c^3*f^2 - 2*a^4*c^5*d*f - 2*a*b^5*c^3*d*e - 4*a^3*b*c^5*d*e +
2*a*b^6*c^2*d*f + 4*a^4*b*c^4*e*f + 6*a^2*b^3*c^4*d*e - 8*a^2*b^4*c^3*d*f +
```

$$\frac{8a^3b^2c^4d^2f + 10a^2b^5c^2e^2f - 14a^3b^3c^3e^2f - 2ab^7c^2e^2f}{c^8} + \left(\frac{(8a^3c^7f - 8a^2c^8d - 24a^2b^2c^6f + 8ab^2c^7d - 8ab^3c^6e + 16a^2b^2c^7e + 8ab^4c^5f)}{c^8} + (8a^2c^2(2b^6f + 8a^2c^4d + 2b^4c^2d - 8a^3c^3f - 2b^5c^2e + 26a^2b^2c^2f - 14ab^4c^2f - 10ab^2c^3d + 12ab^3c^2e - 16a^2b^2c^3e))}{(16a^2c^6 - 4b^2c^5)} \right) \cdot \frac{(2b^6f + 8a^2c^4d + 2b^4c^2d - 8a^3c^3f - 2b^5c^2e + 26a^2b^2c^2f - 14ab^4c^2f - 10ab^2c^3d + 12ab^3c^2e - 16a^2b^2c^3e)}{(2(16a^2c^6 - 4b^2c^5)) - (a(b^5f - 2a^2c^3e + b^3c^2d - b^4c^2e - 3ab^2c^3d - 5ab^3c^2f + 4ab^2c^2e + 5a^2b^2c^2f))^2} / (c^8(4ac - b^2)) / (2a(4ac - b^2)^{1/2}) / (b^{10}f^2 + 4a^4c^6e^2 + b^6c^4d^2 + b^8c^2e^2 - 6ab^4c^5d^2 - 8ab^6c^3e^2 - 2b^9c^2e^2f + 9a^2b^2c^6d^2 + 20a^2b^4c^4e^2 - 16a^3b^2c^5e^2 + 35a^2b^6c^2f^2 - 50a^3b^4c^3f^2 + 25a^4b^2c^4f^2 - 10ab^8c^2f^2 - 2b^7c^3d^2e + 2b^8c^2d^2f + 14ab^5c^4d^2e + 12a^3b^2c^6d^2e - 16ab^6c^3d^2f + 18ab^7c^2e^2f - 20a^4b^2c^5e^2f - 28a^2b^3c^5d^2e + 40a^2b^4c^4d^2f - 30a^3b^2c^5d^2f - 54a^2b^5c^3e^2f + 60a^3b^3c^4e^2f) \cdot (b^5f - 2a^2c^3e + b^3c^2d - b^4c^2e - 3ab^2c^3d - 5ab^3c^2f) / (2c^5(4ac - b^2)^{1/2})$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**7*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)

[Out] Timed out

$$3.46 \quad \int \frac{x^5(d+ex^2+fx^4)}{a+bx^2+cx^4} dx$$

Optimal. Leaf size=203

$$\frac{x^2(-c(af+be)+b^2f+c^2d)}{2c^3} + \frac{\log(a+bx^2+cx^4)(-bc(cd-2af)-ac^2e+b^3(-f)+b^2ce)}{4c^4} + \frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^4\sqrt{b^2-4ac}}$$

Rubi [A] time = 0.42, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1663, 1628, 634, 618, 206, 628}

$$\frac{x^2(-c(af+be)+b^2f+c^2d)}{2c^3} + \frac{\log(a+bx^2+cx^4)(-bc(cd-2af)-ac^2e+b^2ce+b^3(-f))}{4c^4} + \frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(-b^2c(cd-4af)-3abc^2e+2ac^2(cd-af)+b^3ce+b^4(-f))}{2c^4\sqrt{b^2-4ac}} + \frac{x^4(ce-bf)}{4c^2} + \frac{fx^6}{6c}$$

Antiderivative was successfully verified.

[In] Int[(x^5*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]

[Out] ((c^2*d + b^2*f - c*(b*e + a*f))*x^2)/(2*c^3) + ((c*e - b*f)*x^4)/(4*c^2) + (f*x^6)/(6*c) + ((b^3*c*e - 3*a*b*c^2*e - b^4*f - b^2*c*(c*d - 4*a*f) + 2*a*c^2*(c*d - a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^4*Sqrt[b^2 - 4*a*c]) + ((b^2*c*e - a*c^2*e - b^3*f - b*c*(c*d - 2*a*f))*Log[a + b*x^2 + c*x^4])/(4*c^4)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1628

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 1663

Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^p, x], x], x]

p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]

Rubi steps

$$\int \frac{x^5 (d + ex^2 + fx^4)}{a + bx^2 + cx^4} dx = \frac{1}{2} \text{Subst} \left(\int \frac{x^2 (d + ex + fx^2)}{a + bx + cx^2} dx, x, x^2 \right)$$

$$= \frac{1}{2} \text{Subst} \left(\int \left(\frac{c^2d + b^2f - c(be + af)}{c^3} + \frac{(ce - bf)x}{c^2} + \frac{fx^2}{c} - \frac{a(c^2d + b^2f - c(be + af))}{c^3} \right) dx, x, x^2 \right)$$

$$= \frac{(c^2d + b^2f - c(be + af))x^2}{2c^3} + \frac{(ce - bf)x^4}{4c^2} + \frac{fx^6}{6c} - \frac{\text{Subst} \left(\int \frac{a(c^2d + b^2f - c(be + af)) - (b^2ce - a^2)}{a + bx + cx^2} dx, x, x^2 \right)}{2c^3}$$

$$= \frac{(c^2d + b^2f - c(be + af))x^2}{2c^3} + \frac{(ce - bf)x^4}{4c^2} + \frac{fx^6}{6c} + \frac{(b^2ce - ac^2e - b^3f - bc(cd - 2af))}{4c^4}$$

$$= \frac{(c^2d + b^2f - c(be + af))x^2}{2c^3} + \frac{(ce - bf)x^4}{4c^2} + \frac{fx^6}{6c} + \frac{(b^2ce - ac^2e - b^3f - bc(cd - 2af))}{4c^4}$$

$$= \frac{(c^2d + b^2f - c(be + af))x^2}{2c^3} + \frac{(ce - bf)x^4}{4c^2} + \frac{fx^6}{6c} + \frac{(b^3ce - 3abc^2e - b^4f - b^2c(cd - 2af))}{2c^4}$$

Mathematica [A] time = 0.14, size = 193, normalized size = 0.95

$$\frac{6cx^2(-c(af + be) + b^2f + c^2d) - 3 \log(a + bx^2 + cx^4)(bc(cd - 2af) + ac^2e + b^3f - b^2ce) + \frac{6 \tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)(b^2c(cd-4af)+3abc^2e+2ac^2(af-cd)+b^4f-b^3ce)}{\sqrt{4ac-b^2}} + 3c^2x^4(ce - bf) + 2c^3fx^6}{12c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4), x]
[Out] (6*c*(c^2*d + b^2*f - c*(b*e + a*f))*x^2 + 3*c^2*(c*e - b*f)*x^4 + 2*c^3*f*x^6 + (6*(-(b^3*c*e) + 3*a*b*c^2*e + b^4*f + b^2*c*(c*d - 4*a*f) + 2*a*c^2*(-(c*d) + a*f))*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] - 3*(-(b^2*c*e) + a*c^2*e + b^3*f + b*c*(c*d - 2*a*f))*Log[a + b*x^2 + c*x^4])/(12*c^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5 (d + ex^2 + fx^4)}{a + bx^2 + cx^4} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^5*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4), x]
[Out] IntegrateAlgebraic[(x^5*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4), x]
```

fricas [A] time = 2.39, size = 677, normalized size = 3.33



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a), x, algorithm="fricas")
```

```
[Out] [1/12*(2*(b^2*c^3 - 4*a*c^4)*f*x^6 + 3*((b^2*c^3 - 4*a*c^4)*e - (b^3*c^2 - 4*a*b*c^3)*f)*x^4 + 6*((b^2*c^3 - 4*a*c^4)*d - (b^3*c^2 - 4*a*b*c^3)*e + (b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*f)*x^2 + 3*sqrt(b^2 - 4*a*c)*((b^2*c^2 - 2*a*c^3)*d - (b^3*c - 3*a*b*c^2)*e + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*f)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - 3*((b^3*c^2 - 4*a*b*c^3)*d - (b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*e + (b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*f)*log(c*x^4 + b*x^2 + a)/(b^2*c^4 - 4*a*c^5), 1/12*(2*(b^2*c^3 - 4*a*c^4)*f*x^6 + 3*((b^2*c^3 - 4*a*c^4)*e - (b^3*c^2 - 4*a*b*c^3)*f)*x^4 + 6*((b^2*c^3 - 4*a*c^4)*d - (b^3*c^2 - 4*a*b*c^3)*e + (b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*f)*x^2 - 6*sqrt(-b^2 + 4*a*c)*((b^2*c^2 - 2*a*c^3)*d - (b^3*c - 3*a*b*c^2)*e + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*f)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - 3*((b^3*c^2 - 4*a*b*c^3)*d - (b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*e + (b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*f)*log(c*x^4 + b*x^2 + a)/(b^2*c^4 - 4*a*c^5)]
```

giac [A] time = 2.00, size = 214, normalized size = 1.05

$$\frac{2c^2fx^6 - 3bcfx^4 + 3c^2x^4e + 6c^2dx^2 + 6b^2fx^2 - 6acfx^2 - 6bcx^2e}{12c^3} - \frac{(bc^2d + b^3f - 2abc f - b^2ce + ac^2e)\log(cx^4 + bx^2 + a)}{4c^4} + \frac{(b^2c^2d - 2ac^3d + b^4f - 4ab^2cf + 2a^2c^2f - b^3ce + 3abc^2e)\arctan\left(\frac{2cx^2 + b}{\sqrt{-b^2 + 4ac}}\right)}{2\sqrt{-b^2 + 4ac}c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/12*(2*c^2*f*x^6 - 3*b*c*f*x^4 + 3*c^2*x^4*e + 6*c^2*d*x^2 + 6*b^2*f*x^2 - 6*a*c*f*x^2 - 6*b*c*x^2*e)/c^3 - 1/4*(b*c^2*d + b^3*f - 2*a*b*c*f - b^2*c*e + a*c^2*e)*log(c*x^4 + b*x^2 + a)/c^4 + 1/2*(b^2*c^2*d - 2*a*c^3*d + b^4*f - 4*a*b^2*c*f + 2*a^2*c^2*f - b^3*c*e + 3*a*b*c^2*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^4)
```

maple [B] time = 0.01, size = 474, normalized size = 2.33

$$\frac{fx^6}{6c} - \frac{bfx^4}{4c^2} + \frac{cx^4}{2c} - \frac{e^2f\arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}} + \frac{2a^2f\arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}} + \frac{3abc\arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right)}{2\sqrt{-b^2+4ac}} + \frac{ad\arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}} + \frac{afx^2}{2c^2} + \frac{b^4f\arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right)}{2\sqrt{-b^2+4ac}} + \frac{b^2c\arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right)}{2\sqrt{-b^2+4ac}} + \frac{b^2d\arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right)}{2\sqrt{-b^2+4ac}} + \frac{b^2fx^2}{2c^2} - \frac{bx^2}{2c^2} + \frac{dx^2}{2c} - \frac{abf\ln(cx^4+bx^2+a)}{2c^2} - \frac{a\ln(cx^4+bx^2+a)}{4c^2} + \frac{b^3f\ln(cx^4+bx^2+a)}{4c^2} + \frac{b^2c\ln(cx^4+bx^2+a)}{4c^2} + \frac{b^2d\ln(cx^4+bx^2+a)}{4c^2} + \frac{b^2fx^2\ln(cx^4+bx^2+a)}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/6*f*x^6/c-1/4/c^2*x^4*b*f+1/4/c*x^4*e-1/2/c^2*x^2*a*f+1/2/c^3*x^2*b^2*f-1/2/c^2*x^2*b*e+1/2/c*x^2*d+1/2/c^3*ln(c*x^4+b*x^2+a)*a*b*f-1/4/c^2*ln(c*x^4+b*x^2+a)*a*e-1/4/c^4*ln(c*x^4+b*x^2+a)*b^3*f+1/4/c^3*ln(c*x^4+b*x^2+a)*b^2*e-1/4/c^2*ln(c*x^4+b*x^2+a)*b*d+1/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a^2*f-2/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*b^2*f+3/2/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*b*e-1/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*d+1/2/c^4/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^4*f-1/2/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^3*e+1/2/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^2*d
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 1.63, size = 2295, normalized size = 11.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((x^5*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4), x)$

[Out] $x^4*(e/(4*c) - (b*f)/(4*c^2)) - x^2*((b*(e/c - (b*f)/c^2))/(2*c) - d/(2*c) + (a*f)/(2*c^2)) + (\log(a + b*x^2 + c*x^4)*(2*b^5*f - 8*a^2*c^3*e + 2*b^3*c^2*d - 2*b^4*c*e - 8*a*b*c^3*d - 12*a*b^3*c*f + 10*a*b^2*c^2*e + 16*a^2*b*c^2*f))/(2*(16*a*c^5 - 4*b^2*c^4)) + (f*x^6)/(6*c) + (\text{atan}((2*c^6*(4*a*c - b^2)*(x^2*(((((6*b^2*c^6*d + 4*a^2*c^6*f - 6*b^3*c^5*e + 6*b^4*c^4*f - 4*a*c^7*d + 10*a*b*c^6*e - 16*a*b^2*c^5*f)/c^6 + (4*b*c^2*(2*b^5*f - 8*a^2*c^3*e + 2*b^3*c^2*d - 2*b^4*c*e - 8*a*b*c^3*d - 12*a*b^3*c*f + 10*a*b^2*c^2*e + 16*a^2*b*c^2*f))/(16*a*c^5 - 4*b^2*c^4))*(b^4*f + b^2*c^2*d + 2*a^2*c^2*f - 2*a*c^3*d - b^3*c*e + 3*a*b*c^2*e - 4*a*b^2*c*f)))/(8*c^4*(4*a*c - b^2)^(1/2)) + (b*(b^4*f + b^2*c^2*d + 2*a^2*c^2*f - 2*a*c^3*d - b^3*c*e + 3*a*b*c^2*e - 4*a*b^2*c*f)*(2*b^5*f - 8*a^2*c^3*e + 2*b^3*c^2*d - 2*b^4*c*e - 8*a*b*c^3*d - 12*a*b^3*c*f + 10*a*b^2*c^2*e + 16*a^2*b*c^2*f))/(2*c^2*(4*a*c - b^2)^(1/2)*(16*a*c^5 - 4*b^2*c^4)))/a - (b*((b^7*f^2 + b^3*c^4*d^2 + b^5*c^2*e^2 - 3*a*b^3*c^3*e^2 + 2*a^2*b*c^4*e^2 - 2*a^3*b*c^3*f^2 - 2*b^6*c*e*f + 7*a^2*b^3*c^2*f^2 - a*b*c^5*d^2 - 5*a*b^5*c*f^2 - a^2*c^5*d*e - 2*b^4*c^3*d*e + a^3*c^4*e*f + 2*b^5*c^2*d*f + 4*a*b^2*c^4*d*e - 6*a*b^3*c^3*d*f + 3*a^2*b*c^4*d*f + 8*a*b^4*c^2*e*f - 8*a^2*b^2*c^3*e*f)/c^6 + (((6*b^2*c^6*d + 4*a^2*c^6*f - 6*b^3*c^5*e + 6*b^4*c^4*f - 4*a*c^7*d + 10*a*b*c^6*e - 16*a*b^2*c^5*f)/c^6 + (4*b*c^2*(2*b^5*f - 8*a^2*c^3*e + 2*b^3*c^2*d - 2*b^4*c*e - 8*a*b*c^3*d - 12*a*b^3*c*f + 10*a*b^2*c^2*e + 16*a^2*b*c^2*f))/(16*a*c^5 - 4*b^2*c^4))*(2*b^5*f - 8*a^2*c^3*e + 2*b^3*c^2*d - 2*b^4*c*e - 8*a*b*c^3*d - 12*a*b^3*c*f + 10*a*b^2*c^2*e + 16*a^2*b*c^2*f))/(2*(16*a*c^5 - 4*b^2*c^4)) - (b*(b^4*f + b^2*c^2*d + 2*a^2*c^2*f - 2*a*c^3*d - b^3*c*e + 3*a*b*c^2*e - 4*a*b^2*c*f)^2)/(2*c^6*(4*a*c - b^2)))/((2*a*(4*a*c - b^2)^(1/2))) + (((8*a^2*c^6*e + 8*a*b*c^6*d - 8*a*b^2*c^5*e + 8*a*b^3*c^4*f - 16*a^2*b*c^5*f)/c^6 + (8*a*c^2*(2*b^5*f - 8*a^2*c^3*e + 2*b^3*c^2*d - 2*b^4*c*e - 8*a*b*c^3*d - 12*a*b^3*c*f + 10*a*b^2*c^2*e + 16*a^2*b*c^2*f))/(16*a*c^5 - 4*b^2*c^4))*(b^4*f + b^2*c^2*d + 2*a^2*c^2*f - 2*a*c^3*d - b^3*c*e + 3*a*b*c^2*e - 4*a*b^2*c*f))/(8*c^4*(4*a*c - b^2)^(1/2)) + (a*(b^4*f + b^2*c^2*d + 2*a^2*c^2*f - 2*a*c^3*d - b^3*c*e + 3*a*b*c^2*e - 4*a*b^2*c*f)*(2*b^5*f - 8*a^2*c^3*e + 2*b^3*c^2*d - 2*b^4*c*e - 8*a*b*c^3*d - 12*a*b^3*c*f + 10*a*b^2*c^2*e + 16*a^2*b*c^2*f))/(c^2*(4*a*c - b^2)^(1/2)*(16*a*c^5 - 4*b^2*c^4)))/a - (b*((a*b^6*f^2 + a^3*c^4*e^2 + a*b^2*c^4*d^2 + a*b^4*c^2*e^2 - 4*a^2*b^4*c*f^2 - 2*a^2*b^2*c^3*e^2 + 4*a^3*b^2*c^2*f^2 - 2*a*b^3*c^3*d*e + 2*a^2*b*c^4*d*e + 2*a*b^4*c^2*d*f - 4*a^3*b*c^3*e*f - 4*a^2*b^2*c^3*d*f + 6*a^2*b^3*c^2*e*f - 2*a*b^5*c*e*f)/c^6 + (((8*a^2*c^6*e + 8*a*b*c^6*d - 8*a*b^2*c^5*e + 8*a*b^3*c^4*f - 16*a^2*b*c^5*f)/c^6 + (8*a*c^2*(2*b^5*f - 8*a^2*c^3*e + 2*b^3*c^2*d - 2*b^4*c*e - 8*a*b*c^3*d - 12*a*b^3*c*f + 10*a*b^2*c^2*e + 16*a^2*b*c^2*f))/(16*a*c^5 - 4*b^2*c^4))*(2*b^5*f - 8*a^2*c^3*e + 2*b^3*c^2*d - 2*b^4*c*e - 8*a*b*c^3*d - 12*a*b^3*c*f + 10*a*b^2*c^2*e + 16*a^2*b*c^2*f))/(2*(16*a*c^5 - 4*b^2*c^4)) - (a*(b^4*f + b^2*c^2*d + 2*a^2*c^2*f - 2*a*c^3*d - b^3*c*e + 3*a*b*c^2*e - 4*a*b^2*c*f)^2)/(c^6*(4*a*c - b^2)))/((2*a*(4*a*c - b^2)^(1/2)))/((b^8*f^2 + 4*a^2*c^6*d^2 + b^4*c^4*d^2 + 4*a^4*c^4*f^2 + b^6*c^2*e^2 - 4*a*b^2*c^5*d^2 - 6*a*b^4*c^3*e^2 - 2*b^7*c*e*f + 9*a^2*b^2*c^4*e^2 + 20*a^2*b^4*c^2*f^2 - 16*a^3*b^2*c^3*f^2 - 8*a*b^6*c*f^2 - 8*a^3*c^5*d*f - 2*b^5*c^3*d*e + 2*b^6*c^2*d*f + 10*a*b^3*c^4*d*e - 12*a^2*b*c^5*d*e - 12*a*b^4*c^3*d*f + 14*a*b^5*c^2*e*f + 12*a^3*b*c^4*e*f + 20*a^2*b^2*c^4*d*f - 28*a^2*b^3*c^3*e*f))*(b^4*f + b^2*c^2*d + 2*a^2*c^2*f - 2*a*c^3*d - b^3*c*e + 3*a*b*c^2*e - 4*a*b^2*c*f))/(2*c^4*(4*a*c - b^2)^(1/2))$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate(x**5*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

$$3.47 \quad \int \frac{x^3(d+ex^2+fx^4)}{a+bx^2+cx^4} dx$$

Optimal. Leaf size=144

$$\frac{\log(a+bx^2+cx^4)(-c(af+be)+b^2f+c^2d)}{4c^3} - \frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce)}{2c^3\sqrt{b^2-4ac}} + \frac{x^2(c^2e-bf)}{2c^2} + \frac{fx^4}{4c}$$

Rubi [A] time = 0.27, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1663, 1628, 634, 618, 206, 628}

$$\frac{\log(a+bx^2+cx^4)(-c(af+be)+b^2f+c^2d)}{4c^3} - \frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(-bc(cd-3af)-2ac^2e+b^2ce+b^3(-f))}{2c^3\sqrt{b^2-4ac}} + \frac{x^2(c^2e-bf)}{2c^2} + \frac{fx^4}{4c}$$

Antiderivative was successfully verified.

[In] Int[(x^3*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4), x]

[Out] ((c*e - b*f)*x^2)/(2*c^2) + (f*x^4)/(4*c) - ((b^2*c*e - 2*a*c^2*e - b^3*f - b*c*(c*d - 3*a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^3*Sqrt[b^2 - 4*a*c]) + ((c^2*d + b^2*f - c*(b*e + a*f))*Log[a + b*x^2 + c*x^4])/(4*c^3)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1628

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 1663

Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[

(m - 1)/2]

Rubi steps

$$\begin{aligned}
\int \frac{x^3(d + ex^2 + fx^4)}{a + bx^2 + cx^4} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x(d + ex + fx^2)}{a + bx + cx^2} dx, x, x^2 \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \left(\frac{ce - bf}{c^2} + \frac{fx}{c} - \frac{a(ce - bf) - (c^2d - bce + b^2f - acf)x}{c^2(a + bx + cx^2)} \right) dx, x, x^2 \right) \\
&= \frac{(ce - bf)x^2}{2c^2} + \frac{fx^4}{4c} - \frac{\text{Subst} \left(\int \frac{a(ce - bf) - (c^2d - bce + b^2f - acf)x}{a + bx + cx^2} dx, x, x^2 \right)}{2c^2} \\
&= \frac{(ce - bf)x^2}{2c^2} + \frac{fx^4}{4c} - \frac{(-c^2d + bce - b^2f + acf) \text{Subst} \left(\int \frac{b + 2cx}{a + bx + cx^2} dx, x, x^2 \right)}{4c^3} + \frac{(b^2ce - 2ac^2e)}{4c^3} \\
&= \frac{(ce - bf)x^2}{2c^2} + \frac{fx^4}{4c} + \frac{(c^2d - bce + b^2f - acf) \log(a + bx^2 + cx^4)}{4c^3} - \frac{(b^2ce - 2ac^2e)}{4c^3} \\
&= \frac{(ce - bf)x^2}{2c^2} + \frac{fx^4}{4c} - \frac{(b^2ce - 2ac^2e - b^3f - bc(cd - 3af)) \tanh^{-1} \left(\frac{b + 2cx}{\sqrt{b^2 - 4ac}} \right)}{2c^3\sqrt{b^2 - 4ac}} + \frac{(c^2d - bce + b^2f - acf) \log(a + bx^2 + cx^4)}{4c^3}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 136, normalized size = 0.94

$$\frac{\log(a + bx^2 + cx^4)(-c(af + be) + b^2f + c^2d) - \frac{2 \tan^{-1} \left(\frac{b + 2cx}{\sqrt{4ac - b^2}} \right) (bc(cd - 3af) + 2ac^2e + b^3f - b^2ce)}{\sqrt{4ac - b^2}} + 2cx^2(ce - bf) + c^2fx^4}{4c^3}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4), x]

[Out] (2*c*(c*e - b*f)*x^2 + c^2*f*x^4 - (2*(-(b^2*c*e) + 2*a*c^2*e + b^3*f + b*c*(c*d - 3*a*f))*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (c^2*d + b^2*f - c*(b*e + a*f))*Log[a + b*x^2 + c*x^4]/(4*c^3)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(d + ex^2 + fx^4)}{a + bx^2 + cx^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^3*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4), x]**[Out]** IntegrateAlgebraic[(x^3*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4), x]**fricas [A]** time = 1.37, size = 473, normalized size = 3.28

$$\frac{(b^2 - 4ac)^2 d^2 + 2((b^2 - 4ac)^2 - (b^2 - 4ac)f) d - (b^2 - 4ac)^2 (b^2 - 4ac) \log\left(\frac{b^2 + 2cx^2 + a}{\sqrt{b^2 - 4ac}}\right) - ((b^2 - 4ac)^2 - (b^2 - 4ac)f) \log\left(\frac{b^2 + 2cx^2 + a}{\sqrt{b^2 - 4ac}}\right) - ((b^2 - 4ac)^2 - (b^2 - 4ac)f) \log\left(\frac{b^2 + 2cx^2 + a}{\sqrt{b^2 - 4ac}}\right) - ((b^2 - 4ac)^2 - (b^2 - 4ac)f) \log\left(\frac{b^2 + 2cx^2 + a}{\sqrt{b^2 - 4ac}}\right)}{4(b^2 - 4ac)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a), x, algorithm="fricas")

[Out] [1/4*((b^2*c^2 - 4*a*c^3)*f*x^4 + 2*((b^2*c^2 - 4*a*c^3)*e - (b^3*c - 4*a*b*c^2)*f)*x^2 - (b*c^2*d - (b^2*c - 2*a*c^2)*e + (b^3 - 3*a*b*c)*f)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c)))]

$$- 4ac)) / (cx^4 + bx^2 + a) + ((b^2c^2 - 4ac^3)d - (b^3c - 4ab^2c^2)e + (b^4 - 5ab^2c + 4a^2c^2)f) \log(cx^4 + bx^2 + a) / (b^2c^3 - 4ac^4), 1/4 * ((b^2c^2 - 4ac^3)fx^4 + 2 * ((b^2c^2 - 4ac^3)e - (b^3c - 4ab^2c^2)f) * x^2 + 2 * (b^2cd - (b^2c - 2ac^2)e + (b^3 - 3ab^2c)f) * \sqrt{-b^2 + 4ac}) * \arctan(-(2cx^2 + b) * \sqrt{-b^2 + 4ac}) / (b^2 - 4ac)) + ((b^2c^2 - 4ac^3)d - (b^3c - 4ab^2c^2)e + (b^4 - 5ab^2c + 4a^2c^2)f) \log(cx^4 + bx^2 + a) / (b^2c^3 - 4ac^4]$$

giac [A] time = 1.99, size = 141, normalized size = 0.98

$$\frac{cfx^4 - 2bfx^2 + 2cx^2e}{4c^2} + \frac{(c^2d + b^2f - acf - bce) \log(cx^4 + bx^2 + a)}{4c^3} - \frac{(bc^2d + b^3f - 3abcf - b^2ce + 2ac^2e) \arctan\left(\frac{2cx^2 + b}{\sqrt{-b^2 + 4ac}}\right)}{2\sqrt{-b^2 + 4ac}c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] 1/4*(c*f*x^4 - 2*b*f*x^2 + 2*c*x^2*e)/c^2 + 1/4*(c^2*d + b^2*f - a*c*f - b*c*e)*log(c*x^4 + b*x^2 + a)/c^3 - 1/2*(b*c^2*d + b^3*f - 3*a*b*c*f - b^2*c*e + 2*a*c^2*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^3)

maple [B] time = 0.01, size = 321, normalized size = 2.23

$$\frac{fx^4}{4c} + \frac{3abf \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}c^2} - \frac{ae \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}c} - \frac{b^3f \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}c^3} + \frac{b^2e \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}c^2} - \frac{bd \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}c} - \frac{bfx^2}{2c^2} + \frac{ex^2}{2c} - \frac{af \ln(cx^4 + bx^2 + a)}{4c^2} + \frac{b^2f \ln(cx^4 + bx^2 + a)}{4c^3} - \frac{be \ln(cx^4 + bx^2 + a)}{4c^2} + \frac{d \ln(cx^4 + bx^2 + a)}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x)

[Out] 1/4*f*x^4/c - 1/2/c^2*x^2*b*f + 1/2/c*x^2*e - 1/4/c^2*ln(c*x^4+b*x^2+a)*a*f + 1/4/c^3*ln(c*x^4+b*x^2+a)*b^2*f - 1/4/c^2*ln(c*x^4+b*x^2+a)*b*e + 1/4/c*ln(c*x^4+b*x^2+a)*d + 3/2/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*b*f - 1/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*e - 1/2/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^3*f + 1/2/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^2*e - 1/2/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b*d

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 1.30, size = 1689, normalized size = 11.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x)

[Out] x^2*(e/(2*c) - (b*f)/(2*c^2)) + (f*x^4)/(4*c) - (log(a + b*x^2 + c*x^4)*(2*b^4*f + 2*b^2*c^2*d + 8*a^2*c^2*f - 8*a*c^3*d - 2*b^3*c*e + 8*a*b*c^2*e - 10*a*b^2*c*f))/(2*(16*a*c^4 - 4*b^2*c^3)) - (atan((2*c^4*(4*a*c - b^2)*(x^2*(((6*b^3*c^3*f - 6*b^2*c^4*e + 4*a*c^5*e + 6*b*c^5*d - 10*a*b*c^4*f)/c^4 + (4*b*c^2*(2*b^4*f + 2*b^2*c^2*d + 8*a^2*c^2*f - 8*a*c^3*d - 2*b^3*c*e + 8

$$\frac{a^2 b^2 c^2 e - 10 a^2 b^2 c^2 f}{(16 a^2 c^4 - 4 b^2 c^3)} (b^3 f + 2 a^2 c^2 e + b^2 c^2 d - b^2 c^2 e - 3 a^2 b^2 c^2 f) / (8 c^3 (4 a^2 c - b^2)^{1/2}) + (b^2 (b^3 f + 2 a^2 c^2 e + b^2 c^2 d - b^2 c^2 e - 3 a^2 b^2 c^2 f) (2 b^4 f + 2 b^2 c^2 d + 8 a^2 c^2 f - 8 a^2 c^3 d - 2 b^3 c^2 e + 8 a^2 b^2 c^2 e - 10 a^2 b^2 c^2 f)) / (2 c (4 a^2 c - b^2)^{1/2} (16 a^2 c^4 - 4 b^2 c^3)) / a - (b^2 ((b^5 f^2 + b^2 c^4 d^2 + b^3 c^2 e^2 + 2 a^2 b^2 c^2 f^2 + a^2 c^4 d^2 e - 2 b^4 c^2 e f - a^2 b^2 c^3 e^2 - 3 a^2 b^3 c^2 f^2 - 2 b^2 c^3 d^2 e - a^2 c^3 e^2 f + 2 b^3 c^2 d^2 f + 4 a^2 b^2 c^2 e^2 f - 3 a^2 b^2 c^3 d^2 f) / c^4 + (((6 b^3 c^3 f - 6 b^2 c^4 e + 4 a^2 c^5 e + 6 b^2 c^5 d - 10 a^2 b^2 c^4 f) / c^4 + (4 b^2 c^2 (2 b^4 f + 2 b^2 c^2 d + 8 a^2 c^2 f - 8 a^2 c^3 d - 2 b^3 c^2 e + 8 a^2 b^2 c^2 e - 10 a^2 b^2 c^2 f)) / (16 a^2 c^4 - 4 b^2 c^3)) (2 b^4 f + 2 b^2 c^2 d + 8 a^2 c^2 f - 8 a^2 c^3 d - 2 b^3 c^2 e + 8 a^2 b^2 c^2 e - 10 a^2 b^2 c^2 f)) / (2 (16 a^2 c^4 - 4 b^2 c^3)) - (b^2 (b^3 f + 2 a^2 c^2 e + b^2 c^2 d - b^2 c^2 e - 3 a^2 b^2 c^2 f)^2) / (2 c^4 (4 a^2 c - b^2))) / (2 a (4 a^2 c - b^2)^{1/2})) - (((8 a^2 c^4 f - 8 a^2 c^5 d + 8 a^2 b^2 c^4 e - 8 a^2 b^2 c^3 f) / c^4 - (8 a^2 c^2 (2 b^4 f + 2 b^2 c^2 d + 8 a^2 c^2 f - 8 a^2 c^3 d - 2 b^3 c^2 e + 8 a^2 b^2 c^2 e - 10 a^2 b^2 c^2 f)) / (16 a^2 c^4 - 4 b^2 c^3)) (b^3 f + 2 a^2 c^2 e + b^2 c^2 d - b^2 c^2 e - 3 a^2 b^2 c^2 f) / (8 c^3 (4 a^2 c - b^2)^{1/2}) - (a^2 (b^3 f + 2 a^2 c^2 e + b^2 c^2 d - b^2 c^2 e - 3 a^2 b^2 c^2 f) (2 b^4 f + 2 b^2 c^2 d + 8 a^2 c^2 f - 8 a^2 c^3 d - 2 b^3 c^2 e + 8 a^2 b^2 c^2 e - 10 a^2 b^2 c^2 f)) / (c (4 a^2 c - b^2)^{1/2} (16 a^2 c^4 - 4 b^2 c^3))) / a + (b^2 (((8 a^2 c^4 f - 8 a^2 c^5 d + 8 a^2 b^2 c^4 e - 8 a^2 b^2 c^3 f) / c^4 - (8 a^2 c^2 (2 b^4 f + 2 b^2 c^2 d + 8 a^2 c^2 f - 8 a^2 c^3 d - 2 b^3 c^2 e + 8 a^2 b^2 c^2 e - 10 a^2 b^2 c^2 f)) / (16 a^2 c^4 - 4 b^2 c^3)) (2 b^4 f + 2 b^2 c^2 d + 8 a^2 c^2 f - 8 a^2 c^3 d - 2 b^3 c^2 e + 8 a^2 b^2 c^2 e - 10 a^2 b^2 c^2 f)) / (2 (16 a^2 c^4 - 4 b^2 c^3)) - (a^2 c^4 d^2 + a^2 b^4 f^2 + a^3 c^2 f^2 + a^2 b^2 c^2 e^2 - 2 a^2 b^2 c^2 f^2 - 2 a^2 c^3 d^2 f + 2 a^2 b^2 c^2 d^2 f + 2 a^2 b^2 c^2 e^2 f - 2 a^2 b^2 c^3 d^2 e - 2 a^2 b^3 c^2 e^2 f) / c^4 + (a^2 (b^3 f + 2 a^2 c^2 e + b^2 c^2 d - b^2 c^2 e - 3 a^2 b^2 c^2 f)^2) / (c^4 (4 a^2 c - b^2))) / (2 a (4 a^2 c - b^2)^{1/2})) / (b^6 f^2 + 4 a^2 c^4 e^2 + b^2 c^4 d^2 + b^4 c^2 e^2 - 4 a^2 b^2 c^3 e^2 - 2 b^5 c^2 e f + 9 a^2 b^2 c^2 f^2 - 6 a^2 b^4 c^2 f^2 - 2 b^3 c^3 d^2 e + 2 b^4 c^2 d^2 f - 6 a^2 b^2 c^3 d^2 f + 10 a^2 b^3 c^2 e^2 f - 12 a^2 b^2 c^3 e^2 f + 4 a^2 b^2 c^4 d^2 e)) (b^3 f + 2 a^2 c^2 e + b^2 c^2 d - b^2 c^2 e - 3 a^2 b^2 c^2 f) / (2 c^3 (4 a^2 c - b^2)^{1/2})$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a), x)

[Out] Timed out

$$3.48 \quad \int \frac{x(d+ex^2+fx^4)}{a+bx^2+cx^4} dx$$

Optimal. Leaf size=103

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(-2acf+b^2f-bce+2c^2d)}{2c^2\sqrt{b^2-4ac}} + \frac{(ce-bf)\log(a+bx^2+cx^4)}{4c^2} + \frac{fx^2}{2c}$$

Rubi [A] time = 0.18, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1663, 1657, 634, 618, 206, 628}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(-2acf+b^2f-bce+2c^2d)}{2c^2\sqrt{b^2-4ac}} + \frac{(ce-bf)\log(a+bx^2+cx^4)}{4c^2} + \frac{fx^2}{2c}$$

Antiderivative was successfully verified.

[In] Int[(x*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4), x]

[Out] (f*x^2)/(2*c) - ((2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^2*Sqrt[b^2 - 4*a*c]) + ((c*e - b*f)*Log[a + b*x^2 + c*x^4])/(4*c^2)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1657

Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 1663

Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[1/2, Subst[Int[x^((m-1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[

(m - 1)/2]

Rubi steps

$$\begin{aligned}
\int \frac{x(d + ex^2 + fx^4)}{a + bx^2 + cx^4} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{d + ex + fx^2}{a + bx + cx^2} dx, x, x^2 \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \left(\frac{f}{c} + \frac{cd - af + (ce - bf)x}{c(a + bx + cx^2)} \right) dx, x, x^2 \right) \\
&= \frac{fx^2}{2c} + \frac{\text{Subst} \left(\int \frac{cd - af + (ce - bf)x}{a + bx + cx^2} dx, x, x^2 \right)}{2c} \\
&= \frac{fx^2}{2c} + \frac{(ce - bf) \text{Subst} \left(\int \frac{b + 2cx}{a + bx + cx^2} dx, x, x^2 \right)}{4c^2} + \frac{(2c^2d - bce + b^2f - 2acf) \text{Subst} \left(\int \frac{1}{b^2 - 4ac - x^2} dx, x, x^2 \right)}{4c^2} \\
&= \frac{fx^2}{2c} + \frac{(ce - bf) \log(a + bx^2 + cx^4)}{4c^2} - \frac{(2c^2d - bce + b^2f - 2acf) \text{Subst} \left(\int \frac{1}{b^2 - 4ac - x^2} dx, x, x^2 \right)}{2c^2} \\
&= \frac{fx^2}{2c} - \frac{(2c^2d - bce + b^2f - 2acf) \tanh^{-1} \left(\frac{b + 2cx^2}{\sqrt{b^2 - 4ac}} \right)}{2c^2 \sqrt{b^2 - 4ac}} + \frac{(ce - bf) \log(a + bx^2 + cx^4)}{4c^2}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 100, normalized size = 0.97

$$\frac{2 \tan^{-1} \left(\frac{b + 2cx^2}{\sqrt{4ac - b^2}} \right) (-c(2af + be) + b^2f + 2c^2d)}{\sqrt{4ac - b^2}} + \frac{(ce - bf) \log(a + bx^2 + cx^4) + 2cfx^2}{4c^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4), x]

[Out] (2*c*f*x^2 + (2*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (c*e - b*f)*Log[a + b*x^2 + c*x^4])/(4*c^2)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(d + ex^2 + fx^4)}{a + bx^2 + cx^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4), x]

[Out] IntegrateAlgebraic[(x*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4), x]

fricas [A] time = 1.13, size = 318, normalized size = 3.09

$$\frac{2(b^2c - 4ac^2)f^2 - (2c^2d - bce + (b^2 - 2ac)f)\sqrt{b^2 - 4ac} \log\left(\frac{2c^2x^2 + 2bx + a + \sqrt{b^2 - 4ac}}{c^2x^2 + a}\right) + ((b^2c - 4ac^2)e - (b^3 - 4abc)f)\log(cx^2 + a) - 2(b^2c - 4ac^2)f^2 - 2(2c^2d - bce + (b^2 - 2ac)f)\sqrt{b^2 - 4ac} \arctan\left(\frac{2cx + a}{\sqrt{b^2 - 4ac}}\right) + ((b^2c - 4ac^2)e - (b^3 - 4abc)f)\log(cx^2 + bx^2 + a)}{4(b^2c - 4ac^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a), x, algorithm="fricas")

[Out] [1/4*(2*(b^2*c - 4*a*c^2)*f*x^2 - (2*c^2*d - b*c*e + (b^2 - 2*a*c)*f)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + ((b^2*c - 4*a*c^2)*e - (b^3 - 4*a*b*c)

*f)*log(c*x^4 + b*x^2 + a)/(b^2*c^2 - 4*a*c^3), 1/4*(2*(b^2*c - 4*a*c^2)*f*x^2 - 2*(2*c^2*d - b*c*e + (b^2 - 2*a*c)*f)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + ((b^2*c - 4*a*c^2)*e - (b^3 - 4*a*b*c)*f)*log(c*x^4 + b*x^2 + a)/(b^2*c^2 - 4*a*c^3)]

giac [A] time = 1.78, size = 99, normalized size = 0.96

$$\frac{fx^2}{2c} - \frac{(bf - ce) \log(cx^4 + bx^2 + a)}{4c^2} + \frac{(2c^2d + b^2f - 2acf - bce) \arctan\left(\frac{2cx^2 + b}{\sqrt{-b^2 + 4ac}}\right)}{2\sqrt{-b^2 + 4ac}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] 1/2*f*x^2/c - 1/4*(b*f - c*e)*log(c*x^4 + b*x^2 + a)/c^2 + 1/2*(2*c^2*d + b^2*f - 2*a*c*f - b*c*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)

maple [B] time = 0.00, size = 211, normalized size = 2.05

$$-\frac{af \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}c} + \frac{b^2f \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}c^2} - \frac{be \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}c} + \frac{fx^2}{2c} + \frac{d \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} - \frac{bf \ln(cx^4 + bx^2 + a)}{4c^2} + \frac{e \ln(cx^4 + bx^2 + a)}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x)

[Out] 1/2*f*x^2/c-1/4/c^2*ln(c*x^4+b*x^2+a)*b*f+1/4/c*ln(c*x^4+b*x^2+a)*e-1/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*f+1/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*d+1/2/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^2*f-1/2/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b*e

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 1.83, size = 1081, normalized size = 10.50



Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x)

[Out] (f*x^2)/(2*c) + (log(a + b*x^2 + c*x^4)*(2*b^3*f + 8*a*c^2*e - 2*b^2*c*e - 8*a*b*c*f))/(2*(16*a*c^3 - 4*b^2*c^2)) + (atan((2*c^2*(4*a*c - b^2)*(x^2*((((4*c^4*d + 6*b^2*c^2*f - 4*a*c^3*f - 6*b*c^3*e)/c^2 + (4*b*c^2*(2*b^3*f + 8*a*c^2*e - 2*b^2*c*e - 8*a*b*c*f))/(16*a*c^3 - 4*b^2*c^2))*(2*c^2*d + b^2*f - 2*a*c*f - b*c*e))/(8*c^2*(4*a*c - b^2)^(1/2)) + (b*(2*c^2*d + b^2*f - 2*a*c*f - b*c*e)*(2*b^3*f + 8*a*c^2*e - 2*b^2*c*e - 8*a*b*c*f))/(2*(4*a*c -

$$\begin{aligned}
& b^2)^{(1/2)}*(16*a*c^3 - 4*b^2*c^2))/a - (b*((b^3*f^2 + b*c^2*e^2 - c^3*d*e \\
& - a*b*c*f^2 + a*c^2*e*f + b*c^2*d*f - 2*b^2*c*e*f)/c^2 + (((4*c^4*d + 6*b^ \\
& 2*c^2*f - 4*a*c^3*f - 6*b*c^3*e)/c^2 + (4*b*c^2*(2*b^3*f + 8*a*c^2*e - 2*b^ \\
& 2*c*e - 8*a*b*c*f))/(16*a*c^3 - 4*b^2*c^2))*(2*b^3*f + 8*a*c^2*e - 2*b^2*c \\
& e - 8*a*b*c*f))/(2*(16*a*c^3 - 4*b^2*c^2)) - (b*(2*c^2*d + b^2*f - 2*a*c*f \\
& - b*c*e)^2)/(2*c^2*(4*a*c - b^2)))/((2*a*(4*a*c - b^2)^{(1/2)})) - (((((8*a*c^ \\
& 3*e - 8*a*b*c^2*f)/c^2 - (8*a*c^2*(2*b^3*f + 8*a*c^2*e - 2*b^2*c*e - 8*a*b* \\
& c*f))/(16*a*c^3 - 4*b^2*c^2))*(2*c^2*d + b^2*f - 2*a*c*f - b*c*e))/(8*c^2*(\\
& 4*a*c - b^2)^{(1/2)}) - (a*(2*c^2*d + b^2*f - 2*a*c*f - b*c*e)*(2*b^3*f + 8*a \\
& *c^2*e - 2*b^2*c*e - 8*a*b*c*f))/((4*a*c - b^2)^{(1/2)}*(16*a*c^3 - 4*b^2*c^2 \\
&)))/a + (b*(((8*a*c^3*e - 8*a*b*c^2*f)/c^2 - (8*a*c^2*(2*b^3*f + 8*a*c^2*e \\
& - 2*b^2*c*e - 8*a*b*c*f))/(16*a*c^3 - 4*b^2*c^2))*(2*b^3*f + 8*a*c^2*e - 2 \\
& *b^2*c*e - 8*a*b*c*f))/(2*(16*a*c^3 - 4*b^2*c^2)) - (a*b^2*f^2 + a*c^2*e^2 \\
& - 2*a*b*c*e*f)/c^2 + (a*(2*c^2*d + b^2*f - 2*a*c*f - b*c*e)^2)/(c^2*(4*a*c \\
& - b^2)))/((2*a*(4*a*c - b^2)^{(1/2)})))/((4*c^4*d^2 + b^4*f^2 + 4*a^2*c^2*f^2 \\
& + b^2*c^2*e^2 - 8*a*c^3*d*f - 4*b*c^3*d*e - 2*b^3*c*e*f - 4*a*b^2*c*f^2 + 4 \\
& *b^2*c^2*d*f + 4*a*b*c^2*e*f))*(2*c^2*d + b^2*f - 2*a*c*f - b*c*e))/(2*c^2* \\
& (4*a*c - b^2)^{(1/2)})
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)

[Out] Timed out

$$3.49 \quad \int \frac{d+ex^2+fx^4}{x(a+bx^2+cx^4)} dx$$

Optimal. Leaf size=97

$$\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(abf-2ace+bcd)}{2ac\sqrt{b^2-4ac}} - \frac{(cd-af)\log(a+bx^2+cx^4)}{4ac} + \frac{d\log(x)}{a}$$

Rubi [A] time = 0.20, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1663, 1628, 634, 618, 206, 628}

$$\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(abf-2ace+bcd)}{2ac\sqrt{b^2-4ac}} - \frac{(cd-af)\log(a+bx^2+cx^4)}{4ac} + \frac{d\log(x)}{a}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2 + f*x^4)/(x*(a + b*x^2 + c*x^4)),x]

[Out] ((b*c*d - 2*a*c*e + a*b*f)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a*c*Sqrt[b^2 - 4*a*c]) + (d*Log[x])/a - ((c*d - a*f)*Log[a + b*x^2 + c*x^4])/(4*a*c)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1628

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 1663

Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m-1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[

(m - 1)/2]

Rubi steps

$$\begin{aligned}
\int \frac{d + ex^2 + fx^4}{x(a + bx^2 + cx^4)} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{d + ex + fx^2}{x(a + bx + cx^2)} dx, x, x^2 \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \left(\frac{d}{ax} + \frac{-bd + ae - (cd - af)x}{a(a + bx + cx^2)} \right) dx, x, x^2 \right) \\
&= \frac{d \log(x)}{a} + \frac{\text{Subst} \left(\int \frac{-bd + ae - (cd - af)x}{a + bx + cx^2} dx, x, x^2 \right)}{2a} \\
&= \frac{d \log(x)}{a} - \frac{(cd - af) \text{Subst} \left(\int \frac{b + 2cx}{a + bx + cx^2} dx, x, x^2 \right)}{4ac} - \frac{(bcd - 2ace + abf) \text{Subst} \left(\int \frac{1}{a + bx + cx^2} dx, x, x^2 \right)}{4ac} \\
&= \frac{d \log(x)}{a} - \frac{(cd - af) \log(a + bx^2 + cx^4)}{4ac} + \frac{(bcd - 2ace + abf) \text{Subst} \left(\int \frac{1}{b^2 - 4ac - x^2} dx, x, x^2 \right)}{2ac} \\
&= \frac{(bcd - 2ace + abf) \tanh^{-1} \left(\frac{b + 2cx}{\sqrt{b^2 - 4ac}} \right)}{2ac\sqrt{b^2 - 4ac}} + \frac{d \log(x)}{a} - \frac{(cd - af) \log(a + bx^2 + cx^4)}{4ac}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 178, normalized size = 1.84

$$\frac{-\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)\left(cd\sqrt{b^2-4ac}-af\sqrt{b^2-4ac}+abf-2ace+bcd\right)+\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)\left(-cd\sqrt{b^2-4ac}+af\sqrt{b^2-4ac}+abf-2ace+bcd\right)+4cd\log(x)\sqrt{b^2-4ac}}{4ac\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2 + f*x^4)/(x*(a + b*x^2 + c*x^4)), x]

[Out] (4*c*Sqrt[b^2 - 4*a*c]*d*Log[x] - (b*c*d + c*Sqrt[b^2 - 4*a*c]*d - 2*a*c*e + a*b*f - a*Sqrt[b^2 - 4*a*c]*f)*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2] + (b*c*d - c*Sqrt[b^2 - 4*a*c]*d - 2*a*c*e + a*b*f + a*Sqrt[b^2 - 4*a*c]*f)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(4*a*c*Sqrt[b^2 - 4*a*c])

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex^2 + fx^4}{x(a + bx^2 + cx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e*x^2 + f*x^4)/(x*(a + b*x^2 + c*x^4)), x]

[Out] IntegrateAlgebraic[(d + e*x^2 + f*x^4)/(x*(a + b*x^2 + c*x^4)), x]

fricas [A] time = 1.75, size = 309, normalized size = 3.19

$$\frac{4\left(b^2c-4ac^2\right)d\log(x)+\left(bcd-2ace+abf\right)\sqrt{b^2-4ac}\log\left(\frac{b^2+2bx^2+cx^4+\sqrt{b^2-4ac}}{2bx^2+cx^4}\right)-\left(b^2c-4ac^2\right)d-\left(ab^2-4a^2c\right)f\log\left(cx^4+bx^2+a\right)+4\left(b^2c-4ac^2\right)d\log(x)+2\left(bcd-2ace+abf\right)\sqrt{-b^2+4ac}\arctan\left(\frac{b^2+cx^4}{2bx^2+cx^4}\right)-\left(b^2c-4ac^2\right)d-\left(ab^2-4a^2c\right)f\log\left(cx^4+bx^2+a\right)}{4\left(ab^2c-4a^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^4+e*x^2+d)/x/(c*x^4+b*x^2+a), x, algorithm="fricas")

[Out] [1/4*(4*(b^2*c - 4*a*c^2)*d*log(x) + (b*c*d - 2*a*c*e + a*b*f)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c)))/(c*x^4 + b*x^2 + a)) - ((b^2*c - 4*a*c^2)*d - (a*b^2 - 4*a^2*c)*f)*1

$\log(cx^4 + bx^2 + a)/(ab^2c - 4a^2c^2)$, $1/4*(4*(b^2*c - 4*a*c^2)*d*\log(x) + 2*(b*c*d - 2*a*c*e + a*b*f)*\sqrt{-b^2 + 4*a*c}*\arctan(-(2*c*x^2 + b)*\sqrt{-b^2 + 4*a*c})/(b^2 - 4*a*c) - ((b^2*c - 4*a*c^2)*d - (a*b^2 - 4*a^2*c)*f)*\log(cx^4 + bx^2 + a)/(a*b^2*c - 4*a^2*c^2)]$

giac [A] time = 1.90, size = 97, normalized size = 1.00

$$\frac{d \log(x^2)}{2a} - \frac{(cd - af) \log(cx^4 + bx^2 + a)}{4ac} - \frac{(bcd + abf - 2ace) \arctan\left(\frac{2cx^2 + b}{\sqrt{-b^2 + 4ac}}\right)}{2\sqrt{-b^2 + 4ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^4+e*x^2+d)/x/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] $1/2*d*\log(x^2)/a - 1/4*(c*d - a*f)*\log(cx^4 + bx^2 + a)/(a*c) - 1/2*(b*c*d + a*b*f - 2*a*c*e)*\arctan((2*c*x^2 + b)/\sqrt{-b^2 + 4*a*c})/(\sqrt{-b^2 + 4*a*c})*a*c)$

maple [A] time = 0.01, size = 165, normalized size = 1.70

$$-\frac{bd \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}a} - \frac{bf \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}c} + \frac{e \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} + \frac{d \ln(x)}{a} - \frac{d \ln(cx^4 + bx^2 + a)}{4a} + \frac{f \ln(cx^4 + bx^2 + a)}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^4+e*x^2+d)/x/(c*x^4+b*x^2+a),x)

[Out] $d*\ln(x)/a + 1/4/c*\ln(cx^4 + bx^2 + a)*f - 1/4/a*\ln(cx^4 + bx^2 + a)*d + 1/(4*a*c - b^2)^{(1/2)}*\arctan((2*c*x^2 + b)/(4*a*c - b^2)^{(1/2)})*e - 1/2/a/(4*a*c - b^2)^{(1/2)}*\arctan((2*c*x^2 + b)/(4*a*c - b^2)^{(1/2)})*b*d - 1/2/(4*a*c - b^2)^{(1/2)}*\arctan((2*c*x^2 + b)/(4*a*c - b^2)^{(1/2)})*b/c*f$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^4+e*x^2+d)/x/(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see 'assume?' for more details) Is 4*a*c-b^2 positive or negative?

mupad [B] time = 8.88, size = 3927, normalized size = 40.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x^2 + f*x^4)/(x*(a + b*x^2 + c*x^4)),x)

[Out] $(d*\log(x))/a - (\log((b^2*d*f^2 + c^2*d*e^2 - x^2*(b*f - c*e)*(a*f^2 + c*e^2 - b*e*f - c*d*f) + ((c*d - a*f + a*c*(-(a*b*f - 2*a*c*e + b*c*d)^2/(a^2*c^2*(4*a*c - b^2))))^{(1/2)}*(a*b^2*f^2 - x^2*(b*c^2*e^2 - 3*b^3*f^2 + 5*c^3*d*e + 11*a*b*c*f^2 - 9*a*c^2*e*f - 7*b*c^2*d*f + 2*b^2*c*e*f) + a*c^2*e^2 - 4*b*c^2*d*e + 4*b^2*c*d*f + ((c*d - a*f + a*c*(-(a*b*f - 2*a*c*e + b*c*d)^2/(a^2*c^2*(4*a*c - b^2))))^{(1/2)}*(2*c*x^2*(6*b^3*f + 10*a*c^2*e + 5*b*c^2*d - 4*b^2*c*e - 19*a*b*c*f) + 4*b^2*c^2*d - 4*a*b*c^2*e + 4*a*b^2*c*f + (b*c*(c*d - a*f + a*c*(-(a*b*f - 2*a*c*e + b*c*d)^2/(a^2*c^2*(4*a*c - b^2))))^{(1/2)}*(a*b + 3*b^2*x^2 - 10*a*c*x^2))/a))/(4*a*c) - 2*a*b*c*e*f)/(4*a*c) - 2*b*c*d*e*f*(b^2*d*f^2 + c^2*d*e^2 - x^2*(b*f - c*e)*(a*f^2 + c*e^2 - b*e*f$

$$\begin{aligned}
& - c*d*f) + ((a*f - c*d + a*c*(-(a*b*f - 2*a*c*e + b*c*d)^2/(a^2*c^2*(4*a*c - b^2)))^{(1/2)}*(x^2*(b*c^2*e^2 - 3*b^3*f^2 + 5*c^3*d*e + 11*a*b*c*f^2 - 9 \\
& *a*c^2*e*f - 7*b*c^2*d*f + 2*b^2*c*e*f) - a*b^2*f^2 - a*c^2*e^2 + 4*b*c^2*d \\
& *e - 4*b^2*c*d*f + ((a*f - c*d + a*c*(-(a*b*f - 2*a*c*e + b*c*d)^2/(a^2*c^2 \\
& *(4*a*c - b^2)))^{(1/2)}*(2*c*x^2*(6*b^3*f + 10*a*c^2*e + 5*b*c^2*d - 4*b^2*c \\
& *e - 19*a*b*c*f) + 4*b^2*c^2*d - 4*a*b*c^2*e + 4*a*b^2*c*f - (b*c*(a*f - c \\
& *d + a*c*(-(a*b*f - 2*a*c*e + b*c*d)^2/(a^2*c^2*(4*a*c - b^2)))^{(1/2)}*(a*b \\
& + 3*b^2*x^2 - 10*a*c*x^2))/a))/(4*a*c) + 2*a*b*c*e*f))/(4*a*c) - 2*b*c*d*e \\
& *f)*(8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d - 8*a^2*c*f))/(2*(16*a^2*c^2 - 4*a* \\
& b^2*c)) + (atan(((4*a*c - b^2)*(((a*b*f - 2*a*c*e + b*c*d)*(4*b^2*c^2*d - \\
& 4*a*b*c^2*e + 4*a*b^2*c*f + (2*a*b^2*c^2*(8*a*c^2*d + 2*a*b^2*f - 2*b^2*c* \\
& d - 8*a^2*c*f))/(16*a^2*c^2 - 4*a*b^2*c)))/(4*a*c*(4*a*c - b^2)^{(1/2)} + (b \\
& ^2*c*(a*b*f - 2*a*c*e + b*c*d)*(8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d - 8*a^2*c \\
& *f))/(2*(16*a^2*c^2 - 4*a*b^2*c)*(4*a*c - b^2)^{(1/2)})*(8*a*c^2*d + 2*a*b^2 \\
& *f - 2*b^2*c*d - 8*a^2*c*f))/(2*(16*a^2*c^2 - 4*a*b^2*c)) + ((a*b*f - 2*a*c \\
& *e + b*c*d)*(a*b^2*f^2 + a*c^2*e^2 + ((4*b^2*c^2*d - 4*a*b*c^2*e + 4*a*b^2*c \\
& *f + (2*a*b^2*c^2*(8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d - 8*a^2*c*f))/(16*a^2 \\
& *c^2 - 4*a*b^2*c))*(8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d - 8*a^2*c*f))/(2*(16* \\
& a^2*c^2 - 4*a*b^2*c)) - 4*b*c^2*d*e + 4*b^2*c*d*f - 2*a*b*c*e*f))/(4*a*c*(4 \\
& *a*c - b^2)^{(1/2)} - (b^2*(a*b*f - 2*a*c*e + b*c*d)^3)/(16*a^2*c*(4*a*c - b \\
& ^2)^{(3/2)}))*(6*b^4*d + 20*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 4*a^3*c*f - \\
& 28*a*b^2*c*d + 6*a^2*b*c*e))/(c*(a^2*b^2*f^2 + 4*a^2*c^2*e^2 + b^2*c^2*d^2 \\
& - 4*a*b*c^2*d*e + 2*a*b^2*c*d*f - 4*a^2*b*c*e*f)*(a^3*f^2 + 25*a*c^2*d^2 + \\
& a^2*c*e^2 - 6*b^2*c*d^2 + 3*a*b^2*d*f - a^2*b*e*f - 10*a^2*c*d*f - a*b*c*d \\
& *e) + (16*a^3*c*x^2*((3*b^3*d - a*b^2*e + a^2*b*f + a^2*c*e - 8*a*b*c*d)* \\
& (c^2*e^3 + ((8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d - 8*a^2*c*f)*(3*b^3*f^2 - b* \\
& c^2*e^2 + ((8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d - 8*a^2*c*f)*((12*b^3*c^2 - \\
& 40*a*b*c^3)*(8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d - 8*a^2*c*f)))/(2*(16*a^2*c^2 \\
& - 4*a*b^2*c)) - 8*b^2*c^2*e + 20*a*c^3*e + 10*b*c^3*d + 12*b^3*c*f - 38*a* \\
& b*c^2*f))/(2*(16*a^2*c^2 - 4*a*b^2*c)) - 5*c^3*d*e - 11*a*b*c*f^2 + 9*a*c^2 \\
& *e*f + 7*b*c^2*d*f - 2*b^2*c*e*f))/(2*(16*a^2*c^2 - 4*a*b^2*c)) + b^2*e*f^2 \\
& - a*b*f^3 + a*c*e*f^2 + b*c*d*f^2 - 2*b*c*e^2*f - c^2*d*e*f - (((a*b*f - \\
& 2*a*c*e + b*c*d)*((12*b^3*c^2 - 40*a*b*c^3)*(8*a*c^2*d + 2*a*b^2*f - 2*b^2 \\
& *c*d - 8*a^2*c*f))/(2*(16*a^2*c^2 - 4*a*b^2*c)) - 8*b^2*c^2*e + 20*a*c^3*e \\
& + 10*b*c^3*d + 12*b^3*c*f - 38*a*b*c^2*f))/(4*a*c*(4*a*c - b^2)^{(1/2)} + ((\\
& 12*b^3*c^2 - 40*a*b*c^3)*(a*b*f - 2*a*c*e + b*c*d)*(8*a*c^2*d + 2*a*b^2*f - \\
& 2*b^2*c*d - 8*a^2*c*f))/(8*a*c*(16*a^2*c^2 - 4*a*b^2*c)*(4*a*c - b^2)^{(1/2} \\
&))*(a*b*f - 2*a*c*e + b*c*d))/(4*a*c*(4*a*c - b^2)^{(1/2)} - ((12*b^3*c^2 - \\
& 40*a*b*c^3)*(a*b*f - 2*a*c*e + b*c*d)^2*(8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d \\
& - 8*a^2*c*f))/(32*a^2*c^2*(16*a^2*c^2 - 4*a*b^2*c)*(4*a*c - b^2)))/(8*a^3 \\
& *c^2*(a^3*f^2 + 25*a*c^2*d^2 + a^2*c*e^2 - 6*b^2*c*d^2 + 3*a*b^2*d*f - a^2* \\
& b*e*f - 10*a^2*c*d*f - a*b*c*d*e) + ((((((a*b*f - 2*a*c*e + b*c*d)*((12*b \\
& ^3*c^2 - 40*a*b*c^3)*(8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d - 8*a^2*c*f))/(2*(1 \\
& 6*a^2*c^2 - 4*a*b^2*c)) - 8*b^2*c^2*e + 20*a*c^3*e + 10*b*c^3*d + 12*b^3*c* \\
& f - 38*a*b*c^2*f))/(4*a*c*(4*a*c - b^2)^{(1/2)} + ((12*b^3*c^2 - 40*a*b*c^3) \\
& *(a*b*f - 2*a*c*e + b*c*d)*(8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d - 8*a^2*c*f)) \\
& /((8*a*c*(16*a^2*c^2 - 4*a*b^2*c)*(4*a*c - b^2)^{(1/2)}))*(8*a*c^2*d + 2*a*b^2 \\
& *f - 2*b^2*c*d - 8*a^2*c*f))/(2*(16*a^2*c^2 - 4*a*b^2*c)) + ((a*b*f - 2*a*c \\
& *e + b*c*d)*(3*b^3*f^2 - b*c^2*e^2 + ((8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d - \\
& 8*a^2*c*f)*((12*b^3*c^2 - 40*a*b*c^3)*(8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d - \\
& 8*a^2*c*f))/(2*(16*a^2*c^2 - 4*a*b^2*c)) - 8*b^2*c^2*e + 20*a*c^3*e + 10*b \\
& *c^3*d + 12*b^3*c*f - 38*a*b*c^2*f))/(2*(16*a^2*c^2 - 4*a*b^2*c)) - 5*c^3*d \\
& *e - 11*a*b*c*f^2 + 9*a*c^2*e*f + 7*b*c^2*d*f - 2*b^2*c*e*f))/(4*a*c*(4*a*c \\
& - b^2)^{(1/2)} - ((12*b^3*c^2 - 40*a*b*c^3)*(a*b*f - 2*a*c*e + b*c*d)^3)/(6 \\
& 4*a^3*c^3*(4*a*c - b^2)^{(3/2)}))*(6*b^4*d + 20*a^2*c^2*d + 2*a^2*b^2*f - 2*a \\
& *b^3*e - 4*a^3*c*f - 28*a*b^2*c*d + 6*a^2*b*c*e))/(16*a^3*c^2*(4*a*c - b^2) \\
& ^{(1/2)}*(a^3*f^2 + 25*a*c^2*d^2 + a^2*c*e^2 - 6*b^2*c*d^2 + 3*a*b^2*d*f - a^2 \\
& *b*e*f - 10*a^2*c*d*f - a*b*c*d*e)))/(4*a*c - b^2)^{(3/2)})/(a^2*b^2*f^2 + 4 \\
& *a^2*c^2*e^2 + b^2*c^2*d^2 - 4*a*b*c^2*d*e + 2*a*b^2*c*d*f - 4*a^2*b*c*e*f)
\end{aligned}$$

$$\begin{aligned}
& + (2*(4*a*c - b^2)^{(3/2)}*(3*b^3*d - a*b^2*e + a^2*b*f + a^2*c*e - 8*a*b*c*d) \\
& * (b^2*d*f^2 + c^2*d*e^2 + ((8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d - 8*a^2*c*f) \\
&)*(a*b^2*f^2 + a*c^2*e^2 + ((4*b^2*c^2*d - 4*a*b*c^2*e + 4*a*b^2*c*f + (2*a \\
& *b^2*c^2*(8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d - 8*a^2*c*f))/(16*a^2*c^2 - 4*a \\
& *b^2*c))*(8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d - 8*a^2*c*f))/(2*(16*a^2*c^2 - \\
& 4*a*b^2*c)) - 4*b*c^2*d*e + 4*b^2*c*d*f - 2*a*b*c*e*f))/(2*(16*a^2*c^2 - 4* \\
& a*b^2*c)) - (((a*b*f - 2*a*c*e + b*c*d)*(4*b^2*c^2*d - 4*a*b*c^2*e + 4*a*b \\
& ^2*c*f + (2*a*b^2*c^2*(8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d - 8*a^2*c*f))/(16* \\
& a^2*c^2 - 4*a*b^2*c)))/(4*a*c*(4*a*c - b^2)^{(1/2)}) + (b^2*c*(a*b*f - 2*a*c* \\
& e + b*c*d)*(8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d - 8*a^2*c*f))/(2*(16*a^2*c^2 \\
& - 4*a*b^2*c)*(4*a*c - b^2)^{(1/2)}))*(a*b*f - 2*a*c*e + b*c*d))/(4*a*c*(4*a*c \\
& - b^2)^{(1/2)}) - 2*b*c*d*e*f - (b^2*(a*b*f - 2*a*c*e + b*c*d)^2*(8*a*c^2*d \\
& + 2*a*b^2*f - 2*b^2*c*d - 8*a^2*c*f))/(8*a*(16*a^2*c^2 - 4*a*b^2*c)*(4*a*c \\
& - b^2)))/((c*(a^2*b^2*f^2 + 4*a^2*c^2*e^2 + b^2*c^2*d^2 - 4*a*b*c^2*d*e + 2 \\
& *a*b^2*c*d*f - 4*a^2*b*c*e*f)*(a^3*f^2 + 25*a*c^2*d^2 + a^2*c*e^2 - 6*b^2*c \\
& *d^2 + 3*a*b^2*d*f - a^2*b*e*f - 10*a^2*c*d*f - a*b*c*d*e)))*(a*b*f - 2*a*c \\
& *e + b*c*d))/(2*a*c*(4*a*c - b^2)^{(1/2)})
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**4+e*x**2+d)/x/(c*x**4+b*x**2+a), x)

[Out] Timed out

$$3.50 \quad \int \frac{d+ex^2+fx^4}{x^3(a+bx^2+cx^4)} dx$$

Optimal. Leaf size=118

$$\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(-abe-2a(cd-af)+b^2d)}{2a^2\sqrt{b^2-4ac}} + \frac{(bd-ae)\log(a+bx^2+cx^4)}{4a^2} - \frac{\log(x)(bd-ae)}{a^2} - \frac{d}{2ax^2}$$

Rubi [A] time = 0.29, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1663, 1628, 634, 618, 206, 628}

$$\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(-abe-2a(cd-af)+b^2d)}{2a^2\sqrt{b^2-4ac}} + \frac{(bd-ae)\log(a+bx^2+cx^4)}{4a^2} - \frac{\log(x)(bd-ae)}{a^2} - \frac{d}{2ax^2}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2 + f*x^4)/(x^3*(a + b*x^2 + c*x^4)),x]

[Out] -d/(2*a*x^2) - ((b^2*d - a*b*e - 2*a*(c*d - a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^2*Sqrt[b^2 - 4*a*c]) - ((b*d - a*e)*Log[x])/a^2 + ((b*d - a*e)*Log[a + b*x^2 + c*x^4])/(4*a^2)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1628

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 1663

Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[1/2, Subst[Int[x^((m-1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[

(m - 1)/2]

Rubi steps

$$\int \frac{d + ex^2 + fx^4}{x^3(a + bx^2 + cx^4)} dx = \frac{1}{2} \text{Subst} \left(\int \frac{d + ex + fx^2}{x^2(a + bx + cx^2)} dx, x, x^2 \right)$$

$$= \frac{1}{2} \text{Subst} \left(\int \left(\frac{d}{ax^2} + \frac{-bd + ae}{a^2x} + \frac{b^2d - abe - a(cd - af) + c(bd - ae)x}{a^2(a + bx + cx^2)} \right) dx, x, x^2 \right)$$

$$= \frac{d}{2ax^2} - \frac{(bd - ae) \log(x)}{a^2} + \frac{\text{Subst} \left(\int \frac{b^2d - abe - a(cd - af) + c(bd - ae)x}{a + bx + cx^2} dx, x, x^2 \right)}{2a^2}$$

$$= \frac{d}{2ax^2} - \frac{(bd - ae) \log(x)}{a^2} + \frac{(bd - ae) \text{Subst} \left(\int \frac{b + 2cx}{a + bx + cx^2} dx, x, x^2 \right)}{4a^2} + \frac{(b^2d - abe - 2a(cd - af))}{4a^2}$$

$$= \frac{d}{2ax^2} - \frac{(bd - ae) \log(x)}{a^2} + \frac{(bd - ae) \log(a + bx^2 + cx^4)}{4a^2} - \frac{(b^2d - abe - 2a(cd - af))}{4a^2}$$

$$= \frac{d}{2ax^2} - \frac{(b^2d - abe - 2a(cd - af)) \tanh^{-1} \left(\frac{b + 2cx}{\sqrt{b^2 - 4ac}} \right)}{2a^2 \sqrt{b^2 - 4ac}} - \frac{(bd - ae) \log(x)}{a^2} + \frac{(bd - ae) \log(a + bx^2 + cx^4)}{4a^2}$$

Mathematica [A] time = 0.15, size = 203, normalized size = 1.72

$$\frac{\log(-\sqrt{b^2-4ac}+b+2cx^2) \left(a(-e\sqrt{b^2-4ac}+2af-2cd) + b(d\sqrt{b^2-4ac}-ae) + b^2d \right) + \log(\sqrt{b^2-4ac}+b+2cx^2) \left(-a(e\sqrt{b^2-4ac}+2af-2cd) + b(d\sqrt{b^2-4ac}+ae) + b^2(-d) \right) + 4 \log(x)(ae - bd) - \frac{2ad}{x^2}}{4a^2 \sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2 + f*x^4)/(x^3*(a + b*x^2 + c*x^4)), x]
[Out] ((-2*a*d)/x^2 + 4*(-(b*d) + a*e)*Log[x] + ((b^2*d + b*(Sqrt[b^2 - 4*a*c]*d - a*e) + a*(-2*c*d - Sqrt[b^2 - 4*a*c]*e + 2*a*f))*Log[b - Sqrt[b^2 - 4*a*c] ] + 2*c*x^2)/Sqrt[b^2 - 4*a*c] + ((-(b^2*d) + b*(Sqrt[b^2 - 4*a*c]*d + a*e) - a*(-2*c*d + Sqrt[b^2 - 4*a*c]*e + 2*a*f))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c])/(4*a^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex^2 + fx^4}{x^3(a + bx^2 + cx^4)} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x^2 + f*x^4)/(x^3*(a + b*x^2 + c*x^4)), x]
[Out] IntegrateAlgebraic[(d + e*x^2 + f*x^4)/(x^3*(a + b*x^2 + c*x^4)), x]
```

fricas [A] time = 2.15, size = 399, normalized size = 3.38

$$\frac{\left((abx - 2ef - (b^2 - 2ac)d)\sqrt{b^2 - 4ac} \log\left(\frac{2c^2 + 2ax^2 + 2ac - (b^2 - 4ac)\sqrt{b^2 - 4ac}}{2cx^2}\right) - ((b^2 - 4ac)d - (ab^2 - 4ac^2)) \log(cx^2 + b^2 + a) + 4((b^2 - 4ac)d - (ab^2 - 4ac^2)) \log(cx) + 2((abx - 2ef - (b^2 - 2ac)d)\sqrt{b^2 - 4ac} + 4acx^2) \arctan\left(\frac{b + 2cx}{\sqrt{b^2 - 4ac}}\right) + ((b^2 - 4ac)d - (ab^2 - 4ac^2)) \log(cx^2 + b^2 + a) - 4((b^2 - 4ac)d - (ab^2 - 4ac^2)) \log(cx) - 2((ab^2 - 4ac^2)d - (b^2 - 2ac)d^2) \log\left(\frac{b + 2cx}{\sqrt{b^2 - 4ac}}\right) \right)}{4(b^2 - 4ac)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^3/(c*x^4+b*x^2+a), x, algorithm="fricas")
[Out] [-1/4*((a*b*e - 2*a^2*f - (b^2 - 2*a*c)*d)*sqrt(b^2 - 4*a*c)*x^2*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 +
```


$$b*x^2 + a) - ((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*x^2*\log(c*x^4 + b*x^2 + a) + 4*((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*x^2*\log(x) + 2*(a*b^2 - 4*a^2*c)*d)/((a^2*b^2 - 4*a^3*c)*x^2), 1/4*(2*(a*b*e - 2*a^2*f - (b^2 - 2*a*c)*d)*\sqrt{-b^2 + 4*a*c})*x^2*\arctan(-(2*c*x^2 + b)*\sqrt{-b^2 + 4*a*c})/(b^2 - 4*a*c)) + ((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*x^2*\log(c*x^4 + b*x^2 + a) - 4*((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*x^2*\log(x) - 2*(a*b^2 - 4*a^2*c)*d)/((a^2*b^2 - 4*a^3*c)*x^2)]$$

giac [A] time = 1.78, size = 135, normalized size = 1.14

$$\frac{(bd - ae)\log(cx^4 + bx^2 + a)}{4a^2} - \frac{(bd - ae)\log(x^2)}{2a^2} + \frac{(b^2d - 2acd + 2a^2f - abe)\arctan\left(\frac{2cx^2 + b}{\sqrt{-b^2 + 4ac}}\right)}{2\sqrt{-b^2 + 4ac}a^2} + \frac{bdx^2 - ax^2e - ad}{2a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^4+e*x^2+d)/x^3/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] 1/4*(b*d - a*e)*log(c*x^4 + b*x^2 + a)/a^2 - 1/2*(b*d - a*e)*log(x^2)/a^2 + 1/2*(b^2*d - 2*a*c*d + 2*a^2*f - a*b*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2) + 1/2*(b*d*x^2 - a*x^2*e - a*d)/(a^2*x^2)

maple [B] time = 0.01, size = 227, normalized size = 1.92

$$-\frac{be\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}a} - \frac{cd\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}a} + \frac{b^2d\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}a^2} + \frac{f\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} + \frac{e\ln(x)}{a} - \frac{e\ln(cx^4 + bx^2 + a)}{4a} - \frac{bd\ln(x)}{a^2} + \frac{bd\ln(cx^4 + bx^2 + a)}{4a^2} - \frac{d}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^4+e*x^2+d)/x^3/(c*x^4+b*x^2+a),x)

[Out] -1/2*d/a/x^2+1/a*ln(x)*e-1/a^2*ln(x)*b*d-1/4/a*ln(c*x^4+b*x^2+a)*e+1/4/a^2*ln(c*x^4+b*x^2+a)*b*d+1/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*f-1/2/a/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b*e-1/a/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*c*d+1/2/a^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^2*d

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^4+e*x^2+d)/x^3/(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see 'assume?' for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 7.86, size = 4437, normalized size = 37.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x^2 + f*x^4)/(x^3*(a + b*x^2 + c*x^4)),x)

[Out] (log(x)*(a*e - b*d))/a^2 - d/(2*a*x^2) - (log(((c^2*(a*e - b*d)*(a*f - c*d)^2)/a^3 - ((b*d - a*e + a^2*(-(b^2*d + 2*a^2*f - a*b*e - 2*a*c*d))^2/(a^4*(4*a*c - b^2)))^(1/2))*((b*d - a*e + a^2*(-(b^2*d + 2*a^2*f - a*b*e - 2*a*c*d))^2/(a^4*(4*a*c - b^2)))^(1/2))*((2*c^2*x^2*(10*a*c^2*d + 4*a*b^2*f + b^2*c*d - 10*a^2*c*f - 5*a*b*c*e))/a + (4*b*c^2*(b^2*d + a^2*f - a*b*e - a*c*d))/a + (b*c^2*(b*d - a*e + a^2*(-(b^2*d + 2*a^2*f - a*b*e - 2*a*c*d))^2/(a^4*(4*a*c - b^2)))^(1/2))*(a*b + 3*b^2*x^2 - 10*a*c*x^2))/a^2))/(4*a^2) + (c^2

$$\begin{aligned}
& * (a*f - c*d) * (4*b^2*d + a^2*f - 4*a*b*e - a*c*d) / a^2 - (c^2*x^2*(a*f - c*d) \\
&) * (a*b*f + 5*a*c*e - 6*b*c*d) / a^2) / (4*a^2) + (c^2*x^2*(a*f - c*d)^3 / a^3) \\
& * ((c^2*(a*e - b*d) * (a*f - c*d)^2) / a^3 - ((a*e - b*d + a^2*(-(b^2*d + 2*a^2*f \\
& - a*b*e - 2*a*c*d)^2 / (a^4*(4*a*c - b^2)))^{(1/2)}) * (((a*e - b*d + a^2*(-(b^2 \\
& *d + 2*a^2*f - a*b*e - 2*a*c*d)^2 / (a^4*(4*a*c - b^2)))^{(1/2)}) * ((2*c^2*x^2* \\
& (10*a*c^2*d + 4*a*b^2*f + b^2*c*d - 10*a^2*c*f - 5*a*b*c*e)) / a + (4*b*c^2*(\\
& b^2*d + a^2*f - a*b*e - a*c*d)) / a - (b*c^2*(a*e - b*d + a^2*(-(b^2*d + 2*a^2 \\
& *f - a*b*e - 2*a*c*d)^2 / (a^4*(4*a*c - b^2)))^{(1/2)}) * (a*b + 3*b^2*x^2 - 10* \\
& a*c*x^2)) / a^2)) / (4*a^2) - (c^2*(a*f - c*d) * (4*b^2*d + a^2*f - 4*a*b*e - a*c \\
& *d)) / a^2 + (c^2*x^2*(a*f - c*d) * (a*b*f + 5*a*c*e - 6*b*c*d)) / a^2) / (4*a^2) \\
& + (c^2*x^2*(a*f - c*d)^3 / a^3) * (2*b^3*d - 2*a*b^2*e + 8*a^2*c*e - 8*a*b*c*d) / \\
& (2*(16*a^3*c - 4*a^2*b^2)) - (atan((16*a^6*(4*a*c - b^2)^{(3/2)} * (x^2 * (((\\
& (c^5*d^3 - a^3*c^2*f^3 + 3*a^2*c^3*d*f^2 - 3*a*c^4*d^2*f) / a^3 + (((a^3*b*c^2 \\
& *f^2 + 6*a*b*c^4*d^2 - 5*a^2*c^4*d*e + 5*a^3*c^3*e*f - 7*a^2*b*c^3*d*f) / a^3 + \\
& (((20*a^3*c^4*d - 20*a^4*c^3*f + 2*a^2*b^2*c^3*d + 8*a^3*b^2*c^2*f - 10 \\
& *a^3*b*c^3*e) / a^3 + ((40*a^4*b*c^3 - 12*a^3*b^3*c^2) * (2*b^3*d - 2*a*b^2*e + \\
& 8*a^2*c*e - 8*a*b*c*d)) / (2*a^3*(16*a^3*c - 4*a^2*b^2))) * (2*b^3*d - 2*a*b^2 \\
& *e + 8*a^2*c*e - 8*a*b*c*d)) / (2*(16*a^3*c - 4*a^2*b^2))) * (2*b^3*d - 2*a*b^2 \\
& *e + 8*a^2*c*e - 8*a*b*c*d)) / (2*(16*a^3*c - 4*a^2*b^2)) - (((((20*a^3*c^4*d \\
& - 20*a^4*c^3*f + 2*a^2*b^2*c^3*d + 8*a^3*b^2*c^2*f - 10*a^3*b*c^3*e) / a^3 + \\
& ((40*a^4*b*c^3 - 12*a^3*b^3*c^2) * (2*b^3*d - 2*a*b^2*e + 8*a^2*c*e - 8*a*b* \\
& c*d)) / (2*a^3*(16*a^3*c - 4*a^2*b^2))) * (b^2*d + 2*a^2*f - a*b*e - 2*a*c*d)) / \\
& (4*a^2*(4*a*c - b^2)^{(1/2)}) + ((40*a^4*b*c^3 - 12*a^3*b^3*c^2) * (b^2*d + 2*a \\
& ^2*f - a*b*e - 2*a*c*d) * (2*b^3*d - 2*a*b^2*e + 8*a^2*c*e - 8*a*b*c*d)) / (8*a \\
& ^5*(4*a*c - b^2)^{(1/2)} * (16*a^3*c - 4*a^2*b^2))) * (b^2*d + 2*a^2*f - a*b*e - \\
& 2*a*c*d)) / (4*a^2*(4*a*c - b^2)^{(1/2)}) - ((40*a^4*b*c^3 - 12*a^3*b^3*c^2) * (b \\
& ^2*d + 2*a^2*f - a*b*e - 2*a*c*d)^2 * (2*b^3*d - 2*a*b^2*e + 8*a^2*c*e - 8*a* \\
& b*c*d)) / (32*a^7*(4*a*c - b^2) * (16*a^3*c - 4*a^2*b^2))) * (3*b^4*d + a^2*c^2*d \\
& + a^2*b^2*f - 3*a*b^3*e - a^3*c*f - 9*a*b^2*c*d + 8*a^2*b*c*e)) / (8*a^3*c^2 \\
& * (a^4*f^2 - 6*b^4*d^2 + 25*a^3*c*e^2 - 6*a^2*b^2*e^2 + a^2*c^2*d^2 + 12*a*b \\
& ^3*d*e - a^3*b*e*f - 2*a^3*c*d*f + 24*a*b^2*c*d^2 + a^2*b^2*d*f - 49*a^2*b* \\
& c*d*e)) - (((((((20*a^3*c^4*d - 20*a^4*c^3*f + 2*a^2*b^2*c^3*d + 8*a^3*b^2* \\
& c^2*f - 10*a^3*b*c^3*e) / a^3 + ((40*a^4*b*c^3 - 12*a^3*b^3*c^2) * (2*b^3*d - 2 \\
& *a*b^2*e + 8*a^2*c*e - 8*a*b*c*d)) / (2*a^3*(16*a^3*c - 4*a^2*b^2))) * (b^2*d + \\
& 2*a^2*f - a*b*e - 2*a*c*d)) / (4*a^2*(4*a*c - b^2)^{(1/2)}) + ((40*a^4*b*c^3 - \\
& 12*a^3*b^3*c^2) * (b^2*d + 2*a^2*f - a*b*e - 2*a*c*d) * (2*b^3*d - 2*a*b^2*e + \\
& 8*a^2*c*e - 8*a*b*c*d)) / (8*a^5*(4*a*c - b^2)^{(1/2)} * (16*a^3*c - 4*a^2*b^2)) \\
&) * (2*b^3*d - 2*a*b^2*e + 8*a^2*c*e - 8*a*b*c*d)) / (2*(16*a^3*c - 4*a^2*b^2)) \\
& + (((a^3*b*c^2*f^2 + 6*a*b*c^4*d^2 - 5*a^2*c^4*d*e + 5*a^3*c^3*e*f - 7*a^2 \\
& *b*c^3*d*f) / a^3 + (((20*a^3*c^4*d - 20*a^4*c^3*f + 2*a^2*b^2*c^3*d + 8*a^3* \\
& b^2*c^2*f - 10*a^3*b*c^3*e) / a^3 + ((40*a^4*b*c^3 - 12*a^3*b^3*c^2) * (2*b^3*d \\
& - 2*a*b^2*e + 8*a^2*c*e - 8*a*b*c*d)) / (2*a^3*(16*a^3*c - 4*a^2*b^2))) * (2*b \\
& ^3*d - 2*a*b^2*e + 8*a^2*c*e - 8*a*b*c*d)) / (2*(16*a^3*c - 4*a^2*b^2))) * (b^2 \\
& *d + 2*a^2*f - a*b*e - 2*a*c*d)) / (4*a^2*(4*a*c - b^2)^{(1/2)}) - ((40*a^4*b*c \\
& ^3 - 12*a^3*b^3*c^2) * (b^2*d + 2*a^2*f - a*b*e - 2*a*c*d)^3) / (64*a^9*(4*a*c \\
& - b^2)^{(3/2})) * (6*b^5*d + 2*a^2*b^3*f - 20*a^3*c^2*e - 6*a*b^4*e - 30*a*b^3 \\
& *c*d - 6*a^3*b*c*f + 26*a^2*b*c^2*d + 28*a^2*b^2*c*e)) / (16*a^3*c^2*(4*a*c - \\
& b^2)^{(1/2)} * (a^4*f^2 - 6*b^4*d^2 + 25*a^3*c*e^2 - 6*a^2*b^2*e^2 + a^2*c^2*d \\
& ^2 + 12*a*b^3*d*e - a^3*b*e*f - 2*a^3*c*d*f + 24*a*b^2*c*d^2 + a^2*b^2*d*f \\
& - 49*a^2*b*c*d*e)) + (((b*c^4*d^3 - a^3*c^2*e*f^2 - a*c^4*d^2*e - 2*a*b*c^3 \\
& *d^2*f + 2*a^2*c^3*d*e*f + a^2*b*c^2*d*f^2) / a^3 - (((a^2*c^4*d^2 + a^4*c^2 \\
& *f^2 - 4*a*b^2*c^3*d^2 - 2*a^3*c^3*d*f + 4*a^2*b*c^3*d*e - 4*a^3*b*c^2*e*f \\
& + 4*a^2*b^2*c^2*d*f) / a^3 - (((4*a^2*b^3*c^2*d - 4*a^3*b^2*c^2*e - 4*a^3*b*c \\
& ^3*d + 4*a^4*b*c^2*f) / a^3 - (2*a*b^2*c^2*(2*b^3*d - 2*a*b^2*e + 8*a^2*c*e - \\
& 8*a*b*c*d)) / (16*a^3*c - 4*a^2*b^2)) * (2*b^3*d - 2*a*b^2*e + 8*a^2*c*e - 8*a \\
& *b*c*d)) / (2*(16*a^3*c - 4*a^2*b^2)) * (2*b^3*d - 2*a*b^2*e + 8*a^2*c*e - 8*a \\
& *b*c*d)) / (2*(16*a^3*c - 4*a^2*b^2)) - (((((4*a^2*b^3*c^2*d - 4*a^3*b^2*c^2*e - 4*a^3*b*c \\
& ^3*d + 4*a^4*b*c^2*f) / a^3 - (2*a*b^2*c^2*(2*b^3*d - 2*a*b^2*e + 8*a^2*c*e - \\
& 8*a*b*c*d)) / (16*a^3*c - 4*a^2*b^2)) * (b^2*d + 2*a^2*f - a*b*e
\end{aligned}$$

$$\begin{aligned}
& - 2*a*c*d)) / (4*a^2*(4*a*c - b^2)^{(1/2)}) - (b^2*c^2*(b^2*d + 2*a^2*f - a*b*e \\
& - 2*a*c*d)*(2*b^3*d - 2*a*b^2*e + 8*a^2*c*e - 8*a*b*c*d)) / (2*a*(4*a*c - b^2)^{(1/2)}*(16*a^3*c - 4*a^2*b^2)) * (b^2*d + 2*a^2*f - a*b*e - 2*a*c*d)) / (4*a^2*(4*a*c - b^2)^{(1/2)}) + (b^2*c^2*(b^2*d + 2*a^2*f - a*b*e - 2*a*c*d)^2*(2 \\
& *b^3*d - 2*a*b^2*e + 8*a^2*c*e - 8*a*b*c*d)) / (8*a^3*(4*a*c - b^2)*(16*a^3*c - 4*a^2*b^2)) * (3*b^4*d + a^2*c^2*d + a^2*b^2*f - 3*a*b^3*e - a^3*c*f - 9* \\
& a*b^2*c*d + 8*a^2*b*c*e)) / (8*a^3*c^2*(a^4*f^2 - 6*b^4*d^2 + 25*a^3*c*e^2 - 6*a^2*b^2*e^2 + a^2*c^2*d^2 + 12*a*b^3*d*e - a^3*b*e*f - 2*a^3*c*d*f + 24*a*b^2*c*d^2 + a^2*b^2*d*f - 49*a^2*b*c*d*e)) - (((((((((4*a^2*b^3*c^2*d - 4*a^3 \\
& *b^2*c^2*e - 4*a^3*b*c^3*d + 4*a^4*b*c^2*f)/a^3 - (2*a*b^2*c^2*(2*b^3*d - 2*a*b^2*e + 8*a^2*c*e - 8*a*b*c*d)) / (16*a^3*c - 4*a^2*b^2)) * (b^2*d + 2*a^2*f - a*b*e - 2*a*c*d)) / (4*a^2*(4*a*c - b^2)^{(1/2)}) - (b^2*c^2*(b^2*d + 2*a^2*f - a*b*e - 2*a*c*d)*(2*b^3*d - 2*a*b^2*e + 8*a^2*c*e - 8*a*b*c*d)) / (2*a*(4*a*c - b^2)^{(1/2)}*(16*a^3*c - 4*a^2*b^2)) * (2*b^3*d - 2*a*b^2*e + 8*a^2*c*e - 8*a*b*c*d)) / (2*(16*a^3*c - 4*a^2*b^2)) - (((a^2*c^4*d^2 + a^4*c^2*f^2 - 4*a*b^2*c^3*d^2 - 2*a^3*c^3*d*f + 4*a^2*b*c^3*d*e - 4*a^3*b*c^2*e*f + 4*a^2*b^2*c^2*d*f)/a^3 - (((4*a^2*b^3*c^2*d - 4*a^3*b^2*c^2*e - 4*a^3*b*c^3*d + 4*a^4*b*c^2*f)/a^3 - (2*a*b^2*c^2*(2*b^3*d - 2*a*b^2*e + 8*a^2*c*e - 8*a*b*c*d)) / (16*a^3*c - 4*a^2*b^2)) * (2*b^3*d - 2*a*b^2*e + 8*a^2*c*e - 8*a*b*c*d)) / (2*(16*a^3*c - 4*a^2*b^2)) * (b^2*d + 2*a^2*f - a*b*e - 2*a*c*d)) / (4*a^2*(4*a*c - b^2)^{(1/2)}) + (b^2*c^2*(b^2*d + 2*a^2*f - a*b*e - 2*a*c*d)^3) / (16*a^5*(4*a*c - b^2)^{(3/2)))* (6*b^5*d + 2*a^2*b^3*f - 20*a^3*c^2*e - 6*a*b^4*e - 30*a*b^3*c*d - 6*a^3*b*c*f + 26*a^2*b*c^2*d + 28*a^2*b^2*c*e)) / (16*a^3*c^2*(4*a*c - b^2)^{(1/2)}*(a^4*f^2 - 6*b^4*d^2 + 25*a^3*c*e^2 - 6*a^2*b^2*e^2 + a^2*c^2*d^2 + 12*a*b^3*d*e - a^3*b*e*f - 2*a^3*c*d*f + 24*a*b^2*c*d^2 + a^2*b^2*d*f - 49*a^2*b*c*d*e))))) / (4*a^2*c^4*d^2 + b^4*c^2*d^2 + 4*a^4*c^2*f^2 - 4*a*b^2*c^3*d^2 + a^2*b^2*c^2*e^2 - 8*a^3*c^3*d*f - 2*a*b^3*c^2*d*e + 4*a^2*b*c^3*d*e - 4*a^3*b*c^2*e*f + 4*a^2*b^2*c^2*d*f)) * (b^2*d + 2*a^2*f - a*b*e - 2*a*c*d)) / (2*a^2*(4*a*c - b^2)^{(1/2)})
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**4+e*x**2+d)/x**3/(c*x**4+b*x**2+a), x)

[Out] Timed out

$$3.51 \quad \int \frac{d+ex^2+fx^4}{x^5(a+bx^2+cx^4)} dx$$

Optimal. Leaf size=174

$$\frac{\log(a+bx^2+cx^4)(-abe-a(cd-af)+b^2d)}{4a^3} + \frac{\log(x)(-abe-a(cd-af)+b^2d)}{a^3} + \frac{bd-ae}{2a^2x^2} + \frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{a^3}$$

Rubi [A] time = 0.41, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1663, 1628, 634, 618, 206, 628}

$$\frac{\log(a+bx^2+cx^4)(-abe-a(cd-af)+b^2d)}{4a^3} + \frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(2a^2ce-ab^2e-ab(3cd-af)+b^3d)}{2a^3\sqrt{b^2-4ac}} + \frac{\log(x)(-abe-a(cd-af)+b^2d)}{a^3} + \frac{bd-ae}{2a^2x^2} - \frac{d}{4ax^4}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2 + f*x^4)/(x^5*(a + b*x^2 + c*x^4)), x]

[Out] -d/(4*a*x^4) + (b*d - a*e)/(2*a^2*x^2) + ((b^3*d - a*b^2*e + 2*a^2*c*e - a*b*(3*c*d - a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^3*Sqrt[b^2 - 4*a*c]) + ((b^2*d - a*b*e - a*(c*d - a*f))*Log[x])/a^3 - ((b^2*d - a*b*e - a*(c*d - a*f))*Log[a + b*x^2 + c*x^4])/(4*a^3)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1628

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 1663

Int[(Pq_)*(x_)^(m_.)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m-1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[

(m - 1)/2]

Rubi steps

$$\begin{aligned}
\int \frac{d + ex^2 + fx^4}{x^5(a + bx^2 + cx^4)} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{d + ex + fx^2}{x^3(a + bx + cx^2)} dx, x, x^2 \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \left(\frac{d}{ax^3} + \frac{-bd + ae}{a^2x^2} + \frac{b^2d - abe - a(cd - af)}{a^3x} + \frac{-b^3d + ab^2e - a^2ce + ab(2cd - af)}{a^3} \right) dx, x, x^2 \right) \\
&= -\frac{d}{4ax^4} + \frac{bd - ae}{2a^2x^2} + \frac{(b^2d - abe - a(cd - af)) \log(x)}{a^3} + \frac{\text{Subst} \left(\int \frac{-b^3d + ab^2e - a^2ce + ab(2cd - af)}{a^3} dx, x, x^2 \right)}{4a^3} \\
&= -\frac{d}{4ax^4} + \frac{bd - ae}{2a^2x^2} + \frac{(b^2d - abe - a(cd - af)) \log(x)}{a^3} - \frac{(b^2d - abe - a(cd - af)) \text{Subst} \left(\int \frac{1}{x} dx, x, x^2 \right)}{4a^3} \\
&= -\frac{d}{4ax^4} + \frac{bd - ae}{2a^2x^2} + \frac{(b^2d - abe - a(cd - af)) \log(x)}{a^3} - \frac{(b^2d - abe - a(cd - af)) \log(x)}{4a^3} \\
&= -\frac{d}{4ax^4} + \frac{bd - ae}{2a^2x^2} + \frac{(b^3d - ab^2e + 2a^2ce - ab(3cd - af)) \tanh^{-1} \left(\frac{b+2cx^2}{\sqrt{b^2-4ac}} \right)}{2a^3\sqrt{b^2-4ac}} + \frac{(b^2d - abe - a(cd - af)) \log(x)}{4a^3}
\end{aligned}$$

Mathematica [A] time = 0.35, size = 314, normalized size = 1.80

$$\frac{e^2 d}{x^4} - 4 \log(x) (-abe + a(af - cd) + b^2 d) + \frac{\log(-\sqrt{b^2-4ac} + b + 2cx^2) (ab(-c\sqrt{b^2-4ac} + af - 3cd) + (-cd\sqrt{b^2-4ac} + af\sqrt{b^2-4ac} + 2ac)) + b^2(d\sqrt{b^2-4ac} - ab) + b^3 d}{\sqrt{b^2-4ac}} + \frac{\log(\sqrt{b^2-4ac} + b + 2cx^2) (-ab(c\sqrt{b^2-4ac} + af - 3cd) + (-af\sqrt{b^2-4ac} - (d\sqrt{b^2-4ac} + 2ac)) + b^2(d\sqrt{b^2-4ac} + ab) + b^3(-d))}{\sqrt{b^2-4ac}} + \frac{2a(ac - bd)}{x^2}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2 + f*x^4)/(x^5*(a + b*x^2 + c*x^4)), x]

[Out] -1/4*((a^2*d)/x^4 + (2*a*(-(b*d) + a*e))/x^2 - 4*(b^2*d - a*b*e + a*(-(c*d) + a*f))*Log[x] + ((b^3*d + b^2*(Sqrt[b^2 - 4*a*c]*d - a*e) + a*b*(-3*c*d - Sqrt[b^2 - 4*a*c]*e + a*f) + a*(-(c*Sqrt[b^2 - 4*a*c]*d) + 2*a*c*e + a*Sqrt[b^2 - 4*a*c]*f))*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c] + (((-b^3*d) + b^2*(Sqrt[b^2 - 4*a*c]*d + a*e) - a*b*(-3*c*d + Sqrt[b^2 - 4*a*c]*e + a*f) + a*(-(c*(Sqrt[b^2 - 4*a*c]*d + 2*a*e)) + a*Sqrt[b^2 - 4*a*c]*f))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c])/a^3

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex^2 + fx^4}{x^5(a + bx^2 + cx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e*x^2 + f*x^4)/(x^5*(a + b*x^2 + c*x^4)), x]

[Out] IntegrateAlgebraic[(d + e*x^2 + f*x^4)/(x^5*(a + b*x^2 + c*x^4)), x]

fricas [A] time = 3.38, size = 609, normalized size = 3.50

$$\frac{d}{x^5} + \frac{e}{x^3} + \frac{f}{x} + \frac{a^2 d + a b e + a^2 c d + b^2 d - a^2 c d - a b e - a^2 c d}{a^3 x^3} + \frac{a b e + a^2 c d + b^2 d - a^2 c d - a b e - a^2 c d}{a^3 x} + \frac{a b e + a^2 c d + b^2 d - a^2 c d - a b e - a^2 c d}{a^3} \tanh^{-1} \left(\frac{b + 2 c x^2}{\sqrt{b^2 - 4 a c}} \right) + \frac{a b e + a^2 c d + b^2 d - a^2 c d - a b e - a^2 c d}{a^3} \log \left(\frac{b + 2 c x^2 + \sqrt{b^2 - 4 a c}}{b + 2 c x^2 - \sqrt{b^2 - 4 a c}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^4+e*x^2+d)/x^5/(c*x^4+b*x^2+a), x, algorithm="fricas")

```
[Out] [1/4*((a^2*b*f + (b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e)*sqrt(b^2 - 4*a*c)
*x^4*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*
a*c))/(c*x^4 + b*x^2 + a)) - ((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d - (a*b^3 - 4*
a^2*b*c)*e + (a^2*b^2 - 4*a^3*c)*f)*x^4*log(c*x^4 + b*x^2 + a) + 4*((b^4 -
5*a*b^2*c + 4*a^2*c^2)*d - (a*b^3 - 4*a^2*b*c)*e + (a^2*b^2 - 4*a^3*c)*f)*x
^4*log(x) + 2*((a*b^3 - 4*a^2*b*c)*d - (a^2*b^2 - 4*a^3*c)*e)*x^2 - (a^2*b^
2 - 4*a^3*c)*d)/((a^3*b^2 - 4*a^4*c)*x^4), 1/4*(2*(a^2*b*f + (b^3 - 3*a*b*c
)*d - (a*b^2 - 2*a^2*c)*e)*sqrt(-b^2 + 4*a*c)*x^4*arctan(-(2*c*x^2 + b)*sq
rt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d - (a*b^3
- 4*a^2*b*c)*e + (a^2*b^2 - 4*a^3*c)*f)*x^4*log(c*x^4 + b*x^2 + a) + 4*((b^
4 - 5*a*b^2*c + 4*a^2*c^2)*d - (a*b^3 - 4*a^2*b*c)*e + (a^2*b^2 - 4*a^3*c)*
f)*x^4*log(x) + 2*((a*b^3 - 4*a^2*b*c)*d - (a^2*b^2 - 4*a^3*c)*e)*x^2 - (a^
2*b^2 - 4*a^3*c)*d)/((a^3*b^2 - 4*a^4*c)*x^4)]
```

giac [A] time = 1.72, size = 212, normalized size = 1.22

$$\frac{(b^2d - acd + a^2f - abe) \log(cx^4 + bx^2 + a)}{4a^3} + \frac{(b^2d - acd + a^2f - abe) \log(x^2)}{2a^3} - \frac{(b^3d - 3abcd + a^2bf - ab^2e + 2a^2ce) \arctan\left(\frac{2cx^2 + b}{\sqrt{-b^2 + 4ac}}\right)}{2\sqrt{-b^2 + 4ac}a^3} - \frac{3b^2dx^4 - 3acdx^4 + 3a^2fx^4 - 3abx^4e - 2abdx^2 + 2a^2x^2e + a^2d}{4a^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^5/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*(b^2*d - a*c*d + a^2*f - a*b*e)*log(c*x^4 + b*x^2 + a)/a^3 + 1/2*(b^2*
d - a*c*d + a^2*f - a*b*e)*log(x^2)/a^3 - 1/2*(b^3*d - 3*a*b*c*d + a^2*b*f
- a*b^2*e + 2*a^2*c*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2
+ 4*a*c)*a^3) - 1/4*(3*b^2*d*x^4 - 3*a*c*d*x^4 + 3*a^2*f*x^4 - 3*a*b*x^4*e
- 2*a*b*d*x^2 + 2*a^2*x^2*e + a^2*d)/(a^3*x^4)
```

maple [B] time = 0.01, size = 356, normalized size = 2.05

$$\frac{bf \arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right)}{2\sqrt{4ac-b^2}a} - \frac{ce \arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right)}{\sqrt{4ac-b^2}a} + \frac{b^2e \arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right)}{2\sqrt{4ac-b^2}a^2} + \frac{3bcd \arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right)}{2\sqrt{4ac-b^2}a^2} - \frac{b^3d \arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right)}{2\sqrt{4ac-b^2}a^3} + \frac{f \ln(x)}{a} - \frac{f \ln(cx^4 + bx^2 + a)}{4a} - \frac{be \ln(x)}{a^2} + \frac{be \ln(cx^4 + bx^2 + a)}{4a^2} - \frac{cd \ln(x)}{a^2} + \frac{cd \ln(cx^4 + bx^2 + a)}{4a^2} + \frac{b^2d \ln(x)}{a^3} - \frac{b^2d \ln(cx^4 + bx^2 + a)}{4a^3} - \frac{e}{2ax^2} + \frac{bd}{2a^2x^2} - \frac{d}{4ax^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^4+e*x^2+d)/x^5/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/4*d/a/x^4-1/2/a/x^2*e+1/2/a^2/x^2*b*d+1/a*ln(x)*f-1/a^2*ln(x)*b*e-1/a^2*
ln(x)*c*d+1/a^3*ln(x)*b^2*d-1/4/a*ln(c*x^4+b*x^2+a)*f+1/4/a^2*ln(c*x^4+b*x^
2+a)*b*e+1/4/a^2*c*ln(c*x^4+b*x^2+a)*d-1/4/a^3*ln(c*x^4+b*x^2+a)*b^2*d-1/2/
a/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b*f-1/a/(4*a*c-b^
2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*c*e+1/2/a^2/(4*a*c-b^2)^(1/2
)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^2*e+3/2/a^2/(4*a*c-b^2)^(1/2)*arc
tan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b*c*d-1/2/a^3/(4*a*c-b^2)^(1/2)*arctan((
2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^3*d
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^5/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 9.92, size = 6187, normalized size = 35.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x^2 + f*x^4)/(x^5*(a + b*x^2 + c*x^4)),x)

[Out] (log(x)*(b^2*d + a^2*f - a*b*e - a*c*d))/a^3 - (d/(4*a) + (x^2*(a*e - b*d))/(2*a^2))/x^4 + (log(((((((2*c^3*x^2*(b^3*d - a*b^2*e + 5*a^2*b*f - 10*a^2*c*e + 5*a*b*c*d))/a^2 + (4*b*c^2*(b^3*d - a*b^2*e + a^2*b*f + a^2*c*e - 2*a*b*c*d))/a^2 + (b*c^2*(a*b + 3*b^2*x^2 - 10*a*c*x^2)*(b^2*d + a^2*f + a^3*(-(b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 3*a*b*c*d)^2/(a^6*(4*a*c - b^2))))^(1/2) - a*b*e - a*c*d))/a^3)*(b^2*d + a^2*f + a^3*(-(b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 3*a*b*c*d)^2/(a^6*(4*a*c - b^2))))^(1/2) - a*b*e - a*c*d))/(4*a^3) + (c^3*(a*e - b*d)*(4*b^3*d - 4*a*b^2*e + 4*a^2*b*f + a^2*c*e - 5*a*b*c*d))/a^4 + (c^4*x^2*(a*e - b*d)*(6*b^2*d + 5*a^2*f - 6*a*b*e - 5*a*c*d))/a^4*(b^2*d + a^2*f + a^3*(-(b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 3*a*b*c*d)^2/(a^6*(4*a*c - b^2))))^(1/2) - a*b*e - a*c*d)/(4*a^3) + (c^4*(a*e - b*d)^2*(b^2*d + a^2*f - a*b*e - a*c*d))/a^6 - (c^5*x^2*(a*e - b*d)^3)/a^6)*(((c^3*(a*e - b*d)*(4*b^3*d - 4*a*b^2*e + 4*a^2*b*f + a^2*c*e - 5*a*b*c*d))/a^4 - (((2*c^3*x^2*(b^3*d - a*b^2*e + 5*a^2*b*f - 10*a^2*c*e + 5*a*b*c*d))/a^2 + (4*b*c^2*(b^3*d - a*b^2*e + a^2*b*f + a^2*c*e - 2*a*b*c*d))/a^2 - (b*c^2*(a*b + 3*b^2*x^2 - 10*a*c*x^2)*(a^3*(-(b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 3*a*b*c*d)^2/(a^6*(4*a*c - b^2))))^(1/2) - a^2*f - b^2*d + a*b*e + a*c*d))/a^3)*(a^3*(-(b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 3*a*b*c*d)^2/(a^6*(4*a*c - b^2))))^(1/2) - a^2*f - b^2*d + a*b*e + a*c*d))/(4*a^3) + (c^4*x^2*(a*e - b*d)*(6*b^2*d + 5*a^2*f - 6*a*b*e - 5*a*c*d))/a^4*(a^3*(-(b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 3*a*b*c*d)^2/(a^6*(4*a*c - b^2))))^(1/2) - a^2*f - b^2*d + a*b*e + a*c*d))/(4*a^3) - (c^4*(a*e - b*d)^2*(b^2*d + a^2*f - a*b*e - a*c*d))/a^6 + (c^5*x^2*(a*e - b*d)^3)/a^6))*(2*b^4*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a*b^2*c*d + 8*a^2*b*c*e))/((2*(16*a^4*c - 4*a^3*b^2)) - (atan((16*a^9*(4*a*c - b^2)^(3/2)*(x^2*((((a^3*c^5*e^3 - b^3*c^5*d^3 + 3*a*b^2*c^5*d^2*e - 3*a^2*b*c^5*d*e^2)/a^6 - ((6*a^4*b*c^4*e^2 - 5*a^3*b*c^5*d^2 + 6*a^2*b^3*c^4*d^2 + 5*a^4*c^5*d*e - 5*a^5*c^4*e*f + 5*a^4*b*c^4*d*f - 12*a^3*b^2*c^4*d*e)/a^6 + (((2*a^4*b^3*c^3*d - 20*a^6*c^4*e - 2*a^5*b^2*c^3*e + 10*a^5*b*c^4*d + 10*a^6*b*c^3*f)/a^6 + ((40*a^7*b*c^3 - 12*a^6*b^3*c^2)*(2*b^4*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a*b^2*c*d + 8*a^2*b*c*e))/(2*a^6*(16*a^4*c - 4*a^3*b^2)))*(2*b^4*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a*b^2*c*d + 8*a^2*b*c*e))/(2*(16*a^4*c - 4*a^3*b^2)) + (((((2*a^4*b^3*c^3*d - 20*a^6*c^4*e - 2*a^5*b^2*c^3*e + 10*a^5*b*c^4*d + 10*a^6*b*c^3*f)/a^6 + ((40*a^7*b*c^3 - 12*a^6*b^3*c^2)*(2*b^4*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a*b^2*c*d + 8*a^2*b*c*e))/(2*a^6*(16*a^4*c - 4*a^3*b^2)))*(b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 3*a*b*c*d))/(4*a^3*(4*a*c - b^2)^(1/2)) + ((40*a^7*b*c^3 - 12*a^6*b^3*c^2)*(b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 3*a*b*c*d)*(2*b^4*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a*b^2*c*d + 8*a^2*b*c*e))/(8*a^9*(4*a*c - b^2)^(1/2)*(16*a^4*c - 4*a^3*b^2)))*(b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 3*a*b*c*d))/(4*a^3*(4*a*c - b^2)^(1/2)) + ((40*a^7*b*c^3 - 12*a^6*b^3*c^2)*(b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 3*a*b*c*d)^2*(2*b^4*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a*b^2*c*d + 8*a^2*b*c*e))/(32*a^12*(4*a*c - b^2)*(16*a^4*c - 4*a^3*b^2)))*(3*b^5*d + 3*a^2*b^3*f - a^3*c^2*e - 3*a*b^4*e - 12*a*b^3*c*d - 8*a^3*b*c*f + 9*a^2*b*c^2*d + 9*a^2*b^2*c*e))/(8*a^3*c^2*(25*a^5*c*f^2 - 6*b^6*d^2 - 6*a^2*b^4*e^2 + 25*a^3*c^3*d^2 - 6*a^4*b^2*f^2 + a^4*c^2*e^2 + 24*a^3*b^2*c*e^2 + 12*a*b^5*d*e - 54*a^2*b^2*c^2*d^2 + 36*a*b^4*c*d^2 - 12*a^2*b^4*d*f + 12*a^3*b^3*e*f - 50*a^4*c^2*d*f - 60*a^2*b^3*c*d*e + 47*a^3*b*c^2*d*e + 61*a^3*b^2*c*d*f - 49*a^4*b*c*e*f)) + (((((((2*a^4*b^3*c^3*d - 20*a^6*c^4*e - 2*a^5*b^2*c^3*e + 10*a^5*b*c^4*d + 10*a^6*b*c^3*f)/a^6 + ((40*a^7*b*c^3 - 12*a^6*b^3*c^2)*(2*b^4*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a*b^2*c*d + 8*a^2*b*c*e))/(2*a^6*(16*a^4*c - 4*a^3*b^2)))*(b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 3*a*b*c*d))/(4*a^3*(4*a*c - b^2)^(1/2)) + ((40*a^7*b*c^3 - 12*a^6*b^3*c^2)*(b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 3*a*b*c*d)*(2*b^4*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a*b^2*c*d + 8*a^2*b*c*e))/(2*a^6*(16*a^4*c - 4*a^3*b^2)))*(b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 3*a*b*c*d))/(4*a^3*(4*a*c - b^2)^(1/2)) + ((40*a^7*b*c^3 - 12*a^6*b^3*c^2)*(b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 3*a*b*c*d)^2*(2*b^4*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a*b^2*c*d + 8*a^2*b*c*e))/(32*a^12*(4*a*c - b^2)*(16*a^4*c - 4*a^3*b^2)))*(3*b^5*d + 3*a^2*b^3*f - a^3*c^2*e - 3*a*b^4*e - 12*a*b^3*c*d - 8*a^3*b*c*f + 9*a^2*b*c^2*d + 9*a^2*b^2*c*e))/(8*a^3*c^2*(25*a^5*c*f^2 - 6*b^6*d^2 - 6*a^2*b^4*e^2 + 25*a^3*c^3*d^2 - 6*a^4*b^2*f^2 + a^4*c^2*e^2 + 24*a^3*b^2*c*e^2 + 12*a*b^5*d*e - 54*a^2*b^2*c^2*d^2 + 36*a*b^4*c*d^2 - 12*a^2*b^4*d*f + 12*a^3*b^3*e*f - 50*a^4*c^2*d*f - 60*a^2*b^3*c*d*e + 47*a^3*b*c^2*d*e + 61*a^3*b^2*c*d*f - 49*a^4*b*c*e*f)) + (((((((2*a^4*b^3*c^3*d - 20*a^6*c^4*e - 2*a^5*b^2*c^3*e + 10*a^5*b*c^4*d + 10*a^6*b*c^3*f)/a^6 + ((40*a^7*b*c^3 - 12*a^6*b^3*c^2)*(2*b^4*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a*b^2*c*d + 8*a^2*b*c*e))/(2*a^6*(16*a^4*c - 4*a^3*b^2)))*(b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 3*a*b*c*d))/(4*a^3*(4*a*c - b^2)^(1/2)) + ((40*a^7*b*c^3 - 12*a^6*b^3*c^2)*(b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 3*a*b*c*d)^2*(2*b^4*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a*b^2*c*d + 8*a^2*b*c*e))/(32*a^12*(4*a*c - b^2)*(16*a^4*c - 4*a^3*b^2)))*(3*b^5*d + 3*a^2*b^3*f - a^3*c^2*e - 3*a*b^4*e - 12*a*b^3*c*d - 8*a^3*b*c*f + 9*a^2*b*c^2*d + 9*a^2*b^2*c*e))/(8*a^3*c^2*(25*a^5*c*f^2 - 6*b^6*d^2 - 6*a^2*b^4*e^2 + 25*a^3*c^3*d^2 - 6*a^4*b^2*f^2 + a^4*c^2*e^2 + 24*a^3*b^2*c*e^2 + 12*a*b^5*d*e - 54*a^2*b^2*c^2*d^2 + 36*a*b^4*c*d^2 - 12*a^2*b^4*d*f + 12*a^3*b^3*e*f - 50*a^4*c^2*d*f - 60*a^2*b^3*c*d*e + 47*a^3*b*c^2*d*e + 61*a^3*b^2*c*d*f - 49*a^4*b*c*e*f)) + (((((((2*a^4*b^3*c^3*d - 20*a^6*c^4*e - 2*a^5*b^2*c^3*e + 10*a^5*b*c^4*d + 10*a^6*b*c^3*f)/a^6 + ((40*a^7*b*c^3 - 12*a^6*b^3*c^2)*(2*b^4*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a*b^2*c*d + 8*a^2*b*c*e))/(2*a^6*(16*a^4*c - 4*a^3*b^2)))*(b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 3*a*b*c*d))/(4*a^3*(4*a*c - b^2)^(1/2)) + ((40*a^7*b*c^3 - 12*a^6*b^3*c^2)*(b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 3*a*b*c*d)^2*(2*b^4*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a*b^2*c*d + 8*a^2*b*c*e))/(32*a^12*(4*a*c - b^2)*(16*a^4*c - 4*a^3*b^2)))*(3*b^5*d + 3*a^2*b^3*f - a^3*c^2*e - 3*a*b^4*e - 12*a*b^3*c*d - 8*a^3*b*c*f + 9*a^2*b*c^2*d + 9*a^2*b^2*c*e))/(8*a^3*c^2*(25*a^5*c*f^2 - 6*b^6*d^2 - 6*a^2*b^4*e^2 + 25*a^3*c^3*d^2 - 6*a^4*b^2*f^2 + a^4*c^2*e^2 + 24*a^3*b^2*c*e^2 + 12*a*b^5*d*e - 54*a^2*b^2*c^2*d^2 + 36*a*b^4*c*d^2 - 12*a^2*b^4*d*f + 12*a^3*b^3*e*f - 50*a^4*c^2*d*f - 60*a^2*b^3*c*d*e + 47*a^3*b*c^2*d*e + 61*a^3*b^2*c*d*f - 49*a^4*b*c*e*f)) + (((((((2*a^4*b^3*c^3*d - 20*a^6*c^4*e - 2*a^5*b^2*c^3*e + 10*a^5*b*c^4*d + 10*a^6*b*c^3*f)/a^6 + ((40*a^7*b*c^3 - 12*a^6*b^3*c^2)*(2*b^4*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a*b^2*c*d + 8*a^2*b*c*e))/(2*a^6*(16*a^4*c - 4*a^3*b^2)))*(b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 3*a*b*c*d))/(4*a^3*(4*a*c - b^2)^(1/2)) + ((40*a^7*b*c^3 - 12*a^6*b^3*c^2)*(b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 3*a*b*c*d)^2*(2*b^4*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a*b^2*c*d + 8*a^2*b*c*e))/(32*a^12*(4*a*c - b^2)*(16*a^4*c - 4*a^3*b^2)))*(3*b^5*d + 3*a^2*b^3*f - a^3*c^2*e - 3*a*b^4*e - 12*a*b^3*c*d - 8*a^3*b*c*f + 9*a^2*b*c^2*d + 9*a^2*b^2*c*e))/(8*a^3*c^2*(25*a^5*c*f^2 - 6*b^6*d^2 - 6*a^2*b^4*e^2 + 25*a^3*c^3*d^2 - 6*a^4*b^2*f^2 + a^4*c^2*e^2 + 24*a^3*b^2*c*e^2 + 12*a*b^5*d*e - 54*a^2*b^2*c^2*d^2 + 36*a*b^4*c*d^2 - 12*a^2*b^4*d*f + 12*a^3*b^3*e*f - 50*a^4*c^2*d*f - 60*a^2*b^3*c*d*e + 47*a^3*b*c^2*d*e + 61*a^3*b^2*c*d*f - 49*a^4*b*c*e*f)) + ((

$$\begin{aligned}
& \left(2*c*d + 8*a^2*b*c*e \right) / \left(8*a^9*(4*a*c - b^2)^{(1/2)}*(16*a^4*c - 4*a^3*b^2) \right) * \\
& \left(2*b^4*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a*b^2*c*d \right. \\
& \left. + 8*a^2*b*c*e \right) / \left(2*(16*a^4*c - 4*a^3*b^2) \right) + \left((6*a^4*b*c^4*e^2 - 5*a^3*b*c^5*d^2 + 6*a^2*b^3*c^4*d^2 \right. \\
& \left. + 5*a^4*c^5*d*e - 5*a^5*c^4*e*f + 5*a^4*b*c^4*d*f - 12*a^3*b^2*c^4*d*e) / a^6 + \left((2*a^4*b^3*c^3*d - 20*a^6*c^4*e - 2*a^5*b^2*c^3*e \right. \right. \\
& \left. \left. + 10*a^5*b*c^4*d + 10*a^6*b*c^3*f) / a^6 + \left((40*a^7*b*c^3 - 12*a^6*b^3*c^2) * (2*b^4*d + 8*a^2*c^2*d \right. \right. \right. \\
& \left. \left. + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a*b^2*c*d + 8*a^2*b*c*e) \right) / \left(2 * (16*a^4*c - 4*a^3*b^2) \right) \right) * \\
& \left(b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 3*a*b*c*d \right) / \left(4*a^3*(4*a*c - b^2)^{(1/2)} \right) - \left((40*a^7*b*c^3 - 12*a^6*b^3*c^2) * (b^3*d - \right. \\
& \left. a*b^2*e + a^2*b*f + 2*a^2*c*e - 3*a*b*c*d)^3 \right) / \left(64*a^{15}*(4*a*c - b^2)^{(3/2)} \right) * \left(6*b^6*d - 20*a^3*c^3*d \right. \\
& \left. + 6*a^2*b^4*f + 20*a^4*c^2*f - 6*a*b^5*e + 54*a^2*b^2*c^2*d - 36*a*b^4*c*d + 30*a^2*b^3*c*e - 26*a^3*b*c^2*e - 28*a^3*b^2*c \right. \\
& \left. * f \right) / \left(16*a^3*c^2*(4*a*c - b^2)^{(1/2)}*(25*a^5*c*f^2 - 6*b^6*d^2 - 6*a^2*b^4*e^2 + 25*a^3*c^3*d^2 - 6*a^4*b^2*f^2 \right. \\
& \left. + a^4*c^2*e^2 + 24*a^3*b^2*c*e^2 + 12*a*b^5*d*e - 54*a^2*b^2*c^2*d^2 + 36*a*b^4*c*d^2 - 12*a^2*b^4*d*f + 12*a^3*b^3*e*f - 50*a^4*c^2*d*f \right. \\
& \left. - 60*a^2*b^3*c*d*e + 47*a^3*b*c^2*d*e + 61*a^3*b^2*c*d*f - 49*a^4*b*c*e*f) \right) - \left((b^4*c^4*d^3 - a*b^2*c^5*d^3 - a^3*b*c^4*e^3 \right. \\
& \left. - a^3*c^5*d*e^2 + a^4*c^4*e^2*f - 3*a*b^3*c^4*d^2*e + 2*a^2*b*c^5*d^2*e + 3*a^2*b^2*c^4*d*e^2 + a^2*b^2*c^4*d^2*f \right. \\
& \left. - 2*a^3*b*c^4*d*e*f) / a^6 - \left((a^5*c^4*e^2 - 4*a^2*b^4*c^3*d^2 + 5*a^3*b^2*c^4*d^2 - 4*a^4*b^2*c^3*e^2 - 6*a^4*b*c^4*d*e \right. \right. \\
& \left. \left. + 4*a^5*b*c^3*e*f + 8*a^3*b^3*c^3*d*e - 4*a^4*b^2*c^3*d*f) / a^6 - \left((4*a^4*b^4*c^2*d - 8*a^5*b^2*c^3*d - 4*a^5*b^3*c^2*e \right. \right. \right. \\
& \left. \left. + 4*a^6*b^2*c^2*f + 4*a^6*b*c^3*e) / a^6 - (2*a*b^2*c^2*(2*b^4*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a*b^2*c*d \right. \right. \\
& \left. \left. + 8*a^2*b*c*e) \right) / \left(16*a^4*c - 4*a^3*b^2) \right) * \left(2*b^4*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a*b^2*c*d \right. \\
& \left. + 8*a^2*b*c*e \right) / \left(2*(16*a^4*c - 4*a^3*b^2) \right) * \left(2*b^4*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a*b^2*c*d \right. \\
& \left. + 8*a^2*b*c*e \right) / \left(2*(16*a^4*c - 4*a^3*b^2) \right) - \left((4*a^4*b^4*c^2*d - 8*a^5*b^2*c^3*d - 4*a^5*b^3*c^2*e + 4*a^6*b^2*c^2*f + 4 \right. \\
& \left. * a^6*b*c^3*e) / a^6 - (2*a*b^2*c^2*(2*b^4*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a*b^2*c*d + 8*a^2*b*c*e) \right) / \left(16*a^4 \right. \\
& \left. * c - 4*a^3*b^2) \right) * \left(b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 3*a*b*c*d \right) / \left(4*a^3*(4*a*c - b^2)^{(1/2)} \right) - \left(b^2*c^2*(b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - \right. \\
& \left. 3*a*b*c*d) * (2*b^4*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a*b^2*c*d + 8*a^2*b*c*e) \right) / \left(2*a^2*(4*a*c - b^2)^{(1/2)}*(16*a^4*c - 4*a^3*b^2) \right) * \\
& \left(b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 3*a*b*c*d \right) / \left(4*a^3*(4*a*c - b^2)^{(1/2)} \right) + \left(b^2*c^2*(b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 3*a*b*c*d) \right)^2 \\
& * \left(2*b^4*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a*b^2*c*d + 8*a^2*b*c*e \right) / \left(8*a^5*(4*a*c - b^2)*(16*a^4*c - 4*a^3*b^2) \right) * \left(3*b^5*d + \right. \\
& \left. 3*a^2*b^3*f - a^3*c^2*e - 3*a*b^4*e - 12*a*b^3*c*d - 8*a^3*b*c*f + 9*a^2*b*c^2*d + 9*a^2*b^2*c*e \right) / \left(8*a^3*c^2*(25*a^5*c*f^2 - 6*b^6*d^2 - 6*a^2*b^4*e^2 \right. \\
& \left. + 25*a^3*c^3*d^2 - 6*a^4*b^2*f^2 + a^4*c^2*e^2 + 24*a^3*b^2*c*e^2 + 12*a*b^5*d*e - 54*a^2*b^2*c^2*d^2 + 36*a*b^4*c*d^2 - 12*a^2*b^4*d*f + 12*a^3*b^3 \right. \\
& \left. * e*f - 50*a^4*c^2*d*f - 60*a^2*b^3*c*d*e + 47*a^3*b*c^2*d*e + 61*a^3*b^2*c*d*f - 49*a^4*b*c*e*f) \right) + \left((4*a^4*b^4*c^2*d - 8*a^5*b^2*c^3*d - 4*a^5*b^3*c^2*e \right. \\
& \left. + 4*a^6*b^2*c^2*f + 4*a^6*b*c^3*e) / a^6 - (2*a*b^2*c^2*(2*b^4*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a*b^2*c*d + 8*a^2*b*c \right. \\
& \left. * e) \right) / \left(16*a^4*c - 4*a^3*b^2) \right) * \left(b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 3*a*b*c*d \right) / \left(4*a^3*(4*a*c - b^2)^{(1/2)} \right) - \left(b^2*c^2*(b^3*d - a*b^2*e + a^2*b*f + \right. \\
& \left. 2*a^2*c*e - 3*a*b*c*d) * (2*b^4*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a*b^2*c*d + 8*a^2*b*c*e) \right) / \left(2*a^2*(4*a*c - b^2)^{(1/2)}*(16*a^4*c \right. \\
& \left. - 4*a^3*b^2) \right) * \left(2*b^4*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a*b^2*c*d + 8*a^2*b*c*e \right) / \left(2*(16*a^4*c - 4*a^3*b^2) \right) - \left((a^5*c^4*e \right. \\
& \left. ^2 - 4*a^2*b^4*c^3*d^2 + 5*a^3*b^2*c^4*d^2 - 4*a^4*b^2*c^3*e^2 - 6*a^4*b*c^4*d*e + 4*a^5*b*c^3*e*f + 8*a^3*b^3*c^3*d*e - 4*a^4*b^2*c^3*d*f) / a^6 - \left((4 \right. \right. \\
& \left. \left. * a^4*b^4*c^2*d - 8*a^5*b^2*c^3*d - 4*a^5*b^3*c^2*e + 4*a^6*b^2*c^2*f + 4*a^6*b*c^3*e) / a^6 - (2*a*b^2*c^2*(2*b^4*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3 \right. \right. \\
& \left. \left. * e - 8*a^3*c*f - 10*a*b^2*c*d + 8*a^2*b*c*e) \right) / \left(16*a^4*c - 4*a^3*b^2) \right) * \left(2*b \right.
\end{aligned}$$

$$\frac{\begin{aligned} &^4*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a*b^2*c*d + 8 \\ &*a^2*b*c*e)/(2*(16*a^4*c - 4*a^3*b^2)))*(b^3*d - a*b^2*e + a^2*b*f + 2*a^2 \\ &*c*e - 3*a*b*c*d)/(4*a^3*(4*a*c - b^2)^{(1/2)}) + (b^2*c^2*(b^3*d - a*b^2*e \\ &+ a^2*b*f + 2*a^2*c*e - 3*a*b*c*d)^3)/(16*a^8*(4*a*c - b^2)^{(3/2)})*(6*b^6* \\ &d - 20*a^3*c^3*d + 6*a^2*b^4*f + 20*a^4*c^2*f - 6*a*b^5*e + 54*a^2*b^2*c^2* \\ &d - 36*a*b^4*c*d + 30*a^2*b^3*c*e - 26*a^3*b*c^2*e - 28*a^3*b^2*c*f)/(16*a \\ &^3*c^2*(4*a*c - b^2)^{(1/2)}*(25*a^5*c*f^2 - 6*b^6*d^2 - 6*a^2*b^4*e^2 + 25*a \\ &^3*c^3*d^2 - 6*a^4*b^2*f^2 + a^4*c^2*e^2 + 24*a^3*b^2*c*e^2 + 12*a*b^5*d*e \\ &- 54*a^2*b^2*c^2*d^2 + 36*a*b^4*c*d^2 - 12*a^2*b^4*d*f + 12*a^3*b^3*e*f - 5 \\ &0*a^4*c^2*d*f - 60*a^2*b^3*c*d*e + 47*a^3*b*c^2*d*e + 61*a^3*b^2*c*d*f - 49 \\ &*a^4*b*c*e*f)))/(4*a^4*c^4*e^2 + b^6*c^2*d^2 - 6*a*b^4*c^3*d^2 + 9*a^2*b^2 \\ &*c^4*d^2 + a^2*b^4*c^2*e^2 - 4*a^3*b^2*c^3*e^2 + a^4*b^2*c^2*f^2 - 2*a*b^5* \\ &c^2*d*e - 12*a^3*b*c^4*d*e + 4*a^4*b*c^3*e*f + 10*a^2*b^3*c^3*d*e + 2*a^2*b \\ &^4*c^2*d*f - 6*a^3*b^2*c^3*d*f - 2*a^3*b^3*c^2*e*f)*(b^3*d - a*b^2*e + a^2 \\ &*b*f + 2*a^2*c*e - 3*a*b*c*d))/(2*a^3*(4*a*c - b^2)^{(1/2)}) \end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**4+e*x**2+d)/x**5/(c*x**4+b*x**2+a),x)

[Out] Timed out

$$3.52 \quad \int \frac{d+ex^2+fx^4}{x^7(a+bx^2+cx^4)} dx$$

Optimal. Leaf size=244

$$-\frac{-abe - a(cd - af) + b^2d}{2a^3x^2} + \frac{bd - ae}{4a^2x^4} + \frac{\log(a + bx^2 + cx^4)(a^2ce - ab^2e - ab(2cd - af) + b^3d)}{4a^4} - \frac{\log(x)(a^2ce - ab^2e - ab(2cd - af) + b^3d)}{4a^4}$$

Rubi [A] time = 0.57, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1663, 1628, 634, 618, 206, 628}

$$-\frac{-abe - a(cd - af) + b^2d}{2a^3x^2} + \frac{\log(a + bx^2 + cx^4)(a^2ce - ab^2e - ab(2cd - af) + b^3d)}{4a^4} - \frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(3a^2bce + 2a^2c(cd - af) - ab^2(4cd - af) - ab^3e + b^4d)}{2a^4\sqrt{b^2-4ac}} - \frac{\log(x)(a^2ce - ab^2e - ab(2cd - af) + b^3d)}{a^4} + \frac{bd - ae}{4a^2x^4} - \frac{d}{6ax^6}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2 + f*x^4)/(x^7*(a + b*x^2 + c*x^4)),x]

[Out] -d/(6*a*x^6) + (b*d - a*e)/(4*a^2*x^4) - (b^2*d - a*b*e - a*(c*d - a*f))/(2*a^3*x^2) - ((b^4*d - a*b^3*e + 3*a^2*b*c*e + 2*a^2*c*(c*d - a*f) - a*b^2*(4*c*d - a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]]/(2*a^4*Sqrt[b^2 - 4*a*c]) - ((b^3*d - a*b^2*e + a^2*c*e - a*b*(2*c*d - a*f))*Log[x])/a^4 + ((b^3*d - a*b^2*e + a^2*c*e - a*b*(2*c*d - a*f))*Log[a + b*x^2 + c*x^4])/(4*a^4)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1628

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 1663

Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^p, x], x], x]

p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned} \int \frac{d + ex^2 + fx^4}{x^7(a + bx^2 + cx^4)} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{d + ex + fx^2}{x^4(a + bx + cx^2)} dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left(\int \left(\frac{d}{ax^4} + \frac{-bd + ae}{a^2x^3} + \frac{b^2d - abe - a(cd - af)}{a^3x^2} + \frac{-b^3d + ab^2e - a^2ce + ab^2f}{a^4x} \right) dx, x, x^2 \right) \\ &= -\frac{d}{6ax^6} + \frac{bd - ae}{4a^2x^4} - \frac{b^2d - abe - a(cd - af)}{2a^3x^2} - \frac{(b^3d - ab^2e + a^2ce - ab(2cd - af)) \log|x^2 + \frac{bx}{a} + \frac{cx^2}{a}|}{a^4} \\ &= -\frac{d}{6ax^6} + \frac{bd - ae}{4a^2x^4} - \frac{b^2d - abe - a(cd - af)}{2a^3x^2} - \frac{(b^3d - ab^2e + a^2ce - ab(2cd - af)) \log|x^2 + \frac{bx}{a} + \frac{cx^2}{a}|}{a^4} \\ &= -\frac{d}{6ax^6} + \frac{bd - ae}{4a^2x^4} - \frac{b^2d - abe - a(cd - af)}{2a^3x^2} - \frac{(b^3d - ab^2e + a^2ce - ab(2cd - af)) \log|x^2 + \frac{bx}{a} + \frac{cx^2}{a}|}{a^4} \\ &= -\frac{d}{6ax^6} + \frac{bd - ae}{4a^2x^4} - \frac{b^2d - abe - a(cd - af)}{2a^3x^2} - \frac{(b^4d - ab^3e + 3a^2bce + 2a^2c(cd - af)) \log|x^2 + \frac{bx}{a} + \frac{cx^2}{a}|}{2a^4\sqrt{b^2 - 4ac}} \end{aligned}$$

Mathematica [A] time = 0.35, size = 416, normalized size = 1.70

$$\frac{-\frac{2d}{3a^6} - 12 \log(x) (c^2cx - a^2e + ab(f - 2cd) + b^2d) + \frac{3 \log(-\sqrt{b^2 - 4ac} + 2a) \left(\frac{2d}{a^2} (\sqrt{b^2 - 4ac} - 2af + 2a) + ab(-\sqrt{b^2 - 4ac} + f - 4a) \right) + (-2d\sqrt{b^2 - 4ac} + f\sqrt{b^2 - 4ac} + 3ab) \left(\frac{2d}{a^2} (\sqrt{b^2 - 4ac} - a) + b \right) + \frac{3 \log(\sqrt{b^2 - 4ac} + 2a) \left(\frac{2d}{a^2} (\sqrt{b^2 - 4ac} + 2af - 2a) + ab(\sqrt{b^2 - 4ac} + f - 4a) \right) + (-2d\sqrt{b^2 - 4ac} + f\sqrt{b^2 - 4ac} - 3ab) \left(\frac{2d}{a^2} (\sqrt{b^2 - 4ac} + a) + b \right)}{\sqrt{b^2 - 4ac}} + \frac{3c^2bc - ab^3}{a^4} + \frac{6(ab + a^2cd - af) + b^2c - d}{a^2}}{12a^4}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2 + f*x^4)/(x^7*(a + b*x^2 + c*x^4)),x]

[Out] ((-2*a^3*d)/x^6 + (3*a^2*(b*d - a*e))/x^4 + (6*a*(-(b^2*d) + a*b*e + a*(c*d - a*f)))/x^2 - 12*(b^3*d - a*b^2*e + a^2*c*e + a*b*(-2*c*d + a*f))*Log[x] + (3*(b^4*d + b^3*(Sqrt[b^2 - 4*a*c]*d - a*e) + a^2*c*(2*c*d + Sqrt[b^2 - 4*a*c]*e - 2*a*f) + a*b^2*(-4*c*d - Sqrt[b^2 - 4*a*c]*e + a*f) + a*b*(-2*c*Sqrt[b^2 - 4*a*c]*d + 3*a*c*e + a*Sqrt[b^2 - 4*a*c]*f))*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c] + (3*(-(b^4*d) + b^3*(Sqrt[b^2 - 4*a*c]*d + a*e) - a*b^2*(-4*c*d + Sqrt[b^2 - 4*a*c]*e + a*f) + a^2*c*(-2*c*d + Sqrt[b^2 - 4*a*c]*e + 2*a*f) + a*b*(-2*c*Sqrt[b^2 - 4*a*c]*d - 3*a*c*e + a*Sqrt[b^2 - 4*a*c]*f))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c])/(12*a^4)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex^2 + fx^4}{x^7(a + bx^2 + cx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e*x^2 + f*x^4)/(x^7*(a + b*x^2 + c*x^4)),x]

[Out] IntegrateAlgebraic[(d + e*x^2 + f*x^4)/(x^7*(a + b*x^2 + c*x^4)), x]

fricas [A] time = 6.64, size = 834, normalized size = 3.42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^7/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*sqrt(b^2 - 4*a*c))*((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a^2*b*c)*e + (a^2*b^2 - 2*a^3*c)*f)*x^6*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - 3*((b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*d - (a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*e + (a^2*b^3 - 4*a^3*b*c)*f)*x^6*log(c*x^4 + b*x^2 + a) + 12*((b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*d - (a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*e + (a^2*b^3 - 4*a^3*b*c)*f)*x^6*log(x) + 6*((a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*d - (a^2*b^3 - 4*a^3*b*c)*e + (a^3*b^2 - 4*a^4*c)*f)*x^4 - 3*((a^2*b^3 - 4*a^3*b*c)*d - (a^3*b^2 - 4*a^4*c)*e)*x^2 + 2*(a^3*b^2 - 4*a^4*c)*d)/((a^4*b^2 - 4*a^5*c)*x^6), -1/12*(6*sqrt(-b^2 + 4*a*c))*((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a^2*b*c)*e + (a^2*b^2 - 2*a^3*c)*f)*x^6*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - 3*((b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*d - (a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*e + (a^2*b^3 - 4*a^3*b*c)*f)*x^6*log(c*x^4 + b*x^2 + a) + 12*((b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*d - (a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*e + (a^2*b^3 - 4*a^3*b*c)*f)*x^6*log(x) + 6*((a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*d - (a^2*b^3 - 4*a^3*b*c)*e + (a^3*b^2 - 4*a^4*c)*f)*x^4 - 3*((a^2*b^3 - 4*a^3*b*c)*d - (a^3*b^2 - 4*a^4*c)*e)*x^2 + 2*(a^3*b^2 - 4*a^4*c)*d)/((a^4*b^2 - 4*a^5*c)*x^6)]
```

giac [A] time = 1.94, size = 313, normalized size = 1.28

$$\frac{(b^2 d - 2 a b c d + a^2 c d - a b^2 e + a^2 c e) \log(c x^4 + b x^2 + a)}{4 a^2} - \frac{(b^2 d - 2 a b c d + a^2 c d - a b^2 e + a^2 c e) \log(x^2)}{2 a^2} + \frac{(b^5 d - 4 a b^3 c d + 2 a^2 b^2 c^2 d - 2 a^3 c^2 f - a b^2 e + 3 a^2 b c e) \arctan\left(\frac{2 c x^2 + b}{\sqrt{-b^2 + 4 a c}}\right)}{2 \sqrt{-b^2 + 4 a c} a^2} - \frac{11 b^5 d^2 e - 22 a b c d^2 e + 11 a^2 b^2 c^2 e - 11 a b^2 c^2 e + 11 a^2 c^2 e - 6 a^2 d^2 c^4 + 6 a^2 c d^4 - 6 a^2 f^4 + 6 a^2 b c^2 e + 3 a^2 b d^2 - 3 a^2 c^2 e - 2 a^2 d}{12 a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^7/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*(b^3*d - 2*a*b*c*d + a^2*b*f - a*b^2*e + a^2*c*e)*log(c*x^4 + b*x^2 + a)/a^4 - 1/2*(b^3*d - 2*a*b*c*d + a^2*b*f - a*b^2*e + a^2*c*e)*log(x^2)/a^4 + 1/2*(b^4*d - 4*a*b^2*c*d + 2*a^2*c^2*d + a^2*b^2*f - 2*a^3*c*f - a*b^3*e + 3*a^2*b*c*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^4) + 1/12*(11*b^3*d*x^6 - 22*a*b*c*d*x^6 + 11*a^2*b*f*x^6 - 11*a*b^2*c*x^6*e + 11*a^2*c*x^6*e - 6*a*b^2*d*x^4 + 6*a^2*c*d*x^4 - 6*a^3*f*x^4 + 6*a^2*b*x^4*e + 3*a^2*b*d*x^2 - 3*a^3*c*x^2*e - 2*a^3*d)/((a^4*x^6))
```

maple [B] time = 0.01, size = 523, normalized size = 2.14

$$\frac{c \operatorname{arctan}\left(\frac{2 c x^2 + b}{\sqrt{-b^2 + 4 a c}}\right)}{\sqrt{-b^2 + 4 a c} a^4} + \frac{b^2 f \operatorname{arctan}\left(\frac{2 c x^2 + b}{\sqrt{-b^2 + 4 a c}}\right)}{2 \sqrt{-b^2 + 4 a c} a^2} + \frac{3 b c \operatorname{arctan}\left(\frac{2 c x^2 + b}{\sqrt{-b^2 + 4 a c}}\right)}{2 \sqrt{-b^2 + 4 a c} a^2} + \frac{c^2 d \operatorname{arctan}\left(\frac{2 c x^2 + b}{\sqrt{-b^2 + 4 a c}}\right)}{\sqrt{-b^2 + 4 a c} a^2} + \frac{b^2 d \operatorname{arctan}\left(\frac{2 c x^2 + b}{\sqrt{-b^2 + 4 a c}}\right)}{2 \sqrt{-b^2 + 4 a c} a^2} + \frac{b^2 d \operatorname{arctan}\left(\frac{2 c x^2 + b}{\sqrt{-b^2 + 4 a c}}\right)}{\sqrt{-b^2 + 4 a c} a^2} + \frac{b^2 d \operatorname{arctan}\left(\frac{2 c x^2 + b}{\sqrt{-b^2 + 4 a c}}\right)}{2 \sqrt{-b^2 + 4 a c} a^2} + \frac{b^2 d \operatorname{arctan}\left(\frac{2 c x^2 + b}{\sqrt{-b^2 + 4 a c}}\right)}{\sqrt{-b^2 + 4 a c} a^2} + \frac{b^2 d \operatorname{arctan}\left(\frac{2 c x^2 + b}{\sqrt{-b^2 + 4 a c}}\right)}{2 \sqrt{-b^2 + 4 a c} a^2} + \frac{b^2 d \operatorname{arctan}\left(\frac{2 c x^2 + b}{\sqrt{-b^2 + 4 a c}}\right)}{\sqrt{-b^2 + 4 a c} a^2} + \frac{b^2 d \operatorname{arctan}\left(\frac{2 c x^2 + b}{\sqrt{-b^2 + 4 a c}}\right)}{2 \sqrt{-b^2 + 4 a c} a^2} + \frac{b^2 d \operatorname{arctan}\left(\frac{2 c x^2 + b}{\sqrt{-b^2 + 4 a c}}\right)}{\sqrt{-b^2 + 4 a c} a^2} + \frac{b^2 d \operatorname{arctan}\left(\frac{2 c x^2 + b}{\sqrt{-b^2 + 4 a c}}\right)}{2 \sqrt{-b^2 + 4 a c} a^2} + \frac{b^2 d \operatorname{arctan}\left(\frac{2 c x^2 + b}{\sqrt{-b^2 + 4 a c}}\right)}{\sqrt{-b^2 + 4 a c} a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^4+e*x^2+d)/x^7/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/6*d/a/x^6-1/4/a/x^4*e+1/4/a^2/x^4*b*d-1/2/a/x^2*f+1/2/a^2/x^2*b*e+1/2/a^2/x^2*c*d-1/2/a^3/x^2*b^2*d-1/a^2*ln(x)*b*f-1/a^2*ln(x)*c*e+1/a^3*ln(x)*b^2*e+2/a^3*ln(x)*b*c*d-1/a^4*ln(x)*b^3*d+1/4/a^2*ln(c*x^4+b*x^2+a)*b*f+1/4/a^2*c*ln(c*x^4+b*x^2+a)*e-1/4/a^3*ln(c*x^4+b*x^2+a)*b^2*e-1/2/a^3*c*ln(c*x^4+b*x^2+a)*b*d+1/4/a^4*ln(c*x^4+b*x^2+a)*b^3*d-1/a/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*c*f+1/2/a^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^2*f+3/2/a^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b*c*e+1/a^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*c^2*d-1/2/a^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^3*e-2/a^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^2*c*d+1/2/a^4/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^4*d
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^7/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 13.83, size = 9141, normalized size = 37.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2 + f*x^4)/(x^7*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (atan((16*a^12*(4*a*c - b^2)^(3/2)*(x^2*(((a^3*c^8*d^3 - b^6*c^5*d^3 - a^6
*c^5*f^3 + 3*a*b^4*c^6*d^3 - 3*a^4*c^7*d^2*f + 3*a^5*c^6*d*f^2 - 3*a^2*b^2*
c^7*d^3 + a^3*b^3*c^5*e^3 + 3*a*b^5*c^5*d^2*e + 3*a^3*b*c^7*d^2*e + 3*a^5*b
*c^5*e*f^2 - 6*a^2*b^3*c^6*d^2*e - 3*a^2*b^4*c^5*d*e^2 + 3*a^3*b^2*c^6*d*e^
2 - 3*a^2*b^4*c^5*d^2*f + 6*a^3*b^2*c^6*d^2*f - 3*a^4*b^2*c^5*d*f^2 - 3*a^4
*b^2*c^5*e^2*f - 6*a^4*b*c^6*d*e*f + 6*a^3*b^3*c^5*d*e*f)/a^9 - (((11*a^5*b
*c^6*d^2 - 5*a^6*b*c^5*e^2 + 6*a^7*b*c^4*f^2 + 6*a^3*b^5*c^4*d^2 - 17*a^4*b
^3*c^5*d^2 + 6*a^5*b^3*c^4*e^2 - 5*a^6*c^6*d*e + 5*a^7*c^5*e*f - 17*a^6*b*c
^5*d*f - 12*a^4*b^4*c^4*d*e + 22*a^5*b^2*c^5*d*e + 12*a^5*b^3*c^4*d*f - 12*
a^6*b^2*c^4*e*f)/a^9 + (((20*a^9*c^4*f - 20*a^8*c^5*d + 2*a^6*b^4*c^3*d + 8
*a^7*b^2*c^4*d - 2*a^7*b^3*c^3*e + 2*a^8*b^2*c^3*f - 10*a^8*b*c^4*e)/a^9 +
((40*a^10*b*c^3 - 12*a^9*b^3*c^2)*(2*b^5*d + 2*a^2*b^3*f - 8*a^3*c^2*e - 2*
a*b^4*e - 12*a*b^3*c*d - 8*a^3*b*c*f + 16*a^2*b*c^2*d + 10*a^2*b^2*c*e))/(2
*a^9*(16*a^5*c - 4*a^4*b^2)))*(2*b^5*d + 2*a^2*b^3*f - 8*a^3*c^2*e - 2*a*b^
4*e - 12*a*b^3*c*d - 8*a^3*b*c*f + 16*a^2*b*c^2*d + 10*a^2*b^2*c*e))/(2*(16
*a^5*c - 4*a^4*b^2)))*(2*b^5*d + 2*a^2*b^3*f - 8*a^3*c^2*e - 2*a*b^4*e - 12
*a*b^3*c*d - 8*a^3*b*c*f + 16*a^2*b*c^2*d + 10*a^2*b^2*c*e))/(2*(16*a^5*c -
4*a^4*b^2)) + (((((20*a^9*c^4*f - 20*a^8*c^5*d + 2*a^6*b^4*c^3*d + 8*a^7*b
^2*c^4*d - 2*a^7*b^3*c^3*e + 2*a^8*b^2*c^3*f - 10*a^8*b*c^4*e)/a^9 + ((40*a
^10*b*c^3 - 12*a^9*b^3*c^2)*(2*b^5*d + 2*a^2*b^3*f - 8*a^3*c^2*e - 2*a*b^4*
e - 12*a*b^3*c*d - 8*a^3*b*c*f + 16*a^2*b*c^2*d + 10*a^2*b^2*c*e))/(2*a^9*(
16*a^5*c - 4*a^4*b^2)))*(b^4*d + 2*a^2*c^2*d + a^2*b^2*f - a*b^3*e - 2*a^3*
c*f - 4*a*b^2*c*d + 3*a^2*b*c*e))/(4*a^4*(4*a*c - b^2)^(1/2)) + ((40*a^10*b
*c^3 - 12*a^9*b^3*c^2)*(b^4*d + 2*a^2*c^2*d + a^2*b^2*f - a*b^3*e - 2*a^3*c
*f - 4*a*b^2*c*d + 3*a^2*b*c*e))*(2*b^5*d + 2*a^2*b^3*f - 8*a^3*c^2*e - 2*a*
b^4*e - 12*a*b^3*c*d - 8*a^3*b*c*f + 16*a^2*b*c^2*d + 10*a^2*b^2*c*e))/(8*a
^13*(4*a*c - b^2)^(1/2)*(16*a^5*c - 4*a^4*b^2)))*(b^4*d + 2*a^2*c^2*d + a^2
*b^2*f - a*b^3*e - 2*a^3*c*f - 4*a*b^2*c*d + 3*a^2*b*c*e))/(4*a^4*(4*a*c -
b^2)^(1/2)) + ((40*a^10*b*c^3 - 12*a^9*b^3*c^2)*(b^4*d + 2*a^2*c^2*d + a^2*
b^2*f - a*b^3*e - 2*a^3*c*f - 4*a*b^2*c*d + 3*a^2*b*c*e))^2*(2*b^5*d + 2*a^2
*b^3*f - 8*a^3*c^2*e - 2*a*b^4*e - 12*a*b^3*c*d - 8*a^3*b*c*f + 16*a^2*b*c^
2*d + 10*a^2*b^2*c*e))/(32*a^17*(4*a*c - b^2)*(16*a^5*c - 4*a^4*b^2)))*(3*b
^6*d - a^3*c^3*d + 3*a^2*b^4*f + a^4*c^2*f - 3*a*b^5*e + 18*a^2*b^2*c^2*d -
15*a*b^4*c*d + 12*a^2*b^3*c*e - 9*a^3*b*c^2*e - 9*a^3*b^2*c*f))/(8*a^3*c^2
*(a^4*c^4*d^2 - 6*a^2*b^6*e^2 - 6*b^8*d^2 - 6*a^4*b^4*f^2 + 25*a^5*c^3*e^2
+ a^6*c^2*f^2 + 36*a^3*b^4*c*e^2 + 24*a^5*b^2*c*f^2 + 12*a*b^7*d*e - 120*a^
2*b^4*c^2*d^2 + 96*a^3*b^2*c^3*d^2 - 54*a^4*b^2*c^2*e^2 + 48*a*b^6*c*d^2 -
12*a^2*b^6*d*f + 12*a^3*b^5*e*f - 2*a^5*c^3*d*f - 84*a^2*b^5*c*d*e - 97*a^4
*b*c^3*d*e + 72*a^3*b^4*c*d*f - 60*a^4*b^3*c*e*f + 47*a^5*b*c^2*e*f + 168*a
^3*b^3*c^2*d*e - 95*a^4*b^2*c^2*d*f)) + ((((((20*a^9*c^4*f - 20*a^8*c^5*d
+ 2*a^6*b^4*c^3*d + 8*a^7*b^2*c^4*d - 2*a^7*b^3*c^3*e + 2*a^8*b^2*c^3*f - 1
0*a^8*b*c^4*e)/a^9 + ((40*a^10*b*c^3 - 12*a^9*b^3*c^2)*(2*b^5*d + 2*a^2*b^3
*f - 8*a^3*c^2*e - 2*a*b^4*e - 12*a*b^3*c*d - 8*a^3*b*c*f + 16*a^2*b*c^2*d
+ 10*a^2*b^2*c*e))/(2*a^9*(16*a^5*c - 4*a^4*b^2)))*(b^4*d + 2*a^2*c^2*d + a
^2*b^2*f - a*b^3*e - 2*a^3*c*f - 4*a*b^2*c*d + 3*a^2*b*c*e))/(4*a^4*(4*a*c
```

$$\begin{aligned}
& - b^2)^{(1/2)}) + ((40a^{10}b^3c^3 - 12a^9b^3c^2)(b^4d + 2a^2c^2d + a^2b^2f - ab^3e - 2a^3c^2f - 4ab^2cd + 3a^2b^2c^2e)(2b^5d + 2a^2b^3f - 8a^3c^2e - 2ab^4e - 12ab^3cd - 8a^3b^2c^2f + 16a^2b^2c^2d + 10a^2b^2c^2e))/(8a^{13}(4ac - b^2)^{(1/2)}(16a^5c - 4a^4b^2))) \\
& * (2b^5d + 2a^2b^3f - 8a^3c^2e - 2ab^4e - 12ab^3cd - 8a^3b^2c^2f + 16a^2b^2c^2d + 10a^2b^2c^2e))/(2(16a^5c - 4a^4b^2)) + (((11a^5b^6d^2 - 5a^6b^5c^5e^2 + 6a^7b^4c^4f^2 + 6a^3b^5c^4d^2 - 17a^4b^3c^5d^2 + 6a^5b^3c^4e^2 - 5a^6c^6d^2e + 5a^7c^5e^2f - 17a^6b^5c^5d^2f - 12a^4b^4c^4d^2e + 22a^5b^2c^5d^2e + 12a^5b^3c^4d^2f - 12a^6b^2c^4e^2f)/a^9 + (((20a^9c^4f - 20a^8c^5d + 2a^6b^4c^3d + 8a^7b^2c^4d - 2a^7b^3c^3e + 2a^8b^2c^3f - 10a^8b^2c^4e)/a^9 + ((40a^{10}b^3c^3 - 12a^9b^3c^2)(2b^5d + 2a^2b^3f - 8a^3c^2e - 2ab^4e - 12ab^3cd - 8a^3b^2c^2f + 16a^2b^2c^2d + 10a^2b^2c^2e)))/(2a^9(16a^5c - 4a^4b^2)))(2b^5d + 2a^2b^3f - 8a^3c^2e - 2ab^4e - 12ab^3cd - 8a^3b^2c^2f + 16a^2b^2c^2d + 10a^2b^2c^2e))/(2(16a^5c - 4a^4b^2)))(b^4d + 2a^2c^2d + a^2b^2f - ab^3e - 2a^3c^2f - 4ab^2cd + 3a^2b^2c^2e))/(4a^4(4ac - b^2)^{(1/2)}) - ((40a^{10}b^3c^3 - 12a^9b^3c^2)(b^4d + 2a^2c^2d + a^2b^2f - ab^3e - 2a^3c^2f - 4ab^2cd + 3a^2b^2c^2e)^3)/(64a^{21}(4ac - b^2)^{(3/2)))(6b^7d + 6a^2b^5f + 20a^4c^3e - 6ab^6e + 84a^2b^3c^2d - 54a^3b^2c^2e - 42ab^5cd - 46a^3b^3c^3d + 36a^2b^4c^2e - 30a^3b^3c^2f + 26a^4b^2c^2f))/(16a^3c^2(4ac - b^2)^{(1/2)}(a^4c^4d^2 - 6a^2b^6e^2 - 6b^8d^2 - 6a^4b^4f^2 + 25a^5c^3e^2 + a^6c^2f^2 + 36a^3b^4c^2e^2 + 24a^5b^2c^2f^2 + 12ab^7d^2e - 120a^2b^4c^2d^2 + 96a^3b^2c^3d^2 - 54a^4b^2c^2e^2 + 48ab^6c^2d^2 - 12a^2b^6d^2f + 12a^3b^5e^2f - 2a^5c^3d^2f - 84a^2b^5c^2d^2e - 97a^4b^3c^3d^2e + 72a^3b^4c^2d^2f - 60a^4b^3c^2e^2f + 47a^5b^3c^2e^2f + 168a^3b^3c^2d^2e - 95a^4b^2c^2d^2f))) - (((b^7c^4d^3 - 4ab^5c^5d^3 - 2a^3b^3c^7d^3 + a^6b^3c^4f^3 + a^4c^7d^2e + a^6c^5e^2f^2 + 5a^2b^3c^6d^3 - a^3b^4c^4e^3 + a^4b^2c^5e^3 - 2a^5c^6d^2e^2f - 3ab^6c^4d^2e + 2a^4b^3c^6d^2e^2 + 5a^4b^3c^6d^2f - 4a^5b^3c^5d^2f^2 - 2a^5b^3c^5e^2f + 9a^2b^4c^5d^2e + 3a^2b^5c^4d^2e^2 - 7a^3b^2c^6d^2e - 6a^3b^3c^5d^2e^2 + 3a^2b^5c^4d^2f - 8a^3b^3c^5d^2f + 3a^4b^3c^4d^2f^2 + 3a^4b^3c^4e^2f - 3a^5b^2c^4e^2f - 6a^3b^4c^4d^2e^2f + 10a^4b^2c^5d^2e^2f)/a^9 - (((a^6c^6d^2 + a^8c^4f^2 - 4a^3b^6c^3d^2 + 13a^4b^4c^4d^2 - 10a^5b^2c^5d^2 - 4a^5b^4c^3e^2 + 5a^6b^2c^4e^2 - 4a^7b^2c^3f^2 - 2a^7c^5d^2f + 6a^6b^3c^5d^2e - 6a^7b^3c^4e^2f + 8a^4b^5c^3d^2e - 18a^5b^3c^4d^2e - 8a^5b^4c^3d^2f + 14a^6b^2c^4d^2f + 8a^6b^3c^3e^2f)/a^9 - (((4a^6b^5c^2d - 12a^7b^3c^3d - 4a^7b^4c^2e + 8a^8b^2c^3e + 4a^8b^3c^2f + 4a^8b^3c^4d - 4a^9b^3c^3f)/a^9 - (2ab^2c^2(2b^5d + 2a^2b^3f - 8a^3c^2e - 2ab^4e - 12ab^3cd - 8a^3b^2c^2f + 16a^2b^2c^2d + 10a^2b^2c^2e))/(16a^5c - 4a^4b^2)))(2b^5d + 2a^2b^3f - 8a^3c^2e - 2ab^4e - 12ab^3cd - 8a^3b^2c^2f + 16a^2b^2c^2d + 10a^2b^2c^2e))/(2(16a^5c - 4a^4b^2)) - (((((4a^6b^5c^2d - 12a^7b^3c^3d - 4a^7b^4c^2e + 8a^8b^2c^3e + 4a^8b^3c^2f + 4a^8b^3c^4d - 4a^9b^3c^3f)/a^9 - (2ab^2c^2(2b^5d + 2a^2b^3f - 8a^3c^2e - 2ab^4e - 12ab^3cd - 8a^3b^2c^2f + 16a^2b^2c^2d + 10a^2b^2c^2e))/(16a^5c - 4a^4b^2)))(b^4d + 2a^2c^2d + a^2b^2f - ab^3e - 2a^3c^2f - 4ab^2cd + 3a^2b^2c^2e))/(4a^4(4ac - b^2)^{(1/2)}) - (b^2c^2(b^4d + 2a^2c^2d + a^2b^2f - ab^3e - 2a^3c^2f - 4ab^2cd + 3a^2b^2c^2e)(2b^5d + 2a^2b^3f - 8a^3c^2e - 2ab^4e - 12ab^3cd - 8a^3b^2c^2f + 16a^2b^2c^2d + 10a^2b^2c^2e))/(2a^3(4ac - b^2)^{(1/2)}(16a^5c - 4a^4b^2)))(b^4d + 2a^2c^2d + a^2b^2f - ab^3e - 2a^3c^2f - 4ab^2cd + 3a^2b^2c^2e))/(4a^4(4ac - b^2)^{(1/2)}) + (b^2c^2(b^4d + 2a^2c^2d + a^2b^2f - ab^3e - 2a^3c^2f - 4ab^2cd + 3a^2b^2c^2e)^2(2b^5d + 2a^2b^3f - 8a^3c^2e - 2ab^4e - 12ab^3cd - 8a^3b^2c^2f + 16a^2b^2c^2d + 10a^2b^2c^2e))/(8
\end{aligned}$$

$$\begin{aligned}
& *a^7*(4*a*c - b^2)*(16*a^5*c - 4*a^4*b^2)))*(3*b^6*d - a^3*c^3*d + 3*a^2*b^4*f + a^4*c^2*f - 3*a*b^5*e + 18*a^2*b^2*c^2*d - 15*a*b^4*c*d + 12*a^2*b^3*c*e - 9*a^3*b*c^2*e - 9*a^3*b^2*c*f))/(8*a^3*c^2*(a^4*c^4*d^2 - 6*a^2*b^6*e^2 - 6*b^8*d^2 - 6*a^4*b^4*f^2 + 25*a^5*c^3*e^2 + a^6*c^2*f^2 + 36*a^3*b^4*c*e^2 + 24*a^5*b^2*c*f^2 + 12*a*b^7*d*e - 120*a^2*b^4*c^2*d^2 + 96*a^3*b^2*c^3*d^2 - 54*a^4*b^2*c^2*e^2 + 48*a*b^6*c*d^2 - 12*a^2*b^6*d*f + 12*a^3*b^5*e*f - 2*a^5*c^3*d*f - 84*a^2*b^5*c*d*e - 97*a^4*b*c^3*d*e + 72*a^3*b^4*c*d*f - 60*a^4*b^3*c*e*f + 47*a^5*b*c^2*e*f + 168*a^3*b^3*c^2*d*e - 95*a^4*b^2*c^2*d*f)) + (((((((4*a^6*b^5*c^2*d - 12*a^7*b^3*c^3*d - 4*a^7*b^4*c^2*e + 8*a^8*b^2*c^3*e + 4*a^8*b^3*c^2*f + 4*a^8*b*c^4*d - 4*a^9*b*c^3*f)/a^9 - (2*a*b^2*c^2*(2*b^5*d + 2*a^2*b^3*f - 8*a^3*c^2*e - 2*a*b^4*e - 12*a*b^3*c*d - 8*a^3*b*c*f + 16*a^2*b*c^2*d + 10*a^2*b^2*c*e))/(16*a^5*c - 4*a^4*b^2)))*(b^4*d + 2*a^2*c^2*d + a^2*b^2*f - a*b^3*e - 2*a^3*c*f - 4*a*b^2*c*d + 3*a^2*b*c*e))/(4*a^4*(4*a*c - b^2)^(1/2)) - (b^2*c^2*(b^4*d + 2*a^2*c^2*d + a^2*b^2*f - a*b^3*e - 2*a^3*c*f - 4*a*b^2*c*d + 3*a^2*b*c*e)*(2*b^5*d + 2*a^2*b^3*f - 8*a^3*c^2*e - 2*a*b^4*e - 12*a*b^3*c*d - 8*a^3*b*c*f + 16*a^2*b*c^2*d + 10*a^2*b^2*c*e))/(2*a^3*(4*a*c - b^2)^(1/2)*(16*a^5*c - 4*a^4*b^2)))*(2*b^5*d + 2*a^2*b^3*f - 8*a^3*c^2*e - 2*a*b^4*e - 12*a*b^3*c*d - 8*a^3*b*c*f + 16*a^2*b*c^2*d + 10*a^2*b^2*c*e))/(2*(16*a^5*c - 4*a^4*b^2)) - (((a^6*c^6*d^2 + a^8*c^4*f^2 - 4*a^3*b^6*c^3*d^2 + 13*a^4*b^4*c^4*d^2 - 10*a^5*b^2*c^5*d^2 - 4*a^5*b^4*c^3*e^2 + 5*a^6*b^2*c^4*e^2 - 4*a^7*b^2*c^3*f^2 - 2*a^7*c^5*d*f + 6*a^6*b*c^5*d*e - 6*a^7*b*c^4*e*f + 8*a^4*b^5*c^3*d*e - 18*a^5*b^3*c^4*d*e - 8*a^5*b^4*c^3*d*f + 14*a^6*b^2*c^4*d*f + 8*a^6*b^3*c^3*e*f)/a^9 - (((4*a^6*b^5*c^2*d - 12*a^7*b^3*c^3*d - 4*a^7*b^4*c^2*e + 8*a^8*b^2*c^3*e + 4*a^8*b^3*c^2*f + 4*a^8*b*c^4*d - 4*a^9*b*c^3*f)/a^9 - (2*a*b^2*c^2*(2*b^5*d + 2*a^2*b^3*f - 8*a^3*c^2*e - 2*a*b^4*e - 12*a*b^3*c*d - 8*a^3*b*c*f + 16*a^2*b*c^2*d + 10*a^2*b^2*c*e))/(16*a^5*c - 4*a^4*b^2)))*(2*b^5*d + 2*a^2*b^3*f - 8*a^3*c^2*e - 2*a*b^4*e - 12*a*b^3*c*d - 8*a^3*b*c*f + 16*a^2*b*c^2*d + 10*a^2*b^2*c*e))/(2*(16*a^5*c - 4*a^4*b^2)))*(b^4*d + 2*a^2*c^2*d + a^2*b^2*f - a*b^3*e - 2*a^3*c*f - 4*a*b^2*c*d + 3*a^2*b*c*e))/(4*a^4*(4*a*c - b^2)^(1/2)) + (b^2*c^2*(b^4*d + 2*a^2*c^2*d + a^2*b^2*f - a*b^3*e - 2*a^3*c*f - 4*a*b^2*c*d + 3*a^2*b*c*e)^3)/(16*a^11*(4*a*c - b^2)^(3/2)))*(6*b^7*d + 6*a^2*b^5*f + 20*a^4*c^3*e - 6*a*b^6*e + 84*a^2*b^3*c^2*d - 54*a^3*b^2*c^2*e - 42*a*b^5*c*d - 46*a^3*b*c^3*d + 36*a^2*b^4*c*e - 30*a^3*b^3*c*f + 26*a^4*b*c^2*f))/(16*a^3*c^2*(4*a*c - b^2)^(1/2)*(a^4*c^4*d^2 - 6*a^2*b^6*e^2 - 6*b^8*d^2 - 6*a^4*b^4*f^2 + 25*a^5*c^3*e^2 + a^6*c^2*f^2 + 36*a^3*b^4*c*e^2 + 24*a^5*b^2*c*f^2 + 12*a*b^7*d*e - 120*a^2*b^4*c^2*d^2 + 96*a^3*b^2*c^3*d^2 - 54*a^4*b^2*c^2*e^2 + 48*a*b^6*c*d^2 - 12*a^2*b^6*d*f + 12*a^3*b^5*e*f - 2*a^5*c^3*d*f - 84*a^2*b^5*c*d*e - 97*a^4*b*c^3*d*e + 72*a^3*b^4*c*d*f - 60*a^4*b^3*c*e*f + 47*a^5*b*c^2*e*f + 168*a^3*b^3*c^2*d*e - 95*a^4*b^2*c^2*d*f)))/(4*a^4*c^6*d^2 + b^8*c^2*d^2 + 4*a^6*c^4*f^2 - 8*a*b^6*c^3*d^2 + 20*a^2*b^4*c^4*d^2 - 16*a^3*b^2*c^5*d^2 + a^2*b^6*c^2*e^2 - 6*a^3*b^4*c^3*e^2 + 9*a^4*b^2*c^4*e^2 + a^4*b^4*c^2*f^2 - 4*a^5*b^2*c^3*f^2 - 8*a^5*c^5*d*f - 2*a*b^7*c^2*d*e + 12*a^4*b*c^5*d*e - 12*a^5*b*c^4*e*f + 14*a^2*b^5*c^3*d*e - 28*a^3*b^3*c^4*d*e + 2*a^2*b^6*c^2*d*f - 12*a^3*b^4*c^3*d*f + 20*a^4*b^2*c^4*d*f - 2*a^3*b^5*c^2*e*f + 10*a^4*b^3*c^3*e*f)*(b^4*d + 2*a^2*c^2*d + a^2*b^2*f - a*b^3*e - 2*a^3*c*f - 4*a*b^2*c*d + 3*a^2*b*c*e))/(2*a^4*(4*a*c - b^2)^(1/2)) - (log(((c^4*(b^2*d + a^2*f - a*b*e - a*c*d))^2*(b^3*d - a*b^2*e + a^2*b*f + a^2*c*e - 2*a*b*c*d))/a^9 - (((c^3*(4*b^6*d^2 - a^5*c*f^2 + 4*a^2*b^4*e^2 - a^3*c^3*d^2 + 4*a^4*b^2*f^2 - 5*a^3*b^2*c*e^2 - 8*a*b^5*d*e + 10*a^2*b^2*c^2*d^2 - 13*a*b^4*c*d^2 + 8*a^2*b^4*d*f - 8*a^3*b^3*e*f + 2*a^4*c^2*d*f + 18*a^2*b^3*c*d*e - 6*a^3*b*c^2*d*e - 14*a^3*b^2*c*d*f + 6*a^4*b*c*e*f))/a^6 - (((4*b*c^2*(b^4*d + a^2*c^2*d + a^2*b^2*f - a*b^3*e - a^3*c*f - 3*a*b^2*c*d + 2*a^2*b*c*e))/a^3 + (2*c^3*x^2*(b^4*d - 10*a^2*c^2*d + a^2*b^2*f - a*b^3*e + 10*a^3*c*f + 4*a*b^2*c*d - 5*a^2*b*c*e))/a^3 + (b*c^2*(a*b + 3*b^2*x^2 - 10*a*c*x^2)*(b^3*d + a^4*(-(b^4*d + 2*a^2*c^2*d + a^2*b^2*f - a*b^3*e - 2*a^3*c*f - 4*a*b^2*c*d + 3*a^2*b*c*e))^2/(a^8*(4*a*c - b^2)))^(1/2) - a*b^2*e + a^2*b*f + a^2*c*e - 2*a*b*c*d))/a^4)*(b^3*d + a^4*(-(b^4*d + 2*a^2*c^2*d + a^2*b^2*f - a*b^3*e - 2*a^3*c*f - 4*a*b^2*c*d
\end{aligned}$$

$$\begin{aligned}
& + 3*a^2*b*c*e)^2/(a^8*(4*a*c - b^2))^{(1/2)} - a*b^2*e + a^2*b*f + a^2*c*e - \\
& 2*a*b*c*d)/(4*a^4) + (c^4*x^2*(6*b^5*d^2 + 6*a^4*b*f^2 + 6*a^2*b^3*e^2 + \\
& 11*a^2*b*c^2*d^2 - 12*a*b^4*d*e + 5*a^4*c*e*f - 17*a*b^3*c*d^2 - 5*a^3*b*c* \\
& e^2 + 12*a^2*b^3*d*f - 5*a^3*c^2*d*e - 12*a^3*b^2*e*f + 22*a^2*b^2*c*d*e - \\
& 17*a^3*b*c*d*f))/a^6*(b^3*d + a^4*(-(b^4*d + 2*a^2*c^2*d + a^2*b^2*f - a*b \\
& ^3*e - 2*a^3*c*f - 4*a*b^2*c*d + 3*a^2*b*c*e)^2/(a^8*(4*a*c - b^2))^{(1/2)} \\
& - a*b^2*e + a^2*b*f + a^2*c*e - 2*a*b*c*d))/(4*a^4) + (c^5*x^2*(b^2*d + a^2 \\
& *f - a*b*e - a*c*d)^3/a^9)*((c^4*(b^2*d + a^2*f - a*b*e - a*c*d)^2*(b^3*d \\
& - a*b^2*e + a^2*b*f + a^2*c*e - 2*a*b*c*d))/a^9 - (((c^3*(4*b^6*d^2 - a^5*c \\
& *f^2 + 4*a^2*b^4*e^2 - a^3*c^3*d^2 + 4*a^4*b^2*f^2 - 5*a^3*b^2*c*e^2 - 8*a* \\
& b^5*d*e + 10*a^2*b^2*c^2*d^2 - 13*a*b^4*c*d^2 + 8*a^2*b^4*d*f - 8*a^3*b^3*e \\
& *f + 2*a^4*c^2*d*f + 18*a^2*b^3*c*d*e - 6*a^3*b*c^2*d*e - 14*a^3*b^2*c*d*f \\
& + 6*a^4*b*c*e*f))/a^6 - (((4*b*c^2*(b^4*d + a^2*c^2*d + a^2*b^2*f - a*b^3*e \\
& - a^3*c*f - 3*a*b^2*c*d + 2*a^2*b*c*e))/a^3 + (2*c^3*x^2*(b^4*d - 10*a^2*c \\
& ^2*d + a^2*b^2*f - a*b^3*e + 10*a^3*c*f + 4*a*b^2*c*d - 5*a^2*b*c*e))/a^3 + \\
& (b*c^2*(a*b + 3*b^2*x^2 - 10*a*c*x^2)*(b^3*d - a^4*(-(b^4*d + 2*a^2*c^2*d \\
& + a^2*b^2*f - a*b^3*e - 2*a^3*c*f - 4*a*b^2*c*d + 3*a^2*b*c*e)^2/(a^8*(4*a* \\
& c - b^2))^{(1/2)} - a*b^2*e + a^2*b*f + a^2*c*e - 2*a*b*c*d))/a^4)*(b^3*d - \\
& a^4*(-(b^4*d + 2*a^2*c^2*d + a^2*b^2*f - a*b^3*e - 2*a^3*c*f - 4*a*b^2*c*d \\
& + 3*a^2*b*c*e)^2/(a^8*(4*a*c - b^2))^{(1/2)} - a*b^2*e + a^2*b*f + a^2*c*e - \\
& 2*a*b*c*d))/(4*a^4) + (c^4*x^2*(6*b^5*d^2 + 6*a^4*b*f^2 + 6*a^2*b^3*e^2 + \\
& 11*a^2*b*c^2*d^2 - 12*a*b^4*d*e + 5*a^4*c*e*f - 17*a*b^3*c*d^2 - 5*a^3*b*c* \\
& e^2 + 12*a^2*b^3*d*f - 5*a^3*c^2*d*e - 12*a^3*b^2*e*f + 22*a^2*b^2*c*d*e - \\
& 17*a^3*b*c*d*f))/a^6*(b^3*d - a^4*(-(b^4*d + 2*a^2*c^2*d + a^2*b^2*f - a*b \\
& ^3*e - 2*a^3*c*f - 4*a*b^2*c*d + 3*a^2*b*c*e)^2/(a^8*(4*a*c - b^2))^{(1/2)} \\
& - a*b^2*e + a^2*b*f + a^2*c*e - 2*a*b*c*d))/(4*a^4) + (c^5*x^2*(b^2*d + a^2 \\
& *f - a*b*e - a*c*d)^3/a^9)*(2*b^5*d + 2*a^2*b^3*f - 8*a^3*c^2*e - 2*a*b^4 \\
& *e - 12*a*b^3*c*d - 8*a^3*b*c*f + 16*a^2*b*c^2*d + 10*a^2*b^2*c*e))/(2*(16* \\
& a^5*c - 4*a^4*b^2)) - (log(x)*(b^3*d - a*b^2*e + a^2*b*f + a^2*c*e - 2*a*b* \\
& c*d))/a^4 - (d/(6*a) + (x^4*(b^2*d + a^2*f - a*b*e - a*c*d))/(2*a^3) + (x^2 \\
& *(a*e - b*d))/(4*a^2))/x^6
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**4+e*x**2+d)/x**7/(c*x**4+b*x**2+a), x)

[Out] Timed out

$$3.53 \quad \int \frac{x^4(d+ex^2+fx^4)}{a+bx^2+cx^4} dx$$

Optimal. Leaf size=369

$$\frac{x(-c(af+be)+b^2f+c^2d)}{c^3} + \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-b^2c(cd-4af)-3abc^2e+2ac^2(cd-af)+b^4(-f)+b^3ce}{\sqrt{b^2-4ac}} - bc(cd-2af) - a\right)}{\sqrt{2}c^{7/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

Rubi [A] time = 4.58, antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 30, number of rules / integrand size = 0.100, Rules used = {1664, 1166, 205}

$$\frac{x(-c(af+be)+b^2f+c^2d)}{c^3} + \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-b^2c(cd-4af)-3abc^2e+2ac^2(cd-af)+b^4(-f)+b^3ce}{\sqrt{b^2-4ac}} - bc(cd-2af) - a^2e + b^2ce + b^3(-f)\right)}{\sqrt{2}c^{7/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)\left(\frac{-b^2c(cd-4af)-3abc^2e+2ac^2(cd-af)+b^4(-f)+b^3ce}{\sqrt{b^2-4ac}} - bc(cd-2af) - a^2e + b^2ce + b^3(-f)\right)}{\sqrt{2}c^{7/2}\sqrt{b+\sqrt{b^2-4ac}}} + \frac{x^3(cx-bf)}{3c^2} + \frac{fx^5}{5c}$$

Antiderivative was successfully verified.

[In] Int[(x^4*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4), x]

[Out] ((c^2*d + b^2*f - c*(b*e + a*f))*x)/c^3 + ((c*e - b*f)*x^3)/(3*c^2) + (f*x^5)/(5*c) + ((b^2*c*e - a*c^2*e - b^3*f - b*c*(c*d - 2*a*f) - (b^3*c*e - 3*a*b*c^2*e - b^4*f - b^2*c*(c*d - 4*a*f) + 2*a*c^2*(c*d - a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/(Sqrt[2]*c^(7/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^2*c*e - a*c^2*e - b^3*f - b*c*(c*d - 2*a*f) + (b^3*c*e - 3*a*b*c^2*e - b^4*f - b^2*c*(c*d - 4*a*f) + 2*a*c^2*(c*d - a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]/(Sqrt[2]*c^(7/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1664

Int[(Pq_)*((d_.)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]

Rubi steps

$$\int \frac{x^4(d + ex^2 + fx^4)}{a + bx^2 + cx^4} dx = \int \left(\frac{c^2d + b^2f - c(be + af)}{c^3} + \frac{(ce - bf)x^2}{c^2} + \frac{fx^4}{c} - \frac{a(c^2d + b^2f - c(be + af)) - (b^2ce - b^2cf)}{c^3(a + bx^2 + cx^4)} \right) dx$$

$$= \frac{(c^2d + b^2f - c(be + af))x}{c^3} + \frac{(ce - bf)x^3}{3c^2} + \frac{fx^5}{5c} - \frac{\int \frac{a(c^2d + b^2f - c(be + af)) + (-b^2ce + ac^2e + b^3f)}{a + bx^2 + cx^4}}{c^3}$$

$$= \frac{(c^2d + b^2f - c(be + af))x}{c^3} + \frac{(ce - bf)x^3}{3c^2} + \frac{fx^5}{5c} + \frac{(b^2ce - ac^2e - b^3f - bc(cd - 2af))}{c^3}$$

$$= \frac{(c^2d + b^2f - c(be + af))x}{c^3} + \frac{(ce - bf)x^3}{3c^2} + \frac{fx^5}{5c} + \frac{(b^2ce - ac^2e - b^3f - bc(cd - 2af))}{c^3}$$

Mathematica [A] time = 0.51, size = 456, normalized size = 1.24

$$\frac{x(-c(f+bg)+bf+cd)}{c^3} \operatorname{atan}\left(\frac{\sqrt{c}x}{\sqrt{b-cx^2}}\right) \left(a^2(\sqrt{b^2-4ac}-2af+2d) - b^2c(\sqrt{b^2-4ac}-4af+cd) + bc(a\sqrt{b^2-4ac}-2af\sqrt{b^2-4ac}-3ac) + b^2(\sqrt{b^2-4ac}+c) + b^4(-f) \right) \operatorname{atan}\left(\frac{\sqrt{c}x}{\sqrt{b-cx^2}}\right) \left(a^2(\sqrt{b^2-4ac}+2af-2d) + b^2c(-\sqrt{b^2-4ac}-4af+cd) + bc(a\sqrt{b^2-4ac}-2af\sqrt{b^2-4ac}+3ac) + b^2(\sqrt{b^2-4ac}-c) + b^4f \right) \frac{c^2(ce-bf)}{5c} + \frac{c^2}{5c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
[Out] ((c^2*d + b^2*f - c*(b*e + a*f))*x)/c^3 + ((c*e - b*f)*x^3)/(3*c^2) + (f*x^5)/(5*c) - (((b^4*f) - b^2*c*(c*d + Sqrt[b^2 - 4*a*c]*e - 4*a*f) + a*c^2*(2*c*d + Sqrt[b^2 - 4*a*c]*e - 2*a*f) + b^3*(c*e + Sqrt[b^2 - 4*a*c]*f) + b*c*(c*Sqrt[b^2 - 4*a*c]*d - 3*a*c*e - 2*a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/(Sqrt[2]*c^(7/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((b^4*f + b^2*c*(c*d - Sqrt[b^2 - 4*a*c]*e - 4*a*f) + a*c^2*(-2*c*d + Sqrt[b^2 - 4*a*c]*e + 2*a*f) + b^3*(-(c*e) + Sqrt[b^2 - 4*a*c]*f) + b*c*(c*Sqrt[b^2 - 4*a*c]*d + 3*a*c*e - 2*a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]/(Sqrt[2]*c^(7/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4(d + ex^2 + fx^4)}{a + bx^2 + cx^4} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^4*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
[Out] IntegrateAlgebraic[(x^4*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4), x]
```

fricas [B] time = 42.59, size = 15467, normalized size = 41.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] 1/30*(6*c^2*f*x^5 - 15*sqrt(1/2)*c^3*sqrt(-((b^3*c^4 - 3*a*b*c^5)*d^2 - 2*(b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5)*d*e + (b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c^4)*e^2 + (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*f^2 + 2*((b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c^4)*d - (b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a^3*c^4)*e)*f + (b^2*c^7 - 4*a*c^8)*sqrt(((b^4*c^8 - 2*a*b^2*c^9 + a^2*c^10)*d^4 - 4*(b^5*c^7 - 3*a*b^3*c^8 + 2*a^2*b*c^9)*d^3*e + 2*(3*b^6*c^6 - 12*a*b^4*c^7 + 12*a^2*b^2*c^8 - a^3*c^9)*d^2*e^2 - 4*(b^7*c^5 - 5*a*b^5*c^6 +
```

$$\begin{aligned}
&7a^2b^3c^7 - 2a^3b^2c^8) * d^3 + (b^8c^4 - 6a^2b^6c^5 + 11a^2b^4c^6 - 6a^3b^2c^7 + a^4c^8) * e^4 + (b^{12} - 10a^2b^{10}c + 37a^2b^8c^2 - 62a^3b^6c^3 + 46a^4b^4c^4 - 12a^5b^2c^5 + a^6c^6) * f^4 + 4 * ((b^{10}c^2 - 8a^2b^8c^3 + 22a^2b^6c^4 - 24a^3b^4c^5 + 9a^4b^2c^6 - a^5c^7) * d - (b^{11}c - 9a^2b^9c^2 + 29a^2b^7c^3 - 40a^3b^5c^4 + 22a^4b^3c^5 - 3a^5b^2c^6) * e) * f^3 + 2 * ((3b^8c^4 - 18a^2b^6c^5 + 33a^2b^4c^6 - 19a^3b^2c^7 + 3a^4c^8) * d^2 - 2 * (3b^9c^3 - 21a^2b^7c^4 + 48a^2b^5c^5 - 39a^3b^3c^6 + 8a^4b^2c^7) * d * e + (3b^{10}c^2 - 24a^2b^8c^3 + 66a^2b^6c^4 - 72a^3b^4c^5 + 27a^4b^2c^6 - a^5c^7) * e^2) * f^2 + 4 * ((b^6c^6 - 4a^2b^4c^7 + 4a^2b^2c^8 - a^3c^9) * d^3 - (3b^7c^5 - 15a^2b^5c^6 + 21a^2b^3c^7 - 7a^3b^2c^8) * d^2 * e + (3b^8c^4 - 18a^2b^6c^5 + 33a^2b^4c^6 - 18a^3b^2c^7 + a^4c^8) * d * e^2 - (b^9c^3 - 7a^2b^7c^4 + 16a^2b^5c^5 - 13a^3b^3c^6 + 3a^4b^2c^7) * e^3) * f) / (b^2c^{14} - 4a^2c^{15})) / (b^2c^7 - 4a^2c^8) * \log(-2 * ((a^2b^2c^6 - a^2c^7) * d^4 - (3a^2b^3c^5 - 5a^2b^2c^6) * d^3 * e + 3 * (a^2b^4c^4 - 2a^2b^2c^5) * d^2 * e^2 - (a^2b^5c^3 - a^2b^3c^4 - 3a^3b^2c^5) * d * e^3 + (a^2b^4c^3 - 3a^3b^2c^4 + a^4c^5) * e^4 + (a^3b^6 - 5a^4b^4c + 6a^5b^2c^2 - a^6c^3) * f^4 + ((a^2b^8 - 7a^2b^6c + 18a^3b^4c^2 - 19a^4b^2c^3 + 4a^5c^4) * d - (a^2b^7 - 3a^3b^5c - 2a^4b^3c^2 + 5a^5b^2c^3) * e) * f^3 + 3 * ((a^2b^6c^2 - 5a^2b^4c^3 + 7a^3b^2c^4 - 2a^4c^5) * d^2 - (a^2b^7c - 5a^2b^5c^2 + 8a^3b^3c^3 - 5a^4b^2c^4) * d * e + (a^2b^6c - 4a^3b^4c^2 + 3a^4b^2c^3) * e^2) * f^2 + ((3a^2b^4c^4 - 9a^2b^2c^5 + 4a^3c^6) * d^3 - 3 * (2a^2b^5c^3 - 7a^2b^3c^4 + 5a^3b^2c^5) * d^2 * e + 3 * (a^2b^6c^2 - 3a^2b^4c^3 + a^3b^2c^4) * d * e^2 - (3a^2b^5c^2 - 11a^3b^3c^3 + 7a^4b^2c^4) * e^3) * f) * x + \sqrt{1/2} * ((b^4c^6 - 5a^2b^2c^7 + 4a^2c^8) * d^3 - (3b^5c^5 - 17a^2b^3c^6 + 20a^2b^2c^7) * d^2 * e + (3b^6c^4 - 19a^2b^4c^5 + 29a^2b^2c^6 - 4a^3c^7) * d * e^2 - (b^7c^3 - 7a^2b^5c^4 + 13a^2b^3c^5 - 4a^3b^2c^6) * e^3 + (b^{10} - 10a^2b^8c + 35a^2b^6c^2 - 51a^3b^4c^3 + 29a^4b^2c^4 - 4a^5c^5) * f^3 + ((3b^8c^2 - 25a^2b^6c^3 + 66a^2b^4c^4 - 59a^3b^2c^5 + 12a^4c^6) * d - (3b^9c - 27a^2b^7c^2 + 80a^2b^5c^3 - 87a^3b^3c^4 + 28a^4b^2c^5) * e) * f^2 + ((3b^6c^4 - 20a^2b^4c^5 + 35a^2b^2c^6 - 12a^3c^7) * d^2 - 2 * (3b^7c^3 - 22a^2b^5c^4 + 46a^2b^3c^5 - 24a^3b^2c^6) * d * e + (3b^8c^2 - 24a^2b^6c^3 + 58a^2b^4c^4 - 41a^3b^2c^5 + 4a^4c^6) * e^2) * f - ((b^3c^9 - 4a^2b^3c^10) * d - (b^4c^8 - 6a^2b^2c^9 + 8a^2c^10) * e + (b^5c^7 - 7a^2b^3c^8 + 12a^2b^2c^9) * f) * \sqrt{((b^4c^8 - 2a^2b^2c^9 + a^2c^10) * d^4 - 4 * (b^5c^7 - 3a^2b^3c^8 + 2a^2b^2c^9) * d^3 * e + 2 * (3b^6c^6 - 12a^2b^4c^7 + 12a^2b^2c^8 - a^3c^9) * d^2 * e^2 - 4 * (b^7c^5 - 5a^2b^5c^6 + 7a^2b^3c^7 - 2a^3b^2c^8) * d * e^3 + (b^8c^4 - 6a^2b^6c^5 + 11a^2b^4c^6 - 6a^3b^2c^7 + a^4c^8) * e^4 + (b^{12} - 10a^2b^{10}c + 37a^2b^8c^2 - 62a^3b^6c^3 + 46a^4b^4c^4 - 12a^5b^2c^5 + a^6c^6) * f^4 + 4 * ((b^{10}c^2 - 8a^2b^8c^3 + 22a^2b^6c^4 - 24a^3b^4c^5 + 9a^4b^2c^6 - a^5c^7) * d - (b^{11}c - 9a^2b^9c^2 + 29a^2b^7c^3 - 40a^3b^5c^4 + 22a^4b^3c^5 - 3a^5b^2c^6) * e) * f^3 + 2 * ((3b^8c^4 - 18a^2b^6c^5 + 33a^2b^4c^6 - 19a^3b^2c^7 + 3a^4c^8) * d^2 - 2 * (3b^9c^3 - 21a^2b^7c^4 + 48a^2b^5c^5 - 39a^3b^3c^6 + 8a^4b^2c^7) * d * e + (3b^{10}c^2 - 24a^2b^8c^3 + 66a^2b^6c^4 - 72a^3b^4c^5 + 27a^4b^2c^6 - a^5c^7) * e^2) * f^2 + 4 * ((b^6c^6 - 4a^2b^4c^7 + 4a^2b^2c^8 - a^3c^9) * d^3 - (3b^7c^5 - 15a^2b^5c^6 + 21a^2b^3c^7 - 7a^3b^2c^8) * d^2 * e + (3b^8c^4 - 18a^2b^6c^5 + 33a^2b^4c^6 - 18a^3b^2c^7 + a^4c^8) * d * e^2 - (b^9c^3 - 7a^2b^7c^4 + 16a^2b^5c^5 - 13a^3b^3c^6 + 3a^4b^2c^7) * e^3) * f) / (b^2c^{14} - 4a^2c^{15})) * \sqrt{-((b^3c^4 - 3a^2b^2c^5) * d^2 - 2 * (b^4c^3 - 4a^2b^2c^4 + 2a^2c^5) * d * e + (b^5c^2 - 5a^2b^3c^3 + 5a^2b^2c^4) * e^2 + (b^7 - 7a^2b^5c + 14a^2b^3c^2 - 7a^3b^2c^3) * f^2 + 2 * ((b^5c^2 - 5a^2b^3c^3 + 5a^2b^2c^4) * d - (b^6c - 6a^2b^4c^2 + 9a^2b^2c^3 - 2a^3c^4) * e) * f + (b^2c^7 - 4a^2c^8) * \sqrt{((b^4c^8 - 2a^2b^2c^9 + a^2c^10) * d^4 - 4 * (b^5c^7 - 3a^2b^3c^8 + 2a^2b^2c^9) * d^3 * e + 2 * (3b^6c^6 - 12a^2b^4c^7 + 12a^2b^2c^8 - a^3c^9) * d^2 * e^2 - 4 * (b^7c^5 - 5a^2b^5c^6 + 7a^2b^3c^7 - 2a^3b^2c^8) * d * e^3 + (b^8c^4 - 6a^2b^6c^5 + 11a^2b^4c^6 - 6a^3b^2c^7 + a^4c^8) * e^4 + (b^{12} - 10a^2b^{10}c + 37a^2b^8c^2 - 62a^3b^6c^3 + 46a^4b^4c^4}
\end{aligned}$$

$$\begin{aligned}
& c^4 - 12a^5b^2c^5 + a^6c^6) * f^4 + 4 * ((b^{10}c^2 - 8a^2b^8c^3 + 22a^2b^6c^4 - 24a^3b^4c^5 + 9a^4b^2c^6 - a^5c^7) * d - (b^{11}c - 9a^2b^9c^2 + 29a^2b^7c^3 - 40a^3b^5c^4 + 22a^4b^3c^5 - 3a^5b^2c^6) * e) * f^3 \\
& + 2 * ((3b^8c^4 - 18a^2b^6c^5 + 33a^2b^4c^6 - 19a^3b^2c^7 + 3a^4c^8) * d^2 - 2 * (3b^9c^3 - 21a^2b^7c^4 + 48a^2b^5c^5 - 39a^3b^3c^6 + 8a^4b^2c^7) * d * e + (3b^{10}c^2 - 24a^2b^8c^3 + 66a^2b^6c^4 - 72a^3b^4c^5 + 27a^4b^2c^6 - a^5c^7) * e^2) * f^2 + 4 * ((b^6c^6 - 4a^2b^4c^7 + 4a^2b^2c^8 - a^3c^9) * d^3 - (3b^7c^5 - 15a^2b^5c^6 + 21a^2b^3c^7 - 7a^3b^2c^8) * d^2 * e + (3b^8c^4 - 18a^2b^6c^5 + 33a^2b^4c^6 - 18a^3b^2c^7 + a^4c^8) * d * e^2 - (b^9c^3 - 7a^2b^7c^4 + 16a^2b^5c^5 - 13a^3b^3c^6 + 3a^4b^2c^7) * e^3) * f) / (b^2c^{14} - 4a^2c^{15})) / (b^2c^7 - 4a^2c^8)) + 1 \\
& 5 * \sqrt{1/2} * c^3 * \sqrt{-(b^3c^4 - 3a^2b^2c^5) * d^2 - 2 * (b^4c^3 - 4a^2b^2c^4 + 2a^2c^5) * d * e + (b^5c^2 - 5a^2b^3c^3 + 5a^2b^2c^4) * e^2 + (b^7 - 7a^2b^5c + 14a^2b^3c^2 - 7a^3b^2c^3) * f^2 + 2 * ((b^5c^2 - 5a^2b^3c^3 + 5a^2b^2c^4) * d - (b^6c - 6a^2b^4c^2 + 9a^2b^2c^3 - 2a^3c^4) * e) * f + (b^2c^7 - 4a^2c^8) * \sqrt{((b^4c^8 - 2a^2b^2c^9 + a^2c^{10}) * d^4 - 4 * (b^5c^7 - 3a^2b^3c^8 + 2a^2b^2c^9) * d^3 * e + 2 * (3b^6c^6 - 12a^2b^4c^7 + 12a^2b^2c^8 - a^3c^9) * d^2 * e^2 - 4 * (b^7c^5 - 5a^2b^5c^6 + 7a^2b^3c^7 - 2a^3b^2c^8) * d * e^3 + (b^8c^4 - 6a^2b^6c^5 + 11a^2b^4c^6 - 6a^3b^2c^7 + a^4c^8) * e^4 + (b^{12} - 10a^2b^{10}c + 37a^2b^8c^2 - 62a^3b^6c^3 + 46a^4b^4c^4 - 12a^5b^2c^5 + a^6c^6) * f^4 + 4 * ((b^{10}c^2 - 8a^2b^8c^3 + 22a^2b^6c^4 - 24a^3b^4c^5 + 9a^4b^2c^6 - a^5c^7) * d - (b^{11}c - 9a^2b^9c^2 + 29a^2b^7c^3 - 40a^3b^5c^4 + 22a^4b^3c^5 - 3a^5b^2c^6) * e) * f^3 + 2 * ((3b^8c^4 - 18a^2b^6c^5 + 33a^2b^4c^6 - 19a^3b^2c^7 + 3a^4c^8) * d^2 - 2 * (3b^9c^3 - 21a^2b^7c^4 + 48a^2b^5c^5 - 39a^3b^3c^6 + 8a^4b^2c^7) * d * e + (3b^{10}c^2 - 24a^2b^8c^3 + 66a^2b^6c^4 - 72a^3b^4c^5 + 27a^4b^2c^6 - a^5c^7) * e^2) * f^2 + 4 * ((b^6c^6 - 4a^2b^4c^7 + 4a^2b^2c^8 - a^3c^9) * d^3 - (3b^7c^5 - 15a^2b^5c^6 + 21a^2b^3c^7 - 7a^3b^2c^8) * d^2 * e + (3b^8c^4 - 18a^2b^6c^5 + 33a^2b^4c^6 - 18a^3b^2c^7 + a^4c^8) * d * e^2 - (b^9c^3 - 7a^2b^7c^4 + 16a^2b^5c^5 - 13a^3b^3c^6 + 3a^4b^2c^7) * e^3) * f) / (b^2c^{14} - 4a^2c^{15})) / (b^2c^7 - 4a^2c^8) \\
&) * \log(-2 * ((a^2b^2c^6 - a^2c^7) * d^4 - (3a^2b^3c^5 - 5a^2b^2c^6) * d^3 * e + 3 * (a^2b^4c^4 - 2a^2b^2c^5) * d^2 * e^2 - (a^2b^5c^3 - a^2b^3c^4 - 3a^3b^2c^5) * d * e^3 + (a^2b^4c^3 - 3a^3b^2c^4 + a^4c^5) * e^4 + (a^3b^6 - 5a^4b^4c + 6a^5b^2c^2 - a^6c^3) * f^4 + ((a^2b^8 - 7a^2b^6c + 18a^3b^4c^2 - 19a^4b^2c^3 + 4a^5c^4) * d - (a^2b^7 - 3a^3b^5c - 2a^4b^3c^2 + 5a^5b^2c^3) * e) * f^3 + 3 * ((a^2b^6c^2 - 5a^2b^4c^3 + 7a^3b^2c^4 - 2a^4c^5) * d^2 - (a^2b^7c - 5a^2b^5c^2 + 8a^3b^3c^3 - 5a^4b^2c^4) * d * e + (a^2b^6c - 4a^3b^4c^2 + 3a^4b^2c^3) * e^2) * f^2 + ((3a^2b^4c^4 - 9a^2b^2c^5 + 4a^3c^6) * d^3 - 3 * (2a^2b^5c^3 - 7a^2b^3c^4 + 5a^3b^2c^5) * d^2 * e + 3 * (a^2b^6c^2 - 3a^2b^4c^3 + a^3b^2c^4) * d * e^2 - (3a^2b^5c^2 - 11a^3b^3c^3 + 7a^4b^2c^4) * e^3) * f) * x - \sqrt{1/2} * ((b^4c^6 - 5a^2b^2c^7 + 4a^2c^8) * d^3 - (3b^5c^5 - 17a^2b^3c^6 + 20a^2b^2c^7) * d^2 * e + (3b^6c^4 - 19a^2b^4c^5 + 29a^2b^2c^6 - 4a^3c^7) * d * e^2 - (b^7c^3 - 7a^2b^5c^4 + 13a^2b^3c^5 - 4a^3b^2c^6) * e^3 + (b^{10} - 10a^2b^8c + 35a^2b^6c^2 - 51a^3b^4c^3 + 29a^4b^2c^4 - 4a^5c^5) * f^3 + ((3b^8c^2 - 25a^2b^6c^3 + 66a^2b^4c^4 - 59a^3b^2c^5 + 12a^4c^6) * d - (3b^9c - 27a^2b^7c^2 + 80a^2b^5c^3 - 87a^3b^3c^4 + 28a^4b^2c^5) * e) * f^2 + ((3b^6c^4 - 20a^2b^4c^5 + 35a^2b^2c^6 - 12a^3c^7) * d^2 - 2 * (3b^7c^3 - 22a^2b^5c^4 + 46a^2b^3c^5 - 24a^3b^2c^6) * d * e + (3b^8c^2 - 24a^2b^6c^3 + 58a^2b^4c^4 - 41a^3b^2c^5 + 4a^4c^6) * e^2) * f - ((b^3c^9 - 4a^2b^2c^10) * d - (b^4c^8 - 6a^2b^2c^9 + 8a^2c^10) * e + (b^5c^7 - 7a^2b^3c^8 + 12a^2b^2c^9) * f) * \sqrt{((b^4c^8 - 2a^2b^2c^9 + a^2c^{10}) * d^4 - 4 * (b^5c^7 - 3a^2b^3c^8 + 2a^2b^2c^9) * d^3 * e + 2 * (3b^6c^6 - 12a^2b^4c^7 + 12a^2b^2c^8 - a^3c^9) * d^2 * e^2 - 4 * (b^7c^5 - 5a^2b^5c^6 + 7a^2b^3c^7 - 2a^3b^2c^8) * d * e^3 + (b^8c^4 - 6a^2b^6c^5 + 11a^2b^4c^6 - 6a^3b^2c^7 + a^4c^8) * e^4 + (b^{12} - 10a^2b^{10}c + 37a^2b^8c^2 - 62a^3b^6c^3 + 46a^4b^4c^4 - 12a^5b^2c^5 + a^6c^6) * f^4 + 4 * ((b^{10}c^2 - 8a^2b^8c^3 + 22a^2b^6c^4 - 24a^3b^4c^5 + 9a^4b^2c^6 - a^5c^7) * d - (b^{11}c
\end{aligned}$$

$$\begin{aligned}
& c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5* \\
& b*c^6)*e)*f^3 + 2*((3*b^8*c^4 - 18*a*b^6*c^5 + 33*a^2*b^4*c^6 - 19*a^3*b^2* \\
& c^7 + 3*a^4*c^8)*d^2 - 2*(3*b^9*c^3 - 21*a*b^7*c^4 + 48*a^2*b^5*c^5 - 39*a^ \\
& 3*b^3*c^6 + 8*a^4*b*c^7)*d*e + (3*b^10*c^2 - 24*a*b^8*c^3 + 66*a^2*b^6*c^4 \\
& - 72*a^3*b^4*c^5 + 27*a^4*b^2*c^6 - a^5*c^7)*e^2)*f^2 + 4*((b^6*c^6 - 4*a*b \\
& ^4*c^7 + 4*a^2*b^2*c^8 - a^3*c^9)*d^3 - (3*b^7*c^5 - 15*a*b^5*c^6 + 21*a^2* \\
& b^3*c^7 - 7*a^3*b*c^8)*d^2*e + (3*b^8*c^4 - 18*a*b^6*c^5 + 33*a^2*b^4*c^6 - \\
& 18*a^3*b^2*c^7 + a^4*c^8)*d*e^2 - (b^9*c^3 - 7*a*b^7*c^4 + 16*a^2*b^5*c^5 \\
& - 13*a^3*b^3*c^6 + 3*a^4*b*c^7)*e^3)*f)/(b^2*c^14 - 4*a*c^15))*sqrt(-((b^3 \\
& *c^4 - 3*a*b*c^5)*d^2 - 2*(b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5)*d*e + (b^5*c^ \\
& 2 - 5*a*b^3*c^3 + 5*a^2*b*c^4)*e^2 + (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7* \\
& a^3*b*c^3)*f^2 + 2*((b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c^4)*d - (b^6*c - 6*a* \\
& b^4*c^2 + 9*a^2*b^2*c^3 - 2*a^3*c^4)*e)*f + (b^2*c^7 - 4*a*c^8)*sqrt(((b^4* \\
& c^8 - 2*a*b^2*c^9 + a^2*c^10)*d^4 - 4*(b^5*c^7 - 3*a*b^3*c^8 + 2*a^2*b*c^9) \\
& *d^3*e + 2*(3*b^6*c^6 - 12*a*b^4*c^7 + 12*a^2*b^2*c^8 - a^3*c^9)*d^2*e^2 - \\
& 4*(b^7*c^5 - 5*a*b^5*c^6 + 7*a^2*b^3*c^7 - 2*a^3*b*c^8)*d*e^3 + (b^8*c^4 - \\
& 6*a*b^6*c^5 + 11*a^2*b^4*c^6 - 6*a^3*b^2*c^7 + a^4*c^8)*e^4 + (b^12 - 10*a* \\
& b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 \\
& + a^6*c^6)*f^4 + 4*((b^10*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4 - 24*a^3*b^4*c^ \\
& ^5 + 9*a^4*b^2*c^6 - a^5*c^7)*d - (b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - \\
& 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*e)*f^3 + 2*((3*b^8*c^4 - 18* \\
& a*b^6*c^5 + 33*a^2*b^4*c^6 - 19*a^3*b^2*c^7 + 3*a^4*c^8)*d^2 - 2*(3*b^9*c^3 \\
& - 21*a*b^7*c^4 + 48*a^2*b^5*c^5 - 39*a^3*b^3*c^6 + 8*a^4*b*c^7)*d*e + (3*b \\
& ^10*c^2 - 24*a*b^8*c^3 + 66*a^2*b^6*c^4 - 72*a^3*b^4*c^5 + 27*a^4*b^2*c^6 - \\
& a^5*c^7)*e^2)*f^2 + 4*((b^6*c^6 - 4*a*b^4*c^7 + 4*a^2*b^2*c^8 - a^3*c^9)*d \\
& ^3 - (3*b^7*c^5 - 15*a*b^5*c^6 + 21*a^2*b^3*c^7 - 7*a^3*b*c^8)*d^2*e + (3*b \\
& ^8*c^4 - 18*a*b^6*c^5 + 33*a^2*b^4*c^6 - 18*a^3*b^2*c^7 + a^4*c^8)*d*e^2 - \\
& (b^9*c^3 - 7*a*b^7*c^4 + 16*a^2*b^5*c^5 - 13*a^3*b^3*c^6 + 3*a^4*b*c^7)*e^3 \\
&)*f)/(b^2*c^14 - 4*a*c^15)))/(b^2*c^7 - 4*a*c^8))) - 15*sqrt(1/2)*c^3*sqrt(\\
& -((b^3*c^4 - 3*a*b*c^5)*d^2 - 2*(b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5)*d*e + (\\
& b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c^4)*e^2 + (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^ \\
& 2 - 7*a^3*b*c^3)*f^2 + 2*((b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c^4)*d - (b^6*c \\
& - 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a^3*c^4)*e)*f - (b^2*c^7 - 4*a*c^8)*sqrt(\\
& ((b^4*c^8 - 2*a*b^2*c^9 + a^2*c^10)*d^4 - 4*(b^5*c^7 - 3*a*b^3*c^8 + 2*a^2* \\
& b*c^9)*d^3*e + 2*(3*b^6*c^6 - 12*a*b^4*c^7 + 12*a^2*b^2*c^8 - a^3*c^9)*d^2* \\
& e^2 - 4*(b^7*c^5 - 5*a*b^5*c^6 + 7*a^2*b^3*c^7 - 2*a^3*b*c^8)*d*e^3 + (b^8* \\
& c^4 - 6*a*b^6*c^5 + 11*a^2*b^4*c^6 - 6*a^3*b^2*c^7 + a^4*c^8)*e^4 + (b^12 - \\
& 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^ \\
& 2*c^5 + a^6*c^6)*f^4 + 4*((b^10*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4 - 24*a^3 \\
& *b^4*c^5 + 9*a^4*b^2*c^6 - a^5*c^7)*d - (b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7* \\
& c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*e)*f^3 + 2*((3*b^8*c^4 \\
& - 18*a*b^6*c^5 + 33*a^2*b^4*c^6 - 19*a^3*b^2*c^7 + 3*a^4*c^8)*d^2 - 2*(3*b \\
& ^9*c^3 - 21*a*b^7*c^4 + 48*a^2*b^5*c^5 - 39*a^3*b^3*c^6 + 8*a^4*b*c^7)*d*e \\
& + (3*b^10*c^2 - 24*a*b^8*c^3 + 66*a^2*b^6*c^4 - 72*a^3*b^4*c^5 + 27*a^4*b^2 \\
& *c^6 - a^5*c^7)*e^2)*f^2 + 4*((b^6*c^6 - 4*a*b^4*c^7 + 4*a^2*b^2*c^8 - a^3* \\
& c^9)*d^3 - (3*b^7*c^5 - 15*a*b^5*c^6 + 21*a^2*b^3*c^7 - 7*a^3*b*c^8)*d^2*e \\
& + (3*b^8*c^4 - 18*a*b^6*c^5 + 33*a^2*b^4*c^6 - 18*a^3*b^2*c^7 + a^4*c^8)*d* \\
& e^2 - (b^9*c^3 - 7*a*b^7*c^4 + 16*a^2*b^5*c^5 - 13*a^3*b^3*c^6 + 3*a^4*b*c^ \\
& 7)*e^3)*f)/(b^2*c^14 - 4*a*c^15)))/(b^2*c^7 - 4*a*c^8))*log(-2*((a*b^2*c^6 \\
& - a^2*c^7)*d^4 - (3*a*b^3*c^5 - 5*a^2*b*c^6)*d^3*e + 3*(a*b^4*c^4 - 2*a^2*b \\
& ^2*c^5)*d^2*e^2 - (a*b^5*c^3 - a^2*b^3*c^4 - 3*a^3*b*c^5)*d*e^3 + (a^2*b^4* \\
& c^3 - 3*a^3*b^2*c^4 + a^4*c^5)*e^4 + (a^3*b^6 - 5*a^4*b^4*c + 6*a^5*b^2*c^2 \\
& - a^6*c^3)*f^4 + ((a*b^8 - 7*a^2*b^6*c + 18*a^3*b^4*c^2 - 19*a^4*b^2*c^3 + \\
& 4*a^5*c^4)*d - (a^2*b^7 - 3*a^3*b^5*c - 2*a^4*b^3*c^2 + 5*a^5*b*c^3)*e)*f^ \\
& 3 + 3*((a*b^6*c^2 - 5*a^2*b^4*c^3 + 7*a^3*b^2*c^4 - 2*a^4*c^5)*d^2 - (a*b^7 \\
& *c - 5*a^2*b^5*c^2 + 8*a^3*b^3*c^3 - 5*a^4*b*c^4)*d*e + (a^2*b^6*c - 4*a^3* \\
& b^4*c^2 + 3*a^4*b^2*c^3)*e^2)*f^2 + ((3*a*b^4*c^4 - 9*a^2*b^2*c^5 + 4*a^3*c^ \\
& ^6)*d^3 - 3*(2*a*b^5*c^3 - 7*a^2*b^3*c^4 + 5*a^3*b*c^5)*d^2*e + 3*(a*b^6*c^ \\
& 2 - 3*a^2*b^4*c^3 + a^3*b^2*c^4)*d*e^2 - (3*a^2*b^5*c^2 - 11*a^3*b^3*c^3 +
\end{aligned}$$

$$\begin{aligned}
& 7*a^4*b*c^4)*e^3)*f)*x + \text{sqrt}(1/2)*((b^4*c^6 - 5*a*b^2*c^7 + 4*a^2*c^8)*d^3 \\
& - (3*b^5*c^5 - 17*a*b^3*c^6 + 20*a^2*b*c^7)*d^2*e + (3*b^6*c^4 - 19*a*b^4*c^5 \\
& + 29*a^2*b^2*c^6 - 4*a^3*c^7)*d*e^2 - (b^7*c^3 - 7*a*b^5*c^4 + 13*a^2*b^3*c^5 \\
& - 4*a^3*b*c^6)*e^3 + (b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 51*a^3*b^4*c^3 \\
& + 29*a^4*b^2*c^4 - 4*a^5*c^5)*f^3 + ((3*b^8*c^2 - 25*a*b^6*c^3 + 66*a^2*b^4*c^4 \\
& - 59*a^3*b^2*c^5 + 12*a^4*c^6)*d - (3*b^9*c - 27*a*b^7*c^2 + 80*a^2*b^5*c^3 \\
& - 87*a^3*b^3*c^4 + 28*a^4*b*c^5)*e)*f^2 + ((3*b^6*c^4 - 20*a*b^4*c^5 \\
& + 35*a^2*b^2*c^6 - 12*a^3*c^7)*d^2 - 2*(3*b^7*c^3 - 22*a*b^5*c^4 + 46*a^2*b^3*c^5 \\
& - 24*a^3*b*c^6)*d*e + (3*b^8*c^2 - 24*a*b^6*c^3 + 58*a^2*b^4*c^4 - 41*a^3*b^2*c^5 \\
& + 4*a^4*c^6)*e^2)*f + ((b^3*c^9 - 4*a*b*c^10)*d - (b^4*c^8 - 6*a*b^2*c^9 \\
& + 8*a^2*c^10)*e + (b^5*c^7 - 7*a*b^3*c^8 + 12*a^2*b*c^9)*f)*\text{sqrt}(((b^4*c^8 - 2*a*b^2*c^9 \\
& + a^2*c^10)*d^4 - 4*(b^5*c^7 - 3*a*b^3*c^8 + 2*a^2*b*c^9)*d^3*e + 2*(3*b^6*c^6 \\
& - 12*a*b^4*c^7 + 12*a^2*b^2*c^8 - a^3*c^9)*d^2*e^2 - 4*(b^7*c^5 - 5*a*b^5*c^6 \\
& + 7*a^2*b^3*c^7 - 2*a^3*b*c^8)*d*e^3 + (b^8*c^4 - 6*a*b^6*c^5 + 11*a^2*b^4*c^6 \\
& - 6*a^3*b^2*c^7 + a^4*c^8)*e^4 + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 \\
& + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*f^4 + 4*((b^10*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4 \\
& - 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6 - a^5*c^7)*d - (b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 \\
& - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*e)*f^3 + 2*((3*b^8*c^4 - 18*a*b^6*c^5 \\
& + 33*a^2*b^4*c^6 - 19*a^3*b^2*c^7 + 3*a^4*c^8)*d^2 - 2*(3*b^9*c^3 - 21*a*b^7*c^4 \\
& + 48*a^2*b^5*c^5 - 39*a^3*b^3*c^6 + 8*a^4*b*c^7)*d*e + (3*b^10*c^2 - 24*a*b^8*c^3 \\
& + 66*a^2*b^6*c^4 - 72*a^3*b^4*c^5 + 27*a^4*b^2*c^6 - a^5*c^7)*e^2)*f^2 + 4*((b^6*c^6 - 4*a*b^4*c^7 \\
& + 4*a^2*b^2*c^8 - a^3*c^9)*d^3 - (3*b^7*c^5 - 15*a*b^5*c^6 + 21*a^2*b^3*c^7 - 7*a^3*b*c^8) \\
&)*d^2*e + (3*b^8*c^4 - 18*a*b^6*c^5 + 33*a^2*b^4*c^6 - 18*a^3*b^2*c^7 + a^4*c^8)*d*e^2 \\
& - (b^9*c^3 - 7*a*b^7*c^4 + 16*a^2*b^5*c^5 - 13*a^3*b^3*c^6 + 3*a^4*b*c^7)*e^3)*f)/(b^2*c^14 \\
& - 4*a*c^15))*\text{sqrt}(-((b^3*c^4 - 3*a*b*c^5)*d^2 - 2*(b^4*c^3 - 4*a*b^2*c^4 \\
& + 2*a^2*c^5)*d*e + (b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c^4)*e^2 + (b^7 - 7*a*b^5*c \\
& + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*f^2 + 2*((b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c^4)*d \\
& - (b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a^3*c^4)*e)*f - (b^2*c^7 - 4*a*c^8)*\text{sqrt}(((b^4*c^8 \\
& - 2*a*b^2*c^9 + a^2*c^10)*d^4 - 4*(b^5*c^7 - 3*a*b^3*c^8 + 2*a^2*b*c^9)*d^3*e + 2*(3*b^6*c^6 \\
& - 12*a*b^4*c^7 + 12*a^2*b^2*c^8 - a^3*c^9)*d^2*e^2 - 4*(b^7*c^5 - 5*a*b^5*c^6 \\
& + 7*a^2*b^3*c^7 - 2*a^3*b*c^8)*d*e^3 + (b^8*c^4 - 6*a*b^6*c^5 + 11*a^2*b^4*c^6 \\
& - 6*a^3*b^2*c^7 + a^4*c^8)*e^4 + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 \\
& + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*f^4 + 4*((b^10*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4 \\
& - 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6 - a^5*c^7)*d - (b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 \\
& - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*e)*f^3 + 2*((3*b^8*c^4 - 18*a*b^6*c^5 \\
& + 33*a^2*b^4*c^6 - 19*a^3*b^2*c^7 + 3*a^4*c^8)*d^2 - 2*(3*b^9*c^3 - 21*a*b^7*c^4 + 48*a^2*b^5*c^5 \\
& - 39*a^3*b^3*c^6 + 8*a^4*b*c^7)*d*e + (3*b^10*c^2 - 24*a*b^8*c^3 + 66*a^2*b^6*c^4 \\
& - 72*a^3*b^4*c^5 + 27*a^4*b^2*c^6 - a^5*c^7)*e^2)*f^2 + 4*((b^6*c^6 - 4*a*b^4*c^7 \\
& + 4*a^2*b^2*c^8 - a^3*c^9)*d^3 - (3*b^7*c^5 - 15*a*b^5*c^6 + 21*a^2*b^3*c^7 - 7*a^3*b*c^8) \\
&)*d^2*e + (3*b^8*c^4 - 18*a*b^6*c^5 + 33*a^2*b^4*c^6 - 18*a^3*b^2*c^7 + a^4*c^8)*d*e^2 \\
& - (b^9*c^3 - 7*a*b^7*c^4 + 16*a^2*b^5*c^5 - 13*a^3*b^3*c^6 + 3*a^4*b*c^7)*e^3)*f)/(b^2*c^14 \\
& - 4*a*c^15)))/(b^2*c^7 - 4*a*c^8))) + 15*\text{sqrt}(1/2)*c^3*\text{sqrt}(-((b^3*c^4 - 3*a*b*c^5) \\
&)*d^2 - 2*(b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5)*d*e + (b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c^4) \\
&)*e^2 + (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*f^2 + 2*((b^5*c^2 - 5*a*b^3*c^3 \\
& + 5*a^2*b*c^4)*d - (b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a^3*c^4)*e)*f - (b^2*c^7 - 4*a*c^8) \\
&)*\text{sqrt}(((b^4*c^8 - 2*a*b^2*c^9 + a^2*c^10)*d^4 - 4*(b^5*c^7 - 3*a*b^3*c^8 + 2*a^2*b*c^9) \\
&)*d^3*e + 2*(3*b^6*c^6 - 12*a*b^4*c^7 + 12*a^2*b^2*c^8 - a^3*c^9)*d^2*e^2 - 4*(b^7*c^5 - 5*a*b^5*c^6 \\
& + 7*a^2*b^3*c^7 - 2*a^3*b*c^8)*d*e^3 + (b^8*c^4 - 6*a*b^6*c^5 + 11*a^2*b^4*c^6 - 6*a^3*b^2*c^7 \\
& + a^4*c^8)*e^4 + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 \\
& + a^6*c^6)*f^4 + 4*((b^10*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4 - 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6 - a^5*c^7) \\
&)*d - (b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6) \\
&)*e)*f^3 + 2*((3*b^8*c^4 - 18*a*b^6*c^5 + 33*
\end{aligned}$$

$$\begin{aligned}
& a^2b^4c^6 - 19a^3b^2c^7 + 3a^4c^8) * d^2 - 2*(3b^9c^3 - 21a*b^7c^4 \\
& + 48a^2b^5c^5 - 39a^3b^3c^6 + 8a^4b*c^7) * d * e + (3b^{10}c^2 - 24a* \\
& b^8c^3 + 66a^2b^6c^4 - 72a^3b^4c^5 + 27a^4b^2c^6 - a^5c^7) * e^2) * \\
& f^2 + 4*((b^6c^6 - 4a*b^4c^7 + 4a^2b^2c^8 - a^3c^9) * d^3 - (3b^7c^5 \\
& - 15a*b^5c^6 + 21a^2b^3c^7 - 7a^3b*c^8) * d^2 * e + (3b^8c^4 - 18a*b \\
& ^6c^5 + 33a^2b^4c^6 - 18a^3b^2c^7 + a^4c^8) * d * e^2 - (b^9c^3 - 7a* \\
& b^7c^4 + 16a^2b^5c^5 - 13a^3b^3c^6 + 3a^4b*c^7) * e^3) * f) / (b^2c^{14} \\
& - 4a*c^{15})) / (b^2c^7 - 4a*c^8)) * \log(-2*((a*b^2c^6 - a^2c^7) * d^4 - (3a \\
& *b^3c^5 - 5a^2b*c^6) * d^3 * e + 3*(a*b^4c^4 - 2a^2b^2c^5) * d^2 * e^2 - (a \\
& b^5c^3 - a^2b^3c^4 - 3a^3b*c^5) * d * e^3 + (a^2b^4c^3 - 3a^3b^2c^4 + \\
& a^4c^5) * e^4 + (a^3b^6 - 5a^4b^4c + 6a^5b^2c^2 - a^6c^3) * f^4 + ((a \\
& *b^8 - 7a^2b^6c + 18a^3b^4c^2 - 19a^4b^2c^3 + 4a^5c^4) * d - (a^2* \\
& b^7 - 3a^3b^5c - 2a^4b^3c^2 + 5a^5b*c^3) * e) * f^3 + 3*((a*b^6c^2 - 5 \\
& *a^2b^4c^3 + 7a^3b^2c^4 - 2a^4c^5) * d^2 - (a*b^7c - 5a^2b^5c^2 + \\
& 8a^3b^3c^3 - 5a^4b*c^4) * d * e + (a^2b^6c - 4a^3b^4c^2 + 3a^4b^2c \\
& ^3) * e^2) * f^2 + ((3a*b^4c^4 - 9a^2b^2c^5 + 4a^3c^6) * d^3 - 3*(2a*b^5* \\
& c^3 - 7a^2b^3c^4 + 5a^3b*c^5) * d^2 * e + 3*(a*b^6c^2 - 3a^2b^4c^3 + a \\
& ^3b^2c^4) * d * e^2 - (3a^2b^5c^2 - 11a^3b^3c^3 + 7a^4b*c^4) * e^3) * f) * \\
& x - \text{sqrt}(1/2) * ((b^4c^6 - 5a*b^2c^7 + 4a^2c^8) * d^3 - (3b^5c^5 - 17a* \\
& b^3c^6 + 20a^2b*c^7) * d^2 * e + (3b^6c^4 - 19a*b^4c^5 + 29a^2b^2c^6 \\
& - 4a^3c^7) * d * e^2 - (b^7c^3 - 7a*b^5c^4 + 13a^2b^3c^5 - 4a^3b*c^6) \\
& * e^3 + (b^{10} - 10a*b^8c + 35a^2b^6c^2 - 51a^3b^4c^3 + 29a^4b^2c^4 \\
& - 4a^5c^5) * f^3 + ((3b^8c^2 - 25a*b^6c^3 + 66a^2b^4c^4 - 59a^3b \\
& ^2c^5 + 12a^4c^6) * d - (3b^9c - 27a*b^7c^2 + 80a^2b^5c^3 - 87a^3* \\
& b^3c^4 + 28a^4b*c^5) * e) * f^2 + ((3b^6c^4 - 20a*b^4c^5 + 35a^2b^2c^6 \\
& - 12a^3c^7) * d^2 - 2*(3b^7c^3 - 22a*b^5c^4 + 46a^2b^3c^5 - 24a^3 \\
& *b*c^6) * d * e + (3b^8c^2 - 24a*b^6c^3 + 58a^2b^4c^4 - 41a^3b^2c^5 + \\
& 4a^4c^6) * e^2) * f + ((b^3c^9 - 4a*b*c^{10}) * d - (b^4c^8 - 6a*b^2c^9 + 8 \\
& *a^2c^{10}) * e + (b^5c^7 - 7a*b^3c^8 + 12a^2b*c^9) * f) * \text{sqrt}(((b^4c^8 - 2 \\
& *a*b^2c^9 + a^2c^{10}) * d^4 - 4*(b^5c^7 - 3a*b^3c^8 + 2a^2b*c^9) * d^3 * e \\
& + 2*(3b^6c^6 - 12a*b^4c^7 + 12a^2b^2c^8 - a^3c^9) * d^2 * e^2 - 4*(b^7* \\
& c^5 - 5a*b^5c^6 + 7a^2b^3c^7 - 2a^3b*c^8) * d * e^3 + (b^8c^4 - 6a*b^6 \\
& *c^5 + 11a^2b^4c^6 - 6a^3b^2c^7 + a^4c^8) * e^4 + (b^{12} - 10a*b^{10}c \\
& + 37a^2b^8c^2 - 62a^3b^6c^3 + 46a^4b^4c^4 - 12a^5b^2c^5 + a^6c^6) * f^4 + 4*((b^{10}c^2 - 8a*b^8c^3 + 22a^2b^6c^4 - 24a^3b^4c^5 + 9* \\
& a^4b^2c^6 - a^5c^7) * d - (b^{11}c - 9a*b^9c^2 + 29a^2b^7c^3 - 40a^3* \\
& b^5c^4 + 22a^4b^3c^5 - 3a^5b*c^6) * e) * f^3 + 2*((3b^8c^4 - 18a*b^6c^ \\
& ^5 + 33a^2b^4c^6 - 19a^3b^2c^7 + 3a^4c^8) * d^2 - 2*(3b^9c^3 - 21a \\
& *b^7c^4 + 48a^2b^5c^5 - 39a^3b^3c^6 + 8a^4b*c^7) * d * e + (3b^{10}c^2 \\
& - 24a*b^8c^3 + 66a^2b^6c^4 - 72a^3b^4c^5 + 27a^4b^2c^6 - a^5c^7) * e^2) * f^2 + 4*((b^6c^6 - 4a*b^4c^7 + 4a^2b^2c^8 - a^3c^9) * d^3 - (3 \\
& *b^7c^5 - 15a*b^5c^6 + 21a^2b^3c^7 - 7a^3b*c^8) * d^2 * e + (3b^8c^4 \\
& - 18a*b^6c^5 + 33a^2b^4c^6 - 18a^3b^2c^7 + a^4c^8) * d * e^2 - (b^9c^3 \\
& - 7a*b^7c^4 + 16a^2b^5c^5 - 13a^3b^3c^6 + 3a^4b*c^7) * e^3) * f) / (b \\
& ^2c^{14} - 4a*c^{15})) * \text{sqrt}(-((b^3c^4 - 3a*b*c^5) * d^2 - 2*(b^4c^3 - 4a*b \\
& ^2c^4 + 2a^2c^5) * d * e + (b^5c^2 - 5a*b^3c^3 + 5a^2b*c^4) * e^2 + (b^7 \\
& - 7a*b^5c + 14a^2b^3c^2 - 7a^3b*c^3) * f^2 + 2*((b^5c^2 - 5a*b^3c^3 \\
& + 5a^2b*c^4) * d - (b^6c - 6a*b^4c^2 + 9a^2b^2c^3 - 2a^3c^4) * e) * f \\
& - (b^2c^7 - 4a*c^8) * \text{sqrt}(((b^4c^8 - 2a*b^2c^9 + a^2c^{10}) * d^4 - 4*(b^5 \\
& *c^7 - 3a*b^3c^8 + 2a^2b*c^9) * d^3 * e + 2*(3b^6c^6 - 12a*b^4c^7 + 12* \\
& a^2b^2c^8 - a^3c^9) * d^2 * e^2 - 4*(b^7c^5 - 5a*b^5c^6 + 7a^2b^3c^7 - \\
& 2a^3b*c^8) * d * e^3 + (b^8c^4 - 6a*b^6c^5 + 11a^2b^4c^6 - 6a^3b^2c^ \\
& ^7 + a^4c^8) * e^4 + (b^{12} - 10a*b^{10}c + 37a^2b^8c^2 - 62a^3b^6c^3 + \\
& 46a^4b^4c^4 - 12a^5b^2c^5 + a^6c^6) * f^4 + 4*((b^{10}c^2 - 8a*b^8c^ \\
& ^3 + 22a^2b^6c^4 - 24a^3b^4c^5 + 9a^4b^2c^6 - a^5c^7) * d - (b^{11}c \\
& - 9a*b^9c^2 + 29a^2b^7c^3 - 40a^3b^5c^4 + 22a^4b^3c^5 - 3a^5b* \\
& c^6) * e) * f^3 + 2*((3b^8c^4 - 18a*b^6c^5 + 33a^2b^4c^6 - 19a^3b^2c^ \\
& ^7 + 3a^4c^8) * d^2 - 2*(3b^9c^3 - 21a*b^7c^4 + 48a^2b^5c^5 - 39a^3* \\
& b^3c^6 + 8a^4b*c^7) * d * e + (3b^{10}c^2 - 24a*b^8c^3 + 66a^2b^6c^4 -
\end{aligned}$$

$$72*a^3*b^4*c^5 + 27*a^4*b^2*c^6 - a^5*c^7)*e^2)*f^2 + 4*((b^6*c^6 - 4*a*b^4*c^7 + 4*a^2*b^2*c^8 - a^3*c^9)*d^3 - (3*b^7*c^5 - 15*a*b^5*c^6 + 21*a^2*b^3*c^7 - 7*a^3*b*c^8)*d^2*e + (3*b^8*c^4 - 18*a*b^6*c^5 + 33*a^2*b^4*c^6 - 18*a^3*b^2*c^7 + a^4*c^8)*d*e^2 - (b^9*c^3 - 7*a*b^7*c^4 + 16*a^2*b^5*c^5 - 13*a^3*b^3*c^6 + 3*a^4*b*c^7)*e^3)*f)/(b^2*c^14 - 4*a*c^15))/((b^2*c^7 - 4*a*c^8))) + 10*(c^2*e - b*c*f)*x^3 + 30*(c^2*d - b*c*e + (b^2 - a*c)*f)*x)/c^3$$

giac [B] time = 5.03, size = 7243, normalized size = 19.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out]
$$\begin{aligned} & -1/8*((2*b^5*c^4 - 16*a*b^3*c^5 + 32*a^2*b*c^6 - \sqrt{2}*\sqrt{b^2 - 4*a*c})* \\ & \sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*b^5*c^2 + 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)* \\ & a*b^3*c^3 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*b^4*c^3 - 16*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)* \\ & a^2*b*c^4 - 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a*b^2*c^4 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)* \\ & b^3*c^4 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 8*(b^2 - 4*a*c)*a*b*c^5)*c^2*d + \\ & (2*b^7*c^2 - 20*a*b^5*c^3 + 64*a^2*b^3*c^4 - 64*a^3*b*c^5 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*b^7 + 10*\sqrt{2}*\sqrt{b^2 - 4*a*c})* \\ & \sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a*b^5*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*b^6*c - 32*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)* \\ & a^2*b^3*c^2 - 12*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a*b^4*c^2 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)* \\ & b^5*c^2 + 32*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a^3*b*c^3 + 16*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)* \\ & a^2*b^2*c^3 + 6*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a*b^3*c^3 - 8*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)* \\ & a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 + 12*(b^2 - 4*a*c)*a*b^3*c^3 - 16*(b^2 - 4*a*c)*a^2*b*c^4)*c^2*f - (2*b^6*c^3 - 18*a*b^4*c^4 + 48*a^2*b^2*c^5 - 32*a^3*c^6 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)* \\ & b^6*c + 9*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a*b^4*c^2 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*b^5*c^2 - 24*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)* \\ & a^2*b^2*c^3 - 10*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a*b^3*c^3 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*b^4*c^3 + 16*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)* \\ & a^3*c^4 + 8*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a^2*b*c^4 + 5*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a*b^2*c^4 - 4*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)* \\ & a^2*c^5 - 2*(b^2 - 4*a*c)*b^4*c^3 + 10*(b^2 - 4*a*c)*a*b^2*c^4 - 8*(b^2 - 4*a*c)*a^2*c^5)*c^2*e + 2*(\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a*b^4*c^4 - 8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)* \\ & a^2*b^2*c^5 - 2*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a*b^3*c^5 + 2*a*b^4*c^5 + 16*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a^3*c^6 + 8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)* \\ & a^2*b*c^6 + \sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a*b^2*c^6 - 16*a^2*b^2*c^6 - 4*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a^2*c^7 + 32*a^3*c^7 - 2*(b^2 - 4*a*c)*a*b^2*c^5 + 8*(b^2 - 4*a*c)* \\ & a^2*c^6)*d*abs(c) + 2*(\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a*b^6*c^2 - 9*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a^2*b^4*c^3 - 2*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)* \\ & a*b^5*c^3 + 2*a*b^6*c^3 + 24*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a^3*b^2*c^4 + 10*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a^2*b^3*c^4 + \sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)* \\ & a*b^4*c^4 - 18*a^2*b^4*c^4 - 16*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a^4*c^5 - 8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a^3*b*c^5 - 5*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)* \\ & a^2*b^2*c^5 + 48*a^3*b^2*c^5 + 4*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)* \end{aligned}$$

$$\begin{aligned}
& 4*a*c)*c)*a^3*c^6 - 32*a^4*c^6 - 2*(b^2 - 4*a*c)*a*b^4*c^3 + 10*(b^2 - 4*a \\
& *c)*a^2*b^2*c^4 - 8*(b^2 - 4*a*c)*a^3*c^5)*f*abs(c) - 2*(sqrt(2)*sqrt(b*c - \\
& sqrt(b^2 - 4*a*c)*c)*a*b^5*c^3 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c) \\
& *a^2*b^3*c^4 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^4 + 2*a*b^ \\
& 5*c^4 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^5 + 8*sqrt(2)*sq \\
& rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^5 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4 \\
& *a*c)*c)*a*b^3*c^5 - 16*a^2*b^3*c^5 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c \\
&)*c)*a^2*b*c^6 + 32*a^3*b*c^6 - 2*(b^2 - 4*a*c)*a*b^3*c^4 + 8*(b^2 - 4*a*c) \\
& *a^2*b*c^5)*abs(c)*e - (2*b^5*c^6 - 12*a*b^3*c^7 + 16*a^2*b*c^8 - sqrt(2)*s \\
& qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^4 + 6*sqrt(2)*sqrt(b \\
& ^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^5 + 2*sqrt(2)*sqrt(b^2 \\
& - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^5 - 8*sqrt(2)*sqrt(b^2 - 4*a \\
& *c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^6 - 4*sqrt(2)*sqrt(b^2 - 4*a*c) \\
& *sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt \\
& (b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^6 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c \\
& - sqrt(b^2 - 4*a*c)*c)*a*b*c^7 - 2*(b^2 - 4*a*c)*b^3*c^6 + 4*(b^2 - 4*a*c)* \\
& a*b*c^7)*d - (2*b^7*c^4 - 16*a*b^5*c^5 + 36*a^2*b^3*c^6 - 16*a^3*b*c^7 - sq \\
& rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c^2 + 8*sqrt(2) \\
& *sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^3 + 2*sqrt(2)*sq \\
& rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^3 - 18*sqrt(2)*sqrt(b \\
& ^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^4 - 8*sqrt(2)*sqrt(b^ \\
& 2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^4 - sqrt(2)*sqrt(b^2 - 4 \\
& *a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c) \\
& *sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^5 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sq \\
& rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^5 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sq \\
& rt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^5 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b \\
& *c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^6 - 2*(b^2 - 4*a*c)*b^5*c^4 + 8*(b^2 - 4* \\
& a*c)*a*b^3*c^5 - 4*(b^2 - 4*a*c)*a^2*b*c^6)*f + (2*b^6*c^5 - 14*a*b^4*c^6 + \\
& 24*a^2*b^2*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c) \\
& *b^6*c^3 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^ \\
& 4*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^4 \\
& - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^5 \\
& - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^5 - \\
& sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^5 + 3*sqrt(\\
& 2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^6 - 2*(b^2 - 4 \\
& *a*c)*b^4*c^5 + 6*(b^2 - 4*a*c)*a*b^2*c^6)*e)*arctan(2*sqrt(1/2)*x/sqrt((b* \\
& c^5 + sqrt(b^2*c^10 - 4*a*c^11))/c^6))/((a*b^4*c^5 - 8*a^2*b^2*c^6 - 2*a*b^ \\
& 3*c^6 + 16*a^3*c^7 + 8*a^2*b*c^7 + a*b^2*c^7 - 4*a^2*c^8)*c^2) + 1/8*((2*b^ \\
& 5*c^4 - 16*a*b^3*c^5 + 32*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + \\
& sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(\\
& b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 \\
& - 4*a*c)*c)*b^4*c^3 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4 \\
& *a*c)*c)*a^2*b*c^4 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a* \\
& c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c) \\
& *b^3*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b* \\
& c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 8*(b^2 - 4*a*c)*a*b*c^5)*c^2*d + (2*b^7*c^2 \\
& - 20*a*b^5*c^3 + 64*a^2*b^3*c^4 - 64*a^3*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c) \\
& *sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b* \\
& c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s \\
& qrt(b^2 - 4*a*c)*c)*b^6*c - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^ \\
& 2 - 4*a*c)*c)*a^2*b^3*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^ \\
& 2 - 4*a*c)*c)*a*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4 \\
& *a*c)*c)*b^5*c^2 + 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c \\
&)*c)*a^3*b*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)* \\
& c)*a^2*b^2*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c) \\
& *a*b^3*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a \\
& ^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 + 12*(b^2 - 4*a*c)*a*b^3*c^3 - 16*(b^2 - \\
& 4*a*c)*a^2*b*c^4)*c^2*f - (2*b^6*c^3 - 18*a*b^4*c^4 + 48*a^2*b^2*c^5 - 32* \\
& a^3*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c +
\end{aligned}$$

$$\begin{aligned}
& 9\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)ab^4c^2 + 2\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)b^5c^2 - 24\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^2b^2c^3 - 10\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)ab^3c^3 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)b^4c^3 + 16\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^3c^4 + 8\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^2b^2c^4 + 5\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)ab^2c^4 - 4\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^2c^5 - 2(b^2 - 4ac)b^4c^3 + 10(b^2 - 4ac)ab^2c^4 - 8(b^2 - 4ac)a^2c^5)c^2e - 2(\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)ab^4c^4 - 8\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^2b^2c^5 - 2\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)ab^3c^5 - 2ab^4c^5 + 16\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^3c^6 + 8\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^2b^2c^6 + \sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)ab^2c^6 + 16a^2b^2c^6 - 4\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^2c^7 - 32a^3c^7 + 2(b^2 - 4ac)ab^2c^5 - 8(b^2 - 4ac)a^2c^6)d\text{abs}(c) - 2(\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)ab^6c^2 - 9\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^2b^4c^3 - 2\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)ab^5c^3 - 2ab^6c^3 + 24\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^3b^2c^4 + 10\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^2b^3c^4 + \sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)ab^4c^4 + 18a^2b^4c^4 - 16\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^4c^5 - 8\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^3b^2c^5 - 5\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^2b^2c^5 - 48a^3b^2c^5 + 4\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^3c^6 + 32a^4c^6 + 2(b^2 - 4ac)ab^4c^3 - 10(b^2 - 4ac)a^2b^2c^4 + 8(b^2 - 4ac)a^3c^5)f\text{abs}(c) + 2(\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)ab^5c^3 - 8\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^2b^3c^4 - 2\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)ab^4c^4 - 2ab^5c^4 + 16\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^3b^2c^5 + 8\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^2b^2c^5 + \sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)ab^3c^5 + 16a^2b^3c^5 - 4\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^2b^2c^6 - 32a^3b^2c^6 + 2(b^2 - 4ac)ab^3c^4 - 8(b^2 - 4ac)a^2b^2c^5)\text{abs}(c)e - (2b^5c^6 - 12ab^3c^7 + 16a^2b^2c^8 - \sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}}c)b^5c^4 + 6\sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}}c)ab^3c^5 + 2\sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}}c)b^4c^5 - 8\sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^2b^2c^6 - 4\sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}}c)ab^2c^6 - \sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}}c)b^3c^6 + 2\sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}}c)ab^2c^7 - 2(b^2 - 4ac)b^3c^6 + 4(b^2 - 4ac)ab^2c^7)d - (2b^7c^4 - 16ab^5c^5 + 36a^2b^3c^6 - 16a^3b^2c^7 - \sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}}c)b^7c^2 + 8\sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}}c)ab^5c^3 + 2\sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}}c)b^6c^3 - 18\sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^2b^3c^4 - 8\sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}}c)ab^4c^4 - \sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}}c)b^5c^4 + 8\sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^3b^2c^5 + 4\sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^2b^2c^5 + 4\sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}}c)ab^3c^5 - 2\sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^2b^2c^6 - 2(b^2 - 4ac)b^5c^4 + 8(b^2 - 4ac)ab^3c^5 - 4(b^2 - 4ac)a^2b^2c^6)f + (2b^6c^5 - 14ab^4c^6 + 24a^2b^2c^7 - \sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}}c)b^6c^3 + 7\sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}}c)ab^4c^4 + 2\sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}}c)b^5c^4 - 12\sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^2b^2c^5 - 6\sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}}c)ab^3c^5 - \sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}}c)b^4c^5 + 3\sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}}c)ab^2c^6 - 2(b^2 - 4ac)b^4c^5
\end{aligned}$$

[In] integrate(x^4*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] 1/15*(3*c^2*f*x^5 + 5*(c^2*e - b*c*f)*x^3 + 15*(c^2*d - b*c*e + (b^2 - a*c)*f)*x)/c^3 + integrate(-(a*c^2*d - a*b*c*e + (b*c^2*d - (b^2*c - a*c^2)*e + (b^3 - 2*a*b*c)*f)*x^2 + (a*b^2 - a^2*c)*f)/(c*x^4 + b*x^2 + a), x)/c^3

mupad [B] time = 4.91, size = 23332, normalized size = 63.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x)

[Out] $x^3*(e/(3*c) - (b*f)/(3*c^2)) - x*((b*(e/c - (b*f)/c^2))/c - d/c + (a*f)/c^2) + \operatorname{atan}\left(\frac{(16*a^3*c^6*f - 16*a^2*c^7*d - 20*a^2*b^2*c^5*f + 4*a*b^2*c^6*d - 4*a*b^3*c^5*e + 16*a^2*b*c^6*e + 4*a*b^4*c^4*f)/c^5 - (2*x*(4*b^3*c^7 - 16*a*b*c^8)*(-b^9*f^2 + b^5*c^4*d^2 + b^7*c^2*e^2 + b^6*f^2*(-4*a*c - b^2)^3)^{1/2} - 7*a*b^3*c^5*d^2 + 12*a^2*b*c^6*d^2 - a*c^5*d^2*(-4*a*c - b^2)^3)^{1/2} - 9*a*b^5*c^3*e^2 - 20*a^3*b*c^5*e^2 + 28*a^4*b*c^4*f^2 - 2*b^8*c*e*f + 25*a^2*b^3*c^4*e^2 + a^2*c^4*e^2*(-4*a*c - b^2)^3)^{1/2} + b^2*c^4*d^2*(-4*a*c - b^2)^3)^{1/2} + 42*a^2*b^5*c^2*f^2 - 63*a^3*b^3*c^3*f^2 - a^3*c^3*f^2*(-4*a*c - b^2)^3)^{1/2} + b^4*c^2*e^2*(-4*a*c - b^2)^3)^{1/2} - 11*a*b^7*c*f^2 + 16*a^3*c^6*d*e - 2*b^6*c^3*d*e - 16*a^4*c^5*e*f + 2*b^7*c^2*d*f + 16*a*b^4*c^4*d*e - 18*a*b^5*c^3*d*f - 40*a^3*b*c^5*d*f + 20*a*b^6*c^2*e*f - 2*b^5*c*e*f*(-4*a*c - b^2)^3)^{1/2} + 6*a^2*b^2*c^2*f^2*(-4*a*c - b^2)^3)^{1/2} - 5*a*b^4*c*f^2*(-4*a*c - b^2)^3)^{1/2} - 36*a^2*b^2*c^5*d*e + 50*a^2*b^3*c^4*d*f + 2*a^2*c^4*d*f*(-4*a*c - b^2)^3)^{1/2} - 2*b^3*c^3*d*e*(-4*a*c - b^2)^3)^{1/2} - 66*a^2*b^4*c^3*e*f + 76*a^3*b^2*c^4*e*f + 2*b^4*c^2*d*f*(-4*a*c - b^2)^3)^{1/2} - 3*a*b^2*c^3*e^2*(-4*a*c - b^2)^3)^{1/2} + 4*a*b*c^4*d*e*(-4*a*c - b^2)^3)^{1/2} - 6*a*b^2*c^3*d*f*(-4*a*c - b^2)^3)^{1/2} + 8*a*b^3*c^2*e*f*(-4*a*c - b^2)^3)^{1/2} - 6*a^2*b*c^3*e*f*(-4*a*c - b^2)^3)^{1/2})/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8))^{1/2} - (2*x*(b^8*f^2 + 2*a^2*c^6*d^2 - 2*a^3*c^5*e^2 + b^4*c^4*d^2 + 2*a^4*c^4*f^2 + b^6*c^2*e^2 - 4*a*b^2*c^5*d^2 - 6*a*b^4*c^3*e^2 - 2*b^7*c*e*f + 9*a^2*b^2*c^4*e^2 + 20*a^2*b^4*c^2*f^2 - 16*a^3*b^2*c^3*f^2 - 8*a*b^6*c*f^2 - 4*a^3*c^5*d*f - 2*b^5*c^3*d*e + 2*b^6*c^2*d*f + 10*a*b^3*c^4*d*e - 10*a^2*b*c^5*d*e - 12*a*b^4*c^3*d*f + 14*a*b^5*c^2*e*f + 14*a^3*b*c^4*e*f + 18*a^2*b^2*c^4*d*f - 28*a^2*b^3*c^3*e*f))/c^5)*(-b^9*f^2 + b^5*c^4*d^2 + b^7*c^2*e^2 + b^6*f^2*(-4*a*c - b^2)^3)^{1/2} - 7*a*b^3*c^5*d^2 + 12*a^2*b*c^6*d^2 - a*c^5*d^2*(-4*a*c - b^2)^3)^{1/2} - 9*a*b^5*c^3*e^2 - 20*a^3*b*c^5*e^2 + 28*a^4*b*c^4*f^2 - 2*b^8*c*e*f + 25*a^2*b^3*c^4*e^2 + a^2*c^4*e^2*(-4*a*c - b^2)^3)^{1/2} + b^2*c^4*d^2*(-4*a*c - b^2)^3)^{1/2} + 42*a^2*b^5*c^2*f^2 - 63*a^3*b^3*c^3*f^2 - a^3*c^3*f^2*(-4*a*c - b^2)^3)^{1/2} + b^4*c^2*e^2*(-4*a*c - b^2)^3)^{1/2} - 11*a*b^7*c*f^2 + 16*a^3*c^6*d*e - 2*b^6*c^3*d*e - 16*a^4*c^5*e*f + 2*b^7*c^2*d*f + 16*a*b^4*c^4*d*e - 18*a*b^5*c^3*d*f - 40*a^3*b*c^5*d*f + 20*a*b^6*c^2*e*f - 2*b^5*c*e*f*(-4*a*c - b^2)^3)^{1/2} + 6*a^2*b^2*c^2*f^2*(-4*a*c - b^2)^3)^{1/2} - 5*a*b^4*c*f^2*(-4*a*c - b^2)^3)^{1/2} - 36*a^2*b^2*c^5*d*e + 50*a^2*b^3*c^4*d*f + 2*a^2*c^4*d*f*(-4*a*c - b^2)^3)^{1/2} - 2*b^3*c^3*d*e*(-4*a*c - b^2)^3)^{1/2} - 66*a^2*b^4*c^3*e*f + 76*a^3*b^2*c^4*e*f + 2*b^4*c^2*d*f*(-4*a*c - b^2)^3)^{1/2} - 3*a*b^2*c^3*e^2*(-4*a*c - b^2)^3)^{1/2} + 4*a*b*c^4*d*e*(-4*a*c - b^2)^3)^{1/2} - 6*a*b^2*c^3*d*f*(-4*a*c - b^2)^3)^{1/2} + 8*a*b^3*c^2*e*f*(-4*a*c - b^2)^3)^{1/2} - 6*a^2*b*c^3*e*f*(-4*a*c - b^2)^3)^{1/2})/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8))^{1/2} - (2*x*(b^8*f^2 + 2*a^2*c^6*d^2 - 2*a^3*c^5*e^2 + b^4*c^4*d^2 + 2*a^4*c^4*f^2 + b^6*c^2*e^2 - 4*a*b^2*c^5*d^2 - 6*a*b^4*c^3*e^2 - 2*b^7*c*e*f + 9*a^2*b^2*c^4*e^2 + 20*a^2*b^4*c^2*f^2 - 16*a^3*b^2*c^3*f^2 - 8*a*b^6*c*f^2 - 4*a^3*c^5*d*f - 2*b^5*c^3*d*e + 2*b^6*c^2*d*f + 10*a*b^3*c^4*d*e - 10*a^2*b*c^5*d*e - 12*a*b^4*c^3*d*f + 14*a*b^5*c^2*e*f + 14*a^3*b*c^4*e*f + 18*a^2*b^2*c^4*d*f - 28*a^2*b^3*c^3*e*f))/c^5)*(-b^9*f^2 + b^5*c^4*d^2 + b^7*c^2*e^2 + b^6*f^2*(-4*a*c - b^2)^3)^{1/2} - 7*a*b^3*c^5*d^2 + 12*a^2*b*c^6*d^2 - a*c^5*d^2*(-4*a*c - b^2)^3)^{1/2} - 9*a*b^5*c^3*e^2 - 20*a^3*b*c^5*e^2 + 28*a^4*b*c^4*f^2 - 2*b^8*c*e*f + 25*a^2*b^3*c^4*e^2 + a^2*c^4*e^2*(-4*a*c - b^2)^3)^{1/2} + b^2*c^4*d^2*(-4*a*c - b^2)^3)^{1/2} + 42*a^2*b^5*c^2*f^2 - 63*a^3*b^3*c^3*f^2 - a^3*c^3*f^2*(-4*a*c - b^2)^3)^{1/2} + b^4*c^2*e^2*(-4*a*c - b^2)^3)^{1/2} - 11*a*b^7*c*f^2 + 16*a^3*c^6*d*e - 2*b^6*c^3*d*e - 16*a^4*c^5*e*f + 2*b^7*c^2*d*f + 16*a*b^4*c^4*d*e - 18*a*b^5*c^3*d*f - 40*a$

$$\begin{aligned}
&^3b^5c^5d^5f + 20a^2b^6c^2e^5f - 2b^5c^5e^5f(-4ac - b^2)^3)^{(1/2)} + 6a^2b^2c^2f^2(-4ac - b^2)^3)^{(1/2)} - 5ab^4c^4f^2(-4ac - b^2)^3)^{(1/2)} \\
&- 36a^2b^2c^5d^5e + 50a^2b^3c^4d^5f + 2a^2c^4d^5f(-4ac - b^2)^3)^{(1/2)} - 2b^3c^3d^5e(-4ac - b^2)^3)^{(1/2)} - 66a^2b^4c^3e^5f \\
&+ 76a^3b^2c^4e^5f + 2b^4c^2d^5f(-4ac - b^2)^3)^{(1/2)} - 3ab^2c^3e^2(-4ac - b^2)^3)^{(1/2)} + 4ab^2c^4d^5e(-4ac - b^2)^3)^{(1/2)} - \\
&6ab^2c^3d^5f(-4ac - b^2)^3)^{(1/2)} + 8ab^3c^2e^5f(-4ac - b^2)^3)^{(1/2)} - 6a^2b^2c^3e^5f(-4ac - b^2)^3)^{(1/2)} \\
&/ (8(16a^2c^9 + b^4c^7 - 8ab^2c^8))^{(1/2)} * i - (((16a^3c^6f - 16a^2c^7d - 20a^2b^2c^5f + 4ab^2c^6d - 4ab^3c^5e + 16a^2b^2c^6e + 4ab^4c^4f)/c^5 \\
&+ (2x(4b^3c^7 - 16ab^3c^8)) * (-b^9f^2 + b^5c^4d^2 + b^7c^2e^2 + b^6f^2(-4ac - b^2)^3)^{(1/2)} - 7ab^3c^5d^2 + 12a^2b^2c^6d^2 - ac^5d^2(-4ac - b^2)^3)^{(1/2)} - 9ab^5c^3e^2 - 20a^3b^2c^5e^2 + 28a^4b^2c^4f^2 - 2b^8c^5e^5f + 25a^2b^3c^4e^2 + a^2c^4e^2(-4ac - b^2)^3)^{(1/2)} + b^2c^4d^2(-4ac - b^2)^3)^{(1/2)} + 42a^2b^5c^2f^2 - 63a^3b^3c^3f^2 - a^3c^3f^2(-4ac - b^2)^3)^{(1/2)} + b^4c^2e^2(-4ac - b^2)^3)^{(1/2)} - 11ab^7c^4f^2 + 16a^3c^6d^5e - 2b^6c^3d^5e - 16a^4c^5e^5f + 2b^7c^2d^5f + 16ab^4c^4d^5e - 18ab^5c^3d^5f - 40a^3b^2c^5d^5f + 20ab^6c^2e^5f - 2b^5c^5e^5f(-4ac - b^2)^3)^{(1/2)} + 6a^2b^2c^2f^2(-4ac - b^2)^3)^{(1/2)} - 5ab^4c^4f^2(-4ac - b^2)^3)^{(1/2)} - 36a^2b^2c^5d^5e + 50a^2b^3c^4d^5f + 2a^2c^4d^5f(-4ac - b^2)^3)^{(1/2)} - 2b^3c^3d^5e(-4ac - b^2)^3)^{(1/2)} - 66a^2b^4c^3e^5f + 76a^3b^2c^4e^5f + 2b^4c^2d^5f(-4ac - b^2)^3)^{(1/2)} - 3ab^2c^3e^2(-4ac - b^2)^3)^{(1/2)} + 4ab^2c^4d^5e(-4ac - b^2)^3)^{(1/2)} - 6ab^2c^3d^5f(-4ac - b^2)^3)^{(1/2)} + 8ab^3c^2e^5f(-4ac - b^2)^3)^{(1/2)} - 6a^2b^2c^3e^5f(-4ac - b^2)^3)^{(1/2)} \\
&/ (8(16a^2c^9 + b^4c^7 - 8ab^2c^8))^{(1/2)} / c^5 * (-b^9f^2 + b^5c^4d^2 + b^7c^2e^2 + b^6f^2(-4ac - b^2)^3)^{(1/2)} - 7ab^3c^5d^2 + 12a^2b^2c^6d^2 - ac^5d^2(-4ac - b^2)^3)^{(1/2)} - 9ab^5c^3e^2 - 20a^3b^2c^5e^2 + 28a^4b^2c^4f^2 - 2b^8c^5e^5f + 25a^2b^3c^4e^2 + a^2c^4e^2(-4ac - b^2)^3)^{(1/2)} + b^2c^4d^2(-4ac - b^2)^3)^{(1/2)} + 42a^2b^5c^2f^2 - 63a^3b^3c^3f^2 - a^3c^3f^2(-4ac - b^2)^3)^{(1/2)} + b^4c^2e^2(-4ac - b^2)^3)^{(1/2)} - 11ab^7c^4f^2 + 16a^3c^6d^5e - 2b^6c^3d^5e - 16a^4c^5e^5f + 2b^7c^2d^5f + 16ab^4c^4d^5e - 18ab^5c^3d^5f - 40a^3b^2c^5d^5f + 20ab^6c^2e^5f - 2b^5c^5e^5f(-4ac - b^2)^3)^{(1/2)} + 6a^2b^2c^2f^2(-4ac - b^2)^3)^{(1/2)} - 5ab^4c^4f^2(-4ac - b^2)^3)^{(1/2)} - 36a^2b^2c^5d^5e + 50a^2b^3c^4d^5f + 2a^2c^4d^5f(-4ac - b^2)^3)^{(1/2)} - 2b^3c^3d^5e(-4ac - b^2)^3)^{(1/2)} - 66a^2b^4c^3e^5f + 76a^3b^2c^4e^5f + 2b^4c^2d^5f(-4ac - b^2)^3)^{(1/2)} - 3ab^2c^3e^2(-4ac - b^2)^3)^{(1/2)} + 4ab^2c^4d^5e(-4ac - b^2)^3)^{(1/2)} - 6ab^2c^3d^5f(-4ac - b^2)^3)^{(1/2)} + 8ab^3c^2e^5f(-4ac - b^2)^3)^{(1/2)} - 6a^2b^2c^3e^5f(-4ac - b^2)^3)^{(1/2)} \\
&/ (8(16a^2c^9 + b^4c^7 - 8ab^2c^8))^{(1/2)} + (2x(b^8f^2 + 2a^2c^6d^2 - 2a^3c^5e^2 + b^4c^4d^2 + 2a^4c^4f^2 + b^6c^2e^2 - 4ab^2c^5d^2 - 6ab^4c^3e^2 - 2b^7c^5e^5f + 9a^2b^2c^4e^2 + 20a^2b^4c^2f^2 - 16a^3b^2c^3f^2 - 8ab^6c^4f^2 - 4a^3c^5d^5f - 2b^5c^3d^5e + 2b^6c^2d^5f + 10ab^3c^4d^5e - 10a^2b^2c^5d^5e - 12ab^4c^3d^5f + 14ab^5c^2e^5f + 14a^3b^2c^4e^5f + 18a^2b^2c^4d^5f - 28a^2b^3c^3e^5f)) / c^5 * (-b^9f^2 + b^5c^4d^2 + b^7c^2e^2 + b^6f^2(-4ac - b^2)^3)^{(1/2)} - 7ab^3c^5d^2 + 12a^2b^2c^6d^2 - ac^5d^2(-4ac - b^2)^3)^{(1/2)} - 9ab^5c^3e^2 - 20a^3b^2c^5e^2 + 28a^4b^2c^4f^2 - 2b^8c^5e^5f + 25a^2b^3c^4e^2 + a^2c^4e^2(-4ac - b^2)^3)^{(1/2)} + b^2c^4d^2(-4ac - b^2)^3)^{(1/2)} + 42a^2b^5c^2f^2 - 63a^3b^3c^3f^2 - a^3c^3f^2(-4ac - b^2)^3)^{(1/2)} + b^4c^2e^2(-4ac - b^2)^3)^{(1/2)} - 11ab^7c^4f^2 + 16a^3c^6d^5e - 2b^6c^3d^5e - 16a^4c^5e^5f + 2b^7c^2d^5f + 16ab^4c^4d^5e - 18ab^5c^3d^5f - 40a^3b^2c^5d^5f + 20ab^6c^2e^5f - 2b^5c^5e^5f(-4ac - b^2)^3)^{(1/2)} + 6a^2b^2c^2f^2(-4ac - b^2)^3)^{(1/2)} - 5ab^4c^4f^2(-4ac - b^2)^3)^{(1/2)} - 36a^2b^2c^5d^5e + 50a^2b^3c^4d^5f + 2a^2c^4d^5f(-4ac - b^2)^3)^{(1/2)} - 2b^3c^3d^5e(-4ac - b^2)^3)^{(1/2)}
\end{aligned}$$

$$\begin{aligned}
& - 66a^2b^4c^3e^2f + 76a^3b^2c^4e^2f + 2b^4c^2d^2f * (-4ac - b^2)^3)^{(1/2)} - 3ab^2c^3e^2 * (-4ac - b^2)^3)^{(1/2)} + 4ab^2c^4d^2e * (-4ac - b^2)^3)^{(1/2)} - 6ab^2c^3d^2f * (-4ac - b^2)^3)^{(1/2)} + 8ab^3c^2e^2f * (-4ac - b^2)^3)^{(1/2)} - 6a^2b^3c^3e^2f * (-4ac - b^2)^3)^{(1/2)} / (8 * (16a^2c^9 + b^4c^7 - 8ab^2c^8))^{(1/2)} * i) / (((16a^3c^6f - 16a^2 * c^7d - 20a^2b^2c^5f + 4ab^2c^6d - 4ab^3c^5e + 16a^2b^3c^6e + 4ab^4c^4f) / c^5 - (2 * x * (4b^3c^7 - 16ab^3c^8) * (-b^9f^2 + b^5c^4d^2 + b^7c^2e^2 + b^6f^2 * (-4ac - b^2)^3)^{(1/2)} - 7ab^3c^5d^2 + 12a^2b^3c^6d^2 - ac^5d^2 * (-4ac - b^2)^3)^{(1/2)} - 9ab^5c^3e^2 - 20a^3b^3c^5e^2 + 28a^4b^3c^4f^2 - 2b^8c^2e^2f + 25a^2b^3c^4e^2 + a^2c^4e^2 * (-4ac - b^2)^3)^{(1/2)} + b^2c^4d^2 * (-4ac - b^2)^3)^{(1/2)} + 42a^2b^5c^2f^2 - 63a^3b^3c^3f^2 - a^3c^3f^2 * (-4ac - b^2)^3)^{(1/2)} + b^4c^2e^2 * (-4ac - b^2)^3)^{(1/2)} - 11ab^7c^2f^2 + 16a^3c^6d^2e - 2b^6c^3d^2e - 16a^4c^5e^2f + 2b^7c^2d^2f + 16ab^4c^4d^2e - 18ab^5c^3d^2f - 40a^3b^3c^5d^2f + 20ab^6c^2e^2f - 2b^5c^2e^2f * (-4ac - b^2)^3)^{(1/2)} + 6a^2b^2c^2f^2 * (-4ac - b^2)^3)^{(1/2)} - 5ab^4c^2f^2 * (-4ac - b^2)^3)^{(1/2)} - 36a^2b^2c^5d^2e + 50a^2b^3c^4d^2f + 2a^2c^4d^2f * (-4ac - b^2)^3)^{(1/2)} - 2b^3c^3d^2e * (-4ac - b^2)^3)^{(1/2)} - 66a^2b^4c^3e^2f + 76a^3b^2c^4e^2f + 2b^4c^2d^2f * (-4ac - b^2)^3)^{(1/2)} - 3ab^2c^3e^2 * (-4ac - b^2)^3)^{(1/2)} + 4ab^2c^4d^2e * (-4ac - b^2)^3)^{(1/2)} - 6ab^2c^3d^2f * (-4ac - b^2)^3)^{(1/2)} + 8ab^3c^2e^2f * (-4ac - b^2)^3)^{(1/2)} - 6a^2b^3c^3e^2f * (-4ac - b^2)^3)^{(1/2)} / (8 * (16a^2c^9 + b^4c^7 - 8ab^2c^8))^{(1/2)} / c^5 * (-b^9f^2 + b^5c^4d^2 + b^7c^2e^2 + b^6f^2 * (-4ac - b^2)^3)^{(1/2)} - 7ab^3c^5d^2 + 12a^2b^3c^6d^2 - ac^5d^2 * (-4ac - b^2)^3)^{(1/2)} - 9ab^5c^3e^2 - 20a^3b^3c^5e^2 + 28a^4b^3c^4f^2 - 2b^8c^2e^2f + 25a^2b^3c^4e^2 + a^2c^4e^2 * (-4ac - b^2)^3)^{(1/2)} + b^2c^4d^2 * (-4ac - b^2)^3)^{(1/2)} + 42a^2b^5c^2f^2 - 63a^3b^3c^3f^2 - a^3c^3f^2 * (-4ac - b^2)^3)^{(1/2)} + b^4c^2e^2 * (-4ac - b^2)^3)^{(1/2)} - 11ab^7c^2f^2 + 16a^3c^6d^2e - 2b^6c^3d^2e - 16a^4c^5e^2f + 2b^7c^2d^2f + 16ab^4c^4d^2e - 18ab^5c^3d^2f - 40a^3b^3c^5d^2f + 20ab^6c^2e^2f - 2b^5c^2e^2f * (-4ac - b^2)^3)^{(1/2)} + 6a^2b^2c^2f^2 * (-4ac - b^2)^3)^{(1/2)} - 5ab^4c^2f^2 * (-4ac - b^2)^3)^{(1/2)} - 36a^2b^2c^5d^2e + 50a^2b^3c^4d^2f + 2a^2c^4d^2f * (-4ac - b^2)^3)^{(1/2)} - 2b^3c^3d^2e * (-4ac - b^2)^3)^{(1/2)} - 66a^2b^4c^3e^2f + 76a^3b^2c^4e^2f + 2b^4c^2d^2f * (-4ac - b^2)^3)^{(1/2)} - 3ab^2c^3e^2 * (-4ac - b^2)^3)^{(1/2)} + 4ab^2c^4d^2e * (-4ac - b^2)^3)^{(1/2)} - 6ab^2c^3d^2f * (-4ac - b^2)^3)^{(1/2)} + 8ab^3c^2e^2f * (-4ac - b^2)^3)^{(1/2)} - 6a^2b^3c^3e^2f * (-4ac - b^2)^3)^{(1/2)} / (8 * (16a^2c^9 + b^4c^7 - 8ab^2c^8))^{(1/2)} - (2 * x * (b^8f^2 + 2a^2c^6d^2 - 2a^3c^5e^2 + b^4c^4d^2 + 2a^4c^4f^2 + b^6c^2e^2 - 4ab^2c^5d^2 - 6ab^4c^3e^2 - 2b^7c^2e^2f + 9a^2b^2c^4e^2 + 20a^2b^4c^2f^2 - 16a^3b^2c^3f^2 - 8ab^6c^2f^2 - 4a^3c^5d^2f - 2b^5c^3d^2e + 2b^6c^2d^2f + 10ab^3c^4d^2e - 10a^2b^3c^5d^2e - 12ab^4c^3d^2f + 14ab^5c^2e^2f + 14a^3b^3c^4e^2f + 18a^2b^2c^4d^2f - 28a^2b^3c^3e^2f)) / c^5) * (-b^9f^2 + b^5c^4d^2 + b^7c^2e^2 + b^6f^2 * (-4ac - b^2)^3)^{(1/2)} - 7ab^3c^5d^2 + 12a^2b^3c^6d^2 - ac^5d^2 * (-4ac - b^2)^3)^{(1/2)} - 9ab^5c^3e^2 - 20a^3b^3c^5e^2 + 28a^4b^3c^4f^2 - 2b^8c^2e^2f + 25a^2b^3c^4e^2 + a^2c^4e^2 * (-4ac - b^2)^3)^{(1/2)} + b^2c^4d^2 * (-4ac - b^2)^3)^{(1/2)} + 42a^2b^5c^2f^2 - 63a^3b^3c^3f^2 - a^3c^3f^2 * (-4ac - b^2)^3)^{(1/2)} + b^4c^2e^2 * (-4ac - b^2)^3)^{(1/2)} - 11ab^7c^2f^2 + 16a^3c^6d^2e - 2b^6c^3d^2e - 16a^4c^5e^2f + 2b^7c^2d^2f + 16ab^4c^4d^2e - 18ab^5c^3d^2f - 40a^3b^3c^5d^2f + 20ab^6c^2e^2f - 2b^5c^2e^2f * (-4ac - b^2)^3)^{(1/2)} + 6a^2b^2c^2f^2 * (-4ac - b^2)^3)^{(1/2)} - 5ab^4c^2f^2 * (-4ac - b^2)^3)^{(1/2)} - 36a^2b^2c^5d^2e + 50a^2b^3c^4d^2f + 2a^2c^4d^2f * (-4ac - b^2)^3)^{(1/2)} - 2b^3c^3d^2e * (-4ac - b^2)^3)^{(1/2)} - 66a^2b^4c^3e^2f + 76a^3b^2c^4e^2f + 2b^4c^2d^2f * (-4ac - b^2)^3)^{(1/2)} - 3ab^2c^3e^2 * (-4ac - b^2)^3)^{(1/2)} + 4ab^2c^4d^2e * (-4ac - b^2)^3)^{(1/2)} - 6ab^2c^3d^2f * (-4ac - b^2)^3)^{(1/2)} + 8ab^3c^2e^2f * (-4ac - b^2)^3)^{(1/2)} - 6a^2b^3c^3e^2f * (-4ac - b^2)^3)^{(1/2)}
\end{aligned}$$

$$\begin{aligned}
& - b^2)^3)^{(1/2)})/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^{(1/2)} - (2*(a^4 \\
& *b^3*f^3 + a^4*c^3*e^3 + a^2*b*c^4*d^3 + a^2*b^5*d*f^2 + a^3*c^4*d^2*e - a^ \\
& 3*b^4*e*f^2 + a^5*c^2*e*f^2 - a^3*b^2*c^2*e^3 - 2*a^5*b*c*f^3 - 2*a^4*c^3*d \\
& *e*f - 4*a^3*b*c^3*d^2*f - 4*a^3*b^3*c*d*f^2 + 5*a^4*b*c^2*d*f^2 + 2*a^3*b^ \\
& 3*c*e^2*f - 3*a^4*b*c^2*e^2*f + a^4*b^2*c*e*f^2 - 2*a^2*b^2*c^3*d^2*e + a^2 \\
& *b^3*c^2*d*e^2 + 2*a^2*b^3*c^2*d^2*f - 2*a^2*b^4*c*d*e*f + 4*a^3*b^2*c^2*d* \\
& e*f))/c^5 + (((16*a^3*c^6*f - 16*a^2*c^7*d - 20*a^2*b^2*c^5*f + 4*a*b^2*c^6 \\
& *d - 4*a*b^3*c^5*e + 16*a^2*b*c^6*e + 4*a*b^4*c^4*f)/c^5 + (2*x*(4*b^3*c^7 \\
& - 16*a*b*c^8))*(-(b^9*f^2 + b^5*c^4*d^2 + b^7*c^2*e^2 + b^6*f^2*(-(4*a*c - b \\
& ^2)^3)^{(1/2)} - 7*a*b^3*c^5*d^2 + 12*a^2*b*c^6*d^2 - a*c^5*d^2*(-(4*a*c - b \\
& ^2)^3)^{(1/2)} - 9*a*b^5*c^3*e^2 - 20*a^3*b*c^5*e^2 + 28*a^4*b*c^4*f^2 - 2*b^8 \\
& *c*e*f + 25*a^2*b^3*c^4*e^2 + a^2*c^4*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + b^2*c^ \\
& 4*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 42*a^2*b^5*c^2*f^2 - 63*a^3*b^3*c^3*f^2 - \\
& a^3*c^3*f^2*(-(4*a*c - b^2)^3)^{(1/2)} + b^4*c^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} \\
& - 11*a*b^7*c*f^2 + 16*a^3*c^6*d*e - 2*b^6*c^3*d*e - 16*a^4*c^5*e*f + 2*b^7 \\
& *c^2*d*f + 16*a*b^4*c^4*d*e - 18*a*b^5*c^3*d*f - 40*a^3*b*c^5*d*f + 20*a*b^ \\
& 6*c^2*e*f - 2*b^5*c*e*f*(-(4*a*c - b^2)^3)^{(1/2)} + 6*a^2*b^2*c^2*f^2*(-(4*a \\
& *c - b^2)^3)^{(1/2)} - 5*a*b^4*c*f^2*(-(4*a*c - b^2)^3)^{(1/2)} - 36*a^2*b^2*c^ \\
& 5*d*e + 50*a^2*b^3*c^4*d*f + 2*a^2*c^4*d*f*(-(4*a*c - b^2)^3)^{(1/2)} - 2*b^3 \\
& *c^3*d*e*(-(4*a*c - b^2)^3)^{(1/2)} - 66*a^2*b^4*c^3*e*f + 76*a^3*b^2*c^4*e*f \\
& + 2*b^4*c^2*d*f*(-(4*a*c - b^2)^3)^{(1/2)} - 3*a*b^2*c^3*e^2*(-(4*a*c - b^2) \\
& ^3)^{(1/2)} + 4*a*b*c^4*d*e*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a*b^2*c^3*d*f*(-(4*a \\
& *c - b^2)^3)^{(1/2)} + 8*a*b^3*c^2*e*f*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a^2*b*c^3 \\
& *e*f*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^{(1 \\
& /2))/c^5)*(-(b^9*f^2 + b^5*c^4*d^2 + b^7*c^2*e^2 + b^6*f^2*(-(4*a*c - b^2)^ \\
& 3)^{(1/2)} - 7*a*b^3*c^5*d^2 + 12*a^2*b*c^6*d^2 - a*c^5*d^2*(-(4*a*c - b^2)^3 \\
&)^{(1/2)} - 9*a*b^5*c^3*e^2 - 20*a^3*b*c^5*e^2 + 28*a^4*b*c^4*f^2 - 2*b^8*c*e \\
& *f + 25*a^2*b^3*c^4*e^2 + a^2*c^4*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + b^2*c^4*d^ \\
& 2*(-(4*a*c - b^2)^3)^{(1/2)} + 42*a^2*b^5*c^2*f^2 - 63*a^3*b^3*c^3*f^2 - a^3* \\
& c^3*f^2*(-(4*a*c - b^2)^3)^{(1/2)} + b^4*c^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 1 \\
& 1*a*b^7*c*f^2 + 16*a^3*c^6*d*e - 2*b^6*c^3*d*e - 16*a^4*c^5*e*f + 2*b^7*c^2 \\
& *d*f + 16*a*b^4*c^4*d*e - 18*a*b^5*c^3*d*f - 40*a^3*b*c^5*d*f + 20*a*b^6*c^ \\
& 2*e*f - 2*b^5*c*e*f*(-(4*a*c - b^2)^3)^{(1/2)} + 6*a^2*b^2*c^2*f^2*(-(4*a*c - \\
& b^2)^3)^{(1/2)} - 5*a*b^4*c*f^2*(-(4*a*c - b^2)^3)^{(1/2)} - 36*a^2*b^2*c^5*d* \\
& e + 50*a^2*b^3*c^4*d*f + 2*a^2*c^4*d*f*(-(4*a*c - b^2)^3)^{(1/2)} - 2*b^3*c^3 \\
& *d*e*(-(4*a*c - b^2)^3)^{(1/2)} - 66*a^2*b^4*c^3*e*f + 76*a^3*b^2*c^4*e*f + 2 \\
& *b^4*c^2*d*f*(-(4*a*c - b^2)^3)^{(1/2)} - 3*a*b^2*c^3*e^2*(-(4*a*c - b^2)^3) \\
& ^{(1/2)} + 4*a*b*c^4*d*e*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a*b^2*c^3*d*f*(-(4*a*c - \\
& b^2)^3)^{(1/2)} + 8*a*b^3*c^2*e*f*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a^2*b*c^3*e*f \\
& *(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^{(1/2)} \\
& + (2*x*(b^8*f^2 + 2*a^2*c^6*d^2 - 2*a^3*c^5*e^2 + b^4*c^4*d^2 + 2*a^4*c^4*f \\
& ^2 + b^6*c^2*e^2 - 4*a*b^2*c^5*d^2 - 6*a*b^4*c^3*e^2 - 2*b^7*c*e*f + 9*a^2* \\
& b^2*c^4*e^2 + 20*a^2*b^4*c^2*f^2 - 16*a^3*b^2*c^3*f^2 - 8*a*b^6*c*f^2 - 4*a \\
& ^3*c^5*d*f - 2*b^5*c^3*d*e + 2*b^6*c^2*d*f + 10*a*b^3*c^4*d*e - 10*a^2*b*c^ \\
& 5*d*e - 12*a*b^4*c^3*d*f + 14*a*b^5*c^2*e*f + 14*a^3*b*c^4*e*f + 18*a^2*b^2 \\
& *c^4*d*f - 28*a^2*b^3*c^3*e*f))/c^5)*(-(b^9*f^2 + b^5*c^4*d^2 + b^7*c^2*e^2 \\
& + b^6*f^2*(-(4*a*c - b^2)^3)^{(1/2)} - 7*a*b^3*c^5*d^2 + 12*a^2*b*c^6*d^2 - \\
& a*c^5*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 9*a*b^5*c^3*e^2 - 20*a^3*b*c^5*e^2 + 2 \\
& 8*a^4*b*c^4*f^2 - 2*b^8*c*e*f + 25*a^2*b^3*c^4*e^2 + a^2*c^4*e^2*(-(4*a*c - \\
& b^2)^3)^{(1/2)} + b^2*c^4*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 42*a^2*b^5*c^2*f^2 \\
& - 63*a^3*b^3*c^3*f^2 - a^3*c^3*f^2*(-(4*a*c - b^2)^3)^{(1/2)} + b^4*c^2*e^2*(- \\
& (4*a*c - b^2)^3)^{(1/2)} - 11*a*b^7*c*f^2 + 16*a^3*c^6*d*e - 2*b^6*c^3*d*e - \\
& 16*a^4*c^5*e*f + 2*b^7*c^2*d*f + 16*a*b^4*c^4*d*e - 18*a*b^5*c^3*d*f - 40* \\
& a^3*b*c^5*d*f + 20*a*b^6*c^2*e*f - 2*b^5*c*e*f*(-(4*a*c - b^2)^3)^{(1/2)} + 6 \\
& *a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^3)^{(1/2)} - 5*a*b^4*c*f^2*(-(4*a*c - b^2)^3 \\
&)^{(1/2)} - 36*a^2*b^2*c^5*d*e + 50*a^2*b^3*c^4*d*f + 2*a^2*c^4*d*f*(-(4*a*c \\
& - b^2)^3)^{(1/2)} - 2*b^3*c^3*d*e*(-(4*a*c - b^2)^3)^{(1/2)} - 66*a^2*b^4*c^3*e \\
& *f + 76*a^3*b^2*c^4*e*f + 2*b^4*c^2*d*f*(-(4*a*c - b^2)^3)^{(1/2)} - 3*a*b^2* \\
& c^3*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 4*a*b*c^4*d*e*(-(4*a*c - b^2)^3)^{(1/2)} -
\end{aligned}$$

$$\begin{aligned}
& 6*a*b^2*c^3*d*f*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a*b^3*c^2*e*f*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a^2*b*c^3*e*f*(-(4*a*c - b^2)^3)^{(1/2)} / (8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8))^{(1/2)}) * (- (b^9*f^2 + b^5*c^4*d^2 + b^7*c^2*e^2 + b^6*f^2 * (- (4*a*c - b^2)^3)^{(1/2)} - 7*a*b^3*c^5*d^2 + 12*a^2*b*c^6*d^2 - a*c^5*d^2 * (- (4*a*c - b^2)^3)^{(1/2)} - 9*a*b^5*c^3*e^2 - 20*a^3*b*c^5*e^2 + 28*a^4*b*c^4*f^2 - 2*b^8*c*e*f + 25*a^2*b^3*c^4*e^2 + a^2*c^4*e^2 * (- (4*a*c - b^2)^3)^{(1/2)} + b^2*c^4*d^2 * (- (4*a*c - b^2)^3)^{(1/2)} + 42*a^2*b^5*c^2*f^2 - 63*a^3*b^3*c^3*f^2 - a^3*c^3*f^2 * (- (4*a*c - b^2)^3)^{(1/2)} + b^4*c^2*e^2 * (- (4*a*c - b^2)^3)^{(1/2)} - 11*a*b^7*c*f^2 + 16*a^3*c^6*d*e - 2*b^6*c^3*d*e - 16*a^4*c^5*e*f + 2*b^7*c^2*d*f + 16*a*b^4*c^4*d*e - 18*a*b^5*c^3*d*f - 40*a^3*b*c^5*d*f + 20*a*b^6*c^2*e*f - 2*b^5*c*e*f * (- (4*a*c - b^2)^3)^{(1/2)} + 6*a^2*b^2*c^2*f^2 * (- (4*a*c - b^2)^3)^{(1/2)} - 5*a*b^4*c*f^2 * (- (4*a*c - b^2)^3)^{(1/2)} - 36*a^2*b^2*c^5*d*e + 50*a^2*b^3*c^4*d*f + 2*a^2*c^4*d*f * (- (4*a*c - b^2)^3)^{(1/2)} - 2*b^3*c^3*d*e * (- (4*a*c - b^2)^3)^{(1/2)} - 66*a^2*b^4*c^3*e*f + 76*a^3*b^2*c^4*e*f + 2*b^4*c^2*d*f * (- (4*a*c - b^2)^3)^{(1/2)} - 3*a*b^2*c^3*e^2 * (- (4*a*c - b^2)^3)^{(1/2)} + 4*a*b*c^4*d*e * (- (4*a*c - b^2)^3)^{(1/2)} - 6*a*b^2*c^3*d*f * (- (4*a*c - b^2)^3)^{(1/2)} + 8*a*b^3*c^2*e*f * (- (4*a*c - b^2)^3)^{(1/2)} - 6*a^2*b*c^3*e*f * (- (4*a*c - b^2)^3)^{(1/2)} / (8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8))^{(1/2)} * 2i + \operatorname{atan}(\frac{((16*a^3*c^6*f - 16*a^2*c^7*d - 20*a^2*b^2*c^5*f + 4*a*b^2*c^6*d - 4*a*b^3*c^5*e + 16*a^2*b*c^6*e + 4*a*b^4*c^4*f) / c^5 - (2*x*(4*b^3*c^7 - 16*a*b*c^8) * (- (b^9*f^2 + b^5*c^4*d^2 + b^7*c^2*e^2 - b^6*f^2 * (- (4*a*c - b^2)^3)^{(1/2)} - 7*a*b^3*c^5*d^2 + 12*a^2*b*c^6*d^2 + a*c^5*d^2 * (- (4*a*c - b^2)^3)^{(1/2)} - 9*a*b^5*c^3*e^2 - 20*a^3*b*c^5*e^2 + 28*a^4*b*c^4*f^2 - 2*b^8*c*e*f + 25*a^2*b^3*c^4*e^2 - a^2*c^4*e^2 * (- (4*a*c - b^2)^3)^{(1/2)} - b^2*c^4*d^2 * (- (4*a*c - b^2)^3)^{(1/2)} + 42*a^2*b^5*c^2*f^2 - 63*a^3*b^3*c^3*f^2 + a^3*c^3*f^2 * (- (4*a*c - b^2)^3)^{(1/2)} - b^4*c^2*e^2 * (- (4*a*c - b^2)^3)^{(1/2)} - 11*a*b^7*c*f^2 + 16*a^3*c^6*d*e - 2*b^6*c^3*d*e - 16*a^4*c^5*e*f + 2*b^7*c^2*d*f + 16*a*b^4*c^4*d*e - 18*a*b^5*c^3*d*f - 40*a^3*b*c^5*d*f + 20*a*b^6*c^2*e*f + 2*b^5*c*e*f * (- (4*a*c - b^2)^3)^{(1/2)} - 6*a^2*b^2*c^2*f^2 * (- (4*a*c - b^2)^3)^{(1/2)} + 5*a*b^4*c*f^2 * (- (4*a*c - b^2)^3)^{(1/2)} - 36*a^2*b^2*c^5*d*e + 50*a^2*b^3*c^4*d*f - 2*a^2*c^4*d*f * (- (4*a*c - b^2)^3)^{(1/2)} + 2*b^3*c^3*d*e * (- (4*a*c - b^2)^3)^{(1/2)} - 66*a^2*b^4*c^3*e*f + 76*a^3*b^2*c^4*e*f - 2*b^4*c^2*d*f * (- (4*a*c - b^2)^3)^{(1/2)} + 3*a*b^2*c^3*e^2 * (- (4*a*c - b^2)^3)^{(1/2)} - 4*a*b*c^4*d*e * (- (4*a*c - b^2)^3)^{(1/2)} + 6*a*b^2*c^3*d*f * (- (4*a*c - b^2)^3)^{(1/2)} - 8*a*b^3*c^2*e*f * (- (4*a*c - b^2)^3)^{(1/2)} + 6*a^2*b*c^3*e*f * (- (4*a*c - b^2)^3)^{(1/2)} / (8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8))^{(1/2)} / c^5 * (- (b^9*f^2 + b^5*c^4*d^2 + b^7*c^2*e^2 - b^6*f^2 * (- (4*a*c - b^2)^3)^{(1/2)} - 7*a*b^3*c^5*d^2 + 12*a^2*b*c^6*d^2 + a*c^5*d^2 * (- (4*a*c - b^2)^3)^{(1/2)} - 9*a*b^5*c^3*e^2 - 20*a^3*b*c^5*e^2 + 28*a^4*b*c^4*f^2 - 2*b^8*c*e*f + 25*a^2*b^3*c^4*e^2 - a^2*c^4*e^2 * (- (4*a*c - b^2)^3)^{(1/2)} - b^2*c^4*d^2 * (- (4*a*c - b^2)^3)^{(1/2)} + 42*a^2*b^5*c^2*f^2 - 63*a^3*b^3*c^3*f^2 + a^3*c^3*f^2 * (- (4*a*c - b^2)^3)^{(1/2)} - b^4*c^2*e^2 * (- (4*a*c - b^2)^3)^{(1/2)} - 11*a*b^7*c*f^2 + 16*a^3*c^6*d*e - 2*b^6*c^3*d*e - 16*a^4*c^5*e*f + 2*b^7*c^2*d*f + 16*a*b^4*c^4*d*e - 18*a*b^5*c^3*d*f - 40*a^3*b*c^5*d*f + 20*a*b^6*c^2*e*f + 2*b^5*c*e*f * (- (4*a*c - b^2)^3)^{(1/2)} - 6*a^2*b^2*c^2*f^2 * (- (4*a*c - b^2)^3)^{(1/2)} + 5*a*b^4*c*f^2 * (- (4*a*c - b^2)^3)^{(1/2)} - 36*a^2*b^2*c^5*d*e + 50*a^2*b^3*c^4*d*f - 2*a^2*c^4*d*f * (- (4*a*c - b^2)^3)^{(1/2)} + 2*b^3*c^3*d*e * (- (4*a*c - b^2)^3)^{(1/2)} - 66*a^2*b^4*c^3*e*f + 76*a^3*b^2*c^4*e*f - 2*b^4*c^2*d*f * (- (4*a*c - b^2)^3)^{(1/2)} + 3*a*b^2*c^3*e^2 * (- (4*a*c - b^2)^3)^{(1/2)} - 4*a*b*c^4*d*e * (- (4*a*c - b^2)^3)^{(1/2)} + 6*a*b^2*c^3*d*f * (- (4*a*c - b^2)^3)^{(1/2)} - 8*a*b^3*c^2*e*f * (- (4*a*c - b^2)^3)^{(1/2)} + 6*a^2*b*c^3*e*f * (- (4*a*c - b^2)^3)^{(1/2)} / (8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8))^{(1/2)} - (2*x*(b^8*f^2 + 2*a^2*c^6*d^2 - 2*a^3*c^5*e^2 + b^4*c^4*d^2 + 2*a^4*c^4*f^2 + b^6*c^2*e^2 - 4*a*b^2*c^5*d^2 - 6*a*b^4*c^3*e^2 - 2*b^7*c*e*f + 9*a^2*b^2*c^4*e^2 + 20*a^2*b^4*c^2*f^2 - 16*a^3*b^2*c^3*f^2 - 8*a*b^6*c*f^2 - 4*a^3*c^5*d*f - 2*b^5*c^3*d*e + 2*b^6*c^2*d*f + 10*a*b^3*c^4*d*e - 10*a^2*b*c^5*d*e - 12*a*b^4*c^3*d*f + 14*a*b^5*c^2*e*f + 14*a^3*b*c^4*e*f + 18*a^2*b^2*c^4*d*f - 28*a^2*b^3*c^3*e*f)) / c^5 * (- (b^9*f^2 + b^5*c^4*d^2 + b^7*c^2*e^2 - b^6*f^2 * (- (4*a*c - b^2)^3)^{(1/2)} - 7*a*b^3*c^5*d^2 + 1
\end{aligned}$$

$$\begin{aligned}
& 2a^2b^6c^6d^2 + a^5c^5d^2(-4ac - b^2)^3)^{1/2} - 9a^5b^5c^3e^2 - 20 \\
& a^3b^6c^5e^2 + 28a^4b^6c^4f^2 - 2b^8c^6ef + 25a^2b^3c^4e^2 - a^2c^4e^2(-4ac - b^2)^3)^{1/2} - b^2c^4d^2(-4ac - b^2)^3)^{1/2} + 4 \\
& 2a^2b^5c^2f^2 - 63a^3b^3c^3f^2 + a^3c^3f^2(-4ac - b^2)^3)^{1/2} - b^4c^2e^2(-4ac - b^2)^3)^{1/2} - 11a^7b^7c^6d^2e \\
& - 2b^6c^3d^2e - 16a^4c^5eef + 2b^7c^2d^2f + 16a^4b^4c^4d^2e - 18a^5b^5c^3d^2f - 40a^3b^6c^5d^2f \\
& + 20a^6b^6c^2eef + 2b^5c^6eef(-4ac - b^2)^3)^{1/2} - 6a^2b^2c^2f^2(-4ac - b^2)^3)^{1/2} + 5a^4b^4c^4f^2 \\
& (-4ac - b^2)^3)^{1/2} - 36a^2b^2c^5d^2e + 50a^2b^3c^4d^2f - 2a^2c^4d^2f(-4ac - b^2)^3)^{1/2} + 2b^3c^3d^2e(-4ac - b^2)^3)^{1/2} \\
& - 66a^2b^4c^3eef + 76a^3b^2c^4eef - 2b^4c^2d^2f(-4ac - b^2)^3)^{1/2} + 3a^2b^2c^3e^2(-4ac - b^2)^3)^{1/2} - 4a^2b^4c^4d^2e(-4ac - b^2)^3)^{1/2} \\
& + 6a^2b^2c^3d^2f(-4ac - b^2)^3)^{1/2} - 8a^2b^3c^2eef(-4ac - b^2)^3)^{1/2} + 6a^2b^2c^3eef(-4ac - b^2)^3)^{1/2} \\
& - 8a^2b^2c^3eef(-4ac - b^2)^3)^{1/2} / (8(16a^2c^9 + b^4c^7 - 8a^2b^2c^8))^{1/2} * i - ((16a^3c^6f - 16a^2c^7d - 20a^2b^2c^5f + 4a^2b^2c^6d - 4a^2b^3c^5e + 16a^2b^3c^6e + 4a^2b^4c^4f) / c^5 + (2x(4b^3c^7 - 16a^2b^3c^8) * (-b^9f^2 + b^5c^4d^2 + b^7c^2e^2 - b^6f^2(-4ac - b^2)^3)^{1/2} - 7a^2b^3c^5d^2 + 12a^2b^2c^6d^2 + a^5c^5d^2(-4ac - b^2)^3)^{1/2} - 9a^5b^5c^3e^2 - 20a^3b^6c^5e^2 + 28a^4b^6c^4f^2 - 2b^8c^6ef + 25a^2b^3c^4e^2 - a^2c^4e^2(-4ac - b^2)^3)^{1/2} - b^2c^4d^2(-4ac - b^2)^3)^{1/2} + 42a^2b^5c^2f^2 - 63a^3b^3c^3f^2 + a^3c^3f^2(-4ac - b^2)^3)^{1/2} - b^4c^2e^2(-4ac - b^2)^3)^{1/2} - 11a^7b^7c^6d^2e - 2b^6c^3d^2e - 16a^4c^5eef + 2b^7c^2d^2f + 16a^4b^4c^4d^2e - 18a^5b^5c^3d^2f - 40a^3b^6c^5d^2f + 20a^6b^6c^2eef + 2b^5c^6eef(-4ac - b^2)^3)^{1/2} - 6a^2b^2c^2f^2(-4ac - b^2)^3)^{1/2} + 5a^4b^4c^4f^2(-4ac - b^2)^3)^{1/2} - 36a^2b^2c^5d^2e + 50a^2b^3c^4d^2f - 2a^2c^4d^2f(-4ac - b^2)^3)^{1/2} + 2b^3c^3d^2e(-4ac - b^2)^3)^{1/2} - 66a^2b^4c^3eef + 76a^3b^2c^4eef - 2b^4c^2d^2f(-4ac - b^2)^3)^{1/2} + 3a^2b^2c^3e^2(-4ac - b^2)^3)^{1/2} - 4a^2b^4c^4d^2e(-4ac - b^2)^3)^{1/2} + 6a^2b^2c^3d^2f(-4ac - b^2)^3)^{1/2} - 8a^2b^3c^2eef(-4ac - b^2)^3)^{1/2} + 6a^2b^2c^3eef(-4ac - b^2)^3)^{1/2} - 8a^2b^2c^3eef(-4ac - b^2)^3)^{1/2} / (8(16a^2c^9 + b^4c^7 - 8a^2b^2c^8))^{1/2} / c^5 * (-b^9f^2 + b^5c^4d^2 + b^7c^2e^2 - b^6f^2(-4ac - b^2)^3)^{1/2} - 7a^2b^3c^5d^2 + 12a^2b^2c^6d^2 + a^5c^5d^2(-4ac - b^2)^3)^{1/2} - 9a^5b^5c^3e^2 - 20a^3b^6c^5e^2 + 28a^4b^6c^4f^2 - 2b^8c^6ef + 25a^2b^3c^4e^2 - a^2c^4e^2(-4ac - b^2)^3)^{1/2} - b^2c^4d^2(-4ac - b^2)^3)^{1/2} + 42a^2b^5c^2f^2 - 63a^3b^3c^3f^2 + a^3c^3f^2(-4ac - b^2)^3)^{1/2} - b^4c^2e^2(-4ac - b^2)^3)^{1/2} - 11a^7b^7c^6d^2e - 2b^6c^3d^2e - 16a^4c^5eef + 2b^7c^2d^2f + 16a^4b^4c^4d^2e - 18a^5b^5c^3d^2f - 40a^3b^6c^5d^2f + 20a^6b^6c^2eef + 2b^5c^6eef(-4ac - b^2)^3)^{1/2} - 6a^2b^2c^2f^2(-4ac - b^2)^3)^{1/2} + 5a^4b^4c^4f^2(-4ac - b^2)^3)^{1/2} - 36a^2b^2c^5d^2e + 50a^2b^3c^4d^2f - 2a^2c^4d^2f(-4ac - b^2)^3)^{1/2} + 2b^3c^3d^2e(-4ac - b^2)^3)^{1/2} - 66a^2b^4c^3eef + 76a^3b^2c^4eef - 2b^4c^2d^2f(-4ac - b^2)^3)^{1/2} + 3a^2b^2c^3e^2(-4ac - b^2)^3)^{1/2} - 4a^2b^4c^4d^2e(-4ac - b^2)^3)^{1/2} + 6a^2b^2c^3d^2f(-4ac - b^2)^3)^{1/2} - 8a^2b^3c^2eef(-4ac - b^2)^3)^{1/2} + 6a^2b^2c^3eef(-4ac - b^2)^3)^{1/2} - 8a^2b^2c^3eef(-4ac - b^2)^3)^{1/2} / (8(16a^2c^9 + b^4c^7 - 8a^2b^2c^8))^{1/2} + (2x(b^8f^2 + 2a^2c^6d^2 - 2a^3c^5e^2 + b^4c^4d^2 + 2a^4c^4f^2 + b^6c^2e^2 - 4a^2b^2c^5d^2 - 6a^2b^4c^3e^2 - 2b^7c^6eef + 9a^2b^2c^4e^2 + 20a^2b^4c^2f^2 - 16a^3b^2c^3f^2 - 8a^2b^6c^4f^2 - 4a^3c^5d^2f - 2b^5c^3d^2e + 2b^6c^2d^2f + 10a^2b^3c^4d^2e - 10a^2b^6c^5d^2e - 12a^2b^4c^3d^2f + 14a^2b^5c^2eef + 14a^3b^6c^4eef + 18a^2b^2c^4d^2f - 28a^2b^3c^3eef) / c^5) * (-b^9f^2 + b^5c^4d^2 + b^7c^2e^2 - b^6f^2(-4ac - b^2)^3)^{1/2} - 7a^2b^3c^5d^2 + 12a^2b^2c^6d^2 + a^5c^5d^2(-4ac - b^2)^3)^{1/2} - 9a^5b^5c^3e^2 - 20a^3b^6c^5e^2 + 28a^4b^6c^4f^2 - 2b^8c^6ef + 25a^2b^3c^4e^2 - a^2c^4e^2(-4ac - b^2)^3)^{1/2} - b^2c^4d^2(-4ac - b^2)^3)^{1/2} + 42a^2b^5c^2f^2 - 63a^3b^3c^3f^2 + a^3c^3f^2(-
\end{aligned}$$

$$\begin{aligned}
& (4ac - b^2)^3)^{1/2} - b^4c^2e^2(-4ac - b^2)^3)^{1/2} - 11ab^7c^* \\
& f^2 + 16a^3c^6d^*e - 2b^6c^3d^*e - 16a^4c^5e^*f + 2b^7c^2d^*f + 16* \\
& ab^4c^4d^*e - 18ab^5c^3d^*f - 40a^3b^*c^5d^*f + 20ab^6c^2e^*f + 2* \\
& b^5c^*e^*f(-4ac - b^2)^3)^{1/2} - 6a^2b^2c^2f^2(-4ac - b^2)^3)^{(\\
& 1/2) + 5ab^4c^*f^2(-4ac - b^2)^3)^{1/2} - 36a^2b^2c^5d^*e + 50a^2 \\
& *b^3c^4d^*f - 2a^2c^4d^*f(-4ac - b^2)^3)^{1/2} + 2b^3c^3d^*e(-4* \\
& ac - b^2)^3)^{1/2} - 66a^2b^4c^3e^*f + 76a^3b^2c^4e^*f - 2b^4c^2d^* \\
& *f(-4ac - b^2)^3)^{1/2} + 3ab^2c^3e^2(-4ac - b^2)^3)^{1/2} - 4* \\
& ab^*c^4d^*e(-4ac - b^2)^3)^{1/2} + 6ab^2c^3d^*f(-4ac - b^2)^3)^{(\\
& 1/2) - 8ab^3c^2e^*f(-4ac - b^2)^3)^{1/2} + 6a^2b^*c^3e^*f(-4ac \\
& - b^2)^3)^{1/2})/(8(16a^2c^9 + b^4c^7 - 8ab^2c^8)))^{1/2} * i) / (((16 \\
& a^3c^6f - 16a^2c^7d - 20a^2b^2c^5f + 4ab^2c^6d - 4ab^3c^5* \\
& e + 16a^2b^*c^6e + 4ab^4c^4f)/c^5 - (2x*(4b^3c^7 - 16ab^*c^8))*(- \\
& b^9f^2 + b^5c^4d^2 + b^7c^2e^2 - b^6f^2(-4ac - b^2)^3)^{1/2} - 7* \\
& ab^3c^5d^2 + 12a^2b^*c^6d^2 + a^*c^5d^2(-4ac - b^2)^3)^{1/2} - 9a \\
& *b^5c^3e^2 - 20a^3b^*c^5e^2 + 28a^4b^*c^4f^2 - 2b^8c^*e^*f + 25a^2b^ \\
& ^3c^4e^2 - a^2c^4e^2(-4ac - b^2)^3)^{1/2} - b^2c^4d^2(-4ac - \\
& b^2)^3)^{1/2} + 42a^2b^5c^2f^2 - 63a^3b^3c^3f^2 + a^3c^3f^2(-4* \\
& ac - b^2)^3)^{1/2} - b^4c^2e^2(-4ac - b^2)^3)^{1/2} - 11ab^7c^*f^2 \\
& + 16a^3c^6d^*e - 2b^6c^3d^*e - 16a^4c^5e^*f + 2b^7c^2d^*f + 16ab \\
& ^4c^4d^*e - 18ab^5c^3d^*f - 40a^3b^*c^5d^*f + 20ab^6c^2e^*f + 2b^5 \\
& *c^*e^*f(-4ac - b^2)^3)^{1/2} - 6a^2b^2c^2f^2(-4ac - b^2)^3)^{1/2} \\
&) + 5ab^4c^*f^2(-4ac - b^2)^3)^{1/2} - 36a^2b^2c^5d^*e + 50a^2b^ \\
& ^3c^4d^*f - 2a^2c^4d^*f(-4ac - b^2)^3)^{1/2} + 2b^3c^3d^*e(-4ac \\
& - b^2)^3)^{1/2} - 66a^2b^4c^3e^*f + 76a^3b^2c^4e^*f - 2b^4c^2d^*f* \\
& (-4ac - b^2)^3)^{1/2} + 3ab^2c^3e^2(-4ac - b^2)^3)^{1/2} - 4ab \\
& *c^4d^*e(-4ac - b^2)^3)^{1/2} + 6ab^2c^3d^*f(-4ac - b^2)^3)^{1/2} \\
&) - 8ab^3c^2e^*f(-4ac - b^2)^3)^{1/2} + 6a^2b^*c^3e^*f(-4ac - b \\
& ^2)^3)^{1/2})/(8(16a^2c^9 + b^4c^7 - 8ab^2c^8)))^{1/2})/c^5)*(-(b^9* \\
& f^2 + b^5c^4d^2 + b^7c^2e^2 - b^6f^2(-4ac - b^2)^3)^{1/2} - 7ab^ \\
& ^3c^5d^2 + 12a^2b^*c^6d^2 + a^*c^5d^2(-4ac - b^2)^3)^{1/2} - 9ab^5 \\
& *c^3e^2 - 20a^3b^*c^5e^2 + 28a^4b^*c^4f^2 - 2b^8c^*e^*f + 25a^2b^3c^ \\
& ^4e^2 - a^2c^4e^2(-4ac - b^2)^3)^{1/2} - b^2c^4d^2(-4ac - b^2) \\
& ^3)^{1/2} + 42a^2b^5c^2f^2 - 63a^3b^3c^3f^2 + a^3c^3f^2(-4ac \\
& - b^2)^3)^{1/2} - b^4c^2e^2(-4ac - b^2)^3)^{1/2} - 11ab^7c^*f^2 + 1 \\
& 6a^3c^6d^*e - 2b^6c^3d^*e - 16a^4c^5e^*f + 2b^7c^2d^*f + 16ab^4c^ \\
& ^4d^*e - 18ab^5c^3d^*f - 40a^3b^*c^5d^*f + 20ab^6c^2e^*f + 2b^5c^*e \\
& *f(-4ac - b^2)^3)^{1/2} - 6a^2b^2c^2f^2(-4ac - b^2)^3)^{1/2} + \\
& 5ab^4c^*f^2(-4ac - b^2)^3)^{1/2} - 36a^2b^2c^5d^*e + 50a^2b^3c^ \\
& ^4d^*f - 2a^2c^4d^*f(-4ac - b^2)^3)^{1/2} + 2b^3c^3d^*e(-4ac - b \\
& ^2)^3)^{1/2} - 66a^2b^4c^3e^*f + 76a^3b^2c^4e^*f - 2b^4c^2d^*f(-4 \\
& *ac - b^2)^3)^{1/2} + 3ab^2c^3e^2(-4ac - b^2)^3)^{1/2} - 4ab^*c^4 \\
& *d^*e(-4ac - b^2)^3)^{1/2} + 6ab^2c^3d^*f(-4ac - b^2)^3)^{1/2} - \\
& 8ab^3c^2e^*f(-4ac - b^2)^3)^{1/2} + 6a^2b^*c^3e^*f(-4ac - b^2)^ \\
& ^3)^{1/2})/(8(16a^2c^9 + b^4c^7 - 8ab^2c^8)))^{1/2} - (2x*(b^8f^2 + \\
& 2a^2c^6d^2 - 2a^3c^5e^2 + b^4c^4d^2 + 2a^4c^4f^2 + b^6c^2e^2 \\
& - 4ab^2c^5d^2 - 6ab^4c^3e^2 - 2b^7c^*e^*f + 9a^2b^2c^4e^2 + 20* \\
& a^2b^4c^2f^2 - 16a^3b^2c^3f^2 - 8ab^6c^*f^2 - 4a^3c^5d^*f - 2b^ \\
& ^5c^3d^*e + 2b^6c^2d^*f + 10ab^3c^4d^*e - 10a^2b^*c^5d^*e - 12ab^4c^ \\
& ^3d^*f + 14ab^5c^2e^*f + 14a^3b^*c^4e^*f + 18a^2b^2c^4d^*f - 28a^2 \\
& *b^3c^3e^*f)/c^5)*(-(b^9f^2 + b^5c^4d^2 + b^7c^2e^2 - b^6f^2(-4a \\
& *c - b^2)^3)^{1/2} - 7ab^3c^5d^2 + 12a^2b^*c^6d^2 + a^*c^5d^2(-4a* \\
& c - b^2)^3)^{1/2} - 9ab^5c^3e^2 - 20a^3b^*c^5e^2 + 28a^4b^*c^4f^2 - \\
& 2b^8c^*e^*f + 25a^2b^3c^4e^2 - a^2c^4e^2(-4ac - b^2)^3)^{1/2} - \\
& b^2c^4d^2(-4ac - b^2)^3)^{1/2} + 42a^2b^5c^2f^2 - 63a^3b^3c^3* \\
& f^2 + a^3c^3f^2(-4ac - b^2)^3)^{1/2} - b^4c^2e^2(-4ac - b^2)^3) \\
& ^{1/2} - 11ab^7c^*f^2 + 16a^3c^6d^*e - 2b^6c^3d^*e - 16a^4c^5e^*f + \\
& 2b^7c^2d^*f + 16ab^4c^4d^*e - 18ab^5c^3d^*f - 40a^3b^*c^5d^*f + 2 \\
& 0ab^6c^2e^*f + 2b^5c^*e^*f(-4ac - b^2)^3)^{1/2} - 6a^2b^2c^2f^2*
\end{aligned}$$

$$\begin{aligned}
& (- (4ac - b^2)^3)^{1/2} + 5ab^4c^2f^2(- (4ac - b^2)^3)^{1/2} - 36a^2b^2c^5d^2e + 50a^2b^3c^4d^2f - 2a^2c^4d^2f(- (4ac - b^2)^3)^{1/2} + \\
& 2b^3c^3d^2e(- (4ac - b^2)^3)^{1/2} - 66a^2b^4c^3e^2f + 76a^3b^2c^4e^2f - 2b^4c^2d^2f(- (4ac - b^2)^3)^{1/2} + 3ab^2c^3e^2(- (4ac - \\
& b^2)^3)^{1/2} - 4ab^3c^4d^2e(- (4ac - b^2)^3)^{1/2} + 6ab^2c^3d^2f(- (4ac - b^2)^3)^{1/2} - 8ab^3c^2e^2f(- (4ac - b^2)^3)^{1/2} + 6a^2 \\
& b^2c^3e^2f(- (4ac - b^2)^3)^{1/2} / (8(16a^2c^9 + b^4c^7 - 8ab^2c^8))^{1/2} - (2(a^4b^3f^3 + a^4c^3e^3 + a^2b^4c^4d^3 + a^2b^5d^2f^2 + \\
& a^3c^4d^2e - a^3b^4e^2f^2 + a^5c^2e^2f^2 - a^3b^2c^2e^3 - 2a^5b^2c^3f^3 - 2a^4c^3d^2e^2f - 4a^3b^2c^3d^2f - 4a^3b^3c^4d^2f + 5a^4b^2c^2d^2f^2 + \\
& 2a^3b^3c^2e^2f - 3a^4b^2c^2e^2f + a^4b^2c^2e^2f - 2a^2b^2c^3d^2e + a^2b^3c^2d^2e^2 + 2a^2b^3c^2d^2f - 2a^2b^4c^2d^2e^2f + 4a^3b^2c^2d^2e^2f) / c^5 + \\
& ((16a^3c^6f - 16a^2c^7d - 20a^2b^2c^5f + 4ab^2c^6d - 4ab^3c^5e + 16a^2b^2c^6e + 4ab^4c^4f) / c^5 + (2x(4b^3c^7 - 16ab^3c^8) * (- (b^9f^2 + b^5c^4d^2 + b^7c^2e^2 - b^6f^2 * (- (4ac - b^2)^3)^{1/2} - 7ab^3c^5d^2 + 12a^2b^2c^6d^2 + a^5d^2 * (- (4ac - b^2)^3)^{1/2} - 9ab^5c^3e^2 - 20a^3b^2c^5e^2 + 28a^4b^2c^4f^2 - 2b^8c^2e^2f + 25a^2b^3c^4e^2 - a^2c^4e^2 * (- (4ac - b^2)^3)^{1/2} - b^2c^4d^2 * (- (4ac - b^2)^3)^{1/2} + 42a^2b^5c^2f^2 - 63a^3b^3c^3f^2 + a^3c^3f^2 * (- (4ac - b^2)^3)^{1/2} - b^4c^2e^2 * (- (4ac - b^2)^3)^{1/2} - 11ab^7c^2f^2 + 16a^3c^6d^2e - 2b^6c^3d^2e - 16a^4c^5e^2f + 2b^7c^2d^2f + 16ab^4c^4d^2e - 18ab^5c^3d^2f - 40a^3b^2c^5d^2f + 20ab^6c^2e^2f + 2b^5c^2e^2f * (- (4ac - b^2)^3)^{1/2} - 6a^2b^2c^2f^2 * (- (4ac - b^2)^3)^{1/2} + 5ab^4c^2f^2 * (- (4ac - b^2)^3)^{1/2} - 36a^2b^2c^5d^2e + 50a^2b^3c^4d^2f - 2a^2c^4d^2f * (- (4ac - b^2)^3)^{1/2} + 2b^3c^3d^2e * (- (4ac - b^2)^3)^{1/2} - 66a^2b^4c^3e^2f + 76a^3b^2c^4e^2f - 2b^4c^2d^2f * (- (4ac - b^2)^3)^{1/2} + 3ab^2c^3e^2 * (- (4ac - b^2)^3)^{1/2} - 4ab^3c^4d^2e * (- (4ac - b^2)^3)^{1/2} + 6ab^2c^3d^2f * (- (4ac - b^2)^3)^{1/2} - 8ab^3c^2e^2f * (- (4ac - b^2)^3)^{1/2} + 6a^2b^2c^3e^2f * (- (4ac - b^2)^3)^{1/2} / (8(16a^2c^9 + b^4c^7 - 8ab^2c^8))^{1/2} / c^5 * (- (b^9f^2 + b^5c^4d^2 + b^7c^2e^2 - b^6f^2 * (- (4ac - b^2)^3)^{1/2} - 7ab^3c^5d^2 + 12a^2b^2c^6d^2 + a^5d^2 * (- (4ac - b^2)^3)^{1/2} - 9ab^5c^3e^2 - 20a^3b^2c^5e^2 + 28a^4b^2c^4f^2 - 2b^8c^2e^2f + 25a^2b^3c^4e^2 - a^2c^4e^2 * (- (4ac - b^2)^3)^{1/2} - b^2c^4d^2 * (- (4ac - b^2)^3)^{1/2} + 42a^2b^5c^2f^2 - 63a^3b^3c^3f^2 + a^3c^3f^2 * (- (4ac - b^2)^3)^{1/2} - b^4c^2e^2 * (- (4ac - b^2)^3)^{1/2} - 11ab^7c^2f^2 + 16a^3c^6d^2e - 2b^6c^3d^2e - 16a^4c^5e^2f + 2b^7c^2d^2f + 16ab^4c^4d^2e - 18ab^5c^3d^2f - 40a^3b^2c^5d^2f + 20ab^6c^2e^2f + 2b^5c^2e^2f * (- (4ac - b^2)^3)^{1/2} - 6a^2b^2c^2f^2 * (- (4ac - b^2)^3)^{1/2} + 5ab^4c^2f^2 * (- (4ac - b^2)^3)^{1/2} - 36a^2b^2c^5d^2e + 50a^2b^3c^4d^2f - 2a^2c^4d^2f * (- (4ac - b^2)^3)^{1/2} + 2b^3c^3d^2e * (- (4ac - b^2)^3)^{1/2} - 66a^2b^4c^3e^2f + 76a^3b^2c^4e^2f - 2b^4c^2d^2f * (- (4ac - b^2)^3)^{1/2} + 3ab^2c^3e^2 * (- (4ac - b^2)^3)^{1/2} - 4ab^3c^4d^2e * (- (4ac - b^2)^3)^{1/2} + 6ab^2c^3d^2f * (- (4ac - b^2)^3)^{1/2} - 8ab^3c^2e^2f * (- (4ac - b^2)^3)^{1/2} + 6a^2b^2c^3e^2f * (- (4ac - b^2)^3)^{1/2} / (8(16a^2c^9 + b^4c^7 - 8ab^2c^8))^{1/2} + (2x(b^8f^2 + 2a^2c^6d^2 - 2a^3c^5e^2 + b^4c^4d^2 + 2a^4c^4f^2 + b^6c^2e^2 - 4ab^2c^5d^2 - 6ab^4c^3e^2 - 2b^7c^2e^2f + 9a^2b^2c^4e^2 + 20a^2b^4c^2f^2 - 16a^3b^2c^3f^2 - 8ab^6c^2f^2 - 4a^3c^5d^2f - 2b^5c^3d^2e + 2b^6c^2d^2f + 10ab^3c^4d^2e - 10a^2b^2c^5d^2e - 12ab^4c^3d^2f + 14ab^5c^2e^2f + 14a^3b^2c^4e^2f + 18a^2b^2c^4d^2f - 28a^2b^3c^3e^2f) / c^5 * (- (b^9f^2 + b^5c^4d^2 + b^7c^2e^2 - b^6f^2 * (- (4ac - b^2)^3)^{1/2} - 7ab^3c^5d^2 + 12a^2b^2c^6d^2 + a^5d^2 * (- (4ac - b^2)^3)^{1/2} - 9ab^5c^3e^2 - 20a^3b^2c^5e^2 + 28a^4b^2c^4f^2 - 2b^8c^2e^2f + 25a^2b^3c^4e^2 - a^2c^4e^2 * (- (4ac - b^2)^3)^{1/2} - b^2c^4d^2 * (- (4ac - b^2)^3)^{1/2} + 42a^2b^5c^2f^2 - 63a^3b^3c^3f^2 + a^3c^3f^2 * (- (4ac - b^2)^3)^{1/2} - b^4c^2e^2 * (- (4ac - b^2)^3)^{1/2} - 11ab^7c^2f^2 + 16a^3c^6d^2e - 2b^6c^3d^2e - 16a^4c^5e^2f + 2b^7c^2d^2f + 16ab^4c^4d^2e - 18
\end{aligned}$$

$$\begin{aligned}
& a^5 b^3 c^3 d f^2 - 40 a^3 b^3 c^5 d f^2 + 20 a^2 b^6 c^2 e f^2 + 2 b^5 c^3 e f^2 (-4 a^2 c - b^2)^3)^{1/2} - 6 a^2 b^2 c^2 f^2 (-4 a^2 c - b^2)^3)^{1/2} + 5 a^2 b^4 c^2 f^2 (-4 a^2 c - b^2)^3)^{1/2} - 36 a^2 b^2 c^5 d e + 50 a^2 b^3 c^4 d f^2 - 2 a^2 c^4 d f^2 (-4 a^2 c - b^2)^3)^{1/2} + 2 b^3 c^3 d e (-4 a^2 c - b^2)^3)^{1/2} \\
& - 66 a^2 b^4 c^3 e f^2 + 76 a^3 b^2 c^4 e f^2 - 2 b^4 c^2 d f^2 (-4 a^2 c - b^2)^3)^{1/2} + 3 a^2 b^2 c^3 e^2 (-4 a^2 c - b^2)^3)^{1/2} - 4 a^2 b^2 c^4 d e (-4 a^2 c - b^2)^3)^{1/2} + 6 a^2 b^2 c^3 d f^2 (-4 a^2 c - b^2)^3)^{1/2} - 8 a^2 b^3 c^2 e f^2 (-4 a^2 c - b^2)^3)^{1/2} + 6 a^2 b^2 c^3 e f^2 (-4 a^2 c - b^2)^3)^{1/2} / (8 * (16 a^2 c^9 + b^4 c^7 - 8 a^2 b^2 c^8))^{1/2}) * (-b^9 f^2 + b^5 c^4 d^2 + b^7 c^2 e^2 - b^6 f^2 (-4 a^2 c - b^2)^3)^{1/2} - 7 a^2 b^3 c^5 d^2 + 12 a^2 b^2 c^6 d^2 + a^2 c^5 d^2 (-4 a^2 c - b^2)^3)^{1/2} - 9 a^2 b^5 c^3 e^2 - 20 a^3 b^2 c^5 e^2 + 28 a^4 b^2 c^4 f^2 - 2 b^8 c^2 e f^2 + 25 a^2 b^3 c^4 e^2 - a^2 c^4 e^2 (-4 a^2 c - b^2)^3)^{1/2} - b^2 c^4 d^2 (-4 a^2 c - b^2)^3)^{1/2} + 42 a^2 b^5 c^2 f^2 - 63 a^3 b^3 c^3 f^2 + a^3 c^3 f^2 (-4 a^2 c - b^2)^3)^{1/2} - b^4 c^2 e^2 (-4 a^2 c - b^2)^3)^{1/2} - 11 a^2 b^7 c^2 f^2 + 16 a^3 c^6 d e - 2 b^6 c^3 d e - 16 a^4 c^5 e f^2 + 2 b^7 c^2 d f^2 + 16 a^2 b^4 c^4 d e - 18 a^2 b^5 c^3 d f^2 - 40 a^3 b^3 c^5 d f^2 + 20 a^2 b^6 c^2 e f^2 + 2 b^5 c^3 e f^2 (-4 a^2 c - b^2)^3)^{1/2} - 6 a^2 b^2 c^2 f^2 (-4 a^2 c - b^2)^3)^{1/2} + 5 a^2 b^4 c^2 f^2 (-4 a^2 c - b^2)^3)^{1/2} - 36 a^2 b^2 c^5 d e + 50 a^2 b^3 c^4 d f^2 - 2 a^2 c^4 d f^2 (-4 a^2 c - b^2)^3)^{1/2} + 2 b^3 c^3 d e (-4 a^2 c - b^2)^3)^{1/2} - 66 a^2 b^4 c^3 e f^2 + 76 a^3 b^2 c^4 e f^2 - 2 b^4 c^2 d f^2 (-4 a^2 c - b^2)^3)^{1/2} + 3 a^2 b^2 c^3 e^2 (-4 a^2 c - b^2)^3)^{1/2} - 4 a^2 b^2 c^4 d e (-4 a^2 c - b^2)^3)^{1/2} + 6 a^2 b^2 c^3 d f^2 (-4 a^2 c - b^2)^3)^{1/2} - 8 a^2 b^3 c^2 e f^2 (-4 a^2 c - b^2)^3)^{1/2} + 6 a^2 b^2 c^3 e f^2 (-4 a^2 c - b^2)^3)^{1/2} / (8 * (16 a^2 c^9 + b^4 c^7 - 8 a^2 b^2 c^8))^{1/2}) * 2i + (f*x^5)/(5*c)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)

[Out] Timed out

$$3.54 \quad \int \frac{x^2(d+ex^2+fx^4)}{a+bx^2+cx^4} dx$$

Optimal. Leaf size=282

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce}{\sqrt{b^2-4ac}} - acf + b^2f - bce + c^2d\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\frac{-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce}{\sqrt{b^2-4ac}} - acf + b^2f - bce + c^2d\right)}{\sqrt{2}c^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Rubi [A] time = 3.59, antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 30, number of rules / integrand size = 0.100, Rules used = {1664, 1166, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce}{\sqrt{b^2-4ac}} - acf + b^2f - bce + c^2d\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\frac{-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce}{\sqrt{b^2-4ac}} - acf + b^2f - bce + c^2d\right)}{\sqrt{2}c^{5/2}\sqrt{\sqrt{b^2-4ac}+b}} + \frac{x(ce-bf)}{c^2} + \frac{fx^3}{3c}$$

Antiderivative was successfully verified.

[In] Int[(x^2*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4), x]

[Out] ((c*e - b*f)*x)/c^2 + (f*x^3)/(3*c) + ((c^2*d - b*c*e + b^2*f - a*c*f + (b^2*c*e - 2*a*c^2*e - b^3*f - b*c*(c*d - 3*a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/(Sqrt[2]*c^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((c^2*d - b*c*e + b^2*f - a*c*f - (b^2*c*e - 2*a*c^2*e - b^3*f - b*c*(c*d - 3*a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]/(Sqrt[2]*c^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1664

Int[(Pq_)*((d_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned}
\int \frac{x^2 (d + ex^2 + fx^4)}{a + bx^2 + cx^4} dx &= \int \left(\frac{ce - bf}{c^2} + \frac{fx^2}{c} - \frac{a(ce - bf) - (c^2d - bce + b^2f - acf)x^2}{c^2(a + bx^2 + cx^4)} \right) dx \\
&= \frac{(ce - bf)x}{c^2} + \frac{fx^3}{3c} - \frac{\int \frac{a(ce - bf) + (-c^2d + bce - b^2f + acf)x^2}{a + bx^2 + cx^4} dx}{c^2} \\
&= \frac{(ce - bf)x}{c^2} + \frac{fx^3}{3c} + \frac{\left(c^2d - bce + b^2f - acf - \frac{b^2ce - 2ac^2e - b^3f - bc(cd - 3af)}{\sqrt{b^2 - 4ac}} \right) \int \frac{1}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2 - 4ac} + cx}}{2c^2} \\
&= \frac{(ce - bf)x}{c^2} + \frac{fx^3}{3c} + \frac{\left(c^2d - bce + b^2f - acf + \frac{b^2ce - 2ac^2e - b^3f - bc(cd - 3af)}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{\sqrt{2}c^{5/2}\sqrt{b - \sqrt{b^2 - 4ac}}}
\end{aligned}$$

Mathematica [A] time = 0.49, size = 365, normalized size = 1.29

$$\frac{3\sqrt{2} \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right) \left(-bc(e\sqrt{b^2 - 4ac} - 3af + cd) + (cd\sqrt{b^2 - 4ac} - af\sqrt{b^2 - 4ac} - 2ace) + b^2(f\sqrt{b^2 - 4ac} + ce) + b^3(-f) \right)}{\sqrt{b^2 - 4ac}\sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{3\sqrt{2} \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{b^2 - 4ac} + b} \right) \left(bc(-e\sqrt{b^2 - 4ac} - 3af + cd) + (cd\sqrt{b^2 - 4ac} - af\sqrt{b^2 - 4ac} + 2ace) + b^2(f\sqrt{b^2 - 4ac} - ce) + b^3f \right)}{\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac} + b} + 6\sqrt{c}x(ce - bf) + 2c^{3/2}fx^3}{6c^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]

[Out] (6*sqrt[c]*(c*e - b*f)*x + 2*c^(3/2)*f*x^3 + (3*sqrt[2]*(-(b^3*f) - b*c*(c*d + sqrt[b^2 - 4*a*c]*e - 3*a*f) + b^2*(c*e + sqrt[b^2 - 4*a*c]*f) + c*(c*sqrt[b^2 - 4*a*c]*d - 2*a*c*e - a*sqrt[b^2 - 4*a*c]*f))*ArcTan[(sqrt[2]*sqrt[c]*x)/sqrt[b - sqrt[b^2 - 4*a*c]])/(sqrt[b^2 - 4*a*c]*sqrt[b - sqrt[b^2 - 4*a*c]]) + (3*sqrt[2]*(b^3*f + b*c*(c*d - sqrt[b^2 - 4*a*c]*e - 3*a*f) + b^2*(-(c*e) + sqrt[b^2 - 4*a*c]*f) + c*(c*sqrt[b^2 - 4*a*c]*d + 2*a*c*e - a*sqrt[b^2 - 4*a*c]*f))*ArcTan[(sqrt[2]*sqrt[c]*x)/sqrt[b + sqrt[b^2 - 4*a*c]])/(sqrt[b^2 - 4*a*c]*sqrt[b + sqrt[b^2 - 4*a*c]]))/(6*c^(5/2))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 (d + ex^2 + fx^4)}{a + bx^2 + cx^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^2*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]

[Out] IntegrateAlgebraic[(x^2*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4), x]

fricas [B] time = 7.85, size = 9364, normalized size = 33.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] 1/6*(2*c*f*x^3 + 3*sqrt(1/2)*c^2*sqrt(-(b*c^4*d^2 - 2*(b^2*c^3 - 2*a*c^4)*d*e + (b^3*c^2 - 3*a*b*c^3)*e^2 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*f^2 + 2*((b^3*c^2 - 3*a*b*c^3)*d - (b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*e)*f + (b^2*c^5 - 4*a*c^6)*sqrt((c^8*d^4 - 4*b*c^7*d^3*e + 2*(3*b^2*c^6 - a*c^7)*d^2*e^2 - 4*(b^3*c^5 - a*b*c^6)*d*e^3 + (b^4*c^4 - 2*a*b^2*c^5 + a^2*c^6)*e^4 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*f^4 + 4*((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4 - a^3*c^5)*d - (b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*e)*f^3 + 2*((3*b^4*c^4 - 7*a*b^2*c^5 + 3*a^2*c^6)*d^2

$$\begin{aligned}
& - 2*(3*b^5*c^3 - 9*a*b^3*c^4 + 5*a^2*b*c^5)*d*e + (3*b^6*c^2 - 12*a*b^4*c^3 \\
& + 12*a^2*b^2*c^4 - a^3*c^5)*e^2)*f^2 + 4*((b^2*c^6 - a*c^7)*d^3 - (3*b^3*c^5 - 4*a*b*c^6)*d^2*e + (3*b^4*c^4 - 6*a*b^2*c^5 + a^2*c^6)*d*e^2 - (b^5*c^3 - 3*a*b^3*c^4 + 2*a^2*b*c^5)*e^3)*f)/(b^2*c^10 - 4*a*c^11))/((b^2*c^5 - 4*a*c^6))*\log(2*(c^6*d^4 - 3*b*c^5*d^3*e + 3*b^2*c^4*d^2*e^2 - (b^3*c^3 + a*b*c^4)*d*e^3 + (a*b^2*c^3 - a^2*c^4)*e^4 + (a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*f^4 + ((b^6 - 5*a*b^4*c + 9*a^2*b^2*c^2 - 4*a^3*c^3)*d - (a*b^5 - a^2*b^3*c - 3*a^3*b*c^2)*e)*f^3 + 3*((b^4*c^2 - 3*a*b^2*c^3 + 2*a^2*c^4)*d^2 - (b^5*c - 3*a*b^3*c^2 + 3*a^2*b*c^3)*d*e + (a*b^4*c - 2*a^2*b^2*c^2)*e^2)*f^2 + ((3*b^2*c^4 - 4*a*c^5)*d^3 - 3*(2*b^3*c^3 - 3*a*b*c^4)*d^2*e + 3*(b^4*c^2 - a*b^2*c^3)*d*e^2 - (3*a*b^3*c^2 - 5*a^2*b*c^3)*e^3)*f)*x + \sqrt{1/2}*((b^2*c^5 - 4*a*c^6)*d^2*e - 2*(b^3*c^4 - 4*a*b*c^5)*d*e^2 + (b^4*c^3 - 5*a*b^2*c^4 + 4*a^2*c^5)*e^3 - (b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3)*f^3 - (2*(b^5*c^2 - 5*a*b^3*c^3 + 4*a^2*b*c^4)*d - (3*b^6*c - 19*a*b^4*c^2 + 29*a^2*b^2*c^3 - 4*a^3*c^4)*e)*f^2 - ((b^3*c^4 - 4*a*b*c^5)*d^2 - 2*(2*b^4*c^3 - 9*a*b^2*c^4 + 4*a^2*c^5)*d*e + (3*b^5*c^2 - 17*a*b^3*c^3 + 20*a^2*b*c^4)*e^2)*f + (2*(b^2*c^7 - 4*a*c^8)*d - (b^3*c^6 - 4*a*b*c^7)*e + (b^4*c^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*f)*\sqrt{(c^8*d^4 - 4*b*c^7*d^3*e + 2*(3*b^2*c^6 - a*c^7)*d^2*e^2 - 4*(b^3*c^5 - a*b*c^6)*d*e^3 + (b^4*c^4 - 2*a*b^2*c^5 + a^2*c^6)*e^4 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*f^4 + 4*((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4 - a^3*c^5)*d - (b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*e)*f^3 + 2*((3*b^4*c^4 - 7*a*b^2*c^5 + 3*a^2*c^6)*d^2 - 2*(3*b^5*c^3 - 9*a*b^3*c^4 + 5*a^2*b*c^5)*d*e + (3*b^6*c^2 - 12*a*b^4*c^3 + 12*a^2*b^2*c^4 - a^3*c^5)*e^2)*f^2 + 4*((b^2*c^6 - a*c^7)*d^3 - (3*b^3*c^5 - 4*a*b*c^6)*d^2*e + (3*b^4*c^4 - 6*a*b^2*c^5 + a^2*c^6)*d*e^2 - (b^5*c^3 - 3*a*b^3*c^4 + 2*a^2*b*c^5)*e^3)*f)/(b^2*c^10 - 4*a*c^11))*\sqrt{-(b*c^4*d^2 - 2*(b^2*c^3 - 2*a*c^4)*d*e + (b^3*c^2 - 3*a*b*c^3)*e^2 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*f^2 + 2*((b^3*c^2 - 3*a*b*c^3)*d - (b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*e)*f + (b^2*c^5 - 4*a*c^6)*\sqrt{(c^8*d^4 - 4*b*c^7*d^3*e + 2*(3*b^2*c^6 - a*c^7)*d^2*e^2 - 4*(b^3*c^5 - a*b*c^6)*d*e^3 + (b^4*c^4 - 2*a*b^2*c^5 + a^2*c^6)*e^4 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*f^4 + 4*((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4 - a^3*c^5)*d - (b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*e)*f^3 + 2*((3*b^4*c^4 - 7*a*b^2*c^5 + 3*a^2*c^6)*d^2 - 2*(3*b^5*c^3 - 9*a*b^3*c^4 + 5*a^2*b*c^5)*d*e + (3*b^6*c^2 - 12*a*b^4*c^3 + 12*a^2*b^2*c^4 - a^3*c^5)*e^2)*f^2 + 4*((b^2*c^6 - a*c^7)*d^3 - (3*b^3*c^5 - 4*a*b*c^6)*d^2*e + (3*b^4*c^4 - 6*a*b^2*c^5 + a^2*c^6)*d*e^2 - (b^5*c^3 - 3*a*b^3*c^4 + 2*a^2*b*c^5)*e^3)*f)/(b^2*c^10 - 4*a*c^11)))/((b^2*c^5 - 4*a*c^6)) - 3*\sqrt{1/2}*c^2*\sqrt{-(b*c^4*d^2 - 2*(b^2*c^3 - 2*a*c^4)*d*e + (b^3*c^2 - 3*a*b*c^3)*e^2 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*f^2 + 2*((b^3*c^2 - 3*a*b*c^3)*d - (b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*e)*f + (b^2*c^5 - 4*a*c^6)*\sqrt{(c^8*d^4 - 4*b*c^7*d^3*e + 2*(3*b^2*c^6 - a*c^7)*d^2*e^2 - 4*(b^3*c^5 - a*b*c^6)*d*e^3 + (b^4*c^4 - 2*a*b^2*c^5 + a^2*c^6)*e^4 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*f^4 + 4*((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4 - a^3*c^5)*d - (b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*e)*f^3 + 2*((3*b^4*c^4 - 7*a*b^2*c^5 + 3*a^2*c^6)*d^2 - 2*(3*b^5*c^3 - 9*a*b^3*c^4 + 5*a^2*b*c^5)*d*e + (3*b^6*c^2 - 12*a*b^4*c^3 + 12*a^2*b^2*c^4 - a^3*c^5)*e^2)*f^2 + 4*((b^2*c^6 - a*c^7)*d^3 - (3*b^3*c^5 - 4*a*b*c^6)*d^2*e + (3*b^4*c^4 - 6*a*b^2*c^5 + a^2*c^6)*d*e^2 - (b^5*c^3 - 3*a*b^3*c^4 + 2*a^2*b*c^5)*e^3)*f)/(b^2*c^10 - 4*a*c^11)))/((b^2*c^5 - 4*a*c^6))*\log(2*(c^6*d^4 - 3*b*c^5*d^3*e + 3*b^2*c^4*d^2*e^2 - (b^3*c^3 + a*b*c^4)*d*e^3 + (a*b^2*c^3 - a^2*c^4)*e^4 + (a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*f^4 + ((b^6 - 5*a*b^4*c + 9*a^2*b^2*c^2 - 4*a^3*c^3)*d - (a*b^5 - a^2*b^3*c - 3*a^3*b*c^2)*e)*f^3 + 3*((b^4*c^2 - 3*a*b^2*c^3 + 2*a^2*c^4)*d^2 - (b^5*c - 3*a*b^3*c^2 + 3*a^2*b*c^3)*d*e + (a*b^4*c - 2*a^2*b^2*c^2)*e^2)*f^2 + ((3*b^2*c^4 - 4*a*c^5)*d^3 - 3*(2*b^3*c^3 - 3*a*b*c^4)*d^2*e + 3*(b^4*c^2 - a*b^2*c^3)*d*e^2 - (3*a*b^3*c^2 - 5*a^2*b*c^3)*e^3)*f)*x - \sqrt{1/2}*((b^2*c^5 - 4*a*c^6)*d^2*e - 2*(b^3*c^4 - 4*a*b*c^5)*d*e^2 + (b^4*c^3 - 5*a*b^2*c^4 + 4*a^2*c^5)*e^3 - (b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3)*f^3 - (2*(b^5*c^2 - 5*a*b^3*c^3 +
\end{aligned}$$

$$\begin{aligned}
& 4*a^2*b*c^4)*d - (3*b^6*c - 19*a*b^4*c^2 + 29*a^2*b^2*c^3 - 4*a^3*c^4)*e)* \\
& f^2 - ((b^3*c^4 - 4*a*b*c^5)*d^2 - 2*(2*b^4*c^3 - 9*a*b^2*c^4 + 4*a^2*c^5)* \\
& d*e + (3*b^5*c^2 - 17*a*b^3*c^3 + 20*a^2*b*c^4)*e^2)*f + (2*(b^2*c^7 - 4*a* \\
& c^8)*d - (b^3*c^6 - 4*a*b*c^7)*e + (b^4*c^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*f)*s \\
& qrt((c^8*d^4 - 4*b*c^7*d^3*e + 2*(3*b^2*c^6 - a*c^7)*d^2*e^2 - 4*(b^3*c^5 - \\
& a*b*c^6)*d*e^3 + (b^4*c^4 - 2*a*b^2*c^5 + a^2*c^6)*e^4 + (b^8 - 6*a*b^6*c \\
& + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*f^4 + 4*((b^6*c^2 - 4*a*b^4*c^3 \\
& + 4*a^2*b^2*c^4 - a^3*c^5)*d - (b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^ \\
& 3*b*c^4)*e)*f^3 + 2*((3*b^4*c^4 - 7*a*b^2*c^5 + 3*a^2*c^6)*d^2 - 2*(3*b^5*c \\
& ^3 - 9*a*b^3*c^4 + 5*a^2*b*c^5)*d*e + (3*b^6*c^2 - 12*a*b^4*c^3 + 12*a^2*b^ \\
& 2*c^4 - a^3*c^5)*e^2)*f^2 + 4*((b^2*c^6 - a*c^7)*d^3 - (3*b^3*c^5 - 4*a*b*c \\
& ^6)*d^2*e + (3*b^4*c^4 - 6*a*b^2*c^5 + a^2*c^6)*d*e^2 - (b^5*c^3 - 3*a*b^3* \\
& c^4 + 2*a^2*b*c^5)*e^3)*f)/(b^2*c^10 - 4*a*c^11))*sqrt(-(b*c^4*d^2 - 2*(b^ \\
& 2*c^3 - 2*a*c^4)*d*e + (b^3*c^2 - 3*a*b*c^3)*e^2 + (b^5 - 5*a*b^3*c + 5*a^2 \\
& *b*c^2)*f^2 + 2*((b^3*c^2 - 3*a*b*c^3)*d - (b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3) \\
&)*e)*f + (b^2*c^5 - 4*a*c^6)*sqrt((c^8*d^4 - 4*b*c^7*d^3*e + 2*(3*b^2*c^6 - \\
& a*c^7)*d^2*e^2 - 4*(b^3*c^5 - a*b*c^6)*d*e^3 + (b^4*c^4 - 2*a*b^2*c^5 + a^ \\
& 2*c^6)*e^4 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*f \\
& ^4 + 4*((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4 - a^3*c^5)*d - (b^7*c - 5*a* \\
& b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*e)*f^3 + 2*((3*b^4*c^4 - 7*a*b^2*c^5 \\
& + 3*a^2*c^6)*d^2 - 2*(3*b^5*c^3 - 9*a*b^3*c^4 + 5*a^2*b*c^5)*d*e + (3*b^6*c \\
& ^2 - 12*a*b^4*c^3 + 12*a^2*b^2*c^4 - a^3*c^5)*e^2)*f^2 + 4*((b^2*c^6 - a*c \\
& ^7)*d^3 - (3*b^3*c^5 - 4*a*b*c^6)*d^2*e + (3*b^4*c^4 - 6*a*b^2*c^5 + a^2*c^ \\
& 6)*d*e^2 - (b^5*c^3 - 3*a*b^3*c^4 + 2*a^2*b*c^5)*e^3)*f)/(b^2*c^10 - 4*a*c^ \\
& 11)))/(b^2*c^5 - 4*a*c^6))) + 3*sqrt(1/2)*c^2*sqrt(-(b*c^4*d^2 - 2*(b^2*c^3 \\
& - 2*a*c^4)*d*e + (b^3*c^2 - 3*a*b*c^3)*e^2 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^ \\
& 2)*f^2 + 2*((b^3*c^2 - 3*a*b*c^3)*d - (b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*e)* \\
& f - (b^2*c^5 - 4*a*c^6)*sqrt((c^8*d^4 - 4*b*c^7*d^3*e + 2*(3*b^2*c^6 - a*c^ \\
& 7)*d^2*e^2 - 4*(b^3*c^5 - a*b*c^6)*d*e^3 + (b^4*c^4 - 2*a*b^2*c^5 + a^2*c^6) \\
&)*e^4 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*f^4 + \\
& 4*((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4 - a^3*c^5)*d - (b^7*c - 5*a*b^5*c \\
& ^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*e)*f^3 + 2*((3*b^4*c^4 - 7*a*b^2*c^5 + 3* \\
& a^2*c^6)*d^2 - 2*(3*b^5*c^3 - 9*a*b^3*c^4 + 5*a^2*b*c^5)*d*e + (3*b^6*c^2 - \\
& 12*a*b^4*c^3 + 12*a^2*b^2*c^4 - a^3*c^5)*e^2)*f^2 + 4*((b^2*c^6 - a*c^7)*d \\
& ^3 - (3*b^3*c^5 - 4*a*b*c^6)*d^2*e + (3*b^4*c^4 - 6*a*b^2*c^5 + a^2*c^6)*d* \\
& e^2 - (b^5*c^3 - 3*a*b^3*c^4 + 2*a^2*b*c^5)*e^3)*f)/(b^2*c^10 - 4*a*c^11)) \\
& /((b^2*c^5 - 4*a*c^6))*log(2*(c^6*d^4 - 3*b*c^5*d^3*e + 3*b^2*c^4*d^2*e^2 - \\
& (b^3*c^3 + a*b*c^4)*d*e^3 + (a*b^2*c^3 - a^2*c^4)*e^4 + (a^2*b^4 - 3*a^3*b^ \\
& 2*c + a^4*c^2)*f^4 + ((b^6 - 5*a*b^4*c + 9*a^2*b^2*c^2 - 4*a^3*c^3)*d - (a* \\
& b^5 - a^2*b^3*c - 3*a^3*b*c^2)*e)*f^3 + 3*((b^4*c^2 - 3*a*b^2*c^3 + 2*a^2*c^ \\
& 4)*d^2 - (b^5*c - 3*a*b^3*c^2 + 3*a^2*b*c^3)*d*e + (a*b^4*c - 2*a^2*b^2*c^ \\
& 2)*e^2)*f^2 + ((3*b^2*c^4 - 4*a*c^5)*d^3 - 3*(2*b^3*c^3 - 3*a*b*c^4)*d^2*e \\
& + 3*(b^4*c^2 - a*b^2*c^3)*d*e^2 - (3*a*b^3*c^2 - 5*a^2*b*c^3)*e^3)*f)*x + s \\
& qrt(1/2)*((b^2*c^5 - 4*a*c^6)*d^2*e - 2*(b^3*c^4 - 4*a*b*c^5)*d*e^2 + (b^4* \\
& c^3 - 5*a*b^2*c^4 + 4*a^2*c^5)*e^3 - (b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4* \\
& a^3*b*c^3)*f^3 - (2*(b^5*c^2 - 5*a*b^3*c^3 + 4*a^2*b*c^4)*d - (3*b^6*c - 19 \\
& *a*b^4*c^2 + 29*a^2*b^2*c^3 - 4*a^3*c^4)*e)*f^2 - ((b^3*c^4 - 4*a*b*c^5)*d^ \\
& 2 - 2*(2*b^4*c^3 - 9*a*b^2*c^4 + 4*a^2*c^5)*d*e + (3*b^5*c^2 - 17*a*b^3*c^3 \\
& + 20*a^2*b*c^4)*e^2)*f - (2*(b^2*c^7 - 4*a*c^8)*d - (b^3*c^6 - 4*a*b*c^7)* \\
& e + (b^4*c^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*f)*sqrt((c^8*d^4 - 4*b*c^7*d^3*e + \\
& 2*(3*b^2*c^6 - a*c^7)*d^2*e^2 - 4*(b^3*c^5 - a*b*c^6)*d*e^3 + (b^4*c^4 - 2* \\
& a*b^2*c^5 + a^2*c^6)*e^4 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^ \\
& 3 + a^4*c^4)*f^4 + 4*((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4 - a^3*c^5)*d - \\
& (b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*e)*f^3 + 2*((3*b^4*c^4 \\
& - 7*a*b^2*c^5 + 3*a^2*c^6)*d^2 - 2*(3*b^5*c^3 - 9*a*b^3*c^4 + 5*a^2*b*c^5) \\
& *d*e + (3*b^6*c^2 - 12*a*b^4*c^3 + 12*a^2*b^2*c^4 - a^3*c^5)*e^2)*f^2 + 4*(\\
& (b^2*c^6 - a*c^7)*d^3 - (3*b^3*c^5 - 4*a*b*c^6)*d^2*e + (3*b^4*c^4 - 6*a*b^ \\
& 2*c^5 + a^2*c^6)*d*e^2 - (b^5*c^3 - 3*a*b^3*c^4 + 2*a^2*b*c^5)*e^3)*f)/(b^2 \\
& *c^10 - 4*a*c^11))*sqrt(-(b*c^4*d^2 - 2*(b^2*c^3 - 2*a*c^4)*d*e + (b^3*c^2
\end{aligned}$$

$$\begin{aligned}
& - 3*a*b*c^3)*e^2 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*f^2 + 2*((b^3*c^2 - 3*a \\
& *b*c^3)*d - (b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*e)*f - (b^2*c^5 - 4*a*c^6)*sq \\
& rt((c^8*d^4 - 4*b*c^7*d^3*e + 2*(3*b^2*c^6 - a*c^7)*d^2*e^2 - 4*(b^3*c^5 - \\
& a*b*c^6)*d*e^3 + (b^4*c^4 - 2*a*b^2*c^5 + a^2*c^6)*e^4 + (b^8 - 6*a*b^6*c + \\
& 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*f^4 + 4*((b^6*c^2 - 4*a*b^4*c^3 + 4*a \\
& ^2*b^2*c^4 - a^3*c^5)*d - (b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3 \\
& *b*c^4)*e)*f^3 + 2*((3*b^4*c^4 - 7*a*b^2*c^5 + 3*a^2*c^6)*d^2 - 2*(3*b^5*c^ \\
& 3 - 9*a*b^3*c^4 + 5*a^2*b*c^5)*d*e + (3*b^6*c^2 - 12*a*b^4*c^3 + 12*a^2*b^2 \\
& *c^4 - a^3*c^5)*e^2)*f^2 + 4*((b^2*c^6 - a*c^7)*d^3 - (3*b^3*c^5 - 4*a*b*c^ \\
& 6)*d^2*e + (3*b^4*c^4 - 6*a*b^2*c^5 + a^2*c^6)*d*e^2 - (b^5*c^3 - 3*a*b^3*c \\
& ^4 + 2*a^2*b*c^5)*e^3)*f)/(b^2*c^10 - 4*a*c^11))/((b^2*c^5 - 4*a*c^6))) - 3 \\
& *sqrt(1/2)*c^2*sqrt(-(b*c^4*d^2 - 2*(b^2*c^3 - 2*a*c^4)*d*e + (b^3*c^2 - 3* \\
& a*b*c^3)*e^2 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*f^2 + 2*((b^3*c^2 - 3*a*b*c^ \\
& 3)*d - (b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*e)*f - (b^2*c^5 - 4*a*c^6)*sqrt((c \\
& ^8*d^4 - 4*b*c^7*d^3*e + 2*(3*b^2*c^6 - a*c^7)*d^2*e^2 - 4*(b^3*c^5 - a*b*c \\
& ^6)*d*e^3 + (b^4*c^4 - 2*a*b^2*c^5 + a^2*c^6)*e^4 + (b^8 - 6*a*b^6*c + 11*a \\
& ^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*f^4 + 4*((b^6*c^2 - 4*a*b^4*c^3 + 4*a \\
& ^2*b^2*c^4 - a^3*c^5)*d - (b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^ \\
& 4)*e)*f^3 + 2*((3*b^4*c^4 - 7*a*b^2*c^5 + 3*a^2*c^6)*d^2 - 2*(3*b^5*c^3 - 9 \\
& *a*b^3*c^4 + 5*a^2*b*c^5)*d*e + (3*b^6*c^2 - 12*a*b^4*c^3 + 12*a^2*b^2*c^4 \\
& - a^3*c^5)*e^2)*f^2 + 4*((b^2*c^6 - a*c^7)*d^3 - (3*b^3*c^5 - 4*a*b*c^6)*d^ \\
& 2*e + (3*b^4*c^4 - 6*a*b^2*c^5 + a^2*c^6)*d*e^2 - (b^5*c^3 - 3*a*b^3*c^4 + \\
& 2*a^2*b*c^5)*e^3)*f)/(b^2*c^10 - 4*a*c^11))/((b^2*c^5 - 4*a*c^6))*log(2*(c^ \\
& 6*d^4 - 3*b*c^5*d^3*e + 3*b^2*c^4*d^2*e^2 - (b^3*c^3 + a*b*c^4)*d*e^3 + (a* \\
& b^2*c^3 - a^2*c^4)*e^4 + (a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*f^4 + ((b^6 - 5* \\
& a*b^4*c + 9*a^2*b^2*c^2 - 4*a^3*c^3)*d - (a*b^5 - a^2*b^3*c - 3*a^3*b*c^2)* \\
& e)*f^3 + 3*((b^4*c^2 - 3*a*b^2*c^3 + 2*a^2*c^4)*d^2 - (b^5*c - 3*a*b^3*c^2 \\
& + 3*a^2*b*c^3)*d*e + (a*b^4*c - 2*a^2*b^2*c^2)*e^2)*f^2 + ((3*b^2*c^4 - 4*a \\
& *c^5)*d^3 - 3*(2*b^3*c^3 - 3*a*b*c^4)*d^2*e + 3*(b^4*c^2 - a*b^2*c^3)*d*e^2 \\
& - (3*a*b^3*c^2 - 5*a^2*b*c^3)*e^3)*f)*x - sqrt(1/2)*((b^2*c^5 - 4*a*c^6)*d \\
& ^2*e - 2*(b^3*c^4 - 4*a*b*c^5)*d*e^2 + (b^4*c^3 - 5*a*b^2*c^4 + 4*a^2*c^5)* \\
& e^3 - (b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3)*f^3 - (2*(b^5*c^2 - \\
& 5*a*b^3*c^3 + 4*a^2*b*c^4)*d - (3*b^6*c - 19*a*b^4*c^2 + 29*a^2*b^2*c^3 - 4 \\
& *a^3*c^4)*e)*f^2 - ((b^3*c^4 - 4*a*b*c^5)*d^2 - 2*(2*b^4*c^3 - 9*a*b^2*c^4 \\
& + 4*a^2*c^5)*d*e + (3*b^5*c^2 - 17*a*b^3*c^3 + 20*a^2*b*c^4)*e^2)*f - (2*(b \\
& ^2*c^7 - 4*a*c^8)*d - (b^3*c^6 - 4*a*b*c^7)*e + (b^4*c^5 - 6*a*b^2*c^6 + 8* \\
& a^2*c^7)*f)*sqrt((c^8*d^4 - 4*b*c^7*d^3*e + 2*(3*b^2*c^6 - a*c^7)*d^2*e^2 - \\
& 4*(b^3*c^5 - a*b*c^6)*d*e^3 + (b^4*c^4 - 2*a*b^2*c^5 + a^2*c^6)*e^4 + (b^8 \\
& - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*f^4 + 4*((b^6*c^2 \\
& - 4*a*b^4*c^3 + 4*a^2*b^2*c^4 - a^3*c^5)*d - (b^7*c - 5*a*b^5*c^2 + 7*a^2*b \\
& ^3*c^3 - 2*a^3*b*c^4)*e)*f^3 + 2*((3*b^4*c^4 - 7*a*b^2*c^5 + 3*a^2*c^6)*d^2 \\
& - 2*(3*b^5*c^3 - 9*a*b^3*c^4 + 5*a^2*b*c^5)*d*e + (3*b^6*c^2 - 12*a*b^4*c^ \\
& 3 + 12*a^2*b^2*c^4 - a^3*c^5)*e^2)*f^2 + 4*((b^2*c^6 - a*c^7)*d^3 - (3*b^3* \\
& c^5 - 4*a*b*c^6)*d^2*e + (3*b^4*c^4 - 6*a*b^2*c^5 + a^2*c^6)*d*e^2 - (b^5*c \\
& ^3 - 3*a*b^3*c^4 + 2*a^2*b*c^5)*e^3)*f)/(b^2*c^10 - 4*a*c^11))*sqrt(-(b*c^ \\
& 4*d^2 - 2*(b^2*c^3 - 2*a*c^4)*d*e + (b^3*c^2 - 3*a*b*c^3)*e^2 + (b^5 - 5*a* \\
& b^3*c + 5*a^2*b*c^2)*f^2 + 2*((b^3*c^2 - 3*a*b*c^3)*d - (b^4*c - 4*a*b^2*c^ \\
& 2 + 2*a^2*c^3)*e)*f - (b^2*c^5 - 4*a*c^6)*sqrt((c^8*d^4 - 4*b*c^7*d^3*e + 2 \\
& *(3*b^2*c^6 - a*c^7)*d^2*e^2 - 4*(b^3*c^5 - a*b*c^6)*d*e^3 + (b^4*c^4 - 2*a \\
& *b^2*c^5 + a^2*c^6)*e^4 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 \\
& + a^4*c^4)*f^4 + 4*((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4 - a^3*c^5)*d - \\
& (b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*e)*f^3 + 2*((3*b^4*c^4 \\
& - 7*a*b^2*c^5 + 3*a^2*c^6)*d^2 - 2*(3*b^5*c^3 - 9*a*b^3*c^4 + 5*a^2*b*c^5)* \\
& d*e + (3*b^6*c^2 - 12*a*b^4*c^3 + 12*a^2*b^2*c^4 - a^3*c^5)*e^2)*f^2 + 4*((\\
& b^2*c^6 - a*c^7)*d^3 - (3*b^3*c^5 - 4*a*b*c^6)*d^2*e + (3*b^4*c^4 - 6*a*b^2 \\
& *c^5 + a^2*c^6)*d*e^2 - (b^5*c^3 - 3*a*b^3*c^4 + 2*a^2*b*c^5)*e^3)*f)/(b^2* \\
& c^10 - 4*a*c^11))/((b^2*c^5 - 4*a*c^6))) + 6*(c*e - b*f)*x)/c^2
\end{aligned}$$

giac [B] time = 4.75, size = 5461, normalized size = 19.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/8*((2*b^4*c^4 - 16*a*b^2*c^5 + 32*a^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c))*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^5 - 2*(b^2 - 4*a*c)*b^2*c^4 + 8*(b^2 - 4*a*c)*a*c^5)*c^2*d + (2*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2*b^2*c^4 - 32*a^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6 + 9*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2 + 10*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b^2 - 4*a*c)*a^2*c^4)*c^2*f - (2*b^5*c^3 - 16*a*b^3*c^4 + 32*a^2*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^3 + 8*(b^2 - 4*a*c)*a*b*c^4)*c^2*e - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 2*a*b^5*c^3 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - 16*a^2*b^3*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + 32*a^3*b*c^5 - 2*(b^2 - 4*a*c)*a*b^3*c^3 + 8*(b^2 - 4*a*c)*a^2*b*c^4)*f*abs(c) + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 + 2*a*b^4*c^4 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^5 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 - 16*a^2*b^2*c^5 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^6 + 32*a^3*c^6 - 2*(b^2 - 4*a*c)*a*b^2*c^4 + 8*(b^2 - 4*a*c)*a^2*c^5)*abs(c)*e - (2*b^4*c^6 - 8*a*b^2*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^6 - 2*(b^2 - 4*a*c)*b^2*c^6)*d - (2*b^6*c^4 - 14*a*b^4*c^5 + 24*a^2*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^2 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^3 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^4 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 - 2*(b^2 - 4*a*c)*b^4*c^4 + 6*(b^2 - 4*a*c)*a*b^2*c^5)*f + (2*b^5*c^5 - 12*a*b^3*c^6 + 16*a^2*b*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c
```

$$\begin{aligned}
& ^3 + 6*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a*b^3*c^4 \\
& + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*b^4*c^4 - 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^2*b*c^5 - 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a*b^2*c^5 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*b^3*c^5 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a*b*c^6 - 2*(b^2 - 4*a*c)*b^3*c^5 + 4*(b^2 - 4*a*c)*a*b*c^6)*e)*\arctan(2*\sqrt{1/2}*x/\sqrt{(b*c^3 + \sqrt{b^2*c^6 - 4*a*c^7})/c^4))/((a*b^4*c^4 - 8*a^2*b^2*c^5 - 2*a*b^3*c^5 + 16*a^3*c^6 + 8*a^2*b*c^6 + a*b^2*c^6 - 4*a^2*c^7)*c^2) - 1/8*((2*b^4*c^4 - 16*a*b^2*c^5 + 32*a^2*c^6 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*b^4*c^2 + 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^2*c^3 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*b^3*c^3 - 16*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*c^4 - 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b*c^4 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*b^2*c^4 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*c^5 - 2*(b^2 - 4*a*c)*b^2*c^4 + 8*(b^2 - 4*a*c)*a*c^5)*c^2*d + (2*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2*b^2*c^4 - 32*a^3*c^5 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*b^6 + 9*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^4*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*b^5*c - 24*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*b^2*c^2 - 10*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^3*c^2 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*b^4*c^2 + 16*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^3*c^3 + 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*b*c^3 + 5*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^2*c^3 - 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2 + 10*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b^2 - 4*a*c)*a^2*c^4)*c^2*f - (2*b^5*c^3 - 16*a*b^3*c^4 + 32*a^2*b*c^5 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*b^5*c + 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^3*c^2 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*c)*b^4*c^2 - 16*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*b*c^3 - 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^2*c^3 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*b^3*c^3 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^3 + 8*(b^2 - 4*a*c)*a*b*c^4)*c^2*e - 2*(\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^5*c^2 - 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*b^3*c^3 - 2*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^4*c^3 - 2*a*b^5*c^3 + 16*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^3*b*c^4 + 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*b^2*c^4 + \sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^3*c^4 + 16*a^2*b^3*c^4 - 4*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*b*c^5 - 32*a^3*b*c^5 + 2*(b^2 - 4*a*c)*a*b^3*c^3 - 8*(b^2 - 4*a*c)*a^2*b*c^4)*f*\abs(c) + 2*(\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^4*c^3 - 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*b^2*c^4 - 2*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^3*c^4 - 2*a*b^4*c^4 + 16*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^3*c^5 + 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*b*c^5 + \sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^2*c^5 + 16*a^2*b^2*c^5 - 4*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*c^6 - 32*a^3*c^6 + 2*(b^2 - 4*a*c)*a*b^2*c^4 - 8*(b^2 - 4*a*c)*a^2*c^5)*\abs(c)*e - (2*b^4*c^6 - 8*a*b^2*c^7 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*b^4*c^4 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^2*c^5 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*b^3*c^5 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*b^2*c^6 - 2*(b^2 - 4*a*c)*b^2*c^6)*d - (2*b^6*c^4 - 14*a*b^4*c^5 + 24*a^2*b^2*c^6 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*b^6*c^2 + 7*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^4*c^3 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*b^5*c^3 - 12*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*b^2*c^4 - 6*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^3*c^4 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*c^5 - 2*(b^2 - 4*a*c)*a^2*c^5)*e
\end{aligned}$$

$$t(b*c + \sqrt{b^2 - 4*a*c})*c*b^4*c^4 + 3*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c*a*b^2*c^5 - 2*(b^2 - 4*a*c)*b^4*c^4 + 6*(b^2 - 4*a*c)*a*b^2*c^5)*f + (2*b^5*c^5 - 12*a*b^3*c^6 + 16*a^2*b*c^7 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*b^5*c^3 + 6*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*a*b^3*c^4 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*b^4*c^4 - 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*a^2*b*c^5 - 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*a*b^2*c^5 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*b^3*c^5 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*a*b*c^6 - 2*(b^2 - 4*a*c)*b^3*c^5 + 4*(b^2 - 4*a*c)*a*b*c^6)*e)*\arctan(2*\sqrt{1/2}*x/\sqrt{(b*c^3 - \sqrt{b^2*c^6 - 4*a*c^7})/c^4}))/((a*b^4*c^4 - 8*a^2*b^2*c^5 - 2*a*b^3*c^5 + 16*a^3*c^6 + 8*a^2*b*c^6 + a*b^2*c^6 - 4*a^2*c^7)*c^2) + 1/3*(c^2*f*x^3 - 3*b*c*f*x + 3*c^2*x*e)/c^3$$

maple [B] time = 0.03, size = 1035, normalized size = 3.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^2*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a), x)$

[Out] $\frac{1}{3}f*x^3/c - 1/c^2*b*f*x + 1/c*e*x + 1/2/c^2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*a*f - 1/2/c^2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^2*f + 1/2/c^2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b*e - 1/2*d^2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x) - 3/2/c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*a*b*f + 1/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*a*e + 1/2/c^2/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^3*f - 1/2/c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^2*e + 1/2/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b*d - 1/2/c^2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*a*f + 1/2/c^2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^2*f - 1/2/c^2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b*e + 1/2*d^2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x) - 3/2/c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*a*b*f + 1/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*a*e + 1/2/c^2/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^3*f - 1/2/c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^2*e + 1/2/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b*d$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^2*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a), x, \text{algorithm}="maxima")$

[Out] $\frac{1}{3}*(c*f*x^3 + 3*(c*e - b*f)*x)/c^2 - \text{integrate}((a*c*e - a*b*f - (c^2*d - b*c*e + (b^2 - a*c)*f)*x^2)/(c*x^4 + b*x^2 + a), x)/c^2$

mupad [B] time = 3.36, size = 15674, normalized size = 55.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x)

[Out] $x*(e/c - (b*f)/c^2) - \operatorname{atan}\left(\frac{(16a^2c^5e - 4ab^2c^4e + 4ab^3c^3f - 16a^2b^2c^4f)/c^3 - (2x(4b^3c^5 - 16ab^2c^6))*(-(b^7f^2 + b^3c^4d^2 - c^4d^2*(-4ac - b^2)^3)^{1/2} + b^5c^2e^2 - b^4f^2*(-4ac - b^2)^3)^{1/2} - 7ab^3c^3e^2 + 12a^2b^2c^4e^2 + ac^3e^2*(-4ac - b^2)^3)^{1/2} - 20a^3b^2c^3f^2 - 2b^6c^2ef + 25a^2b^3c^2f^2 - a^2c^2f^2*(-4ac - b^2)^3)^{1/2} - b^2c^2e^2*(-4ac - b^2)^3)^{1/2} - 4ab^2c^5d^2 - 9ab^5c^2f^2 - 16a^2c^5de - 2b^4c^3de + 16a^3c^4ef + 2b^5c^2df + 12ab^2c^4de - 14ab^3c^3df + 24a^2b^2c^4df + 2ac^3df*(-4ac - b^2)^3)^{1/2} + 2b^2c^3de*(-4ac - b^2)^3)^{1/2} + 16ab^4c^2ef + 2b^3c^2ef*(-4ac - b^2)^3)^{1/2} + 3ab^2c^2f^2*(-4ac - b^2)^3)^{1/2} - 36a^2b^2c^3ef - 2b^2c^2d*(-4ac - b^2)^3)^{1/2} - 4ab^2c^2ef*(-4ac - b^2)^3)^{1/2}}{(8(16a^2c^7 + b^4c^5 - 8ab^2c^6))^{1/2}}/c^3)*(-(b^7f^2 + b^3c^4d^2 - c^4d^2*(-4ac - b^2)^3)^{1/2} + b^5c^2e^2 - b^4f^2*(-4ac - b^2)^3)^{1/2} - 7ab^3c^3e^2 + 12a^2b^2c^4e^2 + ac^3e^2*(-4ac - b^2)^3)^{1/2} - 20a^3b^2c^3f^2 - 2b^6c^2ef + 25a^2b^3c^2f^2 - a^2c^2f^2*(-4ac - b^2)^3)^{1/2} - b^2c^2e^2*(-4ac - b^2)^3)^{1/2} - 4ab^2c^5d^2 - 9ab^5c^2f^2 - 16a^2c^5de - 2b^4c^3de + 16a^3c^4ef + 2b^5c^2df + 12ab^2c^4de - 14ab^3c^3df + 24a^2b^2c^4df + 2ac^3df*(-4ac - b^2)^3)^{1/2} + 2b^2c^3de*(-4ac - b^2)^3)^{1/2} + 16ab^4c^2ef + 2b^3c^2ef*(-4ac - b^2)^3)^{1/2} + 3ab^2c^2f^2*(-4ac - b^2)^3)^{1/2} - 36a^2b^2c^3ef - 2b^2c^2d*(-4ac - b^2)^3)^{1/2} - 4ab^2c^2ef*(-4ac - b^2)^3)^{1/2}}{(8(16a^2c^7 + b^4c^5 - 8ab^2c^6))^{1/2}} - (2x(b^6f^2 - 2ac^5d^2 + 2a^2c^4e^2 + b^2c^4d^2 - 2a^3c^3f^2 + b^4c^2e^2 - 4ab^2c^3e^2 - 2b^5c^2ef + 9a^2b^2c^2f^2 - 6ab^4c^2f^2 + 4a^2c^4df - 2b^3c^3de + 2b^4c^2df - 8ab^2c^3df + 10ab^3c^2ef - 10a^2b^2c^3ef + 6ab^4c^2de))/c^3)*(-(b^7f^2 + b^3c^4d^2 - c^4d^2*(-4ac - b^2)^3)^{1/2} + b^5c^2e^2 - b^4f^2*(-4ac - b^2)^3)^{1/2} - 7ab^3c^3e^2 + 12a^2b^2c^4e^2 + ac^3e^2*(-4ac - b^2)^3)^{1/2} - 20a^3b^2c^3f^2 - 2b^6c^2ef + 25a^2b^3c^2f^2 - a^2c^2f^2*(-4ac - b^2)^3)^{1/2} - b^2c^2e^2*(-4ac - b^2)^3)^{1/2} - 4ab^2c^5d^2 - 9ab^5c^2f^2 - 16a^2c^5de - 2b^4c^3de + 16a^3c^4ef + 2b^5c^2df + 12ab^2c^4de - 14ab^3c^3df + 24a^2b^2c^4df + 2ac^3df*(-4ac - b^2)^3)^{1/2} + 2b^2c^3de*(-4ac - b^2)^3)^{1/2} + 16ab^4c^2ef + 2b^3c^2ef*(-4ac - b^2)^3)^{1/2} + 3ab^2c^2f^2*(-4ac - b^2)^3)^{1/2} - 36a^2b^2c^3ef - 2b^2c^2d*(-4ac - b^2)^3)^{1/2} - 4ab^2c^2ef*(-4ac - b^2)^3)^{1/2}}{(8(16a^2c^7 + b^4c^5 - 8ab^2c^6))^{1/2}}*i - ((16a^2c^5e - 4ab^2c^4e + 4ab^3c^3f - 16a^2b^2c^4f)/c^3 + (2x(4b^3c^5 - 16ab^2c^6))*(-(b^7f^2 + b^3c^4d^2 - c^4d^2*(-4ac - b^2)^3)^{1/2} + b^5c^2e^2 - b^4f^2*(-4ac - b^2)^3)^{1/2} - 7ab^3c^3e^2 + 12a^2b^2c^4e^2 + ac^3e^2*(-4ac - b^2)^3)^{1/2} - 20a^3b^2c^3f^2 - 2b^6c^2ef + 25a^2b^3c^2f^2 - a^2c^2f^2*(-4ac - b^2)^3)^{1/2} - b^2c^2e^2*(-4ac - b^2)^3)^{1/2} - 4ab^2c^5d^2 - 9ab^5c^2f^2 - 16a^2c^5de - 2b^4c^3de + 16a^3c^4ef + 2b^5c^2df + 12ab^2c^4de - 14ab^3c^3df + 24a^2b^2c^4df + 2ac^3df*(-4ac - b^2)^3)^{1/2} + 2b^2c^3de*(-4ac - b^2)^3)^{1/2} + 16ab^4c^2ef + 2b^3c^2ef*(-4ac - b^2)^3)^{1/2} + 3ab^2c^2f^2*(-4ac - b^2)^3)^{1/2} - 36a^2b^2c^3ef - 2b^2c^2d*(-4ac - b^2)^3)^{1/2} - 4ab^2c^2ef*(-4ac - b^2)^3)^{1/2}}{(8(16a^2c^7 + b^4c^5 - 8ab^2c^6))^{1/2}}/c^3)*(-(b^7f^2 + b^3c^4d^2 - c^4d^2*(-4ac - b^2)^3)^{1/2} + b^5c^2e^2 - b^4f^2*(-4ac - b^2)^3)^{1/2} - 7ab^3c^3e^2 + 12a^2b^2c^4e^2 + ac^3e^2*(-4ac - b^2)^3)^{1/2} - 20a^3b^2c^3f^2 - 2b^6c^2ef + 25a^2b^3c^2f^2 - a^2c^2f^2*(-4ac - b^2)^3)^{1/2} - b^2c^2e^2*(-4ac - b^2)^3)^{1/2} - 4ab^2c^5d^2 - 9ab^5c^2f^2 - 16a^2c^5de - 2b^4c^3de - 4ab^2c^5d^2 - 9ab^5c^2f^2 - 16a^2c^5de - 2b^4c^3de + 16a^3c^4ef + 2b^5c^2df + 12ab^2c^4de - 14ab^3c^3df + 24a^2b^2c^4df + 2ac^3df*(-4ac - b^2)^3)^{1/2} + 2b^2c^3de*(-4ac - b^2)^3)^{1/2} + 16ab^4c^2ef + 2b^3c^2ef*(-4ac - b^2)^3)^{1/2} + 3ab^2c^2f^2*(-4ac - b^2)^3)^{1/2} - 36a^2b^2c^3ef - 2b^2c^2d*(-4ac - b^2)^3)^{1/2} - 4ab^2c^2ef*(-4ac - b^2)^3)^{1/2}}{(8(16a^2c^7 + b^4c^5 - 8ab^2c^6))^{1/2}}/c^3)*(-(b^7f^2 + b^3c^4d^2 - c^4d^2*(-4ac - b^2)^3)^{1/2} + b^5c^2e^2 - b^4f^2*(-4ac - b^2)^3)^{1/2} - 7ab^3c^3e^2 + 12a^2b^2c^4e^2 + ac^3e^2*(-4ac - b^2)^3)^{1/2} - 20a^3b^2c^3f^2 - 2b^6c^2ef + 25a^2b^3c^2f^2 - a^2c^2f^2*(-4ac - b^2)^3)^{1/2} - b^2c^2e^2*(-4ac - b^2)^3)^{1/2} - 4ab^2c^5d^2 - 9ab^5c^2f^2 - 16a^2c^5de - 2b^4c^3de - 4ab^2c^5d^2 - 9ab^5c^2f^2 - 16a^2c^5de - 2b^4c^3de + 16a^3c^4ef + 2b^5c^2df + 12ab^2c^4de - 14ab^3c^3df + 24a^2b^2c^4df + 2ac^3df*(-4ac - b^2)^3)^{1/2} + 2b^2c^3de*(-4ac - b^2)^3)^{1/2} + 16ab^4c^2ef + 2b^3c^2ef*(-4ac - b^2)^3)^{1/2} + 3ab^2c^2f^2*(-4ac - b^2)^3)^{1/2} - 36a^2b^2c^3ef - 2b^2c^2d*(-4ac - b^2)^3)^{1/2} - 4ab^2c^2ef*(-4ac - b^2)^3)^{1/2}}{(8(16a^2c^7 + b^4c^5 - 8ab^2c^6))^{1/2}}/c^3)*(-(b^7f^2 + b^3c^4d^2 - c^4d^2*(-4ac - b^2)^3)^{1/2} + b^5c^2e^2 - b^4f^2*(-4ac - b^2)^3)^{1/2} - 7ab^3c^3e^2 + 12a^2b^2c^4e^2 + ac^3e^2*(-4ac - b^2)^3)^{1/2} - 20a^3b^2c^3f^2 - 2b^6c^2ef + 25a^2b^3c^2f^2 - a^2c^2f^2*(-4ac - b^2)^3)^{1/2} - b^2c^2e^2*(-4ac - b^2)^3)^{1/2} - 4ab^2c^5d^2 - 9ab^5c^2f^2 - 16a^2c^5de - 2b^4c^3de$

$$\begin{aligned}
& *b*c^5*d^2 - 9*a*b^5*c*f^2 - 16*a^2*c^5*d*e - 2*b^4*c^3*d*e + 16*a^3*c^4*e*f \\
& + 2*b^5*c^2*d*f + 12*a*b^2*c^4*d*e - 14*a*b^3*c^3*d*f + 24*a^2*b*c^4*d*f \\
& + 2*a*c^3*d*f*(-(4*a*c - b^2)^3)^{(1/2)} + 2*b*c^3*d*e*(-(4*a*c - b^2)^3)^{(1/2)} \\
& + 16*a*b^4*c^2*e*f + 2*b^3*c*e*f*(-(4*a*c - b^2)^3)^{(1/2)} + 3*a*b^2*c*f^2 \\
& *(-(4*a*c - b^2)^3)^{(1/2)} - 36*a^2*b^2*c^3*e*f - 2*b^2*c^2*d*f*(-(4*a*c - b^2)^3)^{(1/2)} \\
& - 4*a*b*c^2*e*f*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6))^{(1/2)} \\
& + (2*x*(b^6*f^2 - 2*a*c^5*d^2 + 2*a^2*c^4*e^2 + b^2*c^4*d^2 - 2*a^3*c^3*f^2 + b^4*c^2*e^2 - 4*a*b^2*c^3*e^2 - 2*b^5*c*e*f \\
& + 9*a^2*b^2*c^2*f^2 - 6*a*b^4*c*f^2 + 4*a^2*c^4*d*f - 2*b^3*c^3*d*e + 2*b^4*c^2*d*f - 8*a*b^2*c^3*d*f \\
& + 10*a*b^3*c^2*e*f - 10*a^2*b*c^3*e*f + 6*a*b*c^4*d*e))/c^3)*(-(b^7*f^2 + b^3*c^4*d^2 - c^4*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + b^5*c^2*e^2 - b^4*f^2*(-(4*a*c - b^2)^3)^{(1/2)} - 7*a*b^3*c^3*e^2 + 12*a^2*b*c^4*e^2 + a*c^3*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 20*a^3*b*c^3*f^2 - 2*b^6*c*e*f + 25*a^2*b^3*c^2*f^2 - a^2*c^2*f^2*(-(4*a*c - b^2)^3)^{(1/2)} - b^2*c^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a*b*c^5*d^2 - 9*a*b^5*c*f^2 - 16*a^2*c^5*d*e - 2*b^4*c^3*d*e + 16*a^3*c^4*e*f + 2*b^5*c^2*d*f + 12*a*b^2*c^4*d*e - 14*a*b^3*c^3*d*f + 24*a^2*b*c^4*d*f + 2*a*c^3*d*f*(-(4*a*c - b^2)^3)^{(1/2)} + 2*b*c^3*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 16*a*b^4*c^2*e*f + 2*b^3*c*e*f*(-(4*a*c - b^2)^3)^{(1/2)} + 3*a*b^2*c*f^2*(-(4*a*c - b^2)^3)^{(1/2)} - 36*a^2*b^2*c^3*e*f - 2*b^2*c^2*d*f*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a*b*c^2*e*f*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6))^{(1/2)}*i)/((((16*a^2*c^5*e - 4*a*b^2*c^4*e + 4*a*b^3*c^3*f - 16*a^2*b*c^4*f)/c^3 - (2*x*(4*b^3*c^5 - 16*a*b*c^6)*(-(b^7*f^2 + b^3*c^4*d^2 - c^4*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + b^5*c^2*e^2 - b^4*f^2*(-(4*a*c - b^2)^3)^{(1/2)} - 7*a*b^3*c^3*e^2 + 12*a^2*b*c^4*e^2 + a*c^3*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 20*a^3*b*c^3*f^2 - 2*b^6*c*e*f + 25*a^2*b^3*c^2*f^2 - a^2*c^2*f^2*(-(4*a*c - b^2)^3)^{(1/2)} - b^2*c^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a*b*c^5*d^2 - 9*a*b^5*c*f^2 - 16*a^2*c^5*d*e - 2*b^4*c^3*d*e + 16*a^3*c^4*e*f + 2*b^5*c^2*d*f + 12*a*b^2*c^4*d*e - 14*a*b^3*c^3*d*f + 24*a^2*b*c^4*d*f + 2*a*c^3*d*f*(-(4*a*c - b^2)^3)^{(1/2)} + 2*b*c^3*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 16*a*b^4*c^2*e*f + 2*b^3*c*e*f*(-(4*a*c - b^2)^3)^{(1/2)} + 3*a*b^2*c*f^2*(-(4*a*c - b^2)^3)^{(1/2)} - 36*a^2*b^2*c^3*e*f - 2*b^2*c^2*d*f*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a*b*c^2*e*f*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6))^{(1/2)})/c^3)*(-(b^7*f^2 + b^3*c^4*d^2 - c^4*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + b^5*c^2*e^2 - b^4*f^2*(-(4*a*c - b^2)^3)^{(1/2)} - 7*a*b^3*c^3*e^2 + 12*a^2*b*c^4*e^2 + a*c^3*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 20*a^3*b*c^3*f^2 - 2*b^6*c*e*f + 25*a^2*b^3*c^2*f^2 - a^2*c^2*f^2*(-(4*a*c - b^2)^3)^{(1/2)} - b^2*c^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a*b*c^5*d^2 - 9*a*b^5*c*f^2 - 16*a^2*c^5*d*e - 2*b^4*c^3*d*e + 16*a^3*c^4*e*f + 2*b^5*c^2*d*f + 12*a*b^2*c^4*d*e - 14*a*b^3*c^3*d*f + 24*a^2*b*c^4*d*f + 2*a*c^3*d*f*(-(4*a*c - b^2)^3)^{(1/2)} + 2*b*c^3*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 16*a*b^4*c^2*e*f + 2*b^3*c*e*f*(-(4*a*c - b^2)^3)^{(1/2)} + 3*a*b^2*c*f^2*(-(4*a*c - b^2)^3)^{(1/2)} - 36*a^2*b^2*c^3*e*f - 2*b^2*c^2*d*f*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a*b*c^2*e*f*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6))^{(1/2)} - (2*x*(b^6*f^2 - 2*a*c^5*d^2 + 2*a^2*c^4*e^2 + b^2*c^4*d^2 - 2*a^3*c^3*f^2 + b^4*c^2*e^2 - 4*a*b^2*c^3*e^2 - 2*b^5*c*e*f + 9*a^2*b^2*c^2*f^2 - 6*a*b^4*c*f^2 + 4*a^2*c^4*d*f - 2*b^3*c^3*d*e + 2*b^4*c^2*d*f - 8*a*b^2*c^3*d*f + 10*a*b^3*c^2*e*f - 10*a^2*b*c^3*e*f + 6*a*b*c^4*d*e))/c^3)*(-(b^7*f^2 + b^3*c^4*d^2 - c^4*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + b^5*c^2*e^2 - b^4*f^2*(-(4*a*c - b^2)^3)^{(1/2)} - 7*a*b^3*c^3*e^2 + 12*a^2*b*c^4*e^2 + a*c^3*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 20*a^3*b*c^3*f^2 - 2*b^6*c*e*f + 25*a^2*b^3*c^2*f^2 - a^2*c^2*f^2*(-(4*a*c - b^2)^3)^{(1/2)} - b^2*c^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a*b*c^5*d^2 - 9*a*b^5*c*f^2 - 16*a^2*c^5*d*e - 2*b^4*c^3*d*e + 16*a^3*c^4*e*f + 2*b^5*c^2*d*f + 12*a*b^2*c^4*d*e - 14*a*b^3*c^3*d*f + 24*a^2*b*c^4*d*f + 2*a*c^3*d*f*(-(4*a*c - b^2)^3)^{(1/2)} + 2*b*c^3*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 16*a*b^4*c^2*e*f + 2*b^3*c*e*f*(-(4*a*c - b^2)^3)^{(1/2)} + 3*a*b^2*c*f^2*(-(4*a*c - b^2)^3)^{(1/2)} - 36*a^2*b^2*c^3*e*f - 2*b^2*c^2*d*f*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a*b*c^2*e*f*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6))^{(1/2)} + (((16*a^2*c^5*e - 4*a*b^2*c^4*e + 4*a*b^3*c^3*f - 16*a^2*b*c^4*f)
\end{aligned}$$

$$\begin{aligned}
& /c^3 + (2*x*(4*b^3*c^5 - 16*a*b*c^6)*(-(b^7*f^2 + b^3*c^4*d^2 - c^4*d^2*(-(4*a*c - b^2)^3)^{1/2} + b^5*c^2*e^2 - b^4*f^2*(-(4*a*c - b^2)^3)^{1/2} - 7*a*b^3*c^3*e^2 + 12*a^2*b*c^4*e^2 + a*c^3*e^2*(-(4*a*c - b^2)^3)^{1/2} - 20*a^3*b*c^3*f^2 - 2*b^6*c*e*f + 25*a^2*b^3*c^2*f^2 - a^2*c^2*f^2*(-(4*a*c - b^2)^3)^{1/2} - b^2*c^2*e^2*(-(4*a*c - b^2)^3)^{1/2} - 4*a*b*c^5*d^2 - 9*a*b^5*c*f^2 - 16*a^2*c^5*d*e - 2*b^4*c^3*d*e + 16*a^3*c^4*e*f + 2*b^5*c^2*d*f + 12*a*b^2*c^4*d*e - 14*a*b^3*c^3*d*f + 24*a^2*b*c^4*d*f + 2*a*c^3*d*f*(-(4*a*c - b^2)^3)^{1/2} + 2*b*c^3*d*e*(-(4*a*c - b^2)^3)^{1/2} + 16*a*b^4*c^2*e*f + 2*b^3*c*e*f*(-(4*a*c - b^2)^3)^{1/2} + 3*a*b^2*c*f^2*(-(4*a*c - b^2)^3)^{1/2} - 36*a^2*b^2*c^3*e*f - 2*b^2*c^2*d*f*(-(4*a*c - b^2)^3)^{1/2} - 4*a*b*c^2*e*f*(-(4*a*c - b^2)^3)^{1/2}))/((8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^{1/2})/c^3)*(-(b^7*f^2 + b^3*c^4*d^2 - c^4*d^2*(-(4*a*c - b^2)^3)^{1/2} + b^5*c^2*e^2 - b^4*f^2*(-(4*a*c - b^2)^3)^{1/2} - 7*a*b^3*c^3*e^2 + 12*a^2*b*c^4*e^2 + a*c^3*e^2*(-(4*a*c - b^2)^3)^{1/2} - 20*a^3*b*c^3*f^2 - 2*b^6*c*e*f + 25*a^2*b^3*c^2*f^2 - a^2*c^2*f^2*(-(4*a*c - b^2)^3)^{1/2} - b^2*c^2*e^2*(-(4*a*c - b^2)^3)^{1/2} - 4*a*b*c^5*d^2 - 9*a*b^5*c*f^2 - 16*a^2*c^5*d*e - 2*b^4*c^3*d*e + 16*a^3*c^4*e*f + 2*b^5*c^2*d*f + 12*a*b^2*c^4*d*e - 14*a*b^3*c^3*d*f + 24*a^2*b*c^4*d*f + 2*a*c^3*d*f*(-(4*a*c - b^2)^3)^{1/2} + 2*b*c^3*d*e*(-(4*a*c - b^2)^3)^{1/2} + 16*a*b^4*c^2*e*f + 2*b^3*c*e*f*(-(4*a*c - b^2)^3)^{1/2} + 3*a*b^2*c*f^2*(-(4*a*c - b^2)^3)^{1/2} - 36*a^2*b^2*c^3*e*f - 2*b^2*c^2*d*f*(-(4*a*c - b^2)^3)^{1/2} - 4*a*b*c^2*e*f*(-(4*a*c - b^2)^3)^{1/2}))/((8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^{1/2} + (2*x*(b^6*f^2 - 2*a*c^5*d^2 + 2*a^2*c^4*e^2 + b^2*c^4*d^2 - 2*a^3*c^3*f^2 + b^4*c^2*e^2 - 4*a*b^2*c^3*e^2 - 2*b^5*c*e*f + 9*a^2*b^2*c^2*f^2 - 6*a*b^4*c*f^2 + 4*a^2*c^4*d*f - 2*b^3*c^3*d*e + 2*b^4*c^2*d*f - 8*a*b^2*c^3*d*f + 10*a*b^3*c^2*e*f - 10*a^2*b*c^3*e*f + 6*a*b*c^4*d*e))/c^3)*(-(b^7*f^2 + b^3*c^4*d^2 - c^4*d^2*(-(4*a*c - b^2)^3)^{1/2} + b^5*c^2*e^2 - b^4*f^2*(-(4*a*c - b^2)^3)^{1/2} - 7*a*b^3*c^3*e^2 + 12*a^2*b*c^4*e^2 + a*c^3*e^2*(-(4*a*c - b^2)^3)^{1/2} - 20*a^3*b*c^3*f^2 - 2*b^6*c*e*f + 25*a^2*b^3*c^2*f^2 - a^2*c^2*f^2*(-(4*a*c - b^2)^3)^{1/2} - b^2*c^2*e^2*(-(4*a*c - b^2)^3)^{1/2} - 4*a*b*c^5*d^2 - 9*a*b^5*c*f^2 - 16*a^2*c^5*d*e - 2*b^4*c^3*d*e + 16*a^3*c^4*e*f + 2*b^5*c^2*d*f + 12*a*b^2*c^4*d*e - 14*a*b^3*c^3*d*f + 24*a^2*b*c^4*d*f + 2*a*c^3*d*f*(-(4*a*c - b^2)^3)^{1/2} + 2*b*c^3*d*e*(-(4*a*c - b^2)^3)^{1/2} + 16*a*b^4*c^2*e*f + 2*b^3*c*e*f*(-(4*a*c - b^2)^3)^{1/2} + 3*a*b^2*c*f^2*(-(4*a*c - b^2)^3)^{1/2} - 36*a^2*b^2*c^3*e*f - 2*b^2*c^2*d*f*(-(4*a*c - b^2)^3)^{1/2} - 4*a*b*c^2*e*f*(-(4*a*c - b^2)^3)^{1/2}))/((8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^{1/2} - (2*(a*c^4*d^3 - a^4*c*f^3 + a^3*b^2*f^3 - a^2*b*c^2*e^3 + a^2*c^3*d*e^2 - a^2*b^3*e*f^2 - 3*a^2*c^3*d^2*f + 3*a^3*c^2*d*f^2 - a^3*c^2*e^2*f + a*b^4*d*f^2 - 2*a*b*c^3*d^2*e + a*b^2*c^2*d*e^2 + 2*a*b^2*c^2*d^2*f - 3*a^2*b^2*c*d*f^2 + 2*a^2*b^2*c*e^2*f - 2*a*b^3*c*d*e*f + 2*a^2*b*c^2*d*e*f))/c^3)*(-(b^7*f^2 + b^3*c^4*d^2 - c^4*d^2*(-(4*a*c - b^2)^3)^{1/2} + b^5*c^2*e^2 - b^4*f^2*(-(4*a*c - b^2)^3)^{1/2} - 7*a*b^3*c^3*e^2 + 12*a^2*b*c^4*e^2 + a*c^3*e^2*(-(4*a*c - b^2)^3)^{1/2} - 20*a^3*b*c^3*f^2 - 2*b^6*c*e*f + 25*a^2*b^3*c^2*f^2 - a^2*c^2*f^2*(-(4*a*c - b^2)^3)^{1/2} - b^2*c^2*e^2*(-(4*a*c - b^2)^3)^{1/2} - 4*a*b*c^5*d^2 - 9*a*b^5*c*f^2 - 16*a^2*c^5*d*e - 2*b^4*c^3*d*e + 16*a^3*c^4*e*f + 2*b^5*c^2*d*f + 12*a*b^2*c^4*d*e - 14*a*b^3*c^3*d*f + 24*a^2*b*c^4*d*f + 2*a*c^3*d*f*(-(4*a*c - b^2)^3)^{1/2} + 2*b*c^3*d*e*(-(4*a*c - b^2)^3)^{1/2} + 16*a*b^4*c^2*e*f + 2*b^3*c*e*f*(-(4*a*c - b^2)^3)^{1/2} + 3*a*b^2*c*f^2*(-(4*a*c - b^2)^3)^{1/2} - 36*a^2*b^2*c^3*e*f - 2*b^2*c^2*d*f*(-(4*a*c - b^2)^3)^{1/2} - 4*a*b*c^2*e*f*(-(4*a*c - b^2)^3)^{1/2}))/((8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^{1/2})*2i - \operatorname{atan}\left(\frac{(16*a^2*c^5*e - 4*a*b^2*c^4*e + 4*a*b^3*c^3*f - 16*a^2*b*c^4*f)/c^3 - (2*x*(4*b^3*c^5 - 16*a*b*c^6)*(-(b^7*f^2 + b^3*c^4*d^2 + c^4*d^2*(-(4*a*c - b^2)^3)^{1/2} + b^5*c^2*e^2 + b^4*f^2*(-(4*a*c - b^2)^3)^{1/2} - 7*a*b^3*c^3*e^2 + 12*a^2*b*c^4*e^2 - a*c^3*e^2*(-(4*a*c - b^2)^3)^{1/2} - 20*a^3*b*c^3*f^2 - 2*b^6*c*e*f + 25*a^2*b^3*c^2*f^2 + a^2*c^2*f^2*(-(4*a*c - b^2)^3)^{1/2} + b^2*c^2*e^2*(-(4*a*c - b^2)^3)^{1/2} - 4*a*b*c^5*d^2 - 9*a*b^5*c*f^2 - 16*a^2*c^5*d*e - 2*b^4*c^3*d*e + 16*a^3*c^4*e*f + 2*b^5*c^2*d*f + 12*a*b^2*c^4*d*e - 14*a*b^3*c^3*d*f + 24*a^2*b*c^4*d*f - 2*a*c^3*d*f*(-(4*a*c - b^2)^3)^{1/2} + 2*b*c^3*d*e*(-(4*a*c - b^2)^3)^{1/2} + 16*a*b^4*c^2*e*f + 2*b^3*c*e*f*(-(4*a*c - b^2)^3)^{1/2} + 3*a*b^2*c*f^2*(-(4*a*c - b^2)^3)^{1/2} - 36*a^2*b^2*c^3*e*f - 2*b^2*c^2*d*f*(-(4*a*c - b^2)^3)^{1/2} - 4*a*b*c^2*e*f*(-(4*a*c - b^2)^3)^{1/2}))/((8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^{1/2}}{(16*a^2*c^5*e - 4*a*b^2*c^4*e + 4*a*b^3*c^3*f - 16*a^2*b*c^4*f)/c^3}
\end{aligned}$$

$$\begin{aligned}
& *c - b^2)^3)^{(1/2)} - 2*b*c^3*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 16*a*b^4*c^2*e* \\
& f - 2*b^3*c*e*f*(-(4*a*c - b^2)^3)^{(1/2)} - 3*a*b^2*c*f^2*(-(4*a*c - b^2)^3) \\
& ^{(1/2)} - 36*a^2*b^2*c^3*e*f + 2*b^2*c^2*d*f*(-(4*a*c - b^2)^3)^{(1/2)} + 4*a* \\
& b*c^2*e*f*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6) \\
&))^{(1/2)}/c^3)*(-(b^7*f^2 + b^3*c^4*d^2 + c^4*d^2*(-(4*a*c - b^2)^3)^{(1/2)} \\
& + b^5*c^2*e^2 + b^4*f^2*(-(4*a*c - b^2)^3)^{(1/2)} - 7*a*b^3*c^3*e^2 + 12*a^2 \\
& *b*c^4*e^2 - a*c^3*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 20*a^3*b*c^3*f^2 - 2*b^6* \\
& c*e*f + 25*a^2*b^3*c^2*f^2 + a^2*c^2*f^2*(-(4*a*c - b^2)^3)^{(1/2)} + b^2*c^2 \\
& *e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a*b*c^5*d^2 - 9*a*b^5*c*f^2 - 16*a^2*c^5* \\
& d*e - 2*b^4*c^3*d*e + 16*a^3*c^4*e*f + 2*b^5*c^2*d*f + 12*a*b^2*c^4*d*e - 1 \\
& 4*a*b^3*c^3*d*f + 24*a^2*b*c^4*d*f - 2*a*c^3*d*f*(-(4*a*c - b^2)^3)^{(1/2)} - \\
& 2*b*c^3*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 16*a*b^4*c^2*e*f - 2*b^3*c*e*f*(-(4 \\
& *a*c - b^2)^3)^{(1/2)} - 3*a*b^2*c*f^2*(-(4*a*c - b^2)^3)^{(1/2)} - 36*a^2*b^2* \\
& c^3*e*f + 2*b^2*c^2*d*f*(-(4*a*c - b^2)^3)^{(1/2)} + 4*a*b*c^2*e*f*(-(4*a*c - \\
& b^2)^3)^{(1/2)})/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^{(1/2)} - (2*x*(b^6 \\
& *f^2 - 2*a*c^5*d^2 + 2*a^2*c^4*e^2 + b^2*c^4*d^2 - 2*a^3*c^3*f^2 + b^4*c^2* \\
& e^2 - 4*a*b^2*c^3*e^2 - 2*b^5*c*e*f + 9*a^2*b^2*c^2*f^2 - 6*a*b^4*c*f^2 + 4 \\
& *a^2*c^4*d*f - 2*b^3*c^3*d*e + 2*b^4*c^2*d*f - 8*a*b^2*c^3*d*f + 10*a*b^3*c \\
& ^2*e*f - 10*a^2*b*c^3*e*f + 6*a*b*c^4*d*e))/c^3)*(-(b^7*f^2 + b^3*c^4*d^2 + \\
& c^4*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + b^5*c^2*e^2 + b^4*f^2*(-(4*a*c - b^2)^3 \\
&)^{(1/2)} - 7*a*b^3*c^3*e^2 + 12*a^2*b*c^4*e^2 - a*c^3*e^2*(-(4*a*c - b^2)^3) \\
& ^{(1/2)} - 20*a^3*b*c^3*f^2 - 2*b^6*c*e*f + 25*a^2*b^3*c^2*f^2 + a^2*c^2*f^2* \\
& (- (4*a*c - b^2)^3)^{(1/2)} + b^2*c^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a*b*c^5 \\
& *d^2 - 9*a*b^5*c*f^2 - 16*a^2*c^5*d*e - 2*b^4*c^3*d*e + 16*a^3*c^4*e*f + 2* \\
& b^5*c^2*d*f + 12*a*b^2*c^4*d*e - 14*a*b^3*c^3*d*f + 24*a^2*b*c^4*d*f - 2*a* \\
& c^3*d*f*(-(4*a*c - b^2)^3)^{(1/2)} - 2*b*c^3*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 1 \\
& 6*a*b^4*c^2*e*f - 2*b^3*c*e*f*(-(4*a*c - b^2)^3)^{(1/2)} - 3*a*b^2*c*f^2*(-(4 \\
& *a*c - b^2)^3)^{(1/2)} - 36*a^2*b^2*c^3*e*f + 2*b^2*c^2*d*f*(-(4*a*c - b^2)^3) \\
&)^{(1/2)} + 4*a*b*c^2*e*f*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16*a^2*c^7 + b^4*c^5 \\
& - 8*a*b^2*c^6)))^{(1/2)}*i - (((16*a^2*c^5*e - 4*a*b^2*c^4*e + 4*a*b^3*c^3*f \\
& - 16*a^2*b*c^4*f)/c^3 + (2*x*(4*b^3*c^5 - 16*a*b*c^6)*(-(b^7*f^2 + b^3*c^4 \\
& *d^2 + c^4*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + b^5*c^2*e^2 + b^4*f^2*(-(4*a*c - \\
& b^2)^3)^{(1/2)} - 7*a*b^3*c^3*e^2 + 12*a^2*b*c^4*e^2 - a*c^3*e^2*(-(4*a*c - b \\
& ^2)^3)^{(1/2)} - 20*a^3*b*c^3*f^2 - 2*b^6*c*e*f + 25*a^2*b^3*c^2*f^2 + a^2*c^ \\
& 2*f^2*(-(4*a*c - b^2)^3)^{(1/2)} + b^2*c^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a \\
& *b*c^5*d^2 - 9*a*b^5*c*f^2 - 16*a^2*c^5*d*e - 2*b^4*c^3*d*e + 16*a^3*c^4*e* \\
& f + 2*b^5*c^2*d*f + 12*a*b^2*c^4*d*e - 14*a*b^3*c^3*d*f + 24*a^2*b*c^4*d*f \\
& - 2*a*c^3*d*f*(-(4*a*c - b^2)^3)^{(1/2)} - 2*b*c^3*d*e*(-(4*a*c - b^2)^3)^{(1/ \\
& 2)} + 16*a*b^4*c^2*e*f - 2*b^3*c*e*f*(-(4*a*c - b^2)^3)^{(1/2)} - 3*a*b^2*c*f^ \\
& 2*(-(4*a*c - b^2)^3)^{(1/2)} - 36*a^2*b^2*c^3*e*f + 2*b^2*c^2*d*f*(-(4*a*c - \\
& b^2)^3)^{(1/2)} + 4*a*b*c^2*e*f*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16*a^2*c^7 + b^ \\
& 4*c^5 - 8*a*b^2*c^6)))^{(1/2)}/c^3)*(-(b^7*f^2 + b^3*c^4*d^2 + c^4*d^2*(-(4* \\
& a*c - b^2)^3)^{(1/2)} + b^5*c^2*e^2 + b^4*f^2*(-(4*a*c - b^2)^3)^{(1/2)} - 7*a* \\
& b^3*c^3*e^2 + 12*a^2*b*c^4*e^2 - a*c^3*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 20*a^ \\
& 3*b*c^3*f^2 - 2*b^6*c*e*f + 25*a^2*b^3*c^2*f^2 + a^2*c^2*f^2*(-(4*a*c - b^2 \\
&)^3)^{(1/2)} + b^2*c^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a*b*c^5*d^2 - 9*a*b^5 \\
& *c*f^2 - 16*a^2*c^5*d*e - 2*b^4*c^3*d*e + 16*a^3*c^4*e*f + 2*b^5*c^2*d*f + \\
& 12*a*b^2*c^4*d*e - 14*a*b^3*c^3*d*f + 24*a^2*b*c^4*d*f - 2*a*c^3*d*f*(-(4*a \\
& *c - b^2)^3)^{(1/2)} - 2*b*c^3*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 16*a*b^4*c^2*e* \\
& f - 2*b^3*c*e*f*(-(4*a*c - b^2)^3)^{(1/2)} - 3*a*b^2*c*f^2*(-(4*a*c - b^2)^3) \\
& ^{(1/2)} - 36*a^2*b^2*c^3*e*f + 2*b^2*c^2*d*f*(-(4*a*c - b^2)^3)^{(1/2)} + 4*a* \\
& b*c^2*e*f*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6) \\
&))^{(1/2)} + (2*x*(b^6*f^2 - 2*a*c^5*d^2 + 2*a^2*c^4*e^2 + b^2*c^4*d^2 - 2*a^ \\
& 3*c^3*f^2 + b^4*c^2*e^2 - 4*a*b^2*c^3*e^2 - 2*b^5*c*e*f + 9*a^2*b^2*c^2*f^2 \\
& - 6*a*b^4*c*f^2 + 4*a^2*c^4*d*f - 2*b^3*c^3*d*e + 2*b^4*c^2*d*f - 8*a*b^2*c \\
& ^3*d*f + 10*a*b^3*c^2*e*f - 10*a^2*b*c^3*e*f + 6*a*b*c^4*d*e))/c^3)*(-(b^7 \\
& *f^2 + b^3*c^4*d^2 + c^4*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + b^5*c^2*e^2 + b^4*f \\
& ^2*(-(4*a*c - b^2)^3)^{(1/2)} - 7*a*b^3*c^3*e^2 + 12*a^2*b*c^4*e^2 - a*c^3*e^ \\
& 2*(-(4*a*c - b^2)^3)^{(1/2)} - 20*a^3*b*c^3*f^2 - 2*b^6*c*e*f + 25*a^2*b^3*c^
\end{aligned}$$

$$\begin{aligned}
& 2f^2 + a^2c^2f^2(-4ac - b^2)^3)^{1/2} + b^2c^2e^2(-4ac - b^2)^3)^{1/2} - 4abc^5d^2 - 9ab^5cf^2 - 16a^2c^5de - 2b^4c^3de + \\
& 16a^3c^4ef + 2b^5c^2df + 12ab^2c^4de - 14ab^3c^3df + 24a^2bc^4df - 2ac^3d^2f(-4ac - b^2)^3)^{1/2} - 2bc^3d^2e(-4ac - \\
& - b^2)^3)^{1/2} + 16ab^4c^2ef - 2b^3c^2ef(-4ac - b^2)^3)^{1/2} - 3ab^2c^2f^2(-4ac - b^2)^3)^{1/2} - 36a^2b^2c^3ef + 2b^2c^2d \\
& *f(-4ac - b^2)^3)^{1/2} + 4abc^2ef(-4ac - b^2)^3)^{1/2})/(8(16a^2c^7 + b^4c^5 - 8ab^2c^6))^{1/2}*ii)/((((16a^2c^5e - 4ab^2c \\
& ^4e + 4ab^3c^3f - 16a^2bc^4f)/c^3 - (2x(4b^3c^5 - 16abc^6)* \\
& (-b^7f^2 + b^3c^4d^2 + c^4d^2*(-4ac - b^2)^3)^{1/2} + b^5c^2e^2 + \\
& b^4f^2*(-4ac - b^2)^3)^{1/2} - 7ab^3c^3e^2 + 12a^2bc^4e^2 - ac^3e^2*(-4ac - b^2)^3)^{1/2} - 20a^3bc^3f^2 - 2b^6c^2ef + 25a^2b \\
& ^3c^2f^2 + a^2c^2f^2(-4ac - b^2)^3)^{1/2} + b^2c^2e^2(-4ac - b^2)^3)^{1/2} - 4abc^5d^2 - 9ab^5cf^2 - 16a^2c^5de - 2b^4c^3 \\
& *de + 16a^3c^4ef + 2b^5c^2df + 12ab^2c^4de - 14ab^3c^3df + 24a^2bc^4df - 2ac^3d^2f(-4ac - b^2)^3)^{1/2} - 2bc^3d^2e(- \\
& (4ac - b^2)^3)^{1/2} + 16ab^4c^2ef - 2b^3c^2ef(-4ac - b^2)^3)^{1/2} - 3ab^2c^2f^2(-4ac - b^2)^3)^{1/2} - 36a^2b^2c^3ef + 2b^2 \\
& *c^2d^2f(-4ac - b^2)^3)^{1/2} + 4abc^2ef(-4ac - b^2)^3)^{1/2})/(8(16a^2c^7 + b^4c^5 - 8ab^2c^6))^{1/2})/c^3*(-(b^7f^2 + b^3c^4 \\
& *d^2 + c^4d^2*(-4ac - b^2)^3)^{1/2} + b^5c^2e^2 + b^4f^2*(-4ac - b^2)^3)^{1/2} - 7ab^3c^3e^2 + 12a^2bc^4e^2 - ac^3e^2*(-4ac - b \\
& ^2)^3)^{1/2} - 20a^3bc^3f^2 - 2b^6c^2ef + 25a^2b^3c^2f^2 + a^2c^2 \\
& *f^2(-4ac - b^2)^3)^{1/2} + b^2c^2e^2(-4ac - b^2)^3)^{1/2} - 4ab \\
& *c^5d^2 - 9ab^5cf^2 - 16a^2c^5de - 2b^4c^3de + 16a^3c^4ef + 2b^5c^2df + 12ab^2c^4de - 14ab^3c^3df + 24a^2bc^4df \\
& - 2ac^3d^2f(-4ac - b^2)^3)^{1/2} - 2bc^3d^2e(-4ac - b^2)^3)^{1/2} + 16ab^4c^2ef - 2b^3c^2ef(-4ac - b^2)^3)^{1/2} - 3ab^2c^2f^2 \\
& ^2*(-4ac - b^2)^3)^{1/2} - 36a^2b^2c^3ef + 2b^2c^2d^2f(-4ac - b^2)^3)^{1/2} + 4abc^2ef(-4ac - b^2)^3)^{1/2})/(8(16a^2c^7 + b^4 \\
& c^5 - 8ab^2c^6))^{1/2} - (2x*(b^6f^2 - 2ac^5d^2 + 2a^2c^4e^2 + b^2c^4d^2 - 2a^3c^3f^2 + b^4c^2e^2 - 4ab^2c^3e^2 - 2b^5c^2ef \\
& + 9a^2b^2c^2f^2 - 6ab^4cf^2 + 4a^2c^4df - 2b^3c^3d^2e + 2b^4c^2df - 8ab^2c^3df + 10ab^3c^2ef - 10a^2bc^3ef + 6ab^4d^2e)) \\
& /c^3*(-(b^7f^2 + b^3c^4d^2 + c^4d^2*(-4ac - b^2)^3)^{1/2} + b^5c^2e^2 + b^4f^2*(-4ac - b^2)^3)^{1/2} - 7ab^3c^3e^2 + 12a^2b \\
& *c^4e^2 - ac^3e^2*(-4ac - b^2)^3)^{1/2} - 20a^3bc^3f^2 - 2b^6c^2ef + 25a^2b^3c^2f^2 + a^2c^2f^2(-4ac - b^2)^3)^{1/2} + b^2c^2e^2 \\
& ^2*(-4ac - b^2)^3)^{1/2} - 4abc^5d^2 - 9ab^5cf^2 - 16a^2c^5de - 2b^4c^3de + 16a^3c^4ef + 2b^5c^2df + 12ab^2c^4de - 14 \\
& *ab^3c^3df + 24a^2bc^4df - 2ac^3d^2f(-4ac - b^2)^3)^{1/2} - 2bc^3d^2e(-4ac - b^2)^3)^{1/2} + 16ab^4c^2ef - 2b^3c^2ef(-4ac \\
& - b^2)^3)^{1/2} - 3ab^2c^2f^2(-4ac - b^2)^3)^{1/2} - 36a^2b^2c^3ef + 2b^2c^2d^2f(-4ac - b^2)^3)^{1/2} + 4abc^2ef(-4ac - \\
& b^2)^3)^{1/2})/(8(16a^2c^7 + b^4c^5 - 8ab^2c^6))^{1/2} + (((16a^2c^5e - 4ab^2c^4e + 4ab^3c^3f - 16a^2bc^4f)/c^3 + (2x(4b^3c^5 - \\
& 16abc^6)*(-b^7f^2 + b^3c^4d^2 + c^4d^2*(-4ac - b^2)^3)^{1/2} + b^5c^2e^2 + b^4f^2*(-4ac - b^2)^3)^{1/2} - 7ab^3c^3e^2 + 12a^2b \\
& ^2bc^4e^2 - ac^3e^2*(-4ac - b^2)^3)^{1/2} - 20a^3bc^3f^2 - 2b^6c^2ef + 25a^2b^3c^2f^2 + a^2c^2f^2(-4ac - b^2)^3)^{1/2} + b^2c^2e^2 \\
& ^2*(-4ac - b^2)^3)^{1/2} - 4abc^5d^2 - 9ab^5cf^2 - 16a^2c^5de - 2b^4c^3de + 16a^3c^4ef + 2b^5c^2df + 12ab^2c^4de - 14 \\
& *ab^3c^3df + 24a^2bc^4df - 2ac^3d^2f(-4ac - b^2)^3)^{1/2} - 2bc^3d^2e(-4ac - b^2)^3)^{1/2} + 16ab^4c^2ef - 2b^3c^2ef(-4ac \\
& - b^2)^3)^{1/2} - 3ab^2c^2f^2(-4ac - b^2)^3)^{1/2} - 36a^2b^2c^3ef + 2b^2c^2d^2f(-4ac - b^2)^3)^{1/2} + 4abc^2ef(-4ac - \\
& b^2)^3)^{1/2})/(8(16a^2c^7 + b^4c^5 - 8ab^2c^6))^{1/2})/c^3*(-(b^7f^2 + b^3c^4d^2 + c^4d^2*(-4ac - b^2)^3)^{1/2} + b^5c^2e^2 + b^4 \\
& f^2*(-4ac - b^2)^3)^{1/2} - 7ab^3c^3e^2 + 12a^2bc^4e^2 - ac^3e^2
\end{aligned}$$

$$\begin{aligned}
& *e^{2*(-(4*a*c - b^2)^3)^{1/2}} - 20*a^3*b*c^3*f^2 - 2*b^6*c*e*f + 25*a^2*b^3 \\
& *c^2*f^2 + a^2*c^2*f^2*(-(4*a*c - b^2)^3)^{1/2} + b^2*c^2*e^2*(-(4*a*c - b^2)^3)^{1/2} - 4*a*b*c^5*d^2 - 9*a*b^5*c*f^2 - 16*a^2*c^5*d*e - 2*b^4*c^3*d* \\
& e + 16*a^3*c^4*e*f + 2*b^5*c^2*d*f + 12*a*b^2*c^4*d*e - 14*a*b^3*c^3*d*f + \\
& 24*a^2*b*c^4*d*f - 2*a*c^3*d*f*(-(4*a*c - b^2)^3)^{1/2} - 2*b*c^3*d*e*(-(4*a*c - b^2)^3)^{1/2} + 16*a*b^4*c^2*e*f - 2*b^3*c*e*f*(-(4*a*c - b^2)^3)^{1/2} \\
& - 3*a*b^2*c*f^2*(-(4*a*c - b^2)^3)^{1/2} - 36*a^2*b^2*c^3*e*f + 2*b^2*c^2*d*f*(-(4*a*c - b^2)^3)^{1/2} + 4*a*b*c^2*e*f*(-(4*a*c - b^2)^3)^{1/2})/(8 \\
& *(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6))^{1/2} + (2*x*(b^6*f^2 - 2*a*c^5*d^2 \\
& + 2*a^2*c^4*e^2 + b^2*c^4*d^2 - 2*a^3*c^3*f^2 + b^4*c^2*e^2 - 4*a*b^2*c^3* \\
& e^2 - 2*b^5*c*e*f + 9*a^2*b^2*c^2*f^2 - 6*a*b^4*c*f^2 + 4*a^2*c^4*d*f - 2*b \\
& ^3*c^3*d*e + 2*b^4*c^2*d*f - 8*a*b^2*c^3*d*f + 10*a*b^3*c^2*e*f - 10*a^2*b* \\
& c^3*e*f + 6*a*b*c^4*d*e))/c^3)*(-(b^7*f^2 + b^3*c^4*d^2 + c^4*d^2*(-(4*a*c \\
& - b^2)^3)^{1/2} + b^5*c^2*e^2 + b^4*f^2*(-(4*a*c - b^2)^3)^{1/2} - 7*a*b^3* \\
& c^3*e^2 + 12*a^2*b*c^4*e^2 - a*c^3*e^2*(-(4*a*c - b^2)^3)^{1/2} - 20*a^3*b* \\
& c^3*f^2 - 2*b^6*c*e*f + 25*a^2*b^3*c^2*f^2 + a^2*c^2*f^2*(-(4*a*c - b^2)^3)^{1/2} + b^2*c^2*e^2*(-(4*a*c - b^2)^3)^{1/2} - 4*a*b*c^5*d^2 - 9*a*b^5*c*f \\
& ^2 - 16*a^2*c^5*d*e - 2*b^4*c^3*d*e + 16*a^3*c^4*e*f + 2*b^5*c^2*d*f + 12*a \\
& *b^2*c^4*d*e - 14*a*b^3*c^3*d*f + 24*a^2*b*c^4*d*f - 2*a*c^3*d*f*(-(4*a*c - \\
& b^2)^3)^{1/2} - 2*b*c^3*d*e*(-(4*a*c - b^2)^3)^{1/2} + 16*a*b^4*c^2*e*f - \\
& 2*b^3*c*e*f*(-(4*a*c - b^2)^3)^{1/2} - 3*a*b^2*c*f^2*(-(4*a*c - b^2)^3)^{1/2} \\
& - 36*a^2*b^2*c^3*e*f + 2*b^2*c^2*d*f*(-(4*a*c - b^2)^3)^{1/2} + 4*a*b*c^2 \\
& *e*f*(-(4*a*c - b^2)^3)^{1/2})/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6))^{1/2} - (2*(a*c^4*d^3 - a^4*c*f^3 + a^3*b^2*f^3 - a^2*b*c^2*e^3 + a^2*c^3*d* \\
& e^2 - a^2*b^3*e*f^2 - 3*a^2*c^3*d^2*f + 3*a^3*c^2*d*f^2 - a^3*c^2*e^2*f + a \\
& *b^4*d*f^2 - 2*a*b*c^3*d^2*e + a*b^2*c^2*d*e^2 + 2*a*b^2*c^2*d^2*f - 3*a^2* \\
& b^2*c*d*f^2 + 2*a^2*b^2*c*e^2*f - 2*a*b^3*c*d*e*f + 2*a^2*b*c^2*d*e*f))/c^3 \\
&))*(-(b^7*f^2 + b^3*c^4*d^2 + c^4*d^2*(-(4*a*c - b^2)^3)^{1/2} + b^5*c^2*e^2 \\
& + b^4*f^2*(-(4*a*c - b^2)^3)^{1/2} - 7*a*b^3*c^3*e^2 + 12*a^2*b*c^4*e^2 - \\
& a*c^3*e^2*(-(4*a*c - b^2)^3)^{1/2} - 20*a^3*b*c^3*f^2 - 2*b^6*c*e*f + 25*a \\
& ^2*b^3*c^2*f^2 + a^2*c^2*f^2*(-(4*a*c - b^2)^3)^{1/2} + b^2*c^2*e^2*(-(4*a* \\
& c - b^2)^3)^{1/2} - 4*a*b*c^5*d^2 - 9*a*b^5*c*f^2 - 16*a^2*c^5*d*e - 2*b^4* \\
& c^3*d*e + 16*a^3*c^4*e*f + 2*b^5*c^2*d*f + 12*a*b^2*c^4*d*e - 14*a*b^3*c^3* \\
& d*f + 24*a^2*b*c^4*d*f - 2*a*c^3*d*f*(-(4*a*c - b^2)^3)^{1/2} - 2*b*c^3*d*e \\
& *(-(4*a*c - b^2)^3)^{1/2} + 16*a*b^4*c^2*e*f - 2*b^3*c*e*f*(-(4*a*c - b^2)^ \\
& 3)^{1/2} - 3*a*b^2*c*f^2*(-(4*a*c - b^2)^3)^{1/2} - 36*a^2*b^2*c^3*e*f + 2* \\
& b^2*c^2*d*f*(-(4*a*c - b^2)^3)^{1/2} + 4*a*b*c^2*e*f*(-(4*a*c - b^2)^3)^{1/2} \\
& 2))/8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6))^{1/2}*2i + (f*x^3)/(3*c)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a), x)

[Out] Timed out

$$3.55 \quad \int \frac{d+ex^2+fx^4}{a+bx^2+cx^4} dx$$

Optimal. Leaf size=219

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-c(2af+be)+b^2f+2c^2d}{\sqrt{b^2-4ac}}-bf+ce\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\frac{-2acf+b^2f-bce+2c^2d}{\sqrt{b^2-4ac}}-bf+ce\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}} + \frac{fx}{c}$$

Rubi [A] time = 0.64, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1676, 1166, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-c(2af+be)+b^2f+2c^2d}{\sqrt{b^2-4ac}}-bf+ce\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\frac{-2acf+b^2f-bce+2c^2d}{\sqrt{b^2-4ac}}-bf+ce\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}} + \frac{fx}{c}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2 + f*x^4)/(a + b*x^2 + c*x^4), x]

[Out] (f*x)/c + ((c*e - b*f + (2*c^2*d + b^2*f - c*(b*e + 2*a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((c*e - b*f - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1676

Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && Expon[Pq, x^2] > 1

Rubi steps

$$\begin{aligned}
\int \frac{d + ex^2 + fx^4}{a + bx^2 + cx^4} dx &= \int \left(\frac{f}{c} + \frac{cd - af + (ce - bf)x^2}{c(a + bx^2 + cx^4)} \right) dx \\
&= \frac{fx}{c} + \frac{\int \frac{cd - af + (ce - bf)x^2}{a + bx^2 + cx^4} dx}{c} \\
&= \frac{fx}{c} + \frac{\left(ce - bf - \frac{2c^2d - bce + b^2f - 2acf}{\sqrt{b^2 - 4ac}} \right) \int \frac{1}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2 - 4ac} + cx^2} dx}{2c} + \frac{\left(ce - bf + \frac{2c^2d + b^2f - c(be + 2af)}{\sqrt{b^2 - 4ac}} \right) \int \frac{1}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2 - 4ac} + cx^2} dx}{2c} \\
&= \frac{fx}{c} + \frac{\left(ce - bf + \frac{2c^2d + b^2f - c(be + 2af)}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{\sqrt{2}c^{3/2}\sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{\left(ce - bf - \frac{2c^2d - bce + b^2f - 2acf}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b + \sqrt{b^2 - 4ac}}} \right)}{\sqrt{2}c^{3/2}\sqrt{b + \sqrt{b^2 - 4ac}}}
\end{aligned}$$

Mathematica [A] time = 0.33, size = 258, normalized size = 1.18

$$\frac{\sqrt{2} \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right) \left(c \left(e\sqrt{b^2 - 4ac} - 2af - be \right) + bf \left(b - \sqrt{b^2 - 4ac} \right) + 2c^2d \right)}{\sqrt{b^2 - 4ac} \sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{\sqrt{2} \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b^2 - 4ac} + b} \right) \left(-c \left(e\sqrt{b^2 - 4ac} + 2af + be \right) + bf \left(\sqrt{b^2 - 4ac} + b \right) + 2c^2d \right)}{\sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} + b} + 2\sqrt{c}fx}{2c^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2 + f*x^4)/(a + b*x^2 + c*x^4), x]

[Out] (2*sqrt[c]*f*x + (sqrt[2]*(2*c^2*d + b*(b - sqrt[b^2 - 4*a*c]))*f + c*(-(b*e) + sqrt[b^2 - 4*a*c]*e - 2*a*f))*ArcTan[(sqrt[2]*sqrt[c]*x)/sqrt[b - sqrt[b^2 - 4*a*c]])/(sqrt[b^2 - 4*a*c]*sqrt[b - sqrt[b^2 - 4*a*c]]) - (sqrt[2]*(2*c^2*d + b*(b + sqrt[b^2 - 4*a*c]))*f - c*(b*e + sqrt[b^2 - 4*a*c]*e + 2*a*f))*ArcTan[(sqrt[2]*sqrt[c]*x)/sqrt[b + sqrt[b^2 - 4*a*c]])/(sqrt[b^2 - 4*a*c]*sqrt[b + sqrt[b^2 - 4*a*c]])/(2*c^(3/2))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex^2 + fx^4}{a + bx^2 + cx^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e*x^2 + f*x^4)/(a + b*x^2 + c*x^4), x]

[Out] IntegrateAlgebraic[(d + e*x^2 + f*x^4)/(a + b*x^2 + c*x^4), x]

fricas [B] time = 4.07, size = 5788, normalized size = 26.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^4+e*x^2+d)/(c*x^4+b*x^2+a), x, algorithm="fricas")

[Out] -1/2*(sqrt(1/2)*c*sqrt(-(b*c^3*d^2 - 4*a*c^3*d*e + a*b*c^2*e^2 + (a*b^3 - 3*a^2*b*c)*f^2 + 2*(a*b*c^2*d - (a*b^2*c - 2*a^2*c^2)*e)*f + (a*b^2*c^3 - 4*a^2*c^4)*sqrt((c^6*d^4 - 2*a*c^5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*f^4 + 4*((a^2*b^2*c^2 - a^3*c^3)*d - (a^2*b^3*c - a^3*b*c^2)*e)*f^3 - 2*(4*a^2*b*c^3*d*e + (a*b^2*c^3 - 3*a^2*c^4)*d^2 - (3*a^2*b^2*c^2 - a^3*c^3)*e^2)*f^2 - 4*(a*c^5*d^3 - a*b*c^4*d^2*e - a^2*c^4*d*e^2 + a^2*b*c^3*e^3)*f)/(a^2*b^2*c^6 - 4*a^3*c^7)))/(a*b^2*c^3 - 4*a^2*c^4))*log(2*(c^5*d^4 - b*c^4*d^3*e + a*b*c^3*d*e^3 - a^2*c^3*e^4 - (a^3*b^2 - a^4*c)*f^4 - ((a*b^4 - 3*a^2*b^2*c + 4*a^3*c^2)*d - (a^2*b^3 + a^3*b*c)*e)*f^3 - 3*(a^2*b^2*c^3 - 4*a^2*c^4)*sqrt((c^6*d^4 - 2*a*c^5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*f^4 + 4*((a^2*b^2*c^2 - a^3*c^3)*d - (a^2*b^3*c - a^3*b*c^2)*e)*f^3 - 2*(4*a^2*b*c^3*d*e + (a*b^2*c^3 - 3*a^2*c^4)*d^2 - (3*a^2*b^2*c^2 - a^3*c^3)*e^2)*f^2 - 4*(a*c^5*d^3 - a*b*c^4*d^2*e - a^2*c^4*d*e^2 + a^2*b*c^3*e^3)*f)/(a^2*b^2*c^6 - 4*a^3*c^7)))/(a*b^2*c^3 - 4*a^2*c^4))

$$\begin{aligned}
& 2*b^2*c*e^2 + (a*b^2*c^2 - 2*a^2*c^3)*d^2 - (a*b^3*c - a^2*b*c^2)*d*e)*f^2 \\
& + (3*a*b*c^3*d^2*e - 3*a*b^2*c^2*d*e^2 + 3*a^2*b*c^2*e^3 + (b^2*c^3 - 4*a*c^4)*d^3)*f)*x + \sqrt{1/2}*((b^2*c^4 - 4*a*c^5)*d^3 - (a*b^2*c^3 - 4*a^2*c^4)*d*e^2 + (a^2*b^4 - 5*a^3*b^2*c + 4*a^4*c^2)*f^3 - ((a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d + 2*(a^2*b^3*c - 4*a^3*b*c^2)*e)*f^2 - (3*(a*b^2*c^3 - 4*a^2*c^4)*d^2 - 2*(a*b^3*c^2 - 4*a^2*b*c^3)*d*e - (a^2*b^2*c^2 - 4*a^3*c^3)*e^2)*f - ((a*b^3*c^4 - 4*a^2*b*c^5)*d - 2*(a^2*b^2*c^4 - 4*a^3*c^5)*e + (a^2*b^3*c^3 - 4*a^3*b*c^4)*f)*\sqrt{(c^6*d^4 - 2*a*c^5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*f^4 + 4*((a^2*b^2*c^2 - a^3*c^3)*d - (a^2*b^3*c - a^3*b*c^2)*e)*f^3 - 2*(4*a^2*b*c^3*d*e + (a*b^2*c^3 - 3*a^2*c^4)*d^2 - (3*a^2*b^2*c^2 - a^3*c^3)*e^2)*f^2 - 4*(a*c^5*d^3 - a*b*c^4*d^2*e - a^2*c^4*d*e^2 + a^2*b*c^3*e^3)*f)/(a^2*b^2*c^6 - 4*a^3*c^7)})*\sqrt{-(b*c^3*d^2 - 4*a*c^3*d*e + a*b*c^2*e^2 + (a*b^3 - 3*a^2*b*c)*f^2 + 2*(a*b*c^2*d - (a*b^2*c - 2*a^2*c^2)*e)*f + (a*b^2*c^3 - 4*a^2*c^4)*\sqrt{(c^6*d^4 - 2*a*c^5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*f^4 + 4*((a^2*b^2*c^2 - a^3*c^3)*d - (a^2*b^3*c - a^3*b*c^2)*e)*f^3 - 2*(4*a^2*b*c^3*d*e + (a*b^2*c^3 - 3*a^2*c^4)*d^2 - (3*a^2*b^2*c^2 - a^3*c^3)*e^2)*f^2 - 4*(a*c^5*d^3 - a*b*c^4*d^2*e - a^2*c^4*d*e^2 + a^2*b*c^3*e^3)*f)/(a^2*b^2*c^6 - 4*a^3*c^7)}}/(a*b^2*c^3 - 4*a^2*c^4)) - \sqrt{1/2}*c*\sqrt{-(b*c^3*d^2 - 4*a*c^3*d*e + a*b*c^2*e^2 + (a*b^3 - 3*a^2*b*c)*f^2 + 2*(a*b*c^2*d - (a*b^2*c - 2*a^2*c^2)*e)*f + (a*b^2*c^3 - 4*a^2*c^4)*\sqrt{(c^6*d^4 - 2*a*c^5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*f^4 + 4*((a^2*b^2*c^2 - a^3*c^3)*d - (a^2*b^3*c - a^3*b*c^2)*e)*f^3 - 2*(4*a^2*b*c^3*d*e + (a*b^2*c^3 - 3*a^2*c^4)*d^2 - (3*a^2*b^2*c^2 - a^3*c^3)*e^2)*f^2 - 4*(a*c^5*d^3 - a*b*c^4*d^2*e - a^2*c^4*d*e^2 + a^2*b*c^3*e^3)*f)/(a^2*b^2*c^6 - 4*a^3*c^7)}}/(a*b^2*c^3 - 4*a^2*c^4)))*\log(2*(c^5*d^4 - b*c^4*d^3*e + a*b*c^3*d*e^3 - a^2*c^3*e^4 - (a^3*b^2 - a^4*c)*f^4 - ((a*b^4 - 3*a^2*b^2*c + 4*a^3*c^2)*d - (a^2*b^3 + a^3*b*c)*e)*f^3 - 3*(a^2*b^2*c*e^2 + (a*b^2*c^2 - 2*a^2*c^3)*d^2 - (a*b^3*c - a^2*b*c^2)*d*e)*f^2 + (3*a*b*c^3*d^2*e - 3*a*b^2*c^2*d*e^2 + 3*a^2*b*c^2*e^3 + (b^2*c^3 - 4*a*c^4)*d^3)*f)*x - \sqrt{1/2}*((b^2*c^4 - 4*a*c^5)*d^3 - (a*b^2*c^3 - 4*a^2*c^4)*d*e^2 + (a^2*b^4 - 5*a^3*b^2*c + 4*a^4*c^2)*f^3 - ((a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d + 2*(a^2*b^3*c - 4*a^3*b*c^2)*e)*f^2 - (3*(a*b^2*c^3 - 4*a^2*c^4)*d^2 - 2*(a*b^3*c^2 - 4*a^2*b*c^3)*d*e - (a^2*b^2*c^2 - 4*a^3*c^3)*e^2)*f - ((a*b^3*c^4 - 4*a^2*b*c^5)*d - 2*(a^2*b^2*c^4 - 4*a^3*c^5)*e + (a^2*b^3*c^3 - 4*a^3*b*c^4)*f)*\sqrt{(c^6*d^4 - 2*a*c^5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*f^4 + 4*((a^2*b^2*c^2 - a^3*c^3)*d - (a^2*b^3*c - a^3*b*c^2)*e)*f^3 - 2*(4*a^2*b*c^3*d*e + (a*b^2*c^3 - 3*a^2*c^4)*d^2 - (3*a^2*b^2*c^2 - a^3*c^3)*e^2)*f^2 - 4*(a*c^5*d^3 - a*b*c^4*d^2*e - a^2*c^4*d*e^2 + a^2*b*c^3*e^3)*f)/(a^2*b^2*c^6 - 4*a^3*c^7)})*\sqrt{-(b*c^3*d^2 - 4*a*c^3*d*e + a*b*c^2*e^2 + (a*b^3 - 3*a^2*b*c)*f^2 + 2*(a*b*c^2*d - (a*b^2*c - 2*a^2*c^2)*e)*f + (a*b^2*c^3 - 4*a^2*c^4)*\sqrt{(c^6*d^4 - 2*a*c^5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*f^4 + 4*((a^2*b^2*c^2 - a^3*c^3)*d - (a^2*b^3*c - a^3*b*c^2)*e)*f^3 - 2*(4*a^2*b*c^3*d*e + (a*b^2*c^3 - 3*a^2*c^4)*d^2 - (3*a^2*b^2*c^2 - a^3*c^3)*e^2)*f^2 - 4*(a*c^5*d^3 - a*b*c^4*d^2*e - a^2*c^4*d*e^2 + a^2*b*c^3*e^3)*f)/(a^2*b^2*c^6 - 4*a^3*c^7)}}/(a*b^2*c^3 - 4*a^2*c^4)) + \sqrt{1/2}*c*\sqrt{-(b*c^3*d^2 - 4*a*c^3*d*e + a*b*c^2*e^2 + (a*b^3 - 3*a^2*b*c)*f^2 + 2*(a*b*c^2*d - (a*b^2*c - 2*a^2*c^2)*e)*f - (a*b^2*c^3 - 4*a^2*c^4)*\sqrt{(c^6*d^4 - 2*a*c^5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*f^4 + 4*((a^2*b^2*c^2 - a^3*c^3)*d - (a^2*b^3*c - a^3*b*c^2)*e)*f^3 - 2*(4*a^2*b*c^3*d*e + (a*b^2*c^3 - 3*a^2*c^4)*d^2 - (3*a^2*b^2*c^2 - a^3*c^3)*e^2)*f^2 - 4*(a*c^5*d^3 - a*b*c^4*d^2*e - a^2*c^4*d*e^2 + a^2*b*c^3*e^3)*f)/(a^2*b^2*c^6 - 4*a^3*c^7)}}/(a*b^2*c^3 - 4*a^2*c^4)))*\log(2*(c^5*d^4 - b*c^4*d^3*e + a*b*c^3*d*e^3 - a^2*c^3*e^4 - (a^3*b^2 - a^4*c)*f^4 - ((a*b^4 - 3*a^2*b^2*c + 4*a^3*c^2)*d - (a^2*b^3 + a^3*b*c)*e)*f^3 - 3*(a^2*b^2*c*e^2 + (a*b^2*c^2 - 2*a^2*c^3)*d^2 - (a*b^3*c - a^2*b*c^2)*d*e)*f^2 + (3*a*b*c^3*d^2*e - 3*a*b^2*c^2*d*e^2 + 3*a^2*b*c^2*e^3 + (b^2*c^3 - 4*a*c^4)*d^3)*f)*x + \sqrt{1/2}*((b^2*c^4 - 4*a*c^5)*d^3 - (a*b^2*c^3 - 4*a^2*c^4)*d*e^2 + (a^2*b^4 - 5*a^3*b^2*c + 4*a^4*c^2)*f^3 - ((a*b^4*c - 7*a^2*b^2*c^2 + 12*a
\end{aligned}$$

$$\begin{aligned}
& \cdot 3c^3) \cdot d + 2(a^2b^3c - 4a^3b^2c^2) \cdot e) \cdot f^2 - (3(a^2b^2c^3 - 4a^2c^4) \\
& \cdot d^2 - 2(a^2b^3c^2 - 4a^2b^2c^3) \cdot d \cdot e - (a^2b^2c^2 - 4a^3c^3) \cdot e^2) \cdot f + \\
& ((a^2b^3c^4 - 4a^2b^2c^5) \cdot d - 2(a^2b^2c^4 - 4a^3c^5) \cdot e + (a^2b^3c^3 - 4a^3b^2c^4) \cdot f) \cdot \sqrt{(c^6d^4 - 2a^2c^5d^2e^2 + a^2c^4e^4 + (a^2b^4 - 2a^3b^2c + a^4c^2) \cdot f^4 + 4((a^2b^2c^2 - a^3c^3) \cdot d - (a^2b^3c - a^3b^2c^2) \cdot e) \cdot f^3 - 2(4a^2b^2c^3d \cdot e + (a^2b^2c^3 - 3a^2c^4) \cdot d^2 - (3a^2b^2c^2 - a^3c^3) \cdot e^2) \cdot f^2 - 4(a^2c^5d^3 - a^2b^2c^4d^2e - a^2c^4d \cdot e^2 + a^2b^2c^3e^3) \cdot f) / (a^2b^2c^6 - 4a^3c^7))} \cdot \sqrt{-(b^2c^3d^2 - 4a^2c^3d \cdot e + a^2b^2c^2e^2 + (a^2b^3 - 3a^2b^2c) \cdot f^2 + 2(a^2b^2c^2d - (a^2b^2c - 2a^2c^2) \cdot e) \cdot f - (a^2b^2c^3 - 4a^2c^4) \cdot \sqrt{(c^6d^4 - 2a^2c^5d^2e^2 + a^2c^4e^4 + (a^2b^4 - 2a^3b^2c + a^4c^2) \cdot f^4 + 4((a^2b^2c^2 - a^3c^3) \cdot d - (a^2b^3c - a^3b^2c^2) \cdot e) \cdot f^3 - 2(4a^2b^2c^3d \cdot e + (a^2b^2c^3 - 3a^2c^4) \cdot d^2 - (3a^2b^2c^2 - a^3c^3) \cdot e^2) \cdot f^2 - 4(a^2c^5d^3 - a^2b^2c^4d^2e - a^2c^4d \cdot e^2 + a^2b^2c^3e^3) \cdot f) / (a^2b^2c^6 - 4a^3c^7))} / (a^2b^2c^3 - 4a^2c^4))} - \sqrt{1/2} \cdot c \cdot \sqrt{-(b^2c^3d^2 - 4a^2c^3d \cdot e + a^2b^2c^2e^2 + (a^2b^3 - 3a^2b^2c) \cdot f^2 + 2(a^2b^2c^2d - (a^2b^2c - 2a^2c^2) \cdot e) \cdot f - (a^2b^2c^3 - 4a^2c^4) \cdot \sqrt{(c^6d^4 - 2a^2c^5d^2e^2 + a^2c^4e^4 + (a^2b^4 - 2a^3b^2c + a^4c^2) \cdot f^4 + 4((a^2b^2c^2 - a^3c^3) \cdot d - (a^2b^3c - a^3b^2c^2) \cdot e) \cdot f^3 - 2(4a^2b^2c^3d \cdot e + (a^2b^2c^3 - 3a^2c^4) \cdot d^2 - (3a^2b^2c^2 - a^3c^3) \cdot e^2) \cdot f^2 - 4(a^2c^5d^3 - a^2b^2c^4d^2e - a^2c^4d \cdot e^2 + a^2b^2c^3e^3) \cdot f) / (a^2b^2c^6 - 4a^3c^7))} / (a^2b^2c^3 - 4a^2c^4))} \cdot \log(2(c^5d^4 - b^2c^4d^3e + a^2b^2c^3d \cdot e^3 - a^2c^3e^4 - (a^3b^2 - a^4c) \cdot f^4 - ((a^2b^4 - 3a^2b^2c + 4a^3c^2) \cdot d - (a^2b^3 + a^3b^2c) \cdot e) \cdot f^3 - 3(a^2b^2c \cdot e^2 + (a^2b^2c^2 - 2a^2c^3) \cdot d^2 - (a^2b^3c - a^2b^2c^2) \cdot d \cdot e) \cdot f^2 + (3a^2b^2c^3d^2 \cdot e - 3a^2b^2c^2d \cdot e^2 + 3a^2b^2c^2e^3 + (b^2c^3 - 4a^2c^4) \cdot d^3) \cdot f) \cdot x - \sqrt{1/2} \cdot ((b^2c^4 - 4a^2c^5) \cdot d^3 - (a^2b^2c^3 - 4a^2c^4) \cdot d \cdot e^2 + (a^2b^4 - 5a^3b^2c + 4a^4c^2) \cdot f^3 - ((a^2b^4c - 7a^2b^2c^2 + 12a^3c^3) \cdot d + 2(a^2b^3c - 4a^3b^2c^2) \cdot e) \cdot f^2 - (3(a^2b^2c^3 - 4a^2c^4) \cdot d^2 - 2(a^2b^3c^2 - 4a^2b^2c^3) \cdot d \cdot e - (a^2b^2c^2 - 4a^3c^3) \cdot e^2) \cdot f + ((a^2b^3c^4 - 4a^2b^2c^5) \cdot d - 2(a^2b^2c^4 - 4a^3c^5) \cdot e + (a^2b^3c^3 - 4a^3b^2c^4) \cdot f) \cdot \sqrt{(c^6d^4 - 2a^2c^5d^2e^2 + a^2c^4e^4 + (a^2b^4 - 2a^3b^2c + a^4c^2) \cdot f^4 + 4((a^2b^2c^2 - a^3c^3) \cdot d - (a^2b^3c - a^3b^2c^2) \cdot e) \cdot f^3 - 2(4a^2b^2c^3d \cdot e + (a^2b^2c^3 - 3a^2c^4) \cdot d^2 - (3a^2b^2c^2 - a^3c^3) \cdot e^2) \cdot f^2 - 4(a^2c^5d^3 - a^2b^2c^4d^2e - a^2c^4d \cdot e^2 + a^2b^2c^3e^3) \cdot f) / (a^2b^2c^6 - 4a^3c^7))} \cdot \sqrt{-(b^2c^3d^2 - 4a^2c^3d \cdot e + a^2b^2c^2e^2 + (a^2b^3 - 3a^2b^2c) \cdot f^2 + 2(a^2b^2c^2d - (a^2b^2c - 2a^2c^2) \cdot e) \cdot f - (a^2b^2c^3 - 4a^2c^4) \cdot \sqrt{(c^6d^4 - 2a^2c^5d^2e^2 + a^2c^4e^4 + (a^2b^4 - 2a^3b^2c + a^4c^2) \cdot f^4 + 4((a^2b^2c^2 - a^3c^3) \cdot d - (a^2b^3c - a^3b^2c^2) \cdot e) \cdot f^3 - 2(4a^2b^2c^3d \cdot e + (a^2b^2c^3 - 3a^2c^4) \cdot d^2 - (3a^2b^2c^2 - a^3c^3) \cdot e^2) \cdot f^2 - 4(a^2c^5d^3 - a^2b^2c^4d^2e - a^2c^4d \cdot e^2 + a^2b^2c^3e^3) \cdot f) / (a^2b^2c^6 - 4a^3c^7))} / (a^2b^2c^3 - 4a^2c^4))} - 2f \cdot x) / c
\end{aligned}$$

giac [B] time = 3.91, size = 4086, normalized size = 18.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

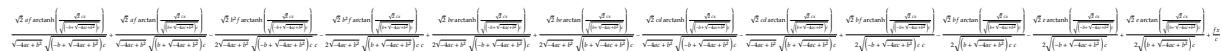
[In] integrate((f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] $f \cdot x / c - 1/8 \cdot ((2b^5c^2 - 16a^2b^3c^3 + 32a^2b^2c^4 - \sqrt{2}) \cdot \sqrt{b^2 - 4ac}) \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{b^2 - 4ac} \cdot b^5 + 8 \cdot \sqrt{2} \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{b^2 - 4ac} \cdot b^4c - 16 \cdot \sqrt{2} \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{b^2 - 4ac} \cdot b^3c^2 + 4 \cdot \sqrt{2} \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{b^2 - 4ac} \cdot b^2c^2 - \sqrt{2} \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{b^2 - 4ac} \cdot b^3c^2 + 4 \cdot \sqrt{2} \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{b^2 - 4ac} \cdot b^2c^2 + 8 \cdot (b^2 - 4ac) \cdot a \cdot b \cdot c^3 \cdot c^2 \cdot f - (2b^4c^3 - 16a^2b^2c^4 + 32a^2c^5 - \sqrt{2}) \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{b^2 - 4ac} \cdot b^4c + 8 \cdot \sqrt{2} \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{b^2 - 4ac} \cdot b^4c$

$$\begin{aligned}
& (b^2 - 4ac)c) * a * b^2 * c^2 + 2 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * b^3 * c^2 - 16 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^2 * c^3 - 8 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a * b * c^3 - \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * b^2 * c^3 + 4 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a * c^4 - 2 * (b^2 - 4ac) * b^2 * c^3 + 8 * (b^2 - 4ac) * a * c^4) * c^2 * e - 2 * (\sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * b^4 * c^3 - 8 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a * b^2 * c^4 - 2 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * b^3 * c^4 + 2 * b^4 * c^4 + 16 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^2 * c^5 + 8 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a * b * c^5 + \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * b^2 * c^5 - 16 * a * b^2 * c^5 - 4 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a * c^6 + 3 * 2 * a^2 * c^6 - 2 * (b^2 - 4ac) * b^2 * c^4 + 8 * (b^2 - 4ac) * a * c^5) * d * \text{abs}(c) + 2 * (\sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a * b^4 * c^2 - 8 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^2 * b^2 * c^3 - 2 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a * b^3 * c^3 + 2 * a * b^4 * c^3 + 16 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^3 * c^4 + 8 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^2 * b * c^4 + \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a * b^2 * c^4 - 16 * a^2 * b^2 * c^4 - 4 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^2 * c^5 + 32 * a^3 * c^5 - 2 * (b^2 - 4ac) * a * b^2 * c^3 + 8 * (b^2 - 4ac) * a^2 * c^4) * f * \text{abs}(c) - 2 * (2 * b^3 * c^6 - 8 * a * b * c^7 - \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * b^3 * c^4 + 4 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a * b * c^5 + 2 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * b^2 * c^5 - \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * b * c^6 - 2 * (b^2 - 4ac) * b * c^6) * d - (2 * b^5 * c^4 - 12 * a * b^3 * c^5 + 16 * a^2 * b * c^6 - \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * b^5 * c^2 + 6 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a * b^3 * c^3 + 2 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * b^4 * c^3 - 8 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^2 * b * c^4 - 4 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a * b^2 * c^4 - \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * b^3 * c^4 + 2 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a * b * c^5 - 2 * (b^2 - 4ac) * b^3 * c^4 + 4 * (b^2 - 4ac) * a * b * c^5) * f + (2 * b^4 * c^5 - 8 * a * b^2 * c^6 - \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * b^4 * c^3 + 4 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a * b^2 * c^4 + 2 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * b^3 * c^4 - \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * b^2 * c^5 - 2 * (b^2 - 4ac) * b^2 * c^5) * e) * \arctan(2 * \sqrt{1/2} * x / \sqrt{(b * c + \sqrt{b^2 * c^2 - 4 * a * c^3}) / c^2}) / ((a * b^4 * c^3 - 8 * a^2 * b^2 * c^4 - 2 * a * b^3 * c^4 + 16 * a^3 * c^5 + 8 * a^2 * b * c^5 + a * b^2 * c^5 - 4 * a^2 * c^6) * c^2) + 1/8 * ((2 * b^5 * c^2 - 16 * a * b^3 * c^3 + 32 * a^2 * b * c^4 - \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c) * b^5 + 8 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c) * a * b^3 * c + 2 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c) * b^4 * c - 16 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c) * a^2 * b * c^2 - 8 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c) * a * b^2 * c^2 - \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c) * b^3 * c^2 + 4 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c) * a * b * c^3 - 2 * (b^2 - 4ac) * b^3 * c^2 + 8 * (b^2 - 4ac) * a * b * c^3) * c^2 * f - (2 * b^4 * c^3 - 16 * a * b^2 * c^4 + 32 * a^2 * c^5 - \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c) * b^4 * c + 8 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c) * a * b^2 * c^2 + 2 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c) * b^3 * c^2 - 16 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c) * a^2 * c^3 - 8 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c) * a * b * c^3 - \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c) * b^2 * c^3 + 4 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c) * a * c^4 - 2 * (b^2 - 4ac) * b^2 * c^3 + 8 * (b^2 - 4ac) * a * c^4) * c^2 * e + 2 * (\sqrt{2} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c) * b^4 * c^3 - 8 * \sqrt{2} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c) * a * b^2 * c^4 - 2 * \sqrt{2} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c) * b^3 * c^4 - 2 * b^4 * c^4 + 16 * \sqrt{2} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c) * a^2 * c^5 + 8 * \sqrt{2} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c) * a * b * c^5 + \sqrt{2} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c) * b^2 * c^5 + 16 * a * b^2 * c^5 - 4 * \sqrt{2} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c) * a * c^6 - 32 * a^2 * c^6 + 2 * (b^2 - 4ac) * b^2 * c^4 - 8 * (b^2 - 4ac) * a * c^5) *
\end{aligned}$$

d*abs(c) - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a*b^4*c^2 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^2*b^2*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a*b^3*c^3 - 2*a*b^4*c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^3*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^2*b*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a*b^2*c^4 + 16*a^2*b^2*c^4 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^2*c^5 - 32*a^3*c^5 + 2*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b^2 - 4*a*c)*a^2*c^4)*f*abs(c) - 2*(2*b^3*c^6 - 8*a*b*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*b^3*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a*b*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*b*c^6 - 2*(b^2 - 4*a*c)*b*c^6)*d - (2*b^5*c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^2*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^5)*f + (2*b^4*c^5 - 8*a*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a*b^2*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*b^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*b^2*c^5 - 2*(b^2 - 4*a*c)*b^2*c^5)*e)*arctan(2*sqrt(1/2)*x/sqrt((b*c - sqrt(b^2*c^2 - 4*a*c^3))/c^2))/((a*b^4*c^3 - 8*a^2*b^2*c^4 - 2*a*b^3*c^4 + 16*a^3*c^5 + 8*a^2*b*c^5 + a*b^2*c^5 - 4*a^2*c^6)*c^2)

maple [B] time = 0.03, size = 676, normalized size = 3.09



Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^4+e*x^2+d)/(c*x^4+b*x^2+a), x)

[Out] f*x/c+1/2/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*f-1/2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*e+1/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*f-1/2/(-4*a*c+b^2)^(1/2)/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*f+1/2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*e-1/(-4*a*c+b^2)^(1/2)*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*d-1/2/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*f+1/2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*e+1/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*f-1/2/(-4*a*c+b^2)^(1/2)/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*f+1/2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*e-1/(-4*a*c+b^2)^(1/2)*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{fx}{c} - \int \frac{(ce-bf)x^2+cd-af}{cx^4+bx^2+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] f*x/c - integrate(-((c*e - b*f)*x^2 + c*d - a*f)/(c*x^4 + b*x^2 + a), x)/c

mupad [B] time = 3.36, size = 10209, normalized size = 46.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x^2 + f*x^4)/(a + b*x^2 + c*x^4),x)

[Out]
$$\frac{f*x}{c} - \operatorname{atan}\left(\frac{\left(\left(\left(4*b^2*c^3*d + 16*a^2*c^3*f - 16*a*c^4*d - 4*a*b^2*c^2*f\right)/c - \left(2*x*(4*b^3*c^3 - 16*a*b*c^4)\right)*\left(-\left(a*b^5*f^2 + b^3*c^3*d^2 - c^3*d^2*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^2 + a*b^2*f^2*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} + a*c^2*e^2*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} - 7*a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 - a^2*c*f^2*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} - 4*a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b^3*c^2*d*f - 8*a^2*b*c^3*d*f + 2*a*c^2*d*f*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} + 12*a^2*b^2*c^2*e*f - 2*a*b^4*c*e*f - 2*a*b*c*e*f*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2}\right)\right)/\left(8*\left(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4\right)\right)^{1/2}\right)/c*\left(-\left(a*b^5*f^2 + b^3*c^3*d^2 - c^3*d^2*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^2 + a*b^2*f^2*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} + a*c^2*e^2*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} - 7*a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 - a^2*c*f^2*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} - 4*a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b^3*c^2*d*f - 8*a^2*b*c^3*d*f + 2*a*c^2*d*f*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} + 12*a^2*b^2*c^2*e*f - 2*a*b^4*c*e*f - 2*a*b*c*e*f*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2}\right)\right)/\left(8*\left(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4\right)\right)^{1/2} - \left(2*x*\left(2*c^4*d^2 + b^4*f^2 - 2*a*c^3*e^2 + 2*a^2*c^2*f^2 + b^2*c^2*e^2 - 4*a*c^3*d*f - 2*b*c^3*d*e - 2*b^3*c*e*f - 4*a*b^2*c*f^2 + 2*b^2*c^2*d*f + 6*a*b*c^2*e*f\right)\right)/c*\left(-\left(a*b^5*f^2 + b^3*c^3*d^2 - c^3*d^2*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^2 + a*b^2*f^2*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} + a*c^2*e^2*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} - 7*a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 - a^2*c*f^2*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} - 4*a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b^3*c^2*d*f - 8*a^2*b*c^3*d*f + 2*a*c^2*d*f*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} + 12*a^2*b^2*c^2*e*f - 2*a*b^4*c*e*f - 2*a*b*c*e*f*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2}\right)\right)/\left(8*\left(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4\right)\right)^{1/2}\right)*1i - \left(\left(\left(4*b^2*c^3*d + 16*a^2*c^3*f - 16*a*c^4*d - 4*a*b^2*c^2*f\right)/c + \left(2*x*\left(4*b^3*c^3 - 16*a*b*c^4\right)\right)*\left(-\left(a*b^5*f^2 + b^3*c^3*d^2 - c^3*d^2*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^2 + a*b^2*f^2*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} + a*c^2*e^2*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} - 7*a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 - a^2*c*f^2*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} - 4*a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b^3*c^2*d*f - 8*a^2*b*c^3*d*f + 2*a*c^2*d*f*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} + 12*a^2*b^2*c^2*e*f - 2*a*b^4*c*e*f - 2*a*b*c*e*f*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2}\right)\right)/\left(8*\left(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4\right)\right)^{1/2}\right)/c + \left(2*x*\left(2*c^4*d^2 + b^4*f^2 - 2*a*c^3*e^2 + 2*a^2*c^2*f^2 + b^2*c^2*e^2 - 4*a*c^3*d*f - 2*b*c^3*d*e - 2*b^3*c*e*f - 4*a*b^2*c*f^2 + 2*b^2*c^2*d*f + 6*a*b*c^2*e*f\right)\right)/c*\left(-\left(a*b^5*f^2 + b^3*c^3*d^2 - c^3*d^2*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^2 + a*b^2*f^2*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} + a*c^2*e^2*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} - 7*a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 - a^2*c*f^2*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} - 4*a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b^3*c^2*d*f - 8*a^2*b*c^3*d*f + 2*a*c^2*d*f*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} + 12*a^2*b^2*c^2*e*f - 2*a*b^4*c*e*f - 2*a*b*c*e*f*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2}\right)\right)/\left(8*\left(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4\right)\right)^{1/2}\right)*1i\right)/\left(\left(\left(4*b^2*c^3*d + 16*a^2*c^3*f - 16*a*c^4*d - 4*a*b^2*c^2*f\right)/c - \left(2*x*\left(4*b^3*c^3 - 16*a*b*c^4\right)\right)*\left(-\left(a*b^5*f^2 + b^3*c^3*d^2 - c^3*d^2*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^2 + a*b^2*f^2*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} + a*c^2*e^2*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} - 7*a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 - a^2*c*f^2*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} - 4*a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b^3*c^2*d*f - 8*a^2*b*c^3*d*f + 2*a*c^2*d*f*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} + 12*a^2*b^2*c^2*e*f - 2*a*b^4*c*e*f - 2*a*b*c*e*f*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2}\right)\right)/\left(8*\left(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4\right)\right)^{1/2}\right)*1i\right)/\left(\left(\left(4*b^2*c^3*d + 16*a^2*c^3*f - 16*a*c^4*d - 4*a*b^2*c^2*f\right)/c - \left(2*x*\left(4*b^3*c^3 - 16*a*b*c^4\right)\right)*\left(-\left(a*b^5*f^2 + b^3*c^3*d^2 - c^3*d^2*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^2 + a*b^2*f^2*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} + a*c^2*e^2*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} - 7*a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 - a^2*c*f^2*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} - 4*a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b^3*c^2*d*f - 8*a^2*b*c^3*d*f + 2*a*c^2*d*f*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} + 12*a^2*b^2*c^2*e*f - 2*a*b^4*c*e*f - 2*a*b*c*e*f*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2}\right)\right)/\left(8*\left(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4\right)\right)^{1/2}\right)*1i\right)$$

$$\begin{aligned}
& *b^5*f^2 + b^3*c^3*d^2 - c^3*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + a*b^3*c^2*e^2 - \\
& 4*a^2*b*c^3*e^2 + a*b^2*f^2*(-(4*a*c - b^2)^3)^{(1/2)} + a*c^2*e^2*(-(4*a*c - \\
& - b^2)^3)^{(1/2)} - 7*a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 - a^2*c*f^2*(-(4*a*c - \\
& - b^2)^3)^{(1/2)} - 4*a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2* \\
& c^3*d*e + 2*a*b^3*c^2*d*f - 8*a^2*b*c^3*d*f + 2*a*c^2*d*f*(-(4*a*c - b^2)^3 \\
&)^{(1/2)} + 12*a^2*b^2*c^2*e*f - 2*a*b^4*c*e*f - 2*a*b*c*e*f*(-(4*a*c - b^2)^ \\
& 3)^{(1/2))}/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4))^{(1/2)}/c)*(-(a*b^5* \\
& f^2 + b^3*c^3*d^2 - c^3*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + a*b^3*c^2*e^2 - 4*a^ \\
& 2*b*c^3*e^2 + a*b^2*f^2*(-(4*a*c - b^2)^3)^{(1/2)} + a*c^2*e^2*(-(4*a*c - b^2 \\
&)^3)^{(1/2)} - 7*a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 - a^2*c*f^2*(-(4*a*c - b^2) \\
& ^3)^{(1/2)} - 4*a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d \\
& *e + 2*a*b^3*c^2*d*f - 8*a^2*b*c^3*d*f + 2*a*c^2*d*f*(-(4*a*c - b^2)^3)^{(1/ \\
& 2)} + 12*a^2*b^2*c^2*e*f - 2*a*b^4*c*e*f - 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^{(1 \\
& /2)))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4))^{(1/2)} - (2*x*(2*c^4*d^2 \\
& + b^4*f^2 - 2*a*c^3*e^2 + 2*a^2*c^2*f^2 + b^2*c^2*e^2 - 4*a*c^3*d*f - 2*b*c \\
& ^3*d*e - 2*b^3*c*e*f - 4*a*b^2*c*f^2 + 2*b^2*c^2*d*f + 6*a*b*c^2*e*f))/c)* \\
& -(a*b^5*f^2 + b^3*c^3*d^2 - c^3*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + a*b^3*c^2*e^ \\
& 2 - 4*a^2*b*c^3*e^2 + a*b^2*f^2*(-(4*a*c - b^2)^3)^{(1/2)} + a*c^2*e^2*(-(4*a \\
& *c - b^2)^3)^{(1/2)} - 7*a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 - a^2*c*f^2*(-(4*a* \\
& c - b^2)^3)^{(1/2)} - 4*a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b \\
& ^2*c^3*d*e + 2*a*b^3*c^2*d*f - 8*a^2*b*c^3*d*f + 2*a*c^2*d*f*(-(4*a*c - b^2 \\
&)^3)^{(1/2)} + 12*a^2*b^2*c^2*e*f - 2*a*b^4*c*e*f - 2*a*b*c*e*f*(-(4*a*c - b^ \\
& 2)^3)^{(1/2)))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4))^{(1/2)} - (2*(a*c^ \\
& 2*e^3 - a^2*b*f^3 - b^3*d*f^2 + c^3*d^2*e + a*b^2*e*f^2 - b*c^2*d*e^2 - b*c \\
& ^2*d^2*f + a^2*c*e*f^2 + 2*a*b*c*d*f^2 - 2*a*b*c*e^2*f - 2*a*c^2*d*e*f + 2* \\
& b^2*c*d*e*f))/c + (((4*b^2*c^3*d + 16*a^2*c^3*f - 16*a*c^4*d - 4*a*b^2*c^2* \\
& f)/c + (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(a*b^5*f^2 + b^3*c^3*d^2 - c^3*d^2*(\\
& -(4*a*c - b^2)^3)^{(1/2)} + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^2 + a*b^2*f^2*(-(4* \\
& a*c - b^2)^3)^{(1/2)} + a*c^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 7*a^2*b^3*c*f^2 \\
& + 12*a^3*b*c^2*f^2 - a^2*c*f^2*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a*b*c^4*d^2 + 1 \\
& 6*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b^3*c^2*d*f - 8*a^2* \\
& b*c^3*d*f + 2*a*c^2*d*f*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b^2*c^2*e*f - 2*a \\
& *b^4*c*e*f - 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^{(1/2)))/(8*(16*a^3*c^5 + a*b^4*c \\
& ^3 - 8*a^2*b^2*c^4))^{(1/2)}/c)*(-(a*b^5*f^2 + b^3*c^3*d^2 - c^3*d^2*(-(4*a \\
& *c - b^2)^3)^{(1/2)} + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^2 + a*b^2*f^2*(-(4*a*c - \\
& b^2)^3)^{(1/2)} + a*c^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 7*a^2*b^3*c*f^2 + 12* \\
& a^3*b*c^2*f^2 - a^2*c*f^2*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a*b*c^4*d^2 + 16*a^2 \\
& *c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b^3*c^2*d*f - 8*a^2*b*c^3 \\
& *d*f + 2*a*c^2*d*f*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b^2*c^2*e*f - 2*a*b^4* \\
& c*e*f - 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^{(1/2)))/(8*(16*a^3*c^5 + a*b^4*c^3 - \\
& 8*a^2*b^2*c^4))^{(1/2)} + (2*x*(2*c^4*d^2 + b^4*f^2 - 2*a*c^3*e^2 + 2*a^2*c^ \\
& 2*f^2 + b^2*c^2*e^2 - 4*a*c^3*d*f - 2*b*c^3*d*e - 2*b^3*c*e*f - 4*a*b^2*c*f \\
& ^2 + 2*b^2*c^2*d*f + 6*a*b*c^2*e*f))/c)*(-(a*b^5*f^2 + b^3*c^3*d^2 - c^3*d^ \\
& 2*(-(4*a*c - b^2)^3)^{(1/2)} + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^2 + a*b^2*f^2*(- \\
& (4*a*c - b^2)^3)^{(1/2)} + a*c^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 7*a^2*b^3*c*f \\
& ^2 + 12*a^3*b*c^2*f^2 - a^2*c*f^2*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a*b*c^4*d^2 \\
& + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b^3*c^2*d*f - 8*a \\
& ^2*b*c^3*d*f + 2*a*c^2*d*f*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b^2*c^2*e*f - \\
& 2*a*b^4*c*e*f - 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^{(1/2)))/(8*(16*a^3*c^5 + a*b^ \\
& 4*c^3 - 8*a^2*b^2*c^4))^{(1/2)))*(-(a*b^5*f^2 + b^3*c^3*d^2 - c^3*d^2*(-(4* \\
& a*c - b^2)^3)^{(1/2)} + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^2 + a*b^2*f^2*(-(4*a*c \\
& - b^2)^3)^{(1/2)} + a*c^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 7*a^2*b^3*c*f^2 + 12 \\
& *a^3*b*c^2*f^2 - a^2*c*f^2*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a*b*c^4*d^2 + 16*a^ \\
& 2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b^3*c^2*d*f - 8*a^2*b*c^ \\
& 3*d*f + 2*a*c^2*d*f*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b^2*c^2*e*f - 2*a*b^4 \\
& *c*e*f - 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^{(1/2)))/(8*(16*a^3*c^5 + a*b^4*c^3 - \\
& 8*a^2*b^2*c^4))^{(1/2)}*i - \operatorname{atan}((((4*b^2*c^3*d + 16*a^2*c^3*f - 16*a*c^4 \\
& *d - 4*a*b^2*c^2*f)/c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(a*b^5*f^2 + b^3*c^ \\
& 3*d^2 + c^3*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^2
\end{aligned}$$

$$\begin{aligned}
& - a*b^2*f^2*(-(4*a*c - b^2)^3)^{(1/2)} - a*c^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - \\
& 7*a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 + a^2*c*f^2*(-(4*a*c - b^2)^3)^{(1/2)} - \\
& 4*a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b^3* \\
& c^2*d*f - 8*a^2*b*c^3*d*f - 2*a*c^2*d*f*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2* \\
& b^2*c^2*e*f - 2*a*b^4*c*e*f + 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16* \\
& a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4))^{(1/2)}/c*(-(a*b^5*f^2 + b^3*c^3*d^2 \\
& + c^3*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^2 - a*b \\
& ^2*f^2*(-(4*a*c - b^2)^3)^{(1/2)} - a*c^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 7*a^ \\
& 2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 + a^2*c*f^2*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a*b \\
& *c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b^3*c^2* \\
& d*f - 8*a^2*b*c^3*d*f - 2*a*c^2*d*f*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b^2*c \\
& ^2*e*f - 2*a*b^4*c*e*f + 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16*a^3*c \\
& ^5 + a*b^4*c^3 - 8*a^2*b^2*c^4))^{(1/2)} - (2*x*(2*c^4*d^2 + b^4*f^2 - 2*a*c \\
& ^3*e^2 + 2*a^2*c^2*f^2 + b^2*c^2*e^2 - 4*a*c^3*d*f - 2*b*c^3*d*e - 2*b^3*c* \\
& e*f - 4*a*b^2*c*f^2 + 2*b^2*c^2*d*f + 6*a*b*c^2*e*f))/c*(-(a*b^5*f^2 + b^3 \\
& *c^3*d^2 + c^3*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e \\
& ^2 - a*b^2*f^2*(-(4*a*c - b^2)^3)^{(1/2)} - a*c^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} \\
&) - 7*a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 + a^2*c*f^2*(-(4*a*c - b^2)^3)^{(1/2)} \\
& - 4*a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a* \\
& b^3*c^2*d*f - 8*a^2*b*c^3*d*f - 2*a*c^2*d*f*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a \\
& ^2*b^2*c^2*e*f - 2*a*b^4*c*e*f + 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(\\
& 16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4))^{(1/2)}*i - (((4*b^2*c^3*d + 16*a^ \\
& 2*c^3*f - 16*a*c^4*d - 4*a*b^2*c^2*f)/c + (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(- \\
& a*b^5*f^2 + b^3*c^3*d^2 + c^3*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + a*b^3*c^2*e^2 \\
& - 4*a^2*b*c^3*e^2 - a*b^2*f^2*(-(4*a*c - b^2)^3)^{(1/2)} - a*c^2*e^2*(-(4*a*c \\
& - b^2)^3)^{(1/2)} - 7*a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 + a^2*c*f^2*(-(4*a*c \\
& - b^2)^3)^{(1/2)} - 4*a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2 \\
& *c^3*d*e + 2*a*b^3*c^2*d*f - 8*a^2*b*c^3*d*f - 2*a*c^2*d*f*(-(4*a*c - b^2)^ \\
& 3)^{(1/2)} + 12*a^2*b^2*c^2*e*f - 2*a*b^4*c*e*f + 2*a*b*c*e*f*(-(4*a*c - b^2) \\
& ^3)^{(1/2)})/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4))^{(1/2)}/c*(-(a*b^5 \\
& *f^2 + b^3*c^3*d^2 + c^3*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + a*b^3*c^2*e^2 - 4*a \\
& ^2*b*c^3*e^2 - a*b^2*f^2*(-(4*a*c - b^2)^3)^{(1/2)} - a*c^2*e^2*(-(4*a*c - b^ \\
& 2)^3)^{(1/2)} - 7*a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 + a^2*c*f^2*(-(4*a*c - b^2) \\
&)^3)^{(1/2)} - 4*a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3* \\
& d*e + 2*a*b^3*c^2*d*f - 8*a^2*b*c^3*d*f - 2*a*c^2*d*f*(-(4*a*c - b^2)^3)^{(1 \\
& /2)} + 12*a^2*b^2*c^2*e*f - 2*a*b^4*c*e*f + 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^{(\\
& 1/2)})/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4))^{(1/2)} + (2*x*(2*c^4*d^2 \\
& + b^4*f^2 - 2*a*c^3*e^2 + 2*a^2*c^2*f^2 + b^2*c^2*e^2 - 4*a*c^3*d*f - 2*b* \\
& c^3*d*e - 2*b^3*c*e*f - 4*a*b^2*c*f^2 + 2*b^2*c^2*d*f + 6*a*b*c^2*e*f))/c* \\
& (- (a*b^5*f^2 + b^3*c^3*d^2 + c^3*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + a*b^3*c^2*e \\
& ^2 - 4*a^2*b*c^3*e^2 - a*b^2*f^2*(-(4*a*c - b^2)^3)^{(1/2)} - a*c^2*e^2*(-(4* \\
& a*c - b^2)^3)^{(1/2)} - 7*a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 + a^2*c*f^2*(-(4*a \\
& *c - b^2)^3)^{(1/2)} - 4*a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a* \\
& b^2*c^3*d*e + 2*a*b^3*c^2*d*f - 8*a^2*b*c^3*d*f - 2*a*c^2*d*f*(-(4*a*c - b^ \\
& 2)^3)^{(1/2)} + 12*a^2*b^2*c^2*e*f - 2*a*b^4*c*e*f + 2*a*b*c*e*f*(-(4*a*c - b \\
& ^2)^3)^{(1/2)})/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4))^{(1/2)}*i)/(((4 \\
& *b^2*c^3*d + 16*a^2*c^3*f - 16*a*c^4*d - 4*a*b^2*c^2*f)/c - (2*x*(4*b^3*c^3 \\
& - 16*a*b*c^4)*(- (a*b^5*f^2 + b^3*c^3*d^2 + c^3*d^2*(-(4*a*c - b^2)^3)^{(1/2)} \\
&) + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^2 - a*b^2*f^2*(-(4*a*c - b^2)^3)^{(1/2)} - \\
& a*c^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 7*a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 + a \\
& ^2*c*f^2*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3 \\
& *c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b^3*c^2*d*f - 8*a^2*b*c^3*d*f - 2*a*c^2*d* \\
& f*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b^2*c^2*e*f - 2*a*b^4*c*e*f + 2*a*b*c*e \\
& *f*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4))^{(\\
& 1/2)}/c*(-(a*b^5*f^2 + b^3*c^3*d^2 + c^3*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + a \\
& *b^3*c^2*e^2 - 4*a^2*b*c^3*e^2 - a*b^2*f^2*(-(4*a*c - b^2)^3)^{(1/2)} - a*c^2 \\
& *e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 7*a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 + a^2*c* \\
& f^2*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3* \\
& e*f - 4*a*b^2*c^3*d*e + 2*a*b^3*c^2*d*f - 8*a^2*b*c^3*d*f - 2*a*c^2*d*f*(-(
\end{aligned}$$

$$\begin{aligned}
& 4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b^2*c^2*e*f - 2*a*b^4*c*e*f + 2*a*b*c*e*f*(- \\
& (4*a*c - b^2)^3)^{(1/2)})/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4))^{(1/2)} \\
& - (2*x*(2*c^4*d^2 + b^4*f^2 - 2*a*c^3*e^2 + 2*a^2*c^2*f^2 + b^2*c^2*e^2 - \\
& 4*a*c^3*d*f - 2*b*c^3*d*e - 2*b^3*c*e*f - 4*a*b^2*c*f^2 + 2*b^2*c^2*d*f + 6 \\
& *a*b*c^2*e*f))/c)*(-(a*b^5*f^2 + b^3*c^3*d^2 + c^3*d^2*(-(4*a*c - b^2)^3)^{(1/2)} \\
& + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^2 - a*b^2*f^2*(-(4*a*c - b^2)^3)^{(1/2)} \\
& - a*c^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 7*a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 \\
& + a^2*c*f^2*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16* \\
& a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b^3*c^2*d*f - 8*a^2*b*c^3*d*f - 2*a*c^2 \\
& *d*f*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b^2*c^2*e*f - 2*a*b^4*c*e*f + 2*a*b* \\
& c*e*f*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4) \\
&))^{(1/2)} - (2*(a*c^2*e^3 - a^2*b*f^3 - b^3*d*f^2 + c^3*d^2*e + a*b^2*e*f^2 \\
& - b*c^2*d*e^2 - b*c^2*d^2*f + a^2*c*e*f^2 + 2*a*b*c*d*f^2 - 2*a*b*c*e^2*f - \\
& 2*a*c^2*d*e*f + 2*b^2*c*d*e*f))/c + (((4*b^2*c^3*d + 16*a^2*c^3*f - 16*a*c \\
& ^4*d - 4*a*b^2*c^2*f)/c + (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(a*b^5*f^2 + b^3* \\
& c^3*d^2 + c^3*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^2 \\
& - a*b^2*f^2*(-(4*a*c - b^2)^3)^{(1/2)} - a*c^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} \\
& - 7*a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 + a^2*c*f^2*(-(4*a*c - b^2)^3)^{(1/2)} \\
& - 4*a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b \\
& ^3*c^2*d*f - 8*a^2*b*c^3*d*f - 2*a*c^2*d*f*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2 \\
& *b^2*c^2*e*f - 2*a*b^4*c*e*f + 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(1 \\
& 6*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4))^{(1/2)})/c)*(-(a*b^5*f^2 + b^3*c^3*d \\
& ^2 + c^3*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^2 - a \\
& *b^2*f^2*(-(4*a*c - b^2)^3)^{(1/2)} - a*c^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 7* \\
& a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 + a^2*c*f^2*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a \\
& *b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b^3*c^2 \\
& *d*f - 8*a^2*b*c^3*d*f - 2*a*c^2*d*f*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b^2 \\
& *c^2*e*f - 2*a*b^4*c*e*f + 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16*a^3 \\
& *c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4))^{(1/2)} + (2*x*(2*c^4*d^2 + b^4*f^2 - 2*a \\
& *c^3*e^2 + 2*a^2*c^2*f^2 + b^2*c^2*e^2 - 4*a*c^3*d*f - 2*b*c^3*d*e - 2*b^3* \\
& c*e*f - 4*a*b^2*c*f^2 + 2*b^2*c^2*d*f + 6*a*b*c^2*e*f))/c)*(-(a*b^5*f^2 + b \\
& ^3*c^3*d^2 + c^3*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + a*b^3*c^2*e^2 - 4*a^2*b*c^3 \\
& *e^2 - a*b^2*f^2*(-(4*a*c - b^2)^3)^{(1/2)} - a*c^2*e^2*(-(4*a*c - b^2)^3)^{(1 \\
& /2)} - 7*a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 + a^2*c*f^2*(-(4*a*c - b^2)^3)^{(1/ \\
& 2)} - 4*a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2* \\
& a*b^3*c^2*d*f - 8*a^2*b*c^3*d*f - 2*a*c^2*d*f*(-(4*a*c - b^2)^3)^{(1/2)} + 12 \\
& *a^2*b^2*c^2*e*f - 2*a*b^4*c*e*f + 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^{(1/2)})/(8 \\
& *(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4))^{(1/2)}))*(-(a*b^5*f^2 + b^3*c^3* \\
& d^2 + c^3*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^2 - \\
& a*b^2*f^2*(-(4*a*c - b^2)^3)^{(1/2)} - a*c^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 7 \\
& *a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 + a^2*c*f^2*(-(4*a*c - b^2)^3)^{(1/2)} - 4* \\
& a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b^3*c^2 \\
& *d*f - 8*a^2*b*c^3*d*f - 2*a*c^2*d*f*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b^2 \\
& *c^2*e*f - 2*a*b^4*c*e*f + 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16*a^3 \\
& *c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4))^{(1/2)}*2i
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)

[Out] Timed out

$$3.56 \quad \int \frac{d+ex^2+fx^4}{x^2(a+bx^2+cx^4)} dx$$

Optimal. Leaf size=213

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{abf-2ace+bcd}{\sqrt{b^2-4ac}}-af+cd\right) - \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{abf-2ace+bcd}{\sqrt{b^2-4ac}}-af+cd\right)}{\sqrt{2}a\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}} - \sqrt{2}a\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}} - \frac{d}{ax}$$

Rubi [A] time = 0.84, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1664, 1166, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{abf-2ace+bcd}{\sqrt{b^2-4ac}}-af+cd\right) - \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{abf-2ace+bcd}{\sqrt{b^2-4ac}}-af+cd\right)}{\sqrt{2}a\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}} - \sqrt{2}a\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}} - \frac{d}{ax}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2 + f*x^4)/(x^2*(a + b*x^2 + c*x^4)),x]

[Out] -(d/(a*x)) - ((c*d - a*f + (b*c*d - 2*a*c*e + a*b*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*a*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((c*d - a*f - (b*c*d - 2*a*c*e + a*b*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*a*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1664

Int[(Pq_)*((d_.)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned}
\int \frac{d + ex^2 + fx^4}{x^2(a + bx^2 + cx^4)} dx &= \int \left(\frac{d}{ax^2} + \frac{-bd + ae - (cd - af)x^2}{a(a + bx^2 + cx^4)} \right) dx \\
&= -\frac{d}{ax} + \frac{\int \frac{-bd + ae + (-cd + af)x^2}{a + bx^2 + cx^4} dx}{a} \\
&= -\frac{d}{ax} - \frac{\left(cd - af - \frac{bcd - 2ace + abf}{\sqrt{b^2 - 4ac}} \right) \int \frac{1}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2 - 4ac} + cx^2} dx}{2a} + \frac{\left(-cd + af + \frac{2ace - b(cd + af)}{\sqrt{b^2 - 4ac}} \right) \int \frac{1}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2 - 4ac} + cx^2} dx}{2a} \\
&= -\frac{d}{ax} - \frac{\left(cd - af - \frac{2ace - b(cd + af)}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{\sqrt{2}a\sqrt{c}\sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{\left(cd - af - \frac{bcd - 2ace + abf}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b + \sqrt{b^2 - 4ac}}} \right)}{\sqrt{2}a\sqrt{c}\sqrt{b + \sqrt{b^2 - 4ac}}}
\end{aligned}$$

Mathematica [A] time = 0.30, size = 253, normalized size = 1.19

$$\frac{\sqrt{2} \tan^{-1} \left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right) \left(cd\sqrt{b^2 - 4ac} - af\sqrt{b^2 - 4ac} + abf - 2ace + bcd \right)}{\sqrt{c}\sqrt{b^2 - 4ac}\sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{\sqrt{2} \tan^{-1} \left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b^2 - 4ac} + b} \right) \left(-cd\sqrt{b^2 - 4ac} + af\sqrt{b^2 - 4ac} + abf - 2ace + bcd \right)}{\sqrt{c}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac} + b} - \frac{2d}{x}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2 + f*x^4)/(x^2*(a + b*x^2 + c*x^4)),x]

[Out] ((-2*d)/x - (Sqrt[2]*(b*c*d + c*Sqrt[b^2 - 4*a*c]*d - 2*a*c*e + a*b*f - a*Sqrt[b^2 - 4*a*c]*f)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b*c*d - c*Sqrt[b^2 - 4*a*c]*d - 2*a*c*e + a*b*f + a*Sqrt[b^2 - 4*a*c]*f)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*a)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex^2 + fx^4}{x^2(a + bx^2 + cx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e*x^2 + f*x^4)/(x^2*(a + b*x^2 + c*x^4)),x]

[Out] IntegrateAlgebraic[(d + e*x^2 + f*x^4)/(x^2*(a + b*x^2 + c*x^4)), x]

fricas [B] time = 2.21, size = 5930, normalized size = 27.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] -1/2*(sqrt(1/2)*a*x*sqrt(-(a^2*b*c*e^2 + a^3*b*f^2 + (b^3*c - 3*a*b*c^2)*d^2 - 2*(a*b^2*c - 2*a^2*c^2)*d*e + 2*(a^2*b*c*d - 2*a^3*c*e)*f + (a^3*b^2*c - 4*a^4*c^2)*sqrt(-(4*a^3*b*c^2*d*e^3 - a^4*c^2*e^4 + 4*a^5*c*d*f^3 - a^6*f^4 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^4 + 4*(a*b^3*c^2 - a^2*b*c^3)*d^3*e - 2*(3*a^2*b^2*c^2 - a^3*c^3)*d^2*e^2 - 2*(2*a^4*b*c*d*e - a^5*c*e^2 - (a^3*b^2*c - 3*a^4*c^2)*d^2)*f^2 + 4*(2*a^3*b*c^2*d^2*e - a^4*c^2*d*e^2 - (a^2*b^2*c^2 - a^3*c^3)*d^3)*f)/(a^6*b^2*c^2 - 4*a^7*c^3)))/(a^3*b^2*c - 4*a^4*c^2))*log(-2*(3*a*b^2*c^2*d^2*e^2 - 3*a^2*b*c^2*d*e^3 + a^3*c^2*e^4 - a^5*c


```

*c^2 + 4*a^2*c^3)*d^3 - 3*(a*b^3*c - a^2*b*c^2)*d^2*e)*f)*x + sqrt(1/2)*((b
^5*c - 5*a*b^3*c^2 + 4*a^2*b*c^3)*d^3 - (3*a*b^4*c - 13*a^2*b^2*c^2 + 4*a^3
*c^3)*d^2*e + 3*(a^2*b^3*c - 4*a^3*b*c^2)*d*e^2 - (a^3*b^2*c - 4*a^4*c^2)*e
^3 - ((a^3*b^3 - 4*a^4*b*c)*d - (a^4*b^2 - 4*a^5*c)*e)*f^2 + 2*((a^2*b^3*c
- 4*a^3*b*c^2)*d^2 - (a^3*b^2*c - 4*a^4*c^2)*d*e)*f + ((a^3*b^4*c - 6*a^4*b
^2*c^2 + 8*a^5*c^3)*d - (a^4*b^3*c - 4*a^5*b*c^2)*e + 2*(a^5*b^2*c - 4*a^6*
c^2)*f)*sqrt(-(4*a^3*b*c^2*d*e^3 - a^4*c^2*e^4 + 4*a^5*c*d*f^3 - a^6*f^4 -
(b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^4 + 4*(a*b^3*c^2 - a^2*b*c^3)*d^3*e - 2
*(3*a^2*b^2*c^2 - a^3*c^3)*d^2*e^2 - 2*(2*a^4*b*c*d*e - a^5*c*e^2 - (a^3*b^
2*c - 3*a^4*c^2)*d^2)*f^2 + 4*(2*a^3*b*c^2*d^2*e - a^4*c^2*d*e^2 - (a^2*b^2
*c^2 - a^3*c^3)*d^3)*f)/(a^6*b^2*c^2 - 4*a^7*c^3))*sqrt(-(a^2*b*c*e^2 + a^
3*b*f^2 + (b^3*c - 3*a*b*c^2)*d^2 - 2*(a*b^2*c - 2*a^2*c^2)*d*e + 2*(a^2*b*
c*d - 2*a^3*c*e)*f - (a^3*b^2*c - 4*a^4*c^2)*sqrt(-(4*a^3*b*c^2*d*e^3 - a^4
*c^2*e^4 + 4*a^5*c*d*f^3 - a^6*f^4 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^4
+ 4*(a*b^3*c^2 - a^2*b*c^3)*d^3*e - 2*(3*a^2*b^2*c^2 - a^3*c^3)*d^2*e^2 - 2
*(2*a^4*b*c*d*e - a^5*c*e^2 - (a^3*b^2*c - 3*a^4*c^2)*d^2)*f^2 + 4*(2*a^3*b
*c^2*d^2*e - a^4*c^2*d*e^2 - (a^2*b^2*c^2 - a^3*c^3)*d^3)*f)/(a^6*b^2*c^2 -
4*a^7*c^3)))/(a^3*b^2*c - 4*a^4*c^2))) - sqrt(1/2)*a*x*sqrt(-(a^2*b*c*e^2
+ a^3*b*f^2 + (b^3*c - 3*a*b*c^2)*d^2 - 2*(a*b^2*c - 2*a^2*c^2)*d*e + 2*(a^
2*b*c*d - 2*a^3*c*e)*f - (a^3*b^2*c - 4*a^4*c^2)*sqrt(-(4*a^3*b*c^2*d*e^3 -
a^4*c^2*e^4 + 4*a^5*c*d*f^3 - a^6*f^4 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*
d^4 + 4*(a*b^3*c^2 - a^2*b*c^3)*d^3*e - 2*(3*a^2*b^2*c^2 - a^3*c^3)*d^2*e^2
- 2*(2*a^4*b*c*d*e - a^5*c*e^2 - (a^3*b^2*c - 3*a^4*c^2)*d^2)*f^2 + 4*(2*a
^3*b*c^2*d^2*e - a^4*c^2*d*e^2 - (a^2*b^2*c^2 - a^3*c^3)*d^3)*f)/(a^6*b^2*c
^2 - 4*a^7*c^3)))/(a^3*b^2*c - 4*a^4*c^2))) *log(-2*(3*a*b^2*c^2*d^2*e^2 - 3*
a^2*b*c^2*d*e^3 + a^3*c^2*e^4 - a^5*f^4 + (b^2*c^3 - a*c^4)*d^4 - (b^3*c^2
+ a*b*c^3)*d^3*e + (a^4*b*e - (a^3*b^2 - 4*a^4*c)*d)*f^3 - 3*(a^3*b*c*d*e -
(a^2*b^2*c - 2*a^3*c^2)*d^2)*f^2 + (3*a^2*b^2*c*d*e^2 - a^3*b*c*e^3 + (b^4
*c - 3*a*b^2*c^2 + 4*a^2*c^3)*d^3 - 3*(a*b^3*c - a^2*b*c^2)*d^2*e)*f)*x - s
qrt(1/2)*((b^5*c - 5*a*b^3*c^2 + 4*a^2*b*c^3)*d^3 - (3*a*b^4*c - 13*a^2*b^2
*c^2 + 4*a^3*c^3)*d^2*e + 3*(a^2*b^3*c - 4*a^3*b*c^2)*d*e^2 - (a^3*b^2*c -
4*a^4*c^2)*e^3 - ((a^3*b^3 - 4*a^4*b*c)*d - (a^4*b^2 - 4*a^5*c)*e)*f^2 + 2*
((a^2*b^3*c - 4*a^3*b*c^2)*d^2 - (a^3*b^2*c - 4*a^4*c^2)*d*e)*f + ((a^3*b^4
*c - 6*a^4*b^2*c^2 + 8*a^5*c^3)*d - (a^4*b^3*c - 4*a^5*b*c^2)*e + 2*(a^5*b^
2*c - 4*a^6*c^2)*f)*sqrt(-(4*a^3*b*c^2*d*e^3 - a^4*c^2*e^4 + 4*a^5*c*d*f^3
- a^6*f^4 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^4 + 4*(a*b^3*c^2 - a^2*b*c^
3)*d^3*e - 2*(3*a^2*b^2*c^2 - a^3*c^3)*d^2*e^2 - 2*(2*a^4*b*c*d*e - a^5*c*
e^2 - (a^3*b^2*c - 3*a^4*c^2)*d^2)*f^2 + 4*(2*a^3*b*c^2*d^2*e - a^4*c^2*d*
e^2 - (a^2*b^2*c^2 - a^3*c^3)*d^3)*f)/(a^6*b^2*c^2 - 4*a^7*c^3))*sqrt(-(a^2*
b*c*e^2 + a^3*b*f^2 + (b^3*c - 3*a*b*c^2)*d^2 - 2*(a*b^2*c - 2*a^2*c^2)*d*e
+ 2*(a^2*b*c*d - 2*a^3*c*e)*f - (a^3*b^2*c - 4*a^4*c^2)*sqrt(-(4*a^3*b*c^2
*d*e^3 - a^4*c^2*e^4 + 4*a^5*c*d*f^3 - a^6*f^4 - (b^4*c^2 - 2*a*b^2*c^3 + a
^2*c^4)*d^4 + 4*(a*b^3*c^2 - a^2*b*c^3)*d^3*e - 2*(3*a^2*b^2*c^2 - a^3*c^3)
*d^2*e^2 - 2*(2*a^4*b*c*d*e - a^5*c*e^2 - (a^3*b^2*c - 3*a^4*c^2)*d^2)*f^2
+ 4*(2*a^3*b*c^2*d^2*e - a^4*c^2*d*e^2 - (a^2*b^2*c^2 - a^3*c^3)*d^3)*f)/(a
^6*b^2*c^2 - 4*a^7*c^3)))/(a^3*b^2*c - 4*a^4*c^2))) + 2*d)/(a*x)

```

giac [B] time = 5.94, size = 3988, normalized size = 18.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -d/(a*x) - 1/8*((2*b^4*c^3 - 16*a*b^2*c^4 + 32*a^2*c^5 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^

```


$$\begin{aligned}
& 2 - 4ac)c)b^2c^3 + 4\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)ac^4 - 2(b^2 - 4ac)b^2c^3 + 8(b^2 - 4ac)a^2c^4)a^2d - (\\
& 2ab^4c^2 - 16a^2b^2c^3 + 32a^3c^4 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)ab^4 + 8\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c) \\
& a^2b^2c + 2\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)ab^3c - 16\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^3c^2 - 8\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c) \\
& a^2b^2c^2 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)ab^2c^2 + 4\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^2c^3 - 2(b^2 - 4ac)ab^2c^2 + 8(b^2 - 4ac)a^2c^3)a^2f + 2(\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)ab^5c - 8\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^2b^3c^2 - 2\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)ab^4c^2 - 2ab^5c^2 + 16\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^3b^2c^3 + 8\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^2b^2c^3 + \sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)ab^3c^3 + 16a^2b^3c^3 - 4\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^2b^2c^4 - 32a^3b^2c^4 + 2(b^2 - 4ac)ab^3c^2 - 8(b^2 - 4ac)a^2b^2c^3)d\text{abs}(a) - 2(\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^2b^4c - 8\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^3b^2c^2 - 2\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^2b^3c^2 - 2a^2b^4c^2 + 16\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^4c^3 + 8\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^3b^2c^3 + \sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^2b^2c^3 + 16a^3b^2c^3 - 4\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^3c^4 - 32a^4c^4 + 2(b^2 - 4ac)a^2b^2c^2 - 8(b^2 - 4ac)a^3c^3)\text{abs}(a) * e + (2a^2b^4c^3 - 8a^3b^2c^4 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^2b^4c + 4\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^3b^2c^2 + 2\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^2b^3c^2 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^2b^2c^3 - 2(b^2 - 4ac)a^2b^2c^3)d + (2a^3b^4c^2 - 8a^4b^2c^3 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^3b^4 + 4\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^4b^2c + 2\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^3b^3c - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^3b^2c^2 - 2(b^2 - 4ac)a^3b^2c^2)f - 2(2a^3b^3c^3 - 8a^4b^2c^4 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^3b^3c + 4\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^4b^2c^2 + 2\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^3b^2c^2 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^3b^2c^3 - 2(b^2 - 4ac)a^3b^2c^3)e) * \arctan(2\sqrt{1/2} * x / \sqrt{(ab + \sqrt{a^2b^2 - 4a^3c}) / (ac)}) / ((a^3b^4c - 8a^4b^2c^2 - 2a^3b^3c^2 + 16a^5c^3 + 8a^4b^2c^3 + a^3b^2c^3 - 4a^4c^4) * \text{abs}(a) * \text{abs}(c)) + 1/8 * ((2b^4c^3 - 16ab^2c^4 + 32a^2c^5 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)b^4c + 8\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)ab^2c^2 + 2\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)b^3c^2 - 16\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^2c^3 - 8\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)ab^2c^3 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^2b^2c^3 + 4\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^2c^3 + 4\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^2c^4 - 2(b^2 - 4ac)b^2c^3 + 8(b^2 - 4ac)a^2c^4)a^2d - (2ab^4c^2 - 16a^2b^2c^3 + 32a^3c^4 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)ab^4 + 8\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^2b^2c + 2\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)ab^3c - 16\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^3c^2 - 8\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^2b^2c^2 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)ab^2c^2 + 4\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^2c^3 - 2(b^2 - 4ac)ab^2c^2 + 8(b^2 - 4ac)a^2c^3)a^2f - 2(\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}c)ab^5c - 8\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^2b^3c^2 - 2\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}c)ab^4c^2 + 2ab^5c^2 + 16\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^3b^2c^3 + 8\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}c)
\end{aligned}$$

$a^2b^2c^3 + \sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}c)ab^3c^3 - 16a^2b^3c^3 - 4\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^2b^4c^4 + 32a^3b^4c^4 - 2(b^2 - 4ac)ab^3c^2 + 8(b^2 - 4ac)a^2b^3c^3)d\text{abs}(a) + 2(\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^2b^4c^4 - 8\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^3b^2c^2 - 2\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^2b^3c^2 + 2a^2b^4c^2 + 16\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^4c^3 + 8\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^3b^3c^3 + \sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^2b^2c^3 - 16a^3b^2c^3 - 4\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^3c^4 + 32a^4c^4 - 2(b^2 - 4ac)a^2b^2c^2 + 8(b^2 - 4ac)a^3c^3)\text{abs}(a)e + (2a^2b^4c^3 - 8a^3b^2c^4 - \sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^2b^4c^4 + 4\sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^3b^2c^2 + 2\sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^2b^3c^2 - \sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^2b^2c^3 - 2(b^2 - 4ac)a^2b^2c^3)d + (2a^3b^4c^2 - 8a^4b^2c^3 - \sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^3b^4 + 4\sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^4b^2c^2 + 2\sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^3b^3c^2 - \sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^3b^2c^2 - 2(b^2 - 4ac)a^3b^2c^2)*f - 2(2a^3b^3c^3 - 8a^4b^2c^4 - \sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^3b^3c^3 + 4\sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^4b^2c^2 + 2\sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^3b^2c^2 - \sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^3b^2c^3 - 2(b^2 - 4ac)a^3b^2c^3)*e)\text{arctan}(2\sqrt{1/2}x/\sqrt{(ab - \sqrt{a^2b^2 - 4a^3c})/(ac)})))/((a^3b^4c - 8a^4b^2c^2 - 2a^3b^3c^2 + 16a^5c^3 + 8a^4b^2c^3 + a^3b^2c^3 - 4a^4c^4)\text{abs}(a)\text{abs}(c))$

maple [B] time = 0.02, size = 563, normalized size = 2.64

$$\frac{\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{2}c}{\sqrt{(b+\sqrt{4ac+2b^2})}}\right)}{2\sqrt{4ac+2b^2}\sqrt{(b+\sqrt{4ac+2b^2})}}c - \frac{\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{2}c}{\sqrt{(b+\sqrt{4ac+2b^2})}}\right)}{2\sqrt{4ac+2b^2}\sqrt{(b+\sqrt{4ac+2b^2})}}c - \frac{\sqrt{2} b f \operatorname{arctanh}\left(\frac{\sqrt{2}c}{\sqrt{(b+\sqrt{4ac+2b^2})}}\right)}{2\sqrt{4ac+2b^2}\sqrt{(b+\sqrt{4ac+2b^2})}}c - \frac{\sqrt{2} b f \operatorname{arctanh}\left(\frac{\sqrt{2}c}{\sqrt{(b+\sqrt{4ac+2b^2})}}\right)}{2\sqrt{4ac+2b^2}\sqrt{(b+\sqrt{4ac+2b^2})}}c - \frac{\sqrt{2} c \operatorname{arctanh}\left(\frac{\sqrt{2}c}{\sqrt{(b+\sqrt{4ac+2b^2})}}\right)}{\sqrt{4ac+2b^2}\sqrt{(b+\sqrt{4ac+2b^2})}}c - \frac{\sqrt{2} c \operatorname{arctanh}\left(\frac{\sqrt{2}c}{\sqrt{(b+\sqrt{4ac+2b^2})}}\right)}{\sqrt{4ac+2b^2}\sqrt{(b+\sqrt{4ac+2b^2})}}c - \frac{\sqrt{2} d f \operatorname{arctanh}\left(\frac{\sqrt{2}c}{\sqrt{(b+\sqrt{4ac+2b^2})}}\right)}{2\sqrt{(b+\sqrt{4ac+2b^2})}}c - \frac{\sqrt{2} d f \operatorname{arctanh}\left(\frac{\sqrt{2}c}{\sqrt{(b+\sqrt{4ac+2b^2})}}\right)}{2\sqrt{(b+\sqrt{4ac+2b^2})}}c - \frac{\sqrt{2} f \operatorname{arctanh}\left(\frac{\sqrt{2}c}{\sqrt{(b+\sqrt{4ac+2b^2})}}\right)}{2\sqrt{(b+\sqrt{4ac+2b^2})}}c - \frac{\sqrt{2} f \operatorname{arctanh}\left(\frac{\sqrt{2}c}{\sqrt{(b+\sqrt{4ac+2b^2})}}\right)}{2\sqrt{(b+\sqrt{4ac+2b^2})}}c \frac{d}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a), x)

[Out] $-d/a/x - 1/2*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*f + 1/2/a*c*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*d + 1/2/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b*f - c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*e + 1/2/a*c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b*d + 1/2*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*f - 1/2/a*c*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*d + 1/2/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b*f - c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*e + 1/2/a*c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b*d$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(cd-af)x^2+bd-ae}{cx^4+bx^2+a} dx - \frac{d}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a), x, algorithm="maxima")

[Out] integrate(-((c*d - a*f)*x^2 + b*d - a*e)/(c*x^4 + b*x^2 + a), x)/a - d/(a*x)

mupad [B] time = 3.52, size = 10170, normalized size = 47.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x^2 + f*x^4)/(x^2*(a + b*x^2 + c*x^4)), x)

[Out] - atan(((x*(4*a^4*c^4*d^2 - 4*a^5*c^3*e^2 + 4*a^6*c^2*f^2 - 2*a^5*b^2*c*f^2 - 2*a^3*b^2*c^3*d^2 - 8*a^5*c^3*d*f + 4*a^4*b*c^3*d*e + 4*a^5*b*c^2*e*f) + (-b^5*c*d^2 + a^3*b^3*f^2 + a^3*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^2*d^2 + 12*a^2*b*c^3*d^2 + a*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*b^3*c*e^2 - 4*a^3*b*c^2*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*f^2 - 16*a^3*c^3*d*e + 16*a^4*c^2*e*f + 2*a^2*b^3*c*d*f - 8*a^3*b*c^2*d*f - 2*a^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b^2*c*e*f + 12*a^2*b^2*c^2*d*e - 2*a*b^4*c*d*e + 2*a*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^5*c^3 + a^3*b^4*c - 8*a^4*b^2*c^2)))^(1/2)*(x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-b^5*c*d^2 + a^3*b^3*f^2 + a^3*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^2*d^2 + 12*a^2*b*c^3*d^2 + a*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*b^3*c*e^2 - 4*a^3*b*c^2*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*f^2 - 16*a^3*c^3*d*e + 16*a^4*c^2*e*f + 2*a^2*b^3*c*d*f - 8*a^3*b*c^2*d*f - 2*a^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b^2*c*e*f + 12*a^2*b^2*c^2*d*e - 2*a*b^4*c*d*e + 2*a*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^5*c^3 + a^3*b^4*c - 8*a^4*b^2*c^2)))^(1/2) - 16*a^6*c^3*e - 4*a^4*b^3*c^2*d + 4*a^5*b^2*c^2*e + 16*a^5*b*c^3*d)*(-b^5*c*d^2 + a^3*b^3*f^2 + a^3*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^2*d^2 + 12*a^2*b*c^3*d^2 + a*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*b^3*c*e^2 - 4*a^3*b*c^2*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*f^2 - 16*a^3*c^3*d*e + 16*a^4*c^2*e*f + 2*a^2*b^3*c*d*f - 8*a^3*b*c^2*d*f - 2*a^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b^2*c*e*f + 12*a^2*b^2*c^2*d*e - 2*a*b^4*c*d*e + 2*a*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^5*c^3 + a^3*b^4*c - 8*a^4*b^2*c^2)))^(1/2)*i + (x*(4*a^4*c^4*d^2 - 4*a^5*c^3*e^2 + 4*a^6*c^2*f^2 - 2*a^5*b^2*c*f^2 - 2*a^3*b^2*c^3*d^2 - 8*a^5*c^3*d*f + 4*a^4*b*c^3*d*e + 4*a^5*b*c^2*e*f) + (-b^5*c*d^2 + a^3*b^3*f^2 + a^3*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^2*d^2 + 12*a^2*b*c^3*d^2 + a*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*b^3*c*e^2 - 4*a^3*b*c^2*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*f^2 - 16*a^3*c^3*d*e + 16*a^4*c^2*e*f + 2*a^2*b^3*c*d*f - 8*a^3*b*c^2*d*f - 2*a^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b^2*c*e*f + 12*a^2*b^2*c^2*d*e - 2*a*b^4*c*d*e + 2*a*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^5*c^3 + a^3*b^4*c - 8*a^4*b^2*c^2)))^(1/2)*i)/((x*(4*a^4*c^4*d^2 - 4*a^5*c^3*e^2 + 4*a^6*c^2*f^2 - 2*a^5*b^2*c*f^2 - 2*a^3*b^2*c^3*d^2 - 8*a^5*c^3*d*f + 4*a^4*b*c^3*d*e + 4*a^5*b*c^2*e*f) +

$$\begin{aligned}
& \left(-(b^5cd^2 + a^3b^3f^2 + a^3f^2(-4ac - b^2)^3)^{1/2} - 7ab^3c^2d^2 + 12a^2b^3c^3d^2 + a^2c^2d^2(-4ac - b^2)^3)^{1/2} + a^2b^3c^2e^2 - 4a^3b^3c^2e^2 - a^2c^2e^2(-4ac - b^2)^3)^{1/2} - b^2cd^2(-4ac - b^2)^3)^{1/2} - 4a^4b^3cf^2 - 16a^3c^3d^2e + 16a^4c^2e^2f + 2a^2b^3c^2d^2f - 8a^3b^3c^2d^2f - 2a^2c^2d^2f(-4ac - b^2)^3)^{1/2} - 4a^3b^2c^2e^2f + 12a^2b^2c^2d^2e - 2ab^4c^2d^2e + 2ab^3c^2d^2e(-4ac - b^2)^3)^{1/2} \right) / (8(16a^5c^3 + a^3b^4c - 8a^4b^2c^2))^{1/2} * (x(32a^6b^3c^3 - 8a^5b^3c^2) * (-b^5cd^2 + a^3b^3f^2 + a^3f^2(-4ac - b^2)^3)^{1/2} - 7ab^3c^2d^2 + 12a^2b^3c^3d^2 + a^2c^2d^2(-4ac - b^2)^3)^{1/2} + a^2b^3c^2e^2 - 4a^3b^3c^2e^2 - a^2c^2e^2(-4ac - b^2)^3)^{1/2} - b^2cd^2(-4ac - b^2)^3)^{1/2} - 4a^4b^3cf^2 - 16a^3c^3d^2e + 16a^4c^2e^2f + 2a^2b^3c^2d^2f - 8a^3b^3c^2d^2f - 2a^2c^2d^2f(-4ac - b^2)^3)^{1/2} - 4a^3b^2c^2e^2f + 12a^2b^2c^2d^2e - 2ab^4c^2d^2e + 2ab^3c^2d^2e(-4ac - b^2)^3)^{1/2} \right) / (8(16a^5c^3 + a^3b^4c - 8a^4b^2c^2))^{1/2} - 16a^6c^3e - 4a^4b^3c^2d + 4a^5b^2c^2e + 16a^5b^3c^3d) * (-b^5cd^2 + a^3b^3f^2 + a^3f^2(-4ac - b^2)^3)^{1/2} - 7ab^3c^2d^2 + 12a^2b^3c^3d^2 + a^2c^2d^2(-4ac - b^2)^3)^{1/2} + a^2b^3c^2e^2 - 4a^3b^3c^2e^2 - a^2c^2e^2(-4ac - b^2)^3)^{1/2} - b^2cd^2(-4ac - b^2)^3)^{1/2} - 4a^4b^3cf^2 - 16a^3c^3d^2e + 16a^4c^2e^2f + 2a^2b^3c^2d^2f - 8a^3b^3c^2d^2f - 2a^2c^2d^2f(-4ac - b^2)^3)^{1/2} - 4a^3b^2c^2e^2f + 12a^2b^2c^2d^2e - 2ab^4c^2d^2e + 2ab^3c^2d^2e(-4ac - b^2)^3)^{1/2} \right) / (8(16a^5c^3 + a^3b^4c - 8a^4b^2c^2))^{1/2} - (x(4a^4c^4d^2 - 4a^5c^3e^2 + 4a^6c^2f^2 - 2a^5b^2c^2f^2 - 2a^3b^2c^3d^2 - 8a^5c^3d^2f + 4a^4b^3c^3d^2e + 4a^5b^3c^2e^2f) + (-b^5cd^2 + a^3b^3f^2 + a^3f^2(-4ac - b^2)^3)^{1/2} - 7ab^3c^2d^2 + 12a^2b^3c^3d^2 + a^2c^2d^2(-4ac - b^2)^3)^{1/2} + a^2b^3c^2e^2 - 4a^3b^3c^2e^2 - a^2c^2e^2(-4ac - b^2)^3)^{1/2} - b^2cd^2(-4ac - b^2)^3)^{1/2} - 4a^4b^3cf^2 - 16a^3c^3d^2e + 16a^4c^2e^2f + 2a^2b^3c^2d^2f - 8a^3b^3c^2d^2f - 2a^2c^2d^2f(-4ac - b^2)^3)^{1/2} - 4a^3b^2c^2e^2f + 12a^2b^2c^2d^2e - 2ab^4c^2d^2e + 2ab^3c^2d^2e(-4ac - b^2)^3)^{1/2} \right) / (8(16a^5c^3 + a^3b^4c - 8a^4b^2c^2))^{1/2} + 16a^6c^3e + 4a^4b^3c^2d - 4a^5b^2c^2e - 16a^5b^3c^3d) * (-b^5cd^2 + a^3b^3f^2 + a^3f^2(-4ac - b^2)^3)^{1/2} - 7ab^3c^2d^2 + 12a^2b^3c^3d^2 + a^2c^2d^2(-4ac - b^2)^3)^{1/2} + a^2b^3c^2e^2 - 4a^3b^3c^2e^2 - a^2c^2e^2(-4ac - b^2)^3)^{1/2} - b^2cd^2(-4ac - b^2)^3)^{1/2} - 4a^4b^3cf^2 - 16a^3c^3d^2e + 16a^4c^2e^2f + 2a^2b^3c^2d^2f - 8a^3b^3c^2d^2f - 2a^2c^2d^2f(-4ac - b^2)^3)^{1/2} - 4a^3b^2c^2e^2f + 12a^2b^2c^2d^2e - 2ab^4c^2d^2e + 2ab^3c^2d^2e(-4ac - b^2)^3)^{1/2} \right) / (8(16a^5c^3 + a^3b^4c - 8a^4b^2c^2))^{1/2} - 2a^6c^2f^3 + 2a^3c^4d^3 + 2a^4c^3d^2e^2 - 6a^4c^3d^2f + 6a^5c^2d^2f^2 - 2a^5c^2e^2f + 2a^5b^3c^3d^2e - 2a^4b^2c^2d^2f) * (-b^5cd^2 + a^3b^3f^2 + a^3f^2(-4ac - b^2)^3)^{1/2} - 7ab^3c^2d^2 + 12a^2b^3c^3d^2 + a^2c^2d^2(-4ac - b^2)^3)^{1/2} + a^2b^3c^2e^2 - 4a^3b^3c^2e^2 - a^2c^2e^2(-4ac - b^2)^3)^{1/2} - b^2cd^2(-4ac - b^2)^3)^{1/2} - 4a^4b^3cf^2 - 16a^3c^3d^2e + 16a^4c^2e^2f + 2a^2b^3c^2d^2f - 8a^3b^3c^2d^2f - 2a^2c^2d^2f(-4ac - b^2)^3)^{1/2} - 4a^3b^2c^2e^2f + 12a^2b^2c^2d^2e - 2ab^4c^2d^2e + 2ab^3c^2d^2e(-4ac - b^2)^3)^{1/2} \right) / (8(16a^5c^3 + a^3b^4c - 8a^4b^2c^2))^{1/2} * 2i - \operatorname{atan}\left(\left(x(4a^4c^4d^2 - 4a^5c^3e^2 + 4a^6c^2f^2 - 2a^5b^2c^2f^2 - 2a^3b^2c^3d^2 - 8a^5c^3d^2f + 4a^4b^3c^3d^2e + 4a^5b^3c^2e^2f) + (-b^5cd^2 + a^3b^3f^2 - a^3f^2(-4ac - b^2)^3)^{1/2} - 7ab^3c^2d^2 + 12a^2b^3c^3d^2 - a^2c^2d^2(-4ac - b^2)^3)^{1/2} - 7ab^3c^2d^2 + 12a^2b^3c^3d^2 - a^2c^2d^2(-4ac - b^2)^3)^{1/2} + a^2b^3c^2e^2 - 4a^3b^3c^2e^2 - a^2c^2e^2(-4ac - b^2)^3)^{1/2} - b^2cd^2(-4ac - b^2)^3)^{1/2} - 4a^4b^3cf^2 - 16a^3c^3d^2e + 16a^4c^2e^2f + 2a^2b^3c^2d^2f - 8a^3b^3c^2d^2f - 2a^2c^2d^2f(-4ac - b^2)^3)^{1/2} - 4a^3b^2c^2e^2f + 12a^2b^2c^2d^2e - 2ab^4c^2d^2e + 2ab^3c^2d^2e(-4ac - b^2)^3)^{1/2} \right) / (8(16a^5c^3 + a^3b^4c - 8a^4b^2c^2))^{1/2}
\end{aligned}$$

$$\begin{aligned}
& b^2)^3)^{(1/2)} + a^2*b^3*c*e^2 - 4*a^3*b*c^2*e^2 + a^2*c*e^2*(-(4*a*c - b^2 \\
&)^3)^{(1/2)} + b^2*c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a^4*b*c*f^2 - 16*a^3*c^ \\
& 3*d*e + 16*a^4*c^2*e*f + 2*a^2*b^3*c*d*f - 8*a^3*b*c^2*d*f + 2*a^2*c*d*f*(- \\
& (4*a*c - b^2)^3)^{(1/2)} - 4*a^3*b^2*c*e*f + 12*a^2*b^2*c^2*d*e - 2*a*b^4*c*d \\
& *e - 2*a*b*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16*a^5*c^3 + a^3*b^4*c - 8*a \\
& ^4*b^2*c^2)))^{(1/2)}*(x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5*c*d^2 + a^3*b^ \\
& 3*f^2 - a^3*f^2*(-(4*a*c - b^2)^3)^{(1/2)} - 7*a*b^3*c^2*d^2 + 12*a^2*b*c^3*d \\
& ^2 - a*c^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + a^2*b^3*c*e^2 - 4*a^3*b*c^2*e^2 + \\
& a^2*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + b^2*c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - \\
& 4*a^4*b*c*f^2 - 16*a^3*c^3*d*e + 16*a^4*c^2*e*f + 2*a^2*b^3*c*d*f - 8*a^3*b \\
& *c^2*d*f + 2*a^2*c*d*f*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a^3*b^2*c*e*f + 12*a^2*b \\
& ^2*c^2*d*e - 2*a*b^4*c*d*e - 2*a*b*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16* \\
& a^5*c^3 + a^3*b^4*c - 8*a^4*b^2*c^2)))^{(1/2)} - 16*a^6*c^3*e - 4*a^4*b^3*c^2 \\
& *d + 4*a^5*b^2*c^2*e + 16*a^5*b*c^3*d))*(-(b^5*c*d^2 + a^3*b^3*f^2 - a^3*f^ \\
& 2*(-(4*a*c - b^2)^3)^{(1/2)} - 7*a*b^3*c^2*d^2 + 12*a^2*b*c^3*d^2 - a*c^2*d^2 \\
& *(-(4*a*c - b^2)^3)^{(1/2)} + a^2*b^3*c*e^2 - 4*a^3*b*c^2*e^2 + a^2*c*e^2*(-(\\
& 4*a*c - b^2)^3)^{(1/2)} + b^2*c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a^4*b*c*f^2 \\
& - 16*a^3*c^3*d*e + 16*a^4*c^2*e*f + 2*a^2*b^3*c*d*f - 8*a^3*b*c^2*d*f + 2*a \\
& ^2*c*d*f*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a^3*b^2*c*e*f + 12*a^2*b^2*c^2*d*e - \\
& 2*a*b^4*c*d*e - 2*a*b*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16*a^5*c^3 + a^3* \\
& b^4*c - 8*a^4*b^2*c^2)))^{(1/2)}*1i + (x*(4*a^4*c^4*d^2 - 4*a^5*c^3*e^2 + 4*a \\
& ^6*c^2*f^2 - 2*a^5*b^2*c*f^2 - 2*a^3*b^2*c^3*d^2 - 8*a^5*c^3*d*f + 4*a^4*b* \\
& c^3*d*e + 4*a^5*b*c^2*e*f) + (-(b^5*c*d^2 + a^3*b^3*f^2 - a^3*f^2*(-(4*a*c \\
& - b^2)^3)^{(1/2)} - 7*a*b^3*c^2*d^2 + 12*a^2*b*c^3*d^2 - a*c^2*d^2*(-(4*a*c \\
& - b^2)^3)^{(1/2)} + a^2*b^3*c*e^2 - 4*a^3*b*c^2*e^2 + a^2*c*e^2*(-(4*a*c - b^2 \\
&)^3)^{(1/2)} + b^2*c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a^4*b*c*f^2 - 16*a^3*c^ \\
& 3*d*e + 16*a^4*c^2*e*f + 2*a^2*b^3*c*d*f - 8*a^3*b*c^2*d*f + 2*a^2*c*d*f*(- \\
& (4*a*c - b^2)^3)^{(1/2)} - 4*a^3*b^2*c*e*f + 12*a^2*b^2*c^2*d*e - 2*a*b^4*c*d \\
& *e - 2*a*b*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16*a^5*c^3 + a^3*b^4*c - 8*a \\
& ^4*b^2*c^2)))^{(1/2)}*(x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5*c*d^2 + a^3*b^ \\
& 3*f^2 - a^3*f^2*(-(4*a*c - b^2)^3)^{(1/2)} - 7*a*b^3*c^2*d^2 + 12*a^2*b*c^3*d \\
& ^2 - a*c^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + a^2*b^3*c*e^2 - 4*a^3*b*c^2*e^2 + \\
& a^2*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + b^2*c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - \\
& 4*a^4*b*c*f^2 - 16*a^3*c^3*d*e + 16*a^4*c^2*e*f + 2*a^2*b^3*c*d*f - 8*a^3*b \\
& *c^2*d*f + 2*a^2*c*d*f*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a^3*b^2*c*e*f + 12*a^2*b \\
& ^2*c^2*d*e - 2*a*b^4*c*d*e - 2*a*b*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16* \\
& a^5*c^3 + a^3*b^4*c - 8*a^4*b^2*c^2)))^{(1/2)} + 16*a^6*c^3*e + 4*a^4*b^3*c^2 \\
& *d - 4*a^5*b^2*c^2*e - 16*a^5*b*c^3*d))*(-(b^5*c*d^2 + a^3*b^3*f^2 - a^3*f^ \\
& 2*(-(4*a*c - b^2)^3)^{(1/2)} - 7*a*b^3*c^2*d^2 + 12*a^2*b*c^3*d^2 - a*c^2*d^2 \\
& *(-(4*a*c - b^2)^3)^{(1/2)} + a^2*b^3*c*e^2 - 4*a^3*b*c^2*e^2 + a^2*c*e^2*(-(\\
& 4*a*c - b^2)^3)^{(1/2)} + b^2*c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a^4*b*c*f^2 \\
& - 16*a^3*c^3*d*e + 16*a^4*c^2*e*f + 2*a^2*b^3*c*d*f - 8*a^3*b*c^2*d*f + 2*a \\
& ^2*c*d*f*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a^3*b^2*c*e*f + 12*a^2*b^2*c^2*d*e - \\
& 2*a*b^4*c*d*e - 2*a*b*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16*a^5*c^3 + a^3* \\
& b^4*c - 8*a^4*b^2*c^2)))^{(1/2)}*1i)/((x*(4*a^4*c^4*d^2 - 4*a^5*c^3*e^2 + 4*a \\
& ^6*c^2*f^2 - 2*a^5*b^2*c*f^2 - 2*a^3*b^2*c^3*d^2 - 8*a^5*c^3*d*f + 4*a^4*b* \\
& c^3*d*e + 4*a^5*b*c^2*e*f) + (-(b^5*c*d^2 + a^3*b^3*f^2 - a^3*f^2*(-(4*a*c \\
& - b^2)^3)^{(1/2)} - 7*a*b^3*c^2*d^2 + 12*a^2*b*c^3*d^2 - a*c^2*d^2*(-(4*a*c \\
& - b^2)^3)^{(1/2)} + a^2*b^3*c*e^2 - 4*a^3*b*c^2*e^2 + a^2*c*e^2*(-(4*a*c - b^2 \\
&)^3)^{(1/2)} + b^2*c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a^4*b*c*f^2 - 16*a^3*c^ \\
& 3*d*e + 16*a^4*c^2*e*f + 2*a^2*b^3*c*d*f - 8*a^3*b*c^2*d*f + 2*a^2*c*d*f*(- \\
& (4*a*c - b^2)^3)^{(1/2)} - 4*a^3*b^2*c*e*f + 12*a^2*b^2*c^2*d*e - 2*a*b^4*c*d \\
& *e - 2*a*b*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16*a^5*c^3 + a^3*b^4*c - 8*a \\
& ^4*b^2*c^2)))^{(1/2)}*(x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5*c*d^2 + a^3*b^ \\
& 3*f^2 - a^3*f^2*(-(4*a*c - b^2)^3)^{(1/2)} - 7*a*b^3*c^2*d^2 + 12*a^2*b*c^3*d \\
& ^2 - a*c^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + a^2*b^3*c*e^2 - 4*a^3*b*c^2*e^2 + \\
& a^2*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + b^2*c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - \\
& 4*a^4*b*c*f^2 - 16*a^3*c^3*d*e + 16*a^4*c^2*e*f + 2*a^2*b^3*c*d*f - 8*a^3*b \\
& *c^2*d*f + 2*a^2*c*d*f*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a^3*b^2*c*e*f + 12*a^2*
\end{aligned}$$

$$\begin{aligned}
& b^2c^2d^2e - 2ab^4c^2d^2e - 2ab^2c^2d^2e^2(-4ac - b^2)^3)^{1/2} / (8(16a^5c^3 + a^3b^4c - 8a^4b^2c^2))^{1/2} - 16a^6c^3e - 4a^4b^3c^2 \\
& *d + 4a^5b^2c^2e + 16a^5b^2c^3d)) * (-b^5cd^2 + a^3b^3f^2 - a^3f^2 \\
& *(-4ac - b^2)^3)^{1/2} - 7ab^3c^2d^2 + 12a^2b^2c^3d^2 - ac^2d^2 \\
& *(-4ac - b^2)^3)^{1/2} + a^2b^3c^2e^2 - 4a^3b^2c^2e^2 + a^2c^2e^2 * (- \\
& 4ac - b^2)^3)^{1/2} + b^2cd^2 * (-4ac - b^2)^3)^{1/2} - 4a^4b^2c^2f^2 \\
& - 16a^3c^3d^2e + 16a^4c^2e^2f + 2a^2b^3c^2d^2f - 8a^3b^2c^2d^2f + 2a \\
& ^2c^2d^2f * (-4ac - b^2)^3)^{1/2} - 4a^3b^2c^2e^2f + 12a^2b^2c^2d^2e - \\
& 2ab^4c^2d^2e - 2ab^2c^2d^2e^2 * (-4ac - b^2)^3)^{1/2} / (8(16a^5c^3 + a^3b^4c \\
& - 8a^4b^2c^2))^{1/2} - (x(4a^4c^4d^2 - 4a^5c^3e^2 + 4a^6c^2f^2 - 2a^5b^2c^2f^2 - 2a^3b^2c^3d^2 - 8a^5c^3d^2f + 4a^4b^2c^3 \\
& *d^2e + 4a^5b^2c^2e^2f) + (-b^5cd^2 + a^3b^3f^2 - a^3f^2 * (-4ac - b \\
& ^2)^3)^{1/2} - 7ab^3c^2d^2 + 12a^2b^2c^3d^2 - ac^2d^2 * (-4ac - b \\
& ^2)^3)^{1/2} + a^2b^3c^2e^2 - 4a^3b^2c^2e^2 + a^2c^2e^2 * (-4ac - b^2)^3 \\
&)^{1/2} + b^2cd^2 * (-4ac - b^2)^3)^{1/2} - 4a^4b^2c^2f^2 - 16a^3c^3d^2 \\
& *e + 16a^4c^2e^2f + 2a^2b^3c^2d^2f - 8a^3b^2c^2d^2f + 2a^2c^2d^2f * (-4 \\
& ac - b^2)^3)^{1/2} - 4a^3b^2c^2e^2f + 12a^2b^2c^2d^2e - 2ab^4c^2d^2e \\
& - 2ab^2c^2d^2e^2 * (-4ac - b^2)^3)^{1/2} / (8(16a^5c^3 + a^3b^4c - 8a^4b^2c^2))^{1/2} * (x(32a^6b^2c^3 - 8a^5b^3c^2) * (-b^5cd^2 + a^3b^3f^2 \\
& ^2 - a^3f^2 * (-4ac - b^2)^3)^{1/2} - 7ab^3c^2d^2 + 12a^2b^2c^3d^2 \\
& - ac^2d^2 * (-4ac - b^2)^3)^{1/2} + a^2b^3c^2e^2 - 4a^3b^2c^2e^2 + a^2c^2e^2 * (-4ac - b^2)^3)^{1/2} + b^2cd^2 * (-4ac - b^2)^3)^{1/2} - 4a \\
& ^4b^2c^2f^2 - 16a^3c^3d^2e + 16a^4c^2e^2f + 2a^2b^3c^2d^2f - 8a^3b^2c^2d^2f + 2a^2c^2d^2f * (-4ac - b^2)^3)^{1/2} - 4a^3b^2c^2e^2f + 12a^2b^2 \\
& *c^2d^2e - 2ab^4c^2d^2e - 2ab^2c^2d^2e^2 * (-4ac - b^2)^3)^{1/2} / (8(16a^5c^3 + a^3b^4c - 8a^4b^2c^2))^{1/2} + 16a^6c^3e + 4a^4b^3c^2d^2 \\
& - 4a^5b^2c^2e - 16a^5b^2c^3d)) * (-b^5cd^2 + a^3b^3f^2 - a^3f^2 * (- \\
& 4ac - b^2)^3)^{1/2} - 7ab^3c^2d^2 + 12a^2b^2c^3d^2 - ac^2d^2 * (- \\
& 4ac - b^2)^3)^{1/2} + a^2b^3c^2e^2 - 4a^3b^2c^2e^2 + a^2c^2e^2 * (-4ac \\
& * - b^2)^3)^{1/2} + b^2cd^2 * (-4ac - b^2)^3)^{1/2} - 4a^4b^2c^2f^2 - 1 \\
& 6a^3c^3d^2e + 16a^4c^2e^2f + 2a^2b^3c^2d^2f - 8a^3b^2c^2d^2f + 2a^2c^2 \\
& *d^2f * (-4ac - b^2)^3)^{1/2} - 4a^3b^2c^2e^2f + 12a^2b^2c^2d^2e - 2a \\
& *b^4c^2d^2e - 2ab^2c^2d^2e^2 * (-4ac - b^2)^3)^{1/2} / (8(16a^5c^3 + a^3b^4c \\
& - 8a^4b^2c^2))^{1/2} - 2a^6c^3f^3 + 2a^3c^4d^3 + 2a^4c^3d^2e^2 \\
& - 6a^4c^3d^2f + 6a^5c^2d^2f^2 - 2a^5c^2e^2f + 2a^5b^2c^2e^2f^2 - \\
& 2a^3b^2c^3d^2e - 2a^4b^2c^2d^2f^2 + 2a^3b^2c^2d^2f)) * (-b^5cd^2 \\
& + a^3b^3f^2 - a^3f^2 * (-4ac - b^2)^3)^{1/2} - 7ab^3c^2d^2 + 12a^2 \\
& *b^2c^3d^2 - ac^2d^2 * (-4ac - b^2)^3)^{1/2} + a^2b^3c^2e^2 - 4a^3b^2c^2 \\
& *e^2 + a^2c^2e^2 * (-4ac - b^2)^3)^{1/2} + b^2cd^2 * (-4ac - b^2)^3)^{1/2} \\
& - 4a^4b^2c^2f^2 - 16a^3c^3d^2e + 16a^4c^2e^2f + 2a^2b^3c^2d^2f - \\
& 8a^3b^2c^2d^2f + 2a^2c^2d^2f * (-4ac - b^2)^3)^{1/2} - 4a^3b^2c^2e^2f + \\
& 12a^2b^2c^2d^2e - 2ab^4c^2d^2e - 2ab^2c^2d^2e^2 * (-4ac - b^2)^3)^{1/2} \\
& / (8(16a^5c^3 + a^3b^4c - 8a^4b^2c^2))^{1/2} * 2i - d/(ax)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**4+e*x**2+d)/x**2/(c*x**4+b*x**2+a),x)

[Out] Timed out

$$3.57 \quad \int \frac{d+ex^2+fx^4}{x^4(a+bx^2+cx^4)} dx$$

Optimal. Leaf size=267

$$\frac{\sqrt{c} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-abe-2a(cd-af)+b^2d}{\sqrt{b^2-4ac}} - ae + bd\right) - \sqrt{c} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-a\left(-e\sqrt{b^2-4ac} - 2af + 2cd\right)\right)}{\sqrt{2}a^2\sqrt{b-\sqrt{b^2-4ac}} - \sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

Rubi [A] time = 1.07, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 30, number of rules / integrand size = 0.100, Rules used = {1664, 1166, 205}

$$\frac{\sqrt{c} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-abe-2a(cd-af)+b^2d}{\sqrt{b^2-4ac}} - ae + bd\right) - \sqrt{c} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-a\left(-e\sqrt{b^2-4ac} - 2af + 2cd\right) - b\left(d\sqrt{b^2-4ac} + ae\right) + b^2d\right)}{\sqrt{2}a^2\sqrt{b-\sqrt{b^2-4ac}} - \sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}} + \frac{bd-ae}{a^2x} - \frac{d}{3ax^3}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2 + f*x^4)/(x^4*(a + b*x^2 + c*x^4)), x]

[Out] -d/(3*a*x^3) + (b*d - a*e)/(a^2*x) + (Sqrt[c]*(b*d - a*e + (b^2*d - a*b*e - 2*a*(c*d - a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*a^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(b^2*d - b*(Sqrt[b^2 - 4*a*c]*d + a*e) - a*(2*c*d - Sqrt[b^2 - 4*a*c]*e - 2*a*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1664

Int[(Pq_)*((d_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned}
\int \frac{d + ex^2 + fx^4}{x^4(a + bx^2 + cx^4)} dx &= \int \left(\frac{d}{ax^4} + \frac{-bd + ae}{a^2x^2} + \frac{b^2d - abe - a(cd - af) + c(bd - ae)x^2}{a^2(a + bx^2 + cx^4)} \right) dx \\
&= -\frac{d}{3ax^3} + \frac{bd - ae}{a^2x} + \frac{\int \frac{b^2d - abe - a(cd - af) + c(bd - ae)x^2}{a + bx^2 + cx^4} dx}{a^2} \\
&= -\frac{d}{3ax^3} + \frac{bd - ae}{a^2x} + \frac{\left(c \left(bd - ae - \frac{b^2d - abe - 2a(cd - af)}{\sqrt{b^2 - 4ac}} \right) \right) \int \frac{1}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2 - 4ac} + cx^2} dx}{2a^2} + \frac{c \left(bd - ae - \frac{b^2d - abe - 2a(cd - af)}{\sqrt{b^2 - 4ac}} \right)}{2a^2} \\
&= -\frac{d}{3ax^3} + \frac{bd - ae}{a^2x} + \frac{\sqrt{c} \left(bd - ae + \frac{b^2d - abe - 2a(cd - af)}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{\sqrt{2} a^2 \sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{\sqrt{c} \left(bd - ae - \frac{b^2d - abe - 2a(cd - af)}{\sqrt{b^2 - 4ac}} \right)}{2a^2}
\end{aligned}$$

Mathematica [A] time = 0.34, size = 284, normalized size = 1.06

$$\frac{3\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(a(-e\sqrt{b^2-4ac}+2af-2cd)+b(d\sqrt{b^2-4ac}-ae)+b^2d\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{3\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b^2-4ac}+b}\right)\left(-a(e\sqrt{b^2-4ac}+2af-2cd)+b(d\sqrt{b^2-4ac}+ae)+b^2(-d)\right)}{\sqrt{b^2-4ac}\sqrt{b^2-4ac}+b} + \frac{6bd-6ae}{x} - \frac{2ad}{x^3}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2 + f*x^4)/(x^4*(a + b*x^2 + c*x^4)),x]

[Out] ((-2*a*d)/x^3 + (6*b*d - 6*a*e)/x + (3*Sqrt[2]*Sqrt[c]*(b^2*d + b*(Sqrt[b^2 - 4*a*c]*d - a*e) + a*(-2*c*d - Sqrt[b^2 - 4*a*c]*e + 2*a*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(-(b^2*d) + b*(Sqrt[b^2 - 4*a*c]*d + a*e) - a*(-2*c*d + Sqrt[b^2 - 4*a*c]*e + 2*a*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(6*a^2)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex^2 + fx^4}{x^4(a + bx^2 + cx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e*x^2 + f*x^4)/(x^4*(a + b*x^2 + c*x^4)),x]

[Out] IntegrateAlgebraic[(d + e*x^2 + f*x^4)/(x^4*(a + b*x^2 + c*x^4)), x]

fricas [B] time = 11.25, size = 9850, normalized size = 36.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] -1/6*(3*sqrt(1/2)*a^2*x^3*sqrt(-(a^4*b*f^2 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d^2 - 2*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*d*e + (a^2*b^3 - 3*a^3*b*c)*e^2 + 2*((a^2*b^3 - 3*a^3*b*c)*d - (a^3*b^2 - 2*a^4*c)*e)*f + (a^5*b^2 - 4*a^6*c)*sqrt((a^8*f^4 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^4 - 4*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^3*e + 2*(3*a^2*b^6 - 12*a^3*b^4*c + 12*a^4*b^2*c^2 - a^5*c^3)*d^2*e^2 - 4*(a^3*b^5 - 3*a^4*b^3*c + 2*a^5*b*c^2)*d*e^3 + (a^4*b^4 - 2*a^5*b^2*c + a^6*c^2)*e^4 - 4*(a^7*b*e - (a^6*b^2 - a^7*c)*d)*f^3 + 2*((3*a^4*b^4 - 7*a^5*b^2*c + 3*

$$\begin{aligned}
& a^6c^2d^2 - 2(3a^5b^3 - 4a^6b^2c)de + (3a^6b^2 - a^7c)e^2f^2 \\
& + 4((a^2b^6 - 4a^3b^4c + 4a^4b^2c^2 - a^5c^3)d^3 - (3a^3b^5 - 9a^4b^3c + 5a^5b^2c^2)d^2e + (3a^4b^4 - 6a^5b^2c + a^6c^2)d^2e^2 - (a^5b^3 - a^6b^2c)e^3)f)/(a^{10}b^2 - 4a^{11}c)))/(a^5b^2 - 4a^6c) \\
&) \log(2(a^6cf^4 + (b^4c^3 - 3ab^2c^4 + a^2c^5)d^4 - (b^5c^2 - ab^3c^3 - 3a^2b^2c^4)d^3e + 3(ab^4c^2 - 2a^2b^2c^3)d^2e^2 - (3a^2b^3c^2 - 5a^3b^2c^3)d^2e^3 + (a^3b^2c^2 - a^4c^3)e^4 - (3a^5b^2c^2 - (3a^4b^2c - 4a^5c^2)d)ef^3 + 3(a^4b^2c^2e^2 + (a^2b^4c - 3a^3b^2c^2 + 2a^4c^3)d^2 - (2a^3b^3c - 3a^4b^2c^2)d^2e)ef^2 + ((b^6c - 5ab^4c^2 + 9a^2b^2c^3 - 4a^3c^4)d^3 - 3(ab^5c - 3a^2b^3c^2 + 3a^3b^2c^3)d^2e + 3(a^2b^4c - a^3b^2c^2)d^2e^2 - (a^3b^3c + a^4b^2c^2)e^3)f)x + \sqrt{1/2}((b^8 - 8ab^6c + 20a^2b^4c^2 - 17a^3b^2c^3 + 4a^4c^4)d^3 - (3ab^7 - 21a^2b^5c + 41a^3b^3c^2 - 20a^4b^2c^3)d^2e + (3a^2b^6 - 18a^3b^4c + 25a^4b^2c^2 - 4a^5c^3)d^2e^2 - (a^3b^5 - 5a^4b^3c + 4a^5b^2c^2)e^3 + (a^6b^2 - 4a^7c)ef^3 + 3((a^4b^4 - 5a^5b^2c + 4a^6c^2)d - (a^5b^3 - 4a^6b^2c)e)ef^2 + ((3a^2b^6 - 19a^3b^4c + 31a^4b^2c^2 - 12a^5c^3)d^2 - 2(3a^3b^5 - 16a^4b^3c + 16a^5b^2c^2)d^2e + (3a^4b^4 - 13a^5b^2c + 4a^6c^2)e^2)f - ((a^5b^5 - 7a^6b^3c + 12a^7b^2c^2)d - (a^6b^4 - 6a^7b^2c + 8a^8c^2)e + (a^7b^3 - 4a^8b^2c)f) \sqrt{(a^8f^4 + (b^8 - 6ab^6c + 11a^2b^4c^2 - 6a^3b^2c^3 + a^4c^4)d^4 - 4(ab^7 - 5a^2b^5c + 7a^3b^3c^2 - 2a^4b^2c^3)d^3e + 2(3a^2b^6 - 12a^3b^4c + 12a^4b^2c^2 - a^5c^3)d^2e^2 - 4(a^3b^5 - 3a^4b^3c + 2a^5b^2c^2)d^2e^3 + (a^4b^4 - 2a^5b^2c + a^6c^2)e^4 - 4(a^7b^2e - (a^6b^2 - a^7c)d)ef^3 + 2(((3a^4b^4 - 7a^5b^2c + 3a^6c^2)d^2 - 2(3a^5b^3 - 4a^6b^2c)d^2e + (3a^6b^2 - a^7c)e^2)ef^2 + 4((a^2b^6 - 4a^3b^4c + 4a^4b^2c^2 - a^5c^3)d^3 - (3a^3b^5 - 9a^4b^3c + 5a^5b^2c^2)d^2e + (3a^4b^4 - 6a^5b^2c + a^6c^2)d^2e^2 - (a^5b^3 - a^6b^2c)e^3)f)/(a^{10}b^2 - 4a^{11}c)) \sqrt{-(a^4b^2f^2 + (b^5 - 5ab^3c + 5a^2b^2c^2)d^2 - 2(ab^4 - 4a^2b^2c + 2a^3c^2)d^2e + (a^2b^3 - 3a^3b^2c)e^2 + 2((a^2b^3 - 3a^3b^2c)d - (a^3b^2 - 2a^4c)e)ef + (a^5b^2 - 4a^6c) \sqrt{(a^8f^4 + (b^8 - 6ab^6c + 11a^2b^4c^2 - 6a^3b^2c^3 + a^4c^4)d^4 - 4(ab^7 - 5a^2b^5c + 7a^3b^3c^2 - 2a^4b^2c^3)d^3e + 2(3a^2b^6 - 12a^3b^4c + 12a^4b^2c^2 - a^5c^3)d^2e^2 - 4(a^3b^5 - 3a^4b^3c + 2a^5b^2c^2)d^2e^3 + (a^4b^4 - 2a^5b^2c + a^6c^2)e^4 - 4(a^7b^2e - (a^6b^2 - a^7c)d)ef^3 + 2(((3a^4b^4 - 7a^5b^2c + 3a^6c^2)d^2 - 2(3a^5b^3 - 4a^6b^2c)d^2e + (3a^6b^2 - a^7c)e^2)ef^2 + 4((a^2b^6 - 4a^3b^4c + 4a^4b^2c^2 - a^5c^3)d^3 - (3a^3b^5 - 9a^4b^3c + 5a^5b^2c^2)d^2e + (3a^4b^4 - 6a^5b^2c + a^6c^2)d^2e^2 - (a^5b^3 - a^6b^2c)e^3)f)/(a^{10}b^2 - 4a^{11}c)))/(a^5b^2 - 4a^6c)) - 3 \sqrt{1/2} a^2 x^3 \sqrt{-(a^4b^2f^2 + (b^5 - 5ab^3c + 5a^2b^2c^2)d^2 - 2(ab^4 - 4a^2b^2c + 2a^3c^2)d^2e + (a^2b^3 - 3a^3b^2c)e^2 + 2((a^2b^3 - 3a^3b^2c)d - (a^3b^2 - 2a^4c)e)ef + (a^5b^2 - 4a^6c) \sqrt{(a^8f^4 + (b^8 - 6ab^6c + 11a^2b^4c^2 - 6a^3b^2c^3 + a^4c^4)d^4 - 4(ab^7 - 5a^2b^5c + 7a^3b^3c^2 - 2a^4b^2c^3)d^3e + 2(3a^2b^6 - 12a^3b^4c + 12a^4b^2c^2 - a^5c^3)d^2e^2 - 4(a^3b^5 - 3a^4b^3c + 2a^5b^2c^2)d^2e^3 + (a^4b^4 - 2a^5b^2c + a^6c^2)e^4 - 4(a^7b^2e - (a^6b^2 - a^7c)d)ef^3 + 2(((3a^4b^4 - 7a^5b^2c + 3a^6c^2)d^2 - 2(3a^5b^3 - 4a^6b^2c)d^2e + (3a^6b^2 - a^7c)e^2)ef^2 + 4((a^2b^6 - 4a^3b^4c + 4a^4b^2c^2 - a^5c^3)d^3 - (3a^3b^5 - 9a^4b^3c + 5a^5b^2c^2)d^2e + (3a^4b^4 - 6a^5b^2c + a^6c^2)d^2e^2 - (a^5b^3 - a^6b^2c)e^3)f)/(a^{10}b^2 - 4a^{11}c)))/(a^5b^2 - 4a^6c)) \log(2(a^6cf^4 + (b^4c^3 - 3ab^2c^4 + a^2c^5)d^4 - (b^5c^2 - ab^3c^3 - 3a^2b^2c^4)d^3e + 3(ab^4c^2 - 2a^2b^2c^3)d^2e^2 - (3a^2b^3c^2 - 5a^3b^2c^3)d^2e^3 + (a^3b^2c^2 - a^4c^3)e^4 - (3a^5b^2c^2 - (3a^4b^2c - 4a^5c^2)d)ef^3 + 3(a^4b^2c^2e^2 + (a^2b^4c - 3a^3b^2c^2 + 2a^4c^3)d^2 - (2a^3b^3c - 3a^4b^2c^2)d^2e)ef^2 + ((b^6c - 5ab^4c^2 + 9a^2b^2c^3 - 4a^3c^4)d^3 - 3(ab^5c - 3a^2b^3c^2 + 3a^3b^2c^3)d^2e + 3(a^2b^4c - a^3b^2c^2)d^2e^2 - (a^3b^3c + a^4b^2c^2)e^3)f)
\end{aligned}$$

$$\begin{aligned}
& c^2) * e^3) * f) * x - \sqrt{1/2} * ((b^8 - 8 * a * b^6 * c + 20 * a^2 * b^4 * c^2 - 17 * a^3 * b^2 * c^3 + 4 * a^4 * c^4) * d^3 - (3 * a * b^7 - 21 * a^2 * b^5 * c + 41 * a^3 * b^3 * c^2 - 20 * a^4 * b * c^3) * d^2 * e + (3 * a^2 * b^6 - 18 * a^3 * b^4 * c + 25 * a^4 * b^2 * c^2 - 4 * a^5 * c^3) * d * e^2 - (a^3 * b^5 - 5 * a^4 * b^3 * c + 4 * a^5 * b * c^2) * e^3 + (a^6 * b^2 - 4 * a^7 * c) * f^3 + 3 * ((a^4 * b^4 - 5 * a^5 * b^2 * c + 4 * a^6 * c^2) * d - (a^5 * b^3 - 4 * a^6 * b * c) * e) * f^2 + ((3 * a^2 * b^6 - 19 * a^3 * b^4 * c + 31 * a^4 * b^2 * c^2 - 12 * a^5 * c^3) * d^2 - 2 * (3 * a^3 * b^5 - 16 * a^4 * b^3 * c + 16 * a^5 * b * c^2) * d * e + (3 * a^4 * b^4 - 13 * a^5 * b^2 * c + 4 * a^6 * c^2) * e^2) * f - ((a^5 * b^5 - 7 * a^6 * b^3 * c + 12 * a^7 * b * c^2) * d - (a^6 * b^4 - 6 * a^7 * b^2 * c + 8 * a^8 * c^2) * e + (a^7 * b^3 - 4 * a^8 * b * c) * f) * \sqrt{(a^8 * f^4 + (b^8 - 6 * a * b^6 * c + 11 * a^2 * b^4 * c^2 - 6 * a^3 * b^2 * c^3 + a^4 * c^4) * d^4 - 4 * (a * b^7 - 5 * a^2 * b^5 * c + 7 * a^3 * b^3 * c^2 - 2 * a^4 * b * c^3) * d^3 * e + 2 * (3 * a^2 * b^6 - 12 * a^3 * b^4 * c + 12 * a^4 * b^2 * c^2 - a^5 * c^3) * d^2 * e^2 - 4 * (a^3 * b^5 - 3 * a^4 * b^3 * c + 2 * a^5 * b * c^2) * d * e^3 + (a^4 * b^4 - 2 * a^5 * b^2 * c + a^6 * c^2) * e^4 - 4 * (a^7 * b * e - (a^6 * b^2 - a^7 * c) * d) * f^3 + 2 * ((3 * a^4 * b^4 - 7 * a^5 * b^2 * c + 3 * a^6 * c^2) * d^2 - 2 * (3 * a^5 * b^3 - 4 * a^6 * b * c) * d * e + (3 * a^6 * b^2 - a^7 * c) * e^2) * f^2 + 4 * ((a^2 * b^6 - 4 * a^3 * b^4 * c + 4 * a^4 * b^2 * c^2 - a^5 * c^3) * d^3 - (3 * a^3 * b^5 - 9 * a^4 * b^3 * c + 5 * a^5 * b * c^2) * d^2 * e + (3 * a^4 * b^4 - 6 * a^5 * b^2 * c + a^6 * c^2) * d * e^2 - (a^5 * b^3 - a^6 * b * c) * e^3) * f) / (a^10 * b^2 - 4 * a^11 * c)) * \sqrt{-(a^4 * b * f^2 + (b^5 - 5 * a * b^3 * c + 5 * a^2 * b * c^2) * d^2 - 2 * (a * b^4 - 4 * a^2 * b^2 * c + 2 * a^3 * c^2) * d * e + (a^2 * b^3 - 3 * a^3 * b * c) * e^2 + 2 * ((a^2 * b^3 - 3 * a^3 * b * c) * d - (a^3 * b^2 - 2 * a^4 * c) * e) * f + (a^5 * b^2 - 4 * a^6 * c) * \sqrt{(a^8 * f^4 + (b^8 - 6 * a * b^6 * c + 11 * a^2 * b^4 * c^2 - 6 * a^3 * b^2 * c^3 + a^4 * c^4) * d^4 - 4 * (a * b^7 - 5 * a^2 * b^5 * c + 7 * a^3 * b^3 * c^2 - 2 * a^4 * b * c^3) * d^3 * e + 2 * (3 * a^2 * b^6 - 12 * a^3 * b^4 * c + 12 * a^4 * b^2 * c^2 - a^5 * c^3) * d^2 * e^2 - 4 * (a^3 * b^5 - 3 * a^4 * b^3 * c + 2 * a^5 * b * c^2) * d * e^3 + (a^4 * b^4 - 2 * a^5 * b^2 * c + a^6 * c^2) * e^4 - 4 * (a^7 * b * e - (a^6 * b^2 - a^7 * c) * d) * f^3 + 2 * ((3 * a^4 * b^4 - 7 * a^5 * b^2 * c + 3 * a^6 * c^2) * d^2 - 2 * (3 * a^5 * b^3 - 4 * a^6 * b * c) * d * e + (3 * a^6 * b^2 - a^7 * c) * e^2) * f^2 + 4 * ((a^2 * b^6 - 4 * a^3 * b^4 * c + 4 * a^4 * b^2 * c^2 - a^5 * c^3) * d^3 - (3 * a^3 * b^5 - 9 * a^4 * b^3 * c + 5 * a^5 * b * c^2) * d^2 * e + (3 * a^4 * b^4 - 6 * a^5 * b^2 * c + a^6 * c^2) * d * e^2 - (a^5 * b^3 - a^6 * b * c) * e^3) * f) / (a^10 * b^2 - 4 * a^11 * c)) / (a^5 * b^2 - 4 * a^6 * c)) + 3 * \sqrt{1/2} * a^2 * x^3 * \sqrt{-(a^4 * b * f^2 + (b^5 - 5 * a * b^3 * c + 5 * a^2 * b * c^2) * d^2 - 2 * (a * b^4 - 4 * a^2 * b^2 * c + 2 * a^3 * c^2) * d * e + (a^2 * b^3 - 3 * a^3 * b * c) * e^2 + 2 * ((a^2 * b^3 - 3 * a^3 * b * c) * d - (a^3 * b^2 - 2 * a^4 * c) * e) * f - (a^5 * b^2 - 4 * a^6 * c) * \sqrt{(a^8 * f^4 + (b^8 - 6 * a * b^6 * c + 11 * a^2 * b^4 * c^2 - 6 * a^3 * b^2 * c^3 + a^4 * c^4) * d^4 - 4 * (a * b^7 - 5 * a^2 * b^5 * c + 7 * a^3 * b^3 * c^2 - 2 * a^4 * b * c^3) * d^3 * e + 2 * (3 * a^2 * b^6 - 12 * a^3 * b^4 * c + 12 * a^4 * b^2 * c^2 - a^5 * c^3) * d^2 * e^2 - 4 * (a^3 * b^5 - 3 * a^4 * b^3 * c + 2 * a^5 * b * c^2) * d * e^3 + (a^4 * b^4 - 2 * a^5 * b^2 * c + a^6 * c^2) * e^4 - 4 * (a^7 * b * e - (a^6 * b^2 - a^7 * c) * d) * f^3 + 2 * ((3 * a^4 * b^4 - 7 * a^5 * b^2 * c + 3 * a^6 * c^2) * d^2 - 2 * (3 * a^5 * b^3 - 4 * a^6 * b * c) * d * e + (3 * a^6 * b^2 - a^7 * c) * e^2) * f^2 + 4 * ((a^2 * b^6 - 4 * a^3 * b^4 * c + 4 * a^4 * b^2 * c^2 - a^5 * c^3) * d^3 - (3 * a^3 * b^5 - 9 * a^4 * b^3 * c + 5 * a^5 * b * c^2) * d^2 * e + (3 * a^4 * b^4 - 6 * a^5 * b^2 * c + a^6 * c^2) * d * e^2 - (a^5 * b^3 - a^6 * b * c) * e^3) * f) / (a^10 * b^2 - 4 * a^11 * c)) / (a^5 * b^2 - 4 * a^6 * c)) * \log(2 * (a^6 * c * f^4 + (b^4 * c^3 - 3 * a * b^2 * c^4 + a^2 * c^5) * d^4 - (b^5 * c^2 - a * b^3 * c^3 - 3 * a^2 * b * c^4) * d^3 * e + 3 * (a * b^4 * c^2 - 2 * a^2 * b^2 * c^3) * d^2 * e^2 - (3 * a^2 * b^3 * c^2 - 5 * a^3 * b * c^3) * d * e^3 + (a^3 * b^2 * c^2 - a^4 * c^3) * e^4 - (3 * a^5 * b * c * e - (3 * a^4 * b^2 * c - 4 * a^5 * c^2) * d) * f^3 + 3 * (a^4 * b^2 * c * e^2 + (a^2 * b^4 * c - 3 * a^3 * b^2 * c^2 + 2 * a^4 * c^3) * d^2 - (2 * a^3 * b^3 * c - 3 * a^4 * b * c^2) * d * e) * f^2 + ((b^6 * c - 5 * a * b^4 * c^2 + 9 * a^2 * b^2 * c^3 - 4 * a^3 * c^4) * d^3 - 3 * (a * b^5 * c - 3 * a^2 * b^3 * c^2 + 3 * a^3 * b * c^3) * d^2 * e + 3 * (a^2 * b^4 * c - a^3 * b^2 * c^2) * d * e^2 - (a^3 * b^3 * c + a^4 * b * c^2) * e^3) * f) * x + \sqrt{1/2} * ((b^8 - 8 * a * b^6 * c + 20 * a^2 * b^4 * c^2 - 17 * a^3 * b^2 * c^3 + 4 * a^4 * c^4) * d^3 - (3 * a * b^7 - 21 * a^2 * b^5 * c + 41 * a^3 * b^3 * c^2 - 20 * a^4 * b * c^3) * d^2 * e + (3 * a^2 * b^6 - 18 * a^3 * b^4 * c + 25 * a^4 * b^2 * c^2 - 4 * a^5 * c^3) * d * e^2 - (a^3 * b^5 - 5 * a^4 * b^3 * c + 4 * a^5 * b * c^2) * e^3 + (a^6 * b^2 - 4 * a^7 * c) * f^3 + 3 * ((a^4 * b^4 - 5 * a^5 * b^2 * c + 4 * a^6 * c^2) * d - (a^5 * b^3 - 4 * a^6 * b * c) * e) * f^2 + ((3 * a^2 * b^6 - 19 * a^3 * b^4 * c + 31 * a^4 * b^2 * c^2 - 12 * a^5 * c^3) * d^2 - 2 * (3 * a^3 * b^5 - 16 * a^4 * b^3 * c + 16 * a^5 * b * c^2) * d * e + (3 * a^4 * b^4 - 13 * a^5 * b^2 * c + 4 * a^6 * c^2) * e^2) * f + ((a^5 * b^5 - 7 * a^6 * b^3 * c + 12 * a^7 * b * c^2) * d - (a^6 * b^4 - 6 * a^7 * b^2 * c + 8 * a^8 * c^2) * e + (a^7 * b^3 - 4 * a^8 * b * c) * f) * \sqrt{(a^8 * f^4 + (b^8 - 6 * a * b^6 * c + 11 * a^2 * b^4 * c^2 - 6 * a^3 * b^2 * c^3 + a^4 * c^4) * d^4 - 4 * (a * b^7 - 5 * a^2 * b^5 * c + 7 * a^3 * b^3 * c^2 - 2 * a^4 * b * c^3) * d^3 * e + 2 * (3 * a^2 * b^6 - 12 * a^3 * b^4 * c + 12 * a^4 * b^2 * c^2 - a^5 * c^3) * d^2 * e^2 - 4 * (a^3 * b^5 - 3 * a^4 * b^3 * c + 2 * a^5 * b * c^2) * d * e^3 + (a^4 * b^4 - 2 * a^5 * b^2 * c + a^6 * c^2) * e^4 - 4 * (a^7 * b * e - (a^6 * b^2 - a^7 * c) * d) * f^3 + 2 * ((3 * a^4 * b^4 - 7 * a^5 * b^2 * c + 3 * a^6 * c^2) * d^2 - 2 * (3 * a^5 * b^3 - 4 * a^6 * b * c) * d * e + (3 * a^6 * b^2 - a^7 * c) * e^2) * f^2 + 4 * ((a^2 * b^6 - 4 * a^3 * b^4 * c + 4 * a^4 * b^2 * c^2 - a^5 * c^3) * d^3 - (3 * a^3 * b^5 - 9 * a^4 * b^3 * c + 5 * a^5 * b * c^2) * d^2 * e + (3 * a^4 * b^4 - 6 * a^5 * b^2 * c + a^6 * c^2) * d * e^2 - (a^5 * b^3 - a^6 * b * c) * e^3) * f) / (a^10 * b^2 - 4 * a^11 * c)) / (a^5 * b^2 - 4 * a^6 * c))
\end{aligned}$$

$$2*a^3*b^4*c + 12*a^4*b^2*c^2 - a^5*c^3)*d^2*e^2 - 4*(a^3*b^5 - 3*a^4*b^3*c + 2*a^5*b*c^2)*d*e^3 + (a^4*b^4 - 2*a^5*b^2*c + a^6*c^2)*e^4 - 4*(a^7*b*e - (a^6*b^2 - a^7*c)*d)*f^3 + 2*((3*a^4*b^4 - 7*a^5*b^2*c + 3*a^6*c^2)*d^2 - 2*(3*a^5*b^3 - 4*a^6*b*c)*d*e + (3*a^6*b^2 - a^7*c)*e^2)*f^2 + 4*((a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2 - a^5*c^3)*d^3 - (3*a^3*b^5 - 9*a^4*b^3*c + 5*a^5*b*c^2)*d^2*e + (3*a^4*b^4 - 6*a^5*b^2*c + a^6*c^2)*d*e^2 - (a^5*b^3 - a^6*b*c)*e^3)*f)/(a^10*b^2 - 4*a^11*c))/(a^5*b^2 - 4*a^6*c)) - 6*(b*d - a*e)*x^2 + 2*a*d)/(a^2*x^3)$$

giac [B] time = 3.44, size = 3813, normalized size = 14.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] $1/4*((\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^6 - 9*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^4*c - 2*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^5*c - 2*b^6*c + 24*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b^2*c^2 + 10*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^3*c^2 + \sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^4*c^2 + 18*a*b^4*c^2 + 2*b^5*c^2 - 16*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^3*c^3 - 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b*c^3 - 5*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c^3 - 48*a^2*b^2*c^3 - 14*a*b^3*c^3 + 4*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*c^4 + 32*a^3*c^4 + 24*a^2*b*c^4 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^5 + 7*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^3*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^4*c - 12*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b*c^2 - 6*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c^2 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^3*c^2 + 3*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b*c^3 + 2*(b^2 - 4*a*c)*b^4*c - 10*(b^2 - 4*a*c)*a*b^2*c^2 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a^2*c^3 + 6*(b^2 - 4*a*c)*a*b*c^3)*d + (\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b^4 - 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^3*b^2*c - 2*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b^3*c - 2*a^2*b^4*c + 16*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^4*c^2 + 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^3*b*c^2 + \sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b^2*c^2 + 16*a^3*b^2*c^2 + 2*a^2*b^3*c^2 - 4*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^3*c^3 - 32*a^4*c^3 - 8*a^3*b*c^3 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b^3 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^3*b*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b^2*c - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b*c^2 + 2*(b^2 - 4*a*c)*a^2*b^2*c - 8*(b^2 - 4*a*c)*a^3*c^2 - 2*(b^2 - 4*a*c)*a^2*b*c^2)*f - (\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^5 - 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b^3*c - 2*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^4*c - 2*a*b^5*c + 16*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^3*b*c^2 + 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b^2*c^2 + \sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^3*c^2 + 16*a^2*b^3*c^2 + 2*a*b^4*c^2 - 4*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b*c^3 - 32*a^3*b*c^3 - 12*a^2*b^2*c^3 + 16*a^3*c^4 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^4 + 6*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b^2*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^3*c - 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^3*c^2 - 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b*c^2 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c^2 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*c^3 + 2*(b^2 - 4*a*c)*a*b^3*c - 8*(b^2 - 4*a*c)*a^2*b*c^2 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + 4*(b^2 - 4*a*c)*a^2*c^3)*e)*arctan(2*\sqrt{1/2}*x/\sqrt{(a^2*b + \sqrt{a^4*b^2 - 4*a^5*c})/(a^2*c)))/((a^3*b^4 - 8*a^4*b^2*c - 2*a^3*b^3*c + 16*a^5*c^2 + 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a^4*c^3)*abs(c)) + 1/4*((\sqrt{2}*$

) * sqrt(b*c - sqrt(b^2 - 4*a*c)*c) * b^6 - 9*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c) * a*b^4*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c) * b^5*c + 2*b^6*c + 24*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c) * a^2*b^2*c^2 + 10*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c) * a*b^3*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c) * b^4*c^2 - 18*a*b^4*c^2 - 2*b^5*c^2 - 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c) * a^3*c^3 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c) * a^2*b*c^3 - 5*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c) * a*b^2*c^3 + 48*a^2*b^2*c^3 + 14*a*b^3*c^3 + 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c) * a^2*c^4 - 32*a^3*c^4 - 2*4*a^2*b*c^4 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c) * b^5 - 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c) * a*b^3*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c) * b^4*c + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c) * a^2*b*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c) * a*b^2*c^2 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c) * b^3*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c) * a*b*c^3 - 2*(b^2 - 4*a*c)*b^4*c + 10*(b^2 - 4*a*c)*a*b^2*c^2 + 2*(b^2 - 4*a*c)*b^3*c^2 - 8*(b^2 - 4*a*c)*a^2*c^3 - 6*(b^2 - 4*a*c)*a*b*c^3)*d + (sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c) * a^2*b^4 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c) * a^3*b^2*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c) * a^2*b^3*c + 2*a^2*b^4*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c) * a^4*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c) * a^3*b*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c) * a^2*b^2*c^2 - 16*a^3*b^2*c^2 - 2*a^2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c) * a^3*c^3 + 32*a^4*c^3 + 8*a^3*b*c^3 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c) * a^2*b^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c) * a^3*b*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c) * a^2*b^2*c + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c) * a^2*b*c^2 - 2*(b^2 - 4*a*c)*a^2*b^2*c + 8*(b^2 - 4*a*c)*a^3*c^2 + 2*(b^2 - 4*a*c)*a^2*b*c^2)*f - (sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c) * a*b^5 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c) * a^2*b^3*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c) * a*b^4*c + 2*a*b^5*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c) * a^3*b*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c) * a^2*b^2*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c) * a*b^3*c^2 - 16*a^2*b^3*c^2 - 2*a*b^4*c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c) * a^2*b*c^3 + 32*a^3*b*c^3 + 12*a^2*b^2*c^3 - 16*a^3*c^4 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c) * a*b^4 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c) * a^2*b^2*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c) * a*b^3*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c) * a^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c) * a^2*b*c^2 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c) * a*b^2*c^2 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c) * a^2*c^3 - 2*(b^2 - 4*a*c)*a*b^3*c + 8*(b^2 - 4*a*c)*a^2*b*c^2 + 2*(b^2 - 4*a*c)*a*b^2*c^2 - 4*(b^2 - 4*a*c)*a^2*c^3)*e)*arctan(2*sqrt(1/2)*x/sqrt((a^2*b - sqrt(a^4*b^2 - 4*a^5*c))/(a^2*c)))/((a^3*b^4 - 8*a^4*b^2*c - 2*a^3*b^3*c + 16*a^5*c^2 + 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a^4*c^3)*abs(c)) + 1/3*(3*b*d*x^2 - 3*a*x^2*e - a*d)/(a^2*x^3)

maple [B] time = 0.03, size = 727, normalized size = 2.72

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a), x)

[Out] -1/3*d/a/x^3-1/a/x*e+1/a^2/x*b*d+1/2/a*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*e-1/2/a^2*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*c*x)*b*d-c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*c*x)*f+1/2/a*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*c*x)*b*e+1/a*c^2/(-4*a*c+b^2)^(1/2)

$$*2^{(1/2)} / ((-b + (-4*a*c + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctanh}(2^{(1/2)} / ((-b + (-4*a*c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * d - 1/2/a^2 * c / (-4*a*c + b^2)^{(1/2)} * 2^{(1/2)} / ((-b + (-4*a*c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * d - 1/2/a^2 * c * 2^{(1/2)} / ((b + (-4*a*c + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctan}(2^{(1/2)} / ((b + (-4*a*c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * e + 1/2/a^2 * c * 2^{(1/2)} / ((b + (-4*a*c + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctan}(2^{(1/2)} / ((b + (-4*a*c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * b * d - c / (-4*a*c + b^2)^{(1/2)} * 2^{(1/2)} / ((b + (-4*a*c + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctan}(2^{(1/2)} / ((b + (-4*a*c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * f + 1/2/a * c / (-4*a*c + b^2)^{(1/2)} * 2^{(1/2)} / ((b + (-4*a*c + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctan}(2^{(1/2)} / ((b + (-4*a*c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * b * e + 1/a * c^2 / (-4*a*c + b^2)^{(1/2)} * 2^{(1/2)} / ((b + (-4*a*c + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctan}(2^{(1/2)} / ((b + (-4*a*c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * d - 1/2/a^2 * c / (-4*a*c + b^2)^{(1/2)} * 2^{(1/2)} / ((b + (-4*a*c + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctan}(2^{(1/2)} / ((b + (-4*a*c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * b^2 * d$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a), x, algorithm="maxima")

[Out] -integrate((a*b*e - a^2*f - (b*c*d - a*c*e)*x^2 - (b^2 - a*c)*d)/(c*x^4 + b*x^2 + a), x)/a^2 + 1/3*(3*(b*d - a*e)*x^2 - a*d)/(a^2*x^3)

mupad [B] time = 4.76, size = 15505, normalized size = 58.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x^2 + f*x^4)/(x^4*(a + b*x^2 + c*x^4)), x)

[Out] atan(((x*(4*a^8*c^5*d^2 - 4*a^9*c^4*e^2 + 4*a^10*c^3*f^2 + 2*a^6*b^4*c^3*d^2 - 8*a^7*b^2*c^4*d^2 + 2*a^8*b^2*c^3*e^2 - 8*a^9*c^4*d*f + 12*a^8*b*c^4*d*e - 4*a^9*b*c^3*e*f - 4*a^7*b^3*c^3*d*e + 4*a^8*b^2*c^3*d*f) - ((b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^3*f^2 + a^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 - 4*a^5*b*c*f^2 + 2*a^2*b^5*d*f + 16*a^4*c^3*d*e - 2*a^3*b^4*e*f - 16*a^5*c^2*e*f - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 14*a^3*b^3*c*d*f + 24*a^4*b*c^2*d*f - 2*a^3*b*e*f*(-(4*a*c - b^2)^3)^(1/2) - 2*a^3*c*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^4*b^2*c*e*f - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 2*a^2*b^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)*(x*(32*a^11*b*c^3 - 8*a^10*b^3*c^2)*(-(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^3*f^2 + a^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 - 4*a^5*b*c*f^2 + 2*a^2*b^5*d*f + 16*a^4*c^3*d*e - 2*a^3*b^4*e*f - 16*a^5*c^2*e*f - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 14*a^3*b^3*c*d*f + 24*a^4*b*c^2*d*f - 2*a^3*b*e*f*(-(4*a*c - b^2)^3)^(1/2) - 2*a^3*c*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^4*b^2*c*e*f - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 2*a^2*b^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) - 16*a^10*c^4*d + 16*a^11*c^3*f - 4*a^8*b^4*c^2*d + 20*a^9*b^2*c^3*d + 4*a^9*b^3*c^2*e - 4*a^10*b^2*c^2*f - 16*a^10*b*c^3*e))*(-(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^3*f^2 + a^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 - 4*a^5*b*c*f^2 + 2*a^2*b^5*d*f + 16*a^4*c^3*d*e - 2*a^3*b^4*e*f - 16*a^5*c^2*e*f - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 14*a^3*b^3*c*d*f + 24*a^4*b*c^2*d*f - 2*a^3*b*e*f*(-(4*a*c - b^2)^3)^(1/2) - 2*a^3*c*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^4*b^2*c*e*f - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 2*a^2*b^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) - 16*a^10*c^4*d + 16*a^11*c^3*f - 4*a^8*b^4*c^2*d + 20*a^9*b^2*c^3*d + 4*a^9*b^3*c^2*e - 4*a^10*b^2*c^2*f - 16*a^10*b*c^3*e))*(-(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^3*f^2 + a^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 - 4*a^5*b*c*f^2 + 2*a^2*b^5*d*f + 16*a^4*c^3*d*e - 2*a^3*b^4*e*f - 16*a^5*c^2*e*f - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 14*a^3*b^3*c*d*f + 24*a^4*b*c^2*d*f - 2*a^3*b*e*f*(-(4*a*c - b^2)^3)^(1/2) - 2*a^3*c*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^4*b^2*c*e*f - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 2*a^2*b^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) - 16*a^10*c^4*d + 16*a^11*c^3*f - 4*a^8*b^4*c^2*d + 20*a^9*b^2*c^3*d + 4*a^9*b^3*c^2*e - 4*a^10*b^2*c^2*f - 16*a^10*b*c^3*e))

$$\begin{aligned}
& a^2c^2d^2(-4ac - b^2)^3)^{(1/2)} - 9ab^5cd^2 - 4a^5b^2cf^2 + 2a^2b^5d^2f + 16a^4c^3d^2e - 2a^3b^4e^2f - 16a^5c^2e^2f - 2ab^3d^2e^2 \\
& (-4ac - b^2)^3)^{(1/2)} + 16a^2b^4cd^2e - 14a^3b^3cd^2f + 24a^4b^2cd^2f - 2a^3b^2e^2f(-4ac - b^2)^3)^{(1/2)} - 2a^3cd^2f(-4ac - b^2)^3)^{(1/2)} \\
& + 12a^4b^2c^2e^2f - 3ab^2cd^2(-4ac - b^2)^3)^{(1/2)} - 36a^3b^2cd^2e + 2a^2b^2d^2f(-4ac - b^2)^3)^{(1/2)} + 4a^2b^2cd^2e(-4ac - b^2)^3)^{(1/2)} \\
& / (8(a^5b^4 + 16a^7c^2 - 8a^6b^2c))^{(1/2)} * 1i + (x(4a^8c^5d^2 - 4a^9c^4e^2 + 4a^{10}c^3f^2 + 2a^6b^4c^3d^2 - 8a^7b^2c^4d^2 \\
& + 2a^8b^2c^3e^2 - 8a^9c^4d^2f + 12a^8b^2c^4d^2e - 4a^9b^2c^3e^2f - 4a^7b^3c^3d^2e + 4a^8b^2c^3d^2f) - (b^7d^2 + a^2b^5e^2 \\
& + b^4d^2(-4ac - b^2)^3)^{(1/2)} + a^4b^3f^2 + a^4f^2(-4ac - b^2)^3)^{(1/2)} - 20a^3b^3cd^2 - 7a^3b^3c^2e^2 + 12a^4b^2c^2e^2 - a^3c^2e^2 \\
& (-4ac - b^2)^3)^{(1/2)} - 2ab^6d^2e + 25a^2b^3c^2d^2 + a^2b^2e^2(-4ac - b^2)^3)^{(1/2)} + a^2c^2d^2(-4ac - b^2)^3)^{(1/2)} - 9ab^5cd^2 \\
& - 4a^5b^2cf^2 + 2a^2b^5d^2f + 16a^4c^3d^2e - 2a^3b^4e^2f - 16a^5c^2e^2f - 2ab^3d^2e^2(-4ac - b^2)^3)^{(1/2)} + 16a^2b^4cd^2e \\
& - 14a^3b^3cd^2f + 24a^4b^2cd^2f - 2a^3b^2e^2f(-4ac - b^2)^3)^{(1/2)} - 2a^3cd^2f(-4ac - b^2)^3)^{(1/2)} + 12a^4b^2c^2e^2f - 3ab^2cd^2 \\
& (-4ac - b^2)^3)^{(1/2)} - 36a^3b^2cd^2e + 2a^2b^2d^2f(-4ac - b^2)^3)^{(1/2)} + 4a^2b^2cd^2e(-4ac - b^2)^3)^{(1/2)} / (8(a^5b^4 + 16a^7c^2 \\
& - 8a^6b^2c))^{(1/2)} * (x(32a^{11}b^2c^3 - 8a^{10}b^3c^2) * (-b^7d^2 + a^2b^5e^2 + b^4d^2(-4ac - b^2)^3)^{(1/2)} + a^4b^3f^2 + a^4f^2 \\
& (-4ac - b^2)^3)^{(1/2)} - 20a^3b^3cd^2 - 7a^3b^3c^2e^2 + 12a^4b^2c^2e^2 - a^3c^2e^2(-4ac - b^2)^3)^{(1/2)} - 2ab^6d^2e + 25a^2b^3c^2d^2 \\
& + a^2b^2e^2(-4ac - b^2)^3)^{(1/2)} + a^2c^2d^2(-4ac - b^2)^3)^{(1/2)} - 9ab^5cd^2 - 4a^5b^2cf^2 + 2a^2b^5d^2f + 16a^4c^3d^2e - 2a^3b^4e^2f \\
& - 16a^5c^2e^2f - 2ab^3d^2e^2(-4ac - b^2)^3)^{(1/2)} + 16a^2b^4cd^2e - 14a^3b^3cd^2f + 24a^4b^2cd^2f - 2a^3b^2e^2f(-4ac - b^2)^3)^{(1/2)} \\
& - 2a^3cd^2f(-4ac - b^2)^3)^{(1/2)} + 12a^4b^2c^2e^2f - 3ab^2cd^2(-4ac - b^2)^3)^{(1/2)} - 36a^3b^2cd^2e + 2a^2b^2d^2f(-4ac - b^2)^3)^{(1/2)} \\
& + 4a^2b^2cd^2e(-4ac - b^2)^3)^{(1/2)} / (8(a^5b^4 + 16a^7c^2 - 8a^6b^2c))^{(1/2)} + 16a^{10}c^4d - 16a^{11}c^3f + 4a^8b^4c^2d - 20a^9b^2c^3d \\
& - 4a^9b^3c^2e + 4a^{10}b^2c^2f + 16a^{10}b^2c^3e) * (-b^7d^2 + a^2b^5e^2 + b^4d^2(-4ac - b^2)^3)^{(1/2)} + a^4b^3f^2 + a^4f^2(-4ac - b^2)^3)^{(1/2)} \\
& - 20a^3b^3cd^2 - 7a^3b^3c^2e^2 + 12a^4b^2c^2e^2 - a^3c^2e^2(-4ac - b^2)^3)^{(1/2)} - 2ab^6d^2e + 25a^2b^3c^2d^2 + a^2b^2e^2(-4ac - b^2)^3)^{(1/2)} \\
& + a^2c^2d^2(-4ac - b^2)^3)^{(1/2)} - 9ab^5cd^2 - 4a^5b^2cf^2 + 2a^2b^5d^2f + 16a^4c^3d^2e - 2a^3b^4e^2f - 16a^5c^2e^2f - 2ab^3d^2e^2(-4ac - b^2)^3)^{(1/2)} \\
& + 16a^2b^4cd^2e - 14a^3b^3cd^2f + 24a^4b^2cd^2f - 2a^3b^2e^2f(-4ac - b^2)^3)^{(1/2)} - 2a^3cd^2f(-4ac - b^2)^3)^{(1/2)} + 12a^4b^2c^2e^2f \\
& - 3ab^2cd^2(-4ac - b^2)^3)^{(1/2)} - 36a^3b^2cd^2e + 2a^2b^2d^2f(-4ac - b^2)^3)^{(1/2)} + 4a^2b^2cd^2e(-4ac - b^2)^3)^{(1/2)} / (8(a^5b^4 + 16a^7c^2 \\
& - 8a^6b^2c))^{(1/2)} * (x(32a^{11}b^2c^3 - 8a^{10}b^3c^2) * (-b^7d^2 + a^2b^5e^2 + b^4d^2(-4ac - b^2)^3)^{(1/2)} + a^4b^3f^2 + a^4f^2(-4ac - b^2)^3)^{(1/2)} \\
& + a^2b^5e^2 + b^4d^2(-4ac - b^2)^3)^{(1/2)} + a^4b^3f^2 + a^4f^2(-4ac - b^2)^3)^{(1/2)}
\end{aligned}$$

$$\begin{aligned}
& -(4ac - b^2)^3)^{(1/2)} - 20a^3b^3c^3d^2 - 7a^3b^3c^3e^2 + 12a^4b^3c^2 \\
& *e^2 - a^3c^3e^2*(-(4ac - b^2)^3)^{(1/2)} - 2a^2b^6d^2e + 25a^2b^3c^2d^2 \\
& + a^2b^2e^2*(-(4ac - b^2)^3)^{(1/2)} + a^2c^2d^2*(-(4ac - b^2)^3)^{(1/2)} - 9a^2b^5c^3d^2 \\
& - 4a^5b^3c^3f^2 + 2a^2b^5d^2f + 16a^4c^3d^2e - 2a^3b^4e^2f - 16a^5c^2e^2f \\
& - 2a^2b^3d^2e*(-(4ac - b^2)^3)^{(1/2)} + 16a^2b^4c^3d^2e - 14a^3b^3c^3d^2f \\
& + 24a^4b^3c^2d^2f - 2a^3b^3e^2f*(-(4ac - b^2)^3)^{(1/2)} - 2a^3c^3d^2f*(-(4ac - b^2)^3)^{(1/2)} \\
& + 12a^4b^2c^3e^2f - 3a^2b^2c^3d^2*(-(4ac - b^2)^3)^{(1/2)} - 36a^3b^2c^2d^2e \\
& + 2a^2b^2d^2f*(-(4ac - b^2)^3)^{(1/2)} + 4a^2b^3c^3d^2e*(-(4ac - b^2)^3)^{(1/2)} \\
& + 4a^2b^3c^3d^2f*(-(4ac - b^2)^3)^{(1/2)}/(8(a^5b^4 + 16a^7c^2 - 8a^6b^2c))^{(1/2)} - 16a^{10}c^4d \\
& + 16a^{11}c^3f - 4a^8b^4c^2d + 20a^9b^2c^3d + 4a^9b^3c^2e - 4a^{10}b^2c^2f - 16a^{10}b^3c^3e \\
&)*(-(b^7d^2 + a^2b^5e^2 + b^4d^2*(-(4ac - b^2)^3)^{(1/2)} + a^4b^3f^2 + a^4f^2*(-(4ac - b^2)^3)^{(1/2)} \\
& - 20a^3b^3c^3d^2 - 7a^3b^3c^3e^2 + 12a^4b^3c^2e^2 - a^3c^3e^2*(-(4ac - b^2)^3)^{(1/2)} - 2a^2b^6d^2e \\
& + 25a^2b^3c^2d^2 + a^2b^2e^2*(-(4ac - b^2)^3)^{(1/2)} + a^2c^2d^2*(-(4ac - b^2)^3)^{(1/2)} - 9a^2b^5c^3d^2 \\
& - 4a^5b^3c^3f^2 + 2a^2b^5d^2f + 16a^4c^3d^2e - 2a^3b^4e^2f - 16a^5c^2e^2f - 2a^2b^3d^2e*(-(4ac - b^2)^3)^{(1/2)} \\
& + 16a^2b^4c^3d^2e - 14a^3b^3c^3d^2f + 24a^4b^3c^2d^2f - 2a^3b^3e^2f*(-(4ac - b^2)^3)^{(1/2)} - 2a^3c^3d^2f*(-(4ac - b^2)^3)^{(1/2)} \\
&) + 12a^4b^2c^3e^2f - 3a^2b^2c^3d^2*(-(4ac - b^2)^3)^{(1/2)} - 36a^3b^2c^2d^2e \\
& + 2a^2b^2d^2f*(-(4ac - b^2)^3)^{(1/2)} + 4a^2b^3c^3d^2e*(-(4ac - b^2)^3)^{(1/2)} \\
& + 4a^2b^3c^3d^2f*(-(4ac - b^2)^3)^{(1/2)}/(8(a^5b^4 + 16a^7c^2 - 8a^6b^2c))^{(1/2)} - (x(4a^8 \\
& *c^5d^2 - 4a^9c^4e^2 + 4a^{10}c^3f^2 + 2a^6b^4c^3d^2 - 8a^7b^2c^4d^2 + 2a^8b^2c^3e^2 \\
& - 8a^9c^4d^2f + 12a^8b^3c^4d^2e - 4a^9b^3c^3e^2f - 4a^7b^3c^3d^2e + 4a^8b^2c^3d^2f) \\
& - (-(b^7d^2 + a^2b^5e^2 + b^4d^2*(-(4ac - b^2)^3)^{(1/2)} + a^4b^3f^2 + a^4f^2*(-(4ac - b^2)^3)^{(1/2)} \\
& - 20a^3b^3c^3d^2 - 7a^3b^3c^3e^2 + 12a^4b^3c^2e^2 - a^3c^3e^2*(-(4ac - b^2)^3)^{(1/2)} - 2a^2b^6d^2e \\
& + 25a^2b^3c^2d^2 + a^2b^2e^2*(-(4ac - b^2)^3)^{(1/2)} + a^2c^2d^2*(-(4ac - b^2)^3)^{(1/2)} - 9a^2b^5c^3d^2 \\
& - 4a^5b^3c^3f^2 + 2a^2b^5d^2f + 16a^4c^3d^2e - 2a^3b^4e^2f - 16a^5c^2e^2f - 2a^2b^3d^2e*(-(4ac - b^2)^3)^{(1/2)} \\
& + 16a^2b^4c^3d^2e - 14a^3b^3c^3d^2f + 24a^4b^3c^2d^2f - 2a^3b^3e^2f*(-(4ac - b^2)^3)^{(1/2)} - 2a^3c^3d^2f*(-(4ac - b^2)^3)^{(1/2)} \\
& - 2a^3c^3d^2f*(-(4ac - b^2)^3)^{(1/2)} + 12a^4b^2c^3e^2f - 3a^2b^2c^3d^2*(-(4ac - b^2)^3)^{(1/2)} \\
& - 36a^3b^2c^2d^2e + 2a^2b^2d^2f*(-(4ac - b^2)^3)^{(1/2)} + 4a^2b^3c^3d^2e*(-(4ac - b^2)^3)^{(1/2)} \\
& + 4a^2b^3c^3d^2f*(-(4ac - b^2)^3)^{(1/2)}/(8(a^5b^4 + 16a^7c^2 - 8a^6b^2c))^{(1/2)} * (x(32a^{11}b^3c^3 - 8a^{10}b^3c^2) \\
&)*(-(b^7d^2 + a^2b^5e^2 + b^4d^2*(-(4ac - b^2)^3)^{(1/2)} + a^4b^3f^2 + a^4f^2*(-(4ac - b^2)^3)^{(1/2)} \\
& - 20a^3b^3c^3d^2 - 7a^3b^3c^3e^2 + 12a^4b^3c^2e^2 - a^3c^3e^2*(-(4ac - b^2)^3)^{(1/2)} - 2a^2b^6d^2e \\
& + 25a^2b^3c^2d^2 + a^2b^2e^2*(-(4ac - b^2)^3)^{(1/2)} + a^2c^2d^2*(-(4ac - b^2)^3)^{(1/2)} - 9a^2b^5c^3d^2 \\
& - 4a^5b^3c^3f^2 + 2a^2b^5d^2f + 16a^4c^3d^2e - 2a^3b^4e^2f - 16a^5c^2e^2f - 2a^2b^3d^2e*(-(4ac - b^2)^3)^{(1/2)} \\
& + 16a^2b^4c^3d^2e - 14a^3b^3c^3d^2f + 24a^4b^3c^2d^2f - 2a^3b^3e^2f*(-(4ac - b^2)^3)^{(1/2)} - 2a^3c^3d^2f*(-(4ac - b^2)^3)^{(1/2)} \\
& + 12a^4b^2c^3e^2f - 3a^2b^2c^3d^2*(-(4ac - b^2)^3)^{(1/2)} - 36a^3b^2c^2d^2e \\
& + 2a^2b^2d^2f*(-(4ac - b^2)^3)^{(1/2)} + 4a^2b^3c^3d^2e*(-(4ac - b^2)^3)^{(1/2)} \\
& + 4a^2b^3c^3d^2f*(-(4ac - b^2)^3)^{(1/2)}/(8(a^5b^4 + 16a^7c^2 - 8a^6b^2c))^{(1/2)} + 2a^8c^4e^3 -
\end{aligned}$$

$$\begin{aligned}
& 2*a^6*b*c^5*d^3 + 2*a^7*c^5*d^2*e + 2*a^9*c^3*e*f^2 - 4*a^8*c^4*d*e*f - 4*a^7*b*c^4*d*e^2 + 4*a^7*b*c^4*d^2*f - 2*a^8*b*c^3*d*f^2 - 2*a^8*b*c^3*e^2*f \\
& + 2*a^6*b^2*c^4*d^2*e - 2*a^6*b^3*c^3*d^2*f + 4*a^7*b^2*c^3*d*e*f)) * (-(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + a^4*b^3*f^2 + a^4*f^2 * (-(4*a*c - b^2)^3)^{(1/2)} - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 9*a*b^5*c*d^2 - 4*a^5*b*c*f^2 + 2*a^2*b^5*d*f + 16*a^4*c^3*d*e - 2*a^3*b^4*e*f - 16*a^5*c^2*e*f - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 16*a^2*b^4*c*d*e - 14*a^3*b^3*c*d*f + 24*a^4*b*c^2*d*f - 2*a^3*b*e*f*(-(4*a*c - b^2)^3)^{(1/2)} - 2*a^3*c*d*f*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^4*b^2*c*e*f - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 36*a^3*b^2*c^2*d*e + 2*a^2*b^2*d*f*(-(4*a*c - b^2)^3)^{(1/2)} + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c))^{(1/2)} * 2i - (d/(3*a) + (x^2*(a*e - b*d))/a^2)/x^3 + atan(((x*(4*a^8*c^5*d^2 - 4*a^9*c^4*e^2 + 4*a^10*c^3*f^2 + 2*a^6*b^4*c^3*d^2 - 8*a^7*b^2*c^4*d^2 + 2*a^8*b^2*c^3*e^2 - 8*a^9*c^4*d*f + 12*a^8*b*c^4*d*e - 4*a^9*b*c^3*e*f - 4*a^7*b^3*c^3*d*e + 4*a^8*b^2*c^3*d*f) - (-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + a^4*b^3*f^2 - a^4*f^2*(-(4*a*c - b^2)^3)^{(1/2)} - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - a^2*c^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 9*a*b^5*c*d^2 - 4*a^5*b*c*f^2 + 2*a^2*b^5*d*f + 16*a^4*c^3*d*e - 2*a^3*b^4*e*f - 16*a^5*c^2*e*f + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 16*a^2*b^4*c*d*e - 14*a^3*b^3*c*d*f + 24*a^4*b*c^2*d*f + 2*a^3*b*e*f*(-(4*a*c - b^2)^3)^{(1/2)} + 2*a^3*c*d*f*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^4*b^2*c*e*f + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 36*a^3*b^2*c^2*d*e - 2*a^2*b^2*d*f*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c))^{(1/2)} * (x*(32*a^11*b*c^3 - 8*a^10*b^3*c^2) * (-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + a^4*b^3*f^2 - a^4*f^2*(-(4*a*c - b^2)^3)^{(1/2)} - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - a^2*c^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 9*a*b^5*c*d^2 - 4*a^5*b*c*f^2 + 2*a^2*b^5*d*f + 16*a^4*c^3*d*e - 2*a^3*b^4*e*f - 16*a^5*c^2*e*f + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 16*a^2*b^4*c*d*e - 14*a^3*b^3*c*d*f + 24*a^4*b*c^2*d*f + 2*a^3*b*e*f*(-(4*a*c - b^2)^3)^{(1/2)} + 2*a^3*c*d*f*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^4*b^2*c*e*f + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 36*a^3*b^2*c^2*d*e - 2*a^2*b^2*d*f*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c))^{(1/2)} - 16*a^10*c^4*d + 16*a^11*c^3*f - 4*a^8*b^4*c^2*d + 20*a^9*b^2*c^3*d + 4*a^9*b^3*c^2*e - 4*a^10*b^2*c^2*f - 16*a^10*b*c^3*e)) * (-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + a^4*b^3*f^2 - a^4*f^2*(-(4*a*c - b^2)^3)^{(1/2)} - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - a^2*c^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 9*a*b^5*c*d^2 - 4*a^5*b*c*f^2 + 2*a^2*b^5*d*f + 16*a^4*c^3*d*e - 2*a^3*b^4*e*f - 16*a^5*c^2*e*f + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 16*a^2*b^4*c*d*e - 14*a^3*b^3*c*d*f + 24*a^4*b*c^2*d*f + 2*a^3*b*e*f*(-(4*a*c - b^2)^3)^{(1/2)} + 2*a^3*c*d*f*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^4*b^2*c*e*f + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 36*a^3*b^2*c^2*d*e - 2*a^2*b^2*d*f*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c))^{(1/2)} * 1i + (x*(4*a^8*c^5*d^2 - 4*a^9*c^4*e^2 + 4*a^10*c^3*f^2 + 2*a^6*b^4*c^3*d^2 - 8*a^7*b^2*c^4*d^2 + 2*a^8*b^2*c^3*e^2 - 8*a^9*c^4*d*f + 12*a^8*b*c^4*d*e - 4*a^9*b*c^3*e*f - 4*a^7*b^3*c^3*d*e + 4*a^8*b^2*c^3*d*f) - (-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + a^4*b^3*f^2 - a^4*f^2*(-(4*a*c - b^2)^3)^{(1/2)} - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - a^2*c^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 9*a*b^5*c*d^2 - 4*a^5*b*c*f^2 + 2*a^2*b^5*d*f + 16*a^4*c^3*d*e
\end{aligned}$$

$$\begin{aligned}
& - 2a^3b^4ef - 16a^5c^2ef + 2ab^3d * (-4ac - b^2)^3)^{(1/2)} + \\
& 16a^2b^4cde - 14a^3b^3c * d * f + 24a^4b^2c^2 * d * f + 2a^3b * e * f * (-4ac - b^2)^3)^{(1/2)} + 2a^3c * d * f * (-4ac - b^2)^3)^{(1/2)} + 12a^4b^2c * e * \\
& f + 3ab^2c * d^2 * (-4ac - b^2)^3)^{(1/2)} - 36a^3b^2c^2 * d * e - 2a^2b^2 \\
& * d * f * (-4ac - b^2)^3)^{(1/2)} - 4a^2b * c * d * e * (-4ac - b^2)^3)^{(1/2)} / (8 * \\
& (a^5b^4 + 16a^7c^2 - 8a^6b^2 * c))^{(1/2)} * (x * (32a^{11}b^3c^3 - 8a^{10}b^3 \\
& * c^2) * (-b^7d^2 + a^2b^5e^2 - b^4d^2 * (-4ac - b^2)^3)^{(1/2)} + a^4b^3 \\
& * f^2 - a^4f^2 * (-4ac - b^2)^3)^{(1/2)} - 20a^3b^3c^3 * d^2 - 7a^3b^3c * e^2 \\
& + 12a^4b^2c^2 * e^2 + a^3c * e^2 * (-4ac - b^2)^3)^{(1/2)} - 2ab^6 * d * e + 2 \\
& 5a^2b^3c^2 * d^2 - a^2b^2 * e^2 * (-4ac - b^2)^3)^{(1/2)} - a^2c^2 * d^2 * (-4 \\
& * ac - b^2)^3)^{(1/2)} - 9ab^5c * d^2 - 4a^5b * c * f^2 + 2a^2b^5 * d * f + 16a \\
& ^4c^3 * d * e - 2a^3b^4 * e * f - 16a^5c^2 * e * f + 2ab^3 * d * e * (-4ac - b^2)^3 \\
&)^{(1/2)} + 16a^2b^4c * d * e - 14a^3b^3c * d * f + 24a^4b^2c^2 * d * f + 2a^3b * \\
& e * f * (-4ac - b^2)^3)^{(1/2)} + 2a^3c * d * f * (-4ac - b^2)^3)^{(1/2)} + 12a^4 \\
& b^2c * e * f + 3ab^2c * d^2 * (-4ac - b^2)^3)^{(1/2)} - 36a^3b^2c^2 * d * e - \\
& 2a^2b^2 * d * f * (-4ac - b^2)^3)^{(1/2)} - 4a^2b * c * d * e * (-4ac - b^2)^3)^{(1/2)} \\
&) / (8 * (a^5b^4 + 16a^7c^2 - 8a^6b^2 * c))^{(1/2)} + 16a^{10}c^4 * d - 16 \\
& a^{11}c^3 * f + 4a^8b^4c^2 * d - 20a^9b^2c^3 * d - 4a^9b^3c^2 * e + 4a^{10} \\
& b^2c^2 * f + 16a^{10}b^3c * e) * (-b^7d^2 + a^2b^5e^2 - b^4d^2 * (-4ac - \\
& - b^2)^3)^{(1/2)} + a^4b^3 * f^2 - a^4f^2 * (-4ac - b^2)^3)^{(1/2)} - 20a^3b \\
& * c^3 * d^2 - 7a^3b^3c * e^2 + 12a^4b^2c^2 * e^2 + a^3c * e^2 * (-4ac - b^2)^3 \\
&)^{(1/2)} - 2ab^6 * d * e + 25a^2b^3c^2 * d^2 - a^2b^2 * e^2 * (-4ac - b^2)^3 \\
&)^{(1/2)} - a^2c^2 * d^2 * (-4ac - b^2)^3)^{(1/2)} - 9ab^5c * d^2 - 4a^5b * c * f \\
& ^2 + 2a^2b^5 * d * f + 16a^4c^3 * d * e - 2a^3b^4 * e * f - 16a^5c^2 * e * f + 2a * \\
& b^3 * d * e * (-4ac - b^2)^3)^{(1/2)} + 16a^2b^4c * d * e - 14a^3b^3c * d * f + 24 \\
& a^4b^2c^2 * d * f + 2a^3b * e * f * (-4ac - b^2)^3)^{(1/2)} + 2a^3c * d * f * (-4ac \\
& - b^2)^3)^{(1/2)} + 12a^4b^2c * e * f + 3ab^2c * d^2 * (-4ac - b^2)^3)^{(1/2)} \\
& - 36a^3b^2c^2 * d * e - 2a^2b^2 * d * f * (-4ac - b^2)^3)^{(1/2)} - 4a^2b * \\
& c * d * e * (-4ac - b^2)^3)^{(1/2)} / (8 * (a^5b^4 + 16a^7c^2 - 8a^6b^2 * c))^{(1/2)} * i) / \\
& ((x * (4a^8c^5 * d^2 - 4a^9c^4 * e^2 + 4a^{10}c^3 * f^2 + 2a^6b^4c^3 \\
& * d^2 - 8a^7b^2c^4 * d^2 + 2a^8b^2c^3 * e^2 - 8a^9c^4 * d * f + 12a^8b^3c^4 \\
& * d * e - 4a^9b^3c^3 * e * f - 4a^7b^3c^3 * d * e + 4a^8b^2c^3 * d * f) - (-b^7d \\
& ^2 + a^2b^5e^2 - b^4d^2 * (-4ac - b^2)^3)^{(1/2)} + a^4b^3 * f^2 - a^4f^2 \\
& * (-4ac - b^2)^3)^{(1/2)} - 20a^3b^3c^3 * d^2 - 7a^3b^3c * e^2 + 12a^4b^2c \\
& ^2 * e^2 + a^3c * e^2 * (-4ac - b^2)^3)^{(1/2)} - 2ab^6 * d * e + 25a^2b^3c^2 * \\
& d^2 - a^2b^2 * e^2 * (-4ac - b^2)^3)^{(1/2)} - a^2c^2 * d^2 * (-4ac - b^2)^3 \\
&)^{(1/2)} - 9ab^5c * d^2 - 4a^5b * c * f^2 + 2a^2b^5 * d * f + 16a^4c^3 * d * e - 2 \\
& a^3b^4 * e * f - 16a^5c^2 * e * f + 2ab^3 * d * e * (-4ac - b^2)^3)^{(1/2)} + 16a \\
& ^2b^4c * d * e - 14a^3b^3c * d * f + 24a^4b^2c^2 * d * f + 2a^3b * e * f * (-4ac - \\
& b^2)^3)^{(1/2)} + 2a^3c * d * f * (-4ac - b^2)^3)^{(1/2)} + 12a^4b^2c * e * f + \\
& 3ab^2c * d^2 * (-4ac - b^2)^3)^{(1/2)} - 36a^3b^2c^2 * d * e - 2a^2b^2 * d * f \\
& * (-4ac - b^2)^3)^{(1/2)} - 4a^2b * c * d * e * (-4ac - b^2)^3)^{(1/2)} / (8 * (a^5 \\
& * b^4 + 16a^7c^2 - 8a^6b^2 * c))^{(1/2)} * (x * (32a^{11}b^3c^3 - 8a^{10}b^3c^2 \\
&) * (-b^7d^2 + a^2b^5e^2 - b^4d^2 * (-4ac - b^2)^3)^{(1/2)} + a^4b^3 * f^2 \\
& - a^4f^2 * (-4ac - b^2)^3)^{(1/2)} - 20a^3b^3c^3 * d^2 - 7a^3b^3c * e^2 + \\
& 12a^4b^2c^2 * e^2 + a^3c * e^2 * (-4ac - b^2)^3)^{(1/2)} - 2ab^6 * d * e + 25a^2 \\
& b^3c^2 * d^2 - a^2b^2 * e^2 * (-4ac - b^2)^3)^{(1/2)} - a^2c^2 * d^2 * (-4ac - \\
& b^2)^3)^{(1/2)} - 9ab^5c * d^2 - 4a^5b * c * f^2 + 2a^2b^5 * d * f + 16a^4c^3 \\
& * d * e - 2a^3b^4 * e * f - 16a^5c^2 * e * f + 2ab^3 * d * e * (-4ac - b^2)^3)^{(1/2)} \\
& + 16a^2b^4c * d * e - 14a^3b^3c * d * f + 24a^4b^2c^2 * d * f + 2a^3b * e * f * \\
& (-4ac - b^2)^3)^{(1/2)} + 2a^3c * d * f * (-4ac - b^2)^3)^{(1/2)} + 12a^4b^2 \\
& c * e * f + 3ab^2c * d^2 * (-4ac - b^2)^3)^{(1/2)} - 36a^3b^2c^2 * d * e - 2a \\
& ^2b^2 * d * f * (-4ac - b^2)^3)^{(1/2)} - 4a^2b * c * d * e * (-4ac - b^2)^3)^{(1/2)} \\
&) / (8 * (a^5b^4 + 16a^7c^2 - 8a^6b^2 * c))^{(1/2)} - 16a^{10}c^4 * d + 16a^{11} \\
& c^3 * f - 4a^8b^4c^2 * d + 20a^9b^2c^3 * d + 4a^9b^3c^2 * e - 4a^{10}b^2 \\
& * c^2 * f - 16a^{10}b^3c * e) * (-b^7d^2 + a^2b^5e^2 - b^4d^2 * (-4ac - b^2)^3 \\
&)^{(1/2)} + a^4b^3 * f^2 - a^4f^2 * (-4ac - b^2)^3)^{(1/2)} - 20a^3b^3c^3 \\
& * d^2 - 7a^3b^3c * e^2 + 12a^4b^2c^2 * e^2 + a^3c * e^2 * (-4ac - b^2)^3)^{(1/2)} \\
& - 2ab^6 * d * e + 25a^2b^3c^2 * d^2 - a^2b^2 * e^2 * (-4ac - b^2)^3)^{(1/2)}
\end{aligned}$$

$$\begin{aligned}
& 2) - a^2c^2d^2(-4ac - b^2)^3)^{(1/2)} - 9ab^5cd^2 - 4a^5b^2cf^2 + \\
& 2a^2b^5df + 16a^4c^3de - 2a^3b^4ef - 16a^5c^2ef + 2ab^3d \\
& de(-4ac - b^2)^3)^{(1/2)} + 16a^2b^4cd^2e - 14a^3b^3cd^2f + 24a^4 \\
& b^2cd^2f + 2a^3b^2ef(-4ac - b^2)^3)^{(1/2)} + 2a^3cd^2f(-4ac - \\
& b^2)^3)^{(1/2)} + 12a^4b^2c^2ef + 3ab^2cd^2(-4ac - b^2)^3)^{(1/2)} - \\
& 36a^3b^2c^2d^2e - 2a^2b^2d^2f(-4ac - b^2)^3)^{(1/2)} - 4a^2b^2cd^2 \\
& e(-4ac - b^2)^3)^{(1/2)} / (8(a^5b^4 + 16a^7c^2 - 8a^6b^2c))^{(1/2)} \\
& - (x(4a^8c^5d^2 - 4a^9c^4e^2 + 4a^10c^3f^2 + 2a^6b^4c^3d^2 - \\
& 8a^7b^2c^4d^2 + 2a^8b^2c^3e^2 - 8a^9c^4d^2f + 12a^8b^2c^4d^2e - \\
& 4a^9b^2c^3ef - 4a^7b^3c^3d^2e + 4a^8b^2c^3d^2f) - ((b^7d^2 + a^2 \\
& b^5e^2 - b^4d^2(-4ac - b^2)^3)^{(1/2)} + a^4b^3f^2 - a^4f^2(-4ac \\
& - b^2)^3)^{(1/2)} - 20a^3b^2cd^2 - 7a^3b^3c^2e^2 + 12a^4b^2c^2e^2 \\
& + a^3c^2e^2(-4ac - b^2)^3)^{(1/2)} - 2ab^6d^2e + 25a^2b^3c^2d^2 - a \\
& ^2b^2e^2(-4ac - b^2)^3)^{(1/2)} - a^2c^2d^2(-4ac - b^2)^3)^{(1/2)} \\
& - 9ab^5cd^2 - 4a^5b^2cf^2 + 2a^2b^5df + 16a^4c^3de - 2a^3b^4 \\
& 4ef - 16a^5c^2ef + 2ab^3d^2ef(-4ac - b^2)^3)^{(1/2)} + 16a^2b^4c \\
& cd^2e - 14a^3b^3cd^2f + 24a^4b^2cd^2f + 2a^3b^2ef(-4ac - b^2)^3 \\
&)^{(1/2)} + 2a^3cd^2f(-4ac - b^2)^3)^{(1/2)} + 12a^4b^2c^2ef + 3ab^2 \\
& cd^2(-4ac - b^2)^3)^{(1/2)} - 36a^3b^2c^2d^2e - 2a^2b^2d^2f(-4ac \\
& - b^2)^3)^{(1/2)} - 4a^2b^2cd^2ef(-4ac - b^2)^3)^{(1/2)} / (8(a^5b^4 + \\
& 16a^7c^2 - 8a^6b^2c))^{(1/2)} * (x(32a^11b^3c^3 - 8a^10b^3c^2) * (-b^7 \\
& d^2 + a^2b^5e^2 - b^4d^2(-4ac - b^2)^3)^{(1/2)} + a^4b^3f^2 - a^4f^2 \\
& (-4ac - b^2)^3)^{(1/2)} - 20a^3b^2cd^2 - 7a^3b^3c^2e^2 + 12a^4b^2 \\
& c^2e^2 + a^3c^2e^2(-4ac - b^2)^3)^{(1/2)} - 2ab^6d^2e + 25a^2b^3c^2 \\
& d^2 - a^2b^2e^2(-4ac - b^2)^3)^{(1/2)} - a^2c^2d^2(-4ac - b^2)^3 \\
&)^{(1/2)} - 9ab^5cd^2 - 4a^5b^2cf^2 + 2a^2b^5df + 16a^4c^3de \\
& - 2a^3b^4ef - 16a^5c^2ef + 2ab^3d^2ef(-4ac - b^2)^3)^{(1/2)} + 1 \\
& 6a^2b^4cd^2e - 14a^3b^3cd^2f + 24a^4b^2cd^2f + 2a^3b^2ef(-4ac \\
& - b^2)^3)^{(1/2)} + 2a^3cd^2f(-4ac - b^2)^3)^{(1/2)} + 12a^4b^2c^2ef \\
& + 3ab^2cd^2(-4ac - b^2)^3)^{(1/2)} - 36a^3b^2c^2d^2e - 2a^2b^2d^2 \\
& d^2f(-4ac - b^2)^3)^{(1/2)} - 4a^2b^2cd^2ef(-4ac - b^2)^3)^{(1/2)} / (8(\\
& a^5b^4 + 16a^7c^2 - 8a^6b^2c))^{(1/2)} + 16a^10c^4d - 16a^11c^3f \\
& + 4a^8b^4c^2d - 20a^9b^2c^3d - 4a^9b^3c^2e + 4a^10b^2c^2f \\
& + 16a^10b^2c^3e) * (-b^7d^2 + a^2b^5e^2 - b^4d^2(-4ac - b^2)^3)^{(1/2)} \\
& + a^4b^3f^2 - a^4f^2(-4ac - b^2)^3)^{(1/2)} - 20a^3b^2cd^2 - \\
& 7a^3b^3c^2e^2 + 12a^4b^2c^2e^2 + a^3c^2e^2(-4ac - b^2)^3)^{(1/2)} - 2 \\
& ab^6d^2e + 25a^2b^3c^2d^2 - a^2b^2e^2(-4ac - b^2)^3)^{(1/2)} - a^2 \\
& c^2d^2(-4ac - b^2)^3)^{(1/2)} - 9ab^5cd^2 - 4a^5b^2cf^2 + 2a^2b^5 \\
& df + 16a^4c^3de - 2a^3b^4ef - 16a^5c^2ef + 2ab^3d^2ef(-4ac - b^2)^3 \\
&)^{(1/2)} + 16a^2b^4cd^2e - 14a^3b^3cd^2f + 24a^4b^2cd^2f + 2a^3b^2 \\
& ef(-4ac - b^2)^3)^{(1/2)} + 2a^3cd^2f(-4ac - b^2)^3)^{(1/2)} + 12a^4 \\
& b^2c^2ef + 3ab^2cd^2(-4ac - b^2)^3)^{(1/2)} - 36a^3b^2c^2d^2e - \\
& 2a^2b^2d^2d^2f(-4ac - b^2)^3)^{(1/2)} - 4a^2b^2cd^2ef(-4ac - b^2)^3 \\
&)^{(1/2)} / (8(a^5b^4 + 16a^7c^2 - 8a^6b^2c))^{(1/2)} + 2a^8 \\
& c^4e^3 - 2a^6b^2c^5d^3 + 2a^7c^5d^2e + 2a^9c^3ef^2 - 4a^8c^4 \\
& d^2ef - 4a^7b^2c^4d^2e + 4a^7b^2c^4d^2f - 2a^8b^2c^3d^2f^2 - 2a^8 \\
& b^2c^3e^2f + 2a^6b^2c^4d^2e - 2a^6b^3c^3d^2f + 4a^7b^2c^3d^2e \\
& f) * (-b^7d^2 + a^2b^5e^2 - b^4d^2(-4ac - b^2)^3)^{(1/2)} + a^4b^3f^2 \\
& - a^4f^2(-4ac - b^2)^3)^{(1/2)} - 20a^3b^2cd^2 - 7a^3b^3c^2e^2 \\
& + 12a^4b^2c^2e^2 + a^3c^2e^2(-4ac - b^2)^3)^{(1/2)} - 2ab^6d^2e + 25 \\
& a^2b^3c^2d^2 - a^2b^2e^2(-4ac - b^2)^3)^{(1/2)} - a^2c^2d^2(-4ac \\
& - b^2)^3)^{(1/2)} - 9ab^5cd^2 - 4a^5b^2cf^2 + 2a^2b^5df + 16a^4 \\
& c^3de - 2a^3b^4ef - 16a^5c^2ef + 2ab^3d^2ef(-4ac - b^2)^3)^{(1/2)} \\
& + 16a^2b^4cd^2e - 14a^3b^3cd^2f + 24a^4b^2cd^2f + 2a^3b^2ef(-4ac \\
& - b^2)^3)^{(1/2)} + 2a^3cd^2f(-4ac - b^2)^3)^{(1/2)} + 12a^4 \\
& b^2c^2ef + 3ab^2cd^2(-4ac - b^2)^3)^{(1/2)} - 36a^3b^2c^2d^2e - \\
& 2a^2b^2d^2d^2f(-4ac - b^2)^3)^{(1/2)} - 4a^2b^2cd^2ef(-4ac - b^2)^3 \\
&)^{(1/2)} / (8(a^5b^4 + 16a^7c^2 - 8a^6b^2c))^{(1/2)} * 2i
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**4+e*x**2+d)/x**4/(c*x**4+b*x**2+a),x)

[Out] Timed out

$$3.58 \quad \int \frac{d+ex^2+fx^4}{x^6(a+bx^2+cx^4)} dx$$

Optimal. Leaf size=329

$$\frac{-abe - a(cd - af) + b^2d}{a^3x} + \frac{bd - ae}{3a^2x^3} - \frac{\sqrt{c} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right) \left(\frac{2a^2ce-ab^2e-ab(3cd-af)+b^3d}{\sqrt{b^2-4ac}} - abe - a(cd - af) + b^2d\right)}{\sqrt{2}a^3\sqrt{b-\sqrt{b^2-4ac}}}$$

Rubi [A] time = 1.94, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 30, number of rules / integrand size = 0.100, Rules used = {1664, 1166, 205}

$$\frac{-abe - a(cd - af) + b^2d}{a^3x} - \frac{\sqrt{c} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right) \left(\frac{2a^2ce-ab^2e-ab(3cd-af)+b^3d}{\sqrt{b^2-4ac}} - abe - a(cd - af) + b^2d\right)}{\sqrt{2}a^3\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b^2-4ac+b}}\right) \left(\frac{2a^2ce-ab^2e-ab(3cd-af)+b^3d}{\sqrt{b^2-4ac}} - abe - a(cd - af) + b^2d\right)}{\sqrt{2}a^3\sqrt{b^2-4ac+b}} + \frac{bd - ae}{3a^2x^3} - \frac{d}{5ax^5}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2 + f*x^4)/(x^6*(a + b*x^2 + c*x^4)), x]

[Out] -d/(5*a*x^5) + (b*d - a*e)/(3*a^2*x^3) - (b^2*d - a*b*e - a*(c*d - a*f))/(a^3*x) - (Sqrt[c]*(b^2*d - a*b*e - a*(c*d - a*f) + (b^3*d - a*b^2*e + 2*a^2*c*e - a*b*(3*c*d - a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*a^3*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(b^2*d - a*b*e - a*(c*d - a*f) - (b^3*d - a*b^2*e + 2*a^2*c*e - a*b*(3*c*d - a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*a^3*Sqrt[b + Sqrt[b^2 - 4*a*c]])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1664

Int[(Pq_)*((d_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]

Rubi steps

$$\int \frac{d + ex^2 + fx^4}{x^6(a + bx^2 + cx^4)} dx = \int \left(\frac{d}{ax^6} + \frac{-bd + ae}{a^2x^4} + \frac{b^2d - abe - a(cd - af)}{a^3x^2} + \frac{-b^3d + ab^2e - a^2ce + ab(2cd - af) - c(b^2d - abe - a(cd - af))}{a^3(a + bx^2 + cx^4)} \right) dx$$

$$= -\frac{d}{5ax^5} + \frac{bd - ae}{3a^2x^3} - \frac{b^2d - abe - a(cd - af)}{a^3x} + \frac{\int \frac{-b^3d + ab^2e - a^2ce + ab(2cd - af) - c(b^2d - abe - a(cd - af))}{a + bx^2 + cx^4} dx}{a^3}$$

$$= -\frac{d}{5ax^5} + \frac{bd - ae}{3a^2x^3} - \frac{b^2d - abe - a(cd - af)}{a^3x} - \frac{\left(c \left(b^2d - abe - a(cd - af) - \frac{b^3d - ab^2e + 2a^2ce}{\sqrt{b^2 - 4ac}} \right) \right)}{2a^3}$$

$$= -\frac{d}{5ax^5} + \frac{bd - ae}{3a^2x^3} - \frac{b^2d - abe - a(cd - af)}{a^3x} - \frac{\sqrt{c} \left(b^2d - abe - a(cd - af) + \frac{b^3d - ab^2e + 2a^2ce}{\sqrt{b^2 - 4ac}} \right)}{\sqrt{2} a^3 \sqrt{b^2 - 4ac}}$$

Mathematica [A] time = 0.55, size = 394, normalized size = 1.20

$$\frac{\frac{6a^2d}{x^5} + \frac{30(ab+e(cd-f)+b^2(-d))}{x} - \frac{15\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(ab(-c\sqrt{b^2-4ac}+af-3cd)+a(-cd\sqrt{b^2-4ac}+af\sqrt{b^2-4ac}+2ace)+b^2(d\sqrt{b^2-4ac}-ae)+b^3d\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{15\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b^2-4ac}+b}\right)\left(ab(c\sqrt{b^2-4ac}+af-3cd)+a(cd\sqrt{b^2-4ac}-af\sqrt{b^2-4ac}+2ace)-b^2(d\sqrt{b^2-4ac}+ae)+b^3d\right)}{\sqrt{b^2-4ac}\sqrt{b^2-4ac}+b}}{30a^3} + \frac{10(ab(-d))}{x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2 + f*x^4)/(x^6*(a + b*x^2 + c*x^4)),x]
[Out] ((-6*a^2*d)/x^5 + (10*a*(b*d - a*e))/x^3 + (30*(-(b^2*d) + a*b*e + a*(c*d - a*f)))/x - (15*sqrt[2]*sqrt[c]*(b^3*d + b^2*(sqrt[b^2 - 4*a*c]*d - a*e) + a*b*(-3*c*d - sqrt[b^2 - 4*a*c]*e + a*f) + a*(-(c*sqrt[b^2 - 4*a*c]*d) + 2*a*c*e + a*sqrt[b^2 - 4*a*c]*f))*ArcTan[(sqrt[2]*sqrt[c]*x)/sqrt[b - sqrt[b^2 - 4*a*c]])/(sqrt[b^2 - 4*a*c]*sqrt[b - sqrt[b^2 - 4*a*c]]) + (15*sqrt[2]*sqrt[c]*(b^3*d - b^2*(sqrt[b^2 - 4*a*c]*d + a*e) + a*b*(-3*c*d + sqrt[b^2 - 4*a*c]*e + a*f) + a*(c*sqrt[b^2 - 4*a*c]*d + 2*a*c*e - a*sqrt[b^2 - 4*a*c]*f))*ArcTan[(sqrt[2]*sqrt[c]*x)/sqrt[b + sqrt[b^2 - 4*a*c]])/(sqrt[b^2 - 4*a*c]*sqrt[b + sqrt[b^2 - 4*a*c]])/(30*a^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex^2 + fx^4}{x^6(a + bx^2 + cx^4)} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x^2 + f*x^4)/(x^6*(a + b*x^2 + c*x^4)),x]
[Out] IntegrateAlgebraic[(d + e*x^2 + f*x^4)/(x^6*(a + b*x^2 + c*x^4)), x]
```

fricas [B] time = 40.46, size = 15830, normalized size = 48.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^6/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] -1/30*(15*sqrt(1/2)*a^3*x^5*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*d^2 - 2*(a*b^6 - 6*a^2*b^4*c + 9*a^3*b^2*c^2 - 2*a^4*c^3)*d*e + (a^2*b^5 - 5*a^3*b^3*c + 5*a^4*b*c^2)*e^2 + (a^4*b^3 - 3*a^5*b*c)*f^2 + 2*((a^2*b^5 - 5*a^3*b^3*c + 5*a^4*b*c^2)*d - (a^3*b^4 - 4*a^4*b^2*c + 2*a^5*c^2)*e)*f + (a^7*b^2 - 4*a^8*c)*sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^4 - 4*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c
```

$$\begin{aligned}
&^5)d^3e + 2*(3*a^2*b^10 - 24*a^3*b^8*c + 66*a^4*b^6*c^2 - 72*a^5*b^4*c^3 \\
&+ 27*a^6*b^2*c^4 - a^7*c^5)*d^2e^2 - 4*(a^3*b^9 - 7*a^4*b^7*c + 16*a^5*b^5 \\
&*c^2 - 13*a^6*b^3*c^3 + 3*a^7*b*c^4)*d*e^3 + (a^4*b^8 - 6*a^5*b^6*c + 11*a^ \\
&6*b^4*c^2 - 6*a^7*b^2*c^3 + a^8*c^4)*e^4 + (a^8*b^4 - 2*a^9*b^2*c + a^10*c^ \\
&2)*f^4 + 4*((a^6*b^6 - 4*a^7*b^4*c + 4*a^8*b^2*c^2 - a^9*c^3)*d - (a^7*b^5 \\
&- 3*a^8*b^3*c + 2*a^9*b*c^2)*e)*f^3 + 2*((3*a^4*b^8 - 18*a^5*b^6*c + 33*a^6 \\
&*b^4*c^2 - 19*a^7*b^2*c^3 + 3*a^8*c^4)*d^2 - 2*(3*a^5*b^7 - 15*a^6*b^5*c + \\
&21*a^7*b^3*c^2 - 7*a^8*b*c^3)*d*e + (3*a^6*b^6 - 12*a^7*b^4*c + 12*a^8*b^2* \\
&c^2 - a^9*c^3)*e^2)*f^2 + 4*((a^2*b^10 - 8*a^3*b^8*c + 22*a^4*b^6*c^2 - 24* \\
&a^5*b^4*c^3 + 9*a^6*b^2*c^4 - a^7*c^5)*d^3 - (3*a^3*b^9 - 21*a^4*b^7*c + 48 \\
&*a^5*b^5*c^2 - 39*a^6*b^3*c^3 + 8*a^7*b*c^4)*d^2*e + (3*a^4*b^8 - 18*a^5*b^ \\
&6*c + 33*a^6*b^4*c^2 - 18*a^7*b^2*c^3 + a^8*c^4)*d*e^2 - (a^5*b^7 - 5*a^6*b \\
&^5*c + 7*a^7*b^3*c^2 - 2*a^8*b*c^3)*e^3)*f)/(a^14*b^2 - 4*a^15*c))/(a^7*b^ \\
&2 - 4*a^8*c))*log(-2*((b^6*c^4 - 5*a*b^4*c^5 + 6*a^2*b^2*c^6 - a^3*c^7)*d^4 \\
&- (b^7*c^3 - 3*a*b^5*c^4 - 2*a^2*b^3*c^5 + 5*a^3*b*c^6)*d^3*e + 3*(a*b^6*c \\
&^3 - 4*a^2*b^4*c^4 + 3*a^3*b^2*c^5)*d^2*e^2 - (3*a^2*b^5*c^3 - 11*a^3*b^3*c \\
&^4 + 7*a^4*b*c^5)*d*e^3 + (a^3*b^4*c^3 - 3*a^4*b^2*c^4 + a^5*c^5)*e^4 + (a^ \\
&6*b^2*c^2 - a^7*c^3)*f^4 + ((3*a^4*b^4*c^2 - 9*a^5*b^2*c^3 + 4*a^6*c^4)*d - \\
&(3*a^5*b^3*c^2 - 5*a^6*b*c^3)*e)*f^3 + 3*((a^2*b^6*c^2 - 5*a^3*b^4*c^3 + 7 \\
&*a^4*b^2*c^4 - 2*a^5*c^5)*d^2 - (2*a^3*b^5*c^2 - 7*a^4*b^3*c^3 + 5*a^5*b*c^ \\
&4)*d*e + (a^4*b^4*c^2 - 2*a^5*b^2*c^3)*e^2)*f^2 + ((b^8*c^2 - 7*a*b^6*c^3 + \\
&18*a^2*b^4*c^4 - 19*a^3*b^2*c^5 + 4*a^4*c^6)*d^3 - 3*(a*b^7*c^2 - 5*a^2*b^ \\
&5*c^3 + 8*a^3*b^3*c^4 - 5*a^4*b*c^5)*d^2*e + 3*(a^2*b^6*c^2 - 3*a^3*b^4*c^3 \\
&+ a^4*b^2*c^4)*d*e^2 - (a^3*b^5*c^2 - a^4*b^3*c^3 - 3*a^5*b*c^4)*e^3)*f)*x \\
&+ sqrt(1/2)*((b^11 - 11*a*b^9*c + 44*a^2*b^7*c^2 - 77*a^3*b^5*c^3 + 54*a^4 \\
&*b^3*c^4 - 8*a^5*b*c^5)*d^3 - (3*a*b^10 - 30*a^2*b^8*c + 105*a^3*b^6*c^2 - \\
&151*a^4*b^4*c^3 + 77*a^5*b^2*c^4 - 4*a^6*c^5)*d^2*e + (3*a^2*b^9 - 27*a^3*b \\
&^7*c + 81*a^4*b^5*c^2 - 92*a^5*b^3*c^3 + 32*a^6*b*c^4)*d*e^2 - (a^3*b^8 - 8 \\
&*a^4*b^6*c + 20*a^5*b^4*c^2 - 17*a^6*b^2*c^3 + 4*a^7*c^4)*e^3 + (a^6*b^5 - \\
&5*a^7*b^3*c + 4*a^8*b*c^2)*f^3 + ((3*a^4*b^7 - 21*a^5*b^5*c + 40*a^6*b^3*c^ \\
&2 - 16*a^7*b*c^3)*d - (3*a^5*b^6 - 18*a^6*b^4*c + 25*a^7*b^2*c^2 - 4*a^8*c^ \\
&3)*e)*f^2 + ((3*a^2*b^9 - 27*a^3*b^7*c + 80*a^4*b^5*c^2 - 85*a^5*b^3*c^3 + \\
&20*a^6*b*c^4)*d^2 - 2*(3*a^3*b^8 - 24*a^4*b^6*c + 59*a^5*b^4*c^2 - 45*a^6*b \\
&^2*c^3 + 4*a^7*c^4)*d*e + (3*a^4*b^7 - 21*a^5*b^5*c + 41*a^6*b^3*c^2 - 20*a \\
&^7*b*c^3)*e^2)*f - ((a^7*b^6 - 8*a^8*b^4*c + 18*a^9*b^2*c^2 - 8*a^10*c^3)*d \\
&- (a^8*b^5 - 7*a^9*b^3*c + 12*a^10*b*c^2)*e + (a^9*b^4 - 6*a^10*b^2*c + 8* \\
&a^11*c^2)*f)*sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + \\
&46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^4 - 4*(a*b^11 - 9*a^2*b^9*c + \\
&29*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d^3*e + 2*(\\
&3*a^2*b^10 - 24*a^3*b^8*c + 66*a^4*b^6*c^2 - 72*a^5*b^4*c^3 + 27*a^6*b^2*c^ \\
&4 - a^7*c^5)*d^2*e^2 - 4*(a^3*b^9 - 7*a^4*b^7*c + 16*a^5*b^5*c^2 - 13*a^6*b \\
&^3*c^3 + 3*a^7*b*c^4)*d*e^3 + (a^4*b^8 - 6*a^5*b^6*c + 11*a^6*b^4*c^2 - 6*a \\
&^7*b^2*c^3 + a^8*c^4)*e^4 + (a^8*b^4 - 2*a^9*b^2*c + a^10*c^2)*f^4 + 4*((a^ \\
&6*b^6 - 4*a^7*b^4*c + 4*a^8*b^2*c^2 - a^9*c^3)*d - (a^7*b^5 - 3*a^8*b^3*c + \\
&2*a^9*b*c^2)*e)*f^3 + 2*((3*a^4*b^8 - 18*a^5*b^6*c + 33*a^6*b^4*c^2 - 19*a \\
&^7*b^2*c^3 + 3*a^8*c^4)*d^2 - 2*(3*a^5*b^7 - 15*a^6*b^5*c + 21*a^7*b^3*c^2 \\
&- 7*a^8*b*c^3)*d*e + (3*a^6*b^6 - 12*a^7*b^4*c + 12*a^8*b^2*c^2 - a^9*c^3)* \\
&e^2)*f^2 + 4*((a^2*b^10 - 8*a^3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9 \\
&*a^6*b^2*c^4 - a^7*c^5)*d^3 - (3*a^3*b^9 - 21*a^4*b^7*c + 48*a^5*b^5*c^2 - \\
&39*a^6*b^3*c^3 + 8*a^7*b*c^4)*d^2*e + (3*a^4*b^8 - 18*a^5*b^6*c + 33*a^6*b^ \\
&4*c^2 - 18*a^7*b^2*c^3 + a^8*c^4)*d*e^2 - (a^5*b^7 - 5*a^6*b^5*c + 7*a^7*b^ \\
&3*c^2 - 2*a^8*b*c^3)*e^3)*f)/(a^14*b^2 - 4*a^15*c))*sqrt(-((b^7 - 7*a*b^5* \\
&c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*d^2 - 2*(a*b^6 - 6*a^2*b^4*c + 9*a^3*b^2* \\
&c^2 - 2*a^4*c^3)*d*e + (a^2*b^5 - 5*a^3*b^3*c + 5*a^4*b*c^2)*e^2 + (a^4*b^3 \\
&- 3*a^5*b*c)*f^2 + 2*((a^2*b^5 - 5*a^3*b^3*c + 5*a^4*b*c^2)*d - (a^3*b^4 - \\
&4*a^4*b^2*c + 2*a^5*c^2)*e)*f + (a^7*b^2 - 4*a^8*c)*sqrt(((b^12 - 10*a*b^1 \\
&0*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a \\
&^6*c^6)*d^4 - 4*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 2 \\
&2*a^5*b^3*c^4 - 3*a^6*b*c^5)*d^3*e + 2*(3*a^2*b^10 - 24*a^3*b^8*c + 66*a^4*
\end{aligned}$$

$$\begin{aligned}
& b^6c^2 - 72a^5b^4c^3 + 27a^6b^2c^4 - a^7c^5) d^2e^2 - 4(a^3b^9 - \\
& 7a^4b^7c + 16a^5b^5c^2 - 13a^6b^3c^3 + 3a^7b^c^4) d^3e^3 + (a^4b^8 - \\
& 6a^5b^6c + 11a^6b^4c^2 - 6a^7b^2c^3 + a^8c^4) e^4 + (a^8b^4 - \\
& 2a^9b^2c + a^{10}c^2) f^4 + 4((a^6b^6 - 4a^7b^4c + 4a^8b^2c^2 - \\
& a^9c^3) d - (a^7b^5 - 3a^8b^3c + 2a^9b^c^2) e) f^3 + 2((3a^4b^8 - \\
& 18a^5b^6c + 33a^6b^4c^2 - 19a^7b^2c^3 + 3a^8c^4) d^2 - 2(3a^5b^7 - \\
& 15a^6b^5c + 21a^7b^3c^2 - 7a^8b^c^3) d^3e + (3a^6b^6 - 12a^7b^4c + \\
& 12a^8b^2c^2 - a^9c^3) e^2) f^2 + 4((a^2b^{10} - 8a^3b^8c + 22a^4b^6c^2 - \\
& 24a^5b^4c^3 + 9a^6b^2c^4 - a^7c^5) d^3 - (3a^3b^9 - 21a^4b^7c + \\
& 48a^5b^5c^2 - 39a^6b^3c^3 + 8a^7b^c^4) d^2e + (3a^4b^8 - 18a^5b^6c + \\
& 33a^6b^4c^2 - 18a^7b^2c^3 + a^8c^4) d^3e^2 - (a^5b^7 - 5a^6b^5c + \\
& 7a^7b^3c^2 - 2a^8b^c^3) e^3) f) / (a^{14}b^2 - 4a^{15}c)) / (a^7b^2 - 4a^8c)) - 15\sqrt{1/2} a^3x^5\sqrt{-(b^7 - \\
& 7a^5b^c + 14a^2b^3c^2 - 7a^3b^c^3) d^2 - 2(a^6b^6 - 6a^2b^4c + \\
& 9a^3b^2c^2 - 2a^4c^3) d^3e + (a^2b^5 - 5a^3b^3c + 5a^4b^c^2) e^2 + \\
& (a^4b^3 - 3a^5b^c) f^2 + 2((a^2b^5 - 5a^3b^3c + 5a^4b^c^2) d - \\
& (a^3b^4 - 4a^4b^2c + 2a^5c^2) e) f + (a^7b^2 - 4a^8c) \sqrt{((b^{12} - \\
& 10a^2b^{10}c + 37a^2b^8c^2 - 62a^3b^6c^3 + 46a^4b^4c^4 - 12a^5b^2c^5 + \\
& a^6c^6) d^4 - 4(a^6b^{11} - 9a^2b^9c + 29a^3b^7c^2 - 40a^4b^5c^3 + \\
& 22a^5b^3c^4 - 3a^6b^c^5) d^3e + 2(3a^2b^{10} - 24a^3b^8c + 66a^4b^6c^2 - \\
& 72a^5b^4c^3 + 27a^6b^2c^4 - a^7c^5) d^2e^2 - 4(a^3b^9 - 7a^4b^7c + \\
& 16a^5b^5c^2 - 13a^6b^3c^3 + 3a^7b^c^4) d^3e^3 + (a^4b^8 - 6a^5b^6c + \\
& 11a^6b^4c^2 - 6a^7b^2c^3 + a^8c^4) e^4 + (a^8b^4 - 2a^9b^2c + a^{10}c^2) f^4 + \\
& 4((a^6b^6 - 4a^7b^4c + 4a^8b^2c^2 - a^9c^3) d - (a^7b^5 - 3a^8b^3c + \\
& 2a^9b^c^2) e) f^3 + 2((3a^4b^8 - 18a^5b^6c + 33a^6b^4c^2 - 19a^7b^2c^3 + \\
& 3a^8c^4) d^2 - 2(3a^5b^7 - 15a^6b^5c + 21a^7b^3c^2 - 7a^8b^c^3) d^3e + \\
& (3a^6b^6 - 12a^7b^4c + 12a^8b^2c^2 - a^9c^3) e^2) f^2 + 4((a^2b^{10} - \\
& 8a^3b^8c + 22a^4b^6c^2 - 24a^5b^4c^3 + 9a^6b^2c^4 - a^7c^5) d^3 - \\
& (3a^3b^9 - 21a^4b^7c + 48a^5b^5c^2 - 39a^6b^3c^3 + 8a^7b^c^4) d^2e + \\
& (3a^4b^8 - 18a^5b^6c + 33a^6b^4c^2 - 18a^7b^2c^3 + a^8c^4) d^3e^2 - \\
& (a^5b^7 - 5a^6b^5c + 7a^7b^3c^2 - 2a^8b^c^3) e^3) f) / (a^{14}b^2 - 4a^{15}c)) / \\
& (a^7b^2 - 4a^8c)) * \log(-2((b^6c^4 - 5a^2b^4c^5 + 6a^2b^2c^6 - a^3c^7) d^4 - \\
& (b^7c^3 - 3a^2b^5c^4 - 2a^2b^3c^5 + 5a^3b^c^6) d^3e + 3(a^6b^c^3 - 4a^2b^4c^4 + \\
& 3a^3b^2c^5) d^2e^2 - (3a^2b^5c^3 - 11a^3b^3c^4 + 7a^4b^c^5) d^3e^3 + (a^3b^4c^3 - \\
& 3a^4b^2c^4 + a^5c^5) e^4 + (a^6b^2c^2 - a^7c^3) f^4 + ((3a^4b^4c^2 - \\
& 9a^5b^2c^3 + 4a^6c^4) d - (3a^5b^3c^2 - 5a^6b^c^3) e) f^3 + 3((a^2b^6c^2 - \\
& 5a^3b^4c^3 + 7a^4b^2c^4 - 2a^5c^5) d^2 - (2a^3b^5c^2 - 7a^4b^3c^3 + \\
& 5a^5b^c^4) d^3e + (a^4b^4c^2 - 2a^5b^2c^3) e^2) f^2 + ((b^8c^2 - 7a^2b^6c^3 + \\
& 18a^2b^4c^4 - 19a^3b^2c^5 + 4a^4b^c^6) d^3 - 3(a^2b^7c^2 - 5a^2b^5c^3 + \\
& 8a^3b^3c^4 - 5a^4b^c^5) d^2e + 3(a^2b^6c^2 - 3a^3b^4c^3 + a^4b^2c^4) d^3e^2 - \\
& (a^3b^5c^2 - a^4b^3c^3 - 3a^5b^c^4) e^3) f) * x - \sqrt{1/2} * ((b^{11} - 11a^2b^9c + \\
& 44a^2b^7c^2 - 77a^3b^5c^3 + 54a^4b^3c^4 - 8a^5b^c^5) d^3 - (3a^2b^{10} - \\
& 30a^2b^8c + 105a^3b^6c^2 - 151a^4b^4c^3 + 77a^5b^2c^4 - 4a^6b^c^5) d^2e + \\
& (3a^2b^9 - 27a^3b^7c + 81a^4b^5c^2 - 92a^5b^3c^3 + 32a^6b^c^4) d^3e^2 - \\
& (a^3b^8 - 8a^4b^6c + 20a^5b^4c^2 - 17a^6b^2c^3 + 4a^7c^4) e^3 + (a^6b^5 - \\
& 5a^7b^3c + 4a^8b^c^2) f^3 + ((3a^4b^7 - 21a^5b^5c + 40a^6b^3c^2 - \\
& 16a^7b^c^3) d - (3a^5b^6 - 18a^6b^4c + 25a^7b^2c^2 - 4a^8c^3) e) f^2 + \\
& ((3a^2b^9 - 27a^3b^7c + 80a^4b^5c^2 - 85a^5b^3c^3 + 20a^6b^c^4) d^2 - \\
& 2(3a^3b^8 - 24a^4b^6c + 59a^5b^4c^2 - 45a^6b^2c^3 + 4a^7c^4) d^3e + (3a^4b^7 - \\
& 21a^5b^5c + 41a^6b^3c^2 - 20a^7b^c^3) e^2) f - ((a^7b^6 - 8a^8b^4c + \\
& 18a^9b^2c^2 - 8a^{10}c^3) d - (a^8b^5 - 7a^9b^3c + 12a^{10}b^c^2) e + \\
& (a^9b^4 - 6a^{10}b^2c + 8a^{11}c^2) f) \sqrt{((b^{12} - 10a^2b^{10}c + 37a^2b^8c^2 - \\
& 62a^3b^6c^3 + 46a^4b^4c^4 - 12a^5b^2c^5 + a^6c^6) d^4 - 4(a^6b^{11} - 9a^2b^9c + \\
& 29a^3b^7c^2 - 40a^4b^5c^3 + 22a^5b^3c^4 - 3a^6b^c^5) d^3e + 2(3a^2b^{10} - 24a^3b^8c + \\
& 66a^4b^6c^2 - 72a^5b^4c^3 + 27a^6b^2c^4 - a^7c^5) d^2e^2 - 4(a^3b^9 - 7a^4b^7c + \\
& 16a^5b^5c^2 - 13a^6b^3c^3 + 3a^7b^c^4) d^3e^3 + (a^4b^8 - 6a^5b^6c + 11a^6b^4c^2 - \\
& 6a^7b^2c^3 + a^8c^4) e^4 + (a^8b^4 - 2a^9b^2c + a^{10}c^2) f^4 + 4((a^6b^6 - 4a^7b^4c + \\
& 4a^8b^2c^2 - a^9c^3) d - (a^7b^5 - 3a^8b^3c + 2a^9b^c^2) e) f^3 + 2((3a^4b^8 - \\
& 18a^5b^6c + 33a^6b^4c^2 - 19a^7b^2c^3 + 3a^8c^4) d^2 - 2(3a^5b^7 - 15a^6b^5c + \\
& 21a^7b^3c^2 - 7a^8b^c^3) d^3e + (3a^6b^6 - 12a^7b^4c + 12a^8b^2c^2 - a^9c^3) e^2) f^2 + \\
& 4((a^2b^{10} - 8a^3b^8c + 22a^4b^6c^2 - 24a^5b^4c^3 + 9a^6b^2c^4 - a^7c^5) d^3 - \\
& (3a^3b^9 - 21a^4b^7c + 48a^5b^5c^2 - 39a^6b^3c^3 + 8a^7b^c^4) d^2e + (3a^4b^8 - \\
& 18a^5b^6c + 33a^6b^4c^2 - 18a^7b^2c^3 + a^8c^4) d^3e^2 - (a^5b^7 - 5a^6b^5c + \\
& 7a^7b^3c^2 - 2a^8b^c^3) e^3) f) / (a^{14}b^2 - 4a^{15}c)) / (a^7b^2 - 4a^8c))
\end{aligned}$$

$$\begin{aligned}
&^2 - 72a^5b^4c^3 + 27a^6b^2c^4 - a^7c^5)d^2e^2 - 4(a^3b^9 - 7a^4b^7c + 16a^5b^5c^2 - 13a^6b^3c^3 + 3a^7b^1c^4)d^2e^3 + (a^4b^8 - 6a^5b^6c + 11a^6b^4c^2 - 6a^7b^2c^3 + a^8c^4)e^4 + (a^8b^4 - 2a^9b^2c + a^{10}c^2)f^4 + 4((a^6b^6 - 4a^7b^4c + 4a^8b^2c^2 - a^9c^3)d - (a^7b^5 - 3a^8b^3c + 2a^9b^1c^2)e)f^3 + 2((3a^4b^8 - 18a^5b^6c + 33a^6b^4c^2 - 19a^7b^2c^3 + 3a^8c^4)d^2 - 2(3a^5b^7 - 15a^6b^5c + 21a^7b^3c^2 - 7a^8b^1c^3)d^2e + (3a^6b^6 - 12a^7b^4c + 12a^8b^2c^2 - a^9c^3)e^2)f^2 + 4((a^2b^{10} - 8a^3b^8c + 22a^4b^6c^2 - 24a^5b^4c^3 + 9a^6b^2c^4 - a^7c^5)d^3 - (3a^3b^9 - 21a^4b^7c + 48a^5b^5c^2 - 39a^6b^3c^3 + 8a^7b^1c^4)d^2e + (3a^4b^8 - 18a^5b^6c + 33a^6b^4c^2 - 18a^7b^2c^3 + a^8c^4)d^2e^2 - (a^5b^7 - 5a^6b^5c + 7a^7b^3c^2 - 2a^8b^1c^3)e^3)f)/(a^{14}b^2 - 4a^{15}c))\sqrt{-((b^7 - 7a^2b^5c + 14a^2b^3c^2 - 7a^3b^1c^3)d^2 - 2(a^2b^6 - 6a^2b^4c + 9a^3b^2c^2 - 2a^4c^3)d^2e + (a^2b^5 - 5a^3b^3c + 5a^4b^1c^2)e^2 + (a^4b^3 - 3a^5b^1c)f^2 + 2((a^2b^5 - 5a^3b^3c + 5a^4b^1c^2)d - (a^3b^4 - 4a^4b^2c + 2a^5c^2)e)f + (a^7b^2 - 4a^8c)\sqrt{((b^{12} - 10a^2b^{10}c + 37a^2b^8c^2 - 62a^3b^6c^3 + 46a^4b^4c^4 - 12a^5b^2c^5 + a^6c^6)d^4 - 4(a^2b^{11} - 9a^2b^9c + 29a^3b^7c^2 - 40a^4b^5c^3 + 22a^5b^3c^4 - 3a^6b^1c^5)d^3e + 2(3a^2b^{10} - 24a^3b^8c + 66a^4b^6c^2 - 72a^5b^4c^3 + 27a^6b^2c^4 - a^7c^5)d^2e^2 - 4(a^3b^9 - 7a^4b^7c + 16a^5b^5c^2 - 13a^6b^3c^3 + 3a^7b^1c^4)d^2e^3 + (a^4b^8 - 6a^5b^6c + 11a^6b^4c^2 - 6a^7b^2c^3 + a^8c^4)e^4 + (a^8b^4 - 2a^9b^2c + a^{10}c^2)f^4 + 4((a^6b^6 - 4a^7b^4c + 4a^8b^2c^2 - a^9c^3)d - (a^7b^5 - 3a^8b^3c + 2a^9b^1c^2)e)f^3 + 2((3a^4b^8 - 18a^5b^6c + 33a^6b^4c^2 - 19a^7b^2c^3 + 3a^8c^4)d^2 - 2(3a^5b^7 - 15a^6b^5c + 21a^7b^3c^2 - 7a^8b^1c^3)d^2e + (3a^6b^6 - 12a^7b^4c + 12a^8b^2c^2 - a^9c^3)e^2)f^2 + 4((a^2b^{10} - 8a^3b^8c + 22a^4b^6c^2 - 24a^5b^4c^3 + 9a^6b^2c^4 - a^7c^5)d^3 - (3a^3b^9 - 21a^4b^7c + 48a^5b^5c^2 - 39a^6b^3c^3 + 8a^7b^1c^4)d^2e + (3a^4b^8 - 18a^5b^6c + 33a^6b^4c^2 - 18a^7b^2c^3 + a^8c^4)d^2e^2 - (a^5b^7 - 5a^6b^5c + 7a^7b^3c^2 - 2a^8b^1c^3)e^3)f)/(a^{14}b^2 - 4a^{15}c)))/(a^7b^2 - 4a^8c)) + 15\sqrt{1/2}a^3x^5\sqrt{-((b^7 - 7a^2b^5c + 14a^2b^3c^2 - 7a^3b^1c^3)d^2 - 2(a^2b^6 - 6a^2b^4c + 9a^3b^2c^2 - 2a^4c^3)d^2e + (a^2b^5 - 5a^3b^3c + 5a^4b^1c^2)e^2 + (a^4b^3 - 3a^5b^1c)f^2 + 2((a^2b^5 - 5a^3b^3c + 5a^4b^1c^2)d - (a^3b^4 - 4a^4b^2c + 2a^5c^2)e)f - (a^7b^2 - 4a^8c)\sqrt{((b^{12} - 10a^2b^{10}c + 37a^2b^8c^2 - 62a^3b^6c^3 + 46a^4b^4c^4 - 12a^5b^2c^5 + a^6c^6)d^4 - 4(a^2b^{11} - 9a^2b^9c + 29a^3b^7c^2 - 40a^4b^5c^3 + 22a^5b^3c^4 - 3a^6b^1c^5)d^3e + 2(3a^2b^{10} - 24a^3b^8c + 66a^4b^6c^2 - 72a^5b^4c^3 + 27a^6b^2c^4 - a^7c^5)d^2e^2 - 4(a^3b^9 - 7a^4b^7c + 16a^5b^5c^2 - 13a^6b^3c^3 + 3a^7b^1c^4)d^2e^3 + (a^4b^8 - 6a^5b^6c + 11a^6b^4c^2 - 6a^7b^2c^3 + a^8c^4)e^4 + (a^8b^4 - 2a^9b^2c + a^{10}c^2)f^4 + 4((a^6b^6 - 4a^7b^4c + 4a^8b^2c^2 - a^9c^3)d - (a^7b^5 - 3a^8b^3c + 2a^9b^1c^2)e)f^3 + 2((3a^4b^8 - 18a^5b^6c + 33a^6b^4c^2 - 19a^7b^2c^3 + 3a^8c^4)d^2 - 2(3a^5b^7 - 15a^6b^5c + 21a^7b^3c^2 - 7a^8b^1c^3)d^2e + (3a^6b^6 - 12a^7b^4c + 12a^8b^2c^2 - a^9c^3)e^2)f^2 + 4((a^2b^{10} - 8a^3b^8c + 22a^4b^6c^2 - 24a^5b^4c^3 + 9a^6b^2c^4 - a^7c^5)d^3 - (3a^3b^9 - 21a^4b^7c + 48a^5b^5c^2 - 39a^6b^3c^3 + 8a^7b^1c^4)d^2e + (3a^4b^8 - 18a^5b^6c + 33a^6b^4c^2 - 18a^7b^2c^3 + a^8c^4)d^2e^2 - (a^5b^7 - 5a^6b^5c + 7a^7b^3c^2 - 2a^8b^1c^3)e^3)f)/(a^{14}b^2 - 4a^{15}c)))/(a^7b^2 - 4a^8c))\log(-2((b^6c^4 - 5a^2b^4c^5 + 6a^2b^2c^6 - a^3c^7)d^4 - (b^7c^3 - 3a^2b^5c^4 - 2a^2b^3c^5 + 5a^3b^1c^6)d^3e + 3(a^2b^6c^3 - 4a^2b^4c^4 + 3a^3b^2c^5)d^2e^2 - (3a^2b^5c^3 - 11a^3b^3c^4 + 7a^4b^1c^5)d^2e^3 + (a^3b^4c^3 - 3a^4b^2c^4 + a^5c^5)e^4 + (a^6b^2c^2 - a^7c^3)f^4 + ((3a^4b^4c^2 - 9a^5b^2c^3 + 4a^6c^4)d - (3a^5b^3c^2 - 5a^6b^1c^3)e)f^3 + 3((a^2b^6c^2 - 5a^3b^4c^3 + 7a^4b^2c^4 - 2a^5c^5)d^2 - (2a^3b^5c^2 - 7a^4b^3c^3 + 5a^5b^1c^4)d^2
\end{aligned}$$

$$\begin{aligned}
& e + (a^4 b^4 c^2 - 2 a^5 b^2 c^3) e^2) f^2 + ((b^8 c^2 - 7 a b^6 c^3 + 18 a^2 b^4 c^4 - 19 a^3 b^2 c^5 + 4 a^4 c^6) d^3 - 3(a b^7 c^2 - 5 a^2 b^5 c^3 + 8 a^3 b^3 c^4 - 5 a^4 b c^5) d^2 e + 3(a^2 b^6 c^2 - 3 a^3 b^4 c^3 + a^4 b^2 c^4) d e^2 - (a^3 b^5 c^2 - a^4 b^3 c^3 - 3 a^5 b c^4) e^3) f) x + \sqrt{1/2} * ((b^{11} - 11 a b^9 c + 44 a^2 b^7 c^2 - 77 a^3 b^5 c^3 + 54 a^4 b^3 c^4 - 8 a^5 b c^5) d^3 - (3 a b^{10} - 30 a^2 b^8 c + 105 a^3 b^6 c^2 - 151 a^4 b^4 c^3 + 77 a^5 b^2 c^4 - 4 a^6 c^5) d^2 e + (3 a^2 b^9 - 27 a^3 b^7 c + 81 a^4 b^5 c^2 - 92 a^5 b^3 c^3 + 32 a^6 b c^4) d e^2 - (a^3 b^8 - 8 a^4 b^6 c + 20 a^5 b^4 c^2 - 17 a^6 b^2 c^3 + 4 a^7 c^4) e^3 + (a^6 b^5 - 5 a^7 b^3 c + 4 a^8 b c^2) f^3 + ((3 a^4 b^7 - 21 a^5 b^5 c + 40 a^6 b^3 c^2 - 16 a^7 b c^3) d - (3 a^5 b^6 - 18 a^6 b^4 c + 25 a^7 b^2 c^2 - 4 a^8 c^3) e) f^2 + ((3 a^2 b^9 - 27 a^3 b^7 c + 80 a^4 b^5 c^2 - 85 a^5 b^3 c^3 + 20 a^6 b c^4) d^2 - 2(3 a^3 b^8 - 24 a^4 b^6 c + 59 a^5 b^4 c^2 - 45 a^6 b^2 c^3 + 4 a^7 c^4) d e + (3 a^4 b^7 - 21 a^5 b^5 c + 41 a^6 b^3 c^2 - 20 a^7 b c^3) e^2) f + ((a^7 b^6 - 8 a^8 b^4 c + 18 a^9 b^2 c^2 - 8 a^{10} c^3) d - (a^8 b^5 - 7 a^9 b^3 c + 12 a^{10} b c^2) e + (a^9 b^4 - 6 a^{10} b^2 c + 8 a^{11} c^2) f) * \sqrt{((b^{12} - 10 a b^{10} c + 37 a^2 b^8 c^2 - 62 a^3 b^6 c^3 + 46 a^4 b^4 c^4 - 12 a^5 b^2 c^5 + a^6 c^6) d^4 - 4(a b^{11} - 9 a^2 b^9 c + 29 a^3 b^7 c^2 - 40 a^4 b^5 c^3 + 22 a^5 b^3 c^4 - 3 a^6 b c^5) d^3 e + 2(3 a^2 b^{10} - 24 a^3 b^8 c + 66 a^4 b^6 c^2 - 72 a^5 b^4 c^3 + 27 a^6 b^2 c^4 - a^7 c^5) d^2 e^2 - 4(a^3 b^9 - 7 a^4 b^7 c + 16 a^5 b^5 c^2 - 13 a^6 b^3 c^3 + 3 a^7 b c^4) d e^3 + (a^4 b^8 - 6 a^5 b^6 c + 11 a^6 b^4 c^2 - 6 a^7 b^2 c^3 + a^8 c^4) e^4 + (a^8 b^4 - 2 a^9 b^2 c + a^{10} c^2) f^4 + 4((a^6 b^6 - 4 a^7 b^4 c + 4 a^8 b^2 c^2 - a^9 c^3) d - (a^7 b^5 - 3 a^8 b^3 c + 2 a^9 b c^2) e) f^3 + 2((3 a^4 b^8 - 18 a^5 b^6 c + 33 a^6 b^4 c^2 - 19 a^7 b^2 c^3 + 3 a^8 c^4) d^2 - 2(3 a^5 b^7 - 15 a^6 b^5 c + 21 a^7 b^3 c^2 - 7 a^8 b c^3) d e + (3 a^6 b^6 - 12 a^7 b^4 c + 12 a^8 b^2 c^2 - a^9 c^3) e^2) f^2 + 4((a^2 b^{10} - 8 a^3 b^8 c + 22 a^4 b^6 c^2 - 24 a^5 b^4 c^3 + 9 a^6 b^2 c^4 - a^7 c^5) d^3 - (3 a^3 b^9 - 21 a^4 b^7 c + 48 a^5 b^5 c^2 - 39 a^6 b^3 c^3 + 8 a^7 b c^4) d^2 e + (3 a^4 b^8 - 18 a^5 b^6 c + 33 a^6 b^4 c^2 - 18 a^7 b^2 c^3 + a^8 c^4) d e^2 - (a^5 b^7 - 5 a^6 b^5 c + 7 a^7 b^3 c^2 - 2 a^8 b c^3) e^3) f) / (a^{14} b^2 - 4 a^{15} c)) * \sqrt{-((b^7 - 7 a b^5 c + 14 a^2 b^3 c^2 - 7 a^3 b c^3) d^2 - 2(a b^6 - 6 a^2 b^4 c + 9 a^3 b^2 c^2 - 2 a^4 c^3) d e + (a^2 b^5 - 5 a^3 b^3 c + 5 a^4 b c^2) e^2 + (a^4 b^3 - 3 a^5 b c) f^2 + 2((a^2 b^5 - 5 a^3 b^3 c + 5 a^4 b c^2) d - (a^3 b^4 - 4 a^4 b^2 c + 2 a^5 c^2) e) f - (a^7 b^2 - 4 a^8 c) * \sqrt{((b^{12} - 10 a b^{10} c + 37 a^2 b^8 c^2 - 62 a^3 b^6 c^3 + 46 a^4 b^4 c^4 - 12 a^5 b^2 c^5 + a^6 c^6) d^4 - 4(a b^{11} - 9 a^2 b^9 c + 29 a^3 b^7 c^2 - 40 a^4 b^5 c^3 + 22 a^5 b^3 c^4 - 3 a^6 b c^5) d^3 e + 2(3 a^2 b^{10} - 24 a^3 b^8 c + 66 a^4 b^6 c^2 - 72 a^5 b^4 c^3 + 27 a^6 b^2 c^4 - a^7 c^5) d^2 e^2 - 4(a^3 b^9 - 7 a^4 b^7 c + 16 a^5 b^5 c^2 - 13 a^6 b^3 c^3 + 3 a^7 b c^4) d e^3 + (a^4 b^8 - 6 a^5 b^6 c + 11 a^6 b^4 c^2 - 6 a^7 b^2 c^3 + a^8 c^4) e^4 + (a^8 b^4 - 2 a^9 b^2 c + a^{10} c^2) f^4 + 4((a^6 b^6 - 4 a^7 b^4 c + 4 a^8 b^2 c^2 - a^9 c^3) d - (a^7 b^5 - 3 a^8 b^3 c + 2 a^9 b c^2) e) f^3 + 2((3 a^4 b^8 - 18 a^5 b^6 c + 33 a^6 b^4 c^2 - 19 a^7 b^2 c^3 + 3 a^8 c^4) d^2 - 2(3 a^5 b^7 - 15 a^6 b^5 c + 21 a^7 b^3 c^2 - 7 a^8 b c^3) d e + (3 a^6 b^6 - 12 a^7 b^4 c + 12 a^8 b^2 c^2 - a^9 c^3) e^2) f^2 + 4((a^2 b^{10} - 8 a^3 b^8 c + 22 a^4 b^6 c^2 - 24 a^5 b^4 c^3 + 9 a^6 b^2 c^4 - a^7 c^5) d^3 - (3 a^3 b^9 - 21 a^4 b^7 c + 48 a^5 b^5 c^2 - 39 a^6 b^3 c^3 + 8 a^7 b c^4) d^2 e + (3 a^4 b^8 - 18 a^5 b^6 c + 33 a^6 b^4 c^2 - 18 a^7 b^2 c^3 + a^8 c^4) d e^2 - (a^5 b^7 - 5 a^6 b^5 c + 7 a^7 b^3 c^2 - 2 a^8 b c^3) e^3) f) / (a^{14} b^2 - 4 a^{15} c)) / (a^7 b^2 - 4 a^8 c)) - 15 * \sqrt{1/2} * a^3 x^5 * \sqrt{-((b^7 - 7 a b^5 c + 14 a^2 b^3 c^2 - 7 a^3 b c^3) d^2 - 2(a b^6 - 6 a^2 b^4 c + 9 a^3 b^2 c^2 - 2 a^4 c^3) d e + (a^2 b^5 - 5 a^3 b^3 c + 5 a^4 b c^2) e^2 + (a^4 b^3 - 3 a^5 b c) f^2 + 2((a^2 b^5 - 5 a^3 b^3 c + 5 a^4 b c^2) d - (a^3 b^4 - 4 a^4 b^2 c + 2 a^5 c^2) e) f - (a^7 b^2 - 4 a^8 c) * \sqrt{((b^{12} - 10 a b^{10} c + 37 a^2 b^8 c^2 - 62 a^3 b^6 c^3 + 46 a^4 b^4 c^4 - 12 a^5 b^2 c^5 + a^6 c^6) d^4 - 4(a b^{11} - 9 a^2 b^9 c + 29 a^3 b^7 c^2 - 40 a^4 b^5 c^3 + 22 a^5 b^3 c^4 - 3 a^6 b c^5) d^3 e + 2(3 a^2 b^{10} - 24 a^3 b^8 c + 66 a^4 b^6 c^2 - 72 a^5 b^4 c^3 + 27 a^6 b^2 c^4 - 3 a^7 b c^4) d^2 e + (3 a^4 b^8 - 18 a^5 b^6 c + 33 a^6 b^4 c^2 - 18 a^7 b^2 c^3 + a^8 c^4) d e^2 - (a^5 b^7 - 5 a^6 b^5 c + 7 a^7 b^3 c^2 - 2 a^8 b c^3) e^3) f) / (a^{14} b^2 - 4 a^{15} c))}
\end{aligned}$$

$$\begin{aligned}
& a^4 b^6 c^2 - 72 a^5 b^4 c^3 + 27 a^6 b^2 c^4 - a^7 c^5) d^2 e^2 - 4 (a^3 b^9 - 7 a^4 b^7 c + 16 a^5 b^5 c^2 - 13 a^6 b^3 c^3 + 3 a^7 b c^4) d e^3 + \\
& (a^4 b^8 - 6 a^5 b^6 c + 11 a^6 b^4 c^2 - 6 a^7 b^2 c^3 + a^8 c^4) e^4 + (a^8 b^4 - 2 a^9 b^2 c + a^{10} c^2) f^4 + 4 ((a^6 b^6 - 4 a^7 b^4 c + 4 a^8 b^2 c^2 - a^9 c^3) d - (a^7 b^5 - 3 a^8 b^3 c + 2 a^9 b c^2) e) f^3 + 2 ((3 a^4 b^8 - 18 a^5 b^6 c + 33 a^6 b^4 c^2 - 19 a^7 b^2 c^3 + 3 a^8 c^4) d^2 - 2 (3 a^5 b^7 - 15 a^6 b^5 c + 21 a^7 b^3 c^2 - 7 a^8 b c^3) d e + (3 a^6 b^6 - 12 a^7 b^4 c + 12 a^8 b^2 c^2 - a^9 c^3) e^2) f^2 + 4 ((a^2 b^{10} - 8 a^3 b^8 c + 22 a^4 b^6 c^2 - 24 a^5 b^4 c^3 + 9 a^6 b^2 c^4 - a^7 c^5) d^3 - (3 a^3 b^9 - 21 a^4 b^7 c + 48 a^5 b^5 c^2 - 39 a^6 b^3 c^3 + 8 a^7 b c^4) d^2 e + (3 a^4 b^8 - 18 a^5 b^6 c + 33 a^6 b^4 c^2 - 18 a^7 b^2 c^3 + a^8 c^4) d e^2 - (a^5 b^7 - 5 a^6 b^5 c + 7 a^7 b^3 c^2 - 2 a^8 b c^3) e^3) f) / (a^{14} b^2 - 4 a^{15} c)) / (a^7 b^2 - 4 a^8 c)) * \log(-2 ((b^6 c^4 - 5 a b^4 c^5 + 6 a^2 b^2 c^6 - a^3 c^7) d^4 - (b^7 c^3 - 3 a b^5 c^4 - 2 a^2 b^3 c^5 + 5 a^3 b c^6) d^3 e + 3 (a b^6 c^3 - 4 a^2 b^4 c^4 + 3 a^3 b^2 c^5) d^2 e^2 - (3 a^2 b^5 c^3 - 11 a^3 b^3 c^4 + 7 a^4 b c^5) d e^3 + (a^3 b^4 c^3 - 3 a^4 b^2 c^4 + a^5 c^5) e^4 + (a^6 b^2 c^2 - a^7 c^3) f^4 + ((3 a^4 b^4 c^2 - 9 a^5 b^2 c^3 + 4 a^6 c^4) d - (3 a^5 b^3 c^2 - 5 a^6 b c^3) e) f^3 + 3 ((a^2 b^6 c^2 - 5 a^3 b^4 c^3 + 7 a^4 b^2 c^4 - 2 a^5 c^5) d^2 - (2 a^3 b^5 c^2 - 7 a^4 b^3 c^3 + 5 a^5 b c^4) d e + (a^4 b^4 c^2 - 2 a^5 b^2 c^3) e^2) f^2 + ((b^8 c^2 - 7 a b^6 c^3 + 18 a^2 b^4 c^4 - 19 a^3 b^2 c^5 + 4 a^4 c^6) d^3 - 3 (a b^7 c^2 - 5 a^2 b^5 c^3 + 8 a^3 b^3 c^4 - 5 a^4 b c^5) d^2 e + 3 (a^2 b^6 c^2 - 3 a^3 b^4 c^3 + a^4 b^2 c^4) d e^2 - (a^3 b^5 c^2 - a^4 b^3 c^3 - 3 a^5 b c^4) e^3) f) * x - \sqrt{1/2} * ((b^{11} - 11 a b^9 c + 44 a^2 b^7 c^2 - 77 a^3 b^5 c^3 + 54 a^4 b^3 c^4 - 8 a^5 b c^5) d^3 - (3 a b^{10} - 30 a^2 b^8 c + 105 a^3 b^6 c^2 - 151 a^4 b^4 c^3 + 77 a^5 b^2 c^4 - 4 a^6 c^5) d^2 e + (3 a^2 b^9 - 27 a^3 b^7 c + 81 a^4 b^5 c^2 - 92 a^5 b^3 c^3 + 32 a^6 b c^4) d e^2 - (a^3 b^8 - 8 a^4 b^6 c + 20 a^5 b^4 c^2 - 17 a^6 b^2 c^3 + 4 a^7 c^4) e^3 + (a^6 b^5 - 5 a^7 b^3 c + 4 a^8 b c^2) f^3 + ((3 a^4 b^7 - 21 a^5 b^5 c + 40 a^6 b^3 c^2 - 16 a^7 b c^3) d - (3 a^5 b^6 - 18 a^6 b^4 c + 25 a^7 b^2 c^2 - 4 a^8 c^3) e) f^2 + ((3 a^2 b^9 - 27 a^3 b^7 c + 80 a^4 b^5 c^2 - 85 a^5 b^3 c^3 + 20 a^6 b c^4) d^2 - 2 (3 a^3 b^8 - 24 a^4 b^6 c + 59 a^5 b^4 c^2 - 45 a^6 b^2 c^3 + 4 a^7 c^4) d e + (3 a^4 b^7 - 21 a^5 b^5 c + 41 a^6 b^3 c^2 - 20 a^7 b c^3) e^2) f + ((a^7 b^6 - 8 a^8 b^4 c + 18 a^9 b^2 c^2 - 8 a^{10} c^3) d - (a^8 b^5 - 7 a^9 b^3 c + 12 a^{10} b c^2) e + (a^9 b^4 - 6 a^{10} b^2 c + 8 a^{11} c^2) f) * \sqrt{((b^{12} - 10 a b^{10} c + 37 a^2 b^8 c^2 - 62 a^3 b^6 c^3 + 46 a^4 b^4 c^4 - 12 a^5 b^2 c^5 + a^6 c^6) d^4 - 4 (a b^{11} - 9 a^2 b^9 c + 29 a^3 b^7 c^2 - 40 a^4 b^5 c^3 + 22 a^5 b^3 c^4 - 3 a^6 b c^5) d^3 e + 2 (3 a^2 b^{10} - 24 a^3 b^8 c + 66 a^4 b^6 c^2 - 72 a^5 b^4 c^3 + 27 a^6 b^2 c^4 - a^7 c^5) d^2 e^2 - 4 (a^3 b^9 - 7 a^4 b^7 c + 16 a^5 b^5 c^2 - 13 a^6 b^3 c^3 + 3 a^7 b c^4) d e^3 + (a^4 b^8 - 6 a^5 b^6 c + 11 a^6 b^4 c^2 - 6 a^7 b^2 c^3 + a^8 c^4) e^4 + (a^8 b^4 - 2 a^9 b^2 c + a^{10} c^2) f^4 + 4 ((a^6 b^6 - 4 a^7 b^4 c + 4 a^8 b^2 c^2 - a^9 c^3) d - (a^7 b^5 - 3 a^8 b^3 c + 2 a^9 b c^2) e) f^3 + 2 ((3 a^4 b^8 - 18 a^5 b^6 c + 33 a^6 b^4 c^2 - 19 a^7 b^2 c^3 + 3 a^8 c^4) d^2 - 2 (3 a^5 b^7 - 15 a^6 b^5 c + 21 a^7 b^3 c^2 - 7 a^8 b c^3) d e + (3 a^6 b^6 - 12 a^7 b^4 c + 12 a^8 b^2 c^2 - a^9 c^3) e^2) f^2 + 4 ((a^2 b^{10} - 8 a^3 b^8 c + 22 a^4 b^6 c^2 - 24 a^5 b^4 c^3 + 9 a^6 b^2 c^4 - a^7 c^5) d^3 - (3 a^3 b^9 - 21 a^4 b^7 c + 48 a^5 b^5 c^2 - 39 a^6 b^3 c^3 + 8 a^7 b c^4) d^2 e + (3 a^4 b^8 - 18 a^5 b^6 c + 33 a^6 b^4 c^2 - 18 a^7 b^2 c^3 + a^8 c^4) d e^2 - (a^5 b^7 - 5 a^6 b^5 c + 7 a^7 b^3 c^2 - 2 a^8 b c^3) e^3) f) / (a^{14} b^2 - 4 a^{15} c)) * \sqrt{-((b^7 - 7 a b^5 c + 14 a^2 b^3 c^2 - 7 a^3 b c^3) d^2 - 2 (a b^6 - 6 a^2 b^4 c + 9 a^3 b^2 c^2 - 2 a^4 c^3) d e + (a^2 b^5 - 5 a^3 b^3 c + 5 a^4 b c^2) e^2 + (a^4 b^3 - 3 a^5 b c) f^2 + 2 ((a^2 b^5 - 5 a^3 b^3 c + 5 a^4 b c^2) d - (a^3 b^4 - 4 a^4 b^2 c + 2 a^5 c^2) e) f - (a^7 b^2 - 4 a^8 c) * \sqrt{((b^{12} - 10 a b^{10} c + 37 a^2 b^8 c^2 - 62 a^3 b^6 c^3 + 46 a^4 b^4 c^4 - 12 a^5 b^2 c^5 + a^6 c^6) d^4 - 4 (a b^{11} - 9 a^2 b^9 c + 29 a^3 b^7 c^2 - 40 a^4 b^5 c^3 + 22 a^5 b^3 c^4 - 3 a^6 b c^5) d^3 e + 2 (3 a^2 b^{10} - 24 a^3 b^8 c + 66 a^4 b^6 c^2 - 72 a^5 b^4 c^3 + 27 a^6 b^2 c^4 - a
\end{aligned}$$

$$\begin{aligned} & ^7*c^5)*d^2*e^2 - 4*(a^3*b^9 - 7*a^4*b^7*c + 16*a^5*b^5*c^2 - 13*a^6*b^3*c^3 \\ & + 3*a^7*b*c^4)*d*e^3 + (a^4*b^8 - 6*a^5*b^6*c + 11*a^6*b^4*c^2 - 6*a^7*b^2*c^3 \\ & + a^8*c^4)*e^4 + (a^8*b^4 - 2*a^9*b^2*c + a^{10}*c^2)*f^4 + 4*((a^6*b^6 - 4*a^7*b^4*c \\ & + 4*a^8*b^2*c^2 - a^9*c^3)*d - (a^7*b^5 - 3*a^8*b^3*c + 2*a^9*b*c^2)*e)*f^3 + 2*((3*a^4*b^8 \\ & - 18*a^5*b^6*c + 33*a^6*b^4*c^2 - 19*a^7*b^2*c^3 + 3*a^8*c^4)*d^2 - 2*(3*a^5*b^7 - 15*a^6*b^5*c \\ & + 21*a^7*b^3*c^2 - 7*a^8*b*c^3)*d*e + (3*a^6*b^6 - 12*a^7*b^4*c + 12*a^8*b^2*c^2 - a^9*c^3)*e^2)*f^2 \\ & + 4*((a^2*b^{10} - 8*a^3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4 - a^7*c^5)*d^3 \\ & - (3*a^3*b^9 - 21*a^4*b^7*c + 48*a^5*b^5*c^2 - 39*a^6*b^3*c^3 + 8*a^7*b*c^4)*d^2*e + (3*a^4*b^8 \\ & - 18*a^5*b^6*c + 33*a^6*b^4*c^2 - 18*a^7*b^2*c^3 + a^8*c^4)*d*e^2 - (a^5*b^7 - 5*a^6*b^5*c + 7*a^7*b^3*c^2 \\ & - 2*a^8*b*c^3)*e^3)*f)/(a^{14}*b^2 - 4*a^{15}*c))/(a^7*b^2 - 4*a^8*c)) - 30*(a*b*e - a^2*f - (b^2 - a*c)*d)*x^4 \\ & + 6*a^2*d - 10*(a*b*d - a^2*e)*x^2)/(a^3*x^5) \end{aligned}$$

giac [B] time = 7.02, size = 6718, normalized size = 20.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^4+e*x^2+d)/x^6/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out]
$$\begin{aligned} & -1/8*((2*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2*b^2*c^4 - 32*a^3*c^5 - \sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*b^6 + 9*\sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a*b^4*c + 2*\sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*b^5*c - 24*\sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a^2*b^2*c^2 - 10*\sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a*b^3*c^2 - \sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*b^4*c^2 + 16*\sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a^3*c^3 + 8*\sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a^2*b*c^3 + 5*\sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a*b^2*c^3 - 4*\sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2 + 10*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b^2 - 4*a*c)*a^2*c^4)*a^2*d + (2*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 32*a^4*c^4 - \sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a^2*b^4 + 8*\sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a^3*b^2*c + 2*\sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a^2*b^3*c - 16*\sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a^4*c^2 - 8*\sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a^3*b*c^2 - \sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a^2*b^2*c^2 + 4*\sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a^3*c^3 - 2*(b^2 - 4*a*c)*a^2*b^2*c^2 + 8*(b^2 - 4*a*c)*a^3*c^3)*a^2*f - (2*a*b^5*c^2 - 16*a^2*b^3*c^3 + 32*a^3*b*c^4 - \sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a*b^5 + 8*\sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a^2*b^3*c + 2*\sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a*b^4*c - 16*\sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a^3*b*c^2 - 8*\sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a^2*b^2*c^2 - \sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a*b^3*c^2 + 4*\sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a^2*b*c^3 - 2*(b^2 - 4*a*c)*a*b^3*c^2 + 8*(b^2 - 4*a*c)*a^2*b*c^3)*a^2*e + 2*(\sqrt{2})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a*b^7 - 10*\sqrt{2})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a^2*b^5*c - 2*\sqrt{2})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a*b^6*c - 2*a*b^7*c + 32*\sqrt{2})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a^3*b^3*c^2 + 12*\sqrt{2})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a^2*b^4*c^2 + \sqrt{2})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a*b^5*c^2 + 20*a^2*b^5*c^2 - 32*\sqrt{2})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a^4*b*c^3 - 16*\sqrt{2})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a^3*b^2*c^3 - 6*\sqrt{2})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a^2*b^3*c^3 - 64*a^3*b^3*c^3 + 8*\sqrt{2})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a^3*b*c^4 + 64*a^4*b*c^4 + 2*(b^2 - 4*a*c)*a*b^5*c - 12*(b^2 - 4*a*c)*a^2*b^3*c^2 + 16*(b^2 - 4*a*c)*a^3*b*c^3)*d*abs(a) + 2*(\sqrt{2})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a^3*b^5 - 8*\sqrt{2})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)* \end{aligned}$$

$$\begin{aligned}
& (b^2 - 4ac)c \cdot a^4 b^3 c - 2\sqrt{2}\sqrt{b^2 - 4ac}c \cdot a^3 b^4 c - 2a^3 b^5 c + 16\sqrt{2}\sqrt{b^2 - 4ac}c \cdot a^5 b^2 c^2 \\
& + 8\sqrt{2}\sqrt{b^2 - 4ac}c \cdot a^4 b^2 c^2 + \sqrt{2}\sqrt{b^2 - 4ac}c \cdot a^3 b^3 c^2 + 16a^4 b^3 c^2 - 4\sqrt{2}\sqrt{b^2 - 4ac}c \\
& + \sqrt{b^2 - 4ac}c \cdot a^4 b^3 c^3 - 32a^5 b^2 c^3 + 2(b^2 - 4ac)a^3 b^3 c - 8(b^2 - 4ac)a^4 b^2 c^2 \cdot f \cdot \operatorname{abs}(a) \\
& - 2(\sqrt{2}\sqrt{b^2 - 4ac}c \cdot a^2 b^6 - 9\sqrt{2}\sqrt{b^2 - 4ac}c \cdot a^3 b^4 c - 2\sqrt{2}\sqrt{b^2 - 4ac}c \cdot a^2 b^5 c \\
& - 2a^2 b^6 c + 24\sqrt{2}\sqrt{b^2 - 4ac}c \cdot a^4 b^2 c^2 + 10\sqrt{2}\sqrt{b^2 - 4ac}c \cdot a^3 b^3 c^2 + \sqrt{2}\sqrt{b^2 - 4ac}c \cdot a^2 b^4 c^2 \\
& + 18a^3 b^4 c^2 - 16\sqrt{2}\sqrt{b^2 - 4ac}c \cdot a^5 c^3 - 8\sqrt{2}\sqrt{b^2 - 4ac}c \cdot a^4 b^2 c^3 - 5\sqrt{2}\sqrt{b^2 - 4ac}c \cdot a^3 b^2 c^3 \\
& - 48a^4 b^2 c^3 + 4\sqrt{2}\sqrt{b^2 - 4ac}c \cdot a^4 c^4 + 32a^5 c^4 + 2(b^2 - 4ac)a^2 b^4 c - 10(b^2 - 4ac)a^3 b^2 c^2 \\
& + 8(b^2 - 4ac)a^4 c^3 \cdot \operatorname{abs}(a) \cdot e + (2a^2 b^6 c^2 - 14a^3 b^4 c^3 + 24a^4 b^2 c^4 - \sqrt{2}\sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \\
& + \sqrt{b^2 - 4ac}c \cdot a^2 b^6 + 7\sqrt{2}\sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \cdot a^3 b^4 c + 2\sqrt{2}\sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \\
& \cdot \sqrt{b^2 - 4ac}c \cdot a^2 b^5 c - 12\sqrt{2}\sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \cdot a^4 b^2 c^2 - 6\sqrt{2}\sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \\
& \cdot \sqrt{b^2 - 4ac}c \cdot a^3 b^3 c^2 - \sqrt{2}\sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \cdot a^2 b^4 c^2 + 3\sqrt{2}\sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \\
& \cdot \sqrt{b^2 - 4ac}c \cdot a^3 b^2 c^3 - 2(b^2 - 4ac)a^2 b^4 c^2 + 6(b^2 - 4ac)a^3 b^2 c^3) \cdot d + (2a^4 b^4 c^2 - 8a^5 b^2 c^3 - \sqrt{2}\sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \\
& \cdot a^4 b^4 + 4\sqrt{2}\sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \cdot a^5 b^2 c + 2\sqrt{2}\sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \cdot a^4 b^3 c \\
& - \sqrt{2}\sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \cdot a^4 b^2 c^2 - 2(b^2 - 4ac)a^4 b^2 c^2) \cdot f - (2a^3 b^5 c^2 - 12a^4 b^3 c^3 \\
& + 16a^5 b^2 c^4 - \sqrt{2}\sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \cdot a^3 b^5 + 6\sqrt{2}\sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \\
& \cdot a^4 b^3 c + 2\sqrt{2}\sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \cdot a^3 b^4 c - 8\sqrt{2}\sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \\
& \cdot a^5 b^2 c^2 - 4\sqrt{2}\sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \cdot a^4 b^2 c^2 - \sqrt{2}\sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \\
& \cdot a^3 b^3 c^2 + 2\sqrt{2}\sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \cdot a^4 b^2 c^3 - 2(b^2 - 4ac)a^3 b^3 c^2 + 4(b^2 - 4ac)a^4 b^2 c^3) \cdot e) \cdot \operatorname{arctan} \\
& (2\sqrt{2}\sqrt{1/2} \cdot x / \sqrt{(a^3 b + \sqrt{a^6 b^2 - 4a^7 c}) / (a^3 c)}) / ((a^5 b^4 - 8a^6 b^2 c - 2a^5 b^3 c + 16a^7 c^2 + 8a^6 b^2 c^2 + a^5 b^2 c^2 - 4a^6 c^3) \\
& \cdot \operatorname{abs}(a) \cdot \operatorname{abs}(c)) + 1/8 \cdot ((2b^6 c^2 - 18a^2 b^4 c^3 + 48a^2 b^2 c^4 - 32a^3 c^5 - \sqrt{2}\sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \cdot b^6 \\
& + 9\sqrt{2}\sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \cdot a^2 b^4 c + 2\sqrt{2}\sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \cdot a^2 b^5 c \\
& - 24\sqrt{2}\sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \cdot a^2 b^2 c^2 - 10\sqrt{2}\sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \cdot a^2 b^3 c^2 \\
& - \sqrt{2}\sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \cdot a^3 b^3 c^2 - \sqrt{2}\sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \cdot b^4 c^2 \\
& + 16\sqrt{2}\sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \cdot a^3 c^3 + 8\sqrt{2}\sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \cdot a^2 b^2 c^3 \\
& + 5\sqrt{2}\sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \cdot a^2 b^3 c^3 - 4\sqrt{2}\sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \\
& \cdot \sqrt{b^2 - 4ac}c \cdot a^2 c^4 - 2(b^2 - 4ac)b^4 c^2 + 10(b^2 - 4ac)a^2 b^2 c^3 - 8(b^2 - 4ac)a^2 c^4) \cdot a^2 \cdot d + (2a^2 b^4 c^2 - 16a^3 b^2 c^3 \\
& + 32a^4 c^4 - \sqrt{2}\sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \cdot a^2 b^4 + 8\sqrt{2}\sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \\
& \cdot \sqrt{b^2 - 4ac}c \cdot a^3 b^2 c + 2\sqrt{2}\sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \cdot a^2 b^3 c - 16\sqrt{2}\sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \\
& \cdot \sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \cdot a^4 c^2 - 8\sqrt{2}\sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \cdot a^3 b^2 c^2 - \sqrt{2}\sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \\
& \cdot \sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \cdot a^2 b^2 c^2 + 4\sqrt{2}\sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \cdot a^3 c^3 - 2(b^2 - 4ac)a^2 b^2 c^2 \\
& + 8(b^2 - 4ac)a^3 c^3) \cdot a^2 \cdot f - (2a^2 b^5 c^2 - 16a^2 b^3 c^3 + 32a^3 b^2 c^4 - \sqrt{2}\sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c \cdot \sqrt{b^2 - 4ac}c) \cdot \operatorname{abs}(a) \cdot \operatorname{abs}(c)
\end{aligned}$$

$$\begin{aligned}
& b^2 - 4ac) * c) * a * b^5 + 8 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} \\
& * a^2 * b^3 * c + 2 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a * b^4 * c - 16 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) \\
& * a^3 * b * c^2 - 8 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^2 * b^2 * c^2 - \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a * b^3 * c^2 \\
& + 4 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^2 * b * c^3 - 2 * (b^2 - 4ac) * a * b^3 * c^2 + 8 * (b^2 - 4ac) * a^2 * b * c^3) * a^2 * e - 2 * (\sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a * b^7 \\
& - 10 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^2 * b^5 * c - 2 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a * b^6 * c + 2 * a * b^7 * c + 32 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^3 * b^3 * c^2 + 12 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^2 * b^4 * c^2 \\
& + \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^2 * b^5 * c^2 - 20 * a^2 * b^5 * c^2 - 32 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^4 * b * c^3 - 16 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^3 * b^2 * c^3 - 6 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^2 * b^3 * c^3 + 64 * a^3 * b^3 * c^3 \\
& + 8 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^3 * b * c^4 - 64 * a^4 * b * c^4 - 2 * (b^2 - 4ac) * a * b^5 * c + 12 * (b^2 - 4ac) * a^2 * b^3 * c^2 - 16 * (b^2 - 4ac) * a^3 * b * c^3) * d * \text{abs}(a) - 2 * (\sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^3 * b^5 - 8 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^4 * b^3 * c - 2 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^3 * b^4 * c + 2 * a^3 * b^5 * c + 16 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^5 * b * c^2 + 8 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^4 * b^2 * c^2 + \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^3 * b^3 * c^2 - 16 * a^4 * b^3 * c^2 - 4 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^4 * b * c^3 + 32 * a^5 * b * c^3 - 2 * (b^2 - 4ac) * a^3 * b^3 * c + 8 * (b^2 - 4ac) * a^4 * b * c^2) * f * \text{abs}(a) + 2 * (\sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^2 * b^6 - 9 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^3 * b^4 * c - 2 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^2 * b^5 * c + 2 * a^2 * b^6 * c + 24 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^4 * b^2 * c^2 + 10 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^3 * b^3 * c^2 + \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^2 * b^4 * c^2 - 18 * a^3 * b^4 * c^2 - 16 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^5 * c^3 - 8 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^4 * b * c^3 - 5 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^3 * b^2 * c^3 + 48 * a^4 * b^2 * c^3 + 4 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^4 * c^4 - 32 * a^5 * c^4 - 2 * (b^2 - 4ac) * a^2 * b^4 * c + 10 * (b^2 - 4ac) * a^3 * b^2 * c^2 - 8 * (b^2 - 4ac) * a^4 * c^3) * \text{abs}(a) * e + (2 * a^2 * b^6 * c^2 - 14 * a^3 * b^4 * c^3 + 24 * a^4 * b^2 * c^4 - \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^2 * b^6 + 7 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^3 * b^4 * c + 2 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^2 * b^5 * c - 12 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^4 * b^2 * c^2 - 6 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^3 * b^3 * c^2 - \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^2 * b^4 * c^2 + 3 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^3 * b^2 * c^3 - 2 * (b^2 - 4ac) * a^2 * b^4 * c^2 + 6 * (b^2 - 4ac) * a^3 * b^2 * c^3) * d + (2 * a^4 * b^4 * c^2 - 8 * a^5 * b^2 * c^3 - \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^4 * b^4 + 4 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^5 * b^2 * c + 2 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^4 * b^3 * c - \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^4 * b^2 * c^2 - 2 * (b^2 - 4ac) * a^4 * b^2 * c^2) * f - (2 * a^3 * b^5 * c^2 - 12 * a^4 * b^3 * c^3 + 16 * a^5 * b * c^4 - \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^3 * b^5 + 6 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^4 * b^3 * c + 2 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^3 * b^4 * c - 8 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^5 * b * c^2 - 4 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^4 * b^2 * c^2 - \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^3 * b^3 * c^2 + 2 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^4 * b * c^3 - 2 * (b^2 - 4ac) * a^3 * b^3 * c^2 + 4 * (b^2 - 4ac) * a^4 * b * c^3) * e) * \arctan(2 * \sqrt{1/2} * x / \sqrt{(a^3 * b - \sqrt{a^6 * b^2 - 4 * a^7 * c}) / (a^3 * c)}) / ((a^5 * b^4 - 8 * a^6 * b^2 * c - 2 * a^5 * b^3 * c + 16 * a^7 * c^2 + 8 * a^6 * b * c^2 + a^5 * b^2 * c^2 - 4 * a^6 * c^3) * \text{abs}(a) * \text{abs}(c))) - 1/15 * (15 * b^2 * d * x^4 - 15 * a * c * d * x^4 + 15 * a^2 * f * x^4 - 15 * a * b * x^4 * e - 5 * a * b * d * x^2 + 5 * a^2 * x^2 * e + 3 * a^2 * d) / (a^3 * x^5)
\end{aligned}$$

maple [B] time = 0.03, size = 1121, normalized size = 3.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((f*x^4+e*x^2+d)/x^6/(c*x^4+b*x^2+a), x)$

[Out]
$$-1/5*d/a/x^5+1/3/a^2/x^3*b*d+1/a^2/x*b*e+1/a^2/x*c*d-1/a^3/x*b^2*d+1/2/a^3*c^2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\text{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^2*d+1/a*c^2/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\text{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*e+1/2/a^2*c^2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\text{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b*e-1/2/a^3*c^2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\text{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^2*d+1/a*c^2/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\text{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*e-1/2/a^2*c^2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\text{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b*e-1/3/a/x^3*e-1/a/x*f+1/2/a*c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\text{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b*f+1/2/a^3*c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\text{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^3*d-3/2/a^2*c^2/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\text{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b*d-1/2/a*c^2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\text{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*f+1/2/a^2*c^2*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\text{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*d+1/2/a*c^2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\text{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*f-1/2/a^2*c^2*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\text{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*d-3/2/a^2*c^2/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\text{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b*d+1/2/a^3*c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\text{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^3*d-1/2/a^2*c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\text{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^2*e-1/2/a^2*c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\text{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^2*e+1/2/a*c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\text{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b*f$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((f*x^4+e*x^2+d)/x^6/(c*x^4+b*x^2+a), x, \text{algorithm}="maxima")$

[Out]
$$-\text{integrate}((a^2*b*f - (a*b*c*e - a^2*c*f - (b^2*c - a*c^2)*d)*x^2 + (b^3 - 2*a*b*c)*d - (a*b^2 - a^2*c)*e)/(c*x^4 + b*x^2 + a), x)/a^3 + 1/15*(15*(a*b*e - a^2*f - (b^2 - a*c)*d)*x^4 - 3*a^2*d + 5*(a*b*d - a^2*e)*x^2)/(a^3*x^5)$$

mupad [B] time = 6.25, size = 23019, normalized size = 69.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((d + e*x^2 + f*x^4)/(x^6*(a + b*x^2 + c*x^4)), x)$

[Out]
$$\text{atan}(((x*(4*a^13*c^5*e^2 - 4*a^12*c^6*d^2 - 4*a^14*c^4*f^2 + 2*a^9*b^6*c^3*d^2 - 12*a^10*b^4*c^4*d^2 + 18*a^11*b^2*c^5*d^2 + 2*a^11*b^4*c^3*e^2 - 8*a^12*b^2*c^4*e^2 + 2*a^13*b^2*c^3*f^2 + 8*a^13*c^5*d*f - 20*a^12*b*c^5*d*e + 12*a^13*b*c^4*e*f - 4*a^10*b^5*c^3*d*e + 20*a^11*b^3*c^4*d*e + 4*a^11*b^4*c^3*d*f - 16*a^12*b^2*c^4*d*f - 4*a^12*b^3*c^3*e*f) - ((b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + a^4*b^5*f^2 + 28*a^4*b*c^4*d^2 - 9*$$

$$\begin{aligned}
& a^3 b^5 c^2 e^2 - 20 a^5 b^3 c^3 e^2 - 7 a^5 b^3 c^3 f^2 + 12 a^6 b^3 c^2 f^2 - a^5 \\
& c^3 f^2 (-4 a^2 c - b^2)^3)^{(1/2)} - 2 a^2 b^8 d^2 e + 42 a^2 b^5 c^2 d^2 - 63 a^3 \\
& b^3 c^3 d^2 + a^2 b^4 e^2 (-4 a^2 c - b^2)^3)^{(1/2)} - a^3 c^3 d^2 (-4 a^2 c \\
& - b^2)^3)^{(1/2)} + 25 a^4 b^3 c^2 e^2 + a^4 b^2 f^2 (-4 a^2 c - b^2)^3)^{(1/2)} \\
& + a^4 c^2 e^2 (-4 a^2 c - b^2)^3)^{(1/2)} - 11 a^2 b^7 c^3 d^2 + 2 a^2 b^7 d^2 f - \\
& 16 a^5 c^4 d^2 e - 2 a^3 b^6 e^2 f + 16 a^6 c^3 e^2 f - 2 a^2 b^5 d^2 e (-4 a^2 c - b^2)^3)^{(1/2)} \\
& + 20 a^2 b^6 c^3 d^2 e - 18 a^3 b^5 c^3 d^2 f - 40 a^5 b^3 c^3 d^2 f + 16 a^4 \\
& b^4 c^3 e^2 f + 6 a^2 b^2 c^2 d^2 (-4 a^2 c - b^2)^3)^{(1/2)} - 5 a^2 b^4 c^3 d^2 (-4 a^2 c \\
& - b^2)^3)^{(1/2)} - 66 a^3 b^4 c^2 d^2 e + 76 a^4 b^2 c^3 d^2 e + 2 a^2 b^4 \\
& d^2 f (-4 a^2 c - b^2)^3)^{(1/2)} + 50 a^4 b^3 c^2 d^2 f - 2 a^3 b^3 e^2 f (-4 a^2 c \\
& - b^2)^3)^{(1/2)} + 2 a^4 c^2 d^2 f (-4 a^2 c - b^2)^3)^{(1/2)} - 36 a^5 b^2 c^2 \\
& e^2 f - 3 a^3 b^2 c^2 e^2 (-4 a^2 c - b^2)^3)^{(1/2)} + 4 a^4 b^3 c^2 e^2 f (-4 a^2 c \\
& - b^2)^3)^{(1/2)} + 8 a^2 b^3 c^3 d^2 e (-4 a^2 c - b^2)^3)^{(1/2)} - 6 a^3 b^3 c^2 d^2 e \\
& (-4 a^2 c - b^2)^3)^{(1/2)} - 6 a^3 b^2 c^3 d^2 f (-4 a^2 c - b^2)^3)^{(1/2)} / (8 (a^7 b^4 + 16 a^9 c^2 - 8 a^8 b^2 c))^{(1/2)} * (x (32 a^16 b^3 c^3 - 8 a^15 b^3 c^2) * (-b^9 d^2 + a^2 b^7 e^2 + b^6 d^2 (-4 a^2 c - b^2)^3)^{(1/2)} + a^4 b^5 f^2 + 28 a^4 b^3 c^4 d^2 - 9 a^3 b^5 c^2 e^2 - 20 a^5 b^3 c^3 e^2 - 7 a^5 b^3 c^3 f^2 + 12 a^6 b^3 c^2 f^2 - a^5 c^3 f^2 (-4 a^2 c - b^2)^3)^{(1/2)} - 2 a^2 b^8 d^2 e + 4 2 a^2 b^5 c^2 d^2 - 63 a^3 b^3 c^3 d^2 + a^2 b^4 e^2 (-4 a^2 c - b^2)^3)^{(1/2)} - a^3 c^3 d^2 (-4 a^2 c - b^2)^3)^{(1/2)} + 25 a^4 b^3 c^2 e^2 + a^4 b^2 f^2 (-4 a^2 c - b^2)^3)^{(1/2)} + a^4 c^2 e^2 (-4 a^2 c - b^2)^3)^{(1/2)} - 11 a^2 b^7 c^3 d^2 + 2 a^2 b^7 d^2 f - 16 a^5 c^4 d^2 e - 2 a^3 b^6 e^2 f + 16 a^6 c^3 e^2 f - 2 a^2 b^5 d^2 e (-4 a^2 c - b^2)^3)^{(1/2)} + 20 a^2 b^6 c^3 d^2 e - 18 a^3 b^5 c^3 d^2 f - 40 a^5 b^3 c^3 d^2 f + 16 a^4 b^4 c^3 e^2 f + 6 a^2 b^2 c^2 d^2 (-4 a^2 c - b^2)^3)^{(1/2)} - 5 a^2 b^4 c^3 d^2 (-4 a^2 c - b^2)^3)^{(1/2)} - 66 a^3 b^4 c^2 d^2 e + 76 a^4 b^2 c^3 d^2 e + 2 a^2 b^4 d^2 f (-4 a^2 c - b^2)^3)^{(1/2)} + 50 a^4 b^3 c^2 d^2 f - 2 a^3 b^3 e^2 f (-4 a^2 c - b^2)^3)^{(1/2)} + 2 a^4 c^2 d^2 f (-4 a^2 c - b^2)^3)^{(1/2)} - 36 a^5 b^2 c^2 e^2 f - 3 a^3 b^2 c^2 e^2 (-4 a^2 c - b^2)^3)^{(1/2)} + 4 a^4 b^3 c^2 e^2 f (-4 a^2 c - b^2)^3)^{(1/2)} + 8 a^2 b^3 c^3 d^2 e (-4 a^2 c - b^2)^3)^{(1/2)} - 6 a^3 b^2 c^3 d^2 f (-4 a^2 c - b^2)^3)^{(1/2)} / (8 (a^7 b^4 + 16 a^9 c^2 - 8 a^8 b^2 c))^{(1/2)} - 16 a^15 c^4 e + 4 a^12 b^5 c^2 d - 24 a^13 b^3 c^3 d - 4 a^13 b^4 c^2 e + 20 a^14 b^2 c^3 e + 4 a^14 b^3 c^2 f + 32 a^14 b^3 c^4 d - 16 a^15 b^3 c^3 f) * (-b^9 d^2 + a^2 b^7 e^2 + b^6 d^2 (-4 a^2 c - b^2)^3)^{(1/2)} + a^4 b^5 f^2 + 28 a^4 b^3 c^4 d^2 - 9 a^3 b^5 c^2 e^2 - 20 a^5 b^3 c^3 e^2 - 7 a^5 b^3 c^3 f^2 + 12 a^6 b^3 c^2 f^2 - a^5 c^3 f^2 (-4 a^2 c - b^2)^3)^{(1/2)} - 2 a^2 b^8 d^2 e + 42 a^2 b^5 c^2 d^2 - 63 a^3 b^3 c^3 d^2 + a^2 b^4 e^2 (-4 a^2 c - b^2)^3)^{(1/2)} - a^3 c^3 d^2 (-4 a^2 c - b^2)^3)^{(1/2)} + 25 a^4 b^3 c^2 e^2 + a^4 b^2 f^2 (-4 a^2 c - b^2)^3)^{(1/2)} + a^4 c^2 e^2 (-4 a^2 c - b^2)^3)^{(1/2)} - 11 a^2 b^7 c^3 d^2 + 2 a^2 b^7 d^2 f - 16 a^5 c^4 d^2 e - 2 a^3 b^6 e^2 f + 16 a^6 c^3 e^2 f - 2 a^2 b^5 d^2 e (-4 a^2 c - b^2)^3)^{(1/2)} + 20 a^2 b^6 c^3 d^2 e - 18 a^3 b^5 c^3 d^2 f - 40 a^5 b^3 c^3 d^2 f + 16 a^4 b^4 c^3 e^2 f + 6 a^2 b^2 c^2 d^2 (-4 a^2 c - b^2)^3)^{(1/2)} - 5 a^2 b^4 c^3 d^2 (-4 a^2 c - b^2)^3)^{(1/2)} - 66 a^3 b^4 c^2 d^2 e + 76 a^4 b^2 c^3 d^2 e + 2 a^2 b^4 d^2 f (-4 a^2 c - b^2)^3)^{(1/2)} + 50 a^4 b^3 c^2 d^2 f - 2 a^3 b^3 e^2 f (-4 a^2 c - b^2)^3)^{(1/2)} + 2 a^4 c^2 d^2 f (-4 a^2 c - b^2)^3)^{(1/2)} - 36 a^5 b^2 c^2 e^2 f - 3 a^3 b^2 c^2 e^2 (-4 a^2 c - b^2)^3)^{(1/2)} + 4 a^4 b^3 c^2 e^2 f (-4 a^2 c - b^2)^3)^{(1/2)} + 8 a^2 b^3 c^3 d^2 e (-4 a^2 c - b^2)^3)^{(1/2)} - 6 a^3 b^2 c^3 d^2 f (-4 a^2 c - b^2)^3)^{(1/2)} / (8 (a^7 b^4 + 16 a^9 c^2 - 8 a^8 b^2 c))^{(1/2)} * i + (x (4 a^13 c^5 e^2 - 4 a^12 c^6 d^2 - 4 a^14 c^4 f^2 + 2 a^9 b^6 c^3 d^2 - 12 a^10 b^4 c^4 d^2 + 18 a^11 b^2 c^5 d^2 + 2 a^11 b^4 c^3 e^2 - 8 a^12 b^2 c^4 e^2 + 2 a^13 b^2 c^3 f^2 + 8 a^13 c^5 d^2 f - 20 a^12 b^3 c^5 d^2 e + 12 a^13 b^3 c^4 e^2 f - 4 a^10 b^5 c^3 d^2 e + 20 a^11 b^3 c^4 d^2 e + 4 a^11 b^4 c^3 d^2 f - 16 a^12 b^2 c^4 d^2 f - 4 a^12 b^3 c^3 e^2 f) - (-b^9 d^2 + a^2 b^7 e^2 + b^6 d^2 (-4 a^2 c - b^2)^3)^{(1/2)} + a^4 b^5 f^2 + 28 a^4 b^3 c^4 d^2 - 9 a^3 b^5 c^2 e^2 - 20 a^5 b^3 c^3 e^2 - 7 a^5 b^3 c^3 f^2 + 12 a^6 b^3 c^2 f^2 - a^5 c^3 f^2 (-4 a^2 c - b^2)^3)^{(1/2)} - 2 a^2 b^8 d^2 e + 42 a^2 b^5 c^2 d^2 - 63 a^3 b^3 c^3 d^2 + a^2 b^4 e^2 (-4 a^2 c - b^2)^3)^{(1/2)} - a^3 c^3 d^2 (-4 a^2 c - b^2)^3)^{(1/2)} + 25 a^4 b^3 c^2 e^2 + a^4 b^2 f^2 (-4 a^2 c - b^2)^3)^{(1/2)} + a^4 c^2 e^2 (-4 a^2 c - b^2)^3)^{(1/2)} + a^4 c^2 e^2 (-4 a^2 c - b^2)^3)^{(1/2)}
\end{aligned}$$

$$\begin{aligned} &-(4ac - b^2)^3)^{(1/2)} - 11ab^7cd^2 + 2a^2b^7d^2f - 16a^5c^4d^2e - \\ &2a^3b^6e^2f + 16a^6c^3e^2f - 2ab^5d^2e^2f*(-(4ac - b^2)^3)^{(1/2)} + 20 \\ &a^2b^6c^2d^2e - 18a^3b^5c^2d^2f - 40a^5b^3c^2d^2f + 16a^4b^4c^2e^2f + 6 \\ &a^2b^2c^2d^2e^2*(-(4ac - b^2)^3)^{(1/2)} - 5ab^4c^2d^2e^2*(-(4ac - b^2)^3 \\ &)^{(1/2)} - 66a^3b^4c^2d^2e + 76a^4b^2c^3d^2e + 2a^2b^4d^2f^2*(-(4ac \\ &- b^2)^3)^{(1/2)} + 50a^4b^3c^2d^2f - 2a^3b^3e^2f^2*(-(4ac - b^2)^3)^{(1/2)} \\ &+ 2a^4c^2d^2f^2*(-(4ac - b^2)^3)^{(1/2)} - 36a^5b^2c^2e^2f - 3a^3b^2 \\ &2c^2e^2f^2*(-(4ac - b^2)^3)^{(1/2)} + 4a^4b^3c^2e^2f^2*(-(4ac - b^2)^3)^{(1/2)} + \\ &8a^2b^3c^2d^2e^2*(-(4ac - b^2)^3)^{(1/2)} - 6a^3b^3c^2d^2e^2*(-(4ac - b^2)^3 \\ &)^{(1/2)} - 6a^3b^2c^2d^2f^2*(-(4ac - b^2)^3)^{(1/2)}/(8(a^7b^4 + 16a^9c^2 \\ & - 8a^8b^2c)))^{(1/2)}*(x*(32a^16b^3c^3 - 8a^15b^3c^2))*(-(b^9d^2 + \\ &a^2b^7e^2 + b^6d^2*(-(4ac - b^2)^3)^{(1/2)} + a^4b^5f^2 + 28a^4b^3c^2 \\ &4d^2 - 9a^3b^5c^2e^2 - 20a^5b^3c^2e^2 - 7a^5b^3c^2f^2 + 12a^6b^3c^2 \\ &f^2 - a^5c^2f^2*(-(4ac - b^2)^3)^{(1/2)} - 2ab^8d^2e + 42a^2b^5c^2d^2 \\ &2 - 63a^3b^3c^3d^2 + a^2b^4e^2*(-(4ac - b^2)^3)^{(1/2)} - a^3c^3d^2 \\ &*(-(4ac - b^2)^3)^{(1/2)} + 25a^4b^3c^2e^2 + a^4b^2f^2*(-(4ac - b^2)^3)^{(1/2)} \\ &+ a^4c^2e^2*(-(4ac - b^2)^3)^{(1/2)} - 11ab^7cd^2 + 2a^2b \\ &b^7d^2f - 16a^5c^4d^2e - 2a^3b^6e^2f + 16a^6c^3e^2f - 2ab^5d^2e^2*(-(4 \\ &4ac - b^2)^3)^{(1/2)} + 20a^2b^6c^2d^2e - 18a^3b^5c^2d^2f - 40a^5b^3c^2 \\ &d^2f + 16a^4b^4c^2e^2f + 6a^2b^2c^2d^2e^2*(-(4ac - b^2)^3)^{(1/2)} - 5ab \\ &b^4c^2d^2e^2*(-(4ac - b^2)^3)^{(1/2)} - 66a^3b^4c^2d^2e + 76a^4b^2c^3d^2e \\ &+ 2a^2b^4d^2f^2*(-(4ac - b^2)^3)^{(1/2)} + 50a^4b^3c^2d^2f - 2a^3b^3e^2 \\ &e^2f^2*(-(4ac - b^2)^3)^{(1/2)} + 2a^4c^2d^2f^2*(-(4ac - b^2)^3)^{(1/2)} - 36 \\ &a^5b^2c^2e^2f - 3a^3b^2c^2e^2f^2*(-(4ac - b^2)^3)^{(1/2)} + 4a^4b^3c^2e^2f^2 \\ &(- (4ac - b^2)^3)^{(1/2)} + 8a^2b^3c^2d^2e^2*(-(4ac - b^2)^3)^{(1/2)} - 6a^3 \\ &b^3c^2d^2e^2*(-(4ac - b^2)^3)^{(1/2)} - 6a^3b^2c^2d^2f^2*(-(4ac - b^2)^3)^{(1/2)}/(8(a \\ &a^7b^4 + 16a^9c^2 - 8a^8b^2c)))^{(1/2)} + 16a^15c^4e - 4a^12 \\ &12b^5c^2d + 24a^13b^3c^3d + 4a^13b^4c^2e - 20a^14b^2c^3e - 4 \\ &a^14b^3c^2f - 32a^14b^3c^4d + 16a^15b^3c^3f)*(-(b^9d^2 + a^2b^7e^2 + \\ &b^6d^2*(-(4ac - b^2)^3)^{(1/2)} + a^4b^5f^2 + 28a^4b^3c^2d^2 - 9 \\ &a^3b^5c^2e^2 - 20a^5b^3c^2e^2 - 7a^5b^3c^2f^2 + 12a^6b^3c^2f^2 - a^5 \\ &5c^2f^2*(-(4ac - b^2)^3)^{(1/2)} - 2ab^8d^2e + 42a^2b^5c^2d^2 - 63a^3 \\ &3b^3c^3d^2 + a^2b^4e^2*(-(4ac - b^2)^3)^{(1/2)} - a^3c^3d^2*(-(4ac \\ &- b^2)^3)^{(1/2)} + 25a^4b^3c^2e^2 + a^4b^2f^2*(-(4ac - b^2)^3)^{(1/2)} \\ &+ a^4c^2e^2*(-(4ac - b^2)^3)^{(1/2)} - 11ab^7cd^2 + 2a^2b^7d^2f - \\ &16a^5c^4d^2e - 2a^3b^6e^2f + 16a^6c^3e^2f - 2ab^5d^2e^2*(-(4ac - b \\ &^2)^3)^{(1/2)} + 20a^2b^6c^2d^2e - 18a^3b^5c^2d^2f - 40a^5b^3c^2d^2f + 16 \\ &a^4b^4c^2e^2f + 6a^2b^2c^2d^2e^2*(-(4ac - b^2)^3)^{(1/2)} - 5ab^4c^2d^2 \\ &2e^2*(-(4ac - b^2)^3)^{(1/2)} - 66a^3b^4c^2d^2e + 76a^4b^2c^3d^2e + 2a^2b \\ &b^4d^2f^2*(-(4ac - b^2)^3)^{(1/2)} + 50a^4b^3c^2d^2f - 2a^3b^3e^2f^2*(-(4 \\ &a^5c - b^2)^3)^{(1/2)} + 2a^4c^2d^2f^2*(-(4ac - b^2)^3)^{(1/2)} - 36a^5b^2c^2 \\ &2e^2f - 3a^3b^2c^2e^2f^2*(-(4ac - b^2)^3)^{(1/2)} + 4a^4b^3c^2e^2f^2 \\ &(- (4ac - b^2)^3)^{(1/2)} + 8a^2b^3c^2d^2e^2*(-(4ac - b^2)^3)^{(1/2)} - 6a^3 \\ &b^3c^2d^2e^2*(-(4ac - b^2)^3)^{(1/2)} - 6a^3b^2c^2d^2f^2*(-(4ac - b^2)^3)^{(1/2)}/(8(\\ &a^7b^4 + 16a^9c^2 - 8a^8b^2c)))^{(1/2)}*ii)/((x*(4a^13c^5e^2 - 4a^1 \\ &2c^6d^2 - 4a^14c^4f^2 + 2a^9b^6c^3d^2 - 12a^10b^4c^4d^2 + 18a^ \\ &^11b^2c^5d^2 + 2a^11b^4c^3e^2 - 8a^12b^2c^4e^2 + 2a^13b^2c^3 \\ &f^2 + 8a^13c^5d^2f - 20a^12b^3c^5d^2e + 12a^13b^3c^4e^2f - 4a^10b^5c \\ &^3d^2e + 20a^11b^3c^4d^2e + 4a^11b^4c^3d^2f - 16a^12b^2c^4d^2f - 4 \\ &a^12b^3c^3e^2f) - ((b^9d^2 + a^2b^7e^2 + b^6d^2*(-(4ac - b^2)^3)^{ \\ &(1/2)} + a^4b^5f^2 + 28a^4b^3c^2d^2 - 9a^3b^5c^2e^2 - 20a^5b^3c^2e^2 \\ &- 7a^5b^3c^2f^2 + 12a^6b^3c^2f^2 - a^5c^2f^2*(-(4ac - b^2)^3)^{(1/2)} \\ &- 2ab^8d^2e + 42a^2b^5c^2d^2 - 63a^3b^3c^3d^2 + a^2b^4e^2*(-(4 \\ &a^3c - b^2)^3)^{(1/2)} - a^3c^3d^2*(-(4ac - b^2)^3)^{(1/2)} + 25a^4b^3c^2 \\ &e^2 + a^4b^2f^2*(-(4ac - b^2)^3)^{(1/2)} + a^4c^2e^2*(-(4ac - b^2)^3 \\ &)^{(1/2)} - 11ab^7cd^2 + 2a^2b^7d^2f - 16a^5c^4d^2e - 2a^3b^6e^2f + \\ &16a^6c^3e^2f - 2ab^5d^2e^2*(-(4ac - b^2)^3)^{(1/2)} + 20a^2b^6c^2d^2e - \\ &18a^3b^5c^2d^2f - 40a^5b^3c^2d^2f + 16a^4b^4c^2e^2f + 6a^2b^2c^2d^2 \\ &2e^2*(-(4ac - b^2)^3)^{(1/2)} - 5ab^4c^2d^2e^2*(-(4ac - b^2)^3)^{(1/2)} - 66a^3 \\ &^3 \end{aligned}$$

$$\begin{aligned}
& *b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 2*a^2*b^4*d*f*(-(4*a*c - b^2)^3)^{(1/2)} \\
& + 50*a^4*b^3*c^2*d*f - 2*a^3*b^3*e*f*(-(4*a*c - b^2)^3)^{(1/2)} + 2*a^4*c^2*d \\
& *f*(-(4*a*c - b^2)^3)^{(1/2)} - 36*a^5*b^2*c^2*e*f - 3*a^3*b^2*c*e^2*(-(4*a*c \\
& - b^2)^3)^{(1/2)} + 4*a^4*b*c*e*f*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a^2*b^3*c*d*e \\
& *(- (4*a*c - b^2)^3)^{(1/2)} - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a^ \\
& 3*b^2*c*d*f*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(a^7*b^4 + 16*a^9*c^2 - 8*a^8*b^2* \\
& c)))^{(1/2)}*(x*(32*a^16*b*c^3 - 8*a^15*b^3*c^2)*(-(b^9*d^2 + a^2*b^7*e^2 + b \\
& ^6*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + a^4*b^5*f^2 + 28*a^4*b*c^4*d^2 - 9*a^3*b^ \\
& 5*c*e^2 - 20*a^5*b*c^3*e^2 - 7*a^5*b^3*c*f^2 + 12*a^6*b*c^2*f^2 - a^5*c*f^2 \\
& *(-(4*a*c - b^2)^3)^{(1/2)} - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c \\
& ^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - a^3*c^3*d^2*(-(4*a*c - b^2) \\
& ^3)^{(1/2)} + 25*a^4*b^3*c^2*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^3)^{(1/2)} + a^4 \\
& *c^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 11*a*b^7*c*d^2 + 2*a^2*b^7*d*f - 16*a^5 \\
& *c^4*d*e - 2*a^3*b^6*e*f + 16*a^6*c^3*e*f - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^{(1/2)} \\
& + 20*a^2*b^6*c*d*e - 18*a^3*b^5*c*d*f - 40*a^5*b*c^3*d*f + 16*a^4*b^4 \\
& *c*e*f + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 5*a*b^4*c*d^2*(-(4*a* \\
& c - b^2)^3)^{(1/2)} - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 2*a^2*b^4*d*f \\
& *(-(4*a*c - b^2)^3)^{(1/2)} + 50*a^4*b^3*c^2*d*f - 2*a^3*b^3*e*f*(-(4*a*c - b \\
& ^2)^3)^{(1/2)} + 2*a^4*c^2*d*f*(-(4*a*c - b^2)^3)^{(1/2)} - 36*a^5*b^2*c^2*e*f \\
& - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 4*a^4*b*c*e*f*(-(4*a*c - b^2)^ \\
& 3)^{(1/2)} + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a^3*b*c^2*d*e*(-(4* \\
& a*c - b^2)^3)^{(1/2)} - 6*a^3*b^2*c*d*f*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(a^7*b^4 \\
& + 16*a^9*c^2 - 8*a^8*b^2*c)))^{(1/2)} - 16*a^15*c^4*e + 4*a^12*b^5*c^2*d - 2 \\
& 4*a^13*b^3*c^3*d - 4*a^13*b^4*c^2*e + 20*a^14*b^2*c^3*e + 4*a^14*b^3*c^2*f \\
& + 32*a^14*b*c^4*d - 16*a^15*b*c^3*f)))*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(- \\
& (4*a*c - b^2)^3)^{(1/2)} + a^4*b^5*f^2 + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - \\
& 20*a^5*b*c^3*e^2 - 7*a^5*b^3*c*f^2 + 12*a^6*b*c^2*f^2 - a^5*c*f^2*(-(4*a*c \\
& - b^2)^3)^{(1/2)} - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + \\
& a^2*b^4*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^{(1/2)} \\
& + 25*a^4*b^3*c^2*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^3)^{(1/2)} + a^4*c^2*e^2* \\
& (- (4*a*c - b^2)^3)^{(1/2)} - 11*a*b^7*c*d^2 + 2*a^2*b^7*d*f - 16*a^5*c^4*d*e \\
& - 2*a^3*b^6*e*f + 16*a^6*c^3*e*f - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 2 \\
& 0*a^2*b^6*c*d*e - 18*a^3*b^5*c*d*f - 40*a^5*b*c^3*d*f + 16*a^4*b^4*c*e*f + \\
& 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^ \\
& 3)^{(1/2)} - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 2*a^2*b^4*d*f*(-(4*a*c \\
& - b^2)^3)^{(1/2)} + 50*a^4*b^3*c^2*d*f - 2*a^3*b^3*e*f*(-(4*a*c - b^2)^3)^{(1/2)} \\
& + 2*a^4*c^2*d*f*(-(4*a*c - b^2)^3)^{(1/2)} - 36*a^5*b^2*c^2*e*f - 3*a^3*b \\
& ^2*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 4*a^4*b*c*e*f*(-(4*a*c - b^2)^3)^{(1/2)} \\
& + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2 \\
&)^3)^{(1/2)} - 6*a^3*b^2*c*d*f*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(a^7*b^4 + 16*a^9 \\
& *c^2 - 8*a^8*b^2*c)))^{(1/2)} - (x*(4*a^13*c^5*e^2 - 4*a^12*c^6*d^2 - 4*a^14* \\
& c^4*f^2 + 2*a^9*b^6*c^3*d^2 - 12*a^10*b^4*c^4*d^2 + 18*a^11*b^2*c^5*d^2 + 2 \\
& *a^11*b^4*c^3*e^2 - 8*a^12*b^2*c^4*e^2 + 2*a^13*b^2*c^3*f^2 + 8*a^13*c^5*d* \\
& f - 20*a^12*b*c^5*d*e + 12*a^13*b*c^4*e*f - 4*a^10*b^5*c^3*d*e + 20*a^11*b^ \\
& 3*c^4*d*e + 4*a^11*b^4*c^3*d*f - 16*a^12*b^2*c^4*d*f - 4*a^12*b^3*c^3*e*f) \\
& - (-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + a^4*b^5*f^2 \\
& + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 7*a^5*b^3*c*f^2 \\
& + 12*a^6*b*c^2*f^2 - a^5*c*f^2*(-(4*a*c - b^2)^3)^{(1/2)} - 2*a*b^8*d*e + 42* \\
& a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^{(1/2)} \\
& - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 25*a^4*b^3*c^2*e^2 + a^4*b^2*f^2* \\
& (- (4*a*c - b^2)^3)^{(1/2)} + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 11*a*b^7* \\
& c*d^2 + 2*a^2*b^7*d*f - 16*a^5*c^4*d*e - 2*a^3*b^6*e*f + 16*a^6*c^3*e*f - 2 \\
& *a*b^5*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 20*a^2*b^6*c*d*e - 18*a^3*b^5*c*d*f - \\
& 40*a^5*b*c^3*d*f + 16*a^4*b^4*c*e*f + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3) \\
& ^{(1/2)} - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 66*a^3*b^4*c^2*d*e + 76*a \\
& ^4*b^2*c^3*d*e + 2*a^2*b^4*d*f*(-(4*a*c - b^2)^3)^{(1/2)} + 50*a^4*b^3*c^2*d* \\
& f - 2*a^3*b^3*e*f*(-(4*a*c - b^2)^3)^{(1/2)} + 2*a^4*c^2*d*f*(-(4*a*c - b^2)^ \\
& 3)^{(1/2)} - 36*a^5*b^2*c^2*e*f - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + \\
& 4*a^4*b*c*e*f*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{1}{2}} - 6a^3bc^2d \sqrt{-(4ac - b^2)^3} - 6a^3b^2cd \sqrt{-(4ac - b^2)^3} / (8(a^7b^4 + 16a^9c^2 - 8a^8b^2c)) \sqrt{\frac{1}{2}} * (x(32a^{16}b^3c^3 - 8a^{15}b^3c^2) * (-(b^9d^2 + a^2b^7e^2 + b^6d^2 * (-(4ac - b^2)^3)^{\frac{1}{2}} + a^4b^5f^2 + 28a^4b^3c^4d^2 - 9a^3b^5c^2e^2 - 20a^5b^3c^3e^2 - 7a^5b^3c^2f^2 + 12a^6b^3c^2f^2 - a^5c^2f^2 * (-(4ac - b^2)^3)^{\frac{1}{2}} - 2ab^8d^2e + 42a^2b^5c^2d^2 - 63a^3b^3c^3d^2 + a^2b^4e^2 * (-(4ac - b^2)^3)^{\frac{1}{2}} - a^3c^3d^2 * (-(4ac - b^2)^3)^{\frac{1}{2}} + 25a^4b^3c^2e^2 + a^4b^2f^2 * (-(4ac - b^2)^3)^{\frac{1}{2}} + a^4c^2e^2 * (-(4ac - b^2)^3)^{\frac{1}{2}} - 11ab^7cd^2 + 2a^2b^7df - 16a^5c^4d^2e - 2a^3b^6ef + 16a^6c^3ef - 2ab^5d^2e * (-(4ac - b^2)^3)^{\frac{1}{2}} + 20a^2b^6c^2d^2e - 18a^3b^5c^2d^2f - 40a^5b^3c^3d^2f + 16a^4b^4c^2ef + 6a^2b^2c^2d^2 * (-(4ac - b^2)^3)^{\frac{1}{2}} - 5ab^4cd^2 * (-(4ac - b^2)^3)^{\frac{1}{2}} - 66a^3b^4c^2d^2e + 76a^4b^2c^3d^2e + 2a^2b^4d^2f * (-(4ac - b^2)^3)^{\frac{1}{2}} + 50a^4b^3c^2d^2f - 2a^3b^3e^2f * (-(4ac - b^2)^3)^{\frac{1}{2}} + 2a^4c^2d^2f * (-(4ac - b^2)^3)^{\frac{1}{2}} - 36a^5b^2c^2e^2f - 3a^3b^2c^2e^2 * (-(4ac - b^2)^3)^{\frac{1}{2}} + 4a^4b^3c^2ef * (-(4ac - b^2)^3)^{\frac{1}{2}} + 8a^2b^3c^2d^2e * (-(4ac - b^2)^3)^{\frac{1}{2}} - 6a^3b^2c^2d^2e * (-(4ac - b^2)^3)^{\frac{1}{2}} - 6a^3b^2cd^2f * (-(4ac - b^2)^3)^{\frac{1}{2}}) / (8(a^7b^4 + 16a^9c^2 - 8a^8b^2c)) \sqrt{\frac{1}{2}} + 16a^{15}c^4e - 4a^{12}b^5c^2d + 24a^{13}b^3c^3d + 4a^{13}b^4c^2e - 20a^{14}b^2c^3e - 4a^{14}b^3c^2f - 32a^{14}b^3c^4d + 16a^{15}b^3c^3f) * (-(b^9d^2 + a^2b^7e^2 + b^6d^2 * (-(4ac - b^2)^3)^{\frac{1}{2}} + a^4b^5f^2 + 28a^4b^3c^4d^2 - 9a^3b^5c^2e^2 - 20a^5b^3c^3e^2 - 7a^5b^3c^2f^2 + 12a^6b^3c^2f^2 - a^5c^2f^2 * (-(4ac - b^2)^3)^{\frac{1}{2}} - 2ab^8d^2e + 42a^2b^5c^2d^2 - 63a^3b^3c^3d^2 + a^2b^4e^2 * (-(4ac - b^2)^3)^{\frac{1}{2}} - a^3c^3d^2 * (-(4ac - b^2)^3)^{\frac{1}{2}} + 25a^4b^3c^2e^2 + a^4b^2f^2 * (-(4ac - b^2)^3)^{\frac{1}{2}} + a^4c^2e^2 * (-(4ac - b^2)^3)^{\frac{1}{2}} - 11ab^7cd^2 + 2a^2b^7df - 16a^5c^4d^2e - 2a^3b^6ef + 16a^6c^3ef - 2ab^5d^2e * (-(4ac - b^2)^3)^{\frac{1}{2}} + 20a^2b^6c^2d^2e - 18a^3b^5c^2d^2f - 40a^5b^3c^3d^2f + 16a^4b^4c^2ef + 6a^2b^2c^2d^2 * (-(4ac - b^2)^3)^{\frac{1}{2}} - 5ab^4cd^2 * (-(4ac - b^2)^3)^{\frac{1}{2}} - 66a^3b^4c^2d^2e + 76a^4b^2c^3d^2e + 2a^2b^4d^2f * (-(4ac - b^2)^3)^{\frac{1}{2}} + 50a^4b^3c^2d^2f - 2a^3b^3e^2f * (-(4ac - b^2)^3)^{\frac{1}{2}} + 2a^4c^2d^2f * (-(4ac - b^2)^3)^{\frac{1}{2}} - 36a^5b^2c^2e^2f - 3a^3b^2c^2e^2 * (-(4ac - b^2)^3)^{\frac{1}{2}} + 4a^4b^3c^2ef * (-(4ac - b^2)^3)^{\frac{1}{2}} + 8a^2b^3c^2d^2e * (-(4ac - b^2)^3)^{\frac{1}{2}} - 6a^3b^2c^2d^2e * (-(4ac - b^2)^3)^{\frac{1}{2}} - 6a^3b^2cd^2f * (-(4ac - b^2)^3)^{\frac{1}{2}}) / (8(a^7b^4 + 16a^9c^2 - 8a^8b^2c)) \sqrt{\frac{1}{2}} - 2a^{10}c^7d^3 + 2a^{13}c^4f^3 - 2a^{11}b^3c^5e^3 - 2a^{11}c^6d^2e^2 + 6a^{11}c^6d^2f - 6a^{12}c^5d^2f^2 + 2a^{12}c^5e^2f + 2a^9b^2c^6d^3 - 4a^{12}b^3c^4ef^2 - 2a^9b^3c^5d^2e + 4a^{10}b^2c^5d^2e^2 + 2a^9b^4c^4d^2f - 6a^{10}b^2c^5d^2f + 4a^{11}b^2c^4d^2f^2 + 2a^{11}b^2c^4e^2f + 4a^{11}b^3c^5d^2ef - 4a^{10}b^3c^4d^2ef) * (-(b^9d^2 + a^2b^7e^2 + b^6d^2 * (-(4ac - b^2)^3)^{\frac{1}{2}} + a^4b^5f^2 + 28a^4b^3c^4d^2 - 9a^3b^5c^2e^2 - 20a^5b^3c^3e^2 - 7a^5b^3c^2f^2 + 12a^6b^3c^2f^2 - a^5c^2f^2 * (-(4ac - b^2)^3)^{\frac{1}{2}} - 2ab^8d^2e + 42a^2b^5c^2d^2 - 63a^3b^3c^3d^2 + a^2b^4e^2 * (-(4ac - b^2)^3)^{\frac{1}{2}} - a^3c^3d^2 * (-(4ac - b^2)^3)^{\frac{1}{2}} + 25a^4b^3c^2e^2 + a^4b^2f^2 * (-(4ac - b^2)^3)^{\frac{1}{2}} + a^4c^2e^2 * (-(4ac - b^2)^3)^{\frac{1}{2}} - 11ab^7cd^2 + 2a^2b^7df - 16a^5c^4d^2e - 2a^3b^6ef + 16a^6c^3ef - 2ab^5d^2e * (-(4ac - b^2)^3)^{\frac{1}{2}} + 20a^2b^6c^2d^2e - 18a^3b^5c^2d^2f - 40a^5b^3c^3d^2f + 16a^4b^4c^2ef + 6a^2b^2c^2d^2 * (-(4ac - b^2)^3)^{\frac{1}{2}} - 5ab^4cd^2 * (-(4ac - b^2)^3)^{\frac{1}{2}} - 66a^3b^4c^2d^2e + 76a^4b^2c^3d^2e + 2a^2b^4d^2f * (-(4ac - b^2)^3)^{\frac{1}{2}} + 50a^4b^3c^2d^2f - 2a^3b^3e^2f * (-(4ac - b^2)^3)^{\frac{1}{2}} + 2a^4c^2d^2f * (-(4ac - b^2)^3)^{\frac{1}{2}} - 36a^5b^2c^2e^2f - 3a^3b^2c^2e^2 * (-(4ac - b^2)^3)^{\frac{1}{2}} + 4a^4b^3c^2ef * (-(4ac - b^2)^3)^{\frac{1}{2}} + 8a^2b^3c^2d^2e * (-(4ac - b^2)^3)^{\frac{1}{2}} - 6a^3b^2c^2d^2e * (-(4ac - b^2)^3)^{\frac{1}{2}}) / (8(a^7b^4 + 16a^9c^2 - 8a^8b^2c)) \sqrt{\frac{1}{2}} * 2i - (d/(5a) + (x^4(b^2d + a^2f - abe - acd))/a^3 + (x^2(ae - bd))/(3a^2))/x^5 + \operatorname{atan}((x(4a^{13}c^5e^2 - 4a^{12}c^6d^2 - 4a^{14}c^4f^2 + 2a^9b^6c^3d^2 -
\end{aligned}$$

$$\begin{aligned}
&12a^{10}b^4c^4d^2 + 18a^{11}b^2c^5d^2 + 2a^{11}b^4c^3e^2 - 8a^{12}b^2 \\
&c^4e^2 + 2a^{13}b^2c^3f^2 + 8a^{13}c^5d^2f - 20a^{12}b^2c^5d^2e + 12a^{13} \\
&b^2c^4e^2f - 4a^{10}b^5c^3d^2e + 20a^{11}b^3c^4d^2e + 4a^{11}b^4c^3d^2f \\
&- 16a^{12}b^2c^4d^2f - 4a^{12}b^3c^3e^2f) - ((b^9d^2 + a^2b^7e^2 - b \\
&^6d^2(-4ac - b^2)^3)^{1/2} + a^4b^5f^2 + 28a^4b^2c^4d^2 - 9a^3b^5 \\
&c^2e^2 - 20a^5b^2c^3e^2 - 7a^5b^3c^2f^2 + 12a^6b^2c^2f^2 + a^5c^2f^2 \\
&(-4ac - b^2)^3)^{1/2} - 2ab^8d^2e + 42a^2b^5c^2d^2 - 63a^3b^3c^3 \\
&d^2 - a^2b^4e^2(-4ac - b^2)^3)^{1/2} + a^3c^3d^2(-4ac - b^2)^3)^{1/2} \\
&+ 25a^4b^3c^2e^2 - a^4b^2f^2(-4ac - b^2)^3)^{1/2} - a^4 \\
&c^2e^2(-4ac - b^2)^3)^{1/2} - 11ab^7c^2d^2 + 2a^2b^7d^2f - 16a^5 \\
&c^4d^2e - 2a^3b^6e^2f + 16a^6c^3e^2f + 2ab^5d^2e(-4ac - b^2)^3)^{1/2} \\
&+ 20a^2b^6c^2d^2e - 18a^3b^5c^2d^2f - 40a^5b^2c^3d^2f + 16a^4b^4 \\
&c^2e^2f - 6a^2b^2c^2d^2(-4ac - b^2)^3)^{1/2} + 5ab^4c^2d^2(-4ac \\
&- b^2)^3)^{1/2} - 66a^3b^4c^2d^2e + 76a^4b^2c^3d^2e - 2a^2b^4d^2f \\
&(-4ac - b^2)^3)^{1/2} + 50a^4b^3c^2d^2f + 2a^3b^3e^2f(-4ac - b^2)^3)^{1/2} \\
&- 2a^4c^2d^2f(-4ac - b^2)^3)^{1/2} - 36a^5b^2c^2e^2f \\
&+ 3a^3b^2c^2e^2(-4ac - b^2)^3)^{1/2} - 4a^4b^2c^2e^2f(-4ac - b^2)^3)^{1/2} \\
&- 8a^2b^3c^2d^2e(-4ac - b^2)^3)^{1/2} + 6a^3b^2c^2d^2e(-4ac - b^2)^3)^{1/2} \\
&+ 6a^3b^2c^2d^2f(-4ac - b^2)^3)^{1/2})/(8(a^7b^4 + 16a^9c^2 - 8a^8b^2c))^{1/2} \\
&+ (x(32a^{16}b^3c^3 - 8a^{15}b^3c^2)(b^9d^2 + a^2b^7e^2 - b^6d^2(-4ac - b^2)^3)^{1/2} \\
&+ a^4b^5f^2 + 28a^4b^2c^4d^2 - 9a^3b^5c^2e^2 - 20a^5b^2c^3e^2 - 7a^5b^3c^2f^2 \\
&+ 12a^6b^2c^2f^2 + a^5c^2f^2(-4ac - b^2)^3)^{1/2} - 2ab^8d^2e + 42a^2 \\
&b^5c^2d^2 - 63a^3b^3c^3d^2 - a^2b^4e^2(-4ac - b^2)^3)^{1/2} + a^3c^3d^2 \\
&(-4ac - b^2)^3)^{1/2} + 25a^4b^3c^2e^2 - a^4b^2f^2(-4ac - b^2)^3)^{1/2} \\
&- a^4c^2e^2(-4ac - b^2)^3)^{1/2} - 11ab^7c^2d^2 + 2a^2b^7d^2f - 16a^5 \\
&c^4d^2e - 2a^3b^6e^2f + 16a^6c^3e^2f + 2ab^5d^2e(-4ac - b^2)^3)^{1/2} \\
&+ 20a^2b^6c^2d^2e - 18a^3b^5c^2d^2f - 40a^5b^2c^3d^2f + 16a^4b^4 \\
&c^2e^2f - 6a^2b^2c^2d^2(-4ac - b^2)^3)^{1/2} + 5ab^4c^2d^2(-4ac - b^2)^3)^{1/2} \\
&- 66a^3b^4c^2d^2e + 76a^4b^2c^3d^2e - 2a^2b^4d^2f(-4ac - b^2)^3)^{1/2} \\
&+ 50a^4b^3c^2d^2f + 2a^3b^3e^2f(-4ac - b^2)^3)^{1/2} - 2a^4c^2d^2f(-4ac - b^2)^3)^{1/2} \\
&- 36a^5b^2c^2e^2f + 3a^3b^2c^2e^2(-4ac - b^2)^3)^{1/2} - 4a^4 \\
&b^2c^2e^2f(-4ac - b^2)^3)^{1/2} - 8a^2b^3c^2d^2e(-4ac - b^2)^3)^{1/2} \\
&+ 6a^3b^2c^2d^2e(-4ac - b^2)^3)^{1/2} + 6a^3b^2c^2d^2f(-4ac - b^2)^3)^{1/2}) \\
&/(8(a^7b^4 + 16a^9c^2 - 8a^8b^2c))^{1/2} - 16a^{15}c^4e + 4a^{12}b^5c^2d \\
&- 24a^{13}b^3c^3d - 4a^{13}b^4c^2e + 20a^{14}b^2c^3e + 4a^{14}b^3c^2f \\
&+ 32a^{14}b^2c^4d - 16a^{15}b^2c^3f) - ((b^9d^2 + a^2b^7e^2 - b^6d^2(-4ac - b^2)^3)^{1/2} \\
&+ a^4b^5f^2 + 28a^4b^2c^4d^2 - 9a^3b^5c^2e^2 - 20a^5b^2c^3e^2 - 7a^5b^3c^2f^2 \\
&+ 12a^6b^2c^2f^2 + a^5c^2f^2(-4ac - b^2)^3)^{1/2} - 2ab^8d^2e + 42a^2 \\
&b^5c^2d^2 - 63a^3b^3c^3d^2 - a^2b^4e^2(-4ac - b^2)^3)^{1/2} + a^3c^3d^2 \\
&(-4ac - b^2)^3)^{1/2} + 25a^4b^3c^2e^2 - a^4b^2f^2(-4ac - b^2)^3)^{1/2} \\
&- a^4c^2e^2(-4ac - b^2)^3)^{1/2} - 11ab^7c^2d^2 + 2a^2 \\
&b^7d^2f - 16a^5c^4d^2e - 2a^3b^6e^2f + 16a^6c^3e^2f + 2ab^5d^2e(-4ac - b^2)^3)^{1/2} \\
&+ 20a^2b^6c^2d^2e - 18a^3b^5c^2d^2f - 40a^5b^2c^3 \\
&d^2f + 16a^4b^4c^2e^2f - 6a^2b^2c^2d^2(-4ac - b^2)^3)^{1/2} + 5ab^4 \\
&c^2d^2(-4ac - b^2)^3)^{1/2} - 66a^3b^4c^2d^2e + 76a^4b^2c^3d^2 \\
&e - 2a^2b^4d^2f(-4ac - b^2)^3)^{1/2} + 50a^4b^3c^2d^2f + 2a^3b^3 \\
&e^2f(-4ac - b^2)^3)^{1/2} - 2a^4c^2d^2f(-4ac - b^2)^3)^{1/2} - 36 \\
&a^5b^2c^2e^2f + 3a^3b^2c^2e^2(-4ac - b^2)^3)^{1/2} - 4a^4b^2c^2e^2 \\
&f(-4ac - b^2)^3)^{1/2} - 8a^2b^3c^2d^2e(-4ac - b^2)^3)^{1/2} + 6a^3 \\
&b^2c^2d^2e(-4ac - b^2)^3)^{1/2} + 6a^3b^2c^2d^2f(-4ac - b^2)^3)^{1/2}) \\
&/(8(a^7b^4 + 16a^9c^2 - 8a^8b^2c))^{1/2} * i + (x(4a^{13}c^5e^2 - 4a^{12}c^6d^2 \\
&- 4a^{14}c^4f^2 + 2a^9b^6c^3d^2 - 12a^{10}b^4c^4d^2 + 18a^{11}b^2c^5d^2 \\
&+ 2a^{11}b^4c^3e^2 - 8a^{12}b^2c^4e^2 + 2a^{13}b^2c^3f^2 + 8a^{13}c^5d^2f \\
&- 20a^{12}b^2c^5d^2e + 12a^{13}b^2c^4e^2f - 4a^{10}b^5c^3d^2e + 20a^{11}b^3c^4d^2e \\
&+ 4a^{11}b^4c^3d^2f - 16a^{12}b^2c^4d^2f - 4a^{12}b^3c^3e^2f) - ((b^9d^2 + a^2b^7e^2 - b^6d^2(-4ac
\end{aligned}$$

$$\begin{aligned}
& - b^2)^3)^{(1/2)} + a^4 b^5 f^2 + 28 a^4 b^3 c^4 d^2 - 9 a^3 b^5 c^3 e^2 - 20 a^5 \\
& * b^3 c^3 e^2 - 7 a^5 b^3 c^3 f^2 + 12 a^6 b^3 c^2 f^2 + a^5 c^3 f^2 * (- (4 a^3 c - b^2) \\
& ^3)^{(1/2)} - 2 a^4 b^8 d^2 e + 42 a^2 b^5 c^2 d^2 - 63 a^3 b^3 c^3 d^2 - a^2 b^4 \\
& * e^2 * (- (4 a^3 c - b^2)^3)^{(1/2)} + a^3 c^3 d^2 * (- (4 a^3 c - b^2)^3)^{(1/2)} + 25 a \\
& ^4 b^3 c^2 e^2 - a^4 b^2 f^2 * (- (4 a^3 c - b^2)^3)^{(1/2)} - a^4 c^2 e^2 * (- (4 a^3 \\
& c - b^2)^3)^{(1/2)} - 11 a^4 b^7 c^4 d^2 + 2 a^2 b^7 d^2 f - 16 a^5 c^4 d^2 e - 2 a^3 \\
& * b^6 e^2 f + 16 a^6 c^3 e^2 f + 2 a^4 b^5 d^2 e * (- (4 a^3 c - b^2)^3)^{(1/2)} + 20 a^2 b \\
& ^6 c^3 d^2 e - 18 a^3 b^5 c^3 d^2 f - 40 a^5 b^3 c^3 d^2 f + 16 a^4 b^4 c^3 e^2 f - 6 a^2 b \\
& ^2 c^2 d^2 * (- (4 a^3 c - b^2)^3)^{(1/2)} + 5 a^4 b^4 c^3 d^2 * (- (4 a^3 c - b^2)^3)^{(1/2)} \\
&) - 66 a^3 b^4 c^2 d^2 e + 76 a^4 b^2 c^3 d^2 e - 2 a^2 b^4 d^2 f * (- (4 a^3 c - b^2) \\
& ^3)^{(1/2)} + 50 a^4 b^3 c^2 d^2 f + 2 a^3 b^3 e^2 f * (- (4 a^3 c - b^2)^3)^{(1/2)} - 2 \\
& * a^4 c^2 d^2 f * (- (4 a^3 c - b^2)^3)^{(1/2)} - 36 a^5 b^2 c^2 e^2 f + 3 a^3 b^2 c^2 e^2 \\
& * (- (4 a^3 c - b^2)^3)^{(1/2)} - 4 a^4 b^3 c^2 e^2 f * (- (4 a^3 c - b^2)^3)^{(1/2)} - 8 a^2 \\
& * b^3 c^2 d^2 e * (- (4 a^3 c - b^2)^3)^{(1/2)} + 6 a^3 b^3 c^2 d^2 e * (- (4 a^3 c - b^2)^3)^{(1 \\
& /2)} + 6 a^3 b^2 c^2 d^2 f * (- (4 a^3 c - b^2)^3)^{(1/2)) / (8 * (a^7 b^4 + 16 a^9 c^2 - \\
& 8 a^8 b^2 c))^{(1/2)} * (x * (32 a^16 b^3 c^3 - 8 a^15 b^3 c^2) * (- (b^9 d^2 + a^2 b \\
& ^7 e^2 - b^6 d^2 * (- (4 a^3 c - b^2)^3)^{(1/2)} + a^4 b^5 f^2 + 28 a^4 b^3 c^4 d^2 \\
& - 9 a^3 b^5 c^3 e^2 - 20 a^5 b^3 c^3 e^2 - 7 a^5 b^3 c^3 f^2 + 12 a^6 b^3 c^2 f^2 + \\
& a^5 c^3 f^2 * (- (4 a^3 c - b^2)^3)^{(1/2)} - 2 a^4 b^8 d^2 e + 42 a^2 b^5 c^2 d^2 - 63 \\
& * a^3 b^3 c^3 d^2 - a^2 b^4 e^2 * (- (4 a^3 c - b^2)^3)^{(1/2)} + a^3 c^3 d^2 * (- (4 a \\
& a^3 c - b^2)^3)^{(1/2)} + 25 a^4 b^3 c^2 e^2 - a^4 b^2 f^2 * (- (4 a^3 c - b^2)^3)^{(\\
& 1/2)} - a^4 c^2 e^2 * (- (4 a^3 c - b^2)^3)^{(1/2)} - 11 a^4 b^7 c^4 d^2 + 2 a^2 b^7 d^2 \\
& f - 16 a^5 c^4 d^2 e - 2 a^3 b^6 e^2 f + 16 a^6 c^3 e^2 f + 2 a^4 b^5 d^2 e * (- (4 a^3 c \\
& - b^2)^3)^{(1/2)} + 20 a^2 b^6 c^3 d^2 e - 18 a^3 b^5 c^3 d^2 f - 40 a^5 b^3 c^3 d^2 f + \\
& 16 a^4 b^4 c^3 e^2 f - 6 a^2 b^2 c^2 d^2 * (- (4 a^3 c - b^2)^3)^{(1/2)} + 5 a^4 b^4 c^3 d \\
& ^2 * (- (4 a^3 c - b^2)^3)^{(1/2)} - 66 a^3 b^4 c^2 d^2 e + 76 a^4 b^2 c^3 d^2 e - 2 a \\
& ^2 b^4 d^2 f * (- (4 a^3 c - b^2)^3)^{(1/2)} + 50 a^4 b^3 c^2 d^2 f + 2 a^3 b^3 e^2 f * (- \\
& (4 a^3 c - b^2)^3)^{(1/2)} - 2 a^4 c^2 d^2 f * (- (4 a^3 c - b^2)^3)^{(1/2)} - 36 a^5 b^ \\
& 2 c^2 e^2 f + 3 a^3 b^2 c^2 e^2 * (- (4 a^3 c - b^2)^3)^{(1/2)} - 4 a^4 b^3 c^2 e^2 f * (- (4 a \\
& * c - b^2)^3)^{(1/2)} - 8 a^2 b^3 c^2 d^2 e * (- (4 a^3 c - b^2)^3)^{(1/2)} + 6 a^3 b^3 c^2 \\
& * d^2 e * (- (4 a^3 c - b^2)^3)^{(1/2)} + 6 a^3 b^2 c^2 d^2 f * (- (4 a^3 c - b^2)^3)^{(1/2)) / (\\
& 8 * (a^7 b^4 + 16 a^9 c^2 - 8 a^8 b^2 c))^{(1/2)} + 16 a^15 c^4 e - 4 a^12 b^5 \\
& * c^2 d + 24 a^13 b^3 c^3 d + 4 a^13 b^4 c^2 e - 20 a^14 b^2 c^3 e - 4 a^14 * \\
& b^3 c^2 f - 32 a^14 b^3 c^4 d + 16 a^15 b^3 c^3 f) * (- (b^9 d^2 + a^2 b^7 e^2 - \\
& b^6 d^2 * (- (4 a^3 c - b^2)^3)^{(1/2)} + a^4 b^5 f^2 + 28 a^4 b^3 c^4 d^2 - 9 a^3 b \\
& ^5 c^3 e^2 - 20 a^5 b^3 c^3 e^2 - 7 a^5 b^3 c^3 f^2 + 12 a^6 b^3 c^2 f^2 + a^5 c^3 f^2 \\
& * (- (4 a^3 c - b^2)^3)^{(1/2)} - 2 a^4 b^8 d^2 e + 42 a^2 b^5 c^2 d^2 - 63 a^3 b^3 c^3 \\
& d^2 - a^2 b^4 e^2 * (- (4 a^3 c - b^2)^3)^{(1/2)} + a^3 c^3 d^2 * (- (4 a^3 c - b^2) \\
& ^3)^{(1/2)} + 25 a^4 b^3 c^2 e^2 - a^4 b^2 f^2 * (- (4 a^3 c - b^2)^3)^{(1/2)} - a^ \\
& 4 c^2 e^2 * (- (4 a^3 c - b^2)^3)^{(1/2)} - 11 a^4 b^7 c^4 d^2 + 2 a^2 b^7 d^2 f - 16 a^ \\
& 5 c^4 d^2 e - 2 a^3 b^6 e^2 f + 16 a^6 c^3 e^2 f + 2 a^4 b^5 d^2 e * (- (4 a^3 c - b^2)^3) \\
& ^{(1/2)} + 20 a^2 b^6 c^3 d^2 e - 18 a^3 b^5 c^3 d^2 f - 40 a^5 b^3 c^3 d^2 f + 16 a^4 b^ \\
& 4 c^3 e^2 f - 6 a^2 b^2 c^2 d^2 * (- (4 a^3 c - b^2)^3)^{(1/2)} + 5 a^4 b^4 c^3 d^2 * (- (4 a \\
& * c - b^2)^3)^{(1/2)} - 66 a^3 b^4 c^2 d^2 e + 76 a^4 b^2 c^3 d^2 e - 2 a^2 b^4 d^2 \\
& f * (- (4 a^3 c - b^2)^3)^{(1/2)} + 50 a^4 b^3 c^2 d^2 f + 2 a^3 b^3 e^2 f * (- (4 a^3 c - \\
& b^2)^3)^{(1/2)} - 2 a^4 c^2 d^2 f * (- (4 a^3 c - b^2)^3)^{(1/2)} - 36 a^5 b^2 c^2 e^2 f \\
& + 3 a^3 b^2 c^2 e^2 * (- (4 a^3 c - b^2)^3)^{(1/2)} - 4 a^4 b^3 c^2 e^2 f * (- (4 a^3 c - b^2) \\
& ^3)^{(1/2)} - 8 a^2 b^3 c^2 d^2 e * (- (4 a^3 c - b^2)^3)^{(1/2)} + 6 a^3 b^3 c^2 d^2 e * (- (4 \\
& * a^3 c - b^2)^3)^{(1/2)} + 6 a^3 b^2 c^2 d^2 f * (- (4 a^3 c - b^2)^3)^{(1/2)) / (8 * (a^7 b^ \\
& 4 + 16 a^9 c^2 - 8 a^8 b^2 c))^{(1/2)} * i) / ((x * (4 a^13 c^5 e^2 - 4 a^12 c^6 * \\
& d^2 - 4 a^14 c^4 f^2 + 2 a^9 b^6 c^3 d^2 - 12 a^10 b^4 c^4 d^2 + 18 a^11 b^ \\
& 2 c^5 d^2 + 2 a^11 b^4 c^3 e^2 - 8 a^12 b^2 c^4 e^2 + 2 a^13 b^2 c^3 f^2 + \\
& 8 a^13 c^5 d^2 f - 20 a^12 b^3 c^5 d^2 e + 12 a^13 b^3 c^4 e^2 f - 4 a^10 b^5 c^3 d^2 e \\
& + 20 a^11 b^3 c^4 d^2 e + 4 a^11 b^4 c^3 d^2 f - 16 a^12 b^2 c^4 d^2 f - 4 a^12 * \\
& b^3 c^3 e^2 f) - (- (b^9 d^2 + a^2 b^7 e^2 - b^6 d^2 * (- (4 a^3 c - b^2)^3)^{(1/2)} \\
& + a^4 b^5 f^2 + 28 a^4 b^3 c^4 d^2 - 9 a^3 b^5 c^3 e^2 - 20 a^5 b^3 c^3 e^2 - 7 a \\
& ^5 b^3 c^3 f^2 + 12 a^6 b^3 c^2 f^2 + a^5 c^3 f^2 * (- (4 a^3 c - b^2)^3)^{(1/2)} - 2 a^4 \\
& b^8 d^2 e + 42 a^2 b^5 c^2 d^2 - 63 a^3 b^3 c^3 d^2 - a^2 b^4 e^2 * (- (4 a^3 c - \\
& b^2)^3)^{(1/2)} + a^3 c^3 d^2 * (- (4 a^3 c - b^2)^3)^{(1/2)} + 25 a^4 b^3 c^2 e^2 -
\end{aligned}$$

$$\begin{aligned}
& a^4 b^2 f^2 (-4ac - b^2)^3^{(1/2)} - a^4 c^2 e^2 (-4ac - b^2)^3^{(1/2)} \\
& - 11 a^3 b^7 c^2 d^2 + 2 a^2 b^7 d^2 f - 16 a^5 c^4 d^2 e - 2 a^3 b^6 e^2 f + 16 a^6 c^3 e^2 f \\
& + 2 a^2 b^5 d^2 e (-4ac - b^2)^3^{(1/2)} + 20 a^2 b^6 c^2 d^2 e - 18 a^3 b^5 c^2 d^2 f \\
& - 40 a^5 b^3 c^3 d^2 f + 16 a^4 b^4 c^2 e^2 f - 6 a^2 b^2 c^2 d^2 (-4ac - b^2)^3^{(1/2)} \\
& + 5 a^2 b^4 c^2 d^2 (-4ac - b^2)^3^{(1/2)} - 66 a^3 b^4 c^2 d^2 e + 76 a^4 b^2 c^3 d^2 e \\
& - 2 a^2 b^4 d^2 f (-4ac - b^2)^3^{(1/2)} + 50 a^4 b^3 c^2 d^2 f + 2 a^3 b^3 e^2 f (-4ac - b^2)^3^{(1/2)} \\
& - 2 a^4 c^2 d^2 f (-4ac - b^2)^3^{(1/2)} - 36 a^5 b^2 c^2 e^2 f + 3 a^3 b^2 c^2 e^2 (-4ac - b^2)^3^{(1/2)} \\
& - 4 a^4 b^2 c^2 e^2 f (-4ac - b^2)^3^{(1/2)} - 8 a^2 b^3 c^2 d^2 e (-4ac - b^2)^3^{(1/2)} \\
& + 6 a^3 b^2 c^2 d^2 e (-4ac - b^2)^3^{(1/2)} + 6 a^3 b^2 c^2 d^2 f (-4ac - b^2)^3^{(1/2)} \\
& / (8 (a^7 b^4 + 16 a^9 c^2 - 8 a^8 b^2 c))^{(1/2)} * (x^2 (32 a^16 b^3 c^3 - 8 a^15 b^3 c^2) * (-b^9 d^2 + a^2 b^7 e^2 - b^6 d^2) \\
& * (-4ac - b^2)^3^{(1/2)} + a^4 b^5 f^2 + 28 a^4 b^3 c^4 d^2 - 9 a^3 b^5 c^2 e^2 - 20 a^5 b^3 c^3 e^2 \\
& - 7 a^5 b^3 c^3 f^2 + 12 a^6 b^3 c^2 f^2 + a^5 c^3 f^2 (-4ac - b^2)^3^{(1/2)} - 2 a^2 b^8 d^2 e \\
& + 42 a^2 b^5 c^2 d^2 - 63 a^3 b^3 c^3 d^2 - a^2 b^4 e^2 (-4ac - b^2)^3^{(1/2)} + a^3 c^3 d^2 (-4ac - b^2)^3^{(1/2)} \\
& + 25 a^4 b^3 c^2 e^2 - a^4 b^2 f^2 (-4ac - b^2)^3^{(1/2)} - a^4 c^2 e^2 (-4ac - b^2)^3^{(1/2)} \\
& - 11 a^3 b^7 c^2 d^2 + 2 a^2 b^7 d^2 f - 16 a^5 c^4 d^2 e - 2 a^3 b^6 e^2 f + 16 a^6 c^3 e^2 f \\
& + 2 a^2 b^5 d^2 e (-4ac - b^2)^3^{(1/2)} + 20 a^2 b^6 c^2 d^2 e - 18 a^3 b^5 c^2 d^2 f \\
& - 40 a^5 b^3 c^3 d^2 f + 16 a^4 b^4 c^2 e^2 f - 6 a^2 b^2 c^2 d^2 (-4ac - b^2)^3^{(1/2)} \\
& + 5 a^2 b^4 c^2 d^2 (-4ac - b^2)^3^{(1/2)} - 66 a^3 b^4 c^2 d^2 e + 76 a^4 b^2 c^3 d^2 e \\
& - 2 a^2 b^4 d^2 f (-4ac - b^2)^3^{(1/2)} + 50 a^4 b^3 c^2 d^2 f + 2 a^3 b^3 e^2 f (-4ac - b^2)^3^{(1/2)} \\
& - 2 a^4 c^2 d^2 f (-4ac - b^2)^3^{(1/2)} - 36 a^5 b^2 c^2 e^2 f + 3 a^3 b^2 c^2 e^2 (-4ac - b^2)^3^{(1/2)} \\
& - 4 a^4 b^2 c^2 e^2 f (-4ac - b^2)^3^{(1/2)} - 8 a^2 b^3 c^2 d^2 e (-4ac - b^2)^3^{(1/2)} \\
& + 6 a^3 b^2 c^2 d^2 e (-4ac - b^2)^3^{(1/2)} + 6 a^3 b^2 c^2 d^2 f (-4ac - b^2)^3^{(1/2)} \\
& / (8 (a^7 b^4 + 16 a^9 c^2 - 8 a^8 b^2 c))^{(1/2)} - 16 a^15 c^4 e + 4 a^12 b^5 c^2 d - 24 a^13 b^3 c^3 d \\
& - 4 a^13 b^4 c^2 e + 20 a^14 b^2 c^3 e + 4 a^14 b^3 c^2 f + 32 a^14 b^3 c^4 d - 16 a^15 b^3 c^3 f) \\
& * (-b^9 d^2 + a^2 b^7 e^2 - b^6 d^2 (-4ac - b^2)^3^{(1/2)} + a^4 b^5 f^2 + 28 a^4 b^3 c^4 d^2 \\
& - 9 a^3 b^5 c^2 e^2 - 20 a^5 b^3 c^3 e^2 - 7 a^5 b^3 c^3 f^2 + 12 a^6 b^3 c^2 f^2 + a^5 c^3 f^2 (-4ac - b^2)^3^{(1/2)} \\
& - 2 a^2 b^8 d^2 e + 42 a^2 b^5 c^2 d^2 - 63 a^3 b^3 c^3 d^2 - a^2 b^4 e^2 (-4ac - b^2)^3^{(1/2)} \\
& + a^3 c^3 d^2 (-4ac - b^2)^3^{(1/2)} + 25 a^4 b^3 c^2 e^2 - a^4 b^2 f^2 (-4ac - b^2)^3^{(1/2)} \\
& - a^4 c^2 e^2 (-4ac - b^2)^3^{(1/2)} - 11 a^3 b^7 c^2 d^2 + 2 a^2 b^7 d^2 f - 16 a^5 c^4 d^2 e \\
& - 2 a^3 b^6 e^2 f + 16 a^6 c^3 e^2 f + 2 a^2 b^5 d^2 e (-4ac - b^2)^3^{(1/2)} + 20 a^2 b^6 c^2 d^2 e \\
& - 18 a^3 b^5 c^2 d^2 f - 40 a^5 b^3 c^3 d^2 f + 16 a^4 b^4 c^2 e^2 f - 6 a^2 b^2 c^2 d^2 (-4ac - b^2)^3^{(1/2)} \\
& + 5 a^2 b^4 c^2 d^2 (-4ac - b^2)^3^{(1/2)} - 66 a^3 b^4 c^2 d^2 e + 76 a^4 b^2 c^3 d^2 e \\
& - 2 a^2 b^4 d^2 f (-4ac - b^2)^3^{(1/2)} + 50 a^4 b^3 c^2 d^2 f + 2 a^3 b^3 e^2 f (-4ac - b^2)^3^{(1/2)} \\
& - 2 a^4 c^2 d^2 f (-4ac - b^2)^3^{(1/2)} - 36 a^5 b^2 c^2 e^2 f + 3 a^3 b^2 c^2 e^2 (-4ac - b^2)^3^{(1/2)} \\
& - 4 a^4 b^2 c^2 e^2 f (-4ac - b^2)^3^{(1/2)} - 8 a^2 b^3 c^2 d^2 e (-4ac - b^2)^3^{(1/2)} \\
& + 6 a^3 b^2 c^2 d^2 e (-4ac - b^2)^3^{(1/2)} + 6 a^3 b^2 c^2 d^2 f (-4ac - b^2)^3^{(1/2)} \\
& / (8 (a^7 b^4 + 16 a^9 c^2 - 8 a^8 b^2 c))^{(1/2)} - (x^2 (4 a^13 c^5 e^2 - 4 a^12 c^6 d^2 - 4 a^14 c^4 f^2 \\
& + 2 a^9 b^6 c^3 d^2 - 12 a^10 b^4 c^4 d^2 + 18 a^11 b^2 c^5 d^2 + 2 a^11 b^4 c^3 e^2 \\
& - 8 a^12 b^2 c^4 e^2 + 2 a^13 b^2 c^3 f^2 + 8 a^13 c^5 d^2 f - 20 a^12 b^3 c^5 d^2 e \\
& + 12 a^13 b^3 c^4 e^2 f - 4 a^10 b^5 c^3 d^2 e + 20 a^11 b^3 c^4 d^2 e + 4 a^11 b^4 c^3 d^2 f \\
& - 16 a^12 b^2 c^4 d^2 f - 4 a^12 b^3 c^3 e^2 f) - (-b^9 d^2 + a^2 b^7 e^2 - b^6 d^2 (-4ac - b^2)^3^{(1/2)} \\
& + a^4 b^5 f^2 + 28 a^4 b^3 c^4 d^2 - 9 a^3 b^5 c^2 e^2 - 20 a^5 b^3 c^3 e^2 - 7 a^5 b^3 c^3 f^2 \\
& + 12 a^6 b^3 c^2 f^2 + a^5 c^3 f^2 (-4ac - b^2)^3^{(1/2)} - 2 a^2 b^8 d^2 e + 42 a^2 b^5 c^2 d^2 \\
& - 63 a^3 b^3 c^3 d^2 - a^2 b^4 e^2 (-4ac - b^2)^3^{(1/2)} + a^3 c^3 d^2 (-4ac - b^2)^3^{(1/2)} \\
& + 25 a^4 b^3 c^2 e^2 - a^4 b^2 f^2 (-4ac - b^2)^3^{(1/2)} - a^4 c^2 e^2 (-4ac - b^2)^3^{(1/2)} \\
& - 11 a^3 b^7 c^2 d^2 + 2 a^2 b^7 d^2 f - 16 a^5 c^4 d^2 e - 2 a^3 b^6 e^2 f + 16 a^6 c^3 e^2 f \\
& + 2 a^2 b^5 d^2 e (-4ac - b^2)^3^{(1/2)} + 20 a^2 b^6 c^2 d^2 e - 18 a^3 b^5 c^2 d^2 f - 40 a^5 b^3 c^3 d^2 f \\
& + 16 a^4 b^4 c^2 e^2 f - 6 a^2 b^2 c^2 d^2 (-4ac - b^2)^3^{(1/2)}
\end{aligned}$$

$$\begin{aligned}
& + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2 \\
& *c^3*d*e - 2*a^2*b^4*d*f*(-(4*a*c - b^2)^3)^{(1/2)} + 50*a^4*b^3*c^2*d*f + 2* \\
& a^3*b^3*e*f*(-(4*a*c - b^2)^3)^{(1/2)} - 2*a^4*c^2*d*f*(-(4*a*c - b^2)^3)^{(1/2)} \\
& - 36*a^5*b^2*c^2*e*f + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a^4* \\
& b*c*e*f*(-(4*a*c - b^2)^3)^{(1/2)} - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)} \\
& + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 6*a^3*b^2*c*d*f*(-(4*a*c - b^2)^3)^{(1/2)} \\
&)/(8*(a^7*b^4 + 16*a^9*c^2 - 8*a^8*b^2*c))^{(1/2)}*(x*(32*a^16*b* \\
& c^3 - 8*a^15*b^3*c^2)*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^{(1/2)} \\
&)^{(1/2)} + a^4*b^5*f^2 + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 \\
& - 7*a^5*b^3*c*f^2 + 12*a^6*b*c^2*f^2 + a^5*c*f^2*(-(4*a*c - b^2)^3)^{(1/2)} \\
& - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4 \\
& *a*c - b^2)^3)^{(1/2)} + a^3*c^3*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 25*a^4*b^3*c^2 \\
& *e^2 - a^4*b^2*f^2*(-(4*a*c - b^2)^3)^{(1/2)} - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} \\
& - 11*a*b^7*c*d^2 + 2*a^2*b^7*d*f - 16*a^5*c^4*d*e - 2*a^3*b^6*e*f \\
& + 16*a^6*c^3*e*f + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 20*a^2*b^6*c*d*e \\
& - 18*a^3*b^5*c*d*f - 40*a^5*b*c^3*d*f + 16*a^4*b^4*c*e*f - 6*a^2*b^2*c^2*d^2 \\
& *(-(4*a*c - b^2)^3)^{(1/2)} + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 66*a^3 \\
& *b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 2*a^2*b^4*d*f*(-(4*a*c - b^2)^3)^{(1/2)} \\
& + 50*a^4*b^3*c^2*d*f + 2*a^3*b^3*e*f*(-(4*a*c - b^2)^3)^{(1/2)} - 2*a^4*c^2* \\
& d*f*(-(4*a*c - b^2)^3)^{(1/2)} - 36*a^5*b^2*c^2*e*f + 3*a^3*b^2*c*e^2*(-(4*a* \\
& c - b^2)^3)^{(1/2)} - 4*a^4*b*c*e*f*(-(4*a*c - b^2)^3)^{(1/2)} - 8*a^2*b^3*c*d* \\
& e*(-(4*a*c - b^2)^3)^{(1/2)} + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 6*a^3 \\
& *b^2*c*d*f*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(a^7*b^4 + 16*a^9*c^2 - 8*a^8*b^2 \\
& *c))^{(1/2)} + 16*a^15*c^4*e - 4*a^12*b^5*c^2*d + 24*a^13*b^3*c^3*d + 4*a^13 \\
& *b^4*c^2*e - 20*a^14*b^2*c^3*e - 4*a^14*b^3*c^2*f - 32*a^14*b*c^4*d + 16*a^15 \\
& *b*c^3*f)*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + \\
& a^4*b^5*f^2 + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 7*a^5 \\
& *b^3*c*f^2 + 12*a^6*b*c^2*f^2 + a^5*c*f^2*(-(4*a*c - b^2)^3)^{(1/2)} - 2*a*b^8 \\
& *d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^{(1/2)} \\
& + a^3*c^3*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 25*a^4*b^3*c^2*e^2 - a^4*b^2*f^2*(-(4*a*c - b^2)^3)^{(1/2)} \\
& - 11*a*b^7*c*d^2 + 2*a^2*b^7*d*f - 16*a^5*c^4*d*e - 2*a^3*b^6*e*f + 16*a^6* \\
& c^3*e*f + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 20*a^2*b^6*c*d*e - 18*a^3* \\
& b^5*c*d*f - 40*a^5*b*c^3*d*f + 16*a^4*b^4*c*e*f - 6*a^2*b^2*c^2*d^2*(-(4*a* \\
& c - b^2)^3)^{(1/2)} + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 66*a^3*b^4*c^2 \\
& *d*e + 76*a^4*b^2*c^3*d*e - 2*a^2*b^4*d*f*(-(4*a*c - b^2)^3)^{(1/2)} + 50*a^4 \\
& *b^3*c^2*d*f + 2*a^3*b^3*e*f*(-(4*a*c - b^2)^3)^{(1/2)} - 2*a^4*c^2*d*f*(-(4* \\
& a*c - b^2)^3)^{(1/2)} - 36*a^5*b^2*c^2*e*f + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} \\
& - 4*a^4*b*c*e*f*(-(4*a*c - b^2)^3)^{(1/2)} - 8*a^2*b^3*c*d*e*(-(4*a* \\
& c - b^2)^3)^{(1/2)} + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 6*a^3*b^2*c* \\
& d*f*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(a^7*b^4 + 16*a^9*c^2 - 8*a^8*b^2*c))^{(1/2)} \\
& - 2*a^10*c^7*d^3 + 2*a^13*c^4*f^3 - 2*a^11*b*c^5*e^3 - 2*a^11*c^6*d*e^2 \\
& + 6*a^11*c^6*d^2*f - 6*a^12*c^5*d*f^2 + 2*a^12*c^5*e^2*f + 2*a^9*b^2*c^6*d^3 \\
& - 4*a^12*b*c^4*e*f^2 - 2*a^9*b^3*c^5*d^2*e + 4*a^10*b^2*c^5*d*e^2 + 2*a^9 \\
& *b^4*c^4*d^2*f - 6*a^10*b^2*c^5*d^2*f + 4*a^11*b^2*c^4*d*f^2 + 2*a^11*b^2*c^4 \\
& *e^2*f + 4*a^11*b*c^5*d*e*f - 4*a^10*b^3*c^4*d*e*f)*(-(b^9*d^2 + a^2*b^7 \\
& *e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + a^4*b^5*f^2 + 28*a^4*b*c^4*d^2 - \\
& 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 7*a^5*b^3*c*f^2 + 12*a^6*b*c^2*f^2 + a^5 \\
& *c*f^2*(-(4*a*c - b^2)^3)^{(1/2)} - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3 \\
& *b^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + a^3*c^3*d^2*(-(4*a* \\
& c - b^2)^3)^{(1/2)} + 25*a^4*b^3*c^2*e^2 - a^4*b^2*f^2*(-(4*a*c - b^2)^3)^{(1/2)} \\
& - 11*a*b^7*c*d^2 + 2*a^2*b^7*d*f - 16*a^5*c^4*d*e - 2*a^3*b^6*e*f + 16*a^6* \\
& c^3*e*f + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 20*a^2*b^6*c*d*e - 18*a^3* \\
& b^5*c*d*f - 40*a^5*b*c^3*d*f + 16*a^4*b^4*c*e*f - 6*a^2*b^2*c^2*d^2*(-(4*a* \\
& c - b^2)^3)^{(1/2)} + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 66*a^3*b^4*c^2 \\
& *d*e + 76*a^4*b^2*c^3*d*e - 2*a^2*b^4*d*f*(-(4*a*c - b^2)^3)^{(1/2)} + 50*a^4 \\
& *b^3*c^2*d*f + 2*a^3*b^3*e*f*(-(4*a*c - b^2)^3)^{(1/2)} - 2*a^4*c^2*d*f*(-(4* \\
& a*c - b^2)^3)^{(1/2)} - 36*a^5*b^2*c^2*e*f + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} \\
& - 4*a^4*b*c*e*f*(-(4*a*c - b^2)^3)^{(1/2)}
\end{aligned}$$

$$- b^2)^3)^{1/2} - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^{1/2} + 6*a^3*b*c^2*d$$

$$*e*(-(4*a*c - b^2)^3)^{1/2} + 6*a^3*b^2*c*d*f*(-(4*a*c - b^2)^3)^{1/2})/(8*$$

$$(a^7*b^4 + 16*a^9*c^2 - 8*a^8*b^2*c))^{1/2}*2i$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**4+e*x**2+d)/x**6/(c*x**4+b*x**2+a),x)

[Out] Timed out

$$3.59 \quad \int \frac{x^7(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2} dx$$

Optimal. Leaf size=320

$$\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(12a^2c^3e - b^3c(cd - 20af) - 12ab^2c^2e + 6abc^2(cd - 5af) - 3b^5f + 2b^4ce)}{2c^4(b^2 - 4ac)^{3/2}} + \frac{x^4(-2c(4af + b^2e) + 2c^2d)}{4c^2(b^2 - 4ac)}$$

Rubi [A] time = 1.23, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {1663, 1644, 800, 634, 618, 206, 628}

$$\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(12a^2c^3e - 12ab^2c^2e - b^3c(cd - 20af) + 6abc^2(cd - 5af) + 2b^4ce - 3b^5f)}{2c^4(b^2 - 4ac)^{3/2}} + \frac{x^4(-2c(-2acf + b^2f - bce + 2c^2d) - b(af + cd) + 2ace)}{2c(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{x^4(-2c(4af + be) + 3b^2f + 4c^2d)}{4c^2(b^2 - 4ac)} + \frac{x^2(-bc(cd - 11af) - 6ac^2e + 2b^2ce - 3b^3f)}{2c^3(b^2 - 4ac)} + \frac{\log(a + bx^2 + cx^4)(-2c(af + be) + 3b^2f + c^2d)}{4c^4}$$

Antiderivative was successfully verified.

[In] Int[(x^7*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]

[Out] ((2*b^2*c*e - 6*a*c^2*e - 3*b^3*f - b*c*(c*d - 11*a*f))*x^2)/(2*c^3*(b^2 - 4*a*c)) + ((4*c^2*d + 3*b^2*f - 2*c*(b*e + 4*a*f))*x^4)/(4*c^2*(b^2 - 4*a*c)) + (x^6*(2*a*c*e - b*(c*d + a*f) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x^2))/(2*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((2*b^4*c*e - 12*a*b^2*c^2*e + 12*a^2*c^3*e - 3*b^5*f - b^3*c*(c*d - 20*a*f) + 6*a*b*c^2*(c*d - 5*a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^4*(b^2 - 4*a*c)^(3/2)) + ((c^2*d + 3*b^2*f - 2*c*(b*e + a*f))*Log[a + b*x^2 + c*x^4])/(4*c^4)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 800

Int((((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]

Rule 1644

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*(f*b - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (IntegerQ[p] || !IntegerQ[m] || !RationalQ[a, b, c, d, e]) && !(IGtQ[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^7 (d + ex^2 + fx^4)}{(a + bx^2 + cx^4)^2} dx = \frac{1}{2} \text{Subst} \left(\int \frac{x^3 (d + ex + fx^2)}{(a + bx + cx^2)^2} dx, x, x^2 \right)$$

$$= \frac{x^6 (2ace - b(cd + af) - (2c^2d - bce + b^2f - 2acf) x^2)}{2c (b^2 - 4ac) (a + bx^2 + cx^4)} - \frac{\text{Subst} \left(\int \frac{x^2 \left(3 \left(2ae - \frac{b(cd + af)}{c} \right) - \frac{4c^2d - bce + b^2f - 2acf}{c} \right)}{a + bx + cx^2} dx, x, x^2 \right)}{2 (b^2 - 4ac)}$$

$$= \frac{x^6 (2ace - b(cd + af) - (2c^2d - bce + b^2f - 2acf) x^2)}{2c (b^2 - 4ac) (a + bx^2 + cx^4)} - \frac{\text{Subst} \left(\int \left(-\frac{2b^2ce - 6ac^2e - 3b^3f - bc(cd - 11af)}{c^3} + \frac{(4c^2d + 3b^2f - 2c(be + 4af)) x^4}{c^3} + \frac{x^6 (2b^2ce - 6ac^2e - 3b^3f - bc(cd - 11af))}{c^3} \right) dx, x, x^2 \right)}{2 (b^2 - 4ac)}$$

$$= \frac{(2b^2ce - 6ac^2e - 3b^3f - bc(cd - 11af)) x^2}{2c^3 (b^2 - 4ac)} + \frac{(4c^2d + 3b^2f - 2c(be + 4af)) x^4}{4c^2 (b^2 - 4ac)} + \frac{x^6 (2b^2ce - 6ac^2e - 3b^3f - bc(cd - 11af))}{2c^3 (b^2 - 4ac)}$$

$$= \frac{(2b^2ce - 6ac^2e - 3b^3f - bc(cd - 11af)) x^2}{2c^3 (b^2 - 4ac)} + \frac{(4c^2d + 3b^2f - 2c(be + 4af)) x^4}{4c^2 (b^2 - 4ac)} + \frac{x^6 (2b^2ce - 6ac^2e - 3b^3f - bc(cd - 11af))}{2c^3 (b^2 - 4ac)}$$

$$= \frac{(2b^2ce - 6ac^2e - 3b^3f - bc(cd - 11af)) x^2}{2c^3 (b^2 - 4ac)} + \frac{(4c^2d + 3b^2f - 2c(be + 4af)) x^4}{4c^2 (b^2 - 4ac)} + \frac{x^6 (2b^2ce - 6ac^2e - 3b^3f - bc(cd - 11af))}{2c^3 (b^2 - 4ac)}$$

Mathematica [A] time = 0.50, size = 309, normalized size = 0.97

$$\frac{2 \tan^{-1} \left(\frac{bx^2}{\sqrt{4ac - b^2}} \right) (-12d^2c^3e + 3b^3c(cd - 20bf) + 12d^2c^2e + 6ab^2c^2(5bf - cd) + 3b^3f - 2b^4ce)}{(4ac - b^2)^{3/2}} + \frac{2(2b^2ce - 6ac^2e - 3b^3f - bc(cd - 11af)) + ab(b^3 - 4c^2c + 5fx^2) + b^2(d + 4cx^2 - 3c^2d^2) + b^3x^2(b^2 - bce + c^2d)}{(b^2 - 4ac)(a + bx^2 + cx^4)} + \log(a + bx^2 + cx^4) (-2c(af + be) + 3b^2f + c^2d) + 2c^2x^2(ce - 2bf) + c^2fx^4}{4c^4}$$

Antiderivative was successfully verified.

[In] Integrate[(x^7*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]

[Out] (2*c*(c*e - 2*b*f)*x^2 + c^2*f*x^4 + (2*(2*a^3*c^2*f + b^3*(c^2*d - b*c*e + b^2*f)*x^2 + a*b*(b^3*f - 3*c^3*d*x^2 + b*c^2*(d + 4*e*x^2) - b^2*c*(e + 5*f*x^2)) + a^2*c*(-4*b^2*f - 2*c^2*(d + e*x^2) + b*c*(3*e + 5*f*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (2*(-2*b^4*c*e + 12*a*b^2*c^2*e - 12*a^2*c^3*e + 3*b^5*f + b^3*c*(c*d - 20*a*f) + 6*a*b*c^2*(-(c*d) + 5*a*f))*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + (c^2*d + 3*b^2*f - 2*c*(b*e + a*f))*Log[a + b*x^2 + c*x^4]/(4*c^4)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7 (d + ex^2 + fx^4)}{(a + bx^2 + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^7*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]

[Out] IntegrateAlgebraic[(x^7*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2, x]

fricas [B] time = 1.87, size = 2111, normalized size = 6.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")

[Out] [1/4*((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*f*x^8 + (2*(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*e - 3*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*f)*x^6 + (2*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*e - (4*b^6*c - 33*a*b^4*c^2 + 72*a^2*b^2*c^3 - 16*a^3*c^4)*f)*x^4 + 2*((b^5*c^2 - 7*a*b^3*c^3 + 12*a^2*b*c^4)*d - (b^6*c - 9*a*b^4*c^2 + 26*a^2*b^2*c^3 - 24*a^3*c^4)*e + (b^7 - 11*a*b^5*c + 41*a^2*b^3*c^2 - 52*a^3*b*c^3)*f)*x^2 - (((b^3*c^3 - 6*a*b*c^4)*d - 2*(b^4*c^2 - 6*a*b^2*c^3 + 6*a^2*c^4)*e + (3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*f)*x^4 + ((b^4*c^2 - 6*a*b^2*c^3)*d - 2*(b^5*c - 6*a*b^3*c^2 + 6*a^2*b*c^3)*e + (3*b^6 - 20*a*b^4*c + 30*a^2*b^2*c^2)*f)*x^2 + (a*b^3*c^2 - 6*a^2*b*c^3)*d - 2*(a*b^4*c - 6*a^2*b^2*c^2 + 6*a^3*c^3)*e + (3*a*b^5 - 20*a^2*b^3*c + 30*a^3*b*c^2)*f)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + 2*(a*b^4*c^2 - 6*a^2*b^2*c^3 + 8*a^3*c^4)*d - 2*(a*b^5*c - 7*a^2*b^3*c^2 + 12*a^3*b*c^3)*e + 2*(a*b^6 - 8*a^2*b^4*c + 18*a^3*b^2*c^2 - 8*a^4*c^3)*f + (((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*d - 2*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*e + (3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*f)*x^4 + ((b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d - 2*(b^6*c - 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*e + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*f)*x^2 + (a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d - 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*e + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*f)*log(c*x^4 + b*x^2 + a))/(a*b^4*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c^6 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*x^4 + (b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*x^2), 1/4*((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*f*x^8 + (2*(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*e - 3*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*f)*x^6 + (2*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*e - (4*b^6*c - 33*a*b^4*c^2 + 72*a^2*b^2*c^3 - 16*a^3*c^4)*f)*x^4 + 2*((b^5*c^2 - 7*a*b^3*c^3 + 12*a^2*b*c^4)*d - (b^6*c - 9*a*b^4*c^2 + 26*a^2*b^2*c^3 - 24*a^3*c^4)*e + (b^7 - 11*a*b^5*c + 41*a^2*b^3*c^2 - 52*a^3*b*c^3)*f)*x^2 + 2*((b^3*c^3 - 6*a*b*c^4)*d - 2*(b^4*c^2 - 6*a*b^2*c^3 + 6*a^2*c^4)*e + (3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*f)*x^4 + ((b^4*c^2 - 6*a*b^2*c^3)*d - 2*(b^5*c - 6*a*b^3*c^2 + 6*a^2*b*c^3)*e + (3*b^6 - 20*a*b^4*c + 30*a^2*b^2*c^2)*f)*x^2 + (a*b^3*c^2 - 6*a^2*b*c^3)*d - 2*(a*b^4*c - 6*a^2*b^2*c^2 + 6*a^3*c^3)*e + (3*a*b^5 - 20*a^2*b^3*c + 30*a^3*b*c^2)*f)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c

)/(b^2 - 4*a*c)) + 2*(a*b^4*c^2 - 6*a^2*b^2*c^3 + 8*a^3*c^4)*d - 2*(a*b^5*c - 7*a^2*b^3*c^2 + 12*a^3*b*c^3)*e + 2*(a*b^6 - 8*a^2*b^4*c + 18*a^3*b^2*c^2 - 8*a^4*c^3)*f + (((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*d - 2*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*e + (3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*f)*x^4 + ((b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d - 2*(b^6*c - 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*e + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*f)*x^2 + (a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d - 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*e + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*f)*log(c*x^4 + b*x^2 + a))/(a*b^4*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c^6 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*x^4 + (b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*x^2)]

giac [A] time = 1.95, size = 424, normalized size = 1.32

$$\frac{(b^2d - 6ab^2c + 3b^3f - 20ab^2f + 30a^2b^2cf - 2b^3c^2 - 12a^2b^2c^2 - 12a^2c^3) \arctan\left(\frac{2cx^2 + b}{\sqrt{-b^2 + 4ac}}\right) + b^2de^4 - 4a^2de^4 + 3b^2f^2e^4 - 14ab^2f^2e^4 + 8a^2c^2f^2e^4 - 2b^2c^2e^4 + 8ab^2c^2e^4 - b^2c^2e^4 + 3a^2b^2c^2e^4 + b^2f^2e^4 - 4ab^2f^2e^4 + 4a^2c^2f^2e^4 - ab^2c^2e^4 + ab^2f^2e^4 - 6a^2b^2f^2e^4 + 4a^2c^2f^2e^4 + 2a^2b^2c^2e^4}{2(b^2 - 4ac)\sqrt{-b^2 + 4ac}} + \frac{b^2de^4 - 4a^2de^4 + 3b^2f^2e^4 - 14ab^2f^2e^4 + 8a^2c^2f^2e^4 - 2b^2c^2e^4 + 8ab^2c^2e^4 - b^2c^2e^4 + 3a^2b^2c^2e^4 + b^2f^2e^4 - 4ab^2f^2e^4 + 4a^2c^2f^2e^4 - ab^2c^2e^4 + ab^2f^2e^4 - 6a^2b^2f^2e^4 + 4a^2c^2f^2e^4 + 2a^2b^2c^2e^4}{4(b^2 - 4ac)\sqrt{-b^2 + 4ac}} + \frac{(2d + 3b^2 - 2ac)\log(cx^4 + bx^2 + a)}{4c^4} + \frac{c^2f^4 - 4abf^2c^2 + 2c^2e^4}{4c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")

[Out] -1/2*(b^3*c^2*d - 6*a*b*c^3*d + 3*b^5*f - 20*a*b^3*c*f + 30*a^2*b*c^2*f - 2*b^4*c*e + 12*a*b^2*c^2*e - 12*a^2*c^3*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^4 - 4*a*c^5)*sqrt(-b^2 + 4*a*c)) - 1/4*(b^2*c^3*d*x^4 - 4*a*c^4*d*x^4 + 3*b^4*c*f*x^4 - 14*a*b^2*c^2*f*x^4 + 8*a^2*c^3*f*x^4 - 2*b^3*c^2*x^4*e + 8*a*b*c^3*x^4*e - b^3*c^2*d*x^2 + 2*a*b*c^3*d*x^2 + b^5*f*x^2 - 4*a*b^3*c*f*x^2 - 2*a^2*b*c^2*f*x^2 + 4*a^2*c^3*x^2*e - a*b^2*c^2*d + a*b^4*f - 6*a^2*b^2*c*f + 4*a^3*c^2*f + 2*a^2*b*c^2*e)/(b^2*c^4 - 4*a*c^5)*(c*x^4 + b*x^2 + a) + 1/4*(c^2*d + 3*b^2*f - 2*a*c*f - 2*b*c*e)*log(c*x^4 + b*x^2 + a)/c^4 + 1/4*(c^2*f*x^4 - 4*b*c*f*x^2 + 2*c^2*x^2*e)/c^4

maple [B] time = 0.02, size = 1167, normalized size = 3.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x)

[Out] -2/c^2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*a*b^2*e+3/2/c/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*a*b*d+5/2/c^3/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*a*b^3*f-5/2/c^2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*a^2*b*f-1/c^3*x^2*b*f-1/c^2/(c*x^4+b*x^2+a)*a^3/(4*a*c-b^2)*f+1/c/(c*x^4+b*x^2+a)*a^2/(4*a*c-b^2)*d+15/c^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a^2*b*f-10/c^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*b^3*f+6/c^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*b^2*e-3/c/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*b*d+7/2/c^3/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*a*b^2*f-2/c^2/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*a*b*e-1/2/c^4/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*b^5*f-1/2/c^2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*b^3*d-3/2/c^2/(c*x^4+b*x^2+a)*a^2/(4*a*c-b^2)*b*e-1/2/c^4/(c*x^4+b*x^2+a)*a/(4*a*c-b^2)*b^4*f-1/2/c^2/(c*x^4+b*x^2+a)*a/(4*a*c-b^2)*b^2*d+1/2/c^3/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*b^4*e+2/c^3/(c*x^4+b*x^2+a)*a^2/(4*a*c-b^2)*b^2*f+1/4/c^2*x^4*f+1/2/c^2*x^2*e+1/2/c^3/(c*x^4+b*x^2+a)*a/(4*a*c-b^2)*b^3*e+1/c/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*a^2*e-1/c^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^4*e+1/2/c^3/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*b^3*e-2/c^2/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*a^2*f+1/c/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*a*d-3/4/c^4/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*b^4*f-1/4/c^2/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*b^2*d-6/c/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a^2*e+3/2/c^4/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^5*f+1/2/c^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^3*d

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 1.33, size = 3499, normalized size = 10.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] x^2*(e/(2*c^2) - (b*f)/c^3) - ((2*a^3*c^2*f - 2*a^2*c^3*d + a*b^4*f - a*b^3*c*e + a*b^2*c^2*d + 3*a^2*b*c^2*e - 4*a^2*b^2*c*f)/(2*c*(4*a*c - b^2)) + (x^2*(b^5*f - 2*a^2*c^3*e + b^3*c^2*d - b^4*c*e - 3*a*b*c^3*d - 5*a*b^3*c*f + 4*a*b^2*c^2*e + 5*a^2*b*c^2*f))/(2*c*(4*a*c - b^2)))/(a*c^3 + c^4*x^4 + b*c^3*x^2) - (log(a + b*x^2 + c*x^4)*(6*b^8*f - 128*a^3*c^5*d + 2*b^6*c^2*d + 256*a^4*c^4*f - 4*b^7*c*e + 96*a^2*b^2*c^4*d - 192*a^2*b^3*c^3*e + 336*a^2*b^4*c^2*f - 576*a^3*b^2*c^3*f - 76*a*b^6*c*f - 24*a*b^4*c^3*d + 48*a*b^5*c^2*e + 256*a^3*b*c^4*e))/(2*(256*a^3*c^7 - 4*b^6*c^4 + 48*a*b^4*c^5 - 192*a^2*b^2*c^6)) + (f*x^4)/(4*c^2) + (atan((((8*a*c^7*(4*a*c - b^2)^3 - 2*b^2*c^6*(4*a*c - b^2)^3)*(((16*a^2*c^5*f - 8*a*c^6*d + 16*a*b*c^5*e - 24*a*b^2*c^4*f)/c^6 - (8*a*c^2*(6*b^8*f - 128*a^3*c^5*d + 2*b^6*c^2*d + 256*a^4*c^4*f - 4*b^7*c*e + 96*a^2*b^2*c^4*d - 192*a^2*b^3*c^3*e + 336*a^2*b^4*c^2*f - 576*a^3*b^2*c^3*f - 76*a*b^6*c*f - 24*a*b^4*c^3*d + 48*a*b^5*c^2*e + 256*a^3*b*c^4*e))/(256*a^3*c^7 - 4*b^6*c^4 + 48*a*b^4*c^5 - 192*a^2*b^2*c^6)))*(3*b^5*f - 12*a^2*c^3*e + b^3*c^2*d - 2*b^4*c*e - 6*a*b*c^3*d - 20*a*b^3*c*f + 12*a*b^2*c^2*e + 30*a^2*b*c^2*f))/(8*c^4*(4*a*c - b^2)^(3/2)) - (a*(3*b^5*f - 12*a^2*c^3*e + b^3*c^2*d - 2*b^4*c*e - 6*a*b*c^3*d - 20*a*b^3*c*f + 12*a*b^2*c^2*e + 30*a^2*b*c^2*f)*(6*b^8*f - 128*a^3*c^5*d + 2*b^6*c^2*d + 256*a^4*c^4*f - 4*b^7*c*e + 96*a^2*b^2*c^4*d - 192*a^2*b^3*c^3*e + 336*a^2*b^4*c^2*f - 576*a^3*b^2*c^3*f - 76*a*b^6*c*f - 24*a*b^4*c^3*d + 48*a*b^5*c^2*e + 256*a^3*b*c^4*e))/(c^2*(4*a*c - b^2)^(3/2)*(256*a^3*c^7 - 4*b^6*c^4 + 48*a*b^4*c^5 - 192*a^2*b^2*c^6)))/(a*(4*a*c - b^2)) - x^2*(((24*a^2*c^7*e - 6*b^3*c^6*d + 12*b^4*c^5*e - 18*b^5*c^4*f + 28*a*b*c^7*d - 56*a*b^2*c^6*e + 96*a*b^3*c^5*f - 92*a^2*b*c^6*f)/(4*a*c^7 - b^2*c^6) - ((8*b^3*c^8 - 32*a*b*c^9)*(6*b^8*f - 128*a^3*c^5*d + 2*b^6*c^2*d + 256*a^4*c^4*f - 4*b^7*c*e + 96*a^2*b^2*c^4*d - 192*a^2*b^3*c^3*e + 336*a^2*b^4*c^2*f - 576*a^3*b^2*c^3*f - 76*a*b^6*c*f - 24*a*b^4*c^3*d + 48*a*b^5*c^2*e + 256*a^3*b*c^4*e))/(2*(4*a*c^7 - b^2*c^6)*(256*a^3*c^7 - 4*b^6*c^4 + 48*a*b^4*c^5 - 192*a^2*b^2*c^6)))*(3*b^5*f - 12*a^2*c^3*e + b^3*c^2*d - 2*b^4*c*e - 6*a*b*c^3*d - 20*a*b^3*c*f + 12*a*b^2*c^2*e + 30*a^2*b*c^2*f))/(8*c^4*(4*a*c - b^2)^(3/2)) - ((8*b^3*c^8 - 32*a*b*c^9)*(3*b^5*f - 12*a^2*c^3*e + b^3*c^2*d - 2*b^4*c*e - 6*a*b*c^3*d - 20*a*b^3*c*f + 12*a*b^2*c^2*e + 30*a^2*b*c^2*f)*(6*b^8*f - 128*a^3*c^5*d + 2*b^6*c^2*d + 256*a^4*c^4*f - 4*b^7*c*e + 96*a^2*b^2*c^4*d - 192*a^2*b^3*c^3*e + 336*a^2*b^4*c^2*f - 576*a^3*b^2*c^3*f - 76*a*b^6*c*f - 24*a*b^4*c^3*d + 48*a*b^5*c^2*e + 256*a^3*b*c^4*e))/(16*c^4*(4*a*c - b^2)^(3/2)*(4*a*c^7 - b^2*c^6)*(256*a^3*c^7 - 4*b^6*c^4 + 48*a*b^4*c^5 - 192*a^2*b^2*c^6)))/(a*(4*a*c - b^2)) + (b*(((24*a^2*c^7*e - 6*b^3*c^6*d + 12*b^4*c^5*e - 18*b^5*c^4*f + 28*a*b*c^7*d - 56*a*b^2*c^6*e + 96*a*b^3*c^5*f - 92*a^2*b*c^6*f)/(4*a*c^7 - b^2*c^6) - ((8*b^3*c^8 - 32*a*b*c^9)*(6*b^8*f - 128*a^3*c^5*d + 2*b^6*c^2*d + 256*a^4*c^4*f - 4*b^7*c*e + 96*a^2*b^2*c^4*d - 192*a^2*b^3*c^3*e + 336*a^2*b^4*c^2*f - 576*a^3*b^2*c^3*f - 76*a*b^6*c*f - 24*a*b^4*c^3*d + 48*a*b^5*c^2*e + 256*a^3*b*c^4*e))/(2*(4*a*c^7 - b^2*c^6)*(256*a^3*c^7 - 4*b^6*c^4 + 48*a*b^4*c^5 - 192*a^2*b^2*c^6)))*(6*b^8*f - 128*a^3*c^5*d + 2*b^6*c^2*d + 256*a^4*c^4*f - 4*b^7*c*e + 96*a^2*b^2*c^4*d - 192*a^2*b^3*c^3*e + 336*a^2*b^4*c^2*f - 576*a^3*b^2*c^3*f - 76*a*b^6*c*f - 24
```

$$\begin{aligned} & *a*b^4*c^3*d + 48*a*b^5*c^2*e + 256*a^3*b*c^4*e)) / (2*(256*a^3*c^7 - 4*b^6*c^4 + 48*a*b^4*c^5 - 192*a^2*b^2*c^6)) - (9*b^7*f^2 + b^3*c^4*d^2 + 4*b^5*c^2*e^2 - 20*a*b^3*c^3*e^2 + 12*a^2*b*c^4*e^2 - 38*a^3*b*c^3*f^2 - 12*b^6*c*e*f + 91*a^2*b^3*c^2*f^2 - 5*a*b*c^5*d^2 - 57*a*b^5*c*f^2 - 6*a^2*c^5*d*e - 4*b^4*c^3*d*e + 12*a^3*c^4*e*f + 6*b^5*c^2*d*f + 20*a*b^2*c^4*d*e - 34*a*b^3*c^3*d*f + 29*a^2*b*c^4*d*f + 68*a*b^4*c^2*e*f - 76*a^2*b^2*c^3*e*f) / (4*a*c^7 - b^2*c^6) + (((b^3*c^8)/2 - 2*a*b*c^9)*(3*b^5*f - 12*a^2*c^3*e + b^3*c^2*d - 2*b^4*c*e - 6*a*b*c^3*d - 20*a*b^3*c*f + 12*a*b^2*c^2*e + 30*a^2*b*c^2*f)^2) / (c^8*(4*a*c - b^2)^3*(4*a*c^7 - b^2*c^6))) / (2*a*(4*a*c - b^2)^(3/2))) + (b*(((16*a^2*c^5*f - 8*a*c^6*d + 16*a*b*c^5*e - 24*a*b^2*c^4*f)/c^6 - (8*a*c^2*(6*b^8*f - 128*a^3*c^5*d + 2*b^6*c^2*d + 256*a^4*c^4*f - 4*b^7*c*e + 96*a^2*b^2*c^4*d - 192*a^2*b^3*c^3*e + 336*a^2*b^4*c^2*f - 576*a^3*b^2*c^3*f - 76*a*b^6*c*f - 24*a*b^4*c^3*d + 48*a*b^5*c^2*e + 256*a^3*b*c^4*e)) / (256*a^3*c^7 - 4*b^6*c^4 + 48*a*b^4*c^5 - 192*a^2*b^2*c^6)) - (a*c^4*d^2 + 9*a*b^4*f^2 + 4*a^3*c^2*f^2 + 4*a*b^2*c^2*e^2 - 12*a^2*b^2*c*f^2 - 4*a^2*c^3*d*f + 6*a*b^2*c^2*d*f + 8*a^2*b*c^2*e*f - 4*a*b*c^3*d*e - 12*a*b^3*c*e*f) / c^6 + (a*(3*b^5*f - 12*a^2*c^3*e + b^3*c^2*d - 2*b^4*c*e - 6*a*b*c^3*d - 20*a*b^3*c*f + 12*a*b^2*c^2*e + 30*a^2*b*c^2*f)^2) / (c^6*(4*a*c - b^2)^3))) / (2*a*(4*a*c - b^2)^(3/2)))) / (9*b^10*f^2 + 144*a^4*c^6*e^2 + b^6*c^4*d^2 + 4*b^8*c^2*e^2 - 12*a*b^4*c^5*d^2 - 48*a*b^6*c^3*e^2 - 12*b^9*c*e*f + 36*a^2*b^2*c^6*d^2 + 192*a^2*b^4*c^4*e^2 - 288*a^3*b^2*c^5*e^2 + 580*a^2*b^6*c^2*f^2 - 1200*a^3*b^4*c^3*f^2 + 900*a^4*b^2*c^4*f^2 - 120*a*b^8*c*f^2 - 4*b^7*c^3*d*e + 6*b^8*c^2*d*f + 48*a*b^5*c^4*d*e + 144*a^3*b*c^6*d*e - 76*a*b^6*c^3*d*f + 152*a*b^7*c^2*e*f - 720*a^4*b*c^5*e*f - 168*a^2*b^3*c^5*d*e + 300*a^2*b^4*c^4*d*f - 360*a^3*b^2*c^5*d*f - 672*a^2*b^5*c^3*e*f + 1200*a^3*b^3*c^4*e*f)) * (3*b^5*f - 12*a^2*c^3*e + b^3*c^2*d - 2*b^4*c*e - 6*a*b*c^3*d - 20*a*b^3*c*f + 12*a*b^2*c^2*e + 30*a^2*b*c^2*f)) / (2*c^4*(4*a*c - b^2)^(3/2)) \end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**7*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)

[Out] Timed out

$$3.60 \quad \int \frac{x^5(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2} dx$$

Optimal. Leaf size=236

$$\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(12a^2c^2f - 2ac(6b^2f - 3bce + 2c^2d) - (b^3(ce - 2bf)))}{2c^3(b^2 - 4ac)^{3/2}} + \frac{x^2(-c(6af + be) + 2b^2f + 2c^2d)}{2c^2(b^2 - 4ac)}$$

Rubi [A] time = 0.44, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {1663, 1644, 773, 634, 618, 206, 628}

$$\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(12a^2c^2f - 2ac(6b^2f - 3bce + 2c^2d) + b^3(-ce - 2bf))}{2c^3(b^2 - 4ac)^{3/2}} + \frac{x^4(x^2(-2acf + b^2f - bce + 2c^2d) - b(af + cd) + 2ace)}{2c(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{x^2(-c(6af + be) + 2b^2f + 2c^2d)}{2c^2(b^2 - 4ac)} + \frac{(ce - 2bf)\log(a + bx^2 + cx^4)}{4c^3}$$

Antiderivative was successfully verified.

[In] Int[(x^5*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]

[Out] ((2*c^2*d + 2*b^2*f - c*(b*e + 6*a*f))*x^2)/(2*c^2*(b^2 - 4*a*c)) + (x^4*(2*a*c*e - b*(c*d + a*f) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x^2))/(2*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((12*a^2*c^2*f - b^3*(c*e - 2*b*f) - 2*a*c*(2*c^2*d - 3*b*c*e + 6*b^2*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^3*(b^2 - 4*a*c)^(3/2)) + ((c*e - 2*b*f)*Log[a + b*x^2 + c*x^4])/(4*c^3)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 773

Int((((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (c*e*f + c*d*g - b*e*g)*x]/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1644

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*(f*b - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (IntegerQ[p] || !IntegerQ[m] || !RationalQ[a, b, c, d, e]) && !(IGtQ[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^5 (d + ex^2 + fx^4)}{(a + bx^2 + cx^4)^2} dx = \frac{1}{2} \text{Subst} \left(\int \frac{x^2 (d + ex + fx^2)}{(a + bx + cx^2)^2} dx, x, x^2 \right)$$

$$= \frac{x^4 (2ace - b(cd + af) - (2c^2d - bce + b^2f - 2acf) x^2)}{2c (b^2 - 4ac) (a + bx^2 + cx^4)} - \text{Subst} \left(\int \frac{x \left(2 \left(2ae - \frac{b(cd+af)}{c} \right) - \frac{(2c^2d - bce + b^2f - 2acf)}{a+bx+c} \right)}{2 (b^2 - 4ac)} dx, x, x^2 \right)$$

$$= \frac{(2c^2d + 2b^2f - c(be + 6af)) x^2}{2c^2 (b^2 - 4ac)} + \frac{x^4 (2ace - b(cd + af) - (2c^2d - bce + b^2f - 2acf))}{2c (b^2 - 4ac) (a + bx^2 + cx^4)}$$

$$= \frac{(2c^2d + 2b^2f - c(be + 6af)) x^2}{2c^2 (b^2 - 4ac)} + \frac{x^4 (2ace - b(cd + af) - (2c^2d - bce + b^2f - 2acf))}{2c (b^2 - 4ac) (a + bx^2 + cx^4)}$$

$$= \frac{(2c^2d + 2b^2f - c(be + 6af)) x^2}{2c^2 (b^2 - 4ac)} + \frac{x^4 (2ace - b(cd + af) - (2c^2d - bce + b^2f - 2acf))}{2c (b^2 - 4ac) (a + bx^2 + cx^4)}$$

$$= \frac{(2c^2d + 2b^2f - c(be + 6af)) x^2}{2c^2 (b^2 - 4ac)} + \frac{x^4 (2ace - b(cd + af) - (2c^2d - bce + b^2f - 2acf))}{2c (b^2 - 4ac) (a + bx^2 + cx^4)}$$

Mathematica [A] time = 0.36, size = 236, normalized size = 1.00

$$\frac{2 \tan^{-1} \left(\frac{b+2cx^2}{\sqrt{4ac-b^2}} \right) (12a^2c^2f - 2ac(6b^2f - 3bce + 2c^2d) + b^3(2bf - ce))}{(4ac - b^2)^{3/2}} - \frac{2(a^2c(2c(e+fx^2) - 3bf) + a(b^3f - b^2c(e+4fx^2) + bc^2(d+3cx^2) - 2c^3dx^2) + b^2x^2(b^2f - bce + c^2d))}{(b^2 - 4ac)(a + bx^2 + cx^4)} + (ce - 2bf) \log(a + bx^2 + cx^4) + 2cfx^2}{4c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
[Out] (2*c*f*x^2 - (2*(b^2*(c^2*d - b*c*e + b^2*f))*x^2 + a^2*c*(-3*b*f + 2*c*(e + f*x^2))) + a*(b^3*f - 2*c^3*d*x^2 + b*c^2*(d + 3*e*x^2) - b^2*c*(e + 4*f*x^2))
```


2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (2*(12*a^2*c^2*f + b^3*(-(c*e) + 2*b*f) - 2*a*c*(2*c^2*d - 3*b*c*e + 6*b^2*f))*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + (c*e - 2*b*f)*Log[a + b*x^2 + c*x^4])/(4*c^3)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5 (d + ex^2 + fx^4)}{(a + bx^2 + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^5*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]

[Out] IntegrateAlgebraic[(x^5*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2, x]

fricas [B] time = 1.15, size = 1455, normalized size = 6.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")

[Out] [1/4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*f*x^6 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*f*x^4 - 2*((b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d - (b^5*c - 7*a*b^3*c^2 + 12*a^2*b*c^3)*e + (b^6 - 9*a*b^4*c + 26*a^2*b^2*c^2 - 24*a^3*c^3)*f)*x^2 + (4*a^2*c^3*d + (4*a*c^4*d + (b^3*c^2 - 6*a*b*c^3)*e - 2*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*f)*x^4 + (4*a*b*c^3*d + (b^4*c - 6*a*b^2*c^2)*e - 2*(b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*f)*x^2 + (a*b^3*c - 6*a^2*b*c^2)*e - 2*(a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*f)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - 2*(a*b^3*c^2 - 4*a^2*b*c^3)*d + 2*(a*b^4*c - 6*a^2*b^2*c^2 + 8*a^3*c^3)*e - 2*(a*b^5 - 7*a^2*b^3*c + 12*a^3*b*c^2)*f + (((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e - 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*f)*x^4 + ((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*e - 2*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*f)*x^2 + (a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*e - 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*f)*log(c*x^4 + b*x^2 + a))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5 + (b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^4 + (b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^2), 1/4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*f*x^6 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*f*x^4 - 2*((b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d - (b^5*c - 7*a*b^3*c^2 + 12*a^2*b*c^3)*e + (b^6 - 9*a*b^4*c + 26*a^2*b^2*c^2 - 24*a^3*c^3)*f)*x^2 + 2*(4*a^2*c^3*d + (4*a*c^4*d + (b^3*c^2 - 6*a*b*c^3)*e - 2*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*f)*x^4 + (4*a*b*c^3*d + (b^4*c - 6*a*b^2*c^2)*e - 2*(b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*f)*x^2 + (a*b^3*c - 6*a^2*b*c^2)*e - 2*(a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*f)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - 2*(a*b^3*c^2 - 4*a^2*b*c^3)*d + 2*(a*b^4*c - 6*a^2*b^2*c^2 + 8*a^3*c^3)*e - 2*(a*b^5 - 7*a^2*b^3*c + 12*a^3*b*c^2)*f + (((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e - 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*f)*x^4 + ((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*e - 2*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*f)*x^2 + (a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*e - 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*f)*log(c*x^4 + b*x^2 + a))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5 + (b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^4 + (b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^2)]

giac [A] time = 1.86, size = 279, normalized size = 1.18

$$\frac{fx^2}{2c^2} - \frac{(4ac^3d - 2b^4f + 12ab^2cf - 12a^2c^2f + b^3ce - 6abc^2e) \arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right)}{2(b^2c^3 - 4ac^4)\sqrt{-b^2+4ac}} + \frac{2b^3fx^4 - 8abcfx^4 - b^2cx^4e + 4ac^2x^4e - 2b^2cdx^2 + 4ac^2dx^2 - 4a^2cfx^2 + b^3x^2e - 2abcx^2e - 2abcd - 2a^2bf + ab^2e}{4(cx^4 + bx^2 + a)(b^2c^2 - 4ac^3)} - \frac{(2bf - ce) \log(cx^4 + bx^2 + a)}{4c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")

```
[Out] 1/2*f*x^2/c^2 - 1/2*(4*a*c^3*d - 2*b^4*f + 12*a*b^2*c*f - 12*a^2*c^2*f + b^3*c*e - 6*a*b*c^2*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^3 - 4*a*c^4)*sqrt(-b^2 + 4*a*c)) + 1/4*(2*b^3*f*x^4 - 8*a*b*c*f*x^4 - b^2*c*x^4*e + 4*a*c^2*x^4*e - 2*b^2*c*d*x^2 + 4*a*c^2*d*x^2 - 4*a^2*c*f*x^2 + b^3*x^2*e - 2*a*b*c*x^2*e - 2*a*b*c*d - 2*a^2*b*f + a*b^2*e)/((c*x^4 + b*x^2 + a)*(b^2*c^2 - 4*a*c^3)) - 1/4*(2*b*f - c*e)*log(c*x^4 + b*x^2 + a)/c^3
```

maple [B] time = 0.02, size = 832, normalized size = 3.53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/2*f*x^2/c^2+1/c/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*a^2*f-2/c^2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*a*b^2*f+3/2/c/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*a*b*e-1/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*a*d+1/2/c^3/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*b^4*f-1/2/c^2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*b^3*e+1/2/c/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*b^2*d-3/2/c^2/(c*x^4+b*x^2+a)*a^2/(4*a*c-b^2)*b*f+1/c/(c*x^4+b*x^2+a)*a^2/(4*a*c-b^2)*e+1/2/c^3/(c*x^4+b*x^2+a)*a/(4*a*c-b^2)*b^3*f-1/2/c^2/(c*x^4+b*x^2+a)*a/(4*a*c-b^2)*b^2*e+1/2/c/(c*x^4+b*x^2+a)*a/(4*a*c-b^2)*b*d-2/c^2/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*a*b*f+1/c/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*a*e+1/2/c^3/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*b^3*f-1/4/c^2/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*b^2*e-6/c/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a^2*f+6/c^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*b^2*f-3/c/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*b*e+2/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*d-1/c^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^4*f+1/2/c^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^3*e
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 1.81, size = 2450, normalized size = 10.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] ((a*(b^3*f + 2*a*c^2*e + b*c^2*d - b^2*c*e - 3*a*b*c*f))/(2*c*(4*a*c - b^2)) + (x^2*(b^4*f + b^2*c^2*d + 2*a^2*c^2*f - 2*a*c^3*d - b^3*c*e + 3*a*b*c^2*e - 4*a*b^2*c*f))/(2*c*(4*a*c - b^2)))/(a*c^2 + c^3*x^4 + b*c^2*x^2) + (f*x^2)/(2*c^2) + (log(a + b*x^2 + c*x^4)*(4*b^7*f + 128*a^3*c^4*e - 2*b^6*c*e - 96*a^2*b^2*c^3*e + 192*a^2*b^3*c^2*f - 48*a*b^5*c*f + 24*a*b^4*c^2*e - 256*a^3*b*c^3*f))/(2*(256*a^3*c^6 - 4*b^6*c^3 + 48*a*b^4*c^4 - 192*a^2*b^2*c^5)) - (atan(((8*a*c^5*(4*a*c - b^2)^3 - 2*b^2*c^4*(4*a*c - b^2)^3)*(x^2*((((24*a^2*c^5*f - 6*b^3*c^4*e + 12*b^4*c^3*f - 8*a*c^6*d + 28*a*b*c^5*e - 56*a*b^2*c^4*f)/(4*a*c^5 - b^2*c^4) + ((8*b^3*c^6 - 32*a*b*c^7)*(4*b^7*f + 128*a^3*c^4*e - 2*b^6*c*e - 96*a^2*b^2*c^3*e + 192*a^2*b^3*c^2*f - 48*a*b^5*c*f + 24*a*b^4*c^2*e - 256*a^3*b*c^3*f))/(2*(4*a*c^5 - b^2*c^4)*(256*a^3*c^6 - 4*b^6*c^3 + 48*a*b^4*c^4 - 192*a^2*b^2*c^5))))))
```

$$\begin{aligned}
& (6 - 4b^6c^3 + 48a^2b^4c^4 - 192a^2b^2c^5)) * (2b^4f + 12a^2c^2f - 4ac^3d - b^3ce + 6abc^2e - 12ab^2cf) / (8c^3(4ac - b^2)^{(3/2)}) \\
& + ((8b^3c^6 - 32abc^7) * (2b^4f + 12a^2c^2f - 4ac^3d - b^3ce + 6abc^2e - 12ab^2cf) * (4b^7f + 128a^3c^4e - 2b^6ce - 96a^2b^2c^3e + 192a^2b^3c^2f - 48ab^5cf + 24ab^4c^2e - 256a^3b^3cf)) / (16c^3(4ac - b^2)^{(3/2)} * (4ac^5 - b^2c^4) * (256a^3c^6 - 4b^6c^3 + 48a^2b^4c^4 - 192a^2b^2c^5)) / (a(4ac - b^2)) + (b((4b^5f^2 + b^3c^2e^2 + 12a^2b^2c^2f^2 + 2ac^4de - 4b^4c^2ef - 5abc^3e^2 - 20ab^3cf^2 - 6a^2c^3e^2f + 20ab^2c^2ef - 4abc^3df) / (4ac^5 - b^2c^4) + ((24a^2c^5f - 6b^3c^4e + 12b^4c^3f - 8ac^6d + 28abc^5e - 56ab^2c^4f) / (4ac^5 - b^2c^4) + ((8b^3c^6 - 32abc^7) * (4b^7f + 128a^3c^4e - 2b^6ce - 96a^2b^2c^3e + 192a^2b^3c^2f - 48ab^5cf + 24ab^4c^2e - 256a^3b^3cf)) / (2(4ac^5 - b^2c^4) * (256a^3c^6 - 4b^6c^3 + 48a^2b^4c^4 - 192a^2b^2c^5))) * (4b^7f + 128a^3c^4e - 2b^6ce - 96a^2b^2c^3e + 192a^2b^3c^2f - 48ab^5cf + 24ab^4c^2e - 256a^3b^3cf)) / (2(256a^3c^6 - 4b^6c^3 + 48a^2b^4c^4 - 192a^2b^2c^5)) - ((b^3c^6)/2 - 2abc^7) * (2b^4f + 12a^2c^2f - 4ac^3d - b^3ce + 6abc^2e - 12ab^2cf)^2 / (c^6(4ac - b^2)^3(4ac^5 - b^2c^4))) / (2a(4ac - b^2)^{(3/2)}) + (((8ac^4e - 16abc^3f) / c^4 - (8ac^2(4b^7f + 128a^3c^4e - 2b^6ce - 96a^2b^2c^3e + 192a^2b^3c^2f - 48ab^5cf + 24ab^4c^2e - 256a^3b^3cf)) / (256a^3c^6 - 4b^6c^3 + 48a^2b^4c^4 - 192a^2b^2c^5)) * (2b^4f + 12a^2c^2f - 4ac^3d - b^3ce + 6abc^2e - 12ab^2cf)) / (8c^3(4ac - b^2)^{(3/2)}) - (a(2b^4f + 12a^2c^2f - 4ac^3d - b^3ce + 6abc^2e - 12ab^2cf) * (4b^7f + 128a^3c^4e - 2b^6ce - 96a^2b^2c^3e + 192a^2b^3c^2f - 48ab^5cf + 24ab^4c^2e - 256a^3b^3cf)) / (c(4ac - b^2)^{(3/2)} * (256a^3c^6 - 4b^6c^3 + 48a^2b^4c^4 - 192a^2b^2c^5))) / (a(4ac - b^2)) + (b((((8ac^4e - 16abc^3f) / c^4 - (8ac^2(4b^7f + 128a^3c^4e - 2b^6ce - 96a^2b^2c^3e + 192a^2b^3c^2f - 48ab^5cf + 24ab^4c^2e - 256a^3b^3cf)) / (256a^3c^6 - 4b^6c^3 + 48a^2b^4c^4 - 192a^2b^2c^5)) * (4b^7f + 128a^3c^4e - 2b^6ce - 96a^2b^2c^3e + 192a^2b^3c^2f - 48ab^5cf + 24ab^4c^2e - 256a^3b^3cf)) / (2(256a^3c^6 - 4b^6c^3 + 48a^2b^4c^4 - 192a^2b^2c^5)) - (4ab^2f^2 + ac^2e^2 - 4abc^2ef) / c^4 + (a(2b^4f + 12a^2c^2f - 4ac^3d - b^3ce + 6abc^2e - 12ab^2cf)^2 / (c^4(4ac - b^2)^3))) / (2a(4ac - b^2)^{(3/2)})) / (4b^8f^2 + 16a^2c^6d^2 + 144a^4c^4f^2 + b^6c^2e^2 - 12ab^4c^3e^2 - 4b^7c^2ef + 36a^2b^2c^4e^2 + 192a^2b^4c^2f^2 - 288a^3b^2c^3f^2 - 48ab^6c^2f^2 - 96a^3c^5df + 8ab^3c^4de - 48a^2b^5c^2de - 16ab^4c^3df + 48ab^5c^2ef + 144a^3b^4c^4ef + 96a^2b^2c^4df - 168a^2b^3c^3ef) * (2b^4f + 12a^2c^2f - 4ac^3d - b^3ce + 6abc^2e - 12ab^2cf)) / (2c^3(4ac - b^2)^{(3/2)})
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)

[Out] Timed out

$$3.61 \quad \int \frac{x^3(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2} dx$$

Optimal. Leaf size=165

$$\frac{x^2 \left(- \left(x^2 (-2acf + b^2f - bce + 2c^2d) \right) - b(af + cd) + 2ace \right) \tanh^{-1} \left(\frac{b+2cx^2}{\sqrt{b^2-4ac}} \right) (-2bc(3af + cd) + 4ac^2e + b^3f)}{2c(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\tanh^{-1} \left(\frac{b+2cx^2}{\sqrt{b^2-4ac}} \right) (-2bc(3af + cd) + 4ac^2e + b^3f)}{2c^2(b^2 - 4ac)^{3/2}}$$

Rubi [A] time = 0.29, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1663, 1644, 634, 618, 206, 628}

$$\frac{x^2 \left(x^2 (-2acf + b^2f - bce + 2c^2d) - b(af + cd) + 2ace \right)}{2c(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\tanh^{-1} \left(\frac{b+2cx^2}{\sqrt{b^2-4ac}} \right) (-2bc(3af + cd) + 4ac^2e + b^3f)}{2c^2(b^2 - 4ac)^{3/2}} + \frac{f \log(a + bx^2 + cx^4)}{4c^2}$$

Antiderivative was successfully verified.

[In] Int[(x^3*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]

[Out] (x^2*(2*a*c*e - b*(c*d + a*f) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x^2))/(2*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((4*a*c^2*e + b^3*f - 2*b*c*(c*d + 3*a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^2*(b^2 - 4*a*c)^(3/2)) + (f*Log[a + b*x^2 + c*x^4])/(4*c^2)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1644

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*(f*b - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2

```
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] || !IntegerQ[m] || !RationalQ[a, b, c, d, e]) && !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\int \frac{x^3 (d + ex^2 + fx^4)}{(a + bx^2 + cx^4)^2} dx = \frac{1}{2} \text{Subst} \left(\int \frac{x (d + ex + fx^2)}{(a + bx + cx^2)^2} dx, x, x^2 \right)$$

$$= \frac{x^2 (2ace - b(cd + af) - (2c^2d - bce + b^2f - 2acf) x^2)}{2c (b^2 - 4ac) (a + bx^2 + cx^4)} - \frac{\text{Subst} \left(\int \frac{2ae - \frac{b(cd+af)}{c} - \frac{(b^2-4ac)}{c}}{a+bx+cx^2} dx, x \right)}{2 (b^2 - 4ac)}$$

$$= \frac{x^2 (2ace - b(cd + af) - (2c^2d - bce + b^2f - 2acf) x^2)}{2c (b^2 - 4ac) (a + bx^2 + cx^4)} + \frac{f \text{Subst} \left(\int \frac{b+2cx}{a+bx+cx^2} dx, x \right)}{4c^2}$$

$$= \frac{x^2 (2ace - b(cd + af) - (2c^2d - bce + b^2f - 2acf) x^2)}{2c (b^2 - 4ac) (a + bx^2 + cx^4)} + \frac{f \log(a + bx^2 + cx^4)}{4c^2} + \dots$$

$$= \frac{x^2 (2ace - b(cd + af) - (2c^2d - bce + b^2f - 2acf) x^2)}{2c (b^2 - 4ac) (a + bx^2 + cx^4)} + \frac{(4ac^2e + b^3f - 2bc(cd + af))}{2c^2 (b^2 - 4ac)}$$

Mathematica [A] time = 0.25, size = 175, normalized size = 1.06

$$\frac{2(-2a^2cf + a(b^2f - bc(e + 3fx^2) + 2c^2(d + ex^2)) + bx^2(b^2f - bce + c^2d))}{(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{2 \tan^{-1} \left(\frac{b + 2cx^2}{\sqrt{4ac - b^2}} \right) (-2bc(3af + cd) + 4ac^2e + b^3f)}{(4ac - b^2)^{3/2}} + f \log(a + bx^2 + cx^4)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*(-2*a^2*c*f + b*(c^2*d - b*c*e + b^2*f)*x^2 + a*(b^2*f + 2*c^2*(d + e*x
^2) - b*c*(e + 3*f*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (2*(4*a*c^
2*e + b^3*f - 2*b*c*(c*d + 3*a*f))*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]
])/(-b^2 + 4*a*c)^(3/2) + f*Log[a + b*x^2 + c*x^4])/(4*c^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 (d + ex^2 + fx^4)}{(a + bx^2 + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^3*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
 [Out] IntegrateAlgebraic[(x^3*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2, x]
fricas [B] time = 1.00, size = 970, normalized size = 5.88

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
 [Out] [1/4*(2*((b^3*c^2 - 4*a*b*c^3)*d - (b^4*c - 6*a*b^2*c^2 + 8*a^2*c^3)*e + (b^5 - 7*a*b^3*c + 12*a^2*b*c^2)*f)*x^2 - (2*a*b*c^2*d - 4*a^2*c^2*e + (2*b*c^3*d - 4*a*c^3*e - (b^3*c - 6*a*b*c^2)*f)*x^4 + (2*b^2*c^2*d - 4*a*b*c^2*e - (b^4 - 6*a*b^2*c)*f)*x^2 - (a*b^3 - 6*a^2*b*c)*f)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + 4*(a*b^2*c^2 - 4*a^2*c^3)*d - 2*(a*b^3*c - 4*a^2*b*c^2)*e + 2*(a*b^4 - 6*a^2*b^2*c + 8*a^3*c^2)*f + ((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*f*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*f*x^2 + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*f)*log(c*x^4 + b*x^2 + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^4 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^2), 1/4*(2*((b^3*c^2 - 4*a*b*c^3)*d - (b^4*c - 6*a*b^2*c^2 + 8*a^2*c^3)*e + (b^5 - 7*a*b^3*c + 12*a^2*b*c^2)*f)*x^2 - 2*(2*a*b*c^2*d - 4*a^2*c^2*e + (2*b*c^3*d - 4*a*c^3*e - (b^3*c - 6*a*b*c^2)*f)*x^4 + (2*b^2*c^2*d - 4*a*b*c^2*e - (b^4 - 6*a*b^2*c)*f)*x^2 - (a*b^3 - 6*a^2*b*c)*f)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + 4*(a*b^2*c^2 - 4*a^2*c^3)*d - 2*(a*b^3*c - 4*a^2*b*c^2)*e + 2*(a*b^4 - 6*a^2*b^2*c + 8*a^3*c^2)*f + ((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*f*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*f*x^2 + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*f)*log(c*x^4 + b*x^2 + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^4 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^2]]

giac [A] time = 1.84, size = 195, normalized size = 1.18

$$\frac{(2bc^2d - b^3f + 6abcf - 4ac^2e) \arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right) + f \log(cx^4 + bx^2 + a) + \frac{2ac^2d + ab^2f - 2a^2cf - abce + (bc^2d + b^3f - 3abcf - b^2ce + 2ac^2e)x^2}{2(cx^4 + bx^2 + a)(b^2 - 4ac)c^2}}{2(b^2c^2 - 4ac^3)\sqrt{-b^2 + 4ac}} + \frac{f \log(cx^4 + bx^2 + a)}{4c^2} + \frac{2ac^2d + ab^2f - 2a^2cf - abce + (bc^2d + b^3f - 3abcf - b^2ce + 2ac^2e)x^2}{2(cx^4 + bx^2 + a)(b^2 - 4ac)c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
 [Out] 1/2*(2*b*c^2*d - b^3*f + 6*a*b*c*f - 4*a*c^2*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^2 - 4*a*c^3)*sqrt(-b^2 + 4*a*c)) + 1/4*f*log(c*x^4 + b*x^2 + a)/c^2 + 1/2*(2*a*c^2*d + a*b^2*f - 2*a^2*c*f - a*b*c*e + (b*c^2*d + b^3*f - 3*a*b*c*f - b^2*c*e + 2*a*c^2*e)*x^2)/((c*x^4 + b*x^2 + a)*(b^2 - 4*a*c)*c^2)

maple [B] time = 0.02, size = 336, normalized size = 2.04

$$\frac{3abf \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right) + 2ae \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right) + b^3f \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right) - bd \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right) + af \ln(cx^4 + bx^2 + a) - b^2f \ln(cx^4 + bx^2 + a) + \frac{(3abcf - 2a^2c^2e - b^3f + b^2ce - b^2d^2)x^2}{(4ac - b^2)^2} + \frac{(2acf - b^2f + bce - 2d^2)a}{(4ac - b^2)^2}}{(4ac - b^2)^{\frac{3}{2}}c} + \frac{2ae \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right) + b^3f \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right) - bd \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right) + af \ln(cx^4 + bx^2 + a) - b^2f \ln(cx^4 + bx^2 + a)}{(4ac - b^2)c} + \frac{(3abcf - 2a^2c^2e - b^3f + b^2ce - b^2d^2)x^2}{4(4ac - b^2)c^2} + \frac{(2acf - b^2f + bce - 2d^2)a}{2cx^4 + 2bx^2 + 2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x)
 [Out] 1/2*((3*a*b*c*f-2*a*c^2*e-b^3*f+b^2*c*e-b*c^2*d)/(4*a*c-b^2)/c^2*x^2+a*(2*a*c*f-b^2*f+b*c*e-2*c^2*d)/(4*a*c-b^2)/c^2)/(c*x^4+b*x^2+a)+1/c/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*a*f-1/4/c^2/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*b^2*f-3/c/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*b*f+2/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*e-1/(4*a*c-b^2)^(3/2)*arctan((2*c

$*x^2+b)/(4*a*c-b^2)^{(1/2)})*b*d+1/2/c^2/(4*a*c-b^2)^{(3/2)}*\arctan((2*c*x^2+b)/(4*a*c-b^2)^{(1/2)})*b^3*f$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 2.72, size = 1651, normalized size = 10.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x)

[Out]
$$- \left(\frac{a(2c^2d + b^2f - 2ac^2f - bc^2e)}{2c^2(4ac - b^2)} + \frac{x^2(b^3f + 2ac^2e + bc^2d - b^2ce - 3ab^2cf)}{2c^2(4ac - b^2)} \right) / (a + b^2x^2 + c^2x^4) - \frac{\log(a + b^2x^2 + c^2x^4)(2b^6f - 128a^3c^3f + 96a^2b^2c^2f - 24ab^4cf)}{2(256a^3c^5 - 4b^6c^2 + 48ab^4c^3 - 192a^2b^2c^4)} - \frac{\operatorname{atan}\left(\frac{(8ac^3(4ac - b^2)^3 - 2b^2c^2(4ac - b^2)^3)((8af + (8ac^2(2b^6f - 128a^3c^3f + 96a^2b^2c^2f - 24ab^4cf)))/(256a^3c^5 - 4b^6c^2 + 48ab^4c^3 - 192a^2b^2c^4))}{(b^3f + 4ac^2e - 2bc^2d - 6ab^2cf)}\right)}{(8c^2(4ac - b^2)^{3/2})} + \frac{a(2b^6f - 128a^3c^3f + 96a^2b^2c^2f - 24ab^4cf)(b^3f + 4ac^2e - 2bc^2d - 6ab^2cf)}{(4ac - b^2)^{3/2}(256a^3c^5 - 4b^6c^2 + 48ab^4c^3 - 192a^2b^2c^4)} / (a(4ac - b^2)) - \frac{x^2(((6b^3c^2f + 8ac^4e - 4bc^4d - 28ab^3cf)/(4ac^3 - b^2c^2) + ((8b^3c^4 - 32ab^2c^5)(2b^6f - 128a^3c^3f + 96a^2b^2c^2f - 24ab^4cf))/(2(4ac^3 - b^2c^2)(256a^3c^5 - 4b^6c^2 + 48ab^4c^3 - 192a^2b^2c^4)))}{(b^3f + 4ac^2e - 2bc^2d - 6ab^2cf)} / (8c^2(4ac - b^2)^{3/2}) + \frac{((8b^3c^4 - 32ab^2c^5)(2b^6f - 128a^3c^3f + 96a^2b^2c^2f - 24ab^4cf)(b^3f + 4ac^2e - 2bc^2d - 6ab^2cf))}{(16c^2(4ac - b^2)^{3/2}(4ac^3 - b^2c^2)(256a^3c^5 - 4b^6c^2 + 48ab^4c^3 - 192a^2b^2c^4))} / (a(4ac - b^2)) + \frac{b((b^3f^2 - 5ab^2cf^2 + 2ac^2ef - bc^2df)/(4ac^3 - b^2c^2) + (((6b^3c^2f + 8ac^4e - 4bc^4d - 28ab^3cf)/(4ac^3 - b^2c^2) + ((8b^3c^4 - 32ab^2c^5)(2b^6f - 128a^3c^3f + 96a^2b^2c^2f - 24ab^4cf))/(2(4ac^3 - b^2c^2)(256a^3c^5 - 4b^6c^2 + 48ab^4c^3 - 192a^2b^2c^4)))}{(b^3f + 4ac^2e - 2bc^2d - 6ab^2cf)^2} / (c^4(4ac - b^2)^3(4ac^3 - b^2c^2)))}{2a(4ac - b^2)^{3/2}} + \frac{b(((8af + (8ac^2(2b^6f - 128a^3c^3f + 96a^2b^2c^2f - 24ab^4cf)))/(256a^3c^5 - 4b^6c^2 + 48ab^4c^3 - 192a^2b^2c^4))}{(2b^6f - 128a^3c^3f + 96a^2b^2c^2f - 24ab^4cf)} / (2(256a^3c^5 - 4b^6c^2 + 48ab^4c^3 - 192a^2b^2c^4)) + \frac{af^2}{c^2} - \frac{a(b^3f + 4ac^2e - 2bc^2d - 6ab^2cf)^2}{c^2(4ac - b^2)^3} / (2a(4ac - b^2)^{3/2})) / (b^6f^2 + 16a^2c^4e^2 + 4b^2c^4d^2 + 36a^2b^2c^2f^2 - 12ab^4cf^2 - 4b^4c^2d^2 + 24ab^2c^3d^2 + 8ab^3c^2e^2 - 48a^2b^2c^3e^2 - 16ab^3c^4d^2) / (b^3f + 4ac^2e - 2bc^2d - 6ab^2cf) / (2c^2(4ac - b^2)^{3/2})$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```


$$3.62 \quad \int \frac{x(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2} dx$$

Optimal. Leaf size=123

$$\frac{-\left(x^2(-2acf + b^2f - bce + 2c^2d)\right) - b(af + cd) + 2ace}{2c(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(2af - be + 2cd)}{(b^2 - 4ac)^{3/2}}$$

Rubi [A] time = 0.18, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {1663, 1660, 12, 618, 206}

$$\frac{x^2\left(-\left(-2acf + b^2f - bce + 2c^2d\right)\right) - b(af + cd) + 2ace}{2c(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(2af - be + 2cd)}{(b^2 - 4ac)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(x*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]

[Out] (2*a*c*e - b*(c*d + a*f) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x^2)/(2*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((2*c*d - b*e + 2*a*f)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1660

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(2*c*f - b*g), x], x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]

Rule 1663

Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^(p, x), x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]

Rubi steps

$$\int \frac{x(d + ex^2 + fx^4)}{(a + bx^2 + cx^4)^2} dx = \frac{1}{2} \text{Subst} \left(\int \frac{d + ex + fx^2}{(a + bx + cx^2)^2} dx, x, x^2 \right)$$

$$= \frac{2ace - b(cd + af) - (2c^2d - bce + b^2f - 2acf)x^2}{2c(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{\text{Subst} \left(\int \frac{2cd - be + 2af}{a + bx + cx^2} dx, x, x^2 \right)}{2(b^2 - 4ac)}$$

$$= \frac{2ace - b(cd + af) - (2c^2d - bce + b^2f - 2acf)x^2}{2c(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{(2cd - be + 2af) \text{Subst} \left(\int \frac{1}{a + bx + cx^2} \right)}{2(b^2 - 4ac)}$$

$$= \frac{2ace - b(cd + af) - (2c^2d - bce + b^2f - 2acf)x^2}{2c(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{(2cd - be + 2af) \text{Subst} \left(\int \frac{1}{b^2 - 4ac - 2bx - 4cx^2} \right)}{b^2 - 4ac}$$

$$= \frac{2ace - b(cd + af) - (2c^2d - bce + b^2f - 2acf)x^2}{2c(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{(2cd - be + 2af) \tanh^{-1} \left(\frac{b + 2cx^2}{\sqrt{b^2 - 4ac}} \right)}{(b^2 - 4ac)^{3/2}}$$

Mathematica [A] time = 0.10, size = 130, normalized size = 1.06

$$\frac{abf - 2ac(e + fx^2) + b^2fx^2 + bc(d - ex^2) + 2c^2dx^2}{2c(4ac - b^2)(a + bx^2 + cx^4)} - \frac{\tan^{-1} \left(\frac{b + 2cx^2}{\sqrt{4ac - b^2}} \right) (-2af + be - 2cd)}{(4ac - b^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (a*b*f + 2*c^2*d*x^2 + b^2*f*x^2 + b*c*(d - e*x^2) - 2*a*c*(e + f*x^2))/(2*c*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)) - ((-2*c*d + b*e - 2*a*f)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(d + ex^2 + fx^4)}{(a + bx^2 + cx^4)^2} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(x*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2, x]
```

fricas [B] time = 0.88, size = 650, normalized size = 5.28

```
[In] integrate(x*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*((2*(b^2*c^2 - 4*a*c^3)*d - (b^3*c - 4*a*b*c^2)*e + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*f)*x^2 + ((2*c^3*d - b*c^2*e + 2*a*c^2*f)*x^4 + 2*a*c^2*d - a*b*c*e + 2*a^2*c*f + (2*b*c^2*d - b^2*c*e + 2*a*b*c*f)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c)))/(c*x^4 + b*x^2 + a)) + (b^3*c - 4*a*b*c^2)*d - 2*(a*b^2*c - 4*a^2*c^2)*e
```

$$+ (a*b^3 - 4*a^2*b*c)*f)/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2), -1/2*((2*(b^2*c^2 - 4*a*c^3)*d - (b^3*c - 4*a*b*c^2)*e + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*f)*x^2 - 2*((2*c^3*d - b*c^2*e + 2*a*c^2*f)*x^4 + 2*a*c^2*d - a*b*c*e + 2*a^2*c*f + (2*b*c^2*d - b^2*c*e + 2*a*b*c*f)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^3*c - 4*a*b*c^2)*d - 2*(a*b^2*c - 4*a^2*c^2)*e + (a*b^3 - 4*a^2*b*c)*f)/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2)]$$

giac [A] time = 2.17, size = 140, normalized size = 1.14

$$\frac{(2cd + 2af - be) \arctan\left(\frac{2cx^2 + b}{\sqrt{-b^2 + 4ac}}\right)}{(b^2 - 4ac)\sqrt{-b^2 + 4ac}} - \frac{2c^2dx^2 + b^2fx^2 - 2acfx^2 - bcx^2e + bcd + abf - 2ace}{2(cx^4 + bx^2 + a)(b^2c - 4ac^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")

[Out] $-(2*c*d + 2*a*f - b*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*c)) - 1/2*(2*c^2*d*x^2 + b^2*f*x^2 - 2*a*c*f*x^2 - b*c*x^2*e + b*c*d + a*b*f - 2*a*c*e)/((c*x^4 + b*x^2 + a)*(b^2*c - 4*a*c^2))$

maple [A] time = 0.01, size = 205, normalized size = 1.67

$$\frac{2af \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}} - \frac{be \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}} + \frac{2cd \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}} + \frac{\frac{(2acf-b^2f+bce-2c^2d)x^2}{(4ac-b^2)c} + \frac{abf-2ace+bcd}{(4ac-b^2)c}}{2cx^4 + 2bx^2 + 2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x)

[Out] $1/2*(-(2*a*c*f-b^2*f+b*c*e-2*c^2*d)/(4*a*c-b^2)/c*x^2+1/c*(a*b*f-2*a*c*e+b*c*d)/(4*a*c-b^2))/(c*x^4+b*x^2+a)+2/(4*a*c-b^2)^{(3/2)}*arctan((2*c*x^2+b)/(4*a*c-b^2)^{(1/2)})*a*f-1/(4*a*c-b^2)^{(3/2)}*arctan((2*c*x^2+b)/(4*a*c-b^2)^{(1/2)})*b*e+2/(4*a*c-b^2)^{(3/2)}*arctan((2*c*x^2+b)/(4*a*c-b^2)^{(1/2)})*c*d$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation may help (example of legal syntax is 'assume(4*a*c-b^2>0)', see 'assume?' for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 0.38, size = 342, normalized size = 2.78

$$\frac{\frac{abf-2ace+bcd}{2c(4ac-b^2)} + \frac{x^2(fb^2-ebc+2d^2-2afc)}{2c(4ac-b^2)}}{cx^4 + bx^2 + a} + \frac{\operatorname{atan}\left(\frac{(4ac-b^2)^4 \left(x^2 \left(\frac{(2c^3d+2a^2f-b^2d)(2af-be+2cd)}{a(4ac-b^2)^{7/2}} + \frac{(2b^3c^2-8abc^3)(b^3-4abc)(2af-be+2cd)^2}{2a(4ac-b^2)^{13/2}} \right) - \frac{2c^2(b^3-4abc)(2af-be+2cd)^2}{(4ac-b^2)^{11/2}} \right)}{8a^2c^2f^2-8abc^2ef+16a^3df+2b^2c^2e^2-8bc^3de+8c^4d^2}}{(4ac-b^2)^{3/2}}\right)}{(4ac-b^2)^{3/2}}(2af-be+2cd)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x)

```
[Out] ((a*b*f - 2*a*c*e + b*c*d)/(2*c*(4*a*c - b^2)) + (x^2*(2*c^2*d + b^2*f - 2*
a*c*f - b*c*e))/(2*c*(4*a*c - b^2)))/(a + b*x^2 + c*x^4) + (atan(((4*a*c -
b^2)^4*(x^2*((2*c^3*d + 2*a*c^2*f - b*c^2*e)*(2*a*f - b*e + 2*c*d))/(a*(4*
a*c - b^2)^(7/2)) + ((2*b^3*c^2 - 8*a*b*c^3)*(b^3 - 4*a*b*c)*(2*a*f - b*e +
2*c*d)^2)/(2*a*(4*a*c - b^2)^(13/2)))) - (2*c^2*(b^3 - 4*a*b*c)*(2*a*f - b*
e + 2*c*d)^2)/(4*a*c - b^2)^(11/2)))/(8*c^4*d^2 + 8*a^2*c^2*f^2 + 2*b^2*c^2
*e^2 + 16*a*c^3*d*f - 8*b*c^3*d*e - 8*a*b*c^2*e*f))*(2*a*f - b*e + 2*c*d))/
(4*a*c - b^2)^(3/2)
```

sympy [B] time = 38.03, size = 474, normalized size = 3.85

$$\frac{\sqrt{\frac{1}{(4ac-b^2)}} (2af - be + 2cd) \log \left(x^2 + \frac{-16a^2c \sqrt{\frac{1}{(4ac-b^2)}} (2af - be + 2cd) + 8a^2c^2 \sqrt{\frac{1}{(4ac-b^2)}} (2af - be + 2cd) - 2af + a^2 \sqrt{\frac{1}{(4ac-b^2)}} (2af - be + 2cd) + 2cd}{4af - 2bc + 4c^2} \right)}{2} + \frac{\sqrt{\frac{1}{(4ac-b^2)}} (2af - be + 2cd) \log \left(x^2 + \frac{-16a^2c \sqrt{\frac{1}{(4ac-b^2)}} (2af - be + 2cd) + 8a^2c^2 \sqrt{\frac{1}{(4ac-b^2)}} (2af - be + 2cd) - 2af + a^2 \sqrt{\frac{1}{(4ac-b^2)}} (2af - be + 2cd) + 2cd}{4af - 2bc + 4c^2} \right)}{2} + \frac{abf - 2ace + bcd + x^2(-2acf + b^2f - be + 2c^2d)}{8a^2c^2 - 2ab^2c + x^2(8ac^3 - 2b^2c^2) + x^2(8ab^2c^2 - 2b^2c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2)**3)*(2*a*f - b*e + 2*c*d)*log(x**2 + (-16*a**2*c**2
*sqrt(-1/(4*a*c - b**2)**3)*(2*a*f - b*e + 2*c*d) + 8*a*b**2*c*sqrt(-1/(4*a
*c - b**2)**3)*(2*a*f - b*e + 2*c*d) + 2*a*b*f - b**4*sqrt(-1/(4*a*c - b**2
)**3)*(2*a*f - b*e + 2*c*d) - b**2*e + 2*b*c*d)/(4*a*c*f - 2*b*c*e + 4*c**2
*d))/2 + sqrt(-1/(4*a*c - b**2)**3)*(2*a*f - b*e + 2*c*d)*log(x**2 + (16*a
**2*c**2*sqrt(-1/(4*a*c - b**2)**3)*(2*a*f - b*e + 2*c*d) - 8*a*b**2*c*sqrt(
-1/(4*a*c - b**2)**3)*(2*a*f - b*e + 2*c*d) + 2*a*b*f + b**4*sqrt(-1/(4*a*c
- b**2)**3)*(2*a*f - b*e + 2*c*d) - b**2*e + 2*b*c*d)/(4*a*c*f - 2*b*c*e +
4*c**2*d))/2 + (a*b*f - 2*a*c*e + b*c*d + x**2*(-2*a*c*f + b**2*f - b*c*e
+ 2*c**2*d))/(8*a**2*c**2 - 2*a*b**2*c + x**4*(8*a*c**3 - 2*b**2*c**2) + x
**2*(8*a*b*c**2 - 2*b**3*c))
```

$$3.63 \quad \int \frac{d+ex^2+fx^4}{x(a+bx^2+cx^4)^2} dx$$

Optimal. Leaf size=166

$$\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(4a^2ce - 2ab(af + 3cd) + b^3d)}{2a^2(b^2 - 4ac)^{3/2}} - \frac{d \log(a + bx^2 + cx^4)}{4a^2} + \frac{d \log(x)}{a^2} + \frac{x^2(abf - 2ace + bcd) - abe}{2a(b^2 - 4ac)(a + bx^2 + cx^4)}$$

Rubi [A] time = 0.39, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {1663, 1646, 800, 634, 618, 206, 628}

$$\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(4a^2ce - 2ab(af + 3cd) + b^3d)}{2a^2(b^2 - 4ac)^{3/2}} - \frac{d \log(a + bx^2 + cx^4)}{4a^2} + \frac{d \log(x)}{a^2} + \frac{x^2(abf - 2ace + bcd) - abe - 2a(cd - af) + b^2d}{2a(b^2 - 4ac)(a + bx^2 + cx^4)}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2 + f*x^4)/(x*(a + b*x^2 + c*x^4)^2), x]

[Out] (b^2*d - a*b*e - 2*a*(c*d - a*f) + (b*c*d - 2*a*c*e + a*b*f)*x^2)/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b^3*d + 4*a^2*c*e - 2*a*b*(3*c*d + a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^2*(b^2 - 4*a*c)^(3/2)) + (d*Log[x])/a^2 - (d*Log[a + b*x^2 + c*x^4])/(4*a^2)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 800

Int((((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{d + ex^2 + fx^4}{x(a + bx^2 + cx^4)^2} dx = \frac{1}{2} \text{Subst} \left(\int \frac{d + ex + fx^2}{x(a + bx + cx^2)^2} dx, x, x^2 \right)$$

$$= \frac{b^2d - abe - 2a(cd - af) + (bcd - 2ace + abf)x^2}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{\text{Subst} \left(\int \frac{-\left(\frac{b^2}{a} - 4c\right)d - \frac{(bcd - 2ace + abf)x}{a}}{x(a + bx + cx^2)} dx, x, x^2 \right)}{2(b^2 - 4ac)}$$

$$= \frac{b^2d - abe - 2a(cd - af) + (bcd - 2ace + abf)x^2}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{\text{Subst} \left(\int \left(\frac{(-b^2 + 4ac)d}{a^2x} + \frac{b^3d + 2a^2ce - ab(3cd + af)}{a^2(a + bx + cx^2)} \right) dx, x, x^2 \right)}{2(b^2 - 4ac)}$$

$$= \frac{b^2d - abe - 2a(cd - af) + (bcd - 2ace + abf)x^2}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{d \log(x)}{a^2} - \frac{\text{Subst} \left(\int \frac{b^3d + 2a^2ce - ab(3cd + af)}{a + bx + cx^2} dx, x, x^2 \right)}{2a^2(b^2 - 4ac)}$$

$$= \frac{b^2d - abe - 2a(cd - af) + (bcd - 2ace + abf)x^2}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{d \log(x)}{a^2} - \frac{d \text{Subst} \left(\int \frac{b + 2cx}{a + bx + cx^2} dx, x, x^2 \right)}{4a^2}$$

$$= \frac{b^2d - abe - 2a(cd - af) + (bcd - 2ace + abf)x^2}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{d \log(x)}{a^2} - \frac{d \log(a + bx^2 + cx^4)}{4a^2} + \frac{\text{Subst} \left(\int \frac{b^3d + 2a^2ce - ab(3cd + af)}{a + bx + cx^2} dx, x, x^2 \right)}{4a^2}$$

$$= \frac{b^2d - abe - 2a(cd - af) + (bcd - 2ace + abf)x^2}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{(b^3d + 4a^2ce - 2ab(3cd + af)) \tan^{-1} \left(\frac{b + 2cx}{\sqrt{b^2 - 4ac}} \right)}{2a^2(b^2 - 4ac)^{3/2}}$$

Mathematica [A] time = 0.45, size = 268, normalized size = 1.61

$$\frac{2a(b(-ae + afx^2 + cdx^2) + 2a(a - c(d + ex^2)) + b^2d)}{(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\log\left(\frac{-\sqrt{b^2 - 4ac} + b + 2cx^2}{4ac(ac - d\sqrt{b^2 - 4ac}) + b^2d\sqrt{b^2 - 4ac} - 2ab(af + 3cd) + b^3d}\right)}{(b^2 - 4ac)^{3/2}} + \frac{\log\left(\frac{\sqrt{b^2 - 4ac} + b + 2cx^2}{-4ac(d\sqrt{b^2 - 4ac} + ae) + b^2d\sqrt{b^2 - 4ac} + 2ab(af + 3cd) + b^3(-d)}\right)}{(b^2 - 4ac)^{3/2}} - 4d \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2 + f*x^4)/(x*(a + b*x^2 + c*x^4)^2), x]
```

```
[Out] -1/4*((-2*a*(b^2*d + b*(-(a*e) + c*d*x^2 + a*f*x^2) + 2*a*(a*f - c*(d + e*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - 4*d*Log[x] + ((b^3*d + b^2*Sqr
```

$t[b^2 - 4ac]d + 4ac(-(\sqrt{b^2 - 4ac})d + ae) - 2ab(3cd + af) \cdot \text{Log}[b - \sqrt{b^2 - 4ac} + 2cx^2]/(b^2 - 4ac)^{3/2} + ((-b^3d) + b^2\sqrt{b^2 - 4ac})d - 4ac(\sqrt{b^2 - 4ac})d + ae + 2ab(3cd + af) \cdot \text{Log}[b + \sqrt{b^2 - 4ac} + 2cx^2]/(b^2 - 4ac)^{3/2})/a^2$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex^2 + fx^4}{x(a + bx^2 + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e*x^2 + f*x^4)/(x*(a + b*x^2 + c*x^4)^2),x]

[Out] IntegrateAlgebraic[(d + e*x^2 + f*x^4)/(x*(a + b*x^2 + c*x^4)^2), x]

fricas [B] time = 3.33, size = 1103, normalized size = 6.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^4+e*x^2+d)/x/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")

[Out] $[1/4*(2*((a*b^3*c - 4*a^2*b*c^2)*d - 2*(a^2*b^2*c - 4*a^3*c^2)*e + (a^2*b^3 - 4*a^3*b*c)*f)*x^2 + (4*a^3*c*e - 2*a^3*b*f + (4*a^2*c^2*e - 2*a^2*b*c*f + (b^3*c - 6*a*b*c^2)*d)*x^4 + (4*a^2*b*c*e - 2*a^2*b^2*f + (b^4 - 6*a*b^2*c)*d)*x^2 + (a*b^3 - 6*a^2*b*c)*d)*\text{sqrt}(b^2 - 4*a*c) \cdot \log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*\text{sqrt}(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + 2*(a*b^4 - 6*a^2*b^2*c + 8*a^3*c^2)*d - 2*(a^2*b^3 - 4*a^3*b*c)*e + 4*(a^3*b^2 - 4*a^4*c)*f - ((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d*x^2 + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*d)*\log(c*x^4 + b*x^2 + a) + 4*((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d*x^2 + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*d)*\log(x)]/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*x^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2), 1/4*(2*((a*b^3*c - 4*a^2*b*c^2)*d - 2*(a^2*b^2*c - 4*a^3*c^2)*e + (a^2*b^3 - 4*a^3*b*c)*f)*x^2 + 2*(4*a^3*c*e - 2*a^3*b*f + (4*a^2*c^2*e - 2*a^2*b*c*f + (b^3*c - 6*a*b*c^2)*d)*x^4 + (4*a^2*b*c*e - 2*a^2*b^2*f + (b^4 - 6*a*b^2*c)*d)*x^2 + (a*b^3 - 6*a^2*b*c)*d)*\text{sqrt}(-b^2 + 4*a*c) \cdot \arctan(-(2*c*x^2 + b)*\text{sqrt}(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + 2*(a*b^4 - 6*a^2*b^2*c + 8*a^3*c^2)*d - 2*(a^2*b^3 - 4*a^3*b*c)*e + 4*(a^3*b^2 - 4*a^4*c)*f - ((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d*x^2 + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*d)*\log(c*x^4 + b*x^2 + a) + 4*((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d*x^2 + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*d)*\log(x)]/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*x^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2)]$

giac [A] time = 2.00, size = 227, normalized size = 1.37

$$\frac{(b^3d - 6abcd - 2a^2bf + 4a^2ce) \arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right) - d \log(cx^4 + bx^2 + a) + \frac{d \log(x^2)}{2a^2} + \frac{b^2cdx^4 - 4acd^2x^4 + b^3d^2x^2 - 2abcdx^2 + 2a^2bf^2x^2 - 4a^2cx^2e + 3ab^2d - 8a^2cd + 4a^3f - 2a^2be}{4(cx^4 + bx^2 + a)(a^2b^2 - 4a^3c)}}{2(a^2b^2 - 4a^3c)\sqrt{-b^2 + 4ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^4+e*x^2+d)/x/(c*x^4+b*x^2+a)^2,x, algorithm="giac")

[Out] $-1/2*(b^3*d - 6*a*b*c*d - 2*a^2*b*f + 4*a^2*c*e) \cdot \arctan((2*c*x^2 + b)/\text{sqrt}(-b^2 + 4*a*c))/((a^2*b^2 - 4*a^3*c)*\text{sqrt}(-b^2 + 4*a*c)) - 1/4*d \cdot \log(c*x^4 + b*x^2 + a)/a^2 + 1/2*d \cdot \log(x^2)/a^2 + 1/4*(b^2*c*d*x^4 - 4*a*c^2*d*x^4 + b^3*d*x^2 - 2*a*b*c*d*x^2 + 2*a^2*b*f*x^2 - 4*a^2*c*x^2*e + 3*a*b^2*d - 8*a^2*c*d + 4*a^3*f - 2*a^2*b*e)/((c*x^4 + b*x^2 + a)*(a^2*b^2 - 4*a^3*c))$

maple [B] time = 0.02, size = 462, normalized size = 2.78

$$\frac{bcx^2}{2(cx^2+bx^2+a)(4ac-b^2)^2} - \frac{bf x^2}{2(cx^2+bx^2+a)(4ac-b^2)} + \frac{cx^2}{(cx^2+bx^2+a)(4ac-b^2)} - \frac{3bcf \arctan\left(\frac{2cx^2+b}{4ac-b^2}\right)}{(4ac-b^2)^2 a} + \frac{b^2 f \arctan\left(\frac{2cx^2+b}{4ac-b^2}\right)}{2(4ac-b^2)^2 c} - \frac{bf \arctan\left(\frac{2cx^2+b}{4ac-b^2}\right)}{(4ac-b^2)^2} + \frac{2bc \arctan\left(\frac{2cx^2+b}{4ac-b^2}\right)}{(4ac-b^2)^2} - \frac{af}{(cx^2+bx^2+a)(4ac-b^2)} - \frac{bd}{2(cx^2+bx^2+a)(4ac-b^2)} - \frac{cd \ln(cx^2+bx^2+a)}{(4ac-b^2)a} + \frac{b^2 d \ln(cx^2+bx^2+a)}{4(4ac-b^2)^2} + \frac{bc}{2(cx^2+bx^2+a)(4ac-b^2)} + \frac{cd}{(cx^2+bx^2+a)(4ac-b^2)} + \frac{d \ln(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^4+e*x^2+d)/x/(c*x^4+b*x^2+a)^2,x)

[Out] $d*\ln(x)/a^2 - 1/2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*b*f + c/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*e - 1/2/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*b*c*d - a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*f + 1/2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*b*e + 1/(c*x^4+b*x^2+a)/(4*a*c-b^2)*c*d - 1/2/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*b^2*d - 1/a/(4*a*c-b^2)*c*\ln(c*x^4+b*x^2+a)*d + 1/4/a^2/(4*a*c-b^2)*\ln(c*x^4+b*x^2+a)*b^2*d - 1/(4*a*c-b^2)^{(3/2)}*arctan((2*c*x^2+b)/(4*a*c-b^2)^{(1/2)})*b*f + 2/(4*a*c-b^2)^{(3/2)}*arctan((2*c*x^2+b)/(4*a*c-b^2)^{(1/2)})*c*e - 3/a/(4*a*c-b^2)^{(3/2)}*arctan((2*c*x^2+b)/(4*a*c-b^2)^{(1/2)})*b*c*d + 1/2/a^2/(4*a*c-b^2)^{(3/2)}*arctan((2*c*x^2+b)/(4*a*c-b^2)^{(1/2)})*b^3*d$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^4+e*x^2+d)/x/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 11.85, size = 8706, normalized size = 52.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x^2 + f*x^4)/(x*(a + b*x^2 + c*x^4)^2),x)

[Out] $(d*\log(x))/a^2 - ((b^2*d + 2*a^2*f - a*b*e - 2*a*c*d)/(2*a*(4*a*c - b^2)) + (x^2*(a*b*f - 2*a*c*e + b*c*d))/(2*a*(4*a*c - b^2)))/(a + b*x^2 + c*x^4) - (\log((((d + a^2*(-(b^3*d - 2*a^2*b*f + 4*a^2*c*e - 6*a*b*c*d)^2/(a^4*(4*a*c - b^2)^3)))^{(1/2)})*(((d + a^2*(-(b^3*d - 2*a^2*b*f + 4*a^2*c*e - 6*a*b*c*d)^2/(a^4*(4*a*c - b^2)^3))^{(1/2)})*((2*c^2*x^2*(20*a^2*c^2*e + 4*a*b^3*f - b^3*c*d + 10*a*b*c^2*d - 8*a*b^2*c*e - 10*a^2*b*c*f))/(a*(4*a*c - b^2)) + (b*c^2*(d + a^2*(-(b^3*d - 2*a^2*b*f + 4*a^2*c*e - 6*a*b*c*d)^2/(a^4*(4*a*c - b^2)^3))^{(1/2)})*(a*b + 3*b^2*x^2 - 10*a*c*x^2))/a^2 - (4*b*c^2*(b^3*d - a^2*b*f + 2*a^2*c*e - 5*a*b*c*d))/(a*(4*a*c - b^2)))))/(4*a^2) + (c^2*(a^3*b^2*f^2 - 4*b^4*c*d^2 + 4*a^3*c^2*e^2 + 17*a*b^2*c^2*d^2 - 4*a*b^4*d*f - 36*a^2*b*c^2*d*e + 18*a^2*b^2*c*d*f + 8*a*b^3*c*d*e - 4*a^3*b*c*e*f))/(a^2*(4*a*c - b^2)^2) - (c^2*x^2*(a^2*b^3*f^2 + 6*b^3*c^2*d^2 + 4*a^2*b*c^2*e^2 - 20*a*b*c^3*d^2 + 40*a^2*c^3*d*e - 14*a*b^2*c^2*d*e - 20*a^2*b*c^2*d*f - 4*a^2*b^2*c*e*f + 7*a*b^3*c*d*f))/(a^2*(4*a*c - b^2)^2)))/(4*a^2) - (c^2*x^2*(a*b*f - 2*a*c*e + b*c*d)^3)/(a^3*(4*a*c - b^2)^3) + (c^2*d*(a*b*f - 2*a*c*e + b*c*d)^2)/(a^3*(4*a*c - b^2)^2)*(((d - a^2*(-(b^3*d - 2*a^2*b*f + 4*a^2*c*e - 6*a*b*c*d)^2/(a^4*(4*a*c - b^2)^3))^{(1/2)})*(((d - a^2*(-(b^3*d - 2*a^2*b*f + 4*a^2*c*e - 6*a*b*c*d)^2/(a^4*(4*a*c - b^2)^3))^{(1/2)})*((2*c^2*x^2*(20*a^2*c^2*e + 4*a*b^3*f - b^3*c*d + 10*a*b*c^2*d - 8*a*b^2*c*e - 10*a^2*b*c*f))/(a*(4*a*c - b^2)) + (b*c^2*(d - a^2*(-(b^3*d - 2*a^2*b*f + 4*a^2*c*e - 6*a*b*c*d)^2/(a^4*(4*a*c - b^2)^3))^{(1/2)})*(a*b + 3*b^2*x^2 - 10*a*c*x^2))/a^2 - (4*b*c^2*(b^3*d - a^2*b*f + 2*a^2*c*e - 5*a*b*c*d))/(a*(4*a*c - b^2)))))/(4*a^2) + (c^2*(a^3*b^2*f^2 - 4*b^4*c*d^2 + 4*a^3*c^2*e^2 + 17*a*b^2*c^2*d^2 - 4*a*b^4*d*f - 36*a^2*b*c^2*d*e + 18*a^2*b^2*c*d*f + 8*a*b^3*c*d*e - 4*a^3*b*c*e*f -$

$$\begin{aligned}
& 4a^3bc^2ef)/(a^2(4ac - b^2)^2) - (c^2x^2(a^2b^3f^2 + 6b^3c^2d^2 + 4a^2b^2c^2e^2 - 20abc^3d^2 + 40a^2c^3de - 14ab^2c^2de \\
& - 20a^2bc^2df - 4a^2b^2c^2ef + 7ab^3cdf)/(a^2(4ac - b^2)^2)))/(4a^2) - (c^2x^2(abf - 2ace + bcd)^3)/(a^3(4ac - b^2)^3) + \\
& (c^2d(abf - 2ace + bcd)^2)/(a^3(4ac - b^2)^2)) * (2b^6d - 12a^3c^3d + 96a^2b^2c^2d - 24ab^4cd)/(2(4a^2b^6 - 256a^5c^3 \\
& - 48a^3b^4c + 192a^4b^2c^2)) - (\operatorname{atan}((x^2(((b^3c^5d^3 - 8a^3c^5e^3 + a^3b^3c^2f^3 - 6ab^2c^5d^2e + 12a^2b^2c^5de^2 + 3ab^3c^4d^2f + 12a^3b^2c^4e^2f + 3a^2b^3c^3df^2 - 6a^3b^2c^3ef^2 - \\
& 12a^2b^2c^4d^2ef)/(a^3b^6 - 64a^6c^3 - 12a^4b^4c + 48a^5b^2c^2) - (((6ab^5c^4d^2 + 80a^3b^2c^6d^2 - 16a^4b^2c^5e^2 - 44a^2b^3c^5d^2 + 4a^3b^3c^4e^2 + a^3b^5c^2f^2 - 4a^4b^3c^3f^2 - 160a^4c^6d^2e + 80a^4b^2c^5d^2f - 14a^2b^4c^4d^2e + 96a^3b^2c^5d^2e + 7a^2b^5c^3d^2f - 48a^3b^3c^4d^2f - 4a^3b^4c^3ef + 16a^4b^2c^4ef)/(a^3b^6 - 64a^6c^3 - 12a^4b^4c + 48a^5b^2c^2) + (((640a^6c^6e - 2a^2b^7c^3d + 36a^3b^5c^4d - 192a^4b^3c^5d - 16a^3b^6c^3e + 168a^4b^4c^4e - 576a^5b^2c^5e + 8a^3b^7c^2f - 84a^4b^5c^3f + 288a^5b^3c^4f + 320a^5b^2c^6d - 320a^6b^2c^5f)/(a^3b^6 - 64a^6c^3 - 12a^4b^4c + 48a^5b^2c^2) - ((2b^6d - 12a^3c^3d + 96a^2b^2c^2d - 24ab^4cd)*(2560a^7b^2c^6 + 12a^3b^9c^2 - 184a^4b^7c^3 + 1056a^5b^5c^4 - 2688a^6b^3c^5))/(2(a^3b^6 - 64a^6c^3 - 12a^4b^4c + 48a^5b^2c^2)) * (4a^2b^6 - 256a^5c^3 - 48a^3b^4c + 192a^4b^2c^2))) * (2b^6d - 128a^3c^3d + 96a^2b^2c^2d - 24ab^4cd)) / (2(4a^2b^6 - 256a^5c^3 - 48a^3b^4c + 192a^4b^2c^2))) * (2b^6d - 128a^3c^3d + 96a^2b^2c^2d - 24ab^4cd)) / (2(4a^2b^6 - 256a^5c^3 - 48a^3b^4c + 192a^4b^2c^2)) + (((((640a^6c^6e - 2a^2b^7c^3d + 36a^3b^5c^4d - 192a^4b^3c^5d - 16a^3b^6c^3e + 168a^4b^4c^4e - 576a^5b^2c^5e + 8a^3b^7c^2f - 84a^4b^5c^3f + 288a^5b^3c^4f + 320a^5b^2c^6d - 320a^6b^2c^5f)/(a^3b^6 - 64a^6c^3 - 12a^4b^4c + 48a^5b^2c^2) - ((2b^6d - 128a^3c^3d + 96a^2b^2c^2d - 24ab^4cd)*(2560a^7b^2c^6 + 12a^3b^9c^2 - 184a^4b^7c^3 + 1056a^5b^5c^4 - 2688a^6b^3c^5))/(2(a^3b^6 - 64a^6c^3 - 12a^4b^4c + 48a^5b^2c^2)) * (4a^2b^6 - 256a^5c^3 - 48a^3b^4c + 192a^4b^2c^2))) * (b^3d - 2a^2b^2f + 4a^2c^2e - 6abc^2d))/(4a^2(4ac - b^2)^(3/2)) - ((2b^6d - 128a^3c^3d + 96a^2b^2c^2d - 24ab^4cd)*(b^3d - 2a^2b^2f + 4a^2c^2e - 6abc^2d)*(2560a^7b^2c^6 + 12a^3b^9c^2 - 184a^4b^7c^3 + 1056a^5b^5c^4 - 2688a^6b^3c^5))/(8a^2(4ac - b^2)^(3/2)*(a^3b^6 - 64a^6c^3 - 12a^4b^4c + 48a^5b^2c^2)) * (4a^2b^6 - 256a^5c^3 - 48a^3b^4c + 192a^4b^2c^2))) * (b^3d - 2a^2b^2f + 4a^2c^2e - 6abc^2d))/(4a^2(4ac - b^2)^(3/2)) - ((2b^6d - 128a^3c^3d + 96a^2b^2c^2d - 24ab^4cd)*(b^3d - 2a^2b^2f + 4a^2c^2e - 6abc^2d)^2*(2560a^7b^2c^6 + 12a^3b^9c^2 - 184a^4b^7c^3 + 1056a^5b^5c^4 - 2688a^6b^3c^5))/(32a^4(4ac - b^2)^3*(a^3b^6 - 64a^6c^3 - 12a^4b^4c + 48a^5b^2c^2)) * (4a^2b^6 - 256a^5c^3 - 48a^3b^4c + 192a^4b^2c^2))) * (3b^5d - a^2b^3f - 2a^3c^2e - 21ab^3cd + a^3b^2cf + 33a^2b^2c^2d + 2a^2b^2c^2e))/(8a^3c^2(4ac - b^2)^3*(400a^3c^3d^2 - 6b^6d^2 + a^4b^2f^2 + 4a^4c^2e^2 - 291a^2b^2c^2d^2 + 72ab^4cd^2 - a^2b^4df + 2a^2b^3c^2de - 12a^3b^2c^2de + 6a^3b^2c^2df - 4a^4b^2c^2ef)) + (((((((640a^6c^6e - 2a^2b^7c^3d + 36a^3b^5c^4d - 192a^4b^3c^5d - 16a^3b^6c^3e + 168a^4b^4c^4e - 576a^5b^2c^5e + 8a^3b^7c^2f - 84a^4b^5c^3f + 288a^5b^3c^4f + 320a^5b^2c^6d - 320a^6b^2c^5f)/(a^3b^6 - 64a^6c^3 - 12a^4b^4c + 48a^5b^2c^2) - ((2b^6d - 128a^3c^3d + 96a^2b^2c^2d - 24ab^4cd)*(2560a^7b^2c^6 + 12a^3b^9c^2 - 184a^4b^7c^3 + 1056a^5b^5c^4 - 2688a^6b^3c^5)) / (2(a^3b^6 - 64a^6c^3 - 12a^4b^4c + 48a^5b^2c^2)) * (4a^2b^6 - 256a^5c^3 - 48a^3b^4c + 192a^4b^2c^2))) * (b^3d - 2a^2b^2f + 4a^2c^2e - 6abc^2d))/(4a^2(4ac - b^2)^(3/2)) - ((2b^6d - 128a^3c^3d + 96a^2b^2c^2d - 24ab^4cd)*(b^3d - 2a^2b^2f + 4a^2c^2e - 6abc^2d) * (2560a^7b^2c^6 + 12a^3b^9c^2 - 184a^4b^7c^3 + 1056a^5b^5c^4 - 268
\end{aligned}$$

$$\begin{aligned}
& 8*a^6*b^3*c^5)) / ((8*a^2*(4*a*c - b^2)^{(3/2)}*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2))) * (2*b^6*d - 128*a^3*c^3*d + 96*a^2*b^2*c^2*d - 24*a*b^4*c*d)) / (2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)) + (((6*a*b^5*c^4*d^2 + 80*a^3*b*c^6*d^2 - 16*a^4*b*c^5*e^2 - 44*a^2*b^3*c^5*d^2 + 4*a^3*b^3*c^4*e^2 + a^3*b^5*c^2*f^2 - 4*a^4*b^3*c^3*f^2 - 160*a^4*c^6*d*e + 80*a^4*b*c^5*d*f - 14*a^2*b^4*c^4*d*e + 96*a^3*b^2*c^5*d*e + 7*a^2*b^5*c^3*d*f - 48*a^3*b^3*c^4*d*f - 4*a^3*b^4*c^3*e*f + 16*a^4*b^2*c^4*e*f) / (a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)) + (((640*a^6*c^6*e - 2*a^2*b^7*c^3*d + 36*a^3*b^5*c^4*d - 192*a^4*b^3*c^5*d - 16*a^3*b^6*c^3*e + 168*a^4*b^4*c^4*e - 576*a^5*b^2*c^5*e + 8*a^3*b^7*c^2*f - 84*a^4*b^5*c^3*f + 288*a^5*b^3*c^4*f + 320*a^5*b*c^6*d - 320*a^6*b*c^5*f) / (a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)) - ((2*b^6*d - 128*a^3*c^3*d + 96*a^2*b^2*c^2*d - 24*a*b^4*c*d)*(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^6*b^3*c^5)) / (2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)) * (4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2))) * (2*b^6*d - 128*a^3*c^3*d + 96*a^2*b^2*c^2*d - 24*a*b^4*c*d)) / (2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2))) * (b^3*d - 2*a^2*b*f + 4*a^2*c*e - 6*a*b*c*d)) / (4*a^2*(4*a*c - b^2)^{(3/2)}) + ((b^3*d - 2*a^2*b*f + 4*a^2*c*e - 6*a*b*c*d)^3*(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^6*b^3*c^5)) / (64*a^6*(4*a*c - b^2)^{(9/2)}*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2))) * (96*b^6*d - 1280*a^3*c^3*d - 32*a^2*b^4*f + 2208*a^2*b^2*c^2*d - 864*a*b^4*c*d + 64*a^2*b^3*c*e - 192*a^3*b*c^2*e + 96*a^3*b^2*c*f)) / (256*a^3*c^2*(4*a*c - b^2)^{(7/2)}*(400*a^3*c^3*d^2 - 6*b^6*d^2 + a^4*b^2*f^2 + 4*a^4*c^2*e^2 - 291*a^2*b^2*c^2*d^2 + 72*a*b^4*c*d^2 - a^2*b^4*d*f + 2*a^2*b^3*c*d*e - 12*a^3*b*c^2*d*e + 6*a^3*b^2*c*d*f - 4*a^4*b*c*e*f)) * (16*a^6*b^6*(4*a*c - b^2)^{(9/2)} - 1024*a^9*c^3*(4*a*c - b^2)^{(9/2)} - 192*a^7*b^4*c*(4*a*c - b^2)^{(9/2)} + 768*a^8*b^2*c^2*(4*a*c - b^2)^{(9/2))) / (16*a^4*c^4*e^2 + b^6*c^2*d^2 - 12*a*b^4*c^3*d^2 + 36*a^2*b^2*c^4*d^2 + 4*a^4*b^2*c^2*f^2 - 48*a^3*b*c^4*d*e - 16*a^4*b*c^3*e*f + 8*a^2*b^3*c^3*d*e - 4*a^2*b^4*c^2*d*f + 24*a^3*b^2*c^3*d*f) + (((16*a^6*b^6*(4*a*c - b^2)^{(9/2)} - 1024*a^9*c^3*(4*a*c - b^2)^{(9/2)} - 192*a^7*b^4*c*(4*a*c - b^2)^{(9/2)} + 768*a^8*b^2*c^2*(4*a*c - b^2)^{(9/2))) * ((b^2*c^4*d^3 + 4*a^2*c^4*d*e^2 - 4*a*b*c^4*d^2*e + 2*a*b^2*c^3*d^2*f + a^2*b^2*c^2*d*f^2 - 4*a^2*b*c^3*d*e*f) / (a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + (((4*a^4*c^4*e^2 - 4*a*b^4*c^3*d^2 + 17*a^2*b^2*c^4*d^2 + a^4*b^2*c^2*f^2 - 36*a^3*b*c^4*d*e - 4*a^4*b*c^3*e*f + 8*a^2*b^3*c^3*d*e - 4*a^2*b^4*c^2*d*f + 18*a^3*b^2*c^3*d*f) / (a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + (((4*a^2*b^6*c^2*d - 36*a^3*b^4*c^3*d + 80*a^4*b^2*c^4*d + 8*a^4*b^3*c^3*e - 4*a^4*b^4*c^2*f + 16*a^5*b^2*c^3*f - 32*a^5*b*c^4*e) / (a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + ((4*a^4*b^6*c^2 - 32*a^5*b^4*c^3 + 64*a^6*b^2*c^4)*(2*b^6*d - 128*a^3*c^3*d + 96*a^2*b^2*c^2*d - 24*a*b^4*c*d)) / (2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)) * (4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2))) * (2*b^6*d - 128*a^3*c^3*d + 96*a^2*b^2*c^2*d - 24*a*b^4*c*d)) / (2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2))) * (2*b^6*d - 128*a^3*c^3*d + 96*a^2*b^2*c^2*d - 24*a*b^4*c*d)) / (2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)) - (((((4*a^2*b^6*c^2*d - 36*a^3*b^4*c^3*d + 80*a^4*b^2*c^4*d + 8*a^4*b^3*c^3*e - 4*a^4*b^4*c^2*f + 16*a^5*b^2*c^3*f - 32*a^5*b*c^4*e) / (a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + ((4*a^4*b^6*c^2 - 32*a^5*b^4*c^3 + 64*a^6*b^2*c^4)*(2*b^6*d - 128*a^3*c^3*d + 96*a^2*b^2*c^2*d - 24*a*b^4*c*d)) / (2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)) * (4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2))) * (b^3*d - 2*a^2*b*f + 4*a^2*c*e - 6*a*b*c*d)) / (8*a^2*(4*a*c - b^2)^{(3/2)}*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)) * (4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2))) * (b^3*d - 2*a^2*b*f + 4*a^2*c*e - 6*a*b*c*d)) / (4*a^2*(4*a*c - b^2)^{(3/2)}) - ((4*a^4*b^6*c^2 - 32*a^5*b^4*c^3 + 64*a^6*b^2*c^4)*(2*b^6*d - 128*a^3*c^3*d + 96*a^2*b^2*c^2*d - 24*a*b^4*c*d)*(b^3*d - 2*a^2*b*f + 4*a^2*c*e - 6*a*b*c*d)^2) / (32*a^4*(
\end{aligned}$$

$$\begin{aligned}
& 4*a*c - b^2)^3*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)*(4*a^2*b^6 - 256*a^5*c^3 \\
& - 48*a^3*b^4*c + 192*a^4*b^2*c^2))*((3*b^5*d - a^2*b^3*f - 2*a^3*c^2*e - \\
& 21*a*b^3*c*d + a^3*b*c*f + 33*a^2*b*c^2*d + 2*a^2*b^2*c*e))/(8*a^3*c^2*(4*a \\
& *c - b^2)^3*(16*a^4*c^4*e^2 + b^6*c^2*d^2 - 12*a*b^4*c^3*d^2 + 36*a^2*b^2*c \\
& ^4*d^2 + 4*a^4*b^2*c^2*f^2 - 48*a^3*b*c^4*d*e - 16*a^4*b*c^3*e*f + 8*a^2*b^ \\
& 3*c^3*d*e - 4*a^2*b^4*c^2*d*f + 24*a^3*b^2*c^3*d*f)*(400*a^3*c^3*d^2 - 6*b^ \\
& 6*d^2 + a^4*b^2*f^2 + 4*a^4*c^2*e^2 - 291*a^2*b^2*c^2*d^2 + 72*a*b^4*c*d^2 \\
& - a^2*b^4*d*f + 2*a^2*b^3*c*d*e - 12*a^3*b*c^2*d*e + 6*a^3*b^2*c*d*f - 4*a^ \\
& 4*b*c*e*f)) - ((((((4*a^2*b^6*c^2*d - 36*a^3*b^4*c^3*d + 80*a^4*b^2*c^4*d \\
& + 8*a^4*b^3*c^3*e - 4*a^4*b^4*c^2*f + 16*a^5*b^2*c^3*f - 32*a^5*b*c^4*e)/(a \\
& ^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + ((4*a^4*b^6*c^2 - 32*a^5*b^4*c^3 + 64* \\
& a^6*b^2*c^4)*(2*b^6*d - 128*a^3*c^3*d + 96*a^2*b^2*c^2*d - 24*a*b^4*c*d))/(\\
& 2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^ \\
& 4*c + 192*a^4*b^2*c^2)))*(b^3*d - 2*a^2*b*f + 4*a^2*c*e - 6*a*b*c*d))/(4*a^ \\
& 2*(4*a*c - b^2)^(3/2)) + ((4*a^4*b^6*c^2 - 32*a^5*b^4*c^3 + 64*a^6*b^2*c^4) \\
& *(2*b^6*d - 128*a^3*c^3*d + 96*a^2*b^2*c^2*d - 24*a*b^4*c*d)*(b^3*d - 2*a^2 \\
& *b*f + 4*a^2*c*e - 6*a*b*c*d))/(8*a^2*(4*a*c - b^2)^(3/2)*(a^3*b^4 + 16*a^5 \\
& *c^2 - 8*a^4*b^2*c)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c \\
& ^2)))*(2*b^6*d - 128*a^3*c^3*d + 96*a^2*b^2*c^2*d - 24*a*b^4*c*d))/(2*(4*a^ \\
& 2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)) + (((4*a^4*c^4*e^2 - \\
& 4*a*b^4*c^3*d^2 + 17*a^2*b^2*c^4*d^2 + a^4*b^2*c^2*f^2 - 36*a^3*b*c^4*d*e \\
& - 4*a^4*b*c^3*e*f + 8*a^2*b^3*c^3*d*e - 4*a^2*b^4*c^2*d*f + 18*a^3*b^2*c^3* \\
& d*f)/(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + (((4*a^2*b^6*c^2*d - 36*a^3*b^4 \\
& *c^3*d + 80*a^4*b^2*c^4*d + 8*a^4*b^3*c^3*e - 4*a^4*b^4*c^2*f + 16*a^5*b^2* \\
& c^3*f - 32*a^5*b*c^4*e)/(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + ((4*a^4*b^6* \\
& c^2 - 32*a^5*b^4*c^3 + 64*a^6*b^2*c^4)*(2*b^6*d - 128*a^3*c^3*d + 96*a^2*b^ \\
& 2*c^2*d - 24*a*b^4*c*d))/(2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)*(4*a^2*b^6 \\
& - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*b^6*d - 128*a^3*c^3*d \\
& + 96*a^2*b^2*c^2*d - 24*a*b^4*c*d))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b \\
& ^4*c + 192*a^4*b^2*c^2)))*(b^3*d - 2*a^2*b*f + 4*a^2*c*e - 6*a*b*c*d))/(4*a \\
& ^2*(4*a*c - b^2)^(3/2)) - ((4*a^4*b^6*c^2 - 32*a^5*b^4*c^3 + 64*a^6*b^2*c^4) \\
& *(b^3*d - 2*a^2*b*f + 4*a^2*c*e - 6*a*b*c*d)^3)/(64*a^6*(4*a*c - b^2)^(9/2) \\
& *(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c))*((16*a^6*b^6*(4*a*c - b^2)^(9/2) - \\
& 1024*a^9*c^3*(4*a*c - b^2)^(9/2) - 192*a^7*b^4*c*(4*a*c - b^2)^(9/2) + 768* \\
& a^8*b^2*c^2*(4*a*c - b^2)^(9/2))*(96*b^6*d - 1280*a^3*c^3*d - 32*a^2*b^4*f \\
& + 2208*a^2*b^2*c^2*d - 864*a*b^4*c*d + 64*a^2*b^3*c*e - 192*a^3*b*c^2*e + 9 \\
& 6*a^3*b^2*c*f))/(256*a^3*c^2*(4*a*c - b^2)^(7/2)*(16*a^4*c^4*e^2 + b^6*c^2* \\
& d^2 - 12*a*b^4*c^3*d^2 + 36*a^2*b^2*c^4*d^2 + 4*a^4*b^2*c^2*f^2 - 48*a^3*b* \\
& c^4*d*e - 16*a^4*b*c^3*e*f + 8*a^2*b^3*c^3*d*e - 4*a^2*b^4*c^2*d*f + 24*a^3 \\
& *b^2*c^3*d*f)*(400*a^3*c^3*d^2 - 6*b^6*d^2 + a^4*b^2*f^2 + 4*a^4*c^2*e^2 - \\
& 291*a^2*b^2*c^2*d^2 + 72*a*b^4*c*d^2 - a^2*b^4*d*f + 2*a^2*b^3*c*d*e - 12*a \\
& ^3*b*c^2*d*e + 6*a^3*b^2*c*d*f - 4*a^4*b*c*e*f)))*(b^3*d - 2*a^2*b*f + 4*a^ \\
& 2*c*e - 6*a*b*c*d))/(2*a^2*(4*a*c - b^2)^(3/2))
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**4+e*x**2+d)/x/(c*x**4+b*x**2+a)**2,x)

[Out] Timed out

$$3.64 \quad \int \frac{d+ex^2+fx^4}{x^3(a+bx^2+cx^4)^2} dx$$

Optimal. Leaf size=234

$$\frac{(2bd - ae) \log(a + bx^2 + cx^4)}{4a^3} - \frac{\log(x)(2bd - ae)}{a^3} - \frac{2a^2ce + cx^2(-abe - 2a(cd - af) + b^2d) - ab^2e - ab(3cd - af)}{2a^2(b^2 - 4ac)(a + bx^2 + cx^4)}$$

Rubi [A] time = 0.73, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 30, number of rules / integrand size = 0.233, Rules used = {1663, 1646, 1628, 634, 618, 206, 628}

$$\frac{2a^2ce + cx^2(-abe - 2a(cd - af) + b^2d) - ab^2e - ab(3cd - af) + b^3d}{2a^2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(6a^2bce + 4a^2c(3cd - af) - 12ab^2cd - ab^3e + 2b^4d)}{2a^3(b^2 - 4ac)^{3/2}} + \frac{(2bd - ae) \log(a + bx^2 + cx^4)}{4a^3} - \frac{\log(x)(2bd - ae)}{a^3} - \frac{d}{2a^2x^2}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2 + f*x^4)/(x^3*(a + b*x^2 + c*x^4)^2), x]

[Out] -d/(2*a^2*x^2) - (b^3*d - a*b^2*e + 2*a^2*c*e - a*b*(3*c*d - a*f) + c*(b^2*d - a*b*e - 2*a*(c*d - a*f))*x^2)/(2*a^2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((2*b^4*d - 12*a*b^2*c*d - a*b^3*e + 6*a^2*b*c*e + 4*a^2*c*(3*c*d - a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^3*(b^2 - 4*a*c)^(3/2)) - ((2*b*d - a*e)*Log[x])/a^3 + ((2*b*d - a*e)*Log[a + b*x^2 + c*x^4])/(4*a^3)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1628

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 1646

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x

```

^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
+ 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m*
- ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x]] /; FreeQ[{a, b, c, d,
e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]

```

Rule 1663

```

Int[(Pq_)*(x_)^(m_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]

```

Rubi steps

$$\begin{aligned}
\int \frac{d + ex^2 + fx^4}{x^3(a + bx^2 + cx^4)^2} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{d + ex + fx^2}{x^2(a + bx + cx^2)^2} dx, x, x^2 \right) \\
&= -\frac{b^3d - ab^2e + 2a^2ce - ab(3cd - af) + c(b^2d - abe - 2a(cd - af))x^2}{2a^2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{\text{Subst} \left(\int \frac{d + ex + fx^2}{x^2(a + bx + cx^2)^2} dx, x, x^2 \right)}{2a^2(b^2 - 4ac)(a + bx^2 + cx^4)} \\
&= -\frac{b^3d - ab^2e + 2a^2ce - ab(3cd - af) + c(b^2d - abe - 2a(cd - af))x^2}{2a^2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{\text{Subst} \left(\int \frac{d + ex + fx^2}{x^2(a + bx + cx^2)^2} dx, x, x^2 \right)}{2a^2(b^2 - 4ac)(a + bx^2 + cx^4)} \\
&= -\frac{d}{2a^2x^2} - \frac{b^3d - ab^2e + 2a^2ce - ab(3cd - af) + c(b^2d - abe - 2a(cd - af))x^2}{2a^2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{\text{Subst} \left(\int \frac{d + ex + fx^2}{x^2(a + bx + cx^2)^2} dx, x, x^2 \right)}{2a^2(b^2 - 4ac)(a + bx^2 + cx^4)} \\
&= -\frac{d}{2a^2x^2} - \frac{b^3d - ab^2e + 2a^2ce - ab(3cd - af) + c(b^2d - abe - 2a(cd - af))x^2}{2a^2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{\text{Subst} \left(\int \frac{d + ex + fx^2}{x^2(a + bx + cx^2)^2} dx, x, x^2 \right)}{2a^2(b^2 - 4ac)(a + bx^2 + cx^4)} \\
&= -\frac{d}{2a^2x^2} - \frac{b^3d - ab^2e + 2a^2ce - ab(3cd - af) + c(b^2d - abe - 2a(cd - af))x^2}{2a^2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{\text{Subst} \left(\int \frac{d + ex + fx^2}{x^2(a + bx + cx^2)^2} dx, x, x^2 \right)}{2a^2(b^2 - 4ac)(a + bx^2 + cx^4)} \\
&= -\frac{d}{2a^2x^2} - \frac{b^3d - ab^2e + 2a^2ce - ab(3cd - af) + c(b^2d - abe - 2a(cd - af))x^2}{2a^2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{\text{Subst} \left(\int \frac{d + ex + fx^2}{x^2(a + bx + cx^2)^2} dx, x, x^2 \right)}{2a^2(b^2 - 4ac)(a + bx^2 + cx^4)}
\end{aligned}$$

Mathematica [A] time = 0.66, size = 403, normalized size = 1.72

$$\frac{\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)\left(4c^2\left(\sqrt{b^2-4ac}-ef+3cd\right)-d^2\left(\sqrt{b^2-4ac}+12cd\right)+2ab\left(3ac-4d\sqrt{b^2-4ac}\right)+d^2\left(2d\sqrt{b^2-4ac}-ad\right)+2d^3d\right)}{(b^2-4ac)^{3/2}} + \frac{\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)\left(4c^2\left(\sqrt{b^2-4ac}+ef-3cd\right)+d^2\left(12cd-\sqrt{b^2-4ac}\right)-2ab\left(4d\sqrt{b^2-4ac}+3ac\right)+d^2\left(2d\sqrt{b^2-4ac}+ad\right)-2d^3d\right)}{(b^2-4ac)^{3/2}} - \frac{2a\left(b^2cd^2-ad\right)+ab\left(ef-\left(3d+ef\right)\right)+2ac\left(\left(ef+3d\right)^2-cd^2\right)+d^3d}{(b^2-4ac)\left(ef+3d+ef+3d\right)} + 4\log(x)(ad-2bd) - \frac{2ad}{x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2 + f*x^4)/(x^3*(a + b*x^2 + c*x^4)^2), x]
```

```
[Out] ((-2*a*d)/x^2 - (2*a*(b^3*d + b^2*(-(a*e) + c*d*x^2) + a*b*(a*f - c*(3*d + e*x^2)) + 2*a*c*(-(c*d*x^2) + a*(e + f*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + 4*(-2*b*d + a*e)*Log[x] + ((2*b^4*d + b^3*(2*sqrt[b^2 - 4*a*c]*d
```

- a*e) + 2*a*b*c*(-4*Sqrt[b^2 - 4*a*c]*d + 3*a*e) - a*b^2*(12*c*d + Sqrt[b^2 - 4*a*c]*e) + 4*a^2*c*(3*c*d + Sqrt[b^2 - 4*a*c]*e - a*f))*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2]/(b^2 - 4*a*c)^(3/2) + ((-2*b^4*d + b^3*(2*Sqrt[b^2 - 4*a*c]*d + a*e) - 2*a*b*c*(4*Sqrt[b^2 - 4*a*c]*d + 3*a*e) + a*b^2*(12*c*d - Sqrt[b^2 - 4*a*c]*e) + 4*a^2*c*(-3*c*d + Sqrt[b^2 - 4*a*c]*e + a*f))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2]/(b^2 - 4*a*c)^(3/2))/(4*a^3)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex^2 + fx^4}{x^3 (a + bx^2 + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e*x^2 + f*x^4)/(x^3*(a + b*x^2 + c*x^4)^2),x]

[Out] IntegrateAlgebraic[(d + e*x^2 + f*x^4)/(x^3*(a + b*x^2 + c*x^4)^2), x]

fricas [B] time = 7.27, size = 1764, normalized size = 7.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^4+e*x^2+d)/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")

[Out] [-1/4*(2*(2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d - (a^2*b^3*c - 4*a^3*b*c^2)*e + 2*(a^3*b^2*c - 4*a^4*c^2)*f)*x^4 + 2*((2*a*b^5 - 15*a^2*b^3*c + 2*8*a^3*b*c^2)*d - (a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*e + (a^3*b^3 - 4*a^4*b*c)*f)*x^2 + ((4*a^3*c^2*f - 2*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d + (a*b^3*c - 6*a^2*b*c^2)*e)*x^6 + (4*a^3*b*c*f - 2*(b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d + (a*b^4 - 6*a^2*b^2*c)*e)*x^4 + (4*a^4*c*f - 2*(a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*d + (a^2*b^3 - 6*a^3*b*c)*e)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + 2*(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*d - ((2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d - (a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*e)*x^6 + (2*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d - (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*e)*x^4 + (2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d - (a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*e)*x^2)*log(c*x^4 + b*x^2 + a) + 4*((2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d - (a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*e)*x^6 + (2*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d - (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*e)*x^4 + (2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d - (a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*e)*x^2)*log(x))/((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*x^6 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^4 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*x^2), -1/4*(2*(2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d - (a^2*b^3*c - 4*a^3*b*c^2)*e + 2*(a^3*b^2*c - 4*a^4*c^2)*f)*x^4 + 2*((2*a*b^5 - 15*a^2*b^3*c + 2*8*a^3*b*c^2)*d - (a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*e + (a^3*b^3 - 4*a^4*b*c)*f)*x^2 - 2*((4*a^3*c^2*f - 2*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d + (a*b^3*c - 6*a^2*b*c^2)*e)*x^6 + (4*a^3*b*c*f - 2*(b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d + (a*b^4 - 6*a^2*b^2*c)*e)*x^4 + (4*a^4*c*f - 2*(a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*d + (a^2*b^3 - 6*a^3*b*c)*e)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + 2*(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*d - ((2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d - (a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*e)*x^6 + (2*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d - (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*e)*x^4 + (2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d - (a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*e)*x^2)*log(x))/((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*x^6 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^4 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*x^2)]

giac [A] time = 1.85, size = 287, normalized size = 1.23

$$\frac{(2bd - 12a^2cd + 12a^2c^2d - 4a^3cf - ab^3e + 6a^2bce) \arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right) - 2b^2cdx^4 - 6ac^2dx^4 + 2a^2cfx^4 - abcx^4e + 2b^3dx^2 - 7abcdx^2 + a^2bfx^2 - ab^2x^2e + 2a^2cx^2e + ab^2d - 4a^2cd + \frac{(2bd - ae) \log(cx^4 + bx^2 + a)}{4a^3} - \frac{(2bd - ae) \log(x^2)}{2a^3}}{2(a^2b^2 - 4a^4c)\sqrt{-b^2 + 4ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^4+e*x^2+d)/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="giac")

[Out] $\frac{1}{2}*(2*b^4*d - 12*a*b^2*c*d + 12*a^2*c^2*d - 4*a^3*c*f - a*b^3*e + 6*a^2*b*c*e)*\arctan((2*c*x^2 + b)/\sqrt{-b^2 + 4*a*c})/((a^3*b^2 - 4*a^4*c)*\sqrt{-b^2 + 4*a*c}) - \frac{1}{2}*(2*b^2*c*d*x^4 - 6*a*c^2*d*x^4 + 2*a^2*c*f*x^4 - a*b*c*x^4*e + 2*b^3*d*x^2 - 7*a*b*c*d*x^2 + a^2*b*f*x^2 - a*b^2*x^2*e + 2*a^2*c*x^2*e + a*b^2*d - 4*a^2*c*d)/((c*x^6 + b*x^4 + a*x^2)*(a^2*b^2 - 4*a^3*c)) + \frac{1}{4}*(2*b*d - a*e)*\log(c*x^4 + b*x^2 + a)/a^3 - \frac{1}{2}*(2*b*d - a*e)*\log(x^2)/a^3$

maple [B] time = 0.02, size = 722, normalized size = 3.09

$$\frac{bd}{2(a^2b^2-4a^4c)\sqrt{-b^2+4ac}} - \frac{2b^2cdx^4-6ac^2dx^4+2a^2cfx^4-abcx^4e+2b^3dx^2-7abcdx^2+a^2bfx^2-ab^2x^2e+2a^2cx^2e+ab^2d-4a^2cd}{2(a^2b^2-4a^4c)\sqrt{-b^2+4ac}} + \frac{(2bd-ae)\log(cx^4+bx^2+a)}{4a^3} - \frac{(2bd-ae)\log(x^2)}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^4+e*x^2+d)/x^3/(c*x^4+b*x^2+a)^2,x)

[Out] $-\frac{1}{2}d/a^2/x^2 + \frac{1}{a^2} \ln(x) * e - \frac{2}{a^3} \ln(x) * b * d + \frac{1}{(c*x^4+b*x^2+a)*c} / (4*a*c-b^2) * x^2 * f - \frac{1}{2} / a / (c*x^4+b*x^2+a) * c / (4*a*c-b^2) * x^2 * b * e - \frac{1}{a} / (c*x^4+b*x^2+a) * c^2 / (4*a*c-b^2) * x^2 * d + \frac{1}{2} / a^2 / (c*x^4+b*x^2+a) * c / (4*a*c-b^2) * x^2 * b^2 * d + \frac{1}{2} / (c*x^4+b*x^2+a) / (4*a*c-b^2) * b * f + \frac{1}{(c*x^4+b*x^2+a) / (4*a*c-b^2) * c * e} - \frac{1}{2} / a / (c*x^4+b*x^2+a) / (4*a*c-b^2) * b^2 * e - \frac{3}{2} / a / (c*x^4+b*x^2+a) / (4*a*c-b^2) * b * c * d + \frac{1}{2} / a^2 / (c*x^4+b*x^2+a) / (4*a*c-b^2) * b^3 * d - \frac{1}{a} / (4*a*c-b^2) * c * \ln(c*x^4+b*x^2+a) * e + \frac{1}{4} / a^2 / (4*a*c-b^2) * \ln(c*x^4+b*x^2+a) * b^2 * e + \frac{2}{a^2} / (4*a*c-b^2) * c * \ln(c*x^4+b*x^2+a) * b * d - \frac{1}{2} / a^3 / (4*a*c-b^2) * \ln(c*x^4+b*x^2+a) * b^3 * d + \frac{2}{(4*a*c-b^2)^{(3/2)}} * \arctan((2*c*x^2+b) / (4*a*c-b^2)^{(1/2)}) * c * f - \frac{3}{a} / (4*a*c-b^2)^{(3/2)} * \arctan((2*c*x^2+b) / (4*a*c-b^2)^{(1/2)}) * b * c * e - \frac{6}{a} / (4*a*c-b^2)^{(3/2)} * \arctan((2*c*x^2+b) / (4*a*c-b^2)^{(1/2)}) * c^2 * d + \frac{1}{2} / a^2 / (4*a*c-b^2)^{(3/2)} * \arctan((2*c*x^2+b) / (4*a*c-b^2)^{(1/2)}) * b^3 * e + \frac{6}{a^2} / (4*a*c-b^2)^{(3/2)} * \arctan((2*c*x^2+b) / (4*a*c-b^2)^{(1/2)}) * b^2 * c * d - \frac{1}{a^3} / (4*a*c-b^2)^{(3/2)} * \arctan((2*c*x^2+b) / (4*a*c-b^2)^{(1/2)}) * b^4 * d$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^4+e*x^2+d)/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details) Is 4*a*c-b^2 positive or negative?

mupad [B] time = 12.98, size = 11879, normalized size = 50.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x^2 + f*x^4)/(x^3*(a + b*x^2 + c*x^4)^2),x)

[Out] $((x^2*(2*b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 7*a*b*c*d))/(2*a^2*(4*a*c - b^2)) - d/(2*a) + (c*x^4*(2*b^2*d + 2*a^2*f - a*b*e - 6*a*c*d))/(2*a^2*(4*a*c - b^2)))/(a*x^2 + b*x^4 + c*x^6) + (\log(x)*(a*e - 2*b*d))/a^3 + (\log(($

$$\begin{aligned}
& \left(\left((a^2e - 2b^2d + a^3(-2b^4d + 12a^2c^2d - ab^3e - 4a^3cf - 12a^2cd + 6a^2bce))^2 / (a^6(4ac - b^2)^3) \right)^{1/2} \right) \left((2c^3x^2(2b^4d - 60a^2c^2d - 8a^2b^2f - ab^3e + 20a^3cf + 4ab^2cd + 10a^2bce)) / (a^2(4ac - b^2)) + (4b^2c^2(2b^4d + 6a^2c^2d - ab^3e - 2a^3cf - 10ab^2cd + 5a^2bce)) / (a^2(4ac - b^2)) + (b^2c^2(ae - 2bd + a^3(-2b^4d + 12a^2c^2d - ab^3e - 4a^3cf - 12a^2cd + 6a^2bce))^2 / (a^6(4ac - b^2)^3) \right)^{1/2} \right) (ab + 3b^2x^2 - 10acx^2) / a^3 \\
& + (c^3(4a^5cf^2 - 16b^6d^2 - 4a^2b^4e^2 + 36a^3c^3d^2 + 17a^3b^2ce^2 + 16ab^5d^2e - 216a^2b^2c^2d^2 + 116ab^4cd^2 - 16a^2b^4d^2f + 8a^3b^3e^2f - 24a^4c^2d^2f - 92a^2b^3cd^2e + 108a^3b^2cd^2e + 72a^3b^2cd^2f - 36a^4b^2c^2e^2f)) / (a^4(4ac - b^2)^2) - (2c^4x^2(12b^5d^2 + 2a^4b^2f^2 + 3a^2b^3e^2 + 138a^2b^2c^2d^2 - 12ab^4d^2e + 20a^4c^2e^2f - 82ab^3cd^2 - 10a^3b^2ce^2 + 14a^2b^3d^2f - 60a^3c^2d^2e - 7a^3b^2e^2f + 61a^2b^2cd^2e - 52a^3b^2cd^2f)) / (a^4(4ac - b^2)^2) (ae - 2bd + a^3(-2b^4d + 12a^2c^2d - ab^3e - 4a^3cf - 12a^2cd + 6a^2bce))^2 / (a^6(4ac - b^2)^3) \right)^{1/2} \\
& + (c^4(ae - 2bd)(2b^2d + 2a^2f - abe - 6ac^2d)^2) / (a^6(4ac - b^2)^2) + (c^5x^2(2b^2d + 2a^2f - abe - 6ac^2d)^3) / (a^6(4ac - b^2)^3) \left((2b^2d - ae + a^3(-2b^4d + 12a^2c^2d - ab^3e - 4a^3cf - 12a^2cd + 6a^2bce))^2 / (a^6(4ac - b^2)^3) \right)^{1/2} \\
& \left((2c^3x^2(2b^4d - 60a^2c^2d - 8a^2b^2f - ab^3e + 20a^3cf + 4ab^2cd + 10a^2bce)) / (a^2(4ac - b^2)) + (4b^2c^2(2b^4d + 6a^2c^2d - ab^3e - 2a^3cf - 10ab^2cd + 5a^2bce)) / (a^2(4ac - b^2)) - (b^2c^2(2bd - ae + a^3(-2b^4d + 12a^2c^2d - ab^3e - 4a^3cf - 12a^2cd + 6a^2bce))^2 / (a^6(4ac - b^2)^3) \right)^{1/2} \\
& (ab + 3b^2x^2 - 10acx^2) / a^3 - (c^3(4a^5cf^2 - 16b^6d^2 - 4a^2b^4e^2 + 36a^3c^3d^2 + 17a^3b^2ce^2 + 16ab^5d^2e - 216a^2b^2c^2d^2 + 116ab^4cd^2 - 16a^2b^4d^2f + 8a^3b^3e^2f - 24a^4c^2d^2f - 92a^2b^3cd^2e + 108a^3b^2cd^2e + 72a^3b^2cd^2f - 36a^4b^2c^2e^2f)) / (a^4(4ac - b^2)^2) + (2c^4x^2(12b^5d^2 + 2a^4b^2f^2 + 3a^2b^3e^2 + 138a^2b^2c^2d^2 - 12ab^4d^2e + 20a^4c^2e^2f - 82ab^3cd^2 - 10a^3b^2ce^2 + 14a^2b^3d^2f - 60a^3c^2d^2e - 7a^3b^2e^2f + 61a^2b^2cd^2e - 52a^3b^2cd^2f)) / (a^4(4ac - b^2)^2) (2bd - ae + a^3(-2b^4d + 12a^2c^2d - ab^3e - 4a^3cf - 12a^2cd + 6a^2bce))^2 / (a^6(4ac - b^2)^3) \right)^{1/2} \\
& + (c^4(ae - 2bd)(2b^2d + 2a^2f - abe - 6ac^2d)^2) / (a^6(4ac - b^2)^2) + (c^5x^2(2b^2d + 2a^2f - abe - 6ac^2d)^3) / (a^6(4ac - b^2)^3) \left((4b^7d + 128a^4c^3e - 2ab^6e + 192a^2b^3c^2d - 96a^3b^2c^2e - 48ab^5cd - 256a^3b^3cd + 24a^2b^4c^2e) \right) / (2(4a^3b^6 - 256a^6c^3 - 48a^4b^4c + 192a^5b^2c^2)) + \left(\operatorname{atan}\left(x^2 \left((216a^3c^8d^3 - 8b^6c^5d^3 - 8a^6c^5f^3 + 72ab^4c^6d^3 - 216a^4c^7d^2f + 72a^5c^6d^2f^2 - 216a^2b^2c^7d^3 + a^3b^3c^5e^3 + 12ab^5c^5d^2e + 108a^3b^3c^7d^2e + 12a^5b^3c^5e^2f - 72a^2b^3c^6d^2e - 6a^2b^4c^5d^2e + 18a^3b^2c^6d^2e - 24a^2b^4c^5d^2f + 144a^3b^2c^6d^2f - 24a^4b^2c^5d^2f - 6a^4b^2c^5e^2f - 72a^4b^2c^6d^2e + 24a^3b^3c^5d^2e) \right) \right) / (a^6b^6 - 64a^9c^3 - 12a^7b^4c + 48a^8b^2c^2) + \left((80a^6b^3c^6e^2 - 1104a^5b^3c^7d^2 - 16a^7b^3c^5f^2 + 24a^2b^7c^4d^2 - 260a^3b^5c^5d^2 + 932a^4b^3c^6d^2 + 6a^4b^5c^4e^2 - 44a^5b^3c^5e^2 + 4a^6b^3c^4f^2 + 480a^6c^7d^2e - 160a^7c^6e^2f + 416a^6b^3c^6d^2f - 24a^3b^6c^4d^2e + 218a^4b^4c^5d^2e - 608a^5b^2c^6d^2e + 28a^4b^5c^4d^2f - 216a^5b^3c^5d^2f - 14a^5b^4c^4e^2f + 96a^6b^2c^5e^2f) \right) / (a^6b^6 - 64a^9c^3 - 12a^7b^4c + 48a^8b^2c^2) + \left((1920a^8c^7d - 640a^9c^6f - 4a^4b^8c^3d + 24a^5b^6c^4d + 120a^6b^4c^5d - 1088a^7b^2c^6d + 2a^5b^7c^3e - 36a^6b^5c^4e + 192a^7b^3c^5e + 16a^6b^6c^3f - 168a^7b^4c^4f + 576a^8b^2c^5f - 320a^8b^3c^6e) \right) / (a^6b^6 - 64a^9c^3 - 12a^7b^4c + 48a^8b^2c^2) - \left((2560a^10b^3c^6 + 12a^6b^9c^2 - 184a^7b^7c^3 + 1056a^8b^5c^4 - 2688a^9b^3c^5) \right) (4b^7d + 128a^4c^3e - 2ab^6e + 192a^2b^3c^2d - 96a^3b^2c^2e - 48ab^5cd - 256a^3b^3cd + 24a^2b^4c^2e) / (2
\end{aligned}$$

$$\begin{aligned}
& (a^6b^6 - 64a^9c^3 - 12a^7b^4c + 48a^8b^2c^2)(4a^3b^6 - 256a^6c^3 - 48a^4b^4c + 192a^5b^2c^2)) * (4b^7d + 128a^4c^3e - 2ab^6e + 192a^2b^3c^2d - 96a^3b^2c^2e - 48ab^5cd - 256a^3b^3cd + 24a^2b^4c^2e) / (2(4a^3b^6 - 256a^6c^3 - 48a^4b^4c + 192a^5b^2c^2)) * (4b^7d + 128a^4c^3e - 2ab^6e + 192a^2b^3c^2d - 96a^3b^2c^2e - 48ab^5cd - 256a^3b^3cd + 24a^2b^4c^2e) / (2(4a^3b^6 - 256a^6c^3 - 48a^4b^4c + 192a^5b^2c^2)) - (((((1920a^8c^7d - 640a^9c^6f - 4a^4b^8c^3d + 24a^5b^6c^4d + 120a^6b^4c^5d - 1088a^7b^2c^6d + 2a^5b^7c^3e - 36a^6b^5c^4e + 192a^7b^3c^5e + 16a^6b^6c^3f - 168a^7b^4c^4f + 576a^8b^2c^5f - 320a^8b^3c^6e) / (a^6b^6 - 64a^9c^3 - 12a^7b^4c + 48a^8b^2c^2) - ((2560a^10b^6c^6 + 12a^6b^9c^2 - 184a^7b^7c^3 + 1056a^8b^5c^4 - 2688a^9b^3c^5) * (4b^7d + 128a^4c^3e - 2ab^6e + 192a^2b^3c^2d - 96a^3b^2c^2e - 48ab^5cd - 256a^3b^3cd + 24a^2b^4c^2e)) / (2(a^6b^6 - 64a^9c^3 - 12a^7b^4c + 48a^8b^2c^2) * (4a^3b^6 - 256a^6c^3 - 48a^4b^4c + 192a^5b^2c^2))) * (2b^4d + 12a^2c^2d - ab^3e - 4a^3cf - 12ab^2cd + 6a^2b^2c^2e) / (4a^3(4ac - b^2)^(3/2)) - ((2560a^10b^6c^6 + 12a^6b^9c^2 - 184a^7b^7c^3 + 1056a^8b^5c^4 - 2688a^9b^3c^5) * (2b^4d + 12a^2c^2d - ab^3e - 4a^3cf - 12ab^2cd + 6a^2b^2c^2e) * (4b^7d + 128a^4c^3e - 2ab^6e + 192a^2b^3c^2d - 96a^3b^2c^2e - 48ab^5cd - 256a^3b^3cd + 24a^2b^4c^2e)) / (8a^3(4ac - b^2)^(3/2)) * (a^6b^6 - 64a^9c^3 - 12a^7b^4c + 48a^8b^2c^2) * (4a^3b^6 - 256a^6c^3 - 48a^4b^4c + 192a^5b^2c^2))) * (2b^4d + 12a^2c^2d - ab^3e - 4a^3cf - 12ab^2cd + 6a^2b^2c^2e) / (4a^3(4ac - b^2)^(3/2)) + ((2560a^10b^6c^6 + 12a^6b^9c^2 - 184a^7b^7c^3 + 1056a^8b^5c^4 - 2688a^9b^3c^5) * (2b^4d + 12a^2c^2d - ab^3e - 4a^3cf - 12ab^2cd + 6a^2b^2c^2e) * (4b^7d + 128a^4c^3e - 2ab^6e + 192a^2b^3c^2d - 96a^3b^2c^2e - 48ab^5cd - 256a^3b^3cd + 24a^2b^4c^2e)) / (32a^6(4ac - b^2)^3(a^6b^6 - 64a^9c^3 - 12a^7b^4c + 48a^8b^2c^2) * (4a^3b^6 - 256a^6c^3 - 48a^4b^4c + 192a^5b^2c^2))) * (6a^3c^3d - 6b^6d - 2a^4c^2f + 3ab^5e - 72a^2b^2c^2d + 42ab^4cd - 21a^2b^3c^2e + 33a^3b^3c^2e + 2a^3b^2c^2f) / (8a^3c^2(4ac - b^2)^3(36a^4c^4d^2 - 6a^2b^6e^2 - 24b^8d^2 + 400a^5c^3e^2 + 4a^6c^2f^2 + 72a^3b^4c^2e^2 + 24ab^7d^2e - 1152a^2b^4c^2d^2 + 1528a^3b^2c^3d^2 - 291a^4b^2c^2e^2 + 288ab^6c^2d^2 - 24a^5c^3d^2f - 288a^2b^5c^2d^2e - 1564a^4b^3c^3d^2e - 4a^3b^4c^2d^2f + 2a^4b^3c^2e^2f - 12a^5b^3c^2e^2f + 1158a^3b^3c^2d^2e + 24a^4b^2c^2d^2f)) + (((((((1920a^8c^7d - 640a^9c^6f - 4a^4b^8c^3d + 24a^5b^6c^4d + 120a^6b^4c^5d - 1088a^7b^2c^6d + 2a^5b^7c^3e - 36a^6b^5c^4e + 192a^7b^3c^5e + 16a^6b^6c^3f - 168a^7b^4c^4f + 576a^8b^2c^5f - 320a^8b^3c^6e) / (a^6b^6 - 64a^9c^3 - 12a^7b^4c + 48a^8b^2c^2) - ((2560a^10b^6c^6 + 12a^6b^9c^2 - 184a^7b^7c^3 + 1056a^8b^5c^4 - 2688a^9b^3c^5) * (4b^7d + 128a^4c^3e - 2ab^6e + 192a^2b^3c^2d - 96a^3b^2c^2e - 48ab^5cd - 256a^3b^3cd + 24a^2b^4c^2e)) / (2(a^6b^6 - 64a^9c^3 - 12a^7b^4c + 48a^8b^2c^2) * (4a^3b^6 - 256a^6c^3 - 48a^4b^4c + 192a^5b^2c^2))) * (2b^4d + 12a^2c^2d - ab^3e - 4a^3cf - 12ab^2cd + 6a^2b^2c^2e) / (4a^3(4ac - b^2)^(3/2)) - ((2560a^10b^6c^6 + 12a^6b^9c^2 - 184a^7b^7c^3 + 1056a^8b^5c^4 - 2688a^9b^3c^5) * (2b^4d + 12a^2c^2d - ab^3e - 4a^3cf - 12ab^2cd + 6a^2b^2c^2e) * (4b^7d + 128a^4c^3e - 2ab^6e + 192a^2b^3c^2d - 96a^3b^2c^2e - 48ab^5cd - 256a^3b^3cd + 24a^2b^4c^2e)) / (8a^3(4ac - b^2)^(3/2)) * (a^6b^6 - 64a^9c^3 - 12a^7b^4c + 48a^8b^2c^2) * (4a^3b^6 - 256a^6c^3 - 48a^4b^4c + 192a^5b^2c^2))) * (4b^7d + 128a^4c^3e - 2ab^6e + 192a^2b^3c^2d - 96a^3b^2c^2e - 48ab^5cd - 256a^3b^3cd + 24a^2b^4c^2e) / (2(4a^3b^6 - 256a^6c^3 - 48a^4b^4c + 192a^5b^2c^2)) + (((80a^6b^6c^6e^2 - 1104a^5b^6c^7d^2 - 16a^7b^6c^5f^2 + 24a^2b^7c^4d^2 - 260a^3b^5c^5d^2 + 932a^4b^3c^6d^2 + 6a^4b^5c^4e^2 - 44a^5b^3c^5e^2 + 4a^6b^3c^4f^2 + 480a^6c^7d^2e - 160a^7c^6e^2f + 416a^6b^6c^6d^2f - 24a^3b^6c^4d^2e + 218a^4b^4c^5
\end{aligned}$$

$$\begin{aligned}
& *d*e - 608*a^5*b^2*c^6*d*e + 28*a^4*b^5*c^4*d*f - 216*a^5*b^3*c^5*d*f - 14* \\
& a^5*b^4*c^4*e*f + 96*a^6*b^2*c^5*e*f)/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c \\
& + 48*a^8*b^2*c^2) + (((1920*a^8*c^7*d - 640*a^9*c^6*f - 4*a^4*b^8*c^3*d + 2 \\
& 4*a^5*b^6*c^4*d + 120*a^6*b^4*c^5*d - 1088*a^7*b^2*c^6*d + 2*a^5*b^7*c^3*e \\
& - 36*a^6*b^5*c^4*e + 192*a^7*b^3*c^5*e + 16*a^6*b^6*c^3*f - 168*a^7*b^4*c^4 \\
& *f + 576*a^8*b^2*c^5*f - 320*a^8*b*c^6*e)/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c \\
& + 48*a^8*b^2*c^2) - ((2560*a^10*b*c^6 + 12*a^6*b^9*c^2 - 184*a^7*b^7*c^3 \\
& + 1056*a^8*b^5*c^4 - 2688*a^9*b^3*c^5)*(4*b^7*d + 128*a^4*c^3*e - 2*a*b^6 \\
& *e + 192*a^2*b^3*c^2*d - 96*a^3*b^2*c^2*e - 48*a*b^5*c*d - 256*a^3*b*c^3*d \\
& + 24*a^2*b^4*c*e))/(2*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 \\
&))*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2))*(4*b^7*d + 1 \\
& 28*a^4*c^3*e - 2*a*b^6*e + 192*a^2*b^3*c^2*d - 96*a^3*b^2*c^2*e - 48*a*b^5* \\
& c*d - 256*a^3*b*c^3*d + 24*a^2*b^4*c*e))/(2*(4*a^3*b^6 - 256*a^6*c^3 - 48*a \\
& ^4*b^4*c + 192*a^5*b^2*c^2))*(2*b^4*d + 12*a^2*c^2*d - a*b^3*e - 4*a^3*c*f \\
& - 12*a*b^2*c*d + 6*a^2*b*c*e))/(4*a^3*(4*a*c - b^2)^(3/2)) + ((2560*a^10*b \\
& *c^6 + 12*a^6*b^9*c^2 - 184*a^7*b^7*c^3 + 1056*a^8*b^5*c^4 - 2688*a^9*b^3*c \\
& ^5)*(2*b^4*d + 12*a^2*c^2*d - a*b^3*e - 4*a^3*c*f - 12*a*b^2*c*d + 6*a^2*b* \\
& c*e)^3)/(64*a^9*(4*a*c - b^2)^(9/2)*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + \\
& 48*a^8*b^2*c^2)))*(768*b^7*d + 5120*a^4*c^3*e - 384*a*b^6*e + 18432*a^2*b^3 \\
& *c^2*d - 8832*a^3*b^2*c^2*e - 6912*a*b^5*c*d - 12544*a^3*b*c^3*d + 3456*a^2 \\
& *b^4*c*e - 256*a^3*b^3*c*f + 768*a^4*b*c^2*f))/(1024*a^3*c^2*(4*a*c - b^2)^(7/2) \\
& *(36*a^4*c^4*d^2 - 6*a^2*b^6*e^2 - 24*b^8*d^2 + 400*a^5*c^3*e^2 + 4*a^6 \\
& *c^2*f^2 + 72*a^3*b^4*c*e^2 + 24*a*b^7*d*e - 1152*a^2*b^4*c^2*d^2 + 1528*a \\
& ^3*b^2*c^3*d^2 - 291*a^4*b^2*c^2*e^2 + 288*a*b^6*c*d^2 - 24*a^5*c^3*d*f - 2 \\
& 88*a^2*b^5*c*d*e - 1564*a^4*b*c^3*d*e - 4*a^3*b^4*c*d*f + 2*a^4*b^3*c*e*f - \\
& 12*a^5*b*c^2*e*f + 1158*a^3*b^3*c^2*d*e + 24*a^4*b^2*c^2*d*f))*(16*a^9*b^6 \\
& *(4*a*c - b^2)^(9/2) - 1024*a^12*c^3*(4*a*c - b^2)^(9/2) - 192*a^10*b^4*c* \\
& (4*a*c - b^2)^(9/2) + 768*a^11*b^2*c^2*(4*a*c - b^2)^(9/2)))/(144*a^4*c^6*d \\
& ^2 + 4*b^8*c^2*d^2 + 16*a^6*c^4*f^2 - 48*a*b^6*c^3*d^2 + 192*a^2*b^4*c^4*d^2 \\
& - 288*a^3*b^2*c^5*d^2 + a^2*b^6*c^2*e^2 - 12*a^3*b^4*c^3*e^2 + 36*a^4*b^2 \\
& *c^4*e^2 - 96*a^5*c^5*d*f - 4*a*b^7*c^2*d*e + 144*a^4*b*c^5*d*e - 48*a^5*b* \\
& c^4*e*f + 48*a^2*b^5*c^3*d*e - 168*a^3*b^3*c^4*d*e - 16*a^3*b^4*c^3*d*f + 9 \\
& 6*a^4*b^2*c^4*d*f + 8*a^4*b^3*c^3*e*f) - ((16*a^9*b^6*(4*a*c - b^2)^(9/2) - \\
& 1024*a^12*c^3*(4*a*c - b^2)^(9/2) - 192*a^10*b^4*c*(4*a*c - b^2)^(9/2) + 7 \\
& 68*a^11*b^2*c^2*(4*a*c - b^2)^(9/2))*((8*b^5*c^4*d^3 - 48*a*b^3*c^5*d^3 + 7 \\
& 2*a^2*b*c^6*d^3 - 36*a^3*c^6*d^2*e - 4*a^5*c^4*e*f^2 - a^3*b^2*c^4*e^3 + 24 \\
& *a^4*c^5*d*e*f - 12*a*b^4*c^4*d^2*e - 12*a^3*b*c^5*d*e^2 - 48*a^3*b*c^5*d^2 \\
& *f + 8*a^4*b*c^4*d*f^2 + 4*a^4*b*c^4*e^2*f + 48*a^2*b^2*c^5*d^2*e + 6*a^2*b \\
& ^3*c^4*d*e^2 + 16*a^2*b^3*c^4*d^2*f - 16*a^3*b^2*c^4*d*e*f)/(a^6*b^4 + 16*a \\
& ^8*c^2 - 8*a^7*b^2*c) + (((36*a^5*c^6*d^2 + 4*a^7*c^4*f^2 - 16*a^2*b^6*c^3* \\
& d^2 + 116*a^3*b^4*c^4*d^2 - 216*a^4*b^2*c^5*d^2 - 4*a^4*b^4*c^3*e^2 + 17*a^5 \\
& *b^2*c^4*e^2 - 24*a^6*c^5*d*f + 108*a^5*b*c^5*d*e - 36*a^6*b*c^4*e*f + 16* \\
& a^3*b^5*c^3*d*e - 92*a^4*b^3*c^4*d*e - 16*a^4*b^4*c^3*d*f + 72*a^5*b^2*c^4* \\
& d*f + 8*a^5*b^3*c^3*e*f)/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) - (((72*a^5*b \\
& ^5*c^3*d - 8*a^4*b^7*c^2*d - 184*a^6*b^3*c^4*d + 4*a^5*b^6*c^2*e - 36*a^6*b \\
& ^4*c^3*e + 80*a^7*b^2*c^4*e + 8*a^7*b^3*c^3*f + 96*a^7*b*c^5*d - 32*a^8*b*c \\
& ^4*f)/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) - ((4*a^7*b^6*c^2 - 32*a^8*b^4*c \\
& ^3 + 64*a^9*b^2*c^4)*(4*b^7*d + 128*a^4*c^3*e - 2*a*b^6*e + 192*a^2*b^3*c^2 \\
& *d - 96*a^3*b^2*c^2*e - 48*a*b^5*c*d - 256*a^3*b*c^3*d + 24*a^2*b^4*c*e))/(\\
& 2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^ \\
& 4*c + 192*a^5*b^2*c^2))*(4*b^7*d + 128*a^4*c^3*e - 2*a*b^6*e + 192*a^2*b^3 \\
& *c^2*d - 96*a^3*b^2*c^2*e - 48*a*b^5*c*d - 256*a^3*b*c^3*d + 24*a^2*b^4*c*e \\
&))/(2*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2))*(4*b^7*d \\
& + 128*a^4*c^3*e - 2*a*b^6*e + 192*a^2*b^3*c^2*d - 96*a^3*b^2*c^2*e - 48*a* \\
& b^5*c*d - 256*a^3*b*c^3*d + 24*a^2*b^4*c*e))/(2*(4*a^3*b^6 - 256*a^6*c^3 - \\
& 48*a^4*b^4*c + 192*a^5*b^2*c^2)) + (((((72*a^5*b^5*c^3*d - 8*a^4*b^7*c^2*d \\
& - 184*a^6*b^3*c^4*d + 4*a^5*b^6*c^2*e - 36*a^6*b^4*c^3*e + 80*a^7*b^2*c^4*e \\
& + 8*a^7*b^3*c^3*f + 96*a^7*b*c^5*d - 32*a^8*b*c^4*f)/(a^6*b^4 + 16*a^8*c^2 \\
& - 8*a^7*b^2*c) - ((4*a^7*b^6*c^2 - 32*a^8*b^4*c^3 + 64*a^9*b^2*c^4)*(4*b^7
\end{aligned}$$

$$\begin{aligned}
& *d + 128*a^4*c^3*e - 2*a*b^6*e + 192*a^2*b^3*c^2*d - 96*a^3*b^2*c^2*e - 48* \\
& a*b^5*c*d - 256*a^3*b*c^3*d + 24*a^2*b^4*c*e)) / (2*(a^6*b^4 + 16*a^8*c^2 - 8 \\
& *a^7*b^2*c)*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)) * (2 \\
& *b^4*d + 12*a^2*c^2*d - a*b^3*e - 4*a^3*c*f - 12*a*b^2*c*d + 6*a^2*b*c*e)) / \\
& (4*a^3*(4*a*c - b^2)^(3/2)) - ((4*a^7*b^6*c^2 - 32*a^8*b^4*c^3 + 64*a^9*b^2 \\
& *c^4)*(2*b^4*d + 12*a^2*c^2*d - a*b^3*e - 4*a^3*c*f - 12*a*b^2*c*d + 6*a^2* \\
& b*c*e)*(4*b^7*d + 128*a^4*c^3*e - 2*a*b^6*e + 192*a^2*b^3*c^2*d - 96*a^3*b^ \\
& 2*c^2*e - 48*a*b^5*c*d - 256*a^3*b*c^3*d + 24*a^2*b^4*c*e)) / (8*a^3*(4*a*c - \\
& b^2)^(3/2)*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(4*a^3*b^6 - 256*a^6*c^3 - \\
& 48*a^4*b^4*c + 192*a^5*b^2*c^2)) * (2*b^4*d + 12*a^2*c^2*d - a*b^3*e - 4*a^ \\
& 3*c*f - 12*a*b^2*c*d + 6*a^2*b*c*e)) / (4*a^3*(4*a*c - b^2)^(3/2)) - ((4*a^7* \\
& b^6*c^2 - 32*a^8*b^4*c^3 + 64*a^9*b^2*c^4)*(2*b^4*d + 12*a^2*c^2*d - a*b^3* \\
& e - 4*a^3*c*f - 12*a*b^2*c*d + 6*a^2*b*c*e))^2*(4*b^7*d + 128*a^4*c^3*e - 2* \\
& a*b^6*e + 192*a^2*b^3*c^2*d - 96*a^3*b^2*c^2*e - 48*a*b^5*c*d - 256*a^3*b*c \\
& ^3*d + 24*a^2*b^4*c*e)) / (32*a^6*(4*a*c - b^2)^3*(a^6*b^4 + 16*a^8*c^2 - 8*a \\
& ^7*b^2*c)*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)) * (6*a \\
& ^3*c^3*d - 6*b^6*d - 2*a^4*c^2*f + 3*a*b^5*e - 72*a^2*b^2*c^2*d + 42*a*b^4* \\
& c*d - 21*a^2*b^3*c*e + 33*a^3*b*c^2*e + 2*a^3*b^2*c*f)) / (8*a^3*c^2*(4*a*c - \\
& b^2)^3*(144*a^4*c^6*d^2 + 4*b^8*c^2*d^2 + 16*a^6*c^4*f^2 - 48*a*b^6*c^3*d^ \\
& 2 + 192*a^2*b^4*c^4*d^2 - 288*a^3*b^2*c^5*d^2 + a^2*b^6*c^2*e^2 - 12*a^3*b^ \\
& 4*c^3*e^2 + 36*a^4*b^2*c^4*e^2 - 96*a^5*c^5*d*f - 4*a*b^7*c^2*d*e + 144*a^4 \\
& *b*c^5*d*e - 48*a^5*b*c^4*e*f + 48*a^2*b^5*c^3*d*e - 168*a^3*b^3*c^4*d*e - \\
& 16*a^3*b^4*c^3*d*f + 96*a^4*b^2*c^4*d*f + 8*a^4*b^3*c^3*e*f)*(36*a^4*c^4*d^ \\
& 2 - 6*a^2*b^6*e^2 - 24*b^8*d^2 + 400*a^5*c^3*e^2 + 4*a^6*c^2*f^2 + 72*a^3*b \\
& ^4*c*e^2 + 24*a*b^7*d*e - 1152*a^2*b^4*c^2*d^2 + 1528*a^3*b^2*c^3*d^2 - 291 \\
& *a^4*b^2*c^2*e^2 + 288*a*b^6*c*d^2 - 24*a^5*c^3*d*f - 288*a^2*b^5*c*d*e - 1 \\
& 564*a^4*b*c^3*d*e - 4*a^3*b^4*c*d*f + 2*a^4*b^3*c*e*f - 12*a^5*b*c^2*e*f + \\
& 1158*a^3*b^3*c^2*d*e + 24*a^4*b^2*c^2*d*f)) + (((((((72*a^5*b^5*c^3*d - 8*a \\
& ^4*b^7*c^2*d - 184*a^6*b^3*c^4*d + 4*a^5*b^6*c^2*e - 36*a^6*b^4*c^3*e + 80* \\
& a^7*b^2*c^4*e + 8*a^7*b^3*c^3*f + 96*a^7*b*c^5*d - 32*a^8*b*c^4*f)) / (a^6*b^4 \\
& + 16*a^8*c^2 - 8*a^7*b^2*c) - ((4*a^7*b^6*c^2 - 32*a^8*b^4*c^3 + 64*a^9*b^ \\
& 2*c^4)*(4*b^7*d + 128*a^4*c^3*e - 2*a*b^6*e + 192*a^2*b^3*c^2*d - 96*a^3*b^ \\
& 2*c^2*e - 48*a*b^5*c*d - 256*a^3*b*c^3*d + 24*a^2*b^4*c*e)) / (2*(a^6*b^4 + 1 \\
& 6*a^8*c^2 - 8*a^7*b^2*c)*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5* \\
& b^2*c^2)) * (2*b^4*d + 12*a^2*c^2*d - a*b^3*e - 4*a^3*c*f - 12*a*b^2*c*d + 6 \\
& *a^2*b*c*e)) / (4*a^3*(4*a*c - b^2)^(3/2)) - ((4*a^7*b^6*c^2 - 32*a^8*b^4*c^3 \\
& + 64*a^9*b^2*c^4)*(2*b^4*d + 12*a^2*c^2*d - a*b^3*e - 4*a^3*c*f - 12*a*b^2 \\
& *c*d + 6*a^2*b*c*e)*(4*b^7*d + 128*a^4*c^3*e - 2*a*b^6*e + 192*a^2*b^3*c^2* \\
& d - 96*a^3*b^2*c^2*e - 48*a*b^5*c*d - 256*a^3*b*c^3*d + 24*a^2*b^4*c*e)) / (8 \\
& *a^3*(4*a*c - b^2)^(3/2)*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(4*a^3*b^6 - \\
& 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)) * (4*b^7*d + 128*a^4*c^3*e - \\
& 2*a*b^6*e + 192*a^2*b^3*c^2*d - 96*a^3*b^2*c^2*e - 48*a*b^5*c*d - 256*a^3*b \\
& *c^3*d + 24*a^2*b^4*c*e)) / (2*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192* \\
& a^5*b^2*c^2)) - (((36*a^5*c^6*d^2 + 4*a^7*c^4*f^2 - 16*a^2*b^6*c^3*d^2 + 11 \\
& 6*a^3*b^4*c^4*d^2 - 216*a^4*b^2*c^5*d^2 - 4*a^4*b^4*c^3*e^2 + 17*a^5*b^2*c^ \\
& 4*e^2 - 24*a^6*c^5*d*f + 108*a^5*b*c^5*d*e - 36*a^6*b*c^4*e*f + 16*a^3*b^5* \\
& c^3*d*e - 92*a^4*b^3*c^4*d*e - 16*a^4*b^4*c^3*d*f + 72*a^5*b^2*c^4*d*f + 8* \\
& a^5*b^3*c^3*e*f)) / (a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) - (((72*a^5*b^5*c^3*d \\
& - 8*a^4*b^7*c^2*d - 184*a^6*b^3*c^4*d + 4*a^5*b^6*c^2*e - 36*a^6*b^4*c^3*e \\
& + 80*a^7*b^2*c^4*e + 8*a^7*b^3*c^3*f + 96*a^7*b*c^5*d - 32*a^8*b*c^4*f)) / (a \\
& ^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) - ((4*a^7*b^6*c^2 - 32*a^8*b^4*c^3 + 64* \\
& a^9*b^2*c^4)*(4*b^7*d + 128*a^4*c^3*e - 2*a*b^6*e + 192*a^2*b^3*c^2*d - 96* \\
& a^3*b^2*c^2*e - 48*a*b^5*c*d - 256*a^3*b*c^3*d + 24*a^2*b^4*c*e)) / (2*(a^6*b \\
& ^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 19 \\
& 2*a^5*b^2*c^2)) * (4*b^7*d + 128*a^4*c^3*e - 2*a*b^6*e + 192*a^2*b^3*c^2*d - \\
& 96*a^3*b^2*c^2*e - 48*a*b^5*c*d - 256*a^3*b*c^3*d + 24*a^2*b^4*c*e)) / (2*(4 \\
& *a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)) * (2*b^4*d + 12*a^ \\
& 2*c^2*d - a*b^3*e - 4*a^3*c*f - 12*a*b^2*c*d + 6*a^2*b*c*e)) / (4*a^3*(4*a*c \\
& - b^2)^(3/2)) + ((4*a^7*b^6*c^2 - 32*a^8*b^4*c^3 + 64*a^9*b^2*c^4)*(2*b^4*d
\end{aligned}$$

$$\begin{aligned}
& + 12a^2c^2d - ab^3e - 4a^3cf - 12ab^2cd + 6a^2bce)^3)/(64a^9(4ac - b^2)^{9/2}(a^6b^4 + 16a^8c^2 - 8a^7b^2c)))(16a^9b^6(4ac - b^2)^{9/2} - 1024a^{12}c^3(4ac - b^2)^{9/2} - 192a^{10}b^4c(4ac - b^2)^{9/2} + 768a^{11}b^2c^2(4ac - b^2)^{9/2}))(768b^7d + 5120a^4c^3e - 384ab^6e + 18432a^2b^3c^2d - 8832a^3b^2c^2e - 6912ab^5cd - 12544a^3b^3c^3d + 3456a^2b^4ce - 256a^3b^3cf + 768a^4b^2c^2f))/(1024a^3c^2(4ac - b^2)^{7/2}(144a^4c^6d^2 + 4b^8c^2d^2 + 16a^6c^4f^2 - 48ab^6c^3d^2 + 192a^2b^4c^4d^2 - 288a^3b^2c^5d^2 + a^2b^6c^2e^2 - 12a^3b^4c^3e^2 + 36a^4b^2c^4e^2 - 96a^5c^5d^2f - 4ab^7c^2de + 144a^4b^3c^5d^2e - 48a^5b^2c^4e^2f + 48a^2b^5c^3d^2e - 168a^3b^3c^4d^2e - 16a^3b^4c^3d^2f + 96a^4b^2c^4d^2f + 8a^4b^3c^3e^2f)(36a^4c^4d^2 - 6a^2b^6e^2 - 24b^8d^2 + 400a^5c^3e^2 + 4a^6c^2f^2 + 72a^3b^4c^2e^2 + 24ab^7d^2e - 1152a^2b^4c^2d^2 + 1528a^3b^2c^3d^2 - 291a^4b^2c^2e^2 + 288ab^6c^2d^2 - 24a^5c^3d^2f - 288a^2b^5c^2d^2e - 1564a^4b^3c^3d^2e - 4a^3b^4c^2d^2f + 2a^4b^3c^2e^2f - 12a^5b^2c^2e^2f + 1158a^3b^3c^2d^2e + 24a^4b^2c^2d^2f)))(2b^4d + 12a^2c^2d - ab^3e - 4a^3cf - 12ab^2cd + 6a^2bce))/(2a^3(4ac - b^2)^{3/2})
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**4+e*x**2+d)/x**3/(c*x**4+b*x**2+a)**2,x)

[Out] Timed out

$$3.65 \quad \int \frac{d+ex^2+fx^4}{x^5(a+bx^2+cx^4)^2} dx$$

Optimal. Leaf size=329

$$\frac{\log(a+bx^2+cx^4)(-2abe-a(2cd-af)+3b^2d)}{4a^4} + \frac{\log(x)(-2abe-a(2cd-af)+3b^2d)}{a^4} + \frac{2bd-ae}{2a^3x^2} - \frac{d}{4a^2x^4} +$$

Rubi [A] time = 1.16, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {1663, 1646, 1628, 634, 618, 206, 628}

$$\frac{\tanh^{-1}\left(\frac{2cd-af}{\sqrt{4ac-b^2}}\right)(12a^2b^2ce+6a^2bc(5cd-af)-12a^2c^2e-ab^3(20cd-af)-2ab^4e+3b^5d)+c^2(2a^2ce-ab^2e-ab(3cd-af)+b^3d)+3a^2bce+2a^2c(4cd-af)-ab^2e+b^3d}{2a^4(b^2-4ac)^{3/2}} - \frac{c^2(2a^2ce-ab^2e-ab(3cd-af)+b^3d)+3a^2bce+2a^2c(4cd-af)-ab^2e+b^3d}{2a^2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\log(a+bx^2+cx^4)(-2abe-a(2cd-af)+3b^2d)}{4a^4} + \frac{\log(x)(-2abe-a(2cd-af)+3b^2d)}{a^4} + \frac{2bd-ae}{2a^3x^2} - \frac{d}{4a^2x^4}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2 + f*x^4)/(x^5*(a + b*x^2 + c*x^4)^2), x]

[Out] -d/(4*a^2*x^4) + (2*b*d - a*e)/(2*a^3*x^2) + (b^4*d - a*b^3*e + 3*a^2*b*c*e + 2*a^2*c*(c*d - a*f) - a*b^2*(4*c*d - a*f) + c*(b^3*d - a*b^2*e + 2*a^2*c*e - a*b*(3*c*d - a*f))*x^2)/(2*a^3*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((3*b^5*d - 2*a*b^4*e + 12*a^2*b^2*c*e - 12*a^3*c^2*e + 6*a^2*b*c*(5*c*d - a*f) - a*b^3*(20*c*d - a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^4*(b^2 - 4*a*c)^(3/2)) + ((3*b^2*d - 2*a*b*e - a*(2*c*d - a*f))*Log[x])/a^4 - ((3*b^2*d - 2*a*b*e - a*(2*c*d - a*f))*Log[a + b*x^2 + c*x^4])/(4*a^4)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1628

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{d + ex^2 + fx^4}{x^5 (a + bx^2 + cx^4)^2} dx = \frac{1}{2} \text{Subst} \left(\int \frac{d + ex + fx^2}{x^3 (a + bx + cx^2)^2} dx, x, x^2 \right)$$

$$= \frac{b^4d - ab^3e + 3a^2bce + 2a^2c(cd - af) - ab^2(4cd - af) + c(b^3d - ab^2e + 2a^2ce - ab(3d - 2e))}{2a^3(b^2 - 4ac)(a + bx^2 + cx^4)}$$

$$= \frac{b^4d - ab^3e + 3a^2bce + 2a^2c(cd - af) - ab^2(4cd - af) + c(b^3d - ab^2e + 2a^2ce - ab(3d - 2e))}{2a^3(b^2 - 4ac)(a + bx^2 + cx^4)}$$

$$= -\frac{d}{4a^2x^4} + \frac{2bd - ae}{2a^3x^2} + \frac{b^4d - ab^3e + 3a^2bce + 2a^2c(cd - af) - ab^2(4cd - af) + c(b^3d - ab^2e + 2a^2ce - ab(3d - 2e))}{2a^3(b^2 - 4ac)(a + bx^2 + cx^4)}$$

$$= -\frac{d}{4a^2x^4} + \frac{2bd - ae}{2a^3x^2} + \frac{b^4d - ab^3e + 3a^2bce + 2a^2c(cd - af) - ab^2(4cd - af) + c(b^3d - ab^2e + 2a^2ce - ab(3d - 2e))}{2a^3(b^2 - 4ac)(a + bx^2 + cx^4)}$$

$$= -\frac{d}{4a^2x^4} + \frac{2bd - ae}{2a^3x^2} + \frac{b^4d - ab^3e + 3a^2bce + 2a^2c(cd - af) - ab^2(4cd - af) + c(b^3d - ab^2e + 2a^2ce - ab(3d - 2e))}{2a^3(b^2 - 4ac)(a + bx^2 + cx^4)}$$

$$= -\frac{d}{4a^2x^4} + \frac{2bd - ae}{2a^3x^2} + \frac{b^4d - ab^3e + 3a^2bce + 2a^2c(cd - af) - ab^2(4cd - af) + c(b^3d - ab^2e + 2a^2ce - ab(3d - 2e))}{2a^3(b^2 - 4ac)(a + bx^2 + cx^4)}$$

Mathematica [A] time = 1.22, size = 592, normalized size = 1.80

```
Integrate[(d + e*x^2 + f*x^4)/(x^5*(a + b*x^2 + c*x^4)^2), x]
```

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2 + f*x^4)/(x^5*(a + b*x^2 + c*x^4)^2), x]
[Out] -1/4*((a^2*d)/x^4 + (2*a*(-2*b*d + a*e))/x^2 + (2*a*(-(b^4*d) + b^3*(a*e - c*d*x^2) + a*b^2*(4*c*d - a*f + c*e*x^2) - a*b*c*(3*a*e - 3*c*d*x^2 + a*f*x
```

$\wedge 2) + 2*a^2*c*(a*f - c*(d + e*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - 4*(3*b^2*d - 2*a*b*e + a*(-2*c*d + a*f))*\text{Log}[x] + ((3*b^5*d + b^4*(3*\text{Sqrt}[b^2 - 4*a*c]*d - 2*a*e) + 2*a^2*b*c*(15*c*d + 4*\text{Sqrt}[b^2 - 4*a*c]*e - 3*a*f) + a*b^3*(-20*c*d - 2*\text{Sqrt}[b^2 - 4*a*c]*e + a*f) - 4*a^2*c*(-2*c*\text{Sqrt}[b^2 - 4*a*c]*d + 3*a*c*e + a*\text{Sqrt}[b^2 - 4*a*c]*f) + a*b^2*(-14*c*\text{Sqrt}[b^2 - 4*a*c]*d + 12*a*c*e + a*\text{Sqrt}[b^2 - 4*a*c]*f))*\text{Log}[b - \text{Sqrt}[b^2 - 4*a*c] + 2*c*x^2)]/(b^2 - 4*a*c)^(3/2) + ((-3*b^5*d + b^4*(3*\text{Sqrt}[b^2 - 4*a*c]*d + 2*a*e) - a*b^3*(-20*c*d + 2*\text{Sqrt}[b^2 - 4*a*c]*e + a*f) + 2*a^2*b*c*(-15*c*d + 4*\text{Sqrt}[b^2 - 4*a*c]*e + 3*a*f) + 4*a^2*c*(2*c*\text{Sqrt}[b^2 - 4*a*c]*d + 3*a*c*e - a*\text{Sqrt}[b^2 - 4*a*c]*f) + a*b^2*(-2*c*(7*\text{Sqrt}[b^2 - 4*a*c]*d + 6*a*e) + a*\text{Sqrt}[b^2 - 4*a*c]*f))*\text{Log}[b + \text{Sqrt}[b^2 - 4*a*c] + 2*c*x^2)]/(b^2 - 4*a*c)^(3/2))/a^4$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex^2 + fx^4}{x^5 (a + bx^2 + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e*x^2 + f*x^4)/(x^5*(a + b*x^2 + c*x^4)^2),x]

[Out] IntegrateAlgebraic[(d + e*x^2 + f*x^4)/(x^5*(a + b*x^2 + c*x^4)^2), x]

fricas [B] time = 15.90, size = 2567, normalized size = 7.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^4+e*x^2+d)/x^5/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")

[Out] [1/4*(2*((3*a*b^5*c - 23*a^2*b^3*c^2 + 44*a^3*b*c^3)*d - 2*(a^2*b^4*c - 7*a^3*b^2*c^2 + 12*a^4*c^3)*e + (a^3*b^3*c - 4*a^4*b*c^2)*f)*x^6 + ((6*a*b^6 - 49*a^2*b^4*c + 108*a^3*b^2*c^2 - 32*a^4*c^3)*d - 2*(2*a^2*b^5 - 15*a^3*b^3*c + 28*a^4*b*c^2)*e + 2*(a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*f)*x^4 + (3*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d - 2*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*e)*x^2 + (((3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*d - 2*(a*b^4*c - 6*a^2*b^2*c^2 + 6*a^3*c^3)*e + (a^2*b^3*c - 6*a^3*b*c^2)*f)*x^8 + ((3*b^6 - 20*a*b^4*c + 30*a^2*b^2*c^2)*d - 2*(a*b^5 - 6*a^2*b^3*c + 6*a^3*b*c^2)*e + (a^2*b^4 - 6*a^3*b^2*c)*f)*x^6 + ((3*a*b^5 - 20*a^2*b^3*c + 30*a^3*b*c^2)*d - 2*(a^2*b^4 - 6*a^3*b^2*c + 6*a^4*c^2)*e + (a^3*b^3 - 6*a^4*b*c)*f)*x^4)*\text{sqrt}(b^2 - 4*a*c)*\text{log}((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*\text{sqrt}(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*d - (((3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*d - 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*e + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*f)*x^8 + ((3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*d - 2*(a*b^6 - 8*a^2*b^4*c + 16*a^3*b^2*c^2)*e + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*f)*x^6 + ((3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*d - 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*e + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*f)*x^4)*\text{log}(c*x^4 + b*x^2 + a) + 4*((3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*d - 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*e + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*f)*x^8 + ((3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*d - 2*(a*b^6 - 8*a^2*b^4*c + 16*a^3*b^2*c^2)*e + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*f)*x^6 + ((3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*d - 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*e + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*f)*x^4)*\text{log}(x))/((a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*x^8 + (a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x^6 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*x^4), 1/4*(2*((3*a*b^5*c - 23*a^2*b^3*c^2 + 44*a^3*b*c^3)*d - 2*(a^2*b^4*c - 7*a^3*b^2*c^2 + 12*a^4*c^3)*e + (a^3*b^3*c - 4*a^4*b*c^2)*f)*x^6 + ((6*a*b^6 - 49*a^2*b^4*c + 108*a^3*b^2*c^2 - 32*a^4*c^3)*d - 2*(2*a^2*b^5 - 15*a^3*b^3*c + 28*a^4*b*c^2)*e + 2*(a^3*

$$\begin{aligned}
& b^4 - 6a^4b^2c + 8a^5c^2) * f) * x^4 + (3(a^2b^5 - 8a^3b^3c + 16a^4b^2c^2) * d - 2(a^3b^4 - 8a^4b^2c + 16a^5c^2) * e) * x^2 + 2(((3b^5c - 20a^2b^3c^2 + 30a^2b^3c^3) * d - 2(a^2b^4c - 6a^3b^2c^2 + 6a^4c^3) * e + (a^2b^3c - 6a^3b^2c^2) * f) * x^8 + ((3b^6 - 20a^2b^4c + 30a^2b^2c^2) * d - 2(a^2b^5 - 6a^3b^3c + 6a^4b^2c^2) * e + (a^2b^4c - 6a^3b^2c^2) * f) * x^6 + ((3a^2b^5 - 20a^2b^3c + 30a^3b^2c^2) * d - 2(a^2b^4 - 6a^3b^2c + 6a^4c^2) * e + (a^3b^3 - 6a^4b^2c) * f) * x^4) * \sqrt{-b^2 + 4ac} * \arctan(-2cx^2 + b) * \sqrt{-b^2 + 4ac} / (b^2 - 4ac)) - (a^3b^4 - 8a^4b^2c + 16a^5c^2) * d - (((3b^6c - 26a^2b^4c^2 + 64a^2b^2c^3 - 32a^3c^4) * d - 2(a^2b^5c - 8a^3b^3c^2 + 16a^4c^3) * e + (a^2b^4c - 8a^3b^2c^2 + 16a^4c^3) * f) * x^8 + ((3b^7 - 26a^2b^5c + 64a^2b^3c^2 - 32a^3b^2c^3) * d - 2(a^2b^6 - 8a^3b^4c + 16a^4b^2c^2) * e + (a^2b^5 - 8a^3b^3c + 16a^4b^2c^2) * f) * x^6 + ((3a^2b^6 - 26a^2b^4c + 64a^3b^2c^2 - 32a^4c^3) * d - 2(a^2b^5 - 8a^3b^3c + 16a^4b^2c^2) * e + (a^3b^4 - 8a^4b^2c + 16a^5c^2) * f) * x^4) * \log(cx^4 + bx^2 + a) + 4(((3b^6c - 26a^2b^4c^2 + 64a^2b^2c^3 - 32a^3c^4) * d - 2(a^2b^5c - 8a^3b^3c^2 + 16a^4c^3) * e + (a^2b^4c - 8a^3b^2c^2 + 16a^4c^3) * f) * x^8 + ((3b^7 - 26a^2b^5c + 64a^2b^3c^2 - 32a^3b^2c^3) * d - 2(a^2b^6 - 8a^3b^4c + 16a^4b^2c^2) * e + (a^2b^5 - 8a^3b^3c + 16a^4b^2c^2) * f) * x^6 + ((3a^2b^6 - 26a^2b^4c + 64a^3b^2c^2 - 32a^4c^3) * d - 2(a^2b^5 - 8a^3b^3c + 16a^4b^2c^2) * e + (a^3b^4 - 8a^4b^2c + 16a^5c^2) * f) * x^4) * \log(x)) / ((a^4b^4c - 8a^5b^2c^2 + 16a^6c^3) * x^8 + (a^4b^5 - 8a^5b^3c + 16a^6b^2c^2) * x^6 + (a^5b^4 - 8a^6b^2c + 16a^7c^2) * x^4)]
\end{aligned}$$

giac [A] time = 1.89, size = 535, normalized size = 1.63

(1/4 - 2a^2b^4 + 3a^2b^2c^2 - 2b^4 - 2a^2b^2c^2 - 12b^2) * arctan(2cx^2 + b) / (4a^2b^2 - 4a^2b^2c^2) * sqrt(-b^2 + 4ac) + 1/4 * (3b^4c * d * x^4 - 14a^2b^2c^2 * d * x^4 + 8a^2c^3 * d * x^4 + a^2b^2c^2 * f * x^4 - 4a^3c^2 * f * x^4 - 2a^2b^3c^2 * x^4 * e + 8a^2b^2c^2 * x^4 * e + 3b^5 * d * x^2 - 12a^2b^3c^2 * d * x^2 + 2a^2b^2c^2 * d * x^2 + a^2b^3 * f * x^2 - 2a^3b^2c^2 * f * x^2 - 2a^2b^4 * x^2 * e + 6a^2b^2c^2 * x^2 * e + 4a^3c^2 * x^2 * e + 5a^2b^4 * d - 22a^2b^2c^2 * d + 12a^3c^2 * d + 3a^3b^2 * f - 8a^4c * f - 4a^2b^3 * e + 14a^3b^2 * c * e) / ((a^4b^2 - 4a^5c) * (cx^4 + bx^2 + a)) - 1/4 * (3b^2 * d - 2a * c * d + a^2 * f - 2a * b * e) * log(cx^4 + bx^2 + a) / a^4 + 1/2 * (3b^2 * d - 2a * c * d + a^2 * f - 2a * b * e) * log(x^2) / a^4 - 1/4 * (9b^2 * d * x^4 - 6a * c * d * x^4 + 3a^2 * f * x^4 - 6a^2 * b * x^4 * e - 4a * b * d * x^2 + 2a^2 * x^2 * e + a^2 * d) / (a^4 * x^4)

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^4+e*x^2+d)/x^5/(c*x^4+b*x^2+a)^2,x, algorithm="giac")

[Out]
$$\begin{aligned}
& -1/2 * (3b^5d - 20a^2b^3cd + 30a^2b^3c^2d + a^2b^3f - 6a^3b^2c^2f - 2a^2b^4e + 12a^2b^2c^2e - 12a^3c^2e) * \arctan((2cx^2 + b) / \sqrt{-b^2 + 4ac}) / ((a^4b^2 - 4a^5c) * \sqrt{-b^2 + 4ac}) + 1/4 * (3b^4cd * x^4 - 14a^2b^2c^2 * d * x^4 + 8a^2c^3 * d * x^4 + a^2b^2c^2 * f * x^4 - 4a^3c^2 * f * x^4 - 2a^2b^3c^2 * x^4 * e + 8a^2b^2c^2 * x^4 * e + 3b^5 * d * x^2 - 12a^2b^3c^2 * d * x^2 + 2a^2b^2c^2 * d * x^2 + a^2b^3 * f * x^2 - 2a^3b^2c^2 * f * x^2 - 2a^2b^4 * x^2 * e + 6a^2b^2c^2 * x^2 * e + 4a^3c^2 * x^2 * e + 5a^2b^4 * d - 22a^2b^2c^2 * d + 12a^3c^2 * d + 3a^3b^2 * f - 8a^4c * f - 4a^2b^3 * e + 14a^3b^2 * c * e) / ((a^4b^2 - 4a^5c) * (cx^4 + bx^2 + a)) - 1/4 * (3b^2 * d - 2a * c * d + a^2 * f - 2a * b * e) * \log(cx^4 + bx^2 + a) / a^4 + 1/2 * (3b^2 * d - 2a * c * d + a^2 * f - 2a * b * e) * \log(x^2) / a^4 - 1/4 * (9b^2 * d * x^4 - 6a * c * d * x^4 + 3a^2 * f * x^4 - 6a^2 * b * x^4 * e - 4a * b * d * x^2 + 2a^2 * x^2 * e + a^2 * d) / (a^4 * x^4)
\end{aligned}$$

maple [B] time = 0.03, size = 1078, normalized size = 3.28

(1/4 * d / a^2 / x^4 + 1/a^3 / x^2 * b * d - 2/a^3 * ln(x) * b * e - 2/a^3 * ln(x) * c * d + 3/a^4 * ln(x) * b^2 * d + 1/(c*x^4 + b*x^2 + a) / (4*a*c - b^2) * c * f + 6/a^2 / (4*a*c - b^2)^(3/2) * arctan((2*c*x^2 + b) / (4*a*c - b^2)^(1/2)) * b^2 * c * e + 15/a^2 / (4*a*c - b^2)^(3/2) * arctan((2*c*x^2 + b) / (4*a*c - b^2)^(1/2)) * b * c^2 * d - 10/a^3 / (4*a*c - b^2)^(3/2) * arctan((2*c*x^2 + b) / (4*a*c - b^2)^(1/2)) * b^3 * c * d - 1/a / (c*x^4 + b*x^2 + a) * c^2 / (4*a*c - b^2) * x^2 * e - 3/2/a / (c*x^4 + b*x^2 + a) / (4*a*c - b^2) * b * c * e + 2/a^2 / (c*x^4 + b*x^2 + a) / (4*a*c - b^2) * b^2 * c * d + 2/a^2 / (4*a*c - b^2) * c * ln(c*x^4 + b*x^2 + a) * b * e - 7/2/a^3 / (4*a*c - b^2) * c * ln(c*x^4 + b*x^2 + a) * b^2 * d - 3/a / (4*a*c - b^2)^(3/2) * arctan((2*c*x^2 + b) / (4*a*c - b^2)^(1/2)) * b * c

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^4+e*x^2+d)/x^5/(c*x^4+b*x^2+a)^2,x)

[Out]
$$\begin{aligned}
& -1/4 * d / a^2 / x^4 + 1/a^3 / x^2 * b * d - 2/a^3 * \ln(x) * b * e - 2/a^3 * \ln(x) * c * d + 3/a^4 * \ln(x) * b^2 * d + 1/(c*x^4 + b*x^2 + a) / (4*a*c - b^2) * c * f + 6/a^2 / (4*a*c - b^2)^(3/2) * \arctan((2*c*x^2 + b) / (4*a*c - b^2)^(1/2)) * b^2 * c * e + 15/a^2 / (4*a*c - b^2)^(3/2) * \arctan((2*c*x^2 + b) / (4*a*c - b^2)^(1/2)) * b * c^2 * d - 10/a^3 / (4*a*c - b^2)^(3/2) * \arctan((2*c*x^2 + b) / (4*a*c - b^2)^(1/2)) * b^3 * c * d - 1/a / (c*x^4 + b*x^2 + a) * c^2 / (4*a*c - b^2) * x^2 * e - 3/2/a / (c*x^4 + b*x^2 + a) / (4*a*c - b^2) * b * c * e + 2/a^2 / (c*x^4 + b*x^2 + a) / (4*a*c - b^2) * b^2 * c * d + 2/a^2 / (4*a*c - b^2) * c * \ln(c*x^4 + b*x^2 + a) * b * e - 7/2/a^3 / (4*a*c - b^2) * c * \ln(c*x^4 + b*x^2 + a) * b^2 * d - 3/a / (4*a*c - b^2)^(3/2) * \arctan((2*c*x^2 + b) / (4*a*c - b^2)^(1/2)) * b * c
\end{aligned}$$

$$\begin{aligned} & *f-1/2/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*b^2*f-1/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)* \\ & c^2*d-1/2/a/(c*x^4+b*x^2+a)*c/(4*a*c-b^2)*x^2*b*f+1/2/a^2/(c*x^4+b*x^2+a)*c \\ & /((4*a*c-b^2)*x^2*b^2*e+3/2/a^2/(c*x^4+b*x^2+a)*c^2/(4*a*c-b^2)*x^2*b*d-1/2/ \\ & a^3/(c*x^4+b*x^2+a)*c/(4*a*c-b^2)*x^2*b^3*d-1/2/a^2/x^2*e+1/a^2*\ln(x)*f+1/2 \\ & /a^2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*b^3*e-1/2/a^3/(c*x^4+b*x^2+a)/(4*a*c-b^2)* \\ & b^4*d+3/4/a^4/(4*a*c-b^2)*\ln(c*x^4+b*x^2+a)*b^4*d+3/2/a^4/(4*a*c-b^2)^(3/2) \\ & *\arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^5*d-6/a/(4*a*c-b^2)^(3/2)*\arctan((\\ & 2*c*x^2+b)/(4*a*c-b^2)^(1/2))*c^2*e+1/2/a^2/(4*a*c-b^2)^(3/2)*\arctan((2*c*x \\ & ^2+b)/(4*a*c-b^2)^(1/2))*b^3*f-1/a^3/(4*a*c-b^2)^(3/2)*\arctan((2*c*x^2+b)/(\\ & 4*a*c-b^2)^(1/2))*b^4*e-1/a/(4*a*c-b^2)*c*\ln(c*x^4+b*x^2+a)*f+1/4/a^2/(4*a* \\ & c-b^2)*\ln(c*x^4+b*x^2+a)*b^2*f+2/a^2/(4*a*c-b^2)*c^2*\ln(c*x^4+b*x^2+a)*d-1/ \\ & 2/a^3/(4*a*c-b^2)*\ln(c*x^4+b*x^2+a)*b^3*e \end{aligned}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^4+e*x^2+d)/x^5/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see 'assume?' for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 21.02, size = 15905, normalized size = 48.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x^2 + f*x^4)/(x^5*(a + b*x^2 + c*x^4)^2),x)

[Out]
$$\begin{aligned} & (\log(x)*(3*b^2*d + a^2*f - 2*a*b*e - 2*a*c*d))/a^4 - (\log(((((((4*b*c^2*(3* \\ & b^5*d + a^2*b^3*f - 6*a^3*c^2*e - 2*a*b^4*e - 17*a*b^3*c*d - 5*a^3*b*c*f + \\ & 19*a^2*b*c^2*d + 10*a^2*b^2*c*e)))/(a^3*(4*a*c - b^2)) - (b*c^2*(a*b + 3*b^2 \\ & *x^2 - 10*a*c*x^2)*(a^4*(-(3*b^5*d + a^2*b^3*f - 12*a^3*c^2*e - 2*a*b^4*e - \\ & 20*a*b^3*c*d - 6*a^3*b*c*f + 30*a^2*b*c^2*d + 12*a^2*b^2*c*e))^2/(a^8*(4*a* \\ & c - b^2)^3))^(1/2) + 3*b^2*d + a^2*f - 2*a*b*e - 2*a*c*d))/a^4 + (2*c^3*x^2 \\ & *(3*b^5*d + a^2*b^3*f + 60*a^3*c^2*e - 2*a*b^4*e + 4*a*b^3*c*d - 10*a^3*b*c \\ & *f - 70*a^2*b*c^2*d - 4*a^2*b^2*c*e))/(a^3*(4*a*c - b^2)))*(a^4*(-(3*b^5*d \\ & + a^2*b^3*f - 12*a^3*c^2*e - 2*a*b^4*e - 20*a*b^3*c*d - 6*a^3*b*c*f + 30*a^ \\ & 2*b*c^2*d + 12*a^2*b^2*c*e))^2/(a^8*(4*a*c - b^2)^3))^(1/2) + 3*b^2*d + a^2* \\ & f - 2*a*b*e - 2*a*c*d)/(4*a^4) + (c^3*(36*b^8*d^2 + 16*a^2*b^6*e^2 + 4*a^4 \\ & *b^4*f^2 - 36*a^5*c^3*e^2 - 116*a^3*b^4*c*e^2 - 17*a^5*b^2*c*f^2 - 48*a*b^7 \\ & *d*e + 778*a^2*b^4*c^2*d^2 - 473*a^3*b^2*c^3*d^2 + 216*a^4*b^2*c^2*e^2 - 30 \\ & 9*a*b^6*c*d^2 + 24*a^2*b^6*d*f - 16*a^3*b^5*e*f + 380*a^2*b^5*c*d*e + 324*a \\ & ^4*b*c^3*d*e - 154*a^3*b^4*c*d*f + 92*a^4*b^3*c*e*f - 108*a^5*b*c^2*e*f - 8 \\ & 32*a^3*b^3*c^2*d*e + 230*a^4*b^2*c^2*d*f))/(a^6*(4*a*c - b^2)^2) + (c^4*x^2 \\ & *(54*b^7*d^2 + 24*a^2*b^5*e^2 + 6*a^4*b^3*f^2 - 440*a^3*b*c^3*d^2 - 164*a^3 \\ & *b^3*c*e^2 + 276*a^4*b*c^2*e^2 - 72*a*b^6*d*e + 1011*a^2*b^3*c^2*d^2 - 441* \\ & a*b^5*c*d^2 - 20*a^5*b*c*f^2 + 36*a^2*b^5*d*f + 240*a^4*c^3*d*e - 24*a^3*b^ \\ & 4*e*f - 120*a^5*c^2*e*f + 540*a^2*b^4*c*d*e - 207*a^3*b^3*c*d*f + 260*a^4*b \\ & *c^2*d*f + 122*a^4*b^2*c*e*f - 1072*a^3*b^2*c^2*d*e))/(a^6*(4*a*c - b^2)^2) \\ &)*(a^4*(-(3*b^5*d + a^2*b^3*f - 12*a^3*c^2*e - 2*a*b^4*e - 20*a*b^3*c*d - 6 \\ & *a^3*b*c*f + 30*a^2*b*c^2*d + 12*a^2*b^2*c*e))^2/(a^8*(4*a*c - b^2)^3))^(1/2) \\ &) + 3*b^2*d + a^2*f - 2*a*b*e - 2*a*c*d)/(4*a^4) - (c^4*(3*b^2*d + a^2*f - \\ & 2*a*b*e - 2*a*c*d)*(3*b^3*d - 2*a*b^2*e + a^2*b*f + 6*a^2*c*e - 11*a*b*c*d \\ &)^2)/(a^9*(4*a*c - b^2)^2) + (c^5*x^2*(3*b^3*d - 2*a*b^2*e + a^2*b*f + 6*a^ \\ & 2*c*e - 11*a*b*c*d)^3)/(a^9*(4*a*c - b^2)^3))*((((c^3*(36*b^8*d^2 + 16*a^2* \\ & b^6*e^2 + 4*a^4*b^4*f^2 - 36*a^5*c^3*e^2 - 116*a^3*b^4*c*e^2 - 17*a^5*b^2*c \\ & *f^2 - 48*a*b^7*d*e + 778*a^2*b^4*c^2*d^2 - 473*a^3*b^2*c^3*d^2 + 216*a^4*b \end{aligned}$$

$$\begin{aligned}
& ^2c^2e^2 - 309ab^6cd^2 + 24a^2b^6d^2f - 16a^3b^5e^2f + 380a^2b^5cd^2e + 324a^4b^3cd^2e - 154a^3b^4cd^2f + 92a^4b^3c^2e^2f - 108a^5b^3c^2e^2f - 832a^3b^3c^2d^2e + 230a^4b^2c^2d^2f) / (a^6(4ac - b^2)^2) - (((b^2c^2(ab + 3b^2x^2 - 10acx^2))(a^4(-(3b^5d + a^2b^3f - 12a^3c^2e - 2ab^4e - 20ab^3cd - 6a^3b^2cf + 30a^2b^2cd + 12a^2b^2c^2e))^2 / (a^8(4ac - b^2)^3))^{1/2} - 3b^2d - a^2f + 2ab^2e + 2ac^2d)) / a^4 + (4b^2c^2(3b^5d + a^2b^3f - 6a^3c^2e - 2ab^4e - 17ab^3cd - 5a^3b^2cf + 19a^2b^2cd + 10a^2b^2c^2e)) / (a^3(4ac - b^2)) + (2c^3x^2(3b^5d + a^2b^3f + 60a^3c^2e - 2ab^4e + 4ab^3cd - 10a^3b^2cf - 70a^2b^2cd - 4a^2b^2c^2e)) / (a^3(4ac - b^2))) * (a^4(-(3b^5d + a^2b^3f - 12a^3c^2e - 2ab^4e - 20ab^3cd - 6a^3b^2cf + 30a^2b^2cd + 12a^2b^2c^2e))^2 / (a^8(4ac - b^2)^3))^{1/2} - 3b^2d - a^2f + 2ab^2e + 2ac^2d)) / (4a^4) + (c^4x^2(54b^7d^2 + 24a^2b^5e^2 + 6a^4b^3f^2 - 440a^3b^2cd^2 - 164a^3b^3c^2e^2 + 276a^4b^2c^2e^2 - 72ab^6d^2e + 1011a^2b^3c^2d^2 - 441ab^5cd^2 - 20a^5b^2c^2f^2 + 36a^2b^5d^2f + 240a^4c^3d^2e - 24a^3b^4e^2f - 120a^5c^2e^2f + 540a^2b^4cd^2e - 207a^3b^3cd^2f + 260a^4b^2cd^2f + 122a^4b^2c^2e^2f - 1072a^3b^2c^2d^2e)) / (a^6(4ac - b^2)^2)) * (a^4(-(3b^5d + a^2b^3f - 12a^3c^2e - 2ab^4e - 20ab^3cd - 6a^3b^2cf + 30a^2b^2cd + 12a^2b^2c^2e))^2 / (a^8(4ac - b^2)^3))^{1/2} - 3b^2d - a^2f + 2ab^2e + 2ac^2d)) / (4a^4) + (c^4(3b^2d + a^2f - 2ab^2e - 2ac^2d)) * (3b^3d - 2ab^2e + a^2bf + 6a^2c^2e - 11ab^2cd)^2 / (a^9(4ac - b^2)^2) - (c^5x^2(3b^3d - 2ab^2e + a^2bf + 6a^2c^2e - 11ab^2cd)^3) / (a^9(4ac - b^2)^3)) * (6b^8d + 256a^4c^4d + 2a^2b^6f - 128a^5c^3f - 4ab^7e + 336a^2b^4c^2d - 576a^3b^2c^3d - 192a^3b^3c^2e + 96a^4b^2c^2f - 76ab^6cd + 48a^2b^5c^2e + 256a^4b^2c^3e - 24a^3b^4c^2f)) / (2(4a^4b^6 - 256a^7c^3 - 48a^5b^4c + 192a^6b^2c^2)) - (d/(4a) + (x^2(2ae - 3bd)) / (4a^2) + (x^4(6b^4d + 8a^2c^2d + 2a^2b^2f - 4ab^3e - 4a^3cf - 25ab^2cd + 14a^2b^2c^2e)) / (4a^3(4ac - b^2)) + (cx^6(3b^3d - 2ab^2e + a^2bf + 6a^2c^2e - 11ab^2cd)) / (2a^3(4ac - b^2))) / (ax^4 + bx^6 + cx^8) + (atan(x^2(((((((1760a^7b^8cd^2 - 1104a^8b^7c^2e^2 + 80a^9b^6c^2f^2 + 54a^3b^9c^4d^2 - 657a^4b^7c^5d^2 + 2775a^5b^5c^6d^2 - 4484a^6b^3c^7d^2 + 24a^5b^7c^4e^2 - 260a^6b^5c^5e^2 + 932a^7b^3c^6e^2 + 6a^7b^5c^4f^2 - 44a^8b^3c^5f^2 - 960a^8c^8d^2e + 480a^9c^7e^2f - 1040a^8b^7c^7d^2f - 72a^4b^8c^4d^2e + 828a^5b^6c^5d^2e - 3232a^6b^4c^6d^2e + 4528a^7b^2c^7d^2e + 36a^5b^7c^4d^2f - 351a^6b^5c^5d^2f + 1088a^7b^3c^6d^2f - 24a^6b^6c^4e^2f + 218a^7b^4c^5e^2f - 608a^8b^2c^6e^2f)) / (a^9b^6 - 64a^12c^3 - 12a^10b^4c + 48a^11b^2c^2) - (((1920a^11c^7e + 6a^6b^9c^3d - 40a^7b^7c^4d - 108a^8b^5c^5d + 1248a^9b^3c^6d - 4a^7b^8c^3e + 24a^8b^6c^4e + 120a^9b^4c^5e - 1088a^10b^2c^6e + 2a^8b^7c^3f - 36a^9b^5c^4f + 192a^10b^3c^5f - 2240a^10b^2c^7d - 320a^11b^2c^6f)) / (a^9b^6 - 64a^12c^3 - 12a^10b^4c + 48a^11b^2c^2) + ((2560a^13b^6c^6 + 12a^9b^9c^2 - 184a^10b^7c^3 + 1056a^11b^5c^4 - 2688a^12b^3c^5) * (6b^8d + 256a^4c^4d + 2a^2b^6f - 128a^5c^3f - 4ab^7e + 336a^2b^4c^2d - 576a^3b^2c^3d - 192a^3b^3c^2e + 96a^4b^2c^2f - 76ab^6cd + 48a^2b^5c^2e + 256a^4b^2c^3e - 24a^3b^4c^2f)) / (2(a^9b^6 - 64a^12c^3 - 12a^10b^4c + 48a^11b^2c^2)) * (4a^4b^6 - 256a^7c^3 - 48a^5b^4c + 192a^6b^2c^2))) * (6b^8d + 256a^4c^4d + 2a^2b^6f - 128a^5c^3f - 4ab^7e + 336a^2b^4c^2d - 576a^3b^2c^3d - 192a^3b^3c^2e + 96a^4b^2c^2f - 76ab^6cd + 48a^2b^5c^2e + 256a^4b^2c^3e - 24a^3b^4c^2f)) / (2(4a^4b^6 - 256a^7c^3 - 48a^5b^4c + 192a^6b^2c^2)) - (216a^6c^8e^3 + 27b^9c^5d^3 - 297ab^7c^6d^3 + 1089a^2b^5c^7d^3 - 1331a^3b^3c^8d^3 - 8a^3b^6c^5e^3 + 72a^4b^4c^6e^3 - 216a^5b^2c^7e^3 + a^6b^3
\end{aligned}$$

$$\begin{aligned}
& *c^5*f^3 - 54*a*b^8*c^5*d^2*e - 1188*a^5*b*c^8*d*e^2 + 108*a^6*b*c^7*e^2*f \\
& + 558*a^2*b^6*c^6*d^2*e + 36*a^2*b^7*c^5*d*e^2 - 1914*a^3*b^4*c^7*d^2*e - 3 \\
& 48*a^3*b^5*c^6*d*e^2 + 2178*a^4*b^2*c^8*d^2*e + 1116*a^4*b^3*c^7*d*e^2 + 27 \\
& *a^2*b^7*c^5*d^2*f - 198*a^3*b^5*c^6*d^2*f + 363*a^4*b^3*c^7*d^2*f + 9*a^4* \\
& b^5*c^5*d*f^2 - 33*a^5*b^3*c^6*d*f^2 + 12*a^4*b^5*c^5*e^2*f - 72*a^5*b^3*c^ \\
& 6*e^2*f - 6*a^5*b^4*c^5*e*f^2 + 18*a^6*b^2*c^6*e*f^2 - 36*a^3*b^6*c^5*d*e*f \\
& + 240*a^4*b^4*c^6*d*e*f - 396*a^5*b^2*c^7*d*e*f)/(a^9*b^6 - 64*a^12*c^3 - \\
& 12*a^10*b^4*c + 48*a^11*b^2*c^2) + (((((1920*a^11*c^7*e + 6*a^6*b^9*c^3*d - \\
& 40*a^7*b^7*c^4*d - 108*a^8*b^5*c^5*d + 1248*a^9*b^3*c^6*d - 4*a^7*b^8*c^3* \\
& e + 24*a^8*b^6*c^4*e + 120*a^9*b^4*c^5*e - 1088*a^10*b^2*c^6*e + 2*a^8*b^7* \\
& c^3*f - 36*a^9*b^5*c^4*f + 192*a^10*b^3*c^5*f - 2240*a^10*b*c^7*d - 320*a^1 \\
& 1*b*c^6*f)/(a^9*b^6 - 64*a^12*c^3 - 12*a^10*b^4*c + 48*a^11*b^2*c^2) + ((25 \\
& 60*a^13*b*c^6 + 12*a^9*b^9*c^2 - 184*a^10*b^7*c^3 + 1056*a^11*b^5*c^4 - 268 \\
& 8*a^12*b^3*c^5)*(6*b^8*d + 256*a^4*c^4*d + 2*a^2*b^6*f - 128*a^5*c^3*f - 4* \\
& a*b^7*e + 336*a^2*b^4*c^2*d - 576*a^3*b^2*c^3*d - 192*a^3*b^3*c^2*e + 96*a^ \\
& 4*b^2*c^2*f - 76*a*b^6*c*d + 48*a^2*b^5*c*e + 256*a^4*b*c^3*e - 24*a^3*b^4* \\
& c*f))/(2*(a^9*b^6 - 64*a^12*c^3 - 12*a^10*b^4*c + 48*a^11*b^2*c^2)*(4*a^4*b \\
& ^6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b^2*c^2)))*(3*b^5*d + a^2*b^3*f - \\
& 12*a^3*c^2*e - 2*a*b^4*e - 20*a*b^3*c*d - 6*a^3*b*c*f + 30*a^2*b*c^2*d + 1 \\
& 2*a^2*b^2*c*e))/(4*a^4*(4*a*c - b^2)^(3/2)) + ((2560*a^13*b*c^6 + 12*a^9*b^ \\
& 9*c^2 - 184*a^10*b^7*c^3 + 1056*a^11*b^5*c^4 - 2688*a^12*b^3*c^5)*(3*b^5*d \\
& + a^2*b^3*f - 12*a^3*c^2*e - 2*a*b^4*e - 20*a*b^3*c*d - 6*a^3*b*c*f + 30*a^ \\
& 2*b*c^2*d + 12*a^2*b^2*c*e)*(6*b^8*d + 256*a^4*c^4*d + 2*a^2*b^6*f - 128*a^ \\
& 5*c^3*f - 4*a*b^7*e + 336*a^2*b^4*c^2*d - 576*a^3*b^2*c^3*d - 192*a^3*b^3*c^ \\
& ^2*e + 96*a^4*b^2*c^2*f - 76*a*b^6*c*d + 48*a^2*b^5*c*e + 256*a^4*b*c^3*e - \\
& 24*a^3*b^4*c*f))/(8*a^4*(4*a*c - b^2)^(3/2)*(a^9*b^6 - 64*a^12*c^3 - 12*a^ \\
& 10*b^4*c + 48*a^11*b^2*c^2)*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192*a^ \\
& ^6*b^2*c^2)))*(3*b^5*d + a^2*b^3*f - 12*a^3*c^2*e - 2*a*b^4*e - 20*a*b^3*c* \\
& d - 6*a^3*b*c*f + 30*a^2*b*c^2*d + 12*a^2*b^2*c*e))/(4*a^4*(4*a*c - b^2)^(3 \\
& /2)) + ((2560*a^13*b*c^6 + 12*a^9*b^9*c^2 - 184*a^10*b^7*c^3 + 1056*a^11*b^ \\
& 5*c^4 - 2688*a^12*b^3*c^5)*(3*b^5*d + a^2*b^3*f - 12*a^3*c^2*e - 2*a*b^4*e \\
& - 20*a*b^3*c*d - 6*a^3*b*c*f + 30*a^2*b*c^2*d + 12*a^2*b^2*c*e)^2*(6*b^8*d \\
& + 256*a^4*c^4*d + 2*a^2*b^6*f - 128*a^5*c^3*f - 4*a*b^7*e + 336*a^2*b^4*c^2 \\
& *d - 576*a^3*b^2*c^3*d - 192*a^3*b^3*c^2*e + 96*a^4*b^2*c^2*f - 76*a*b^6*c* \\
& d + 48*a^2*b^5*c*e + 256*a^4*b*c^3*e - 24*a^3*b^4*c*f))/(32*a^8*(4*a*c - b^ \\
& 2)^3*(a^9*b^6 - 64*a^12*c^3 - 12*a^10*b^4*c + 48*a^11*b^2*c^2)*(4*a^4*b^6 - \\
& 256*a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b^2*c^2)))*(9*b^7*d + 3*a^2*b^5*f + 6 \\
& *a^4*c^3*e - 6*a*b^6*e + 150*a^2*b^3*c^2*d - 72*a^3*b^2*c^2*e - 69*a*b^5*c* \\
& d - 75*a^3*b*c^3*d + 42*a^2*b^4*c*e - 21*a^3*b^3*c*f + 33*a^4*b*c^2*f))/(8* \\
& a^3*c^2*(4*a*c - b^2)^3*(1600*a^5*c^5*d^2 - 24*a^2*b^8*e^2 - 54*b^10*d^2 - \\
& 6*a^4*b^6*f^2 + 36*a^6*c^4*e^2 + 400*a^7*c^3*f^2 + 288*a^3*b^6*c*e^2 + 72*a^ \\
& ^5*b^4*c*f^2 + 72*a*b^9*d*e - 3480*a^2*b^6*c^2*d^2 + 7200*a^3*b^4*c^3*d^2 - \\
& 5775*a^4*b^2*c^4*d^2 - 1152*a^4*b^4*c^2*e^2 + 1528*a^5*b^2*c^3*e^2 - 291*a^ \\
& ^6*b^2*c^2*f^2 + 720*a*b^8*c*d^2 - 36*a^2*b^8*d*f + 24*a^3*b^7*e*f - 1600*a^ \\
& ^6*c^4*d*f - 912*a^2*b^7*c*d*e + 3020*a^5*b*c^4*d*e + 456*a^3*b^6*c*d*f - 2 \\
& 88*a^4*b^5*c*e*f - 1564*a^6*b*c^3*e*f + 4032*a^3*b^5*c^2*d*e - 6900*a^4*b^3 \\
& *c^3*d*e - 2025*a^4*b^4*c^2*d*f + 3510*a^5*b^2*c^3*d*f + 1158*a^5*b^3*c^2*e \\
& *f)) - (((((1760*a^7*b*c^8*d^2 - 1104*a^8*b*c^7*e^2 + 80*a^9*b*c^6*f^2 + 54 \\
& *a^3*b^9*c^4*d^2 - 657*a^4*b^7*c^5*d^2 + 2775*a^5*b^5*c^6*d^2 - 4484*a^6*b^ \\
& 3*c^7*d^2 + 24*a^5*b^7*c^4*e^2 - 260*a^6*b^5*c^5*e^2 + 932*a^7*b^3*c^6*e^2 \\
& + 6*a^7*b^5*c^4*f^2 - 44*a^8*b^3*c^5*f^2 - 960*a^8*c^8*d*e + 480*a^9*c^7*e* \\
& f - 1040*a^8*b*c^7*d*f - 72*a^4*b^8*c^4*d*e + 828*a^5*b^6*c^5*d*e - 3232*a^ \\
& 6*b^4*c^6*d*e + 4528*a^7*b^2*c^7*d*e + 36*a^5*b^7*c^4*d*f - 351*a^6*b^5*c^5 \\
& *d*f + 1088*a^7*b^3*c^6*d*f - 24*a^6*b^6*c^4*e*f + 218*a^7*b^4*c^5*e*f - 60 \\
& 8*a^8*b^2*c^6*e*f)/(a^9*b^6 - 64*a^12*c^3 - 12*a^10*b^4*c + 48*a^11*b^2*c^2 \\
&) - (((1920*a^11*c^7*e + 6*a^6*b^9*c^3*d - 40*a^7*b^7*c^4*d - 108*a^8*b^5*c^ \\
& ^5*d + 1248*a^9*b^3*c^6*d - 4*a^7*b^8*c^3*e + 24*a^8*b^6*c^4*e + 120*a^9*b^ \\
& 4*c^5*e - 1088*a^10*b^2*c^6*e + 2*a^8*b^7*c^3*f - 36*a^9*b^5*c^4*f + 192*a^ \\
& 10*b^3*c^5*f - 2240*a^10*b*c^7*d - 320*a^11*b*c^6*f)/(a^9*b^6 - 64*a^12*c^3
\end{aligned}$$

$$\begin{aligned}
& - 12a^{10}b^4c + 48a^{11}b^2c^2) + ((2560a^{13}b^6c^6 + 12a^9b^9c^2 - 184a^{10}b^7c^3 + 1056a^{11}b^5c^4 - 2688a^{12}b^3c^5) * (6b^8d + 256a^4c^4d + 2a^2b^6f - 128a^5c^3f - 4a^7e + 336a^2b^4c^2d - 576a^3b^2c^3d - 192a^3b^3c^2e + 96a^4b^2c^2f - 76a^6c^3d + 48a^2b^5c^3e + 256a^4b^3c^3e - 24a^3b^4c^3f)) / (2(a^9b^6 - 64a^{12}c^3 - 12a^{10}b^4c + 48a^{11}b^2c^2) * (4a^4b^6 - 256a^7c^3 - 48a^5b^4c + 192a^6b^2c^2)) * (6b^8d + 256a^4c^4d + 2a^2b^6f - 128a^5c^3f - 4a^7e + 336a^2b^4c^2d - 576a^3b^2c^3d - 192a^3b^3c^2e + 96a^4b^2c^2f - 76a^6c^3d + 48a^2b^5c^3e + 256a^4b^3c^3e - 24a^3b^4c^3f)) / (2(4a^4b^6 - 256a^7c^3 - 48a^5b^4c + 192a^6b^2c^2)) * (3b^5d + a^2b^3f - 12a^3c^2e - 2a^4e - 20a^3c^3d - 6a^3b^3c^3f + 30a^2b^3c^2d + 12a^2b^2c^3e)) / (4a^4(4a^3c - b^2)^{3/2}) - (((((1920a^{11}c^7e + 6a^6b^9c^3d - 40a^7b^7c^4d - 108a^8b^5c^5d + 1248a^9b^3c^6d - 4a^7b^8c^3e + 24a^8b^6c^4e + 120a^9b^4c^5e - 1088a^{10}b^2c^6e + 2a^8b^7c^3f - 36a^9b^5c^4f + 192a^{10}b^3c^5f - 2240a^{10}b^3c^7d - 320a^{11}b^3c^6f)) / (a^9b^6 - 64a^{12}c^3 - 12a^{10}b^4c + 48a^{11}b^2c^2) + ((2560a^{13}b^6c^6 + 12a^9b^9c^2 - 184a^{10}b^7c^3 + 1056a^{11}b^5c^4 - 2688a^{12}b^3c^5) * (6b^8d + 256a^4c^4d + 2a^2b^6f - 128a^5c^3f - 4a^7e + 336a^2b^4c^2d - 576a^3b^2c^3d - 192a^3b^3c^2e + 96a^4b^2c^2f - 76a^6c^3d + 48a^2b^5c^3e + 256a^4b^3c^3e - 24a^3b^4c^3f)) / (2(a^9b^6 - 64a^{12}c^3 - 12a^{10}b^4c + 48a^{11}b^2c^2) * (4a^4b^6 - 256a^7c^3 - 48a^5b^4c + 192a^6b^2c^2)) * (3b^5d + a^2b^3f - 12a^3c^2e - 2a^4e - 20a^3c^3d - 6a^3b^3c^3f + 30a^2b^3c^2d + 12a^2b^2c^3e)) / (4a^4(4a^3c - b^2)^{3/2}) + ((2560a^{13}b^6c^6 + 12a^9b^9c^2 - 184a^{10}b^7c^3 + 1056a^{11}b^5c^4 - 2688a^{12}b^3c^5) * (3b^5d + a^2b^3f - 12a^3c^2e - 2a^4e - 20a^3c^3d - 6a^3b^3c^3f + 30a^2b^3c^2d + 12a^2b^2c^3e)) * (6b^8d + 256a^4c^4d + 2a^2b^6f - 128a^5c^3f - 4a^7e + 336a^2b^4c^2d - 576a^3b^2c^3d - 192a^3b^3c^2e + 96a^4b^2c^2f - 76a^6c^3d + 48a^2b^5c^3e + 256a^4b^3c^3e - 24a^3b^4c^3f)) / (8a^4(4a^3c - b^2)^{3/2}) * (a^9b^6 - 64a^{12}c^3 - 12a^{10}b^4c + 48a^{11}b^2c^2) * (4a^4b^6 - 256a^7c^3 - 48a^5b^4c + 192a^6b^2c^2)) * (6b^8d + 256a^4c^4d + 2a^2b^6f - 128a^5c^3f - 4a^7e + 336a^2b^4c^2d - 576a^3b^2c^3d - 192a^3b^3c^2e + 96a^4b^2c^2f - 76a^6c^3d + 48a^2b^5c^3e + 256a^4b^3c^3e - 24a^3b^4c^3f)) / (2(4a^4b^6 - 256a^7c^3 - 48a^5b^4c + 192a^6b^2c^2)) + ((2560a^{13}b^6c^6 + 12a^9b^9c^2 - 184a^{10}b^7c^3 + 1056a^{11}b^5c^4 - 2688a^{12}b^3c^5) * (3b^5d + a^2b^3f - 12a^3c^2e - 2a^4e - 20a^3c^3d - 6a^3b^3c^3f + 30a^2b^3c^2d + 12a^2b^2c^3e)^3) / (64a^{12}(4a^3c - b^2)^{9/2} * (a^9b^6 - 64a^{12}c^3 - 12a^{10}b^4c + 48a^{11}b^2c^2)) * (4608b^8d + 40960a^4c^4d + 1536a^2b^6f - 20480a^5c^3f - 3072a^7e + 138240a^2b^4c^2d - 145920a^3b^2c^3d - 73728a^3b^3c^2e + 35328a^4b^2c^2f - 44544a^6c^3d + 27648a^2b^5c^3e + 50176a^4b^3c^3e - 13824a^3b^4c^3f)) / (4096a^3c^2(4a^3c - b^2)^{7/2} * (1600a^5c^5d^2 - 24a^2b^8e^2 - 54b^10d^2 - 6a^4b^6f^2 + 36a^6c^4e^2 + 400a^7c^3f^2 + 288a^3b^6c^3e^2 + 72a^5b^4c^3f^2 + 72a^6b^9d^2e - 3480a^2b^6c^2d^2 + 7200a^3b^4c^3d^2 - 5775a^4b^2c^4d^2 - 1152a^4b^4c^2e^2 + 1528a^5b^2c^3e^2 - 291a^6b^2c^2f^2 + 720a^6b^8c^3d^2 - 36a^2b^8d^2f + 24a^3b^7e^2f - 1600a^6c^4d^2f - 912a^2b^7c^3d^2e + 3020a^5b^3c^4d^2e + 456a^3b^6c^3d^2f - 288a^4b^5c^3e^2f - 1564a^6b^3c^3e^2f + 4032a^3b^5c^2d^2e - 6900a^4b^3c^3d^2e - 2025a^4b^4c^2d^2f + 3510a^5b^2c^3d^2f + 1158a^5b^3c^2e^2f)) * (16a^{12}b^6(4a^3c - b^2)^{9/2} - 1024a^{15}c^3(4a^3c - b^2)^{9/2} - 192a^{13}b^4c(4a^3c - b^2)^{9/2} + 768a^{14}b^2c^2(4a^3c - b^2)^{9/2})) / (144a^6c^6e^2 + 9b^10c^2d^2 - 120a^6b^8c^3d^2 + 580a^2b^6c^4d^2 - 1200a^3b^4c^5d^2 + 900a^4b^2c^6d^2 + 4a^2b^8c^2e^2 - 48a^3b^6c^3e^2 + 192a^4b^4c^4e^2 - 288a^5b^2c^5e^2 + a^4b^6c^2f^2 - 12a^5b^4c^3f^2 + 36a^6b^2c^4f^2 - 12a^6b^9c^2d^2e - 720a^5b^3c^6d^2e + 144a^6b^3c^5e^2f + 152a^2b^7c^3d^2e - 672a^3b^5c^4d^2e + 1200a^4b^3c^5d^2e + 6a^2b^8c^2d^2f - 76a^3b^6c^3d^2f + 300a^4b^4c^4d^2f - 360a
\end{aligned}$$

$$\begin{aligned}
& ^5b^2c^5d^f - 4a^3b^7c^2e^f + 48a^4b^5c^3e^f - 168a^5b^3c^4e \\
& *f) - ((16a^{12}b^6(4a^*c - b^2)^{(9/2)} - 1024a^{15}c^3(4a^*c - b^2)^{(9/2)} \\
& - 192a^{13}b^4c^*(4a^*c - b^2)^{(9/2)} + 768a^{14}b^2c^2(4a^*c - b^2)^{(9/2)} \\
&))*((27b^8c^4d^3 - 216a*b^6c^5d^3 - 72a^5b*c^6e^3 - 72a^5c^7d^e \\
& ^2 + 36a^6c^6e^2*f + 495a^2b^4c^6d^3 - 242a^3b^2c^7d^3 - 8a^3b \\
& ^5c^4e^3 + 48a^4b^3c^5e^3 + a^6b^2c^4f^3 - 54a*b^7c^4d^2e + 26 \\
& 4a^4b*c^7d^2e + 12a^6b*c^5e^f^2 + 396a^2b^5c^5d^2e + 36a^2b^6 \\
& *c^4d^e^2 - 798a^3b^3c^6d^2e - 240a^3b^4c^5d^e^2 + 420a^4b^2c^ \\
& 6d^e^2 + 27a^2b^6c^4d^2*f - 144a^3b^4c^5d^2*f + 165a^4b^2c^6d^ \\
& 2*f + 9a^4b^4c^4d*f^2 - 24a^5b^2c^5d*f^2 + 12a^4b^4c^4e^2*f - 4 \\
& 8a^5b^2c^5e^2*f - 6a^5b^3c^4e^f^2 - 156a^5b*c^6d^e^f - 36a^3b^ \\
& 5c^4d^e^f + 168a^4b^3c^5d^e^f)/(a^9b^4 + 16a^{11}c^2 - 8a^{10}b^2c) \\
& + (((36a^8c^6e^2 - 36a^3b^8c^3d^2 + 309a^4b^6c^4d^2 - 778a^5b \\
& ^4c^5d^2 + 473a^6b^2c^6d^2 - 16a^5b^6c^3e^2 + 116a^6b^4c^4e^2 \\
& - 216a^7b^2c^5e^2 - 4a^7b^4c^3f^2 + 17a^8b^2c^4f^2 - 324a^7b \\
& *c^6d^e + 108a^8b*c^5e^f + 48a^4b^7c^3d^e - 380a^5b^5c^4d^e + 8 \\
& 32a^6b^3c^5d^e - 24a^5b^6c^3d^f + 154a^6b^4c^4d^f - 230a^7b^2 \\
& *c^5d^f + 16a^6b^5c^3e^f - 92a^7b^3c^4e^f)/(a^9b^4 + 16a^{11}c^2 \\
& - 8a^{10}b^2c) + (((12a^6b^8c^2d - 116a^7b^6c^3d + 348a^8b^4c^4 \\
& *d - 304a^9b^2c^5d - 8a^7b^7c^2e + 72a^8b^5c^3e - 184a^9b^3c^ \\
& ^4e + 4a^8b^6c^2f - 36a^9b^4c^3f + 80a^{10}b^2c^4f + 96a^{10}b*c \\
& ^5e)/(a^9b^4 + 16a^{11}c^2 - 8a^{10}b^2c) + ((4a^{10}b^6c^2 - 32a^{11}b \\
& ^4c^3 + 64a^{12}b^2c^4)*(6b^8d + 256a^4c^4d + 2a^2b^6f - 128a^5c^ \\
& ^3f - 4a*b^7e + 336a^2b^4c^2d - 576a^3b^2c^3d - 192a^3b^3c^2 \\
& *e + 96a^4b^2c^2f - 76a*b^6c*d + 48a^2b^5c*e + 256a^4b*c^3e - 2 \\
& 4a^3b^4c*f))/(2*(a^9b^4 + 16a^{11}c^2 - 8a^{10}b^2c)*(4a^4b^6 - 256* \\
& a^7c^3 - 48a^5b^4c + 192a^6b^2c^2))*(6b^8d + 256a^4c^4d + 2a^ \\
& 2b^6f - 128a^5c^3f - 4a*b^7e + 336a^2b^4c^2d - 576a^3b^2c^3d \\
& - 192a^3b^3c^2e + 96a^4b^2c^2f - 76a*b^6c*d + 48a^2b^5c*e + 2 \\
& 56a^4b*c^3e - 24a^3b^4c*f))/(2*(4a^4b^6 - 256a^7c^3 - 48a^5b^4c \\
& + 192a^6b^2c^2))*(6b^8d + 256a^4c^4d + 2a^2b^6f - 128a^5c^3 \\
& *f - 4a*b^7e + 336a^2b^4c^2d - 576a^3b^2c^3d - 192a^3b^3c^2e \\
& + 96a^4b^2c^2f - 76a*b^6c*d + 48a^2b^5c*e + 256a^4b*c^3e - 24a \\
& ^3b^4c*f))/(2*(4a^4b^6 - 256a^7c^3 - 48a^5b^4c + 192a^6b^2c^2)) \\
& - ((((((12a^6b^8c^2d - 116a^7b^6c^3d + 348a^8b^4c^4d - 304a^9 \\
& b^2c^5d - 8a^7b^7c^2e + 72a^8b^5c^3e - 184a^9b^3c^4e + 4a^8b \\
& ^6c^2f - 36a^9b^4c^3f + 80a^{10}b^2c^4f + 96a^{10}b*c^5e)/(a^9b^ \\
& 4 + 16a^{11}c^2 - 8a^{10}b^2c) + ((4a^{10}b^6c^2 - 32a^{11}b^4c^3 + 64a \\
& ^{12}b^2c^4)*(6b^8d + 256a^4c^4d + 2a^2b^6f - 128a^5c^3f - 4a*b \\
& ^7e + 336a^2b^4c^2d - 576a^3b^2c^3d - 192a^3b^3c^2e + 96a^4b \\
& ^2c^2f - 76a*b^6c*d + 48a^2b^5c*e + 256a^4b*c^3e - 24a^3b^4c*f \\
&))/(2*(a^9b^4 + 16a^{11}c^2 - 8a^{10}b^2c)*(4a^4b^6 - 256a^7c^3 - 48 \\
& a^5b^4c + 192a^6b^2c^2))*(3b^5d + a^2b^3f - 12a^3c^2e - 2a*b^ \\
& 4e - 20a*b^3c*d - 6a^3b*c*f + 30a^2b*c^2d + 12a^2b^2c*e))/(4a^4 \\
& *(4a^*c - b^2)^{(3/2)}) + (((4a^{10}b^6c^2 - 32a^{11}b^4c^3 + 64a^{12}b^2c^ \\
& 4)*(3b^5d + a^2b^3f - 12a^3c^2e - 2a*b^4e - 20a*b^3c*d - 6a^3b \\
& *c*f + 30a^2b*c^2d + 12a^2b^2c*e)*(6b^8d + 256a^4c^4d + 2a^2b^ \\
& 6f - 128a^5c^3f - 4a*b^7e + 336a^2b^4c^2d - 576a^3b^2c^3d - 1 \\
& 92a^3b^3c^2e + 96a^4b^2c^2f - 76a*b^6c*d + 48a^2b^5c*e + 256a \\
& ^4b*c^3e - 24a^3b^4c*f))/(8a^4*(4a^*c - b^2)^{(3/2)}*(a^9b^4 + 16a^{11} \\
& *c^2 - 8a^{10}b^2c)*(4a^4b^6 - 256a^7c^3 - 48a^5b^4c + 192a^6b^2 \\
& c^2))*(3b^5d + a^2b^3f - 12a^3c^2e - 2a*b^4e - 20a*b^3c*d - 6a \\
& ^3b*c*f + 30a^2b*c^2d + 12a^2b^2c*e))/(4a^4*(4a^*c - b^2)^{(3/2)}) - \\
& ((4a^{10}b^6c^2 - 32a^{11}b^4c^3 + 64a^{12}b^2c^4)*(3b^5d + a^2b^3f \\
& - 12a^3c^2e - 2a*b^4e - 20a*b^3c*d - 6a^3b*c*f + 30a^2b*c^2d + \\
& 12a^2b^2c*e)^2*(6b^8d + 256a^4c^4d + 2a^2b^6f - 128a^5c^3f - \\
& 4a*b^7e + 336a^2b^4c^2d - 576a^3b^2c^3d - 192a^3b^3c^2e + 96 \\
& a^4b^2c^2f - 76a*b^6c*d + 48a^2b^5c*e + 256a^4b*c^3e - 24a^3b^ \\
& 4c*f))/(32a^8*(4a^*c - b^2)^3*(a^9b^4 + 16a^{11}c^2 - 8a^{10}b^2c)*(4a
\end{aligned}$$

$$\begin{aligned}
& ^4b^6 - 256a^7c^3 - 48a^5b^4c + 192a^6b^2c^2)))(9b^7d + 3a^2b \\
& ^5f + 6a^4c^3e - 6ab^6e + 150a^2b^3c^2d - 72a^3b^2c^2e - 69* \\
& ab^5cd - 75a^3b^3c^3d + 42a^2b^4c^3e - 21a^3b^3c^3f + 33a^4b^3c^2 \\
& *f)))/(8a^3c^2(4ac - b^2)^3(144a^6c^6e^2 + 9b^10c^2d^2 - 120ab \\
& ^8c^3d^2 + 580a^2b^6c^4d^2 - 1200a^3b^4c^5d^2 + 900a^4b^2c^6d \\
& ^2 + 4a^2b^8c^2e^2 - 48a^3b^6c^3e^2 + 192a^4b^4c^4e^2 - 288a^5 \\
& b^2c^5e^2 + a^4b^6c^2f^2 - 12a^5b^4c^3f^2 + 36a^6b^2c^4f^2 - \\
& 12ab^9c^2d^2e - 720a^5b^6c^6d^2e + 144a^6b^3c^5d^2e + 152a^2b^7c^3 \\
& d^2e - 672a^3b^5c^4d^2e + 1200a^4b^3c^5d^2e + 6a^2b^8c^2d^2f - 76a \\
& ^3b^6c^3d^2f + 300a^4b^4c^4d^2f - 360a^5b^2c^5d^2f - 4a^3b^7c^2* \\
& e^2f + 48a^4b^5c^3e^2f - 168a^5b^3c^4e^2f)(1600a^5c^5d^2 - 24a^2* \\
& b^8e^2 - 54b^10d^2 - 6a^4b^6f^2 + 36a^6c^4e^2 + 400a^7c^3f^2 + \\
& 288a^3b^6c^3e^2 + 72a^5b^4c^3f^2 + 72a^2b^9d^2e - 3480a^2b^6c^2d^2 \\
& + 7200a^3b^4c^3d^2 - 5775a^4b^2c^4d^2 - 1152a^4b^4c^2e^2 + 1528 \\
& a^5b^2c^3e^2 - 291a^6b^2c^2f^2 + 720a^2b^8c^3d^2 - 36a^2b^8d^2f + \\
& 24a^3b^7e^2f - 1600a^6c^4d^2f - 912a^2b^7c^3d^2e + 3020a^5b^3c^4d^2e \\
& + 456a^3b^6c^3d^2f - 288a^4b^5c^3e^2f - 1564a^6b^3c^3e^2f + 4032a^3b^ \\
& 5c^2d^2e - 6900a^4b^3c^3d^2e - 2025a^4b^4c^2d^2f + 3510a^5b^2c^3* \\
& d^2f + 1158a^5b^3c^2e^2f)) + (((((((12a^6b^8c^2d - 116a^7b^6c^3d \\
& + 348a^8b^4c^4d - 304a^9b^2c^5d - 8a^7b^7c^2e + 72a^8b^5c^3* \\
& e - 184a^9b^3c^4e + 4a^8b^6c^2f - 36a^9b^4c^3f + 80a^10b^2c^ \\
& 4f + 96a^10b^3c^5e)/(a^9b^4 + 16a^11c^2 - 8a^10b^2c) + ((4a^10b^ \\
& 6c^2 - 32a^11b^4c^3 + 64a^12b^2c^4)*(6b^8d + 256a^4c^4d + 2a^2 \\
& *b^6f - 128a^5c^3f - 4ab^7e + 336a^2b^4c^2d - 576a^3b^2c^3d \\
& - 192a^3b^3c^2e + 96a^4b^2c^2f - 76a^2b^6c^3d + 48a^2b^5c^3e + 25 \\
& 6a^4b^3c^3e - 24a^3b^4c^3f)))/(2(a^9b^4 + 16a^11c^2 - 8a^10b^2c)* \\
& (4a^4b^6 - 256a^7c^3 - 48a^5b^4c + 192a^6b^2c^2)))(3b^5d + a^2 \\
& *b^3f - 12a^3c^2e - 2ab^4e - 20a^2b^3c^2d - 6a^3b^3c^2f + 30a^2b^3c \\
& ^2d + 12a^2b^2c^2e))/(4a^4(4ac - b^2)^(3/2)) + ((4a^10b^6c^2 - 32 \\
& a^11b^4c^3 + 64a^12b^2c^4)*(3b^5d + a^2b^3f - 12a^3c^2e - 2a* \\
& b^4e - 20a^2b^3c^2d - 6a^3b^3c^2f + 30a^2b^3c^2d + 12a^2b^2c^2e)*(6b^ \\
& 8d + 256a^4c^4d + 2a^2b^6f - 128a^5c^3f - 4ab^7e + 336a^2b^4 \\
& c^2d - 576a^3b^2c^3d - 192a^3b^3c^2e + 96a^4b^2c^2f - 76a^2b^ \\
& 6c^3d + 48a^2b^5c^3e + 256a^4b^3c^3e - 24a^3b^4c^3f))/(8a^4(4ac - \\
& b^2)^(3/2)*(a^9b^4 + 16a^11c^2 - 8a^10b^2c)*(4a^4b^6 - 256a^7c^3 \\
& - 48a^5b^4c + 192a^6b^2c^2)))(6b^8d + 256a^4c^4d + 2a^2b^6f \\
& - 128a^5c^3f - 4ab^7e + 336a^2b^4c^2d - 576a^3b^2c^3d - 192* \\
& a^3b^3c^2e + 96a^4b^2c^2f - 76a^2b^6c^3d + 48a^2b^5c^3e + 256a^4* \\
& b^3c^3e - 24a^3b^4c^3f))/(2(4a^4b^6 - 256a^7c^3 - 48a^5b^4c + 192 \\
& a^6b^2c^2)) + (((36a^8c^6e^2 - 36a^3b^8c^3d^2 + 309a^4b^6c^4d \\
& ^2 - 778a^5b^4c^5d^2 + 473a^6b^2c^6d^2 - 16a^5b^6c^3e^2 + 116a \\
& ^6b^4c^4e^2 - 216a^7b^2c^5e^2 - 4a^7b^4c^3f^2 + 17a^8b^2c^4f \\
& ^2 - 324a^7b^3c^6d^2e + 108a^8b^3c^5e^2f + 48a^4b^7c^3d^2e - 380a^5b \\
& ^5c^4d^2e + 832a^6b^3c^5d^2e - 24a^5b^6c^3d^2f + 154a^6b^4c^4d^2f \\
& - 230a^7b^2c^5d^2f + 16a^6b^5c^3e^2f - 92a^7b^3c^4e^2f)/(a^9b^4 \\
& + 16a^11c^2 - 8a^10b^2c) + (((12a^6b^8c^2d - 116a^7b^6c^3d + 3 \\
& 48a^8b^4c^4d - 304a^9b^2c^5d - 8a^7b^7c^2e + 72a^8b^5c^3e - \\
& 184a^9b^3c^4e + 4a^8b^6c^2f - 36a^9b^4c^3f + 80a^10b^2c^4f \\
& + 96a^10b^3c^5e)/(a^9b^4 + 16a^11c^2 - 8a^10b^2c) + ((4a^10b^6c \\
& ^2 - 32a^11b^4c^3 + 64a^12b^2c^4)*(6b^8d + 256a^4c^4d + 2a^2b^ \\
& 6f - 128a^5c^3f - 4ab^7e + 336a^2b^4c^2d - 576a^3b^2c^3d - 1 \\
& 92a^3b^3c^2e + 96a^4b^2c^2f - 76a^2b^6c^3d + 48a^2b^5c^3e + 256a \\
& ^4b^3c^3e - 24a^3b^4c^3f))/(2(a^9b^4 + 16a^11c^2 - 8a^10b^2c)*(4* \\
& a^4b^6 - 256a^7c^3 - 48a^5b^4c + 192a^6b^2c^2)))(6b^8d + 256a^ \\
& 4c^4d + 2a^2b^6f - 128a^5c^3f - 4ab^7e + 336a^2b^4c^2d - 576 \\
& a^3b^2c^3d - 192a^3b^3c^2e + 96a^4b^2c^2f - 76a^2b^6c^3d + 48a \\
& ^2b^5c^3e + 256a^4b^3c^3e - 24a^3b^4c^3f))/(2(4a^4b^6 - 256a^7c^3 \\
& - 48a^5b^4c + 192a^6b^2c^2)))(3b^5d + a^2b^3f - 12a^3c^2e - \\
& 2a^2b^4e - 20a^2b^3c^2d - 6a^3b^3c^2f + 30a^2b^3c^2d + 12a^2b^2c^2e))/
\end{aligned}$$

$$\begin{aligned}
& (4a^4(4ac - b^2)^{3/2}) - ((4a^{10}b^6c^2 - 32a^{11}b^4c^3 + 64a^{12}b^2c^4)(3b^5d + a^2b^3f - 12a^3c^2e - 2ab^4e - 20ab^3cd - 6 \\
& a^3b^2c^2d + 30a^2b^2c^2d + 12a^2b^2c^2e)^3)/(64a^{12}(4ac - b^2)^{9/2})(a^9b^4 + 16a^{11}c^2 - 8a^{10}b^2c)) * (16a^{12}b^6(4ac - b^2)^{9/2} \\
& - 1024a^{15}c^3(4ac - b^2)^{9/2} - 192a^{13}b^4c(4ac - b^2)^{9/2} \\
& + 768a^{14}b^2c^2(4ac - b^2)^{9/2}) * (4608b^8d + 40960a^4c^4d + 153 \\
& 6a^2b^6f - 20480a^5c^3f - 3072ab^7e + 138240a^2b^4c^2d - 14592 \\
& 0a^3b^2c^3d - 73728a^3b^3c^2e + 35328a^4b^2c^2f - 44544ab^6c \\
& *d + 27648a^2b^5c^2e + 50176a^4b^3c^3e - 13824a^3b^4c^2f) / (4096a^3c \\
& ^2(4ac - b^2)^{7/2})(144a^6c^6e^2 + 9b^{10}c^2d^2 - 120ab^8c^3d \\
& ^2 + 580a^2b^6c^4d^2 - 1200a^3b^4c^5d^2 + 900a^4b^2c^6d^2 + 4a \\
& ^2b^8c^2e^2 - 48a^3b^6c^3e^2 + 192a^4b^4c^4e^2 - 288a^5b^2c^5 \\
& *e^2 + a^4b^6c^2f^2 - 12a^5b^4c^3f^2 + 36a^6b^2c^4f^2 - 12ab^9 \\
& *c^2d^2e - 720a^5b^3c^6d^2e + 144a^6b^2c^5e^2f + 152a^2b^7c^3d^2e - 67 \\
& 2a^3b^5c^4d^2e + 1200a^4b^3c^5d^2e + 6a^2b^8c^2d^2f - 76a^3b^6c \\
& ^3d^2f + 300a^4b^4c^4d^2f - 360a^5b^2c^5d^2f - 4a^3b^7c^2e^2f + 48 \\
& a^4b^5c^3e^2f - 168a^5b^3c^4e^2f) * (1600a^5c^5d^2 - 24a^2b^8e^2 \\
& - 54b^{10}d^2 - 6a^4b^6f^2 + 36a^6c^4e^2 + 400a^7c^3f^2 + 288a^3b \\
& ^6c^2e^2 + 72a^5b^4c^2f^2 + 72ab^9d^2e - 3480a^2b^6c^2d^2 + 7200a \\
& ^3b^4c^3d^2 - 5775a^4b^2c^4d^2 - 1152a^4b^4c^2e^2 + 1528a^5b^2 \\
& *c^3e^2 - 291a^6b^2c^2f^2 + 720ab^8c^2d^2 - 36a^2b^8d^2f + 24a^3b \\
& ^7e^2f - 1600a^6c^4d^2f - 912a^2b^7c^2d^2e + 3020a^5b^3c^4d^2e + 456a \\
& ^3b^6c^2d^2f - 288a^4b^5c^2e^2f - 1564a^6b^3c^3e^2f + 4032a^3b^5c^2d^2 \\
& e - 6900a^4b^3c^3d^2e - 2025a^4b^4c^2d^2f + 3510a^5b^2c^3d^2f + 11 \\
& 58a^5b^3c^2e^2f) * (3b^5d + a^2b^3f - 12a^3c^2e - 2ab^4e - 20 \\
& ab^3cd - 6a^3b^2c^2d + 12a^2b^2c^2e) / (2a^4(4ac - \\
& b^2)^{3/2})
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**4+e*x**2+d)/x**5/(c*x**4+b*x**2+a)**2,x)

[Out] Timed out

$$3.66 \int \frac{x^6(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2} dx$$

Optimal. Leaf size=550

$$\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right) \left(\frac{-20a^2c^3e-b^3c(cd-34af)-19ab^2c^2e+4abc^2(2cd-13af)-5b^5f+3b^4ce}{\sqrt{b^2-4ac}} - b^2c(cd-24af) - 13abc^2e + 2ac^2(3cd - 2\sqrt{2}c^{7/2}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}} \right)$$

Rubi [A] time = 13.23, antiderivative size = 550, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1668, 1676, 1166, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right) \left(\frac{-20a^2c^3e-b^3c(cd-34af)-19ab^2c^2e+4abc^2(2cd-13af)-5b^5f+3b^4ce}{\sqrt{b^2-4ac}} - b^2c(cd-24af) - 13abc^2e + 2ac^2(3cd - 2\sqrt{2}c^{7/2}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}} \right)}{2\sqrt{2}c^{7/2}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

[In] Int[(x^6*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]

[Out] ((c*e - 2*b*f)*x)/c^3 + (f*x^3)/(3*c^2) + (x*(a*(b^2*c*e - 2*a*c^2*e - b^3*f - b*c*(c*d - 3*a*f)) + (b^3*c*e - 3*a*b*c^2*e - b^4*f - b^2*c*(c*d - 4*a*f) + 2*a*c^2*(c*d - a*f))*x^2))/(2*c^3*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((3*b^3*c*e - 13*a*b*c^2*e - 5*b^4*f - b^2*c*(c*d - 24*a*f) + 2*a*c^2*(3*c*d - 7*a*f) - (3*b^4*c*e - 19*a*b^2*c^2*e + 20*a^2*c^3*e - 5*b^5*f - b^3*c*(c*d - 34*a*f) + 4*a*b*c^2*(2*c*d - 13*a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/(2*Sqrt[2]*c^(7/2)*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((3*b^3*c*e - 13*a*b*c^2*e - 5*b^4*f - b^2*c*(c*d - 24*a*f) + 2*a*c^2*(3*c*d - 7*a*f) + (3*b^4*c*e - 19*a*b^2*c^2*e + 20*a^2*c^3*e - 5*b^5*f - b^3*c*(c*d - 34*a*f) + 4*a*b*c^2*(2*c*d - 13*a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]/(2*Sqrt[2]*c^(7/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1668

Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]

Rule 1676

`Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1`

Rubi steps

$$\begin{aligned} \int \frac{x^6(d + ex^2 + fx^4)}{(a + bx^2 + cx^4)^2} dx &= \frac{x(a(b^2ce - 2ac^2e - b^3f - bc(cd - 3af)) + (b^3ce - 3abc^2e - b^4f - b^2c(cd - 4af)) +}{2c^3(b^2 - 4ac)(a + bx^2 + cx^4)} \\ &= \frac{x(a(b^2ce - 2ac^2e - b^3f - bc(cd - 3af)) + (b^3ce - 3abc^2e - b^4f - b^2c(cd - 4af)) +}{2c^3(b^2 - 4ac)(a + bx^2 + cx^4)} \\ &= \frac{(ce - 2bf)x}{c^3} + \frac{fx^3}{3c^2} + \frac{x(a(b^2ce - 2ac^2e - b^3f - bc(cd - 3af)) + (b^3ce - 3abc^2e - b^4f - b^2c(cd - 4af)) +}{2c^3(b^2 - 4ac)(a + bx^2 + cx^4)} \\ &= \frac{(ce - 2bf)x}{c^3} + \frac{fx^3}{3c^2} + \frac{x(a(b^2ce - 2ac^2e - b^3f - bc(cd - 3af)) + (b^3ce - 3abc^2e - b^4f - b^2c(cd - 4af)) +}{2c^3(b^2 - 4ac)(a + bx^2 + cx^4)} \\ &= \frac{(ce - 2bf)x}{c^3} + \frac{fx^3}{3c^2} + \frac{x(a(b^2ce - 2ac^2e - b^3f - bc(cd - 3af)) + (b^3ce - 3abc^2e - b^4f - b^2c(cd - 4af)) +}{2c^3(b^2 - 4ac)(a + bx^2 + cx^4)} \end{aligned}$$

Mathematica [A] time = 2.13, size = 648, normalized size = 1.18

Integrate[(x^6*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2, x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && Expon[Pq, x^2] > 1

Antiderivative was successfully verified.

[In] Integrate[(x^6*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2, x]

[Out] (12* \sqrt{c} *(c*e - 2*b*f)*x + 4*c^(3/2)*f*x^3 - (6* \sqrt{c} *x*(b^2*(c^2*d - b*c*e + b^2*f)*x^2 + a^2*c*(-3*b*f + 2*c*(e + f*x^2)) + a*(b^3*f - 2*c^3*d*x^2 + b*c^2*(d + 3*e*x^2) - b^2*c*(e + 4*f*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (3* $\sqrt{2}$ *(-5*b^5*f + a*b*c^2*(8*c*d + 13* $\sqrt{b^2 - 4*a*c}$)*e - 52*a*f) - b^3*c*(c*d + 3* $\sqrt{b^2 - 4*a*c}$ *e - 34*a*f) + b^4*(3*c*e + 5* $\sqrt{b^2 - 4*a*c}$ *f) + b^2*c*(c* $\sqrt{b^2 - 4*a*c}$ *d - 19*a*c*e - 24*a* $\sqrt{b^2 - 4*a*c}$ *f) + 2*a*c^2*(-3*c* $\sqrt{b^2 - 4*a*c}$ *d + 10*a*c*e + 7*a* $\sqrt{b^2 - 4*a*c}$ *f))*ArcTan[($\sqrt{2}$ * \sqrt{c} *x)/ $\sqrt{b - \sqrt{b^2 - 4*a*c}}$])/((b^2 - 4*a*c)^(3/2)* $\sqrt{b - \sqrt{b^2 - 4*a*c}}$) + (3* $\sqrt{2}$ *(5*b^5*f + b^3*c*(c*d - 3* $\sqrt{b^2 - 4*a*c}$ *e - 34*a*f) + a*b*c^2*(-8*c*d + 13* $\sqrt{b^2 - 4*a*c}$ *e + 52*a*f) + b^4*(-3*c*e + 5* $\sqrt{b^2 - 4*a*c}$ *f) + b^2*c*(c* $\sqrt{b^2 - 4*a*c}$ *d + 19*a*c*e - 24*a* $\sqrt{b^2 - 4*a*c}$ *f) - 2*a*c^2*(3*c* $\sqrt{b^2 - 4*a*c}$ *d + 10*a*c*e - 7*a* $\sqrt{b^2 - 4*a*c}$ *f))*ArcTan[($\sqrt{2}$ * \sqrt{c} *x)/ $\sqrt{b + \sqrt{b^2 - 4*a*c}}$])/((b^2 - 4*a*c)^(3/2)* $\sqrt{b + \sqrt{b^2 - 4*a*c}}$))/(12*c^(7/2))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6(d + ex^2 + fx^4)}{(a + bx^2 + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^6*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]

[Out] IntegrateAlgebraic[(x^6*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2, x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")

[Out] Timed out

giac [B] time = 9.04, size = 8957, normalized size = 16.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")

[Out]
$$-1/2*(b^2*c^2*d*x^3 - 2*a*c^3*d*x^3 + b^4*f*x^3 - 4*a*b^2*c*f*x^3 + 2*a^2*c^2*f*x^3 - b^3*c*x^3*e + 3*a*b*c^2*x^3*e + a*b*c^2*d*x + a*b^3*f*x - 3*a^2*b*c*f*x - a*b^2*c*x*e + 2*a^2*c^2*x*e)/((b^2*c^3 - 4*a*c^4)*(c*x^4 + b*x^2 + a)) + 1/16*((2*b^4*c^4 - 20*a*b^2*c^5 + 48*a^2*c^6 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*b^4*c^2 + 10*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*a*b^2*c^3 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*b^3*c^3 - 24*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*a^2*c^4 - 12*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*a*b*c^4 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*b^2*c^4 + 6*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*a*c^5 - 2*(b^2 - 4*a*c)*b^2*c^4 + 12*(b^2 - 4*a*c)*a*c^5)*(b^2*c^3 - 4*a*c^4)^2*d + (10*b^6*c^2 - 88*a*b^4*c^3 + 220*a^2*b^2*c^4 - 112*a^3*c^5 - 5*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*b^6 + 44*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*a*b^4*c + 10*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*b^5*c - 110*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*a^2*b^2*c^2 - 48*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*a*b^3*c^2 - 5*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*b^4*c^2 + 56*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*a^3*c^3 + 28*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*a^2*b*c^3 + 24*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*a*b^2*c^3 - 14*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*a^2*c^4 - 10*(b^2 - 4*a*c)*b^4*c^2 + 48*(b^2 - 4*a*c)*a*b^2*c^3 - 28*(b^2 - 4*a*c)*a^2*c^4)*(b^2*c^3 - 4*a*c^4)^2*f - (6*b^5*c^3 - 50*a*b^3*c^4 + 104*a^2*b*c^5 - 3*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*b^5*c + 25*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*a*b^3*c^2 + 6*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*b^4*c^2 - 52*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*a^2*b*c^3 - 26*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*a*b^2*c^3 - 3*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*b^3*c^3 + 13*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*a*b*c^4 - 6*(b^2 - 4*a*c)*b^3*c^3 + 26*(b^2 - 4*a*c)*a*b*c^4)*(b^2*c^3 - 4*a*c^4)^2*e + 2*(\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*a*b^5*c^6 - 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*a^2*b^3*c^7 - 2*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*a*b^4*c^7 - 2*a*b^5*c^7 + 16*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*a^3*b*c^8 + 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*a^2*b^2*c^8 + \sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*a*b^3*c^8 + 16*a^2*b^3*c^8 - 4*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*a^2*b*c^9 - 32*a^3*b*c^9 + 2*(b^2 - 4*a*c)*a*b^3*c^7 - 8*(b^2 - 4*a*c)*a^2*b*c^8)*d*abs(b^2*c^3 - 4*a*c^4) + 2*(5*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*a*b^7*c^4 - 59*\sqrt{2})*$$

$$\begin{aligned}
& \text{rt}(b^2 - 4ac) \cdot c \cdot a^2 b^3 c^{11} + 80 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}} \cdot c \cdot a^3 b^3 c^{12} - 6(b^2 - 4ac) b^7 c^9 + 62(b^2 - 4ac) \\
& \cdot a^2 b^5 c^{10} - 192(b^2 - 4ac) a^2 b^3 c^{11} + 160(b^2 - 4ac) a^3 b^3 c^{12} \\
& \cdot e \cdot \arctan\left(\frac{2\sqrt{\frac{1}{2}} \sqrt{x} / \sqrt{(b^3 c^3 - 4ab^2 c^4 + \sqrt{(b^3 c^3 - 4ab^2 c^4)^2 - 4(a^2 b^2 c^3 - 4a^2 c^4)(b^2 c^4 - 4ac^5)})}}}{(b^2 c^4 - 4ac^5)}\right) \\
& \cdot \left((ab^6 c^7 - 12a^2 b^4 c^8 - 2ab^5 c^8 + 48a^3 b^2 c^9 + 16a^2 b^3 c^9 + ab^4 c^9 - 64a^4 c^{10} - 32a^3 b^3 c^{10} - 8a^2 b^2 c^{10} + 16a^3 c^{11}) \cdot \text{abs}(b^2 c^3 - 4ac^4) \cdot \text{abs}(c) \right) + \frac{1}{16} \cdot \left((2b^4 c^4 - 20ab^2 c^5 + 48 \\
& a^2 c^6 - \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c) b^4 c^2 + 10\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c) a^2 b^2 c^3 + 2\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c) b^3 c^3 - 24\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c) a^2 c^4 - 12\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c) a^2 b^2 c^4 - \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c) b^2 c^4 + 6\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c) a^2 c^5 - 2(b^2 - 4ac) b^2 c^4 + 12(b^2 - 4ac) a^2 c^5) \cdot (b^2 c^3 - 4ac^4)^2 d + (10b^6 c^2 - 88ab^4 c^3 + 220a^2 b^2 c^4 - 112a^3 c^5 - 5\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c) b^6 + 44\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c) a^2 b^4 c + 10\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c) b^5 c - 110\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c) a^2 b^2 c^2 - 48\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c) a^2 b^2 c^2 - 5\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c) b^4 c^2 + 56\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c) a^3 c^3 + 28\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c) a^2 b^3 c^3 + 24\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c) a^2 b^2 c^3 - 14\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c) a^2 c^4 - 10(b^2 - 4ac) b^4 c^2 + 48(b^2 - 4ac) a^2 b^2 c^3 - 28(b^2 - 4ac) a^2 c^4) \cdot (b^2 c^3 - 4ac^4)^2 f - (6b^5 c^3 - 50ab^3 c^4 + 104a^2 b^3 c^5 - 3\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c) b^5 c + 25\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c) a^2 b^3 c^2 + 6\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c) b^4 c^2 - 52\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c) a^2 b^2 c^3 - 26\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c) a^2 b^2 c^3 - 3\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c) b^3 c^3 + 13\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c) a^2 b^2 c^3 - 6(b^2 - 4ac) b^3 c^3 + 26(b^2 - 4ac) a^2 b^2 c^3) \cdot (b^2 c^3 - 4ac^4)^2 e + 2(\sqrt{2} \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c) a^2 b^3 c^7 - 2\sqrt{2} \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c) a^2 b^4 c^7 + 2a^2 b^5 c^7 + 16\sqrt{2} \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c) a^3 b^3 c^8 + 8\sqrt{2} \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c) a^2 b^2 c^8 + \sqrt{2} \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c) a^2 b^3 c^8 - 16a^2 b^3 c^8 - 4\sqrt{2} \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c) a^2 b^3 c^8 + 32a^3 b^3 c^8 - 2(b^2 - 4ac) a^2 b^3 c^7 + 8(b^2 - 4ac) a^2 b^2 c^8) \cdot d \cdot \text{abs}(b^2 c^3 - 4ac^4) + 2(5\sqrt{2} \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c) a^2 b^5 c^5 - 10\sqrt{2} \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c) a^2 b^6 c^5 + 10a^2 b^7 c^5 + 232\sqrt{2} \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c) a^3 b^3 c^6 + 78\sqrt{2} \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c) a^2 b^4 c^6 + 5\sqrt{2} \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c) a^2 b^5 c^6 - 118a^2 b^5 c^6 - 304\sqrt{2} \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c) a^4 b^2 c^7 - 152\sqrt{2} \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c) a^3 b^2 c^7 - 39\sqrt{2} \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c) a^2 b^3 c^7 + 464a^3 b^3 c^7 + 76\sqrt{2} \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c) a^3 b^2 c^8 - 608a^4 b^2 c^8 - 10(b^2 - 4ac) a^2 b^5 c^5 + 78(b^2 - 4ac) a^2 b^3 c^6 - 152(b^2 - 4ac) a^3 b^2 c^7) \cdot f \cdot \text{abs}(b^2 c^3 - 4ac^4) - 2(3\sqrt{2} \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c) a^2 b^4 c^6 - 6\sqrt{2} \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c) a^2 b^5 c^6 + 6a^2 b^6 c^6 + 128\sqrt{2} \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c) a^3 b^2 c^7 + 44\sqrt{2} \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c) a^2 b^3 c^7 + 3\sqrt{2} \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c) a^2 b^4 c^7 - 68a^2 b^4 c^7 - 160\sqrt{2} \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c) a^4 c^8 - 80\sqrt{2} \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c) a^3 b^2 c^8 - 22\sqrt{2} \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c)
\end{aligned}$$

$$\begin{aligned} & \text{rt}(b*c - \sqrt{b^2 - 4*a*c})*c)*a^2*b^2*c^8 + 256*a^3*b^2*c^8 + 40*\sqrt{2}*\text{sq} \\ & \text{rt}(b*c - \sqrt{b^2 - 4*a*c})*c)*a^3*c^9 - 320*a^4*c^9 - 6*(b^2 - 4*a*c)*a*b^4 \\ & *c^6 + 44*(b^2 - 4*a*c)*a^2*b^2*c^7 - 80*(b^2 - 4*a*c)*a^3*c^8)*\text{abs}(b^2*c^3 \\ & - 4*a*c^4)*e - (2*b^8*c^{10} - 32*a*b^6*c^{11} + 160*a^2*b^4*c^{12} - 256*a^3*b^ \\ & 2*c^{13} - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*b^8*c^8 \\ & + 16*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a*b^6*c^9 + \\ & 2*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*b^7*c^9 - 80*\text{sq} \\ & \text{rt}(2)*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a^2*b^4*c^{10} - 24*\text{s} \\ & \text{qrt}(2)*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a*b^5*c^{10} - \sqrt{(} \\ & 2)*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*b^6*c^{10} + 128*\sqrt{2} \\ & *\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a^3*b^2*c^{11} + 64*\sqrt{2} \\ &)*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a^2*b^3*c^{11} + 12*\sqrt{2} \\ &)*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a*b^4*c^{11} - 32*\sqrt{2} \\ &)*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a^2*b^2*c^{12} - 2*(b^2 - \\ & 4*a*c)*b^6*c^{10} + 24*(b^2 - 4*a*c)*a*b^4*c^{11} - 64*(b^2 - 4*a*c)*a^2*b^2*c \\ & ^{12})*d - (10*b^{10}*c^8 - 148*a*b^8*c^9 + 808*a^2*b^6*c^{10} - 1920*a^3*b^4*c^{1} \\ & 1 + 1664*a^4*b^2*c^{12} - 5*\sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4} \\ & *a*c})*c)*b^{10}*c^6 + 74*\sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a} \\ & *c})*c)*a*b^8*c^7 + 10*\sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*a} \\ & *c})*c)*b^9*c^7 - 404*\sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*a} \\ & *c})*c)*a^2*b^6*c^8 - 108*\sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*a} \\ & *c})*c)*a*b^7*c^8 - 5*\sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*a} \\ & *c})*c)*b^8*c^8 + 960*\sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*a} \\ & *c})*c)*a^3* \\ & b^4*c^9 + 376*\sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*a} \\ & *c})*c)*a^2 \\ & *b^5*c^9 + 54*\sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*a} \\ & *c})*c)*a*b \\ & ^6*c^9 - 832*\sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*a} \\ & *c})*c)*a^4* \\ & b^2*c^{10} - 416*\sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*a} \\ & *c})*c)*a^ \\ & 3*b^3*c^{10} - 188*\sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*a} \\ & *c})*c)* \\ & a^2*b^4*c^{10} + 208*\sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*a} \\ & *c})*c)* \\ & a^3*b^2*c^{11} - 10*(b^2 - 4*a*c)*b^8*c^8 + 108*(b^2 - 4*a*c)*a*b^6*c^9 - 3 \\ & 76*(b^2 - 4*a*c)*a^2*b^4*c^{10} + 416*(b^2 - 4*a*c)*a^3*b^2*c^{11})*f + (6*b^9* \\ & c^9 - 86*a*b^7*c^{10} + 440*a^2*b^5*c^{11} - 928*a^3*b^3*c^{12} + 640*a^4*b*c^{13} \\ & - 3*\sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*a} \\ & *c})*c)*b^9*c^7 + 43* \\ & \sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*a} \\ & *c})*c)*a*b^7*c^8 + 6*\text{sq} \\ & \text{rt}(2)*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*a} \\ & *c})*c)*b^8*c^8 - 220*\sqrt{2} \\ &)*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*a} \\ & *c})*c)*a^2*b^5*c^9 - 62*\sqrt{2} \\ &)*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*a} \\ & *c})*c)*a*b^6*c^9 - 3*\sqrt{2})*\text{s} \\ & \text{qrt}(b^2 - 4*a*c)*\sqrt{b*c - \sqrt{b^2 - 4*a*a} \\ & *c})*c)*b^7*c^9 + 464*\sqrt{2})*\sqrt{2})*\sqrt{ \\ & (b^2 - 4*a*c)*\sqrt{b*c - \sqrt{b^2 - 4*a*a} \\ & *c})*c)*a^3*b^3*c^{10} + 192*\sqrt{2})*\text{s} \\ & \text{qrt}(b^2 - 4*a*c)*\sqrt{b*c - \sqrt{b^2 - 4*a*a} \\ & *c})*c)*a^2*b^4*c^{10} + 31*\sqrt{2})*\text{s} \\ & \text{qrt}(b^2 - 4*a*c)*\sqrt{b*c - \sqrt{b^2 - 4*a*a} \\ & *c})*c)*a*b^5*c^{10} - 320*\sqrt{2})*\text{s} \\ & \text{qrt}(b^2 - 4*a*c)*\sqrt{b*c - \sqrt{b^2 - 4*a*a} \\ & *c})*c)*a^4*b*c^{11} - 160*\sqrt{2})*\text{s} \\ & \text{qrt}(b^2 - 4*a*c)*\sqrt{b*c - \sqrt{b^2 - 4*a*a} \\ & *c})*c)*a^3*b^2*c^{11} - 96*\sqrt{2})* \\ & \sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*a} \\ & *c})*c)*a^2*b^3*c^{11} + 80*\sqrt{2} \\ &)*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*a} \\ & *c})*c)*a^3*b*c^{12} - 6*(b^2 - 4* \\ & a*c)*b^7*c^9 + 62*(b^2 - 4*a*c)*a*b^5*c^{10} - 192*(b^2 - 4*a*c)*a^2*b^3*c^{11} \\ & + 160*(b^2 - 4*a*c)*a^3*b*c^{12})*e)*\arctan(2*\sqrt{1/2})*x/\sqrt{((b^3*c^3 - 4* \\ & a*b*c^4 - \sqrt{(b^3*c^3 - 4*a*b*c^4)^2 - 4*(a*b^2*c^3 - 4*a^2*c^4)*(b^2*c^4 \\ & - 4*a*c^5)))/(b^2*c^4 - 4*a*c^5)))/((a*b^6*c^7 - 12*a^2*b^4*c^8 - 2*a*b^5* \\ & c^8 + 48*a^3*b^2*c^9 + 16*a^2*b^3*c^9 + a*b^4*c^9 - 64*a^4*c^{10} - 32*a^3*b* \\ & c^{10} - 8*a^2*b^2*c^{10} + 16*a^3*c^{11})*\text{abs}(b^2*c^3 - 4*a*c^4)*\text{abs}(c)) + 1/3*(\\ & c^4*f*x^3 - 6*b*c^3*f*x + 3*c^4*x*e)/c^6 \end{aligned}$$

maple [B] time = 0.06, size = 2558, normalized size = 4.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^6*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2, x)$

$b + (-4ac + b^2)^{1/2} \cdot c^{1/2} \cdot cx \cdot b^4 \cdot f$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")

[Out]
$$-1/2 * (((b^2*c^2 - 2*a*c^3)*d - (b^3*c - 3*a*b*c^2)*e + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*f)*x^3 + (a*b*c^2*d - (a*b^2*c - 2*a^2*c^2)*e + (a*b^3 - 3*a^2*b*c)*f)*x) / (a*b^2*c^3 - 4*a^2*c^4 + (b^2*c^4 - 4*a*c^5)*x^4 + (b^3*c^3 - 4*a*b*c^4)*x^2) + 1/2 * integrate((a*b*c^2*d + ((b^2*c^2 - 6*a*c^3)*d - (3*b^3*c - 13*a*b*c^2)*e + (5*b^4 - 24*a*b^2*c + 14*a^2*c^2)*f)*x^2 - (3*a*b^2*c - 10*a^2*c^2)*e + (5*a*b^3 - 19*a^2*b*c)*f) / (c*x^4 + b*x^2 + a), x) / (b^2*c^3 - 4*a*c^4) + 1/3 * (c*f*x^3 + 3*(c*e - 2*b*f)*x) / c^3$$

mupad [B] time = 4.10, size = 33799, normalized size = 61.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^6*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x)

[Out]
$$x * (e/c^2 - (2*b*f)/c^3) + ((x^3*(b^4*f + b^2*c^2*d + 2*a^2*c^2*f - 2*a*c^3*d - b^3*c*e + 3*a*b*c^2*e - 4*a*b^2*c*f)) / (2*(4*a*c - b^2)) + (x*(2*a^2*c^2*e + a*b^3*f + a*b*c^2*d - a*b^2*c*e - 3*a^2*b*c*f)) / (2*(4*a*c - b^2))) / (a*c^3 + c^4*x^4 + b*c^3*x^2) - \operatorname{atan}\left(\frac{(10240*a^5*c^9*e + 192*a^2*b^5*c^7*d - 768*a^3*b^3*c^8*d - 736*a^2*b^6*c^6*e + 4224*a^3*b^4*c^7*e - 10752*a^4*b^2*c^8*e + 1264*a^2*b^7*c^5*f - 7488*a^3*b^5*c^6*f + 19712*a^4*b^3*c^7*f - 16*a*b^7*c^6*d + 1024*a^4*b*c^9*d + 48*a*b^8*c^5*e - 80*a*b^9*c^4*f - 19456*a^5*b*c^8*f) / (8*(64*a^3*c^8 - b^6*c^5 + 12*a*b^4*c^6 - 48*a^2*b^2*c^7)) - (x * (-25*b^15*f^2 + b^11*c^4*d^2 + 9*b^13*c^2*e^2 + 25*b^6*f^2*(-(4*a*c - b^2)^9)^{1/2} - 27*a*b^9*c^5*d^2 - 3840*a^5*b*c^9*d^2 - 9*a*c^5*d^2*(-(4*a*c - b^2)^9)^{1/2} - 213*a*b^11*c^3*e^2 + 26880*a^6*b*c^8*e^2 - 80640*a^7*b*c^7*f^2 - 30*b^14*c*e*f + 288*a^2*b^7*c^6*d^2 - 1504*a^3*b^5*c^7*d^2 + 3840*a^4*b^3*c^8*d^2 + 2077*a^2*b^9*c^4*e^2 - 10656*a^3*b^7*c^5*e^2 + 30240*a^4*b^5*c^6*e^2 - 44800*a^5*b^3*c^7*e^2 + 25*a^2*c^4*e^2*(-(4*a*c - b^2)^9)^{1/2} + b^2*c^4*d^2*(-(4*a*c - b^2)^9)^{1/2} + 6366*a^2*b^11*c^2*f^2 - 35767*a^3*b^9*c^3*f^2 + 116928*a^4*b^7*c^4*f^2 - 219744*a^5*b^5*c^5*f^2 + 215040*a^6*b^3*c^6*f^2 - 49*a^3*c^3*f^2*(-(4*a*c - b^2)^9)^{1/2} + 9*b^4*c^2*e^2*(-(4*a*c - b^2)^9)^{1/2} - 615*a*b^13*c*f^2 - 15360*a^6*c^9*d*e - 6*b^12*c^3*d*e + 35840*a^7*c^8*e*f + 10*b^13*c^2*d*f + 152*a*b^10*c^4*d*e - 258*a*b^11*c^3*d*f + 43520*a^6*b*c^8*d*f + 724*a*b^12*c^2*e*f - 30*b^5*c*e*f*(-(4*a*c - b^2)^9)^{1/2} + 246*a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^{1/2} - 165*a*b^4*c*f^2*(-(4*a*c - b^2)^9)^{1/2} - 1548*a^2*b^8*c^5*d*e + 8064*a^3*b^6*c^6*d*e - 22400*a^4*b^4*c^7*d*e + 30720*a^5*b^2*c^8*d*e + 2706*a^2*b^9*c^4*d*f - 14784*a^3*b^7*c^5*d*f + 44352*a^4*b^5*c^6*d*f - 69120*a^5*b^3*c^7*d*f + 42*a^2*c^4*d*f*(-(4*a*c - b^2)^9)^{1/2} - 6*b^3*c^3*d*e*(-(4*a*c - b^2)^9)^{1/2} - 7278*a^2*b^10*c^3*e*f + 39132*a^3*b^8*c^4*e*f - 119616*a^4*b^6*c^5*e*f + 201600*a^5*b^4*c^6*e*f - 161280*a^6*b^2*c^7*e*f + 10*b^4*c^2*d*f*(-(4*a*c - b^2)^9)^{1/2} - 51*a*b^2*c^3*e^2*(-(4*a*c - b^2)^9)^{1/2} + 44*a*b*c^4*d*e*(-(4*a*c - b^2)^9)^{1/2} - 78*a*b^2*c^3*d*f*(-(4*a*c - b^2)^9)^{1/2} + 184*a*b^3*c^2*e*f*(-(4*a*c - b^2)^9)^{1/2} - 186*a^2*b*c^3*e*f*(-(4*a*c - b^2)^9)^{1/2}) / (32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12))^{1/2} * (16*b^7*c^7 - 192*a*b^5*c^8 - 1024*a^3*b*c^10 + 768*a^2*b^3*c^9) / (2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)) * (-25*b^15*f^2 + b^11*c^4*d^2 + 9*b^13*c^2*e^2 + 25*b^6*f^2*(-(4*a*c - b^2)^9)^{1/2} - 27*a*b^9*c^5*d^2 - 3840*a^5*b*c^9*d^2 - 9*a*c^5*d^2*(-(4*a*c - b^2)^9)^{1/2} - 213*a*b^11*c^3*e^2 + 26880*a^6*b*c^8*e^2 - 80640*a^7*b*c^7*f^2 - 30*b^14*c*e*f + 288*a^2*b^7*c^6*d^2 - 15$$

$$\begin{aligned}
& 04*a^3*b^5*c^7*d^2 + 3840*a^4*b^3*c^8*d^2 + 2077*a^2*b^9*c^4*e^2 - 10656*a^3*b^7*c^5*e^2 + 30240*a^4*b^5*c^6*e^2 - 44800*a^5*b^3*c^7*e^2 + 25*a^2*c^4* \\
& e^2*(-(4*a*c - b^2)^9)^{(1/2)} + b^2*c^4*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + 6366* \\
& a^2*b^11*c^2*f^2 - 35767*a^3*b^9*c^3*f^2 + 116928*a^4*b^7*c^4*f^2 - 219744* \\
& a^5*b^5*c^5*f^2 + 215040*a^6*b^3*c^6*f^2 - 49*a^3*c^3*f^2*(-(4*a*c - b^2)^9 \\
&)^{(1/2)} + 9*b^4*c^2*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 615*a*b^13*c*f^2 - 15360 \\
& *a^6*c^9*d*e - 6*b^12*c^3*d*e + 35840*a^7*c^8*e*f + 10*b^13*c^2*d*f + 152*a \\
& *b^10*c^4*d*e - 258*a*b^11*c^3*d*f + 43520*a^6*b*c^8*d*f + 724*a*b^12*c^2*e \\
& *f - 30*b^5*c*e*f*(-(4*a*c - b^2)^9)^{(1/2)} + 246*a^2*b^2*c^2*f^2*(-(4*a*c - \\
& b^2)^9)^{(1/2)} - 165*a*b^4*c*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 1548*a^2*b^8*c^ \\
& 5*d*e + 8064*a^3*b^6*c^6*d*e - 22400*a^4*b^4*c^7*d*e + 30720*a^5*b^2*c^8*d* \\
& e + 2706*a^2*b^9*c^4*d*f - 14784*a^3*b^7*c^5*d*f + 44352*a^4*b^5*c^6*d*f - \\
& 69120*a^5*b^3*c^7*d*f + 42*a^2*c^4*d*f*(-(4*a*c - b^2)^9)^{(1/2)} - 6*b^3*c^3 \\
& *d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 7278*a^2*b^10*c^3*e*f + 39132*a^3*b^8*c^4*e \\
& *f - 119616*a^4*b^6*c^5*e*f + 201600*a^5*b^4*c^6*e*f - 161280*a^6*b^2*c^7*e \\
& *f + 10*b^4*c^2*d*f*(-(4*a*c - b^2)^9)^{(1/2)} - 51*a*b^2*c^3*e^2*(-(4*a*c - \\
& b^2)^9)^{(1/2)} + 44*a*b*c^4*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 78*a*b^2*c^3*d*f* \\
& (- (4*a*c - b^2)^9)^{(1/2)} + 184*a*b^3*c^2*e*f*(-(4*a*c - b^2)^9)^{(1/2)} - 186 \\
& *a^2*b*c^3*e*f*(-(4*a*c - b^2)^9)^{(1/2)}/(32*(4096*a^6*c^13 + b^12*c^7 - 24 \\
& *a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 614 \\
& 4*a^5*b^2*c^12)))^{(1/2)} - (x*(25*b^10*f^2 - 72*a^3*c^7*d^2 + 200*a^4*c^6*e^ \\
& 2 + b^6*c^4*d^2 - 392*a^5*c^5*f^2 + 9*b^8*c^2*e^2 - 16*a*b^4*c^5*d^2 - 114* \\
& a*b^6*c^3*e^2 - 30*b^9*c*e*f + 74*a^2*b^2*c^6*d^2 + 481*a^2*b^4*c^4*e^2 - 7 \\
& 18*a^3*b^2*c^5*e^2 + 1676*a^2*b^6*c^2*f^2 - 3536*a^3*b^4*c^3*f^2 + 2794*a^4 \\
& *b^2*c^4*f^2 - 340*a*b^8*c*f^2 + 336*a^4*c^6*d*f - 6*b^7*c^3*d*e + 10*b^8*c \\
& ^2*d*f + 86*a*b^5*c^4*d*e + 472*a^3*b*c^6*d*e - 148*a*b^6*c^3*d*f + 394*a*b \\
& ^7*c^2*e*f - 1768*a^4*b*c^5*e*f - 374*a^2*b^3*c^5*d*e + 698*a^2*b^4*c^4*d*f \\
& - 1132*a^3*b^2*c^5*d*f - 1804*a^2*b^5*c^3*e*f + 3266*a^3*b^3*c^4*e*f))/ (2* \\
& (16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))*(-(25*b^15*f^2 + b^11*c^4*d^2 + 9*b^ \\
& 13*c^2*e^2 + 25*b^6*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 27*a*b^9*c^5*d^2 - 3840* \\
& a^5*b*c^9*d^2 - 9*a*c^5*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - 213*a*b^11*c^3*e^2 + \\
& 26880*a^6*b*c^8*e^2 - 80640*a^7*b*c^7*f^2 - 30*b^14*c*e*f + 288*a^2*b^7*c^ \\
& 6*d^2 - 1504*a^3*b^5*c^7*d^2 + 3840*a^4*b^3*c^8*d^2 + 2077*a^2*b^9*c^4*e^2 \\
& - 10656*a^3*b^7*c^5*e^2 + 30240*a^4*b^5*c^6*e^2 - 44800*a^5*b^3*c^7*e^2 + 2 \\
& 5*a^2*c^4*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + b^2*c^4*d^2*(-(4*a*c - b^2)^9)^{(1/ \\
& 2)} + 6366*a^2*b^11*c^2*f^2 - 35767*a^3*b^9*c^3*f^2 + 116928*a^4*b^7*c^4*f^2 \\
& - 219744*a^5*b^5*c^5*f^2 + 215040*a^6*b^3*c^6*f^2 - 49*a^3*c^3*f^2*(-(4*a* \\
& c - b^2)^9)^{(1/2)} + 9*b^4*c^2*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 615*a*b^13*c*f \\
& ^2 - 15360*a^6*c^9*d*e - 6*b^12*c^3*d*e + 35840*a^7*c^8*e*f + 10*b^13*c^2*d \\
& *f + 152*a*b^10*c^4*d*e - 258*a*b^11*c^3*d*f + 43520*a^6*b*c^8*d*f + 724*a* \\
& b^12*c^2*e*f - 30*b^5*c*e*f*(-(4*a*c - b^2)^9)^{(1/2)} + 246*a^2*b^2*c^2*f^2* \\
& (- (4*a*c - b^2)^9)^{(1/2)} - 165*a*b^4*c*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 1548* \\
& a^2*b^8*c^5*d*e + 8064*a^3*b^6*c^6*d*e - 22400*a^4*b^4*c^7*d*e + 30720*a^5* \\
& b^2*c^8*d*e + 2706*a^2*b^9*c^4*d*f - 14784*a^3*b^7*c^5*d*f + 44352*a^4*b^5* \\
& c^6*d*f - 69120*a^5*b^3*c^7*d*f + 42*a^2*c^4*d*f*(-(4*a*c - b^2)^9)^{(1/2)} - \\
& 6*b^3*c^3*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 7278*a^2*b^10*c^3*e*f + 39132*a^3 \\
& *b^8*c^4*e*f - 119616*a^4*b^6*c^5*e*f + 201600*a^5*b^4*c^6*e*f - 161280*a^6 \\
& *b^2*c^7*e*f + 10*b^4*c^2*d*f*(-(4*a*c - b^2)^9)^{(1/2)} - 51*a*b^2*c^3*e^2*(\\
& - (4*a*c - b^2)^9)^{(1/2)} + 44*a*b*c^4*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 78*a*b^ \\
& 2*c^3*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 184*a*b^3*c^2*e*f*(-(4*a*c - b^2)^9)^{(\\
& 1/2)} - 186*a^2*b*c^3*e*f*(-(4*a*c - b^2)^9)^{(1/2)}/(32*(4096*a^6*c^13 + b^1 \\
& 2*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4* \\
& c^11 - 6144*a^5*b^2*c^12)))^{(1/2)}*i - (((10240*a^5*c^9*e + 192*a^2*b^5*c^7 \\
& *d - 768*a^3*b^3*c^8*d - 736*a^2*b^6*c^6*e + 4224*a^3*b^4*c^7*e - 10752*a^4 \\
& *b^2*c^8*e + 1264*a^2*b^7*c^5*f - 7488*a^3*b^5*c^6*f + 19712*a^4*b^3*c^7*f \\
& - 16*a*b^7*c^6*d + 1024*a^4*b*c^9*d + 48*a*b^8*c^5*e - 80*a*b^9*c^4*f - 194 \\
& 56*a^5*b*c^8*f)/(8*(64*a^3*c^8 - b^6*c^5 + 12*a*b^4*c^6 - 48*a^2*b^2*c^7)) \\
& + (x*(-(25*b^15*f^2 + b^11*c^4*d^2 + 9*b^13*c^2*e^2 + 25*b^6*f^2*(-(4*a*c - \\
& b^2)^9)^{(1/2)} - 27*a*b^9*c^5*d^2 - 3840*a^5*b*c^9*d^2 - 9*a*c^5*d^2*(-(4*a
\end{aligned}$$

$$\begin{aligned}
& *c - b^2)^9)^{(1/2)} - 213*a*b^{11}*c^3*e^2 + 26880*a^6*b*c^8*e^2 - 80640*a^7*b \\
& *c^7*f^2 - 30*b^{14}*c*e*f + 288*a^2*b^7*c^6*d^2 - 1504*a^3*b^5*c^7*d^2 + 384 \\
& 0*a^4*b^3*c^8*d^2 + 2077*a^2*b^9*c^4*e^2 - 10656*a^3*b^7*c^5*e^2 + 30240*a^4 \\
& *b^5*c^6*e^2 - 44800*a^5*b^3*c^7*e^2 + 25*a^2*c^4*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + b^2*c^4*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + 6366*a^2*b^{11}*c^2*f^2 - 35767 \\
& *a^3*b^9*c^3*f^2 + 116928*a^4*b^7*c^4*f^2 - 219744*a^5*b^5*c^5*f^2 + 215040 \\
& *a^6*b^3*c^6*f^2 - 49*a^3*c^3*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + 9*b^4*c^2*e^2* \\
& (- (4*a*c - b^2)^9)^{(1/2)} - 615*a*b^{13}*c*f^2 - 15360*a^6*c^9*d*e - 6*b^{12}*c^3 \\
& *d*e + 35840*a^7*c^8*e*f + 10*b^{13}*c^2*d*f + 152*a*b^{10}*c^4*d*e - 258*a*b^{11} \\
& *c^3*d*f + 43520*a^6*b*c^8*d*f + 724*a*b^{12}*c^2*e*f - 30*b^5*c*e*f*(-(4*a \\
& *c - b^2)^9)^{(1/2)} + 246*a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 165*a*b \\
& ^4*c*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 1548*a^2*b^8*c^5*d*e + 8064*a^3*b^6*c^6 \\
& *d*e - 22400*a^4*b^4*c^7*d*e + 30720*a^5*b^2*c^8*d*e + 2706*a^2*b^9*c^4*d*f \\
& - 14784*a^3*b^7*c^5*d*f + 44352*a^4*b^5*c^6*d*f - 69120*a^5*b^3*c^7*d*f + \\
& 42*a^2*c^4*d*f*(-(4*a*c - b^2)^9)^{(1/2)} - 6*b^3*c^3*d*e*(-(4*a*c - b^2)^9)^{(1/2)} \\
& - 7278*a^2*b^{10}*c^3*e*f + 39132*a^3*b^8*c^4*e*f - 119616*a^4*b^6*c^5* \\
& e*f + 201600*a^5*b^4*c^6*e*f - 161280*a^6*b^2*c^7*e*f + 10*b^4*c^2*d*f*(-(4 \\
& *a*c - b^2)^9)^{(1/2)} - 51*a*b^2*c^3*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 44*a*b*c \\
& ^4*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 78*a*b^2*c^3*d*f*(-(4*a*c - b^2)^9)^{(1/2)} \\
& + 184*a*b^3*c^2*e*f*(-(4*a*c - b^2)^9)^{(1/2)} - 186*a^2*b*c^3*e*f*(-(4*a*c \\
& - b^2)^9)^{(1/2)})/(32*(4096*a^6*c^13 + b^{12}*c^7 - 24*a*b^{10}*c^8 + 240*a^2*b^8 \\
& *c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^{(1/2)}* \\
& (16*b^7*c^7 - 192*a*b^5*c^8 - 1024*a^3*b*c^10 + 768*a^2*b^3*c^9))/(2*(16*a^2 \\
& *c^7 + b^4*c^5 - 8*a*b^2*c^6)))*(-(25*b^{15}*f^2 + b^{11}*c^4*d^2 + 9*b^{13}*c^2 \\
& *e^2 + 25*b^6*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 27*a*b^9*c^5*d^2 - 3840*a^5*b* \\
& c^9*d^2 - 9*a*c^5*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - 213*a*b^{11}*c^3*e^2 + 26880 \\
& *a^6*b*c^8*e^2 - 80640*a^7*b*c^7*f^2 - 30*b^{14}*c*e*f + 288*a^2*b^7*c^6*d^2 \\
& - 1504*a^3*b^5*c^7*d^2 + 3840*a^4*b^3*c^8*d^2 + 2077*a^2*b^9*c^4*e^2 - 1065 \\
& 6*a^3*b^7*c^5*e^2 + 30240*a^4*b^5*c^6*e^2 - 44800*a^5*b^3*c^7*e^2 + 25*a^2*c^4 \\
& *e^2*(-(4*a*c - b^2)^9)^{(1/2)} + b^2*c^4*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + 6 \\
& 366*a^2*b^{11}*c^2*f^2 - 35767*a^3*b^9*c^3*f^2 + 116928*a^4*b^7*c^4*f^2 - 219 \\
& 744*a^5*b^5*c^5*f^2 + 215040*a^6*b^3*c^6*f^2 - 49*a^3*c^3*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + 9*b^4*c^2*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 615*a*b^{13}*c*f^2 - 1 \\
& 5360*a^6*c^9*d*e - 6*b^{12}*c^3*d*e + 35840*a^7*c^8*e*f + 10*b^{13}*c^2*d*f + 1 \\
& 52*a*b^{10}*c^4*d*e - 258*a*b^{11}*c^3*d*f + 43520*a^6*b*c^8*d*f + 724*a*b^{12}*c^2 \\
& *e*f - 30*b^5*c*e*f*(-(4*a*c - b^2)^9)^{(1/2)} + 246*a^2*b^2*c^2*f^2*(-(4*a \\
& *c - b^2)^9)^{(1/2)} - 165*a*b^4*c*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 1548*a^2*b^8 \\
& *c^5*d*e + 8064*a^3*b^6*c^6*d*e - 22400*a^4*b^4*c^7*d*e + 30720*a^5*b^2*c^8 \\
& *d*e + 2706*a^2*b^9*c^4*d*f - 14784*a^3*b^7*c^5*d*f + 44352*a^4*b^5*c^6*d* \\
& f - 69120*a^5*b^3*c^7*d*f + 42*a^2*c^4*d*f*(-(4*a*c - b^2)^9)^{(1/2)} - 6*b^3 \\
& *c^3*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 7278*a^2*b^{10}*c^3*e*f + 39132*a^3*b^8*c^4 \\
& *e*f - 119616*a^4*b^6*c^5*e*f + 201600*a^5*b^4*c^6*e*f - 161280*a^6*b^2*c^7 \\
& *e*f + 10*b^4*c^2*d*f*(-(4*a*c - b^2)^9)^{(1/2)} - 51*a*b^2*c^3*e^2*(-(4*a* \\
& c - b^2)^9)^{(1/2)} + 44*a*b*c^4*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 78*a*b^2*c^3* \\
& d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 184*a*b^3*c^2*e*f*(-(4*a*c - b^2)^9)^{(1/2)} - \\
& 186*a^2*b*c^3*e*f*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(4096*a^6*c^13 + b^{12}*c^7 \\
& - 24*a*b^{10}*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - \\
& 6144*a^5*b^2*c^12)))^{(1/2)} + (x*(25*b^{10}*f^2 - 72*a^3*c^7*d^2 + 200*a^4*c^6 \\
& *e^2 + b^6*c^4*d^2 - 392*a^5*c^5*f^2 + 9*b^8*c^2*e^2 - 16*a*b^4*c^5*d^2 - \\
& 114*a*b^6*c^3*e^2 - 30*b^9*c*e*f + 74*a^2*b^2*c^6*d^2 + 481*a^2*b^4*c^4*e^2 \\
& - 718*a^3*b^2*c^5*e^2 + 1676*a^2*b^6*c^2*f^2 - 3536*a^3*b^4*c^3*f^2 + 2794 \\
& *a^4*b^2*c^4*f^2 - 340*a*b^8*c*f^2 + 336*a^4*c^6*d*f - 6*b^7*c^3*d*e + 10*b^8 \\
& *c^2*d*f + 86*a*b^5*c^4*d*e + 472*a^3*b*c^6*d*e - 148*a*b^6*c^3*d*f + 394 \\
& *a*b^7*c^2*e*f - 1768*a^4*b*c^5*e*f - 374*a^2*b^3*c^5*d*e + 698*a^2*b^4*c^4 \\
& *d*f - 1132*a^3*b^2*c^5*d*f - 1804*a^2*b^5*c^3*e*f + 3266*a^3*b^3*c^4*e*f)) \\
& / (2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))*(-(25*b^{15}*f^2 + b^{11}*c^4*d^2 + \\
& 9*b^{13}*c^2*e^2 + 25*b^6*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 27*a*b^9*c^5*d^2 - 3 \\
& 840*a^5*b*c^9*d^2 - 9*a*c^5*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - 213*a*b^{11}*c^3*e \\
& ^2 + 26880*a^6*b*c^8*e^2 - 80640*a^7*b*c^7*f^2 - 30*b^{14}*c*e*f + 288*a^2*b^
\end{aligned}$$

$$\begin{aligned}
& 7*c^6*d^2 - 1504*a^3*b^5*c^7*d^2 + 3840*a^4*b^3*c^8*d^2 + 2077*a^2*b^9*c^4* \\
& e^2 - 10656*a^3*b^7*c^5*e^2 + 30240*a^4*b^5*c^6*e^2 - 44800*a^5*b^3*c^7*e^2 \\
& + 25*a^2*c^4*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + b^2*c^4*d^2*(-(4*a*c - b^2)^9) \\
& ^{(1/2)} + 6366*a^2*b^11*c^2*f^2 - 35767*a^3*b^9*c^3*f^2 + 116928*a^4*b^7*c^4 \\
& *f^2 - 219744*a^5*b^5*c^5*f^2 + 215040*a^6*b^3*c^6*f^2 - 49*a^3*c^3*f^2*(-(4 \\
& *a*c - b^2)^9)^{(1/2)} + 9*b^4*c^2*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 615*a*b^13 \\
& *c*f^2 - 15360*a^6*c^9*d*e - 6*b^12*c^3*d*e + 35840*a^7*c^8*e*f + 10*b^13*c \\
& ^2*d*f + 152*a*b^10*c^4*d*e - 258*a*b^11*c^3*d*f + 43520*a^6*b*c^8*d*f + 72 \\
& 4*a*b^12*c^2*e*f - 30*b^5*c*e*f*(-(4*a*c - b^2)^9)^{(1/2)} + 246*a^2*b^2*c^2* \\
& f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 165*a*b^4*c*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 1 \\
& 548*a^2*b^8*c^5*d*e + 8064*a^3*b^6*c^6*d*e - 22400*a^4*b^4*c^7*d*e + 30720* \\
& a^5*b^2*c^8*d*e + 2706*a^2*b^9*c^4*d*f - 14784*a^3*b^7*c^5*d*f + 44352*a^4* \\
& b^5*c^6*d*f - 69120*a^5*b^3*c^7*d*f + 42*a^2*c^4*d*f*(-(4*a*c - b^2)^9)^{(1/ \\
& 2)} - 6*b^3*c^3*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 7278*a^2*b^10*c^3*e*f + 39132 \\
& *a^3*b^8*c^4*e*f - 119616*a^4*b^6*c^5*e*f + 201600*a^5*b^4*c^6*e*f - 161280 \\
& *a^6*b^2*c^7*e*f + 10*b^4*c^2*d*f*(-(4*a*c - b^2)^9)^{(1/2)} - 51*a*b^2*c^3*e \\
& ^2*(-(4*a*c - b^2)^9)^{(1/2)} + 44*a*b*c^4*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 78* \\
& a*b^2*c^3*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 184*a*b^3*c^2*e*f*(-(4*a*c - b^2)^ \\
& 9)^{(1/2)} - 186*a^2*b*c^3*e*f*(-(4*a*c - b^2)^9)^{(1/2))}/(32*(4096*a^6*c^13 + \\
& b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3840*a^4* \\
& b^4*c^11 - 6144*a^5*b^2*c^12)))^{(1/2)}*i)/((((10240*a^5*c^9*e + 192*a^2*b^5 \\
& *c^7*d - 768*a^3*b^3*c^8*d - 736*a^2*b^6*c^6*e + 4224*a^3*b^4*c^7*e - 10752 \\
& *a^4*b^2*c^8*e + 1264*a^2*b^7*c^5*f - 7488*a^3*b^5*c^6*f + 19712*a^4*b^3*c^ \\
& 7*f - 16*a*b^7*c^6*d + 1024*a^4*b*c^9*d + 48*a*b^8*c^5*e - 80*a*b^9*c^4*f - \\
& 19456*a^5*b*c^8*f)/(8*(64*a^3*c^8 - b^6*c^5 + 12*a*b^4*c^6 - 48*a^2*b^2*c^ \\
& 7)) - (x*(-(25*b^15*f^2 + b^11*c^4*d^2 + 9*b^13*c^2*e^2 + 25*b^6*f^2*(-(4*a \\
& *c - b^2)^9)^{(1/2)} - 27*a*b^9*c^5*d^2 - 3840*a^5*b*c^9*d^2 - 9*a*c^5*d^2*(- \\
& (4*a*c - b^2)^9)^{(1/2)} - 213*a*b^11*c^3*e^2 + 26880*a^6*b*c^8*e^2 - 80640*a \\
& ^7*b*c^7*f^2 - 30*b^14*c*e*f + 288*a^2*b^7*c^6*d^2 - 1504*a^3*b^5*c^7*d^2 + \\
& 3840*a^4*b^3*c^8*d^2 + 2077*a^2*b^9*c^4*e^2 - 10656*a^3*b^7*c^5*e^2 + 3024 \\
& 0*a^4*b^5*c^6*e^2 - 44800*a^5*b^3*c^7*e^2 + 25*a^2*c^4*e^2*(-(4*a*c - b^2)^ \\
& 9)^{(1/2)} + b^2*c^4*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + 6366*a^2*b^11*c^2*f^2 - 3 \\
& 5767*a^3*b^9*c^3*f^2 + 116928*a^4*b^7*c^4*f^2 - 219744*a^5*b^5*c^5*f^2 + 21 \\
& 5040*a^6*b^3*c^6*f^2 - 49*a^3*c^3*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + 9*b^4*c^2* \\
& e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 615*a*b^13*c*f^2 - 15360*a^6*c^9*d*e - 6*b^1 \\
& 2*c^3*d*e + 35840*a^7*c^8*e*f + 10*b^13*c^2*d*f + 152*a*b^10*c^4*d*e - 258* \\
& a*b^11*c^3*d*f + 43520*a^6*b*c^8*d*f + 724*a*b^12*c^2*e*f - 30*b^5*c*e*f*(- \\
& (4*a*c - b^2)^9)^{(1/2)} + 246*a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 165 \\
& *a*b^4*c*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 1548*a^2*b^8*c^5*d*e + 8064*a^3*b^6 \\
& *c^6*d*e - 22400*a^4*b^4*c^7*d*e + 30720*a^5*b^2*c^8*d*e + 2706*a^2*b^9*c^4 \\
& *d*f - 14784*a^3*b^7*c^5*d*f + 44352*a^4*b^5*c^6*d*f - 69120*a^5*b^3*c^7*d* \\
& f + 42*a^2*c^4*d*f*(-(4*a*c - b^2)^9)^{(1/2)} - 6*b^3*c^3*d*e*(-(4*a*c - b^2) \\
& ^9)^{(1/2)} - 7278*a^2*b^10*c^3*e*f + 39132*a^3*b^8*c^4*e*f - 119616*a^4*b^6* \\
& c^5*e*f + 201600*a^5*b^4*c^6*e*f - 161280*a^6*b^2*c^7*e*f + 10*b^4*c^2*d*f* \\
& (- (4*a*c - b^2)^9)^{(1/2)} - 51*a*b^2*c^3*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 44*a \\
& *b*c^4*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 78*a*b^2*c^3*d*f*(-(4*a*c - b^2)^9)^{(\\
& 1/2)} + 184*a*b^3*c^2*e*f*(-(4*a*c - b^2)^9)^{(1/2)} - 186*a^2*b*c^3*e*f*(-(4* \\
& a*c - b^2)^9)^{(1/2))}/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^ \\
& 2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^{(1 \\
& /2)}*(16*b^7*c^7 - 192*a*b^5*c^8 - 1024*a^3*b*c^10 + 768*a^2*b^3*c^9))/(2*(1 \\
& 6*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))*(-(25*b^15*f^2 + b^11*c^4*d^2 + 9*b^13 \\
& *c^2*e^2 + 25*b^6*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 27*a*b^9*c^5*d^2 - 3840*a^ \\
& 5*b*c^9*d^2 - 9*a*c^5*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - 213*a*b^11*c^3*e^2 + 2 \\
& 6880*a^6*b*c^8*e^2 - 80640*a^7*b*c^7*f^2 - 30*b^14*c*e*f + 288*a^2*b^7*c^6* \\
& d^2 - 1504*a^3*b^5*c^7*d^2 + 3840*a^4*b^3*c^8*d^2 + 2077*a^2*b^9*c^4*e^2 - \\
& 10656*a^3*b^7*c^5*e^2 + 30240*a^4*b^5*c^6*e^2 - 44800*a^5*b^3*c^7*e^2 + 25* \\
& a^2*c^4*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + b^2*c^4*d^2*(-(4*a*c - b^2)^9)^{(1/2)} \\
& + 6366*a^2*b^11*c^2*f^2 - 35767*a^3*b^9*c^3*f^2 + 116928*a^4*b^7*c^4*f^2 - \\
& 219744*a^5*b^5*c^5*f^2 + 215040*a^6*b^3*c^6*f^2 - 49*a^3*c^3*f^2*(-(4*a*c
\end{aligned}$$

$$\begin{aligned}
& - b^2)^9)^{(1/2)} + 9*b^4*c^2*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 615*a*b^{13}*c*f^2 \\
& - 15360*a^6*c^9*d*e - 6*b^{12}*c^3*d*e + 35840*a^7*c^8*e*f + 10*b^{13}*c^2*d*f \\
& + 152*a*b^{10}*c^4*d*e - 258*a*b^{11}*c^3*d*f + 43520*a^6*b*c^8*d*f + 724*a*b^{12}*c^2*e*f \\
& - 30*b^5*c*e*f*(-(4*a*c - b^2)^9)^{(1/2)} + 246*a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} \\
& - 165*a*b^4*c*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 1548*a^2*b^8*c^5*d*e + 8064*a^3*b^6*c^6*d*e \\
& - 22400*a^4*b^4*c^7*d*e + 30720*a^5*b^2*c^8*d*e + 2706*a^2*b^9*c^4*d*f - 14784*a^3*b^7*c^5*d*f \\
& + 44352*a^4*b^5*c^6*d*f - 69120*a^5*b^3*c^7*d*f + 42*a^2*c^4*d*f*(-(4*a*c - b^2)^9)^{(1/2)} - 6 \\
& *b^3*c^3*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 7278*a^2*b^{10}*c^3*e*f + 39132*a^3*b^8*c^4*e*f \\
& - 119616*a^4*b^6*c^5*e*f + 201600*a^5*b^4*c^6*e*f - 161280*a^6*b^2*c^7*e*f \\
& + 10*b^4*c^2*d*f*(-(4*a*c - b^2)^9)^{(1/2)} - 51*a*b^2*c^3*e^2*(-(4*a*c - b^2)^9)^{(1/2)} \\
& + 44*a*b*c^4*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 78*a*b^2*c^3*d*f*(-(4*a*c - b^2)^9)^{(1/2)} \\
& + 184*a*b^3*c^2*e*f*(-(4*a*c - b^2)^9)^{(1/2)} - 186*a^2*b*c^3*e*f*(-(4*a*c - b^2)^9)^{(1/2)}) \\
& / (32*(4096*a^6*c^{13} + b^{12}*c^7 - 24*a*b^{10}*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^{10} + 3840*a^4*b^4*c^{11} \\
& - 6144*a^5*b^2*c^{12}))^{(1/2)} - (x*(25*b^{10}*f^2 - 72*a^3*c^7*d^2 + 200*a^4*c^6*e^2 + b^6*c^4*d^2 \\
& - 392*a^5*c^5*f^2 + 9*b^8*c^2*e^2 - 16*a*b^4*c^5*d^2 - 114*a*b^6*c^3*e^2 - 30*b^9*c*e*f \\
& + 74*a^2*b^2*c^6*d^2 + 481*a^2*b^4*c^4*e^2 - 718*a^3*b^2*c^5*e^2 + 1676*a^2*b^6*c^2*f^2 \\
& - 3536*a^3*b^4*c^3*f^2 + 2794*a^4*b^2*c^4*f^2 - 340*a*b^8*c*f^2 + 336*a^4*c^6*d*f - 6*b^7*c^3*d*e \\
& + 10*b^8*c^2*d*f + 86*a*b^5*c^4*d*e + 472*a^3*b*c^6*d*e - 148*a*b^6*c^3*d*f + 394*a*b^7*c^2*e*f \\
& - 1768*a^4*b*c^5*e*f - 374*a^2*b^3*c^5*d*e + 698*a^2*b^4*c^4*d*f - 1132*a^3*b^2*c^5*d*f \\
& - 1804*a^2*b^5*c^3*e*f + 3266*a^3*b^3*c^4*e*f)) / (2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6))) \\
& * (- (25*b^{15}*f^2 + b^{11}*c^4*d^2 + 9*b^{13}*c^2*e^2 + 25*b^6*f^2*(-(4*a*c - b^2)^9)^{(1/2)} \\
& - 27*a*b^9*c^5*d^2 - 3840*a^5*b*c^9*d^2 - 9*a*c^5*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - 213*a*b^{11}*c^3 \\
& *e^2 + 26880*a^6*b*c^8*e^2 - 80640*a^7*b*c^7*f^2 - 30*b^{14}*c*e*f + 288*a^2*b^7*c^6*d^2 \\
& - 1504*a^3*b^5*c^7*d^2 + 3840*a^4*b^3*c^8*d^2 + 2077*a^2*b^9*c^4*e^2 - 10656*a^3*b^7*c^5*e^2 \\
& + 30240*a^4*b^5*c^6*e^2 - 44800*a^5*b^3*c^7*e^2 + 25*a^2*c^4*e^2*(-(4*a*c - b^2)^9)^{(1/2)} \\
& + b^2*c^4*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + 6366*a^2*b^{11}*c^2*f^2 - 35767*a^3*b^9*c^3*f^2 \\
& + 116928*a^4*b^7*c^4*f^2 - 219744*a^5*b^5*c^5*f^2 + 215040*a^6*b^3*c^6*f^2 - 49*a^3*c^3*f^2 \\
& * (- (4*a*c - b^2)^9)^{(1/2)} + 9*b^4*c^2*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 615*a*b^{13}*c*f^2 \\
& - 15360*a^6*c^9*d*e - 6*b^{12}*c^3*d*e + 35840*a^7*c^8*e*f + 10*b^{13}*c^2*d*f + 152*a*b^{10}*c^4*d*e \\
& - 258*a*b^{11}*c^3*d*f + 43520*a^6*b*c^8*d*f + 724*a*b^{12}*c^2*e*f - 30*b^5*c*e*f*(-(4*a*c - b^2)^9)^{(1/2)} \\
& + 246*a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 165*a*b^4*c*f^2*(-(4*a*c - b^2)^9)^{(1/2)} \\
& - 1548*a^2*b^8*c^5*d*e + 8064*a^3*b^6*c^6*d*e - 22400*a^4*b^4*c^7*d*e + 30720*a^5*b^2*c^8*d*e \\
& + 2706*a^2*b^9*c^4*d*f - 14784*a^3*b^7*c^5*d*f + 44352*a^4*b^5*c^6*d*f - 69120*a^5*b^3*c^7*d*f \\
& + 42*a^2*c^4*d*f*(-(4*a*c - b^2)^9)^{(1/2)} - 6*b^3*c^3*d*e*(-(4*a*c - b^2)^9)^{(1/2)} \\
& - 7278*a^2*b^{10}*c^3*e*f + 39132*a^3*b^8*c^4*e*f - 119616*a^4*b^6*c^5*e*f + 201600*a^5*b^4*c^6*e*f \\
& - 161280*a^6*b^2*c^7*e*f + 10*b^4*c^2*d*f*(-(4*a*c - b^2)^9)^{(1/2)} - 51*a*b^2*c^3*e^2*(-(4*a*c - b^2)^9)^{(1/2)} \\
& + 44*a*b*c^4*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 78*a*b^2*c^3*d*f*(-(4*a*c - b^2)^9)^{(1/2)} \\
& + 184*a*b^3*c^2*e*f*(-(4*a*c - b^2)^9)^{(1/2)} - 186*a^2*b*c^3*e*f*(-(4*a*c - b^2)^9)^{(1/2)}) \\
& / (32*(4096*a^6*c^{13} + b^{12}*c^7 - 24*a*b^{10}*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^{10} + 3840*a^4*b^4*c^{11} \\
& - 6144*a^5*b^2*c^{12}))^{(1/2)} + (((10240*a^5*c^9*e + 192*a^2*b^5*c^7*d - 768*a^3*b^3*c^8*d \\
& - 736*a^2*b^6*c^6*e + 4224*a^3*b^4*c^7*e - 10752*a^4*b^2*c^8*e + 1264*a^2*b^7*c^5*f \\
& - 7488*a^3*b^5*c^6*f + 19712*a^4*b^3*c^7*f - 16*a*b^7*c^6*d + 1024*a^4*b*c^9*d \\
& + 48*a*b^8*c^5*e - 80*a*b^9*c^4*f - 19456*a^5*b*c^8*f) / (8*(64*a^3*c^8 - b^6*c^5 \\
& + 12*a*b^4*c^6 - 48*a^2*b^2*c^7)) + (x*(- (25*b^{15}*f^2 + b^{11}*c^4*d^2 + 9*b^{13}*c^2*e^2 \\
& + 25*b^6*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 27*a*b^9*c^5*d^2 - 3840*a^5*b*c^9*d^2 \\
& - 9*a*c^5*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - 213*a*b^{11}*c^3*e^2 + 26880*a^6*b*c^8*e^2 \\
& - 80640*a^7*b*c^7*f^2 - 30*b^{14}*c*e*f + 288*a^2*b^7*c^6*d^2 - 1504*a^3*b^5*c^7*d^2 \\
& + 3840*a^4*b^3*c^8*d^2 + 2077*a^2*b^9*c^4*e^2 - 10656*a^3*b^7*c^5*e^2 + 30240*a^4*b^5*c^6*e^2 \\
& - 44800*a^5*b^3*c^7*e^2 + 25*a^2*c^4*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + b^2*c^4*d^2*(-(4*a*c - b^2)^9)^{(1/2)} \\
& + 6366*a^2*b^{11}*c^2*f^2 -
\end{aligned}$$

$$\begin{aligned}
& 35767a^3b^9c^3f^2 + 116928a^4b^7c^4f^2 - 219744a^5b^5c^5f^2 + 2 \\
& 15040a^6b^3c^6f^2 - 49a^3c^3f^2(-4ac - b^2)^9)^{(1/2)} + 9b^4c^2 \\
& *e^2(-4ac - b^2)^9)^{(1/2)} - 615ab^{13}c^3f^2 - 15360a^6c^9d^2e - 6b^ \\
& 12c^3d^2e + 35840a^7c^8e^2f + 10b^{13}c^2d^2f + 152ab^{10}c^4d^2e - 258 \\
& *ab^{11}c^3d^2f + 43520a^6b^8c^8d^2f + 724ab^{12}c^2e^2f - 30b^5c^5e^2f * \\
& (-4ac - b^2)^9)^{(1/2)} + 246a^2b^2c^2f^2(-4ac - b^2)^9)^{(1/2)} - 16 \\
& 5ab^4c^2f^2(-4ac - b^2)^9)^{(1/2)} - 1548a^2b^8c^5d^2e + 8064a^3b^ \\
& 6c^6d^2e - 22400a^4b^4c^7d^2e + 30720a^5b^2c^8d^2e + 2706a^2b^9c^ \\
& 4d^2f - 14784a^3b^7c^5d^2f + 44352a^4b^5c^6d^2f - 69120a^5b^3c^7d \\
& *f + 42a^2c^4d^2f(-4ac - b^2)^9)^{(1/2)} - 6b^3c^3d^2e^2(-4ac - b^2 \\
&)^9)^{(1/2)} - 7278a^2b^{10}c^3e^2f + 39132a^3b^8c^4e^2f - 119616a^4b^6 \\
& *c^5e^2f + 201600a^5b^4c^6e^2f - 161280a^6b^2c^7e^2f + 10b^4c^2d^2f \\
& *(-4ac - b^2)^9)^{(1/2)} - 51ab^2c^3e^2(-4ac - b^2)^9)^{(1/2)} + 44a \\
& *b^4c^2d^2e^2(-4ac - b^2)^9)^{(1/2)} - 78ab^2c^3d^2f(-4ac - b^2)^9)^{(\\
& 1/2)} + 184ab^3c^2e^2f(-4ac - b^2)^9)^{(1/2)} - 186a^2b^3c^3e^2f(-4 \\
& *ac - b^2)^9)^{(1/2))/(32(4096a^6c^{13} + b^{12}c^7 - 24ab^{10}c^8 + 240a \\
& ^2b^8c^9 - 1280a^3b^6c^{10} + 3840a^4b^4c^{11} - 6144a^5b^2c^{12}))^{(\\
& 1/2)}*(16b^7c^7 - 192ab^5c^8 - 1024a^3b^3c^{10} + 768a^2b^3c^9))/(2*(\\
& 16a^2c^7 + b^4c^5 - 8ab^2c^6)))*(-25b^{15}f^2 + b^{11}c^4d^2 + 9b^1 \\
& 3c^2e^2 + 25b^6f^2(-4ac - b^2)^9)^{(1/2)} - 27ab^9c^5d^2 - 3840a \\
& ^5b^3c^9d^2 - 9ac^5d^2(-4ac - b^2)^9)^{(1/2)} - 213ab^{11}c^3e^2 + \\
& 26880a^6b^3c^8e^2 - 80640a^7b^3c^7f^2 - 30b^{14}c^2e^2f + 288a^2b^7c^6 \\
& *d^2 - 1504a^3b^5c^7d^2 + 3840a^4b^3c^8d^2 + 2077a^2b^9c^4e^2 - \\
& 10656a^3b^7c^5e^2 + 30240a^4b^5c^6e^2 - 44800a^5b^3c^7e^2 + 25 \\
& *a^2c^4e^2(-4ac - b^2)^9)^{(1/2)} + b^2c^4d^2(-4ac - b^2)^9)^{(1/2)} \\
&) + 6366a^2b^{11}c^2f^2 - 35767a^3b^9c^3f^2 + 116928a^4b^7c^4f^2 \\
& - 219744a^5b^5c^5f^2 + 215040a^6b^3c^6f^2 - 49a^3c^3f^2(-4ac \\
& - b^2)^9)^{(1/2)} + 9b^4c^2e^2(-4ac - b^2)^9)^{(1/2)} - 615ab^{13}c^3f^ \\
& 2 - 15360a^6c^9d^2e - 6b^{12}c^3d^2e + 35840a^7c^8e^2f + 10b^{13}c^2d^2 \\
& *f + 152ab^{10}c^4d^2e - 258ab^{11}c^3d^2f + 43520a^6b^8c^8d^2f + 724ab \\
& ^{12}c^2e^2f - 30b^5c^5e^2f(-4ac - b^2)^9)^{(1/2)} + 246a^2b^2c^2f^2 * \\
& (-4ac - b^2)^9)^{(1/2)} - 165ab^4c^2f^2(-4ac - b^2)^9)^{(1/2)} - 1548a \\
& ^2b^8c^5d^2e + 8064a^3b^6c^6d^2e - 22400a^4b^4c^7d^2e + 30720a^5b \\
& ^2c^8d^2e + 2706a^2b^9c^4d^2f - 14784a^3b^7c^5d^2f + 44352a^4b^5c \\
& ^6d^2f - 69120a^5b^3c^7d^2f + 42a^2c^4d^2f(-4ac - b^2)^9)^{(1/2)} - \\
& 6b^3c^3d^2e^2(-4ac - b^2)^9)^{(1/2)} - 7278a^2b^{10}c^3e^2f + 39132a^3 * \\
& b^8c^4e^2f - 119616a^4b^6c^5e^2f + 201600a^5b^4c^6e^2f - 161280a^6 * \\
& b^2c^7e^2f + 10b^4c^2d^2f(-4ac - b^2)^9)^{(1/2)} - 51ab^2c^3e^2 * \\
& (-4ac - b^2)^9)^{(1/2)} + 44ab^4c^2d^2e^2(-4ac - b^2)^9)^{(1/2)} - 78ab^2 \\
& *c^3d^2f(-4ac - b^2)^9)^{(1/2)} + 184ab^3c^2e^2f(-4ac - b^2)^9)^{(1 \\
& /2)} - 186a^2b^3c^3e^2f(-4ac - b^2)^9)^{(1/2))/(32(4096a^6c^{13} + b^{12} \\
& *c^7 - 24ab^{10}c^8 + 240a^2b^8c^9 - 1280a^3b^6c^{10} + 3840a^4b^4c \\
& ^{11} - 6144a^5b^2c^{12}))^{(1/2)} + (x(25b^{10}f^2 - 72a^3c^7d^2 + 200a \\
& ^4c^6e^2 + b^6c^4d^2 - 392a^5c^5f^2 + 9b^8c^2e^2 - 16ab^4c^5d \\
& ^2 - 114ab^6c^3e^2 - 30b^9c^2e^2f + 74a^2b^2c^6d^2 + 481a^2b^4c^ \\
& 4e^2 - 718a^3b^2c^5e^2 + 1676a^2b^6c^2f^2 - 3536a^3b^4c^3f^2 + \\
& 2794a^4b^2c^4f^2 - 340ab^8c^2f^2 + 336a^4c^6d^2f - 6b^7c^3d^2e + \\
& 10b^8c^2d^2f + 86ab^5c^4d^2e + 472a^3b^3c^6d^2e - 148ab^6c^3d^2f \\
& + 394ab^7c^2e^2f - 1768a^4b^3c^5e^2f - 374a^2b^3c^5d^2e + 698a^2b^ \\
& 4c^4d^2f - 1132a^3b^2c^5d^2f - 1804a^2b^5c^3e^2f + 3266a^3b^3c^4 * \\
& e^2f))/(2(16a^2c^7 + b^4c^5 - 8ab^2c^6)))*(-25b^{15}f^2 + b^{11}c^4d \\
& ^2 + 9b^{13}c^2e^2 + 25b^6f^2(-4ac - b^2)^9)^{(1/2)} - 27ab^9c^5d^ \\
& 2 - 3840a^5b^3c^9d^2 - 9ac^5d^2(-4ac - b^2)^9)^{(1/2)} - 213ab^{11} * \\
& c^3e^2 + 26880a^6b^3c^8e^2 - 80640a^7b^3c^7f^2 - 30b^{14}c^2e^2f + 288a \\
& ^2b^7c^6d^2 - 1504a^3b^5c^7d^2 + 3840a^4b^3c^8d^2 + 2077a^2b^9 \\
& *c^4e^2 - 10656a^3b^7c^5e^2 + 30240a^4b^5c^6e^2 - 44800a^5b^3c^ \\
& 7e^2 + 25a^2c^4e^2(-4ac - b^2)^9)^{(1/2)} + b^2c^4d^2(-4ac - b^ \\
& 2)^9)^{(1/2)} + 6366a^2b^{11}c^2f^2 - 35767a^3b^9c^3f^2 + 116928a^4b^ \\
& 7c^4f^2 - 219744a^5b^5c^5f^2 + 215040a^6b^3c^6f^2 - 49a^3c^3f^
\end{aligned}$$

$$\begin{aligned}
& 2*(-(4*a*c - b^2)^9)^{(1/2)} + 9*b^4*c^2*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 615*a \\
& *b^{13}*c*f^2 - 15360*a^6*c^9*d*e - 6*b^{12}*c^3*d*e + 35840*a^7*c^8*e*f + 10*b \\
& ^{13}*c^2*d*f + 152*a*b^{10}*c^4*d*e - 258*a*b^{11}*c^3*d*f + 43520*a^6*b*c^8*d*f \\
& + 724*a*b^{12}*c^2*e*f - 30*b^5*c*e*f*(-(4*a*c - b^2)^9)^{(1/2)} + 246*a^2*b^2 \\
& *c^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 165*a*b^4*c*f^2*(-(4*a*c - b^2)^9)^{(1/2)} \\
&) - 1548*a^2*b^8*c^5*d*e + 8064*a^3*b^6*c^6*d*e - 22400*a^4*b^4*c^7*d*e + 3 \\
& 0720*a^5*b^2*c^8*d*e + 2706*a^2*b^9*c^4*d*f - 14784*a^3*b^7*c^5*d*f + 44352 \\
& *a^4*b^5*c^6*d*f - 69120*a^5*b^3*c^7*d*f + 42*a^2*c^4*d*f*(-(4*a*c - b^2)^9 \\
&)^{(1/2)} - 6*b^3*c^3*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 7278*a^2*b^10*c^3*e*f + \\
& 39132*a^3*b^8*c^4*e*f - 119616*a^4*b^6*c^5*e*f + 201600*a^5*b^4*c^6*e*f - 1 \\
& 61280*a^6*b^2*c^7*e*f + 10*b^4*c^2*d*f*(-(4*a*c - b^2)^9)^{(1/2)} - 51*a*b^2*c \\
& ^3*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 44*a*b*c^4*d*e*(-(4*a*c - b^2)^9)^{(1/2)} \\
& - 78*a*b^2*c^3*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 184*a*b^3*c^2*e*f*(-(4*a*c - \\
& b^2)^9)^{(1/2)} - 186*a^2*b*c^3*e*f*(-(4*a*c - b^2)^9)^{(1/2)}/(32*(4096*a^6*c \\
& ^{13} + b^{12}*c^7 - 24*a*b^{10}*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^{10} + 3840 \\
& *a^4*b^4*c^{11} - 6144*a^5*b^2*c^{12}))^{(1/2)} - (216*a^4*c^6*d^3 + 225*a^4*b^6 \\
& *f^3 - 2744*a^7*c^3*f^3 - 1300*a^5*b*c^4*e^3 - 2060*a^5*b^4*c*f^3 + 125*a^2 \\
& *b^8*d*f^2 + 600*a^5*c^5*d*e^2 - 175*a^3*b^7*e*f^2 - 1512*a^5*c^5*d^2*f + 3 \\
& 528*a^6*c^4*d*f^2 - 1400*a^6*c^4*e^2*f + 5*a^2*b^4*c^4*d^3 - 66*a^3*b^2*c^5 \\
& *d^3 - 63*a^3*b^5*c^2*e^3 + 573*a^4*b^3*c^3*e^3 + 5334*a^6*b^2*c^2*f^3 - 92 \\
& 4*a^4*b*c^5*d^2*e - 1350*a^3*b^6*c*d*f^2 + 210*a^3*b^6*c*e^2*f + 1485*a^4*b \\
& ^5*c*e*f^2 - 364*a^6*b*c^3*e*f^2 - 30*a^2*b^5*c^3*d^2*e + 45*a^2*b^6*c^2*d* \\
& e^2 + 339*a^3*b^3*c^4*d^2*e - 402*a^3*b^4*c^3*d*e^2 + 762*a^4*b^2*c^4*d*e^2 \\
& + 50*a^2*b^6*c^2*d^2*f - 600*a^3*b^4*c^3*d^2*f + 2002*a^4*b^2*c^4*d^2*f + \\
& 4835*a^4*b^4*c^2*d*f^2 - 6598*a^5*b^2*c^3*d*f^2 - 1927*a^4*b^4*c^2*e^2*f + \\
& 4722*a^5*b^2*c^3*e^2*f - 3061*a^5*b^3*c^2*e*f^2 - 150*a^2*b^7*c*d*e*f + 231 \\
& 2*a^5*b*c^4*d*e*f + 1480*a^3*b^5*c^2*d*e*f - 4122*a^4*b^3*c^3*d*e*f)/(4*(64 \\
& *a^3*c^8 - b^6*c^5 + 12*a*b^4*c^6 - 48*a^2*b^2*c^7))))*(-(25*b^{15}*f^2 + b^{11} \\
& *c^4*d^2 + 9*b^{13}*c^2*e^2 + 25*b^6*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 27*a*b^9 \\
& *c^5*d^2 - 3840*a^5*b*c^9*d^2 - 9*a*c^5*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - 213* \\
& a*b^{11}*c^3*e^2 + 26880*a^6*b*c^8*e^2 - 80640*a^7*b*c^7*f^2 - 30*b^{14}*c*e*f \\
& + 288*a^2*b^7*c^6*d^2 - 1504*a^3*b^5*c^7*d^2 + 3840*a^4*b^3*c^8*d^2 + 2077* \\
& a^2*b^9*c^4*e^2 - 10656*a^3*b^7*c^5*e^2 + 30240*a^4*b^5*c^6*e^2 - 44800*a^5 \\
& *b^3*c^7*e^2 + 25*a^2*c^4*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + b^2*c^4*d^2*(-(4*a \\
& *c - b^2)^9)^{(1/2)} + 6366*a^2*b^{11}*c^2*f^2 - 35767*a^3*b^9*c^3*f^2 + 116928 \\
& *a^4*b^7*c^4*f^2 - 219744*a^5*b^5*c^5*f^2 + 215040*a^6*b^3*c^6*f^2 - 49*a^3 \\
& *c^3*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + 9*b^4*c^2*e^2*(-(4*a*c - b^2)^9)^{(1/2)} \\
& - 615*a*b^{13}*c*f^2 - 15360*a^6*c^9*d*e - 6*b^{12}*c^3*d*e + 35840*a^7*c^8*e*f \\
& + 10*b^{13}*c^2*d*f + 152*a*b^{10}*c^4*d*e - 258*a*b^{11}*c^3*d*f + 43520*a^6*b* \\
& c^8*d*f + 724*a*b^{12}*c^2*e*f - 30*b^5*c*e*f*(-(4*a*c - b^2)^9)^{(1/2)} + 246* \\
& a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 165*a*b^4*c*f^2*(-(4*a*c - b^2)^ \\
& 9)^{(1/2)} - 1548*a^2*b^8*c^5*d*e + 8064*a^3*b^6*c^6*d*e - 22400*a^4*b^4*c^7* \\
& d*e + 30720*a^5*b^2*c^8*d*e + 2706*a^2*b^9*c^4*d*f - 14784*a^3*b^7*c^5*d*f \\
& + 44352*a^4*b^5*c^6*d*f - 69120*a^5*b^3*c^7*d*f + 42*a^2*c^4*d*f*(-(4*a*c - \\
& b^2)^9)^{(1/2)} - 6*b^3*c^3*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 7278*a^2*b^10*c^3 \\
& *e*f + 39132*a^3*b^8*c^4*e*f - 119616*a^4*b^6*c^5*e*f + 201600*a^5*b^4*c^6* \\
& e*f - 161280*a^6*b^2*c^7*e*f + 10*b^4*c^2*d*f*(-(4*a*c - b^2)^9)^{(1/2)} - 51 \\
& *a*b^2*c^3*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 44*a*b*c^4*d*e*(-(4*a*c - b^2)^9) \\
& ^{(1/2)} - 78*a*b^2*c^3*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 184*a*b^3*c^2*e*f*(-(4 \\
& *a*c - b^2)^9)^{(1/2)} - 186*a^2*b*c^3*e*f*(-(4*a*c - b^2)^9)^{(1/2)}/(32*(409 \\
& 6*a^6*c^{13} + b^{12}*c^7 - 24*a*b^{10}*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^{10} \\
& + 3840*a^4*b^4*c^{11} - 6144*a^5*b^2*c^{12}))^{(1/2)}*2i - atan((((10240*a^5*c \\
& ^9*e + 192*a^2*b^5*c^7*d - 768*a^3*b^3*c^8*d - 736*a^2*b^6*c^6*e + 4224*a^3 \\
& *b^4*c^7*e - 10752*a^4*b^2*c^8*e + 1264*a^2*b^7*c^5*f - 7488*a^3*b^5*c^6*f \\
& + 19712*a^4*b^3*c^7*f - 16*a*b^7*c^6*d + 1024*a^4*b*c^9*d + 48*a*b^8*c^5*e \\
& - 80*a*b^9*c^4*f - 19456*a^5*b*c^8*f)/(8*(64*a^3*c^8 - b^6*c^5 + 12*a*b^4*c \\
& ^6 - 48*a^2*b^2*c^7)) - (x*(-(25*b^{15}*f^2 + b^{11}*c^4*d^2 + 9*b^{13}*c^2*e^2 - \\
& 25*b^6*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 27*a*b^9*c^5*d^2 - 3840*a^5*b*c^9*d^ \\
& 2 + 9*a*c^5*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - 213*a*b^{11}*c^3*e^2 + 26880*a^6*b
\end{aligned}$$

$$\begin{aligned}
& *c^8e^2 - 80640a^7b^7c^7f^2 - 30b^{14}c^7e^2 + 288a^2b^7c^6d^2 - 1504 \\
& a^3b^5c^7d^2 + 3840a^4b^3c^8d^2 + 2077a^2b^9c^4e^2 - 10656a^3b^7c^5e^2 + 30240a^4b^5c^6e^2 - 44800a^5b^3c^7e^2 - 25a^2c^4e^2 \\
& 2*(-(4ac - b^2)^9)^{(1/2)} - b^2c^4d^2*(-(4ac - b^2)^9)^{(1/2)} + 6366a^2b^{11}c^2f^2 - 35767a^3b^9c^3f^2 + 116928a^4b^7c^4f^2 - 219744a^5b^5c^5f^2 \\
& + 215040a^6b^3c^6f^2 + 49a^3c^3f^2*(-(4ac - b^2)^9)^{(1/2)} - 9b^4c^2e^2*(-(4ac - b^2)^9)^{(1/2)} - 615ab^{13}c^3f^2 - 15360a^6c^9d^2e \\
& - 6b^{12}c^3d^2e + 35840a^7c^8e^2f + 10b^{13}c^2d^2f + 152ab^{10}c^4d^2e - 258ab^{11}c^3d^2f + 43520a^6b^7c^8d^2f + 724ab^{12}c^2e^2f \\
& + 30b^5c^7e^2f*(-(4ac - b^2)^9)^{(1/2)} - 246a^2b^2c^2f^2*(-(4ac - b^2)^9)^{(1/2)} + 165ab^4c^2f^2*(-(4ac - b^2)^9)^{(1/2)} - 1548a^2b^8c^5d^2e \\
& + 8064a^3b^6c^6d^2e - 22400a^4b^4c^7d^2e + 30720a^5b^2c^8d^2e + 2706a^2b^9c^4d^2f - 14784a^3b^7c^5d^2f + 44352a^4b^5c^6d^2f - 69 \\
& 120a^5b^3c^7d^2f - 42a^2c^4d^2f*(-(4ac - b^2)^9)^{(1/2)} + 6b^3c^3d^2e*(-(4ac - b^2)^9)^{(1/2)} - 7278a^2b^{10}c^3e^2f + 39132a^3b^8c^4e^2f \\
& - 119616a^4b^6c^5e^2f + 201600a^5b^4c^6e^2f - 161280a^6b^2c^7e^2f - 10b^4c^2d^2f*(-(4ac - b^2)^9)^{(1/2)} + 51ab^2c^3e^2*(-(4ac - b^2)^9)^{(1/2)} \\
& - 44ab^2c^4d^2e*(-(4ac - b^2)^9)^{(1/2)} + 78ab^2c^3d^2f*(-(4ac - b^2)^9)^{(1/2)} - 184ab^3c^2e^2f*(-(4ac - b^2)^9)^{(1/2)} + 186a^2b^3c^3e^2f \\
& (- (4ac - b^2)^9)^{(1/2)} / (32(4096a^6c^{13} + b^{12}c^7 - 24ab^{10}c^8 + 240a^2b^8c^9 - 1280a^3b^6c^{10} + 3840a^4b^4c^{11} - 6144a^5b^2c^{12})) \\
&)^{(1/2)} * (16b^7c^7 - 192ab^5c^8 - 1024a^3b^3c^{10} + 768a^2b^3c^9) / (2(16a^2c^7 + b^4c^5 - 8ab^2c^6)) * (- (25b^{15}f^2 + b^{11}c^4d^2 \\
& + 9b^{13}c^2e^2 - 25b^6f^2*(-(4ac - b^2)^9)^{(1/2)} - 27ab^9c^5d^2 - 3840a^5b^7c^9d^2 + 9a^2c^5d^2*(-(4ac - b^2)^9)^{(1/2)} - 213ab^{11}c^3e^2 \\
& + 26880a^6b^7c^8e^2 - 80640a^7b^7c^7f^2 - 30b^{14}c^7e^2 + 288a^2b^7c^6d^2 - 1504a^3b^5c^7d^2 + 3840a^4b^3c^8d^2 + 2077a^2b^9c^4e^2 \\
& - 10656a^3b^7c^5e^2 + 30240a^4b^5c^6e^2 - 44800a^5b^3c^7e^2 - 25a^2c^4e^2*(-(4ac - b^2)^9)^{(1/2)} - b^2c^4d^2*(-(4ac - b^2)^9)^{(1/2)} \\
& + 6366a^2b^{11}c^2f^2 - 35767a^3b^9c^3f^2 + 116928a^4b^7c^4f^2 - 219744a^5b^5c^5f^2 + 215040a^6b^3c^6f^2 + 49a^3c^3f^2*(-(4ac - b^2)^9)^{(1/2)} \\
& - 9b^4c^2e^2*(-(4ac - b^2)^9)^{(1/2)} - 615ab^{13}c^3f^2 - 15360a^6c^9d^2e - 6b^{12}c^3d^2e + 35840a^7c^8e^2f + 10b^{13}c^2d^2f + 152ab^{10}c^4d^2e \\
& - 258ab^{11}c^3d^2f + 43520a^6b^7c^8d^2f + 724ab^{12}c^2e^2f + 30b^5c^7e^2f*(-(4ac - b^2)^9)^{(1/2)} - 246a^2b^2c^2f^2*(-(4ac - b^2)^9)^{(1/2)} \\
& + 165ab^4c^2f^2*(-(4ac - b^2)^9)^{(1/2)} - 1548a^2b^8c^5d^2e + 8064a^3b^6c^6d^2e - 22400a^4b^4c^7d^2e + 30720a^5b^2c^8d^2e + 2706a^2b^9c^4d^2f \\
& - 14784a^3b^7c^5d^2f + 44352a^4b^5c^6d^2f - 69120a^5b^3c^7d^2f - 42a^2c^4d^2f*(-(4ac - b^2)^9)^{(1/2)} + 6b^3c^3d^2e*(-(4ac - b^2)^9)^{(1/2)} \\
& - 7278a^2b^{10}c^3e^2f + 39132a^3b^8c^4e^2f - 119616a^4b^6c^5e^2f + 201600a^5b^4c^6e^2f - 161280a^6b^2c^7e^2f - 10b^4c^2d^2f*(-(4ac - b^2)^9)^{(1/2)} \\
& + 51ab^2c^3e^2*(-(4ac - b^2)^9)^{(1/2)} - 44ab^2c^4d^2e*(-(4ac - b^2)^9)^{(1/2)} + 78ab^2c^3d^2f*(-(4ac - b^2)^9)^{(1/2)} - 184ab^3c^2e^2f \\
& (- (4ac - b^2)^9)^{(1/2)} + 186a^2b^3c^3e^2f*(-(4ac - b^2)^9)^{(1/2)} / (32(4096a^6c^{13} + b^{12}c^7 - 24ab^{10}c^8 + 240a^2b^8c^9 - 1280a^3b^6c^{10} \\
& + 3840a^4b^4c^{11} - 6144a^5b^2c^{12})) \\
&)^{(1/2)} - (x(25b^{10}f^2 - 72a^3c^7d^2 + 200a^4c^6e^2 + b^6c^4d^2 - 392a^5c^5f^2 + 9b^8c^2e^2 - 16ab^4c^5d^2 - 114ab^6c^3e^2 - 30b^9c^7e^2 \\
& + 74a^2b^2c^6d^2 + 481a^2b^4c^4e^2 - 718a^3b^2c^5e^2 + 1676a^2b^6c^2f^2 - 3536a^3b^4c^3f^2 + 2794a^4b^2c^4f^2 - 340ab^8c^3f^2 + 336a^4c^6d^2f \\
& - 6b^7c^3d^2e + 10b^8c^2d^2f + 86ab^5c^4d^2e + 472a^3b^3c^6d^2e - 148ab^6c^3d^2f + 394ab^7c^2e^2f - 1768a^4b^5c^5e^2f - 374a^2b^3c^5d^2e \\
& + 698a^2b^4c^4d^2f - 1132a^3b^2c^5d^2f - 1804a^2b^5c^3e^2f + 3266a^3b^3c^4e^2f) / (2(16a^2c^7 + b^4c^5 - 8ab^2c^6)) * (- (25b^{15}f^2 + b^{11}c^4d^2 \\
& + 9b^{13}c^2e^2 - 25b^6f^2*(-(4ac - b^2)^9)^{(1/2)} - 27ab^9c^5d^2 - 3840a^5b^7c^9d^2 + 9a^2c^5d^2*(-(4ac - b^2)^9)^{(1/2)} - 213ab^{11}c^3e^2 \\
& + 26880a^6b^7c^8e^2 - 80640a^7b^7c^7f^2 - 30b^{14}c^7e^2 + 288a^2b^7c^6d^2 - 1504a^3b^5c^7d^2 + 3840a^4b^3c^8d^2
\end{aligned}$$

$$\begin{aligned}
&^2 + 2077a^2b^9c^4e^2 - 10656a^3b^7c^5e^2 + 30240a^4b^5c^6e^2 - \\
&44800a^5b^3c^7e^2 - 25a^2c^4e^2(-4ac - b^2)^9)^{(1/2)} - b^2c^4* \\
&d^2(-4ac - b^2)^9)^{(1/2)} + 6366a^2b^{11}c^2f^2 - 35767a^3b^9c^3f^2 \\
&2 + 116928a^4b^7c^4f^2 - 219744a^5b^5c^5f^2 + 215040a^6b^3c^6f^2 \\
&2 + 49a^3c^3f^2(-4ac - b^2)^9)^{(1/2)} - 9b^4c^2e^2(-4ac - b^2) \\
&^9)^{(1/2)} - 615a^2b^{13}c^3f^2 - 15360a^6c^9d^2e - 6b^{12}c^3d^2e + 35840a \\
&^7c^8e^2f + 10b^{13}c^2d^2f + 152a^2b^{10}c^4d^2e - 258a^2b^{11}c^3d^2f + 43 \\
&520a^6b^8c^8d^2f + 724a^2b^{12}c^2e^2f + 30b^5c^5e^2f(-4ac - b^2)^9)^{(1 \\
&/2)} - 246a^2b^2c^2f^2(-4ac - b^2)^9)^{(1/2)} + 165a^2b^4c^2f^2(-4ac \\
&*c - b^2)^9)^{(1/2)} - 1548a^2b^8c^5d^2e + 8064a^3b^6c^6d^2e - 22400a^4 \\
&4b^4c^7d^2e + 30720a^5b^2c^8d^2e + 2706a^2b^9c^4d^2f - 14784a^3b^7 \\
&7c^5d^2f + 44352a^4b^5c^6d^2f - 69120a^5b^3c^7d^2f - 42a^2c^4d^2f * \\
&(-4ac - b^2)^9)^{(1/2)} + 6b^3c^3d^2e(-4ac - b^2)^9)^{(1/2)} - 7278a^2 \\
&2b^{10}c^3e^2f + 39132a^3b^8c^4e^2f - 119616a^4b^6c^5e^2f + 201600a^5 \\
&5b^4c^6e^2f - 161280a^6b^2c^7e^2f - 10b^4c^2d^2f(-4ac - b^2)^9)^{(\\
&/2)} + 51a^2b^2c^3e^2(-4ac - b^2)^9)^{(1/2)} - 44a^2b^4c^4d^2e(-4ac \\
&- b^2)^9)^{(1/2)} + 78a^2b^2c^3d^2f(-4ac - b^2)^9)^{(1/2)} - 184a^2b^3c^2 \\
&2e^2f(-4ac - b^2)^9)^{(1/2)} + 186a^2b^2c^3e^2f(-4ac - b^2)^9)^{(1/2)} \\
&)/(32(4096a^6c^{13} + b^{12}c^7 - 24a^2b^{10}c^8 + 240a^2b^8c^9 - 1280a^3 \\
&3b^6c^{10} + 3840a^4b^4c^{11} - 6144a^5b^2c^{12}))^{(1/2)}*i - (((10240a \\
&^5c^9e + 192a^2b^5c^7d - 768a^3b^3c^8d - 736a^2b^6c^6e + 4224 \\
&a^3b^4c^7e - 10752a^4b^2c^8e + 1264a^2b^7c^5f - 7488a^3b^5c^6 \\
&6f + 19712a^4b^3c^7f - 16a^2b^7c^6d + 1024a^4b^6c^9d + 48a^2b^8c^5 \\
&5e - 80a^2b^9c^4f - 19456a^5b^6c^8f)/(8(64a^3c^8 - b^6c^5 + 12a^2b \\
&^4c^6 - 48a^2b^2c^7)) + (x(-(25b^{15}f^2 + b^{11}c^4d^2 + 9b^{13}c^2e \\
&^2 - 25b^6f^2(-4ac - b^2)^9)^{(1/2)} - 27a^2b^9c^5d^2 - 3840a^5b^6c^9 \\
&9d^2 + 9a^2c^5d^2(-4ac - b^2)^9)^{(1/2)} - 213a^2b^{11}c^3e^2 + 26880a \\
&^6b^8c^8e^2 - 80640a^7b^7c^7f^2 - 30b^{14}c^4e^2f + 288a^2b^7c^6d^2 - \\
&1504a^3b^5c^7d^2 + 3840a^4b^3c^8d^2 + 2077a^2b^9c^4e^2 - 10656a \\
&a^3b^7c^5e^2 + 30240a^4b^5c^6e^2 - 44800a^5b^3c^7e^2 - 25a^2c^4 \\
&4e^2(-4ac - b^2)^9)^{(1/2)} - b^2c^4d^2(-4ac - b^2)^9)^{(1/2)} + 636 \\
&6a^2b^{11}c^2f^2 - 35767a^3b^9c^3f^2 + 116928a^4b^7c^4f^2 - 21974 \\
&4a^5b^5c^5f^2 + 215040a^6b^3c^6f^2 + 49a^3c^3f^2(-4ac - b^2) \\
&^9)^{(1/2)} - 9b^4c^2e^2(-4ac - b^2)^9)^{(1/2)} - 615a^2b^{13}c^3f^2 - 153 \\
&60a^6c^9d^2e - 6b^{12}c^3d^2e + 35840a^7c^8e^2f + 10b^{13}c^2d^2f + 152 \\
&a^2b^{10}c^4d^2e - 258a^2b^{11}c^3d^2f + 43520a^6b^8c^8d^2f + 724a^2b^{12}c^2 \\
&e^2f + 30b^5c^5e^2f(-4ac - b^2)^9)^{(1/2)} - 246a^2b^2c^2f^2(-4ac \\
&- b^2)^9)^{(1/2)} + 165a^2b^4c^2f^2(-4ac - b^2)^9)^{(1/2)} - 1548a^2b^8 \\
&c^5d^2e + 8064a^3b^6c^6d^2e - 22400a^4b^4c^7d^2e + 30720a^5b^2c^8 \\
&d^2e + 2706a^2b^9c^4d^2f - 14784a^3b^7c^5d^2f + 44352a^4b^5c^6d^2f \\
&- 69120a^5b^3c^7d^2f - 42a^2c^4d^2f(-4ac - b^2)^9)^{(1/2)} + 6b^3c^3 \\
&^3d^2e(-4ac - b^2)^9)^{(1/2)} - 7278a^2b^{10}c^3e^2f + 39132a^3b^8c^4 \\
&e^2f - 119616a^4b^6c^5e^2f + 201600a^5b^4c^6e^2f - 161280a^6b^2c^7 \\
&e^2f - 10b^4c^2d^2f(-4ac - b^2)^9)^{(1/2)} + 51a^2b^2c^3e^2(-4ac \\
&- b^2)^9)^{(1/2)} - 44a^2b^4c^4d^2e(-4ac - b^2)^9)^{(1/2)} + 78a^2b^2c^3d^2 \\
&f(-4ac - b^2)^9)^{(1/2)} - 184a^2b^3c^2e^2f(-4ac - b^2)^9)^{(1/2)} + 1 \\
&86a^2b^2c^3e^2f(-4ac - b^2)^9)^{(1/2)})/(32(4096a^6c^{13} + b^{12}c^7 - \\
&24a^2b^{10}c^8 + 240a^2b^8c^9 - 1280a^3b^6c^{10} + 3840a^4b^4c^{11} - 6 \\
&144a^5b^2c^{12}))^{(1/2)}*(16b^7c^7 - 192a^2b^5c^8 - 1024a^3b^6c^{10} + 7 \\
&68a^2b^3c^9))/(2*(16a^2c^7 + b^4c^5 - 8a^2b^2c^6)))*(-(25b^{15}f^2 + \\
&b^{11}c^4d^2 + 9b^{13}c^2e^2 - 25b^6f^2(-4ac - b^2)^9)^{(1/2)} - 27a \\
&b^9c^5d^2 - 3840a^5b^6c^9d^2 + 9a^2c^5d^2(-4ac - b^2)^9)^{(1/2)} - \\
&213a^2b^{11}c^3e^2 + 26880a^6b^8c^8e^2 - 80640a^7b^7c^7f^2 - 30b^{14}c^4 \\
&e^2f + 288a^2b^7c^6d^2 - 1504a^3b^5c^7d^2 + 3840a^4b^3c^8d^2 + 2 \\
&077a^2b^9c^4e^2 - 10656a^3b^7c^5e^2 + 30240a^4b^5c^6e^2 - 44800 \\
&a^5b^3c^7e^2 - 25a^2c^4e^2(-4ac - b^2)^9)^{(1/2)} - b^2c^4d^2(- \\
&(4ac - b^2)^9)^{(1/2)} + 6366a^2b^{11}c^2f^2 - 35767a^3b^9c^3f^2 + 11 \\
&6928a^4b^7c^4f^2 - 219744a^5b^5c^5f^2 + 215040a^6b^3c^6f^2 + 49 \\
&a^3c^3f^2(-4ac - b^2)^9)^{(1/2)} - 9b^4c^2e^2(-4ac - b^2)^9)^{(1
\end{aligned}$$

$$\begin{aligned}
& /2) - 615*a*b^{13}*c*f^2 - 15360*a^6*c^9*d*e - 6*b^{12}*c^3*d*e + 35840*a^7*c^8 \\
& *e*f + 10*b^{13}*c^2*d*f + 152*a*b^{10}*c^4*d*e - 258*a*b^{11}*c^3*d*f + 43520*a^ \\
& 6*b*c^8*d*f + 724*a*b^{12}*c^2*e*f + 30*b^5*c*e*f*(-(4*a*c - b^2)^9)^{(1/2)} - \\
& 246*a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + 165*a*b^4*c*f^2*(-(4*a*c - b \\
& ^2)^9)^{(1/2)} - 1548*a^2*b^8*c^5*d*e + 8064*a^3*b^6*c^6*d*e - 22400*a^4*b^4* \\
& c^7*d*e + 30720*a^5*b^2*c^8*d*e + 2706*a^2*b^9*c^4*d*f - 14784*a^3*b^7*c^5* \\
& d*f + 44352*a^4*b^5*c^6*d*f - 69120*a^5*b^3*c^7*d*f - 42*a^2*c^4*d*f*(-(4*a \\
& *c - b^2)^9)^{(1/2)} + 6*b^3*c^3*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 7278*a^2*b^10 \\
& *c^3*e*f + 39132*a^3*b^8*c^4*e*f - 119616*a^4*b^6*c^5*e*f + 201600*a^5*b^4* \\
& c^6*e*f - 161280*a^6*b^2*c^7*e*f - 10*b^4*c^2*d*f*(-(4*a*c - b^2)^9)^{(1/2)} \\
& + 51*a*b^2*c^3*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 44*a*b*c^4*d*e*(-(4*a*c - b^2 \\
&)^9)^{(1/2)} + 78*a*b^2*c^3*d*f*(-(4*a*c - b^2)^9)^{(1/2)} - 184*a*b^3*c^2*e*f* \\
& (- (4*a*c - b^2)^9)^{(1/2)} + 186*a^2*b*c^3*e*f*(-(4*a*c - b^2)^9)^{(1/2)))/(32* \\
& (4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6* \\
& c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^{(1/2)} + (x*(25*b^10*f^2 - 7 \\
& 2*a^3*c^7*d^2 + 200*a^4*c^6*e^2 + b^6*c^4*d^2 - 392*a^5*c^5*f^2 + 9*b^8*c^2 \\
& *e^2 - 16*a*b^4*c^5*d^2 - 114*a*b^6*c^3*e^2 - 30*b^9*c*e*f + 74*a^2*b^2*c^6 \\
& *d^2 + 481*a^2*b^4*c^4*e^2 - 718*a^3*b^2*c^5*e^2 + 1676*a^2*b^6*c^2*f^2 - 3 \\
& 536*a^3*b^4*c^3*f^2 + 2794*a^4*b^2*c^4*f^2 - 340*a*b^8*c*f^2 + 336*a^4*c^6* \\
& d*f - 6*b^7*c^3*d*e + 10*b^8*c^2*d*f + 86*a*b^5*c^4*d*e + 472*a^3*b*c^6*d*e \\
& - 148*a*b^6*c^3*d*f + 394*a*b^7*c^2*e*f - 1768*a^4*b*c^5*e*f - 374*a^2*b^3 \\
& *c^5*d*e + 698*a^2*b^4*c^4*d*f - 1132*a^3*b^2*c^5*d*f - 1804*a^2*b^5*c^3*e* \\
& f + 3266*a^3*b^3*c^4*e*f))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))*(-(25* \\
& b^15*f^2 + b^11*c^4*d^2 + 9*b^13*c^2*e^2 - 25*b^6*f^2*(-(4*a*c - b^2)^9)^{(1 \\
& /2)} - 27*a*b^9*c^5*d^2 - 3840*a^5*b*c^9*d^2 + 9*a*c^5*d^2*(-(4*a*c - b^2)^9 \\
&)^{(1/2)} - 213*a*b^11*c^3*e^2 + 26880*a^6*b*c^8*e^2 - 80640*a^7*b*c^7*f^2 - \\
& 30*b^14*c*e*f + 288*a^2*b^7*c^6*d^2 - 1504*a^3*b^5*c^7*d^2 + 3840*a^4*b^3*c \\
& ^8*d^2 + 2077*a^2*b^9*c^4*e^2 - 10656*a^3*b^7*c^5*e^2 + 30240*a^4*b^5*c^6*e \\
& ^2 - 44800*a^5*b^3*c^7*e^2 - 25*a^2*c^4*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - b^2* \\
& c^4*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + 6366*a^2*b^11*c^2*f^2 - 35767*a^3*b^9*c^ \\
& 3*f^2 + 116928*a^4*b^7*c^4*f^2 - 219744*a^5*b^5*c^5*f^2 + 215040*a^6*b^3*c^ \\
& 6*f^2 + 49*a^3*c^3*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 9*b^4*c^2*e^2*(-(4*a*c - \\
& b^2)^9)^{(1/2)} - 615*a*b^{13}*c*f^2 - 15360*a^6*c^9*d*e - 6*b^{12}*c^3*d*e + 358 \\
& 40*a^7*c^8*e*f + 10*b^{13}*c^2*d*f + 152*a*b^{10}*c^4*d*e - 258*a*b^{11}*c^3*d*f \\
& + 43520*a^6*b*c^8*d*f + 724*a*b^{12}*c^2*e*f + 30*b^5*c*e*f*(-(4*a*c - b^2)^9 \\
&)^{(1/2)} - 246*a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + 165*a*b^4*c*f^2*(- \\
& (4*a*c - b^2)^9)^{(1/2)} - 1548*a^2*b^8*c^5*d*e + 8064*a^3*b^6*c^6*d*e - 2240 \\
& 0*a^4*b^4*c^7*d*e + 30720*a^5*b^2*c^8*d*e + 2706*a^2*b^9*c^4*d*f - 14784*a^ \\
& 3*b^7*c^5*d*f + 44352*a^4*b^5*c^6*d*f - 69120*a^5*b^3*c^7*d*f - 42*a^2*c^4* \\
& d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 6*b^3*c^3*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 727 \\
& 8*a^2*b^10*c^3*e*f + 39132*a^3*b^8*c^4*e*f - 119616*a^4*b^6*c^5*e*f + 20160 \\
& 0*a^5*b^4*c^6*e*f - 161280*a^6*b^2*c^7*e*f - 10*b^4*c^2*d*f*(-(4*a*c - b^2) \\
& ^9)^{(1/2)} + 51*a*b^2*c^3*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 44*a*b*c^4*d*e*(-(4 \\
& *a*c - b^2)^9)^{(1/2)} + 78*a*b^2*c^3*d*f*(-(4*a*c - b^2)^9)^{(1/2)} - 184*a*b^ \\
& 3*c^2*e*f*(-(4*a*c - b^2)^9)^{(1/2)} + 186*a^2*b*c^3*e*f*(-(4*a*c - b^2)^9)^{(\\
& 1/2)))/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 128 \\
& 0*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^{(1/2)}*i)/(((102 \\
& 40*a^5*c^9*e + 192*a^2*b^5*c^7*d - 768*a^3*b^3*c^8*d - 736*a^2*b^6*c^6*e + \\
& 4224*a^3*b^4*c^7*e - 10752*a^4*b^2*c^8*e + 1264*a^2*b^7*c^5*f - 7488*a^3*b^ \\
& 5*c^6*f + 19712*a^4*b^3*c^7*f - 16*a*b^7*c^6*d + 1024*a^4*b*c^9*d + 48*a*b^ \\
& 8*c^5*e - 80*a*b^9*c^4*f - 19456*a^5*b*c^8*f)/(8*(64*a^3*c^8 - b^6*c^5 + 12 \\
& *a*b^4*c^6 - 48*a^2*b^2*c^7)) - (x*(-(25*b^15*f^2 + b^11*c^4*d^2 + 9*b^13*c \\
& ^2*e^2 - 25*b^6*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 27*a*b^9*c^5*d^2 - 3840*a^5* \\
& b*c^9*d^2 + 9*a*c^5*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - 213*a*b^11*c^3*e^2 + 268 \\
& 80*a^6*b*c^8*e^2 - 80640*a^7*b*c^7*f^2 - 30*b^14*c*e*f + 288*a^2*b^7*c^6*d^ \\
& 2 - 1504*a^3*b^5*c^7*d^2 + 3840*a^4*b^3*c^8*d^2 + 2077*a^2*b^9*c^4*e^2 - 10 \\
& 656*a^3*b^7*c^5*e^2 + 30240*a^4*b^5*c^6*e^2 - 44800*a^5*b^3*c^7*e^2 - 25*a^ \\
& 2*c^4*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - b^2*c^4*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + \\
& 6366*a^2*b^11*c^2*f^2 - 35767*a^3*b^9*c^3*f^2 + 116928*a^4*b^7*c^4*f^2 - 2
\end{aligned}$$

$$\begin{aligned}
& 19744a^5b^5c^5f^2 + 215040a^6b^3c^6f^2 + 49a^3c^3f^2(-4ac - b^2)^9)^{(1/2)} - 9b^4c^2e^2(-4ac - b^2)^9)^{(1/2)} - 615ab^{13}cf^2 - \\
& 15360a^6c^9d^e - 6b^{12}c^3d^e + 35840a^7c^8ef + 10b^{13}c^2df + 152ab^{10}c^4d^e - 258ab^{11}c^3d^f + 43520a^6b^8d^f + 724ab^{12} \\
& c^2ef + 30b^5c^eef(-4ac - b^2)^9)^{(1/2)} - 246a^2b^2c^2f^2(-4ac - b^2)^9)^{(1/2)} + 165ab^4cf^2(-4ac - b^2)^9)^{(1/2)} - 1548a^2b^8 \\
& c^5d^e + 8064a^3b^6c^6d^e - 22400a^4b^4c^7d^e + 30720a^5b^2c^8d^e + 2706a^2b^9c^4d^f - 14784a^3b^7c^5d^f + 44352a^4b^5c^6d^f \\
& - 69120a^5b^3c^7d^f - 42a^2c^4d^f(-4ac - b^2)^9)^{(1/2)} + 6b^3c^3d^e(-4ac - b^2)^9)^{(1/2)} - 7278a^2b^{10}c^3ef + 39132a^3b^8 \\
& c^4ef - 119616a^4b^6c^5eef + 201600a^5b^4c^6eef - 161280a^6b^2c^7ef - 10b^4c^2d^f(-4ac - b^2)^9)^{(1/2)} + 51ab^2c^3e^2(-4ac - b^2)^9)^{(1/2)} \\
& - 44ab^2c^3d^e(-4ac - b^2)^9)^{(1/2)} + 78ab^2c^3d^f(-4ac - b^2)^9)^{(1/2)} - 184ab^3c^2eef(-4ac - b^2)^9)^{(1/2)} \\
& + 186a^2b^2c^3eef(-4ac - b^2)^9)^{(1/2))} / (32(4096a^6c^{13} + b^{12}c^7 - 24ab^{10}c^8 + 240a^2b^8c^9 - 1280a^3b^6c^{10} + 3840a^4b^4c^{11} \\
& - 6144a^5b^2c^{12}))^{(1/2)} * (16b^7c^7 - 192ab^5c^8 - 1024a^3b^6c^{10} + 768a^2b^3c^9) / (2(16a^2c^7 + b^4c^5 - 8ab^2c^6)) * (-25b^{15}f^2 \\
& + b^{11}c^4d^2 + 9b^{13}c^2e^2 - 25b^6f^2(-4ac - b^2)^9)^{(1/2)} - 27ab^9c^5d^2 - 3840a^5b^8c^9d^2 + 9a^5c^5d^2(-4ac - b^2)^9)^{(1/2)} \\
& - 213ab^{11}c^3e^2 + 26880a^6b^8c^8e^2 - 80640a^7b^7c^7f^2 - 30b^{14}c^eef + 288a^2b^7c^6d^2 - 1504a^3b^5c^7d^2 + 3840a^4b^3c^8d^2 \\
& + 2077a^2b^9c^4e^2 - 10656a^3b^7c^5e^2 + 30240a^4b^5c^6e^2 - 44800a^5b^3c^7e^2 - 25a^2c^4e^2(-4ac - b^2)^9)^{(1/2)} - b^2c^4d^2 \\
& (-4ac - b^2)^9)^{(1/2)} + 6366a^2b^{11}c^2f^2 - 35767a^3b^9c^3f^2 + 116928a^4b^7c^4f^2 - 219744a^5b^5c^5f^2 + 215040a^6b^3c^6f^2 \\
& + 49a^3c^3f^2(-4ac - b^2)^9)^{(1/2)} - 9b^4c^2e^2(-4ac - b^2)^9)^{(1/2)} - 615ab^{13}cf^2 - 15360a^6c^9d^e - 6b^{12}c^3d^e + 35840a^7 \\
& c^8ef + 10b^{13}c^2df + 152ab^{10}c^4d^e - 258ab^{11}c^3d^f + 43520a^6b^8d^f + 724ab^{12}c^2ef + 30b^5c^eef(-4ac - b^2)^9)^{(1/2)} \\
& - 246a^2b^2c^2f^2(-4ac - b^2)^9)^{(1/2)} + 165ab^4cf^2(-4ac - b^2)^9)^{(1/2)} - 1548a^2b^8c^5d^e + 8064a^3b^6c^6d^e - 22400a^4b^4 \\
& c^7d^e + 30720a^5b^2c^8d^e + 2706a^2b^9c^4d^f - 14784a^3b^7c^5d^f + 44352a^4b^5c^6d^f - 69120a^5b^3c^7d^f - 42a^2c^4d^f(-4ac - b^2)^9)^{(1/2)} \\
& + 6b^3c^3d^e(-4ac - b^2)^9)^{(1/2)} - 7278a^2b^{10}c^3ef + 39132a^3b^8c^4ef - 119616a^4b^6c^5eef + 201600a^5b^4c^6eef - 161280a^6b^2c^7ef \\
& - 10b^4c^2d^f(-4ac - b^2)^9)^{(1/2)} + 51ab^2c^3e^2(-4ac - b^2)^9)^{(1/2)} - 44ab^2c^3d^e(-4ac - b^2)^9)^{(1/2)} + 78ab^2c^3d^f(-4ac - b^2)^9)^{(1/2)} \\
& - 184ab^3c^2eef(-4ac - b^2)^9)^{(1/2)} + 186a^2b^2c^3eef(-4ac - b^2)^9)^{(1/2))} / (32(4096a^6c^{13} + b^{12}c^7 - 24ab^{10}c^8 + 240a^2b^8c^9 - 1280a^3b^6 \\
& c^{10} + 3840a^4b^4c^{11} - 6144a^5b^2c^{12}))^{(1/2)} - (x(25b^{10}f^2 - 72a^3c^7d^2 + 200a^4c^6e^2 + b^6c^4d^2 - 392a^5c^5f^2 + 9b^8 \\
& c^2e^2 - 16ab^4c^5d^2 - 114ab^6c^3e^2 - 30b^9c^eef + 74a^2b^2c^6d^2 + 481a^2b^4c^4e^2 - 718a^3b^2c^5e^2 + 1676a^2b^6c^2f^2 \\
& - 3536a^3b^4c^3f^2 + 2794a^4b^2c^4f^2 - 340ab^8c^f^2 + 336a^4c^6d^f - 6b^7c^3d^e + 10b^8c^2d^f + 86ab^5c^4d^e + 472a^3b^c^6 \\
& d^e - 148ab^6c^3d^f + 394ab^7c^2eef - 1768a^4b^c^5eef - 374a^2b^3c^5d^e + 698a^2b^4c^4d^f - 1132a^3b^2c^5d^f - 1804a^2b^5c^3 \\
& eef + 3266a^3b^3c^4eef) / (2(16a^2c^7 + b^4c^5 - 8ab^2c^6)) * (-25b^{15}f^2 + b^{11}c^4d^2 + 9b^{13}c^2e^2 - 25b^6f^2(-4ac - b^2)^9)^{(1/2)} \\
& - 27ab^9c^5d^2 - 3840a^5b^8c^9d^2 + 9a^5c^5d^2(-4ac - b^2)^9)^{(1/2)} - 213ab^{11}c^3e^2 + 26880a^6b^8c^8e^2 - 80640a^7b^7c^7f^2 \\
& - 30b^{14}c^eef + 288a^2b^7c^6d^2 - 1504a^3b^5c^7d^2 + 3840a^4b^3c^8d^2 + 2077a^2b^9c^4e^2 - 10656a^3b^7c^5e^2 + 30240a^4b^5c^6 \\
& e^2 - 44800a^5b^3c^7e^2 - 25a^2c^4e^2(-4ac - b^2)^9)^{(1/2)} - b^2c^4d^2(-4ac - b^2)^9)^{(1/2)} + 6366a^2b^{11}c^2f^2 - 35767a^3b^9 \\
& c^3f^2 + 116928a^4b^7c^4f^2 - 219744a^5b^5c^5f^2 + 215040a^6b^3c^6f^2 + 49a^3c^3f^2(-4ac - b^2)^9)^{(1/2)} - 9b^4c^2e^2(-4ac - b^2)^9)^{(1/2)}
\end{aligned}$$

$$\begin{aligned}
& c - b^2)^9)^{(1/2)} - 615*a*b^{13}*c*f^2 - 15360*a^6*c^9*d*e - 6*b^{12}*c^3*d*e + \\
& 35840*a^7*c^8*e*f + 10*b^{13}*c^2*d*f + 152*a*b^{10}*c^4*d*e - 258*a*b^{11}*c^3* \\
& d*f + 43520*a^6*b*c^8*d*f + 724*a*b^{12}*c^2*e*f + 30*b^5*c*e*f*(-(4*a*c - b^ \\
& 2)^9)^{(1/2)} - 246*a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + 165*a*b^4*c*f^ \\
& 2*(-(4*a*c - b^2)^9)^{(1/2)} - 1548*a^2*b^8*c^5*d*e + 8064*a^3*b^6*c^6*d*e - \\
& 22400*a^4*b^4*c^7*d*e + 30720*a^5*b^2*c^8*d*e + 2706*a^2*b^9*c^4*d*f - 1478 \\
& 4*a^3*b^7*c^5*d*f + 44352*a^4*b^5*c^6*d*f - 69120*a^5*b^3*c^7*d*f - 42*a^2*c^ \\
& 4*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 6*b^3*c^3*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - \\
& 7278*a^2*b^{10}*c^3*e*f + 39132*a^3*b^8*c^4*e*f - 119616*a^4*b^6*c^5*e*f + 2 \\
& 01600*a^5*b^4*c^6*e*f - 161280*a^6*b^2*c^7*e*f - 10*b^4*c^2*d*f*(-(4*a*c - \\
& b^2)^9)^{(1/2)} + 51*a*b^2*c^3*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 44*a*b*c^4*d*e* \\
& (- (4*a*c - b^2)^9)^{(1/2)} + 78*a*b^2*c^3*d*f*(-(4*a*c - b^2)^9)^{(1/2)} - 184* \\
& a*b^3*c^2*e*f*(-(4*a*c - b^2)^9)^{(1/2)} + 186*a^2*b*c^3*e*f*(-(4*a*c - b^2)^ \\
& 9)^{(1/2)))/(32*(4096*a^6*c^13 + b^{12}*c^7 - 24*a*b^{10}*c^8 + 240*a^2*b^8*c^9 - \\
& 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^{(1/2)} + (((10 \\
& 240*a^5*c^9*e + 192*a^2*b^5*c^7*d - 768*a^3*b^3*c^8*d - 736*a^2*b^6*c^6*e + \\
& 4224*a^3*b^4*c^7*e - 10752*a^4*b^2*c^8*e + 1264*a^2*b^7*c^5*f - 7488*a^3*b \\
& ^5*c^6*f + 19712*a^4*b^3*c^7*f - 16*a*b^7*c^6*d + 1024*a^4*b*c^9*d + 48*a*b \\
& ^8*c^5*e - 80*a*b^9*c^4*f - 19456*a^5*b*c^8*f)/(8*(64*a^3*c^8 - b^6*c^5 + 1 \\
& 2*a*b^4*c^6 - 48*a^2*b^2*c^7)) + (x*(-(25*b^{15}*f^2 + b^{11}*c^4*d^2 + 9*b^{13}* \\
& c^2*e^2 - 25*b^6*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 27*a*b^9*c^5*d^2 - 3840*a^5 \\
& *b*c^9*d^2 + 9*a*c^5*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - 213*a*b^{11}*c^3*e^2 + 26 \\
& 880*a^6*b*c^8*e^2 - 80640*a^7*b*c^7*f^2 - 30*b^{14}*c*e*f + 288*a^2*b^7*c^6*d \\
& ^2 - 1504*a^3*b^5*c^7*d^2 + 3840*a^4*b^3*c^8*d^2 + 2077*a^2*b^9*c^4*e^2 - 1 \\
& 0656*a^3*b^7*c^5*e^2 + 30240*a^4*b^5*c^6*e^2 - 44800*a^5*b^3*c^7*e^2 - 25*a \\
& ^2*c^4*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - b^2*c^4*d^2*(-(4*a*c - b^2)^9)^{(1/2)} \\
& + 6366*a^2*b^{11}*c^2*f^2 - 35767*a^3*b^9*c^3*f^2 + 116928*a^4*b^7*c^4*f^2 - \\
& 219744*a^5*b^5*c^5*f^2 + 215040*a^6*b^3*c^6*f^2 + 49*a^3*c^3*f^2*(-(4*a*c - \\
& b^2)^9)^{(1/2)} - 9*b^4*c^2*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 615*a*b^{13}*c*f^2 \\
& - 15360*a^6*c^9*d*e - 6*b^{12}*c^3*d*e + 35840*a^7*c^8*e*f + 10*b^{13}*c^2*d*f \\
& + 152*a*b^{10}*c^4*d*e - 258*a*b^{11}*c^3*d*f + 43520*a^6*b*c^8*d*f + 724*a*b^1 \\
& 2*c^2*e*f + 30*b^5*c*e*f*(-(4*a*c - b^2)^9)^{(1/2)} - 246*a^2*b^2*c^2*f^2*(-(\\
& 4*a*c - b^2)^9)^{(1/2)} + 165*a*b^4*c*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 1548*a^2 \\
& *b^8*c^5*d*e + 8064*a^3*b^6*c^6*d*e - 22400*a^4*b^4*c^7*d*e + 30720*a^5*b^2 \\
& *c^8*d*e + 2706*a^2*b^9*c^4*d*f - 14784*a^3*b^7*c^5*d*f + 44352*a^4*b^5*c^6 \\
& *d*f - 69120*a^5*b^3*c^7*d*f - 42*a^2*c^4*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 6* \\
& b^3*c^3*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 7278*a^2*b^{10}*c^3*e*f + 39132*a^3*b^ \\
& 8*c^4*e*f - 119616*a^4*b^6*c^5*e*f + 201600*a^5*b^4*c^6*e*f - 161280*a^6*b^ \\
& 2*c^7*e*f - 10*b^4*c^2*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 51*a*b^2*c^3*e^2*(-(4 \\
& *a*c - b^2)^9)^{(1/2)} - 44*a*b*c^4*d*e*(-(4*a*c - b^2)^9)^{(1/2)} + 78*a*b^2*c \\
& ^3*d*f*(-(4*a*c - b^2)^9)^{(1/2)} - 184*a*b^3*c^2*e*f*(-(4*a*c - b^2)^9)^{(1/2} \\
&) + 186*a^2*b*c^3*e*f*(-(4*a*c - b^2)^9)^{(1/2)))/(32*(4096*a^6*c^13 + b^{12}*c \\
& ^7 - 24*a*b^{10}*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^1 \\
& 1 - 6144*a^5*b^2*c^12)))^{(1/2)}*(16*b^7*c^7 - 192*a*b^5*c^8 - 1024*a^3*b*c^1 \\
& 0 + 768*a^2*b^3*c^9))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))*(-(25*b^{15}* \\
& f^2 + b^{11}*c^4*d^2 + 9*b^{13}*c^2*e^2 - 25*b^6*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - \\
& 27*a*b^9*c^5*d^2 - 3840*a^5*b*c^9*d^2 + 9*a*c^5*d^2*(-(4*a*c - b^2)^9)^{(1/ \\
& 2)} - 213*a*b^{11}*c^3*e^2 + 26880*a^6*b*c^8*e^2 - 80640*a^7*b*c^7*f^2 - 30*b^ \\
& 14*c*e*f + 288*a^2*b^7*c^6*d^2 - 1504*a^3*b^5*c^7*d^2 + 3840*a^4*b^3*c^8*d^ \\
& 2 + 2077*a^2*b^9*c^4*e^2 - 10656*a^3*b^7*c^5*e^2 + 30240*a^4*b^5*c^6*e^2 - \\
& 44800*a^5*b^3*c^7*e^2 - 25*a^2*c^4*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - b^2*c^4*d \\
& ^2*(-(4*a*c - b^2)^9)^{(1/2)} + 6366*a^2*b^{11}*c^2*f^2 - 35767*a^3*b^9*c^3*f^2 \\
& + 116928*a^4*b^7*c^4*f^2 - 219744*a^5*b^5*c^5*f^2 + 215040*a^6*b^3*c^6*f^2 \\
& + 49*a^3*c^3*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 9*b^4*c^2*e^2*(-(4*a*c - b^2)^ \\
& 9)^{(1/2)} - 615*a*b^{13}*c*f^2 - 15360*a^6*c^9*d*e - 6*b^{12}*c^3*d*e + 35840*a^ \\
& 7*c^8*e*f + 10*b^{13}*c^2*d*f + 152*a*b^{10}*c^4*d*e - 258*a*b^{11}*c^3*d*f + 435 \\
& 20*a^6*b*c^8*d*f + 724*a*b^{12}*c^2*e*f + 30*b^5*c*e*f*(-(4*a*c - b^2)^9)^{(1/ \\
& 2)} - 246*a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + 165*a*b^4*c*f^2*(-(4*a* \\
& c - b^2)^9)^{(1/2)} - 1548*a^2*b^8*c^5*d*e + 8064*a^3*b^6*c^6*d*e - 22400*a^4
\end{aligned}$$

$$\begin{aligned}
& *b^4*c^7*d*e + 30720*a^5*b^2*c^8*d*e + 2706*a^2*b^9*c^4*d*f - 14784*a^3*b^7 \\
& *c^5*d*f + 44352*a^4*b^5*c^6*d*f - 69120*a^5*b^3*c^7*d*f - 42*a^2*c^4*d*f*(\\
& -(4*a*c - b^2)^9)^{(1/2)} + 6*b^3*c^3*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 7278*a^2 \\
& *b^10*c^3*e*f + 39132*a^3*b^8*c^4*e*f - 119616*a^4*b^6*c^5*e*f + 201600*a^5 \\
& *b^4*c^6*e*f - 161280*a^6*b^2*c^7*e*f - 10*b^4*c^2*d*f*(-(4*a*c - b^2)^9)^{(\\
& 1/2)} + 51*a*b^2*c^3*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 44*a*b*c^4*d*e*(-(4*a*c \\
& - b^2)^9)^{(1/2)} + 78*a*b^2*c^3*d*f*(-(4*a*c - b^2)^9)^{(1/2)} - 184*a*b^3*c^2 \\
& *e*f*(-(4*a*c - b^2)^9)^{(1/2)} + 186*a^2*b*c^3*e*f*(-(4*a*c - b^2)^9)^{(1/2)} \\
& / (32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3 \\
& *b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^{(1/2)} + (x*(25*b^10*f^ \\
& 2 - 72*a^3*c^7*d^2 + 200*a^4*c^6*e^2 + b^6*c^4*d^2 - 392*a^5*c^5*f^2 + 9*b^ \\
& 8*c^2*e^2 - 16*a*b^4*c^5*d^2 - 114*a*b^6*c^3*e^2 - 30*b^9*c*e*f + 74*a^2*b^ \\
& 2*c^6*d^2 + 481*a^2*b^4*c^4*e^2 - 718*a^3*b^2*c^5*e^2 + 1676*a^2*b^6*c^2*f^ \\
& 2 - 3536*a^3*b^4*c^3*f^2 + 2794*a^4*b^2*c^4*f^2 - 340*a*b^8*c*f^2 + 336*a^4 \\
& *c^6*d*f - 6*b^7*c^3*d*e + 10*b^8*c^2*d*f + 86*a*b^5*c^4*d*e + 472*a^3*b*c^ \\
& 6*d*e - 148*a*b^6*c^3*d*f + 394*a*b^7*c^2*e*f - 1768*a^4*b*c^5*e*f - 374*a^ \\
& 2*b^3*c^5*d*e + 698*a^2*b^4*c^4*d*f - 1132*a^3*b^2*c^5*d*f - 1804*a^2*b^5*c \\
& ^3*e*f + 3266*a^3*b^3*c^4*e*f)) / (2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)) * (\\
& -(25*b^15*f^2 + b^11*c^4*d^2 + 9*b^13*c^2*e^2 - 25*b^6*f^2*(-(4*a*c - b^2)^ \\
& 9)^{(1/2)} - 27*a*b^9*c^5*d^2 - 3840*a^5*b*c^9*d^2 + 9*a*c^5*d^2*(-(4*a*c - b \\
& ^2)^9)^{(1/2)} - 213*a*b^11*c^3*e^2 + 26880*a^6*b*c^8*e^2 - 80640*a^7*b*c^7*f \\
& ^2 - 30*b^14*c*e*f + 288*a^2*b^7*c^6*d^2 - 1504*a^3*b^5*c^7*d^2 + 3840*a^4* \\
& b^3*c^8*d^2 + 2077*a^2*b^9*c^4*e^2 - 10656*a^3*b^7*c^5*e^2 + 30240*a^4*b^5* \\
& c^6*e^2 - 44800*a^5*b^3*c^7*e^2 - 25*a^2*c^4*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - \\
& b^2*c^4*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + 6366*a^2*b^11*c^2*f^2 - 35767*a^3*b \\
& ^9*c^3*f^2 + 116928*a^4*b^7*c^4*f^2 - 219744*a^5*b^5*c^5*f^2 + 215040*a^6*b \\
& ^3*c^6*f^2 + 49*a^3*c^3*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 9*b^4*c^2*e^2*(-(4*a \\
& *c - b^2)^9)^{(1/2)} - 615*a*b^13*c*f^2 - 15360*a^6*c^9*d*e - 6*b^12*c^3*d*e \\
& + 35840*a^7*c^8*e*f + 10*b^13*c^2*d*f + 152*a*b^10*c^4*d*e - 258*a*b^11*c^3 \\
& *d*f + 43520*a^6*b*c^8*d*f + 724*a*b^12*c^2*e*f + 30*b^5*c*e*f*(-(4*a*c - b \\
& ^2)^9)^{(1/2)} - 246*a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + 165*a*b^4*c*f \\
& ^2*(-(4*a*c - b^2)^9)^{(1/2)} - 1548*a^2*b^8*c^5*d*e + 8064*a^3*b^6*c^6*d*e - \\
& 22400*a^4*b^4*c^7*d*e + 30720*a^5*b^2*c^8*d*e + 2706*a^2*b^9*c^4*d*f - 147 \\
& 84*a^3*b^7*c^5*d*f + 44352*a^4*b^5*c^6*d*f - 69120*a^5*b^3*c^7*d*f - 42*a^2 \\
& *c^4*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 6*b^3*c^3*d*e*(-(4*a*c - b^2)^9)^{(1/2)} \\
& - 7278*a^2*b^10*c^3*e*f + 39132*a^3*b^8*c^4*e*f - 119616*a^4*b^6*c^5*e*f + \\
& 201600*a^5*b^4*c^6*e*f - 161280*a^6*b^2*c^7*e*f - 10*b^4*c^2*d*f*(-(4*a*c - \\
& b^2)^9)^{(1/2)} + 51*a*b^2*c^3*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 44*a*b*c^4*d*e \\
& *(- (4*a*c - b^2)^9)^{(1/2)} + 78*a*b^2*c^3*d*f*(-(4*a*c - b^2)^9)^{(1/2)} - 184 \\
& *a*b^3*c^2*e*f*(-(4*a*c - b^2)^9)^{(1/2)} + 186*a^2*b*c^3*e*f*(-(4*a*c - b^2) \\
& ^9)^{(1/2)} / (32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 \\
& - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^{(1/2)} - (216 \\
& *a^4*c^6*d^3 + 225*a^4*b^6*f^3 - 2744*a^7*c^3*f^3 - 1300*a^5*b*c^4*e^3 - 20 \\
& 60*a^5*b^4*c*f^3 + 125*a^2*b^8*d*f^2 + 600*a^5*c^5*d*e^2 - 175*a^3*b^7*e*f^ \\
& 2 - 1512*a^5*c^5*d^2*f + 3528*a^6*c^4*d*f^2 - 1400*a^6*c^4*e^2*f + 5*a^2*b^ \\
& 4*c^4*d^3 - 66*a^3*b^2*c^5*d^3 - 63*a^3*b^5*c^2*e^3 + 573*a^4*b^3*c^3*e^3 + \\
& 5334*a^6*b^2*c^2*f^3 - 924*a^4*b*c^5*d^2*e - 1350*a^3*b^6*c*d*f^2 + 210*a^ \\
& 3*b^6*c*e^2*f + 1485*a^4*b^5*c*e*f^2 - 364*a^6*b*c^3*e*f^2 - 30*a^2*b^5*c^3 \\
& *d^2*e + 45*a^2*b^6*c^2*d*e^2 + 339*a^3*b^3*c^4*d^2*e - 402*a^3*b^4*c^3*d*e \\
& ^2 + 762*a^4*b^2*c^4*d*e^2 + 50*a^2*b^6*c^2*d^2*f - 600*a^3*b^4*c^3*d^2*f + \\
& 2002*a^4*b^2*c^4*d^2*f + 4835*a^4*b^4*c^2*d*f^2 - 6598*a^5*b^2*c^3*d*f^2 - \\
& 1927*a^4*b^4*c^2*e^2*f + 4722*a^5*b^2*c^3*e^2*f - 3061*a^5*b^3*c^2*e*f^2 - \\
& 150*a^2*b^7*c*d*e*f + 2312*a^5*b*c^4*d*e*f + 1480*a^3*b^5*c^2*d*e*f - 4122 \\
& *a^4*b^3*c^3*d*e*f) / (4*(64*a^3*c^8 - b^6*c^5 + 12*a*b^4*c^6 - 48*a^2*b^2*c^ \\
& 7))) * (-(25*b^15*f^2 + b^11*c^4*d^2 + 9*b^13*c^2*e^2 - 25*b^6*f^2*(-(4*a*c \\
& - b^2)^9)^{(1/2)} - 27*a*b^9*c^5*d^2 - 3840*a^5*b*c^9*d^2 + 9*a*c^5*d^2*(-(4* \\
& a*c - b^2)^9)^{(1/2)} - 213*a*b^11*c^3*e^2 + 26880*a^6*b*c^8*e^2 - 80640*a^7* \\
& b*c^7*f^2 - 30*b^14*c*e*f + 288*a^2*b^7*c^6*d^2 - 1504*a^3*b^5*c^7*d^2 + 38 \\
& 40*a^4*b^3*c^8*d^2 + 2077*a^2*b^9*c^4*e^2 - 10656*a^3*b^7*c^5*e^2 + 30240*a
\end{aligned}$$

$$\begin{aligned}
& ^4b^5c^6e^2 - 44800a^5b^3c^7e^2 - 25a^2c^4e^2(-4ac - b^2)^9)^{1/2} - b^2c^4d^2(-4ac - b^2)^9)^{1/2} + 6366a^2b^{11}c^2f^2 - 3576 \\
& 7a^3b^9c^3f^2 + 116928a^4b^7c^4f^2 - 219744a^5b^5c^5f^2 + 21504 \\
& 0a^6b^3c^6f^2 + 49a^3c^3f^2(-4ac - b^2)^9)^{1/2} - 9b^4c^2e^2 \\
& *(-4ac - b^2)^9)^{1/2} - 615ab^{13}c^3f^2 - 15360a^6c^9d^2e - 6b^{12}c^3d^2e + 35840a^7c^8e^2f + 10b^{13}c^2d^2f + 152ab^{10}c^4d^2e - 258ab \\
& ^{11}c^3d^2f + 43520a^6b^2c^8d^2f + 724ab^{12}c^2e^2f + 30b^5c^2e^2f(-4ac - b^2)^9)^{1/2} - 246a^2b^2c^2f^2(-4ac - b^2)^9)^{1/2} + 165ab \\
& b^4c^2f^2(-4ac - b^2)^9)^{1/2} - 1548a^2b^8c^5d^2e + 8064a^3b^6c^6d^2e - 22400a^4b^4c^7d^2e + 30720a^5b^2c^8d^2e + 2706a^2b^9c^4d^2 \\
& f - 14784a^3b^7c^5d^2f + 44352a^4b^5c^6d^2f - 69120a^5b^3c^7d^2f - \\
& 42a^2c^4d^2f(-4ac - b^2)^9)^{1/2} + 6b^3c^3d^2e(-4ac - b^2)^9)^{1/2} - 7278a^2b^{10}c^3e^2f + 39132a^3b^8c^4e^2f - 119616a^4b^6c^5 \\
& e^2f + 201600a^5b^4c^6e^2f - 161280a^6b^2c^7e^2f - 10b^4c^2d^2f(-4ac - b^2)^9)^{1/2} + 51ab^2c^3e^2(-4ac - b^2)^9)^{1/2} - 44ab^2c^4d^2e(-4ac - b^2)^9)^{1/2} + 78ab^2c^3d^2f(-4ac - b^2)^9)^{1/2} \\
&) - 184ab^3c^2e^2f(-4ac - b^2)^9)^{1/2} + 186a^2b^2c^3e^2f(-4ac - b^2)^9)^{1/2})/(32(4096a^6c^{13} + b^{12}c^7 - 24ab^{10}c^8 + 240a^2b^8c^9 - 1280a^3b^6c^{10} + 3840a^4b^4c^{11} - 6144a^5b^2c^{12}))^{1/2} \\
& *2i + (f*x^3)/(3*c^2)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**6*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)

[Out] Timed out

$$3.67 \quad \int \frac{x^4(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2} dx$$

Optimal. Leaf size=436

$$\frac{x(a(-2acf + b^2f - bce + 2c^2d) - x^2(-bc(cd - 3af) - 2ac^2e + b^3(-f) + b^2ce))}{2c^2(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)\left(-\frac{b^2c(19a}{\right)}{2\sqrt{2}c^{5/2}(b^2 - 4ac)\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{2\sqrt{2}c^{5/2}(b^2 - 4ac)\sqrt{b - \sqrt{b^2 - 4ac}}}$$

Rubi [A] time = 5.54, antiderivative size = 436, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 30, number of rules / integrand size = 0.133, Rules used = {1668, 1676, 1166, 205}

$$\frac{x(a(-2acf + b^2f - bce + 2c^2d) - x^2(-bc(cd - 3af) - 2ac^2e + b^3(-f) + b^2ce))}{2c^2(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)\left(\frac{b^2(19af + cd) - 6ac^2e + b^3(-f) + b^2ce}{\sqrt{b^2 - 4ac}}\right)}{2\sqrt{2}c^{5/2}(b^2 - 4ac)\sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b + \sqrt{b^2 - 4ac}}}\right)\left(\frac{b^2(19af + cd) - 6ac^2e + b^3(-f) + b^2ce}{\sqrt{b^2 - 4ac}}\right)}{2\sqrt{2}c^{5/2}(b^2 - 4ac)\sqrt{b + \sqrt{b^2 - 4ac}}} + \frac{fx}{c^2}$$

Antiderivative was successfully verified.

[In] Int[(x^4*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]

[Out] (f*x)/c^2 + (x*(a*(2*c^2*d - b*c*e + b^2*f - 2*a*c*f) - (b^2*c*e - 2*a*c^2*e - b^3*f - b*c*(c*d - 3*a*f))*x^2))/(2*c^2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b^2*c*e - 6*a*c^2*e - 3*b^3*f + b*c*(c*d + 13*a*f) - (b^3*c*e - 8*a*b*c^2*e - 3*b^4*f + 4*a*c^2*(c*d - 5*a*f) + b^2*c*(c*d + 19*a*f)))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/(2*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^2*c*e - 6*a*c^2*e - 3*b^3*f + b*c*(c*d + 13*a*f) + (b^3*c*e - 8*a*b*c^2*e - 3*b^4*f + 4*a*c^2*(c*d - 5*a*f) + b^2*c*(c*d + 19*a*f)))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]/(2*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1668

Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]

Rule 1676

Int[(Pq_)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] := Int[ExpandIntegrand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && Expon[Pq, x^2] > 1

Rubi steps

$$\int \frac{x^4(d + ex^2 + fx^4)}{(a + bx^2 + cx^4)^2} dx = \frac{x(a(2c^2d - bce + b^2f - 2acf) - (b^2ce - 2ac^2e - b^3f - bc(cd - 3af))x^2)}{2c^2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{a^2(2c^2d + b^2f)}{2c^2(b^2 - 4ac)(a + bx^2 + cx^4)}$$

$$= \frac{x(a(2c^2d - bce + b^2f - 2acf) - (b^2ce - 2ac^2e - b^3f - bc(cd - 3af))x^2)}{2c^2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{2a(b^2 - 4ac)}{2c^2(b^2 - 4ac)(a + bx^2 + cx^4)}$$

$$= \frac{fx}{c^2} + \frac{x(a(2c^2d - bce + b^2f - 2acf) - (b^2ce - 2ac^2e - b^3f - bc(cd - 3af))x^2)}{2c^2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{2a(b^2 - 4ac)}{2c^2(b^2 - 4ac)(a + bx^2 + cx^4)}$$

$$= \frac{fx}{c^2} + \frac{x(a(2c^2d - bce + b^2f - 2acf) - (b^2ce - 2ac^2e - b^3f - bc(cd - 3af))x^2)}{2c^2(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{b^2 - 4ac}{2c^2(b^2 - 4ac)(a + bx^2 + cx^4)}$$

$$= \frac{fx}{c^2} + \frac{x(a(2c^2d - bce + b^2f - 2acf) - (b^2ce - 2ac^2e - b^3f - bc(cd - 3af))x^2)}{2c^2(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{b^2 - 4ac}{2c^2(b^2 - 4ac)(a + bx^2 + cx^4)}$$

Mathematica [A] time = 1.54, size = 511, normalized size = 1.17

$$\frac{2\sqrt{2}(-2b^2f + d(b^2 + 4c^2) + 2b^2d + 4c^2d) + \sqrt{2}\arctan\left(\frac{\sqrt{2}bx}{\sqrt{b^2 - 4ac}}\right)2c^2(3x\sqrt{b^2 - 4ac} - 10bf + 2d) + b^2(-\sqrt{b^2 - 4ac} + 19bf + d) - b(d\sqrt{b^2 - 4ac} + 13f\sqrt{b^2 - 4ac} + 8ac) + b^2(3f\sqrt{b^2 - 4ac} + c) - 3b^2f}{(b^2 - 4ac)^2\sqrt{b^2 - 4ac}} - \frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}bx}{\sqrt{b^2 - 4ac}}\right)2c^2(3x\sqrt{b^2 - 4ac} + 10bf - 2d) + b^2(\sqrt{b^2 - 4ac} + 19bf + d) - b(d\sqrt{b^2 - 4ac} + 13f\sqrt{b^2 - 4ac} + 8ac) + b^2(3f\sqrt{b^2 - 4ac} + c) - 3b^2f}{(b^2 - 4ac)^2\sqrt{b^2 - 4ac}} + 4\sqrt{c}fx$$

Antiderivative was successfully verified.

[In] Integrate[(x^4*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]

[Out] (4*sqrt[c]*f*x + (2*sqrt[c]*x*(-2*a^2*c*f + b*(c^2*d - b*c*e + b^2*f))*x^2 + a*(b^2*f + 2*c^2*(d + e*x^2) - b*c*(e + 3*f*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (sqrt[2]*(-3*b^4*f + 2*a*c^2*(2*c*d + 3*sqrt[b^2 - 4*a*c]*e - 10*a*f) + b^2*c*(c*d - sqrt[b^2 - 4*a*c]*e + 19*a*f) + b^3*(c*e + 3*sqrt[b^2 - 4*a*c]*f) - b*c*(c*sqrt[b^2 - 4*a*c]*d + 8*a*c*e + 13*a*sqrt[b^2 - 4*a*c]*f))*ArcTan[(sqrt[2]*sqrt[c]*x)/sqrt[b - sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(3/2)*sqrt[b - sqrt[b^2 - 4*a*c]]) - (sqrt[2]*(3*b^4*f + 2*a*c^2*(-2*c*d + 3*sqrt[b^2 - 4*a*c]*e + 10*a*f) - b^2*c*(c*d + sqrt[b^2 - 4*a*c]*e + 19*a*f) + b^3*(-(c*e) + 3*sqrt[b^2 - 4*a*c]*f) - b*c*(c*sqrt[b^2 - 4*a*c]*d - 8*a*c*e + 13*a*sqrt[b^2 - 4*a*c]*f))*ArcTan[(sqrt[2]*sqrt[c]*x)/sqrt[b + sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(3/2)*sqrt[b + sqrt[b^2 - 4*a*c]])/(4*c^(5/2))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4(d + ex^2 + fx^4)}{(a + bx^2 + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^4*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]

[Out] IntegrateAlgebraic[(x^4*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2, x]

fricas [B] time = 31.02, size = 12597, normalized size = 28.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")

[Out]
$$\frac{1}{4} \cdot (4 \cdot (b^2 \cdot c - 4 \cdot a \cdot c^2) \cdot f \cdot x^5 + 2 \cdot (b \cdot c^2 \cdot d - (b^2 \cdot c - 2 \cdot a \cdot c^2) \cdot e + (3 \cdot b^3 - 11 \cdot a \cdot b \cdot c) \cdot f) \cdot x^3 + \sqrt{1/2} \cdot (a \cdot b^2 \cdot c^2 - 4 \cdot a^2 \cdot c^3 + (b^2 \cdot c^3 - 4 \cdot a \cdot c^4) \cdot x^4 + (b^3 \cdot c^2 - 4 \cdot a \cdot b \cdot c^3) \cdot x^2) \cdot \sqrt{-(b^3 \cdot c^4 + 12 \cdot a \cdot b \cdot c^5) \cdot d^2 + 2 \cdot (b^4 \cdot c^3 - 6 \cdot a \cdot b^2 \cdot c^4 - 24 \cdot a^2 \cdot c^5) \cdot d \cdot e + (b^5 \cdot c^2 - 15 \cdot a \cdot b^3 \cdot c^3 + 60 \cdot a^2 \cdot b \cdot c^4) \cdot e^2 + (9 \cdot b^7 - 105 \cdot a \cdot b^5 \cdot c + 385 \cdot a^2 \cdot b^3 \cdot c^2 - 420 \cdot a^3 \cdot b \cdot c^3) \cdot f^2 - 2 \cdot ((3 \cdot b^5 \cdot c^2 - 13 \cdot a \cdot b^3 \cdot c^3 - 12 \cdot a^2 \cdot b \cdot c^4) \cdot d + (3 \cdot b^6 \cdot c - 40 \cdot a \cdot b^4 \cdot c^2 + 15 \cdot 0 \cdot a^2 \cdot b^2 \cdot c^3 - 120 \cdot a^3 \cdot c^4) \cdot e) \cdot f + (b^6 \cdot c^5 - 12 \cdot a \cdot b^4 \cdot c^6 + 48 \cdot a^2 \cdot b^2 \cdot c^7 - 64 \cdot a^3 \cdot c^8) \cdot \sqrt{(c^8 \cdot d^4 + 4 \cdot b \cdot c^7 \cdot d^3 \cdot e + 6 \cdot (b^2 \cdot c^6 - 3 \cdot a \cdot c^7) \cdot d^2 \cdot e^2 + 4 \cdot (b^3 \cdot c^5 - 9 \cdot a \cdot b \cdot c^6) \cdot d \cdot e^3 + (b^4 \cdot c^4 - 18 \cdot a \cdot b^2 \cdot c^5 + 81 \cdot a^2 \cdot c^6) \cdot e^4 + (81 \cdot b^8 - 918 \cdot a \cdot b^6 \cdot c + 3051 \cdot a^2 \cdot b^4 \cdot c^2 - 2550 \cdot a^3 \cdot b^2 \cdot c^3 + 625 \cdot a^4 \cdot c^4) \cdot f^4 - 4 \cdot ((27 \cdot b^6 \cdot c^2 - 108 \cdot a \cdot b^4 \cdot c^3 - 180 \cdot a^2 \cdot b^2 \cdot c^4 + 125 \cdot a^3 \cdot c^5) \cdot d + (27 \cdot b^7 \cdot c - 351 \cdot a \cdot b^5 \cdot c^2 + 1197 \cdot a^2 \cdot b^3 \cdot c^3 - 550 \cdot a^3 \cdot b \cdot c^4) \cdot e) \cdot f^3 + 6 \cdot ((9 \cdot b^4 \cdot c^4 + 3 \cdot a \cdot b^2 \cdot c^5 + 25 \cdot a^2 \cdot c^6) \cdot d^2 + 2 \cdot (9 \cdot b^5 \cdot c^3 - 51 \cdot a \cdot b^3 \cdot c^4 - 65 \cdot a^2 \cdot b \cdot c^5) \cdot d \cdot e + (9 \cdot b^6 \cdot c^2 - 132 \cdot a \cdot b^4 \cdot c^3 + 484 \cdot a^2 \cdot b^2 \cdot c^4 - 75 \cdot a^3 \cdot c^5) \cdot e^2) \cdot f^2 - 4 \cdot ((3 \cdot b^2 \cdot c^6 + 5 \cdot a \cdot c^7) \cdot d^3 + 3 \cdot (3 \cdot b^3 \cdot c^5 - 4 \cdot a \cdot b \cdot c^6) \cdot d^2 \cdot e + 3 \cdot (3 \cdot b^4 \cdot c^4 - 22 \cdot a \cdot b^2 \cdot c^5 - 15 \cdot a^2 \cdot c^6) \cdot d \cdot e^2 + (3 \cdot b^5 \cdot c^3 - 49 \cdot a \cdot b^3 \cdot c^4 + 198 \cdot a^2 \cdot b \cdot c^5) \cdot e^3) \cdot f) / (b^6 \cdot c^{10} - 12 \cdot a \cdot b^4 \cdot c^{11} + 48 \cdot a^2 \cdot b^2 \cdot c^{12} - 64 \cdot a^3 \cdot c^{13})) / (b^6 \cdot c^5 - 12 \cdot a \cdot b^4 \cdot c^6 + 48 \cdot a^2 \cdot b^2 \cdot c^7 - 64 \cdot a^3 \cdot c^8) \cdot \log(((3 \cdot b^2 \cdot c^6 + 4 \cdot a \cdot c^7) \cdot d^4 + (9 \cdot b^3 \cdot c^5 - 20 \cdot a \cdot b \cdot c^6) \cdot d^3 \cdot e + 3 \cdot (3 \cdot b^4 \cdot c^4 - 28 \cdot a \cdot b^2 \cdot c^5) \cdot d^2 \cdot e^2 + (3 \cdot b^5 \cdot c^3 - 65 \cdot a \cdot b^3 \cdot c^4 + 324 \cdot a^2 \cdot b \cdot c^5) \cdot d \cdot e^3 - (5 \cdot a \cdot b^4 \cdot c^3 - 81 \cdot a^2 \cdot b^2 \cdot c^4 + 324 \cdot a^3 \cdot c^5) \cdot e^4 - (189 \cdot a^2 \cdot b^6 - 1971 \cdot a^3 \cdot b^4 \cdot c + 5625 \cdot a^4 \cdot b^2 \cdot c^2 - 2500 \cdot a^5 \cdot c^3) \cdot f^4 - ((81 \cdot b^8 - 945 \cdot a \cdot b^6 \cdot c + 3213 \cdot a^2 \cdot b^4 \cdot c^2 - 3000 \cdot a^3 \cdot b^2 \cdot c^3 + 2000 \cdot a^4 \cdot c^4) \cdot d - (135 \cdot a \cdot b^7 - 1323 \cdot a^2 \cdot b^5 \cdot c + 2727 \cdot a^3 \cdot b^3 \cdot c^2 + 2500 \cdot a^4 \cdot b \cdot c^3) \cdot e) \cdot f^3 + 3 \cdot ((27 \cdot b^6 \cdot c^2 - 117 \cdot a \cdot b^4 \cdot c^3 - 150 \cdot a^2 \cdot b^2 \cdot c^4 + 200 \cdot a^3 \cdot c^5) \cdot d^2 + (27 \cdot b^7 \cdot c - 405 \cdot a \cdot b^5 \cdot c^2 + 1461 \cdot a^2 \cdot b^3 \cdot c^3 - 500 \cdot a^3 \cdot b \cdot c^4) \cdot d \cdot e - (45 \cdot a \cdot b^6 \cdot c - 558 \cdot a^2 \cdot b^4 \cdot c^2 + 1672 \cdot a^3 \cdot b^2 \cdot c^3) \cdot e^2) \cdot f^2 - ((27 \cdot b^4 \cdot c^4 + 80 \cdot a^2 \cdot c^6) \cdot d^3 + 3 \cdot (18 \cdot b^5 \cdot c^3 - 123 \cdot a \cdot b^3 \cdot c^4 - 100 \cdot a^2 \cdot b \cdot c^5) \cdot d^2 \cdot e + 3 \cdot (9 \cdot b^6 \cdot c^2 - 165 \cdot a \cdot b^4 \cdot c^3 + 692 \cdot a^2 \cdot b^2 \cdot c^4) \cdot d \cdot e^2 - (45 \cdot a \cdot b^5 \cdot c^2 - 647 \cdot a^2 \cdot b^3 \cdot c^3 + 2268 \cdot a^3 \cdot b \cdot c^4) \cdot e^3) \cdot f) \cdot x + 1/2 \cdot \sqrt{1/2} \cdot (2 \cdot (b^4 \cdot c^6 - 8 \cdot a \cdot b^2 \cdot c^7 + 16 \cdot a^2 \cdot c^8) \cdot d^3 + 3 \cdot (b^5 \cdot c^5 - 8 \cdot a \cdot b^3 \cdot c^6 + 16 \cdot a^2 \cdot b \cdot c^7) \cdot d^2 \cdot e - 18 \cdot (a \cdot b^4 \cdot c^5 - 8 \cdot a^2 \cdot b^2 \cdot c^6 + 16 \cdot a^3 \cdot c^7) \cdot d \cdot e^2 - (b^7 \cdot c^3 - 17 \cdot a \cdot b^5 \cdot c^4 + 88 \cdot a^2 \cdot b^3 \cdot c^5 - 144 \cdot a^3 \cdot b \cdot c^6) \cdot e^3 + (27 \cdot b^{10} - 459 \cdot a \cdot b^8 \cdot c + 2961 \cdot a^2 \cdot b^6 \cdot c^2 - 8818 \cdot a^3 \cdot b^4 \cdot c^3 + 11360 \cdot a^4 \cdot b^2 \cdot c^4 - 4000 \cdot a^5 \cdot c^5) \cdot f^3 - 3 \cdot (2 \cdot (12 \cdot a \cdot b^6 \cdot c^3 - 121 \cdot a^2 \cdot b^4 \cdot c^4 + 392 \cdot a^3 \cdot b^2 \cdot c^5 - 400 \cdot a^4 \cdot c^6) \cdot d + (9 \cdot b^9 \cdot c - 153 \cdot a \cdot b^7 \cdot c^2 + 947 \cdot a^2 \cdot b^5 \cdot c^3 - 2536 \cdot a^3 \cdot b^3 \cdot c^4 + 2480 \cdot a^4 \cdot b \cdot c^5) \cdot e) \cdot f^2 - 3 \cdot ((3 \cdot b^6 \cdot c^4 - 14 \cdot a \cdot b^4 \cdot c^5 - 32 \cdot a^2 \cdot b^2 \cdot c^6 + 160 \cdot a^3 \cdot c^7) \cdot d^2 - 26 \cdot (a \cdot b^5 \cdot c^4 - 8 \cdot a^2 \cdot b^3 \cdot c^5 + 16 \cdot a^3 \cdot b \cdot c^6) \cdot d \cdot e - 3 \cdot (b^8 \cdot c^2 - 17 \cdot a \cdot b^6 \cdot c^3 + 98 \cdot a^2 \cdot b^4 \cdot c^4 - 224 \cdot a^3 \cdot b^2 \cdot c^5 + 160 \cdot a^4 \cdot c^6) \cdot e^2) \cdot f + (4 \cdot (b^7 \cdot c^7 - 12 \cdot a \cdot b^5 \cdot c^8 + 48 \cdot a^2 \cdot b^3 \cdot c^9 - 64 \cdot a^3 \cdot b \cdot c^{10}) \cdot d + (b^8 \cdot c^6 - 24 \cdot a \cdot b^6 \cdot c^7 + 192 \cdot a^2 \cdot b^4 \cdot c^8 - 640 \cdot a^3 \cdot b^2 \cdot c^9 + 768 \cdot a^4 \cdot c^{10}) \cdot e - (3 \cdot b^9 \cdot c^5 - 52 \cdot a \cdot b^7 \cdot c^6 + 336 \cdot a^2 \cdot b^5 \cdot c^7 - 960 \cdot a^3 \cdot b^3 \cdot c^8 + 1024 \cdot a^4 \cdot b \cdot c^9) \cdot f) \cdot \sqrt{(c^8 \cdot d^4 + 4 \cdot b \cdot c^7 \cdot d^3 \cdot e + 6 \cdot (b^2 \cdot c^6 - 3 \cdot a \cdot c^7) \cdot d^2 \cdot e^2 + 4 \cdot (b^3 \cdot c^5 - 9 \cdot a \cdot b \cdot c^6) \cdot d \cdot e^3 + (b^4 \cdot c^4 - 18 \cdot a \cdot b^2 \cdot c^5 + 81 \cdot a^2 \cdot c^6) \cdot e^4 + (81 \cdot b^8 - 918 \cdot a \cdot b^6 \cdot c + 3051 \cdot a^2 \cdot b^4 \cdot c^2 - 2550 \cdot a^3 \cdot b^2 \cdot c^3 + 625 \cdot a^4 \cdot c^4) \cdot f^4 - 4 \cdot ((27 \cdot b^6 \cdot c^2 - 108 \cdot a \cdot b^4 \cdot c^3 - 180 \cdot a^2 \cdot b^2 \cdot c^4 + 125 \cdot a^3 \cdot c^5) \cdot d + (27 \cdot b^7 \cdot c - 351 \cdot a \cdot b^5 \cdot c^2 + 1197 \cdot a^2 \cdot b^3 \cdot c^3 - 550 \cdot a^3 \cdot b \cdot c^4) \cdot e) \cdot f^3 + 6 \cdot ((9 \cdot b^4 \cdot c^4 + 3 \cdot a \cdot b^2 \cdot c^5 + 25 \cdot a^2 \cdot c^6) \cdot d^2 + 2 \cdot (9 \cdot b^5 \cdot c^3 - 51 \cdot a \cdot b^3 \cdot c^4 - 65 \cdot a^2 \cdot b \cdot c^5) \cdot d \cdot e + (9 \cdot b^6 \cdot c^2 - 132 \cdot a \cdot b^4 \cdot c^3 + 484 \cdot a^2 \cdot b^2 \cdot c^4 - 75 \cdot a^3 \cdot c^5) \cdot e^2) \cdot f^2 - 4 \cdot ((3 \cdot b^2 \cdot c^6 + 5 \cdot a \cdot c^7) \cdot d^3 + 3 \cdot (3 \cdot b^3 \cdot c^5 - 4 \cdot a \cdot b \cdot c^6) \cdot d^2 \cdot e + 3 \cdot (3 \cdot b^4 \cdot c^4 - 22 \cdot a \cdot b^2 \cdot c^5 - 15 \cdot a^2 \cdot c^6) \cdot d \cdot e^2 + (3 \cdot b^5 \cdot c^3 - 49 \cdot a \cdot b^3 \cdot c^4 + 198 \cdot a^2 \cdot b \cdot c^5) \cdot e^3) \cdot f) / (b^6 \cdot c^{10} - 12 \cdot a \cdot b^4 \cdot c^{11} + 48 \cdot a^2 \cdot b^2 \cdot c^{12} - 64 \cdot a^3 \cdot c^{13})$$

$$\begin{aligned}
&^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13}))*\text{sqrt}(-((b^3*c^4 + 12*a*b*c^5)*d^2 + 2*(b^4*c^3 - 6*a*b^2*c^4 - 24*a^2*c^5)*d*e + (b^5*c^2 - 15*a*b^3*c^3 + 60*a^2*b*c^4)*e^2 + (9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3)*f^2 - 2*((3*b^5*c^2 - 13*a*b^3*c^3 - 12*a^2*b*c^4)*d + (3*b^6*c - 40*a*b^4*c^2 + 150*a^2*b^2*c^3 - 120*a^3*c^4)*e)*f + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*\text{sqrt}((c^8*d^4 + 4*b*c^7*d^3*e + 6*(b^2*c^6 - 3*a*c^7)*d^2*e^2 + 4*(b^3*c^5 - 9*a*b*c^6)*d*e^3 + (b^4*c^4 - 18*a*b^2*c^5 + 81*a^2*c^6)*e^4 + (81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)*f^4 - 4*((27*b^6*c^2 - 108*a*b^4*c^3 - 180*a^2*b^2*c^4 + 125*a^3*c^5)*d + (27*b^7*c - 351*a*b^5*c^2 + 1197*a^2*b^3*c^3 - 550*a^3*b*c^4)*e)*f^3 + 6*((9*b^4*c^4 + 3*a*b^2*c^5 + 25*a^2*c^6)*d^2 + 2*(9*b^5*c^3 - 51*a*b^3*c^4 - 65*a^2*b*c^5)*d*e + (9*b^6*c^2 - 132*a*b^4*c^3 + 484*a^2*b^2*c^4 - 75*a^3*c^5)*e^2)*f^2 - 4*((3*b^2*c^6 + 5*a*c^7)*d^3 + 3*(3*b^3*c^5 - 4*a*b*c^6)*d^2*e + 3*(3*b^4*c^4 - 22*a*b^2*c^5 - 15*a^2*c^6)*d*e^2 + (3*b^5*c^3 - 49*a*b^3*c^4 + 198*a^2*b*c^5)*e^3)*f)/(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13}))/((b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))) - \text{sqrt}(1/2)*(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2)*\text{sqrt}(-((b^3*c^4 + 12*a*b*c^5)*d^2 + 2*(b^4*c^3 - 6*a*b^2*c^4 - 24*a^2*c^5)*d*e + (b^5*c^2 - 15*a*b^3*c^3 + 60*a^2*b*c^4)*e^2 + (9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3)*f^2 - 2*((3*b^5*c^2 - 13*a*b^3*c^3 - 12*a^2*b*c^4)*d + (3*b^6*c - 40*a*b^4*c^2 + 150*a^2*b^2*c^3 - 120*a^3*c^4)*e)*f + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*\text{sqrt}((c^8*d^4 + 4*b*c^7*d^3*e + 6*(b^2*c^6 - 3*a*c^7)*d^2*e^2 + 4*(b^3*c^5 - 9*a*b*c^6)*d*e^3 + (b^4*c^4 - 18*a*b^2*c^5 + 81*a^2*c^6)*e^4 + (81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)*f^4 - 4*((27*b^6*c^2 - 108*a*b^4*c^3 - 180*a^2*b^2*c^4 + 125*a^3*c^5)*d + (27*b^7*c - 351*a*b^5*c^2 + 1197*a^2*b^3*c^3 - 550*a^3*b*c^4)*e)*f^3 + 6*((9*b^4*c^4 + 3*a*b^2*c^5 + 25*a^2*c^6)*d^2 + 2*(9*b^5*c^3 - 51*a*b^3*c^4 - 65*a^2*b*c^5)*d*e + (9*b^6*c^2 - 132*a*b^4*c^3 + 484*a^2*b^2*c^4 - 75*a^3*c^5)*e^2)*f^2 - 4*((3*b^2*c^6 + 5*a*c^7)*d^3 + 3*(3*b^3*c^5 - 4*a*b*c^6)*d^2*e + 3*(3*b^4*c^4 - 22*a*b^2*c^5 - 15*a^2*c^6)*d*e^2 + (3*b^5*c^3 - 49*a*b^3*c^4 + 198*a^2*b*c^5)*e^3)*f)/(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13}))/((b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))*\log(((3*b^2*c^6 + 4*a*c^7)*d^4 + (9*b^3*c^5 - 20*a*b*c^6)*d^3*e + 3*(3*b^4*c^4 - 28*a*b^2*c^5)*d^2*e^2 + (3*b^5*c^3 - 65*a*b^3*c^4 + 324*a^2*b*c^5)*d*e^3 - (5*a*b^4*c^3 - 81*a^2*b^2*c^4 + 324*a^3*c^5)*e^4 - (189*a^2*b^6 - 1971*a^3*b^4*c + 5625*a^4*b^2*c^2 - 2500*a^5*c^3)*f^4 - ((81*b^8 - 945*a*b^6*c + 3213*a^2*b^4*c^2 - 3000*a^3*b^2*c^3 + 2000*a^4*c^4)*d - (135*a*b^7 - 1323*a^2*b^5*c + 2727*a^3*b^3*c^2 + 2500*a^4*b*c^3)*e)*f^3 + 3*((27*b^6*c^2 - 117*a*b^4*c^3 - 150*a^2*b^2*c^4 + 200*a^3*c^5)*d^2 + (27*b^7*c - 405*a*b^5*c^2 + 1461*a^2*b^3*c^3 - 500*a^3*b*c^4)*d*e - (45*a*b^6*c - 558*a^2*b^4*c^2 + 1672*a^3*b^2*c^3)*e^2)*f^2 - ((27*b^4*c^4 + 80*a^2*c^6)*d^3 + 3*(18*b^5*c^3 - 123*a*b^3*c^4 - 100*a^2*b*c^5)*d^2*e + 3*(9*b^6*c^2 - 165*a*b^4*c^3 + 692*a^2*b^2*c^4)*d*e^2 - (45*a*b^5*c^2 - 647*a^2*b^3*c^3 + 2268*a^3*b*c^4)*e^3)*f)*x - 1/2*\text{sqrt}(1/2)*(2*(b^4*c^6 - 8*a*b^2*c^7 + 16*a^2*c^8)*d^3 + 3*(b^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*d^2*e - 18*(a*b^4*c^5 - 8*a^2*b^2*c^6 + 16*a^3*c^7)*d*e^2 - (b^7*c^3 - 17*a*b^5*c^4 + 88*a^2*b^3*c^5 - 144*a^3*b*c^6)*e^3 + (27*b^{10} - 459*a*b^8*c + 2961*a^2*b^6*c^2 - 8818*a^3*b^4*c^3 + 11360*a^4*b^2*c^4 - 4000*a^5*c^5)*f^3 - 3*(2*(12*a*b^6*c^3 - 121*a^2*b^4*c^4 + 392*a^3*b^2*c^5 - 400*a^4*c^6)*d + (9*b^9*c - 153*a*b^7*c^2 + 947*a^2*b^5*c^3 - 2536*a^3*b^3*c^4 + 2480*a^4*b*c^5)*e)*f^2 - 3*((3*b^6*c^4 - 14*a*b^4*c^5 - 32*a^2*b^2*c^6 + 160*a^3*c^7)*d^2 - 26*(a*b^5*c^4 - 8*a^2*b^3*c^5 + 16*a^3*b*c^6)*d*e - 3*(b^8*c^2 - 17*a*b^6*c^3 + 98*a^2*b^4*c^4 - 224*a^3*b^2*c^5 + 160*a^4*c^6)*e^2)*f + (4*(b^7*c^7 - 12*a*b^5*c^8 + 48*a^2*b^3*c^9 - 64*a^3*b*c^10)*d + (b^8*c^6 - 24*a*b^6*c^7 + 192*a^2*b^4*c^8 - 640*a^3*b^2*c^9 + 768*a^4*c^10)*e - (3*b^9*c^5 - 52*a*b^7*c^6 + 336*a^2*b^5*c^7 - 960*a^3*b^3*c^8 + 1024*a^4*b*c^9)*f)*\text{sqrt}((c^8*d^4 + 4*b*c^7*d^3*e + 6*(b^2*c^6 - 3*a*c^7)*d^2*e^2 + 4*(b^3*c^5 - 9*a*b*c^6)*d*e^3 + (b^4*c^4 - 18*a*b^2*c^5 + 81*a^2*c^6)*e^4 + (81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^
\end{aligned}$$

$$\begin{aligned}
& 2*c^3 + 625*a^4*c^4)*f^4 - 4*((27*b^6*c^2 - 108*a*b^4*c^3 - 180*a^2*b^2*c^4 \\
& + 125*a^3*c^5)*d + (27*b^7*c - 351*a*b^5*c^2 + 1197*a^2*b^3*c^3 - 550*a^3* \\
& b*c^4)*e)*f^3 + 6*((9*b^4*c^4 + 3*a*b^2*c^5 + 25*a^2*c^6)*d^2 + 2*(9*b^5*c^ \\
& 3 - 51*a*b^3*c^4 - 65*a^2*b*c^5)*d*e + (9*b^6*c^2 - 132*a*b^4*c^3 + 484*a^2 \\
& *b^2*c^4 - 75*a^3*c^5)*e^2)*f^2 - 4*((3*b^2*c^6 + 5*a*c^7)*d^3 + 3*(3*b^3*c \\
& ^5 - 4*a*b*c^6)*d^2*e + 3*(3*b^4*c^4 - 22*a*b^2*c^5 - 15*a^2*c^6)*d*e^2 + (\\
& 3*b^5*c^3 - 49*a*b^3*c^4 + 198*a^2*b*c^5)*e^3)*f)/(b^6*c^10 - 12*a*b^4*c^11 \\
& + 48*a^2*b^2*c^12 - 64*a^3*c^13))*sqrt(-((b^3*c^4 + 12*a*b*c^5)*d^2 + 2*(\\
& b^4*c^3 - 6*a*b^2*c^4 - 24*a^2*c^5)*d*e + (b^5*c^2 - 15*a*b^3*c^3 + 60*a^2* \\
& b*c^4)*e^2 + (9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3)*f^2 - \\
& 2*((3*b^5*c^2 - 13*a*b^3*c^3 - 12*a^2*b*c^4)*d + (3*b^6*c - 40*a*b^4*c^2 + \\
& 150*a^2*b^2*c^3 - 120*a^3*c^4)*e)*f + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2* \\
& c^7 - 64*a^3*c^8)*sqrt((c^8*d^4 + 4*b*c^7*d^3*e + 6*(b^2*c^6 - 3*a*c^7)*d^2 \\
& *e^2 + 4*(b^3*c^5 - 9*a*b*c^6)*d*e^3 + (b^4*c^4 - 18*a*b^2*c^5 + 81*a^2*c^6 \\
&)*e^4 + (81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a \\
& ^4*c^4)*f^4 - 4*((27*b^6*c^2 - 108*a*b^4*c^3 - 180*a^2*b^2*c^4 + 125*a^3*c^ \\
& 5)*d + (27*b^7*c - 351*a*b^5*c^2 + 1197*a^2*b^3*c^3 - 550*a^3*b*c^4)*e)*f^3 \\
& + 6*((9*b^4*c^4 + 3*a*b^2*c^5 + 25*a^2*c^6)*d^2 + 2*(9*b^5*c^3 - 51*a*b^3* \\
& c^4 - 65*a^2*b*c^5)*d*e + (9*b^6*c^2 - 132*a*b^4*c^3 + 484*a^2*b^2*c^4 - 75 \\
& *a^3*c^5)*e^2)*f^2 - 4*((3*b^2*c^6 + 5*a*c^7)*d^3 + 3*(3*b^3*c^5 - 4*a*b*c^ \\
& 6)*d^2*e + 3*(3*b^4*c^4 - 22*a*b^2*c^5 - 15*a^2*c^6)*d*e^2 + (3*b^5*c^3 - 4 \\
& 9*a*b^3*c^4 + 198*a^2*b*c^5)*e^3)*f)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2 \\
& *c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^ \\
& 8))) + sqrt(1/2)*(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^ \\
& 2 - 4*a*b*c^3)*x^2)*sqrt(-((b^3*c^4 + 12*a*b*c^5)*d^2 + 2*(b^4*c^3 - 6*a*b^ \\
& 2*c^4 - 24*a^2*c^5)*d*e + (b^5*c^2 - 15*a*b^3*c^3 + 60*a^2*b*c^4)*e^2 + (9* \\
& b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3)*f^2 - 2*((3*b^5*c^2 - \\
& 13*a*b^3*c^3 - 12*a^2*b*c^4)*d + (3*b^6*c - 40*a*b^4*c^2 + 150*a^2*b^2*c^3 \\
& - 120*a^3*c^4)*e)*f - (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8 \\
&)*sqrt((c^8*d^4 + 4*b*c^7*d^3*e + 6*(b^2*c^6 - 3*a*c^7)*d^2*e^2 + 4*(b^3*c^ \\
& 5 - 9*a*b*c^6)*d*e^3 + (b^4*c^4 - 18*a*b^2*c^5 + 81*a^2*c^6)*e^4 + (81*b^8 \\
& - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)*f^4 - 4* \\
& ((27*b^6*c^2 - 108*a*b^4*c^3 - 180*a^2*b^2*c^4 + 125*a^3*c^5)*d + (27*b^7*c \\
& - 351*a*b^5*c^2 + 1197*a^2*b^3*c^3 - 550*a^3*b*c^4)*e)*f^3 + 6*((9*b^4*c^4 \\
& + 3*a*b^2*c^5 + 25*a^2*c^6)*d^2 + 2*(9*b^5*c^3 - 51*a*b^3*c^4 - 65*a^2*b*c \\
& ^5)*d*e + (9*b^6*c^2 - 132*a*b^4*c^3 + 484*a^2*b^2*c^4 - 75*a^3*c^5)*e^2)*f \\
& ^2 - 4*((3*b^2*c^6 + 5*a*c^7)*d^3 + 3*(3*b^3*c^5 - 4*a*b*c^6)*d^2*e + 3*(3* \\
& b^4*c^4 - 22*a*b^2*c^5 - 15*a^2*c^6)*d*e^2 + (3*b^5*c^3 - 49*a*b^3*c^4 + 19 \\
& 8*a^2*b*c^5)*e^3)*f)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c \\
& ^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))*log(((3*b^2* \\
& c^6 + 4*a*c^7)*d^4 + (9*b^3*c^5 - 20*a*b*c^6)*d^3*e + 3*(3*b^4*c^4 - 28*a*b \\
& ^2*c^5)*d^2*e^2 + (3*b^5*c^3 - 65*a*b^3*c^4 + 324*a^2*b*c^5)*d*e^3 - (5*a*b \\
& ^4*c^3 - 81*a^2*b^2*c^4 + 324*a^3*c^5)*e^4 - (189*a^2*b^6 - 1971*a^3*b^4*c \\
& + 5625*a^4*b^2*c^2 - 2500*a^5*c^3)*f^4 - ((81*b^8 - 945*a*b^6*c + 3213*a^2* \\
& b^4*c^2 - 3000*a^3*b^2*c^3 + 2000*a^4*c^4)*d - (135*a*b^7 - 1323*a^2*b^5*c \\
& + 2727*a^3*b^3*c^2 + 2500*a^4*b*c^3)*e)*f^3 + 3*((27*b^6*c^2 - 117*a*b^4*c^ \\
& 3 - 150*a^2*b^2*c^4 + 200*a^3*c^5)*d^2 + (27*b^7*c - 405*a*b^5*c^2 + 1461*a \\
& ^2*b^3*c^3 - 500*a^3*b*c^4)*d*e - (45*a*b^6*c - 558*a^2*b^4*c^2 + 1672*a^3* \\
& b^2*c^3)*e^2)*f^2 - ((27*b^4*c^4 + 80*a^2*c^6)*d^3 + 3*(18*b^5*c^3 - 123*a* \\
& b^3*c^4 - 100*a^2*b*c^5)*d^2*e + 3*(9*b^6*c^2 - 165*a*b^4*c^3 + 692*a^2*b^2 \\
& *c^4)*d*e^2 - (45*a*b^5*c^2 - 647*a^2*b^3*c^3 + 2268*a^3*b*c^4)*e^3)*f)*x + \\
& 1/2*sqrt(1/2)*(2*(b^4*c^6 - 8*a*b^2*c^7 + 16*a^2*c^8)*d^3 + 3*(b^5*c^5 - 8 \\
& *a*b^3*c^6 + 16*a^2*b*c^7)*d^2*e - 18*(a*b^4*c^5 - 8*a^2*b^2*c^6 + 16*a^3*c \\
& ^7)*d*e^2 - (b^7*c^3 - 17*a*b^5*c^4 + 88*a^2*b^3*c^5 - 144*a^3*b*c^6)*e^3 + \\
& (27*b^10 - 459*a*b^8*c + 2961*a^2*b^6*c^2 - 8818*a^3*b^4*c^3 + 11360*a^4*b \\
& ^2*c^4 - 4000*a^5*c^5)*f^3 - 3*(2*(12*a*b^6*c^3 - 121*a^2*b^4*c^4 + 392*a^3 \\
& *b^2*c^5 - 400*a^4*c^6)*d + (9*b^9*c - 153*a*b^7*c^2 + 947*a^2*b^5*c^3 - 25 \\
& 36*a^3*b^3*c^4 + 2480*a^4*b*c^5)*e)*f^2 - 3*((3*b^6*c^4 - 14*a*b^4*c^5 - 32 \\
& *a^2*b^2*c^6 + 160*a^3*c^7)*d^2 - 26*(a*b^5*c^4 - 8*a^2*b^3*c^5 + 16*a^3*b
\end{aligned}$$

$$\begin{aligned}
& c^6) * d * e - 3 * (b^8 * c^2 - 17 * a * b^6 * c^3 + 98 * a^2 * b^4 * c^4 - 224 * a^3 * b^2 * c^5 + 160 * a^4 * c^6) * e^2 * f - (4 * (b^7 * c^7 - 12 * a * b^5 * c^8 + 48 * a^2 * b^3 * c^9 - 64 * a^3 * b * c^{10}) * d + (b^8 * c^6 - 24 * a * b^6 * c^7 + 192 * a^2 * b^4 * c^8 - 640 * a^3 * b^2 * c^9 + 768 * a^4 * c^{10}) * e - (3 * b^9 * c^5 - 52 * a * b^7 * c^6 + 336 * a^2 * b^5 * c^7 - 960 * a^3 * b^3 * c^8 + 1024 * a^4 * b * c^9) * f) * \sqrt{(c^8 * d^4 + 4 * b * c^7 * d^3 * e + 6 * (b^2 * c^6 - 3 * a * c^7) * d^2 * e^2 + 4 * (b^3 * c^5 - 9 * a * b * c^6) * d * e^3 + (b^4 * c^4 - 18 * a * b^2 * c^5 + 81 * a^2 * c^6) * e^4 + (81 * b^8 - 918 * a * b^6 * c + 3051 * a^2 * b^4 * c^2 - 2550 * a^3 * b^2 * c^3 + 625 * a^4 * c^4) * f^4 - 4 * ((27 * b^6 * c^2 - 108 * a * b^4 * c^3 - 180 * a^2 * b^2 * c^4 + 125 * a^3 * c^5) * d + (27 * b^7 * c - 351 * a * b^5 * c^2 + 1197 * a^2 * b^3 * c^3 - 550 * a^3 * b * c^4) * e) * f^3 + 6 * ((9 * b^4 * c^4 + 3 * a * b^2 * c^5 + 25 * a^2 * c^6) * d^2 + 2 * (9 * b^5 * c^3 - 51 * a * b^3 * c^4 - 65 * a^2 * b * c^5) * d * e + (9 * b^6 * c^2 - 132 * a * b^4 * c^3 + 484 * a^2 * b^2 * c^4 - 75 * a^3 * c^5) * e^2) * f^2 - 4 * ((3 * b^2 * c^6 + 5 * a * c^7) * d^3 + 3 * (3 * b^3 * c^5 - 4 * a * b * c^6) * d^2 * e + 3 * (3 * b^4 * c^4 - 22 * a * b^2 * c^5 - 15 * a^2 * c^6) * d * e^2 + (3 * b^5 * c^3 - 49 * a * b^3 * c^4 + 198 * a^2 * b * c^5) * e^3) * f) / (b^6 * c^{10} - 12 * a * b^4 * c^{11} + 48 * a^2 * b^2 * c^{12} - 64 * a^3 * c^{13})) * \sqrt{-((b^3 * c^4 + 12 * a * b * c^5) * d^2 + 2 * (b^4 * c^3 - 6 * a * b^2 * c^4 - 24 * a^2 * c^5) * d * e + (b^5 * c^2 - 15 * a * b^3 * c^3 + 60 * a^2 * b * c^4) * e^2 + (9 * b^7 - 105 * a * b^5 * c + 385 * a^2 * b^3 * c^2 - 420 * a^3 * b * c^3) * f^2 - 2 * ((3 * b^5 * c^2 - 13 * a * b^3 * c^3 - 12 * a^2 * b * c^4) * d + (3 * b^6 * c - 40 * a * b^4 * c^2 + 150 * a^2 * b^2 * c^3 - 120 * a^3 * c^4) * e) * f - (b^6 * c^5 - 12 * a * b^4 * c^6 + 48 * a^2 * b^2 * c^7 - 64 * a^3 * c^8) * \sqrt{(c^8 * d^4 + 4 * b * c^7 * d^3 * e + 6 * (b^2 * c^6 - 3 * a * c^7) * d^2 * e^2 + 4 * (b^3 * c^5 - 9 * a * b * c^6) * d * e^3 + (b^4 * c^4 - 18 * a * b^2 * c^5 + 81 * a^2 * c^6) * e^4 + (81 * b^8 - 918 * a * b^6 * c + 3051 * a^2 * b^4 * c^2 - 2550 * a^3 * b^2 * c^3 + 625 * a^4 * c^4) * f^4 - 4 * ((27 * b^6 * c^2 - 108 * a * b^4 * c^3 - 180 * a^2 * b^2 * c^4 + 125 * a^3 * c^5) * d + (27 * b^7 * c - 351 * a * b^5 * c^2 + 1197 * a^2 * b^3 * c^3 - 550 * a^3 * b * c^4) * e) * f^3 + 6 * ((9 * b^4 * c^4 + 3 * a * b^2 * c^5 + 25 * a^2 * c^6) * d^2 + 2 * (9 * b^5 * c^3 - 51 * a * b^3 * c^4 - 65 * a^2 * b * c^5) * d * e + (9 * b^6 * c^2 - 132 * a * b^4 * c^3 + 484 * a^2 * b^2 * c^4 - 75 * a^3 * c^5) * e^2) * f^2 - 4 * ((3 * b^2 * c^6 + 5 * a * c^7) * d^3 + 3 * (3 * b^3 * c^5 - 4 * a * b * c^6) * d^2 * e + 3 * (3 * b^4 * c^4 - 22 * a * b^2 * c^5 - 15 * a^2 * c^6) * d * e^2 + (3 * b^5 * c^3 - 49 * a * b^3 * c^4 + 198 * a^2 * b * c^5) * e^3) * f) / (b^6 * c^{10} - 12 * a * b^4 * c^{11} + 48 * a^2 * b^2 * c^{12} - 64 * a^3 * c^{13})) / (b^6 * c^5 - 12 * a * b^4 * c^6 + 48 * a^2 * b^2 * c^7 - 64 * a^3 * c^8)) - \sqrt{1/2} * (a * b^2 * c^2 - 4 * a^2 * c^3 + (b^2 * c^3 - 4 * a * c^4) * x^4 + (b^3 * c^2 - 4 * a * b * c^3) * x^2) * \sqrt{-((b^3 * c^4 + 12 * a * b * c^5) * d^2 + 2 * (b^4 * c^3 - 6 * a * b^2 * c^4 - 24 * a^2 * c^5) * d * e + (b^5 * c^2 - 15 * a * b^3 * c^3 + 60 * a^2 * b * c^4) * e^2 + (9 * b^7 - 105 * a * b^5 * c + 385 * a^2 * b^3 * c^2 - 420 * a^3 * b * c^3) * f^2 - 2 * ((3 * b^5 * c^2 - 13 * a * b^3 * c^3 - 12 * a^2 * b * c^4) * d + (3 * b^6 * c - 40 * a * b^4 * c^2 + 150 * a^2 * b^2 * c^3 - 120 * a^3 * c^4) * e) * f - (b^6 * c^5 - 12 * a * b^4 * c^6 + 48 * a^2 * b^2 * c^7 - 64 * a^3 * c^8) * \sqrt{(c^8 * d^4 + 4 * b * c^7 * d^3 * e + 6 * (b^2 * c^6 - 3 * a * c^7) * d^2 * e^2 + 4 * (b^3 * c^5 - 9 * a * b * c^6) * d * e^3 + (b^4 * c^4 - 18 * a * b^2 * c^5 + 81 * a^2 * c^6) * e^4 + (81 * b^8 - 918 * a * b^6 * c + 3051 * a^2 * b^4 * c^2 - 2550 * a^3 * b^2 * c^3 + 625 * a^4 * c^4) * f^4 - 4 * ((27 * b^6 * c^2 - 108 * a * b^4 * c^3 - 180 * a^2 * b^2 * c^4 + 125 * a^3 * c^5) * d + (27 * b^7 * c - 351 * a * b^5 * c^2 + 1197 * a^2 * b^3 * c^3 - 550 * a^3 * b * c^4) * e) * f^3 + 6 * ((9 * b^4 * c^4 + 3 * a * b^2 * c^5 + 25 * a^2 * c^6) * d^2 + 2 * (9 * b^5 * c^3 - 51 * a * b^3 * c^4 - 65 * a^2 * b * c^5) * d * e + (9 * b^6 * c^2 - 132 * a * b^4 * c^3 + 484 * a^2 * b^2 * c^4 - 75 * a^3 * c^5) * e^2) * f^2 - 4 * ((3 * b^2 * c^6 + 5 * a * c^7) * d^3 + 3 * (3 * b^3 * c^5 - 4 * a * b * c^6) * d^2 * e + 3 * (3 * b^4 * c^4 - 22 * a * b^2 * c^5 - 15 * a^2 * c^6) * d * e^2 + (3 * b^5 * c^3 - 49 * a * b^3 * c^4 + 198 * a^2 * b * c^5) * e^3) * f) / (b^6 * c^{10} - 12 * a * b^4 * c^{11} + 48 * a^2 * b^2 * c^{12} - 64 * a^3 * c^{13})) / (b^6 * c^5 - 12 * a * b^4 * c^6 + 48 * a^2 * b^2 * c^7 - 64 * a^3 * c^8)) * \log(((3 * b^2 * c^6 + 4 * a * c^7) * d^4 + (9 * b^3 * c^5 - 20 * a * b * c^6) * d^3 * e + 3 * (3 * b^4 * c^4 - 28 * a * b^2 * c^5) * d^2 * e^2 + (3 * b^5 * c^3 - 65 * a * b^3 * c^4 + 324 * a^2 * b * c^5) * d * e^3 - (5 * a * b^4 * c^3 - 81 * a^2 * b^2 * c^4 + 324 * a^3 * c^5) * e^4 - (189 * a^2 * b^6 - 1971 * a^3 * b^4 * c + 5625 * a^4 * b^2 * c^2 - 2500 * a^5 * c^3) * f^4 - ((81 * b^8 - 945 * a * b^6 * c + 3213 * a^2 * b^4 * c^2 - 3000 * a^3 * b^2 * c^3 + 2000 * a^4 * c^4) * d - (135 * a * b^7 - 1323 * a^2 * b^5 * c + 2727 * a^3 * b^3 * c^2 + 2500 * a^4 * b * c^3) * e) * f^3 + 3 * ((27 * b^6 * c^2 - 117 * a * b^4 * c^3 - 150 * a^2 * b^2 * c^4 + 200 * a^3 * c^5) * d^2 + (27 * b^7 * c - 405 * a * b^5 * c^2 + 1461 * a^2 * b^3 * c^3 - 500 * a^3 * b * c^4) * d * e - (45 * a * b^6 * c - 558 * a^2 * b^4 * c^2 + 1672 * a^3 * b^2 * c^3) * e^2) * f^2 - ((27 * b^4 * c^4 + 80 * a^2 * c^6) * d^3 + 3 * (18 * b^5 * c^3 - 123 * a * b^3 * c^4 - 100 * a^2 * b * c^5) * d^2 * e + 3 * (9 * b^6 * c^2 - 165 * a * b^4 * c^3 + 692 * a^2 * b^2 * c^4) * d * e^2 - (45 * a * b^5 * c^2 - 647 * a^2 * b^3 * c^3 + 2268 * a^3 * b * c^4) * e^3) * f) * x - 1/2 * \sqrt{1/2} * (2 * (b^4 * c^6 - 8 * a * b^2 * c^7 + 16 * a^2 * c^8) * d^3 + 3 * (b^5 * c^5 - 8 * a * b^3 *
\end{aligned}$$

$$\begin{aligned}
& c^6 + 16a^2bc^7)d^2e - 18*(a^4b^5c^5 - 8a^2b^2c^6 + 16a^3c^7)*de \\
& ^2 - (b^7c^3 - 17ab^5c^4 + 88a^2b^3c^5 - 144a^3bc^6)*e^3 + (27b^10 - 459ab^8c + 2961a^2b^6c^2 - 8818a^3b^4c^3 + 11360a^4b^2c^4 \\
& - 4000a^5c^5)*f^3 - 3*(2*(12a^2b^6c^3 - 121a^2b^4c^4 + 392a^3b^2c^5 - 400a^4c^6)*d + (9b^9c - 153ab^7c^2 + 947a^2b^5c^3 - 2536a^3b^3c^4 + 2480a^4bc^5)*e)*f^2 - 3*((3b^6c^4 - 14ab^4c^5 - 32a^2b^2c^6 + 160a^3c^7)*d^2 - 26*(a^4b^5c^4 - 8a^2b^3c^5 + 16a^3bc^6)*de - 3*(b^8c^2 - 17ab^6c^3 + 98a^2b^4c^4 - 224a^3b^2c^5 + 160a^4c^6)*e^2)*f - (4*(b^7c^7 - 12ab^5c^8 + 48a^2b^3c^9 - 64a^3bc^10)*d + (b^8c^6 - 24ab^6c^7 + 192a^2b^4c^8 - 640a^3b^2c^9 + 768a^4c^10)*e - (3b^9c^5 - 52ab^7c^6 + 336a^2b^5c^7 - 960a^3b^3c^8 + 1024a^4bc^9)*f)*sqrt((c^8d^4 + 4b^7c^7d^3e + 6*(b^2c^6 - 3ac^7)*d^2e^2 + 4*(b^3c^5 - 9ab^2c^6)*de^3 + (b^4c^4 - 18ab^2c^5 + 81a^2c^6)*e^4 + (81b^8 - 918ab^6c + 3051a^2b^4c^2 - 2550a^3b^2c^3 + 625a^4c^4)*f^4 - 4*((27b^6c^2 - 108ab^4c^3 - 180a^2b^2c^4 + 125a^3c^5)*d + (27b^7c - 351ab^5c^2 + 1197a^2b^3c^3 - 550a^3bc^4)*e)*f^3 + 6*((9b^4c^4 + 3ab^2c^5 + 25a^2c^6)*d^2 + 2*(9b^5c^3 - 51ab^3c^4 - 65a^2bc^5)*de + (9b^6c^2 - 132ab^4c^3 + 484a^2b^2c^4 - 75a^3c^5)*e^2)*f^2 - 4*((3b^2c^6 + 5ac^7)*d^3 + 3*(3b^3c^5 - 4ab^2c^6)*d^2e + 3*(3b^4c^4 - 22ab^2c^5 - 15a^2c^6)*de^2 + (3b^5c^3 - 49ab^3c^4 + 198a^2bc^5)*e^3)*f)/(b^6c^10 - 12ab^4c^11 + 48a^2b^2c^12 - 64a^3c^13))*sqrt(-((b^3c^4 + 12ab^2c^5)*d^2 + 2*(b^4c^3 - 6ab^2c^4 - 24a^2c^5)*de + (b^5c^2 - 15ab^3c^3 + 60a^2bc^4)*e^2 + (9b^7 - 105ab^5c + 385a^2b^3c^2 - 420a^3bc^3)*f^2 - 2*((3b^5c^2 - 13ab^3c^3 - 12a^2bc^4)*d + (3b^6c - 40ab^4c^2 + 150a^2b^2c^3 - 120a^3c^4)*e)*f - (b^6c^5 - 12ab^4c^6 + 48a^2b^2c^7 - 64a^3c^8)*sqrt((c^8d^4 + 4b^7c^7d^3e + 6*(b^2c^6 - 3ac^7)*d^2e^2 + 4*(b^3c^5 - 9ab^2c^6)*de^3 + (b^4c^4 - 18ab^2c^5 + 81a^2c^6)*e^4 + (81b^8 - 918ab^6c + 3051a^2b^4c^2 - 2550a^3b^2c^3 + 625a^4c^4)*f^4 - 4*((27b^6c^2 - 108ab^4c^3 - 180a^2b^2c^4 + 125a^3c^5)*d + (27b^7c - 351ab^5c^2 + 1197a^2b^3c^3 - 550a^3bc^4)*e)*f^3 + 6*((9b^4c^4 + 3ab^2c^5 + 25a^2c^6)*d^2 + 2*(9b^5c^3 - 51ab^3c^4 - 65a^2bc^5)*de + (9b^6c^2 - 132ab^4c^3 + 484a^2b^2c^4 - 75a^3c^5)*e^2)*f^2 - 4*((3b^2c^6 + 5ac^7)*d^3 + 3*(3b^3c^5 - 4ab^2c^6)*d^2e + 3*(3b^4c^4 - 22ab^2c^5 - 15a^2c^6)*de^2 + (3b^5c^3 - 49ab^3c^4 + 198a^2bc^5)*e^3)*f)/(b^6c^10 - 12ab^4c^11 + 48a^2b^2c^12 - 64a^3c^13)))/((b^6c^5 - 12ab^4c^6 + 48a^2b^2c^7 - 64a^3c^8))) + 2*(2ac^2d - abc^2e + (3ab^2 - 10a^2c)*f)*x)/(a^2b^2c^2 - 4a^2c^3 + (b^2c^3 - 4ac^4)*x^4 + (b^3c^2 - 4abc^3)*x^2)
\end{aligned}$$

giac [B] time = 8.25, size = 7496, normalized size = 17.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")

[Out] $f*x/c^2 + 1/2*(b*c^2*d*x^3 + b^3*f*x^3 - 3*a*b*c*f*x^3 - b^2*c*x^3*e + 2*a*c^2*x^3*e + 2*a*c^2*d*x + a*b^2*f*x - 2*a^2*c*f*x - a*b*c*x*e)/((c*x^4 + b*x^2 + a)*(b^2*c^2 - 4*a*c^3)) + 1/16*((2*b^3*c^4 - 8*a*b*c^5 - \sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c}*b^3*c^2 + 4*\sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c}*a*b*c^3 + 2*\sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c}*b^2*c^3 - \sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c}*b*c^4 - 2*(b^2 - 4*a*c)*b*c^4)*(b^2*c^2 - 4*a*c^3)^2*d - (6*b^5*c^2 - 50*a*b^3*c^3 + 104*a^2*b*c^4 - 3*\sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c}*b^5 + 25*\sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c}*a*b^3*c + 6*\sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c}*b^4*c - 52*\sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c}*a^2*b*c^2 - 26*\sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c}*a*b^2*c^2 - 3*\sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c}$

$$\begin{aligned}
& b^2 - 4ac) * c) * b^3 * c^2 + 13 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a * b * c^3 - 6 * (b^2 - 4ac) * b^3 * c^2 + 26 * (b^2 - 4ac) * a * b * c^3) * \\
& (b^2 * c^2 - 4a * c^3)^2 * f + (2 * b^4 * c^3 - 20 * a * b^2 * c^4 + 48 * a^2 * c^5 - \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * b^4 * c + 10 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a * b^2 * c^2 + 2 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * b^3 * c^2 - 24 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^2 * c^3 - 12 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a * b * c^3 - \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * b^2 * c^3 + 6 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a * c^4 - 2 * (b^2 - 4ac) * b^2 * c^3 + 12 * (b^2 - 4ac) * a * c^4) * (b^2 * c^2 - 4a * c^3)^2 * e - 4 * (\sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a * b^4 * c^5 - 8 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^2 * b^2 * c^6 - 2 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a * b^3 * c^6 + 2 * a * b^4 * c^6 + 16 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^3 * c^7 + 8 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^2 * b * c^7 + \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a * b^2 * c^7 - 16 * a^2 * b^2 * c^7 - 4 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^2 * c^8 + 32 * a^3 * c^8 - 2 * (b^2 - 4ac) * a * b^2 * c^6 + 8 * (b^2 - 4ac) * a^2 * c^7) * d * \text{abs}(-b^2 * c^2 + 4a * c^3) - 2 * (3 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a * b^6 * c^3 - 34 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^2 * b^4 * c^4 - 6 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a * b^5 * c^4 + 6 * a * b^6 * c^4 + 128 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^3 * b^2 * c^5 + 44 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^2 * b^3 * c^5 + 3 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a * b^4 * c^5 - 68 * a^2 * b^4 * c^5 - 160 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^4 * c^6 - 80 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^3 * b * c^6 - 22 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^2 * b^2 * c^6 + 256 * a^3 * b^2 * c^6 + 40 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^3 * c^7 - 320 * a^4 * c^7 - 6 * (b^2 - 4ac) * a * b^4 * c^4 + 44 * (b^2 - 4ac) * a^2 * b^2 * c^5 - 80 * (b^2 - 4ac) * a^3 * c^6) * f * \text{abs}(-b^2 * c^2 + 4a * c^3) + 2 * (\sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a * b^5 * c^4 - 8 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^2 * b^3 * c^5 - 2 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a * b^4 * c^5 + 2 * a * b^5 * c^5 + 16 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^3 * b * c^6 + 8 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^2 * b^2 * c^6 + \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a * b^3 * c^6 - 16 * a^2 * b^3 * c^6 - 4 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^2 * b * c^7 + 32 * a^3 * b * c^7 - 2 * (b^2 - 4ac) * a * b^3 * c^5 + 8 * (b^2 - 4ac) * a^2 * b * c^6) * \text{abs}(-b^2 * c^2 + 4a * c^3) * e - (2 * b^7 * c^8 - 8 * a * b^5 * c^9 - 32 * a^2 * b^3 * c^10 + 128 * a^3 * b * c^11 - \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * b^7 * c^6 + 4 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a * b^5 * c^7 + 2 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * b^6 * c^7 + 16 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^2 * b^3 * c^8 - \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * b^5 * c^8 - 64 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^3 * b * c^9 - 32 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^2 * b^2 * c^9 + 16 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^2 * b * c^10 - 2 * (b^2 - 4ac) * b^5 * c^8 + 32 * (b^2 - 4ac) * a^2 * b * c^10) * d + (6 * b^9 * c^6 - 86 * a * b^7 * c^7 + 440 * a^2 * b^5 * c^8 - 928 * a^3 * b^3 * c^9 + 640 * a^4 * b * c^10 - 3 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * b^9 * c^4 + 43 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a * b^7 * c^5 + 6 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * b^8 * c^5 - 220 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^2 * b^5 * c^6 - 62 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a * b^6 * c^6 - 3 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * b^7 * c^6 + 464 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^3 * b^3 * c^7 + 192 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^2 * b^4 * c^7 + 31 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a * b^5 * c^7 - 320 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^4 * b * c^8 - 160 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^3 * b^2 * c^8 - 96 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^2 * b^3 * c^8 + 80 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^3 * b * c^9 - 6 * (b^2 - 4ac) * b^7 * c^6 + 62 * (b^2 - 4ac) * a * b^5 * c^7 - 192 * (b^2 - 4ac) * a^2 * b^3 * c^8 + 160 * (b^2 - 4ac) * a^3 * b * c^9) * f - (2 * b^8 * c^7 - 32 * a * b^6 * c^8 + 160 * a^2 * b^4 * c^9 - 256 *
\end{aligned}$$

$$\begin{aligned}
& a^3 b^2 c^{10} - \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc - \sqrt{b^2 - 4ac} c} b^8 c^5 + 16 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc - \sqrt{b^2 - 4ac} c} a b^6 c^6 \\
& + 2 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc - \sqrt{b^2 - 4ac} c} b^7 c^6 - 80 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc - \sqrt{b^2 - 4ac} c} a^2 b^4 c^7 - 24 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc - \sqrt{b^2 - 4ac} c} a b^5 c^7 - \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc - \sqrt{b^2 - 4ac} c} b^6 c^7 + 128 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc - \sqrt{b^2 - 4ac} c} a^3 b^2 c^8 + 64 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc - \sqrt{b^2 - 4ac} c} a^2 b^3 c^8 + 12 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc - \sqrt{b^2 - 4ac} c} a b^4 c^8 - 32 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc - \sqrt{b^2 - 4ac} c} a^2 b^2 c^9 - 2 (b^2 - 4ac) b^6 c^7 + 24 (b^2 - 4ac) a b^4 c^8 - 64 (b^2 - 4ac) a^2 b^2 c^9) \\
& * e) \arctan(2 \sqrt{1/2} x / \sqrt{(b^3 c^2 - 4 a b c^3 + \sqrt{(b^3 c^2 - 4 a b c^3)^2 - 4 (a b^2 c^2 - 4 a^2 c^3) (b^2 c^3 - 4 a c^4)}) / (b^2 c^3 - 4 a c^4)) / ((a b^6 c^5 - 12 a^2 b^4 c^6 - 2 a b^5 c^6 + 48 a^3 b^2 c^7 + 16 a^2 b^3 c^7 + a b^4 c^7 - 64 a^4 c^8 - 32 a^3 b c^8 - 8 a^2 b^2 c^8 + 16 a^3 c^9) * \text{abs}(-b^2 c^2 + 4 a c^3) * \text{abs}(c)) - 1/16 ((2 b^3 c^4 - 8 a b c^5 - \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac} c} b^3 c^2 + 4 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac} c} a b c^3 + 2 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac} c} b^2 c^3 - \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac} c} b c^4 - 2 (b^2 - 4ac) b c^4) (b^2 c^2 - 4 a c^3)^2 d - (6 b^5 c^2 - 50 a b^3 c^3 + 104 a^2 b c^4 - 3 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac} c} b^5 + 25 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac} c} a b^3 c + 6 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac} c} b^4 c - 52 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac} c} a^2 b c^2 - 26 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac} c} a b^2 c^2 - 3 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac} c} b^3 c^2 + 13 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac} c} a b c^3 - 6 (b^2 - 4ac) b^3 c^2 + 26 (b^2 - 4ac) a b c^3) (b^2 c^2 - 4 a c^3)^2 f + (2 b^4 c^3 - 20 a b^2 c^4 + 48 a^2 c^5 - \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac} c} b^4 c + 10 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac} c} a b^2 c^2 + 2 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac} c} b^3 c^2 - 24 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac} c} a^2 c^3 - 12 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac} c} a b c^3 - \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac} c} b^2 c^3 + 6 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac} c} a c^4 - 2 (b^2 - 4ac) b^2 c^3 + 12 (b^2 - 4ac) a c^4) (b^2 c^2 - 4 a c^3)^2 e + 4 (\sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac} c} a b^4 c^5 - 8 \sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac} c} a^2 b^2 c^6 - 2 \sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac} c} a b^3 c^6 - 2 a b^4 c^6 + 16 \sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac} c} a^3 c^7 + 8 \sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac} c} a^2 b c^7 + \sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac} c} a b^2 c^7 + 16 a^2 b^2 c^7 - 4 \sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac} c} a^2 c^8 - 32 a^3 c^8 + 2 (b^2 - 4ac) a b^2 c^6 - 8 (b^2 - 4ac) a^2 c^7) d * \text{abs}(-b^2 c^2 + 4 a c^3) + 2 (3 \sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac} c} a b^6 c^3 - 34 \sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac} c} a^2 b^4 c^4 - 6 \sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac} c} a b^5 c^4 - 6 a b^6 c^4 + 128 \sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac} c} a^3 b^2 c^5 + 44 \sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac} c} a^2 b^3 c^5 + 3 \sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac} c} a b^4 c^5 + 68 a^2 b^4 c^5 - 160 \sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac} c} a^4 c^6 - 80 \sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac} c} a^3 b c^6 - 22 \sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac} c} a^2 b^2 c^6 - 256 a^3 b^2 c^6 + 40 \sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac} c} a^3 c^7 + 320 a^4 c^7 + 6 (b^2 - 4ac) a b^4 c^4 - 44 (b^2 - 4ac) a^2 b^2 c^5 + 80 (b^2 - 4ac) a^3 c^6) * f * \text{abs}(-b^2 c^2 + 4 a c^3) - 2 (\sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac} c} a b^5 c^4 - 8 \sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac} c} a^2 b^3 c^5 - 2 \sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac} c} a b^4 c^5 - 2 a b^5 c^5 + 16 \sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac} c} a^3 b c^6 + 8 \sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac} c} a^2 b^2 c^6 + \sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac} c} a b^3 c^6 + 16 a^2 b^3 c^6 - 4 \sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac} c} a^2 b c^7 - 32 a^3 b c^7 + 2 (b^2 - 4ac) a b^3 c^5 - 8 (b
\end{aligned}$$

$$\begin{aligned} &^2 - 4*a*c)*a^2*b*c^6)*abs(-b^2*c^2 + 4*a*c^3)*e - (2*b^7*c^8 - 8*a*b^5*c^9 \\ &- 32*a^2*b^3*c^10 + 128*a^3*b*c^11 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \\ &\sqrt{b^2 - 4*a*c}*c}*b^7*c^6 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c} \\ &)*a*b^5*c^7 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c} \\ &)*b^6*c^7 + 16*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c} \\ &)*a^2*b^3*c^8 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c} \\ &)*b^5*c^8 - 64*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c} \\ &)*a^3*b*c^9 - 32*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c} \\ &)*a^2*b^2*c^9 + 16*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c} \\ &)*a^2*b*c^10 - 2*(b^2 - 4*a*c)*b^5*c^8 + 32*(b^2 - 4*a*c)*a^2*b*c^10)*d + (6* \\ &b^9*c^6 - 86*a*b^7*c^7 + 440*a^2*b^5*c^8 - 928*a^3*b^3*c^9 + 640*a^4*b*c^10 \\ &- 3*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*b^9*c^4 + 43 \\ &*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^7*c^5 + 6*\sqrt{2} \\ &*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*b^8*c^5 - 220*\sqrt{2} \\ &*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*b^5*c^6 - 62*\sqrt{2} \\ &*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^6*c^6 - 3*\sqrt{2} \\ &*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*b^7*c^6 + 464*\sqrt{2} \\ &*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^3*b^3*c^7 + 192*\sqrt{2} \\ &*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*b^4*c^7 + 31*\sqrt{2} \\ &*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^5*c^7 - 320*\sqrt{2} \\ &*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^4*b*c^8 - 160*\sqrt{2} \\ &*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^3*b^2*c^8 - 96*\sqrt{2} \\ &*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*b^3*c^8 + 80*\sqrt{2} \\ &*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^3*b*c^9 - 6*(b^2 - 4*a*c) \\ &)*b^7*c^6 + 62*(b^2 - 4*a*c)*a*b^5*c^7 - 192*(b^2 - 4*a*c)*a^2*b^3*c^8 + 160* \\ &(b^2 - 4*a*c)*a^3*b*c^9)*f - (2*b^8*c^7 - 32*a*b^6*c^8 + 160*a^2*b^4*c^9 - 2 \\ &56*a^3*b^2*c^10 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c} \\ &)*b^8*c^5 + 16*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b \\ &^6*c^6 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*b^7*c^6 \\ &- 80*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*b^4*c^7 \\ &- 24*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^5*c^7 \\ &- \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*b^6*c^7 + 128* \\ &\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^3*b^2*c^8 + 64* \\ &\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*b^3*c^8 + 12* \\ &\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^4*c^8 - 32* \\ &\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*b^2*c^9 - 2*(b^2 \\ &- 4*a*c)*b^6*c^7 + 24*(b^2 - 4*a*c)*a*b^4*c^8 - 64*(b^2 - 4*a*c)*a^2*b^2*c^9 \\ &)*e)*\arctan(2*\sqrt{1/2}*x/\sqrt{(b^3*c^2 - 4*a*b*c^3 - \sqrt{(b^3*c^2 - 4*a \\ &)*b*c^3})^2 - 4*(a*b^2*c^2 - 4*a^2*c^3)*(b^2*c^3 - 4*a*c^4)})/(b^2*c^3 - 4*a \\ &)*c^4))/((a*b^6*c^5 - 12*a^2*b^4*c^6 - 2*a*b^5*c^6 + 48*a^3*b^2*c^7 + 16*a^2 \\ &)*b^3*c^7 + a*b^4*c^7 - 64*a^4*c^8 - 32*a^3*b*c^8 - 8*a^2*b^2*c^8 + 16*a^3*c^9) \\ &)*abs(-b^2*c^2 + 4*a*c^3)*abs(c)) \end{aligned}$$

maple [B] time = 0.05, size = 1977, normalized size = 4.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^4*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x)$

[Out]
$$\begin{aligned} &-1/4/c/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} \\ &)*\arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^3*e+13/4/c/(4*a*c-b^2) \\ &)*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)}) \\ &)*c)^{(1/2)}*c*x)*a*b*f-13/4/c/(4*a*c-b^2)*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)}) \\ &)*c)^{(1/2)}*\arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*a \\ &)*b*f+f*x/c^2+1/2/c/(c*x^4+b*x^2+a)*a/(4*a*c-b^2)*x*b*e+3/2/c/(c*x^4+b*x^2+a) \\ &/((4*a*c-b^2)*x^3*a*b*f-1/2/c^2/(c*x^4+b*x^2+a)*a/(4*a*c-b^2)*x*b^2*f-3/4/c^2 \\ &/((4*a*c-b^2)*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)}) \\ &)*c)^{(1/2)}*c*x)*b^3*f+1/4/c/(4*a*c-b^2)*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)}) \\ &)*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} \end{aligned}$$

) * c * x) * b^2 * e + 3/4 / c^2 / (4 * a * c - b^2) * 2^(1/2) / ((b + (-4 * a * c + b^2)^(1/2)) * c)^(1/2) * arctan(2^(1/2) / ((b + (-4 * a * c + b^2)^(1/2)) * c)^(1/2) * c * x) * b^3 * f - 1/4 / c / (4 * a * c - b^2) * 2^(1/2) / ((b + (-4 * a * c + b^2)^(1/2)) * c)^(1/2) * arctan(2^(1/2) / ((b + (-4 * a * c + b^2)^(1/2)) * c)^(1/2) * c * x) * b^2 * e - 1/4 / (4 * a * c - b^2) / (-4 * a * c + b^2)^(1/2) * 2^(1/2) / ((-b + (-4 * a * c + b^2)^(1/2)) * c)^(1/2) * arctanh(2^(1/2) / ((-b + (-4 * a * c + b^2)^(1/2)) * c)^(1/2) * c * x) * b^2 * d - 1/4 / (4 * a * c - b^2) / (-4 * a * c + b^2)^(1/2) * 2^(1/2) / ((b + (-4 * a * c + b^2)^(1/2)) * c)^(1/2) * arctan(2^(1/2) / ((b + (-4 * a * c + b^2)^(1/2)) * c)^(1/2) * c * x) * b^2 * d + 5 / (4 * a * c - b^2) / (-4 * a * c + b^2)^(1/2) * 2^(1/2) / ((-b + (-4 * a * c + b^2)^(1/2)) * c)^(1/2) * arctanh(2^(1/2) / ((-b + (-4 * a * c + b^2)^(1/2)) * c)^(1/2) * c * x) * a^2 * f + 3/4 / c^2 / (4 * a * c - b^2) / (-4 * a * c + b^2)^(1/2) * 2^(1/2) / ((-b + (-4 * a * c + b^2)^(1/2)) * c)^(1/2) * arctanh(2^(1/2) / ((-b + (-4 * a * c + b^2)^(1/2)) * c)^(1/2) * c * x) * b^4 * f - 1/4 / c / (4 * a * c - b^2) / (-4 * a * c + b^2)^(1/2) * 2^(1/2) / ((-b + (-4 * a * c + b^2)^(1/2)) * c)^(1/2) * arctanh(2^(1/2) / ((-b + (-4 * a * c + b^2)^(1/2)) * c)^(1/2) * c * x) * b^3 * e + 2 / (4 * a * c - b^2) / (-4 * a * c + b^2)^(1/2) * 2^(1/2) / ((b + (-4 * a * c + b^2)^(1/2)) * c)^(1/2) * arctan(2^(1/2) / ((b + (-4 * a * c + b^2)^(1/2)) * c)^(1/2) * c * x) * a * b * e - 1 / (c * x^4 + b * x^2 + a) * a / (4 * a * c - b^2) * x * d - 1 / (c * x^4 + b * x^2 + a) / (4 * a * c - b^2) * x^3 * a * e - 1/2 / (c * x^4 + b * x^2 + a) / (4 * a * c - b^2) * x^3 * b * d - c / (4 * a * c - b^2) / (-4 * a * c + b^2)^(1/2) * 2^(1/2) / ((b + (-4 * a * c + b^2)^(1/2)) * c)^(1/2) * arctan(2^(1/2) / ((b + (-4 * a * c + b^2)^(1/2)) * c)^(1/2) * c * x) * a * d + 3/4 / c^2 / (4 * a * c - b^2) / (-4 * a * c + b^2)^(1/2) * 2^(1/2) / ((b + (-4 * a * c + b^2)^(1/2)) * c)^(1/2) * arctan(2^(1/2) / ((b + (-4 * a * c + b^2)^(1/2)) * c)^(1/2) * c * x) * b^4 * f + 2 / (4 * a * c - b^2) / (-4 * a * c + b^2)^(1/2) * 2^(1/2) / ((-b + (-4 * a * c + b^2)^(1/2)) * c)^(1/2) * arctanh(2^(1/2) / ((-b + (-4 * a * c + b^2)^(1/2)) * c)^(1/2) * c * x) * a * b * e - c / (4 * a * c - b^2) / (-4 * a * c + b^2)^(1/2) * 2^(1/2) / ((-b + (-4 * a * c + b^2)^(1/2)) * c)^(1/2) * arctanh(2^(1/2) / ((-b + (-4 * a * c + b^2)^(1/2)) * c)^(1/2) * c * x) * a * d - 19/4 / c / (4 * a * c - b^2) / (-4 * a * c + b^2)^(1/2) * 2^(1/2) / ((-b + (-4 * a * c + b^2)^(1/2)) * c)^(1/2) * arctanh(2^(1/2) / ((-b + (-4 * a * c + b^2)^(1/2)) * c)^(1/2) * c * x) * a * b^2 * f - 19/4 / c / (4 * a * c - b^2) / (-4 * a * c + b^2)^(1/2) * 2^(1/2) / ((b + (-4 * a * c + b^2)^(1/2)) * c)^(1/2) * arctan(2^(1/2) / ((b + (-4 * a * c + b^2)^(1/2)) * c)^(1/2) * c * x) * a * b^2 * f - 1/2 / c^2 / (c * x^4 + b * x^2 + a) / (4 * a * c - b^2) * x^3 * b^3 * f + 1/2 / c / (c * x^4 + b * x^2 + a) / (4 * a * c - b^2) * x^3 * b^2 * e + 1 / c / (c * x^4 + b * x^2 + a) * a^2 / (4 * a * c - b^2) * x * f - 3/2 / (4 * a * c - b^2) * 2^(1/2) / ((-b + (-4 * a * c + b^2)^(1/2)) * c)^(1/2) * arctanh(2^(1/2) / ((-b + (-4 * a * c + b^2)^(1/2)) * c)^(1/2) * c * x) * a * e + 1/4 / (4 * a * c - b^2) * 2^(1/2) / ((-b + (-4 * a * c + b^2)^(1/2)) * c)^(1/2) * arctanh(2^(1/2) / ((-b + (-4 * a * c + b^2)^(1/2)) * c)^(1/2) * c * x) * b * d + 3/2 / (4 * a * c - b^2) * 2^(1/2) / ((b + (-4 * a * c + b^2)^(1/2)) * c)^(1/2) * arctan(2^(1/2) / ((b + (-4 * a * c + b^2)^(1/2)) * c)^(1/2) * c * x) * a * e - 1/4 / (4 * a * c - b^2) * 2^(1/2) / ((b + (-4 * a * c + b^2)^(1/2)) * c)^(1/2) * arctan(2^(1/2) / ((b + (-4 * a * c + b^2)^(1/2)) * c)^(1/2) * c * x) * b * d + 5 / (4 * a * c - b^2) / (-4 * a * c + b^2)^(1/2) * 2^(1/2) / ((b + (-4 * a * c + b^2)^(1/2)) * c)^(1/2) * arctan(2^(1/2) / ((b + (-4 * a * c + b^2)^(1/2)) * c)^(1/2) * c * x) * a^2 * f

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(bc^2d - (b^2c - 2ac^2)e + (b^3 - 3abc)f)x^3 + (2ac^2d - abce + (ab^2 - 2a^2c)f)x}{2(ab^2c^2 - 4a^2c^3 + (b^2c^3 - 4ac^4)x^4 + (b^3c^2 - 4abc^3)x^2)} + \frac{fx}{c^2} + \frac{-\int \frac{2ac^2d - abce - (bc^2d + (b^2c - 6ac^2)e - (3b^3 - 13abc)f)x^2 + (3ab^2 - 10a^2c)f}{cx^4 + bx^2 + a} dx}{2(b^2c^2 - 4ac^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")

[Out] 1/2*((b*c^2*d - (b^2*c - 2*a*c^2)*e + (b^3 - 3*a*b*c)*f)*x^3 + (2*a*c^2*d - a*b*c*e + (a*b^2 - 2*a^2*c)*f)*x)/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2) + f*x/c^2 + 1/2*integrate(-(2*a*c^2*d - a*b*c*e - (b*c^2*d + (b^2*c - 6*a*c^2)*e - (3*b^3 - 13*a*b*c)*f)*x^2 + (3*a*b^2 - 10*a^2*c)*f)/(c*x^4 + b*x^2 + a), x)/(b^2*c^2 - 4*a*c^3)

mupad [B] time = 2.65, size = 25862, normalized size = 59.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x)

[Out] (f*x)/c^2 - atan((((10240*a^5*c^7*f - 2048*a^4*c^8*d - 384*a^2*b^4*c^6*d + 1536*a^3*b^2*c^7*d + 192*a^2*b^5*c^5*e - 768*a^3*b^3*c^6*e - 736*a^2*b^6*c

$$\begin{aligned}
& ^4*f + 4224*a^3*b^4*c^5*f - 10752*a^4*b^2*c^6*f + 32*a*b^6*c^5*d - 16*a*b^7* \\
& *c^4*e + 1024*a^4*b*c^7*e + 48*a*b^8*c^3*f)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a \\
& *b^4*c^4 - 48*a^2*b^2*c^5)) - (x*((768*a^4*b*c^8*d^2 - b^9*c^4*d^2 - c^4*d^2 \\
& *(-4*a*c - b^2)^9)^{(1/2)} - b^{11}*c^2*e^2 - 9*b^4*f^2*(-(4*a*c - b^2)^9)^{(1 \\
& /2)} - 9*b^{13}*f^2 + 27*a*b^9*c^3*e^2 + 3840*a^5*b*c^7*e^2 + 9*a*c^3*e^2*(-(4 \\
& *a*c - b^2)^9)^{(1/2)} - 26880*a^6*b*c^6*f^2 + 6*b^{12}*c*e*f + 96*a^2*b^5*c^6* \\
& d^2 - 512*a^3*b^3*c^7*d^2 - 288*a^2*b^7*c^4*e^2 + 1504*a^3*b^5*c^5*e^2 - 38 \\
& 40*a^4*b^3*c^6*e^2 - 2077*a^2*b^9*c^2*f^2 + 10656*a^3*b^7*c^3*f^2 - 30240*a \\
& ^4*b^5*c^4*f^2 + 44800*a^5*b^3*c^5*f^2 - 25*a^2*c^2*f^2*(-(4*a*c - b^2)^9)^{(\\
& 1/2)} - b^2*c^2*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 213*a*b^{11}*c*f^2 - 3072*a^5* \\
& c^8*d*e - 2*b^{10}*c^3*d*e + 15360*a^6*c^7*e*f + 6*b^{11}*c^2*d*f + 36*a*b^8*c^ \\
& 4*d*e - 98*a*b^9*c^3*d*f + 1536*a^5*b*c^7*d*f + 10*a*c^3*d*f*(-(4*a*c - b^2 \\
&)^9)^{(1/2)} - 2*b*c^3*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 152*a*b^{10}*c^2*e*f + 6* \\
& b^3*c*e*f*(-(4*a*c - b^2)^9)^{(1/2)} + 51*a*b^2*c*f^2*(-(4*a*c - b^2)^9)^{(1/2 \\
&)} - 192*a^2*b^6*c^5*d*e + 128*a^3*b^4*c^6*d*e + 1536*a^4*b^2*c^7*d*e + 576* \\
& a^2*b^7*c^4*d*f - 1344*a^3*b^5*c^5*d*f + 512*a^4*b^3*c^6*d*f + 1548*a^2*b^8 \\
& *c^3*e*f - 8064*a^3*b^6*c^4*e*f + 22400*a^4*b^4*c^5*e*f - 30720*a^5*b^2*c^6 \\
& *e*f + 6*b^2*c^2*d*f*(-(4*a*c - b^2)^9)^{(1/2)} - 44*a*b*c^2*e*f*(-(4*a*c - b \\
& ^2)^9)^{(1/2)))/(32*(4096*a^6*c^11 + b^{12}*c^5 - 24*a*b^{10}*c^6 + 240*a^2*b^8*c \\
& ^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^{10}))^{(1/2)}*(16*b \\
& ^7*c^5 - 192*a*b^5*c^6 - 1024*a^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 \\
& + b^4*c^3 - 8*a*b^2*c^4)))*((768*a^4*b*c^8*d^2 - b^9*c^4*d^2 - c^4*d^2*(-(4 \\
& *a*c - b^2)^9)^{(1/2)} - b^{11}*c^2*e^2 - 9*b^4*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - \\
& 9*b^{13}*f^2 + 27*a*b^9*c^3*e^2 + 3840*a^5*b*c^7*e^2 + 9*a*c^3*e^2*(-(4*a*c - \\
& b^2)^9)^{(1/2)} - 26880*a^6*b*c^6*f^2 + 6*b^{12}*c*e*f + 96*a^2*b^5*c^6*d^2 - \\
& 512*a^3*b^3*c^7*d^2 - 288*a^2*b^7*c^4*e^2 + 1504*a^3*b^5*c^5*e^2 - 3840*a^4 \\
& *b^3*c^6*e^2 - 2077*a^2*b^9*c^2*f^2 + 10656*a^3*b^7*c^3*f^2 - 30240*a^4*b^5 \\
& *c^4*f^2 + 44800*a^5*b^3*c^5*f^2 - 25*a^2*c^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} \\
& - b^2*c^2*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 213*a*b^{11}*c*f^2 - 3072*a^5*c^8*d* \\
& e - 2*b^{10}*c^3*d*e + 15360*a^6*c^7*e*f + 6*b^{11}*c^2*d*f + 36*a*b^8*c^4*d*e \\
& - 98*a*b^9*c^3*d*f + 1536*a^5*b*c^7*d*f + 10*a*c^3*d*f*(-(4*a*c - b^2)^9)^{(\\
& 1/2)} - 2*b*c^3*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 152*a*b^{10}*c^2*e*f + 6*b^3*c* \\
& e*f*(-(4*a*c - b^2)^9)^{(1/2)} + 51*a*b^2*c*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 19 \\
& 2*a^2*b^6*c^5*d*e + 128*a^3*b^4*c^6*d*e + 1536*a^4*b^2*c^7*d*e + 576*a^2*b^ \\
& 7*c^4*d*f - 1344*a^3*b^5*c^5*d*f + 512*a^4*b^3*c^6*d*f + 1548*a^2*b^8*c^3*e \\
& *f - 8064*a^3*b^6*c^4*e*f + 22400*a^4*b^4*c^5*e*f - 30720*a^5*b^2*c^6*e*f + \\
& 6*b^2*c^2*d*f*(-(4*a*c - b^2)^9)^{(1/2)} - 44*a*b*c^2*e*f*(-(4*a*c - b^2)^9 \\
&)^{(1/2)))/(32*(4096*a^6*c^11 + b^{12}*c^5 - 24*a*b^{10}*c^6 + 240*a^2*b^8*c^7 - 1 \\
& 280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^{10}))^{(1/2)} - (x*(9*b^8 \\
& *f^2 + 8*a^2*c^6*d^2 - 72*a^3*c^5*e^2 + b^4*c^4*d^2 + 200*a^4*c^4*f^2 + b^6 \\
& *c^2*e^2 + 2*a*b^2*c^5*d^2 - 16*a*b^4*c^3*e^2 - 6*b^7*c*e*f + 74*a^2*b^2*c^ \\
& 4*e^2 + 481*a^2*b^4*c^2*f^2 - 718*a^3*b^2*c^3*f^2 - 114*a*b^6*c*f^2 - 80*a^ \\
& 3*c^5*d*f + 2*b^5*c^3*d*e - 6*b^6*c^2*d*f - 14*a*b^3*c^4*d*e - 8*a^2*b*c^5* \\
& d*e + 32*a*b^4*c^3*d*f + 86*a*b^5*c^2*e*f + 472*a^3*b*c^4*e*f + 4*a^2*b^2*c \\
& ^4*d*f - 374*a^2*b^3*c^3*e*f))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((\\
& 768*a^4*b*c^8*d^2 - b^9*c^4*d^2 - c^4*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - b^{11}*c \\
& ^2*e^2 - 9*b^4*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 9*b^{13}*f^2 + 27*a*b^9*c^3*e^2 \\
& + 3840*a^5*b*c^7*e^2 + 9*a*c^3*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 26880*a^6*b* \\
& c^6*f^2 + 6*b^{12}*c*e*f + 96*a^2*b^5*c^6*d^2 - 512*a^3*b^3*c^7*d^2 - 288*a^2 \\
& *b^7*c^4*e^2 + 1504*a^3*b^5*c^5*e^2 - 3840*a^4*b^3*c^6*e^2 - 2077*a^2*b^9*c \\
& ^2*f^2 + 10656*a^3*b^7*c^3*f^2 - 30240*a^4*b^5*c^4*f^2 + 44800*a^5*b^3*c^5* \\
& f^2 - 25*a^2*c^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - b^2*c^2*e^2*(-(4*a*c - b^2) \\
& ^9)^{(1/2)} + 213*a*b^{11}*c*f^2 - 3072*a^5*c^8*d*e - 2*b^{10}*c^3*d*e + 15360*a^ \\
& 6*c^7*e*f + 6*b^{11}*c^2*d*f + 36*a*b^8*c^4*d*e - 98*a*b^9*c^3*d*f + 1536*a^5 \\
& *b*c^7*d*f + 10*a*c^3*d*f*(-(4*a*c - b^2)^9)^{(1/2)} - 2*b*c^3*d*e*(-(4*a*c - \\
& b^2)^9)^{(1/2)} - 152*a*b^{10}*c^2*e*f + 6*b^3*c*e*f*(-(4*a*c - b^2)^9)^{(1/2)} \\
& + 51*a*b^2*c*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 192*a^2*b^6*c^5*d*e + 128*a^3*b \\
& ^4*c^6*d*e + 1536*a^4*b^2*c^7*d*e + 576*a^2*b^7*c^4*d*f - 1344*a^3*b^5*c^5* \\
& d*f + 512*a^4*b^3*c^6*d*f + 1548*a^2*b^8*c^3*e*f - 8064*a^3*b^6*c^4*e*f + 2
\end{aligned}$$

$$\begin{aligned}
& 2400a^4b^4c^5eef - 30720a^5b^2c^6eef + 6b^2c^2d*ff*(-(4a*c - b^2)^9)^{(1/2)} - 44a*b*c^2e*ff*(-(4a*c - b^2)^9)^{(1/2)} / (32*(4096a^6c^{11} + \\
& b^{12}c^5 - 24a*b^{10}c^6 + 240a^2b^8c^7 - 1280a^3b^6c^8 + 3840a^4b^4c^9 - 6144a^5b^2c^{10}))^{(1/2)} * i - (((10240a^5c^7*ff - 2048a^4c^8*d \\
& - 384a^2b^4c^6*d + 1536a^3b^2c^7*d + 192a^2b^5c^5*e - 768a^3b^3 \\
& *c^6*e - 736a^2b^6c^4*f + 4224a^3b^4c^5*f - 10752a^4b^2c^6*f + 32 \\
& a*b^6c^5*d - 16a*b^7c^4*e + 1024a^4b*c^7*e + 48a*b^8c^3*f) / (8*(64a^3c^6 - b^6c^3 + 12a*b^4c^4 - 48a^2b^2c^5)) + (x*((768a^4b*c^8*d^2 \\
& - b^9c^4*d^2 - c^4*d^2*(-(4a*c - b^2)^9)^{(1/2)} - b^{11}c^2e^2 - 9b^4*f^2 \\
& *(-(4a*c - b^2)^9)^{(1/2)} - 9b^{13}f^2 + 27a*b^9c^3e^2 + 3840a^5b*c^7 \\
& e^2 + 9a*c^3e^2*(-(4a*c - b^2)^9)^{(1/2)} - 26880a^6b*c^6*f^2 + 6b^{12}c \\
& *e*f + 96a^2b^5c^6*d^2 - 512a^3b^3c^7*d^2 - 288a^2b^7c^4e^2 + 150 \\
& 4a^3b^5c^5e^2 - 3840a^4b^3c^6e^2 - 2077a^2b^9c^2*f^2 + 10656a^3 \\
& *b^7c^3*f^2 - 30240a^4b^5c^4*f^2 + 44800a^5b^3c^5*f^2 - 25a^2c^2*f \\
& ^2*(-(4a*c - b^2)^9)^{(1/2)} - b^2c^2e^2*(-(4a*c - b^2)^9)^{(1/2)} + 213a* \\
& b^{11}c*f^2 - 3072a^5c^8*d*e - 2b^{10}c^3*d*e + 15360a^6c^7*e*ff + 6b^{11} \\
& *c^2*d*ff + 36a*b^8c^4*d*e - 98a*b^9c^3*d*ff + 1536a^5b*c^7*d*ff + 10a* \\
& c^3*d*ff*(-(4a*c - b^2)^9)^{(1/2)} - 2b*c^3*d*e*(-(4a*c - b^2)^9)^{(1/2)} - 1 \\
& 52a*b^{10}c^2*e*ff + 6b^3c*e*ff*(-(4a*c - b^2)^9)^{(1/2)} + 51a*b^2c*f^2*(\\
& -(4a*c - b^2)^9)^{(1/2)} - 192a^2b^6c^5*d*e + 128a^3b^4c^6*d*e + 1536* \\
& a^4b^2c^7*d*e + 576a^2b^7c^4*d*ff - 1344a^3b^5c^5*d*ff + 512a^4b^3c^6*d \\
& *ff + 1548a^2b^8c^3e*ff - 8064a^3b^6c^4e*ff + 22400a^4b^4c^5e \\
& *ff - 30720a^5b^2c^6e*ff + 6b^2c^2d*ff*(-(4a*c - b^2)^9)^{(1/2)} - 44a* \\
& b*c^2e*ff*(-(4a*c - b^2)^9)^{(1/2)} / (32*(4096a^6c^{11} + b^{12}c^5 - 24a*b^{10}c^6 + 240a^2b^8c^7 - 1280a^3b^6c^8 + 3840a^4b^4c^9 - 6144a^5b^2c^{10}))^{(1/2)} * (16b^7c^5 - 192a*b^5c^6 - 1024a^3b*c^8 + 768a^2b^3 \\
& *c^7)) / (2*(16a^2c^5 + b^4c^3 - 8a*b^2c^4)) * ((768a^4b*c^8*d^2 - b^9c^4d^2 - c^4d^2*(-(4a*c - b^2)^9)^{(1/2)} - b^{11}c^2e^2 - 9b^4*f^2*(-(4a \\
& *c - b^2)^9)^{(1/2)} - 9b^{13}f^2 + 27a*b^9c^3e^2 + 3840a^5b*c^7e^2 + \\
& 9a*c^3e^2*(-(4a*c - b^2)^9)^{(1/2)} - 26880a^6b*c^6*f^2 + 6b^{12}c*e*ff + \\
& 96a^2b^5c^6*d^2 - 512a^3b^3c^7*d^2 - 288a^2b^7c^4e^2 + 1504a^3b^5c^5e^2 - 3840a^4b^3c^6e^2 - 2077a^2b^9c^2*f^2 + 10656a^3b^7c^3f^2 - 30240a^4b^5c^4f^2 + 44800a^5b^3c^5f^2 - 25a^2c^2f^2*(-(4a*c - b^2)^9)^{(1/2)} - b^2c^2e^2*(-(4a*c - b^2)^9)^{(1/2)} + 213a*b^{11}c \\
& *f^2 - 3072a^5c^8*d*e - 2b^{10}c^3*d*e + 15360a^6c^7*e*ff + 6b^{11}c^2*d \\
& *ff + 36a*b^8c^4*d*e - 98a*b^9c^3*d*ff + 1536a^5b*c^7*d*ff + 10a*c^3*d \\
& *ff*(-(4a*c - b^2)^9)^{(1/2)} - 2b*c^3*d*e*(-(4a*c - b^2)^9)^{(1/2)} - 152a*b \\
& ^{10}c^2*e*ff + 6b^3c*e*ff*(-(4a*c - b^2)^9)^{(1/2)} + 51a*b^2c*f^2*(-(4a* \\
& c - b^2)^9)^{(1/2)} - 192a^2b^6c^5*d*e + 128a^3b^4c^6*d*e + 1536a^4b^2c^7d \\
& *e + 576a^2b^7c^4*d*ff - 1344a^3b^5c^5*d*ff + 512a^4b^3c^6*d* \\
& f + 1548a^2b^8c^3e*ff - 8064a^3b^6c^4e*ff + 22400a^4b^4c^5e*ff - 3 \\
& 0720a^5b^2c^6e*ff + 6b^2c^2d*ff*(-(4a*c - b^2)^9)^{(1/2)} - 44a*b*c^2e \\
& *ff*(-(4a*c - b^2)^9)^{(1/2)} / (32*(4096a^6c^{11} + b^{12}c^5 - 24a*b^{10}c^6 + 240a^2b^8c^7 - 1280a^3b^6c^8 + 3840a^4b^4c^9 - 6144a^5b^2c^{10}))^{(1/2)} + (x*(9b^8f^2 + 8a^2c^6d^2 - 72a^3c^5e^2 + b^4c^4d^2 + \\
& 200a^4c^4f^2 + b^6c^2e^2 + 2a*b^2c^5d^2 - 16a*b^4c^3e^2 - 6b^7 \\
& *c*e*ff + 74a^2b^2c^4e^2 + 481a^2b^4c^2*f^2 - 718a^3b^2c^3f^2 - 1 \\
& 14a*b^6c*f^2 - 80a^3c^5*d*ff + 2b^5c^3*d*e - 6b^6c^2*d*ff - 14a*b^3c \\
& ^4*d*e - 8a^2b*c^5*d*e + 32a*b^4c^3*d*ff + 86a*b^5c^2e*ff + 472a^3b \\
& *c^4e*ff + 4a^2b^2c^4*d*ff - 374a^2b^3c^3e*ff)) / (2*(16a^2c^5 + b^4c^3 - 8a*b^2c^4)) * ((768a^4b*c^8*d^2 - b^9c^4d^2 - c^4d^2*(-(4a*c - b^2)^9)^{(1/2)} - b^{11}c^2e^2 - 9b^4*f^2*(-(4a*c - b^2)^9)^{(1/2)} - 9b^{13}f^2 + 27a*b^9c^3e^2 + 3840a^5b*c^7e^2 + 9a*c^3e^2*(-(4a*c - b^2)^9)^{(1/2)} - 26880a^6b*c^6*f^2 + 6b^{12}c*e*ff + 96a^2b^5c^6*d^2 - 512a^3b^3c^7*d^2 - 288a^2b^7c^4e^2 + 1504a^3b^5c^5e^2 - 3840a^4b^3c^6e^2 - 2077a^2b^9c^2*f^2 + 10656a^3b^7c^3f^2 - 30240a^4b^5c^4f^2 + 44800a^5b^3c^5f^2 - 25a^2c^2f^2*(-(4a*c - b^2)^9)^{(1/2)} - b^2c^2e^2*(-(4a*c - b^2)^9)^{(1/2)} + 213a*b^{11}c*f^2 - 3072a^5c^8*d*e - 2b^{10}c^3*d*e + 15360a^6c^7*e*ff + 6b^{11}c^2*d*ff + 36a*b^8c^4*d*e - 98a*
\end{aligned}$$

$$\begin{aligned}
& b^9 c^3 d^2 f + 1536 a^5 b^3 c^7 d^2 f + 10 a^3 c^3 d^2 f * (-4 a^2 c - b^2)^9)^{(1/2)} - \\
& 2 b^3 c^3 d^2 e * (-4 a^2 c - b^2)^9)^{(1/2)} - 152 a^3 b^10 c^2 e^2 f + 6 b^3 c^3 e^2 f * (- \\
& 4 a^2 c - b^2)^9)^{(1/2)} + 51 a^3 b^2 c^3 f^2 * (-4 a^2 c - b^2)^9)^{(1/2)} - 192 a^2 b^6 \\
& c^5 d^2 e + 128 a^3 b^4 c^6 d^2 e + 1536 a^4 b^2 c^7 d^2 e + 576 a^2 b^7 c^4 d^2 \\
& e f - 1344 a^3 b^5 c^5 d^2 e f + 512 a^4 b^3 c^6 d^2 e f + 1548 a^2 b^8 c^3 e^2 f - 80 \\
& 64 a^3 b^6 c^4 e^2 f + 22400 a^4 b^4 c^5 e^2 f - 30720 a^5 b^2 c^6 e^2 f + 6 b^2 c^2 \\
& d^2 f * (-4 a^2 c - b^2)^9)^{(1/2)} - 44 a^3 b^3 c^2 e^2 f * (-4 a^2 c - b^2)^9)^{(1/2)} \\
& / (32 * (4096 a^6 c^11 + b^12 c^5 - 24 a^3 b^10 c^6 + 240 a^2 b^8 c^7 - 1280 a^3 \\
& b^6 c^8 + 3840 a^4 b^4 c^9 - 6144 a^5 b^2 c^10)))^{(1/2)} * i) / ((63 a^3 b^5 f^3 - \\
& 216 a^4 c^4 e^3 + 3 a^3 b^3 c^4 d^3 + 4 a^2 b^2 c^5 d^3 - 573 a^4 b^3 c^3 f^3 + \\
& 1300 a^5 b^2 c^2 f^3 - 24 a^3 c^5 d^2 e - 45 a^2 b^6 e^2 f - 600 a^5 c^3 e^2 f - \\
& 5 a^2 b^4 c^2 e^3 + 66 a^3 b^2 c^3 e^3 + 27 a^3 b^7 d^2 f^2 + 240 a^4 c^4 d^2 e^2 f + \\
& 6 a^3 b^4 c^3 d^2 e + 3 a^3 b^5 c^2 d^2 e^2 + 204 a^3 b^3 c^4 d^2 e^2 - 18 \\
& a^3 b^5 c^2 d^2 e^2 f - 279 a^2 b^5 c^3 d^2 e^2 f + 12 a^3 b^3 c^4 d^2 e^2 f - 420 a^4 b^3 c^3 \\
& d^2 e^2 f^2 + 30 a^2 b^5 c^3 e^2 f^2 + 402 a^3 b^4 c^3 e^2 f^2 + 924 a^4 b^3 c^3 e^2 f^2 - 4 \\
& 2 a^2 b^2 c^4 d^2 e - 51 a^2 b^3 c^3 d^2 e^2 + 81 a^2 b^3 c^3 d^2 e^2 f + 801 a^3 b^3 c^2 \\
& d^2 e^2 f^2 - 339 a^3 b^3 c^2 e^2 f^2 - 762 a^4 b^2 c^2 e^2 f^2 - 18 a^3 b^6 c^3 \\
& d^2 e^2 f + 246 a^2 b^4 c^2 d^2 e^2 f - 804 a^3 b^2 c^3 d^2 e^2 f) / (4 * (64 a^3 c^6 - b^6 \\
& c^3 + 12 a^2 b^4 c^4 - 48 a^2 b^2 c^5)) + (((10240 a^5 c^7 f - 2048 a^4 c^8 \\
& d - 384 a^2 b^4 c^6 d + 1536 a^3 b^2 c^7 d + 192 a^2 b^5 c^5 e - 768 a^3 b^3 c^6 e - \\
& 736 a^2 b^6 c^4 f + 4224 a^3 b^4 c^5 f - 10752 a^4 b^2 c^6 f + 32 a^2 b^6 c^5 d - \\
& 16 a^2 b^7 c^4 e + 1024 a^4 b^3 c^7 e + 48 a^2 b^8 c^3 f) / (8 * (64 a^3 c^6 - b^6 c^3 + \\
& 12 a^2 b^4 c^4 - 48 a^2 b^2 c^5)) - (x * ((768 a^4 b^3 c^8 d^2 - b^9 c^4 d^2 - c^4 d^2 * \\
& (-4 a^2 c - b^2)^9)^{(1/2)} - b^11 c^2 e^2 - 9 b^4 f^2 * (-4 a^2 c - b^2)^9)^{(1/2)} - \\
& 9 b^13 f^2 + 27 a^3 b^9 c^3 e^2 + 3840 a^5 b^3 c^7 e^2 + 9 a^3 c^3 e^2 * (-4 a^2 c - \\
& b^2)^9)^{(1/2)} - 26880 a^6 b^3 c^6 f^2 + 6 b^12 c^3 e^2 f + 96 a^2 b^5 c^6 d^2 - \\
& 512 a^3 b^3 c^7 d^2 - 288 a^2 b^7 c^4 e^2 + 1504 a^3 b^5 c^5 e^2 - 3840 a^4 b^3 c^6 e^2 - \\
& 2077 a^2 b^9 c^2 f^2 + 10656 a^3 b^7 c^3 f^2 - 30240 a^4 b^5 c^4 f^2 + 44800 a^5 b^3 c^5 f^2 - \\
& 25 a^2 c^2 f^2 * (-4 a^2 c - b^2)^9)^{(1/2)} - b^2 c^2 e^2 * (-4 a^2 c - b^2)^9)^{(1/2)} + \\
& 213 a^3 b^11 c^3 f^2 - 3072 a^5 c^8 d^2 e - 2 b^10 c^3 d^2 e + 15360 a^6 c^7 e^2 f + 6 b^11 \\
& c^2 d^2 e f + 36 a^3 b^8 c^4 d^2 e - 98 a^3 b^9 c^3 d^2 e f + 1536 a^5 b^3 c^7 d^2 e f + 10 \\
& a^3 c^3 d^2 e f * (-4 a^2 c - b^2)^9)^{(1/2)} - 2 b^3 c^3 d^2 e * (-4 a^2 c - b^2)^9)^{(1/2)} - \\
& 152 a^3 b^10 c^2 e^2 f + 6 b^3 c^3 e^2 f * (-4 a^2 c - b^2)^9)^{(1/2)} + 51 a^3 b^2 c^3 f^2 * \\
& (-4 a^2 c - b^2)^9)^{(1/2)} - 192 a^2 b^6 c^5 d^2 e + 128 a^3 b^4 c^6 d^2 e + 153 \\
& 6 a^4 b^2 c^7 d^2 e + 576 a^2 b^7 c^4 d^2 e f - 1344 a^3 b^5 c^5 d^2 e f + 512 a^4 b^3 \\
& c^6 d^2 e f + 1548 a^2 b^8 c^3 e^2 f - 8064 a^3 b^6 c^4 e^2 f + 22400 a^4 b^4 c^5 e^2 \\
& f - 30720 a^5 b^2 c^6 e^2 f + 6 b^2 c^2 d^2 e f * (-4 a^2 c - b^2)^9)^{(1/2)} - 44 \\
& a^3 b^3 c^2 e^2 f * (-4 a^2 c - b^2)^9)^{(1/2)} / (32 * (4096 a^6 c^11 + b^12 c^5 - 24 a^3 \\
& b^10 c^6 + 240 a^2 b^8 c^7 - 1280 a^3 b^6 c^8 + 3840 a^4 b^4 c^9 - 6144 a^5 b^2 c^10)))^{(1/2)} \\
& * (16 b^7 c^5 - 192 a^3 b^5 c^6 - 1024 a^3 b^3 c^8 + 768 a^2 b^3 c^7) / (2 * (16 a^2 c^5 + \\
& b^4 c^3 - 8 a^2 b^2 c^4)) * ((768 a^4 b^3 c^8 d^2 - b^9 c^4 d^2 - c^4 d^2 * (-4 a^2 c - \\
& b^2)^9)^{(1/2)} - b^11 c^2 e^2 - 9 b^4 f^2 * (-4 a^2 c - b^2)^9)^{(1/2)} - 9 b^13 f^2 + \\
& 27 a^3 b^9 c^3 e^2 + 3840 a^5 b^3 c^7 e^2 + 9 a^3 c^3 e^2 * (-4 a^2 c - b^2)^9)^{(1/2)} - \\
& 26880 a^6 b^3 c^6 f^2 + 6 b^12 c^3 e^2 f + 96 a^2 b^5 c^6 d^2 - 512 a^3 b^3 c^7 d^2 - \\
& 288 a^2 b^7 c^4 e^2 + 1504 a^3 b^5 c^5 e^2 - 3840 a^4 b^3 c^6 e^2 - 2077 a^2 b^9 c^2 f^2 + \\
& 10656 a^3 b^7 c^3 f^2 - 30240 a^4 b^5 c^4 f^2 + 44800 a^5 b^3 c^5 f^2 - 25 a^2 c^2 f^2 * \\
& (-4 a^2 c - b^2)^9)^{(1/2)} - b^2 c^2 e^2 * (-4 a^2 c - b^2)^9)^{(1/2)} + 213 a^3 b^11 \\
& c^3 f^2 - 3072 a^5 c^8 d^2 e - 2 b^10 c^3 d^2 e + 15360 a^6 c^7 e^2 f + 6 b^11 c^2 \\
& d^2 e f + 36 a^3 b^8 c^4 d^2 e - 98 a^3 b^9 c^3 d^2 e f + 1536 a^5 b^3 c^7 d^2 e f + 10 \\
& a^3 c^3 d^2 e f * (-4 a^2 c - b^2)^9)^{(1/2)} - 2 b^3 c^3 d^2 e * (-4 a^2 c - b^2)^9)^{(1/2)} - \\
& 152 a^3 b^10 c^2 e^2 f + 6 b^3 c^3 e^2 f * (-4 a^2 c - b^2)^9)^{(1/2)} + 51 a^3 b^2 c^3 f^2 * \\
& (-4 a^2 c - b^2)^9)^{(1/2)} - 192 a^2 b^6 c^5 d^2 e + 128 a^3 b^4 c^6 d^2 e + 1536 a^4 b^2 \\
& c^7 d^2 e + 576 a^2 b^7 c^4 d^2 e f - 1344 a^3 b^5 c^5 d^2 e f + 512 a^4 b^3 c^6 d^2 \\
& e f + 1548 a^2 b^8 c^3 e^2 f - 8064 a^3 b^6 c^4 e^2 f + 22400 a^4 b^4 c^5 e^2 f - \\
& 30720 a^5 b^2 c^6 e^2 f + 6 b^2 c^2 d^2 e f * (-4 a^2 c - b^2)^9)^{(1/2)} - 44 a^3 b^3 c^2 \\
& e^2 f * (-4 a^2 c - b^2)^9)^{(1/2)} / (32 * (4096 a^6 c^11 + b^12 c^5 - 24 a^3 b^10 c^6 + \\
& 240 a^2 b^8 c^7 - 1280 a^3 b^6 c^8 + 3840 a^4 b^4 c^9 - 6144 a^5 b^2 c^10)))^{(1/2)}
\end{aligned}$$

$$\begin{aligned}
& \dots)^{(1/2)} - (x*(9*b^8*f^2 + 8*a^2*c^6*d^2 - 72*a^3*c^5*e^2 + b^4*c^4*d^2 \\
& + 200*a^4*c^4*f^2 + b^6*c^2*e^2 + 2*a*b^2*c^5*d^2 - 16*a*b^4*c^3*e^2 - 6*b \\
& ^7*c*e*f + 74*a^2*b^2*c^4*e^2 + 481*a^2*b^4*c^2*f^2 - 718*a^3*b^2*c^3*f^2 - \\
& 114*a*b^6*c*f^2 - 80*a^3*c^5*d*f + 2*b^5*c^3*d*e - 6*b^6*c^2*d*f - 14*a*b^ \\
& 3*c^4*d*e - 8*a^2*b*c^5*d*e + 32*a*b^4*c^3*d*f + 86*a*b^5*c^2*e*f + 472*a^3 \\
& *b*c^4*e*f + 4*a^2*b^2*c^4*d*f - 374*a^2*b^3*c^3*e*f))/(2*(16*a^2*c^5 + b^4 \\
& *c^3 - 8*a*b^2*c^4)))*((768*a^4*b*c^8*d^2 - b^9*c^4*d^2 - c^4*d^2*(-(4*a*c \\
& - b^2)^9)^{(1/2)} - b^{11}*c^2*e^2 - 9*b^4*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 9*b^1 \\
& 3*f^2 + 27*a*b^9*c^3*e^2 + 3840*a^5*b*c^7*e^2 + 9*a*c^3*e^2*(-(4*a*c - b^2) \\
& ^9)^{(1/2)} - 26880*a^6*b*c^6*f^2 + 6*b^12*c*e*f + 96*a^2*b^5*c^6*d^2 - 512*a \\
& ^3*b^3*c^7*d^2 - 288*a^2*b^7*c^4*e^2 + 1504*a^3*b^5*c^5*e^2 - 3840*a^4*b^3* \\
& c^6*e^2 - 2077*a^2*b^9*c^2*f^2 + 10656*a^3*b^7*c^3*f^2 - 30240*a^4*b^5*c^4* \\
& f^2 + 44800*a^5*b^3*c^5*f^2 - 25*a^2*c^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - b^2 \\
& *c^2*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 213*a*b^{11}*c*f^2 - 3072*a^5*c^8*d*e - 2 \\
& *b^{10}*c^3*d*e + 15360*a^6*c^7*e*f + 6*b^{11}*c^2*d*f + 36*a*b^8*c^4*d*e - 98* \\
& a*b^9*c^3*d*f + 1536*a^5*b*c^7*d*f + 10*a*c^3*d*f*(-(4*a*c - b^2)^9)^{(1/2)} \\
& - 2*b*c^3*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 152*a*b^{10}*c^2*e*f + 6*b^3*c*e*f*(\\
& -(4*a*c - b^2)^9)^{(1/2)} + 51*a*b^2*c*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 192*a^2 \\
& *b^6*c^5*d*e + 128*a^3*b^4*c^6*d*e + 1536*a^4*b^2*c^7*d*e + 576*a^2*b^7*c^4 \\
& *d*f - 1344*a^3*b^5*c^5*d*f + 512*a^4*b^3*c^6*d*f + 1548*a^2*b^8*c^3*e*f - \\
& 8064*a^3*b^6*c^4*e*f + 22400*a^4*b^4*c^5*e*f - 30720*a^5*b^2*c^6*e*f + 6*b^ \\
& 2*c^2*d*f*(-(4*a*c - b^2)^9)^{(1/2)} - 44*a*b*c^2*e*f*(-(4*a*c - b^2)^9)^{(1/2)} \\
&))/(32*(4096*a^6*c^{11} + b^{12}*c^5 - 24*a*b^{10}*c^6 + 240*a^2*b^8*c^7 - 1280*a \\
& ^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^{10}))^{(1/2)} + (((10240*a^5*c \\
& ^7*f - 2048*a^4*c^8*d - 384*a^2*b^4*c^6*d + 1536*a^3*b^2*c^7*d + 192*a^2*b^ \\
& 5*c^5*e - 768*a^3*b^3*c^6*e - 736*a^2*b^6*c^4*f + 4224*a^3*b^4*c^5*f - 1075 \\
& 2*a^4*b^2*c^6*f + 32*a*b^6*c^5*d - 16*a*b^7*c^4*e + 1024*a^4*b*c^7*e + 48*a \\
& *b^8*c^3*f)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) + (x \\
& *((768*a^4*b*c^8*d^2 - b^9*c^4*d^2 - c^4*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - b^{1 \\
& 1}*c^2*e^2 - 9*b^4*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 9*b^{13}*f^2 + 27*a*b^9*c^3* \\
& e^2 + 3840*a^5*b*c^7*e^2 + 9*a*c^3*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 26880*a^6 \\
& *b*c^6*f^2 + 6*b^12*c*e*f + 96*a^2*b^5*c^6*d^2 - 512*a^3*b^3*c^7*d^2 - 288* \\
& a^2*b^7*c^4*e^2 + 1504*a^3*b^5*c^5*e^2 - 3840*a^4*b^3*c^6*e^2 - 2077*a^2*b^ \\
& 9*c^2*f^2 + 10656*a^3*b^7*c^3*f^2 - 30240*a^4*b^5*c^4*f^2 + 44800*a^5*b^3*c \\
& ^5*f^2 - 25*a^2*c^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - b^2*c^2*e^2*(-(4*a*c - b \\
& ^2)^9)^{(1/2)} + 213*a*b^{11}*c*f^2 - 3072*a^5*c^8*d*e - 2*b^{10}*c^3*d*e + 15360 \\
& *a^6*c^7*e*f + 6*b^{11}*c^2*d*f + 36*a*b^8*c^4*d*e - 98*a*b^9*c^3*d*f + 1536* \\
& a^5*b*c^7*d*f + 10*a*c^3*d*f*(-(4*a*c - b^2)^9)^{(1/2)} - 2*b*c^3*d*e*(-(4*a* \\
& c - b^2)^9)^{(1/2)} - 152*a*b^{10}*c^2*e*f + 6*b^3*c*e*f*(-(4*a*c - b^2)^9)^{(1/ \\
& 2)} + 51*a*b^2*c*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 192*a^2*b^6*c^5*d*e + 128*a^ \\
& 3*b^4*c^6*d*e + 1536*a^4*b^2*c^7*d*e + 576*a^2*b^7*c^4*d*f - 1344*a^3*b^5*c \\
& ^5*d*f + 512*a^4*b^3*c^6*d*f + 1548*a^2*b^8*c^3*e*f - 8064*a^3*b^6*c^4*e*f \\
& + 22400*a^4*b^4*c^5*e*f - 30720*a^5*b^2*c^6*e*f + 6*b^2*c^2*d*f*(-(4*a*c - \\
& b^2)^9)^{(1/2)} - 44*a*b*c^2*e*f*(-(4*a*c - b^2)^9)^{(1/2)}))/(32*(4096*a^6*c^{11} \\
& + b^{12}*c^5 - 24*a*b^{10}*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4 \\
& *b^4*c^9 - 6144*a^5*b^2*c^{10}))^{(1/2)}*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^ \\
& 3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((768 \\
& *a^4*b*c^8*d^2 - b^9*c^4*d^2 - c^4*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - b^{11}*c^2* \\
& e^2 - 9*b^4*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 9*b^{13}*f^2 + 27*a*b^9*c^3*e^2 + \\
& 3840*a^5*b*c^7*e^2 + 9*a*c^3*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 26880*a^6*b*c^6 \\
& *f^2 + 6*b^12*c*e*f + 96*a^2*b^5*c^6*d^2 - 512*a^3*b^3*c^7*d^2 - 288*a^2*b^ \\
& 7*c^4*e^2 + 1504*a^3*b^5*c^5*e^2 - 3840*a^4*b^3*c^6*e^2 - 2077*a^2*b^9*c^2* \\
& f^2 + 10656*a^3*b^7*c^3*f^2 - 30240*a^4*b^5*c^4*f^2 + 44800*a^5*b^3*c^5*f^2 \\
& - 25*a^2*c^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - b^2*c^2*e^2*(-(4*a*c - b^2)^9) \\
& ^{(1/2)} + 213*a*b^{11}*c*f^2 - 3072*a^5*c^8*d*e - 2*b^{10}*c^3*d*e + 15360*a^6*c \\
& ^7*e*f + 6*b^{11}*c^2*d*f + 36*a*b^8*c^4*d*e - 98*a*b^9*c^3*d*f + 1536*a^5*b* \\
& c^7*d*f + 10*a*c^3*d*f*(-(4*a*c - b^2)^9)^{(1/2)} - 2*b*c^3*d*e*(-(4*a*c - b^ \\
& 2)^9)^{(1/2)} - 152*a*b^{10}*c^2*e*f + 6*b^3*c*e*f*(-(4*a*c - b^2)^9)^{(1/2)} + 5 \\
& 1*a*b^2*c*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 192*a^2*b^6*c^5*d*e + 128*a^3*b^4*
\end{aligned}$$

$$\begin{aligned}
& c^6 d e + 1536 a^4 b^2 c^7 d e + 576 a^2 b^7 c^4 d f - 1344 a^3 b^5 c^5 d f \\
& + 512 a^4 b^3 c^6 d f + 1548 a^2 b^8 c^3 e f - 8064 a^3 b^6 c^4 e f + 2240 \\
& 0 a^4 b^4 c^5 e f - 30720 a^5 b^2 c^6 e f + 6 b^2 c^2 d f (-4 a c - b^2)^9 \\
&)^{(1/2)} - 44 a b c^2 e f (-4 a c - b^2)^9)^{(1/2)} / (32 (4096 a^6 c^{11} + b^{12} \\
& 2 c^5 - 24 a b^{10} c^6 + 240 a^2 b^8 c^7 - 1280 a^3 b^6 c^8 + 3840 a^4 b^4 c^9 \\
& ^9 - 6144 a^5 b^2 c^{10}))^{(1/2)} + (x (9 b^8 f^2 + 8 a^2 c^6 d^2 - 72 a^3 c^5 \\
& 5 e^2 + b^4 c^4 d^2 + 200 a^4 c^4 f^2 + b^6 c^2 e^2 + 2 a b^2 c^5 d^2 - 16 a \\
& a b^4 c^3 e^2 - 6 b^7 c e f + 74 a^2 b^2 c^4 e^2 + 481 a^2 b^4 c^2 f^2 - 71 \\
& 8 a^3 b^2 c^3 f^2 - 114 a b^6 c f^2 - 80 a^3 c^5 d f + 2 b^5 c^3 d e - 6 b^6 \\
& 6 c^2 d f - 14 a b^3 c^4 d e - 8 a^2 b c^5 d e + 32 a b^4 c^3 d f + 86 a b^5 \\
& 5 c^2 e f + 472 a^3 b c^4 e f + 4 a^2 b^2 c^4 d f - 374 a^2 b^3 c^3 e f) / (\\
& 2 (16 a^2 c^5 + b^4 c^3 - 8 a b^2 c^4)) * ((768 a^4 b c^8 d^2 - b^9 c^4 d^2 \\
& - c^4 d^2 (-4 a c - b^2)^9)^{(1/2)} - b^{11} c^2 e^2 - 9 b^4 f^2 (-4 a c - b^2)^9)^{(1/2)} \\
& - 9 b^{13} f^2 + 27 a b^9 c^3 e^2 + 3840 a^5 b c^7 e^2 + 9 a c^3 e^2 (-4 a c - b^2)^9)^{(1/2)} \\
& - 26880 a^6 b c^6 f^2 + 6 b^{12} c e f + 96 a^2 b^5 c^6 d^2 - 512 a^3 b^3 c^7 d^2 - \\
& 288 a^2 b^7 c^4 e^2 + 1504 a^3 b^5 c^5 e^2 - 3840 a^4 b^3 c^6 e^2 - 2077 a^2 b^9 c^2 f^2 \\
& + 10656 a^3 b^7 c^3 f^2 - 30240 a^4 b^5 c^4 f^2 + 44800 a^5 b^3 c^5 f^2 - 25 a^2 c^2 f^2 (-4 a c \\
& - b^2)^9)^{(1/2)} - b^2 c^2 e^2 (-4 a c - b^2)^9)^{(1/2)} + 213 a b^{11} c f^2 - 3 \\
& 072 a^5 c^8 d e - 2 b^{10} c^3 d e + 15360 a^6 c^7 e f + 6 b^{11} c^2 d f + 36 a \\
& a b^8 c^4 d e - 98 a b^9 c^3 d f + 1536 a^5 b c^7 d f + 10 a c^3 d f (-4 a c \\
& c - b^2)^9)^{(1/2)} - 2 b c^3 d e (-4 a c - b^2)^9)^{(1/2)} - 152 a b^{10} c^2 \\
& e f + 6 b^3 c e f (-4 a c - b^2)^9)^{(1/2)} + 51 a b^2 c f^2 (-4 a c - b^2)^9)^{(1/2)} \\
& - 192 a^2 b^6 c^5 d e + 128 a^3 b^4 c^6 d e + 1536 a^4 b^2 c^7 d e \\
& e + 576 a^2 b^7 c^4 d f - 1344 a^3 b^5 c^5 d f + 512 a^4 b^3 c^6 d f + 1548 \\
& a^2 b^8 c^3 e f - 8064 a^3 b^6 c^4 e f + 22400 a^4 b^4 c^5 e f - 30720 a^5 \\
& b^2 c^6 e f + 6 b^2 c^2 d f (-4 a c - b^2)^9)^{(1/2)} - 44 a b c^2 e f (-4 \\
& a c - b^2)^9)^{(1/2)} / (32 (4096 a^6 c^{11} + b^{12} c^5 - 24 a b^{10} c^6 + 240 a \\
& ^2 b^8 c^7 - 1280 a^3 b^6 c^8 + 3840 a^4 b^4 c^9 - 6144 a^5 b^2 c^{10}))^{(1/2)} \\
&)) * ((768 a^4 b c^8 d^2 - b^9 c^4 d^2 - c^4 d^2 (-4 a c - b^2)^9)^{(1/2)} - \\
& b^{11} c^2 e^2 - 9 b^4 f^2 (-4 a c - b^2)^9)^{(1/2)} - 9 b^{13} f^2 + 27 a b^9 c^3 \\
& e^2 + 3840 a^5 b c^7 e^2 + 9 a c^3 e^2 (-4 a c - b^2)^9)^{(1/2)} - 26880 \\
& a^6 b c^6 f^2 + 6 b^{12} c e f + 96 a^2 b^5 c^6 d^2 - 512 a^3 b^3 c^7 d^2 - \\
& 288 a^2 b^7 c^4 e^2 + 1504 a^3 b^5 c^5 e^2 - 3840 a^4 b^3 c^6 e^2 - 2077 a^2 \\
& b^9 c^2 f^2 + 10656 a^3 b^7 c^3 f^2 - 30240 a^4 b^5 c^4 f^2 + 44800 a^5 b \\
& ^3 c^5 f^2 - 25 a^2 c^2 f^2 (-4 a c - b^2)^9)^{(1/2)} - b^2 c^2 e^2 (-4 a c \\
& - b^2)^9)^{(1/2)} + 213 a b^{11} c f^2 - 3072 a^5 c^8 d e - 2 b^{10} c^3 d e + 1 \\
& 5360 a^6 c^7 e f + 6 b^{11} c^2 d f + 36 a b^8 c^4 d e - 98 a b^9 c^3 d f + 1 \\
& 536 a^5 b c^7 d f + 10 a c^3 d f (-4 a c - b^2)^9)^{(1/2)} - 2 b c^3 d e (- \\
& 4 a c - b^2)^9)^{(1/2)} - 152 a b^{10} c^2 e f + 6 b^3 c e f (-4 a c - b^2)^9)^{(1/2)} \\
& + 51 a b^2 c f^2 (-4 a c - b^2)^9)^{(1/2)} - 192 a^2 b^6 c^5 d e + 12 \\
& 8 a^3 b^4 c^6 d e + 1536 a^4 b^2 c^7 d e + 576 a^2 b^7 c^4 d f - 1344 a^3 b \\
& ^5 c^5 d f + 512 a^4 b^3 c^6 d f + 1548 a^2 b^8 c^3 e f - 8064 a^3 b^6 c^4 e \\
& e f + 22400 a^4 b^4 c^5 e f - 30720 a^5 b^2 c^6 e f + 6 b^2 c^2 d f (-4 a c \\
& c - b^2)^9)^{(1/2)} - 44 a b c^2 e f (-4 a c - b^2)^9)^{(1/2)} / (32 (4096 a^6 \\
& c^{11} + b^{12} c^5 - 24 a b^{10} c^6 + 240 a^2 b^8 c^7 - 1280 a^3 b^6 c^8 + 3840 \\
& a^4 b^4 c^9 - 6144 a^5 b^2 c^{10}))^{(1/2)} * 2i - ((x^3 (b^3 f + 2 a c^2 e + b \\
& c^2 d - b^2 c e - 3 a b c f)) / (2 (4 a c - b^2))) + (x (2 a c^2 d + a b^2 f \\
& - 2 a^2 c f - a b c e)) / (2 (4 a c - b^2))) / (a c^2 + c^3 x^4 + b c^2 x^2) - \\
& \operatorname{atan}((((10240 a^5 c^7 f - 2048 a^4 c^8 d - 384 a^2 b^4 c^6 d + 1536 a^3 b^2 \\
& 2 c^7 d + 192 a^2 b^5 c^5 e - 768 a^3 b^3 c^6 e - 736 a^2 b^6 c^4 f + 4224 a \\
& ^3 b^4 c^5 f - 10752 a^4 b^2 c^6 f + 32 a b^6 c^5 d - 16 a b^7 c^4 e + 102 \\
& 4 a^4 b c^7 e + 48 a b^8 c^3 f) / (8 (64 a^3 c^6 - b^6 c^3 + 12 a b^4 c^4 - 4 \\
& 8 a^2 b^2 c^5)) - (x ((c^4 d^2 (-4 a c - b^2)^9)^{(1/2)} - b^9 c^4 d^2 - 9 b \\
& ^{13} f^2 - b^{11} c^2 e^2 + 9 b^4 f^2 (-4 a c - b^2)^9)^{(1/2)} + 768 a^4 b c^8 \\
& d^2 + 27 a b^9 c^3 e^2 + 3840 a^5 b c^7 e^2 - 9 a c^3 e^2 (-4 a c - b^2)^9)^{(1/2)} \\
& - 26880 a^6 b c^6 f^2 + 6 b^{12} c e f + 96 a^2 b^5 c^6 d^2 - 512 a^3 b^3 c^7 d^2 - \\
& 288 a^2 b^7 c^4 e^2 + 1504 a^3 b^5 c^5 e^2 - 3840 a^4 b^3 c^6 e^2 - 2077 a^2 b^9 c^2 f^2 \\
& + 10656 a^3 b^7 c^3 f^2 - 30240 a^4 b^5 c^4 f^2 - 30240 a^4 b^5 c^4 f^2
\end{aligned}$$

$$\begin{aligned}
& \quad ^2 + 44800*a^5*b^3*c^5*f^2 + 25*a^2*c^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + b^2*c^2*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 213*a*b^11*c*f^2 - 3072*a^5*c^8*d*e - 2*b^10*c^3*d*e + 15360*a^6*c^7*e*f + 6*b^11*c^2*d*f + 36*a*b^8*c^4*d*e - 98*a*b^9*c^3*d*f + 1536*a^5*b*c^7*d*f - 10*a*c^3*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + \\
& \quad 2*b*c^3*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 152*a*b^10*c^2*e*f - 6*b^3*c*e*f*(-(4*a*c - b^2)^9)^{(1/2)} - 51*a*b^2*c*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 192*a^2*b^6*c^5*d*e + 128*a^3*b^4*c^6*d*e + 1536*a^4*b^2*c^7*d*e + 576*a^2*b^7*c^4*d*f - 1344*a^3*b^5*c^5*d*f + 512*a^4*b^3*c^6*d*f + 1548*a^2*b^8*c^3*e*f - 8064*a^3*b^6*c^4*e*f + 22400*a^4*b^4*c^5*e*f - 30720*a^5*b^2*c^6*e*f - 6*b^2*c^2*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 44*a*b*c^2*e*f*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^{(1/2)}*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4))*((c^4*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - b^9*c^4*d^2 - 9*b^13*f^2 - b^11*c^2*e^2 + 9*b^4*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + 768*a^4*b*c^8*d^2 + 27*a*b^9*c^3*e^2 + 3840*a^5*b*c^7*e^2 - 9*a*c^3*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 26880*a^6*b*c^6*f^2 + 6*b^12*c*e*f + 96*a^2*b^5*c^6*d^2 - 512*a^3*b^3*c^7*d^2 - 288*a^2*b^7*c^4*e^2 + 1504*a^3*b^5*c^5*e^2 - 3840*a^4*b^3*c^6*e^2 - 2077*a^2*b^9*c^2*f^2 + 10656*a^3*b^7*c^3*f^2 - 30240*a^4*b^5*c^4*f^2 + 44800*a^5*b^3*c^5*f^2 + 25*a^2*c^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + b^2*c^2*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 213*a*b^11*c*f^2 - 3072*a^5*c^8*d*e - 2*b^10*c^3*d*e + 15360*a^6*c^7*e*f + 6*b^11*c^2*d*f + 36*a*b^8*c^4*d*e - 98*a*b^9*c^3*d*f + 1536*a^5*b*c^7*d*f - 10*a*c^3*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 2*b*c^3*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 152*a*b^10*c^2*e*f - 6*b^3*c*e*f*(-(4*a*c - b^2)^9)^{(1/2)} - 51*a*b^2*c*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 192*a^2*b^6*c^5*d*e + 128*a^3*b^4*c^6*d*e + 1536*a^4*b^2*c^7*d*e + 576*a^2*b^7*c^4*d*f - 1344*a^3*b^5*c^5*d*f + 512*a^4*b^3*c^6*d*f + 1548*a^2*b^8*c^3*e*f - 8064*a^3*b^6*c^4*e*f + 22400*a^4*b^4*c^5*e*f - 30720*a^5*b^2*c^6*e*f - 6*b^2*c^2*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 44*a*b*c^2*e*f*(-(4*a*c - b^2)^9)^{(1/2)))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^{(1/2)} - (x*(9*b^8*f^2 + 8*a^2*c^6*d^2 - 72*a^3*c^5*e^2 + b^4*c^4*d^2 + 200*a^4*c^4*f^2 + b^6*c^2*e^2 + 2*a*b^2*c^5*d^2 - 16*a*b^4*c^3*e^2 - 6*b^7*c*e*f + 74*a^2*b^2*c^4*e^2 + 481*a^2*b^4*c^2*f^2 - 718*a^3*b^2*c^3*f^2 - 114*a*b^6*c*f^2 - 80*a^3*c^5*d*f + 2*b^5*c^3*d*e - 6*b^6*c^2*d*f - 14*a*b^3*c^4*d*e - 8*a^2*b*c^5*d*e + 32*a*b^4*c^3*d*f + 86*a*b^5*c^2*e*f + 472*a^3*b*c^4*e*f + 4*a^2*b^2*c^4*d*f - 374*a^2*b^3*c^3*e*f))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4))*((c^4*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - b^9*c^4*d^2 - 9*b^13*f^2 - b^11*c^2*e^2 + 9*b^4*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + 768*a^4*b*c^8*d^2 + 27*a*b^9*c^3*e^2 + 3840*a^5*b*c^7*e^2 - 9*a*c^3*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 26880*a^6*b*c^6*f^2 + 6*b^12*c*e*f + 96*a^2*b^5*c^6*d^2 - 512*a^3*b^3*c^7*d^2 - 288*a^2*b^7*c^4*e^2 + 1504*a^3*b^5*c^5*e^2 - 3840*a^4*b^3*c^6*e^2 - 2077*a^2*b^9*c^2*f^2 + 10656*a^3*b^7*c^3*f^2 - 30240*a^4*b^5*c^4*f^2 + 44800*a^5*b^3*c^5*f^2 + 25*a^2*c^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + b^2*c^2*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 213*a*b^11*c*f^2 - 3072*a^5*c^8*d*e - 2*b^10*c^3*d*e + 15360*a^6*c^7*e*f + 6*b^11*c^2*d*f + 36*a*b^8*c^4*d*e - 98*a*b^9*c^3*d*f + 1536*a^5*b*c^7*d*f - 10*a*c^3*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 2*b*c^3*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 152*a*b^10*c^2*e*f - 6*b^3*c*e*f*(-(4*a*c - b^2)^9)^{(1/2)} - 51*a*b^2*c*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 192*a^2*b^6*c^5*d*e + 128*a^3*b^4*c^6*d*e + 1536*a^4*b^2*c^7*d*e + 576*a^2*b^7*c^4*d*f - 1344*a^3*b^5*c^5*d*f + 512*a^4*b^3*c^6*d*f + 1548*a^2*b^8*c^3*e*f - 8064*a^3*b^6*c^4*e*f + 22400*a^4*b^4*c^5*e*f - 30720*a^5*b^2*c^6*e*f - 6*b^2*c^2*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 44*a*b*c^2*e*f*(-(4*a*c - b^2)^9)^{(1/2)))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^{(1/2)}*i1 - (((10240*a^5*c^7*f - 2048*a^4*c^8*d - 384*a^2*b^4*c^6*d + 1536*a^3*b^2*c^7*d + 192*a^2*b^5*c^5*e - 768*a^3*b^3*c^6*e - 736*a^2*b^6*c^4*f + 4224*a^3*b^4*c^5*f - 10752*a^4*b^2*c^6*f + 32*a*b^6*c^5*d - 16*a*b^7*c^4*e + 1024*a^4*b*c^7*e + 48*a*b^8*c^3*f)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) + (x*((c^4*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - b^9*c^4*d^2 - 9*b^13*f^2 - b^11*c^2*e^2 + 9*b^4*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + 768*a^4*b*c^8*d^2 + 27*a*b^9*c^3*e^2 + 3840*a^5*b*c^7*e^2 - 9*a*c^3*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 26880*a^6*b*c^6*f^2 + 6*b^12*c*e*f + 96*a^2*b^5*c^6*d^2 - 512*a^3*b^3*c^7*d^2 - 288*a^2*b^7*c^4*e^2 + 1504*a^3*b^5*c^5*e^2 - 3840*a^4*b^3*c^6*e^2 - 2077*a^2*b^9*c^2*f^2 + 10656*a^3*b^7*c^3*f^2 - 30240*a^4*b^5*c^4*f^2 + 44800*a^5*b^3*c^5*f^2 + 25*a^2*c^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + b^2*c^2*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 213*a*b^11*c*f^2 - 3072*a^5*c^8*d*e - 2*b^10*c^3*d*e + 15360*a^6*c^7*e*f + 6*b^11*c^2*d*f + 36*a*b^8*c^4*d*e - 98*a*b^9*c^3*d*f + 1536*a^5*b*c^7*d*f - 10*a*c^3*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 2*b*c^3*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 152*a*b^10*c^2*e*f - 6*b^3*c*e*f*(-(4*a*c - b^2)^9)^{(1/2)} - 51*a*b^2*c*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 192*a^2*b^6*c^5*d*e + 128*a^3*b^4*c^6*d*e + 1536*a^4*b^2*c^7*d*e + 576*a^2*b^7*c^4*d*f - 1344*a^3*b^5*c^5*d*f + 512*a^4*b^3*c^6*d*f + 1548*a^2*b^8*c^3*e*f - 8064*a^3*b^6*c^4*e*f + 22400*a^4*b^4*c^5*e*f - 30720*a^5*b^2*c^6*e*f - 6*b^2*c^2*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 44*a*b*c^2*e*f*(-(4*a*c - b^2)^9)^{(1/2)))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^{(1/2)}
\end{aligned}$$

$$\begin{aligned}
& 2) - b^9 c^4 d^2 - 9 b^{13} f^2 - b^{11} c^2 e^2 + 9 b^4 f^2 (-4 a c - b^2)^9 \\
& ^{(1/2)} + 768 a^4 b c^8 d^2 + 27 a b^9 c^3 e^2 + 3840 a^5 b c^7 e^2 - 9 a c^3 e^2 (-4 a c - b^2)^9)^{(1/2)} - 26880 a^6 b c^6 f^2 + 6 b^{12} c e f + 96 a^2 b^5 c^6 d^2 - 512 a^3 b^3 c^7 d^2 - 288 a^2 b^7 c^4 e^2 + 1504 a^3 b^5 c^5 e^2 - 3840 a^4 b^3 c^6 e^2 - 2077 a^2 b^9 c^2 f^2 + 10656 a^3 b^7 c^3 f^2 - 30240 a^4 b^5 c^4 f^2 + 44800 a^5 b^3 c^5 f^2 + 25 a^2 c^2 f^2 (-4 a c - b^2)^9)^{(1/2)} + b^2 c^2 e^2 (-4 a c - b^2)^9)^{(1/2)} + 213 a b^{11} c f^2 - 3072 a^5 c^8 d e - 2 b^{10} c^3 d e + 15360 a^6 c^7 e f + 6 b^{11} c^2 d f + 36 a b^8 c^4 d e - 98 a b^9 c^3 d f + 1536 a^5 b c^7 d f - 10 a c^3 d f (-4 a c - b^2)^9)^{(1/2)} + 2 b c^3 d e (-4 a c - b^2)^9)^{(1/2)} - 152 a b^{10} c^2 e f - 6 b^3 c e f (-4 a c - b^2)^9)^{(1/2)} - 51 a b^2 c f^2 (-4 a c - b^2)^9)^{(1/2)} - 192 a^2 b^6 c^5 d e + 128 a^3 b^4 c^6 d e + 1536 a^4 b^2 c^7 d e + 576 a^2 b^7 c^4 d f - 1344 a^3 b^5 c^5 d f + 512 a^4 b^3 c^6 d f + 1548 a^2 b^8 c^3 e f - 8064 a^3 b^6 c^4 e f + 22400 a^4 b^4 c^5 e f - 30720 a^5 b^2 c^6 e f - 6 b^2 c^2 d f (-4 a c - b^2)^9)^{(1/2)} + 44 a b c^2 e f (-4 a c - b^2)^9)^{(1/2)} / (32 (4096 a^6 c^{11} + b^{12} c^5 - 24 a b^{10} c^6 + 240 a^2 b^8 c^7 - 1280 a^3 b^6 c^8 + 3840 a^4 b^4 c^9 - 6144 a^5 b^2 c^{10})))^{(1/2)} * (16 b^7 c^5 - 192 a b^5 c^6 - 1024 a^3 b c^8 + 768 a^2 b^3 c^7) / (2 (16 a^2 c^5 + b^4 c^3 - 8 a b^2 c^4)) * ((c^4 d^2 (-4 a c - b^2)^9)^{(1/2)} - b^9 c^4 d^2 - 9 b^{13} f^2 - b^{11} c^2 e^2 + 9 b^4 f^2 (-4 a c - b^2)^9)^{(1/2)} + 768 a^4 b c^8 d^2 + 27 a b^9 c^3 e^2 + 3840 a^5 b c^7 e^2 - 9 a c^3 e^2 (-4 a c - b^2)^9)^{(1/2)} - 26880 a^6 b c^6 f^2 + 6 b^{12} c e f + 96 a^2 b^5 c^6 d^2 - 512 a^3 b^3 c^7 d^2 - 288 a^2 b^7 c^4 e^2 + 1504 a^3 b^5 c^5 e^2 - 3840 a^4 b^3 c^6 e^2 - 2077 a^2 b^9 c^2 f^2 + 10656 a^3 b^7 c^3 f^2 - 30240 a^4 b^5 c^4 f^2 + 44800 a^5 b^3 c^5 f^2 + 25 a^2 c^2 f^2 (-4 a c - b^2)^9)^{(1/2)} + b^2 c^2 e^2 (-4 a c - b^2)^9)^{(1/2)} + 213 a b^{11} c f^2 - 3072 a^5 c^8 d e - 2 b^{10} c^3 d e + 15360 a^6 c^7 e f + 6 b^{11} c^2 d f + 36 a b^8 c^4 d e - 98 a b^9 c^3 d f + 1536 a^5 b c^7 d f - 10 a c^3 d f (-4 a c - b^2)^9)^{(1/2)} + 2 b c^3 d e (-4 a c - b^2)^9)^{(1/2)} - 152 a b^{10} c^2 e f - 6 b^3 c e f (-4 a c - b^2)^9)^{(1/2)} - 51 a b^2 c f^2 (-4 a c - b^2)^9)^{(1/2)} - 192 a^2 b^6 c^5 d e + 128 a^3 b^4 c^6 d e + 1536 a^4 b^2 c^7 d e + 576 a^2 b^7 c^4 d f - 1344 a^3 b^5 c^5 d f + 512 a^4 b^3 c^6 d f + 1548 a^2 b^8 c^3 e f - 8064 a^3 b^6 c^4 e f + 22400 a^4 b^4 c^5 e f - 30720 a^5 b^2 c^6 e f - 6 b^2 c^2 d f (-4 a c - b^2)^9)^{(1/2)} + 44 a b c^2 e f (-4 a c - b^2)^9)^{(1/2)} / (32 (4096 a^6 c^{11} + b^{12} c^5 - 24 a b^{10} c^6 + 240 a^2 b^8 c^7 - 1280 a^3 b^6 c^8 + 3840 a^4 b^4 c^9 - 6144 a^5 b^2 c^{10})))^{(1/2)} + (x (9 b^8 f^2 + 8 a^2 c^6 d^2 - 72 a^3 c^5 e^2 + b^4 c^4 d^2 + 200 a^4 c^4 f^2 + b^6 c^2 e^2 + 2 a b^2 c^5 d^2 - 16 a b^4 c^3 e^2 - 6 b^7 c e f + 74 a^2 b^2 c^4 e^2 + 481 a^2 b^4 c^2 f^2 - 718 a^3 b^2 c^3 f^2 - 114 a b^6 c f^2 - 80 a^3 c^5 d f + 2 b^5 c^3 d e - 6 b^6 c^2 d f - 14 a b^3 c^4 d e - 8 a^2 b c^5 d e + 32 a b^4 c^3 d f + 86 a b^5 c^2 e f + 472 a^3 b c^4 e f + 4 a^2 b^2 c^4 d f - 374 a^2 b^3 c^3 e f)) / (2 (16 a^2 c^5 + b^4 c^3 - 8 a b^2 c^4)) * ((c^4 d^2 (-4 a c - b^2)^9)^{(1/2)} - b^9 c^4 d^2 - 9 b^{13} f^2 - b^{11} c^2 e^2 + 9 b^4 f^2 (-4 a c - b^2)^9)^{(1/2)} + 768 a^4 b c^8 d^2 + 27 a b^9 c^3 e^2 + 3840 a^5 b c^7 e^2 - 9 a c^3 e^2 (-4 a c - b^2)^9)^{(1/2)} - 26880 a^6 b c^6 f^2 + 6 b^{12} c e f + 96 a^2 b^5 c^6 d^2 - 512 a^3 b^3 c^7 d^2 - 288 a^2 b^7 c^4 e^2 + 1504 a^3 b^5 c^5 e^2 - 3840 a^4 b^3 c^6 e^2 - 2077 a^2 b^9 c^2 f^2 + 10656 a^3 b^7 c^3 f^2 - 30240 a^4 b^5 c^4 f^2 + 44800 a^5 b^3 c^5 f^2 + 25 a^2 c^2 f^2 (-4 a c - b^2)^9)^{(1/2)} + b^2 c^2 e^2 (-4 a c - b^2)^9)^{(1/2)} + 213 a b^{11} c f^2 - 3072 a^5 c^8 d e - 2 b^{10} c^3 d e + 15360 a^6 c^7 e f + 6 b^{11} c^2 d f + 36 a b^8 c^4 d e - 98 a b^9 c^3 d f + 1536 a^5 b c^7 d f - 10 a c^3 d f (-4 a c - b^2)^9)^{(1/2)} + 2 b c^3 d e (-4 a c - b^2)^9)^{(1/2)} - 152 a b^{10} c^2 e f - 6 b^3 c e f (-4 a c - b^2)^9)^{(1/2)} - 51 a b^2 c f^2 (-4 a c - b^2)^9)^{(1/2)} - 192 a^2 b^6 c^5 d e + 128 a^3 b^4 c^6 d e + 1536 a^4 b^2 c^7 d e + 576 a^2 b^7 c^4 d f - 1344 a^3 b^5 c^5 d f + 512 a^4 b^3 c^6 d f + 1548 a^2 b^8 c^3 e f - 8064 a^3 b^6 c^4 e f + 22400 a^4 b^4 c^5 e f - 30720 a^5 b^2 c^6 e f - 6 b^2 c^2 d f (-4 a c - b^2)^9)^{(1/2)} + 44 a b c^2 e f (-4 a c - b^2)^9)^{(1/2)} / (32 (4096 a^6 c^{11} + b^{12} c^5 - 24 a b^{10} c^6 + 240 a^2 b^8 c^7 - 1280 a^3 b^6 c^8 + 3840 a^4 b^4 c^9 - 6144 a^5 b^2 c^{10})))^{(1/2)} +
\end{aligned}$$

$$\begin{aligned}
& (840a^4b^4c^9 - 6144a^5b^2c^{10}))^{(1/2)} \cdot i / ((63a^3b^5f^3 - 216a^4 \\
& *c^4e^3 + 3a^3b^3c^4d^3 + 4a^2b^2c^5d^3 - 573a^4b^3c^4f^3 + 1300a^5 \\
& *b^2c^2f^3 - 24a^3c^5d^2e - 45a^2b^6e^2f - 600a^5c^3e^2f - 5a^2 \\
& *b^4c^2e^3 + 66a^3b^2c^3e^3 + 27a^2b^7d^2f + 240a^4c^4d^2e^2f + 6 \\
& *a^2b^4c^3d^2e + 3a^3b^5c^2d^2e^2 + 204a^3b^2c^4d^2e^2 - 18a^2b^5c^2d \\
& ^2e^2f - 279a^2b^5c^2d^2e^2 + 12a^3b^2c^4d^2e^2f - 420a^4b^2c^3d^2e^2f + 30 \\
& *a^2b^5c^2e^2f + 402a^3b^4c^2e^2f + 924a^4b^2c^3e^2f - 42a^2b^2c^4 \\
& *d^2e^2 - 51a^2b^3c^3d^2e^2 + 81a^2b^3c^3d^2e^2f + 801a^3b^3c^2d^2e^2f \\
& ^2 - 339a^3b^3c^2e^2f - 762a^4b^2c^2e^2f - 18a^2b^6c^2d^2e^2f + 246 \\
& *a^2b^4c^2d^2e^2f - 804a^3b^2c^3d^2e^2f) / (4 * (64a^3c^6 - b^6c^3 + 12a \\
& *b^4c^4 - 48a^2b^2c^5)) + (((10240a^5c^7f - 2048a^4c^8d - 384a^2 \\
& *b^4c^6d + 1536a^3b^2c^7d + 192a^2b^5c^5e - 768a^3b^3c^6e - 7 \\
& 36a^2b^6c^4f + 4224a^3b^4c^5f - 10752a^4b^2c^6f + 32a^2b^6c^5 \\
& d - 16a^2b^7c^4e + 1024a^4b^2c^7e + 48a^2b^8c^3f) / (8 * (64a^3c^6 - b^6 \\
& c^3 + 12a^2b^4c^4 - 48a^2b^2c^5)) - (x * ((c^4d^2 * (-4a^2c - b^2)^9)^{1/2} - b^9c^4d^2 \\
& - 9b^13f^2 - b^11c^2e^2 + 9b^4f^2 * (-4a^2c - b^2)^9)^{1/2} + 768a^4b^2c^8d^2 \\
& + 27a^2b^9c^3e^2 + 3840a^5b^2c^7e^2 - 9a^2c^3e^2 * (-4a^2c - b^2)^9)^{1/2} - 26880a^6b^2c^6f^2 \\
& + 6b^12c^2e^2f + 96a^2b^5c^6d^2 - 512a^3b^3c^7d^2 - 288a^2b^7c^4e^2 + 1504a^3b^5c^5e^2 \\
& - 3840a^4b^3c^6e^2 - 2077a^2b^9c^2f^2 + 10656a^3b^7c^3f^2 - 30240a^4b^5c^4f^2 \\
& + 44800a^5b^3c^5f^2 + 25a^2c^2f^2 * (-4a^2c - b^2)^9)^{1/2} + b^2c^2e^2 * (-4a^2c - b^2)^9)^{1/2} \\
& + 213a^2b^11c^2f^2 - 3072a^5c^8d^2e - 2b^10c^3d^2e + 15360a^6c^7e^2f + 6b^11c^2d^2f \\
& + 36a^2b^8c^4d^2e - 98a^2b^9c^3d^2f + 1536a^5b^2c^7d^2f - 10a^2c^3d^2f * (-4a^2c \\
& - b^2)^9)^{1/2} + 2b^2c^3d^2e * (-4a^2c - b^2)^9)^{1/2} - 152a^2b^10c^2e^2f \\
& - 6b^3c^2e^2f * (-4a^2c - b^2)^9)^{1/2} - 51a^2b^2c^2f^2 * (-4a^2c - b^2)^9)^{1/2} \\
& - 192a^2b^6c^5d^2e + 128a^3b^4c^6d^2e + 1536a^4b^2c^7d^2e + 576a^2b^7c^4d^2f \\
& - 1344a^3b^5c^5d^2f + 512a^4b^3c^6d^2f + 1548a^2b^8c^3e^2f - 8064a^3b^6c^4e^2f \\
& + 22400a^4b^4c^5e^2f - 30720a^5b^2c^6e^2f - 6b^2c^2d^2f * (-4a^2c - b^2)^9)^{1/2} \\
& + 44a^2b^2c^2e^2f * (-4a^2c - b^2)^9)^{1/2} / (32 * (4096a^6c^11 + b^12c^5 - 24a^2b^10c^6 \\
& + 240a^2b^8c^7 - 1280a^3b^6c^8 + 3840a^4b^4c^9 - 6144a^5b^2c^{10}))^{1/2} * (16b^7c^5 \\
& - 192a^2b^5c^6 - 1024a^3b^2c^8 + 768a^2b^3c^7) / (2 * (16a^2c^5 + b^4c^3 - 8a^2b^2c^4)) \\
& * ((c^4d^2 * (-4a^2c - b^2)^9)^{1/2} - b^9c^4d^2 - 9b^13f^2 - b^11c^2e^2 + 9b^4f^2 * (-4a^2c \\
& - b^2)^9)^{1/2} + 768a^4b^2c^8d^2 + 27a^2b^9c^3e^2 + 3840a^5b^2c^7e^2 - 9a^2c^3e^2 \\
& * (-4a^2c - b^2)^9)^{1/2} - 26880a^6b^2c^6f^2 + 6b^12c^2e^2f + 96a^2b^5c^6d^2 \\
& - 512a^3b^3c^7d^2 - 288a^2b^7c^4e^2 + 1504a^3b^5c^5e^2 - 3840a^4b^3c^6e^2 \\
& - 2077a^2b^9c^2f^2 + 10656a^3b^7c^3f^2 - 30240a^4b^5c^4f^2 + 44800a^5b^3c^5f^2 \\
& + 25a^2c^2f^2 * (-4a^2c - b^2)^9)^{1/2} + b^2c^2e^2 * (-4a^2c - b^2)^9)^{1/2} \\
& + 213a^2b^11c^2f^2 - 3072a^5c^8d^2e - 2b^10c^3d^2e + 15360a^6c^7e^2f + 6b^11c^2d^2f \\
& + 36a^2b^8c^4d^2e - 98a^2b^9c^3d^2f + 1536a^5b^2c^7d^2f - 10a^2c^3d^2f * (-4a^2c \\
& - b^2)^9)^{1/2} + 2b^2c^3d^2e * (-4a^2c - b^2)^9)^{1/2} - 152a^2b^10c^2e^2f \\
& - 6b^3c^2e^2f * (-4a^2c - b^2)^9)^{1/2} - 51a^2b^2c^2f^2 * (-4a^2c - b^2)^9)^{1/2} \\
& - 192a^2b^6c^5d^2e + 128a^3b^4c^6d^2e + 1536a^4b^2c^7d^2e + 576a^2b^7c^4d^2f \\
& - 1344a^3b^5c^5d^2f + 512a^4b^3c^6d^2f + 1548a^2b^8c^3e^2f - 8064a^3b^6c^4e^2f \\
& + 22400a^4b^4c^5e^2f - 30720a^5b^2c^6e^2f - 6b^2c^2d^2f * (-4a^2c - b^2)^9)^{1/2} \\
& + 44a^2b^2c^2e^2f * (-4a^2c - b^2)^9)^{1/2} / (32 * (4096a^6c^11 + b^12c^5 - 24a^2b^10c^6 \\
& + 240a^2b^8c^7 - 1280a^3b^6c^8 + 3840a^4b^4c^9 - 6144a^5b^2c^{10}))^{1/2} \\
& - (x * (9b^8f^2 + 8a^2c^6d^2 - 72a^3c^5e^2 + b^4c^4d^2 + 200a^4c^4f^2 + b^6c^2e^2 \\
& + 2a^2b^2c^5d^2 - 16a^2b^4c^3e^2 - 6b^7c^2e^2f + 74a^2b^2c^4e^2 + 481a^2b^4c^2f^2 \\
& - 718a^3b^2c^3f^2 - 114a^2b^6c^2f^2 - 80a^3c^5d^2f + 2b^5c^3d^2e - 6b^6c^2d^2f \\
& - 14a^2b^3c^4d^2e - 8a^2b^2c^5d^2e + 32a^2b^4c^3d^2f + 86a^2b^5c^2e^2f + 472a^3b^2c^4e^2f \\
& + 4a^2b^2c^4d^2f - 374a^2b^3c^3e^2f) / (2 * (16a^2c^5 + b^4c^3 - 8a^2b^2c^4))) \\
& * ((c^4d^2 * (-4a^2c - b^2)^9)^{1/2} - b^9c^4d^2 - 9b^13f^2 - b^11c^2e^2 + 9b^4f^2 * (-4a^2c \\
& - b^2)^9)^{1/2} + 768a^4b^2c^8d^2 + 27a^2b^9c^3e^2
\end{aligned}$$

$$\begin{aligned}
& *b^9*c^3*e^2 + 3840*a^5*b*c^7*e^2 - 9*a*c^3*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - \\
& 26880*a^6*b*c^6*f^2 + 6*b^{12}*c*e*f + 96*a^2*b^5*c^6*d^2 - 512*a^3*b^3*c^7*d \\
& ^2 - 288*a^2*b^7*c^4*e^2 + 1504*a^3*b^5*c^5*e^2 - 3840*a^4*b^3*c^6*e^2 - 20 \\
& 77*a^2*b^9*c^2*f^2 + 10656*a^3*b^7*c^3*f^2 - 30240*a^4*b^5*c^4*f^2 + 44800* \\
& a^5*b^3*c^5*f^2 + 25*a^2*c^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + b^2*c^2*e^2*(-(\\
& 4*a*c - b^2)^9)^{(1/2)} + 213*a*b^{11}*c*f^2 - 3072*a^5*c^8*d*e - 2*b^{10}*c^3*d* \\
& e + 15360*a^6*c^7*e*f + 6*b^{11}*c^2*d*f + 36*a*b^8*c^4*d*e - 98*a*b^9*c^3*d* \\
& f + 1536*a^5*b*c^7*d*f - 10*a*c^3*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 2*b*c^3*d* \\
& e*(-(4*a*c - b^2)^9)^{(1/2)} - 152*a*b^{10}*c^2*e*f - 6*b^3*c*e*f*(-(4*a*c - b^ \\
& 2)^9)^{(1/2)} - 51*a*b^2*c*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 192*a^2*b^6*c^5*d*e \\
& + 128*a^3*b^4*c^6*d*e + 1536*a^4*b^2*c^7*d*e + 576*a^2*b^7*c^4*d*f - 1344* \\
& a^3*b^5*c^5*d*f + 512*a^4*b^3*c^6*d*f + 1548*a^2*b^8*c^3*e*f - 8064*a^3*b^6 \\
& *c^4*e*f + 22400*a^4*b^4*c^5*e*f - 30720*a^5*b^2*c^6*e*f - 6*b^2*c^2*d*f*(- \\
& (4*a*c - b^2)^9)^{(1/2)} + 44*a*b*c^2*e*f*(-(4*a*c - b^2)^9)^{(1/2)}/(32*(4096 \\
& *a^6*c^{11} + b^{12}*c^5 - 24*a*b^{10}*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + \\
& 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^{10}))^{(1/2)} + (((10240*a^5*c^7*f - 2048* \\
& a^4*c^8*d - 384*a^2*b^4*c^6*d + 1536*a^3*b^2*c^7*d + 192*a^2*b^5*c^5*e - 76 \\
& 8*a^3*b^3*c^6*e - 736*a^2*b^6*c^4*f + 4224*a^3*b^4*c^5*f - 10752*a^4*b^2*c^ \\
& 6*f + 32*a*b^6*c^5*d - 16*a*b^7*c^4*e + 1024*a^4*b*c^7*e + 48*a*b^8*c^3*f)/ \\
& (8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) + (x*((c^4*d^2*(\\
& -(4*a*c - b^2)^9)^{(1/2)} - b^9*c^4*d^2 - 9*b^{13}*f^2 - b^{11}*c^2*e^2 + 9*b^4*f \\
& ^2*(-(4*a*c - b^2)^9)^{(1/2)} + 768*a^4*b*c^8*d^2 + 27*a*b^9*c^3*e^2 + 3840*a \\
& ^5*b*c^7*e^2 - 9*a*c^3*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 26880*a^6*b*c^6*f^2 + \\
& 6*b^{12}*c*e*f + 96*a^2*b^5*c^6*d^2 - 512*a^3*b^3*c^7*d^2 - 288*a^2*b^7*c^4* \\
& e^2 + 1504*a^3*b^5*c^5*e^2 - 3840*a^4*b^3*c^6*e^2 - 2077*a^2*b^9*c^2*f^2 + \\
& 10656*a^3*b^7*c^3*f^2 - 30240*a^4*b^5*c^4*f^2 + 44800*a^5*b^3*c^5*f^2 + 25* \\
& a^2*c^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + b^2*c^2*e^2*(-(4*a*c - b^2)^9)^{(1/2)} \\
& + 213*a*b^{11}*c*f^2 - 3072*a^5*c^8*d*e - 2*b^{10}*c^3*d*e + 15360*a^6*c^7*e*f \\
& + 6*b^{11}*c^2*d*f + 36*a*b^8*c^4*d*e - 98*a*b^9*c^3*d*f + 1536*a^5*b*c^7*d* \\
& f - 10*a*c^3*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 2*b*c^3*d*e*(-(4*a*c - b^2)^9)^{(\\
& 1/2)} - 152*a*b^{10}*c^2*e*f - 6*b^3*c*e*f*(-(4*a*c - b^2)^9)^{(1/2)} - 51*a*b^ \\
& 2*c*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 192*a^2*b^6*c^5*d*e + 128*a^3*b^4*c^6*d* \\
& e + 1536*a^4*b^2*c^7*d*e + 576*a^2*b^7*c^4*d*f - 1344*a^3*b^5*c^5*d*f + 512 \\
& *a^4*b^3*c^6*d*f + 1548*a^2*b^8*c^3*e*f - 8064*a^3*b^6*c^4*e*f + 22400*a^4* \\
& b^4*c^5*e*f - 30720*a^5*b^2*c^6*e*f - 6*b^2*c^2*d*f*(-(4*a*c - b^2)^9)^{(1/2)} \\
&) + 44*a*b*c^2*e*f*(-(4*a*c - b^2)^9)^{(1/2)}/(32*(4096*a^6*c^{11} + b^{12}*c^5 \\
& - 24*a*b^{10}*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6 \\
& 144*a^5*b^2*c^{10}))^{(1/2)}*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^3*b*c^8 + 76 \\
& 8*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((c^4*d^2*(-(4*a* \\
& c - b^2)^9)^{(1/2)} - b^9*c^4*d^2 - 9*b^{13}*f^2 - b^{11}*c^2*e^2 + 9*b^4*f^2*(-(\\
& 4*a*c - b^2)^9)^{(1/2)} + 768*a^4*b*c^8*d^2 + 27*a*b^9*c^3*e^2 + 3840*a^5*b*c \\
& ^7*e^2 - 9*a*c^3*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 26880*a^6*b*c^6*f^2 + 6*b^{1 \\
& 2}*c*e*f + 96*a^2*b^5*c^6*d^2 - 512*a^3*b^3*c^7*d^2 - 288*a^2*b^7*c^4*e^2 + \\
& 1504*a^3*b^5*c^5*e^2 - 3840*a^4*b^3*c^6*e^2 - 2077*a^2*b^9*c^2*f^2 + 10656* \\
& a^3*b^7*c^3*f^2 - 30240*a^4*b^5*c^4*f^2 + 44800*a^5*b^3*c^5*f^2 + 25*a^2*c^ \\
& 2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + b^2*c^2*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 213 \\
& *a*b^{11}*c*f^2 - 3072*a^5*c^8*d*e - 2*b^{10}*c^3*d*e + 15360*a^6*c^7*e*f + 6*b \\
& ^{11}*c^2*d*f + 36*a*b^8*c^4*d*e - 98*a*b^9*c^3*d*f + 1536*a^5*b*c^7*d*f - 10 \\
& *a*c^3*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 2*b*c^3*d*e*(-(4*a*c - b^2)^9)^{(1/2)} \\
& - 152*a*b^{10}*c^2*e*f - 6*b^3*c*e*f*(-(4*a*c - b^2)^9)^{(1/2)} - 51*a*b^2*c*f^ \\
& 2*(-(4*a*c - b^2)^9)^{(1/2)} - 192*a^2*b^6*c^5*d*e + 128*a^3*b^4*c^6*d*e + 15 \\
& 36*a^4*b^2*c^7*d*e + 576*a^2*b^7*c^4*d*f - 1344*a^3*b^5*c^5*d*f + 512*a^4*b \\
& ^3*c^6*d*f + 1548*a^2*b^8*c^3*e*f - 8064*a^3*b^6*c^4*e*f + 22400*a^4*b^4*c^ \\
& 5*e*f - 30720*a^5*b^2*c^6*e*f - 6*b^2*c^2*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 44 \\
& *a*b*c^2*e*f*(-(4*a*c - b^2)^9)^{(1/2)}/(32*(4096*a^6*c^{11} + b^{12}*c^5 - 24*a \\
& *b^{10}*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^ \\
& 5*b^2*c^{10}))^{(1/2)} + (x*(9*b^8*f^2 + 8*a^2*c^6*d^2 - 72*a^3*c^5*e^2 + b^4* \\
& c^4*d^2 + 200*a^4*c^4*f^2 + b^6*c^2*e^2 + 2*a*b^2*c^5*d^2 - 16*a*b^4*c^3*e^ \\
& 2 - 6*b^7*c*e*f + 74*a^2*b^2*c^4*e^2 + 481*a^2*b^4*c^2*f^2 - 718*a^3*b^2*c^
\end{aligned}$$

$$\begin{aligned}
& 3f^2 - 114ab^6c^2f^2 - 80a^3c^5d^2f + 2b^5c^3d^2e - 6b^6c^2d^2f - \\
& 14ab^3c^4d^2e - 8a^2b^2c^5d^2e + 32ab^4c^3d^2f + 86ab^5c^2e^2f + \\
& 472a^3b^2c^4e^2f + 4a^2b^2c^4d^2f - 374a^2b^3c^3e^2f) / (2(16a^2c^5 + b^4c^3 - 8ab^2c^4)) * ((c^4d^2(-4ac - b^2)^9)^{1/2} - b^9c^4d^2 \\
& ^2 - 9b^{13}f^2 - b^{11}c^2e^2 + 9b^4f^2(-4ac - b^2)^9)^{1/2} + 768a^4b^2c^8d^2 + 27ab^9c^3e^2 + 3840a^5b^2c^7e^2 - 9ac^3e^2(-4ac \\
& - b^2)^9)^{1/2} - 26880a^6b^2c^6f^2 + 6b^{12}c^2e^2f + 96a^2b^5c^6d^2 \\
& - 512a^3b^3c^7d^2 - 288a^2b^7c^4e^2 + 1504a^3b^5c^5e^2 - 3840a^4b^3c^6e^2 - 2077a^2b^9c^2f^2 + 10656a^3b^7c^3f^2 - 30240a^4b^5c^4f^2 \\
& + 44800a^5b^3c^5f^2 + 25a^2c^2f^2(-4ac - b^2)^9)^{1/2} + b^2c^2e^2(-4ac - b^2)^9)^{1/2} + 213ab^{11}c^2f^2 - 3072a^5c^8d^2e \\
& - 2b^{10}c^3d^2e + 15360a^6c^7e^2f + 6b^{11}c^2d^2f + 36ab^8c^4d^2e - 98ab^9c^3d^2f + 1536a^5b^2c^7d^2f - 10ac^3d^2f(-4ac - b^2)^9)^{1/2} \\
& + 2b^2c^3d^2e(-4ac - b^2)^9)^{1/2} - 152ab^{10}c^2e^2f - 6b^3c^2e^2f(-4ac - b^2)^9)^{1/2} - 51ab^2c^2f^2(-4ac - b^2)^9)^{1/2} - \\
& 192a^2b^6c^5d^2e + 128a^3b^4c^6d^2e + 1536a^4b^2c^7d^2e + 576a^2b^7c^4d^2f - 1344a^3b^5c^5d^2f + 512a^4b^3c^6d^2f + 1548a^2b^8c^3e^2f \\
& - 8064a^3b^6c^4e^2f + 22400a^4b^4c^5e^2f - 30720a^5b^2c^6e^2f - 6b^2c^2d^2f(-4ac - b^2)^9)^{1/2} + 44ab^2c^2e^2f(-4ac - b^2)^9)^{1/2} \\
&) / (32(4096a^6c^{11} + b^{12}c^5 - 24ab^{10}c^6 + 240a^2b^8c^7 - 1280a^3b^6c^8 + 3840a^4b^4c^9 - 6144a^5b^2c^{10}))^{1/2}) * ((c^4d^2(-4ac - b^2)^9)^{1/2} - b^9c^4d^2 \\
& - 9b^{13}f^2 - b^{11}c^2e^2 + 9b^4f^2(-4ac - b^2)^9)^{1/2} + 768a^4b^2c^8d^2 + 27ab^9c^3e^2 + 3840a^5b^2c^7e^2 - 9ac^3e^2(-4ac - b^2)^9)^{1/2} - 26880a^6b^2c^6f^2 \\
& ^2 + 6b^{12}c^2e^2f + 96a^2b^5c^6d^2 - 512a^3b^3c^7d^2 - 288a^2b^7c^4e^2 + 1504a^3b^5c^5e^2 - 3840a^4b^3c^6e^2 - 2077a^2b^9c^2f^2 + 10656a^3b^7c^3f^2 - 30240a^4b^5c^4f^2 \\
& + 44800a^5b^3c^5f^2 + 25a^2c^2f^2(-4ac - b^2)^9)^{1/2} + b^2c^2e^2(-4ac - b^2)^9)^{1/2} + 213ab^{11}c^2f^2 - 3072a^5c^8d^2e - 2b^{10}c^3d^2e + 15360a^6c^7e^2f \\
& + 6b^{11}c^2d^2f + 36ab^8c^4d^2e - 98ab^9c^3d^2f + 1536a^5b^2c^7d^2f - 10ac^3d^2f(-4ac - b^2)^9)^{1/2} + 2b^2c^3d^2e(-4ac - b^2)^9)^{1/2} - 152ab^{10}c^2e^2f \\
& - 6b^3c^2e^2f(-4ac - b^2)^9)^{1/2} - 51ab^2c^2f^2(-4ac - b^2)^9)^{1/2} - 192a^2b^6c^5d^2e + 128a^3b^4c^6d^2e + 1536a^4b^2c^7d^2e + 576a^2b^7c^4d^2f - 1344a^3b^5c^5d^2f \\
& + 512a^4b^3c^6d^2f + 1548a^2b^8c^3e^2f - 8064a^3b^6c^4e^2f + 22400a^4b^4c^5e^2f - 30720a^5b^2c^6e^2f - 6b^2c^2d^2f(-4ac - b^2)^9)^{1/2} + 44ab^2c^2e^2f(-4ac - b^2)^9)^{1/2} \\
&) / (32(4096a^6c^{11} + b^{12}c^5 - 24ab^{10}c^6 + 240a^2b^8c^7 - 1280a^3b^6c^8 + 3840a^4b^4c^9 - 6144a^5b^2c^{10}))^{1/2}) * 2i
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)

[Out] Timed out

$$3.68 \quad \int \frac{x^2(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2} dx$$

Optimal. Leaf size=362

$$\frac{x(x^2(-2acf + b^2f - bce + 2c^2d) + abf - 2ace + bcd)}{2c(b^2 - 4ac)(a + bx^2 + cx^4)} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right) \left(\frac{-4bc(2af + cd) + 4ac^2e + b^3f + b^2ce}{c\sqrt{b^2 - 4ac}} + 6af - \frac{b^2f}{c} - be + 2cd\right) \frac{1}{2\sqrt{2}\sqrt{c}(b^2 - 4ac)\sqrt{b - \sqrt{b^2 - 4ac}}}$$

Rubi [A] time = 2.50, antiderivative size = 362, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, number of rules / integrand size = 0.100, Rules used = {1668, 1166, 205}

$$\frac{x(x^2(-2acf + b^2f - bce + 2c^2d) + abf - 2ace + bcd)}{2c(b^2 - 4ac)(a + bx^2 + cx^4)} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right) \left(\frac{-4bc(2af + cd) + 4ac^2e + b^3f + b^2ce}{c\sqrt{b^2 - 4ac}} + 6af - \frac{b^2f}{c} - be + 2cd\right) \frac{1}{2\sqrt{2}\sqrt{c}(b^2 - 4ac)\sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b + \sqrt{b^2 - 4ac}}}\right) \left(\frac{-4bc(2af + cd) + 4ac^2e + b^3f + b^2ce}{c\sqrt{b^2 - 4ac}} + 6af - \frac{b^2f}{c} - be + 2cd\right)}{2\sqrt{2}\sqrt{c}(b^2 - 4ac)\sqrt{b + \sqrt{b^2 - 4ac}}}$$

Antiderivative was successfully verified.

[In] Int[(x^2*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2, x]

[Out] -(x*(b*c*d - 2*a*c*e + a*b*f + (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x^2))/(2*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((2*c*d - b*e + 6*a*f - (b^2*f)/c + (b^2*c*e + 4*a*c^2*e + b^3*f - 4*b*c*(c*d + 2*a*f))/(c*sqrt[b^2 - 4*a*c]))*ArcTan[(sqrt[2]*sqrt[c]*x)/sqrt[b - sqrt[b^2 - 4*a*c]]]/(2*sqrt[2]*sqrt[c]*(b^2 - 4*a*c)*sqrt[b - sqrt[b^2 - 4*a*c]]) - ((2*c*d - b*e + 6*a*f - (b^2*f)/c - (b^2*c*e + 4*a*c^2*e + b^3*f - 4*b*c*(c*d + 2*a*f))/(c*sqrt[b^2 - 4*a*c]))*ArcTan[(sqrt[2]*sqrt[c]*x)/sqrt[b + sqrt[b^2 - 4*a*c]]]/(2*sqrt[2]*sqrt[c]*(b^2 - 4*a*c)*sqrt[b + sqrt[b^2 - 4*a*c]])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1668

Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]

Rubi steps

$$\int \frac{x^2 (d + ex^2 + fx^4)}{(a + bx^2 + cx^4)^2} dx = -\frac{x (bcd - 2ace + abf + (2c^2d - bce + b^2f - 2acf) x^2)}{2c (b^2 - 4ac) (a + bx^2 + cx^4)} - \frac{\frac{-\frac{a(bcd-2ace+abf)}{c} + a(2cd-be+6af)}{a+bx^2+cx^4}}{2a (b^2 - 4ac)}$$

$$= -\frac{x (bcd - 2ace + abf + (2c^2d - bce + b^2f - 2acf) x^2)}{2c (b^2 - 4ac) (a + bx^2 + cx^4)} - \frac{(2cd - be + 6af - \frac{b^2f}{c})}{2a (b^2 - 4ac)}$$

$$= -\frac{x (bcd - 2ace + abf + (2c^2d - bce + b^2f - 2acf) x^2)}{2c (b^2 - 4ac) (a + bx^2 + cx^4)} - \frac{(2cd - be + 6af - \frac{b^2f}{c})}{2\sqrt{2}\sqrt{c}}$$

Mathematica [A] time = 1.10, size = 414, normalized size = 1.14

$$\frac{2\sqrt{c}(abf-2ac(e+fx^2)+b^2f^2+bc(d-ex^2)+2c^2d^2)}{(b^2-4ac)(e+bx^2+cx^4)} + \frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{b^2-4ac}}\right)\left(bc(e\sqrt{b^2-4ac}+8af+4cd)-2(cd\sqrt{b^2-4ac}+3af\sqrt{b^2-4ac}+2acc)+b^2(\sqrt{b^2-4ac}-ce)+b^3(-f)\right)}{(b^2-4ac)^{3/2}\sqrt{b^2-4ac}} + \frac{\sqrt{2}\tan^{-1}\left(\frac{-\sqrt{2}\sqrt{c}}{\sqrt{b^2-4ac}}\right)\left(bc(e\sqrt{b^2-4ac}-8af-4cd)-2(cd\sqrt{b^2-4ac}+3af\sqrt{b^2-4ac}-2acc)+b^2(\sqrt{b^2-4ac}+ce)+b^3(f)\right)}{(b^2-4ac)^{3/2}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]

[Out] $((-2*\text{Sqrt}[c]*x*(a*b*f + 2*c^2*d*x^2 + b^2*f*x^2 + b*c*(d - e*x^2) - 2*a*c*(e + f*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (\text{Sqrt}[2]*(-(b^3*f) + b*c*(4*c*d + \text{Sqrt}[b^2 - 4*a*c]*e + 8*a*f) + b^2*(-(c*e) + \text{Sqrt}[b^2 - 4*a*c]*f) - 2*c*(c*\text{Sqrt}[b^2 - 4*a*c]*d + 2*a*c*e + 3*a*\text{Sqrt}[b^2 - 4*a*c]*f))*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]])]/((b^2 - 4*a*c)^{(3/2)}*\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]]) + (\text{Sqrt}[2]*(b^3*f + b*c*(-4*c*d + \text{Sqrt}[b^2 - 4*a*c]*e - 8*a*f) + b^2*(c*e + \text{Sqrt}[b^2 - 4*a*c]*f) - 2*c*(c*\text{Sqrt}[b^2 - 4*a*c]*d - 2*a*c*e + 3*a*\text{Sqrt}[b^2 - 4*a*c]*f))*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]])]/((b^2 - 4*a*c)^{(3/2)}*\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]]))/(4*c^{(3/2)})$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 (d + ex^2 + fx^4)}{(a + bx^2 + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^2*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]

[Out] IntegrateAlgebraic[(x^2*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2, x]

fricas [B] time = 15.05, size = 8951, normalized size = 24.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")

[Out] $-1/4*(2*(2*c^2*d - b*c*e + (b^2 - 2*a*c)*f)*x^3 + \text{sqrt}(1/2)*((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)*\text{sqrt}(-((b^3*c^3 + 12*a*b*c^4)*d^2 - 4*(3*a*b^2*c^3 + 4*a^2*c^4)*d*e + (a*b^3*c^2 + 12*a^2*b*c^3)*e^2 + (a*b^5 - 15*a^2*b^3*c + 60*a^3*b*c^2)*f^2 - 2*((3*a*b^3*c^2 - 2*8*a^2*b*c^3)*d - (a*b^4*c - 6*a^2*b^2*c^2 - 24*a^3*c^3)*e)*f + (a*b^6*c^3 - 12*a^2*b^4*c^4 + 48*a^3*b^2*c^5 - 64*a^4*c^6)*\text{sqrt}((c^6*d^4 - 2*a*c^5*d^2$

$$\begin{aligned}
& e^2 + a^2c^4e^4 + (a^2b^4 - 18a^3b^2c + 81a^4c^2)*f^4 - 4*(3*(a^2b^2c^2 - 9a^3c^3)*d - (a^2b^3c - 9a^3b*c^2)*e)*f^3 - 2*(12a^2b*c^3*d*e + (a*b^2*c^3 - 27a^2*c^4)*d^2 - 3*(a^2b^2*c^2 - 3a^3*c^3)*e^2)*f^2 + \\
& 4*(3*a*c^5*d^3 - a*b*c^4*d^2*e - 3*a^2*c^4*d*e^2 + a^2*b*c^3*e^3)*f)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))/(a*b^6*c^3 - 12*a^2*b^4*c^4 + 48*a^3*b^2*c^5 - 64*a^4*c^6))*\log(((3*b^2*c^5 + 4*a*c^6)*d^4 - \\
& (b^3*c^4 + 12*a*b*c^5)*d^3*e + (a*b^3*c^3 + 12*a^2*b*c^4)*d*e^3 - (3*a^2*b^2*c^3 + 4*a^3*c^4)*e^4 + (5*a^3*b^4 - 81*a^4*b^2*c + 324*a^5*c^2)*f^4 + ((\\
& a*b^6 - 15*a^2*b^4*c + 432*a^4*c^3)*d - (3*a^2*b^5 - 65*a^3*b^3*c + 324*a^4*b*c^2)*e)*f^3 - 3*(3*(a*b^4*c^2 - 6*a^2*b^2*c^3 - 24*a^3*c^4)*d^2 - (a*b^5*c + 3*a^2*b^3*c^2 - 108*a^3*b*c^3)*d*e + (3*a^2*b^4*c - 28*a^3*b^2*c^2)*e^2)*f^2 - \\
& ((b^4*c^3 - 24*a*b^2*c^4 - 48*a^2*c^5)*d^3 + 9*(a*b^3*c^3 + 12*a^2*b*c^4)*d^2*e - 3*(a*b^4*c^2 + 12*a^2*b^2*c^3)*d*e^2 + (9*a^2*b^3*c^2 - 20*a^3*b*c^3)*e^3)*f)*x + 1/2*\sqrt{1/2}*((b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*d^3 - 2*(a*b^4*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c^6)*d^2*e - (a*b^5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5)*d*e^2 + 2*(a^2*b^4*c^3 - 8*a^3*b^2*c^4 + 16*a^4*c^5)*e^3 - (a^2*b^7 - 17*a^3*b^5*c + 88*a^4*b^3*c^2 - 144*a^5*b*c^3)*f^3 - \\
& ((a*b^7*c - 23*a^2*b^5*c^2 + 136*a^3*b^3*c^3 - 240*a^4*b*c^4)*d + 18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e)*f^2 + (7*(a*b^5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5)*d^2 - 2*(a*b^6*c^2 - 2*a^2*b^4*c^3 - 32*a^3*b^2*c^4 + 96*a^4*c^5)*d*e + 3*(a^2*b^5*c^2 - 8*a^3*b^3*c^3 + 16*a^4*b*c^4)*e^2)*f - ((a*b^8*c^4 - 8*a^2*b^6*c^5 + 128*a^4*b^2*c^7 - 256*a^5*c^8)*d - 4*(a^2*b^7*c^4 - 12*a^3*b^5*c^5 + 48*a^4*b^3*c^6 - 64*a^5*b*c^7)*e - (a^2*b^8*c^3 - 24*a^3*b^6*c^4 + 192*a^4*b^4*c^5 - 640*a^5*b^2*c^6 + 768*a^6*c^7)*f)*\sqrt{(c^6*d^4 - 2*a*c^5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b^4 - 18*a^3*b^2*c + 81*a^4*c^2)*f^4 - 4*(3*(a^2*b^2*c^2 - 9a^3*c^3)*d - (a^2*b^3*c - 9a^3*b*c^2)*e)*f^3 - 2*(12*a^2*b*c^3*d*e + (a*b^2*c^3 - 27*a^2*c^4)*d^2 - 3*(a^2*b^2*c^2 - 3a^3*c^3)*e^2)*f^2 + 4*(3*a*c^5*d^3 - a*b*c^4*d^2*e - 3*a^2*c^4*d*e^2 + a^2*b*c^3*e^3)*f)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))*\sqrt{-((b^3*c^3 + 12*a*b*c^4)*d^2 - 4*(3*a*b^2*c^3 + 4*a^2*c^4)*d*e + (a*b^3*c^2 + 12*a^2*b*c^3)*e^2 + (a*b^5 - 15*a^2*b^3*c + 60*a^3*b*c^2)*f^2 - 2*((3*a*b^3*c^2 - 28*a^2*b*c^3)*d - (a*b^4*c - 6*a^2*b^2*c^2 - 24*a^3*c^3)*e)*f + (a*b^6*c^3 - 12*a^2*b^4*c^4 + 48*a^3*b^2*c^5 - 64*a^4*c^6)*\sqrt{(c^6*d^4 - 2*a*c^5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b^4 - 18*a^3*b^2*c + 81*a^4*c^2)*f^4 - 4*(3*(a^2*b^2*c^2 - 9a^3*c^3)*d - (a^2*b^3*c - 9a^3*b*c^2)*e)*f^3 - 2*(12*a^2*b*c^3*d*e + (a*b^2*c^3 - 27*a^2*c^4)*d^2 - 3*(a^2*b^2*c^2 - 3a^3*c^3)*e^2)*f^2 + 4*(3*a*c^5*d^3 - a*b*c^4*d^2*e - 3*a^2*c^4*d*e^2 + a^2*b*c^3*e^3)*f)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))/(a*b^6*c^3 - 12*a^2*b^4*c^4 + 48*a^3*b^2*c^5 - 64*a^4*c^6))*\log(((3*b^2*c^5 + 4*a*c^6)*d^4 - (b^3*c^4 + 12*a*b*c^5)*d^3*e + (a*b^3*c^3 + 12*a^2*b*c^4)*d*e^3 - (3*a^2*b^2*c^3 + 4*a^3*c^4)*e^4 + (5*a^3*b^4 - 81*a^4*b^2*c + 324*a^5*c^2)*f^4 + ((a*b^6 - 15*a^2*b^4*c + 432*a^4*c^3)*d - (3*a^2*b^5 - 65*a^3*b^3*c + 324*a^4*b*c^2)*e)*f^3 - 3*(3*(a*b^4*c^2 - 6*a^2*b^2*c^3 - 24*a^3*c^4)*d^2 - (a*b^5*c + 3*a^2*b^3*c^2 - 108*a^3*b*c^3)*d*e + (3*a^2*b^4*c - 28*a^3*b^2*c^2)*e^2)*f^2 - ((b^4*c^3 - 24*a*b^2*c^4 - 48*a^2*c^5)*d^3 + 9*(a*b^3*c^3 + 12*a^2*b*c^4)*d^2*e - 3*(a*b^4*c^2 + 12*a^2*b^2*c^3)*d*e^2 + (9*a^2*b^3*c^2 - 20*a^3*b*c^3)*e^3)*f)*x - 1/2*\sqrt{1/2}*((b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*d^3 - 2*(a*b^4*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c^6)*d^2*e - (a*b^
\end{aligned}$$

$$\begin{aligned}
& 5c^3 - 8a^2b^3c^4 + 16a^3b^2c^5)de^2 + 2(a^2b^4c^3 - 8a^3b^2c^4 + 16a^4c^5)e^3 - (a^2b^7 - 17a^3b^5c + 88a^4b^3c^2 - 144a^5b^2c^3) * f^3 - ((a^2b^7c - 23a^2b^5c^2 + 136a^3b^3c^3 - 240a^4b^2c^4) * d + 18(a^3b^4c^2 - 8a^4b^2c^3 + 16a^5c^4) * e) * f^2 + (7(a^2b^5c^3 - 8a^2b^3c^4 + 16a^3b^2c^5) * d^2 - 2(a^2b^6c^2 - 2a^2b^4c^3 - 32a^3b^2c^4 + 96a^4c^5) * d * e + 3(a^2b^5c^2 - 8a^3b^3c^3 + 16a^4b^2c^4) * e^2) * f - ((a^2b^8c^4 - 8a^2b^6c^5 + 128a^4b^2c^7 - 256a^5c^8) * d - 4(a^2b^7c^4 - 12a^3b^5c^5 + 48a^4b^3c^6 - 64a^5b^2c^7) * e - (a^2b^8c^3 - 24a^3b^6c^4 + 192a^4b^4c^5 - 640a^5b^2c^6 + 768a^6c^7) * f) * \text{sqrt}((c^6d^4 - 2a^2c^5d^2e^2 + a^2c^4e^4 + (a^2b^4 - 18a^3b^2c + 81a^4c^2) * f^4 - 4(3(a^2b^2c^2 - 9a^3c^3) * d - (a^2b^3c - 9a^3b^2c^2) * e) * f^3 - 2(12a^2b^2c^3 * d * e + (a^2b^2c^3 - 27a^2c^4) * d^2 - 3(a^2b^2c^2 - 3a^3c^3) * e^2) * f^2 + 4(3a^2c^5d^3 - a^2b^2c^4 * d^2 * e - 3a^2c^4 * d * e^2 + a^2b^2c^3 * e^3) * f) / (a^2b^6c^6 - 12a^3b^4c^7 + 48a^4b^2c^8 - 64a^5c^9)) * \text{sqrt}(-(b^3c^3 + 12a^2b^2c^4) * d^2 - 4(3a^2b^2c^3 + 4a^2c^4) * d * e + (a^2b^3c^2 + 12a^2b^2c^3) * e^2 + (a^2b^5 - 15a^2b^3c + 60a^3b^2c^2) * f^2 - 2((3a^2b^3c^2 - 28a^2b^2c^3) * d - (a^2b^4c - 6a^2b^2c^2 - 24a^3c^3) * e) * f + (a^2b^6c^3 - 12a^2b^4c^4 + 48a^3b^2c^5 - 64a^4c^6) * \text{sqrt}((c^6d^4 - 2a^2c^5d^2e^2 + a^2c^4e^4 + (a^2b^4 - 18a^3b^2c + 81a^4c^2) * f^4 - 4(3(a^2b^2c^2 - 9a^3c^3) * d - (a^2b^3c - 9a^3b^2c^2) * e) * f^3 - 2(12a^2b^2c^3 * d * e + (a^2b^2c^3 - 27a^2c^4) * d^2 - 3(a^2b^2c^2 - 3a^3c^3) * e^2) * f^2 + 4(3a^2c^5d^3 - a^2b^2c^4 * d^2 * e - 3a^2c^4 * d * e^2 + a^2b^2c^3 * e^3) * f) / (a^2b^6c^6 - 12a^3b^4c^7 + 48a^4b^2c^8 - 64a^5c^9))) / (a^2b^6c^3 - 12a^2b^4c^4 + 48a^3b^2c^5 - 64a^4c^6)) + \text{sqrt}(1/2) * ((b^2c^2 - 4a^2c^3) * x^4 + a^2b^2c - 4a^2c^2 + (b^3c - 4a^2b^2c^2) * x^2) * \text{sqrt}(-(b^3c^3 + 12a^2b^2c^4) * d^2 - 4(3a^2b^2c^3 + 4a^2c^4) * d * e + (a^2b^3c^2 + 12a^2b^2c^3) * e^2 + (a^2b^5 - 15a^2b^3c + 60a^3b^2c^2) * f^2 - 2((3a^2b^3c^2 - 28a^2b^2c^3) * d - (a^2b^4c - 6a^2b^2c^2 - 24a^3c^3) * e) * f - (a^2b^6c^3 - 12a^2b^4c^4 + 48a^3b^2c^5 - 64a^4c^6) * \text{sqrt}((c^6d^4 - 2a^2c^5d^2e^2 + a^2c^4e^4 + (a^2b^4 - 18a^3b^2c + 81a^4c^2) * f^4 - 4(3(a^2b^2c^2 - 9a^3c^3) * d - (a^2b^3c - 9a^3b^2c^2) * e) * f^3 - 2(12a^2b^2c^3 * d * e + (a^2b^2c^3 - 27a^2c^4) * d^2 - 3(a^2b^2c^2 - 3a^3c^3) * e^2) * f^2 + 4(3a^2c^5d^3 - a^2b^2c^4 * d^2 * e - 3a^2c^4 * d * e^2 + a^2b^2c^3 * e^3) * f) / (a^2b^6c^6 - 12a^3b^4c^7 + 48a^4b^2c^8 - 64a^5c^9))) / (a^2b^6c^3 - 12a^2b^4c^4 + 48a^3b^2c^5 - 64a^4c^6)) * \log(((3b^2c^5 + 4a^2c^6) * d^4 - (b^3c^4 + 12a^2b^2c^5) * d^3 * e + (a^2b^3c^3 + 12a^2b^2c^4) * d * e^3 - (3a^2b^2c^3 + 4a^3c^4) * e^4 + (5a^3b^4 - 81a^4b^2c + 324a^5c^2) * f^4 + ((a^2b^6 - 15a^2b^4c + 432a^4c^3) * d - (3a^2b^5 - 65a^3b^3c + 324a^4b^2c^2) * e) * f^3 - 3(3(a^2b^4c^2 - 6a^2b^2c^3 - 24a^3c^4) * d^2 - (a^2b^5c + 3a^2b^3c^2 - 108a^3b^2c^3) * d * e + (3a^2b^4c - 28a^3b^2c^2) * e^2) * f^2 - ((b^4c^3 - 24a^2b^2c^4 - 48a^2c^5) * d^3 + 9(a^2b^3c^3 + 12a^2b^2c^4) * d^2 * e - 3(a^2b^4c^2 + 12a^2b^2c^3) * d * e^2 + (9a^2b^3c^2 - 20a^3b^2c^3) * e^3) * f) * x + 1/2 * \text{sqrt}(1/2) * ((b^5c^4 - 8a^2b^3c^5 + 16a^2b^2c^6) * d^3 - 2(a^2b^4c^4 - 8a^2b^2c^5 + 16a^3c^6) * d^2 * e - (a^2b^5c^3 - 8a^2b^3c^4 + 16a^3b^2c^5) * d * e^2 + 2(a^2b^4c^3 - 8a^3b^2c^4 + 16a^4c^5) * e^3 - (a^2b^7 - 17a^3b^5c + 88a^4b^3c^2 - 144a^5b^2c^3) * f^3 - ((a^2b^7c - 23a^2b^5c^2 + 136a^3b^3c^3 - 240a^4b^2c^4) * d + 18(a^3b^4c^2 - 8a^4b^2c^3 + 16a^5c^4) * e) * f^2 + (7(a^2b^5c^3 - 8a^2b^3c^4 + 16a^3b^2c^5) * d^2 - 2(a^2b^6c^2 - 2a^2b^4c^3 - 32a^3b^2c^4 + 96a^4c^5) * d * e + 3(a^2b^5c^2 - 8a^3b^3c^3 + 16a^4b^2c^4) * e^2) * f + ((a^2b^8c^4 - 8a^2b^6c^5 + 128a^4b^2c^7 - 256a^5c^8) * d - 4(a^2b^7c^4 - 12a^3b^5c^5 + 48a^4b^3c^6 - 64a^5b^2c^7) * e - (a^2b^8c^3 - 24a^3b^6c^4 + 192a^4b^4c^5 - 640a^5b^2c^6 + 768a^6c^7) * f) * \text{sqrt}((c^6d^4 - 2a^2c^5d^2e^2 + a^2c^4e^4 + (a^2b^4 - 18a^3b^2c + 81a^4c^2) * f^4 - 4(3(a^2b^2c^2 - 9a^3c^3) * d - (a^2b^3c - 9a^3b^2c^2) * e) * f^3 - 2(12a^2b^2c^3 * d * e + (a^2b^2c^3 - 27a^2c^4) * d^2 - 3(a^2b^2c^2 - 3a^3c^3) * e^2) * f^2 + 4(3a^2c^5d^3 - a^2b^2c^4 * d^2 * e - 3a^2c^4 * d * e^2 + a^2b^2c^3 * e^3) * f) / (a^2b^6c^6 - 12a^3b^4c^7 + 48a^4b^2c^8 - 64a^5c^9)) * \text{sqrt}(-(b^3c^3 + 12a^2b^2c^4) * d^2 - 4(3a^2b^2c^3 + 4
\end{aligned}$$

$$\begin{aligned}
& *a^2*c^4)*d*e + (a*b^3*c^2 + 12*a^2*b*c^3)*e^2 + (a*b^5 - 15*a^2*b^3*c + 60 \\
& *a^3*b*c^2)*f^2 - 2*((3*a*b^3*c^2 - 28*a^2*b*c^3)*d - (a*b^4*c - 6*a^2*b^2* \\
& c^2 - 24*a^3*c^3)*e)*f - (a*b^6*c^3 - 12*a^2*b^4*c^4 + 48*a^3*b^2*c^5 - 64* \\
& a^4*c^6)*\sqrt{(c^6*d^4 - 2*a*c^5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b^4 - 18*a^3* \\
& b^2*c + 81*a^4*c^2)*f^4 - 4*(3*(a^2*b^2*c^2 - 9*a^3*c^3)*d - (a^2*b^3*c - 9 \\
& *a^3*b*c^2)*e)*f^3 - 2*(12*a^2*b*c^3*d*e + (a*b^2*c^3 - 27*a^2*c^4)*d^2 - 3 \\
& *(a^2*b^2*c^2 - 3*a^3*c^3)*e^2)*f^2 + 4*(3*a*c^5*d^3 - a*b*c^4*d^2*e - 3*a^ \\
& 2*c^4*d*e^2 + a^2*b*c^3*e^3)*f)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^ \\
& c^8 - 64*a^5*c^9)))/(a*b^6*c^3 - 12*a^2*b^4*c^4 + 48*a^3*b^2*c^5 - 64*a^4*c^ \\
& ^6))) - \sqrt{1/2}*((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - \\
& 4*a*b*c^2)*x^2)*\sqrt{-((b^3*c^3 + 12*a*b*c^4)*d^2 - 4*(3*a*b^2*c^3 + 4*a^2 \\
& *c^4)*d*e + (a*b^3*c^2 + 12*a^2*b*c^3)*e^2 + (a*b^5 - 15*a^2*b^3*c + 60*a^3 \\
& *b*c^2)*f^2 - 2*((3*a*b^3*c^2 - 28*a^2*b*c^3)*d - (a*b^4*c - 6*a^2*b^2*c^2 \\
& - 24*a^3*c^3)*e)*f - (a*b^6*c^3 - 12*a^2*b^4*c^4 + 48*a^3*b^2*c^5 - 64*a^4* \\
& c^6)*\sqrt{(c^6*d^4 - 2*a*c^5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b^4 - 18*a^3*b^2* \\
& c + 81*a^4*c^2)*f^4 - 4*(3*(a^2*b^2*c^2 - 9*a^3*c^3)*d - (a^2*b^3*c - 9*a^3 \\
& *b*c^2)*e)*f^3 - 2*(12*a^2*b*c^3*d*e + (a*b^2*c^3 - 27*a^2*c^4)*d^2 - 3*(a^ \\
& 2*b^2*c^2 - 3*a^3*c^3)*e^2)*f^2 + 4*(3*a*c^5*d^3 - a*b*c^4*d^2*e - 3*a^2*c^ \\
& 4*d*e^2 + a^2*b*c^3*e^3)*f)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 \\
& - 64*a^5*c^9)))/(a*b^6*c^3 - 12*a^2*b^4*c^4 + 48*a^3*b^2*c^5 - 64*a^4*c^6)) \\
& * \log(((3*b^2*c^5 + 4*a*c^6)*d^4 - (b^3*c^4 + 12*a*b*c^5)*d^3*e + (a*b^3*c^3 \\
& + 12*a^2*b*c^4)*d*e^3 - (3*a^2*b^2*c^3 + 4*a^3*c^4)*e^4 + (5*a^3*b^4 - 81* \\
& a^4*b^2*c + 324*a^5*c^2)*f^4 + ((a*b^6 - 15*a^2*b^4*c + 432*a^4*c^3)*d - (3 \\
& *a^2*b^5 - 65*a^3*b^3*c + 324*a^4*b*c^2)*e)*f^3 - 3*(3*(a*b^4*c^2 - 6*a^2*b^ \\
& ^2*c^3 - 24*a^3*c^4)*d^2 - (a*b^5*c + 3*a^2*b^3*c^2 - 108*a^3*b*c^3)*d*e + \\
& (3*a^2*b^4*c - 28*a^3*b^2*c^2)*e^2)*f^2 - ((b^4*c^3 - 24*a*b^2*c^4 - 48*a^2 \\
& *c^5)*d^3 + 9*(a*b^3*c^3 + 12*a^2*b*c^4)*d^2*e - 3*(a*b^4*c^2 + 12*a^2*b^2* \\
& c^3)*d*e^2 + (9*a^2*b^3*c^2 - 20*a^3*b*c^3)*e^3)*f)*x - 1/2*\sqrt{1/2}*((b^5 \\
& *c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*d^3 - 2*(a*b^4*c^4 - 8*a^2*b^2*c^5 + 16* \\
& a^3*c^6)*d^2*e - (a*b^5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5)*d*e^2 + 2*(a^2* \\
& b^4*c^3 - 8*a^3*b^2*c^4 + 16*a^4*c^5)*e^3 - (a^2*b^7 - 17*a^3*b^5*c + 88*a^ \\
& 4*b^3*c^2 - 144*a^5*b*c^3)*f^3 - ((a*b^7*c - 23*a^2*b^5*c^2 + 136*a^3*b^3*c^ \\
& ^3 - 240*a^4*b*c^4)*d + 18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e)*f^ \\
& 2 + (7*(a*b^5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5)*d^2 - 2*(a*b^6*c^2 - 2*a^ \\
& 2*b^4*c^3 - 32*a^3*b^2*c^4 + 96*a^4*c^5)*d*e + 3*(a^2*b^5*c^2 - 8*a^3*b^3*c^ \\
& ^3 + 16*a^4*b*c^4)*e^2)*f + ((a*b^8*c^4 - 8*a^2*b^6*c^5 + 128*a^4*b^2*c^7 - \\
& 256*a^5*c^8)*d - 4*(a^2*b^7*c^4 - 12*a^3*b^5*c^5 + 48*a^4*b^3*c^6 - 64*a^5 \\
& *b*c^7)*e - (a^2*b^8*c^3 - 24*a^3*b^6*c^4 + 192*a^4*b^4*c^5 - 640*a^5*b^2*c^ \\
& ^6 + 768*a^6*c^7)*f)*\sqrt{(c^6*d^4 - 2*a*c^5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b^ \\
& ^4 - 18*a^3*b^2*c + 81*a^4*c^2)*f^4 - 4*(3*(a^2*b^2*c^2 - 9*a^3*c^3)*d - (a \\
& ^2*b^3*c - 9*a^3*b*c^2)*e)*f^3 - 2*(12*a^2*b*c^3*d*e + (a*b^2*c^3 - 27*a^2* \\
& c^4)*d^2 - 3*(a^2*b^2*c^2 - 3*a^3*c^3)*e^2)*f^2 + 4*(3*a*c^5*d^3 - a*b*c^4* \\
& d^2*e - 3*a^2*c^4*d*e^2 + a^2*b*c^3*e^3)*f)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + \\
& 48*a^4*b^2*c^8 - 64*a^5*c^9))*\sqrt{-((b^3*c^3 + 12*a*b*c^4)*d^2 - 4*(3*a* \\
& b^2*c^3 + 4*a^2*c^4)*d*e + (a*b^3*c^2 + 12*a^2*b*c^3)*e^2 + (a*b^5 - 15*a^2 \\
& *b^3*c + 60*a^3*b*c^2)*f^2 - 2*((3*a*b^3*c^2 - 28*a^2*b*c^3)*d - (a*b^4*c - \\
& 6*a^2*b^2*c^2 - 24*a^3*c^3)*e)*f - (a*b^6*c^3 - 12*a^2*b^4*c^4 + 48*a^3*b^ \\
& 2*c^5 - 64*a^4*c^6)*\sqrt{(c^6*d^4 - 2*a*c^5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b^ \\
& ^4 - 18*a^3*b^2*c + 81*a^4*c^2)*f^4 - 4*(3*(a^2*b^2*c^2 - 9*a^3*c^3)*d - (a^ \\
& 2*b^3*c - 9*a^3*b*c^2)*e)*f^3 - 2*(12*a^2*b*c^3*d*e + (a*b^2*c^3 - 27*a^2*c^ \\
& ^4)*d^2 - 3*(a^2*b^2*c^2 - 3*a^3*c^3)*e^2)*f^2 + 4*(3*a*c^5*d^3 - a*b*c^4*d \\
& ^2*e - 3*a^2*c^4*d*e^2 + a^2*b*c^3*e^3)*f)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + \\
& 48*a^4*b^2*c^8 - 64*a^5*c^9)))/(a*b^6*c^3 - 12*a^2*b^4*c^4 + 48*a^3*b^2*c^5 \\
& - 64*a^4*c^6))) + 2*(b*c*d - 2*a*c*e + a*b*f)*x)/((b^2*c^2 - 4*a*c^3)*x^4 \\
& + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)
\end{aligned}$$

giac [B] time = 6.81, size = 6208, normalized size = 17.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")

[Out]
$$-1/2*(2*c^2*d*x^3 + b^2*f*x^3 - 2*a*c*f*x^3 - b*c*x^3*e + b*c*d*x + a*b*f*x - 2*a*c*x*e)/((c*x^4 + b*x^2 + a)*(b^2*c - 4*a*c^2)) - 1/16*(2*(2*b^2*c^4 - 8*a*c^5 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*b^2*c^2 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*c^3 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*b*c^3 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*c^4 - 2*(b^2 - 4*a*c)*c^4)*(b^2*c - 4*a*c^2)^2*d - (2*b^4*c^2 - 20*a*b^2*c^3 + 48*a^2*c^4 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*b^4 + 10*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^2*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*b^3*c - 24*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*c^2 - 12*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b*c^2 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*b^2*c^2 + 6*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 12*(b^2 - 4*a*c)*a*c^3)*(b^2*c - 4*a*c^2)^2*f - (2*b^3*c^3 - 8*a*b*c^4 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*b^3*c + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b*c^2 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*b^2*c^2 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*b*c^3 - 2*(b^2 - 4*a*c)*b*c^3)*(b^2*c - 4*a*c^2)^2*e - 2*(\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*b^5*c^3 - 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^3*c^4 - 2*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*b^4*c^4 - 2*b^5*c^4 + 16*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*b*c^5 + 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^2*c^5 + \sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*b^3*c^5 + 16*a*b^3*c^5 - 4*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b*c^6 - 32*a^2*b*c^6 + 2*(b^2 - 4*a*c)*b^3*c^4 - 8*(b^2 - 4*a*c)*a*b*c^5)*d*abs(b^2*c - 4*a*c^2) - 2*(\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^5*c^2 - 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*b^3*c^3 - 2*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^4*c^3 - 2*a*b^5*c^3 + 16*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^3*b*c^4 + 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*b^2*c^4 + \sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^3*c^4 + 16*a^2*b^3*c^4 - 4*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*b*c^5 - 32*a^3*b*c^5 + 2*(b^2 - 4*a*c)*a*b^3*c^3 - 8*(b^2 - 4*a*c)*a^2*b*c^4)*f*abs(b^2*c - 4*a*c^2) + 4*(\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^4*c^3 - 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*b^2*c^4 - 2*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^3*c^4 - 2*a*b^4*c^4 + 16*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^3*c^5 + 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*b*c^5 + \sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^2*c^5 + 16*a^2*b^2*c^5 - 4*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*c^6 - 32*a^3*c^6 + 2*(b^2 - 4*a*c)*a*b^2*c^4 - 8*(b^2 - 4*a*c)*a^2*c^5)*abs(b^2*c - 4*a*c^2)*e - 4*(2*b^6*c^6 - 16*a*b^4*c^7 + 32*a^2*b^2*c^8 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*b^6*c^4 + 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^4*c^5 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*b^5*c^5 - 16*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*b^2*c^6 - 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^3*c^6 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*b^4*c^6 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^2*c^7 - 2*(b^2 - 4*a*c)*b^4*c^6 + 8*(b^2 - 4*a*c)*a*b^2*c^7)*d + (2*b^8*c^4 - 32*a*b^6*c^5 + 160*a^2*b^4*c^6 - 256*a^3*b^2*c^7 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*b^8*c^2 + 16*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^6*c^3 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*b^7*c^3 - 80*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*b^4*c^4 - 24*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^5*c^4 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*b^6*c^4 + 128*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^3*b^2*c^5 + 64*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*b^3*c^5 + 12*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^4*c^5 - 32*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*b^2*c^6 - 2*(b^2 - 4*a*c)*b^6*c^4 + 24*(b^2 - 4*a*c)*a*b^4*c$$

$$\begin{aligned}
&^5 - 64*(b^2 - 4*a*c)*a^2*b^2*c^6)*f + (2*b^7*c^5 - 8*a*b^5*c^6 - 32*a^2*b^3*c^7 + 128*a^3*b*c^8 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^7*c^3 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^5*c^4 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^6*c^4 + 16*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b^3*c^5 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^5*c^5 - 64*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^3*b*c^6 - 32*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b^2*c^6 + 16*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b*c^7 - 2*(b^2 - 4*a*c)*b^5*c^5 + 32*(b^2 - 4*a*c)*a^2*b*c^7)*e)*\arctan(2*\sqrt{1/2}*x/\sqrt{(b^3*c - 4*a*b*c^2 + \sqrt{(b^3*c - 4*a*b*c^2)^2 - 4*(a*b^2*c - 4*a^2*c^2)*(b^2*c^2 - 4*a*c^3)})/(b^2*c^2 - 4*a*c^3)))/((a*b^6*c^3 - 12*a^2*b^4*c^4 - 2*a*b^5*c^4 + 48*a^3*b^2*c^5 + 16*a^2*b^3*c^5 + a*b^4*c^5 - 64*a^4*c^6 - 32*a^3*b*c^6 - 8*a^2*b^2*c^6 + 16*a^3*c^7)*\text{abs}(b^2*c - 4*a*c^2)*\text{abs}(c)) + 1/16*(2*(2*b^2*c^4 - 8*a*c^5 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^2*c^2 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*c^3 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b*c^3 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*c^4 - 2*(b^2 - 4*a*c)*c^4*(b^2*c - 4*a*c^2)^2*d - (2*b^4*c^2 - 20*a*b^2*c^3 + 48*a^2*c^4 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^4 + 10*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^3*c - 24*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*c^2 - 12*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b*c^2 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^2*c^2 + 6*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 12*(b^2 - 4*a*c)*a*c^3*(b^2*c - 4*a*c^2)^2*f - (2*b^3*c^3 - 8*a*b*c^4 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^3*c + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b*c^2 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^2*c^2 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^2*c^2 - 2*(b^2 - 4*a*c)*b*c^3*(b^2*c - 4*a*c^2)^2*e + 2*(\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^5*c^3 - 8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^3*c^4 - 2*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^4*c^4 + 2*b^5*c^4 + 16*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b*c^5 + 8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c^5 + \sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^3*c^5 - 16*a*b^3*c^5 - 4*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b*c^6 + 32*a^2*b*c^6 - 2*(b^2 - 4*a*c)*b^3*c^4 + 8*(b^2 - 4*a*c)*a*b*c^5)*d*\text{abs}(b^2*c - 4*a*c^2) + 2*(\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^5*c^2 - 8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b^3*c^3 - 2*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^4*c^3 + 2*a*b^5*c^3 + 16*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*b*c^4 + 8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b^2*c^4 + \sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^3*c^4 - 16*a^2*b^3*c^4 - 4*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b*c^5 + 32*a^3*b*c^5 - 2*(b^2 - 4*a*c)*a*b^3*c^3 + 8*(b^2 - 4*a*c)*a^2*b*c^4)*f*\text{abs}(b^2*c - 4*a*c^2) - 4*(\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^4*c^3 - 8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b^2*c^4 - 2*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^3*c^4 + 2*a*b^4*c^4 + 16*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*c^5 + 8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b*c^5 + \sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c^5 - 16*a^2*b^2*c^5 - 4*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*c^6 + 32*a^3*c^6 - 2*(b^2 - 4*a*c)*a*b^2*c^4 + 8*(b^2 - 4*a*c)*a^2*c^5)*\text{abs}(b^2*c - 4*a*c^2)*e - 4*(2*b^6*c^6 - 16*a*b^4*c^7 + 32*a^2*b^2*c^8 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^6*c^4 + 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^4*c^5 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^5*c^5 - 16*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b^2*c^6 - 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^3*c^6 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^4*c^6 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c^7 - 2*(b^2 - 4*a*c)*b^4*c^6 + 8*(b^2 - 4*a*c)*a*b^2*c^7
\end{aligned}$$

)d + (2*b^8*c^4 - 32*a*b^6*c^5 + 160*a^2*b^4*c^6 - 256*a^3*b^2*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*b^8*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a*b^6*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*b^7*c^3 - 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^2*b^4*c^4 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a*b^5*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*b^6*c^4 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^3*b^2*c^5 + 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^2*b^3*c^5 + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a*b^4*c^5 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^2*b^2*c^6 - 2*(b^2 - 4*a*c)*b^6*c^4 + 24*(b^2 - 4*a*c)*a*b^4*c^5 - 64*(b^2 - 4*a*c)*a^2*b^2*c^6)*f + (2*b^7*c^5 - 8*a*b^5*c^6 - 32*a^2*b^3*c^7 + 128*a^3*b*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*b^7*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a*b^5*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*b^6*c^4 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^2*b^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*b^5*c^5 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^3*b*c^6 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^2*b^2*c^6 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^2*b*c^7 - 2*(b^2 - 4*a*c)*b^5*c^5 + 32*(b^2 - 4*a*c)*a^2*b*c^7)*e) *arctan(2*sqrt(1/2)*x/sqrt((b^3*c - 4*a*b*c^2 - sqrt((b^3*c - 4*a*b*c^2)^2 - 4*(a*b^2*c - 4*a^2*c^2)*(b^2*c^2 - 4*a*c^3)))/(b^2*c^2 - 4*a*c^3)))/((a*b^6*c^3 - 12*a^2*b^4*c^4 - 2*a*b^5*c^4 + 48*a^3*b^2*c^5 + 16*a^2*b^3*c^5 + a*b^4*c^5 - 64*a^4*c^6 - 32*a^3*b*c^6 - 8*a^2*b^2*c^6 + 16*a^3*c^7)*abs(b^2*c - 4*a*c^2)*abs(c))

maple [B] time = 0.04, size = 1300, normalized size = 3.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x)

[Out] (-1/2*(2*a*c*f-b^2*f+b*c*e-2*c^2*d)/(4*a*c-b^2)/c*x^3+1/2*(a*b*f-2*a*c*e+b*c*d)/(4*a*c-b^2)/c*x)/(c*x^4+b*x^2+a)-3/2/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*c*x)*a*f+1/4/(4*a*c-b^2)/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*c*x)*b^2*f+1/4/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*c*x)*b*e-1/2/(4*a*c-b^2)*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*c*x)*d+2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*c*x)*a*b*f-1/(4*a*c-b^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*c*x)*a*e-1/4/(4*a*c-b^2)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*c*x)*a*f-1/4/(4*a*c-b^2)/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*c*x)*b^2*f-1/4/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*c*x)*b*e+1/2/(4*a*c-b^2)*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*c*x)*d+2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/

$$\frac{1/2}{((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*a*e^{-1/4}/(4*a*c-b^2)/c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^3*f-1/4/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^2*e+1/(4*a*c-b^2)*c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b*d}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(2c^2d - bce + (b^2 - 2ac)f)x^3 + (bcd - 2ace + abf)x}{2((b^2c^2 - 4ac^3)x^4 + ab^2c - 4a^2c^2 + (b^3c - 4abc^2)x^2)} - \int \frac{bcd - 2ace + abf - (2c^2d - bce - (b^2 - 6ac)f)x^2}{cx^4 + bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")

[Out] -1/2*((2*c^2*d - b*c*e + (b^2 - 2*a*c)*f)*x^3 + (b*c*d - 2*a*c*e + a*b*f)*x^2)/((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2) - 1/2*integrate(-(b*c*d - 2*a*c*e + a*b*f - (2*c^2*d - b*c*e - (b^2 - 6*a*c)*f)*x^2)/(c*x^4 + b*x^2 + a), x)/(b^2*c - 4*a*c^2)

mupad [B] time = 6.54, size = 19494, normalized size = 53.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x)

[Out] ((x^3*(2*c^2*d + b^2*f - 2*a*c*f - b*c*e))/(2*c*(4*a*c - b^2)) + (x*(a*b*f - 2*a*c*e + b*c*d))/(2*c*(4*a*c - b^2)))/(a + b*x^2 + c*x^4) - atan((((204*8*a^4*c^6*e + 16*b^7*c^3*d + 768*a^2*b^3*c^5*d + 384*a^2*b^4*c^4*e - 1536*a^3*b^2*c^5*e - 192*a^2*b^5*c^3*f + 768*a^3*b^3*c^4*f - 192*a*b^5*c^4*d - 1024*a^3*b*c^6*d - 32*a*b^6*c^3*e + 16*a*b^7*c^2*f - 1024*a^4*b*c^5*f)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)) - (x*((768*a^4*b*c^7*d^2 - b^9*c^3*d^2 - c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - a*b^11*f^2 - a*b^9*c^2*e^2 + 768*a^5*b*c^6*e^2 + a*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + a*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^2*b^9*c*f^2 + 3840*a^6*b*c^5*f^2 - 9*a^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 96*a^2*b^5*c^5*d^2 - 512*a^3*b^3*c^6*d^2 + 96*a^3*b^5*c^4*e^2 - 512*a^4*b^3*c^5*e^2 - 288*a^3*b^7*c^2*f^2 + 1504*a^4*b^5*c^3*f^2 - 3840*a^5*b^3*c^4*f^2 - 1024*a^5*c^7*d*e - 3072*a^6*c^6*e*f + 12*a*b^8*c^3*d*e + 6*a*b^9*c^2*d*f + 3584*a^5*b*c^6*d*f - 6*a*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 128*a^2*b^6*c^4*d*e + 384*a^3*b^4*c^5*d*e - 128*a^2*b^7*c^3*d*f + 960*a^3*b^5*c^4*d*f - 3072*a^4*b^3*c^5*d*f + 36*a^2*b^8*c^2*e*f - 192*a^3*b^6*c^3*e*f + 128*a^4*b^4*c^4*e*f + 1536*a^5*b^2*c^5*e*f - 2*a*b^10*c*e*f + 2*a*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2)))/(32*(4096*a^7*c^9 + a*b^12*c^3 - 24*a^2*b^10*c^4 + 240*a^3*b^8*c^5 - 1280*a^4*b^6*c^6 + 3840*a^5*b^4*c^7 - 6144*a^6*b^2*c^8)))^(1/2)*(16*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*((768*a^4*b*c^7*d^2 - b^9*c^3*d^2 - c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - a*b^11*f^2 - a*b^9*c^2*e^2 + 768*a^5*b*c^6*e^2 + a*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + a*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^2*b^9*c*f^2 + 3840*a^6*b*c^5*f^2 - 9*a^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 96*a^2*b^5*c^5*d^2 - 512*a^3*b^3*c^6*d^2 + 96*a^3*b^5*c^4*e^2 - 512*a^4*b^3*c^5*e^2 - 288*a^3*b^7*c^2*f^2 + 1504*a^4*b^5*c^3*f^2 - 3840*a^5*b^3*c^4*f^2 - 1024*a^5*c^7*d*e - 3072*a^6*c^6*e*f + 12*a*b^8*c^3*d*e + 6*a*b^9*c^2*d*f + 3584*a^5*b*c^6*d*f - 6*a*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 128*a^2*b^6*c^4*d*e + 384*a^3*b^4*c^5*d*e - 128*a^2*b^7*c^3*d*f + 960*a^3*b^5*c^4*d*f - 3072*a^4*b^3*c^5*d*f + 36*a^2*b^8*c^2*e*f - 192*a^3*b^6*c^3*e*f + 128*a^4*b^4*c^4*e*f + 1536*a^5*b^2*c^5*e*f - 2*a*b^10*c*e*f + 2*a*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2)))/(32*(4096*a^7*c^9 + a*b^12*c^3 - 24*a^2*b^10*c^4 + 240*a^3*b^8*c^5 - 1280*a^4*b^6*c^6 + 3840*a^5*b^4*c^7 - 6144*a^6*b^2*c^8)))^(1/2)

$$\begin{aligned}
& 4*c^7 - 6144*a^6*b^2*c^8)))^{(1/2)} + (x*(8*a*c^5*d^2 - b^6*f^2 - 8*a^2*c^4*e \\
& ^2 - 10*b^2*c^4*d^2 + 72*a^3*c^3*f^2 - b^4*c^2*e^2 - 2*a*b^2*c^3*e^2 - 2*b^ \\
& 5*c*e*f - 74*a^2*b^2*c^2*f^2 + 16*a*b^4*c*f^2 + 48*a^2*c^4*d*f + 6*b^3*c^3* \\
& d*e + 6*b^4*c^2*d*f - 52*a*b^2*c^3*d*f + 14*a*b^3*c^2*e*f + 8*a^2*b*c^3*e*f \\
& + 8*a*b*c^4*d*e))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*((768*a^4*b*c^7* \\
& d^2 - b^9*c^3*d^2 - c^3*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - a*b^11*f^2 - a*b^9*c \\
& ^2*e^2 + 768*a^5*b*c^6*e^2 + a*b^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + a*c^2*e^2 \\
& *(-(4*a*c - b^2)^9)^{(1/2)} + 27*a^2*b^9*c*f^2 + 3840*a^6*b*c^5*f^2 - 9*a^2*c \\
& *f^2*(-(4*a*c - b^2)^9)^{(1/2)} + 96*a^2*b^5*c^5*d^2 - 512*a^3*b^3*c^6*d^2 + \\
& 96*a^3*b^5*c^4*e^2 - 512*a^4*b^3*c^5*e^2 - 288*a^3*b^7*c^2*f^2 + 1504*a^4*b \\
& ^5*c^3*f^2 - 3840*a^5*b^3*c^4*f^2 - 1024*a^5*c^7*d*e - 3072*a^6*c^6*e*f + 1 \\
& 2*a*b^8*c^3*d*e + 6*a*b^9*c^2*d*f + 3584*a^5*b*c^6*d*f - 6*a*c^2*d*f*(-(4*a \\
& *c - b^2)^9)^{(1/2)} - 128*a^2*b^6*c^4*d*e + 384*a^3*b^4*c^5*d*e - 128*a^2*b^ \\
& 7*c^3*d*f + 960*a^3*b^5*c^4*d*f - 3072*a^4*b^3*c^5*d*f + 36*a^2*b^8*c^2*e*f \\
& - 192*a^3*b^6*c^3*e*f + 128*a^4*b^4*c^4*e*f + 1536*a^5*b^2*c^5*e*f - 2*a*b \\
& ^10*c*e*f + 2*a*b*c*e*f*(-(4*a*c - b^2)^9)^{(1/2)))/(32*(4096*a^7*c^9 + a*b^1 \\
& 2*c^3 - 24*a^2*b^10*c^4 + 240*a^3*b^8*c^5 - 1280*a^4*b^6*c^6 + 3840*a^5*b^4 \\
& *c^7 - 6144*a^6*b^2*c^8)))^{(1/2)}*i - (((2048*a^4*c^6*e + 16*b^7*c^3*d + 76 \\
& 8*a^2*b^3*c^5*d + 384*a^2*b^4*c^4*e - 1536*a^3*b^2*c^5*e - 192*a^2*b^5*c^3* \\
& f + 768*a^3*b^3*c^4*f - 192*a*b^5*c^4*d - 1024*a^3*b*c^6*d - 32*a*b^6*c^3*e \\
& + 16*a*b^7*c^2*f - 1024*a^4*b*c^5*f)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 \\
& + 48*a^2*b^2*c^3)) + (x*((768*a^4*b*c^7*d^2 - b^9*c^3*d^2 - c^3*d^2*(-(4*a \\
& *c - b^2)^9)^{(1/2)} - a*b^11*f^2 - a*b^9*c^2*e^2 + 768*a^5*b*c^6*e^2 + a*b^2 \\
& *f^2*(-(4*a*c - b^2)^9)^{(1/2)} + a*c^2*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 27*a^2 \\
& *b^9*c*f^2 + 3840*a^6*b*c^5*f^2 - 9*a^2*c*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + 96 \\
& *a^2*b^5*c^5*d^2 - 512*a^3*b^3*c^6*d^2 + 96*a^3*b^5*c^4*e^2 - 512*a^4*b^3*c \\
& ^5*e^2 - 288*a^3*b^7*c^2*f^2 + 1504*a^4*b^5*c^3*f^2 - 3840*a^5*b^3*c^4*f^2 \\
& - 1024*a^5*c^7*d*e - 3072*a^6*c^6*e*f + 12*a*b^8*c^3*d*e + 6*a*b^9*c^2*d*f \\
& + 3584*a^5*b*c^6*d*f - 6*a*c^2*d*f*(-(4*a*c - b^2)^9)^{(1/2)} - 128*a^2*b^6*c \\
& ^4*d*e + 384*a^3*b^4*c^5*d*e - 128*a^2*b^7*c^3*d*f + 960*a^3*b^5*c^4*d*f - \\
& 3072*a^4*b^3*c^5*d*f + 36*a^2*b^8*c^2*e*f - 192*a^3*b^6*c^3*e*f + 128*a^4*b \\
& ^4*c^4*e*f + 1536*a^5*b^2*c^5*e*f - 2*a*b^10*c*e*f + 2*a*b*c*e*f*(-(4*a*c - \\
& b^2)^9)^{(1/2)))/(32*(4096*a^7*c^9 + a*b^12*c^3 - 24*a^2*b^10*c^4 + 240*a^3* \\
& b^8*c^5 - 1280*a^4*b^6*c^6 + 3840*a^5*b^4*c^7 - 6144*a^6*b^2*c^8)))^{(1/2)}*(\\
& 16*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(2*(b^4*c + \\
& 16*a^2*c^3 - 8*a*b^2*c^2)))*((768*a^4*b*c^7*d^2 - b^9*c^3*d^2 - c^3*d^2*(-(\\
& 4*a*c - b^2)^9)^{(1/2)} - a*b^11*f^2 - a*b^9*c^2*e^2 + 768*a^5*b*c^6*e^2 + a \\
& *b^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + a*c^2*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 27 \\
& *a^2*b^9*c*f^2 + 3840*a^6*b*c^5*f^2 - 9*a^2*c*f^2*(-(4*a*c - b^2)^9)^{(1/2)} \\
& + 96*a^2*b^5*c^5*d^2 - 512*a^3*b^3*c^6*d^2 + 96*a^3*b^5*c^4*e^2 - 512*a^4*b \\
& ^3*c^5*e^2 - 288*a^3*b^7*c^2*f^2 + 1504*a^4*b^5*c^3*f^2 - 3840*a^5*b^3*c^4* \\
& f^2 - 1024*a^5*c^7*d*e - 3072*a^6*c^6*e*f + 12*a*b^8*c^3*d*e + 6*a*b^9*c^2* \\
& d*f + 3584*a^5*b*c^6*d*f - 6*a*c^2*d*f*(-(4*a*c - b^2)^9)^{(1/2)} - 128*a^2*b \\
& ^6*c^4*d*e + 384*a^3*b^4*c^5*d*e - 128*a^2*b^7*c^3*d*f + 960*a^3*b^5*c^4*d* \\
& f - 3072*a^4*b^3*c^5*d*f + 36*a^2*b^8*c^2*e*f - 192*a^3*b^6*c^3*e*f + 128*a \\
& ^4*b^4*c^4*e*f + 1536*a^5*b^2*c^5*e*f - 2*a*b^10*c*e*f + 2*a*b*c*e*f*(-(4*a \\
& *c - b^2)^9)^{(1/2)))/(32*(4096*a^7*c^9 + a*b^12*c^3 - 24*a^2*b^10*c^4 + 240* \\
& a^3*b^8*c^5 - 1280*a^4*b^6*c^6 + 3840*a^5*b^4*c^7 - 6144*a^6*b^2*c^8)))^{(1/ \\
& 2)} - (x*(8*a*c^5*d^2 - b^6*f^2 - 8*a^2*c^4*e^2 - 10*b^2*c^4*d^2 + 72*a^3*c^ \\
& 3*f^2 - b^4*c^2*e^2 - 2*a*b^2*c^3*e^2 - 2*b^5*c*e*f - 74*a^2*b^2*c^2*f^2 + \\
& 16*a*b^4*c*f^2 + 48*a^2*c^4*d*f + 6*b^3*c^3*d*e + 6*b^4*c^2*d*f - 52*a*b^2* \\
& c^3*d*f + 14*a*b^3*c^2*e*f + 8*a^2*b*c^3*e*f + 8*a*b*c^4*d*e))/(2*(b^4*c + \\
& 16*a^2*c^3 - 8*a*b^2*c^2)))*((768*a^4*b*c^7*d^2 - b^9*c^3*d^2 - c^3*d^2*(-(\\
& 4*a*c - b^2)^9)^{(1/2)} - a*b^11*f^2 - a*b^9*c^2*e^2 + 768*a^5*b*c^6*e^2 + a \\
& *b^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + a*c^2*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 27* \\
& a^2*b^9*c*f^2 + 3840*a^6*b*c^5*f^2 - 9*a^2*c*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + \\
& 96*a^2*b^5*c^5*d^2 - 512*a^3*b^3*c^6*d^2 + 96*a^3*b^5*c^4*e^2 - 512*a^4*b^ \\
& 3*c^5*e^2 - 288*a^3*b^7*c^2*f^2 + 1504*a^4*b^5*c^3*f^2 - 3840*a^5*b^3*c^4*f \\
& ^2 - 1024*a^5*c^7*d*e - 3072*a^6*c^6*e*f + 12*a*b^8*c^3*d*e + 6*a*b^9*c^2*d
\end{aligned}$$

$$\begin{aligned}
&^2 + a*b^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + a*c^2*e^2*(-(4*a*c - b^2)^9)^{(1/2)} \\
&+ 27*a^2*b^9*c*f^2 + 3840*a^6*b*c^5*f^2 - 9*a^2*c*f^2*(-(4*a*c - b^2)^9)^{(1/2)} \\
&+ 96*a^2*b^5*c^5*d^2 - 512*a^3*b^3*c^6*d^2 + 96*a^3*b^5*c^4*e^2 - 512 \\
&*a^4*b^3*c^5*e^2 - 288*a^3*b^7*c^2*f^2 + 1504*a^4*b^5*c^3*f^2 - 3840*a^5*b^ \\
&3*c^4*f^2 - 1024*a^5*c^7*d*e - 3072*a^6*c^6*e*f + 12*a*b^8*c^3*d*e + 6*a*b^ \\
&9*c^2*d*f + 3584*a^5*b*c^6*d*f - 6*a*c^2*d*f*(-(4*a*c - b^2)^9)^{(1/2)} - 128 \\
&*a^2*b^6*c^4*d*e + 384*a^3*b^4*c^5*d*e - 128*a^2*b^7*c^3*d*f + 960*a^3*b^5* \\
&c^4*d*f - 3072*a^4*b^3*c^5*d*f + 36*a^2*b^8*c^2*e*f - 192*a^3*b^6*c^3*e*f + \\
&128*a^4*b^4*c^4*e*f + 1536*a^5*b^2*c^5*e*f - 2*a*b^10*c*e*f + 2*a*b*c*e*f* \\
&(-(4*a*c - b^2)^9)^{(1/2)})/(32*(4096*a^7*c^9 + a*b^12*c^3 - 24*a^2*b^10*c^4 \\
&+ 240*a^3*b^8*c^5 - 1280*a^4*b^6*c^6 + 3840*a^5*b^4*c^7 - 6144*a^6*b^2*c^8) \\
&))^{(1/2)}*(16*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(\\
&2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2))*((768*a^4*b*c^7*d^2 - b^9*c^3*d^2 - \\
&c^3*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - a*b^11*f^2 - a*b^9*c^2*e^2 + 768*a^5*b*c^ \\
&6*e^2 + a*b^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + a*c^2*e^2*(-(4*a*c - b^2)^9)^{(1/2)} \\
&+ 27*a^2*b^9*c*f^2 + 3840*a^6*b*c^5*f^2 - 9*a^2*c*f^2*(-(4*a*c - b^2)^9)^{(1/2)} \\
&+ 96*a^2*b^5*c^5*d^2 - 512*a^3*b^3*c^6*d^2 + 96*a^3*b^5*c^4*e^2 - \\
&512*a^4*b^3*c^5*e^2 - 288*a^3*b^7*c^2*f^2 + 1504*a^4*b^5*c^3*f^2 - 3840*a^5 \\
&5*b^3*c^4*f^2 - 1024*a^5*c^7*d*e - 3072*a^6*c^6*e*f + 12*a*b^8*c^3*d*e + 6* \\
&a*b^9*c^2*d*f + 3584*a^5*b*c^6*d*f - 6*a*c^2*d*f*(-(4*a*c - b^2)^9)^{(1/2)} - \\
&128*a^2*b^6*c^4*d*e + 384*a^3*b^4*c^5*d*e - 128*a^2*b^7*c^3*d*f + 960*a^3* \\
&b^5*c^4*d*f - 3072*a^4*b^3*c^5*d*f + 36*a^2*b^8*c^2*e*f - 192*a^3*b^6*c^3*e \\
&>*f + 128*a^4*b^4*c^4*e*f + 1536*a^5*b^2*c^5*e*f - 2*a*b^10*c*e*f + 2*a*b*c* \\
&>e*f*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(4096*a^7*c^9 + a*b^12*c^3 - 24*a^2*b^10* \\
&c^4 + 240*a^3*b^8*c^5 - 1280*a^4*b^6*c^6 + 3840*a^5*b^4*c^7 - 6144*a^6*b^2* \\
&c^8))^{(1/2)} - (x*(8*a*c^5*d^2 - b^6*f^2 - 8*a^2*c^4*e^2 - 10*b^2*c^4*d^2 + \\
&72*a^3*c^3*f^2 - b^4*c^2*e^2 - 2*a*b^2*c^3*e^2 - 2*b^5*c*e*f - 74*a^2*b^2* \\
&c^2*f^2 + 16*a*b^4*c*f^2 + 48*a^2*c^4*d*f + 6*b^3*c^3*d*e + 6*b^4*c^2*d*f - \\
&52*a*b^2*c^3*d*f + 14*a*b^3*c^2*e*f + 8*a^2*b*c^3*e*f + 8*a*b*c^4*d*e))/(2 \\
&*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2))*((768*a^4*b*c^7*d^2 - b^9*c^3*d^2 - c^ \\
&3*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - a*b^11*f^2 - a*b^9*c^2*e^2 + 768*a^5*b*c^ \\
&6*e^2 + a*b^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + a*c^2*e^2*(-(4*a*c - b^2)^9)^{(1/2)} \\
&+ 27*a^2*b^9*c*f^2 + 3840*a^6*b*c^5*f^2 - 9*a^2*c*f^2*(-(4*a*c - b^2)^9)^{(1/2)} \\
&+ 96*a^2*b^5*c^5*d^2 - 512*a^3*b^3*c^6*d^2 + 96*a^3*b^5*c^4*e^2 - \\
&512*a^4*b^3*c^5*e^2 - 288*a^3*b^7*c^2*f^2 + 1504*a^4*b^5*c^3*f^2 - 3840*a^5 \\
&*b^3*c^4*f^2 - 1024*a^5*c^7*d*e - 3072*a^6*c^6*e*f + 12*a*b^8*c^3*d*e + 6*a \\
&*b^9*c^2*d*f + 3584*a^5*b*c^6*d*f - 6*a*c^2*d*f*(-(4*a*c - b^2)^9)^{(1/2)} - \\
&128*a^2*b^6*c^4*d*e + 384*a^3*b^4*c^5*d*e - 128*a^2*b^7*c^3*d*f + 960*a^3*b \\
&^5*c^4*d*f - 3072*a^4*b^3*c^5*d*f + 36*a^2*b^8*c^2*e*f - 192*a^3*b^6*c^3*e* \\
&>f + 128*a^4*b^4*c^4*e*f + 1536*a^5*b^2*c^5*e*f - 2*a*b^10*c*e*f + 2*a*b*c*e \\
&>*f*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(4096*a^7*c^9 + a*b^12*c^3 - 24*a^2*b^10*c^ \\
&^4 + 240*a^3*b^8*c^5 - 1280*a^4*b^6*c^6 + 3840*a^5*b^4*c^7 - 6144*a^6*b^2*c^ \\
&^8))^{(1/2)} + (8*a*c^5*d^3 + b^6*d*f^2 + 5*a^2*b^4*f^3 + 6*b^2*c^4*d^3 + 21 \\
&6*a^4*c^2*f^3 - 3*a*b^3*c^2*e^3 - 4*a^2*b*c^3*e^3 - 66*a^3*b^2*c*f^3 + 8*a^ \\
&2*c^4*d*e^2 + 72*a^2*c^4*d^2*f + 216*a^3*c^3*d*f^2 - 5*b^3*c^3*d^2*e + b^4* \\
&c^2*d*e^2 + 24*a^3*c^3*e^2*f - 5*b^4*c^2*d^2*f - 3*a*b^5*e*f^2 - 28*a*b*c^4 \\
&*d^2*e - 12*a*b^4*c*d*f^2 - 6*a*b^4*c*e^2*f + 18*a*b^2*c^3*d*e^2 + 26*a*b^2 \\
&*c^3*d^2*f + 51*a^2*b^3*c*e*f^2 - 204*a^3*b*c^2*e*f^2 + 2*b^5*c*d*e*f + 2*a \\
&^2*b^2*c^2*d*f^2 + 42*a^2*b^2*c^2*e^2*f + 6*a*b^3*c^2*d*e*f - 152*a^2*b*c^3 \\
&*d*e*f)/(4*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)))*((768*a^ \\
&4*b*c^7*d^2 - b^9*c^3*d^2 - c^3*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - a*b^11*f^2 - \\
&a*b^9*c^2*e^2 + 768*a^5*b*c^6*e^2 + a*b^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + a \\
&*c^2*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 27*a^2*b^9*c*f^2 + 3840*a^6*b*c^5*f^2 - \\
&9*a^2*c*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + 96*a^2*b^5*c^5*d^2 - 512*a^3*b^3*c^ \\
&6*d^2 + 96*a^3*b^5*c^4*e^2 - 512*a^4*b^3*c^5*e^2 - 288*a^3*b^7*c^2*f^2 + 15 \\
&04*a^4*b^5*c^3*f^2 - 3840*a^5*b^3*c^4*f^2 - 1024*a^5*c^7*d*e - 3072*a^6*c^6 \\
&*e*f + 12*a*b^8*c^3*d*e + 6*a*b^9*c^2*d*f + 3584*a^5*b*c^6*d*f - 6*a*c^2*d* \\
&>f*(-(4*a*c - b^2)^9)^{(1/2)} - 128*a^2*b^6*c^4*d*e + 384*a^3*b^4*c^5*d*e - 12 \\
&8*a^2*b^7*c^3*d*f + 960*a^3*b^5*c^4*d*f - 3072*a^4*b^3*c^5*d*f + 36*a^2*b^8
\end{aligned}$$

$$\begin{aligned}
& *c^2*ef - 192*a^3*b^6*c^3*ef + 128*a^4*b^4*c^4*ef + 1536*a^5*b^2*c^5*ef \\
& - 2*a*b^{10}*c*ef + 2*a*b*c*ef*(-(4*a*c - b^2)^9)^{(1/2)}/(32*(4096*a^7*c^9 \\
& + a*b^{12}*c^3 - 24*a^2*b^{10}*c^4 + 240*a^3*b^8*c^5 - 1280*a^4*b^6*c^6 + 3840 \\
& *a^5*b^4*c^7 - 6144*a^6*b^2*c^8)))^{(1/2)}*2i - \operatorname{atan}((((2048*a^4*c^6*e + 16* \\
& b^7*c^3*d + 768*a^2*b^3*c^5*d + 384*a^2*b^4*c^4*e - 1536*a^3*b^2*c^5*e - 19 \\
& 2*a^2*b^5*c^3*f + 768*a^3*b^3*c^4*f - 192*a*b^5*c^4*d - 1024*a^3*b*c^6*d - \\
& 32*a*b^6*c^3*e + 16*a*b^7*c^2*f - 1024*a^4*b*c^5*f)/(8*(b^6*c - 64*a^3*c^4 \\
& - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)) - (x*((c^3*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - \\
& b^9*c^3*d^2 - a*b^{11}*f^2 + 768*a^4*b*c^7*d^2 - a*b^9*c^2*e^2 + 768*a^5*b*c \\
& ^6*e^2 - a*b^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - a*c^2*e^2*(-(4*a*c - b^2)^9)^ \\
& (1/2) + 27*a^2*b^9*c*f^2 + 3840*a^6*b*c^5*f^2 + 9*a^2*c*f^2*(-(4*a*c - b^2) \\
& ^9)^{(1/2)} + 96*a^2*b^5*c^5*d^2 - 512*a^3*b^3*c^6*d^2 + 96*a^3*b^5*c^4*e^2 - \\
& 512*a^4*b^3*c^5*e^2 - 288*a^3*b^7*c^2*f^2 + 1504*a^4*b^5*c^3*f^2 - 3840*a^ \\
& 5*b^3*c^4*f^2 - 1024*a^5*c^7*d*e - 3072*a^6*c^6*ef + 12*a*b^8*c^3*d*e + 6* \\
& a*b^9*c^2*d*f + 3584*a^5*b*c^6*d*f + 6*a*c^2*d*f*(-(4*a*c - b^2)^9)^{(1/2)} - \\
& 128*a^2*b^6*c^4*d*e + 384*a^3*b^4*c^5*d*e - 128*a^2*b^7*c^3*d*f + 960*a^3* \\
& b^5*c^4*d*f - 3072*a^4*b^3*c^5*d*f + 36*a^2*b^8*c^2*ef - 192*a^3*b^6*c^3* \\
& *f + 128*a^4*b^4*c^4*ef + 1536*a^5*b^2*c^5*ef - 2*a*b^{10}*c*ef - 2*a*b*c* \\
& ef*(-(4*a*c - b^2)^9)^{(1/2)}/(32*(4096*a^7*c^9 + a*b^{12}*c^3 - 24*a^2*b^{10}* \\
& c^4 + 240*a^3*b^8*c^5 - 1280*a^4*b^6*c^6 + 3840*a^5*b^4*c^7 - 6144*a^6*b^2* \\
& c^8)))^{(1/2)}*(16*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5 \\
&))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2))*((c^3*d^2*(-(4*a*c - b^2)^9)^{(1/ \\
& 2)} - b^9*c^3*d^2 - a*b^{11}*f^2 + 768*a^4*b*c^7*d^2 - a*b^9*c^2*e^2 + 768*a^5 \\
& *b*c^6*e^2 - a*b^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - a*c^2*e^2*(-(4*a*c - b^2) \\
& ^9)^{(1/2)} + 27*a^2*b^9*c*f^2 + 3840*a^6*b*c^5*f^2 + 9*a^2*c*f^2*(-(4*a*c - \\
& b^2)^9)^{(1/2)} + 96*a^2*b^5*c^5*d^2 - 512*a^3*b^3*c^6*d^2 + 96*a^3*b^5*c^4*e \\
& ^2 - 512*a^4*b^3*c^5*e^2 - 288*a^3*b^7*c^2*f^2 + 1504*a^4*b^5*c^3*f^2 - 384 \\
& 0*a^5*b^3*c^4*f^2 - 1024*a^5*c^7*d*e - 3072*a^6*c^6*ef + 12*a*b^8*c^3*d*e \\
& + 6*a*b^9*c^2*d*f + 3584*a^5*b*c^6*d*f + 6*a*c^2*d*f*(-(4*a*c - b^2)^9)^{(1/ \\
& 2)} - 128*a^2*b^6*c^4*d*e + 384*a^3*b^4*c^5*d*e - 128*a^2*b^7*c^3*d*f + 960* \\
& a^3*b^5*c^4*d*f - 3072*a^4*b^3*c^5*d*f + 36*a^2*b^8*c^2*ef - 192*a^3*b^6*c \\
& ^3*ef + 128*a^4*b^4*c^4*ef + 1536*a^5*b^2*c^5*ef - 2*a*b^{10}*c*ef - 2*a* \\
& b*c*ef*(-(4*a*c - b^2)^9)^{(1/2)}/(32*(4096*a^7*c^9 + a*b^{12}*c^3 - 24*a^2*b \\
& ^{10}*c^4 + 240*a^3*b^8*c^5 - 1280*a^4*b^6*c^6 + 3840*a^5*b^4*c^7 - 6144*a^6* \\
& b^2*c^8)))^{(1/2)} + (x*(8*a*c^5*d^2 - b^6*f^2 - 8*a^2*c^4*e^2 - 10*b^2*c^4*d \\
& ^2 + 72*a^3*c^3*f^2 - b^4*c^2*e^2 - 2*a*b^2*c^3*e^2 - 2*b^5*c*ef - 74*a^2* \\
& b^2*c^2*f^2 + 16*a*b^4*c*f^2 + 48*a^2*c^4*d*f + 6*b^3*c^3*d*e + 6*b^4*c^2*d \\
& *f - 52*a*b^2*c^3*d*f + 14*a*b^3*c^2*ef + 8*a^2*b*c^3*ef + 8*a*b*c^4*d*e) \\
&)/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2))*((c^3*d^2*(-(4*a*c - b^2)^9)^{(1/2 \\
&)} - b^9*c^3*d^2 - a*b^{11}*f^2 + 768*a^4*b*c^7*d^2 - a*b^9*c^2*e^2 + 768*a^5* \\
& b*c^6*e^2 - a*b^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - a*c^2*e^2*(-(4*a*c - b^2) \\
& ^9)^{(1/2)} + 27*a^2*b^9*c*f^2 + 3840*a^6*b*c^5*f^2 + 9*a^2*c*f^2*(-(4*a*c - b \\
& ^2)^9)^{(1/2)} + 96*a^2*b^5*c^5*d^2 - 512*a^3*b^3*c^6*d^2 + 96*a^3*b^5*c^4*e \\
& ^2 - 512*a^4*b^3*c^5*e^2 - 288*a^3*b^7*c^2*f^2 + 1504*a^4*b^5*c^3*f^2 - 3840 \\
& *a^5*b^3*c^4*f^2 - 1024*a^5*c^7*d*e - 3072*a^6*c^6*ef + 12*a*b^8*c^3*d*e + \\
& 6*a*b^9*c^2*d*f + 3584*a^5*b*c^6*d*f + 6*a*c^2*d*f*(-(4*a*c - b^2)^9)^{(1/2 \\
&)} - 128*a^2*b^6*c^4*d*e + 384*a^3*b^4*c^5*d*e - 128*a^2*b^7*c^3*d*f + 960*a \\
& ^3*b^5*c^4*d*f - 3072*a^4*b^3*c^5*d*f + 36*a^2*b^8*c^2*ef - 192*a^3*b^6*c^ \\
& 3*ef + 128*a^4*b^4*c^4*ef + 1536*a^5*b^2*c^5*ef - 2*a*b^{10}*c*ef - 2*a*b \\
& *c*ef*(-(4*a*c - b^2)^9)^{(1/2)}/(32*(4096*a^7*c^9 + a*b^{12}*c^3 - 24*a^2*b^ \\
& ^{10}*c^4 + 240*a^3*b^8*c^5 - 1280*a^4*b^6*c^6 + 3840*a^5*b^4*c^7 - 6144*a^6*b \\
& ^2*c^8)))^{(1/2)}*1i - (((2048*a^4*c^6*e + 16*b^7*c^3*d + 768*a^2*b^3*c^5*d + \\
& 384*a^2*b^4*c^4*e - 1536*a^3*b^2*c^5*e - 192*a^2*b^5*c^3*f + 768*a^3*b^3*c \\
& ^4*f - 192*a*b^5*c^4*d - 1024*a^3*b*c^6*d - 32*a*b^6*c^3*e + 16*a*b^7*c^2*f \\
& - 1024*a^4*b*c^5*f)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 \\
&)) + (x*((c^3*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - b^9*c^3*d^2 - a*b^{11}*f^2 + 768 \\
& *a^4*b*c^7*d^2 - a*b^9*c^2*e^2 + 768*a^5*b*c^6*e^2 - a*b^2*f^2*(-(4*a*c - b \\
& ^2)^9)^{(1/2)} - a*c^2*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 27*a^2*b^9*c*f^2 + 3840 \\
& *a^6*b*c^5*f^2 + 9*a^2*c*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + 96*a^2*b^5*c^5*d^2
\end{aligned}$$

$$\begin{aligned}
& - 512a^3b^3c^6d^2 + 96a^3b^5c^4e^2 - 512a^4b^3c^5e^2 - 288a^3b^7c^2f^2 + 1504a^4b^5c^3f^2 - 3840a^5b^3c^4f^2 - 1024a^5c^7d^* \\
& e - 3072a^6c^6e^*f + 12a^*b^8c^3d^*e + 6a^*b^9c^2d^*f + 3584a^5b^*c^6d^* \\
& d^*f + 6a^*c^2d^*f^*(-(4a^*c - b^2)^9)^{(1/2)} - 128a^2b^6c^4d^*e + 384a^3b^4c^5d^*e - 128a^2b^7c^3d^*f + 960a^3b^5c^4d^*f - 3072a^4b^3c^5d^* \\
& d^*f + 36a^2b^8c^2e^*f - 192a^3b^6c^3e^*f + 128a^4b^4c^4e^*f + 1536a^5b^2c^5e^*f - 2a^*b^{10}c^*e^*f - 2a^*b^*c^*e^*f^*(-(4a^*c - b^2)^9)^{(1/2)})/(\\
& 32(4096a^7c^9 + a^*b^{12}c^3 - 24a^2b^{10}c^4 + 240a^3b^8c^5 - 1280a^4b^6c^6 + 3840a^5b^4c^7 - 6144a^6b^2c^8))^{(1/2)}(16b^7c^3 - 192a^* \\
& a^*b^5c^4 - 1024a^3b^*c^6 + 768a^2b^3c^5)/(2(b^4c + 16a^2c^3 - 8a^* \\
& b^2c^2))((c^3d^2*(-(4a^*c - b^2)^9)^{(1/2)} - b^9c^3d^2 - a^*b^{11}f^2 + \\
& 768a^4b^*c^7d^2 - a^*b^9c^2e^2 + 768a^5b^*c^6e^2 - a^*b^2f^2*(-(4a^*c - b^2)^9)^{(1/2)} - a^*c^2e^2*(-(4a^*c - b^2)^9)^{(1/2)} + 27a^2b^9c^*f^2 + \\
& 3840a^6b^*c^5f^2 + 9a^2c^*f^2*(-(4a^*c - b^2)^9)^{(1/2)} + 96a^2b^5c^5d^2 - 512a^3b^3c^6d^2 + 96a^3b^5c^4e^2 - 512a^4b^3c^5e^2 - 288a^3b^7c^2f^2 + 1504a^4b^5c^3f^2 - 3840a^5b^3c^4f^2 - 1024a^5c^7d^* \\
& e - 3072a^6c^6e^*f + 12a^*b^8c^3d^*e + 6a^*b^9c^2d^*f + 3584a^5b^*c^6d^* \\
& c^6d^*f + 6a^*c^2d^*f^*(-(4a^*c - b^2)^9)^{(1/2)} - 128a^2b^6c^4d^*e + 384a^3b^4c^5d^*e - 128a^2b^7c^3d^*f + 960a^3b^5c^4d^*f - 3072a^4b^3c^5d^* \\
& c^5d^*f + 36a^2b^8c^2e^*f - 192a^3b^6c^3e^*f + 128a^4b^4c^4e^*f + 1536a^5b^2c^5e^*f - 2a^*b^{10}c^*e^*f - 2a^*b^*c^*e^*f^*(-(4a^*c - b^2)^9)^{(1/2)})/(32(4096a^7c^9 + a^*b^{12}c^3 - 24a^2b^{10}c^4 + 240a^3b^8c^5 - 1280a^4b^6c^6 + 3840a^5b^4c^7 - 6144a^6b^2c^8))^{(1/2)} - (x^*(8a^*c^5d^2 - b^6f^2 - 8a^2c^4e^2 - 10b^2c^4d^2 + 72a^3c^3f^2 - b^4c^2e^2 - 2a^*b^2c^3e^2 - 2b^5c^*e^*f - 74a^2b^2c^2f^2 + 16a^*b^4c^*f^2 + 48a^2c^4d^*f + 6b^3c^3d^*e + 6b^4c^2d^*f - 52a^*b^2c^3d^*f + 14a^*b^3c^2e^*f + 8a^2b^*c^3e^*f + 8a^*b^*c^4d^*e))/(2(b^4c + 16a^2c^3 - 8a^*b^2c^2))((c^3d^2*(-(4a^*c - b^2)^9)^{(1/2)} - b^9c^3d^2 - a^*b^{11}f^2 + 768a^4b^*c^7d^2 - a^*b^9c^2e^2 + 768a^5b^*c^6e^2 - a^*b^2f^2*(-(4a^*c - b^2)^9)^{(1/2)} - a^*c^2e^2*(-(4a^*c - b^2)^9)^{(1/2)} + 27a^2b^9c^*f^2 + 3840a^6b^*c^5f^2 + 9a^2c^*f^2*(-(4a^*c - b^2)^9)^{(1/2)} + 96a^2b^5c^5d^2 - 512a^3b^3c^6d^2 + 96a^3b^5c^4e^2 - 512a^4b^3c^5e^2 - 288a^3b^7c^2f^2 + 1504a^4b^5c^3f^2 - 3840a^5b^3c^4f^2 - 1024a^5c^7d^*e - 3072a^6c^6e^*f + 12a^*b^8c^3d^*e + 6a^*b^9c^2d^*f + 3584a^5b^*c^6d^*c^6d^*f + 6a^*c^2d^*f^*(-(4a^*c - b^2)^9)^{(1/2)} - 128a^2b^6c^4d^*e + 384a^3b^4c^5d^*e - 128a^2b^7c^3d^*f + 960a^3b^5c^4d^*f - 3072a^4b^3c^5d^*c^5d^*f + 36a^2b^8c^2e^*f - 192a^3b^6c^3e^*f + 128a^4b^4c^4e^*f + 1536a^5b^2c^5e^*f - 2a^*b^{10}c^*e^*f - 2a^*b^*c^*e^*f^*(-(4a^*c - b^2)^9)^{(1/2)})/(32(4096a^7c^9 + a^*b^{12}c^3 - 24a^2b^{10}c^4 + 240a^3b^8c^5 - 1280a^4b^6c^6 + 3840a^5b^4c^7 - 6144a^6b^2c^8))^{(1/2)}*1i)/(((2048a^4c^6e + 16b^7c^3d + 768a^2b^3c^5d + 384a^2b^4c^4e - 1536a^3b^2c^5e - 192a^2b^5c^3f + 768a^3b^3c^4f - 192a^*b^5c^4d - 1024a^3b^*c^6d - 32a^*b^6c^3e + 16a^*b^7c^2f - 1024a^4b^*c^5f)/(8(b^6c - 64a^3c^4 - 12a^*b^4c^2 + 48a^2b^2c^3)) - (x^*((c^3d^2*(-(4a^*c - b^2)^9)^{(1/2)} - b^9c^3d^2 - a^*b^{11}f^2 + 768a^4b^*c^7d^2 - a^*b^9c^2e^2 + 768a^5b^*c^6e^2 - a^*b^2f^2*(-(4a^*c - b^2)^9)^{(1/2)} - a^*c^2e^2*(-(4a^*c - b^2)^9)^{(1/2)} + 27a^2b^9c^*f^2 + 3840a^6b^*c^5f^2 + 9a^2c^*f^2*(-(4a^*c - b^2)^9)^{(1/2)} + 96a^2b^5c^5d^2 - 512a^3b^3c^6d^2 + 96a^3b^5c^4e^2 - 512a^4b^3c^5e^2 - 288a^3b^7c^2f^2 + 1504a^4b^5c^3f^2 - 3840a^5b^3c^4f^2 - 1024a^5c^7d^*e - 3072a^6c^6e^*f + 12a^*b^8c^3d^*e + 6a^*b^9c^2d^*f + 3584a^5b^*c^6d^*f + 6a^*c^2d^*f^*(-(4a^*c - b^2)^9)^{(1/2)} - 128a^2b^6c^4d^*e + 384a^3b^4c^5d^*e - 128a^2b^7c^3d^*f + 960a^3b^5c^4d^*f - 3072a^4b^3c^5d^*f + 36a^2b^8c^2e^*f - 192a^3b^6c^3e^*f + 128a^4b^4c^4e^*f + 1536a^5b^2c^5e^*f - 2a^*b^{10}c^*e^*f - 2a^*b^*c^*e^*f^*(-(4a^*c - b^2)^9)^{(1/2)})/(32(4096a^7c^9 + a^*b^{12}c^3 - 24a^2b^{10}c^4 + 240a^3b^8c^5 - 1280a^4b^6c^6 + 3840a^5b^4c^7 - 6144a^6b^2c^8))^{(1/2)}(16b^7c^3 - 192a^*b^5c^4 - 1024a^3b^*c^6 + 768a^2b^3c^5)/(2(b^4c + 16a^2c^3 - 8a^*b^2c^2))((c^3d^2*(-(4a^*c - b^2)^9)^{(1/2)} - b^9c^3d^2 - a^*b^{11}f^2 + 768a^4b^*c^7d^2 - a^*b^9c^2e^2
\end{aligned}$$

$$\begin{aligned}
& e^2 + 768a^5b^3c^6e^2 - a^2b^2f^2(-4ac - b^2)^9)^{(1/2)} - a^2c^2e^2(- \\
& (4ac - b^2)^9)^{(1/2)} + 27a^2b^9c^5f^2 + 3840a^6b^3c^5f^2 + 9a^2c^5f^2 \\
& 2(-4ac - b^2)^9)^{(1/2)} + 96a^2b^5c^5d^2 - 512a^3b^3c^6d^2 + 96a^3 \\
& b^5c^4e^2 - 512a^4b^3c^5e^2 - 288a^3b^7c^2f^2 + 1504a^4b^5c^3 \\
& f^2 - 3840a^5b^3c^4f^2 - 1024a^5c^7d^2e - 3072a^6c^6e^2f + 12a^ \\
& b^8c^3d^2e + 6a^2b^9c^2d^2f + 3584a^5b^3c^6d^2f + 6a^2c^2d^2f(-4ac \\
& - b^2)^9)^{(1/2)} - 128a^2b^6c^4d^2e + 384a^3b^4c^5d^2e - 128a^2b^7c^3 \\
& d^2f + 960a^3b^5c^4d^2f - 3072a^4b^3c^5d^2f + 36a^2b^8c^2e^2f - \\
& 192a^3b^6c^3e^2f + 128a^4b^4c^4e^2f + 1536a^5b^2c^5e^2f - 2a^2b^10 \\
& c^2e^2f - 2a^2b^2c^2e^2f(-4ac - b^2)^9)^{(1/2)} / (32(4096a^7c^9 + a^2b^12c^3 \\
& - 24a^2b^10c^4 + 240a^3b^8c^5 - 1280a^4b^6c^6 + 3840a^5b^4c^7 - \\
& 6144a^6b^2c^8))^{(1/2)} + (x(8a^5c^5d^2 - b^6f^2 - 8a^2c^4e^2 - \\
& 10b^2c^4d^2 + 72a^3c^3f^2 - b^4c^2e^2 - 2a^2b^2c^3e^2 - 2b^5c^2 \\
& e^2f - 74a^2b^2c^2f^2 + 16a^2b^4c^2f^2 + 48a^2c^4d^2f + 6b^3c^3d^2e \\
& + 6b^4c^2d^2f - 52a^2b^2c^3d^2f + 14a^2b^3c^2e^2f + 8a^2b^3c^3e^2f + 8 \\
& a^2b^3c^4d^2e)) / (2(b^4c + 16a^2c^3 - 8a^2b^2c^2)) * ((c^3d^2(-4ac - \\
& b^2)^9)^{(1/2)} - b^9c^3d^2 - a^2b^11f^2 + 768a^4b^3c^7d^2 - a^2b^9c^2e^2 \\
& + 768a^5b^3c^6e^2 - a^2b^2f^2(-4ac - b^2)^9)^{(1/2)} - a^2c^2e^2(- \\
& (4ac - b^2)^9)^{(1/2)} + 27a^2b^9c^5f^2 + 3840a^6b^3c^5f^2 + 9a^2c^5f^2 \\
& (-4ac - b^2)^9)^{(1/2)} + 96a^2b^5c^5d^2 - 512a^3b^3c^6d^2 + 96a^3 \\
& b^5c^4e^2 - 512a^4b^3c^5e^2 - 288a^3b^7c^2f^2 + 1504a^4b^5c^3 \\
& f^2 - 3840a^5b^3c^4f^2 - 1024a^5c^7d^2e - 3072a^6c^6e^2f + 12a^ \\
& b^8c^3d^2e + 6a^2b^9c^2d^2f + 3584a^5b^3c^6d^2f + 6a^2c^2d^2f(-4ac \\
& - b^2)^9)^{(1/2)} - 128a^2b^6c^4d^2e + 384a^3b^4c^5d^2e - 128a^2b^7c^3 \\
& d^2f + 960a^3b^5c^4d^2f - 3072a^4b^3c^5d^2f + 36a^2b^8c^2e^2f - 1 \\
& 92a^3b^6c^3e^2f + 128a^4b^4c^4e^2f + 1536a^5b^2c^5e^2f - 2a^2b^10 \\
& c^2e^2f - 2a^2b^2c^2e^2f(-4ac - b^2)^9)^{(1/2)} / (32(4096a^7c^9 + a^2b^12c^3 \\
& - 24a^2b^10c^4 + 240a^3b^8c^5 - 1280a^4b^6c^6 + 3840a^5b^4c^7 - \\
& 6144a^6b^2c^8))^{(1/2)} + (((2048a^4c^6e + 16b^7c^3d + 768a^2b^3 \\
& c^5d + 384a^2b^4c^4e - 1536a^3b^2c^5e - 192a^2b^5c^3f + 768 \\
& a^3b^3c^4f - 192a^2b^5c^4d - 1024a^3b^3c^6d - 32a^2b^6c^3e + 16a^ \\
& b^7c^2f - 1024a^4b^3c^5f) / (8(b^6c - 64a^3c^4 - 12a^2b^4c^2 + 48a^2 \\
& b^2c^3)) + (x((c^3d^2(-4ac - b^2)^9)^{(1/2)} - b^9c^3d^2 - a^2b^11 \\
& f^2 + 768a^4b^3c^7d^2 - a^2b^9c^2e^2 + 768a^5b^3c^6e^2 - a^2b^2f^2(- \\
& (4ac - b^2)^9)^{(1/2)} - a^2c^2e^2(-4ac - b^2)^9)^{(1/2)} + 27a^2b^9 \\
& c^5f^2 + 3840a^6b^3c^5f^2 + 9a^2c^5f^2(-4ac - b^2)^9)^{(1/2)} + 96a^2 \\
& b^5c^5d^2 - 512a^3b^3c^6d^2 + 96a^3b^5c^4e^2 - 512a^4b^3c^5e^2 - \\
& 288a^3b^7c^2f^2 + 1504a^4b^5c^3f^2 - 3840a^5b^3c^4f^2 - 1024a^5 \\
& c^7d^2e - 3072a^6c^6e^2f + 12a^2b^8c^3d^2e + 6a^2b^9c^2d^2f + 3584a^5 \\
& b^3c^6d^2f + 6a^2c^2d^2f(-4ac - b^2)^9)^{(1/2)} - 128a^2b^6c^4d^2e \\
& + 384a^3b^4c^5d^2e - 128a^2b^7c^3d^2f + 960a^3b^5c^4d^2f - 3072a^4 \\
& b^3c^5d^2f + 36a^2b^8c^2e^2f - 192a^3b^6c^3e^2f + 128a^4b^4c^4 \\
& e^2f + 1536a^5b^2c^5e^2f - 2a^2b^10c^2e^2f - 2a^2b^2c^2e^2f(-4ac - b^2)^9 \\
&)^{(1/2)} / (32(4096a^7c^9 + a^2b^12c^3 - 24a^2b^10c^4 + 240a^3b^8c^5 - \\
& 1280a^4b^6c^6 + 3840a^5b^4c^7 - 6144a^6b^2c^8))^{(1/2)} * (16b^7c^3 - \\
& 192a^2b^5c^4 - 1024a^3b^3c^6 + 768a^2b^3c^5) / (2(b^4c + 16a^2 \\
& c^3 - 8a^2b^2c^2)) * ((c^3d^2(-4ac - b^2)^9)^{(1/2)} - b^9c^3d^2 - a^2 \\
& b^11f^2 + 768a^4b^3c^7d^2 - a^2b^9c^2e^2 + 768a^5b^3c^6e^2 - a^2b^2f^2 \\
& (-4ac - b^2)^9)^{(1/2)} - a^2c^2e^2(-4ac - b^2)^9)^{(1/2)} + 27a^2b^9 \\
& c^5f^2 + 3840a^6b^3c^5f^2 + 9a^2c^5f^2(-4ac - b^2)^9)^{(1/2)} + 96a^2 \\
& b^5c^5d^2 - 512a^3b^3c^6d^2 + 96a^3b^5c^4e^2 - 512a^4b^3c^5e^2 - \\
& 288a^3b^7c^2f^2 + 1504a^4b^5c^3f^2 - 3840a^5b^3c^4f^2 - 1 \\
& 024a^5c^7d^2e - 3072a^6c^6e^2f + 12a^2b^8c^3d^2e + 6a^2b^9c^2d^2f + 3 \\
& 584a^5b^3c^6d^2f + 6a^2c^2d^2f(-4ac - b^2)^9)^{(1/2)} - 128a^2b^6c^4 \\
& d^2e + 384a^3b^4c^5d^2e - 128a^2b^7c^3d^2f + 960a^3b^5c^4d^2f - 307 \\
& 2a^4b^3c^5d^2f + 36a^2b^8c^2e^2f - 192a^3b^6c^3e^2f + 128a^4b^4c^4 \\
& e^2f + 1536a^5b^2c^5e^2f - 2a^2b^10c^2e^2f - 2a^2b^2c^2e^2f(-4ac - b^2)^9 \\
&)^{(1/2)} / (32(4096a^7c^9 + a^2b^12c^3 - 24a^2b^10c^4 + 240a^3b^8 \\
& c^5 - 1280a^4b^6c^6 + 3840a^5b^4c^7 - 6144a^6b^2c^8))^{(1/2)} - (x
\end{aligned}$$

$$\begin{aligned} &*(8*a*c^5*d^2 - b^6*f^2 - 8*a^2*c^4*e^2 - 10*b^2*c^4*d^2 + 72*a^3*c^3*f^2 - \\ &b^4*c^2*e^2 - 2*a*b^2*c^3*e^2 - 2*b^5*c*e*f - 74*a^2*b^2*c^2*f^2 + 16*a*b^4*c*f^2 + 48*a^2*c^4*d*f + 6*b^3*c^3*d*e + 6*b^4*c^2*d*f - 52*a*b^2*c^3*d*f \\ &+ 14*a*b^3*c^2*e*f + 8*a^2*b*c^3*e*f + 8*a*b*c^4*d*e))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*((c^3*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - b^9*c^3*d^2 - a*b^11*f^2 + 768*a^4*b*c^7*d^2 - a*b^9*c^2*e^2 + 768*a^5*b*c^6*e^2 - a*b^2*f^2 \\ &*(-(4*a*c - b^2)^9)^{(1/2)} - a*c^2*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 27*a^2*b^9*c*f^2 + 3840*a^6*b*c^5*f^2 + 9*a^2*c*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + 96*a^2*b^5*c^5*d^2 - 512*a^3*b^3*c^6*d^2 + 96*a^3*b^5*c^4*e^2 - 512*a^4*b^3*c^5*e^2 - 288*a^3*b^7*c^2*f^2 + 1504*a^4*b^5*c^3*f^2 - 3840*a^5*b^3*c^4*f^2 - 1024*a^5*c^7*d*e - 3072*a^6*c^6*e*f + 12*a*b^8*c^3*d*e + 6*a*b^9*c^2*d*f + 3584*a^5*b*c^6*d*f + 6*a*c^2*d*f*(-(4*a*c - b^2)^9)^{(1/2)} - 128*a^2*b^6*c^4*d*e + 384*a^3*b^4*c^5*d*e - 128*a^2*b^7*c^3*d*f + 960*a^3*b^5*c^4*d*f - 3072*a^4*b^3*c^5*d*f + 36*a^2*b^8*c^2*e*f - 192*a^3*b^6*c^3*e*f + 128*a^4*b^4*c^4*e*f + 1536*a^5*b^2*c^5*e*f - 2*a*b^10*c*e*f - 2*a*b*c*e*f*(-(4*a*c - b^2)^9)^{(1/2)))/(32*(4096*a^7*c^9 + a*b^12*c^3 - 24*a^2*b^10*c^4 + 240*a^3*b^8*c^5 - 1280*a^4*b^6*c^6 + 3840*a^5*b^4*c^7 - 6144*a^6*b^2*c^8))^(1/2) + (8*a*c^5*d^3 + b^6*d*f^2 + 5*a^2*b^4*f^3 + 6*b^2*c^4*d^3 + 216*a^4*c^2*f^3 - 3*a*b^3*c^2*e^3 - 4*a^2*b*c^3*e^3 - 66*a^3*b^2*c*f^3 + 8*a^2*c^4*d*e^2 + 72*a^2*c^4*d^2*f + 216*a^3*c^3*d*f^2 - 5*b^3*c^3*d^2*e + b^4*c^2*d*e^2 + 24*a^3*c^3*e^2*f - 5*b^4*c^2*d^2*f - 3*a*b^5*e*f^2 - 28*a*b*c^4*d^2*e - 12*a*b^4*c*d*f^2 - 6*a*b^4*c*e^2*f + 18*a*b^2*c^3*d*e^2 + 26*a*b^2*c^3*d^2*f + 51*a^2*b^3*c*e*f^2 - 204*a^3*b*c^2*e*f^2 + 2*b^5*c*d*e*f + 2*a^2*b^2*c^2*d*f^2 + 42*a^2*b^2*c^2*e^2*f + 6*a*b^3*c^2*d*e*f - 152*a^2*b*c^3*d*e*f)/(4*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3))))*((c^3*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - b^9*c^3*d^2 - a*b^11*f^2 + 768*a^4*b*c^7*d^2 - a*b^9*c^2*e^2 + 768*a^5*b*c^6*e^2 - a*b^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - a*c^2*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 27*a^2*b^9*c*f^2 + 3840*a^6*b*c^5*f^2 + 9*a^2*c*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + 96*a^2*b^5*c^5*d^2 - 512*a^3*b^3*c^6*d^2 + 96*a^3*b^5*c^4*e^2 - 512*a^4*b^3*c^5*e^2 - 288*a^3*b^7*c^2*f^2 + 1504*a^4*b^5*c^3*f^2 - 3840*a^5*b^3*c^4*f^2 - 1024*a^5*c^7*d*e - 3072*a^6*c^6*e*f + 12*a*b^8*c^3*d*e + 6*a*b^9*c^2*d*f + 3584*a^5*b*c^6*d*f + 6*a*c^2*d*f*(-(4*a*c - b^2)^9)^{(1/2)} - 128*a^2*b^6*c^4*d*e + 384*a^3*b^4*c^5*d*e - 128*a^2*b^7*c^3*d*f + 960*a^3*b^5*c^4*d*f - 3072*a^4*b^3*c^5*d*f + 36*a^2*b^8*c^2*e*f - 192*a^3*b^6*c^3*e*f + 128*a^4*b^4*c^4*e*f + 1536*a^5*b^2*c^5*e*f - 2*a*b^10*c*e*f - 2*a*b*c*e*f*(-(4*a*c - b^2)^9)^{(1/2)))/(32*(4096*a^7*c^9 + a*b^12*c^3 - 24*a^2*b^10*c^4 + 240*a^3*b^8*c^5 - 1280*a^4*b^6*c^6 + 3840*a^5*b^4*c^7 - 6144*a^6*b^2*c^8))^(1/2)*2i \end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)

[Out] Timed out

$$3.69 \quad \int \frac{d+ex^2+fx^4}{(a+bx^2+cx^4)^2} dx$$

Optimal. Leaf size=346

$$\frac{x(x^2(abf - 2ace + bcd) - abe - 2a(cd - af) + b^2d)}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right) \left(\frac{b^2(cd - af) + 4abce - 4ac(af + 3cd)}{\sqrt{b^2 - 4ac}} + abf - 2ace\right)}{2\sqrt{2}a\sqrt{c}(b^2 - 4ac)\sqrt{b - \sqrt{b^2 - 4ac}}}$$

Rubi [A] time = 1.90, antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1678, 1166, 205}

$$\frac{x(x^2(abf - 2ace + bcd) - abe - 2a(cd - af) + b^2d)}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right) \left(\frac{b^2(cd - af) + 4abce - 4ac(af + 3cd)}{\sqrt{b^2 - 4ac}} + abf - 2ace + bcd\right)}{2\sqrt{2}a\sqrt{c}(b^2 - 4ac)\sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b^2 - 4ac + b}}\right) \left(\frac{b^2(cd - af) + 4abce - 4ac(af + 3cd)}{\sqrt{b^2 - 4ac}} + abf - 2ace + bcd\right)}{2\sqrt{2}a\sqrt{c}(b^2 - 4ac)\sqrt{\sqrt{b^2 - 4ac} + b}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2 + f*x^4)/(a + b*x^2 + c*x^4)^2, x]

[Out] (x*(b^2*d - a*b*e - 2*a*(c*d - a*f) + (b*c*d - 2*a*c*e + a*b*f)*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b*c*d - 2*a*c*e + a*b*f + (4*a*b*c*e + b^2*(c*d - a*f) - 4*a*c*(3*c*d + a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(2*Sqrt[2]*a*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b*c*d - 2*a*c*e + a*b*f - (4*a*b*c*e + b^2*(c*d - a*f) - 4*a*c*(3*c*d + a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(2*Sqrt[2]*a*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1166

Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1678

Int[(Pq_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := With[{d = Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]

Rubi steps

$$\int \frac{d + ex^2 + fx^4}{(a + bx^2 + cx^4)^2} dx = \frac{x(b^2d - abe - 2a(cd - af) + (bcd - 2ace + abf)x^2)}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{\int \frac{-b^2d - abe + 2a(3cd + af) + (-bcd + 2ace)}{a + bx^2 + cx^4}}{2a(b^2 - 4ac)}$$

$$= \frac{x(b^2d - abe - 2a(cd - af) + (bcd - 2ace + abf)x^2)}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{(bcd - 2ace + abf - \frac{4abce + b^2(c)}{4a})}{2a(b^2 - 4ac)}$$

$$= \frac{x(b^2d - abe - 2a(cd - af) + (bcd - 2ace + abf)x^2)}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{(bcd - 2ace + abf + \frac{4abce + b^2(c)}{2\sqrt{2}a\sqrt{c}})}{2\sqrt{2}a\sqrt{c}(b^2 - 4ac)}$$

Mathematica [A] time = 1.08, size = 382, normalized size = 1.10

$$\frac{2x(b(-ae+afx^2+cdx^2)+2a(af-c(d+ex^2))+b^2d)}{(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{2} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(b\left(cd\sqrt{b^2-4ac}+af\sqrt{b^2-4ac}+4ace\right)-2a\left(c\sqrt{b^2-4ac}+2af+6cd\right)+b^2(cd-af)\right)}{\sqrt{c}(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(b\left(cd\sqrt{b^2-4ac}+af\sqrt{b^2-4ac}-4ace\right)+2ac\left(-c\sqrt{b^2-4ac}+2af+6cd\right)+b^2(af-cd)\right)}{\sqrt{c}(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2 + f*x^4)/(a + b*x^2 + c*x^4)^2,x]

[Out] ((2*x*(b^2*d + b*(-(a*e) + c*d*x^2 + a*f*x^2) + 2*a*(a*f - c*(d + e*x^2))))/(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(b^2*(c*d - a*f) - 2*a*c*(6*c*d + Sqrt[b^2 - 4*a*c]*e + 2*a*f) + b*(c*Sqrt[b^2 - 4*a*c]*d + 4*a*c*e + a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b^2*(-(c*d) + a*f) + 2*a*c*(6*c*d - Sqrt[b^2 - 4*a*c]*e + 2*a*f) + b*(c*Sqrt[b^2 - 4*a*c]*d - 4*a*c*e + a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex^2 + fx^4}{(a + bx^2 + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e*x^2 + f*x^4)/(a + b*x^2 + c*x^4)^2,x]

[Out] IntegrateAlgebraic[(d + e*x^2 + f*x^4)/(a + b*x^2 + c*x^4)^2, x]

fricas [B] time = 17.14, size = 8991, normalized size = 25.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")

[Out] 1/4*(2*(b*c*d - 2*a*c*e + a*b*f)*x^3 + sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-((b^5*c - 15*a*b^3*c^2 + 60*a^2*b*c^3)*d^2 + 2*(a*b^4*c - 6*a^2*b^2*c^2 - 24*a^3*c^3)*d*e + (a^2*b^3*c + 12*a^3*b*c^2)*e^2 + (a^3*b^3 + 12*a^4*b*c)*f^2 - 2*((3*a^2*b^3*c - 28*a^3*b*c^2)*d + 2*(3*a^3*b^2*c + 4*a^4*c^2)*e)*f + (a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4)*sqrt((4*a^3*b*c^2*d*e^3 + a^4*c^2*e^4 + 12*a^5*c*d*f^3 + a^6*f^4 + (b^4*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4)*d^4 + 4*(a*b^3*c^2 - 9*a^2*b*c^3)*d^3*e + 6*(a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^2 - 2*

$$\begin{aligned}
& (2a^4b^2c^2d^2e + a^5c^2d^2e^2 + (a^3b^2c - 27a^4c^2)d^2)f^2 - 12(2a^3b^2c^2d^2e + a^4c^2d^2e^2 + (a^2b^2c^2 - 9a^3c^3)d^3)f)/(a^6b^6c^2 - 12a^7b^4c^3 + 48a^8b^2c^4 - 64a^9c^5)))/(a^3b^6c - 12a^4b^4c^2 + 48a^5b^2c^3 - 64a^6c^4)*\log(((5b^4c^3 - 81ab^2c^4 + 324a^2c^5)d^4 - (3b^5c^2 - 65ab^3c^3 + 324a^2b^2c^4)d^3e - 3(3ab^4c^2 - 28a^2b^2c^3)d^2e^2 - (9a^2b^3c^2 - 20a^3b^2c^3)d^2e^3 - (3a^3b^2c^2 + 4a^4c^3)e^4 + (3a^5b^2 + 4a^6c)f^4 - ((a^3b^4 - 24a^4b^2c - 48a^5c^2)d + (a^4b^3 + 12a^5b^2c)e)f^3 - 9((a^2b^4c - 6a^3b^2c^2 - 24a^4c^3)d^2 + (a^3b^3c + 12a^4b^2c^2)d^2e)f^2 + ((b^6c - 15ab^4c^2 + 432a^3c^4)d^3 + 3(ab^5c + 3a^2b^3c^2 - 108a^3b^2c^3)d^2e + 3(a^2b^4c + 12a^3b^2c^2)d^2e^2 + (a^3b^3c + 12a^4b^2c^2)e^3)f)*x + 1/2*\sqrt{1/2}*((b^8c - 23ab^6c^2 + 190a^2b^4c^3 - 672a^3b^2c^4 + 864a^4c^5)d^3 + 3(ab^7c - 15a^2b^5c^2 + 72a^3b^3c^3 - 112a^4b^2c^4)d^2e + 3(a^2b^6c - 10a^3b^4c^2 + 32a^4b^2c^3 - 32a^5c^4)d^2e^2 + (a^3b^5c - 8a^4b^3c^2 + 16a^5b^2c^3)e^3 + 2(a^5b^4 - 8a^6b^2c + 16a^7c^2)f^3 - ((a^3b^6 - 26a^4b^4c + 160a^5b^2c^2 - 288a^6c^3)d + (a^4b^5 - 8a^5b^3c + 16a^6b^2c^2)e)f^2 - 2((4a^2b^6c - 59a^3b^4c^2 + 280a^4b^2c^3 - 432a^5c^4)d^2 + 5(a^3b^5c - 8a^4b^3c^2 + 16a^5b^2c^3)d^2e + (a^4b^4c - 8a^5b^2c^2 + 16a^6c^3)e^2)f - ((a^3b^9c - 20a^4b^7c^2 + 144a^5b^5c^3 - 448a^6b^3c^4 + 512a^7b^2c^5)d + (a^4b^8c - 8a^5b^6c^2 + 128a^7b^2c^4 - 256a^8c^5)e - 4(a^5b^7c - 12a^6b^5c^2 + 48a^7b^3c^3 - 64a^8b^2c^4)f)*\sqrt{(4a^3b^2c^2d^2e^3 + a^4c^2e^4 + 12a^5c^2d^2f^3 + a^6f^4 + (b^4c^2 - 18ab^2c^3 + 81a^2c^4)d^4 + 4(ab^3c^2 - 9a^2b^2c^3)d^3e + 6(a^2b^2c^2 - 3a^3c^3)d^2e^2 - 2(2a^4b^2c^2d^2e + a^5c^2d^2e^2 + (a^3b^2c - 27a^4c^2)d^2)f^2 - 12(2a^3b^2c^2d^2e + a^4c^2d^2e^2 + (a^2b^2c^2 - 9a^3c^3)d^3)f)/(a^6b^6c^2 - 12a^7b^4c^3 + 48a^8b^2c^4 - 64a^9c^5)))*\sqrt{-((b^5c - 15ab^3c^2 + 60a^2b^2c^3)d^2 + 2(ab^4c - 6a^2b^2c^2 - 24a^3c^3)d^2e + (a^2b^3c + 12a^3b^2c^2)e^2 + (a^3b^3 + 12a^4b^2c)f^2 - 2((3a^2b^3c - 28a^3b^2c^2)d + 2(3a^3b^2c + 4a^4c^2)e)f + (a^3b^6c - 12a^4b^4c^2 + 48a^5b^2c^3 - 64a^6c^4)*\sqrt{(4a^3b^2c^2d^2e^3 + a^4c^2e^4 + 12a^5c^2d^2f^3 + a^6f^4 + (b^4c^2 - 18ab^2c^3 + 81a^2c^4)d^4 + 4(ab^3c^2 - 9a^2b^2c^3)d^3e + 6(a^2b^2c^2 - 3a^3c^3)d^2e^2 - 2(2a^4b^2c^2d^2e + a^5c^2d^2e^2 + (a^3b^2c - 27a^4c^2)d^2)f^2 - 12(2a^3b^2c^2d^2e + a^4c^2d^2e^2 + (a^2b^2c^2 - 9a^3c^3)d^3)f)/(a^6b^6c^2 - 12a^7b^4c^3 + 48a^8b^2c^4 - 64a^9c^5)))/\sqrt{1/2}*((ab^2c - 4a^2c^2)x^4 + a^2b^2 - 4a^3c + (ab^3 - 4a^2b^2c)x^2)*\sqrt{-((b^5c - 15ab^3c^2 + 60a^2b^2c^3)d^2 + 2(ab^4c - 6a^2b^2c^2 - 24a^3c^3)d^2e + (a^2b^3c + 12a^3b^2c^2)e^2 + (a^3b^3 + 12a^4b^2c)f^2 - 2((3a^2b^3c - 28a^3b^2c^2)d + 2(3a^3b^2c + 4a^4c^2)e)f + (a^3b^6c - 12a^4b^4c^2 + 48a^5b^2c^3 - 64a^6c^4)*\sqrt{(4a^3b^2c^2d^2e^3 + a^4c^2e^4 + 12a^5c^2d^2f^3 + a^6f^4 + (b^4c^2 - 18ab^2c^3 + 81a^2c^4)d^4 + 4(ab^3c^2 - 9a^2b^2c^3)d^3e + 6(a^2b^2c^2 - 3a^3c^3)d^2e^2 - 2(2a^4b^2c^2d^2e + a^5c^2d^2e^2 + (a^3b^2c - 27a^4c^2)d^2)f^2 - 12(2a^3b^2c^2d^2e + a^4c^2d^2e^2 + (a^2b^2c^2 - 9a^3c^3)d^3)f)/(a^6b^6c^2 - 12a^7b^4c^3 + 48a^8b^2c^4 - 64a^9c^5)))/\sqrt{1/2}*((b^8c - 23ab^6c^2 + 190a^2b^4c^3 - 672a^3b^2c^4 + 864a^4c^5)d^3 + 3(ab^7c - 15a^2b^5c^2 + 72a^3b^3c^3 - 112a^4b^2c^4)d^2e + 3(a^2b^6c - 10a^3b^4c^2 + 32a^4b^2c^3 - 32
\end{aligned}$$

$$\begin{aligned}
& a^5 c^4 d e^2 + (a^3 b^5 c - 8 a^4 b^3 c^2 + 16 a^5 b c^3) e^3 + 2(a^5 b^4 - 8 a^6 b^2 c + 16 a^7 c^2) f^3 - ((a^3 b^6 - 26 a^4 b^4 c + 160 a^5 b^2 c^2 - 288 a^6 c^3) d + (a^4 b^5 - 8 a^5 b^3 c + 16 a^6 b c^2) e) f^2 - 2((4 a^2 b^6 c - 59 a^3 b^4 c^2 + 280 a^4 b^2 c^3 - 432 a^5 c^4) d^2 + 5(a^3 b^5 c - 8 a^4 b^3 c^2 + 16 a^5 b c^3) d e + (a^4 b^4 c - 8 a^5 b^2 c^2 + 16 a^6 c^3) e^2) f - ((a^3 b^9 c - 20 a^4 b^7 c^2 + 144 a^5 b^5 c^3 - 448 a^6 b^3 c^4 + 512 a^7 b c^5) d + (a^4 b^8 c - 8 a^5 b^6 c^2 + 128 a^7 b^2 c^4 - 256 a^8 c^5) e - 4(a^5 b^7 c - 12 a^6 b^5 c^2 + 48 a^7 b^3 c^3 - 64 a^8 b c^4) f) \sqrt{(4 a^3 b c^2 d e^3 + a^4 c^2 e^4 + 12 a^5 c d f^3 + a^6 f^4 + (b^4 c^2 - 18 a b^2 c^3 + 81 a^2 c^4) d^4 + 4(a b^3 c^2 - 9 a^2 b c^3) d^3 e + 6(a^2 b^2 c^2 - 3 a^3 c^3) d^2 e^2 - 2(2 a^4 b c d e + a^5 c e^2 + (a^3 b^2 c - 27 a^4 c^2) d^2) f^2 - 12(2 a^3 b c^2 d^2 e + a^4 c^2 d e^2 + (a^2 b^2 c^2 - 9 a^3 c^3) d^3) f) / (a^6 b^6 c^2 - 12 a^7 b^4 c^3 + 48 a^8 b^2 c^4 - 64 a^9 c^5))} \sqrt{-((b^5 c - 15 a b^3 c^2 + 60 a^2 b c^3) d^2 + 2(a b^4 c - 6 a^2 b^2 c^2 - 24 a^3 c^3) d e + (a^2 b^3 c + 12 a^3 b c^2) e^2 + (a^3 b^3 + 12 a^4 b c) f^2 - 2((3 a^2 b^3 c - 28 a^3 b c^2) d + 2(3 a^3 b^2 c + 4 a^4 c^2) e) f + (a^3 b^6 c - 12 a^4 b^4 c^2 + 48 a^5 b^2 c^3 - 64 a^6 c^4) \sqrt{(4 a^3 b c^2 d e^3 + a^4 c^2 e^4 + 12 a^5 c d f^3 + a^6 f^4 + (b^4 c^2 - 18 a b^2 c^3 + 81 a^2 c^4) d^4 + 4(a b^3 c^2 - 9 a^2 b c^3) d^3 e + 6(a^2 b^2 c^2 - 3 a^3 c^3) d^2 e^2 - 2(2 a^4 b c d e + a^5 c e^2 + (a^3 b^2 c - 27 a^4 c^2) d^2) f^2 - 12(2 a^3 b c^2 d^2 e + a^4 c^2 d e^2 + (a^2 b^2 c^2 - 9 a^3 c^3) d^3) f) / (a^6 b^6 c^2 - 12 a^7 b^4 c^3 + 48 a^8 b^2 c^4 - 64 a^9 c^5))} / (a^3 b^6 c - 12 a^4 b^4 c^2 + 48 a^5 b^2 c^3 - 64 a^6 c^4)) + \sqrt{1/2} * ((a b^2 c - 4 a^2 c^2) x^4 + a^2 b^2 - 4 a^3 c + (a b^3 - 4 a^2 b c) x^2) \sqrt{-((b^5 c - 15 a b^3 c^2 + 60 a^2 b c^3) d^2 + 2(a b^4 c - 6 a^2 b^2 c^2 - 24 a^3 c^3) d e + (a^2 b^3 c + 12 a^3 b c^2) e^2 + (a^3 b^3 + 12 a^4 b c) f^2 - 2((3 a^2 b^3 c - 28 a^3 b c^2) d + 2(3 a^3 b^2 c + 4 a^4 c^2) e) f - (a^3 b^6 c - 12 a^4 b^4 c^2 + 48 a^5 b^2 c^3 - 64 a^6 c^4) \sqrt{(4 a^3 b c^2 d e^3 + a^4 c^2 e^4 + 12 a^5 c d f^3 + a^6 f^4 + (b^4 c^2 - 18 a b^2 c^3 + 81 a^2 c^4) d^4 + 4(a b^3 c^2 - 9 a^2 b c^3) d^3 e + 6(a^2 b^2 c^2 - 3 a^3 c^3) d^2 e^2 - 2(2 a^4 b c d e + a^5 c e^2 + (a^3 b^2 c - 27 a^4 c^2) d^2) f^2 - 12(2 a^3 b c^2 d^2 e + a^4 c^2 d e^2 + (a^2 b^2 c^2 - 9 a^3 c^3) d^3) f) / (a^6 b^6 c^2 - 12 a^7 b^4 c^3 + 48 a^8 b^2 c^4 - 64 a^9 c^5))} / (a^3 b^6 c - 12 a^4 b^4 c^2 + 48 a^5 b^2 c^3 - 64 a^6 c^4)) * \log(((5 b^4 c^3 - 81 a b^2 c^4 + 324 a^2 c^5) d^4 - (3 b^5 c^2 - 65 a b^3 c^3 + 324 a^2 b c^4) d^3 e - 3(3 a b^4 c^2 - 28 a^2 b^2 c^3) d^2 e^2 - (9 a^2 b^3 c^2 - 20 a^3 b c^3) d e^3 - (3 a^3 b^2 c^2 + 4 a^4 c^3) e^4 + (3 a^5 b^2 + 4 a^6 c) f^4 - ((a^3 b^4 - 24 a^4 b^2 c - 48 a^5 c^2) d + (a^4 b^3 + 12 a^5 b c) e) f^3 - 9((a^2 b^4 c - 6 a^3 b^2 c^2 - 24 a^4 c^3) d^2 + (a^3 b^3 c + 12 a^4 b c^2) d e) f^2 + ((b^6 c - 15 a b^4 c^2 + 432 a^3 c^4) d^3 + 3(a b^5 c + 3 a^2 b^3 c^2 - 108 a^3 b c^3) d^2 e + 3(a^2 b^4 c + 12 a^3 b^2 c^2) d e^2 + (a^3 b^3 c + 12 a^4 b c^2) e^3) f) x + 1/2 \sqrt{1/2} * ((b^8 c - 23 a b^6 c^2 + 190 a^2 b^4 c^3 - 672 a^3 b^2 c^4 + 864 a^4 c^5) d^3 + 3(a b^7 c - 15 a^2 b^5 c^2 + 72 a^3 b^3 c^3 - 112 a^4 b c^4) d^2 e + 3(a^2 b^6 c - 10 a^3 b^4 c^2 + 32 a^4 b^2 c^3 - 32 a^5 c^4) d e^2 + (a^3 b^5 c - 8 a^4 b^3 c^2 + 16 a^5 b c^3) e^3 + 2(a^5 b^4 - 8 a^6 b^2 c + 16 a^7 c^2) f^3 - ((a^3 b^6 - 26 a^4 b^4 c + 160 a^5 b^2 c^2 - 288 a^6 c^3) d + (a^4 b^5 - 8 a^5 b^3 c + 16 a^6 b c^2) e) f^2 - 2((4 a^2 b^6 c - 59 a^3 b^4 c^2 + 280 a^4 b^2 c^3 - 432 a^5 c^4) d^2 + 5(a^3 b^5 c - 8 a^4 b^3 c^2 + 16 a^5 b c^3) d e + (a^4 b^4 c - 8 a^5 b^2 c^2 + 16 a^6 c^3) e^2) f + ((a^3 b^9 c - 20 a^4 b^7 c^2 + 144 a^5 b^5 c^3 - 448 a^6 b^3 c^4 + 512 a^7 b c^5) d + (a^4 b^8 c - 8 a^5 b^6 c^2 + 128 a^7 b^2 c^4 - 256 a^8 c^5) e - 4(a^5 b^7 c - 12 a^6 b^5 c^2 + 48 a^7 b^3 c^3 - 64 a^8 b c^4) f) \sqrt{(4 a^3 b c^2 d e^3 + a^4 c^2 e^4 + 12 a^5 c d f^3 + a^6 f^4 + (b^4 c^2 - 18 a b^2 c^3 + 81 a^2 c^4) d^4 + 4(a b^3 c^2 - 9 a^2 b c^3) d^3 e + 6(a^2 b^2 c^2 - 3 a^3 c^3) d^2 e^2 - 2(2 a^4 b c d e + a^5 c e^2 + (a^3 b^2 c - 27 a^4 c^2) d^2) f^2 - 12(2 a^3 b c^2 d^2 e + a^4 c^2 d e^2 + (a^2 b^2 c^2 - 9 a^3 c^3) d^3) f) / (a^6 b^6 c^2 - 12 a^7 b^4 c^3 + 48 a^8 b^2 c^4 - 64 a^9 c^5))} \sqrt{-((b^5 c - 15 a b^3 c^2 + 60 a^2 b c^3) d^2 + 2(a b^4 c - 6 a^2 b^2 c^2 - 24 a^3 c^3) d e + (a^2 b^3 c + 12 a^3 b c^2) e^2 + (a^3 b^3 + 12 a^4 b c) f^2 - 2((3 a^2 b^3 c - 28 a^3 b c^2) d + 2(3 a^3 b^2 c + 4 a^4 c^2) e) f + (a^3 b^6 c - 12 a^4 b^4 c^2 + 48 a^5 b^2 c^3 - 64 a^6 c^4) \sqrt{(4 a^3 b c^2 d e^3 + a^4 c^2 e^4 + 12 a^5 c d f^3 + a^6 f^4 + (b^4 c^2 - 18 a b^2 c^3 + 81 a^2 c^4) d^4 + 4(a b^3 c^2 - 9 a^2 b c^3) d^3 e + 6(a^2 b^2 c^2 - 3 a^3 c^3) d^2 e^2 - 2(2 a^4 b c d e + a^5 c e^2 + (a^3 b^2 c - 27 a^4 c^2) d^2) f^2 - 12(2 a^3 b c^2 d^2 e + a^4 c^2 d e^2 + (a^2 b^2 c^2 - 9 a^3 c^3) d^3) f) / (a^6 b^6 c^2 - 12 a^7 b^4 c^3 + 48 a^8 b^2 c^4 - 64 a^9 c^5))} / (a^3 b^6 c - 12 a^4 b^4 c^2 + 48 a^5 b^2 c^3 - 64 a^6 c^4)) + \sqrt{1/2} * ((a b^2 c - 4 a^2 c^2) x^4 + a^2 b^2 - 4 a^3 c + (a b^3 - 4 a^2 b c) x^2) \sqrt{-((b^5 c - 15 a b^3 c^2 + 60 a^2 b c^3) d^2 + 2(a b^4 c - 6 a^2 b^2 c^2 - 24 a^3 c^3) d e + (a^2 b^3 c + 12 a^3 b c^2) e^2 + (a^3 b^3 + 12 a^4 b c) f^2 - 2((3 a^2 b^3 c - 28 a^3 b c^2) d + 2(3 a^3 b^2 c + 4 a^4 c^2) e) f - (a^3 b^6 c - 12 a^4 b^4 c^2 + 48 a^5 b^2 c^3 - 64 a^6 c^4) \sqrt{(4 a^3 b c^2 d e^3 + a^4 c^2 e^4 + 12 a^5 c d f^3 + a^6 f^4 + (b^4 c^2 - 18 a b^2 c^3 + 81 a^2 c^4) d^4 + 4(a b^3 c^2 - 9 a^2 b c^3) d^3 e + 6(a^2 b^2 c^2 - 3 a^3 c^3) d^2 e^2 - 2(2 a^4 b c d e + a^5 c e^2 + (a^3 b^2 c - 27 a^4 c^2) d^2) f^2 - 12(2 a^3 b c^2 d^2 e + a^4 c^2 d e^2 + (a^2 b^2 c^2 - 9 a^3 c^3) d^3) f) / (a^6 b^6 c^2 - 12 a^7 b^4 c^3 + 48 a^8 b^2 c^4 - 64 a^9 c^5))} / (a^3 b^6 c - 12 a^4 b^4 c^2 + 48 a^5 b^2 c^3 - 64 a^6 c^4))
\end{aligned}$$

$$\begin{aligned}
& - 6a^2b^2c^2 - 24a^3c^3)d^2e + (a^2b^3c + 12a^3b^2c^2)e^2 + (a^3b^3 + 12a^4b^2c^2)e^2 - 2((3a^2b^3c - 28a^3b^2c^2)d + 2(3a^3b^2c^2 + 4a^4c^2)e)f - (a^3b^6c - 12a^4b^4c^2 + 48a^5b^2c^3 - 64a^6c^4) \sqrt{(4a^3b^2c^2d^2e^3 + a^4c^2e^4 + 12a^5c^2d^2f^3 + a^6f^4 + (b^4c^2 - 18a^2b^2c^3 + 81a^2c^4)d^4 + 4(a^2b^3c^2 - 9a^2b^2c^3)d^3e + 6(a^2b^2c^2 - 3a^3c^3)d^2e^2 - 2(2a^4b^2c^2d^2e + a^5c^2e^2 + (a^3b^2c - 27a^4c^2)d^2)f^2 - 12(2a^3b^2c^2d^2e + a^4c^2d^2e^2 + (a^2b^2c^2 - 9a^3c^3)d^3)f)} / (a^6b^6c^2 - 12a^7b^4c^3 + 48a^8b^2c^4 - 64a^9c^5)) / (a^3b^6c - 12a^4b^4c^2 + 48a^5b^2c^3 - 64a^6c^4) \\
&)) - \sqrt{1/2}((a^2b^2c - 4a^2c^2)x^4 + a^2b^2 - 4a^3c + (a^2b^3 - 4a^2b^2c)x^2) \sqrt{-(b^5c - 15a^2b^3c^2 + 60a^2b^2c^3)d^2 + 2(a^2b^4c - 6a^2b^2c^2 - 24a^3c^3)d^2e + (a^2b^3c + 12a^3b^2c^2)e^2 + (a^3b^3 + 12a^4b^2c^2)e^2 - 2((3a^2b^3c - 28a^3b^2c^2)d + 2(3a^3b^2c^2 + 4a^4c^2)e)f - (a^3b^6c - 12a^4b^4c^2 + 48a^5b^2c^3 - 64a^6c^4) \sqrt{(4a^3b^2c^2d^2e^3 + a^4c^2e^4 + 12a^5c^2d^2f^3 + a^6f^4 + (b^4c^2 - 18a^2b^2c^3 + 81a^2c^4)d^4 + 4(a^2b^3c^2 - 9a^2b^2c^3)d^3e + 6(a^2b^2c^2 - 3a^3c^3)d^2e^2 - 2(2a^4b^2c^2d^2e + a^5c^2e^2 + (a^3b^2c - 27a^4c^2)d^2)f^2 - 12(2a^3b^2c^2d^2e + a^4c^2d^2e^2 + (a^2b^2c^2 - 9a^3c^3)d^3)f)} / (a^6b^6c^2 - 12a^7b^4c^3 + 48a^8b^2c^4 - 64a^9c^5)) / (a^3b^6c - 12a^4b^4c^2 + 48a^5b^2c^3 - 64a^6c^4) \\
&) * \log(((5b^4c^3 - 81a^2b^2c^4 + 324a^2c^5)d^4 - (3b^5c^2 - 65a^2b^3c^3 + 324a^2b^2c^4)d^3e - 3(3a^2b^4c^2 - 28a^2b^2c^3)d^2e^2 - (9a^2b^3c^2 - 20a^3b^2c^3)d^2e^3 - (3a^3b^2c^2 + 4a^4c^3)e^4 + (3a^5b^2 + 4a^6c^2)f^4 - ((a^3b^4 - 24a^4b^2c - 48a^5c^2)d + (a^4b^3 + 12a^5b^2c^2)e)f^3 - 9((a^2b^4c - 6a^3b^2c^2 - 24a^4c^3)d^2 + (a^3b^3c + 12a^4b^2c^2)d^2e)f^2 + ((b^6c - 15a^2b^4c^2 + 432a^3c^4)d^3 + 3(a^2b^5c + 3a^2b^3c^2 - 108a^3b^2c^3)d^2e + 3(a^2b^4c + 12a^3b^2c^2)d^2e^2 + (a^3b^3c + 12a^4b^2c^2)e^3)f) * x - 1/2 \sqrt{1/2} * ((b^8c - 23a^2b^6c^2 + 190a^2b^4c^3 - 672a^3b^2c^4 + 864a^4c^5)d^3 + 3(a^2b^7c - 15a^2b^5c^2 + 72a^3b^3c^3 - 112a^4b^2c^4)d^2e + 3(a^2b^6c - 10a^3b^4c^2 + 32a^4b^2c^3 - 32a^5c^4)d^2e^2 + (a^3b^5c - 8a^4b^3c^2 + 16a^5b^2c^3)e^3 + 2(a^5b^4 - 8a^6b^2c + 16a^7c^2)f^3 - ((a^3b^6 - 26a^4b^4c + 160a^5b^2c^2 - 288a^6c^3)d + (a^4b^5 - 8a^5b^3c + 16a^6b^2c^2)e)f^2 - 2((4a^2b^6c - 59a^3b^4c^2 + 280a^4b^2c^3 - 432a^5c^4)d^2 + 5(a^3b^5c - 8a^4b^3c^2 + 16a^5b^2c^3)d^2e + (a^4b^4c - 8a^5b^2c^2 + 16a^6c^3)e^2)f + ((a^3b^9c - 20a^4b^7c^2 + 144a^5b^5c^3 - 448a^6b^3c^4 + 512a^7b^2c^5)d + (a^4b^8c - 8a^5b^6c^2 + 128a^7b^2c^4 - 256a^8c^5)e - 4(a^5b^7c - 12a^6b^5c^2 + 48a^7b^3c^3 - 64a^8b^2c^4)f) \sqrt{(4a^3b^2c^2d^2e^3 + a^4c^2e^4 + 12a^5c^2d^2f^3 + a^6f^4 + (b^4c^2 - 18a^2b^2c^3 + 81a^2c^4)d^4 + 4(a^2b^3c^2 - 9a^2b^2c^3)d^3e + 6(a^2b^2c^2 - 3a^3c^3)d^2e^2 - 2(2a^4b^2c^2d^2e + a^5c^2e^2 + (a^3b^2c - 27a^4c^2)d^2)f^2 - 12(2a^3b^2c^2d^2e + a^4c^2d^2e^2 + (a^2b^2c^2 - 9a^3c^3)d^3)f)} / (a^6b^6c^2 - 12a^7b^4c^3 + 48a^8b^2c^4 - 64a^9c^5)) / (a^3b^6c - 12a^4b^4c^2 + 48a^5b^2c^3 - 64a^6c^4) \\
&)) - 2(a^2b^2c - 4a^2c^2)x^4 + a^2b^2 - 4a^3c + (a^2b^3 - 4a^2b^2c)x^2)
\end{aligned}$$

giac [B] time = 6.97, size = 6356, normalized size = 18.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")

[Out] $\frac{1}{2} \cdot (b \cdot c \cdot d \cdot x^3 + a \cdot b \cdot f \cdot x^3 - 2 \cdot a \cdot c \cdot x^3 \cdot e + b^2 \cdot d \cdot x - 2 \cdot a \cdot c \cdot d \cdot x + 2 \cdot a^2 \cdot f \cdot x - a \cdot b \cdot x \cdot e) / ((c \cdot x^4 + b \cdot x^2 + a) \cdot (a \cdot b^2 - 4 \cdot a^2 \cdot c)) + \frac{1}{16} \cdot ((2 \cdot b^3 \cdot c^3 - 8 \cdot a \cdot b \cdot c^4 - \sqrt{2} \cdot \sqrt{b^2 - 4 \cdot a \cdot c}) \cdot \sqrt{b \cdot c + \sqrt{b^2 - 4 \cdot a \cdot c}} \cdot c) \cdot b^3 \cdot c + 4 \cdot \sqrt{2} \cdot \sqrt{b^2 - 4 \cdot a \cdot c} \cdot \sqrt{b \cdot c + \sqrt{b^2 - 4 \cdot a \cdot c}} \cdot c) \cdot a \cdot b \cdot c^2 + 2 \cdot \sqrt{2} \cdot \sqrt{b^2 - 4 \cdot a \cdot c} \cdot \sqrt{b \cdot c + \sqrt{b^2 - 4 \cdot a \cdot c}} \cdot c) \cdot b^2 \cdot c^2 - \sqrt{2} \cdot \sqrt{b^2 - 4 \cdot a \cdot c} \cdot \sqrt{b \cdot c + \sqrt{b^2 - 4 \cdot a \cdot c}} \cdot c) \cdot b \cdot c^3 - 2 \cdot (b^2 - 4 \cdot a \cdot c) \cdot b \cdot c^3 \cdot (a \cdot b^2 - 4 \cdot a^2 \cdot c)^2 \cdot d + (2 \cdot a \cdot b^3 \cdot c^2 - 8 \cdot a^2 \cdot b \cdot c^3 - \sqrt{2} \cdot \sqrt{b^2 - 4 \cdot a \cdot c}) \cdot \sqrt{b \cdot c + \sqrt{b^2 - 4 \cdot a \cdot c}} \cdot c) \cdot a \cdot b^3 + 4 \cdot \sqrt{2} \cdot \sqrt{b^2 - 4 \cdot a \cdot c} \cdot \sqrt{b \cdot c + \sqrt{b^2 - 4 \cdot a \cdot c}} \cdot c) \cdot a^2 \cdot b \cdot c + 2 \cdot \sqrt{2} \cdot \sqrt{b^2 - 4 \cdot a \cdot c} \cdot \sqrt{b \cdot c + \sqrt{b^2 - 4 \cdot a \cdot c}} \cdot c) \cdot a \cdot b^2 \cdot c - \sqrt{2} \cdot \sqrt{b^2 - 4 \cdot a \cdot c} \cdot \sqrt{b \cdot c + \sqrt{b^2 - 4 \cdot a \cdot c}} \cdot c) \cdot a \cdot b \cdot c^2 - 2 \cdot (b^2 - 4 \cdot a \cdot c) \cdot a \cdot b \cdot c^2 \cdot (a \cdot b^2 - 4 \cdot a^2 \cdot c)^2 \cdot f - 2 \cdot (2 \cdot a \cdot b^2 \cdot c^3 - 8 \cdot a^2 \cdot c^4 - \sqrt{2} \cdot \sqrt{b^2 - 4 \cdot a \cdot c}) \cdot \sqrt{b \cdot c + \sqrt{b^2 - 4 \cdot a \cdot c}} \cdot c) \cdot a \cdot b^2 \cdot c + 4 \cdot \sqrt{2} \cdot \sqrt{b^2 - 4 \cdot a \cdot c} \cdot \sqrt{b \cdot c + \sqrt{b^2 - 4 \cdot a \cdot c}} \cdot c) \cdot a^2 \cdot c^2 + 2 \cdot \sqrt{2} \cdot \sqrt{b^2 - 4 \cdot a \cdot c} \cdot \sqrt{b \cdot c + \sqrt{b^2 - 4 \cdot a \cdot c}} \cdot c) \cdot a \cdot b \cdot c^2 - \sqrt{2} \cdot \sqrt{b^2 - 4 \cdot a \cdot c} \cdot \sqrt{b \cdot c + \sqrt{b^2 - 4 \cdot a \cdot c}} \cdot c) \cdot a \cdot c^3 - 2 \cdot (b^2 - 4 \cdot a \cdot c) \cdot a \cdot c^3 \cdot (a \cdot b^2 - 4 \cdot a^2 \cdot c)^2 \cdot e + 2 \cdot (\sqrt{2} \cdot \sqrt{b \cdot c + \sqrt{b^2 - 4 \cdot a \cdot c}} \cdot c) \cdot a \cdot b^6 \cdot c - 14 \cdot \sqrt{2} \cdot \sqrt{b \cdot c + \sqrt{b^2 - 4 \cdot a \cdot c}} \cdot c) \cdot a^2 \cdot b^4 \cdot c^2 - 2 \cdot \sqrt{2} \cdot \sqrt{b \cdot c + \sqrt{b^2 - 4 \cdot a \cdot c}} \cdot c) \cdot a \cdot b^5 \cdot c^2 - 2 \cdot a \cdot b^6 \cdot c^2 + 64 \cdot \sqrt{2} \cdot \sqrt{b \cdot c + \sqrt{b^2 - 4 \cdot a \cdot c}} \cdot c) \cdot a^3 \cdot b^2 \cdot c^3 + 20 \cdot \sqrt{2} \cdot \sqrt{b \cdot c + \sqrt{b^2 - 4 \cdot a \cdot c}} \cdot c) \cdot a^2 \cdot b^3 \cdot c^3 + \sqrt{2} \cdot \sqrt{b \cdot c + \sqrt{b^2 - 4 \cdot a \cdot c}} \cdot c) \cdot a \cdot b^4 \cdot c^3 + 28 \cdot a^2 \cdot b^4 \cdot c^3 - 96 \cdot \sqrt{2} \cdot \sqrt{b \cdot c + \sqrt{b^2 - 4 \cdot a \cdot c}} \cdot c) \cdot a^4 \cdot c^4 - 48 \cdot \sqrt{2} \cdot \sqrt{b \cdot c + \sqrt{b^2 - 4 \cdot a \cdot c}} \cdot c) \cdot a^3 \cdot b \cdot c^4 - 10 \cdot \sqrt{2} \cdot \sqrt{b \cdot c + \sqrt{b^2 - 4 \cdot a \cdot c}} \cdot c) \cdot a^2 \cdot b^2 \cdot c^4 - 128 \cdot a^3 \cdot b^2 \cdot c^4 + 24 \cdot \sqrt{2} \cdot \sqrt{b \cdot c + \sqrt{b^2 - 4 \cdot a \cdot c}} \cdot c) \cdot a^3 \cdot c^5 + 192 \cdot a^4 \cdot c^5 + 2 \cdot (b^2 - 4 \cdot a \cdot c) \cdot a \cdot b^4 \cdot c^2 - 20 \cdot (b^2 - 4 \cdot a \cdot c) \cdot a^2 \cdot b^2 \cdot c^3 + 48 \cdot (b^2 - 4 \cdot a \cdot c) \cdot a^3 \cdot c^4) \cdot d \cdot \text{abs}(a \cdot b^2 - 4 \cdot a^2 \cdot c) - 4 \cdot (\sqrt{2} \cdot \sqrt{b \cdot c + \sqrt{b^2 - 4 \cdot a \cdot c}} \cdot c) \cdot a^3 \cdot b^4 \cdot c - 8 \cdot \sqrt{2} \cdot \sqrt{b \cdot c + \sqrt{b^2 - 4 \cdot a \cdot c}} \cdot c) \cdot a^4 \cdot b^2 \cdot c^2 - 2 \cdot \sqrt{2} \cdot \sqrt{b \cdot c + \sqrt{b^2 - 4 \cdot a \cdot c}} \cdot c) \cdot a^3 \cdot b^3 \cdot c^2 - 2 \cdot a^3 \cdot b^4 \cdot c^2 + 16 \cdot \sqrt{2} \cdot \sqrt{b \cdot c + \sqrt{b^2 - 4 \cdot a \cdot c}} \cdot c) \cdot a^5 \cdot c^3 + 8 \cdot \sqrt{2} \cdot \sqrt{b \cdot c + \sqrt{b^2 - 4 \cdot a \cdot c}} \cdot c) \cdot a^4 \cdot b \cdot c^3 + \sqrt{2} \cdot \sqrt{b \cdot c + \sqrt{b^2 - 4 \cdot a \cdot c}} \cdot c) \cdot a^3 \cdot b^2 \cdot c^3 + 16 \cdot a^4 \cdot b^2 \cdot c^3 - 4 \cdot \sqrt{2} \cdot \sqrt{b \cdot c + \sqrt{b^2 - 4 \cdot a \cdot c}} \cdot c) \cdot a^4 \cdot c^4 - 32 \cdot a^5 \cdot c^4 + 2 \cdot (b^2 - 4 \cdot a \cdot c) \cdot a^3 \cdot b^2 \cdot c^2 - 8 \cdot (b^2 - 4 \cdot a \cdot c) \cdot a^4 \cdot c^3) \cdot f \cdot \text{abs}(a \cdot b^2 - 4 \cdot a^2 \cdot c) + 2 \cdot (\sqrt{2} \cdot \sqrt{b \cdot c + \sqrt{b^2 - 4 \cdot a \cdot c}} \cdot c) \cdot a^2 \cdot b^5 \cdot c - 8 \cdot \sqrt{2} \cdot \sqrt{b \cdot c + \sqrt{b^2 - 4 \cdot a \cdot c}} \cdot c) \cdot a^3 \cdot b^3 \cdot c^2 - 2 \cdot \sqrt{2} \cdot \sqrt{b \cdot c + \sqrt{b^2 - 4 \cdot a \cdot c}} \cdot c) \cdot a^2 \cdot b^4 \cdot c^2 - 2 \cdot a^2 \cdot b^5 \cdot c^2 + 16 \cdot \sqrt{2} \cdot \sqrt{b \cdot c + \sqrt{b^2 - 4 \cdot a \cdot c}} \cdot c) \cdot a^4 \cdot b \cdot c^3 + 8 \cdot \sqrt{2} \cdot \sqrt{b \cdot c + \sqrt{b^2 - 4 \cdot a \cdot c}} \cdot c) \cdot a^3 \cdot b^2 \cdot c^3 + \sqrt{2} \cdot \sqrt{b \cdot c + \sqrt{b^2 - 4 \cdot a \cdot c}} \cdot c) \cdot a^2 \cdot b^3 \cdot c^3 + 16 \cdot a^3 \cdot b^3 \cdot c^3 - 4 \cdot \sqrt{2} \cdot \sqrt{b \cdot c + \sqrt{b^2 - 4 \cdot a \cdot c}} \cdot c) \cdot a^3 \cdot b \cdot c^4 - 32 \cdot a^4 \cdot b \cdot c^4 + 2 \cdot (b^2 - 4 \cdot a \cdot c) \cdot a^2 \cdot b^3 \cdot c^2 - 8 \cdot (b^2 - 4 \cdot a \cdot c) \cdot a^3 \cdot b \cdot c^3) \cdot \text{abs}(a \cdot b^2 - 4 \cdot a^2 \cdot c) \cdot e + (2 \cdot a^2 \cdot b^7 \cdot c^3 - 40 \cdot a^3 \cdot b^5 \cdot c^4 + 224 \cdot a^4 \cdot b^3 \cdot c^5 - 384 \cdot a^5 \cdot b \cdot c^6 - \sqrt{2} \cdot \sqrt{b^2 - 4 \cdot a \cdot c}) \cdot \sqrt{b \cdot c + \sqrt{b^2 - 4 \cdot a \cdot c}} \cdot c) \cdot a^2 \cdot b^7 \cdot c + 20 \cdot \sqrt{2} \cdot \sqrt{b^2 - 4 \cdot a \cdot c}) \cdot \sqrt{b \cdot c + \sqrt{b^2 - 4 \cdot a \cdot c}} \cdot c) \cdot a^3 \cdot b^5 \cdot c^2 + 2 \cdot \sqrt{2} \cdot \sqrt{b^2 - 4 \cdot a \cdot c}) \cdot \sqrt{b \cdot c + \sqrt{b^2 - 4 \cdot a \cdot c}} \cdot c) \cdot a^2 \cdot b^6 \cdot c^2 - 112 \cdot \sqrt{2} \cdot \sqrt{b^2 - 4 \cdot a \cdot c}) \cdot \sqrt{b \cdot c + \sqrt{b^2 - 4 \cdot a \cdot c}} \cdot c) \cdot a^4 \cdot b^3 \cdot c^3 - 32 \cdot \sqrt{2} \cdot \sqrt{b^2 - 4 \cdot a \cdot c}) \cdot \sqrt{b \cdot c + \sqrt{b^2 - 4 \cdot a \cdot c}} \cdot c) \cdot a^3 \cdot b^4 \cdot c^3 - \sqrt{2} \cdot \sqrt{b^2 - 4 \cdot a \cdot c}) \cdot \sqrt{b \cdot c + \sqrt{b^2 - 4 \cdot a \cdot c}} \cdot c) \cdot a^2 \cdot b^5 \cdot c^3 + 192 \cdot \sqrt{2} \cdot \sqrt{b^2 - 4 \cdot a \cdot c}) \cdot \sqrt{b \cdot c + \sqrt{b^2 - 4 \cdot a \cdot c}} \cdot c) \cdot a^5 \cdot b \cdot c^4 + 96 \cdot \sqrt{2} \cdot \sqrt{b^2 - 4 \cdot a \cdot c}) \cdot \sqrt{b \cdot c + \sqrt{b^2 - 4 \cdot a \cdot c}} \cdot c) \cdot a^4 \cdot b^2 \cdot c^4 + 16 \cdot \sqrt{2} \cdot \sqrt{b^2 - 4 \cdot a \cdot c}) \cdot \sqrt{b \cdot c + \sqrt{b^2 - 4 \cdot a \cdot c}} \cdot c) \cdot a^3 \cdot b^3 \cdot c^4 - 48 \cdot \sqrt{2} \cdot \sqrt{b^2 - 4 \cdot a \cdot c}) \cdot \sqrt{b \cdot c + \sqrt{b^2 - 4 \cdot a \cdot c}} \cdot c) \cdot a^4 \cdot b \cdot c^5 - 2 \cdot (b^2 - 4 \cdot a \cdot c) \cdot a^2 \cdot b^5 \cdot c^3 + 32 \cdot (b^2 - 4 \cdot a \cdot c) \cdot a^3 \cdot b^3 \cdot c^4 - 96 \cdot (b^2 - 4 \cdot a \cdot c) \cdot a^4 \cdot b \cdot c^5) \cdot d - (2 \cdot a^3 \cdot b^7 \cdot c^2 - 8 \cdot a^4 \cdot b^5 \cdot c^3 - 32 \cdot a^5 \cdot b^3 \cdot c^4 + 128 \cdot a^6 \cdot b \cdot c^5 - \sqrt{2} \cdot \sqrt{b^2 - 4 \cdot a \cdot c}) \cdot \sqrt{b^2 - 4 \cdot a \cdot c}) \cdot \sqrt{b \cdot c + \sqrt{b^2 - 4 \cdot a \cdot c}} \cdot c) \cdot a^3 \cdot b^7 + 4 \cdot \sqrt{2} \cdot \sqrt{b^2 - 4 \cdot a \cdot c}) \cdot \sqrt{b^2 - 4 \cdot a \cdot c}) \cdot \sqrt{b \cdot c + \sqrt{b^2 - 4 \cdot a \cdot c}} \cdot c) \cdot a^4 \cdot b^5 \cdot c + 2 \cdot \sqrt{2} \cdot \sqrt{b^2 - 4 \cdot a \cdot c}) \cdot \sqrt{b^2 - 4 \cdot a \cdot c}) \cdot \sqrt{b \cdot c + \sqrt{b^2 - 4 \cdot a \cdot c}} \cdot c) \cdot a^3 \cdot b^6 \cdot c + 16 \cdot \sqrt{2} \cdot \sqrt{b^2 - 4 \cdot a \cdot c}) \cdot \sqrt{b^2 - 4 \cdot a \cdot c}) \cdot \sqrt{b \cdot c + \sqrt{b^2 - 4 \cdot a \cdot c}} \cdot c) \cdot a^5 \cdot b^3 \cdot c^2 - \sqrt{2} \cdot \sqrt{b^2 - 4 \cdot a \cdot c}) \cdot \sqrt{b^2 - 4 \cdot a \cdot c})$

$$\begin{aligned}
&)*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^3*b^5*c^2 - 64*\sqrt{2}*\sqrt{b^2 - 4*a*c} \\
&)*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^6*b*c^3 - 32*\sqrt{2}*\sqrt{b^2 - 4*a*c}* \\
& \sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^5*b^2*c^3 + 16*\sqrt{2}*\sqrt{b^2 - 4*a*c}* \\
& \sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^5*b*c^4 - 2*(b^2 - 4*a*c)*a^3*b^5*c^2 + 3 \\
& 2*(b^2 - 4*a*c)*a^5*b*c^4)*f + 4*(2*a^3*b^6*c^3 - 16*a^4*b^4*c^4 + 32*a^5*b \\
& ^2*c^5 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^3*b^6* \\
& c + 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^4*b^4*c^2 \\
& + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^3*b^5*c^2 \\
& - 16*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^5*b^2*c^3 \\
& - 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^4*b^3*c^3 - \\
& \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^3*b^4*c^3 + 4* \\
& \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^4*b^2*c^4 - 2*(\\
& b^2 - 4*a*c)*a^3*b^4*c^3 + 8*(b^2 - 4*a*c)*a^4*b^2*c^4)*e)*\arctan(2*\sqrt{1/ \\
& 2}*x/\sqrt{((a*b^3 - 4*a^2*b*c + \sqrt{((a*b^3 - 4*a^2*b*c)^2 - 4*(a^2*b^2 - 4* \\
& a^3*c)*(a*b^2*c - 4*a^2*c^2)))/((a^3*b^6*c - 12*a^4 \\
& *b^4*c^2 - 2*a^3*b^5*c^2 + 48*a^5*b^2*c^3 + 16*a^4*b^3*c^3 + a^3*b^4*c^3 - \\
& 64*a^6*c^4 - 32*a^5*b*c^4 - 8*a^4*b^2*c^4 + 16*a^5*c^5)*\text{abs}(a*b^2 - 4*a^2*c \\
&)*\text{abs}(c)) - 1/16*((2*b^3*c^3 - 8*a*b*c^4 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b \\
& *c - \sqrt{b^2 - 4*a*c}}*c)*b^3*c + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{ \\
& b^2 - 4*a*c}}*c)*a*b*c^2 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^ \\
& 2 - 4*a*c}}*c)*b^2*c^2 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a \\
& *c}}*c)*b*c^3 - 2*(b^2 - 4*a*c)*b*c^3)*(a*b^2 - 4*a^2*c)^2*d + (2*a*b^3*c^2 \\
& - 8*a^2*b*c^3 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a \\
& *b^3 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b*c \\
& + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c - \sqrt{ \\
& 2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b*c^2 - 2*(b^2 - 4 \\
& *a*c)*a*b*c^2)*(a*b^2 - 4*a^2*c)^2*f - 2*(2*a*b^2*c^3 - 8*a^2*c^4 - \sqrt{2} \\
& *\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c + 4*\sqrt{2}*\sqrt{ \\
& b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*c^2 + 2*\sqrt{2}*\sqrt{b^2 \\
& - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b*c^2 - \sqrt{2}*\sqrt{b^2 - 4*a*c} \\
&)*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*c^3 - 2*(b^2 - 4*a*c)*a*c^3)*(a*b^2 - 4 \\
& *a^2*c)^2*e - 2*(\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^6*c - 14*\sqrt{2} \\
& *\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b^4*c^2 - 2*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - \\
& 4*a*c}}*c)*a*b^5*c^2 + 2*a*b^6*c^2 + 64*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4 \\
& *a*c}}*c)*a^3*b^2*c^3 + 20*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b^3* \\
& c^3 + \sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^4*c^3 - 28*a^2*b^4*c^3 - \\
& 96*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^4*c^4 - 48*\sqrt{2}*\sqrt{b*c - \\
& \sqrt{b^2 - 4*a*c}}*c)*a^3*b*c^4 - 10*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c) \\
& *a^2*b^2*c^4 + 128*a^3*b^2*c^4 + 24*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c) \\
& *a^3*c^5 - 192*a^4*c^5 - 2*(b^2 - 4*a*c)*a*b^4*c^2 + 20*(b^2 - 4*a*c)*a^2*b \\
& ^2*c^3 - 48*(b^2 - 4*a*c)*a^3*c^4)*d*\text{abs}(a*b^2 - 4*a^2*c) + 4*(\sqrt{2}*\sqrt{ \\
& b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*b^4*c - 8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a \\
& *c}}*c)*a^4*b^2*c^2 - 2*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*b^3*c^2 \\
& + 2*a^3*b^4*c^2 + 16*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^5*c^3 + 8*\sqrt{ \\
& 2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^4*b*c^3 + \sqrt{2}*\sqrt{b*c - \sqrt{b^ \\
& ^2 - 4*a*c}}*c)*a^3*b^2*c^3 - 16*a^4*b^2*c^3 - 4*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 \\
& - 4*a*c}}*c)*a^4*c^4 + 32*a^5*c^4 - 2*(b^2 - 4*a*c)*a^3*b^2*c^2 + 8*(b^2 - \\
& 4*a*c)*a^4*c^3)*f*\text{abs}(a*b^2 - 4*a^2*c) - 2*(\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4 \\
& *a*c}}*c)*a^2*b^5*c - 8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*b^3*c^2 \\
& - 2*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b^4*c^2 + 2*a^2*b^5*c^2 + 1 \\
& 6*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^4*b*c^3 + 8*\sqrt{2}*\sqrt{b*c - \\
& \sqrt{b^2 - 4*a*c}}*c)*a^3*b^2*c^3 + \sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)* \\
& a^2*b^3*c^3 - 16*a^3*b^3*c^3 - 4*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^ \\
& 3*b*c^4 + 32*a^4*b*c^4 - 2*(b^2 - 4*a*c)*a^2*b^3*c^2 + 8*(b^2 - 4*a*c)*a^3* \\
& b*c^3)*\text{abs}(a*b^2 - 4*a^2*c)*e + (2*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 224*a^4*b \\
& ^3*c^5 - 384*a^5*b*c^6 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4* \\
& a*c}}*c)*a^2*b^7*c + 20*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a* \\
& c}}*c)*a^3*b^5*c^2 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c} \\
&)*c)*a^2*b^6*c^2 - 112*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*
\end{aligned}$$

$$\begin{aligned}
& c) * c) * a^4 * b^3 * c^3 - 32 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c - \sqrt{b^2 - 4 * a * c}} \\
& c) * a^3 * b^4 * c^3 - \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c - \sqrt{b^2 - 4 * a * c}} * \\
& c) * a^2 * b^5 * c^3 + 192 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c - \sqrt{b^2 - 4 * a * c}} \\
& * c) * a^5 * b * c^4 + 96 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c - \sqrt{b^2 - 4 * a * c}} * c \\
&) * a^4 * b^2 * c^4 + 16 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c - \sqrt{b^2 - 4 * a * c}} * c \\
&) * a^3 * b^3 * c^4 - 48 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c - \sqrt{b^2 - 4 * a * c}} * c \\
&) * a^4 * b * c^5 - 2 * (b^2 - 4 * a * c) * a^2 * b^5 * c^3 + 32 * (b^2 - 4 * a * c) * a^3 * b^3 * c^4 - \\
& 96 * (b^2 - 4 * a * c) * a^4 * b * c^5) * d - (2 * a^3 * b^7 * c^2 - 8 * a^4 * b^5 * c^3 - 32 * a^5 * b^3 * \\
& * c^4 + 128 * a^6 * b * c^5 - \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c - \sqrt{b^2 - 4 * a * c}} \\
& c) * a^3 * b^7 + 4 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c - \sqrt{b^2 - 4 * a * c}} * c) \\
& * a^4 * b^5 * c + 2 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c - \sqrt{b^2 - 4 * a * c}} * c) * a^5 * \\
& b^3 * c^2 - \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c - \sqrt{b^2 - 4 * a * c}} * c) * a^3 * b^5 \\
& * c^2 - 64 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c - \sqrt{b^2 - 4 * a * c}} * c) * a^6 * b * c \\
& ^3 - 32 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c - \sqrt{b^2 - 4 * a * c}} * c) * a^5 * b^2 * c \\
& ^3 + 16 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c - \sqrt{b^2 - 4 * a * c}} * c) * a^5 * b * c^4 \\
& - 2 * (b^2 - 4 * a * c) * a^3 * b^5 * c^2 + 32 * (b^2 - 4 * a * c) * a^5 * b * c^4) * f + 4 * (2 * a^3 * b \\
& ^6 * c^3 - 16 * a^4 * b^4 * c^4 + 32 * a^5 * b^2 * c^5 - \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c - \sqrt{b^2 - 4 * a * c}} * c) * a^3 * b^6 * c + 8 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c - \sqrt{b^2 - 4 * a * c}} * c) * a^4 * b^4 * c^2 + 2 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c - \sqrt{b^2 - 4 * a * c}} * c) * a^3 * b^5 * c^2 - 16 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c - \sqrt{b^2 - 4 * a * c}} * c) * a^5 * b^2 * c^3 - 8 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c - \sqrt{b^2 - 4 * a * c}} * c) * a^4 * b^3 * c^3 - \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c - \sqrt{b^2 - 4 * a * c}} * c) * a^3 * b^4 * c^3 + 4 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c - \sqrt{b^2 - 4 * a * c}} * c) * a^4 * b^2 * c^4 - 2 * (b^2 - 4 * a * c) * a^3 * b^4 * c^3 + 8 * (b^2 - 4 * a * c) * a^4 * b^2 * c^4) * e) * \arctan(2 * \sqrt{1/2} * x / \sqrt{(a * b^3 - 4 * a^2 * b * c - \sqrt{(a * b^3 - 4 * a^2 * b * c)^2 - 4 * (a^2 * b^2 - 4 * a^3 * c) * (a * b^2 * c - 4 * a^2 * c^2)})}) / ((a * b^2 * c - 4 * a^2 * c^2))) / ((a^3 * b^6 * c - 12 * a^4 * b^4 * c^2 - 2 * a^3 * b^5 * c^2 + 48 * a^5 * b^2 * c^3 + 16 * a^4 * b^3 * c^3 + a^3 * b^4 * c^3 - 64 * a^6 * c^4 - 32 * a^5 * b * c^4 - 8 * a^4 * b^2 * c^4 + 16 * a^5 * c^5) * \text{abs}(a * b^2 - 4 * a^2 * c) * \text{abs}(c))
\end{aligned}$$

maple [B] time = 0.04, size = 1182, normalized size = 3.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2, x)$

[Out]
$$\begin{aligned}
& (-1/2/a*(a*b*f-2*a*c*e+b*c*d)/(4*a*c-b^2)*x^3-1/2*(2*a^2*f-a*b*e-2*a*c*d+b^2*d)/a/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+1/4/(4*a*c-b^2)*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x) \\
& *b*f-1/2/(4*a*c-b^2)*c*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*e+1/4/a/(4*a*c-b^2)*c*2^{(1/2)}/ \\
& ((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b*d-a/(4*a*c-b^2)*c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)* \\
& f-1/4/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^2*f+1/(4*a*c-b^2)*c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b*e-3/(4*a*c-b^2)*c^2/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*d+1/4/a/(4*a*c-b^2)*c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^2*d-1/4/(4*a*c-b^2)*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b*f+1/2/(4*a*c-b^2)*c*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*e-1/4/a/(4*a*c-b^2)*c*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b*d-a/(4*a*c-b^2)*c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*f-1/4/(4*a*c-b^2)/(-4
\end{aligned}$$

$$\begin{aligned} & *a*c+b^2)^{(1/2)}*2^{(1/2)} / ((b+(-4*a*c+b^2)^{(1/2)}) *c)^{(1/2)} * \arctan(2^{(1/2)} / ((b \\ & +(-4*a*c+b^2)^{(1/2)}) *c)^{(1/2)} *c*x) *b^2*f+1 / (4*a*c-b^2) *c / (-4*a*c+b^2)^{(1/2)} \\ & *2^{(1/2)} / ((b+(-4*a*c+b^2)^{(1/2)}) *c)^{(1/2)} * \arctan(2^{(1/2)} / ((b+(-4*a*c+b^2)^{(1/2)}) *c)^{(1/2)} *c*x) *b^2 *e-3 / (4*a*c-b^2) *c^2 / (-4*a*c+b^2)^{(1/2)} *2^{(1/2)} / ((b+(-4*a*c+b^2)^{(1/2)}) *c)^{(1/2)} * \arctan(2^{(1/2)} / ((b+(-4*a*c+b^2)^{(1/2)}) *c)^{(1/2)} *c*x) *d+1/4/a / (4*a*c-b^2) *c / (-4*a*c+b^2)^{(1/2)} *2^{(1/2)} / ((b+(-4*a*c+b^2)^{(1/2)}) *c)^{(1/2)} * \arctan(2^{(1/2)} / ((b+(-4*a*c+b^2)^{(1/2)}) *c)^{(1/2)} *c*x) *b^2 *d \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")

[Out] $\frac{1}{2} * ((b*c*d - 2*a*c*e + a*b*f) * x^3 - (a*b*e - 2*a^2*f - (b^2 - 2*a*c) * d) * x) / ((a*b^2*c - 4*a^2*c^2) * x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c) * x^2) + \frac{1}{2} * \integrate((a*b*e - 2*a^2*f + (b*c*d - 2*a*c*e + a*b*f) * x^2 + (b^2 - 6*a*c) * d) / (c*x^4 + b*x^2 + a), x) / (a*b^2 - 4*a^2*c)$

mupad [B] time = 6.55, size = 19589, normalized size = 56.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x^2 + f*x^4)/(a + b*x^2 + c*x^4)^2,x)

[Out] $\operatorname{atan}\left(\frac{\left(\left(\left(6144*a^5*c^6*d + 2048*a^6*c^5*f - 288*a^2*b^6*c^3*d + 1920*a^3*b^4*c^4*d - 5632*a^4*b^2*c^5*d + 16*a^2*b^7*c^2*e - 192*a^3*b^5*c^3*e + 768*a^4*b^3*c^4*e - 32*a^3*b^6*c^2*f + 384*a^4*b^4*c^3*f - 1536*a^5*b^2*c^4*f + 16*a*b^8*c^2*d - 1024*a^5*b*c^5*e\right)\right)\right) / \left(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)\right) - \left(x*\left(\left(27*a*b^9*c^2*d^2 - a^3*b^9*f^2 - a^3*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - b^{11}*c*d^2 + 3840*a^5*b*c^6*d^2 - 9*a*c^2*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - a^2*b^9*c*e^2 + 768*a^6*b*c^5*e^2 + a^2*c*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + b^2*c*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + 768*a^7*b*c^4*f^2 - 288*a^2*b^7*c^3*d^2 + 1504*a^3*b^5*c^4*d^2 - 3840*a^4*b^3*c^5*d^2 + 96*a^4*b^5*c^3*e^2 - 512*a^5*b^3*c^4*e^2 + 96*a^5*b^5*c^2*f^2 - 512*a^6*b^3*c^3*f^2 - 3072*a^6*c^6*d*e - 1024*a^7*c^5*e*f + 6*a^2*b^9*c*d*f + 3584*a^6*b*c^5*d*f - 6*a^2*c*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 12*a^3*b^8*c*e*f + 36*a^2*b^8*c^2*d*e - 192*a^3*b^6*c^3*d*e + 128*a^4*b^4*c^4*d*e + 1536*a^5*b^2*c^5*d*e - 128*a^3*b^7*c^2*d*f + 960*a^4*b^5*c^3*d*f - 3072*a^5*b^3*c^4*d*f - 128*a^4*b^6*c^2*e*f + 384*a^5*b^4*c^3*e*f - 2*a*b^{10}*c*d*e + 2*a*b*c*d*e*(-(4*a*c - b^2)^9)^{(1/2)}\right)\right) / \left(32*(4096*a^9*c^7 + a^3*b^{12}*c - 24*a^4*b^{10}*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)\right)^{(1/2)} * \left(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4\right) / \left(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)\right) * \left(\left(27*a*b^9*c^2*d^2 - a^3*b^9*f^2 - a^3*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - b^{11}*c*d^2 + 3840*a^5*b*c^6*d^2 - 9*a*c^2*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - a^2*b^9*c*e^2 + 768*a^6*b*c^5*e^2 + a^2*c*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + b^2*c*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + 768*a^7*b*c^4*f^2 - 288*a^2*b^7*c^3*d^2 + 1504*a^3*b^5*c^4*d^2 - 3840*a^4*b^3*c^5*d^2 + 96*a^4*b^5*c^3*e^2 - 512*a^5*b^3*c^4*e^2 + 96*a^5*b^5*c^2*f^2 - 512*a^6*b^3*c^3*f^2 - 3072*a^6*c^6*d*e - 1024*a^7*c^5*e*f + 6*a^2*b^9*c*d*f + 3584*a^6*b*c^5*d*f - 6*a^2*c*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 12*a^3*b^8*c*e*f + 36*a^2*b^8*c^2*d*e - 192*a^3*b^6*c^3*d*e + 128*a^4*b^4*c^4*d*e + 1536*a^5*b^2*c^5*d*e - 128*a^3*b^7*c^2*d*f + 960*a^4*b^5*c^3*d*f - 3072*a^5*b^3*c^4*d*f - 128*a^4*b^6*c^2*e*f + 384*a^5*b^4*c^3*e*f - 2*a*b^{10}*c*d*e + 2*a*b*c*d*e*(-(4*a*c - b^2)^9)^{(1/2)}\right) / \left(32*(4096*a^9*c^7 + a^3*b^{12}*c - 24*a^4*b^{10}*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)\right)^{(1/2)} + \left(x*\left(72*a^2*c^5*d^2 - 8*a^3*c^4*e^2 + b^4*c^3*d^2 + 8*a^4*c^3*f^2 - 14*a*b^2*c^4*d^2 + a^2*b^4*c*f^2 + 10*a^2*b^2*c^3*e^2 + 2*a^3*b^2*c^2*f^2 + 48*a^3*c^4*d*f + 2*a*b^3*c^3*d*e - 40*a^2*b*c^4*d*e - 8*a^3*b*c^3*e*f + \right.\right.$

$$\begin{aligned}
& (4a^2b^2c^3d^2f - 6a^2b^3c^2e^2f)) / (2(a^2b^4 + 16a^4c^2 - 8a^3b^2c)) \\
& * ((27a^9b^9c^2d^2 - a^3b^9f^2 - a^3f^2(-4ac - b^2)^9)^{(1/2)} - b^{11}c^2d^2 + 3840a^5b^6c^6d^2 - 9a^2c^2d^2(-4ac - b^2)^9)^{(1/2)} - a^2b^9c^2e^2 + 768a^6b^6c^5e^2 + a^2c^2e^2(-4ac - b^2)^9)^{(1/2)} + b^2c^2d^2(-4ac - b^2)^9)^{(1/2)} + 768a^7b^6c^4f^2 - 288a^2b^7c^3d^2 + 1504a^3b^5c^4d^2 - 3840a^4b^3c^5d^2 + 96a^4b^5c^3e^2 - 512a^5b^3c^4e^2 + 96a^5b^5c^2f^2 - 512a^6b^3c^3f^2 - 3072a^6c^6d^2e - 1024a^7c^5e^2f + 6a^2b^9c^2d^2f + 3584a^6b^6c^5d^2f - 6a^2c^2d^2f(-4ac - b^2)^9)^{(1/2)} + 12a^3b^8c^2e^2f + 36a^2b^8c^2d^2e - 192a^3b^6c^3d^2e + 128a^4b^4c^4d^2e + 1536a^5b^2c^5d^2e - 128a^3b^7c^2d^2f + 960a^4b^5c^3d^2f - 3072a^5b^3c^4d^2f - 128a^4b^6c^2e^2f + 384a^5b^4c^3e^2f - 2a^2b^10c^2d^2e + 2a^2b^10c^2d^2e(-4ac - b^2)^9)^{(1/2)} / (32(4096a^9c^7 + a^3b^12c - 24a^4b^10c^2 + 240a^5b^8c^3 - 1280a^6b^6c^4 + 3840a^7b^4c^5 - 6144a^8b^2c^6))^{(1/2)} * i - ((6144a^5c^6d + 2048a^6c^5f - 288a^2b^6c^3d + 1920a^3b^4c^4d - 5632a^4b^2c^5d + 16a^2b^7c^2e - 192a^3b^5c^3e + 768a^4b^3c^4e - 32a^3b^6c^2f + 384a^4b^4c^3f - 1536a^5b^2c^4f + 16a^2b^8c^2d - 1024a^5b^6c^5e) / (8(a^2b^6 - 64a^5c^3 - 12a^3b^4c + 48a^4b^2c^2)) + (x((27a^9b^9c^2d^2 - a^3b^9f^2 - a^3f^2(-4ac - b^2)^9)^{(1/2)} - b^{11}c^2d^2 + 3840a^5b^6c^6d^2 - 9a^2c^2d^2(-4ac - b^2)^9)^{(1/2)} - a^2b^9c^2e^2 + 768a^6b^6c^5e^2 + a^2c^2e^2(-4ac - b^2)^9)^{(1/2)} + b^2c^2d^2(-4ac - b^2)^9)^{(1/2)} + 768a^7b^6c^4f^2 - 288a^2b^7c^3d^2 + 1504a^3b^5c^4d^2 - 3840a^4b^3c^5d^2 + 96a^4b^5c^3e^2 - 512a^5b^3c^4e^2 + 96a^5b^5c^2f^2 - 512a^6b^3c^3f^2 - 3072a^6c^6d^2e - 1024a^7c^5e^2f + 6a^2b^9c^2d^2f + 3584a^6b^6c^5d^2f - 6a^2c^2d^2f(-4ac - b^2)^9)^{(1/2)} + 12a^3b^8c^2e^2f + 36a^2b^8c^2d^2e - 192a^3b^6c^3d^2e + 128a^4b^4c^4d^2e + 1536a^5b^2c^5d^2e - 128a^3b^7c^2d^2f + 960a^4b^5c^3d^2f - 3072a^5b^3c^4d^2f - 128a^4b^6c^2e^2f + 384a^5b^4c^3e^2f - 2a^2b^10c^2d^2e + 2a^2b^10c^2d^2e(-4ac - b^2)^9)^{(1/2)} / (32(4096a^9c^7 + a^3b^12c - 24a^4b^10c^2 + 240a^5b^8c^3 - 1280a^6b^6c^4 + 3840a^7b^4c^5 - 6144a^8b^2c^6))^{(1/2)} * (1024a^5b^6c^5 - 16a^2b^7c^2 + 192a^3b^5c^3 - 768a^4b^3c^4) / (2(a^2b^4 + 16a^4c^2 - 8a^3b^2c)) * ((27a^9b^9c^2d^2 - a^3b^9f^2 - a^3f^2(-4ac - b^2)^9)^{(1/2)} - b^{11}c^2d^2 + 3840a^5b^6c^6d^2 - 9a^2c^2d^2(-4ac - b^2)^9)^{(1/2)} - a^2b^9c^2e^2 + 768a^6b^6c^5e^2 + a^2c^2e^2(-4ac - b^2)^9)^{(1/2)} + b^2c^2d^2(-4ac - b^2)^9)^{(1/2)} + 768a^7b^6c^4f^2 - 288a^2b^7c^3d^2 + 1504a^3b^5c^4d^2 - 3840a^4b^3c^5d^2 + 96a^4b^5c^3e^2 - 512a^5b^3c^4e^2 + 96a^5b^5c^2f^2 - 512a^6b^3c^3f^2 - 3072a^6c^6d^2e - 1024a^7c^5e^2f + 6a^2b^9c^2d^2f + 3584a^6b^6c^5d^2f - 6a^2c^2d^2f(-4ac - b^2)^9)^{(1/2)} + 12a^3b^8c^2e^2f + 36a^2b^8c^2d^2e - 192a^3b^6c^3d^2e + 128a^4b^4c^4d^2e + 1536a^5b^2c^5d^2e - 128a^3b^7c^2d^2f + 960a^4b^5c^3d^2f - 3072a^5b^3c^4d^2f - 128a^4b^6c^2e^2f + 384a^5b^4c^3e^2f - 2a^2b^10c^2d^2e + 2a^2b^10c^2d^2e(-4ac - b^2)^9)^{(1/2)} / (32(4096a^9c^7 + a^3b^12c - 24a^4b^10c^2 + 240a^5b^8c^3 - 1280a^6b^6c^4 + 3840a^7b^4c^5 - 6144a^8b^2c^6))^{(1/2)} - (x(72a^2c^5d^2 - 8a^3c^4e^2 + b^4c^3d^2 + 8a^4c^3f^2 - 14a^2b^2c^4d^2 + a^2b^4c^3f^2 + 10a^2b^2c^3e^2 + 2a^3b^2c^2f^2 + 48a^3c^4d^2f + 2a^2b^3c^3d^2e - 40a^2b^3c^4d^2e - 8a^3b^3c^3e^2f + 4a^2b^2c^3d^2f - 6a^2b^3c^2e^2f)) / (2(a^2b^4 + 16a^4c^2 - 8a^3b^2c)) * ((27a^9b^9c^2d^2 - a^3b^9f^2 - a^3f^2(-4ac - b^2)^9)^{(1/2)} - b^{11}c^2d^2 + 3840a^5b^6c^6d^2 - 9a^2c^2d^2(-4ac - b^2)^9)^{(1/2)} - a^2b^9c^2e^2 + 768a^6b^6c^5e^2 + a^2c^2e^2(-4ac - b^2)^9)^{(1/2)} + b^2c^2d^2(-4ac - b^2)^9)^{(1/2)} + 768a^7b^6c^4f^2 - 288a^2b^7c^3d^2 + 1504a^3b^5c^4d^2 - 3840a^4b^3c^5d^2 + 96a^4b^5c^3e^2 - 512a^5b^3c^4e^2 + 96a^5b^5c^2f^2 - 512a^6b^3c^3f^2 - 3072a^6c^6d^2e - 1024a^7c^5e^2f + 6a^2b^9c^2d^2f + 3584a^6b^6c^5d^2f - 6a^2c^2d^2f(-4ac - b^2)^9)^{(1/2)} + 12a^3b^8c^2e^2f + 36a^2b^8c^2d^2e - 192a^3b^6c^3d^2e + 128a^4b^4c^4d^2e + 1536a^5b^2c^5d^2e - 128a^3b^7c^2d^2f + 960a^4b^5c^3d^2f - 3072a^5b^3c^4d^2f - 128a^4b^6c^2e^2f + 384a^5b^4c^3e^2f -
\end{aligned}$$

$$\begin{aligned}
& 2*a*b^{10}*c*d*e + 2*a*b*c*d*e*(-(4*a*c - b^2)^9)^{(1/2)}/(32*(4096*a^9*c^7 + \\
& a^3*b^{12}*c - 24*a^4*b^{10}*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7* \\
& b^4*c^5 - 6144*a^8*b^2*c^6)))^{(1/2)*i}/((8*a^3*c^4*e^3 + 5*b^3*c^4*d^3 - \\
& 3*a^3*b^3*c*f^3 - 4*a^4*b*c^2*f^3 + 72*a^2*c^5*d^2*e - 3*b^4*c^3*d^2*e + 8 \\
& *a^4*c^3*e*f^2 + b^5*c^2*d^2*f + 6*a^2*b^2*c^3*e^3 - 36*a*b*c^5*d^3 + a*b^5 \\
& *c*d*f^2 + 48*a^3*c^4*d*e*f + 18*a*b^2*c^4*d^2*e + 3*a*b^3*c^3*d*e^2 - 60*a \\
& ^2*b*c^4*d*e^2 - a*b^3*c^3*d^2*f - 60*a^2*b*c^4*d^2*f - 28*a^3*b*c^3*d*f^2 \\
& + a^2*b^4*c*e*f^2 - 28*a^3*b*c^3*e^2*f - 9*a^2*b^3*c^2*d*f^2 - 5*a^2*b^3*c^2 \\
& *e^2*f + 18*a^3*b^2*c^2*e*f^2 - 4*a*b^4*c^2*d*e*f + 52*a^2*b^2*c^3*d*e*f)/ \\
& (4*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + (((6144*a^5*c^6*d + \\
& 2048*a^6*c^5*f - 288*a^2*b^6*c^3*d + 1920*a^3*b^4*c^4*d - 5632*a^4*b^2*c^5*d + \\
& 16*a^2*b^7*c^2*e - 192*a^3*b^5*c^3*e + 768*a^4*b^3*c^4*e - 32*a^3*b^6*c^2*f + \\
& 384*a^4*b^4*c^3*f - 1536*a^5*b^2*c^4*f + 16*a*b^8*c^2*d - 1024*a^5*b*c^5*e)/ \\
& (8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*((27*a*b^9*c^2*d^2 - \\
& a^3*b^9*f^2 - a^3*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - b^11*c*d^2 + 3840*a^5*b*c^6*d^2 - \\
& 9*a*c^2*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - a^2*b^9*c*e^2 + 768*a^6*b*c^5*e^2 + a^2*c*e^2*(- \\
& (4*a*c - b^2)^9)^{(1/2)} + b^2*c*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + 768*a^7*b*c^4*f^2 - \\
& 288*a^2*b^7*c^3*d^2 + 1504*a^3*b^5*c^4*d^2 - 3840*a^4*b^3*c^5*d^2 + 96*a^4*b^5*c^3*e^2 - \\
& 512*a^5*b^3*c^4*e^2 + 96*a^5*b^5*c^2*f^2 - 512*a^6*b^3*c^3*f^2 - 3072*a^6*c^6*d*e - \\
& 1024*a^7*c^5*e*f + 6*a^2*b^9*c*d*f + 3584*a^6*b*c^5*d*f - 6*a^2*c*d*f*(-(4*a*c - \\
& b^2)^9)^{(1/2)} + 12*a^3*b^8*c*e*f + 36*a^2*b^8*c^2*d*e - 192*a^3*b^6*c^3*d*e + \\
& 128*a^4*b^4*c^4*d*e + 1536*a^5*b^2*c^5*d*e - 128*a^3*b^7*c^2*d*f + 960*a^4*b^5*c^3*d*f - \\
& 3072*a^5*b^3*c^4*d*f - 128*a^4*b^6*c^2*e*f + 384*a^5*b^4*c^3*e*f - 2*a*b^{10}*c*d*e + \\
& 2*a*b*c*d*e*(-(4*a*c - b^2)^9)^{(1/2)}/(32*(4096*a^9*c^7 + a^3*b^{12}*c - 24*a^4*b^{10}*c^2 + \\
& 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^{(1/2)}*(1024*a^5*b*c^5 - \\
& 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) \\
& *((27*a*b^9*c^2*d^2 - a^3*b^9*f^2 - a^3*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - b^11*c*d^2 + \\
& 3840*a^5*b*c^6*d^2 - 9*a*c^2*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - a^2*b^9*c*e^2 + 768*a^6*b*c^5*e^2 + \\
& a^2*c*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + b^2*c*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + 768*a^7*b*c^4*f^2 - \\
& 288*a^2*b^7*c^3*d^2 + 1504*a^3*b^5*c^4*d^2 - 3840*a^4*b^3*c^5*d^2 + 96*a^4*b^5*c^3*e^2 - \\
& 512*a^5*b^3*c^4*e^2 + 96*a^5*b^5*c^2*f^2 - 512*a^6*b^3*c^3*f^2 - 3072*a^6*c^6*d*e - \\
& 1024*a^7*c^5*e*f + 6*a^2*b^9*c*d*f + 3584*a^6*b*c^5*d*f - 6*a^2*c*d*f*(-(4*a*c - \\
& b^2)^9)^{(1/2)} + 12*a^3*b^8*c*e*f + 36*a^2*b^8*c^2*d*e - 192*a^3*b^6*c^3*d*e + 128*a^4* \\
& b^4*c^4*d*e + 1536*a^5*b^2*c^5*d*e - 128*a^3*b^7*c^2*d*f + 960*a^4*b^5*c^3*d*f - \\
& 3072*a^5*b^3*c^4*d*f - 128*a^4*b^6*c^2*e*f + 384*a^5*b^4*c^3*e*f - 2*a*b^{10}*c*d*e + \\
& 2*a*b*c*d*e*(-(4*a*c - b^2)^9)^{(1/2)}/(32*(4096*a^9*c^7 + a^3*b^{12}*c - 24*a^4*b^{10}*c^2 + \\
& 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^{(1/2)} + (x*(72*a^2* \\
& c^5*d^2 - 8*a^3*c^4*e^2 + b^4*c^3*d^2 + 8*a^4*c^3*f^2 - 14*a*b^2*c^4*d^2 + a^2*b^4*c*f^2 + \\
& 10*a^2*b^2*c^3*e^2 + 2*a^3*b^2*c^2*f^2 + 48*a^3*c^4*d*f + 2*a*b^3*c^3*d*e - 40*a^2*b*c^4*d*e - \\
& 8*a^3*b*c^3*e*f + 4*a^2*b^2*c^3*d*f - 6*a^2*b^3*c^2*e*f))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) \\
& *((27*a*b^9*c^2*d^2 - a^3*b^9*f^2 - a^3*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - b^11*c*d^2 + 3840* \\
& a^5*b*c^6*d^2 - 9*a*c^2*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - a^2*b^9*c*e^2 + 768*a^6*b*c^5*e^2 + \\
& a^2*c*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + b^2*c*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + 768*a^7*b*c^4*f^2 - \\
& 288*a^2*b^7*c^3*d^2 + 1504*a^3*b^5*c^4*d^2 - 3840*a^4*b^3*c^5*d^2 + 96*a^4*b^5*c^3*e^2 - \\
& 512*a^5*b^3*c^4*e^2 + 96*a^5*b^5*c^2*f^2 - 512*a^6*b^3*c^3*f^2 - 3072*a^6*c^6*d*e - 1024*a^7*c^5*e*f \\
& + 6*a^2*b^9*c*d*f + 3584*a^6*b*c^5*d*f - 6*a^2*c*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 12*a^3*b^8*c* \\
& e*f + 36*a^2*b^8*c^2*d*e - 192*a^3*b^6*c^3*d*e + 128*a^4*b^4*c^4*d*e + 1536*a^5*b^2*c^5*d*e - \\
& 128*a^3*b^7*c^2*d*f + 960*a^4*b^5*c^3*d*f - 3072*a^5*b^3*c^4*d*f - 128*a^4*b^6*c^2*e*f + 384*a^5*b^4*c^3* \\
& e*f - 2*a*b^{10}*c*d*e + 2*a*b*c*d*e*(-(4*a*c - b^2)^9)^{(1/2)}/(32*(4096*a^9*c^7 + a^3*b^{12}*c - \\
& 24*a^4*b^{10}*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^{(1/2)} + \\
& (((6144*a^5*c^6*d + 2048*a^6*c^5*f - 288*a^2*b^6*c^3*d + 1920*a^3*b^4*c^4*d - 5632*a^4*b^2*c^5*d + 16*a^2*b^7*c
\end{aligned}$$

$$\begin{aligned}
&^2e - 192a^3b^5c^3e + 768a^4b^3c^4e - 32a^3b^6c^2f + 384a^4b^4c^3f - 1536a^5b^2c^4f + 16ab^8c^2d - 1024a^5b^3c^5e)/(8(a^2b^6 - 64a^5c^3 - 12a^3b^4c + 48a^4b^2c^2)) + (x((27ab^9c^2d^2 - a^3b^9f^2 - a^3f^2(-4ac - b^2)^9)^{(1/2)} - b^{11}cd^2 + 3840a^5b^3c^6d^2 - 9a^2c^2d^2(-4ac - b^2)^9)^{(1/2)} - a^2b^9c^2e^2 + 768a^6b^3c^5e^2 + a^2c^2e^2(-4ac - b^2)^9)^{(1/2)} + b^2cd^2(-4ac - b^2)^9)^{(1/2)} + 768a^7b^3c^4f^2 - 288a^2b^7c^3d^2 + 1504a^3b^5c^4d^2 - 3840a^4b^3c^5d^2 + 96a^4b^5c^3e^2 - 512a^5b^3c^4e^2 + 96a^5b^5c^2f^2 - 512a^6b^3c^3f^2 - 3072a^6c^6d^2e - 1024a^7c^5e^2f + 6a^2b^9c^2d^2 + 3584a^6b^3c^5d^2f - 6a^2c^2d^2f(-4ac - b^2)^9)^{(1/2)} + 12a^3b^8c^2d^2e - 192a^3b^6c^3d^2e + 128a^4b^4c^4d^2e + 1536a^5b^2c^5d^2e - 128a^3b^7c^2d^2f + 960a^4b^5c^3d^2f - 3072a^5b^3c^4d^2f - 128a^4b^6c^2e^2f + 384a^5b^4c^3e^2f - 2ab^{10}c^2d^2e + 2ab^8c^2d^2e(-4ac - b^2)^9)^{(1/2)})/(32(4096a^9c^7 + a^3b^{12}c - 24a^4b^{10}c^2 + 240a^5b^8c^3 - 1280a^6b^6c^4 + 3840a^7b^4c^5 - 6144a^8b^2c^6))^{(1/2)}(1024a^5b^3c^5 - 16a^2b^7c^2 + 192a^3b^5c^3 - 768a^4b^3c^4))/(2(a^2b^4 + 16a^4c^2 - 8a^3b^2c))((27ab^9c^2d^2 - a^3b^9f^2 - a^3f^2(-4ac - b^2)^9)^{(1/2)} - b^{11}cd^2 + 3840a^5b^3c^6d^2 - 9a^2c^2d^2(-4ac - b^2)^9)^{(1/2)} - a^2b^9c^2e^2 + 768a^6b^3c^5e^2 + a^2c^2e^2(-4ac - b^2)^9)^{(1/2)} + b^2cd^2(-4ac - b^2)^9)^{(1/2)} + 768a^7b^3c^4f^2 - 288a^2b^7c^3d^2 + 1504a^3b^5c^4d^2 - 3840a^4b^3c^5d^2 + 96a^4b^5c^3e^2 - 512a^5b^3c^4e^2 + 96a^5b^5c^2f^2 - 512a^6b^3c^3f^2 - 3072a^6c^6d^2e - 1024a^7c^5e^2f + 6a^2b^9c^2d^2 + 3584a^6b^3c^5d^2f - 6a^2c^2d^2f(-4ac - b^2)^9)^{(1/2)} + 12a^3b^8c^2d^2e - 192a^3b^6c^3d^2e + 128a^4b^4c^4d^2e + 1536a^5b^2c^5d^2e - 128a^3b^7c^2d^2f + 960a^4b^5c^3d^2f - 3072a^5b^3c^4d^2f - 128a^4b^6c^2e^2f + 384a^5b^4c^3e^2f - 2ab^{10}c^2d^2e + 2ab^8c^2d^2e(-4ac - b^2)^9)^{(1/2)})/(32(4096a^9c^7 + a^3b^{12}c - 24a^4b^{10}c^2 + 240a^5b^8c^3 - 1280a^6b^6c^4 + 3840a^7b^4c^5 - 6144a^8b^2c^6))^{(1/2)} - (x(72a^2c^5d^2 - 8a^3c^4e^2 + b^4c^3d^2 + 8a^4c^3f^2 - 14ab^2c^4d^2 + a^2b^4c^2f^2 + 10a^2b^2c^3e^2 + 2a^3b^2c^2f^2 + 48a^3c^4d^2f + 2ab^3c^3d^2e - 40a^2b^4c^2d^2e - 8a^3b^3c^3e^2f + 4a^2b^2c^3d^2f - 6a^2b^3c^2e^2f)))/(2(a^2b^4 + 16a^4c^2 - 8a^3b^2c))((27ab^9c^2d^2 - a^3b^9f^2 - a^3f^2(-4ac - b^2)^9)^{(1/2)} - b^{11}cd^2 + 3840a^5b^3c^6d^2 - 9a^2c^2d^2(-4ac - b^2)^9)^{(1/2)} - a^2b^9c^2e^2 + 768a^6b^3c^5e^2 + a^2c^2e^2(-4ac - b^2)^9)^{(1/2)} + b^2cd^2(-4ac - b^2)^9)^{(1/2)} + 768a^7b^3c^4f^2 - 288a^2b^7c^3d^2 + 1504a^3b^5c^4d^2 - 3840a^4b^3c^5d^2 + 96a^4b^5c^3e^2 - 512a^5b^3c^4e^2 + 96a^5b^5c^2f^2 - 512a^6b^3c^3f^2 - 3072a^6c^6d^2e - 1024a^7c^5e^2f + 6a^2b^9c^2d^2 + 3584a^6b^3c^5d^2f - 6a^2c^2d^2f(-4ac - b^2)^9)^{(1/2)} + 12a^3b^8c^2d^2e - 192a^3b^6c^3d^2e + 128a^4b^4c^4d^2e + 1536a^5b^2c^5d^2e - 128a^3b^7c^2d^2f + 960a^4b^5c^3d^2f - 3072a^5b^3c^4d^2f - 128a^4b^6c^2e^2f + 384a^5b^4c^3e^2f - 2ab^{10}c^2d^2e + 2ab^8c^2d^2e(-4ac - b^2)^9)^{(1/2)})/(32(4096a^9c^7 + a^3b^{12}c - 24a^4b^{10}c^2 + 240a^5b^8c^3 - 1280a^6b^6c^4 + 3840a^7b^4c^5 - 6144a^8b^2c^6))^{(1/2)}*2i - ((x*
\end{aligned}$$

$$\begin{aligned}
& (b^2*d + 2*a^2*f - a*b*e - 2*a*c*d)/(2*a*(4*a*c - b^2)) + (x^3*(a*b*f - 2* \\
& a*c*e + b*c*d))/(2*a*(4*a*c - b^2))/(a + b*x^2 + c*x^4) + \operatorname{atan}\left(\frac{(6144*a^5*c^6*d + 2048*a^6*c^5*f - 288*a^2*b^6*c^3*d + 1920*a^3*b^4*c^4*d - 5632*a^4*b^2*c^5*d + 16*a^2*b^7*c^2*e - 192*a^3*b^5*c^3*e + 768*a^4*b^3*c^4*e - 32*a^3*b^6*c^2*f + 384*a^4*b^4*c^3*f - 1536*a^5*b^2*c^4*f + 16*a*b^8*c^2*d - 1024*a^5*b*c^5*e)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2))}{(x*((a^3*f^2*(-(4*a*c - b^2)^9)^{1/2} - a^3*b^9*f^2 - b^{11}*c*d^2 + 27*a*b^9*c^2*d^2 + 3840*a^5*b*c^6*d^2 + 9*a*c^2*d^2*(-(4*a*c - b^2)^9)^{1/2} - a^2*b^9*c*e^2 + 768*a^6*b*c^5*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^9)^{1/2} - b^2*c*d^2*(-(4*a*c - b^2)^9)^{1/2} + 768*a^7*b*c^4*f^2 - 288*a^2*b^7*c^3*d^2 + 1504*a^3*b^5*c^4*d^2 - 3840*a^4*b^3*c^5*d^2 + 96*a^4*b^5*c^3*e^2 - 512*a^5*b^3*c^4*e^2 + 96*a^5*b^5*c^2*f^2 - 512*a^6*b^3*c^3*f^2 - 3072*a^6*c^6*d*e - 1024*a^7*c^5*e*f + 6*a^2*b^9*c*d*f + 3584*a^6*b*c^5*d*f + 6*a^2*c*d*f*(-(4*a*c - b^2)^9)^{1/2} + 12*a^3*b^8*c*e*f + 36*a^2*b^8*c^2*d*e - 192*a^3*b^6*c^3*d*e + 128*a^4*b^4*c^4*d*e + 1536*a^5*b^2*c^5*d*e - 128*a^3*b^7*c^2*d*f + 960*a^4*b^5*c^3*d*f - 3072*a^5*b^3*c^4*d*f - 128*a^4*b^6*c^2*e*f + 384*a^5*b^4*c^3*e*f - 2*a*b^10*c*d*e - 2*a*b*c*d*e*(-(4*a*c - b^2)^9)^{1/2})/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6))^{1/2}*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c))\right) * ((a^3*f^2*(-(4*a*c - b^2)^9)^{1/2} - a^3*b^9*f^2 - b^{11}*c*d^2 + 27*a*b^9*c^2*d^2 + 3840*a^5*b*c^6*d^2 + 9*a*c^2*d^2*(-(4*a*c - b^2)^9)^{1/2} - a^2*b^9*c*e^2 + 768*a^6*b*c^5*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^9)^{1/2} - b^2*c*d^2*(-(4*a*c - b^2)^9)^{1/2} + 768*a^7*b*c^4*f^2 - 288*a^2*b^7*c^3*d^2 + 1504*a^3*b^5*c^4*d^2 - 3840*a^4*b^3*c^5*d^2 + 96*a^4*b^5*c^3*e^2 - 512*a^5*b^3*c^4*e^2 + 96*a^5*b^5*c^2*f^2 - 512*a^6*b^3*c^3*f^2 - 3072*a^6*c^6*d*e - 1024*a^7*c^5*e*f + 6*a^2*b^9*c*d*f + 3584*a^6*b*c^5*d*f + 6*a^2*c*d*f*(-(4*a*c - b^2)^9)^{1/2} + 12*a^3*b^8*c*e*f + 36*a^2*b^8*c^2*d*e - 192*a^3*b^6*c^3*d*e + 128*a^4*b^4*c^4*d*e + 1536*a^5*b^2*c^5*d*e - 128*a^3*b^7*c^2*d*f + 960*a^4*b^5*c^3*d*f - 3072*a^5*b^3*c^4*d*f - 128*a^4*b^6*c^2*e*f + 384*a^5*b^4*c^3*e*f - 2*a*b^10*c*d*e - 2*a*b*c*d*e*(-(4*a*c - b^2)^9)^{1/2})/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6))^{1/2} + (x*(72*a^2*c^5*d^2 - 8*a^3*c^4*e^2 + b^4*c^3*d^2 + 8*a^4*c^3*f^2 - 14*a*b^2*c^4*d^2 + a^2*b^4*c*f^2 + 10*a^2*b^2*c^3*e^2 + 2*a^3*b^2*c^2*f^2 + 48*a^3*c^4*d*f + 2*a*b^3*c^3*d*e - 40*a^2*b*c^4*d*e - 8*a^3*b*c^3*e*f + 4*a^2*b^2*c^3*d*f - 6*a^2*b^3*c^2*e*f))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c))\right) * ((a^3*f^2*(-(4*a*c - b^2)^9)^{1/2} - a^3*b^9*f^2 - b^{11}*c*d^2 + 27*a*b^9*c^2*d^2 + 3840*a^5*b*c^6*d^2 + 9*a*c^2*d^2*(-(4*a*c - b^2)^9)^{1/2} - a^2*b^9*c*e^2 + 768*a^6*b*c^5*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^9)^{1/2} - b^2*c*d^2*(-(4*a*c - b^2)^9)^{1/2} + 768*a^7*b*c^4*f^2 - 288*a^2*b^7*c^3*d^2 + 1504*a^3*b^5*c^4*d^2 - 3840*a^4*b^3*c^5*d^2 + 96*a^4*b^5*c^3*e^2 - 512*a^5*b^3*c^4*e^2 + 96*a^5*b^5*c^2*f^2 - 512*a^6*b^3*c^3*f^2 - 3072*a^6*c^6*d*e - 1024*a^7*c^5*e*f + 6*a^2*b^9*c*d*f + 3584*a^6*b*c^5*d*f + 6*a^2*c*d*f*(-(4*a*c - b^2)^9)^{1/2} + 12*a^3*b^8*c*e*f + 36*a^2*b^8*c^2*d*e - 192*a^3*b^6*c^3*d*e + 128*a^4*b^4*c^4*d*e + 1536*a^5*b^2*c^5*d*e - 128*a^3*b^7*c^2*d*f + 960*a^4*b^5*c^3*d*f - 3072*a^5*b^3*c^4*d*f - 128*a^4*b^6*c^2*e*f + 384*a^5*b^4*c^3*e*f - 2*a*b^10*c*d*e - 2*a*b*c*d*e*(-(4*a*c - b^2)^9)^{1/2})/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6))^{1/2} * i - \left(\frac{(6144*a^5*c^6*d + 2048*a^6*c^5*f - 288*a^2*b^6*c^3*d + 1920*a^3*b^4*c^4*d - 5632*a^4*b^2*c^5*d + 16*a^2*b^7*c^2*e - 192*a^3*b^5*c^3*e + 768*a^4*b^3*c^4*e - 32*a^3*b^6*c^2*f + 384*a^4*b^4*c^3*f - 1536*a^5*b^2*c^4*f + 16*a*b^8*c^2*d - 1024*a^5*b*c^5*e)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2))}{(x*((a^3*f^2*(-(4*a*c - b^2)^9)^{1/2} - a^3*b^9*f^2 - b^{11}*c*d^2 + 27*a*b^9*c^2*d^2 + 3840*a^5*b*c^6*d^2 + 9*a*c^2*d^2*(-(4*a*c - b^2)^9)^{1/2} - a^2*b^9*c*e^2 + 768*a^6*b*c^5*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^9)^{1/2} - b^2*c*d^2*(-(4*a*c - b^2)^9)^{1/2} + 768*a^7*b*c^4*f^2 - 288*a^2*b^7*c^3*d^2 + 1504*a^3*b^5*c^4*d^2 - 3840*a^4*b^3*c^5*d^2 + 96*a^4*b^5*c^3*e^2 - 512*a^5*b^3*c^4*e^2 + 96*
\end{aligned}$$

$$\begin{aligned}
& a^5 b^5 c^2 f^2 - 512 a^6 b^3 c^3 f^2 - 3072 a^6 c^6 d e - 1024 a^7 c^5 e f \\
& + 6 a^2 b^9 c d f + 3584 a^6 b^3 c^5 d f + 6 a^2 c^2 d f (-4 a c - b^2)^9)^{(1/2)} \\
& + 12 a^3 b^8 c e f + 36 a^2 b^8 c^2 d e - 192 a^3 b^6 c^3 d e + 128 a^4 \\
& b^4 c^4 d e + 1536 a^5 b^2 c^5 d e - 128 a^3 b^7 c^2 d f + 960 a^4 b^5 c^3 \\
& d f - 3072 a^5 b^3 c^4 d f - 128 a^4 b^6 c^2 e f + 384 a^5 b^4 c^3 e f - 2 \\
& a b^{10} c d e - 2 a b^2 c^2 d e (-4 a c - b^2)^9)^{(1/2)} / (32 (4096 a^9 c^7 + a \\
& ^3 b^{12} c - 24 a^4 b^{10} c^2 + 240 a^5 b^8 c^3 - 1280 a^6 b^6 c^4 + 3840 a^7 \\
& b^4 c^5 - 6144 a^8 b^2 c^6))^{(1/2)} * (1024 a^5 b^3 c^5 - 16 a^2 b^7 c^2 + 192 \\
& a^3 b^5 c^3 - 768 a^4 b^3 c^4) / (2 (a^2 b^4 + 16 a^4 c^2 - 8 a^3 b^2 c)) * \\
& ((a^3 f^2 (-4 a c - b^2)^9)^{(1/2)} - a^3 b^9 f^2 - b^{11} c d^2 + 27 a b^9 c^2 \\
& d^2 + 3840 a^5 b^3 c^6 d^2 + 9 a c^2 d^2 (-4 a c - b^2)^9)^{(1/2)} - a^2 b^9 \\
& c e^2 + 768 a^6 b^3 c^5 e^2 - a^2 c e^2 (-4 a c - b^2)^9)^{(1/2)} - b^2 c d^2 \\
& (-4 a c - b^2)^9)^{(1/2)} + 768 a^7 b^3 c^4 f^2 - 288 a^2 b^7 c^3 d^2 + 1504 a^3 \\
& b^5 c^4 d^2 - 3840 a^4 b^3 c^5 d^2 + 96 a^4 b^5 c^3 e^2 - 512 a^5 b^3 c^4 \\
& e^2 + 96 a^5 b^5 c^2 f^2 - 512 a^6 b^3 c^3 f^2 - 3072 a^6 c^6 d e - 1024 \\
& a^7 c^5 e f + 6 a^2 b^9 c d f + 3584 a^6 b^3 c^5 d f + 6 a^2 c^2 d f (-4 a c \\
& - b^2)^9)^{(1/2)} + 12 a^3 b^8 c e f + 36 a^2 b^8 c^2 d e - 192 a^3 b^6 c^3 d \\
& e + 128 a^4 b^4 c^4 d e + 1536 a^5 b^2 c^5 d e - 128 a^3 b^7 c^2 d f + 960 \\
& a^4 b^5 c^3 d f - 3072 a^5 b^3 c^4 d f - 128 a^4 b^6 c^2 e f + 384 a^5 b^4 \\
& c^3 e f - 2 a b^{10} c d e - 2 a b^2 c^2 d e (-4 a c - b^2)^9)^{(1/2)} / (32 (4096 \\
& a^9 c^7 + a^3 b^{12} c - 24 a^4 b^{10} c^2 + 240 a^5 b^8 c^3 - 1280 a^6 b^6 c^4 \\
& + 3840 a^7 b^4 c^5 - 6144 a^8 b^2 c^6))^{(1/2)} - (x (72 a^2 c^5 d^2 - 8 a^3 \\
& c^4 e^2 + b^4 c^3 d^2 + 8 a^4 c^3 f^2 - 14 a b^2 c^4 d^2 + a^2 b^4 c^3 f^2 \\
& + 10 a^2 b^2 c^3 e^2 + 2 a^3 b^2 c^2 f^2 + 48 a^3 c^4 d f + 2 a b^3 c^3 d e \\
& - 40 a^2 b^3 c^4 d e - 8 a^3 b^3 c^3 e f + 4 a^2 b^2 c^3 d f - 6 a^2 b^3 c^2 e \\
& f)) / (2 (a^2 b^4 + 16 a^4 c^2 - 8 a^3 b^2 c)) * ((a^3 f^2 (-4 a c - b^2)^9)^{(1/2)} \\
& - a^3 b^9 f^2 - b^{11} c d^2 + 27 a b^9 c^2 d^2 + 3840 a^5 b^3 c^6 d^2 \\
& + 9 a c^2 d^2 (-4 a c - b^2)^9)^{(1/2)} - a^2 b^9 c e^2 + 768 a^6 b^3 c^5 e^2 \\
& - a^2 c e^2 (-4 a c - b^2)^9)^{(1/2)} - b^2 c d^2 (-4 a c - b^2)^9)^{(1/2)} + \\
& 768 a^7 b^3 c^4 f^2 - 288 a^2 b^7 c^3 d^2 + 1504 a^3 b^5 c^4 d^2 - 3840 a^4 b^3 \\
& c^5 d^2 + 96 a^4 b^5 c^3 e^2 - 512 a^5 b^3 c^4 e^2 + 96 a^5 b^5 c^2 f^2 \\
& - 512 a^6 b^3 c^3 f^2 - 3072 a^6 c^6 d e - 1024 a^7 c^5 e f + 6 a^2 b^9 c \\
& d f + 3584 a^6 b^3 c^5 d f + 6 a^2 c^2 d f (-4 a c - b^2)^9)^{(1/2)} + 12 a^3 b^8 \\
& c e f + 36 a^2 b^8 c^2 d e - 192 a^3 b^6 c^3 d e + 128 a^4 b^4 c^4 d e + \\
& 1536 a^5 b^2 c^5 d e - 128 a^3 b^7 c^2 d f + 960 a^4 b^5 c^3 d f - 3072 a^5 \\
& b^3 c^4 d f - 128 a^4 b^6 c^2 e f + 384 a^5 b^4 c^3 e f - 2 a b^{10} c d e - \\
& 2 a b^2 c^2 d e (-4 a c - b^2)^9)^{(1/2)} / (32 (4096 a^9 c^7 + a^3 b^{12} c - 24 a^4 \\
& b^{10} c^2 + 240 a^5 b^8 c^3 - 1280 a^6 b^6 c^4 + 3840 a^7 b^4 c^5 - 6144 \\
& a^8 b^2 c^6))^{(1/2)} * i) / ((8 a^3 c^4 e^3 + 5 b^3 c^4 d^3 - 3 a^3 b^3 c^3 f^3 \\
& - 4 a^4 b^3 c^2 f^3 + 72 a^2 c^5 d^2 e - 3 b^4 c^3 d^2 e + 8 a^4 c^3 e f^2 + \\
& b^5 c^2 d^2 f + 6 a^2 b^2 c^3 e^3 - 36 a b^3 c^5 d^3 + a b^5 c^3 d f^2 + 48 a^3 \\
& c^4 d e f + 18 a b^2 c^4 d^2 e + 3 a b^3 c^3 d e^2 - 60 a^2 b^3 c^4 d e^2 - \\
& a b^3 c^3 d^2 f - 60 a^2 b^3 c^4 d^2 f - 28 a^3 b^3 c^3 d f^2 + a^2 b^4 c^3 e f^2 \\
& - 28 a^3 b^3 c^3 e^2 f - 9 a^2 b^3 c^2 d f^2 - 5 a^2 b^3 c^2 e^2 f + 18 a^3 \\
& b^2 c^2 e f^2 - 4 a b^4 c^2 d e f + 52 a^2 b^2 c^3 d e f) / (4 (a^2 b^6 - 64 \\
& a^5 c^3 - 12 a^3 b^4 c + 48 a^4 b^2 c^2)) + (((6144 a^5 c^6 d + 2048 a^6 c^5 \\
& f - 288 a^2 b^6 c^3 d + 1920 a^3 b^4 c^4 d - 5632 a^4 b^2 c^5 d + 16 a^2 \\
& b^7 c^2 e - 192 a^3 b^5 c^3 e + 768 a^4 b^3 c^4 e - 32 a^3 b^6 c^2 f + 384 \\
& a^4 b^4 c^3 f - 1536 a^5 b^2 c^4 f + 16 a b^8 c^2 d - 1024 a^5 b^3 c^5 e) / (8 \\
& (a^2 b^6 - 64 a^5 c^3 - 12 a^3 b^4 c + 48 a^4 b^2 c^2)) - (x ((a^3 f^2 (-4 \\
& a c - b^2)^9)^{(1/2)} - a^3 b^9 f^2 - b^{11} c d^2 + 27 a b^9 c^2 d^2 + 3840 \\
& a^5 b^3 c^6 d^2 + 9 a c^2 d^2 (-4 a c - b^2)^9)^{(1/2)} - a^2 b^9 c e^2 + 768 \\
& a^6 b^3 c^5 e^2 - a^2 c e^2 (-4 a c - b^2)^9)^{(1/2)} - b^2 c d^2 (-4 a c - b^2)^9)^{(1/2)} \\
& + 768 a^7 b^3 c^4 f^2 - 288 a^2 b^7 c^3 d^2 + 1504 a^3 b^5 c^4 d^2 - 3840 a^4 \\
& b^3 c^5 d^2 + 96 a^4 b^5 c^3 e^2 - 512 a^5 b^3 c^4 e^2 + 96 a^5 b^5 c^2 f^2 - \\
& 512 a^6 b^3 c^3 f^2 - 3072 a^6 c^6 d e - 1024 a^7 c^5 e f + 6 a^2 b^9 c d f + \\
& 3584 a^6 b^3 c^5 d f + 6 a^2 c^2 d f (-4 a c - b^2)^9)^{(1/2)} + 12 a^3 b^8 c e f \\
& + 36 a^2 b^8 c^2 d e - 192 a^3 b^6 c^3 d e + 128 a^4 b^4 c^4 d e + 1536 a^5 b^2 c^5 \\
& d e - 128 a^3 b^7 c^2 d f + 960 a^4 b^5 c^3 d f - 3072 a^5 b^3 c^4 d f + 960 a^4 b^5 c^3
\end{aligned}$$

$$\begin{aligned}
& d*f - 3072*a^5*b^3*c^4*d*f - 128*a^4*b^6*c^2*e*f + 384*a^5*b^4*c^3*e*f - 2* \\
& a*b^{10}*c*d*e - 2*a*b*c*d*e*(-(4*a*c - b^2)^9)^{(1/2)}/(32*(4096*a^9*c^7 + a^ \\
& 3*b^{12}*c - 24*a^4*b^{10}*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7* \\
& b^4*c^5 - 6144*a^8*b^2*c^6)))^{(1/2)}*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192* \\
& a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(\\
& (a^3*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - a^3*b^9*f^2 - b^{11}*c*d^2 + 27*a*b^9*c^2 \\
& *d^2 + 3840*a^5*b*c^6*d^2 + 9*a*c^2*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - a^2*b^9* \\
& c*e^2 + 768*a^6*b*c^5*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - b^2*c*d^2* \\
& (- (4*a*c - b^2)^9)^{(1/2)} + 768*a^7*b*c^4*f^2 - 288*a^2*b^7*c^3*d^2 + 1504*a \\
& ^3*b^5*c^4*d^2 - 3840*a^4*b^3*c^5*d^2 + 96*a^4*b^5*c^3*e^2 - 512*a^5*b^3*c^4 \\
& *e^2 + 96*a^5*b^5*c^2*f^2 - 512*a^6*b^3*c^3*f^2 - 3072*a^6*c^6*d*e - 1024* \\
& a^7*c^5*e*f + 6*a^2*b^9*c*d*f + 3584*a^6*b*c^5*d*f + 6*a^2*c*d*f*(-(4*a*c - \\
& b^2)^9)^{(1/2)} + 12*a^3*b^8*c*e*f + 36*a^2*b^8*c^2*d*e - 192*a^3*b^6*c^3*d* \\
& e + 128*a^4*b^4*c^4*d*e + 1536*a^5*b^2*c^5*d*e - 128*a^3*b^7*c^2*d*f + 960* \\
& a^4*b^5*c^3*d*f - 3072*a^5*b^3*c^4*d*f - 128*a^4*b^6*c^2*e*f + 384*a^5*b^4* \\
& c^3*e*f - 2*a*b^{10}*c*d*e - 2*a*b*c*d*e*(-(4*a*c - b^2)^9)^{(1/2)}/(32*(4096* \\
& a^9*c^7 + a^3*b^{12}*c - 24*a^4*b^{10}*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 \\
& + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^{(1/2)} + (x*(72*a^2*c^5*d^2 - 8*a^ \\
& 3*c^4*e^2 + b^4*c^3*d^2 + 8*a^4*c^3*f^2 - 14*a*b^2*c^4*d^2 + a^2*b^4*c*f^2 \\
& + 10*a^2*b^2*c^3*e^2 + 2*a^3*b^2*c^2*f^2 + 48*a^3*c^4*d*f + 2*a*b^3*c^3*d*e \\
& - 40*a^2*b*c^4*d*e - 8*a^3*b*c^3*e*f + 4*a^2*b^2*c^3*d*f - 6*a^2*b^3*c^2*e \\
& *f))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((a^3*f^2*(-(4*a*c - b^2)^9) \\
& ^{(1/2)} - a^3*b^9*f^2 - b^{11}*c*d^2 + 27*a*b^9*c^2*d^2 + 3840*a^5*b*c^6*d^2 + \\
& 9*a*c^2*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - a^2*b^9*c*e^2 + 768*a^6*b*c^5*e^2 - \\
& a^2*c*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - b^2*c*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + \\
& 768*a^7*b*c^4*f^2 - 288*a^2*b^7*c^3*d^2 + 1504*a^3*b^5*c^4*d^2 - 3840*a^4*b \\
& ^3*c^5*d^2 + 96*a^4*b^5*c^3*e^2 - 512*a^5*b^3*c^4*e^2 + 96*a^5*b^5*c^2*f^2 \\
& - 512*a^6*b^3*c^3*f^2 - 3072*a^6*c^6*d*e - 1024*a^7*c^5*e*f + 6*a^2*b^9*c*d \\
& *f + 3584*a^6*b*c^5*d*f + 6*a^2*c*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 12*a^3*b^8 \\
& *c*e*f + 36*a^2*b^8*c^2*d*e - 192*a^3*b^6*c^3*d*e + 128*a^4*b^4*c^4*d*e + 1 \\
& 536*a^5*b^2*c^5*d*e - 128*a^3*b^7*c^2*d*f + 960*a^4*b^5*c^3*d*f - 3072*a^5* \\
& b^3*c^4*d*f - 128*a^4*b^6*c^2*e*f + 384*a^5*b^4*c^3*e*f - 2*a*b^{10}*c*d*e - \\
& 2*a*b*c*d*e*(-(4*a*c - b^2)^9)^{(1/2)}/(32*(4096*a^9*c^7 + a^3*b^{12}*c - 24*a \\
& ^4*b^{10}*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144* \\
& a^8*b^2*c^6)))^{(1/2)} + (((6144*a^5*c^6*d + 2048*a^6*c^5*f - 288*a^2*b^6*c^3 \\
& *d + 1920*a^3*b^4*c^4*d - 5632*a^4*b^2*c^5*d + 16*a^2*b^7*c^2*e - 192*a^3*b \\
& ^5*c^3*e + 768*a^4*b^3*c^4*e - 32*a^3*b^6*c^2*f + 384*a^4*b^4*c^3*f - 1536* \\
& a^5*b^2*c^4*f + 16*a*b^8*c^2*d - 1024*a^5*b*c^5*e))/(8*(a^2*b^6 - 64*a^5*c^3 \\
& - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + (x*((a^3*f^2*(-(4*a*c - b^2)^9)^{(1/2)} \\
& - a^3*b^9*f^2 - b^{11}*c*d^2 + 27*a*b^9*c^2*d^2 + 3840*a^5*b*c^6*d^2 + 9*a*c^ \\
& 2*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - a^2*b^9*c*e^2 + 768*a^6*b*c^5*e^2 - a^2*c* \\
& e^2*(-(4*a*c - b^2)^9)^{(1/2)} - b^2*c*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + 768*a^7 \\
& *b*c^4*f^2 - 288*a^2*b^7*c^3*d^2 + 1504*a^3*b^5*c^4*d^2 - 3840*a^4*b^3*c^5* \\
& d^2 + 96*a^4*b^5*c^3*e^2 - 512*a^5*b^3*c^4*e^2 + 96*a^5*b^5*c^2*f^2 - 512*a \\
& ^6*b^3*c^3*f^2 - 3072*a^6*c^6*d*e - 1024*a^7*c^5*e*f + 6*a^2*b^9*c*d*f + 35 \\
& 84*a^6*b*c^5*d*f + 6*a^2*c*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 12*a^3*b^8*c*e*f \\
& + 36*a^2*b^8*c^2*d*e - 192*a^3*b^6*c^3*d*e + 128*a^4*b^4*c^4*d*e + 1536*a^5 \\
& *b^2*c^5*d*e - 128*a^3*b^7*c^2*d*f + 960*a^4*b^5*c^3*d*f - 3072*a^5*b^3*c^4 \\
& *d*f - 128*a^4*b^6*c^2*e*f + 384*a^5*b^4*c^3*e*f - 2*a*b^{10}*c*d*e - 2*a*b*c \\
& *d*e*(-(4*a*c - b^2)^9)^{(1/2)}/(32*(4096*a^9*c^7 + a^3*b^{12}*c - 24*a^4*b^{10} \\
& *c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2 \\
& *c^6)))^{(1/2)}*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4* \\
& b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((a^3*f^2*(-(4*a*c - b^ \\
& 2)^9)^{(1/2)} - a^3*b^9*f^2 - b^{11}*c*d^2 + 27*a*b^9*c^2*d^2 + 3840*a^5*b*c^6* \\
& d^2 + 9*a*c^2*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - a^2*b^9*c*e^2 + 768*a^6*b*c^5* \\
& e^2 - a^2*c*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - b^2*c*d^2*(-(4*a*c - b^2)^9)^{(1/ \\
& 2)} + 768*a^7*b*c^4*f^2 - 288*a^2*b^7*c^3*d^2 + 1504*a^3*b^5*c^4*d^2 - 3840* \\
& a^4*b^3*c^5*d^2 + 96*a^4*b^5*c^3*e^2 - 512*a^5*b^3*c^4*e^2 + 96*a^5*b^5*c^2 \\
& *f^2 - 512*a^6*b^3*c^3*f^2 - 3072*a^6*c^6*d*e - 1024*a^7*c^5*e*f + 6*a^2*b^
\end{aligned}$$

$$\begin{aligned}
& 9*c*d*f + 3584*a^6*b*c^5*d*f + 6*a^2*c*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 12*a^3*b^8*c*e*f + 36*a^2*b^8*c^2*d*e - 192*a^3*b^6*c^3*d*e + 128*a^4*b^4*c^4*d*e \\
& + 1536*a^5*b^2*c^5*d*e - 128*a^3*b^7*c^2*d*f + 960*a^4*b^5*c^3*d*f - 3072*a^5*b^3*c^4*d*f - 128*a^4*b^6*c^2*e*f + 384*a^5*b^4*c^3*e*f - 2*a*b^10*c*d \\
& *e - 2*a*b*c*d*e*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - \\
& 6144*a^8*b^2*c^6)))^{(1/2)} - (x*(72*a^2*c^5*d^2 - 8*a^3*c^4*e^2 + b^4*c^3*d^2 + 8*a^4*c^3*f^2 - 14*a*b^2*c^4*d^2 + a^2*b^4*c*f^2 + 10*a^2*b^2*c^3*e^2 + \\
& 2*a^3*b^2*c^2*f^2 + 48*a^3*c^4*d*f + 2*a*b^3*c^3*d*e - 40*a^2*b*c^4*d*e - 8*a^3*b*c^3*e*f + 4*a^2*b^2*c^3*d*f - 6*a^2*b^3*c^2*e*f))/(2*(a^2*b^4 + 16* \\
& a^4*c^2 - 8*a^3*b^2*c)))*((a^3*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - a^3*b^9*f^2 - b^11*c*d^2 + 27*a*b^9*c^2*d^2 + 3840*a^5*b*c^6*d^2 + 9*a*c^2*d^2*(-(4*a*c \\
& - b^2)^9)^{(1/2)} - a^2*b^9*c*e^2 + 768*a^6*b*c^5*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - b^2*c*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + 768*a^7*b*c^4*f^2 - 28 \\
& 8*a^2*b^7*c^3*d^2 + 1504*a^3*b^5*c^4*d^2 - 3840*a^4*b^3*c^5*d^2 + 96*a^4*b^5*c^3*e^2 - 512*a^5*b^3*c^4*e^2 + 96*a^5*b^5*c^2*f^2 - 512*a^6*b^3*c^3*f^2 \\
& - 3072*a^6*c^6*d*e - 1024*a^7*c^5*e*f + 6*a^2*b^9*c*d*f + 3584*a^6*b*c^5*d*f + 6*a^2*c*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 12*a^3*b^8*c*e*f + 36*a^2*b^8*c^2 \\
& *d*e - 192*a^3*b^6*c^3*d*e + 128*a^4*b^4*c^4*d*e + 1536*a^5*b^2*c^5*d*e - 128*a^3*b^7*c^2*d*f + 960*a^4*b^5*c^3*d*f - 3072*a^5*b^3*c^4*d*f - 128*a^4*b^6 \\
& *c^2*e*f + 384*a^5*b^4*c^3*e*f - 2*a*b^10*c*d*e - 2*a*b*c*d*e*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^{(1/2)})) \\
& *(((a^3*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - a^3*b^9*f^2 - b^11*c*d^2 + 27*a*b^9*c^2*d^2 + 3840*a^5*b*c^6*d^2 + 9*a*c^2*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - a^2*b^9 \\
& *c*e^2 + 768*a^6*b*c^5*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - b^2*c*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + 768*a^7*b*c^4*f^2 - 288*a^2*b^7*c^3*d^2 + 1504 \\
& *a^3*b^5*c^4*d^2 - 3840*a^4*b^3*c^5*d^2 + 96*a^4*b^5*c^3*e^2 - 512*a^5*b^3*c^4*e^2 + 96*a^5*b^5*c^2*f^2 - 512*a^6*b^3*c^3*f^2 - 3072*a^6*c^6*d*e - 102 \\
& 4*a^7*c^5*e*f + 6*a^2*b^9*c*d*f + 3584*a^6*b*c^5*d*f + 6*a^2*c*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 12*a^3*b^8*c*e*f + 36*a^2*b^8*c^2*d*e - 192*a^3*b^6*c^3* \\
& d*e + 128*a^4*b^4*c^4*d*e + 1536*a^5*b^2*c^5*d*e - 128*a^3*b^7*c^2*d*f + 960*a^4*b^5*c^3*d*f - 3072*a^5*b^3*c^4*d*f - 128*a^4*b^6*c^2*e*f + 384*a^5*b^4 \\
& *c^3*e*f - 2*a*b^10*c*d*e - 2*a*b*c*d*e*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^{(1/2)}*2i
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)

[Out] Timed out

$$3.70 \quad \int \frac{d+ex^2+fx^4}{x^2(a+bx^2+cx^4)^2} dx$$

Optimal. Leaf size=399

$$\frac{x \left(a \left(\frac{b^3 d}{a} + a(bf + 2ce) - b(be + 3cd) \right) + cx^2 (-abe - 2a(cd - af) + b^2 d) \right) \sqrt{c} \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right) \left(\frac{12a^2 ce - ab^2 e - 4a^2 d}{\sqrt{b^2 - 4ac}} \right)}{2a^2 (b^2 - 4ac) (a + bx^2 + cx^4)} \quad 2\sqrt{2} a^2 (b^2 - 4ac)$$

Rubi [A] time = 2.20, antiderivative size = 399, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1669, 1664, 1166, 205}

$$\frac{x \left(cx^2 (-abe - 2a(cd - af) + b^2 d) + a \left(\frac{b^3 d}{a} + a(bf + 2ce) - b(be + 3cd) \right) \right) \sqrt{c} \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right) \left(\frac{12a^2 ce - ab^2 e - 4a^2 d}{\sqrt{b^2 - 4ac}} \right) - \frac{\sqrt{c} \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right) \left(\frac{-12a^2 ce - ab^2 e - 4a^2 d + 3b^2 d}{\sqrt{b^2 - 4ac}} - abe - 2a(5cd - af) + 3b^2 d \right)}{2a^2 (b^2 - 4ac) (a + bx^2 + cx^4)} \quad \frac{\sqrt{c} \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right) \left(\frac{-12a^2 ce - ab^2 e - 4a^2 d + 3b^2 d}{\sqrt{b^2 - 4ac}} - abe - 2a(5cd - af) + 3b^2 d \right)}{2\sqrt{2} a^2 (b^2 - 4ac) \sqrt{b - \sqrt{b^2 - 4ac}}} \quad \frac{d}{a^2 x}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2 + f*x^4)/(x^2*(a + b*x^2 + c*x^4)^2), x]

[Out] -(d/(a^2*x)) - (x*(a*((b^3*d)/a - b*(3*c*d + b*e) + a*(2*c*e + b*f)) + c*(b^2*d - a*b*e - 2*a*(c*d - a*f))*x^2)/(2*a^2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (Sqrt[c]*(3*b^2*d - a*b*e - 2*a*(5*c*d - a*f) + (3*b^3*d - a*b^2*e + 12*a^2*c*e - 4*a*b*(4*c*d + a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(3*b^2*d - a*b*e - 2*a*(5*c*d - a*f) - (3*b^3*d - a*b^2*e + 12*a^2*c*e - 4*a*b*(4*c*d + a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1664

Int[(Pq_)*((d_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]

Rule 1669

Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]

, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]

Rubi steps

$$\begin{aligned} \int \frac{d + ex^2 + fx^4}{x^2 (a + bx^2 + cx^4)^2} dx &= \frac{x \left(a \left(\frac{b^3 d}{a} - b(3cd + be) + a(2ce + bf) \right) + c(b^2 d - abe - 2a(cd - af)) x^2 \right)}{2a^2 (b^2 - 4ac) (a + bx^2 + cx^4)} - \int \frac{-2(}{x} \\ &= -\frac{x \left(a \left(\frac{b^3 d}{a} - b(3cd + be) + a(2ce + bf) \right) + c(b^2 d - abe - 2a(cd - af)) x^2 \right)}{2a^2 (b^2 - 4ac) (a + bx^2 + cx^4)} - \int \frac{2(}{x} \\ &= -\frac{d}{a^2 x} - \frac{x \left(a \left(\frac{b^3 d}{a} - b(3cd + be) + a(2ce + bf) \right) + c(b^2 d - abe - 2a(cd - af)) x^2 \right)}{2a^2 (b^2 - 4ac) (a + bx^2 + cx^4)} \\ &= -\frac{d}{a^2 x} - \frac{x \left(a \left(\frac{b^3 d}{a} - b(3cd + be) + a(2ce + bf) \right) + c(b^2 d - abe - 2a(cd - af)) x^2 \right)}{2a^2 (b^2 - 4ac) (a + bx^2 + cx^4)} \\ &= -\frac{d}{a^2 x} - \frac{x \left(a \left(\frac{b^3 d}{a} - b(3cd + be) + a(2ce + bf) \right) + c(b^2 d - abe - 2a(cd - af)) x^2 \right)}{2a^2 (b^2 - 4ac) (a + bx^2 + cx^4)} \end{aligned}$$

Mathematica [A] time = 1.32, size = 444, normalized size = 1.11

$$\frac{-2a(b^2(cd^2 - af) + ab(af - (3d + ce^2)) + 2ac(a(c + f^2) - cd^2) + b^2d)}{(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\sqrt{2} \sqrt{c} \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c}}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right) \left(ab(\sqrt{b^2 - 4ac} + 4af + 16cd) - 2(-5cd\sqrt{b^2 - 4ac} + af\sqrt{b^2 - 4ac} + 6acx) + b^2(a - 3d\sqrt{b^2 - 4ac}) - 3b^3d \right)}{(b^2 - 4ac)^{3/2} \sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{\sqrt{2} \sqrt{c} \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c}}{\sqrt{b^2 - 4ac + b}} \right) \left(ab(\sqrt{b^2 - 4ac} - 4af - 16cd) + 2(5cd\sqrt{b^2 - 4ac} - af\sqrt{b^2 - 4ac} + 6acx) - b^2(3d\sqrt{b^2 - 4ac} + ax) + 3b^3d \right)}{(b^2 - 4ac)^{3/2} \sqrt{b^2 - 4ac + b}} - \frac{4d}{x}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2 + f*x^4)/(x^2*(a + b*x^2 + c*x^4)^2), x]

[Out] ((-4*d)/x - (2*x*(b^3*d + b^2*(-(a*e) + c*d*x^2) + a*b*(a*f - c*(3*d + e*x^2)) + 2*a*c*(-(c*d*x^2) + a*(e + f*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(-3*b^3*d + b^2*(-3*Sqrt[b^2 - 4*a*c]*d + a*e) + a*b*(16*c*d + Sqrt[b^2 - 4*a*c]*e + 4*a*f) - 2*a*(-5*c*Sqrt[b^2 - 4*a*c]*d + 6*a*c*e + a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(3*b^3*d - b^2*(3*Sqrt[b^2 - 4*a*c]*d + a*e) + a*b*(-16*c*d + Sqrt[b^2 - 4*a*c]*e - 4*a*f) + 2*a*(5*c*Sqrt[b^2 - 4*a*c]*d + 6*a*c*e - a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])/(4*a^2)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex^2 + fx^4}{x^2 (a + bx^2 + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e*x^2 + f*x^4)/(x^2*(a + b*x^2 + c*x^4)^2), x]

[Out] IntegrateAlgebraic[(d + e*x^2 + f*x^4)/(x^2*(a + b*x^2 + c*x^4)^2), x]

fricas [B] time = 40.37, size = 13111, normalized size = 32.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(a*b*c*e - 2*a^2*c*f - (3*b^2*c - 10*a*c^2)*d)*x^4 - 2*(a^2*b*f + (3*
*b^3 - 11*a*b*c)*d - (a*b^2 - 2*a^2*c)*e)*x^2 + sqrt(1/2)*((a^2*b^2*c - 4*a
^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)*sqrt(-((9*
b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3)*d^2 - 2*(3*a*b^6 - 40*
a^2*b^4*c + 150*a^3*b^2*c^2 - 120*a^4*c^3)*d*e + (a^2*b^5 - 15*a^3*b^3*c +
60*a^4*b*c^2)*e^2 + (a^4*b^3 + 12*a^5*b*c)*f^2 - 2*((3*a^2*b^5 - 13*a^3*b^3
*c - 12*a^4*b*c^2)*d - (a^3*b^4 - 6*a^4*b^2*c - 24*a^5*c^2)*e)*f + (a^5*b^6
- 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((a^8*f^4 + (81*b^8 - 91
8*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)*d^4 - 4*(27*
a*b^7 - 351*a^2*b^5*c + 1197*a^3*b^3*c^2 - 550*a^4*b*c^3)*d^3*e + 6*(9*a^2*
b^6 - 132*a^3*b^4*c + 484*a^4*b^2*c^2 - 75*a^5*c^3)*d^2*e^2 - 4*(3*a^3*b^5
- 49*a^4*b^3*c + 198*a^5*b*c^2)*d*e^3 + (a^4*b^4 - 18*a^5*b^2*c + 81*a^6*c^
2)*e^4 + 4*(a^7*b*e - (3*a^6*b^2 + 5*a^7*c)*d)*f^3 + 6*((9*a^4*b^4 + 3*a^5*
b^2*c + 25*a^6*c^2)*d^2 - 2*(3*a^5*b^3 - 4*a^6*b*c)*d*e + (a^6*b^2 - 3*a^7*
c)*e^2)*f^2 - 4*((27*a^2*b^6 - 108*a^3*b^4*c - 180*a^4*b^2*c^2 + 125*a^5*c^
3)*d^3 - 3*(9*a^3*b^5 - 51*a^4*b^3*c - 65*a^5*b*c^2)*d^2*e + 3*(3*a^4*b^4 -
22*a^5*b^2*c - 15*a^6*c^2)*d*e^2 - (a^5*b^3 - 9*a^6*b*c)*e^3)*f)/(a^10*b^6
- 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3))/((189*b^6*c^3 - 1971*a*b^4*c^4 + 5625
*a^2*b^2*c^5 - 2500*a^3*c^6)*d^4 - (135*b^7*c^2 - 1323*a*b^5*c^3 + 2727*a^2
*b^3*c^4 + 2500*a^3*b*c^5)*d^3*e + 3*(45*a*b^6*c^2 - 558*a^2*b^4*c^3 + 1672
*a^3*b^2*c^4)*d^2*e^2 - (45*a^2*b^5*c^2 - 647*a^3*b^3*c^3 + 2268*a^4*b*c^4)
*d*e^3 + (5*a^3*b^4*c^2 - 81*a^4*b^2*c^3 + 324*a^5*c^4)*e^4 - (3*a^6*b^2*c
+ 4*a^7*c^2)*f^4 + ((27*a^4*b^4*c + 80*a^6*c^3)*d - (9*a^5*b^3*c - 20*a^6*b
*c^2)*e)*f^3 - 3*((27*a^2*b^6*c - 117*a^3*b^4*c^2 - 150*a^4*b^2*c^3 + 200*a
^5*c^4)*d^2 - (18*a^3*b^5*c - 123*a^4*b^3*c^2 - 100*a^5*b*c^3)*d*e + (3*a^4
*b^4*c - 28*a^5*b^2*c^2)*e^2)*f^2 + ((81*b^8*c - 945*a*b^6*c^2 + 3213*a^2*b
^4*c^3 - 3000*a^3*b^2*c^4 + 2000*a^4*c^5)*d^3 - 3*(27*a*b^7*c - 405*a^2*b^5
*c^2 + 1461*a^3*b^3*c^3 - 500*a^4*b*c^4)*d^2*e + 3*(9*a^2*b^6*c - 165*a^3*b
^4*c^2 + 692*a^4*b^2*c^3)*d*e^2 - (3*a^3*b^5*c - 65*a^4*b^3*c^2 + 324*a^5*b
*c^3)*e^3)*f)*x + 1/2*sqrt(1/2)*((27*b^11 - 486*a*b^9*c + 3330*a^2*b^7*c^2
- 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5*b*c^5)*d^3 - 3*(9*a*b^10
- 177*a^2*b^8*c + 1285*a^3*b^6*c^2 - 4138*a^4*b^4*c^3 + 5216*a^5*b^2*c^4 -
800*a^6*c^5)*d^2*e + 3*(3*a^2*b^9 - 64*a^3*b^7*c + 495*a^4*b^5*c^2 - 1656*
a^5*b^3*c^3 + 2032*a^6*b*c^4)*d*e^2 - (a^3*b^8 - 23*a^4*b^6*c + 190*a^5*b^4
*c^2 - 672*a^6*b^2*c^3 + 864*a^7*c^4)*e^3 - (a^6*b^5 - 8*a^7*b^3*c + 16*a^8
*b*c^2)*f^3 + 3*((3*a^4*b^7 - 25*a^5*b^5*c + 56*a^6*b^3*c^2 - 16*a^7*b*c^3)
*d - (a^5*b^6 - 10*a^6*b^4*c + 32*a^7*b^2*c^2 - 32*a^8*c^3)*e)*f^2 - 3*((9*
a^2*b^9 - 105*a^3*b^7*c + 373*a^4*b^5*c^2 - 248*a^5*b^3*c^3 - 560*a^6*b*c^4
)*d^2 - 2*(3*a^3*b^8 - 40*a^4*b^6*c + 166*a^5*b^4*c^2 - 176*a^6*b^2*c^3 - 1
60*a^7*c^4)*d*e + (a^4*b^7 - 15*a^5*b^5*c + 72*a^6*b^3*c^2 - 112*a^7*b*c^3)
*e^2)*f - ((3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*c^2 - 1344*a^8*b^4*c^3
+ 2176*a^9*b^2*c^4 - 1280*a^10*c^5)*d - (a^6*b^9 - 20*a^7*b^7*c + 144*a^8*b
^5*c^2 - 448*a^9*b^3*c^3 + 512*a^10*b*c^4)*e - (a^7*b^8 - 8*a^8*b^6*c + 128
*a^10*b^2*c^3 - 256*a^11*c^4)*f)*sqrt((a^8*f^4 + (81*b^8 - 918*a*b^6*c + 30
51*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)*d^4 - 4*(27*a*b^7 - 351*a^
2*b^5*c + 1197*a^3*b^3*c^2 - 550*a^4*b*c^3)*d^3*e + 6*(9*a^2*b^6 - 132*a^3*
b^4*c + 484*a^4*b^2*c^2 - 75*a^5*c^3)*d^2*e^2 - 4*(3*a^3*b^5 - 49*a^4*b^3*c
+ 198*a^5*b*c^2)*d*e^3 + (a^4*b^4 - 18*a^5*b^2*c + 81*a^6*c^2)*e^4 + 4*(a^
7*b*e - (3*a^6*b^2 + 5*a^7*c)*d)*f^3 + 6*((9*a^4*b^4 + 3*a^5*b^2*c + 25*a^6
*c^2)*d^2 - 2*(3*a^5*b^3 - 4*a^6*b*c)*d*e + (a^6*b^2 - 3*a^7*c)*e^2)*f^2 -
4*((27*a^2*b^6 - 108*a^3*b^4*c - 180*a^4*b^2*c^2 + 125*a^5*c^3)*d^3 - 3*(9*
a^3*b^5 - 51*a^4*b^3*c - 65*a^5*b*c^2)*d^2*e + 3*(3*a^4*b^4 - 22*a^5*b^2*c
- 15*a^6*c^2)*d*e^2 - (a^5*b^3 - 9*a^6*b*c)*e^3)*f)/(a^10*b^6 - 12*a^11*b^4
```

$$\begin{aligned}
& *c + 48*a^{12}*b^2*c^2 - 64*a^{13}*c^3)) * \text{sqrt}(-((9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3)*d^2 - 2*(3*a*b^6 - 40*a^2*b^4*c + 150*a^3*b^2*c^2 - 120*a^4*c^3)*d*e + (a^2*b^5 - 15*a^3*b^3*c + 60*a^4*b*c^2)*e^2 + (a^4*b^3 + 12*a^5*b*c)*f^2 - 2*((3*a^2*b^5 - 13*a^3*b^3*c - 12*a^4*b*c^2)*d - (a^3*b^4 - 6*a^4*b^2*c - 24*a^5*c^2)*e)*f + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*\text{sqrt}((a^8*f^4 + (81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)*d^4 - 4*(27*a*b^7 - 351*a^2*b^5*c + 1197*a^3*b^3*c^2 - 550*a^4*b*c^3)*d^3*e + 6*(9*a^2*b^6 - 132*a^3*b^4*c + 484*a^4*b^2*c^2 - 75*a^5*c^3)*d^2*e^2 - 4*(3*a^3*b^5 - 49*a^4*b^3*c + 198*a^5*b*c^2)*d*e^3 + (a^4*b^4 - 18*a^5*b^2*c + 81*a^6*c^2)*e^4 + 4*(a^7*b*e - (3*a^6*b^2 + 5*a^7*c)*d)*f^3 + 6*((9*a^4*b^4 + 3*a^5*b^2*c + 25*a^6*c^2)*d^2 - 2*(3*a^5*b^3 - 4*a^6*b*c)*d*e + (a^6*b^2 - 3*a^7*c)*e^2)*f^2 - 4*((27*a^2*b^6 - 108*a^3*b^4*c - 180*a^4*b^2*c^2 + 125*a^5*c^3)*d^3 - 3*(9*a^3*b^5 - 51*a^4*b^3*c - 65*a^5*b*c^2)*d^2*e + 3*(3*a^4*b^4 - 22*a^5*b^2*c - 15*a^6*c^2)*d*e^2 - (a^5*b^3 - 9*a^6*b*c)*e^3)*f)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)) - \text{sqrt}(1/2)*((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)*\text{sqrt}(-((9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3)*d^2 - 2*(3*a*b^6 - 40*a^2*b^4*c + 150*a^3*b^2*c^2 - 120*a^4*c^3)*d*e + (a^2*b^5 - 15*a^3*b^3*c + 60*a^4*b*c^2)*e^2 + (a^4*b^3 + 12*a^5*b*c)*f^2 - 2*((3*a^2*b^5 - 13*a^3*b^3*c - 12*a^4*b*c^2)*d - (a^3*b^4 - 6*a^4*b^2*c - 24*a^5*c^2)*e)*f + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*\text{sqrt}((a^8*f^4 + (81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)*d^4 - 4*(27*a*b^7 - 351*a^2*b^5*c + 1197*a^3*b^3*c^2 - 550*a^4*b*c^3)*d^3*e + 6*(9*a^2*b^6 - 132*a^3*b^4*c + 484*a^4*b^2*c^2 - 75*a^5*c^3)*d^2*e^2 - 4*(3*a^3*b^5 - 49*a^4*b^3*c + 198*a^5*b*c^2)*d*e^3 + (a^4*b^4 - 18*a^5*b^2*c + 81*a^6*c^2)*e^4 + 4*(a^7*b*e - (3*a^6*b^2 + 5*a^7*c)*d)*f^3 + 6*((9*a^4*b^4 + 3*a^5*b^2*c + 25*a^6*c^2)*d^2 - 2*(3*a^5*b^3 - 4*a^6*b*c)*d*e + (a^6*b^2 - 3*a^7*c)*e^2)*f^2 - 4*((27*a^2*b^6 - 108*a^3*b^4*c - 180*a^4*b^2*c^2 + 125*a^5*c^3)*d^3 - 3*(9*a^3*b^5 - 51*a^4*b^3*c - 65*a^5*b*c^2)*d^2*e + 3*(3*a^4*b^4 - 22*a^5*b^2*c - 15*a^6*c^2)*d*e^2 - (a^5*b^3 - 9*a^6*b*c)*e^3)*f)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))*\log(-((189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*d^4 - (135*b^7*c^2 - 1323*a*b^5*c^3 + 2727*a^2*b^3*c^4 + 2500*a^3*b*c^5)*d^3*e + 3*(45*a*b^6*c^2 - 558*a^2*b^4*c^3 + 1672*a^3*b^2*c^4)*d^2*e^2 - (45*a^2*b^5*c^2 - 647*a^3*b^3*c^3 + 2268*a^4*b*c^4)*d*e^3 + (5*a^3*b^4*c^2 - 81*a^4*b^2*c^3 + 324*a^5*c^4)*e^4 - (3*a^6*b^2*c + 4*a^7*c^2)*f^4 + ((27*a^4*b^4*c + 80*a^6*c^3)*d - (9*a^5*b^3*c - 20*a^6*b*c^2)*e)*f^3 - 3*((27*a^2*b^6*c - 117*a^3*b^4*c^2 - 150*a^4*b^2*c^3 + 200*a^5*c^4)*d^2 - (18*a^3*b^5*c - 123*a^4*b^3*c^2 - 100*a^5*b*c^3)*d*e + (3*a^4*b^4*c - 28*a^5*b^2*c^2)*e^2)*f^2 + ((81*b^8*c - 945*a*b^6*c^2 + 3213*a^2*b^4*c^3 - 3000*a^3*b^2*c^4 + 2000*a^4*c^5)*d^3 - 3*(27*a*b^7*c - 405*a^2*b^5*c^2 + 1461*a^3*b^3*c^3 - 500*a^4*b*c^4)*d^2*e + 3*(9*a^2*b^6*c - 165*a^3*b^4*c^2 + 692*a^4*b^2*c^3)*d*e^2 - (3*a^3*b^5*c - 65*a^4*b^3*c^2 + 324*a^5*b*c^3)*e^3)*f)*x - 1/2*\text{sqrt}(1/2)*((27*b^11 - 486*a*b^9*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5*b*c^5)*d^3 - 3*(9*a*b^10 - 177*a^2*b^8*c + 1285*a^3*b^6*c^2 - 4138*a^4*b^4*c^3 + 5216*a^5*b^2*c^4 - 800*a^6*c^5)*d^2*e + 3*(3*a^2*b^9 - 64*a^3*b^7*c + 495*a^4*b^5*c^2 - 1656*a^5*b^3*c^3 + 2032*a^6*b*c^4)*d*e^2 - (a^3*b^8 - 23*a^4*b^6*c + 190*a^5*b^4*c^2 - 672*a^6*b^2*c^3 + 864*a^7*c^4)*e^3 - (a^6*b^5 - 8*a^7*b^3*c + 16*a^8*b*c^2)*f^3 + 3*((3*a^4*b^7 - 25*a^5*b^5*c + 56*a^6*b^3*c^2 - 16*a^7*b*c^3)*d - (a^5*b^6 - 10*a^6*b^4*c + 32*a^7*b^2*c^2 - 32*a^8*c^3)*e)*f^2 - 3*((9*a^2*b^9 - 105*a^3*b^7*c + 373*a^4*b^5*c^2 - 248*a^5*b^3*c^3 - 560*a^6*b*c^4)*d^2 - 2*(3*a^3*b^8 - 40*a^4*b^6*c + 166*a^5*b^4*c^2 - 176*a^6*b^2*c^3 - 160*a^7*c^4)*d*e + (a^4*b^7 - 15*a^5*b^5*c + 72*a^6*b^3*c^2 - 112*a^7*b*c^3)*e^2)*f - ((3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*c^2 - 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^5)*d - (a^6*b^9 - 20*a^7*b^7*c + 144*a^8*b^5*c^2 - 448*a^9*b^3*c^3 + 512*a^10*b*c^4)*e - (a^7*b^8 - 8*a^8*b^6*c + 128*a^10*b^2*c^3 - 256*a^11*c^4)*f)*\text{sqrt}((
\end{aligned}$$

$$\begin{aligned}
& a^8 f^4 + (81 b^8 - 918 a b^6 c + 3051 a^2 b^4 c^2 - 2550 a^3 b^2 c^3 + 625 a^4 c^4) d^4 - 4(27 a^2 b^7 - 351 a^2 b^5 c + 1197 a^3 b^3 c^2 - 550 a^4 b c^3) d^3 e + 6(9 a^2 b^6 - 132 a^3 b^4 c + 484 a^4 b^2 c^2 - 75 a^5 c^3) d^2 e^2 - 4(3 a^3 b^5 - 49 a^4 b^3 c + 198 a^5 b c^2) d e^3 + (a^4 b^4 - 18 a^5 b^2 c + 81 a^6 c^2) e^4 + 4(a^7 b e - (3 a^6 b^2 + 5 a^7 c) d) f^3 + 6((9 a^4 b^4 + 3 a^5 b^2 c + 25 a^6 c^2) d^2 - 2(3 a^5 b^3 - 4 a^6 b c) d e + (a^6 b^2 - 3 a^7 c) e^2) f^2 - 4((27 a^2 b^6 - 108 a^3 b^4 c - 180 a^4 b^2 c^2 + 125 a^5 c^3) d^3 - 3(9 a^3 b^5 - 51 a^4 b^3 c - 65 a^5 b c^2) d^2 e + 3(3 a^4 b^4 - 22 a^5 b^2 c - 15 a^6 c^2) d e^2 - (a^5 b^3 - 9 a^6 b c) e^3) f) / (a^{10} b^6 - 12 a^{11} b^4 c + 48 a^{12} b^2 c^2 - 64 a^{13} c^3)) * \text{sqrt}(-((9 b^7 - 105 a b^5 c + 385 a^2 b^3 c^2 - 420 a^3 b c^3) d^2 - 2(3 a b^6 - 40 a^2 b^4 c + 150 a^3 b^2 c^2 - 120 a^4 c^3) d e + (a^2 b^5 - 15 a^3 b^3 c + 60 a^4 b c^2) e^2 + (a^4 b^3 + 12 a^5 b c) f^2 - 2((3 a^2 b^5 - 13 a^3 b^3 c - 12 a^4 b c^2) d - (a^3 b^4 - 6 a^4 b^2 c - 24 a^5 c^2) e) f + (a^5 b^6 - 12 a^6 b^4 c + 48 a^7 b^2 c^2 - 64 a^8 c^3) \text{sqrt}((a^8 f^4 + (81 b^8 - 918 a b^6 c + 3051 a^2 b^4 c^2 - 2550 a^3 b^2 c^3 + 625 a^4 c^4) d^4 - 4(27 a^2 b^7 - 351 a^2 b^5 c + 1197 a^3 b^3 c^2 - 550 a^4 b c^3) d^3 e + 6(9 a^2 b^6 - 132 a^3 b^4 c + 484 a^4 b^2 c^2 - 75 a^5 c^3) d^2 e^2 - 4(3 a^3 b^5 - 49 a^4 b^3 c + 198 a^5 b c^2) d e^3 + (a^4 b^4 - 18 a^5 b^2 c + 81 a^6 c^2) e^4 + 4(a^7 b e - (3 a^6 b^2 + 5 a^7 c) d) f^3 + 6((9 a^4 b^4 + 3 a^5 b^2 c + 25 a^6 c^2) d^2 - 2(3 a^5 b^3 - 4 a^6 b c) d e + (a^6 b^2 - 3 a^7 c) e^2) f^2 - 4((27 a^2 b^6 - 108 a^3 b^4 c - 180 a^4 b^2 c^2 + 125 a^5 c^3) d^3 - 3(9 a^3 b^5 - 51 a^4 b^3 c - 65 a^5 b c^2) d^2 e + 3(3 a^4 b^4 - 22 a^5 b^2 c - 15 a^6 c^2) d e^2 - (a^5 b^3 - 9 a^6 b c) e^3) f) / (a^{10} b^6 - 12 a^{11} b^4 c + 48 a^{12} b^2 c^2 - 64 a^{13} c^3))) / (a^5 b^6 - 12 a^6 b^4 c + 48 a^7 b^2 c^2 - 64 a^8 c^3)) + \text{sqrt}(1/2) * ((a^2 b^2 c - 4 a^3 c^2) x^5 + (a^2 b^3 - 4 a^3 b c) x^3 + (a^3 b^2 - 4 a^4 c) x) * \text{sqrt}(-((9 b^7 - 105 a b^5 c + 385 a^2 b^3 c^2 - 420 a^3 b c^3) d^2 - 2(3 a b^6 - 40 a^2 b^4 c + 150 a^3 b^2 c^2 - 120 a^4 c^3) d e + (a^2 b^5 - 15 a^3 b^3 c + 60 a^4 b c^2) e^2 + (a^4 b^3 + 12 a^5 b c) f^2 - 2((3 a^2 b^5 - 13 a^3 b^3 c - 12 a^4 b c^2) d - (a^3 b^4 - 6 a^4 b^2 c - 24 a^5 c^2) e) f - (a^5 b^6 - 12 a^6 b^4 c + 48 a^7 b^2 c^2 - 64 a^8 c^3) \text{sqrt}((a^8 f^4 + (81 b^8 - 918 a b^6 c + 3051 a^2 b^4 c^2 - 2550 a^3 b^2 c^3 + 625 a^4 c^4) d^4 - 4(27 a^2 b^7 - 351 a^2 b^5 c + 1197 a^3 b^3 c^2 - 550 a^4 b c^3) d^3 e + 6(9 a^2 b^6 - 132 a^3 b^4 c + 484 a^4 b^2 c^2 - 75 a^5 c^3) d^2 e^2 - 4(3 a^3 b^5 - 49 a^4 b^3 c + 198 a^5 b c^2) d e^3 + (a^4 b^4 - 18 a^5 b^2 c + 81 a^6 c^2) e^4 + 4(a^7 b e - (3 a^6 b^2 + 5 a^7 c) d) f^3 + 6((9 a^4 b^4 + 3 a^5 b^2 c + 25 a^6 c^2) d^2 - 2(3 a^5 b^3 - 4 a^6 b c) d e + (a^6 b^2 - 3 a^7 c) e^2) f^2 - 4((27 a^2 b^6 - 108 a^3 b^4 c - 180 a^4 b^2 c^2 + 125 a^5 c^3) d^3 - 3(9 a^3 b^5 - 51 a^4 b^3 c - 65 a^5 b c^2) d^2 e + 3(3 a^4 b^4 - 22 a^5 b^2 c - 15 a^6 c^2) d e^2 - (a^5 b^3 - 9 a^6 b c) e^3) f) / (a^{10} b^6 - 12 a^{11} b^4 c + 48 a^{12} b^2 c^2 - 64 a^{13} c^3))) / (a^5 b^6 - 12 a^6 b^4 c + 48 a^7 b^2 c^2 - 64 a^8 c^3)) * \log(-((189 b^6 c^3 - 1971 a b^4 c^4 + 5625 a^2 b^2 c^5 - 2500 a^3 c^6) d^4 - (135 b^7 c^2 - 1323 a b^5 c^3 + 2727 a^2 b^3 c^4 + 2500 a^3 b c^5) d^3 e + 3(45 a b^6 c^2 - 558 a^2 b^4 c^3 + 1672 a^3 b^2 c^4) d^2 e^2 - (45 a^2 b^5 c^2 - 647 a^3 b^3 c^3 + 2268 a^4 b c^4) d e^3 + (5 a^3 b^4 c^2 - 81 a^4 b^2 c^3 + 324 a^5 c^4) e^4 - (3 a^6 b^2 c + 4 a^7 c^2) f^4 + ((27 a^4 b^4 c + 80 a^6 c^3) d - (9 a^5 b^3 c - 20 a^6 b c^2) e) f^3 - 3((27 a^2 b^6 c - 117 a^3 b^4 c^2 - 150 a^4 b^2 c^3 + 200 a^5 c^4) d^2 - (18 a^3 b^5 c - 123 a^4 b^3 c^2 - 100 a^5 b c^3) d e + (3 a^4 b^4 c - 28 a^5 b^2 c^2) e^2) f^2 + ((81 b^8 c - 945 a b^6 c^2 + 3213 a^2 b^4 c^3 - 3000 a^3 b^2 c^4 + 2000 a^4 c^5) d^3 - 3(27 a^2 b^7 c - 405 a^2 b^5 c^2 + 1461 a^3 b^3 c^3 - 500 a^4 b c^4) d^2 e + 3(9 a^2 b^6 c - 165 a^3 b^4 c^2 + 692 a^4 b^2 c^3) d e^2 - (3 a^3 b^5 c - 65 a^4 b^3 c^2 + 324 a^5 b c^3) e^3) f) x + 1/2 * \text{sqrt}(1/2) * ((27 b^{11} - 486 a b^9 c + 3330 a^2 b^7 c^2 - 10549 a^3 b^5 c^3 + 14408 a^4 b^3 c^4 - 5200 a^5 b c^5) d^3 - 3(9 a b^{10} - 177 a^2 b^8 c + 1285 a^3 b^6 c^2 - 4138 a^4 b^4 c^3 + 5216 a^5 b^2 c^4 - 800 a^6 c^5) d^2 e + 3(3 a^2 b^9 - 64 a^3 b^7 c + 495 a^4 b^5 c^2 - 1656 a^5 b^3 c^3 + 2032 a^6 b c^4) d e^2 - (a^3 b^8 - 23 a^4 b^6 c + 190 a^5 b^4 c^2
\end{aligned}$$

$$\begin{aligned}
& 2 - 672a^6b^2c^3 + 864a^7c^4)e^3 - (a^6b^5 - 8a^7b^3c + 16a^8bc^2)*f^3 + 3*((3a^4b^7 - 25a^5b^5c + 56a^6b^3c^2 - 16a^7bc^3)*d \\
& - (a^5b^6 - 10a^6b^4c + 32a^7b^2c^2 - 32a^8c^3)*e)*f^2 - 3*((9a^2b^9 - 105a^3b^7c + 373a^4b^5c^2 - 248a^5b^3c^3 - 560a^6b^2c^4)*d \\
& ^2 - 2*(3a^3b^8 - 40a^4b^6c + 166a^5b^4c^2 - 176a^6b^2c^3 - 160a^7c^4)*d*e + (a^4b^7 - 15a^5b^5c + 72a^6b^3c^2 - 112a^7bc^3)*e^2 \\
&)*f + ((3a^5b^10 - 55a^6b^8c + 392a^7b^6c^2 - 1344a^8b^4c^3 + 2176a^9b^2c^4 - 1280a^10c^5)*d - (a^6b^9 - 20a^7b^7c + 144a^8b^5c^2 - 448a^9b^3c^3 + 512a^10bc^4)*e - (a^7b^8 - 8a^8b^6c + 128a^10b^2c^3 - 256a^11c^4)*f)*\sqrt{(a^8f^4 + (81b^8 - 918ab^6c + 3051a^2b^4c^2 - 2550a^3b^2c^3 + 625a^4c^4)*d^4 - 4*(27ab^7 - 351a^2b^5c + 1197a^3b^3c^2 - 550a^4bc^3)*d^3e + 6*(9a^2b^6 - 132a^3b^4c + 484a^4b^2c^2 - 75a^5c^3)*d^2e^2 - 4*(3a^3b^5 - 49a^4b^3c + 198a^5b^2c^2)*d^2e^3 + (a^4b^4 - 18a^5b^2c + 81a^6c^2)*e^4 + 4*(a^7b^8 - 3a^6b^2 + 5a^7c)*d)*f^3 + 6*((9a^4b^4 + 3a^5b^2c + 25a^6c^2)*d^2 - 2*(3a^5b^3 - 4a^6bc)*d*e + (a^6b^2 - 3a^7c)*e^2)*f^2 - 4*((27a^2b^6 - 108a^3b^4c - 180a^4b^2c^2 + 125a^5c^3)*d^3 - 3*(9a^3b^5 - 51a^4b^3c - 65a^5bc^2)*d^2e + 3*(3a^4b^4 - 22a^5b^2c - 15a^6c^2)*d^2e^2 - (a^5b^3 - 9a^6bc)*e^3)*f)/(a^10b^6 - 12a^11b^4c + 48a^12b^2c^2 - 64a^13c^3))*\sqrt{-((9b^7 - 105ab^5c + 385a^2b^3c^2 - 420a^3b^2c^3)*d^2 - 2*(3ab^6 - 40a^2b^4c + 150a^3b^2c^2 - 120a^4c^3)*d*e + (a^2b^5 - 15a^3b^3c + 60a^4bc^2)*e^2 + (a^4b^3 + 12a^5bc)*f^2 - 2*((3a^2b^5 - 13a^3b^3c - 12a^4bc^2)*d - (a^3b^4 - 6a^4b^2c - 24a^5c^2)*e)*f - (a^5b^6 - 12a^6b^4c + 48a^7b^2c^2 - 64a^8c^3)*\sqrt{(a^8f^4 + (81b^8 - 918ab^6c + 3051a^2b^4c^2 - 2550a^3b^2c^3 + 625a^4c^4)*d^4 - 4*(27ab^7 - 351a^2b^5c + 1197a^3b^3c^2 - 550a^4bc^3)*d^3e + 6*(9a^2b^6 - 132a^3b^4c + 484a^4b^2c^2 - 75a^5c^3)*d^2e^2 - 4*(3a^3b^5 - 49a^4b^3c + 198a^5b^2c^2)*d^2e^3 + (a^4b^4 - 18a^5b^2c + 81a^6c^2)*e^4 + 4*(a^7b^8 - 3a^6b^2 + 5a^7c)*d)*f^3 + 6*((9a^4b^4 + 3a^5b^2c + 25a^6c^2)*d^2 - 2*(3a^5b^3 - 4a^6bc)*d*e + (a^6b^2 - 3a^7c)*e^2)*f^2 - 4*((27a^2b^6 - 108a^3b^4c - 180a^4b^2c^2 + 125a^5c^3)*d^3 - 3*(9a^3b^5 - 51a^4b^3c - 65a^5bc^2)*d^2e + 3*(3a^4b^4 - 22a^5b^2c - 15a^6c^2)*d^2e^2 - (a^5b^3 - 9a^6bc)*e^3)*f)/(a^10b^6 - 12a^11b^4c + 48a^12b^2c^2 - 64a^13c^3)))/(a^5b^6 - 12a^6b^4c + 48a^7b^2c^2 - 64a^8c^3)) - \sqrt{1/2}*((a^2b^2c - 4a^3c^2)*x^5 + (a^2b^3 - 4a^3bc)*x^3 + (a^3b^2 - 4a^4c)*x)*\sqrt{-((9b^7 - 105ab^5c + 385a^2b^3c^2 - 420a^3b^2c^3)*d^2 - 2*(3ab^6 - 40a^2b^4c + 150a^3b^2c^2 - 120a^4c^3)*d*e + (a^2b^5 - 15a^3b^3c + 60a^4bc^2)*e^2 + (a^4b^3 + 12a^5bc)*f^2 - 2*((3a^2b^5 - 13a^3b^3c - 12a^4bc^2)*d - (a^3b^4 - 6a^4b^2c - 24a^5c^2)*e)*f - (a^5b^6 - 12a^6b^4c + 48a^7b^2c^2 - 64a^8c^3)*\sqrt{(a^8f^4 + (81b^8 - 918ab^6c + 3051a^2b^4c^2 - 2550a^3b^2c^3 + 625a^4c^4)*d^4 - 4*(27ab^7 - 351a^2b^5c + 1197a^3b^3c^2 - 550a^4bc^3)*d^3e + 6*(9a^2b^6 - 132a^3b^4c + 484a^4b^2c^2 - 75a^5c^3)*d^2e^2 - 4*(3a^3b^5 - 49a^4b^3c + 198a^5b^2c^2)*d^2e^3 + (a^4b^4 - 18a^5b^2c + 81a^6c^2)*e^4 + 4*(a^7b^8 - 3a^6b^2 + 5a^7c)*d)*f^3 + 6*((9a^4b^4 + 3a^5b^2c + 25a^6c^2)*d^2 - 2*(3a^5b^3 - 4a^6bc)*d*e + (a^6b^2 - 3a^7c)*e^2)*f^2 - 4*((27a^2b^6 - 108a^3b^4c - 180a^4b^2c^2 + 125a^5c^3)*d^3 - 3*(9a^3b^5 - 51a^4b^3c - 65a^5bc^2)*d^2e + 3*(3a^4b^4 - 22a^5b^2c - 15a^6c^2)*d^2e^2 - (a^5b^3 - 9a^6bc)*e^3)*f)/(a^10b^6 - 12a^11b^4c + 48a^12b^2c^2 - 64a^13c^3)))/(a^5b^6 - 12a^6b^4c + 48a^7b^2c^2 - 64a^8c^3))*\log(-((189b^6c^3 - 1971ab^4c^4 + 5625a^2b^2c^5 - 2500a^3c^6)*d^4 - (135b^7c^2 - 1323ab^5c^3 + 2727a^2b^3c^4 + 2500a^3b^2c^5)*d^3e + 3*(45ab^6c^2 - 558a^2b^4c^3 + 1672a^3b^2c^4)*d^2e^2 - (45a^2b^5c^2 - 647a^3b^3c^3 + 2268a^4b^2c^4)*d^2e^3 + (5a^3b^4c^2 - 81a^4b^2c^3 + 324a^5c^4)*e^4 - (3a^6b^2c + 4a^7c^2)*f^4 + ((27a^4b^4c + 80a^6c^3)*d - (9a^5b^3c - 20a^6bc^2)*e)*f^3 - 3*((27a^2b^6c - 117a^3b^4c^2 - 150a^4b^2c^3 + 200a^5c^4)*d^2 - (18a^3b^5c - 123a^4b^3c
\end{aligned}$$

$$\begin{aligned}
& c^2 - 100a^5b^3c^3) * d * e + (3a^4b^4c - 28a^5b^2c^2) * e^2 * f^2 + ((81b^8c - 945a^2b^6c^2 + 3213a^2b^4c^3 - 3000a^3b^2c^4 + 2000a^4c^5) * \\
& d^3 - 3(27a^2b^7c - 405a^2b^5c^2 + 1461a^3b^3c^3 - 500a^4b^3c^4) * d^2 * e + 3(9a^2b^6c - 165a^3b^4c^2 + 692a^4b^2c^3) * d * e^2 - (3a^3b^5c - 65a^4b^3c^2 + 324a^5b^3c^3) * e^3) * f) * x - 1/2 * \sqrt{1/2} * ((27b^{11} \\
& - 486a^2b^9c + 3330a^2b^7c^2 - 10549a^3b^5c^3 + 14408a^4b^3c^4 - 5200a^5b^3c^5) * d^3 - 3(9a^2b^{10} - 177a^2b^8c + 1285a^3b^6c^2 - 4138 \\
& a^4b^4c^3 + 5216a^5b^2c^4 - 800a^6c^5) * d^2 * e + 3(3a^2b^9 - 64a^3b^7c + 495a^4b^5c^2 - 1656a^5b^3c^3 + 2032a^6b^3c^4) * d * e^2 - (a^3b^8 - 23a^4b^6c + 190a^5b^4c^2 - 672a^6b^2c^3 + 864a^7c^4) * e^3 \\
& - (a^6b^5 - 8a^7b^3c + 16a^8b^3c^2) * f^3 + 3((3a^4b^7 - 25a^5b^5c + 56a^6b^3c^2 - 16a^7b^3c^3) * d - (a^5b^6 - 10a^6b^4c + 32a^7b^2c^2 - 32a^8c^3) * e) * f^2 - 3((9a^2b^9 - 105a^3b^7c + 373a^4b^5c^2 - 248a^5b^3c^3 - 560a^6b^3c^4) * d^2 - 2(3a^3b^8 - 40a^4b^6c + 166a^5b^4c^2 - 176a^6b^2c^3 - 160a^7c^4) * d * e + (a^4b^7 - 15a^5b^5c + 72a^6b^3c^2 - 112a^7b^3c^3) * e^2) * f + ((3a^5b^{10} - 55a^6b^8c + 392a^7b^6c^2 - 1344a^8b^4c^3 + 2176a^9b^2c^4 - 1280a^{10}c^5) * d - (a^6b^9 - 20a^7b^7c + 144a^8b^5c^2 - 448a^9b^3c^3 + 512a^{10}b^3c^4) * e - (a^7b^8 - 8a^8b^6c + 128a^{10}b^2c^3 - 256a^{11}c^4) * f) * \sqrt{(a^8 * f^4 + (81b^8 - 918a^2b^6c + 3051a^2b^4c^2 - 2550a^3b^2c^3 + 625a^4c^4) * d^4 - 4(27a^2b^7 - 351a^2b^5c + 1197a^3b^3c^2 - 550a^4b^3c^3) * d^3 * e + 6(9a^2b^6 - 132a^3b^4c + 484a^4b^2c^2 - 75a^5c^3) * d^2 * e^2 - 4(3a^3b^5 - 49a^4b^3c + 198a^5b^3c^2) * d * e^3 + (a^4b^4 - 18a^5b^2c + 81a^6c^2) * e^4 + 4(a^7b^3e - (3a^6b^2 + 5a^7c) * d) * f^3 + 6((9a^4b^4 + 3a^5b^2c + 25a^6c^2) * d^2 - 2(3a^5b^3 - 4a^6b^3c) * d * e + (a^6b^2 - 3a^7c) * e^2) * f^2 - 4((27a^2b^6 - 108a^3b^4c - 180a^4b^2c^2 + 125a^5c^3) * d^3 - 3(9a^3b^5 - 51a^4b^3c - 65a^5b^3c^2) * d^2 * e + 3(3a^4b^4 - 22a^5b^2c - 15a^6c^2) * d * e^2 - (a^5b^3 - 9a^6b^3c) * e^3) * f) / (a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3)) * \sqrt{-(9b^7 - 105a^2b^5c + 385a^2b^3c^2 - 420a^3b^3c^3) * d^2 - 2(3a^2b^6 - 40a^2b^4c + 150a^3b^2c^2 - 120a^4c^3) * d * e + (a^2b^5 - 15a^3b^3c + 60a^4b^3c^2) * e^2 + (a^4b^3 + 12a^5b^3c) * f^2 - 2((3a^2b^5 - 13a^3b^3c - 12a^4b^3c^2) * d - (a^3b^4 - 6a^4b^2c - 24a^5c^2) * e) * f - (a^5b^6 - 12a^6b^4c + 48a^7b^2c^2 - 64a^8c^3) * \sqrt{(a^8 * f^4 + (81b^8 - 918a^2b^6c + 3051a^2b^4c^2 - 2550a^3b^2c^3 + 625a^4c^4) * d^4 - 4(27a^2b^7 - 351a^2b^5c + 1197a^3b^3c^2 - 550a^4b^3c^3) * d^3 * e + 6(9a^2b^6 - 132a^3b^4c + 484a^4b^2c^2 - 75a^5c^3) * d^2 * e^2 - 4(3a^3b^5 - 49a^4b^3c + 198a^5b^3c^2) * d * e^3 + (a^4b^4 - 18a^5b^2c + 81a^6c^2) * e^4 + 4(a^7b^3e - (3a^6b^2 + 5a^7c) * d) * f^3 + 6((9a^4b^4 + 3a^5b^2c + 25a^6c^2) * d^2 - 2(3a^5b^3 - 4a^6b^3c) * d * e + (a^6b^2 - 3a^7c) * e^2) * f^2 - 4((27a^2b^6 - 108a^3b^4c - 180a^4b^2c^2 + 125a^5c^3) * d^3 - 3(9a^3b^5 - 51a^4b^3c - 65a^5b^3c^2) * d^2 * e + 3(3a^4b^4 - 22a^5b^2c - 15a^6c^2) * d * e^2 - (a^5b^3 - 9a^6b^3c) * e^3) * f) / (a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3)) / (a^5b^6 - 12a^6b^4c + 48a^7b^2c^2 - 64a^8c^3)) - 4(a^2b^2 - 4a^2c) * d) / ((a^2b^2 * c - 4a^3c^2) * x^5 + (a^2b^3 - 4a^3b^3c) * x^3 + (a^3b^2 - 4a^4c) * x)
\end{aligned}$$

giac [B] time = 7.09, size = 7182, normalized size = 18.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="giac")

[Out]
$$\begin{aligned}
& -1/2 * (3b^2c * d * x^4 - 10a * c^2 * d * x^4 + 2a^2 * c * f * x^4 - a * b * c * x^4 * e + 3b^3 * \\
& d * x^2 - 11a * b * c * d * x^2 + a^2 * b * f * x^2 - a * b^2 * x^2 * e + 2a^2 * c * x^2 * e + 2a * b^2 * d - 8a^2 * c * d) / ((c * x^5 + b * x^3 + a * x) * (a^2 * b^2 - 4a^3 * c)) - 1/16 * ((6b^4 \\
& * c^2 - 44a * b^2 * c^3 + 80a^2 * c^4 - 3 * \sqrt{2} * \sqrt{b^2 - 4a * c}) * \sqrt{b * c + \sqrt{b^2 - 4a * c}} * \\
& \sqrt{b^2 - 4a * c}) * b^4 + 22 * \sqrt{2} * \sqrt{b^2 - 4a * c}) * \sqrt{b * c + \sqrt{b^2 - 4a * c}} * \\
& \sqrt{b^2 - 4a * c}) * a * b^2 * c + 6 * \sqrt{2} * \sqrt{b^2 - 4a * c}) * \sqrt{b * c + \sqrt{b^2 - 4a * c}}
\end{aligned}$$

$$\begin{aligned}
& *c) *c) *b^3c - 40\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}} *c) \\
& *a^2c^2 - 20\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}} *c) *ab \\
& *c^2 - 3\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}} *c) *b^2c^2 \\
& + 10\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}} *c) *ac^3 - 6*(b \\
& ^2 - 4ac) *b^2c^2 + 20*(b^2 - 4ac) *ac^3) * (a^2b^2 - 4a^3c)^2 *d + 2*(\\
& 2a^2b^2c^2 - 8a^3c^3 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - \\
& 4ac}} *c) *a^2b^2 + 4\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}} \\
& c) *c) *a^3c + 2\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}} *c) *a \\
& ^2bc - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}} *c) *a^2c^2 \\
& - 2*(b^2 - 4ac) *a^2c^2) * (a^2b^2 - 4a^3c)^2 *f - (2ab^3c^2 - 8a^2b \\
& *c^3 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}} *c) *ab^3 + 4* \\
& \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}} *c) *a^2bc + 2\sqrt{2}\sqrt{ \\
& 2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}} *c) *ab^2c - \sqrt{2}\sqrt{ \\
& (b^2 - 4ac)\sqrt{bc + \sqrt{b^2 - 4ac}} *c) *ab^2c^2 - 2*(b^2 - 4ac) *ab \\
& *c^2) * (a^2b^2 - 4a^3c)^2 *e + 2*(3\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}} *c \\
&) *a^2b^7 - 37\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}} *c) *a^3b^5c - 6\sqrt{2} \\
&) * \sqrt{bc + \sqrt{b^2 - 4ac}} *c) *a^2b^6c - 6a^2b^7c + 152\sqrt{2}\sqrt{ \\
& t(bc + \sqrt{b^2 - 4ac}) *c) *a^4b^3c^2 + 50\sqrt{2}\sqrt{bc + \sqrt{b^2 - \\
& 4ac}} *c) *a^3b^4c^2 + 3\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}} *c) *a^2b^5* \\
& c^2 + 74a^3b^5c^2 - 208\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}} *c) *a^5b^3c^ \\
& 3 - 104\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}} *c) *a^4b^2c^3 - 25\sqrt{2}\sqrt{ \\
& rt(bc + \sqrt{b^2 - 4ac}) *c) *a^3b^3c^3 - 304a^4b^3c^3 + 52\sqrt{2}\sqrt{ \\
& rt(bc + \sqrt{b^2 - 4ac}) *c) *a^4b^3c^4 + 416a^5b^3c^4 + 6*(b^2 - 4ac) *a \\
& ^2b^5c - 50*(b^2 - 4ac) *a^3b^3c^2 + 104*(b^2 - 4ac) *a^4b^3c^3) *d *ab \\
& s(a^2b^2 - 4a^3c) - 2*(\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}} *c) *a^4b^5 - \\
& 8\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}} *c) *a^5b^3c - 2\sqrt{2}\sqrt{bc + \\
& \sqrt{b^2 - 4ac}} *c) *a^4b^4c - 2a^4b^5c + 16\sqrt{2}\sqrt{bc + \sqrt{ \\
& b^2 - 4ac}} *c) *a^6b^2c^2 + 8\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}} *c) *a^5b \\
& ^2c^2 + \sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}} *c) *a^4b^3c^2 + 16a^5b^3c \\
& ^2 - 4\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}} *c) *a^5b^3c^3 - 32a^6b^3c^3 + 2 \\
& *(b^2 - 4ac) *a^4b^3c - 8*(b^2 - 4ac) *a^5b^2c^2) *f *abs(a^2b^2 - 4a^3 \\
& *c) - 2*(\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}} *c) *a^3b^6 - 14\sqrt{2}\sqrt{ \\
& bc + \sqrt{b^2 - 4ac}} *c) *a^4b^4c - 2\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}} \\
& c) *c) *a^3b^5c - 2a^3b^6c + 64\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}} *c) * \\
& a^5b^2c^2 + 20\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}} *c) *a^4b^3c^2 + \sqrt{ \\
& 2}\sqrt{bc + \sqrt{b^2 - 4ac}} *c) *a^3b^4c^2 + 28a^4b^4c^2 - 96\sqrt{2}\sqrt{ \\
& 2}\sqrt{bc + \sqrt{b^2 - 4ac}} *c) *a^6c^3 - 48\sqrt{2}\sqrt{bc + \sqrt{b^2 \\
& - 4ac}} *c) *a^5b^3c^3 - 10\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}} *c) *a^4b^2 \\
& *c^3 - 128a^5b^2c^3 + 24\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}} *c) *a^5c^4 \\
& + 192a^6c^4 + 2*(b^2 - 4ac) *a^3b^4c - 20*(b^2 - 4ac) *a^4b^2c^2 + \\
& 48*(b^2 - 4ac) *a^5c^3) *abs(a^2b^2 - 4a^3c) *e + (6a^4b^8c^2 - 80a \\
& ^5b^6c^3 + 352a^6b^4c^4 - 512a^7b^2c^5 - 3\sqrt{2}\sqrt{b^2 - 4ac} \\
&) * \sqrt{bc + \sqrt{b^2 - 4ac}} *c) *a^4b^8 + 40\sqrt{2}\sqrt{b^2 - 4ac} * \sqrt{ \\
& rt(bc + \sqrt{b^2 - 4ac}) *c) *a^5b^6c + 6\sqrt{2}\sqrt{b^2 - 4ac} * \sqrt{ \\
& bc + \sqrt{b^2 - 4ac}} *c) *a^4b^7c - 176\sqrt{2}\sqrt{b^2 - 4ac} * \sqrt{ \\
& bc + \sqrt{b^2 - 4ac}} *c) *a^6b^4c^2 - 56\sqrt{2}\sqrt{b^2 - 4ac} * \sqrt{ \\
& bc + \sqrt{b^2 - 4ac}} *c) *a^5b^5c^2 - 3\sqrt{2}\sqrt{b^2 - 4ac} * \sqrt{ \\
& bc + \sqrt{b^2 - 4ac}} *c) *a^4b^6c^2 + 256\sqrt{2}\sqrt{b^2 - 4ac} * \sqrt{ \\
& bc + \sqrt{b^2 - 4ac}} *c) *a^7b^2c^3 + 128\sqrt{2}\sqrt{b^2 - 4ac} * \sqrt{ \\
& bc + \sqrt{b^2 - 4ac}} *c) *a^6b^3c^3 + 28\sqrt{2}\sqrt{b^2 - 4ac} * \sqrt{ \\
& bc + \sqrt{b^2 - 4ac}} *c) *a^5b^4c^3 - 64\sqrt{2}\sqrt{b^2 - 4ac} * \sqrt{ \\
& bc + \sqrt{b^2 - 4ac}} *c) *a^6b^2c^4 - 6*(b^2 - 4ac) *a^4b^6c^2 + 56* \\
& (b^2 - 4ac) *a^5b^4c^3 - 128*(b^2 - 4ac) *a^6b^2c^4) *d - 4*(2a^6b^6* \\
& c^2 - 16a^7b^4c^3 + 32a^8b^2c^4 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \\
& \sqrt{b^2 - 4ac}} *c) *a^6b^6 + 8\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{ \\
& t(b^2 - 4ac)} *c) *a^7b^4c + 2\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{ \\
& ^2 - 4ac}} *c) *a^6b^5c - 16\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 \\
& - 4ac}} *c) *a^8b^2c^2 - 8\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 \\
& - 4ac}} *c) *a^7b^3c^2 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4
\end{aligned}$$

$$\begin{aligned}
& *a*c)*c)*a^6*b^4*c^2 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*a^5*b^7*c^3 - 2*(b^2 - 4*a*c)*a^6*b^4*c^2 + 8*(b^2 - 4*a*c)*a^7*b^2*c^3)*f - (2*a^5*b^7*c^2 - 40*a^6*b^5*c^3 + 224*a^7*b^3*c^4 - 384*a^8*b*c^5 \\
& - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*a^5*b^7 + 20*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*a^6*b^5*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*a^5*b^6*c - 112*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*a^7*b^3*c^2 - 32*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*a^6*b^4*c^2 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*a^5*b^5*c^2 + 192*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*a^8*b*c^3 + 96*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*a^7*b^2*c^3 + 16*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*a^6*b^3*c^3 - 48*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*a^7*b*c^4 - 2*(b^2 - 4*a*c)*a^5*b^5*c^2 + 32*(b^2 - 4*a*c)*a^6*b^3*c^3 - 96*(b^2 - 4*a*c)*a^7*b*c^4)* \\
& e)*\arctan(2*\sqrt{1/2}*x/\sqrt{(a^2*b^3 - 4*a^3*b*c + \sqrt{(a^2*b^3 - 4*a^3*b*c)^2 - 4*(a^3*b^2 - 4*a^4*c)*(a^2*b^2*c - 4*a^3*c^2)})))/(a^2*b^2*c - 4*a^3*c^2)))/(a^5*b^6 - 12*a^6*b^4*c - 2*a^5*b^5*c + 48*a^7*b^2*c^2 + 16*a^6*b^3*c^2 + a^5*b^4*c^2 - 64*a^8*c^3 - 32*a^7*b*c^3 - 8*a^6*b^2*c^3 + 16*a^7*c^4)*\text{abs}(a^2*b^2 - 4*a^3*c)*\text{abs}(c) + 1/16*((6*b^4*c^2 - 44*a*b^2*c^3 + 80*a^2*c^4 - 3*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*b^4 + 22*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a*b^2*c + 6*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*b^3*c - 40*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a^2*c^2 - 20*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a*b*c^2 - 3*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*b^2*c^2 + 10*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a*c^3 - 6*(b^2 - 4*a*c)*b^2*c^2 + 20*(b^2 - 4*a*c)*a*c^3)*(a^2*b^2 - 4*a^3*c)^2*d + 2*(2*a^2*b^2*c^2 - 8*a^3*c^3 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a^2*b^2 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a^3*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a^2*b*c - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a^2*c^2 - 2*(b^2 - 4*a*c)*a^2*c^2)*(a^2*b^2 - 4*a^3*c)^2*f - (2*a*b^3*c^2 - 8*a^2*b*c^3 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a*b^3 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a^2*b*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a*b^2*c - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a*b*c^2 - 2*(b^2 - 4*a*c)*a*b*c^2)*(a^2*b^2 - 4*a^3*c)^2*e - 2*(3*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a^2*b^7 - 37*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a^3*b^5*c - 6*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a^2*b^6*c + 6*a^2*b^7*c + 152*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a^4*b^3*c^2 + 50*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a^3*b^4*c^2 + 3*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a^2*b^5*c^2 - 74*a^3*b^5*c^2 - 208*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a^5*b*c^3 - 104*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a^4*b^2*c^3 - 25*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a^3*b^3*c^3 + 304*a^4*b^3*c^3 + 52*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a^4*b*c^4 - 416*a^5*b*c^4 - 6*(b^2 - 4*a*c)*a^2*b^5*c + 50*(b^2 - 4*a*c)*a^3*b^3*c^2 - 104*(b^2 - 4*a*c)*a^4*b*c^3)*d*\text{abs}(a^2*b^2 - 4*a^3*c) + 2*(\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a^4*b^5 - 8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a^5*b^3*c - 2*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a^4*b^4*c + 2*a^4*b^5*c + 16*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a^6*b*c^2 + 8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a^5*b^2*c^2 + \sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a^4*b^3*c^2 - 16*a^5*b^3*c^2 - 4*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a^5*b*c^3 + 32*a^6*b*c^3 - 2*(b^2 - 4*a*c)*a^4*b^3*c + 8*(b^2 - 4*a*c)*a^5*b*c^2)*f*\text{abs}(a^2*b^2 - 4*a^3*c) + 2*(\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a^3*b^6 - 14*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a^4*b^4*c - 2*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a^3*b^5*c + 2*a^3*b^6*c + 64*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a^5*b^2*c^2 + 20*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a^4*b^3*c^2 + \sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a^3*b^4*c^2 - 28*a^4*b^4*c^2 - 96*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a^6*c^3 - 48*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a^5*b*c^3 - 10*
\end{aligned}$$

```

sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^4*b^2*c^3 + 128*a^5*b^2*c^3 + 24*
sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^5*c^4 - 192*a^6*c^4 - 2*(b^2 - 4*
a*c)*a^3*b^4*c + 20*(b^2 - 4*a*c)*a^4*b^2*c^2 - 48*(b^2 - 4*a*c)*a^5*c^3)*a
bs(a^2*b^2 - 4*a^3*c)*e + (6*a^4*b^8*c^2 - 80*a^5*b^6*c^3 + 352*a^6*b^4*c^4
- 512*a^7*b^2*c^5 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c))*a^4*b^8 + 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*
a^5*b^6*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a
^4*b^7*c - 176*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^
6*b^4*c^2 - 56*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^
5*b^5*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^4
*b^6*c^2 + 256*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^
7*b^2*c^3 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a
^6*b^3*c^3 + 28*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a
^5*b^4*c^3 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a
^6*b^2*c^4 - 6*(b^2 - 4*a*c)*a^4*b^6*c^2 + 56*(b^2 - 4*a*c)*a^5*b^4*c^3 - 1
28*(b^2 - 4*a*c)*a^6*b^2*c^4)*d - 4*(2*a^6*b^6*c^2 - 16*a^7*b^4*c^3 + 32*a^
8*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^6*b
^6 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^7*b^4*c
+ 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^6*b^5*c - 1
6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^8*b^2*c^2 - 8
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^7*b^3*c^2 - sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^6*b^4*c^2 + 4*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^7*b^2*c^3 - 2*(b^2
- 4*a*c)*a^6*b^4*c^2 + 8*(b^2 - 4*a*c)*a^7*b^2*c^3)*f - (2*a^5*b^7*c^2 - 4
0*a^6*b^5*c^3 + 224*a^7*b^3*c^4 - 384*a^8*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^5*b^7 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c))*a^6*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c))*a^5*b^6*c - 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c))*a^7*b^3*c^2 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c))*a^6*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c))*a^5*b^5*c^2 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c))*a^8*b*c^3 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c))*a^7*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c))*a^6*b^3*c^3 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c))*a^7*b*c^4 - 2*(b^2 - 4*a*c)*a^5*b^5*c^2 + 32*(b^2 - 4*
a*c)*a^6*b^3*c^3 - 96*(b^2 - 4*a*c)*a^7*b*c^4)*e)*arctan(2*sqrt(1/2)*x/sqrt
((a^2*b^3 - 4*a^3*b*c - sqrt((a^2*b^3 - 4*a^3*b*c)^2 - 4*(a^3*b^2 - 4*a^4*c
)*(a^2*b^2*c - 4*a^3*c^2)))/(a^2*b^2*c - 4*a^3*c^2)))/((a^5*b^6 - 12*a^6*b^
4*c - 2*a^5*b^5*c + 48*a^7*b^2*c^2 + 16*a^6*b^3*c^2 + a^5*b^4*c^2 - 64*a^8*
c^3 - 32*a^7*b*c^3 - 8*a^6*b^2*c^3 + 16*a^7*c^4)*abs(a^2*b^2 - 4*a^3*c)*abs
(c))

```

maple [B] time = 0.05, size = 1575, normalized size = 3.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a)^2,x)

```

[Out] 4/a*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*d+4/a*c^2/(4*a*
c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh
(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*d-3/4/a^2*c/(4*a*c-b^2)/
(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)
/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3*d-3/4/a^2*c/(4*a*c-b^2)/(-4*a*c
+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3*d+1/4/a*c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)
*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)
)^(1/2))*c)^(1/2)*c*x)*b^2*e+1/4/a*c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)
/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)

```

$$\begin{aligned} & \sqrt{c}x) * b^2 e^{-d/a^2/x + 1/4/a*c/(4*a*c - b^2)} * 2^{(1/2)} / ((-b + (-4*a*c + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctanh}(2^{(1/2)} / ((-b + (-4*a*c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * b * e^{-1/a} \\ & / (c * x^4 + b * x^2 + a) * c^2 / (4 * a * c - b^2) * x^3 d - 1/2/a / (c * x^4 + b * x^2 + a) / (4 * a * c - b^2) * x * \\ & b^2 e + 1/2/a^2 / (c * x^4 + b * x^2 + a) / (4 * a * c - b^2) * x * b^3 d - 1/2 * c / (4 * a * c - b^2) * 2^{(1/2)} \\ & / ((-b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctanh}(2^{(1/2)} / ((-b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * f + 1/2 * c / (4 * a * c - b^2) * 2^{(1/2)} / ((b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} \\ & * \operatorname{arctan}(2^{(1/2)} / ((b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * f - 1/4/a * c / (4 * a * c - b^2) \\ & * 2^{(1/2)} / ((b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctan}(2^{(1/2)} / ((b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * b * e + c / (4 * a * c - b^2) / (-4 * a * c + b^2)^{(1/2)} * 2^{(1/2)} / ((-b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctanh}(2^{(1/2)} / ((-b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * b * f + c / (4 * a * c - b^2) / (-4 * a * c + b^2)^{(1/2)} * 2^{(1/2)} / ((b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctan}(2^{(1/2)} / ((b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * b * f - 3/4/a^2 * c / (4 * a * c - b^2) * 2^{(1/2)} / ((-b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctanh}(2^{(1/2)} / ((-b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * b^2 d + 3/4/a^2 * c / (4 * a * c - b^2) * 2^{(1/2)} / ((b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctan}(2^{(1/2)} / ((b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * b^2 d + 1 / (c * x^4 + b * x^2 + a) / (4 * a * c - b^2) * x * c * e + 1/2 / (c * x^4 + b * x^2 + a) / (4 * a * c - b^2) * x * b * f + c / (c * x^4 + b * x^2 + a) / (4 * a * c - b^2) * x^3 * f + 1/2/a^2 / (c * x^4 + b * x^2 + a) * c / (4 * a * c - b^2) * x^3 * b^2 d + 5/2/a * c^2 / (4 * a * c - b^2) * 2^{(1/2)} / ((-b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctanh}(2^{(1/2)} / ((-b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * d - 5/2/a * c^2 / (4 * a * c - b^2) * 2^{(1/2)} / ((b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctan}(2^{(1/2)} / ((b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * d - 3 * c^2 / (4 * a * c - b^2) / (-4 * a * c + b^2)^{(1/2)} * 2^{(1/2)} / ((b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctan}(2^{(1/2)} / ((b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * e - 3 * c^2 / (4 * a * c - b^2) / (-4 * a * c + b^2)^{(1/2)} * 2^{(1/2)} / ((-b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctanh}(2^{(1/2)} / ((-b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * e - 1/2/a / (c * x^4 + b * x^2 + a) * c / (4 * a * c - b^2) * x^3 * b * e - 3/2/a / (c * x^4 + b * x^2 + a) / (4 * a * c - b^2) * x * b * c * d \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(abce - 2a^2cf - (3b^2c - 10ac^2)d)x^4 - (a^2bf + (3b^3 - 11abc)d - (ab^2 - 2a^2c)e)x^2 - 2(ab^2 - 4a^2c)d}{2((a^2b^2c - 4a^3c^2)x^5 + (a^2b^3 - 4a^3bc)x^3 + (a^3b^2 - 4a^4c)x)} - \int \frac{a^2bf + (abce - 2a^2cf - (3b^2c - 10ac^2)d)x^2 - (3b^3 - 11abc)d + (ab^2 - 6a^2c)e}{c x^4 + b x^2 + a} dx}{2(a^2b^2 - 4a^3c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")

[Out] 1/2*((a*b*c*e - 2*a^2*c*f - (3*b^2*c - 10*a*c^2)*d)*x^4 - (a^2*b*f + (3*b^3 - 11*a*b*c)*d - (a*b^2 - 2*a^2*c)*e)*x^2 - 2*(a*b^2 - 4*a^2*c)*d)/((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x) - 1/2*integrate(-(a^2*b*f + (a*b*c*e - 2*a^2*c*f - (3*b^2*c - 10*a*c^2)*d)*x^2 - (3*b^3 - 13*a*b*c)*d + (a*b^2 - 6*a^2*c)*e)/(c*x^4 + b*x^2 + a), x)/(a^2*b^2 - 4*a^3*c)

mupad [B] time = 6.86, size = 28164, normalized size = 70.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x^2 + f*x^4)/(x^2*(a + b*x^2 + c*x^4)^2),x)

[Out] ((x^2*(3*b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 11*a*b*c*d))/(2*a^2*(4*a*c - b^2)) - d/a + (c*x^4*(3*b^2*d + 2*a^2*f - a*b*e - 10*a*c*d))/(2*a^2*(4*a*c - b^2)))/(a*x + b*x^3 + c*x^5) - atan(((x*(204800*a^12*c^9*d^2 - 73728*a^13*c^8*e^2 + 8192*a^14*c^7*f^2 + 144*a^6*b^12*c^3*d^2 - 3264*a^7*b^10*c^4*d^2 + 30112*a^8*b^8*c^5*d^2 - 143360*a^9*b^6*c^6*d^2 + 365568*a^10*b^4*c^7*d^2 - 458752*a^11*b^2*c^8*d^2 + 16*a^8*b^10*c^3*e^2 - 416*a^9*b^8*c^4*e^2 + 4608*a^10*b^6*c^5*e^2 - 25600*a^11*b^4*c^6*e^2 + 69632*a^12*b^2*c^7*e^2 + 160*a^10*b^8*c^3*f^2 - 2048*a^11*b^6*c^4*f^2 + 9216*a^12*b^4*c^5*f^2 - 16384*a^13*b^2*c^6*f^2 - 81920*a^13*c^8*d*f + 237568*a^12*b*c^8*d*e + 40960*a^13*b*c^7*e*f - 96*a^7*b^11*c^3*d*e + 2336*a^8*b^9*c^4*d*e - 22528*a^9*b^7*c^5*d*e + 107520*a^10*b^5*c^6*d*e - 253952*a^11*b^3*c^7*d*e - 96*a^8*b^10*c^3*d*f + 1472*a^9*b^8*c^4*d*f - 7168*a^10*b^6*c^5*d*f + 6144*a^11*b^4*c^6*d*f

$$\begin{aligned}
& + 40960a^{12}b^2c^7d^2f + 32a^9b^9c^3e^2f - 1024a^{10}b^7c^4e^2f + 92 \\
& 16a^{11}b^5c^5e^2f - 32768a^{12}b^3c^6e^2f) + ((27a^3b^9c^2e^2 - a^2b^{11}e^2 - 9b^4d^2(-4ac - b^2)^9)^{1/2} - a^4b^9f^2 - a^4f^2(-4ac - b^2)^9)^{1/2} - 26880a^6b^3c^6d^2 - 9b^{13}d^2 + 3840a^7b^3c^5e^2 + \\
& 9a^3c^2e^2(-4ac - b^2)^9)^{1/2} + 768a^8b^3c^4f^2 + 6a^2b^{12}d^2e - 2077a^2b^9c^2d^2 + 10656a^3b^7c^3d^2 - 30240a^4b^5c^4d^2 + 4480 \\
& 0a^5b^3c^5d^2 - a^2b^2e^2(-4ac - b^2)^9)^{1/2} - 25a^2c^2d^2(-4ac - b^2)^9)^{1/2} - 288a^4b^7c^2e^2 + 1504a^5b^5c^3e^2 - 3840 \\
& a^6b^3c^4e^2 + 96a^6b^5c^2f^2 - 512a^7b^3c^3f^2 + 213a^2b^{11}c^2d^2 + 6a^2b^{11}d^2f + 15360a^7c^6d^2e - 2a^3b^{10}e^2f - 3072a^8c^5e^2f \\
& + 6a^2b^3d^2e(-4ac - b^2)^9)^{1/2} - 152a^2b^{10}c^2d^2e - 98a^3b^9c^2d^2e + 1536a^7b^3c^5d^2e - 2a^3b^2e^2f(-4ac - b^2)^9)^{1/2} + 10a^3c^2d^2e^2f(-4ac - b^2)^9)^{1/2} + 36a^4b^8c^2e^2f + 51a^2b^2c^2d^2(-4ac - b^2)^9)^{1/2} + 1548a^3b^8c^2d^2e - 8064a^4b^6c^3d^2e + 22400a^5b^4c^4d^2e - 30720a^6b^2c^5d^2e + 6a^2b^2d^2e^2(-4ac - b^2)^9)^{1/2} + 576a^4b^7c^2d^2e - 1344a^5b^5c^3d^2e + 512a^6b^3c^4d^2e - 192a^5b^6c^2e^2f + 128a^6b^4c^3e^2f + 1536a^7b^2c^4e^2f - 44a^2b^3c^2d^2e^2(-4ac - b^2)^9)^{1/2} / (32(a^5b^{12} + 4096a^{11}c^6 - 24a^6b^{10}c^3 + 240a^7b^8c^2 - 1280a^8b^6c^3 + 3840a^9b^4c^4 - 6144a^{10}b^2c^5))^{1/2} * (x((27a^3b^9c^2e^2 - a^2b^{11}e^2 - 9b^4d^2(-4ac - b^2)^9)^{1/2} - a^4b^9f^2 - a^4f^2(-4ac - b^2)^9)^{1/2} - 26880a^6b^3c^6d^2 - 9b^{13}d^2 + 3840a^7b^3c^5e^2 + 9a^3c^2e^2(-4ac - b^2)^9)^{1/2} + 768a^8b^3c^4f^2 + 6a^2b^{12}d^2e - 2077a^2b^9c^2d^2 + 10656a^3b^7c^3d^2 - 30240a^4b^5c^4d^2 + 44800a^5b^3c^5d^2 - a^2b^2e^2(-4ac - b^2)^9)^{1/2} - 25a^2c^2d^2(-4ac - b^2)^9)^{1/2} - 288a^4b^7c^2e^2 + 1504a^5b^5c^3e^2 - 3840a^6b^3c^4e^2 + 96a^6b^5c^2f^2 - 512a^7b^3c^3f^2 + 213a^2b^{11}c^2d^2 + 6a^2b^{11}d^2f + 15360a^7c^6d^2e - 2a^3b^{10}e^2f - 3072a^8c^5e^2f + 6a^2b^3d^2e(-4ac - b^2)^9)^{1/2} - 152a^2b^{10}c^2d^2e - 98a^3b^9c^2d^2e + 1536a^7b^3c^5d^2e - 2a^3b^2e^2f(-4ac - b^2)^9)^{1/2} + 10a^3c^2d^2e^2f(-4ac - b^2)^9)^{1/2} + 36a^4b^8c^2e^2f + 51a^2b^2c^2d^2(-4ac - b^2)^9)^{1/2} + 1548a^3b^8c^2d^2e - 8064a^4b^6c^3d^2e + 22400a^5b^4c^4d^2e - 30720a^6b^2c^5d^2e + 6a^2b^2d^2e^2(-4ac - b^2)^9)^{1/2} + 576a^4b^7c^2d^2e - 1344a^5b^5c^3d^2e + 512a^6b^3c^4d^2e - 192a^5b^6c^2e^2f + 128a^6b^4c^3e^2f + 1536a^7b^2c^4e^2f - 44a^2b^3c^2d^2e^2(-4ac - b^2)^9)^{1/2} / (32(a^5b^{12} + 4096a^{11}c^6 - 24a^6b^{10}c^3 + 240a^7b^8c^2 - 1280a^8b^6c^3 + 3840a^9b^4c^4 - 6144a^{10}b^2c^5))^{1/2} * (1048576a^{16}b^8c^8 + 256a^{10}b^{13}c^2 - 6144a^{11}b^{11}c^3 + 61440a^{12}b^9c^4 - 327680a^{13}b^7c^5 + 983040a^{14}b^5c^6 - 1572864a^{15}b^3c^7) - 393216a^{15}c^8e + 192a^8b^{13}c^2d - 4672a^9b^{11}c^3d + 47360a^{10}b^9c^4d - 256000a^{11}b^7c^5d + 778240a^{12}b^5c^6d - 1261568a^{13}b^3c^7d - 64a^9b^{12}c^2e + 1664a^{10}b^{10}c^3e - 17920a^{11}b^8c^4e + 102400a^{12}b^6c^5e - 327680a^{13}b^4c^6e + 557056a^{14}b^2c^7e - 64a^{10}b^{11}c^2f + 1280a^{11}b^9c^3f - 10240a^{12}b^7c^4f + 40960a^{13}b^5c^5f - 81920a^{14}b^3c^6f + 851968a^{14}b^3c^8d + 65536a^{15}b^3c^7f) * ((27a^3b^9c^2e^2 - a^2b^{11}e^2 - 9b^4d^2(-4ac - b^2)^9)^{1/2} - a^4b^9f^2 - a^4f^2(-4ac - b^2)^9)^{1/2} - 26880a^6b^3c^6d^2 - 9b^{13}d^2 + 3840a^7b^3c^5e^2 + 9a^3c^2e^2(-4ac - b^2)^9)^{1/2} + 768a^8b^3c^4f^2 + 6a^2b^{12}d^2e - 2077a^2b^9c^2d^2 + 10656a^3b^7c^3d^2 - 30240a^4b^5c^4d^2 + 44800a^5b^3c^5d^2 - a^2b^2e^2(-4ac - b^2)^9)^{1/2} - 25a^2c^2d^2(-4ac - b^2)^9)^{1/2} - 288a^4b^7c^2e^2 + 1504a^5b^5c^3e^2 - 3840a^6b^3c^4e^2 + 96a^6b^5c^2f^2 - 512a^7b^3c^3f^2 + 213a^2b^{11}c^2d^2 + 6a^2b^{11}d^2f + 15360a^7c^6d^2e - 2a^3b^{10}e^2f - 3072a^8c^5e^2f + 6a^2b^3d^2e(-4ac - b^2)^9)^{1/2} - 152a^2b^{10}c^2d^2e - 98a^3b^9c^2d^2e + 1536a^7b^3c^5d^2e - 2a^3b^2e^2f(-4ac - b^2)^9)^{1/2} + 10a^3c^2d^2e^2f(-4ac - b^2)^9)^{1/2} + 36a^4b^8c^2e^2f + 51a^2b^2c^2d^2(-4ac - b^2)^9)^{1/2} + 1548a^3b^8c^2d^2e - 8064a^4b^6c^3d^2e + 22400a^5b^4c^4d^2e - 30720a^6b^2c^5d^2e + 6a^2b^2d^2e^2(-4ac - b^2)^9)^{1/2} + 576a^4b^7c^2d^2e - 1344a^5b^5c^3d^2e + 512a^6b^3c^4d^2e
\end{aligned}$$

$$\begin{aligned}
& - 192a^5b^6c^2e^f + 128a^6b^4c^3e^f + 1536a^7b^2c^4e^f - 44a^2 \\
& *b*c*d*e*(-(4*a*c - b^2)^9)^{(1/2)}/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b \\
& ^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5))^{(1/2)}*1i + (x*(204800*a^12*c^9*d^2 - 73728*a^13*c^8*e^2 + 8192*a^ \\
& 14*c^7*f^2 + 144*a^6*b^12*c^3*d^2 - 3264*a^7*b^10*c^4*d^2 + 30112*a^8*b^8*c \\
& ^5*d^2 - 143360*a^9*b^6*c^6*d^2 + 365568*a^10*b^4*c^7*d^2 - 458752*a^11*b^2 \\
& *c^8*d^2 + 16*a^8*b^10*c^3*e^2 - 416*a^9*b^8*c^4*e^2 + 4608*a^10*b^6*c^5*e^ \\
& 2 - 25600*a^11*b^4*c^6*e^2 + 69632*a^12*b^2*c^7*e^2 + 160*a^10*b^8*c^3*f^2 \\
& - 2048*a^11*b^6*c^4*f^2 + 9216*a^12*b^4*c^5*f^2 - 16384*a^13*b^2*c^6*f^2 - \\
& 81920*a^13*c^8*d*f + 237568*a^12*b*c^8*d*e + 40960*a^13*b*c^7*e^f - 96*a^7* \\
& b^11*c^3*d*e + 2336*a^8*b^9*c^4*d*e - 22528*a^9*b^7*c^5*d*e + 107520*a^10*b \\
& ^5*c^6*d*e - 253952*a^11*b^3*c^7*d*e - 96*a^8*b^10*c^3*d*f + 1472*a^9*b^8*c \\
& ^4*d*f - 7168*a^10*b^6*c^5*d*f + 6144*a^11*b^4*c^6*d*f + 40960*a^12*b^2*c^7 \\
& *d*f + 32*a^9*b^9*c^3*e^f - 1024*a^10*b^7*c^4*e^f + 9216*a^11*b^5*c^5*e^f - \\
& 32768*a^12*b^3*c^6*e^f) + ((27*a^3*b^9*c*e^2 - a^2*b^11*e^2 - 9*b^4*d^2*(- \\
& (4*a*c - b^2)^9)^{(1/2)} - a^4*b^9*f^2 - a^4*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 2 \\
& 6880*a^6*b*c^6*d^2 - 9*b^13*d^2 + 3840*a^7*b*c^5*e^2 + 9*a^3*c*e^2*(-(4*a*c \\
& - b^2)^9)^{(1/2)} + 768*a^8*b*c^4*f^2 + 6*a*b^12*d*e - 2077*a^2*b^9*c^2*d^2 \\
& + 10656*a^3*b^7*c^3*d^2 - 30240*a^4*b^5*c^4*d^2 + 44800*a^5*b^3*c^5*d^2 - a \\
& ^2*b^2*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 25*a^2*c^2*d^2*(-(4*a*c - b^2)^9)^{(1/ \\
& 2)} - 288*a^4*b^7*c^2*e^2 + 1504*a^5*b^5*c^3*e^2 - 3840*a^6*b^3*c^4*e^2 + 96 \\
& *a^6*b^5*c^2*f^2 - 512*a^7*b^3*c^3*f^2 + 213*a*b^11*c*d^2 + 6*a^2*b^11*d*f \\
& + 15360*a^7*c^6*d*e - 2*a^3*b^10*e^f - 3072*a^8*c^5*e^f + 6*a*b^3*d*e*(-(4* \\
& a*c - b^2)^9)^{(1/2)} - 152*a^2*b^10*c*d*e - 98*a^3*b^9*c*d*f + 1536*a^7*b*c^ \\
& 5*d*f - 2*a^3*b*e^f*(-(4*a*c - b^2)^9)^{(1/2)} + 10*a^3*c*d*f*(-(4*a*c - b^2) \\
& ^9)^{(1/2)} + 36*a^4*b^8*c*e^f + 51*a*b^2*c*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + 15 \\
& 48*a^3*b^8*c^2*d*e - 8064*a^4*b^6*c^3*d*e + 22400*a^5*b^4*c^4*d*e - 30720*a \\
& ^6*b^2*c^5*d*e + 6*a^2*b^2*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 576*a^4*b^7*c^2*d \\
& *f - 1344*a^5*b^5*c^3*d*f + 512*a^6*b^3*c^4*d*f - 192*a^5*b^6*c^2*e^f + 128 \\
& *a^6*b^4*c^3*e^f + 1536*a^7*b^2*c^4*e^f - 44*a^2*b*c*d*e*(-(4*a*c - b^2)^9) \\
& ^{(1/2)}/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1 \\
& 280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5))^{(1/2)}*(393216*a^1 \\
& 5*c^8*e + x*((27*a^3*b^9*c*e^2 - a^2*b^11*e^2 - 9*b^4*d^2*(-(4*a*c - b^2)^9) \\
&)^{(1/2)} - a^4*b^9*f^2 - a^4*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 26880*a^6*b*c^6* \\
& d^2 - 9*b^13*d^2 + 3840*a^7*b*c^5*e^2 + 9*a^3*c*e^2*(-(4*a*c - b^2)^9)^{(1/2) \\
&) + 768*a^8*b*c^4*f^2 + 6*a*b^12*d*e - 2077*a^2*b^9*c^2*d^2 + 10656*a^3*b^7 \\
& *c^3*d^2 - 30240*a^4*b^5*c^4*d^2 + 44800*a^5*b^3*c^5*d^2 - a^2*b^2*e^2*(-(4 \\
& *a*c - b^2)^9)^{(1/2)} - 25*a^2*c^2*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - 288*a^4*b^ \\
& 7*c^2*e^2 + 1504*a^5*b^5*c^3*e^2 - 3840*a^6*b^3*c^4*e^2 + 96*a^6*b^5*c^2*f^ \\
& 2 - 512*a^7*b^3*c^3*f^2 + 213*a*b^11*c*d^2 + 6*a^2*b^11*d*f + 15360*a^7*c^6 \\
& *d*e - 2*a^3*b^10*e^f - 3072*a^8*c^5*e^f + 6*a*b^3*d*e*(-(4*a*c - b^2)^9)^{(\\
& 1/2)} - 152*a^2*b^10*c*d*e - 98*a^3*b^9*c*d*f + 1536*a^7*b*c^5*d*f - 2*a^3*b \\
& *e^f*(-(4*a*c - b^2)^9)^{(1/2)} + 10*a^3*c*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 36* \\
& a^4*b^8*c*e^f + 51*a*b^2*c*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + 1548*a^3*b^8*c^2* \\
& d*e - 8064*a^4*b^6*c^3*d*e + 22400*a^5*b^4*c^4*d*e - 30720*a^6*b^2*c^5*d*e \\
& + 6*a^2*b^2*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 576*a^4*b^7*c^2*d*f - 1344*a^5*b \\
& ^5*c^3*d*f + 512*a^6*b^3*c^4*d*f - 192*a^5*b^6*c^2*e^f + 128*a^6*b^4*c^3*e* \\
& f + 1536*a^7*b^2*c^4*e^f - 44*a^2*b*c*d*e*(-(4*a*c - b^2)^9)^{(1/2)}/(32*(a^ \\
& 5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 \\
& + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5))^{(1/2)}*(1048576*a^16*b*c^8 + 256* \\
& a^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c \\
& ^5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b^3*c^7) - 192*a^8*b^13*c^2*d + 467 \\
& 2*a^9*b^11*c^3*d - 47360*a^10*b^9*c^4*d + 256000*a^11*b^7*c^5*d - 778240*a^ \\
& 12*b^5*c^6*d + 1261568*a^13*b^3*c^7*d + 64*a^9*b^12*c^2*e - 1664*a^10*b^10* \\
& c^3*e + 17920*a^11*b^8*c^4*e - 102400*a^12*b^6*c^5*e + 327680*a^13*b^4*c^6* \\
& e - 557056*a^14*b^2*c^7*e + 64*a^10*b^11*c^2*f - 1280*a^11*b^9*c^3*f + 1024 \\
& 0*a^12*b^7*c^4*f - 40960*a^13*b^5*c^5*f + 81920*a^14*b^3*c^6*f - 851968*a^1 \\
& 4*b*c^8*d - 65536*a^15*b*c^7*f))*((27*a^3*b^9*c*e^2 - a^2*b^11*e^2 - 9*b^4* \\
& d^2*(-(4*a*c - b^2)^9)^{(1/2)} - a^4*b^9*f^2 - a^4*f^2*(-(4*a*c - b^2)^9)^{(1/
\end{aligned}$$

$$\begin{aligned}
& 2) - 26880a^6b^6c^6d^2 - 9b^{13}d^2 + 3840a^7b^6c^5e^2 + 9a^3c^3e^2(- \\
& (4ac - b^2)^9)^{(1/2)} + 768a^8b^6c^4f^2 + 6a^6b^{12}de - 2077a^2b^9c^2 \\
& d^2 + 10656a^3b^7c^3d^2 - 30240a^4b^5c^4d^2 + 44800a^5b^3c^5d^2 \\
& - a^2b^2e^2(-(4ac - b^2)^9)^{(1/2)} - 25a^2c^2d^2(-(4ac - b^2)^9)^{(1/2)} \\
& - 288a^4b^7c^2e^2 + 1504a^5b^5c^3e^2 - 3840a^6b^3c^4e^2 \\
& + 96a^6b^5c^2f^2 - 512a^7b^3c^3f^2 + 213a^6b^{11}cd^2 + 6a^2b^{11} \\
& d^2 + 15360a^7c^6d^2e - 2a^3b^{10}ef - 3072a^8c^5ef + 6a^6b^3d^2e \\
& *(-(4ac - b^2)^9)^{(1/2)} - 152a^2b^{10}cde - 98a^3b^9cdf + 1536a^7 \\
& b^6c^5d^2ef - 2a^3b^8ef*(-(4ac - b^2)^9)^{(1/2)} + 10a^3c^2d^2ef*(-(4ac \\
& - b^2)^9)^{(1/2)} + 36a^4b^8c^2ef + 51a^6b^2c^2d^2*(-(4ac - b^2)^9)^{(1/2)} \\
&) + 1548a^3b^8c^2de - 8064a^4b^6c^3de + 22400a^5b^4c^4de - 3 \\
& 0720a^6b^2c^5de + 6a^2b^2d^2ef*(-(4ac - b^2)^9)^{(1/2)} + 576a^4b^7 \\
& c^2d^2ef - 1344a^5b^5c^3d^2ef + 512a^6b^3c^4d^2ef - 192a^5b^6c^2e^2ef \\
& + 128a^6b^4c^3e^2ef + 1536a^7b^2c^4e^2ef - 44a^2b^6c^2d^2ef*(-(4ac - b \\
& ^2)^9)^{(1/2)})/(32*(a^5b^{12} + 4096a^{11}c^6 - 24a^6b^{10}c + 240a^7b^8c^2 \\
& - 1280a^8b^6c^3 + 3840a^9b^4c^4 - 6144a^{10}b^2c^5)))^{(1/2)}*1i)/(\\
& (x*(204800a^{12}c^9d^2 - 73728a^{13}c^8e^2 + 8192a^{14}c^7f^2 + 144a^6b^{12} \\
& c^3d^2 - 3264a^7b^{10}c^4d^2 + 30112a^8b^8c^5d^2 - 143360a^9b^6 \\
& c^6d^2 + 365568a^{10}b^4c^7d^2 - 458752a^{11}b^2c^8d^2 + 16a^8b^{11} \\
& 0c^3e^2 - 416a^9b^8c^4e^2 + 4608a^{10}b^6c^5e^2 - 25600a^{11}b^4c^6 \\
& e^2 + 69632a^{12}b^2c^7e^2 + 160a^{10}b^8c^3f^2 - 2048a^{11}b^6c^4f^2 \\
& + 9216a^{12}b^4c^5f^2 - 16384a^{13}b^2c^6f^2 - 81920a^{13}c^8d^2ef + \\
& 237568a^{12}b^6c^8d^2e + 40960a^{13}b^4c^7e^2ef - 96a^7b^{11}c^3d^2e + 2336a^8 \\
& b^9c^4d^2e - 22528a^9b^7c^5d^2e + 107520a^{10}b^5c^6d^2e - 253952a^{11} \\
& b^3c^7d^2e - 96a^8b^{10}c^3d^2ef + 1472a^9b^8c^4d^2ef - 7168a^{10}b^6 \\
& c^5d^2ef + 6144a^{11}b^4c^6d^2ef + 40960a^{12}b^2c^7d^2ef + 32a^9b^9c^3 \\
& e^2ef - 1024a^{10}b^7c^4e^2ef + 9216a^{11}b^5c^5e^2ef - 32768a^{12}b^3c^6e^2 \\
& ef) + ((27a^3b^9c^3e^2 - a^2b^{11}e^2 - 9b^4d^2*(-(4ac - b^2)^9)^{(1/2)} \\
&) - a^4b^9f^2 - a^4f^2*(-(4ac - b^2)^9)^{(1/2)} - 26880a^6b^6c^6d^2 - \\
& 9b^{13}d^2 + 3840a^7b^6c^5e^2 + 9a^3c^3e^2*(-(4ac - b^2)^9)^{(1/2)} + 76 \\
& 8a^8b^6c^4f^2 + 6a^6b^{12}de - 2077a^2b^9c^2d^2 + 10656a^3b^7c^3d^2 \\
& - 30240a^4b^5c^4d^2 + 44800a^5b^3c^5d^2 - a^2b^2e^2*(-(4ac - b^2)^9)^{(1/2)} \\
& - 25a^2c^2d^2*(-(4ac - b^2)^9)^{(1/2)} - 288a^4b^7c^2e^2 \\
& + 1504a^5b^5c^3e^2 - 3840a^6b^3c^4e^2 + 96a^6b^5c^2f^2 - 51 \\
& 2a^7b^3c^3f^2 + 213a^6b^{11}cd^2 + 6a^2b^{11}d^2 + 15360a^7c^6d^2e - \\
& 2a^3b^{10}ef - 3072a^8c^5ef + 6a^6b^3d^2e*(-(4ac - b^2)^9)^{(1/2)} - \\
& 152a^2b^{10}cde - 98a^3b^9cdf + 1536a^7b^6c^5d^2ef - 2a^3b^8ef*(-(4ac \\
& - b^2)^9)^{(1/2)} + 10a^3c^2d^2ef*(-(4ac - b^2)^9)^{(1/2)} + 36a^4b^8 \\
& c^2ef + 51a^6b^2c^2d^2*(-(4ac - b^2)^9)^{(1/2)} + 1548a^3b^8c^2de - \\
& 8064a^4b^6c^3de + 22400a^5b^4c^4de - 30720a^6b^2c^5de + 6a^2 \\
& b^2d^2ef*(-(4ac - b^2)^9)^{(1/2)} + 576a^4b^7c^2d^2ef - 1344a^5b^5c^3 \\
& d^2ef + 512a^6b^3c^4d^2ef - 192a^5b^6c^2e^2ef + 128a^6b^4c^3e^2ef + 15 \\
& 36a^7b^2c^4e^2ef - 44a^2b^6c^2d^2ef*(-(4ac - b^2)^9)^{(1/2)})/(32*(a^5b^{12} \\
& + 4096a^{11}c^6 - 24a^6b^{10}c + 240a^7b^8c^2 - 1280a^8b^6c^3 + 384 \\
& 0a^9b^4c^4 - 6144a^{10}b^2c^5)))^{(1/2)}*(x*((27a^3b^9c^3e^2 - a^2b^{11} \\
& e^2 - 9b^4d^2*(-(4ac - b^2)^9)^{(1/2)} - a^4b^9f^2 - a^4f^2*(-(4ac \\
& - b^2)^9)^{(1/2)} - 26880a^6b^6c^6d^2 - 9b^{13}d^2 + 3840a^7b^6c^5e^2 + 9 \\
& a^3c^3e^2*(-(4ac - b^2)^9)^{(1/2)} + 768a^8b^6c^4f^2 + 6a^6b^{12}de - 20 \\
& 77a^2b^9c^2d^2 + 10656a^3b^7c^3d^2 - 30240a^4b^5c^4d^2 + 44800a^5 \\
& b^3c^5d^2 - a^2b^2e^2*(-(4ac - b^2)^9)^{(1/2)} - 25a^2c^2d^2*(-(4 \\
& 4ac - b^2)^9)^{(1/2)} - 288a^4b^7c^2e^2 + 1504a^5b^5c^3e^2 - 3840a^6 \\
& b^3c^4e^2 + 96a^6b^5c^2f^2 - 512a^7b^3c^3f^2 + 213a^6b^{11}cd^2 \\
& + 6a^2b^{11}d^2 + 15360a^7c^6d^2e - 2a^3b^{10}ef - 3072a^8c^5ef \\
& + 6a^6b^3d^2e*(-(4ac - b^2)^9)^{(1/2)} - 152a^2b^{10}cde - 98a^3b^9cdf \\
& + 1536a^7b^6c^5d^2ef - 2a^3b^8ef*(-(4ac - b^2)^9)^{(1/2)} + 10a^3c^2 \\
& d^2ef*(-(4ac - b^2)^9)^{(1/2)} + 36a^4b^8c^2ef + 51a^6b^2c^2d^2*(-(4ac - \\
& b^2)^9)^{(1/2)} + 1548a^3b^8c^2de - 8064a^4b^6c^3de + 22400a^5b^4 \\
& c^4de - 30720a^6b^2c^5de + 6a^2b^2d^2ef*(-(4ac - b^2)^9)^{(1/2)} \\
& + 576a^4b^7c^2d^2ef - 1344a^5b^5c^3d^2ef + 512a^6b^3c^4d^2ef - 192a^
\end{aligned}$$

$$\begin{aligned}
& 5*b^6*c^2*e*f + 128*a^6*b^4*c^3*e*f + 1536*a^7*b^2*c^4*e*f - 44*a^2*b*c*d*e \\
& *(-4*a*c - b^2)^9)^{(1/2)}/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + \\
& 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)) \\
&)^{(1/2)}*(1048576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 6144 \\
& 0*a^12*b^9*c^4 - 327680*a^13*b^7*c^5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b \\
& ^3*c^7) - 393216*a^15*c^8*e + 192*a^8*b^13*c^2*d - 4672*a^9*b^11*c^3*d + 47 \\
& 360*a^10*b^9*c^4*d - 256000*a^11*b^7*c^5*d + 778240*a^12*b^5*c^6*d - 126156 \\
& 8*a^13*b^3*c^7*d - 64*a^9*b^12*c^2*e + 1664*a^10*b^10*c^3*e - 17920*a^11*b^ \\
& 8*c^4*e + 102400*a^12*b^6*c^5*e - 327680*a^13*b^4*c^6*e + 557056*a^14*b^2*c \\
& ^7*e - 64*a^10*b^11*c^2*f + 1280*a^11*b^9*c^3*f - 10240*a^12*b^7*c^4*f + 40 \\
& 960*a^13*b^5*c^5*f - 81920*a^14*b^3*c^6*f + 851968*a^14*b*c^8*d + 65536*a^1 \\
& 5*b*c^7*f))*((27*a^3*b^9*c*e^2 - a^2*b^11*e^2 - 9*b^4*d^2*(-4*a*c - b^2)^9 \\
&)^{(1/2)} - a^4*b^9*f^2 - a^4*f^2*(-4*a*c - b^2)^9)^{(1/2)} - 26880*a^6*b*c^6* \\
& d^2 - 9*b^13*d^2 + 3840*a^7*b*c^5*e^2 + 9*a^3*c*e^2*(-4*a*c - b^2)^9)^{(1/2} \\
&) + 768*a^8*b*c^4*f^2 + 6*a*b^12*d*e - 2077*a^2*b^9*c^2*d^2 + 10656*a^3*b^7 \\
& *c^3*d^2 - 30240*a^4*b^5*c^4*d^2 + 44800*a^5*b^3*c^5*d^2 - a^2*b^2*e^2*(-4 \\
& *a*c - b^2)^9)^{(1/2)} - 25*a^2*c^2*d^2*(-4*a*c - b^2)^9)^{(1/2)} - 288*a^4*b^ \\
& 7*c^2*e^2 + 1504*a^5*b^5*c^3*e^2 - 3840*a^6*b^3*c^4*e^2 + 96*a^6*b^5*c^2*f^ \\
& 2 - 512*a^7*b^3*c^3*f^2 + 213*a*b^11*c*d^2 + 6*a^2*b^11*d*f + 15360*a^7*c^6 \\
& *d*e - 2*a^3*b^10*e*f - 3072*a^8*c^5*e*f + 6*a*b^3*d*e*(-4*a*c - b^2)^9)^{(\\
& 1/2)} - 152*a^2*b^10*c*d*e - 98*a^3*b^9*c*d*f + 1536*a^7*b*c^5*d*f - 2*a^3*b \\
& *e*f*(-4*a*c - b^2)^9)^{(1/2)} + 10*a^3*c*d*f*(-4*a*c - b^2)^9)^{(1/2)} + 36* \\
& a^4*b^8*c*e*f + 51*a*b^2*c*d^2*(-4*a*c - b^2)^9)^{(1/2)} + 1548*a^3*b^8*c^2* \\
& d*e - 8064*a^4*b^6*c^3*d*e + 22400*a^5*b^4*c^4*d*e - 30720*a^6*b^2*c^5*d*e \\
& + 6*a^2*b^2*d*f*(-4*a*c - b^2)^9)^{(1/2)} + 576*a^4*b^7*c^2*d*f - 1344*a^5*b \\
& ^5*c^3*d*f + 512*a^6*b^3*c^4*d*f - 192*a^5*b^6*c^2*e*f + 128*a^6*b^4*c^3*e* \\
& f + 1536*a^7*b^2*c^4*e*f - 44*a^2*b*c*d*e*(-4*a*c - b^2)^9)^{(1/2)}/(32*(a^ \\
& 5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 \\
& + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5))^{(1/2)} - (x*(204800*a^12*c^9*d^2 \\
& - 73728*a^13*c^8*e^2 + 8192*a^14*c^7*f^2 + 144*a^6*b^12*c^3*d^2 - 3264*a^7* \\
& b^10*c^4*d^2 + 30112*a^8*b^8*c^5*d^2 - 143360*a^9*b^6*c^6*d^2 + 365568*a^10 \\
& *b^4*c^7*d^2 - 458752*a^11*b^2*c^8*d^2 + 16*a^8*b^10*c^3*e^2 - 416*a^9*b^8* \\
& c^4*e^2 + 4608*a^10*b^6*c^5*e^2 - 25600*a^11*b^4*c^6*e^2 + 69632*a^12*b^2*c \\
& ^7*e^2 + 160*a^10*b^8*c^3*f^2 - 2048*a^11*b^6*c^4*f^2 + 9216*a^12*b^4*c^5*f \\
& ^2 - 16384*a^13*b^2*c^6*f^2 - 81920*a^13*c^8*d*f + 237568*a^12*b*c^8*d*e + \\
& 40960*a^13*b*c^7*e*f - 96*a^7*b^11*c^3*d*e + 2336*a^8*b^9*c^4*d*e - 22528*a \\
& ^9*b^7*c^5*d*e + 107520*a^10*b^5*c^6*d*e - 253952*a^11*b^3*c^7*d*e - 96*a^8 \\
& *b^10*c^3*d*f + 1472*a^9*b^8*c^4*d*f - 7168*a^10*b^6*c^5*d*f + 6144*a^11*b^ \\
& 4*c^6*d*f + 40960*a^12*b^2*c^7*d*f + 32*a^9*b^9*c^3*e*f - 1024*a^10*b^7*c^4 \\
& *e*f + 9216*a^11*b^5*c^5*e*f - 32768*a^12*b^3*c^6*e*f) + ((27*a^3*b^9*c*e^2 \\
& - a^2*b^11*e^2 - 9*b^4*d^2*(-4*a*c - b^2)^9)^{(1/2)} - a^4*b^9*f^2 - a^4*f^ \\
& 2*(-4*a*c - b^2)^9)^{(1/2)} - 26880*a^6*b*c^6*d^2 - 9*b^13*d^2 + 3840*a^7*b* \\
& c^5*e^2 + 9*a^3*c*e^2*(-4*a*c - b^2)^9)^{(1/2)} + 768*a^8*b*c^4*f^2 + 6*a*b^ \\
& 12*d*e - 2077*a^2*b^9*c^2*d^2 + 10656*a^3*b^7*c^3*d^2 - 30240*a^4*b^5*c^4*d \\
& ^2 + 44800*a^5*b^3*c^5*d^2 - a^2*b^2*e^2*(-4*a*c - b^2)^9)^{(1/2)} - 25*a^2* \\
& c^2*d^2*(-4*a*c - b^2)^9)^{(1/2)} - 288*a^4*b^7*c^2*e^2 + 1504*a^5*b^5*c^3*e \\
& ^2 - 3840*a^6*b^3*c^4*e^2 + 96*a^6*b^5*c^2*f^2 - 512*a^7*b^3*c^3*f^2 + 213* \\
& a*b^11*c*d^2 + 6*a^2*b^11*d*f + 15360*a^7*c^6*d*e - 2*a^3*b^10*e*f - 3072*a \\
& ^8*c^5*e*f + 6*a*b^3*d*e*(-4*a*c - b^2)^9)^{(1/2)} - 152*a^2*b^10*c*d*e - 98 \\
& *a^3*b^9*c*d*f + 1536*a^7*b*c^5*d*f - 2*a^3*b*e*f*(-4*a*c - b^2)^9)^{(1/2} \\
& + 10*a^3*c*d*f*(-4*a*c - b^2)^9)^{(1/2)} + 36*a^4*b^8*c*e*f + 51*a*b^2*c*d^2 \\
& *(-4*a*c - b^2)^9)^{(1/2)} + 1548*a^3*b^8*c^2*d*e - 8064*a^4*b^6*c^3*d*e + 2 \\
& 2400*a^5*b^4*c^4*d*e - 30720*a^6*b^2*c^5*d*e + 6*a^2*b^2*d*f*(-4*a*c - b^2 \\
&)^9)^{(1/2)} + 576*a^4*b^7*c^2*d*f - 1344*a^5*b^5*c^3*d*f + 512*a^6*b^3*c^4*d \\
& *f - 192*a^5*b^6*c^2*e*f + 128*a^6*b^4*c^3*e*f + 1536*a^7*b^2*c^4*e*f - 44* \\
& a^2*b*c*d*e*(-4*a*c - b^2)^9)^{(1/2)}/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^ \\
& 6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^1 \\
& 0*b^2*c^5))^{(1/2)}*(393216*a^15*c^8*e + x*((27*a^3*b^9*c*e^2 - a^2*b^11*e^2 \\
& - 9*b^4*d^2*(-4*a*c - b^2)^9)^{(1/2)} - a^4*b^9*f^2 - a^4*f^2*(-4*a*c - b^
\end{aligned}$$

$$\begin{aligned}
& 2)^9)^{(1/2)} - 26880*a^6*b*c^6*d^2 - 9*b^{13}*d^2 + 3840*a^7*b*c^5*e^2 + 9*a^3 \\
& *c*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 768*a^8*b*c^4*f^2 + 6*a*b^{12}*d*e - 2077*a \\
& ^2*b^9*c^2*d^2 + 10656*a^3*b^7*c^3*d^2 - 30240*a^4*b^5*c^4*d^2 + 44800*a^5* \\
& b^3*c^5*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 25*a^2*c^2*d^2*(-(4*a* \\
& c - b^2)^9)^{(1/2)} - 288*a^4*b^7*c^2*e^2 + 1504*a^5*b^5*c^3*e^2 - 3840*a^6*b \\
& ^3*c^4*e^2 + 96*a^6*b^5*c^2*f^2 - 512*a^7*b^3*c^3*f^2 + 213*a*b^{11}*c*d^2 + \\
& 6*a^2*b^{11}*d*f + 15360*a^7*c^6*d*e - 2*a^3*b^{10}*e*f - 3072*a^8*c^5*e*f + 6* \\
& a*b^3*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 152*a^2*b^{10}*c*d*e - 98*a^3*b^9*c*d*f \\
& + 1536*a^7*b*c^5*d*f - 2*a^3*b*e*f*(-(4*a*c - b^2)^9)^{(1/2)} + 10*a^3*c*d*f* \\
& (- (4*a*c - b^2)^9)^{(1/2)} + 36*a^4*b^8*c*e*f + 51*a*b^2*c*d^2*(-(4*a*c - b^2 \\
&)^9)^{(1/2)} + 1548*a^3*b^8*c^2*d*e - 8064*a^4*b^6*c^3*d*e + 22400*a^5*b^4*c^ \\
& 4*d*e - 30720*a^6*b^2*c^5*d*e + 6*a^2*b^2*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 57 \\
& 6*a^4*b^7*c^2*d*f - 1344*a^5*b^5*c^3*d*f + 512*a^6*b^3*c^4*d*f - 192*a^5*b^ \\
& 6*c^2*e*f + 128*a^6*b^4*c^3*e*f + 1536*a^7*b^2*c^4*e*f - 44*a^2*b*c*d*e*(-(\\
& 4*a*c - b^2)^9)^{(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240* \\
& a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^((1 \\
& /2)*(1048576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 61440*a^ \\
& 12*b^9*c^4 - 327680*a^13*b^7*c^5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b^3*c \\
& ^7) - 192*a^8*b^13*c^2*d + 4672*a^9*b^11*c^3*d - 47360*a^10*b^9*c^4*d + 256 \\
& 000*a^11*b^7*c^5*d - 778240*a^12*b^5*c^6*d + 1261568*a^13*b^3*c^7*d + 64*a^ \\
& 9*b^12*c^2*e - 1664*a^10*b^10*c^3*e + 17920*a^11*b^8*c^4*e - 102400*a^12*b^ \\
& 6*c^5*e + 327680*a^13*b^4*c^6*e - 557056*a^14*b^2*c^7*e + 64*a^10*b^11*c^2* \\
& f - 1280*a^11*b^9*c^3*f + 10240*a^12*b^7*c^4*f - 40960*a^13*b^5*c^5*f + 819 \\
& 20*a^14*b^3*c^6*f - 851968*a^14*b*c^8*d - 65536*a^15*b*c^7*f))*((27*a^3*b^9 \\
& *c*e^2 - a^2*b^11*e^2 - 9*b^4*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - a^4*b^9*f^2 - \\
& a^4*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 26880*a^6*b*c^6*d^2 - 9*b^{13}*d^2 + 3840* \\
& a^7*b*c^5*e^2 + 9*a^3*c*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 768*a^8*b*c^4*f^2 + \\
& 6*a*b^{12}*d*e - 2077*a^2*b^9*c^2*d^2 + 10656*a^3*b^7*c^3*d^2 - 30240*a^4*b^5 \\
& *c^4*d^2 + 44800*a^5*b^3*c^5*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 2 \\
& 5*a^2*c^2*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - 288*a^4*b^7*c^2*e^2 + 1504*a^5*b^5 \\
& *c^3*e^2 - 3840*a^6*b^3*c^4*e^2 + 96*a^6*b^5*c^2*f^2 - 512*a^7*b^3*c^3*f^2 \\
& + 213*a*b^{11}*c*d^2 + 6*a^2*b^{11}*d*f + 15360*a^7*c^6*d*e - 2*a^3*b^{10}*e*f - \\
& 3072*a^8*c^5*e*f + 6*a*b^3*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 152*a^2*b^{10}*c*d* \\
& e - 98*a^3*b^9*c*d*f + 1536*a^7*b*c^5*d*f - 2*a^3*b*e*f*(-(4*a*c - b^2)^9)^ \\
& (1/2) + 10*a^3*c*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 36*a^4*b^8*c*e*f + 51*a*b^2 \\
& *c*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + 1548*a^3*b^8*c^2*d*e - 8064*a^4*b^6*c^3*d \\
& *e + 22400*a^5*b^4*c^4*d*e - 30720*a^6*b^2*c^5*d*e + 6*a^2*b^2*d*f*(-(4*a*c \\
& - b^2)^9)^{(1/2)} + 576*a^4*b^7*c^2*d*f - 1344*a^5*b^5*c^3*d*f + 512*a^6*b^3 \\
& *c^4*d*f - 192*a^5*b^6*c^2*e*f + 128*a^6*b^4*c^3*e*f + 1536*a^7*b^2*c^4*e*f \\
& - 44*a^2*b*c*d*e*(-(4*a*c - b^2)^9)^{(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - \\
& 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 61 \\
& 44*a^10*b^2*c^5)))^((1/2) + 128000*a^10*c^9*d^3 - 1024*a^13*c^6*f^3 + 4608*a \\
& ^11*b*c^7*e^3 + 46080*a^11*c^8*d*e^2 - 76800*a^11*c^8*d^2*f + 15360*a^12*c^ \\
& 7*d*f^2 - 9216*a^12*c^7*e^2*f + 504*a^6*b^8*c^5*d^3 - 8112*a^7*b^6*c^6*d^3 \\
& + 48704*a^8*b^4*c^7*d^3 - 129280*a^9*b^2*c^8*d^3 - 40*a^8*b^7*c^4*e^3 + 608 \\
& *a^9*b^5*c^5*e^3 - 2944*a^10*b^3*c^6*e^3 - 48*a^10*b^6*c^3*f^3 + 320*a^11*b \\
& ^4*c^4*f^3 - 256*a^12*b^2*c^5*f^3 - 84480*a^10*b*c^8*d^2*e + 7680*a^12*b*c^ \\
& 6*e*f^2 - 360*a^6*b^9*c^4*d^2*e + 5736*a^7*b^7*c^5*d^2*e + 240*a^7*b^8*c^4* \\
& d*e^2 - 33888*a^8*b^5*c^6*d^2*e - 3792*a^8*b^6*c^5*d*e^2 + 87936*a^9*b^3*c^ \\
& 7*d^2*e + 21696*a^9*b^4*c^6*d*e^2 - 52992*a^10*b^2*c^7*d*e^2 + 216*a^6*b^10 \\
& *c^3*d^2*f - 3744*a^7*b^8*c^4*d^2*f + 25200*a^8*b^6*c^5*d^2*f + 72*a^8*b^8* \\
& c^3*d*f^2 - 81984*a^9*b^4*c^6*d^2*f - 1296*a^9*b^6*c^4*d*f^2 + 128256*a^10* \\
& b^2*c^7*d^2*f + 7872*a^10*b^4*c^5*d*f^2 - 19200*a^11*b^2*c^6*d*f^2 + 24*a^8 \\
& *b^8*c^3*e^2*f - 336*a^9*b^6*c^4*e^2*f - 24*a^9*b^7*c^3*e*f^2 + 960*a^10*b^ \\
& 4*c^5*e^2*f + 672*a^10*b^5*c^4*e*f^2 + 2304*a^11*b^2*c^6*e^2*f - 4224*a^11* \\
& b^3*c^5*e*f^2 - 21504*a^11*b*c^7*d*e*f - 144*a^7*b^9*c^3*d*e*f + 2256*a^8*b \\
& ^7*c^4*d*e*f - 12480*a^9*b^5*c^5*d*e*f + 28416*a^10*b^3*c^6*d*e*f))*((27*a^ \\
& 3*b^9*c*e^2 - a^2*b^11*e^2 - 9*b^4*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - a^4*b^9*f \\
& ^2 - a^4*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 26880*a^6*b*c^6*d^2 - 9*b^{13}*d^2 +
\end{aligned}$$

$$\begin{aligned}
& 3840a^7b^5c^5e^2 + 9a^3c^5e^2(-4ac - b^2)^9)^{(1/2)} + 768a^8b^5c^4f^2 + 6a^8b^5c^4d^2 + 6a^8b^5c^4e^2 - 2077a^2b^9c^2d^2 + 10656a^3b^7c^3d^2 - 30240a^4b^5c^4d^2 + 44800a^5b^3c^5d^2 - a^2b^2e^2(-4ac - b^2)^9)^{(1/2)} \\
& - 25a^2c^2d^2(-4ac - b^2)^9)^{(1/2)} - 288a^4b^7c^2e^2 + 1504a^5b^5c^3e^2 - 3840a^6b^3c^4e^2 + 96a^6b^5c^2f^2 - 512a^7b^3c^3f^2 + 213a^8b^11c^d^2 + 6a^2b^11d^2 + 15360a^7c^6d^2e - 2a^3b^10e^2 \\
& - 3072a^8c^5e^2 + 6a^8b^3d^2e(-4ac - b^2)^9)^{(1/2)} - 152a^2b^10c^d^2e - 98a^3b^9c^d^2e + 1536a^7b^5c^d^2e - 2a^3b^5e^2(-4ac - b^2)^9)^{(1/2)} + 10a^3c^d^2e(-4ac - b^2)^9)^{(1/2)} + 36a^4b^8c^d^2e + 51a^8b^2c^d^2e(-4ac - b^2)^9)^{(1/2)} + 1548a^3b^8c^2d^2e - 8064a^4b^6c^3d^2e + 22400a^5b^4c^4d^2e - 30720a^6b^2c^5d^2e + 6a^2b^2d^2e(-4ac - b^2)^9)^{(1/2)} + 576a^4b^7c^2d^2e - 1344a^5b^5c^3d^2e + 512a^6b^3c^4d^2e - 192a^5b^6c^2e^2 + 128a^6b^4c^3e^2 + 1536a^7b^2c^4e^2 - 44a^2b^3c^d^2e(-4ac - b^2)^9)^{(1/2)} \\
& / (32(a^5b^12 + 4096a^11c^6 - 24a^6b^10c + 240a^7b^8c^2 - 1280a^8b^6c^3 + 3840a^9b^4c^4 - 6144a^10b^2c^5))^{(1/2)} * 2i - \operatorname{atan}\left(\frac{(x(204800a^{12}c^9d^2 - 73728a^{13}c^8e^2 + 8192a^{14}c^7f^2 + 144a^6b^{12}c^3d^2 - 3264a^7b^{10}c^4d^2 + 30112a^8b^8c^5d^2 - 143360a^9b^6c^6d^2 + 365568a^{10}b^4c^7d^2 - 458752a^{11}b^2c^8d^2 + 16a^8b^{10}c^3e^2 - 416a^9b^8c^4e^2 + 4608a^{10}b^6c^5e^2 - 25600a^{11}b^4c^6e^2 + 69632a^{12}b^2c^7e^2 + 160a^{10}b^8c^3f^2 - 2048a^{11}b^6c^4f^2 + 9216a^{12}b^4c^5f^2 - 16384a^{13}b^2c^6f^2 - 81920a^{13}c^8d^2e + 237568a^{12}b^5c^8d^2e + 40960a^{13}b^3c^7e^2 - 96a^7b^{11}c^3d^2e + 2336a^8b^9c^4d^2e - 22528a^9b^7c^5d^2e + 107520a^{10}b^5c^6d^2e - 253952a^{11}b^3c^7d^2e - 96a^8b^{10}c^3d^2e + 1472a^9b^8c^4d^2e - 7168a^{10}b^6c^5d^2e + 6144a^{11}b^4c^6d^2e + 40960a^{12}b^2c^7d^2e + 32a^9b^9c^3e^2 - 1024a^{10}b^7c^4e^2 + 9216a^{11}b^5c^5e^2 - 32768a^{12}b^3c^6e^2)}{(9b^4d^2(-4ac - b^2)^9)^{(1/2)} - a^2b^{11}e^2 - 9b^{13}d^2 - a^4b^9f^2 + a^4f^2(-4ac - b^2)^9)^{(1/2)} - 26880a^6b^5c^6d^2 + 27a^3b^9c^5e^2 + 3840a^7b^5c^5e^2 - 9a^3c^5e^2(-4ac - b^2)^9)^{(1/2)} + 768a^8b^5c^4f^2 + 6a^8b^5c^4d^2 - 2077a^2b^9c^2d^2 + 10656a^3b^7c^3d^2 - 30240a^4b^5c^4d^2 + 44800a^5b^3c^5d^2 + a^2b^2e^2(-4ac - b^2)^9)^{(1/2)} + 25a^2c^2d^2(-4ac - b^2)^9)^{(1/2)} - 288a^4b^7c^2e^2 + 1504a^5b^5c^3e^2 - 3840a^6b^3c^4e^2 + 96a^6b^5c^2f^2 - 512a^7b^3c^3f^2 + 213a^8b^{11}c^d^2 + 6a^2b^{11}d^2 + 15360a^7c^6d^2e - 2a^3b^{10}e^2 - 3072a^8c^5e^2 - 6a^8b^3d^2e(-4ac - b^2)^9)^{(1/2)} - 152a^2b^{10}c^d^2e - 98a^3b^9c^d^2e + 1536a^7b^5c^d^2e + 2a^3b^5e^2(-4ac - b^2)^9)^{(1/2)} - 10a^3c^d^2e(-4ac - b^2)^9)^{(1/2)} + 36a^4b^8c^d^2e - 51a^8b^2c^d^2e(-4ac - b^2)^9)^{(1/2)} + 1548a^3b^8c^2d^2e - 8064a^4b^6c^3d^2e + 22400a^5b^4c^4d^2e - 30720a^6b^2c^5d^2e - 6a^2b^2d^2e(-4ac - b^2)^9)^{(1/2)} + 576a^4b^7c^2d^2e - 1344a^5b^5c^3d^2e + 512a^6b^3c^4d^2e - 192a^5b^6c^2e^2 + 128a^6b^4c^3e^2 + 1536a^7b^2c^4e^2 + 44a^2b^3c^d^2e(-4ac - b^2)^9)^{(1/2)} \\
& / (32(a^5b^12 + 4096a^11c^6 - 24a^6b^10c + 240a^7b^8c^2 - 1280a^8b^6c^3 + 3840a^9b^4c^4 - 6144a^{10}b^2c^5))^{(1/2)} * (x((9b^4d^2(-4ac - b^2)^9)^{(1/2)} - a^2b^{11}e^2 - 9b^{13}d^2 - a^4b^9f^2 + a^4f^2(-4ac - b^2)^9)^{(1/2)} - 26880a^6b^5c^6d^2 + 27a^3b^9c^5e^2 + 3840a^7b^5c^5e^2 - 9a^3c^5e^2(-4ac - b^2)^9)^{(1/2)} + 768a^8b^5c^4f^2 + 6a^8b^5c^4d^2 - 2077a^2b^9c^2d^2 + 10656a^3b^7c^3d^2 - 30240a^4b^5c^4d^2 + 44800a^5b^3c^5d^2 + a^2b^2e^2(-4ac - b^2)^9)^{(1/2)} + 25a^2c^2d^2(-4ac - b^2)^9)^{(1/2)} - 288a^4b^7c^2e^2 + 1504a^5b^5c^3e^2 - 3840a^6b^3c^4e^2 + 96a^6b^5c^2f^2 - 512a^7b^3c^3f^2 + 213a^8b^{11}c^d^2 + 6a^2b^{11}d^2 + 15360a^7c^6d^2e - 2a^3b^{10}e^2 - 3072a^8c^5e^2 - 6a^8b^3d^2e(-4ac - b^2)^9)^{(1/2)} - 152a^2b^{10}c^d^2e - 98a^3b^9c^d^2e + 1536a^7b^5c^d^2e + 2a^3b^5e^2(-4ac - b^2)^9)^{(1/2)} - 10a^3c^d^2e(-4ac - b^2)^9)^{(1/2)} + 36a^4b^8c^d^2e - 51a^8b^2c^d^2e(-4ac - b^2)^9)^{(1/2)} + 1548a^3b^8c^2d^2e - 8064a^4b^6c^3d^2e + 22400a^5b^4c^4d^2e - 30720a^6b^2c^5d^2e - 6a^2b^2d^2e(-4ac - b^2)^9)^{(1/2)} + 576a^4b^7c^2d^2e - 1344a^5b^5c^3d^2e + 512a^6b^3c^4d^2e - 192a^5b^6c^2e^2 + 128a^6b^4c^3e^2
\end{aligned}$$

$$\begin{aligned}
& f + 1536a^7b^2c^4e^f + 44a^2b^*c^*d^*e^*(-(4ac - b^2)^9)^{(1/2)} / (32(a^5b^{12} + 4096a^{11}c^6 - 24a^6b^{10}c + 240a^7b^8c^2 - 1280a^8b^6c^3 \\
& + 3840a^9b^4c^4 - 6144a^{10}b^2c^5))^{(1/2)} * (1048576a^{16}b^*c^*e^8 + 256a^{10}b^{13}c^2 - 6144a^{11}b^{11}c^3 + 61440a^{12}b^9c^4 - 327680a^{13}b^7c^5 \\
& + 983040a^{14}b^5c^6 - 1572864a^{15}b^3c^7) - 393216a^{15}c^8e + 192a^8b^{13}c^2d - 4672a^9b^{11}c^3d + 47360a^{10}b^9c^4d - 256000a^{11}b^7c^5d \\
& + 778240a^{12}b^5c^6d - 1261568a^{13}b^3c^7d - 64a^9b^{12}c^2e + 1664a^{10}b^{10}c^3e - 17920a^{11}b^8c^4e + 102400a^{12}b^6c^5e - \\
& 327680a^{13}b^4c^6e + 557056a^{14}b^2c^7e - 64a^{10}b^{11}c^2f + 1280a^{11}b^9c^3f - 10240a^{12}b^7c^4f + 40960a^{13}b^5c^5f - 81920a^{14}b^3c^6f \\
& + 851968a^{14}b^*c^*e^8d + 65536a^{15}b^*c^*e^7f) * ((9b^4d^2 * (-(4ac - b^2)^9)^{(1/2)} - a^2b^{11}e^2 - 9b^{13}d^2 - a^4b^9f^2 + a^4f^2 * (-(4ac - b^2)^9)^{(1/2)} \\
& - 26880a^6b^*c^*e^6d^2 + 27a^3b^9c^*e^2 + 3840a^7b^*c^*e^5e^2 - 9a^3c^*e^2 * (-(4ac - b^2)^9)^{(1/2)} + 768a^8b^*c^*e^4f^2 + 6a^*b^{12}d^*e \\
& - 2077a^2b^9c^2d^2 + 10656a^3b^7c^3d^2 - 30240a^4b^5c^4d^2 + 44800a^5b^3c^5d^2 + a^2b^2e^2 * (-(4ac - b^2)^9)^{(1/2)} + 25a^2c^2d^2 * (-(4ac - b^2)^9)^{(1/2)} \\
& - 288a^4b^7c^2e^2 + 1504a^5b^5c^3e^2 - 3840a^6b^3c^4e^2 + 96a^6b^5c^2f^2 - 512a^7b^3c^3f^2 + 213a^*b^{11}c^*d^2 + 6a^2b^{11}d^*f + 15360a^7c^6d^*e \\
& - 2a^3b^{10}e^*f - 3072a^8c^5e^*f - 6a^*b^3d^*e^* * (-(4ac - b^2)^9)^{(1/2)} - 152a^2b^{10}c^*d^*e - 98a^3b^9c^*d^*f + 1536a^7b^*c^*e^5d^*f + 2a^3b^*e^*f * (-(4ac - b^2)^9)^{(1/2)} \\
& - 10a^3c^*d^*f * (-(4ac - b^2)^9)^{(1/2)} + 36a^4b^8c^*e^*f - 51a^*b^2c^*d^2 * (-(4ac - b^2)^9)^{(1/2)} + 1548a^3b^8c^2d^*e - 8064a^4b^6c^3d^*e + 22400a^5b^4c^4d^*e \\
& - 30720a^6b^2c^5d^*e - 6a^2b^2d^*f * (-(4ac - b^2)^9)^{(1/2)} + 576a^4b^7c^2d^*f - 1344a^5b^5c^3d^*f + 512a^6b^3c^4d^*f - 192a^5b^6c^2e^*f + 128a^6b^4c^3e^*f \\
& + 1536a^7b^2c^4e^*f + 44a^2b^*c^*d^*e^* * (-(4ac - b^2)^9)^{(1/2)} / (32(a^5b^{12} + 4096a^{11}c^6 - 24a^6b^{10}c + 240a^7b^8c^2 - 1280a^8b^6c^3 + 3840a^9b^4c^4 - 6144a^{10}b^2c^5))^{(1/2)} \\
& * i + (x * (204800a^{12}c^9d^2 - 73728a^{13}c^8e^2 + 8192a^{14}c^7f^2 + 144a^6b^{12}c^3d^2 - 3264a^7b^{10}c^4d^2 + 30112a^8b^8c^5d^2 - 143360a^9b^6c^6d^2 + 365568a^{10}b^4c^7d^2 \\
& - 458752a^{11}b^2c^8d^2 + 16a^8b^{10}c^3e^2 - 416a^9b^8c^4e^2 + 4608a^{10}b^6c^5e^2 - 25600a^{11}b^4c^6e^2 + 69632a^{12}b^2c^7e^2 + 160a^{10}b^8c^3f^2 - 2048a^{11}b^6c^4f^2 \\
& + 9216a^{12}b^4c^5f^2 - 16384a^{13}b^2c^6f^2 - 81920a^{13}c^8d^*f + 237568a^{12}b^*c^*e^8d^*e + 40960a^{13}b^*c^*e^7e^*f - 96a^7b^{11}c^3d^*e + 2336a^8b^9c^4d^*e \\
& - 22528a^9b^7c^5d^*e + 107520a^{10}b^5c^6d^*e - 253952a^{11}b^3c^7d^*e - 96a^8b^{10}c^3d^*f + 1472a^9b^8c^4d^*f - 7168a^{10}b^6c^5d^*f + 6144a^{11}b^4c^6d^*f + 40960a^{12}b^2c^7d^*f \\
& + 32a^9b^9c^3e^*f - 1024a^{10}b^7c^4e^*f + 9216a^{11}b^5c^5e^*f - 32768a^{12}b^3c^6e^*f) + ((9b^4d^2 * (-(4ac - b^2)^9)^{(1/2)} - a^2b^{11}e^2 - 9b^{13}d^2 - a^4b^9f^2 + a^4f^2 * (-(4ac - b^2)^9)^{(1/2)} \\
& - 26880a^6b^*c^*e^6d^2 + 27a^3b^9c^*e^2 + 3840a^7b^*c^*e^5e^2 - 9a^3c^*e^2 * (-(4ac - b^2)^9)^{(1/2)} + 768a^8b^*c^*e^4f^2 + 6a^*b^{12}d^*e - 2077a^2b^9c^2d^2 + 10656a^3b^7c^3d^2 \\
& - 30240a^4b^5c^4d^2 + 44800a^5b^3c^5d^2 + a^2b^2e^2 * (-(4ac - b^2)^9)^{(1/2)} + 25a^2c^2d^2 * (-(4ac - b^2)^9)^{(1/2)} - 288a^4b^7c^2e^2 + 1504a^5b^5c^3e^2 - 3840a^6b^3c^4e^2 \\
& + 96a^6b^5c^2f^2 - 512a^7b^3c^3f^2 + 213a^*b^{11}c^*d^2 + 6a^2b^{11}d^*f + 15360a^7c^6d^*e - 2a^3b^{10}e^*f - 3072a^8c^5e^*f - 6a^*b^3d^*e^* * (-(4ac - b^2)^9)^{(1/2)} \\
& - 152a^2b^{10}c^*d^*e - 98a^3b^9c^*d^*f + 1536a^7b^*c^*e^5d^*f + 2a^3b^*e^*f * (-(4ac - b^2)^9)^{(1/2)} - 10a^3c^*d^*f * (-(4ac - b^2)^9)^{(1/2)} + 36a^4b^8c^*e^*f \\
& - 51a^*b^2c^*d^2 * (-(4ac - b^2)^9)^{(1/2)} + 1548a^3b^8c^2d^*e - 8064a^4b^6c^3d^*e + 22400a^5b^4c^4d^*e - 30720a^6b^2c^5d^*e - 6a^2b^2d^*f * (-(4ac - b^2)^9)^{(1/2)} + 576a^4b^7c^2d^*f \\
& - 1344a^5b^5c^3d^*f + 512a^6b^3c^4d^*f - 192a^5b^6c^2e^*f + 128a^6b^4c^3e^*f + 1536a^7b^2c^4e^*f + 44a^2b^*c^*d^*e^* * (-(4ac - b^2)^9)^{(1/2)} / (32(a^5b^{12} + 4096a^{11}c^6 - 24a^6b^{10}c + 240a^7b^8c^2 - 1280a^8b^6c^3 \\
& + 3840a^9b^4c^4 - 6144a^{10}b^2c^5))^{(1/2)} * (393216a^{15}c^8e + x * ((9b^4d^2 * (-(4ac - b^2)^9)^{(1/2)} - a^2b^{11}e^2 - 9b^{13}d^2 - a^4b^9f^2 + a^4f^2 * (-(4ac - b^2)^9)^{(1/2)} - 26880a^6b^*c^*e^6d^2 + 2
\end{aligned}$$

$$\begin{aligned}
& 7a^3b^9c^2e^2 + 3840a^7b^5c^5e^2 - 9a^3c^2e^2(-4ac - b^2)^9)^{(1/2)} \\
& + 768a^8b^4c^4f^2 + 6a^8b^12d^2e - 2077a^2b^9c^2d^2 + 10656a^3b^7c^3d^2 \\
& - 30240a^4b^5c^4d^2 + 44800a^5b^3c^5d^2 + a^2b^2e^2(-4ac - b^2)^9)^{(1/2)} + 25a^2c^2d^2(-4ac - b^2)^9)^{(1/2)} - 288a^4b^7c^2e^2 \\
& + 1504a^5b^5c^3e^2 - 3840a^6b^3c^4e^2 + 96a^6b^5c^2f^2 - 512a^7b^3c^3f^2 + 213a^8b^11c^2d^2 + 6a^2b^11d^2f \\
& + 15360a^7c^6d^2e - 2a^3b^10e^2f - 3072a^8c^5e^2f - 6a^8b^3d^2e(-4ac - b^2)^9)^{(1/2)} - 152a^2b^10c^2d^2e \\
& - 98a^3b^9c^2d^2f + 1536a^7b^5c^4d^2f + 2a^3b^8e^2f(-4ac - b^2)^9)^{(1/2)} - 10a^3c^2d^2f(-4ac - b^2)^9)^{(1/2)} + 36a^4b^8c^2e^2f \\
& - 51a^4b^2c^2d^2(-4ac - b^2)^9)^{(1/2)} + 1548a^3b^8c^2d^2e - 8064a^4b^6c^3d^2e + 22400a^5b^4c^4d^2e - 30720a^6b^2c^5d^2e \\
& - 6a^2b^2d^2f(-4ac - b^2)^9)^{(1/2)} + 576a^4b^7c^2d^2f - 1344a^5b^5c^3d^2f + 512a^6b^3c^4d^2f - 192a^5b^6c^2e^2f \\
& + 128a^6b^4c^3e^2f + 1536a^7b^2c^4e^2f + 44a^2b^8c^2d^2e(-4ac - b^2)^9)^{(1/2))}/(32(a^5b^12 + 4096a^11c^6 - 24a^6b^10c + 240a^7b^8c^2 - 1280a^8b^6c^3 \\
& + 3840a^9b^4c^4 - 6144a^10b^2c^5))^{(1/2)}*(1048576a^16b^8c^8 + 256a^10b^13c^2 - 6144a^11b^11c^3 + 61440a^12b^9c^4 - 327680a^13b^7c^5 \\
& + 983040a^14b^5c^6 - 1572864a^15b^3c^7) - 192a^8b^13c^2d + 4672a^9b^11c^3d - 47360a^10b^9c^4d + 256000a^11b^7c^5d - 778240a^12b^5c^6d \\
& + 1261568a^13b^3c^7d + 64a^9b^12c^2e - 1664a^10b^10c^3e + 17920a^11b^8c^4e - 102400a^12b^6c^5e + 327680a^13b^4c^6e \\
& - 557056a^14b^2c^7e + 64a^10b^11c^2f - 1280a^11b^9c^3f + 10240a^12b^7c^4f - 40960a^13b^5c^5f + 81920a^14b^3c^6f - 851968a^14b^8c^2d \\
& - 65536a^15b^6c^7f)*((9b^4d^2(-4ac - b^2)^9)^{(1/2)} - a^2b^11e^2 - 9b^13d^2 - a^4b^9f^2 + a^4f^2(-4ac - b^2)^9)^{(1/2)} - 26880a^6b^6c^6d^2 \\
& + 27a^3b^9c^2e^2 + 3840a^7b^5c^5e^2 - 9a^3c^2e^2(-4ac - b^2)^9)^{(1/2)} + 768a^8b^4c^4f^2 + 6a^8b^12d^2e - 2077a^2b^9c^2d^2 \\
& + 10656a^3b^7c^3d^2 - 30240a^4b^5c^4d^2 + 44800a^5b^3c^5d^2 + a^2b^2e^2(-4ac - b^2)^9)^{(1/2)} + 25a^2c^2d^2(-4ac - b^2)^9)^{(1/2)} - 288a^4b^7c^2e^2 \\
& + 1504a^5b^5c^3e^2 - 3840a^6b^3c^4e^2 + 96a^6b^5c^2f^2 - 512a^7b^3c^3f^2 + 213a^8b^11c^2d^2 + 6a^2b^11d^2f + 15360a^7c^6d^2e \\
& - 2a^3b^10e^2f - 3072a^8c^5e^2f - 6a^8b^3d^2e(-4ac - b^2)^9)^{(1/2)} - 152a^2b^10c^2d^2e - 98a^3b^9c^2d^2f + 1536a^7b^5c^4d^2f \\
& + 2a^3b^8e^2f(-4ac - b^2)^9)^{(1/2)} - 10a^3c^2d^2f(-4ac - b^2)^9)^{(1/2)} + 36a^4b^8c^2e^2f - 51a^4b^2c^2d^2(-4ac - b^2)^9)^{(1/2)} \\
& + 1548a^3b^8c^2d^2e - 8064a^4b^6c^3d^2e + 22400a^5b^4c^4d^2e - 30720a^6b^2c^5d^2e - 6a^2b^2d^2f(-4ac - b^2)^9)^{(1/2)} + 576a^4b^7c^2d^2f \\
& - 1344a^5b^5c^3d^2f + 512a^6b^3c^4d^2f - 192a^5b^6c^2e^2f + 128a^6b^4c^3e^2f + 1536a^7b^2c^4e^2f + 44a^2b^8c^2d^2e(-4ac - b^2)^9)^{(1/2))}/(32(a^5b^12 + 4096a^11c^6 - 24a^6b^10c + 240a^7b^8c^2 - 1280a^8b^6c^3 \\
& + 3840a^9b^4c^4 - 6144a^10b^2c^5))^{(1/2)}*1i)/((x*(204800a^12c^9d^2 - 73728a^13c^8e^2 + 8192a^14c^7f^2 + 144a^6b^12c^3d^2 - 3264a^7b^10c^4d^2 + 30112a^8b^8c^5d^2 - 143360a^9b^6c^6d^2 \\
& + 365568a^10b^4c^7d^2 - 458752a^11b^2c^8d^2 + 16a^8b^10c^3e^2 - 416a^9b^8c^4e^2 + 4608a^10b^6c^5e^2 - 25600a^11b^4c^6e^2 + 69632a^12b^2c^7e^2 + 160a^10b^8c^3f^2 - 2048a^11b^6c^4f^2 \\
& + 9216a^12b^4c^5f^2 - 16384a^13b^2c^6f^2 - 81920a^13c^8d^2f + 237568a^12b^6c^8d^2e + 40960a^13b^4c^7e^2f - 96a^7b^11c^3d^2e + 2336a^8b^9c^4d^2e \\
& - 22528a^9b^7c^5d^2e + 107520a^10b^5c^6d^2e - 253952a^11b^3c^7d^2e - 96a^8b^10c^3d^2f + 1472a^9b^8c^4d^2f - 7168a^10b^6c^5d^2f + 6144a^11b^4c^6d^2f + 40960a^12b^2c^7d^2f + 32a^9b^9c^3e^2f \\
& - 1024a^10b^7c^4e^2f + 9216a^11b^5c^5e^2f - 32768a^12b^3c^6e^2f) + ((9b^4d^2(-4ac - b^2)^9)^{(1/2)} - a^2b^11e^2 - 9b^13d^2 - a^4b^9f^2 + a^4f^2(-4ac - b^2)^9)^{(1/2)} - 26880a^6b^6c^6d^2 \\
& + 27a^3b^9c^2e^2 + 3840a^7b^5c^5e^2 - 9a^3c^2e^2(-4ac - b^2)^9)^{(1/2)} + 768a^8b^4c^4f^2 + 6a^8b^12d^2e - 2077a^2b^9c^2d^2 + 10656a^3b^7c^3d^2 - 30240a^4b^5c^4d^2 \\
& + 44800a^5b^3c^5d^2 + a^2b^2e^2(-4ac - b^2)^9)^{(1/2)} + 25a^2c^2d^2(-4ac - b^2)^9)^{(1/2)} - 288a^4b^7c^2e^2 + 1504a^5b^5c^3e^2 - 3840a^6b^3c^4e^2 + 96a^6b^5c^2f^2 - 512
\end{aligned}$$

$$\begin{aligned}
& a^7 b^3 c^3 f^2 + 213 a^2 b^{11} c d^2 + 6 a^2 b^{11} d f + 15360 a^7 c^6 d e - \\
& 2 a^3 b^{10} e f - 3072 a^8 c^5 e f - 6 a b^3 d e (-4 a c - b^2)^9)^{(1/2)} - \\
& 152 a^2 b^{10} c d e - 98 a^3 b^9 c d f + 1536 a^7 b c^5 d f + 2 a^3 b e f (- \\
& (4 a c - b^2)^9)^{(1/2)} - 10 a^3 c d f (-4 a c - b^2)^9)^{(1/2)} + 36 a^4 b^8 \\
& c e f - 51 a b^2 c d^2 (-4 a c - b^2)^9)^{(1/2)} + 1548 a^3 b^8 c^2 d e - 8 \\
& 064 a^4 b^6 c^3 d e + 22400 a^5 b^4 c^4 d e - 30720 a^6 b^2 c^5 d e - 6 a^2 \\
& b^2 d f (-4 a c - b^2)^9)^{(1/2)} + 576 a^4 b^7 c^2 d f - 1344 a^5 b^5 c^3 \\
& d f + 512 a^6 b^3 c^4 d f - 192 a^5 b^6 c^2 e f + 128 a^6 b^4 c^3 e f + 153 \\
& 6 a^7 b^2 c^4 e f + 44 a^2 b c d e (-4 a c - b^2)^9)^{(1/2)}) / (32 (a^5 b^{12} \\
& + 4096 a^{11} c^6 - 24 a^6 b^{10} c + 240 a^7 b^8 c^2 - 1280 a^8 b^6 c^3 + 3840 \\
& a^9 b^4 c^4 - 6144 a^{10} b^2 c^5)))^{(1/2)} * (x ((9 b^4 d^2 (-4 a c - b^2)^9)^{(1/2)} \\
& - a^2 b^{11} e^2 - 9 b^{13} d^2 - a^4 b^9 f^2 + a^4 f^2 (-4 a c - b^2)^9)^{(1/2)} \\
& - 26880 a^6 b c^6 d^2 + 27 a^3 b^9 c e^2 + 3840 a^7 b c^5 e^2 - 9 a^3 c e^2 (-4 a c - b^2)^9)^{(1/2)} \\
& + 768 a^8 b c^4 f^2 + 6 a b^{12} d e - 207 \\
& 7 a^2 b^9 c^2 d^2 + 10656 a^3 b^7 c^3 d^2 - 30240 a^4 b^5 c^4 d^2 + 44800 a^5 \\
& b^3 c^5 d^2 + a^2 b^2 e^2 (-4 a c - b^2)^9)^{(1/2)} + 25 a^2 c^2 d^2 (-4 \\
& a c - b^2)^9)^{(1/2)} - 288 a^4 b^7 c^2 e^2 + 1504 a^5 b^5 c^3 e^2 - 3840 a^6 \\
& b^3 c^4 e^2 + 96 a^6 b^5 c^2 f^2 - 512 a^7 b^3 c^3 f^2 + 213 a^2 b^{11} c d^2 \\
& + 6 a^2 b^{11} d f + 15360 a^7 c^6 d e - 2 a^3 b^{10} e f - 3072 a^8 c^5 e f - \\
& 6 a b^3 d e (-4 a c - b^2)^9)^{(1/2)} - 152 a^2 b^{10} c d e - 98 a^3 b^9 c d \\
& f + 1536 a^7 b c^5 d f + 2 a^3 b e f (-4 a c - b^2)^9)^{(1/2)} - 10 a^3 c d \\
& f (-4 a c - b^2)^9)^{(1/2)} + 36 a^4 b^8 c e f - 51 a b^2 c d^2 (-4 a c - \\
& b^2)^9)^{(1/2)} + 1548 a^3 b^8 c^2 d e - 8064 a^4 b^6 c^3 d e + 22400 a^5 b^4 \\
& c^4 d e - 30720 a^6 b^2 c^5 d e - 6 a^2 b^2 d f (-4 a c - b^2)^9)^{(1/2)} + \\
& 576 a^4 b^7 c^2 d f - 1344 a^5 b^5 c^3 d f + 512 a^6 b^3 c^4 d f - 192 a^5 \\
& b^6 c^2 e f + 128 a^6 b^4 c^3 e f + 1536 a^7 b^2 c^4 e f + 44 a^2 b c d e (- \\
& (4 a c - b^2)^9)^{(1/2)}) / (32 (a^5 b^{12} + 4096 a^{11} c^6 - 24 a^6 b^{10} c + 2 \\
& 40 a^7 b^8 c^2 - 1280 a^8 b^6 c^3 + 3840 a^9 b^4 c^4 - 6144 a^{10} b^2 c^5)))^{(1/2)} * (1048576 a^{16} b c^8 \\
& + 256 a^{10} b^{13} c^2 - 6144 a^{11} b^{11} c^3 + 61440 \\
& a^{12} b^9 c^4 - 327680 a^{13} b^7 c^5 + 983040 a^{14} b^5 c^6 - 1572864 a^{15} b^3 \\
& c^7) - 393216 a^{15} c^8 e + 192 a^8 b^{13} c^2 d - 4672 a^9 b^{11} c^3 d + 473 \\
& 60 a^{10} b^9 c^4 d - 256000 a^{11} b^7 c^5 d + 778240 a^{12} b^5 c^6 d - 1261568 \\
& a^{13} b^3 c^7 d - 64 a^9 b^{12} c^2 e + 1664 a^{10} b^{10} c^3 e - 17920 a^{11} b^8 \\
& c^4 e + 102400 a^{12} b^6 c^5 e - 327680 a^{13} b^4 c^6 e + 557056 a^{14} b^2 c^7 \\
& e - 64 a^{10} b^{11} c^2 f + 1280 a^{11} b^9 c^3 f - 10240 a^{12} b^7 c^4 f + 409 \\
& 60 a^{13} b^5 c^5 f - 81920 a^{14} b^3 c^6 f + 851968 a^{14} b c^8 d + 65536 a^{15} \\
& b c^7 f)) * ((9 b^4 d^2 (-4 a c - b^2)^9)^{(1/2)} - a^2 b^{11} e^2 - 9 b^{13} d^2 \\
& - a^4 b^9 f^2 + a^4 f^2 (-4 a c - b^2)^9)^{(1/2)} - 26880 a^6 b c^6 d^2 + 2 \\
& 7 a^3 b^9 c e^2 + 3840 a^7 b c^5 e^2 - 9 a^3 c e^2 (-4 a c - b^2)^9)^{(1/2)} \\
& + 768 a^8 b c^4 f^2 + 6 a b^{12} d e - 2077 a^2 b^9 c^2 d^2 + 10656 a^3 b^7 \\
& c^3 d^2 - 30240 a^4 b^5 c^4 d^2 + 44800 a^5 b^3 c^5 d^2 + a^2 b^2 e^2 (-4 a \\
& a c - b^2)^9)^{(1/2)} + 25 a^2 c^2 d^2 (-4 a c - b^2)^9)^{(1/2)} - 288 a^4 b^7 \\
& c^2 e^2 + 1504 a^5 b^5 c^3 e^2 - 3840 a^6 b^3 c^4 e^2 + 96 a^6 b^5 c^2 f^2 \\
& - 512 a^7 b^3 c^3 f^2 + 213 a^2 b^{11} c d^2 + 6 a^2 b^{11} d f + 15360 a^7 c^6 d \\
& e - 2 a^3 b^{10} e f - 3072 a^8 c^5 e f - 6 a b^3 d e (-4 a c - b^2)^9)^{(1 \\
& /2)} - 152 a^2 b^{10} c d e - 98 a^3 b^9 c d f + 1536 a^7 b c^5 d f + 2 a^3 b \\
& e f (-4 a c - b^2)^9)^{(1/2)} - 10 a^3 c d f (-4 a c - b^2)^9)^{(1/2)} + 36 a \\
& ^4 b^8 c e f - 51 a b^2 c d^2 (-4 a c - b^2)^9)^{(1/2)} + 1548 a^3 b^8 c^2 d \\
& e - 8064 a^4 b^6 c^3 d e + 22400 a^5 b^4 c^4 d e - 30720 a^6 b^2 c^5 d e - \\
& 6 a^2 b^2 d f (-4 a c - b^2)^9)^{(1/2)} + 576 a^4 b^7 c^2 d f - 1344 a^5 b^ \\
& 5 c^3 d f + 512 a^6 b^3 c^4 d f - 192 a^5 b^6 c^2 e f + 128 a^6 b^4 c^3 e f \\
& + 1536 a^7 b^2 c^4 e f + 44 a^2 b c d e (-4 a c - b^2)^9)^{(1/2)}) / (32 (a^5 \\
& b^{12} + 4096 a^{11} c^6 - 24 a^6 b^{10} c + 240 a^7 b^8 c^2 - 1280 a^8 b^6 c^3 \\
& + 3840 a^9 b^4 c^4 - 6144 a^{10} b^2 c^5)))^{(1/2)} - (x (204800 a^{12} c^9 d^2 - \\
& 73728 a^{13} c^8 e^2 + 8192 a^{14} c^7 f^2 + 144 a^6 b^{12} c^3 d^2 - 3264 a^7 b \\
& ^{10} c^4 d^2 + 30112 a^8 b^8 c^5 d^2 - 143360 a^9 b^6 c^6 d^2 + 365568 a^{10} \\
& b^4 c^7 d^2 - 458752 a^{11} b^2 c^8 d^2 + 16 a^8 b^{10} c^3 e^2 - 416 a^9 b^8 c^ \\
& ^4 e^2 + 4608 a^{10} b^6 c^5 e^2 - 25600 a^{11} b^4 c^6 e^2 + 69632 a^{12} b^2 c^ \\
& ^7 e^2 + 160 a^{10} b^8 c^3 f^2 - 2048 a^{11} b^6 c^4 f^2 + 9216 a^{12} b^4 c^5 f^
\end{aligned}$$

$$\begin{aligned}
& 2 - 16384a^{13}b^2c^6f^2 - 81920a^{13}c^8d^2f + 237568a^{12}b^2c^8d^2e + 4 \\
& 0960a^{13}b^2c^7e^2f - 96a^7b^{11}c^3d^2e + 2336a^8b^9c^4d^2e - 22528a^9 \\
& b^7c^5d^2e + 107520a^{10}b^5c^6d^2e - 253952a^{11}b^3c^7d^2e - 96a^8b^{10} \\
& c^3d^2f + 1472a^9b^8c^4d^2f - 7168a^{10}b^6c^5d^2f + 6144a^{11}b^4c^6d^2f \\
& + 40960a^{12}b^2c^7d^2f + 32a^9b^9c^3e^2f - 1024a^{10}b^7c^4e^2f + 9216a^{11} \\
& b^5c^5e^2f - 32768a^{12}b^3c^6e^2f) + ((9b^4d^2(-4ac - b^2)^9)^{(1/2)} - a^2b^{11}e^2 \\
& - 9b^{13}d^2 - a^4b^9f^2 + a^4f^2(-4ac - b^2)^9)^{(1/2)} - 26880a^6b^2c^6d^2 \\
& + 27a^3b^9c^2e^2 + 3840a^7b^2c^5e^2 - 9a^3c^2e^2(-4ac - b^2)^9)^{(1/2)} + 768a^8 \\
& b^2c^4f^2 + 6a^2b^{12}d^2e - 2077a^2b^9c^2d^2 + 10656a^3b^7c^3d^2 - 30240a^4b^5c^4d^2 \\
& + 44800a^5b^3c^5d^2 + a^2b^2e^2(-4ac - b^2)^9)^{(1/2)} + 25a^2c^2d^2(-4ac - b^2)^9)^{(1/2)} \\
& - 288a^4b^7c^2e^2 + 1504a^5b^5c^3e^2 - 3840a^6b^3c^4e^2 + 96a^6b^5c^2f^2 - 512a^7b^3c^3f^2 \\
& + 213a^2b^{11}c^2d^2 + 6a^2b^{11}d^2f + 15360a^7c^6d^2e - 2a^3b^{10}e^2f - 3072a^8c^5e^2f \\
& - 6a^2b^3d^2e(-4ac - b^2)^9)^{(1/2)} - 152a^2b^{10}c^2d^2e - 98a^3b^9c^2d^2f \\
& + 1536a^7b^2c^5d^2f + 2a^3b^2e^2f(-4ac - b^2)^9)^{(1/2)} - 10a^3c^2d^2f(-4ac - b^2)^9)^{(1/2)} \\
& + 36a^4b^8c^2e^2f - 51a^2b^2c^2d^2(-4ac - b^2)^9)^{(1/2)} + 1548a^3b^8c^2d^2e - 8064a^4b^6c^3d^2e \\
& + 22400a^5b^4c^4d^2e - 30720a^6b^2c^5d^2e - 6a^2b^2d^2f(-4ac - b^2)^9)^{(1/2)} \\
& + 576a^4b^7c^2d^2f - 1344a^5b^5c^3d^2f + 512a^6b^3c^4d^2f - 192a^5b^6c^2e^2f \\
& + 128a^6b^4c^3e^2f + 1536a^7b^2c^4e^2f + 44a^2b^2c^2d^2e(-4ac - b^2)^9)^{(1/2)} \\
& / (32(a^5b^{12} + 4096a^{11}c^6 - 24a^6b^{10}c + 240a^7b^8c^2 - 1280a^8b^6c^3 + 3840a^9b^4c^4 \\
& - 6144a^{10}b^2c^5)))^{(1/2)} * (393216a^{15}c^8e + x((9b^4d^2(-4ac - b^2)^9)^{(1/2)} - a^2b^{11}e^2 \\
& - 9b^{13}d^2 - a^4b^9f^2 + a^4f^2(-4ac - b^2)^9)^{(1/2)} - 26880a^6b^2c^6d^2 \\
& + 27a^3b^9c^2e^2 + 3840a^7b^2c^5e^2 - 9a^3c^2e^2(-4ac - b^2)^9)^{(1/2)} + 768a^8 \\
& b^2c^4f^2 + 6a^2b^{12}d^2e - 2077a^2b^9c^2d^2 + 10656a^3b^7c^3d^2 - 30240a^4b^5c^4d^2 \\
& + 44800a^5b^3c^5d^2 + a^2b^2e^2(-4ac - b^2)^9)^{(1/2)} + 25a^2c^2d^2(-4ac - b^2)^9)^{(1/2)} \\
& - 288a^4b^7c^2e^2 + 1504a^5b^5c^3e^2 - 3840a^6b^3c^4e^2 + 96a^6b^5c^2f^2 - 512a^7b^3c^3f^2 \\
& + 213a^2b^{11}c^2d^2 + 6a^2b^{11}d^2f + 15360a^7c^6d^2e - 2a^3b^{10}e^2f - 3072a^8c^5e^2f \\
& - 6a^2b^3d^2e(-4ac - b^2)^9)^{(1/2)} - 152a^2b^{10}c^2d^2e - 98a^3b^9c^2d^2f + 1536a^7b^2c^5d^2f \\
& + 2a^3b^2e^2f(-4ac - b^2)^9)^{(1/2)} - 10a^3c^2d^2f(-4ac - b^2)^9)^{(1/2)} + 36a^4b^8c^2e^2f \\
& - 51a^2b^2c^2d^2(-4ac - b^2)^9)^{(1/2)} + 1548a^3b^8c^2d^2e - 8064a^4b^6c^3d^2e + 22400a^5b^4c^4 \\
& d^2e - 30720a^6b^2c^5d^2e - 6a^2b^2d^2f(-4ac - b^2)^9)^{(1/2)} + 576a^4b^7c^2d^2f \\
& - 1344a^5b^5c^3d^2f + 512a^6b^3c^4d^2f - 192a^5b^6c^2e^2f + 128a^6b^4c^3e^2f \\
& + 1536a^7b^2c^4e^2f + 44a^2b^2c^2d^2e(-4ac - b^2)^9)^{(1/2)} / (32(a^5b^{12} + 4096a^{11}c^6 - 24a^6b^{10}c \\
& + 240a^7b^8c^2 - 1280a^8b^6c^3 + 3840a^9b^4c^4 - 6144a^{10}b^2c^5)))^{(1/2)} * (1048576a^{16}b^2c^8 \\
& + 256a^{10}b^{13}c^2 - 6144a^{11}b^{11}c^3 + 61440a^{12}b^9c^4 - 327680a^{13}b^7c^5 + 983040a^{14}b^5c^6 \\
& - 1572864a^{15}b^3c^7) - 192a^8b^{13}c^2d + 4672a^9b^{11}c^3d - 47360a^{10}b^9c^4d + 2560 \\
& 00a^{11}b^7c^5d - 778240a^{12}b^5c^6d + 1261568a^{13}b^3c^7d + 64a^9b^{12}c^2e - 1664a^{10}b^{10}c^3e \\
& + 17920a^{11}b^8c^4e - 102400a^{12}b^6c^5e + 327680a^{13}b^4c^6e - 557056a^{14}b^2c^7e + 64a^{10}b^{11}c^2f \\
& - 1280a^{11}b^9c^3f + 10240a^{12}b^7c^4f - 40960a^{13}b^5c^5f + 81920a^{14}b^3c^6f - 851968a^{14} \\
& b^2c^8d - 65536a^{15}b^2c^7f) * ((9b^4d^2(-4ac - b^2)^9)^{(1/2)} - a^2b^{11}e^2 - 9b^{13}d^2 - a^4b^9f^2 \\
& + a^4f^2(-4ac - b^2)^9)^{(1/2)} - 26880a^6b^2c^6d^2 + 27a^3b^9c^2e^2 + 3840a^7b^2c^5e^2 - 9a^3c^2e^2 \\
& (-4ac - b^2)^9)^{(1/2)} + 768a^8b^2c^4f^2 + 6a^2b^{12}d^2e - 2077a^2b^9c^2d^2 + 10656a^3b^7c^3d^2 \\
& - 30240a^4b^5c^4d^2 + 44800a^5b^3c^5d^2 + a^2b^2e^2(-4ac - b^2)^9)^{(1/2)} + 25a^2c^2d^2(-4ac - b^2)^9)^{(1/2)} \\
& - 288a^4b^7c^2e^2 + 1504a^5b^5c^3e^2 - 3840a^6b^3c^4e^2 + 96a^6b^5c^2f^2 - 512a^7b^3c^3f^2 + 213a^2b^{11} \\
& c^2d^2 + 6a^2b^{11}d^2f + 15360a^7c^6d^2e - 2a^3b^{10}e^2f - 3072a^8c^5e^2f - 6a^2b^3d^2e(-4ac - b^2)^9)^{(1/2)} \\
& - 152a^2b^{10}c^2d^2e - 98a^3b^9c^2d^2f + 1536a^7b^2c^5d^2f + 2a^3b^2e^2f(-4ac - b^2)^9)^{(1/2)}
\end{aligned}$$

$$\begin{aligned}
& 1/2) - 10*a^3*c*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 36*a^4*b^8*c*e*f - 51*a*b^2* \\
& c*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + 1548*a^3*b^8*c^2*d*e - 8064*a^4*b^6*c^3*d* \\
& e + 22400*a^5*b^4*c^4*d*e - 30720*a^6*b^2*c^5*d*e - 6*a^2*b^2*d*f*(-(4*a*c \\
& - b^2)^9)^{(1/2)} + 576*a^4*b^7*c^2*d*f - 1344*a^5*b^5*c^3*d*f + 512*a^6*b^3* \\
& c^4*d*f - 192*a^5*b^6*c^2*e*f + 128*a^6*b^4*c^3*e*f + 1536*a^7*b^2*c^4*e*f \\
& + 44*a^2*b*c*d*e*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^5*b^12 + 4096*a^11*c^6 - \\
& 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 614 \\
& 4*a^10*b^2*c^5)))^{(1/2)} + 128000*a^10*c^9*d^3 - 1024*a^13*c^6*f^3 + 4608*a^ \\
& 11*b*c^7*e^3 + 46080*a^11*c^8*d*e^2 - 76800*a^11*c^8*d^2*f + 15360*a^12*c^7 \\
& *d*f^2 - 9216*a^12*c^7*e^2*f + 504*a^6*b^8*c^5*d^3 - 8112*a^7*b^6*c^6*d^3 + \\
& 48704*a^8*b^4*c^7*d^3 - 129280*a^9*b^2*c^8*d^3 - 40*a^8*b^7*c^4*e^3 + 608* \\
& a^9*b^5*c^5*e^3 - 2944*a^10*b^3*c^6*e^3 - 48*a^10*b^6*c^3*f^3 + 320*a^11*b^ \\
& 4*c^4*f^3 - 256*a^12*b^2*c^5*f^3 - 84480*a^10*b*c^8*d^2*e + 7680*a^12*b*c^6 \\
& *e*f^2 - 360*a^6*b^9*c^4*d^2*e + 5736*a^7*b^7*c^5*d^2*e + 240*a^7*b^8*c^4*d \\
& *e^2 - 33888*a^8*b^5*c^6*d^2*e - 3792*a^8*b^6*c^5*d*e^2 + 87936*a^9*b^3*c^7 \\
& *d^2*e + 21696*a^9*b^4*c^6*d*e^2 - 52992*a^10*b^2*c^7*d*e^2 + 216*a^6*b^10* \\
& c^3*d^2*f - 3744*a^7*b^8*c^4*d^2*f + 25200*a^8*b^6*c^5*d^2*f + 72*a^8*b^8*c \\
& ^3*d*f^2 - 81984*a^9*b^4*c^6*d^2*f - 1296*a^9*b^6*c^4*d*f^2 + 128256*a^10*b \\
& ^2*c^7*d^2*f + 7872*a^10*b^4*c^5*d*f^2 - 19200*a^11*b^2*c^6*d*f^2 + 24*a^8* \\
& b^8*c^3*e^2*f - 336*a^9*b^6*c^4*e^2*f - 24*a^9*b^7*c^3*e*f^2 + 960*a^10*b^4 \\
& *c^5*e^2*f + 672*a^10*b^5*c^4*e*f^2 + 2304*a^11*b^2*c^6*e^2*f - 4224*a^11*b \\
& ^3*c^5*e*f^2 - 21504*a^11*b*c^7*d*e*f - 144*a^7*b^9*c^3*d*e*f + 2256*a^8*b^ \\
& 7*c^4*d*e*f - 12480*a^9*b^5*c^5*d*e*f + 28416*a^10*b^3*c^6*d*e*f))((9*b^4* \\
& d^2*(-(4*a*c - b^2)^9)^{(1/2)} - a^2*b^11*e^2 - 9*b^13*d^2 - a^4*b^9*f^2 + a^ \\
& 4*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 26880*a^6*b*c^6*d^2 + 27*a^3*b^9*c*e^2 + 3 \\
& 840*a^7*b*c^5*e^2 - 9*a^3*c*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 768*a^8*b*c^4*f^ \\
& 2 + 6*a*b^12*d*e - 2077*a^2*b^9*c^2*d^2 + 10656*a^3*b^7*c^3*d^2 - 30240*a^4 \\
& *b^5*c^4*d^2 + 44800*a^5*b^3*c^5*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^9)^{(1/2)} \\
& + 25*a^2*c^2*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - 288*a^4*b^7*c^2*e^2 + 1504*a^5 \\
& *b^5*c^3*e^2 - 3840*a^6*b^3*c^4*e^2 + 96*a^6*b^5*c^2*f^2 - 512*a^7*b^3*c^3* \\
& f^2 + 213*a*b^11*c*d^2 + 6*a^2*b^11*d*f + 15360*a^7*c^6*d*e - 2*a^3*b^10*e* \\
& f - 3072*a^8*c^5*e*f - 6*a*b^3*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 152*a^2*b^10* \\
& c*d*e - 98*a^3*b^9*c*d*f + 1536*a^7*b*c^5*d*f + 2*a^3*b*e*f*(-(4*a*c - b^2) \\
& ^9)^{(1/2)} - 10*a^3*c*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 36*a^4*b^8*c*e*f - 51*a \\
& *b^2*c*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + 1548*a^3*b^8*c^2*d*e - 8064*a^4*b^6*c \\
& ^3*d*e + 22400*a^5*b^4*c^4*d*e - 30720*a^6*b^2*c^5*d*e - 6*a^2*b^2*d*f*(-(4 \\
& *a*c - b^2)^9)^{(1/2)} + 576*a^4*b^7*c^2*d*f - 1344*a^5*b^5*c^3*d*f + 512*a^6 \\
& *b^3*c^4*d*f - 192*a^5*b^6*c^2*e*f + 128*a^6*b^4*c^3*e*f + 1536*a^7*b^2*c^4 \\
& *e*f + 44*a^2*b*c*d*e*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^5*b^12 + 4096*a^11*c^ \\
& ^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 \\
& - 6144*a^10*b^2*c^5)))^{(1/2)}*2i
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**4+e*x**2+d)/x**2/(c*x**4+b*x**2+a)**2,x)

[Out] Timed out

3.71 $\int \frac{d+ex^2+fx^4}{x^4(a+bx^2+cx^4)^2} dx$

Optimal. Leaf size=575

$$\frac{2bd - ae}{a^3x} - \frac{d}{3a^2x^3} + \frac{x \left(a^2 \left(\frac{b^4d}{a^2} - \frac{b^2(be+4cd)}{a} - 2acf + b^2f + 3bce + 2c^2d \right) + cx^2 (2a^2ce - ab^2e - ab(3cd - af) + b^3d) \right)}{2a^3 (b^2 - 4ac) (a + bx^2 + cx^4)}$$

Rubi [A] time = 9.91, antiderivative size = 575, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1669, 1664, 1166, 205}

$\frac{-(\frac{d}{a^3} - \frac{2bd-ae}{a^3x}) - \frac{d}{3a^2x^3} + \frac{x \left(a^2 \left(\frac{b^4d}{a^2} - \frac{b^2(be+4cd)}{a} - 2acf + b^2f + 3bce + 2c^2d \right) + cx^2 (2a^2ce - ab^2e - ab(3cd - af) + b^3d) \right)}{2a^3 (b^2 - 4ac) (a + bx^2 + cx^4)}}{2a^3 (b^2 - 4ac) (a + bx^2 + cx^4)}$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2 + f*x^4)/(x^4*(a + b*x^2 + c*x^4)^2), x]
[Out] -d/(3*a^2*x^3) + (2*b*d - a*e)/(a^3*x) + (x*(a^2*((b^4*d)/a^2 + 2*c^2*d + 3*b*c*e - (b^2*(4*c*d + b*e))/a + b^2*f - 2*a*c*f) + c*(b^3*d - a*b^2*e + 2*a^2*c*e - a*b*(3*c*d - a*f))*x^2)/(2*a^3*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[c]*(5*b^4*d + b^3*(5*Sqrt[b^2 - 4*a*c]*d - 3*a*e) + 2*a^2*c*(14*c*d + 5*Sqrt[b^2 - 4*a*c]*e - 6*a*f) - a*b^2*(29*c*d + 3*Sqrt[b^2 - 4*a*c]*e - a*f) - a*b*(19*c*Sqrt[b^2 - 4*a*c]*d - 16*a*c*e - a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(2*Sqrt[2]*a^3*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(5*b^4*d - b^3*(5*Sqrt[b^2 - 4*a*c]*d + 3*a*e) + 2*a^2*c*(14*c*d - 5*Sqrt[b^2 - 4*a*c]*e - 6*a*f) - a*b^2*(29*c*d - 3*Sqrt[b^2 - 4*a*c]*e - a*f) + a*b*(19*c*Sqrt[b^2 - 4*a*c]*d + 16*a*c*e - a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(2*Sqrt[2]*a^3*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1664

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1669

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
```

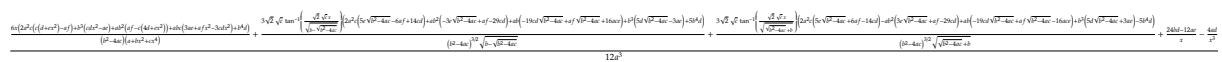


```
Int[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{d + ex^2 + fx^4}{x^4(a + bx^2 + cx^4)^2} dx &= \frac{x \left(a^2 \left(\frac{b^4d}{a^2} + 2c^2d + 3bce - \frac{b^2(4cd+be)}{a} + b^2f - 2acf \right) + c(b^3d - ab^2e + 2a^2ce - ab(3d - ae)) \right)}{2a^3(b^2 - 4ac)(a + bx^2 + cx^4)} \\ &= \frac{x \left(a^2 \left(\frac{b^4d}{a^2} + 2c^2d + 3bce - \frac{b^2(4cd+be)}{a} + b^2f - 2acf \right) + c(b^3d - ab^2e + 2a^2ce - ab(3d - ae)) \right)}{2a^3(b^2 - 4ac)(a + bx^2 + cx^4)} \\ &= -\frac{d}{3a^2x^3} + \frac{2bd - ae}{a^3x} + \frac{x \left(a^2 \left(\frac{b^4d}{a^2} + 2c^2d + 3bce - \frac{b^2(4cd+be)}{a} + b^2f - 2acf \right) + c(b^3d - ab^2e + 2a^2ce - ab(3d - ae)) \right)}{2a^3(b^2 - 4ac)(a + bx^2 + cx^4)} \\ &= -\frac{d}{3a^2x^3} + \frac{2bd - ae}{a^3x} + \frac{x \left(a^2 \left(\frac{b^4d}{a^2} + 2c^2d + 3bce - \frac{b^2(4cd+be)}{a} + b^2f - 2acf \right) + c(b^3d - ab^2e + 2a^2ce - ab(3d - ae)) \right)}{2a^3(b^2 - 4ac)(a + bx^2 + cx^4)} \\ &= -\frac{d}{3a^2x^3} + \frac{2bd - ae}{a^3x} + \frac{x \left(a^2 \left(\frac{b^4d}{a^2} + 2c^2d + 3bce - \frac{b^2(4cd+be)}{a} + b^2f - 2acf \right) + c(b^3d - ab^2e + 2a^2ce - ab(3d - ae)) \right)}{2a^3(b^2 - 4ac)(a + bx^2 + cx^4)} \end{aligned}$$

Mathematica [A] time = 1.80, size = 548, normalized size = 0.95



Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2 + f*x^4)/(x^4*(a + b*x^2 + c*x^4)^2), x]
[Out] ((-4*a*d)/x^3 + (24*b*d - 12*a*e)/x + (6*x*(b^4*d + b^3*(-(a*e) + c*d*x^2) + a*b*c*(3*a*e - 3*c*d*x^2 + a*f*x^2) + 2*a^2*c*(-(a*f) + c*(d + e*x^2))) + a*b^2*(a*f - c*(4*d + e*x^2))))/(b^2 - 4*a*c)*(a + b*x^2 + c*x^4) + (3*sqrt[2]*sqrt[c]*(5*b^4*d + b^3*(5*sqrt[b^2 - 4*a*c]*d - 3*a*e) + 2*a^2*c*(14*c*d + 5*sqrt[b^2 - 4*a*c]*e - 6*a*f) + a*b^2*(-29*c*d - 3*sqrt[b^2 - 4*a*c]*e + a*f) + a*b*(-19*c*sqrt[b^2 - 4*a*c]*d + 16*a*c*e + a*sqrt[b^2 - 4*a*c]*f))*ArcTan[(sqrt[2]*sqrt[c]*x)/sqrt[b - sqrt[b^2 - 4*a*c]]]/((b^2 - 4*a*c)^(3/2)*sqrt[b - sqrt[b^2 - 4*a*c]]) + (3*sqrt[2]*sqrt[c]*(-5*b^4*d + b^3*(5*sqrt[b^2 - 4*a*c]*d + 3*a*e) - a*b^2*(-29*c*d + 3*sqrt[b^2 - 4*a*c]*e + a*f) + 2*a^2*c*(-14*c*d + 5*sqrt[b^2 - 4*a*c]*e + 6*a*f) + a*b*(-19*c*sqrt[b^2 - 4*a*c]*d - 16*a*c*e + a*sqrt[b^2 - 4*a*c]*f))*ArcTan[(sqrt[2]*sqrt[c]*x)/sqrt[b + sqrt[b^2 - 4*a*c]]]/((b^2 - 4*a*c)^(3/2)*sqrt[b + sqrt[b^2 - 4*a*c]]))/(12*a^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex^2 + fx^4}{x^4(a + bx^2 + cx^4)^2} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x^2 + f*x^4)/(x^4*(a + b*x^2 + c*x^4)^2), x]
[Out] IntegrateAlgebraic[(d + e*x^2 + f*x^4)/(x^4*(a + b*x^2 + c*x^4)^2), x]
fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
[Out] Timed out
giac [B] time = 8.59, size = 8660, normalized size = 15.06
```

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
[Out] 1/2*(b^3*c*d*x^3 - 3*a*b*c^2*d*x^3 + a^2*b*c*f*x^3 - a*b^2*c*x^3*e + 2*a^2*c^2*x^3*e + b^4*d*x - 4*a*b^2*c*d*x + 2*a^2*c^2*d*x + a^2*b^2*f*x - 2*a^3*c*f*x - a*b^3*x*e + 3*a^2*b*c*x*e)/((a^3*b^2 - 4*a^4*c)*(c*x^4 + b*x^2 + a))
+ 1/16*((10*b^5*c^2 - 78*a*b^3*c^3 + 152*a^2*b*c^4 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*b^5 + 39*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a*b^3*c + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*b^4*c - 76*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^2*b*c^2 - 38*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a*b^2*c^2 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*b^3*c^2 + 19*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a*b*c^3 - 10*(b^2 - 4*a*c)*b^3*c^2 + 38*(b^2 - 4*a*c)*a*b*c^3)*(a^3*b^2 - 4*a^4*c)^2*d + (2*a^2*b^3*c^2 - 8*a^3*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^2*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^3*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^2*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^2*b*c^2 - 2*(b^2 - 4*a*c)*a^2*b*c^2)*(a^3*b^2 - 4*a^4*c)^2*f - (6*a*b^4*c^2 - 44*a^2*b^2*c^3 + 80*a^3*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a*b^4 + 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^2*b^2*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a*b^3*c - 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^3*c^2 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^2*b*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a*b^2*c^2 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^2*c^3 - 6*(b^2 - 4*a*c)*a*b^2*c^2 + 20*(b^2 - 4*a*c)*a^2*c^3)*(a^3*b^2 - 4*a^4*c)^2*e + 2*(5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^3*b^8 - 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^4*b^6*c - 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^3*b^7*c - 10*a^3*b^8*c + 286*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^5*b^4*c^2 + 88*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^4*b^5*c^2 + 5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^3*b^6*c^2 + 128*a^4*b^6*c^2 - 496*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^6*b^2*c^3 - 220*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^5*b^3*c^3 - 44*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^4*b^4*c^3 - 572*a^5*b^4*c^3 + 224*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^7*c^4 + 112*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^6*b*c^4 + 110*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^5*b^2*c^4 + 992*a^6*b^2*c^4 - 56*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^6*c^5 - 448*a^7*c^5 + 10*(b^2 - 4*a*c)*a^3*b^6*c - 88*(b^2 - 4*a*c)*a^4*b^4*c^2 + 220*(b^2 - 4*a*c)*a^5*b^2*c^3 - 112*(b^2 - 4*a*c)*a^6*c^4)*d*abs(a^3*b^2 - 4*a^4*c) + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^5*b^6 - 14*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^6*b^4*c - 2*sqrt(2)*sqrt(b*c +
```

$$\begin{aligned}
& \sqrt{b^2 - 4ac} \cdot c \cdot a^5 b^5 c - 2a^5 b^6 c + 64\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^7 b^2 c^2 + 20\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^6 b^3 c^2 \\
& + \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^5 b^4 c^2 + 28a^6 b^4 c^2 - 96\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^8 c^3 - 48\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \\
& \cdot c \cdot a^7 b^2 c^3 - 10\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^6 b^2 c^3 - 128a^7 b^2 c^3 + 24\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^7 c^4 \\
& + 192a^8 c^4 + 2(b^2 - 4ac) a^5 b^4 c - 20(b^2 - 4ac) a^6 b^2 c^2 + 48(b^2 - 4ac) a^7 c^3 \cdot f \cdot \text{abs}(a^3 b^2 - 4a^4 c) - 2(3\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^4 b^7 \\
& - 37\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^5 b^5 c - 6\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^4 b^6 c - 6a^4 b^7 c + 152\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^6 b^3 c^2 \\
& + 50\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^5 b^4 c^2 + 3\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^4 b^5 c^2 + 74a^5 b^5 c^2 - 208\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \\
& \cdot c \cdot a^7 b^2 c^3 - 104\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^6 b^2 c^3 - 25\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^5 b^3 c^3 - 304a^6 b^3 c^3 + 52\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^6 b^4 c^4 \\
& + 416a^7 b^4 c^4 + 6(b^2 - 4ac) a^4 b^5 c - 50(b^2 - 4ac) a^5 b^3 c^2 + 104(b^2 - 4ac) a^6 b^3 c^3 \cdot \text{abs}(a^3 b^2 - 4a^4 c) \cdot e + (10a^6 b^9 c^2 - 138a^7 b^7 c^3 \\
& + 680a^8 b^5 c^4 - 1376a^9 b^3 c^5 + 896a^{10} b^2 c^6 - 5\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^6 b^9 + 69\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^7 b^7 c \\
& + 10\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^6 b^8 c - 340\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^8 b^5 c^2 - 98\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^7 b^6 c^2 - 5\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \\
& \cdot c \cdot a^6 b^7 c^2 + 688\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^9 b^3 c^3 + 288\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^8 b^4 c^3 + 49\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^7 b^5 c^3 - 448\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \\
& \cdot c \cdot a^{10} b^2 c^4 - 224\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^9 b^2 c^4 - 144\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^8 b^3 c^4 + 112\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^9 b^3 c^5 - 10(b^2 - 4ac) a^6 b^7 c^2 \\
& + 98(b^2 - 4ac) a^7 b^5 c^3 - 288(b^2 - 4ac) a^8 b^3 c^4 + 224(b^2 - 4ac) a^9 b^2 c^5) \cdot d + (2a^8 b^7 c^2 - 40a^9 b^5 c^3 + 224a^{10} b^3 c^4 - 384a^{11} b^2 c^5 - \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^8 b^7 \\
& + 20\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^9 b^5 c + 2\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^8 b^6 c - 112\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^{10} b^3 c^2 - 32\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^9 b^4 c^2 \\
& - \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^8 b^5 c^2 + 192\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^{11} b^2 c^3 + 16\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^9 b^3 c^3 - 48\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^{10} b^2 c^4 \\
& - 2(b^2 - 4ac) a^8 b^5 c^2 + 32(b^2 - 4ac) a^9 b^3 c^3 - 96(b^2 - 4ac) a^{10} b^2 c^4) \cdot f - (6a^7 b^8 c^2 - 80a^8 b^6 c^3 + 352a^9 b^4 c^4 - 512a^{10} b^2 c^5 - 3\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^7 b^8 \\
& + 40\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^8 b^6 c + 6\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^7 b^7 c - 176\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^9 b^4 c^2 - 56\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^8 b^5 c^2 \\
& - 3\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^7 b^6 c^2 + 256\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^{10} b^2 c^3 + 128\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^9 b^3 c^3 + 28\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^8 b^4 c^3 \\
& - 64\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^9 b^2 c^4 - 6(b^2 - 4ac) a^7 b^6 c^2 + 56(b^2 - 4ac) a^8 b^4 c^3 - 128(b^2 - 4ac) a^9 b^2 c^4) \cdot e) \cdot \arctan(2\sqrt{1/2} \cdot x / \sqrt{(a^3 b^3 - 4a^4 b^2 c + \sqrt{(a^3 b^3 - 4a^4 b^2 c)^2 - 4(a^4 b^2 -
\end{aligned}$$

$$\begin{aligned}
& 4*a^5*c)*(a^3*b^2*c - 4*a^4*c^2))/(a^3*b^2*c - 4*a^4*c^2))/((a^7*b^6 - 12 \\
& *a^8*b^4*c - 2*a^7*b^5*c + 48*a^9*b^2*c^2 + 16*a^8*b^3*c^2 + a^7*b^4*c^2 - \\
& 64*a^10*c^3 - 32*a^9*b*c^3 - 8*a^8*b^2*c^3 + 16*a^9*c^4)*abs(a^3*b^2 - 4*a^ \\
& 4*c)*abs(c)) - 1/16*((10*b^5*c^2 - 78*a*b^3*c^3 + 152*a^2*b*c^4 - 5*sqrt(2) \\
& *sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*b^5 + 39*sqrt(2)*sqrt(b^ \\
& 2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a*b^3*c + 10*sqrt(2)*sqrt(b^2 - \\
& 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*b^4*c - 76*sqrt(2)*sqrt(b^2 - 4*a*c) \\
& *sqrt(b*c - sqrt(b^2 - 4*a*c))*a^2*b*c^2 - 38*sqrt(2)*sqrt(b^2 - 4*a*c)*s \\
& qrt(b*c - sqrt(b^2 - 4*a*c))*a*b^2*c^2 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt \\
& (b*c - sqrt(b^2 - 4*a*c))*b^3*c^2 + 19*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c \\
& - sqrt(b^2 - 4*a*c))*a*b*c^3 - 10*(b^2 - 4*a*c)*b^3*c^2 + 38*(b^2 - 4*a* \\
& c)*a*b*c^3)*(a^3*b^2 - 4*a^4*c)^2*d + (2*a^2*b^3*c^2 - 8*a^3*b*c^3 - sqrt(2) \\
&)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^2*b^3 + 4*sqrt(2)*sqr \\
& t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^3*b*c + 2*sqrt(2)*sqrt(b^2 \\
& - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^2*b^2*c - sqrt(2)*sqrt(b^2 - 4* \\
& a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^2*b*c^2 - 2*(b^2 - 4*a*c)*a^2*b*c^2) \\
& *(a^3*b^2 - 4*a^4*c)^2*f - (6*a*b^4*c^2 - 44*a^2*b^2*c^3 + 80*a^3*c^4 - 3*s \\
& qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a*b^4 + 22*sqrt(2) \\
& *sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^2*b^2*c + 6*sqrt(2)*sqr \\
& t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a*b^3*c - 40*sqrt(2)*sqrt(b \\
& ^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^3*c^2 - 20*sqrt(2)*sqrt(b^2 - \\
& 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^2*b*c^2 - 3*sqrt(2)*sqrt(b^2 - 4* \\
& a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a*b^2*c^2 + 10*sqrt(2)*sqrt(b^2 - 4*a* \\
& c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^2*c^3 - 6*(b^2 - 4*a*c)*a*b^2*c^2 + 20 \\
& *(b^2 - 4*a*c)*a^2*c^3)*(a^3*b^2 - 4*a^4*c)^2*e - 2*(5*sqrt(2)*sqrt(b*c - s \\
& qrt(b^2 - 4*a*c))*a^3*b^8 - 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^ \\
& 4*b^6*c - 10*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^3*b^7*c + 10*a^3*b^8 \\
& *c + 286*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^5*b^4*c^2 + 88*sqrt(2)*s \\
& qrt(b*c - sqrt(b^2 - 4*a*c))*a^4*b^5*c^2 + 5*sqrt(2)*sqrt(b*c - sqrt(b^2 \\
& - 4*a*c))*a^3*b^6*c^2 - 128*a^4*b^6*c^2 - 496*sqrt(2)*sqrt(b*c - sqrt(b^2 \\
& - 4*a*c))*a^6*b^2*c^3 - 220*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^5* \\
& b^3*c^3 - 44*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^4*b^4*c^3 + 572*a^5* \\
& b^4*c^3 + 224*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^7*c^4 + 112*sqrt(2) \\
& *sqrt(b*c - sqrt(b^2 - 4*a*c))*a^6*b*c^4 + 110*sqrt(2)*sqrt(b*c - sqrt(b^ \\
& 2 - 4*a*c))*a^5*b^2*c^4 - 992*a^6*b^2*c^4 - 56*sqrt(2)*sqrt(b*c - sqrt(b^ \\
& 2 - 4*a*c))*a^6*c^5 + 448*a^7*c^5 - 10*(b^2 - 4*a*c)*a^3*b^6*c + 88*(b^2 \\
& - 4*a*c)*a^4*b^4*c^2 - 220*(b^2 - 4*a*c)*a^5*b^2*c^3 + 112*(b^2 - 4*a*c)*a^ \\
& 6*c^4)*d*abs(a^3*b^2 - 4*a^4*c) - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))* \\
&)*a^5*b^6 - 14*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^6*b^4*c - 2*sqrt(2) \\
&)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^5*b^5*c + 2*a^5*b^6*c + 64*sqrt(2)*sqrt \\
& (b*c - sqrt(b^2 - 4*a*c))*a^7*b^2*c^2 + 20*sqrt(2)*sqrt(b*c - sqrt(b^2 - \\
& 4*a*c))*a^6*b^3*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^5*b^4*c^2 \\
& - 28*a^6*b^4*c^2 - 96*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^8*c^3 - 48 \\
& *sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^7*b*c^3 - 10*sqrt(2)*sqrt(b*c - \\
& sqrt(b^2 - 4*a*c))*a^6*b^2*c^3 + 128*a^7*b^2*c^3 + 24*sqrt(2)*sqrt(b*c - \\
& sqrt(b^2 - 4*a*c))*a^7*c^4 - 192*a^8*c^4 - 2*(b^2 - 4*a*c)*a^5*b^4*c + 20 \\
& *(b^2 - 4*a*c)*a^6*b^2*c^2 - 48*(b^2 - 4*a*c)*a^7*c^3)*f*abs(a^3*b^2 - 4*a^ \\
& 4*c) + 2*(3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^4*b^7 - 37*sqrt(2)*sq \\
& rt(b*c - sqrt(b^2 - 4*a*c))*a^5*b^5*c - 6*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4 \\
& *a*c))*a^4*b^6*c + 6*a^4*b^7*c + 152*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c) \\
&)*a^6*b^3*c^2 + 50*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^5*b^4*c^2 + \\
& 3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^4*b^5*c^2 - 74*a^5*b^5*c^2 - 20 \\
& 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^7*b*c^3 - 104*sqrt(2)*sqrt(b*c \\
& - sqrt(b^2 - 4*a*c))*a^6*b^2*c^3 - 25*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c) \\
&)*a^5*b^3*c^3 + 304*a^6*b^3*c^3 + 52*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c) \\
&)*a^6*b*c^4 - 416*a^7*b*c^4 - 6*(b^2 - 4*a*c)*a^4*b^5*c + 50*(b^2 - 4*a* \\
& c)*a^5*b^3*c^2 - 104*(b^2 - 4*a*c)*a^6*b*c^3)*abs(a^3*b^2 - 4*a^4*c)*e + (1 \\
& 0*a^6*b^9*c^2 - 138*a^7*b^7*c^3 + 680*a^8*b^5*c^4 - 1376*a^9*b^3*c^5 + 896* \\
& a^10*b*c^6 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^
\end{aligned}$$

$$\begin{aligned}
& 6*b^9 + 69*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^7*b^8 \\
& 7*c + 10*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^6*b^8* \\
& c - 340*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^8*b^5*c \\
& ^2 - 98*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^7*b^6*c \\
& ^2 - 5*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^6*b^7*c^2 \\
& + 688*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^9*b^3*c \\
& ^3 + 288*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^8*b^4* \\
& c^3 + 49*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^7*b^5* \\
& c^3 - 448*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^{10}*b* \\
& c^4 - 224*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^9*b^2 \\
& *c^4 - 144*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^8*b^3* \\
& c^4 + 112*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^9*b \\
& *c^5 - 10*(b^2 - 4*a*c)*a^6*b^7*c^2 + 98*(b^2 - 4*a*c)*a^7*b^5*c^3 - 288*(b \\
& ^2 - 4*a*c)*a^8*b^3*c^4 + 224*(b^2 - 4*a*c)*a^9*b*c^5)*d + (2*a^8*b^7*c^2 - \\
& 40*a^9*b^5*c^3 + 224*a^{10}*b^3*c^4 - 384*a^{11}*b*c^5 - \sqrt{2}*\sqrt{b^2 - 4* \\
& a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^8*b^7 + 20*\sqrt{2}*\sqrt{b^2 - 4*a*c} \\
& *\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^9*b^5*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{ \\
& b*c - \sqrt{b^2 - 4*a*c}*c})*a^8*b^6*c - 112*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{ \\
& b*c - \sqrt{b^2 - 4*a*c}*c})*a^{10}*b^3*c^2 - 32*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{ \\
& b*c - \sqrt{b^2 - 4*a*c}*c})*a^9*b^4*c^2 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{ \\
& b*c - \sqrt{b^2 - 4*a*c}*c})*a^8*b^5*c^2 + 192*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{ \\
& b*c - \sqrt{b^2 - 4*a*c}*c})*a^{11}*b*c^3 + 96*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{ \\
& b*c - \sqrt{b^2 - 4*a*c}*c})*a^{10}*b^2*c^3 + 16*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{ \\
& b*c - \sqrt{b^2 - 4*a*c}*c})*a^9*b^3*c^3 - 48*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{ \\
& b*c - \sqrt{b^2 - 4*a*c}*c})*a^{10}*b*c^4 - 2*(b^2 - 4*a*c)*a^8*b^5*c^2 + 32*(\\
& b^2 - 4*a*c)*a^9*b^3*c^3 - 96*(b^2 - 4*a*c)*a^{10}*b*c^4)*f - (6*a^7*b^8*c^2 \\
& - 80*a^8*b^6*c^3 + 352*a^9*b^4*c^4 - 512*a^{10}*b^2*c^5 - 3*\sqrt{2}*\sqrt{b^2 \\
& - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^7*b^8 + 40*\sqrt{2}*\sqrt{b^2 - 4* \\
& a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^8*b^6*c + 6*\sqrt{2}*\sqrt{b^2 - 4*a*c} \\
&)*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^7*b^7*c - 176*\sqrt{2}*\sqrt{b^2 - 4*a*c} \\
& *\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^9*b^4*c^2 - 56*\sqrt{2}*\sqrt{b^2 - 4*a*c} \\
& *\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^8*b^5*c^2 - 3*\sqrt{2}*\sqrt{b^2 - 4*a*c}* \\
& \sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^7*b^6*c^2 + 256*\sqrt{2}*\sqrt{b^2 - 4*a*c} \\
& *\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^{10}*b^2*c^3 + 128*\sqrt{2}*\sqrt{b^2 - 4*a* \\
& c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^9*b^3*c^3 + 28*\sqrt{2}*\sqrt{b^2 - 4*a* \\
& c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^8*b^4*c^3 - 64*\sqrt{2}*\sqrt{b^2 - 4*a* \\
& c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^9*b^2*c^4 - 6*(b^2 - 4*a*c)*a^7*b^6*c^2 \\
& + 56*(b^2 - 4*a*c)*a^8*b^4*c^3 - 128*(b^2 - 4*a*c)*a^9*b^2*c^4)*e)*\arctan \\
& (2*\sqrt{1/2}*x/\sqrt{(a^3*b^3 - 4*a^4*b*c - \sqrt{(a^3*b^3 - 4*a^4*b*c)^2 - 4 \\
& *(a^4*b^2 - 4*a^5*c)*(a^3*b^2*c - 4*a^4*c^2)}))/((a^3*b^2*c - 4*a^4*c^2)))/((\\
& a^7*b^6 - 12*a^8*b^4*c - 2*a^7*b^5*c + 48*a^9*b^2*c^2 + 16*a^8*b^3*c^2 + a^7* \\
& b^4*c^2 - 64*a^{10}*c^3 - 32*a^9*b*c^3 - 8*a^8*b^2*c^3 + 16*a^9*c^4)*\text{abs}(a^3* \\
& b^2 - 4*a^4*c)*\text{abs}(c)) + 1/3*(6*b*d*x^2 - 3*a*x^2*e - a*d)/(a^3*x^3)
\end{aligned}$$

maple [B] time = 0.06, size = 2180, normalized size = 3.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a)^2,x)$

[Out]
$$\begin{aligned}
& -1/3*d/a^2/x^3+2/a^3/x*b*d-5/4/a^3*c/(4*a*c-b^2)*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)}) \\
& *c)^{(1/2)}*\arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^3*d+7 \\
& /a*c^3/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} \\
& *\arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*d+1/4/a*c/(4*a* \\
& c-b^2)*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctanh(2^{(1/2)}/((-b+(-4*a* \\
& c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b*f-3/4/a^2*c/(4*a*c-b^2)*2^{(1/2)}/((-b+(-4*a*c \\
& +b^2)^{(1/2)})*c)^{(1/2)}*\arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x \\
&)*b^2*e-1/4/a*c/(4*a*c-b^2)*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctan \\
& (2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b*f+3/4/a^2*c/(4*a*c-b^2)*2^
\end{aligned}$$

$$\begin{aligned} & (1/2)/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctan(2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^2*e+19/4/a^2*c^2/(4*a*c-b^2)*2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b*d+7/a*c^3/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*d-19/4/a^2*c^2/(4*a*c-b^2)*2^{(1/2)/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b*d+5/4/a^3*c/(4*a*c-b^2)*2^{(1/2)/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^3*d-29/4/a^2*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^2*d+1/4/a*c/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^2*f+4/a*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b*e-3/4/a^2*c/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^3*e+4/a*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^3*e-3/4/a^2*c/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^3*e+1/4/a*c/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^2*f+5/4/a^3*c/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^4*d+5/4/a^3*c/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^4*d-29/4/a^2*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^2*d-1/2/a/(c*x^4+b*x^2+a)*c/(4*a*c-b^2)*x^3*b*f+1/2/a^2/(c*x^4+b*x^2+a)*c/(4*a*c-b^2)*x^3*b^2*e+3/2/a^2/(c*x^4+b*x^2+a)*c^2/(4*a*c-b^2)*x^3*b*d-3/2/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*b*c*e+2/a^2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*b^2*c*d-1/2/a^3/(c*x^4+b*x^2+a)*c/(4*a*c-b^2)*x^3*b^3*d+5/2/a*c^2/(4*a*c-b^2)*2^{(1/2)/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*e-5/2/a*c^2/(4*a*c-b^2)*2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*e-3*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*f-3*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*f+1/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*c*f-1/a/(c*x^4+b*x^2+a)*c^2/(4*a*c-b^2)*x^3*e-1/2/a^3/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*b^4*d-1/2/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*b^2*f-1/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*c^2*d+1/2/a^2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*b^3*e-1/a^2/x*e \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")

[Out] $\frac{1}{6}*(3*(a^2*b*c*f + (5*b^3*c - 19*a*b*c^2)*d - (3*a*b^2*c - 10*a^2*c^2)*e)*x^6 + ((15*b^4 - 62*a*b^2*c + 14*a^2*c^2)*d - 3*(3*a*b^3 - 11*a^2*b*c)*e + 3*(a^2*b^2 - 2*a^3*c)*f)*x^4 + 2*(5*(a*b^3 - 4*a^2*b*c)*d - 3*(a^2*b^2 - 4*a^3*c)*e)*x^2 - 2*(a^2*b^2 - 4*a^3*c)*d)/((a^3*b^2*c - 4*a^4*c^2)*x^7 + (a^3*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3) + 1/2*integrate(((a^2*b*c*f + (5*b^3*c - 19*a*b*c^2)*d - (3*a*b^2*c - 10*a^2*c^2)*e)*x^2 + (5*b^4 - 24*a*b^2*c + 14*a^2*c^2)*d - (3*a*b^3 - 13*a^2*b*c)*e + (a^2*b^2 - 6*a^3*c)*f)/(c*x^4 + b*x^2 + a), x)/(a^3*b^2 - 4*a^4*c)$

mupad [B] time = 7.37, size = 36097, normalized size = 62.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((d + e*x^2 + f*x^4)/(x^4*(a + b*x^2 + c*x^4)^2), x)$

[Out] $\text{atan}\left(\frac{(x(204800a^{17}c^9e^2 - 401408a^{16}c^{10}d^2 - 73728a^{18}c^8f^2 + 400a^9b^{14}c^3d^2 - 9440a^{10}b^{12}c^4d^2 + 92816a^{11}b^{10}c^5d^2 - 488096a^{12}b^8c^6d^2 + 1458688a^{13}b^6c^7d^2 - 2401280a^{14}b^4c^8d^2 + 1871872a^{15}b^2c^9d^2 + 144a^{11}b^{12}c^3e^2 - 3264a^{12}b^{10}c^4e^2 + 30112a^{13}b^8c^5e^2 - 143360a^{14}b^6c^6e^2 + 365568a^{15}b^4c^7e^2 - 458752a^{16}b^2c^8e^2 + 16a^{13}b^{10}c^3f^2 - 416a^{14}b^8c^4f^2 + 4608a^{15}b^6c^5f^2 - 25600a^{16}b^4c^6f^2 + 69632a^{17}b^2c^7f^2 + 344064a^{17}c^9d^2 - 1236992a^{16}b^2c^9d^2 + 237568a^{17}b^2c^8e^2 - 480a^{10}b^{13}c^3d^2 + 11104a^{11}b^{11}c^4d^2 - 105824a^{12}b^9c^5d^2 + 530432a^{13}b^7c^6d^2 - 1469440a^{14}b^5c^7d^2 + 2121728a^{15}b^3c^8d^2 + 160a^{11}b^{12}c^3d^2 - 3968a^{12}b^{10}c^4d^2 + 39488a^{13}b^8c^5d^2 - 200704a^{14}b^6c^6d^2 + 542720a^{15}b^4c^7d^2 - 720896a^{16}b^2c^8d^2 - 96a^{12}b^{11}c^3e^2 + 2336a^{13}b^9c^4e^2 - 22528a^{14}b^7c^5e^2 + 107520a^{15}b^5c^6e^2 - 253952a^{16}b^3c^7e^2 + (-25b^{15}d^2 + 9a^2b^{13}e^2 + 25b^6d^2(-4ac - b^2)^9)^{1/2} + a^4b^{11}f^2 - 80640a^7b^7c^7d^2 - 213a^3b^{11}c^3e^2 + 26880a^8b^6c^6e^2 - 27a^5b^9c^3f^2 - 3840a^9b^5c^5f^2 - 9a^5c^3f^2(-4ac - b^2)^9)^{1/2} - 30a^2b^{14}d^2 + 6366a^2b^{11}c^2d^2 - 35767a^3b^9c^3d^2 + 116928a^4b^7c^4d^2 - 219744a^5b^5c^5d^2 + 215040a^6b^3c^6d^2 + 9a^2b^4e^2(-4ac - b^2)^9)^{1/2} - 49a^3c^3d^2(-4ac - b^2)^9)^{1/2} + 2077a^4b^9c^2e^2 - 10656a^5b^7c^3e^2 + 30240a^6b^5c^4e^2 - 44800a^7b^3c^5e^2 + a^4b^2f^2(-4ac - b^2)^9)^{1/2} + 25a^4c^2e^2(-4ac - b^2)^9)^{1/2} + 288a^6b^7c^2f^2 - 1504a^7b^5c^3f^2 + 3840a^8b^3c^4f^2 - 615a^2b^{13}c^3d^2 + 10a^2b^{13}d^2 + 35840a^8c^7d^2 - 6a^3b^{12}e^2 - 15360a^9c^6e^2 - 30a^2b^5d^2(-4ac - b^2)^9)^{1/2} + 724a^2b^{12}c^3d^2 - 258a^3b^{11}c^3d^2 + 43520a^8b^6c^6d^2 + 152a^4b^{10}c^3e^2 + 246a^2b^2c^2d^2(-4ac - b^2)^9)^{1/2} - 165a^2b^4c^3d^2(-4ac - b^2)^9)^{1/2} - 7278a^3b^{10}c^2d^2 + 39132a^4b^8c^3d^2 - 119616a^5b^6c^4d^2 + 201600a^6b^4c^5d^2 - 161280a^7b^2c^6d^2 + 10a^2b^4d^2f(-4ac - b^2)^9)^{1/2} + 2706a^4b^9c^2d^2 - 14784a^5b^7c^3d^2 + 44352a^6b^5c^4d^2 - 69120a^7b^3c^5d^2 - 6a^3b^3e^2f(-4ac - b^2)^9)^{1/2} + 42a^4c^2d^2f(-4ac - b^2)^9)^{1/2} - 1548a^5b^8c^2e^2 + 8064a^6b^6c^3e^2 - 22400a^7b^4c^4e^2 + 30720a^8b^2c^5e^2 - 51a^3b^2c^2e^2(-4ac - b^2)^9)^{1/2} + 44a^4b^3c^2e^2f(-4ac - b^2)^9)^{1/2} + 184a^2b^3c^3d^2e^2(-4ac - b^2)^9)^{1/2} - 186a^3b^2c^2d^2e^2(-4ac - b^2)^9)^{1/2} - 78a^3b^2c^3d^2f(-4ac - b^2)^9)^{1/2})/(32(a^7b^{12} + 4096a^{13}c^6 - 24a^8b^{10}c + 240a^9b^8c^2 - 1280a^{10}b^6c^3 + 3840a^{11}b^4c^4 - 6144a^{12}b^2c^5))^{1/2} * (393216a^{20}c^8f - 917504a^{19}c^9d + x(-25b^{15}d^2 + 9a^2b^{13}e^2 + 25b^6d^2(-4ac - b^2)^9)^{1/2} + a^4b^{11}f^2 - 80640a^7b^7c^7d^2 - 213a^3b^{11}c^3e^2 + 26880a^8b^6c^6e^2 - 27a^5b^9c^3f^2 - 3840a^9b^5c^5f^2 - 9a^5c^3f^2(-4ac - b^2)^9)^{1/2} - 30a^2b^{14}d^2 + 6366a^2b^{11}c^2d^2 - 35767a^3b^9c^3d^2 + 116928a^4b^7c^4d^2 - 219744a^5b^5c^5d^2 + 215040a^6b^3c^6d^2 + 9a^2b^4e^2(-4ac - b^2)^9)^{1/2} - 49a^3c^3d^2(-4ac - b^2)^9)^{1/2} + 2077a^4b^9c^2e^2 - 10656a^5b^7c^3e^2 + 30240a^6b^5c^4e^2 - 44800a^7b^3c^5e^2 + a^4b^2f^2(-4ac - b^2)^9)^{1/2} + 25a^4c^2e^2(-4ac - b^2)^9)^{1/2} + 288a^6b^7c^2f^2 - 1504a^7b^5c^3f^2 + 3840a^8b^3c^4f^2 - 615a^2b^{13}c^3d^2 + 10a^2b^{13}d^2 + 35840a^8c^7d^2 - 6a^3b^{12}e^2 - 15360a^9c^6e^2 - 30a^2b^5d^2(-4ac - b^2)^9)^{1/2} + 724a^2b^{12}c^3d^2 - 258a^3b^{11}c^3d^2 + 43520a^8b^6c^6d^2 + 152a^4b^{10}c^3e^2 + 246a^2b^2c^2d^2(-4ac - b^2)^9)^{1/2} - 165a^2b^4c^3d^2(-4ac - b^2)^9)^{1/2} - 7278a^3b^{10}c^2d^2 + 39132a^4b^8c^3d^2 - 119616a^5b^6c^4d^2 + 201600a^6b^4c^5d^2 - 161280a^7b^2c^6d^2 + 10a^2b^4d^2f(-4ac - b^2)^9)^{1/2} + 2706a^4b^9c^2d^2 - 14784a^5b^7c^3d^2 + 44352a^6b^5c^4d^2 - 69120a^7b^3c^5d^2 - 6a^3b^3e^2f(-4ac - b^2)^9)^{1/2} + 42a^4c^2d^2f(-4ac - b^2)^9)^{1/2}$

$$\begin{aligned}
& c - b^2)^9)^{(1/2)} - 1548*a^5*b^8*c^2*e*f + 8064*a^6*b^6*c^3*e*f - 22400*a^7* \\
& *b^4*c^4*e*f + 30720*a^8*b^2*c^5*e*f - 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^{ \\
& (1/2)} + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^{(1/2)} + 184*a^2*b^3*c*d*e*(-(4*a* \\
& c - b^2)^9)^{(1/2)} - 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 78*a^3*b^2 \\
& *c*d*f*(-(4*a*c - b^2)^9)^{(1/2)))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^1 \\
& 0*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b \\
& ^2*c^5)))^{(1/2)}*(1048576*a^21*b*c^8 + 256*a^15*b^13*c^2 - 6144*a^16*b^11*c^ \\
& 3 + 61440*a^17*b^9*c^4 - 327680*a^18*b^7*c^5 + 983040*a^19*b^5*c^6 - 157286 \\
& 4*a^20*b^3*c^7) + 320*a^12*b^14*c^2*d - 7936*a^13*b^12*c^3*d + 82816*a^14*b \\
& ^10*c^4*d - 468480*a^15*b^8*c^5*d + 1536000*a^16*b^6*c^6*d - 2867200*a^17*b \\
& ^4*c^7*d + 2719744*a^18*b^2*c^8*d - 192*a^13*b^13*c^2*e + 4672*a^14*b^11*c^ \\
& 3*e - 47360*a^15*b^9*c^4*e + 256000*a^16*b^7*c^5*e - 778240*a^17*b^5*c^6*e \\
& + 1261568*a^18*b^3*c^7*e + 64*a^14*b^12*c^2*f - 1664*a^15*b^10*c^3*f + 1792 \\
& 0*a^16*b^8*c^4*f - 102400*a^17*b^6*c^5*f + 327680*a^18*b^4*c^6*f - 557056*a \\
& ^19*b^2*c^7*f - 851968*a^19*b*c^8*e))*(-(25*b^15*d^2 + 9*a^2*b^13*e^2 + 25* \\
& b^6*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + a^4*b^11*f^2 - 80640*a^7*b*c^7*d^2 - 213 \\
& *a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^2 - 3840*a^9*b*c^5*f \\
& ^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 30*a*b^14*d*e + 6366*a^2*b^11*c \\
& ^2*d^2 - 35767*a^3*b^9*c^3*d^2 + 116928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^ \\
& 5*d^2 + 215040*a^6*b^3*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 4 \\
& 9*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*b \\
& ^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^2 - 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(- \\
& (4*a*c - b^2)^9)^{(1/2)} + 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 288*a^6* \\
& b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^2 + 3840*a^8*b^3*c^4*f^2 - 615*a*b^13*c*d^ \\
& 2 + 10*a^2*b^13*d*f + 35840*a^8*c^7*d*e - 6*a^3*b^12*e*f - 15360*a^9*c^6*e* \\
& f - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^{(1/2)} + 724*a^2*b^12*c*d*e - 258*a^3*b^ \\
& 11*c*d*f + 43520*a^8*b*c^6*d*f + 152*a^4*b^10*c*e*f + 246*a^2*b^2*c^2*d^2*(- \\
& (4*a*c - b^2)^9)^{(1/2)} - 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - 7278*a \\
& ^3*b^10*c^2*d*e + 39132*a^4*b^8*c^3*d*e - 119616*a^5*b^6*c^4*d*e + 201600*a \\
& ^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9) \\
& ^{(1/2)} + 2706*a^4*b^9*c^2*d*f - 14784*a^5*b^7*c^3*d*f + 44352*a^6*b^5*c^4*d \\
& *f - 69120*a^7*b^3*c^5*d*f - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^{(1/2)} + 42*a^ \\
& 4*c^2*d*f*(-(4*a*c - b^2)^9)^{(1/2)} - 1548*a^5*b^8*c^2*e*f + 8064*a^6*b^6*c^ \\
& 3*e*f - 22400*a^7*b^4*c^4*e*f + 30720*a^8*b^2*c^5*e*f - 51*a^3*b^2*c*e^2*(- \\
& (4*a*c - b^2)^9)^{(1/2)} + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^{(1/2)} + 184*a^2* \\
& b^3*c*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^{(\\
& 1/2)} - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^{(1/2)))/(32*(a^7*b^12 + 4096*a^13 \\
& *c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4* \\
& c^4 - 6144*a^12*b^2*c^5)))^{(1/2)}*i + (x*(204800*a^17*c^9*e^2 - 401408*a^16 \\
& *c^10*d^2 - 73728*a^18*c^8*f^2 + 400*a^9*b^14*c^3*d^2 - 9440*a^10*b^12*c^4* \\
& d^2 + 92816*a^11*b^10*c^5*d^2 - 488096*a^12*b^8*c^6*d^2 + 1458688*a^13*b^6* \\
& c^7*d^2 - 2401280*a^14*b^4*c^8*d^2 + 1871872*a^15*b^2*c^9*d^2 + 144*a^11*b^ \\
& 12*c^3*e^2 - 3264*a^12*b^10*c^4*e^2 + 30112*a^13*b^8*c^5*e^2 - 143360*a^14* \\
& b^6*c^6*e^2 + 365568*a^15*b^4*c^7*e^2 - 458752*a^16*b^2*c^8*e^2 + 16*a^13*b \\
& ^10*c^3*f^2 - 416*a^14*b^8*c^4*f^2 + 4608*a^15*b^6*c^5*f^2 - 25600*a^16*b^4 \\
& *c^6*f^2 + 69632*a^17*b^2*c^7*f^2 + 344064*a^17*c^9*d*f - 1236992*a^16*b*c^ \\
& 9*d*e + 237568*a^17*b*c^8*e*f - 480*a^10*b^13*c^3*d*e + 11104*a^11*b^11*c^4 \\
& *d*e - 105824*a^12*b^9*c^5*d*e + 530432*a^13*b^7*c^6*d*e - 1469440*a^14*b^5 \\
& *c^7*d*e + 2121728*a^15*b^3*c^8*d*e + 160*a^11*b^12*c^3*d*f - 3968*a^12*b^1 \\
& 0*c^4*d*f + 39488*a^13*b^8*c^5*d*f - 200704*a^14*b^6*c^6*d*f + 542720*a^15* \\
& b^4*c^7*d*f - 720896*a^16*b^2*c^8*d*f - 96*a^12*b^11*c^3*e*f + 2336*a^13*b^ \\
& 9*c^4*e*f - 22528*a^14*b^7*c^5*e*f + 107520*a^15*b^5*c^6*e*f - 253952*a^16* \\
& b^3*c^7*e*f) + (-(25*b^15*d^2 + 9*a^2*b^13*e^2 + 25*b^6*d^2*(-(4*a*c - b^2) \\
& ^9)^{(1/2)} + a^4*b^11*f^2 - 80640*a^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880 \\
& *a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^2 - 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a \\
& *c - b^2)^9)^{(1/2)} - 30*a*b^14*d*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9* \\
& c^3*d^2 + 116928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3* \\
& c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 49*a^3*c^3*d^2*(-(4*a*c \\
& - b^2)^9)^{(1/2)} + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*
\end{aligned}$$

$$\begin{aligned}
& b^5c^4e^2 - 44800a^7b^3c^5e^2 + a^4b^2f^2(-4ac - b^2)^9)^{(1/2)} \\
& + 25a^4c^2e^2(-4ac - b^2)^9)^{(1/2)} + 288a^6b^7c^2f^2 - 1504a^7b^5c^3f^2 + 3840a^8b^3c^4f^2 - 615ab^{13}cd^2 + 10a^2b^{13}df + 3 \\
& 5840a^8c^7d^2e - 6a^3b^{12}e^2f - 15360a^9c^6e^2f - 30ab^5d^2e(-4ac - b^2)^9)^{(1/2)} + 724a^2b^{12}c^2d^2e - 258a^3b^{11}c^2d^2f + 43520a^8b^3c^6d^2f \\
& + 152a^4b^{10}c^2e^2f + 246a^2b^2c^2d^2(-4ac - b^2)^9)^{(1/2)} - 165ab^4c^2d^2(-4ac - b^2)^9)^{(1/2)} - 7278a^3b^{10}c^2d^2e + 39132 \\
& a^4b^8c^3d^2e - 119616a^5b^6c^4d^2e + 201600a^6b^4c^5d^2e - 161280a^7b^2c^6d^2e + 10a^2b^4d^2f(-4ac - b^2)^9)^{(1/2)} + 2706a^4b^9c^2d^2f \\
& - 14784a^5b^7c^3d^2f + 44352a^6b^5c^4d^2f - 69120a^7b^3c^5d^2f - 6a^3b^3e^2f(-4ac - b^2)^9)^{(1/2)} + 42a^4c^2d^2f(-4ac - b^2)^9)^{(1/2)} \\
& - 1548a^5b^8c^2e^2f + 8064a^6b^6c^3e^2f - 22400a^7b^4c^4e^2f + 30720a^8b^2c^5e^2f - 51a^3b^2c^2e^2(-4ac - b^2)^9)^{(1/2)} \\
& + 44a^4b^2c^2e^2f(-4ac - b^2)^9)^{(1/2)} + 184a^2b^3c^2d^2e(-4ac - b^2)^9)^{(1/2)} - 186a^3b^2c^2d^2e(-4ac - b^2)^9)^{(1/2)} - 78a^3b^2c^2d^2f \\
& (-4ac - b^2)^9)^{(1/2)} / (32(a^7b^{12} + 4096a^{13}c^6 - 24a^8b^{10}c + 240a^9b^8c^2 - 1280a^{10}b^6c^3 + 3840a^{11}b^4c^4 - 6144a^{12}b^2c^5)) \\
&)^{(1/2)}(917504a^{19}c^9d - 393216a^{20}c^8f + x(-25b^{15}d^2 + 9a^2b^{13}e^2 + 25b^6d^2(-4ac - b^2)^9)^{(1/2)} + a^4b^{11}f^2 - 80640a^7b^3c^7d^2 \\
& - 213a^3b^{11}c^2e^2 + 26880a^8b^3c^6e^2 - 27a^5b^9c^2f^2 - 3840a^9b^3c^5f^2 - 9a^5c^2f^2(-4ac - b^2)^9)^{(1/2)} - 30ab^{14}d^2e + \\
& 6366a^2b^{11}c^2d^2 - 35767a^3b^9c^3d^2 + 116928a^4b^7c^4d^2 - 219744a^5b^5c^5d^2 + 215040a^6b^3c^6d^2 + 9a^2b^4e^2(-4ac - b^2)^9)^{(1/2)} \\
& - 49a^3c^3d^2(-4ac - b^2)^9)^{(1/2)} + 2077a^4b^9c^2e^2 - 10656a^5b^7c^3e^2 + 30240a^6b^5c^4e^2 - 44800a^7b^3c^5e^2 + a^4b^2f^2(-4ac - b^2)^9)^{(1/2)} \\
& + 25a^4c^2e^2(-4ac - b^2)^9)^{(1/2)} + 288a^6b^7c^2f^2 - 1504a^7b^5c^3f^2 + 3840a^8b^3c^4f^2 - 615ab^{13}cd^2 + 10a^2b^{13}df + 35840a^8c^7d^2e \\
& - 6a^3b^{12}e^2f - 15360a^9c^6e^2f - 30ab^5d^2e(-4ac - b^2)^9)^{(1/2)} + 724a^2b^{12}c^2d^2e - 258a^3b^{11}c^2d^2f + 43520a^8b^3c^6d^2f \\
& + 152a^4b^{10}c^2e^2f + 246a^2b^2c^2d^2(-4ac - b^2)^9)^{(1/2)} - 165ab^4c^2d^2(-4ac - b^2)^9)^{(1/2)} - 7278a^3b^{10}c^2d^2e + 39132a^4b^8c^3d^2e \\
& - 119616a^5b^6c^4d^2e + 201600a^6b^4c^5d^2e - 161280a^7b^2c^6d^2e + 10a^2b^4d^2f(-4ac - b^2)^9)^{(1/2)} + 2706a^4b^9c^2d^2f - 14784a^5b^7c^3d^2f \\
& + 44352a^6b^5c^4d^2f - 69120a^7b^3c^5d^2f - 6a^3b^3e^2f(-4ac - b^2)^9)^{(1/2)} + 42a^4c^2d^2f(-4ac - b^2)^9)^{(1/2)} - 1548a^5b^8c^2e^2f + \\
& 8064a^6b^6c^3e^2f - 22400a^7b^4c^4e^2f + 30720a^8b^2c^5e^2f - 51a^3b^2c^2e^2(-4ac - b^2)^9)^{(1/2)} + 44a^4b^2c^2e^2f(-4ac - b^2)^9)^{(1/2)} \\
& + 184a^2b^3c^2d^2e(-4ac - b^2)^9)^{(1/2)} - 186a^3b^2c^2d^2e(-4ac - b^2)^9)^{(1/2)} - 78a^3b^2c^2d^2f(-4ac - b^2)^9)^{(1/2)} / (32(a^7b^{12} + 4096a^{13}c^6 - 24a^8b^{10}c + 240a^9b^8c^2 \\
& - 1280a^{10}b^6c^3 + 3840a^{11}b^4c^4 - 6144a^{12}b^2c^5))^{(1/2)}(1048576a^{21}b^8c^8 + 256a^{15}b^{13}c^2 - 6144a^{16}b^{11}c^3 + 61440a^{17}b^9c^4 - 327680a^{18}b^7c^5 \\
& + 983040a^{19}b^5c^6 - 1572864a^{20}b^3c^7) - 320a^{12}b^{14}c^2d + 7936a^{13}b^{12}c^3d - 82816a^{14}b^{10}c^4d + 468480a^{15}b^8c^5d - 1536000 \\
& a^{16}b^6c^6d + 2867200a^{17}b^4c^7d - 2719744a^{18}b^2c^8d + 192a^{13}b^{13}c^2e - 4672a^{14}b^{11}c^3e + 47360a^{15}b^9c^4e - 256000a^{16}b^7c^5e \\
& + 778240a^{17}b^5c^6e - 1261568a^{18}b^3c^7e - 64a^{14}b^{12}c^2f + 1664a^{15}b^{10}c^3f - 17920a^{16}b^8c^4f + 102400a^{17}b^6c^5f - 327680a^{18}b^4c^6f \\
& + 557056a^{19}b^2c^7f + 851968a^{19}b^2c^8e))(-25b^{15}d^2 + 9a^2b^{13}e^2 + 25b^6d^2(-4ac - b^2)^9)^{(1/2)} + a^4b^{11}f^2 - 80640a^7b^3c^7d^2 \\
& - 213a^3b^{11}c^2e^2 + 26880a^8b^3c^6e^2 - 27a^5b^9c^2f^2 - 3840a^9b^3c^5f^2 - 9a^5c^2f^2(-4ac - b^2)^9)^{(1/2)} - 30ab^{14}d^2e + 6366a^2b^{11}c^2d^2 \\
& - 35767a^3b^9c^3d^2 + 116928a^4b^7c^4d^2 - 219744a^5b^5c^5d^2 + 215040a^6b^3c^6d^2 + 9a^2b^4e^2(-4ac - b^2)^9)^{(1/2)} - 49a^3c^3d^2(-4ac - b^2)^9)^{(1/2)} + 20 \\
& 77a^4b^9c^2e^2 - 10656a^5b^7c^3e^2 + 30240a^6b^5c^4e^2 - 44800a^7b^3c^5e^2 + a^4b^2f^2(-4ac - b^2)^9)^{(1/2)} + 25a^4c^2e^2(-4ac - b^2)^9)^{(1/2)} \\
& + 288a^6b^7c^2f^2 - 1504a^7b^5c^3f^2 + 3840a^8b^3c^4f^2 + 3840a^8b^3c^4f^2
\end{aligned}$$

$$\begin{aligned}
& \cdot 8b^3c^4f^2 - 615a^2b^13cd^2 + 10a^2b^13d^2f + 35840a^8c^7d^2e - 6 \\
& a^3b^12ef - 15360a^9c^6e^2f - 30a^2b^5d^2e^2(-4ac - b^2)^9)^{(1/2)} + \\
& 724a^2b^12cd^2e - 258a^3b^11cd^2f + 43520a^8b^6cd^2f + 152a^4b^10 \\
& c^2e^2f + 246a^2b^2c^2d^2(-4ac - b^2)^9)^{(1/2)} - 165a^2b^4cd^2(- \\
& -4ac - b^2)^9)^{(1/2)} - 7278a^3b^10c^2d^2e + 39132a^4b^8c^3d^2e - 1 \\
& 19616a^5b^6c^4d^2e + 201600a^6b^4c^5d^2e - 161280a^7b^2c^6d^2e + 1 \\
& 0a^2b^4d^2f(-4ac - b^2)^9)^{(1/2)} + 2706a^4b^9c^2d^2f - 14784a^5b^7 \\
& c^3d^2f + 44352a^6b^5c^4d^2f - 69120a^7b^3c^5d^2f - 6a^3b^3e^2f^2 \\
& (-4ac - b^2)^9)^{(1/2)} + 42a^4c^2d^2f(-4ac - b^2)^9)^{(1/2)} - 1548a^5 \\
& b^8c^2e^2f + 8064a^6b^6c^3e^2f - 22400a^7b^4c^4e^2f + 30720a^8b^2 \\
& c^5e^2f - 51a^3b^2c^2e^2(-4ac - b^2)^9)^{(1/2)} + 44a^4b^2c^2e^2(- \\
& -4ac - b^2)^9)^{(1/2)} + 184a^2b^3cd^2e(-4ac - b^2)^9)^{(1/2)} - 186a^3 \\
& b^2c^2d^2e(-4ac - b^2)^9)^{(1/2)} - 78a^3b^2cd^2e(-4ac - b^2)^9)^{(1/2)} \\
& (1/2))/(32(a^7b^12 + 4096a^13c^6 - 24a^8b^10c + 240a^9b^8c^2 - 12 \\
& 80a^10b^6c^3 + 3840a^11b^4c^4 - 6144a^12b^2c^5))^{(1/2)} * i) / ((x(2 \\
& 04800a^17c^9e^2 - 401408a^16c^10d^2 - 73728a^18c^8f^2 + 400a^9b^14 \\
& c^3d^2 - 9440a^10b^12c^4d^2 + 92816a^11b^10c^5d^2 - 488096a^12 \\
& b^8c^6d^2 + 1458688a^13b^6c^7d^2 - 2401280a^14b^4c^8d^2 + 187187 \\
& 2a^15b^2c^9d^2 + 144a^11b^12c^3e^2 - 3264a^12b^10c^4e^2 + 30112 \\
& a^13b^8c^5e^2 - 143360a^14b^6c^6e^2 + 365568a^15b^4c^7e^2 - 458 \\
& 752a^16b^2c^8e^2 + 16a^13b^10c^3f^2 - 416a^14b^8c^4f^2 + 4608a^15 \\
& b^6c^5f^2 - 25600a^16b^4c^6f^2 + 69632a^17b^2c^7f^2 + 344064a^17 \\
& c^9d^2f - 1236992a^16b^6c^9d^2e + 237568a^17b^6c^8e^2f - 480a^10b^13 \\
& c^3d^2e + 11104a^11b^11c^4d^2e - 105824a^12b^9c^5d^2e + 530432a^13 \\
& b^7c^6d^2e - 1469440a^14b^5c^7d^2e + 2121728a^15b^3c^8d^2e + 160a^11 \\
& b^12c^3d^2f - 3968a^12b^10c^4d^2f + 39488a^13b^8c^5d^2f - 200704 \\
& a^14b^6c^6d^2f + 542720a^15b^4c^7d^2f - 720896a^16b^2c^8d^2f - 96a^12 \\
& b^11c^3e^2f + 2336a^13b^9c^4e^2f - 22528a^14b^7c^5e^2f + 107520 \\
& a^15b^5c^6e^2f - 253952a^16b^3c^7e^2f) + (-25b^15d^2 + 9a^2b^13e^2 \\
& + 25b^6d^2(-4ac - b^2)^9)^{(1/2)} + a^4b^11f^2 - 80640a^7b^6c^7d^2 \\
& - 213a^3b^11c^2e^2 + 26880a^8b^6c^6e^2 - 27a^5b^9c^2f^2 - 3840a^9 \\
& b^5c^5f^2 - 9a^5c^2f^2(-4ac - b^2)^9)^{(1/2)} - 30a^2b^14d^2e + 6366a^2 \\
& b^11c^2d^2 - 35767a^3b^9c^3d^2 + 116928a^4b^7c^4d^2 - 219744a^5 \\
& b^5c^5d^2 + 215040a^6b^3c^6d^2 + 9a^2b^4e^2(-4ac - b^2)^9)^{(1/2)} - \\
& 49a^3c^3d^2(-4ac - b^2)^9)^{(1/2)} + 2077a^4b^9c^2e^2 - 10 \\
& 656a^5b^7c^3e^2 + 30240a^6b^5c^4e^2 - 44800a^7b^3c^5e^2 + a^4b^2 \\
& f^2(-4ac - b^2)^9)^{(1/2)} + 25a^4c^2e^2(-4ac - b^2)^9)^{(1/2)} + \\
& 288a^6b^7c^2f^2 - 1504a^7b^5c^3f^2 + 3840a^8b^3c^4f^2 - 615a^2 \\
& b^13cd^2 + 10a^2b^13d^2f + 35840a^8c^7d^2e - 6a^3b^12ef - 15360a^9 \\
& c^6e^2f - 30a^2b^5d^2e^2(-4ac - b^2)^9)^{(1/2)} + 724a^2b^12cd^2e - 2 \\
& 58a^3b^11cd^2f + 43520a^8b^6cd^2f + 152a^4b^10c^2e^2f + 246a^2b^2c^2 \\
& d^2(-4ac - b^2)^9)^{(1/2)} - 165a^2b^4cd^2(-4ac - b^2)^9)^{(1/2)} - \\
& 7278a^3b^10c^2d^2e + 39132a^4b^8c^3d^2e - 119616a^5b^6c^4d^2e + \\
& 201600a^6b^4c^5d^2e - 161280a^7b^2c^6d^2e + 10a^2b^4d^2f(-4ac - \\
& - b^2)^9)^{(1/2)} + 2706a^4b^9c^2d^2f - 14784a^5b^7c^3d^2f + 44352a^6 \\
& b^5c^4d^2f - 69120a^7b^3c^5d^2f - 6a^3b^3e^2f^2(-4ac - b^2)^9)^{(1/2)} \\
&) + 42a^4c^2d^2f(-4ac - b^2)^9)^{(1/2)} - 1548a^5b^8c^2e^2f + 8064a^6 \\
& b^6c^3e^2f - 22400a^7b^4c^4e^2f + 30720a^8b^2c^5e^2f - 51a^3b^2 \\
& c^2e^2(-4ac - b^2)^9)^{(1/2)} + 44a^4b^2c^2e^2(-4ac - b^2)^9)^{(1/2)} + \\
& 184a^2b^3cd^2e(-4ac - b^2)^9)^{(1/2)} - 186a^3b^2c^2d^2e(-4ac - \\
& - b^2)^9)^{(1/2)} - 78a^3b^2cd^2e(-4ac - b^2)^9)^{(1/2)} / (32(a^7b^12 + \\
& 4096a^13c^6 - 24a^8b^10c + 240a^9b^8c^2 - 1280a^10b^6c^3 + 3840a^11 \\
& b^4c^4 - 6144a^12b^2c^5))^{(1/2)} * (393216a^20c^8f - 917504a^19c^9 \\
& d + x(-25b^15d^2 + 9a^2b^13e^2 + 25b^6d^2(-4ac - b^2)^9)^{(1/2)} + \\
& a^4b^11f^2 - 80640a^7b^6c^7d^2 - 213a^3b^11c^2e^2 + 26880a^8b^6c^6 \\
& e^2 - 27a^5b^9c^2f^2 - 3840a^9b^5c^5f^2 - 9a^5c^2f^2(-4ac - b^2)^9)^{(1/2)} \\
& - 30a^2b^14d^2e + 6366a^2b^11c^2d^2 - 35767a^3b^9c^3d^2 + 116928a^4 \\
& b^7c^4d^2 - 219744a^5b^5c^5d^2 + 215040a^6b^3c^6d^2 + 9a^2b^4e^2(-4ac - \\
& - b^2)^9)^{(1/2)} - 49a^3c^3d^2(-4ac - b^2)^9)^{(1/2)} - 2077a^4b^9c^2e^2
\end{aligned}$$

$$\begin{aligned}
&)^9)^{(1/2)} + 2077a^4b^9c^2e^2 - 10656a^5b^7c^3e^2 + 30240a^6b^5c^4e^2 - 44800a^7b^3c^5e^2 + a^4b^2f^2(-4ac - b^2)^9)^{(1/2)} + 25a^4c^2e^2(-4ac - b^2)^9)^{(1/2)} + 288a^6b^7c^2f^2 - 1504a^7b^5c^3f^2 + 3840a^8b^3c^4f^2 - 615ab^{13}cd^2 + 10a^2b^{13}df + 35840a^8c^7d^2e - 6a^3b^{12}ef - 15360a^9c^6ef - 30ab^5d^2e(-4ac - b^2)^9)^{(1/2)} + 724a^2b^{12}cde - 258a^3b^{11}cdf + 43520a^8b^6c^6d^2ef + 152a^4b^{10}c^2ef + 246a^2b^2c^2d^2(-4ac - b^2)^9)^{(1/2)} - 165ab^4cd^2(-4ac - b^2)^9)^{(1/2)} - 7278a^3b^{10}c^2d^2e + 39132a^4b^8c^3d^2e - 119616a^5b^6c^4d^2e + 201600a^6b^4c^5d^2e - 161280a^7b^2c^6d^2e + 10a^2b^4d^2f(-4ac - b^2)^9)^{(1/2)} + 2706a^4b^9c^2d^2f - 14784a^5b^7c^3d^2f + 44352a^6b^5c^4d^2f - 69120a^7b^3c^5d^2f - 6a^3b^3e^2f(-4ac - b^2)^9)^{(1/2)} + 42a^4c^2d^2f(-4ac - b^2)^9)^{(1/2)} - 1548a^5b^8c^2e^2f + 8064a^6b^6c^3e^2f - 22400a^7b^4c^4e^2f + 30720a^8b^2c^5e^2f - 51a^3b^2c^2e^2(-4ac - b^2)^9)^{(1/2)} + 44a^4b^2c^2e^2f(-4ac - b^2)^9)^{(1/2)} + 184a^2b^3c^2d^2e^2(-4ac - b^2)^9)^{(1/2)} - 186a^3b^2c^2d^2e^2(-4ac - b^2)^9)^{(1/2)} - 78a^3b^2c^2d^2e^2(-4ac - b^2)^9)^{(1/2))} / (32(a^7b^{12} + 4096a^{13}c^6 - 24a^8b^{10}c + 240a^9b^8c^2 - 1280a^{10}b^6c^3 + 3840a^{11}b^4c^4 - 6144a^{12}b^2c^5))^{(1/2)} * (1048576a^{21}b^8c^8 + 256a^{15}b^{13}c^2 - 6144a^{16}b^{11}c^3 + 61440a^{17}b^9c^4 - 327680a^{18}b^7c^5 + 983040a^{19}b^5c^6 - 1572864a^{20}b^3c^7) + 320a^{12}b^{14}c^2d - 7936a^{13}b^{12}c^3d + 82816a^{14}b^{10}c^4d - 468480a^{15}b^8c^5d + 1536000a^{16}b^6c^6d - 2867200a^{17}b^4c^7d + 2719744a^{18}b^2c^8d - 192a^{13}b^{13}c^2e + 4672a^{14}b^{11}c^3e - 47360a^{15}b^9c^4e + 256000a^{16}b^7c^5e - 778240a^{17}b^5c^6e + 1261568a^{18}b^3c^7e + 64a^{14}b^{12}c^2f - 1664a^{15}b^{10}c^3f + 17920a^{16}b^8c^4f - 102400a^{17}b^6c^5f + 327680a^{18}b^4c^6f - 557056a^{19}b^2c^7f - 851968a^{19}b^2c^8e) * (-25b^{15}d^2 + 9a^2b^{13}e^2 + 25b^6d^2(-4ac - b^2)^9)^{(1/2)} + a^4b^{11}f^2 - 80640a^7b^6c^7d^2 - 213a^3b^{11}c^2e^2 + 26880a^8b^6c^6e^2 - 27a^5b^9c^2f^2 - 3840a^9b^5c^5f^2 - 9a^5c^2f^2(-4ac - b^2)^9)^{(1/2)} - 30ab^{14}d^2e + 6366a^2b^{11}c^2d^2 - 35767a^3b^9c^3d^2 + 116928a^4b^7c^4d^2 - 219744a^5b^5c^5d^2 + 215040a^6b^3c^6d^2 + 9a^2b^4e^2(-4ac - b^2)^9)^{(1/2)} - 49a^3c^3d^2(-4ac - b^2)^9)^{(1/2)} + 2077a^4b^9c^2e^2 - 10656a^5b^7c^3e^2 + 30240a^6b^5c^4e^2 - 44800a^7b^3c^5e^2 + a^4b^2f^2(-4ac - b^2)^9)^{(1/2)} + 25a^4c^2e^2(-4ac - b^2)^9)^{(1/2)} + 288a^6b^7c^2f^2 - 1504a^7b^5c^3f^2 + 3840a^8b^3c^4f^2 - 615ab^{13}cd^2 + 10a^2b^{13}df + 35840a^8c^7d^2e - 6a^3b^{12}ef - 15360a^9c^6ef - 30ab^5d^2e(-4ac - b^2)^9)^{(1/2)} + 724a^2b^{12}cde - 258a^3b^{11}cdf + 43520a^8b^6c^6d^2ef + 152a^4b^{10}c^2ef + 246a^2b^2c^2d^2(-4ac - b^2)^9)^{(1/2)} - 165ab^4cd^2(-4ac - b^2)^9)^{(1/2)} - 7278a^3b^{10}c^2d^2e + 39132a^4b^8c^3d^2e - 119616a^5b^6c^4d^2e + 201600a^6b^4c^5d^2e - 161280a^7b^2c^6d^2e + 10a^2b^4d^2f(-4ac - b^2)^9)^{(1/2)} + 2706a^4b^9c^2d^2f - 14784a^5b^7c^3d^2f + 44352a^6b^5c^4d^2f - 69120a^7b^3c^5d^2f - 6a^3b^3e^2f(-4ac - b^2)^9)^{(1/2)} + 42a^4c^2d^2f(-4ac - b^2)^9)^{(1/2)} - 1548a^5b^8c^2e^2f + 8064a^6b^6c^3e^2f - 22400a^7b^4c^4e^2f + 30720a^8b^2c^5e^2f - 51a^3b^2c^2e^2(-4ac - b^2)^9)^{(1/2)} + 44a^4b^2c^2e^2f(-4ac - b^2)^9)^{(1/2)} + 184a^2b^3c^2d^2e^2(-4ac - b^2)^9)^{(1/2)} - 186a^3b^2c^2d^2e^2(-4ac - b^2)^9)^{(1/2)} - 78a^3b^2c^2d^2e^2(-4ac - b^2)^9)^{(1/2))} / (32(a^7b^{12} + 4096a^{13}c^6 - 24a^8b^{10}c + 240a^9b^8c^2 - 1280a^{10}b^6c^3 + 3840a^{11}b^4c^4 - 6144a^{12}b^2c^5))^{(1/2)} - (x(204800a^{17}c^9e^2 - 401408a^{16}c^{10}d^2 - 73728a^{18}c^8f^2 + 400a^9b^{14}c^3d^2 - 9440a^{10}b^{12}c^4d^2 + 92816a^{11}b^{10}c^5d^2 - 488096a^{12}b^8c^6d^2 + 1458688a^{13}b^6c^7d^2 - 2401280a^{14}b^4c^8d^2 + 1871872a^{15}b^2c^9d^2 + 144a^{11}b^{12}c^3e^2 - 3264a^{12}b^{10}c^4e^2 + 30112a^{13}b^8c^5e^2 - 143360a^{14}b^6c^6e^2 + 365568a^{15}b^4c^7e^2 - 458752a^{16}b^2c^8e^2 + 16a^{13}b^{10}c^3f^2 - 416a^{14}b^8c^4f^2 + 4608a^{15}b^6c^5f^2 - 25600a^{16}b^4c^6f^2 + 69632a^{17}b^2c^7f^2 + 344064a^{17}c^9d^2f - 1236992a^{16}b^6c^9d^2e + 237568a^{17}b^6c^8e^2f - 480a^{10}b^{13}c^3d^2e + 11104a^{11}b^{11}c^4d^2e - 105824a
\end{aligned}$$

$$\begin{aligned}
& a^{12}b^9c^5d^5e + 530432a^{13}b^7c^6d^5e - 1469440a^{14}b^5c^7d^5e + 212 \\
& 1728a^{15}b^3c^8d^5e + 160a^{11}b^{12}c^3d^5f - 3968a^{12}b^{10}c^4d^5f + 39 \\
& 488a^{13}b^8c^5d^5f - 200704a^{14}b^6c^6d^5f + 542720a^{15}b^4c^7d^5f - \\
& 720896a^{16}b^2c^8d^5f - 96a^{12}b^{11}c^3e^5f + 2336a^{13}b^9c^4e^5f - 22 \\
& 528a^{14}b^7c^5e^5f + 107520a^{15}b^5c^6e^5f - 253952a^{16}b^3c^7e^5f) + \\
& (- (25b^{15}d^2 + 9a^2b^{13}e^2 + 25b^6d^2*(-(4ac - b^2)^9)^{(1/2)} + a^4 \\
& 4b^{11}f^2 - 80640a^7b^7c^7d^2 - 213a^3b^{11}c^3e^2 + 26880a^8b^7c^6e^2 \\
& - 27a^5b^9c^3f^2 - 3840a^9b^5c^5f^2 - 9a^5c^3f^2*(-(4ac - b^2)^9)^{(1/2)} - \\
& 30a^2b^{14}d^5e + 6366a^2b^{11}c^2d^2 - 35767a^3b^9c^3d^2 + 1169 \\
& 28a^4b^7c^4d^2 - 219744a^5b^5c^5d^2 + 215040a^6b^3c^6d^2 + 9a^2 \\
& 2b^4e^2*(-(4ac - b^2)^9)^{(1/2)} - 49a^3c^3d^2*(-(4ac - b^2)^9)^{(1/2)} \\
&) + 2077a^4b^9c^2e^2 - 10656a^5b^7c^3e^2 + 30240a^6b^5c^4e^2 - \\
& 44800a^7b^3c^5e^2 + a^4b^2f^2*(-(4ac - b^2)^9)^{(1/2)} + 25a^4c^2e^2 \\
& 2*(-(4ac - b^2)^9)^{(1/2)} + 288a^6b^7c^2f^2 - 1504a^7b^5c^3f^2 + \\
& 3840a^8b^3c^4f^2 - 615a^2b^{13}c^4d^2 + 10a^2b^{13}d^5f + 35840a^8c^7d^5 \\
& e - 6a^3b^{12}e^5f - 15360a^9c^6e^5f - 30a^2b^5d^5e*(-(4ac - b^2)^9)^{(1/2)} \\
& + 724a^2b^{12}c^4d^5e - 258a^3b^{11}c^4d^5f + 43520a^8b^7c^6d^5f + 152a^4 \\
& b^{10}c^4e^5f + 246a^2b^2c^2d^2*(-(4ac - b^2)^9)^{(1/2)} - 165a^2b^4c^4 \\
& d^2*(-(4ac - b^2)^9)^{(1/2)} - 7278a^3b^{10}c^2d^5e + 39132a^4b^8c^3d^5 \\
& e - 119616a^5b^6c^4d^5e + 201600a^6b^4c^5d^5e - 161280a^7b^2c^6d^5 \\
& e + 10a^2b^4d^5f*(-(4ac - b^2)^9)^{(1/2)} + 2706a^4b^9c^2d^5f - 14784 \\
& a^5b^7c^3d^5f + 44352a^6b^5c^4d^5f - 69120a^7b^3c^5d^5f - 6a^3b^3 \\
& 3e^5f*(-(4ac - b^2)^9)^{(1/2)} + 42a^4c^2d^5f*(-(4ac - b^2)^9)^{(1/2)} - \\
& 1548a^5b^8c^2e^5f + 8064a^6b^6c^3e^5f - 22400a^7b^4c^4e^5f + 30720 \\
& a^8b^2c^5e^5f - 51a^3b^2c^5e^2*(-(4ac - b^2)^9)^{(1/2)} + 44a^4b^3c^5 \\
& e^5f*(-(4ac - b^2)^9)^{(1/2)} + 184a^2b^3c^4d^5e*(-(4ac - b^2)^9)^{(1/2)} - \\
& 186a^3b^3c^2d^5e*(-(4ac - b^2)^9)^{(1/2)} - 78a^3b^2c^4d^5f*(-(4ac - b^2)^9)^{(1/2)} \\
&)/(32*(a^7b^{12} + 4096a^{13}c^6 - 24a^8b^{10}c + 240a^9b^8c^2 - 1280a^{10}b^6c^3 \\
& + 3840a^{11}b^4c^4 - 6144a^{12}b^2c^5)))^{(1/2)}*(917 \\
& 504a^{19}c^9d - 393216a^{20}c^8f + x*(-(25b^{15}d^2 + 9a^2b^{13}e^2 + 25 \\
& b^6d^2*(-(4ac - b^2)^9)^{(1/2)} + a^4b^{11}f^2 - 80640a^7b^7c^7d^2 - 21 \\
& 3a^3b^{11}c^3e^2 + 26880a^8b^7c^6e^2 - 27a^5b^9c^3f^2 - 3840a^9b^5c^5 \\
& f^2 - 9a^5c^3f^2*(-(4ac - b^2)^9)^{(1/2)} - 30a^2b^{14}d^5e + 6366a^2b^{11} \\
& c^2d^2 - 35767a^3b^9c^3d^2 + 116928a^4b^7c^4d^2 - 219744a^5b^5c^5 \\
& d^2 + 215040a^6b^3c^6d^2 + 9a^2b^4e^2*(-(4ac - b^2)^9)^{(1/2)} - \\
& 49a^3c^3d^2*(-(4ac - b^2)^9)^{(1/2)} + 2077a^4b^9c^2e^2 - 10656a^5b^7 \\
& c^3e^2 + 30240a^6b^5c^4e^2 - 44800a^7b^3c^5e^2 + a^4b^2f^2*(-(4ac - b^2)^9)^{(1/2)} \\
& + 25a^4c^2e^2*(-(4ac - b^2)^9)^{(1/2)} + 288a^6b^7c^2f^2 - 1504a^7b^5c^3 \\
& f^2 + 3840a^8b^3c^4f^2 - 615a^2b^{13}c^4 \\
& d^2 + 10a^2b^{13}d^5f + 35840a^8c^7d^5e - 6a^3b^{12}e^5f - 15360a^9c^6e^5 \\
& f - 30a^2b^5d^5e*(-(4ac - b^2)^9)^{(1/2)} + 724a^2b^{12}c^4d^5e - 258a^3b \\
& ^{11}c^4d^5f + 43520a^8b^7c^6d^5f + 152a^4b^{10}c^4e^5f + 246a^2b^2c^2d^2 \\
& 2*(-(4ac - b^2)^9)^{(1/2)} - 165a^2b^4c^4d^2*(-(4ac - b^2)^9)^{(1/2)} - 7278 \\
& a^3b^{10}c^2d^5e + 39132a^4b^8c^3d^5e - 119616a^5b^6c^4d^5e + 201600 \\
& a^6b^4c^5d^5e - 161280a^7b^2c^6d^5e + 10a^2b^4d^5f*(-(4ac - b^2)^9 \\
&)^{(1/2)} + 2706a^4b^9c^2d^5f - 14784a^5b^7c^3d^5f + 44352a^6b^5c^4 \\
& d^5f - 69120a^7b^3c^5d^5f - 6a^3b^3e^5f*(-(4ac - b^2)^9)^{(1/2)} + 42a \\
& ^4c^2d^5f*(-(4ac - b^2)^9)^{(1/2)} - 1548a^5b^8c^2e^5f + 8064a^6b^6c^3 \\
& e^5f - 22400a^7b^4c^4e^5f + 30720a^8b^2c^5e^5f - 51a^3b^2c^5e^2*(-(4ac - b^2)^9)^{(1/2)} \\
& + 44a^4b^3c^5e^5f*(-(4ac - b^2)^9)^{(1/2)} + 184a^2b^3c^4d^5e*(-(4ac - b^2)^9)^{(1/2)} \\
& - 186a^3b^3c^2d^5e*(-(4ac - b^2)^9)^{(1/2)} - 78a^3b^2c^4d^5f*(-(4ac - b^2)^9)^{(1/2)} \\
&)/(32*(a^7b^{12} + 4096a^{13}c^6 - 24a^8b^{10}c + 240a^9b^8c^2 - 1280a^{10}b^6c^3 + 3840a^{11}b^4 \\
& c^4 - 6144a^{12}b^2c^5)))^{(1/2)}*(1048576a^{21}b^8c^8 + 256a^{15}b^{13}c^2 - \\
& 6144a^{16}b^{11}c^3 + 61440a^{17}b^9c^4 - 327680a^{18}b^7c^5 + 983040a^{19}b^5c^6 - 1572864a^{20}b^3c^7) \\
& - 320a^{12}b^{14}c^2d + 7936a^{13}b^{12}c^3d - 82816a^{14}b^{10}c^4d + 468480a^{15}b^8c^5d - 1536000a^{16}b^6c^6 \\
& d + 2867200a^{17}b^4c^7d - 2719744a^{18}b^2c^8d + 192a^{13}b^{13}c^2e - 4672a^{14}b^{11}c^3e + 47360a^{15}b^9c^4e - 256000a^{16}b^7c^5e + 7782
\end{aligned}$$

$$\begin{aligned}
&40a^{17}b^5c^6e - 1261568a^{18}b^3c^7e - 64a^{14}b^{12}c^2f + 1664a^{15} \\
&b^{10}c^3f - 17920a^{16}b^8c^4f + 102400a^{17}b^6c^5f - 327680a^{18}b^4 \\
&c^6f + 557056a^{19}b^2c^7f + 851968a^{19}b^3c^8e) * (- (25b^{15}d^2 + 9 \\
&a^2b^{13}e^2 + 25b^6d^2 * (- (4ac - b^2)^9)^{(1/2)} + a^4b^{11}f^2 - 80640a \\
&^7b^3c^7d^2 - 213a^3b^{11}c^2e^2 + 26880a^8b^3c^6e^2 - 27a^5b^9c^2f^2 \\
&- 3840a^9b^3c^5f^2 - 9a^5c^2f^2 * (- (4ac - b^2)^9)^{(1/2)} - 30a^2b^{14}d^2e \\
&+ 6366a^2b^{11}c^2d^2 - 35767a^3b^9c^3d^2 + 116928a^4b^7c^4d^2 - \\
&219744a^5b^5c^5d^2 + 215040a^6b^3c^6d^2 + 9a^2b^4e^2 * (- (4ac - \\
&b^2)^9)^{(1/2)} - 49a^3c^3d^2 * (- (4ac - b^2)^9)^{(1/2)} + 2077a^4b^9c^2 \\
&*e^2 - 10656a^5b^7c^3e^2 + 30240a^6b^5c^4e^2 - 44800a^7b^3c^5e^2 \\
&+ a^4b^2f^2 * (- (4ac - b^2)^9)^{(1/2)} + 25a^4c^2e^2 * (- (4ac - b^2)^9 \\
&)^{(1/2)} + 288a^6b^7c^2f^2 - 1504a^7b^5c^3f^2 + 3840a^8b^3c^4f^2 \\
&- 615a^2b^{13}c^2d^2 + 10a^2b^{13}d^2f + 35840a^8c^7d^2e - 6a^3b^{12}e^2f \\
&- 15360a^9c^6e^2f - 30a^2b^5d^2e * (- (4ac - b^2)^9)^{(1/2)} + 724a^2b^{12}c \\
&d^2e - 258a^3b^{11}c^2d^2f + 43520a^8b^3c^6d^2f + 152a^4b^{10}c^2e^2f + 246 \\
&a^2b^2c^2d^2 * (- (4ac - b^2)^9)^{(1/2)} - 165a^2b^4c^2d^2 * (- (4ac - b^2)^ \\
&^9)^{(1/2)} - 7278a^3b^{10}c^2d^2e + 39132a^4b^8c^3d^2e - 119616a^5b^6c \\
&^4d^2e + 201600a^6b^4c^5d^2e - 161280a^7b^2c^6d^2e + 10a^2b^4d^2f * \\
&(- (4ac - b^2)^9)^{(1/2)} + 2706a^4b^9c^2d^2f - 14784a^5b^7c^3d^2f + 4 \\
&4352a^6b^5c^4d^2f - 69120a^7b^3c^5d^2f - 6a^3b^3e^2f * (- (4ac - b^2 \\
&)^9)^{(1/2)} + 42a^4c^2d^2f * (- (4ac - b^2)^9)^{(1/2)} - 1548a^5b^8c^2e^2f \\
&+ 8064a^6b^6c^3e^2f - 22400a^7b^4c^4e^2f + 30720a^8b^2c^5e^2f - 5 \\
&1a^3b^2c^2e^2 * (- (4ac - b^2)^9)^{(1/2)} + 44a^4b^3c^2e^2f * (- (4ac - b^2)^9 \\
&)^{(1/2)} + 184a^2b^3c^2d^2e * (- (4ac - b^2)^9)^{(1/2)} - 186a^3b^3c^2d^2e * (- \\
&(4ac - b^2)^9)^{(1/2)} - 78a^3b^2c^2d^2f * (- (4ac - b^2)^9)^{(1/2))} / (32(a^ \\
&7b^{12} + 4096a^{13}c^6 - 24a^8b^{10}c + 240a^9b^8c^2 - 1280a^{10}b^6c^3 \\
&+ 3840a^{11}b^4c^4 - 6144a^{12}b^2c^5))^{(1/2)} - 128000a^{15}c^9e^3 + \\
&476672a^{13}b^3c^{10}d^3 - 4608a^{16}b^3c^7f^3 - 250880a^{14}c^{10}d^2e - 460 \\
&80a^{16}c^8e^2f^2 + 1800a^9b^9c^6d^3 - 29080a^{10}b^7c^7d^3 + 176032a \\
&a^{11}b^5c^8d^3 - 473216a^{12}b^3c^9d^3 - 504a^{11}b^8c^5e^3 + 8112a^{12}b^6c^6e^3 \\
&- 48704a^{13}b^4c^7e^3 + 129280a^{14}b^2c^8e^3 + 40a^{13} \\
&b^7c^4f^3 - 608a^{14}b^5c^5f^3 + 2944a^{15}b^3c^6f^3 + 215040a^{15}c \\
&^9d^2e^2f + 442880a^{14}b^3c^9d^2e^2 - 433664a^{14}b^3c^9d^2f + 109056a^{15} \\
&b^3c^8d^2f^2 + 84480a^{15}b^3c^8e^2f - 1400a^9b^{10}c^5d^2e + 21680a^{10} \\
&b^8c^6d^2e + 1680a^{10}b^9c^5d^2e - 121648a^{11}b^6c^7d^2e - 2717 \\
&6a^{11}b^7c^6d^2e + 275264a^{12}b^4c^8d^2e + 164448a^{12}b^5c^7d^2e^2 \\
&- 121088a^{13}b^2c^9d^2e - 441216a^{13}b^3c^8d^2e^2 + 1000a^9b^{11}c \\
&^4d^2f - 17800a^{10}b^9c^5d^2f + 124280a^{11}b^7c^6d^2f + 400a^{11} \\
&b^9c^4d^2f^2 - 422944a^{12}b^5c^7d^2f - 6600a^{12}b^7c^5d^2f^2 + 69491 \\
&2a^{13}b^3c^8d^2f + 40416a^{13}b^5c^6d^2f^2 - 108928a^{14}b^3c^7d^2f^2 \\
&+ 360a^{11}b^9c^4e^2f - 5736a^{12}b^7c^5e^2f - 240a^{12}b^8c^4e^2f^2 \\
&+ 33888a^{13}b^5c^6e^2f + 3792a^{13}b^6c^5e^2f^2 - 87936a^{14}b^3c^7 \\
&*e^2f - 21696a^{14}b^4c^6e^2f^2 + 52992a^{15}b^2c^7e^2f^2 - 1200a^{10}b^ \\
&10c^4d^2e^2f + 20240a^{11}b^8c^5d^2e^2f - 130656a^{12}b^6c^6d^2e^2f + 39436 \\
&8a^{13}b^4c^7d^2e^2f - 528896a^{14}b^2c^8d^2e^2f) * (- (25b^{15}d^2 + 9a^2b \\
&^13e^2 + 25b^6d^2 * (- (4ac - b^2)^9)^{(1/2)} + a^4b^{11}f^2 - 80640a^7b^3 \\
&c^7d^2 - 213a^3b^{11}c^2e^2 + 26880a^8b^3c^6e^2 - 27a^5b^9c^2f^2 - 384 \\
&0a^9b^3c^5f^2 - 9a^5c^2f^2 * (- (4ac - b^2)^9)^{(1/2)} - 30a^2b^{14}d^2e + 63 \\
&66a^2b^{11}c^2d^2 - 35767a^3b^9c^3d^2 + 116928a^4b^7c^4d^2 - 2197 \\
&44a^5b^5c^5d^2 + 215040a^6b^3c^6d^2 + 9a^2b^4e^2 * (- (4ac - b^2)^ \\
&^9)^{(1/2)} - 49a^3c^3d^2 * (- (4ac - b^2)^9)^{(1/2)} + 2077a^4b^9c^2e^2 \\
&- 10656a^5b^7c^3e^2 + 30240a^6b^5c^4e^2 - 44800a^7b^3c^5e^2 + a \\
&^4b^2f^2 * (- (4ac - b^2)^9)^{(1/2)} + 25a^4c^2e^2 * (- (4ac - b^2)^9)^{(1/ \\
&2)} + 288a^6b^7c^2f^2 - 1504a^7b^5c^3f^2 + 3840a^8b^3c^4f^2 - 61 \\
&5a^2b^{13}c^2d^2 + 10a^2b^{13}d^2f + 35840a^8c^7d^2e - 6a^3b^{12}e^2f - 153 \\
&60a^9c^6e^2f - 30a^2b^5d^2e * (- (4ac - b^2)^9)^{(1/2)} + 724a^2b^{12}c^2d^2e \\
&- 258a^3b^{11}c^2d^2f + 43520a^8b^3c^6d^2f + 152a^4b^{10}c^2e^2f + 246a^2b^2 \\
&c^2d^2 * (- (4ac - b^2)^9)^{(1/2)} - 165a^2b^4c^2d^2 * (- (4ac - b^2)^9)^{(\\
&1/2)} - 7278a^3b^{10}c^2d^2e + 39132a^4b^8c^3d^2e - 119616a^5b^6c^4d^2
\end{aligned}$$

$$\begin{aligned}
& *e + 201600*a^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 2706*a^4*b^9*c^2*d*f - 14784*a^5*b^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^{(1/2)} \\
& + 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^{(1/2)} - 1548*a^5*b^8*c^2*e*f + 8064*a^6*b^6*c^3*e*f - 22400*a^7*b^4*c^4*e*f + 30720*a^8*b^2*c^5*e*f - 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^{(1/2)} \\
& + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^{(1/2)}/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^{(1/2)}*2i - (d/(3*a) + (x^2*(3*a*e - 5*b*d))/(3*a^2) + (x^4*(15*b^4*d + 14*a^2*c^2*d + 3*a^2*b^2*f - 9*a*b^3*e - 6*a^3*c*f - 62*a*b^2*c*d + 33*a^2*b*c*e))/(6*a^3*(4*a*c - b^2)) + (c*x^6*(5*b^3*d - 3*a*b^2*e + a^2*b*f + 10*a^2*c*e - 19*a*b*c*d))/(2*a^3*(4*a*c - b^2)))/(a*x^3 + b*x^5 + c*x^7) + \text{atan}(((x*(204800*a^17*c^9*e^2 - 401408*a^16*c^10*d^2 - 73728*a^18*c^8*f^2 + 400*a^9*b^14*c^3*d^2 - 9440*a^10*b^12*c^4*d^2 + 92816*a^11*b^10*c^5*d^2 - 488096*a^12*b^8*c^6*d^2 + 1458688*a^13*b^6*c^7*d^2 - 2401280*a^14*b^4*c^8*d^2 + 1871872*a^15*b^2*c^9*d^2 + 144*a^11*b^12*c^3*e^2 - 3264*a^12*b^10*c^4*e^2 + 30112*a^13*b^8*c^5*e^2 - 143360*a^14*b^6*c^6*e^2 + 365568*a^15*b^4*c^7*e^2 - 458752*a^16*b^2*c^8*e^2 + 16*a^13*b^10*c^3*f^2 - 416*a^14*b^8*c^4*f^2 + 4608*a^15*b^6*c^5*f^2 - 25600*a^16*b^4*c^6*f^2 + 69632*a^17*b^2*c^7*f^2 + 344064*a^17*c^9*d*f - 1236992*a^16*b*c^9*d*e + 237568*a^17*b*c^8*e*f - 480*a^10*b^13*c^3*d*e + 11104*a^11*b^11*c^4*d*e - 105824*a^12*b^9*c^5*d*e + 530432*a^13*b^7*c^6*d*e - 1469440*a^14*b^5*c^7*d*e + 2121728*a^15*b^3*c^8*d*e + 160*a^11*b^12*c^3*d*f - 3968*a^12*b^10*c^4*d*f + 39488*a^13*b^8*c^5*d*f - 200704*a^14*b^6*c^6*d*f + 542720*a^15*b^4*c^7*d*f - 720896*a^16*b^2*c^8*d*f - 96*a^12*b^11*c^3*e*f + 2336*a^13*b^9*c^4*e*f - 22528*a^14*b^7*c^5*e*f + 107520*a^15*b^5*c^6*e*f - 253952*a^16*b^3*c^7*e*f) + (-(25*b^15*d^2 + 9*a^2*b^13*e^2 - 25*b^6*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + a^4*b^11*f^2 - 80640*a^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^2 - 3840*a^9*b*c^5*f^2 + 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 30*a*b^14*d*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^2 + 116928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2 - 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^2 - 44800*a^7*b^3*c^5*e^2 - a^4*b^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^2 + 3840*a^8*b^3*c^4*f^2 - 615*a*b^13*c*d^2 + 10*a^2*b^13*d*f + 35840*a^8*c^7*d*e - 6*a^3*b^12*e*f - 15360*a^9*c^6*e*f + 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^{(1/2)} + 724*a^2*b^12*c*d*e - 258*a^3*b^11*c*d*f + 43520*a^8*b*c^6*d*f + 152*a^4*b^10*c*e*f - 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - 7278*a^3*b^10*c^2*d*e + 39132*a^4*b^8*c^3*d*e - 119616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d*e - 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 2706*a^4*b^9*c^2*d*f - 14784*a^5*b^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f + 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^{(1/2)} - 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^{(1/2)} - 1548*a^5*b^8*c^2*e*f + 8064*a^6*b^6*c^3*e*f - 22400*a^7*b^4*c^4*e*f + 30720*a^8*b^2*c^5*e*f + 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^{(1/2)} - 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^{(1/2)} + 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^{(1/2)} + 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^{(1/2)}/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^{(1/2)}*(393216*a^20*c^8*f - 917504*a^19*c^9*d + x*(-(25*b^15*d^2 + 9*a^2*b^13*e^2 - 25*b^6*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + a^4*b^11*f^2 - 80640*a^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^2 - 3840*a^9*b*c^5*f^2 + 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 30*a*b^14*d*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^2 + 116928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2 - 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^2 - 44800*a^7*b^3*c^5*e^2
\end{aligned}$$

$$\begin{aligned}
& - a^4 b^2 f^2 (-4ac - b^2)^9)^{(1/2)} - 25a^4 c^2 e^2 (-4ac - b^2)^9)^{(1/2)} + 288a^6 b^7 c^2 f^2 - 1504a^7 b^5 c^3 f^2 + 3840a^8 b^3 c^4 f^2 - \\
& 615a^9 b^13 c^4 d^2 + 10a^2 b^13 d^2 f + 35840a^8 c^7 d^2 e - 6a^3 b^12 e^2 f - 15360a^9 c^6 e^2 f + 30a^3 b^5 d^2 e (-4ac - b^2)^9)^{(1/2)} + 724a^2 b^12 c^2 d^2 e - 258a^3 b^11 c^2 d^2 f + 43520a^8 b^6 c^6 d^2 f + 152a^4 b^10 c^2 e^2 f - 246a^2 b^2 c^2 d^2 (-4ac - b^2)^9)^{(1/2)} + 165a^3 b^4 c^2 d^2 (-4ac - b^2)^9)^{(1/2)} - 7278a^3 b^10 c^2 d^2 e + 39132a^4 b^8 c^3 d^2 e - 119616a^5 b^6 c^4 d^2 e + 201600a^6 b^4 c^5 d^2 e - 161280a^7 b^2 c^6 d^2 e - 10a^2 b^4 d^2 f (-4ac - b^2)^9)^{(1/2)} + 2706a^4 b^9 c^2 d^2 f - 14784a^5 b^7 c^3 d^2 f + 44352a^6 b^5 c^4 d^2 f - 69120a^7 b^3 c^5 d^2 f + 6a^3 b^3 c^2 e^2 f (-4ac - b^2)^9)^{(1/2)} - 42a^4 c^2 d^2 f (-4ac - b^2)^9)^{(1/2)} - 1548a^5 b^8 c^2 e^2 f + 8064a^6 b^6 c^3 e^2 f - 22400a^7 b^4 c^4 e^2 f + 30720a^8 b^2 c^5 e^2 f + 51a^3 b^2 c^2 e^2 (-4ac - b^2)^9)^{(1/2)} - 44a^4 b^2 c^2 e^2 f (-4ac - b^2)^9)^{(1/2)} - 184a^2 b^3 c^2 d^2 e (-4ac - b^2)^9)^{(1/2)} + 186a^3 b^2 c^2 d^2 e (-4ac - b^2)^9)^{(1/2)} + 78a^3 b^2 c^2 d^2 f (-4ac - b^2)^9)^{(1/2)} / (32(a^7 b^12 + 4096a^13 c^6 - 24a^8 b^10 c + 240a^9 b^8 c^2 - 1280a^10 b^6 c^3 + 3840a^11 b^4 c^4 - 6144a^12 b^2 c^5)))^{(1/2)} * (1048576a^21 b^8 c^8 + 256a^15 b^13 c^2 - 6144a^16 b^11 c^3 + 61440a^17 b^9 c^4 - 327680a^18 b^7 c^5 + 983040a^19 b^5 c^6 - 1572864a^20 b^3 c^7) + 320a^12 b^14 c^2 d - 7936a^13 b^12 c^3 d + 82816a^14 b^10 c^4 d - 468480a^15 b^8 c^5 d + 153600a^16 b^6 c^6 d - 2867200a^17 b^4 c^7 d + 2719744a^18 b^2 c^8 d - 192a^13 b^13 c^2 e + 4672a^14 b^11 c^3 e - 47360a^15 b^9 c^4 e + 256000a^16 b^7 c^5 e - 778240a^17 b^5 c^6 e + 1261568a^18 b^3 c^7 e + 64a^14 b^12 c^2 f - 1664a^15 b^10 c^3 f + 17920a^16 b^8 c^4 f - 102400a^17 b^6 c^5 f + 327680a^18 b^4 c^6 f - 557056a^19 b^2 c^7 f - 851968a^19 b^2 c^8 e) * (-25b^15 d^2 + 9a^2 b^13 e^2 - 25b^6 d^2 (-4ac - b^2)^9)^{(1/2)} + a^4 b^11 f^2 - 80640a^7 b^6 c^7 d^2 - 213a^3 b^11 c^2 e^2 + 26880a^8 b^6 c^6 e^2 - 27a^5 b^9 c^2 f^2 - 3840a^9 b^5 c^5 f^2 + 9a^5 c^2 f^2 (-4ac - b^2)^9)^{(1/2)} - 30a^3 b^14 d^2 e + 6366a^2 b^11 c^2 d^2 - 35767a^3 b^9 c^3 d^2 + 116928a^4 b^7 c^4 d^2 - 219744a^5 b^5 c^5 d^2 + 215040a^6 b^3 c^6 d^2 - 9a^2 b^4 e^2 (-4ac - b^2)^9)^{(1/2)} + 49a^3 c^3 d^2 (-4ac - b^2)^9)^{(1/2)} + 2077a^4 b^9 c^2 e^2 - 10656a^5 b^7 c^3 e^2 + 30240a^6 b^5 c^4 e^2 - 44800a^7 b^3 c^5 e^2 - a^4 b^2 f^2 (-4ac - b^2)^9)^{(1/2)} - 25a^4 c^2 e^2 (-4ac - b^2)^9)^{(1/2)} + 288a^6 b^7 c^2 f^2 - 1504a^7 b^5 c^3 f^2 + 3840a^8 b^3 c^4 f^2 - 615a^9 b^13 c^4 d^2 + 10a^2 b^13 d^2 f + 35840a^8 c^7 d^2 e - 6a^3 b^12 e^2 f - 15360a^9 c^6 e^2 f + 30a^3 b^5 d^2 e (-4ac - b^2)^9)^{(1/2)} + 724a^2 b^12 c^2 d^2 e - 258a^3 b^11 c^2 d^2 f + 43520a^8 b^6 c^6 d^2 f + 152a^4 b^10 c^2 e^2 f - 246a^2 b^2 c^2 d^2 (-4ac - b^2)^9)^{(1/2)} + 165a^3 b^4 c^2 d^2 (-4ac - b^2)^9)^{(1/2)} - 7278a^3 b^10 c^2 d^2 e + 39132a^4 b^8 c^3 d^2 e - 119616a^5 b^6 c^4 d^2 e + 201600a^6 b^4 c^5 d^2 e - 161280a^7 b^2 c^6 d^2 e - 10a^2 b^4 d^2 f (-4ac - b^2)^9)^{(1/2)} + 2706a^4 b^9 c^2 d^2 f - 14784a^5 b^7 c^3 d^2 f + 44352a^6 b^5 c^4 d^2 f - 69120a^7 b^3 c^5 d^2 f + 6a^3 b^3 c^2 e^2 f (-4ac - b^2)^9)^{(1/2)} - 42a^4 c^2 d^2 f (-4ac - b^2)^9)^{(1/2)} - 1548a^5 b^8 c^2 e^2 f + 8064a^6 b^6 c^3 e^2 f - 22400a^7 b^4 c^4 e^2 f + 30720a^8 b^2 c^5 e^2 f + 51a^3 b^2 c^2 e^2 (-4ac - b^2)^9)^{(1/2)} - 44a^4 b^2 c^2 e^2 f (-4ac - b^2)^9)^{(1/2)} - 184a^2 b^3 c^2 d^2 e (-4ac - b^2)^9)^{(1/2)} + 186a^3 b^2 c^2 d^2 e (-4ac - b^2)^9)^{(1/2)} + 78a^3 b^2 c^2 d^2 f (-4ac - b^2)^9)^{(1/2)} / (32(a^7 b^12 + 4096a^13 c^6 - 24a^8 b^10 c + 240a^9 b^8 c^2 - 1280a^10 b^6 c^3 + 3840a^11 b^4 c^4 - 6144a^12 b^2 c^5)))^{(1/2)} * 1i + (x(204800a^17 c^9 e^2 - 401408a^16 c^10 d^2 - 73728a^18 c^8 f^2 + 400a^9 b^14 c^3 d^2 - 9440a^10 b^12 c^4 d^2 + 92816a^11 b^10 c^5 d^2 - 488096a^12 b^8 c^6 d^2 + 1458688a^13 b^6 c^7 d^2 - 2401280a^14 b^4 c^8 d^2 + 1871872a^15 b^2 c^9 d^2 + 144a^11 b^12 c^3 e^2 - 3264a^12 b^10 c^4 e^2 + 30112a^13 b^8 c^5 e^2 - 143360a^14 b^6 c^6 e^2 + 365568a^15 b^4 c^7 e^2 - 458752a^16 b^2 c^8 e^2 + 16a^13 b^10 c^3 f^2 - 416a^14 b^8 c^4 f^2 + 4608a^15 b^6 c^5 f^2 - 25600a^16 b^4 c^6 f^2 + 69632a^17 b^2 c^7 f^2 + 344064a^17 c^9 d^2 f - 1236992a^16 b^6 c^9 d^2 e + 237568a^17 b^6 c^8 e^2 f - 480a^10 b^13 c^3 d^2 e + 11104a^11 b^11 c^4 d^2 e - 105824a^12 b^9 c^5 d^2 e + 530432a^13 b^7 c^6 d^2 e - 1469440a^14 b^5 c^7 d^2 e + 2121728a^15 b^3 c^8 d^2 e + 160*
\end{aligned}$$

$$\begin{aligned}
& a^{11}b^{12}c^3d^3f - 3968a^{12}b^{10}c^4d^3f + 39488a^{13}b^8c^5d^3f - 20070 \\
& 4a^{14}b^6c^6d^3f + 542720a^{15}b^4c^7d^3f - 720896a^{16}b^2c^8d^3f - 96 \\
& *a^{12}b^{11}c^3e^3f + 2336a^{13}b^9c^4e^3f - 22528a^{14}b^7c^5e^3f + 10752 \\
& 0a^{15}b^5c^6e^3f - 253952a^{16}b^3c^7e^3f) + (- (25b^{15}d^2 + 9a^2b^{13} \\
& *e^2 - 25b^6d^2*(-(4ac - b^2)^9)^{(1/2)} + a^4b^{11}f^2 - 80640a^7b^3c^7 \\
& *d^2 - 213a^3b^{11}c^3e^2 + 26880a^8b^3c^6e^2 - 27a^5b^9c^3f^2 - 3840a \\
& ^9b^3c^5f^2 + 9a^5c^3f^2*(-(4ac - b^2)^9)^{(1/2)} - 30ab^{14}d^3e + 6366* \\
& a^2b^{11}c^2d^2 - 35767a^3b^9c^3d^2 + 116928a^4b^7c^4d^2 - 219744* \\
& a^5b^5c^5d^2 + 215040a^6b^3c^6d^2 - 9a^2b^4e^2*(-(4ac - b^2)^9) \\
& ^{(1/2)} + 49a^3c^3d^2*(-(4ac - b^2)^9)^{(1/2)} + 2077a^4b^9c^2e^2 - 1 \\
& 0656a^5b^7c^3e^2 + 30240a^6b^5c^4e^2 - 44800a^7b^3c^5e^2 - a^4* \\
& b^2f^2*(-(4ac - b^2)^9)^{(1/2)} - 25a^4c^2e^2*(-(4ac - b^2)^9)^{(1/2)} \\
& + 288a^6b^7c^2f^2 - 1504a^7b^5c^3f^2 + 3840a^8b^3c^4f^2 - 615a \\
& *b^{13}c^3d^2 + 10a^2b^{13}d^3f + 35840a^8c^7d^3e - 6a^3b^{12}e^3f - 15360* \\
& a^9c^6e^3f + 30ab^5d^3e*(-(4ac - b^2)^9)^{(1/2)} + 724a^2b^{12}c^3d^3e - \\
& 258a^3b^{11}c^3d^3f + 43520a^8b^3c^6d^3f + 152a^4b^{10}c^3e^3f - 246a^2b^2 \\
& *c^2d^2*(-(4ac - b^2)^9)^{(1/2)} + 165ab^4c^3d^2*(-(4ac - b^2)^9)^{(1/2)} \\
&) - 7278a^3b^{10}c^2d^3e + 39132a^4b^8c^3d^3e - 119616a^5b^6c^4d^3e \\
& + 201600a^6b^4c^5d^3e - 161280a^7b^2c^6d^3e - 10a^2b^4d^3f*(-(4ac \\
& - b^2)^9)^{(1/2)} + 2706a^4b^9c^2d^3f - 14784a^5b^7c^3d^3f + 44352a^6 \\
& *b^5c^4d^3f - 69120a^7b^3c^5d^3f + 6a^3b^3e^3f*(-(4ac - b^2)^9)^{(1/ \\
& 2)} - 42a^4c^2d^3f*(-(4ac - b^2)^9)^{(1/2)} - 1548a^5b^8c^2e^3f + 8064* \\
& a^6b^6c^3e^3f - 22400a^7b^4c^4e^3f + 30720a^8b^2c^5e^3f + 51a^3b^ \\
& 2c^3e^2*(-(4ac - b^2)^9)^{(1/2)} - 44a^4b^3c^3e^3f*(-(4ac - b^2)^9)^{(1/2)} \\
& - 184a^2b^3c^3d^3e*(-(4ac - b^2)^9)^{(1/2)} + 186a^3b^3c^2d^3e*(-(4ac - \\
& b^2)^9)^{(1/2)} + 78a^3b^2c^3d^3f*(-(4ac - b^2)^9)^{(1/2))}/(32*(a^7b^{12} + \\
& 4096a^{13}c^6 - 24a^8b^{10}c + 240a^9b^8c^2 - 1280a^{10}b^6c^3 + 3840 \\
& *a^{11}b^4c^4 - 6144a^{12}b^2c^5)))^{(1/2)}*(917504a^{19}c^9d - 393216a^{20} \\
& *c^8f + x*(-(25b^{15}d^2 + 9a^2b^{13}e^2 - 25b^6d^2*(-(4ac - b^2)^9)^{(1/2)} + a^4b^{11}f^2 - 80640a^7b^3c^7 \\
& *d^2 - 213a^3b^{11}c^3e^2 + 26880a^8 \\
& *b^3c^6e^2 - 27a^5b^9c^3f^2 - 3840a^9b^3c^5f^2 + 9a^5c^3f^2*(-(4ac - \\
& b^2)^9)^{(1/2)} - 30ab^{14}d^3e + 6366a^2b^{11}c^2d^2 - 35767a^3b^9c^3* \\
& d^2 + 116928a^4b^7c^4d^2 - 219744a^5b^5c^5d^2 + 215040a^6b^3c^6* \\
& d^2 - 9a^2b^4e^2*(-(4ac - b^2)^9)^{(1/2)} + 49a^3c^3d^2*(-(4ac - b^ \\
& 2)^9)^{(1/2)} + 2077a^4b^9c^2e^2 - 10656a^5b^7c^3e^2 + 30240a^6b^5* \\
& c^4e^2 - 44800a^7b^3c^5e^2 - a^4b^2f^2*(-(4ac - b^2)^9)^{(1/2)} - 25 \\
& *a^4c^2e^2*(-(4ac - b^2)^9)^{(1/2)} + 288a^6b^7c^2f^2 - 1504a^7b^5* \\
& c^3f^2 + 3840a^8b^3c^4f^2 - 615ab^{13}c^3d^2 + 10a^2b^{13}d^3f + 35840 \\
& *a^8c^7d^3e - 6a^3b^{12}e^3f - 15360a^9c^6e^3f + 30ab^5d^3e*(-(4ac - \\
& b^2)^9)^{(1/2)} + 724a^2b^{12}c^3d^3e - 258a^3b^{11}c^3d^3f + 43520a^8b^3c^6* \\
& d^3f + 152a^4b^{10}c^3e^3f - 246a^2b^2c^2d^2*(-(4ac - b^2)^9)^{(1/2)} + 1 \\
& 65ab^4c^3d^2*(-(4ac - b^2)^9)^{(1/2)} - 7278a^3b^{10}c^2d^3e + 39132a^4 \\
& *b^8c^3d^3e - 119616a^5b^6c^4d^3e + 201600a^6b^4c^5d^3e - 161280a^7 \\
& *b^2c^6d^3e - 10a^2b^4d^3f*(-(4ac - b^2)^9)^{(1/2)} + 2706a^4b^9c^2d^3 \\
& *f - 14784a^5b^7c^3d^3f + 44352a^6b^5c^4d^3f - 69120a^7b^3c^5d^3f \\
& + 6a^3b^3e^3f*(-(4ac - b^2)^9)^{(1/2)} - 42a^4c^2d^3f*(-(4ac - b^2)^9 \\
&)^{(1/2)} - 1548a^5b^8c^2e^3f + 8064a^6b^6c^3e^3f - 22400a^7b^4c^4e \\
& *f + 30720a^8b^2c^5e^3f + 51a^3b^2c^3e^2*(-(4ac - b^2)^9)^{(1/2)} - 44 \\
& *a^4b^3c^3e^3f*(-(4ac - b^2)^9)^{(1/2)} - 184a^2b^3c^3d^3e*(-(4ac - b^2)^9 \\
&)^{(1/2)} + 186a^3b^3c^2d^3e*(-(4ac - b^2)^9)^{(1/2)} + 78a^3b^2c^3d^3f*(-(\\
& 4ac - b^2)^9)^{(1/2))}/(32*(a^7b^{12} + 4096a^{13}c^6 - 24a^8b^{10}c + 240* \\
& a^9b^8c^2 - 1280a^{10}b^6c^3 + 3840a^{11}b^4c^4 - 6144a^{12}b^2c^5)))^{(1/2)}*(1048576a^{21}b^3c^8 + 256a^{15}b^{13}c^2 - 6144a^{16}b^{11}c^3 + 61440* \\
& a^{17}b^9c^4 - 327680a^{18}b^7c^5 + 983040a^{19}b^5c^6 - 1572864a^{20}b^3 \\
& *c^7) - 320a^{12}b^{14}c^2d + 7936a^{13}b^{12}c^3d - 82816a^{14}b^{10}c^4d \\
& + 468480a^{15}b^8c^5d - 1536000a^{16}b^6c^6d + 2867200a^{17}b^4c^7d - \\
& 2719744a^{18}b^2c^8d + 192a^{13}b^{13}c^2e - 4672a^{14}b^{11}c^3e + 4736 \\
& 0a^{15}b^9c^4e - 256000a^{16}b^7c^5e + 778240a^{17}b^5c^6e - 1261568* \\
& a^{18}b^3c^7e - 64a^{14}b^{12}c^2f + 1664a^{15}b^{10}c^3f - 17920a^{16}b^8
\end{aligned}$$

$$\begin{aligned}
& *c^4f + 102400a^{17}b^6c^5f - 327680a^{18}b^4c^6f + 557056a^{19}b^2c^7f + 851968a^{19}b^3c^8e)) * (- (25b^{15}d^2 + 9a^2b^{13}e^2 - 25b^6d^2 * (- \\
& (4ac - b^2)^9)^{1/2} + a^4b^{11}f^2 - 80640a^7b^3c^7d^2 - 213a^3b^{11}c^e^2 + 26880a^8b^3c^6e^2 - 27a^5b^9c^5f^2 - 3840a^9b^3c^5f^2 + 9a^5 \\
& *c^5f^2 * (- (4ac - b^2)^9)^{1/2} - 30ab^{14}d^2e + 6366a^2b^{11}c^2d^2 - 35767a^3b^9c^3d^2 + 116928a^4b^7c^4d^2 - 219744a^5b^5c^5d^2 + 21 \\
& 5040a^6b^3c^6d^2 - 9a^2b^4e^2 * (- (4ac - b^2)^9)^{1/2} + 49a^3c^3d^2 * (- (4ac - b^2)^9)^{1/2} + 2077a^4b^9c^2e^2 - 10656a^5b^7c^3e^2 \\
& + 30240a^6b^5c^4e^2 - 44800a^7b^3c^5e^2 - a^4b^2f^2 * (- (4ac - b^2)^9)^{1/2} - 25a^4c^2e^2 * (- (4ac - b^2)^9)^{1/2} + 288a^6b^7c^2f^2 \\
& - 1504a^7b^5c^3f^2 + 3840a^8b^3c^4f^2 - 615ab^{13}cd^2 + 10a^2b^{13}d^2f + 35840a^8c^7d^2e - 6a^3b^{12}e^2f - 15360a^9c^6e^2f + 30ab^5d^2e * (- (4ac - b^2)^9)^{1/2} \\
& + 724a^2b^{12}cd^2e - 258a^3b^{11}cd^2f + 43520a^8b^3c^6d^2f + 152a^4b^{10}c^2e^2f - 246a^2b^2c^2d^2 * (- (4ac - b^2)^9)^{1/2} + 165ab^4cd^2 * (- (4ac - b^2)^9)^{1/2} - 7278a^3b^{10}c^2d^2e \\
& + 39132a^4b^8c^3d^2e - 119616a^5b^6c^4d^2e + 201600a^6b^4c^5d^2e - 161280a^7b^2c^6d^2e - 10a^2b^4d^2f * (- (4ac - b^2)^9)^{1/2} + 2706a^4b^9c^2d^2f \\
& - 14784a^5b^7c^3d^2f + 44352a^6b^5c^4d^2f - 69120a^7b^3c^5d^2f + 6a^3b^3e^2f * (- (4ac - b^2)^9)^{1/2} - 42a^4c^2d^2f * (- (4ac - b^2)^9)^{1/2} - 1548a^5b^8c^2e^2f \\
& + 8064a^6b^6c^3e^2f - 22400a^7b^4c^4e^2f + 30720a^8b^2c^5e^2f + 51a^3b^2c^2e^2 * (- (4ac - b^2)^9)^{1/2} - 44a^4b^3c^2e^2f * (- (4ac - b^2)^9)^{1/2} - 184a^2b^3cd^2e * (- (4ac - b^2)^9)^{1/2} \\
& + 186a^3b^3c^2d^2e * (- (4ac - b^2)^9)^{1/2} + 78a^3b^2cd^2f * (- (4ac - b^2)^9)^{1/2}) / (32(a^7b^{12} + 4096a^{13}c^6 - 24a^8b^{10}c + 240a^9b^8c^2 - 1280a^{10}b^6c^3 + 3840a^{11}b^4c^4 - 6144a^{12}b^2c^5))^{1/2} * i) / ((x * (204800a^{17}c^9e^2 - 401408a^{16}c^{10}d^2 \\
& - 73728a^{18}c^8f^2 + 400a^9b^{14}c^3d^2 - 9440a^{10}b^{12}c^4d^2 + 92816a^{11}b^{10}c^5d^2 - 488096a^{12}b^8c^6d^2 + 1458688a^{13}b^6c^7d^2 - 2401280a^{14}b^4c^8d^2 + 1871872a^{15}b^2c^9d^2 + 144a^{11}b^{12}c^3e^2 \\
& - 3264a^{12}b^{10}c^4e^2 + 30112a^{13}b^8c^5e^2 - 143360a^{14}b^6c^6e^2 + 365568a^{15}b^4c^7e^2 - 458752a^{16}b^2c^8e^2 + 16a^{13}b^{10}c^3f^2 - 416a^{14}b^8c^4f^2 + 4608a^{15}b^6c^5f^2 - 25600a^{16}b^4c^6f^2 + 69632a^{17}b^2c^7f^2 \\
& + 344064a^{17}c^9d^2f - 1236992a^{16}b^3c^9d^2e + 237568a^{17}b^3c^8e^2f - 480a^{10}b^{13}c^3d^2e + 11104a^{11}b^{11}c^4d^2e - 105824a^{12}b^9c^5d^2e + 530432a^{13}b^7c^6d^2e - 1469440a^{14}b^5c^7d^2e + 2121728a^{15}b^3c^8d^2e + 160a^{11}b^{12}c^3d^2f - 3968a^{12}b^{10}c^4d^2f \\
& + 39488a^{13}b^8c^5d^2f - 200704a^{14}b^6c^6d^2f + 542720a^{15}b^4c^7d^2f - 720896a^{16}b^2c^8d^2f - 96a^{12}b^{11}c^3e^2f + 2336a^{13}b^9c^4e^2f - 22528a^{14}b^7c^5e^2f + 107520a^{15}b^5c^6e^2f - 253952a^{16}b^3c^7e^2f \\
& + (- (25b^{15}d^2 + 9a^2b^{13}e^2 - 25b^6d^2 * (- (4ac - b^2)^9)^{1/2} + a^4b^{11}f^2 - 80640a^7b^3c^7d^2 - 213a^3b^{11}c^e^2 + 26880a^8b^3c^6e^2 - 27a^5b^9c^5f^2 - 3840a^9b^3c^5f^2 + 9a^5c^5f^2 * (- (4ac - b^2)^9)^{1/2} \\
& - 30ab^{14}d^2e + 6366a^2b^{11}c^2d^2 - 35767a^3b^9c^3d^2 + 116928a^4b^7c^4d^2 - 219744a^5b^5c^5d^2 + 215040a^6b^3c^6d^2 - 9a^2b^4e^2 * (- (4ac - b^2)^9)^{1/2} + 49a^3c^3d^2 * (- (4ac - b^2)^9)^{1/2} \\
& + 2077a^4b^9c^2e^2 - 10656a^5b^7c^3e^2 + 30240a^6b^5c^4e^2 - 44800a^7b^3c^5e^2 - a^4b^2f^2 * (- (4ac - b^2)^9)^{1/2} - 25a^4c^2e^2 * (- (4ac - b^2)^9)^{1/2} + 288a^6b^7c^2f^2 - 1504a^7b^5c^3f^2 \\
& + 3840a^8b^3c^4f^2 - 615ab^{13}cd^2 + 10a^2b^{13}d^2f + 35840a^8c^7d^2e - 6a^3b^{12}e^2f - 15360a^9c^6e^2f + 30ab^5d^2e * (- (4ac - b^2)^9)^{1/2} + 724a^2b^{12}cd^2e - 258a^3b^{11}cd^2f + 43520a^8b^3c^6d^2f + 152a^4b^{10}c^2e^2f \\
& - 246a^2b^2c^2d^2 * (- (4ac - b^2)^9)^{1/2} + 165ab^4cd^2 * (- (4ac - b^2)^9)^{1/2} - 7278a^3b^{10}c^2d^2e + 39132a^4b^8c^3d^2e - 119616a^5b^6c^4d^2e + 201600a^6b^4c^5d^2e - 161280a^7b^2c^6d^2e - 10a^2b^4d^2f * (- (4ac - b^2)^9)^{1/2} \\
& + 2706a^4b^9c^2d^2f - 14784a^5b^7c^3d^2f + 44352a^6b^5c^4d^2f - 69120a^7b^3c^5d^2f + 6a^3b^3e^2f * (- (4ac - b^2)^9)^{1/2} - 42a^4c^2d^2f * (- (4ac - b^2)^9)^{1/2} - 1548a^5b^8c^2e^2f + 8064a^6b^6c^3e^2f - 22400a^7b^4c^4e^2f + 30720a^8b^2c^5e^2f \\
& + 51a^3b^2c^2e^2 * (- (4ac - b^2)^9)^{1/2} - 44a^4b^3c^2e^2f * (- (4ac - b^2)^9)^{1/2} - 184a^2b^3cd^2e * (- (4ac - b^2)^9)^{1/2} + 186a^3b^3c^2d^2e * (- (4ac - b^2)^9)^{1/2} + 78a^3b^2cd^2f * (- (4ac - b^2)^9)^{1/2})
\end{aligned}$$

$$\begin{aligned}
& *c*e*f*(-(4*a*c - b^2)^9)^{(1/2)} - 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^{(1/2)} \\
& + 186*a^3*b^2*c*d*e*(-(4*a*c - b^2)^9)^{(1/2)} + 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^{(1/2)} \\
&)/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5))^{(1/2)} * \\
& (393216*a^20*c^8*f - 917504*a^19*c^9*d + x*(-(25*b^15*d^2 + 9*a^2*b^13*e^2 - 25*b^6*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + a^4*b^11*f^2 - 80640*a^7*b*c^7*d^2 \\
& - 213*a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^2 - 3840*a^9*b*c^5*f^2 + 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 30*a*b^14*d*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^2 + 116928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2 - 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^{(1/2)} \\
&) + 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^2 - 44800*a^7*b^3*c^5*e^2 - a^4*b^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 288 \\
& *a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^2 + 3840*a^8*b^3*c^4*f^2 - 615*a*b^13*c*d^2 + 10*a^2*b^13*d*f + 35840*a^8*c^7*d*e - 6*a^3*b^12*e*f - 15360*a^9*c^6*e*f + 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^{(1/2)} + 724*a^2*b^12*c*d*e - 258*a^3*b^11*c*d*f + 43520*a^8*b*c^6*d*f + 152*a^4*b^10*c*e*f - 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - 7 \\
& 278*a^3*b^10*c^2*d*e + 39132*a^4*b^8*c^3*d*e - 119616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d*e - 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 2706*a^4*b^9*c^2*d*f - 14784*a^5*b^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f + 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^{(1/2)} - 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^{(1/2)} - 1548*a^5*b^8*c^2*e*f + 8064*a^6*b^6*c^3*e*f - 22400*a^7*b^4*c^4*e*f + 30720*a^8*b^2*c^5*e*f + 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^{(1/2)} - 184 \\
& *a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^{(1/2)} + 186*a^3*b^2*c*d*e*(-(4*a*c - b^2)^9)^{(1/2)} + 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^7*b^12 + 4096 \\
& *a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5))^{(1/2)} * (1048576*a^21*b*c^8 + 256*a^15*b^13*c^2 - 6144*a^16*b^11*c^3 + 61440*a^17*b^9*c^4 - 327680*a^18*b^7*c^5 + 983040 \\
& *a^19*b^5*c^6 - 1572864*a^20*b^3*c^7) + 320*a^12*b^14*c^2*d - 7936*a^13*b^12*c^3*d + 82816*a^14*b^10*c^4*d - 468480*a^15*b^8*c^5*d + 1536000*a^16*b^6*c^6*d - 2867200*a^17*b^4*c^7*d + 2719744*a^18*b^2*c^8*d - 192*a^13*b^13*c^2 \\
& *e + 4672*a^14*b^11*c^3*e - 47360*a^15*b^9*c^4*e + 256000*a^16*b^7*c^5*e - 778240*a^17*b^5*c^6*e + 1261568*a^18*b^3*c^7*e + 64*a^14*b^12*c^2*f - 1664*a^15*b^10*c^3*f + 17920*a^16*b^8*c^4*f - 102400*a^17*b^6*c^5*f + 327680*a^18*b^4*c^6*f - 557056*a^19*b^2*c^7*f - 851968*a^19*b*c^8*e) * (- (25*b^15*d^2 \\
& + 9*a^2*b^13*e^2 - 25*b^6*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + a^4*b^11*f^2 - 80640*a^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^2 - 3840*a^9*b*c^5*f^2 + 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 30*a*b^14 \\
& *d*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^2 + 116928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2 - 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + 2077*a^4*b^9 \\
& *c^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^2 - 44800*a^7*b^3*c^5*e^2 - a^4*b^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^2 + 3840*a^8*b^3*c^4 \\
& *f^2 - 615*a*b^13*c*d^2 + 10*a^2*b^13*d*f + 35840*a^8*c^7*d*e - 6*a^3*b^12*e*f - 15360*a^9*c^6*e*f + 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^{(1/2)} + 724*a^2*b^12*c*d*e - 258*a^3*b^11*c*d*f + 43520*a^8*b*c^6*d*f + 152*a^4*b^10*c*e*f - \\
& 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - 7278*a^3*b^10*c^2*d*e + 39132*a^4*b^8*c^3*d*e - 119616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d*e - 10*a^2*b^4 \\
& d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 2706*a^4*b^9*c^2*d*f - 14784*a^5*b^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f + 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^{(1/2)} - 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^{(1/2)} - 1548*a^5*b^8*c^2 \\
& *e*f + 8064*a^6*b^6*c^3*e*f - 22400*a^7*b^4*c^4*e*f + 30720*a^8*b^2*c^5*e*f + 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^{(1/2)} - 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^{(1/2)} + 186*a^3*b^2*c*d \\
& e*(-(4*a*c - b^2)^9)^{(1/2)} + 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^{(1/2)})/(32
\end{aligned}$$

$$\begin{aligned}
& * (a^7 b^{12} + 4096 a^{13} c^6 - 24 a^8 b^{10} c + 240 a^9 b^8 c^2 - 1280 a^{10} b^6 c^3 + 3840 a^{11} b^4 c^4 - 6144 a^{12} b^2 c^5))^{(1/2)} - (x * (204800 a^{17} c^9 e^2 - 401408 a^{16} c^{10} d^2 - 73728 a^{18} c^8 f^2 + 400 a^9 b^{14} c^3 d^2 - 9440 a^{10} b^{12} c^4 d^2 + 92816 a^{11} b^{10} c^5 d^2 - 488096 a^{12} b^8 c^6 d^2 + 1458688 a^{13} b^6 c^7 d^2 - 2401280 a^{14} b^4 c^8 d^2 + 1871872 a^{15} b^2 c^9 d^2 + 144 a^{11} b^{12} c^3 e^2 - 3264 a^{12} b^{10} c^4 e^2 + 30112 a^{13} b^8 c^5 e^2 - 143360 a^{14} b^6 c^6 e^2 + 365568 a^{15} b^4 c^7 e^2 - 458752 a^{16} b^2 c^8 e^2 + 16 a^{13} b^{10} c^3 f^2 - 416 a^{14} b^8 c^4 f^2 + 4608 a^{15} b^6 c^5 f^2 - 25600 a^{16} b^4 c^6 f^2 + 69632 a^{17} b^2 c^7 f^2 + 344064 a^{17} c^9 d f - 1236992 a^{16} b c^9 d e + 237568 a^{17} b c^8 e f - 480 a^{10} b^{13} c^3 d e + 11104 a^{11} b^{11} c^4 d e - 105824 a^{12} b^9 c^5 d e + 530432 a^{13} b^7 c^6 d e - 1469440 a^{14} b^5 c^7 d e + 2121728 a^{15} b^3 c^8 d e + 160 a^{11} b^{12} c^3 d f - 3968 a^{12} b^{10} c^4 d f + 39488 a^{13} b^8 c^5 d f - 200704 a^{14} b^6 c^6 d f + 542720 a^{15} b^4 c^7 d f - 720896 a^{16} b^2 c^8 d f - 96 a^{12} b^{11} c^3 e e f + 2336 a^{13} b^9 c^4 e e f - 22528 a^{14} b^7 c^5 e e f + 107520 a^{15} b^5 c^6 e e f - 253952 a^{16} b^3 c^7 e e f) + (- (25 b^{15} d^2 + 9 a^2 b^{13} e^2 - 25 b^6 d^2 * (- (4 a c - b^2)^9)^{(1/2)} + a^4 b^{11} f^2 - 80640 a^7 b c^7 d^2 - 213 a^3 b^{11} c e^2 + 26880 a^8 b c^6 e^2 - 27 a^5 b^9 c f^2 - 3840 a^9 b c^5 f^2 + 9 a^5 c f^2 * (- (4 a c - b^2)^9)^{(1/2)} - 30 a b^{14} d e + 6366 a^2 b^{11} c^2 d^2 - 35767 a^3 b^9 c^3 d^2 + 116928 a^4 b^7 c^4 d^2 - 219744 a^5 b^5 c^5 d^2 + 215040 a^6 b^3 c^6 d^2 - 9 a^2 b^4 e^2 * (- (4 a c - b^2)^9)^{(1/2)} + 49 a^3 c^3 d^2 * (- (4 a c - b^2)^9)^{(1/2)} + 2077 a^4 b^9 c^2 e^2 - 10656 a^5 b^7 c^3 e^2 + 30240 a^6 b^5 c^4 e^2 - 44800 a^7 b^3 c^5 e^2 - a^4 b^2 f^2 * (- (4 a c - b^2)^9)^{(1/2)} - 25 a^4 c^2 e^2 * (- (4 a c - b^2)^9)^{(1/2)} + 288 a^6 b^7 c^2 f^2 - 1504 a^7 b^5 c^3 f^2 + 3840 a^8 b^3 c^4 f^2 - 615 a b^{13} c d^2 + 10 a^2 b^{13} d f + 35840 a^8 c^7 d e - 6 a^3 b^{12} e f - 15360 a^9 c^6 e e f + 30 a b^5 d e * (- (4 a c - b^2)^9)^{(1/2)} + 724 a^2 b^{12} c d e - 258 a^3 b^{11} c d f + 43520 a^8 b c^6 d f + 152 a^4 b^{10} c e f - 246 a^2 b^2 c^2 d^2 * (- (4 a c - b^2)^9)^{(1/2)} + 165 a a b^4 c d^2 * (- (4 a c - b^2)^9)^{(1/2)} - 7278 a^3 b^{10} c^2 d e + 39132 a^4 b^8 c^3 d e - 119616 a^5 b^6 c^4 d e + 201600 a^6 b^4 c^5 d e - 161280 a^7 b^2 c^6 d e - 10 a^2 b^4 d f * (- (4 a c - b^2)^9)^{(1/2)} + 2706 a^4 b^9 c^2 d f - 14784 a^5 b^7 c^3 d f + 44352 a^6 b^5 c^4 d f - 69120 a^7 b^3 c^5 d f + 6 a^3 b^3 e e f * (- (4 a c - b^2)^9)^{(1/2)} - 42 a^4 c^2 d f * (- (4 a c - b^2)^9)^{(1/2)} - 1548 a^5 b^8 c^2 e e f + 8064 a^6 b^6 c^3 e e f - 22400 a^7 b^4 c^4 e e f + 30720 a^8 b^2 c^5 e e f + 51 a^3 b^2 c e^2 * (- (4 a c - b^2)^9)^{(1/2)} - 44 a^4 b c e e f * (- (4 a c - b^2)^9)^{(1/2)} - 184 a^2 b^3 c d e * (- (4 a c - b^2)^9)^{(1/2)} + 186 a^3 b c^2 d e * (- (4 a c - b^2)^9)^{(1/2)} + 78 a^3 b^2 c d f * (- (4 a c - b^2)^9)^{(1/2)) / (32 * (a^7 b^{12} + 4096 a^{13} c^6 - 24 a^8 b^{10} c + 240 a^9 b^8 c^2 - 1280 a^{10} b^6 c^3 + 3840 a^{11} b^4 c^4 - 6144 a^{12} b^2 c^5))^{(1/2)} * (917504 a^{19} c^9 d - 393216 a^{20} c^8 f + x * (- (25 b^{15} d^2 + 9 a^2 b^{13} e^2 - 25 b^6 d^2 * (- (4 a c - b^2)^9)^{(1/2)} + a^4 b^{11} f^2 - 80640 a^7 b c^7 d^2 - 213 a^3 b^{11} c e^2 + 26880 a^8 b c^6 e^2 - 27 a^5 b^9 c f^2 - 3840 a^9 b c^5 f^2 + 9 a^5 c f^2 * (- (4 a c - b^2)^9)^{(1/2)} - 30 a b^{14} d e + 6366 a^2 b^{11} c^2 d^2 - 35767 a^3 b^9 c^3 d^2 + 116928 a^4 b^7 c^4 d^2 - 219744 a^5 b^5 c^5 d^2 + 215040 a^6 b^3 c^6 d^2 - 9 a^2 b^4 e^2 * (- (4 a c - b^2)^9)^{(1/2)} + 49 a^3 c^3 d^2 * (- (4 a c - b^2)^9)^{(1/2)} + 2077 a^4 b^9 c^2 e^2 - 10656 a^5 b^7 c^3 e^2 + 30240 a^6 b^5 c^4 e^2 - 44800 a^7 b^3 c^5 e^2 - a^4 b^2 f^2 * (- (4 a c - b^2)^9)^{(1/2)} - 25 a^4 c^2 e^2 * (- (4 a c - b^2)^9)^{(1/2)} + 288 a^6 b^7 c^2 f^2 - 1504 a^7 b^5 c^3 f^2 + 3840 a^8 b^3 c^4 f^2 - 615 a b^{13} c d^2 + 10 a^2 b^{13} d f + 35840 a^8 c^7 d e - 6 a^3 b^{12} e f - 15360 a^9 c^6 e e f + 30 a b^5 d e * (- (4 a c - b^2)^9)^{(1/2)} + 724 a^2 b^{12} c d e - 258 a^3 b^{11} c d f + 43520 a^8 b c^6 d f + 152 a^4 b^{10} c e f - 246 a^2 b^2 c^2 d^2 * (- (4 a c - b^2)^9)^{(1/2)} + 165 a a b^4 c d^2 * (- (4 a c - b^2)^9)^{(1/2)} - 7278 a^3 b^{10} c^2 d e + 39132 a^4 b^8 c^3 d e - 119616 a^5 b^6 c^4 d e + 201600 a^6 b^4 c^5 d e - 161280 a^7 b^2 c^6 d e - 10 a^2 b^4 d f * (- (4 a c - b^2)^9)^{(1/2)} + 2706 a^4 b^9 c^2 d f - 14784 a^5 b^7 c^3 d f + 44352 a^6 b^5 c^4 d f - 69120 a^7 b^3 c^5 d f + 6 a^3 b^3 e e f * (- (4 a c - b^2)^9)^{(1/2)} - 42 a^4 c^2 d f * (- (4 a c - b^2)^9)^{(1/2)} - 1548 a^5 b^8 c^2 e e f + 8064 a^6 b^6 c^3 e e f - 22400 a^7 b^4 c^4 e e f + 30720 a^8
\end{aligned}$$

$$\begin{aligned}
& *b^2*c^5*e*f + 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 44*a^4*b*c*e*f*(\\
& -(4*a*c - b^2)^9)^{(1/2)} - 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^{(1/2)} + 186* \\
& a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^{(1/2)} + 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9 \\
&)^{(1/2)})/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - \\
& 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5))^{(1/2)}*(1048576 \\
& *a^21*b*c^8 + 256*a^15*b^13*c^2 - 6144*a^16*b^11*c^3 + 61440*a^17*b^9*c^4 - \\
& 327680*a^18*b^7*c^5 + 983040*a^19*b^5*c^6 - 1572864*a^20*b^3*c^7) - 320*a^ \\
& 12*b^14*c^2*d + 7936*a^13*b^12*c^3*d - 82816*a^14*b^10*c^4*d + 468480*a^15* \\
& b^8*c^5*d - 1536000*a^16*b^6*c^6*d + 2867200*a^17*b^4*c^7*d - 2719744*a^18* \\
& b^2*c^8*d + 192*a^13*b^13*c^2*e - 4672*a^14*b^11*c^3*e + 47360*a^15*b^9*c^4 \\
& *e - 256000*a^16*b^7*c^5*e + 778240*a^17*b^5*c^6*e - 1261568*a^18*b^3*c^7*e \\
& - 64*a^14*b^12*c^2*f + 1664*a^15*b^10*c^3*f - 17920*a^16*b^8*c^4*f + 10240 \\
& 0*a^17*b^6*c^5*f - 327680*a^18*b^4*c^6*f + 557056*a^19*b^2*c^7*f + 851968*a \\
& ^19*b*c^8*e))*(-(25*b^15*d^2 + 9*a^2*b^13*e^2 - 25*b^6*d^2*(-(4*a*c - b^2)^ \\
& 9)^{(1/2)} + a^4*b^11*f^2 - 80640*a^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880* \\
& a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^2 - 3840*a^9*b*c^5*f^2 + 9*a^5*c*f^2*(-(4*a* \\
& c - b^2)^9)^{(1/2)} - 30*a*b^14*d*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c \\
& ^3*d^2 + 116928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c \\
& ^6*d^2 - 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 49*a^3*c^3*d^2*(-(4*a*c - \\
& b^2)^9)^{(1/2)} + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b \\
& ^5*c^4*e^2 - 44800*a^7*b^3*c^5*e^2 - a^4*b^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - \\
& 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b \\
& ^5*c^3*f^2 + 3840*a^8*b^3*c^4*f^2 - 615*a*b^13*c*d^2 + 10*a^2*b^13*d*f + 35 \\
& 840*a^8*c^7*d*e - 6*a^3*b^12*e*f - 15360*a^9*c^6*e*f + 30*a*b^5*d*e*(-(4*a* \\
& c - b^2)^9)^{(1/2)} + 724*a^2*b^12*c*d*e - 258*a^3*b^11*c*d*f + 43520*a^8*b*c \\
& ^6*d*f + 152*a^4*b^10*c*e*f - 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^{(1/2)} \\
& + 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - 7278*a^3*b^10*c^2*d*e + 39132* \\
& a^4*b^8*c^3*d*e - 119616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5*d*e - 161280* \\
& a^7*b^2*c^6*d*e - 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 2706*a^4*b^9*c^ \\
& 2*d*f - 14784*a^5*b^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d \\
& *f + 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^{(1/2)} - 42*a^4*c^2*d*f*(-(4*a*c - b^2 \\
&)^9)^{(1/2)} - 1548*a^5*b^8*c^2*e*f + 8064*a^6*b^6*c^3*e*f - 22400*a^7*b^4*c^ \\
& 4*e*f + 30720*a^8*b^2*c^5*e*f + 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - \\
& 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^{(1/2)} - 184*a^2*b^3*c*d*e*(-(4*a*c - b^2 \\
&)^9)^{(1/2)} + 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^{(1/2)} + 78*a^3*b^2*c*d*f* \\
& (- (4*a*c - b^2)^9)^{(1/2)})/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 2 \\
& 40*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5) \\
&))^{(1/2)} - 128000*a^15*c^9*e^3 + 476672*a^13*b*c^10*d^3 - 4608*a^16*b*c^7*f \\
& ^3 - 250880*a^14*c^10*d^2*e - 46080*a^16*c^8*e*f^2 + 1800*a^9*b^9*c^6*d^3 - \\
& 29080*a^10*b^7*c^7*d^3 + 176032*a^11*b^5*c^8*d^3 - 473216*a^12*b^3*c^9*d^3 \\
& - 504*a^11*b^8*c^5*e^3 + 8112*a^12*b^6*c^6*e^3 - 48704*a^13*b^4*c^7*e^3 + \\
& 129280*a^14*b^2*c^8*e^3 + 40*a^13*b^7*c^4*f^3 - 608*a^14*b^5*c^5*f^3 + 2944 \\
& *a^15*b^3*c^6*f^3 + 215040*a^15*c^9*d*e*f + 442880*a^14*b*c^9*d*e^2 - 43366 \\
& 4*a^14*b*c^9*d^2*f + 109056*a^15*b*c^8*d*f^2 + 84480*a^15*b*c^8*e^2*f - 140 \\
& 0*a^9*b^10*c^5*d^2*e + 21680*a^10*b^8*c^6*d^2*e + 1680*a^10*b^9*c^5*d*e^2 - \\
& 121648*a^11*b^6*c^7*d^2*e - 27176*a^11*b^7*c^6*d*e^2 + 275264*a^12*b^4*c^8 \\
& *d^2*e + 164448*a^12*b^5*c^7*d*e^2 - 121088*a^13*b^2*c^9*d^2*e - 441216*a^1 \\
& 3*b^3*c^8*d*e^2 + 1000*a^9*b^11*c^4*d^2*f - 17800*a^10*b^9*c^5*d^2*f + 1242 \\
& 80*a^11*b^7*c^6*d^2*f + 400*a^11*b^9*c^4*d*f^2 - 422944*a^12*b^5*c^7*d^2*f \\
& - 6600*a^12*b^7*c^5*d*f^2 + 694912*a^13*b^3*c^8*d^2*f + 40416*a^13*b^5*c^6* \\
& d*f^2 - 108928*a^14*b^3*c^7*d*f^2 + 360*a^11*b^9*c^4*e^2*f - 5736*a^12*b^7* \\
& c^5*e^2*f - 240*a^12*b^8*c^4*e*f^2 + 33888*a^13*b^5*c^6*e^2*f + 3792*a^13*b \\
& ^6*c^5*e*f^2 - 87936*a^14*b^3*c^7*e^2*f - 21696*a^14*b^4*c^6*e*f^2 + 52992* \\
& a^15*b^2*c^7*e*f^2 - 1200*a^10*b^10*c^4*d*e*f + 20240*a^11*b^8*c^5*d*e*f - \\
& 130656*a^12*b^6*c^6*d*e*f + 394368*a^13*b^4*c^7*d*e*f - 528896*a^14*b^2*c^8 \\
& *d*e*f))*(-(25*b^15*d^2 + 9*a^2*b^13*e^2 - 25*b^6*d^2*(-(4*a*c - b^2)^9)^{(1 \\
& /2)} + a^4*b^11*f^2 - 80640*a^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880*a^8*b \\
& *c^6*e^2 - 27*a^5*b^9*c*f^2 - 3840*a^9*b*c^5*f^2 + 9*a^5*c*f^2*(-(4*a*c - b \\
& ^2)^9)^{(1/2)} - 30*a*b^14*d*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^
\end{aligned}$$

$$\begin{aligned}
& 2 + 116928a^4b^7c^4d^2 - 219744a^5b^5c^5d^2 + 215040a^6b^3c^6d^2 \\
& - 9a^2b^4e^2(-4ac - b^2)^9)^{(1/2)} + 49a^3c^3d^2(-4ac - b^2)^9)^{(1/2)} + 2077a^4b^9c^2e^2 - 10656a^5b^7c^3e^2 + 30240a^6b^5c^4e^2 \\
& - 44800a^7b^3c^5e^2 - a^4b^2f^2(-4ac - b^2)^9)^{(1/2)} - 25a^4c^2e^2(-4ac - b^2)^9)^{(1/2)} + 288a^6b^7c^2f^2 - 1504a^7b^5c^3f^2 \\
& + 3840a^8b^3c^4f^2 - 615a^2b^13cd^2 + 10a^2b^13d^2f + 35840a^8c^7de - 6a^3b^12ef - 15360a^9c^6ef \\
& + 30ab^5d^2ef(-4ac - b^2)^9)^{(1/2)} + 724a^2b^12cde - 258a^3b^11cdf + 43520a^8b^6c^6d^2f \\
& + 152a^4b^10c^2ef - 246a^2b^2c^2d^2(-4ac - b^2)^9)^{(1/2)} + 165ab^4c^2d^2(-4ac - b^2)^9)^{(1/2)} \\
& - 7278a^3b^10c^2de + 39132a^4b^8c^3de - 119616a^5b^6c^4de + 201600a^6b^4c^5de - 161280a^7b^2c^6de \\
& - 10a^2b^4d^2f(-4ac - b^2)^9)^{(1/2)} + 2706a^4b^9c^2d^2f - 14784a^5b^7c^3d^2f + 44352a^6b^5c^4d^2f \\
& - 69120a^7b^3c^5d^2f + 6a^3b^3e^2f(-4ac - b^2)^9)^{(1/2)} - 42a^4c^2d^2f(-4ac - b^2)^9)^{(1/2)} \\
& - 1548a^5b^8c^2ef + 8064a^6b^6c^3ef - 22400a^7b^4c^4ef + 30720a^8b^2c^5ef + 51a^3b^2c^2e^2(-4ac - b^2)^9)^{(1/2)} \\
& - 44a^4b^3c^2e^2f(-4ac - b^2)^9)^{(1/2)} - 184a^2b^3c^2d^2ef(-4ac - b^2)^9)^{(1/2)} + 186a^3b^3c^2d^2ef(-4ac - b^2)^9)^{(1/2)} \\
& + 78a^3b^2c^2d^2f(-4ac - b^2)^9)^{(1/2)}/(32(a^7b^12 + 4096a^13c^6 - 24a^8b^10c + 240a^9b^8c^2 - 1280a^10b^6c^3 + 3840a^11b^4c^4 - 6144a^12b^2c^5)))^{(1/2)}*2i
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**4+e*x**2+d)/x**4/(c*x**4+b*x**2+a)**2,x)

[Out] Timed out

$$3.72 \quad \int \frac{x^9(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} dx$$

Optimal. Leaf size=68

$$\frac{5x^8}{8} - \frac{9x^6}{2} + \frac{49x^4}{2} - \frac{293x^2}{2} + 2 \log(x^2 + 1) + 392 \log(x^2 + 2) + \frac{415x^2 + 414}{2(x^4 + 3x^2 + 2)}$$

Rubi [A] time = 0.13, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {1663, 1660, 1657, 632, 31}

$$\frac{5x^8}{8} - \frac{9x^6}{2} + \frac{49x^4}{2} - \frac{293x^2}{2} + \frac{415x^2 + 414}{2(x^4 + 3x^2 + 2)} + 2 \log(x^2 + 1) + 392 \log(x^2 + 2)$$

Antiderivative was successfully verified.

[In] Int[(x^9*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]

[Out] (-293*x^2)/2 + (49*x^4)/2 - (9*x^6)/2 + (5*x^8)/8 + (414 + 415*x^2)/(2*(2 + 3*x^2 + x^4)) + 2*Log[1 + x^2] + 392*Log[2 + x^2]

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 632

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]

Rule 1657

Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Int[Expand[Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 1660

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(2*c*f - b*g), x], x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]

Rule 1663

Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned}
\int \frac{x^9(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x^4(4+x+3x^2+5x^3)}{(2+3x+x^2)^2} dx, x, x^2 \right) \\
&= \frac{414+415x^2}{2(2+3x^2+x^4)} - \frac{1}{2} \text{Subst} \left(\int \frac{-206-105x+53x^2-27x^3+12x^4-5x^5}{2+3x+x^2} dx, x, x^2 \right) \\
&= \frac{414+415x^2}{2(2+3x^2+x^4)} - \frac{1}{2} \text{Subst} \left(\int \left(293-98x+27x^2-5x^3 - \frac{4(198+197x)}{2+3x+x^2} \right) dx, x, x^2 \right) \\
&= -\frac{293x^2}{2} + \frac{49x^4}{2} - \frac{9x^6}{2} + \frac{5x^8}{8} + \frac{414+415x^2}{2(2+3x^2+x^4)} + 2 \text{Subst} \left(\int \frac{198+197x}{2+3x+x^2} dx, x, x^2 \right) \\
&= -\frac{293x^2}{2} + \frac{49x^4}{2} - \frac{9x^6}{2} + \frac{5x^8}{8} + \frac{414+415x^2}{2(2+3x^2+x^4)} + 2 \text{Subst} \left(\int \frac{1}{1+x} dx, x, x^2 \right) \\
&= -\frac{293x^2}{2} + \frac{49x^4}{2} - \frac{9x^6}{2} + \frac{5x^8}{8} + \frac{414+415x^2}{2(2+3x^2+x^4)} + 2 \log(1+x^2) + 392 \log(x^2+2)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 62, normalized size = 0.91

$$\frac{1}{8} \left(5x^8 - 36x^6 + 196x^4 - 1172x^2 + 16 \log(x^2 + 1) + 3136 \log(x^2 + 2) + \frac{4(415x^2 + 414)}{x^4 + 3x^2 + 2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x^9*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]

[Out] (-1172*x^2 + 196*x^4 - 36*x^6 + 5*x^8 + (4*(414 + 415*x^2)))/(2 + 3*x^2 + x^4) + 16*Log[1 + x^2] + 3136*Log[2 + x^2])/8

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^9*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]

[Out] IntegrateAlgebraic[(x^9*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2, x]

fricas [A] time = 1.42, size = 82, normalized size = 1.21

$$\frac{5x^{12} - 21x^{10} + 98x^8 - 656x^6 - 3124x^4 - 684x^2 + 3136(x^4 + 3x^2 + 2)\log(x^2 + 2) + 16(x^4 + 3x^2 + 2)\log(x^2 + 1) + 1656}{8(x^4 + 3x^2 + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")

[Out] 1/8*(5*x^12 - 21*x^10 + 98*x^8 - 656*x^6 - 3124*x^4 - 684*x^2 + 3136*(x^4 + 3*x^2 + 2)*log(x^2 + 2) + 16*(x^4 + 3*x^2 + 2)*log(x^2 + 1) + 1656)/(x^4 + 3*x^2 + 2)

giac [A] time = 0.36, size = 63, normalized size = 0.93

$$\frac{5}{8}x^8 - \frac{9}{2}x^6 + \frac{49}{2}x^4 - \frac{293}{2}x^2 - \frac{394x^4 + 767x^2 + 374}{2(x^4 + 3x^2 + 2)} + 392 \log(x^2 + 2) + 2 \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")

[Out] $5/8*x^8 - 9/2*x^6 + 49/2*x^4 - 293/2*x^2 - 1/2*(394*x^4 + 767*x^2 + 374)/(x^4 + 3*x^2 + 2) + 392*\log(x^2 + 2) + 2*\log(x^2 + 1)$

maple [A] time = 0.02, size = 56, normalized size = 0.82

$$\frac{5x^8}{8} - \frac{9x^6}{2} + \frac{49x^4}{2} - \frac{293x^2}{2} + 2\ln(x^2 + 1) + 392\ln(x^2 + 2) - \frac{1}{2(x^2 + 1)} + \frac{208}{x^2 + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^9*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x)

[Out] $5/8*x^8 - 9/2*x^6 + 49/2*x^4 - 293/2*x^2 + 2*\ln(x^2 + 1) - 1/2/(x^2 + 1) + 208/(x^2 + 2) + 392*\ln(x^2 + 2)$

maxima [A] time = 0.60, size = 58, normalized size = 0.85

$$\frac{5}{8}x^8 - \frac{9}{2}x^6 + \frac{49}{2}x^4 - \frac{293}{2}x^2 + \frac{415x^2 + 414}{2(x^4 + 3x^2 + 2)} + 392\log(x^2 + 2) + 2\log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")

[Out] $5/8*x^8 - 9/2*x^6 + 49/2*x^4 - 293/2*x^2 + 1/2*(415*x^2 + 414)/(x^4 + 3*x^2 + 2) + 392*\log(x^2 + 2) + 2*\log(x^2 + 1)$

mupad [B] time = 0.06, size = 57, normalized size = 0.84

$$2\ln(x^2 + 1) + 392\ln(x^2 + 2) + \frac{\frac{415x^2}{2} + 207}{x^4 + 3x^2 + 2} - \frac{293x^2}{2} + \frac{49x^4}{2} - \frac{9x^6}{2} + \frac{5x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^9*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^2,x)

[Out] $2*\log(x^2 + 1) + 392*\log(x^2 + 2) + ((415*x^2)/2 + 207)/(3*x^2 + x^4 + 2) - (293*x^2)/2 + (49*x^4)/2 - (9*x^6)/2 + (5*x^8)/8$

sympy [A] time = 0.17, size = 61, normalized size = 0.90

$$\frac{5x^8}{8} - \frac{9x^6}{2} + \frac{49x^4}{2} - \frac{293x^2}{2} + \frac{415x^2 + 414}{2x^4 + 6x^2 + 4} + 2\log(x^2 + 1) + 392\log(x^2 + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**9*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)

[Out] $5*x**8/8 - 9*x**6/2 + 49*x**4/2 - 293*x**2/2 + (415*x**2 + 414)/(2*x**4 + 6*x**2 + 4) + 2*\log(x**2 + 1) + 392*\log(x**2 + 2)$

$$3.73 \quad \int \frac{x^7(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} dx$$

Optimal. Leaf size=61

$$\frac{5x^6}{6} - \frac{27x^4}{4} + 49x^2 - \frac{5}{2} \log(x^2 + 1) - 144 \log(x^2 + 2) - \frac{207x^2 + 206}{2(x^4 + 3x^2 + 2)}$$

Rubi [A] time = 0.12, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {1663, 1660, 1657, 632, 31}

$$\frac{5x^6}{6} - \frac{27x^4}{4} + 49x^2 - \frac{207x^2 + 206}{2(x^4 + 3x^2 + 2)} - \frac{5}{2} \log(x^2 + 1) - 144 \log(x^2 + 2)$$

Antiderivative was successfully verified.

[In] Int[(x^7*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]

[Out] 49*x^2 - (27*x^4)/4 + (5*x^6)/6 - (206 + 207*x^2)/(2*(2 + 3*x^2 + x^4)) - (5*Log[1 + x^2])/2 - 144*Log[2 + x^2]

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 632

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]

Rule 1657

Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 1660

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(2*c*f - b*g), x], x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]

Rule 1663

Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned}
\int \frac{x^7(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x^3(4+x+3x^2+5x^3)}{(2+3x+x^2)^2} dx, x, x^2 \right) \\
&= -\frac{206+207x^2}{2(2+3x^2+x^4)} - \frac{1}{2} \text{Subst} \left(\int \frac{102+53x-27x^2+12x^3-5x^4}{2+3x+x^2} dx, x, x^2 \right) \\
&= -\frac{206+207x^2}{2(2+3x^2+x^4)} - \frac{1}{2} \text{Subst} \left(\int \left(-98+27x-5x^2 + \frac{298+293x}{2+3x+x^2} \right) dx, x, x^2 \right) \\
&= 49x^2 - \frac{27x^4}{4} + \frac{5x^6}{6} - \frac{206+207x^2}{2(2+3x^2+x^4)} - \frac{1}{2} \text{Subst} \left(\int \frac{298+293x}{2+3x+x^2} dx, x, x^2 \right) \\
&= 49x^2 - \frac{27x^4}{4} + \frac{5x^6}{6} - \frac{206+207x^2}{2(2+3x^2+x^4)} - \frac{5}{2} \text{Subst} \left(\int \frac{1}{1+x} dx, x, x^2 \right) - 144 \text{Sub} \\
&= 49x^2 - \frac{27x^4}{4} + \frac{5x^6}{6} - \frac{206+207x^2}{2(2+3x^2+x^4)} - \frac{5}{2} \log(1+x^2) - 144 \log(2+x^2)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 61, normalized size = 1.00

$$\frac{5x^6}{6} - \frac{27x^4}{4} + 49x^2 - \frac{5}{2} \log(x^2+1) - 144 \log(x^2+2) + \frac{-207x^2-206}{2(x^4+3x^2+2)}$$

Antiderivative was successfully verified.

[In] Integrate[(x^7*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]

[Out] 49*x^2 - (27*x^4)/4 + (5*x^6)/6 + (-206 - 207*x^2)/(2*(2 + 3*x^2 + x^4)) - (5*Log[1 + x^2])/2 - 144*Log[2 + x^2]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^7*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]

[Out] IntegrateAlgebraic[(x^7*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2, x]

fricas [A] time = 1.64, size = 77, normalized size = 1.26

$$\frac{10x^{10} - 51x^8 + 365x^6 + 1602x^4 - 66x^2 - 1728(x^4 + 3x^2 + 2) \log(x^2 + 2) - 30(x^4 + 3x^2 + 2) \log(x^2 + 1) - 1236}{12(x^4 + 3x^2 + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")

[Out] 1/12*(10*x^10 - 51*x^8 + 365*x^6 + 1602*x^4 - 66*x^2 - 1728*(x^4 + 3*x^2 + 2)*log(x^2 + 2) - 30*(x^4 + 3*x^2 + 2)*log(x^2 + 1) - 1236)/(x^4 + 3*x^2 + 2)

giac [A] time = 0.32, size = 58, normalized size = 0.95

$$\frac{5}{6}x^6 - \frac{27}{4}x^4 + 49x^2 + \frac{293x^4 + 465x^2 + 174}{4(x^4 + 3x^2 + 2)} - 144 \log(x^2 + 2) - \frac{5}{2} \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")

[Out] $\frac{5}{6}x^6 - \frac{27}{4}x^4 + 49x^2 + \frac{1}{4}(293x^4 + 465x^2 + 174)/(x^4 + 3x^2 + 2) - 144\log(x^2 + 2) - \frac{5}{2}\log(x^2 + 1)$

maple [A] time = 0.02, size = 51, normalized size = 0.84

$$\frac{5x^6}{6} - \frac{27x^4}{4} + 49x^2 - \frac{5 \ln(x^2 + 1)}{2} - 144 \ln(x^2 + 2) + \frac{1}{2x^2 + 2} - \frac{104}{x^2 + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x)

[Out] $\frac{5}{6}x^6 - 27/4x^4 + 49x^2 - 5/2\ln(x^2+1) + 1/2/(x^2+1) - 104/(x^2+2) - 144\ln(x^2+2)$

maxima [A] time = 0.72, size = 53, normalized size = 0.87

$$\frac{5}{6}x^6 - \frac{27}{4}x^4 + 49x^2 - \frac{207x^2 + 206}{2(x^4 + 3x^2 + 2)} - 144 \log(x^2 + 2) - \frac{5}{2} \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")

[Out] $\frac{5}{6}x^6 - \frac{27}{4}x^4 + 49x^2 - \frac{1}{2}(207x^2 + 206)/(x^4 + 3x^2 + 2) - 144\log(x^2 + 2) - \frac{5}{2}\log(x^2 + 1)$

mupad [B] time = 0.04, size = 53, normalized size = 0.87

$$49x^2 - 144 \ln(x^2 + 2) - \frac{\frac{207x^2}{2} + 103}{x^4 + 3x^2 + 2} - \frac{5 \ln(x^2 + 1)}{2} - \frac{27x^4}{4} + \frac{5x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^7*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^2,x)

[Out] $49x^2 - 144\log(x^2 + 2) - ((207x^2)/2 + 103)/(3x^2 + x^4 + 2) - (5\log(x^2 + 1))/2 - (27x^4)/4 + (5x^6)/6$

sympy [A] time = 0.17, size = 56, normalized size = 0.92

$$\frac{5x^6}{6} - \frac{27x^4}{4} + 49x^2 + \frac{-207x^2 - 206}{2x^4 + 6x^2 + 4} - \frac{5 \log(x^2 + 1)}{2} - 144 \log(x^2 + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**7*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)

[Out] $5x^{**6}/6 - 27x^{**4}/4 + 49x^{**2} + (-207x^{**2} - 206)/(2x^{**4} + 6x^{**2} + 4) - 5\log(x^{**2} + 1)/2 - 144\log(x^{**2} + 2)$

$$3.74 \quad \int \frac{x^5(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} dx$$

Optimal. Leaf size=54

$$\frac{5x^4}{4} - \frac{27x^2}{2} + 3 \log(x^2 + 1) + 46 \log(x^2 + 2) + \frac{103x^2 + 102}{2(x^4 + 3x^2 + 2)}$$

Rubi [A] time = 0.11, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {1663, 1660, 1657, 632, 31}

$$\frac{5x^4}{4} - \frac{27x^2}{2} + \frac{103x^2 + 102}{2(x^4 + 3x^2 + 2)} + 3 \log(x^2 + 1) + 46 \log(x^2 + 2)$$

Antiderivative was successfully verified.

[In] Int[(x^5*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]

[Out] (-27*x^2)/2 + (5*x^4)/4 + (102 + 103*x^2)/(2*(2 + 3*x^2 + x^4)) + 3*Log[1 + x^2] + 46*Log[2 + x^2]

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 632

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]

Rule 1657

Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Int[Expand[Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 1660

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(2*c*f - b*g), x], x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]

Rule 1663

Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned}
\int \frac{x^5(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x^2(4+x+3x^2+5x^3)}{(2+3x+x^2)^2} dx, x, x^2 \right) \\
&= \frac{102+103x^2}{2(2+3x^2+x^4)} - \frac{1}{2} \text{Subst} \left(\int \frac{-50-27x+12x^2-5x^3}{2+3x+x^2} dx, x, x^2 \right) \\
&= \frac{102+103x^2}{2(2+3x^2+x^4)} - \frac{1}{2} \text{Subst} \left(\int \left(27-5x - \frac{2(52+49x)}{2+3x+x^2} \right) dx, x, x^2 \right) \\
&= -\frac{27x^2}{2} + \frac{5x^4}{4} + \frac{102+103x^2}{2(2+3x^2+x^4)} + \text{Subst} \left(\int \frac{52+49x}{2+3x+x^2} dx, x, x^2 \right) \\
&= -\frac{27x^2}{2} + \frac{5x^4}{4} + \frac{102+103x^2}{2(2+3x^2+x^4)} + 3 \text{Subst} \left(\int \frac{1}{1+x} dx, x, x^2 \right) + 46 \text{Subst} \left(\int \frac{1}{1+x} dx, x, x^2 \right) \\
&= -\frac{27x^2}{2} + \frac{5x^4}{4} + \frac{102+103x^2}{2(2+3x^2+x^4)} + 3 \log(1+x^2) + 46 \log(2+x^2)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 54, normalized size = 1.00

$$\frac{5x^4}{4} - \frac{27x^2}{2} + 3 \log(x^2 + 1) + 46 \log(x^2 + 2) + \frac{103x^2 + 102}{2(x^4 + 3x^2 + 2)}$$

Antiderivative was successfully verified.

[In] Integrate[(x^5*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]

[Out] (-27*x^2)/2 + (5*x^4)/4 + (102 + 103*x^2)/(2*(2 + 3*x^2 + x^4)) + 3*Log[1 + x^2] + 46*Log[2 + x^2]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^5*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]

[Out] IntegrateAlgebraic[(x^5*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2, x]

fricas [A] time = 1.22, size = 72, normalized size = 1.33

$$\frac{5x^8 - 39x^6 - 152x^4 + 98x^2 + 184(x^4 + 3x^2 + 2) \log(x^2 + 2) + 12(x^4 + 3x^2 + 2) \log(x^2 + 1) + 204}{4(x^4 + 3x^2 + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")

[Out] 1/4*(5*x^8 - 39*x^6 - 152*x^4 + 98*x^2 + 184*(x^4 + 3*x^2 + 2)*log(x^2 + 2) + 12*(x^4 + 3*x^2 + 2)*log(x^2 + 1) + 204)/(x^4 + 3*x^2 + 2)

giac [A] time = 0.37, size = 53, normalized size = 0.98

$$\frac{5}{4}x^4 - \frac{27}{2}x^2 - \frac{49x^4 + 44x^2 - 4}{2(x^4 + 3x^2 + 2)} + 46 \log(x^2 + 2) + 3 \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")

[Out] 5/4*x^4 - 27/2*x^2 - 1/2*(49*x^4 + 44*x^2 - 4)/(x^4 + 3*x^2 + 2) + 46*log(x^2 + 2) + 3*log(x^2 + 1)

maple [A] time = 0.02, size = 46, normalized size = 0.85

$$\frac{5x^4}{4} - \frac{27x^2}{2} + 3\ln(x^2 + 1) + 46\ln(x^2 + 2) - \frac{1}{2(x^2 + 1)} + \frac{52}{x^2 + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x)

[Out] 5/4*x^4-27/2*x^2+3*ln(x^2+1)-1/2/(x^2+1)+52/(x^2+2)+46*ln(x^2+2)

maxima [A] time = 1.07, size = 48, normalized size = 0.89

$$\frac{5}{4}x^4 - \frac{27}{2}x^2 + \frac{103x^2 + 102}{2(x^4 + 3x^2 + 2)} + 46\log(x^2 + 2) + 3\log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")

[Out] 5/4*x^4 - 27/2*x^2 + 1/2*(103*x^2 + 102)/(x^4 + 3*x^2 + 2) + 46*log(x^2 + 2) + 3*log(x^2 + 1)

mupad [B] time = 0.90, size = 47, normalized size = 0.87

$$3\ln(x^2 + 1) + 46\ln(x^2 + 2) + \frac{\frac{103x^2}{2} + 51}{x^4 + 3x^2 + 2} - \frac{27x^2}{2} + \frac{5x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^5*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^2,x)

[Out] 3*log(x^2 + 1) + 46*log(x^2 + 2) + ((103*x^2)/2 + 51)/(3*x^2 + x^4 + 2) - (27*x^2)/2 + (5*x^4)/4

sympy [A] time = 0.17, size = 48, normalized size = 0.89

$$\frac{5x^4}{4} - \frac{27x^2}{2} + \frac{103x^2 + 102}{2x^4 + 6x^2 + 4} + 3\log(x^2 + 1) + 46\log(x^2 + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)

[Out] 5*x**4/4 - 27*x**2/2 + (103*x**2 + 102)/(2*x**4 + 6*x**2 + 4) + 3*log(x**2 + 1) + 46*log(x**2 + 2)

$$3.75 \quad \int \frac{x^3(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} dx$$

Optimal. Leaf size=49

$$\frac{5x^2}{2} - \frac{7}{2} \log(x^2 + 1) - 10 \log(x^2 + 2) - \frac{51x^2 + 50}{2(x^4 + 3x^2 + 2)}$$

Rubi [A] time = 0.09, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {1663, 1660, 1657, 632, 31}

$$\frac{5x^2}{2} - \frac{51x^2 + 50}{2(x^4 + 3x^2 + 2)} - \frac{7}{2} \log(x^2 + 1) - 10 \log(x^2 + 2)$$

Antiderivative was successfully verified.

[In] Int[(x^3*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]

[Out] (5*x^2)/2 - (50 + 51*x^2)/(2*(2 + 3*x^2 + x^4)) - (7*Log[1 + x^2])/2 - 10*Log[2 + x^2]

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 632

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]

Rule 1657

Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 1660

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(2*c*f - b*g), x], x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]

Rule 1663

Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned}
\int \frac{x^3(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x(4+x+3x^2+5x^3)}{(2+3x+x^2)^2} dx, x, x^2 \right) \\
&= -\frac{50+51x^2}{2(2+3x^2+x^4)} - \frac{1}{2} \text{Subst} \left(\int \frac{24+12x-5x^2}{2+3x+x^2} dx, x, x^2 \right) \\
&= -\frac{50+51x^2}{2(2+3x^2+x^4)} - \frac{1}{2} \text{Subst} \left(\int \left(-5 + \frac{34+27x}{2+3x+x^2} \right) dx, x, x^2 \right) \\
&= \frac{5x^2}{2} - \frac{50+51x^2}{2(2+3x^2+x^4)} - \frac{1}{2} \text{Subst} \left(\int \frac{34+27x}{2+3x+x^2} dx, x, x^2 \right) \\
&= \frac{5x^2}{2} - \frac{50+51x^2}{2(2+3x^2+x^4)} - \frac{7}{2} \text{Subst} \left(\int \frac{1}{1+x} dx, x, x^2 \right) - 10 \text{Subst} \left(\int \frac{1}{2+x} dx, x, x^2 \right) \\
&= \frac{5x^2}{2} - \frac{50+51x^2}{2(2+3x^2+x^4)} - \frac{7}{2} \log(1+x^2) - 10 \log(2+x^2)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 1.00

$$\frac{5x^2}{2} - \frac{7}{2} \log(x^2+1) - 10 \log(x^2+2) + \frac{-51x^2-50}{2(x^4+3x^2+2)}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2, x]

[Out] (5*x^2)/2 + (-50 - 51*x^2)/(2*(2 + 3*x^2 + x^4)) - (7*Log[1 + x^2])/2 - 10*Log[2 + x^2]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^3*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2, x]

[Out] IntegrateAlgebraic[(x^3*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2, x]

fricas [A] time = 1.18, size = 67, normalized size = 1.37

$$\frac{5x^6 + 15x^4 - 41x^2 - 20(x^4 + 3x^2 + 2) \log(x^2 + 2) - 7(x^4 + 3x^2 + 2) \log(x^2 + 1) - 50}{2(x^4 + 3x^2 + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2, x, algorithm="fricas")

[Out] 1/2*(5*x^6 + 15*x^4 - 41*x^2 - 20*(x^4 + 3*x^2 + 2)*log(x^2 + 2) - 7*(x^4 + 3*x^2 + 2)*log(x^2 + 1) - 50)/(x^4 + 3*x^2 + 2)

giac [A] time = 0.39, size = 45, normalized size = 0.92

$$\frac{5}{2} x^2 - \frac{51x^2 + 50}{2(x^2 + 2)(x^2 + 1)} - 10 \log(x^2 + 2) - \frac{7}{2} \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")

[Out] $\frac{5}{2}x^2 - \frac{1}{2}(51x^2 + 50)/((x^2 + 2)(x^2 + 1)) - 10\log(x^2 + 2) - \frac{7}{2}\log(x^2 + 1)$

maple [A] time = 0.02, size = 41, normalized size = 0.84

$$\frac{5x^2}{2} - \frac{7\ln(x^2 + 1)}{2} - 10\ln(x^2 + 2) + \frac{1}{2x^2 + 2} - \frac{26}{x^2 + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x)

[Out] $\frac{5}{2}x^2 - 7/2*\ln(x^2+1) + 1/2/(x^2+1) - 26/(x^2+2) - 10*\ln(x^2+2)$

maxima [A] time = 0.51, size = 43, normalized size = 0.88

$$\frac{5}{2}x^2 - \frac{51x^2 + 50}{2(x^4 + 3x^2 + 2)} - 10\log(x^2 + 2) - \frac{7}{2}\log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")

[Out] $\frac{5}{2}x^2 - \frac{1}{2}(51x^2 + 50)/(x^4 + 3x^2 + 2) - 10\log(x^2 + 2) - \frac{7}{2}\log(x^2 + 1)$

mupad [B] time = 0.04, size = 43, normalized size = 0.88

$$\frac{5x^2}{2} - 10\ln(x^2 + 2) - \frac{\frac{51x^2}{2} + 25}{x^4 + 3x^2 + 2} - \frac{7\ln(x^2 + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^2,x)

[Out] $(5x^2)/2 - 10*\log(x^2 + 2) - ((51x^2)/2 + 25)/(3x^2 + x^4 + 2) - (7*\log(x^2 + 1))/2$

sympy [A] time = 0.17, size = 44, normalized size = 0.90

$$\frac{5x^2}{2} + \frac{-51x^2 - 50}{2x^4 + 6x^2 + 4} - \frac{7\log(x^2 + 1)}{2} - 10\log(x^2 + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)

[Out] $5*x**2/2 + (-51*x**2 - 50)/(2*x**4 + 6*x**2 + 4) - 7*\log(x**2 + 1)/2 - 10*\log(x**2 + 2)$

$$3.76 \quad \int \frac{x(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} dx$$

Optimal. Leaf size=42

$$4 \log(x^2 + 1) - \frac{3}{2} \log(x^2 + 2) + \frac{25x^2 + 24}{2(x^4 + 3x^2 + 2)}$$

Rubi [A] time = 0.05, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1663, 1660, 632, 31}

$$\frac{25x^2 + 24}{2(x^4 + 3x^2 + 2)} + 4 \log(x^2 + 1) - \frac{3}{2} \log(x^2 + 2)$$

Antiderivative was successfully verified.

[In] Int[(x*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]

[Out] (24 + 25*x^2)/(2*(2 + 3*x^2 + x^4)) + 4*Log[1 + x^2] - (3*Log[2 + x^2])/2

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 632

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]

Rule 1660

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(2*c*f - b*g), x], x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]

Rule 1663

Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned}
\int \frac{x(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{4+x+3x^2+5x^3}{(2+3x+x^2)^2} dx, x, x^2 \right) \\
&= \frac{24+25x^2}{2(2+3x^2+x^4)} - \frac{1}{2} \text{Subst} \left(\int \frac{-13-5x}{2+3x+x^2} dx, x, x^2 \right) \\
&= \frac{24+25x^2}{2(2+3x^2+x^4)} - \frac{3}{2} \text{Subst} \left(\int \frac{1}{2+x} dx, x, x^2 \right) + 4 \text{Subst} \left(\int \frac{1}{1+x} dx, x, x^2 \right) \\
&= \frac{24+25x^2}{2(2+3x^2+x^4)} + 4 \log(1+x^2) - \frac{3}{2} \log(2+x^2)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 1.00

$$4 \log(x^2+1) - \frac{3}{2} \log(x^2+2) + \frac{25x^2+24}{2(x^4+3x^2+2)}$$

Antiderivative was successfully verified.

[In] Integrate[(x*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]

[Out] (24 + 25*x^2)/(2*(2 + 3*x^2 + x^4)) + 4*Log[1 + x^2] - (3*Log[2 + x^2])/2

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]

[Out] IntegrateAlgebraic[(x*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2, x]

fricas [A] time = 1.23, size = 57, normalized size = 1.36

$$\frac{25x^2 - 3(x^4 + 3x^2 + 2) \log(x^2 + 2) + 8(x^4 + 3x^2 + 2) \log(x^2 + 1) + 24}{2(x^4 + 3x^2 + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")

[Out] 1/2*(25*x^2 - 3*(x^4 + 3*x^2 + 2)*log(x^2 + 2) + 8*(x^4 + 3*x^2 + 2)*log(x^2 + 1) + 24)/(x^4 + 3*x^2 + 2)

giac [A] time = 0.35, size = 40, normalized size = 0.95

$$\frac{25x^2+24}{2(x^2+2)(x^2+1)} - \frac{3}{2} \log(x^2+2) + 4 \log(x^2+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")

[Out] 1/2*(25*x^2 + 24)/((x^2 + 2)*(x^2 + 1)) - 3/2*log(x^2 + 2) + 4*log(x^2 + 1)

maple [A] time = 0.02, size = 36, normalized size = 0.86

$$4 \ln(x^2 + 1) - \frac{3 \ln(x^2 + 2)}{2} - \frac{1}{2(x^2 + 1)} + \frac{13}{x^2 + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x)`

[Out] `4*ln(x^2+1)-1/2/(x^2+1)+13/(x^2+2)-3/2*ln(x^2+2)`

maxima [A] time = 0.52, size = 38, normalized size = 0.90

$$\frac{25x^2 + 24}{2(x^4 + 3x^2 + 2)} - \frac{3}{2} \log(x^2 + 2) + 4 \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")`

[Out] `1/2*(25*x^2 + 24)/(x^4 + 3*x^2 + 2) - 3/2*log(x^2 + 2) + 4*log(x^2 + 1)`

mupad [B] time = 0.05, size = 37, normalized size = 0.88

$$4 \ln(x^2 + 1) - \frac{3 \ln(x^2 + 2)}{2} + \frac{\frac{25x^2}{2} + 12}{x^4 + 3x^2 + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^2,x)`

[Out] `4*log(x^2 + 1) - (3*log(x^2 + 2))/2 + ((25*x^2)/2 + 12)/(3*x^2 + x^4 + 2)`

sympy [A] time = 0.17, size = 36, normalized size = 0.86

$$\frac{25x^2 + 24}{2x^4 + 6x^2 + 4} + 4 \log(x^2 + 1) - \frac{3 \log(x^2 + 2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)`

[Out] `(25*x**2 + 24)/(2*x**4 + 6*x**2 + 4) + 4*log(x**2 + 1) - 3*log(x**2 + 2)/2`

$$3.77 \quad \int \frac{4+x^2+3x^4+5x^6}{x(2+3x^2+x^4)^2} dx$$

Optimal. Leaf size=44

$$-\frac{9}{2} \log(x^2 + 1) + 4 \log(x^2 + 2) - \frac{12x^2 + 11}{2(x^4 + 3x^2 + 2)} + \log(x)$$

Rubi [A] time = 0.08, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {1663, 1646, 800}

$$-\frac{12x^2 + 11}{2(x^4 + 3x^2 + 2)} - \frac{9}{2} \log(x^2 + 1) + 4 \log(x^2 + 2) + \log(x)$$

Antiderivative was successfully verified.

[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x*(2 + 3*x^2 + x^4)^2),x]

[Out] -(11 + 12*x^2)/(2*(2 + 3*x^2 + x^4)) + Log[x] - (9*Log[1 + x^2])/2 + 4*Log[2 + x^2]

Rule 800

Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]

Rule 1646

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]

Rule 1663

Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned}
\int \frac{4 + x^2 + 3x^4 + 5x^6}{x(2 + 3x^2 + x^4)^2} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{4 + x + 3x^2 + 5x^3}{x(2 + 3x + x^2)^2} dx, x, x^2 \right) \\
&= -\frac{11 + 12x^2}{2(2 + 3x^2 + x^4)} - \frac{1}{2} \text{Subst} \left(\int \frac{-2 + 7x}{x(2 + 3x + x^2)} dx, x, x^2 \right) \\
&= -\frac{11 + 12x^2}{2(2 + 3x^2 + x^4)} - \frac{1}{2} \text{Subst} \left(\int \left(-\frac{1}{x} + \frac{9}{1 + x} - \frac{8}{2 + x} \right) dx, x, x^2 \right) \\
&= -\frac{11 + 12x^2}{2(2 + 3x^2 + x^4)} + \log(x) - \frac{9}{2} \log(1 + x^2) + 4 \log(2 + x^2)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 44, normalized size = 1.00

$$-\frac{9}{2} \log(x^2 + 1) + 4 \log(x^2 + 2) + \frac{-12x^2 - 11}{2(x^4 + 3x^2 + 2)} + \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x*(2 + 3*x^2 + x^4)^2), x]

[Out] (-11 - 12*x^2)/(2*(2 + 3*x^2 + x^4)) + Log[x] - (9*Log[1 + x^2])/2 + 4*Log[2 + x^2]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4 + x^2 + 3x^4 + 5x^6}{x(2 + 3x^2 + x^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(4 + x^2 + 3*x^4 + 5*x^6)/(x*(2 + 3*x^2 + x^4)^2), x]

[Out] IntegrateAlgebraic[(4 + x^2 + 3*x^4 + 5*x^6)/(x*(2 + 3*x^2 + x^4)^2), x]

fricas [A] time = 1.47, size = 71, normalized size = 1.61

$$\frac{12x^2 - 8(x^4 + 3x^2 + 2) \log(x^2 + 2) + 9(x^4 + 3x^2 + 2) \log(x^2 + 1) - 2(x^4 + 3x^2 + 2) \log(x) + 11}{2(x^4 + 3x^2 + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^6+3*x^4+x^2+4)/x/(x^4+3*x^2+2)^2,x, algorithm="fricas")

[Out] -1/2*(12*x^2 - 8*(x^4 + 3*x^2 + 2)*log(x^2 + 2) + 9*(x^4 + 3*x^2 + 2)*log(x^2 + 1) - 2*(x^4 + 3*x^2 + 2)*log(x) + 11)/(x^4 + 3*x^2 + 2)

giac [A] time = 0.38, size = 47, normalized size = 1.07

$$\frac{x^4 - 21x^2 - 20}{4(x^4 + 3x^2 + 2)} + 4 \log(x^2 + 2) - \frac{9}{2} \log(x^2 + 1) + \frac{1}{2} \log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^6+3*x^4+x^2+4)/x/(x^4+3*x^2+2)^2,x, algorithm="giac")

[Out] 1/4*(x^4 - 21*x^2 - 20)/(x^4 + 3*x^2 + 2) + 4*log(x^2 + 2) - 9/2*log(x^2 + 1) + 1/2*log(x^2)

maple [A] time = 0.02, size = 38, normalized size = 0.86

$$\ln(x) - \frac{9 \ln(x^2 + 1)}{2} + 4 \ln(x^2 + 2) + \frac{1}{2x^2 + 2} - \frac{13}{2(x^2 + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^6+3*x^4+x^2+4)/x/(x^4+3*x^2+2)^2,x)

[Out] ln(x)-9/2*ln(x^2+1)+1/2/(x^2+1)-13/2/(x^2+2)+4*ln(x^2+2)

maxima [A] time = 0.72, size = 44, normalized size = 1.00

$$-\frac{12x^2 + 11}{2(x^4 + 3x^2 + 2)} + 4 \log(x^2 + 2) - \frac{9}{2} \log(x^2 + 1) + \frac{1}{2} \log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^6+3*x^4+x^2+4)/x/(x^4+3*x^2+2)^2,x, algorithm="maxima")

[Out] -1/2*(12*x^2 + 11)/(x^4 + 3*x^2 + 2) + 4*log(x^2 + 2) - 9/2*log(x^2 + 1) + 1/2*log(x^2)

mupad [B] time = 0.04, size = 40, normalized size = 0.91

$$4 \ln(x^2 + 2) - \frac{9 \ln(x^2 + 1)}{2} + \ln(x) - \frac{6x^2 + \frac{11}{2}}{x^4 + 3x^2 + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(x*(3*x^2 + x^4 + 2)^2),x)

[Out] 4*log(x^2 + 2) - (9*log(x^2 + 1))/2 + log(x) - (6*x^2 + 11/2)/(3*x^2 + x^4 + 2)

sympy [A] time = 0.18, size = 41, normalized size = 0.93

$$\frac{-12x^2 - 11}{2x^4 + 6x^2 + 4} + \log(x) - \frac{9 \log(x^2 + 1)}{2} + 4 \log(x^2 + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**6+3*x**4+x**2+4)/x/(x**4+3*x**2+2)**2,x)

[Out] (-12*x**2 - 11)/(2*x**4 + 6*x**2 + 4) + log(x) - 9*log(x**2 + 1)/2 + 4*log(x**2 + 2)

$$3.78 \quad \int \frac{4+x^2+3x^4+5x^6}{x^3(2+3x^2+x^4)^2} dx$$

Optimal. Leaf size=55

$$-\frac{1}{2x^2} + 5 \log(x^2 + 1) - \frac{29}{8} \log(x^2 + 2) + \frac{11x^2 + 9}{4(x^4 + 3x^2 + 2)} - \frac{11 \log(x)}{4}$$

Rubi [A] time = 0.10, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {1663, 1646, 1628}

$$\frac{11x^2 + 9}{4(x^4 + 3x^2 + 2)} - \frac{1}{2x^2} + 5 \log(x^2 + 1) - \frac{29}{8} \log(x^2 + 2) - \frac{11 \log(x)}{4}$$

Antiderivative was successfully verified.

[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^3*(2 + 3*x^2 + x^4)^2), x]

[Out] -1/(2*x^2) + (9 + 11*x^2)/(4*(2 + 3*x^2 + x^4)) - (11*Log[x])/4 + 5*Log[1 + x^2] - (29*Log[2 + x^2])/8

Rule 1628

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 1646

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]

Rule 1663

Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned}
\int \frac{4 + x^2 + 3x^4 + 5x^6}{x^3(2 + 3x^2 + x^4)^2} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{4 + x + 3x^2 + 5x^3}{x^2(2 + 3x + x^2)^2} dx, x, x^2 \right) \\
&= \frac{9 + 11x^2}{4(2 + 3x^2 + x^4)} - \frac{1}{2} \text{Subst} \left(\int \frac{-2 + \frac{5x}{2} - \frac{11x^2}{2}}{x^2(2 + 3x + x^2)} dx, x, x^2 \right) \\
&= \frac{9 + 11x^2}{4(2 + 3x^2 + x^4)} - \frac{1}{2} \text{Subst} \left(\int \left(-\frac{1}{x^2} + \frac{11}{4x} - \frac{10}{1+x} + \frac{29}{4(2+x)} \right) dx, x, x^2 \right) \\
&= -\frac{1}{2x^2} + \frac{9 + 11x^2}{4(2 + 3x^2 + x^4)} - \frac{11 \log(x)}{4} + 5 \log(1 + x^2) - \frac{29}{8} \log(2 + x^2)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 50, normalized size = 0.91

$$\frac{1}{8} \left(-\frac{4}{x^2} + 40 \log(x^2 + 1) - 29 \log(x^2 + 2) + \frac{22x^2 + 18}{x^4 + 3x^2 + 2} - 22 \log(x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^3*(2 + 3*x^2 + x^4)^2), x]

[Out] (-4/x^2 + (18 + 22*x^2)/(2 + 3*x^2 + x^4) - 22*Log[x] + 40*Log[1 + x^2] - 29*Log[2 + x^2])/8

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4 + x^2 + 3x^4 + 5x^6}{x^3(2 + 3x^2 + x^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(4 + x^2 + 3*x^4 + 5*x^6)/(x^3*(2 + 3*x^2 + x^4)^2), x]

[Out] IntegrateAlgebraic[(4 + x^2 + 3*x^4 + 5*x^6)/(x^3*(2 + 3*x^2 + x^4)^2), x]

fricas [A] time = 1.34, size = 92, normalized size = 1.67

$$\frac{18x^4 + 6x^2 - 29(x^6 + 3x^4 + 2x^2) \log(x^2 + 2) + 40(x^6 + 3x^4 + 2x^2) \log(x^2 + 1) - 22(x^6 + 3x^4 + 2x^2) \log(x) - 8}{8(x^6 + 3x^4 + 2x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^6+3*x^4+x^2+4)/x^3/(x^4+3*x^2+2)^2,x, algorithm="fricas")

[Out] 1/8*(18*x^4 + 6*x^2 - 29*(x^6 + 3*x^4 + 2*x^2)*log(x^2 + 2) + 40*(x^6 + 3*x^4 + 2*x^2)*log(x^2 + 1) - 22*(x^6 + 3*x^4 + 2*x^2)*log(x) - 8)/(x^6 + 3*x^4 + 2*x^2)

giac [A] time = 0.37, size = 53, normalized size = 0.96

$$\frac{9x^4 + 3x^2 - 4}{4(x^6 + 3x^4 + 2x^2)} - \frac{29}{8} \log(x^2 + 2) + 5 \log(x^2 + 1) - \frac{11}{8} \log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^6+3*x^4+x^2+4)/x^3/(x^4+3*x^2+2)^2,x, algorithm="giac")

[Out] $\frac{1}{4} \cdot (9x^4 + 3x^2 - 4) / (x^6 + 3x^4 + 2x^2) - \frac{29}{8} \log(x^2 + 2) + 5 \log(x^2 + 1) - \frac{11}{8} \log(x^2)$

maple [A] time = 0.02, size = 45, normalized size = 0.82

$$-\frac{11 \ln(x)}{4} + 5 \ln(x^2 + 1) - \frac{29 \ln(x^2 + 2)}{8} - \frac{1}{2x^2} - \frac{1}{2(x^2 + 1)} + \frac{13}{4(x^2 + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((5*x^6+3*x^4+x^2+4)/x^3/(x^4+3*x^2+2)^2,x)`

[Out] $-1/2/x^2 - 11/4 \ln(x) + 5 \ln(x^2 + 1) - 1/2/(x^2 + 1) + 13/4/(x^2 + 2) - 29/8 \ln(x^2 + 2)$

maxima [A] time = 0.79, size = 53, normalized size = 0.96

$$\frac{9x^4 + 3x^2 - 4}{4(x^6 + 3x^4 + 2x^2)} - \frac{29}{8} \log(x^2 + 2) + 5 \log(x^2 + 1) - \frac{11}{8} \log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^6+3*x^4+x^2+4)/x^3/(x^4+3*x^2+2)^2,x, algorithm="maxima")`

[Out] $\frac{1}{4} \cdot (9x^4 + 3x^2 - 4) / (x^6 + 3x^4 + 2x^2) - \frac{29}{8} \log(x^2 + 2) + 5 \log(x^2 + 1) - \frac{11}{8} \log(x^2)$

mupad [B] time = 0.04, size = 50, normalized size = 0.91

$$5 \ln(x^2 + 1) - \frac{29 \ln(x^2 + 2)}{8} - \frac{11 \ln(x)}{4} + \frac{\frac{9x^4}{4} + \frac{3x^2}{4} - 1}{x^6 + 3x^4 + 2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^3*(3*x^2 + x^4 + 2)^2),x)`

[Out] $5 \log(x^2 + 1) - (29 \log(x^2 + 2))/8 - (11 \log(x))/4 + ((3x^2)/4 + (9x^4)/4 - 1)/(2x^2 + 3x^4 + x^6)$

sympy [A] time = 0.20, size = 51, normalized size = 0.93

$$\frac{9x^4 + 3x^2 - 4}{4x^6 + 12x^4 + 8x^2} - \frac{11 \log(x)}{4} + 5 \log(x^2 + 1) - \frac{29 \log(x^2 + 2)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x**6+3*x**4+x**2+4)/x**3/(x**4+3*x**2+2)**2,x)`

[Out] $(9x^4 + 3x^2 - 4) / (4x^6 + 12x^4 + 8x^2) - 11 \log(x) / 4 + 5 \log(x^2 + 1) - 29 \log(x^2 + 2) / 8$

$$3.79 \quad \int \frac{4+x^2+3x^4+5x^6}{x^5(2+3x^2+x^4)^2} dx$$

Optimal. Leaf size=64

$$-\frac{1}{4x^4} + \frac{11}{8x^2} - \frac{11}{2} \log(x^2 + 1) + \frac{21}{8} \log(x^2 + 2) - \frac{9x^2 + 5}{8(x^4 + 3x^2 + 2)} + \frac{23 \log(x)}{4}$$

Rubi [A] time = 0.11, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {1663, 1646, 1628}

$$-\frac{9x^2 + 5}{8(x^4 + 3x^2 + 2)} + \frac{11}{8x^2} - \frac{1}{4x^4} - \frac{11}{2} \log(x^2 + 1) + \frac{21}{8} \log(x^2 + 2) + \frac{23 \log(x)}{4}$$

Antiderivative was successfully verified.

[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^5*(2 + 3*x^2 + x^4)^2), x]

[Out] -1/(4*x^4) + 11/(8*x^2) - (5 + 9*x^2)/(8*(2 + 3*x^2 + x^4)) + (23*Log[x])/4 - (11*Log[1 + x^2])/2 + (21*Log[2 + x^2])/8

Rule 1628

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 1646

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]

Rule 1663

Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned}
\int \frac{4 + x^2 + 3x^4 + 5x^6}{x^5(2 + 3x^2 + x^4)^2} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{4 + x + 3x^2 + 5x^3}{x^3(2 + 3x + x^2)^2} dx, x, x^2 \right) \\
&= -\frac{5 + 9x^2}{8(2 + 3x^2 + x^4)} - \frac{1}{2} \text{Subst} \left(\int \frac{-2 + \frac{5x}{2} - \frac{17x^2}{4} + \frac{9x^3}{4}}{x^3(2 + 3x + x^2)} dx, x, x^2 \right) \\
&= -\frac{5 + 9x^2}{8(2 + 3x^2 + x^4)} - \frac{1}{2} \text{Subst} \left(\int \left(-\frac{1}{x^3} + \frac{11}{4x^2} - \frac{23}{4x} + \frac{11}{1+x} - \frac{21}{4(2+x)} \right) dx, x, x^2 \right) \\
&= -\frac{1}{4x^4} + \frac{11}{8x^2} - \frac{5 + 9x^2}{8(2 + 3x^2 + x^4)} + \frac{23 \log(x)}{4} - \frac{11}{2} \log(1 + x^2) + \frac{21}{8} \log(2 + x^2)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 56, normalized size = 0.88

$$\frac{1}{8} \left(-\frac{2}{x^4} + \frac{11}{x^2} - 44 \log(x^2 + 1) + 21 \log(x^2 + 2) - \frac{9x^2 + 5}{x^4 + 3x^2 + 2} + 46 \log(x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^5*(2 + 3*x^2 + x^4)^2), x]

[Out] (-2/x^4 + 11/x^2 - (5 + 9*x^2)/(2 + 3*x^2 + x^4) + 46*Log[x] - 44*Log[1 + x^2] + 21*Log[2 + x^2])/8

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4 + x^2 + 3x^4 + 5x^6}{x^5(2 + 3x^2 + x^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(4 + x^2 + 3*x^4 + 5*x^6)/(x^5*(2 + 3*x^2 + x^4)^2), x]

[Out] IntegrateAlgebraic[(4 + x^2 + 3*x^4 + 5*x^6)/(x^5*(2 + 3*x^2 + x^4)^2), x]

fricas [A] time = 1.22, size = 97, normalized size = 1.52

$$\frac{2x^6 + 26x^4 + 16x^2 + 21(x^8 + 3x^6 + 2x^4) \log(x^2 + 2) - 44(x^8 + 3x^6 + 2x^4) \log(x^2 + 1) + 46(x^8 + 3x^6 + 2x^4) \log(x) - 4}{8(x^8 + 3x^6 + 2x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^6+3*x^4+x^2+4)/x^5/(x^4+3*x^2+2)^2,x, algorithm="fricas")

[Out] 1/8*(2*x^6 + 26*x^4 + 16*x^2 + 21*(x^8 + 3*x^6 + 2*x^4)*log(x^2 + 2) - 44*(x^8 + 3*x^6 + 2*x^4)*log(x^2 + 1) + 46*(x^8 + 3*x^6 + 2*x^4)*log(x) - 4)/(x^8 + 3*x^6 + 2*x^4)

giac [A] time = 0.34, size = 66, normalized size = 1.03

$$\frac{23x^4 + 51x^2 + 36}{16(x^4 + 3x^2 + 2)} - \frac{69x^4 - 22x^2 + 4}{16x^4} + \frac{21}{8} \log(x^2 + 2) - \frac{11}{2} \log(x^2 + 1) + \frac{23}{8} \log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^6+3*x^4+x^2+4)/x^5/(x^4+3*x^2+2)^2,x, algorithm="giac")

[Out] $1/16*(23*x^4 + 51*x^2 + 36)/(x^4 + 3*x^2 + 2) - 1/16*(69*x^4 - 22*x^2 + 4)/x^4 + 21/8*\log(x^2 + 2) - 11/2*\log(x^2 + 1) + 23/8*\log(x^2)$

maple [A] time = 0.02, size = 50, normalized size = 0.78

$$\frac{23 \ln(x)}{4} - \frac{11 \ln(x^2 + 1)}{2} + \frac{21 \ln(x^2 + 2)}{8} + \frac{11}{8x^2} - \frac{1}{4x^4} + \frac{1}{2x^2 + 2} - \frac{13}{8(x^2 + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((5*x^6+3*x^4+x^2+4)/x^5/(x^4+3*x^2+2)^2,x)`

[Out] $-1/4/x^4+11/8/x^2+23/4*\ln(x)-11/2*\ln(x^2+1)+1/2/(x^2+1)-13/8/(x^2+2)+21/8*\ln(x^2+2)$

maxima [A] time = 0.68, size = 56, normalized size = 0.88

$$\frac{x^6 + 13x^4 + 8x^2 - 2}{4(x^8 + 3x^6 + 2x^4)} + \frac{21}{8} \log(x^2 + 2) - \frac{11}{2} \log(x^2 + 1) + \frac{23}{8} \log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^6+3*x^4+x^2+4)/x^5/(x^4+3*x^2+2)^2,x, algorithm="maxima")`

[Out] $1/4*(x^6 + 13*x^4 + 8*x^2 - 2)/(x^8 + 3*x^6 + 2*x^4) + 21/8*\log(x^2 + 2) - 11/2*\log(x^2 + 1) + 23/8*\log(x^2)$

mupad [B] time = 0.92, size = 55, normalized size = 0.86

$$\frac{21 \ln(x^2 + 2)}{8} - \frac{11 \ln(x^2 + 1)}{2} + \frac{23 \ln(x)}{4} + \frac{\frac{x^6}{4} + \frac{13x^4}{4} + 2x^2 - \frac{1}{2}}{x^8 + 3x^6 + 2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^5*(3*x^2 + x^4 + 2)^2),x)`

[Out] $(21*\log(x^2 + 2))/8 - (11*\log(x^2 + 1))/2 + (23*\log(x))/4 + (2*x^2 + (13*x^4)/4 + x^6/4 - 1/2)/(2*x^4 + 3*x^6 + x^8)$

sympy [A] time = 0.21, size = 56, normalized size = 0.88

$$\frac{23 \log(x)}{4} - \frac{11 \log(x^2 + 1)}{2} + \frac{21 \log(x^2 + 2)}{8} + \frac{x^6 + 13x^4 + 8x^2 - 2}{4x^8 + 12x^6 + 8x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x**6+3*x**4+x**2+4)/x**5/(x**4+3*x**2+2)**2,x)`

[Out] $23*\log(x)/4 - 11*\log(x**2 + 1)/2 + 21*\log(x**2 + 2)/8 + (x**6 + 13*x**4 + 8*x**2 - 2)/(4*x**8 + 12*x**6 + 8*x**4)$

$$3.80 \quad \int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} dx$$

Optimal. Leaf size=70

$$\frac{5x^7}{7} - \frac{27x^5}{5} + \frac{98x^3}{3} - \frac{(207x^2 + 206)x}{2(x^4 + 3x^2 + 2)} - 293x + \frac{9}{2} \tan^{-1}(x) + 340\sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Rubi [A] time = 0.08, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {1668, 1676, 1166, 203}

$$\frac{5x^7}{7} - \frac{27x^5}{5} + \frac{98x^3}{3} - \frac{(207x^2 + 206)x}{2(x^4 + 3x^2 + 2)} - 293x + \frac{9}{2} \tan^{-1}(x) + 340\sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

[In] Int[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]

[Out] -293*x + (98*x^3)/3 - (27*x^5)/5 + (5*x^7)/7 - (x*(206 + 207*x^2))/(2*(2 + 3*x^2 + x^4)) + (9*ArcTan[x])/2 + 340*sqrt[2]*ArcTan[x/sqrt[2]]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1668

Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]

Rule 1676

Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && Expon[Pq, x^2] > 1

Rubi steps

$$\begin{aligned}
\int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} dx &= -\frac{x(206+207x^2)}{2(2+3x^2+x^4)} - \frac{1}{4} \int \frac{-412-6x^2+212x^4-108x^6+48x^8-20x^{10}}{2+3x^2+x^4} dx \\
&= -\frac{x(206+207x^2)}{2(2+3x^2+x^4)} - \frac{1}{4} \int \left(1172 - 392x^2 + 108x^4 - 20x^6 - \frac{2(1378+1369x^2)}{2+3x^2+x^4} \right) dx \\
&= -293x + \frac{98x^3}{3} - \frac{27x^5}{5} + \frac{5x^7}{7} - \frac{x(206+207x^2)}{2(2+3x^2+x^4)} + \frac{1}{2} \int \frac{1378+1369x^2}{2+3x^2+x^4} dx \\
&= -293x + \frac{98x^3}{3} - \frac{27x^5}{5} + \frac{5x^7}{7} - \frac{x(206+207x^2)}{2(2+3x^2+x^4)} + \frac{9}{2} \int \frac{1}{1+x^2} dx + 680 \int \frac{1}{2+3x^2+x^4} dx \\
&= -293x + \frac{98x^3}{3} - \frac{27x^5}{5} + \frac{5x^7}{7} - \frac{x(206+207x^2)}{2(2+3x^2+x^4)} + \frac{9}{2} \tan^{-1}(x) + 340\sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 71, normalized size = 1.01

$$\frac{5x^7}{7} - \frac{27x^5}{5} + \frac{98x^3}{3} + \frac{-207x^3 - 206x}{2(x^4 + 3x^2 + 2)} - 293x + \frac{9}{2} \tan^{-1}(x) + 340\sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]

[Out] -293*x + (98*x^3)/3 - (27*x^5)/5 + (5*x^7)/7 + (-206*x - 207*x^3)/(2*(2 + 3*x^2 + x^4)) + (9*ArcTan[x])/2 + 340*sqrt[2]*ArcTan[x/sqrt[2]]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]

[Out] IntegrateAlgebraic[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2, x]

fricas [A] time = 1.34, size = 79, normalized size = 1.13

$$\frac{150x^{11} - 684x^9 + 3758x^7 - 43218x^5 - 192605x^3 + 71400\sqrt{2}(x^4 + 3x^2 + 2)\arctan\left(\frac{1}{2}\sqrt{2}x\right) + 945(x^4 + 3x^2 + 2)\arctan(x) - 144690x}{210(x^4 + 3x^2 + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")

[Out] 1/210*(150*x^11 - 684*x^9 + 3758*x^7 - 43218*x^5 - 192605*x^3 + 71400*sqrt(2)*(x^4 + 3*x^2 + 2)*arctan(1/2*sqrt(2)*x) + 945*(x^4 + 3*x^2 + 2)*arctan(x) - 144690*x)/(x^4 + 3*x^2 + 2)

giac [A] time = 0.33, size = 58, normalized size = 0.83

$$\frac{5}{7}x^7 - \frac{27}{5}x^5 + \frac{98}{3}x^3 + 340\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - 293x - \frac{207x^3 + 206x}{2(x^4 + 3x^2 + 2)} + \frac{9}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")

[Out] $\frac{5}{7}x^7 - \frac{27}{5}x^5 + \frac{98}{3}x^3 + 340\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - 293x - \frac{1}{2}\frac{(207x^3 + 206x)}{(x^4 + 3x^2 + 2)} + \frac{9}{2}\arctan(x)$

maple [A] time = 0.01, size = 56, normalized size = 0.80

$$\frac{5x^7}{7} - \frac{27x^5}{5} + \frac{98x^3}{3} - 293x + \frac{x}{2x^2 + 2} - \frac{104x}{x^2 + 2} + \frac{9\arctan(x)}{2} + 340\sqrt{2}\arctan\left(\frac{\sqrt{2}x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x)

[Out] $\frac{5}{7}x^7 - \frac{27}{5}x^5 + \frac{98}{3}x^3 - 293x + \frac{1}{2}x/(x^2+1) + \frac{9}{2}\arctan(x) - 104x/(x^2+2) + 340\arctan\left(\frac{1}{2}\sqrt{2}x\right)$

maxima [A] time = 1.64, size = 58, normalized size = 0.83

$$\frac{5}{7}x^7 - \frac{27}{5}x^5 + \frac{98}{3}x^3 + 340\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - 293x - \frac{207x^3 + 206x}{2(x^4 + 3x^2 + 2)} + \frac{9}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")

[Out] $\frac{5}{7}x^7 - \frac{27}{5}x^5 + \frac{98}{3}x^3 + 340\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - 293x - \frac{1}{2}\frac{(207x^3 + 206x)}{(x^4 + 3x^2 + 2)} + \frac{9}{2}\arctan(x)$

mupad [B] time = 0.95, size = 58, normalized size = 0.83

$$\frac{9\operatorname{atan}(x)}{2} - 293x + 340\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right) - \frac{\frac{207x^3}{2} + 103x}{x^4 + 3x^2 + 2} + \frac{98x^3}{3} - \frac{27x^5}{5} + \frac{5x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^8*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^2,x)

[Out] $\frac{9\operatorname{atan}(x)}{2} - 293x + 340\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right) - \frac{(103x + (207x^3)/2)}{(3x^2 + x^4 + 2)} + \frac{98x^3}{3} - \frac{27x^5}{5} + \frac{5x^7}{7}$

sympy [A] time = 0.21, size = 68, normalized size = 0.97

$$\frac{5x^7}{7} - \frac{27x^5}{5} + \frac{98x^3}{3} - 293x + \frac{-207x^3 - 206x}{2x^4 + 6x^2 + 4} + \frac{9\operatorname{atan}(x)}{2} + 340\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**8*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)

[Out] $\frac{5x^7}{7} - \frac{27x^5}{5} + \frac{98x^3}{3} - 293x + \frac{(-207x^3 - 206x)}{(2x^4 + 6x^2 + 4)} + \frac{9\operatorname{atan}(x)}{2} + 340\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)$

$$3.81 \quad \int \frac{x^6(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} dx$$

Optimal. Leaf size=57

$$x^5 - 9x^3 + \frac{(103x^2 + 102)x}{2(x^4 + 3x^2 + 2)} + 98x - \frac{11}{2} \tan^{-1}(x) - 118\sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Rubi [A] time = 0.08, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {1668, 1676, 1166, 203}

$$x^5 - 9x^3 + \frac{(103x^2 + 102)x}{2(x^4 + 3x^2 + 2)} + 98x - \frac{11}{2} \tan^{-1}(x) - 118\sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

[In] Int[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]

[Out] 98*x - 9*x^3 + x^5 + (x*(102 + 103*x^2))/(2*(2 + 3*x^2 + x^4)) - (11*ArcTan[x])/2 - 118*sqrt[2]*ArcTan[x/sqrt[2]]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1668

Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :=> With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p+1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p+1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p+1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p+1)*ExpandToSum[2*a*(p+1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p+3) - 2*a*c*d*(4*p+5) - a*b*e + c*(4*p+7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]

Rule 1676

Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :=> Int[ExpandIntegrand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && Expon[Pq, x^2] > 1

Rubi steps

$$\begin{aligned}
\int \frac{x^6(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} dx &= \frac{x(102+103x^2)}{2(2+3x^2+x^4)} - \frac{1}{4} \int \frac{204+6x^2-108x^4+48x^6-20x^8}{2+3x^2+x^4} dx \\
&= \frac{x(102+103x^2)}{2(2+3x^2+x^4)} - \frac{1}{4} \int \left(-392+108x^2-20x^4 + \frac{2(494+483x^2)}{2+3x^2+x^4} \right) dx \\
&= 98x - 9x^3 + x^5 + \frac{x(102+103x^2)}{2(2+3x^2+x^4)} - \frac{1}{2} \int \frac{494+483x^2}{2+3x^2+x^4} dx \\
&= 98x - 9x^3 + x^5 + \frac{x(102+103x^2)}{2(2+3x^2+x^4)} - \frac{11}{2} \int \frac{1}{1+x^2} dx - 236 \int \frac{1}{2+x^2} dx \\
&= 98x - 9x^3 + x^5 + \frac{x(102+103x^2)}{2(2+3x^2+x^4)} - \frac{11}{2} \tan^{-1}(x) - 118\sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.05, size = 58, normalized size = 1.02

$$x^5 - 9x^3 + \frac{103x^3 + 102x}{2(x^4 + 3x^2 + 2)} + 98x - \frac{11}{2} \tan^{-1}(x) - 118\sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]

[Out] 98*x - 9*x^3 + x^5 + (102*x + 103*x^3)/(2*(2 + 3*x^2 + x^4)) - (11*ArcTan[x])/2 - 118*sqrt[2]*ArcTan[x/Sqrt[2]]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]

[Out] IntegrateAlgebraic[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2, x]

fricas [A] time = 1.31, size = 74, normalized size = 1.30

$$\frac{2x^9 - 12x^7 + 146x^5 + 655x^3 - 236\sqrt{2}(x^4 + 3x^2 + 2)\arctan\left(\frac{1}{2}\sqrt{2}x\right) - 11(x^4 + 3x^2 + 2)\arctan(x) + 494x}{2(x^4 + 3x^2 + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")

[Out] 1/2*(2*x^9 - 12*x^7 + 146*x^5 + 655*x^3 - 236*sqrt(2)*(x^4 + 3*x^2 + 2)*arctan(1/2*sqrt(2)*x) - 11*(x^4 + 3*x^2 + 2)*arctan(x) + 494*x)/(x^4 + 3*x^2 + 2)

giac [A] time = 0.31, size = 51, normalized size = 0.89

$$x^5 - 9x^3 - 118\sqrt{2} \arctan\left(\frac{1}{2}\sqrt{2}x\right) + 98x + \frac{103x^3 + 102x}{2(x^4 + 3x^2 + 2)} - \frac{11}{2} \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")

[Out] $x^5 - 9x^3 - 118\sqrt{2}\arctan(1/2\sqrt{2}x) + 98x + 1/2(103x^3 + 102x)/(x^4 + 3x^2 + 2) - 11/2\arctan(x)$

maple [A] time = 0.01, size = 49, normalized size = 0.86

$$x^5 - 9x^3 + 98x - \frac{x}{2(x^2 + 1)} + \frac{52x}{x^2 + 2} - \frac{11 \arctan(x)}{2} - 118\sqrt{2} \arctan\left(\frac{\sqrt{2}x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x)

[Out] $x^5 - 9x^3 + 98x - 1/2/(x^2 + 1)x - 11/2\arctan(x) + 52/(x^2 + 2)x - 118*2^{(1/2)}\arctan(1/2*2^{(1/2)}x)$

maxima [A] time = 1.62, size = 51, normalized size = 0.89

$$x^5 - 9x^3 - 118\sqrt{2} \arctan\left(\frac{1}{2}\sqrt{2}x\right) + 98x + \frac{103x^3 + 102x}{2(x^4 + 3x^2 + 2)} - \frac{11}{2} \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")

[Out] $x^5 - 9x^3 - 118\sqrt{2}\arctan(1/2\sqrt{2}x) + 98x + 1/2(103x^3 + 102x)/(x^4 + 3x^2 + 2) - 11/2\arctan(x)$

mupad [B] time = 0.05, size = 50, normalized size = 0.88

$$98x - \frac{11 \operatorname{atan}(x)}{2} - 118\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right) + \frac{\frac{103x^3}{2} + 51x}{x^4 + 3x^2 + 2} - 9x^3 + x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^6*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^2,x)

[Out] $98x - (11*\operatorname{atan}(x))/2 - 118*2^{(1/2)}*\operatorname{atan}((2^{(1/2)}x)/2) + (51*x + (103*x^3)/2)/(3*x^2 + x^4 + 2) - 9*x^3 + x^5$

sympy [A] time = 0.21, size = 54, normalized size = 0.95

$$x^5 - 9x^3 + 98x + \frac{103x^3 + 102x}{2x^4 + 6x^2 + 4} - \frac{11 \operatorname{atan}(x)}{2} - 118\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**6*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)

[Out] $x**5 - 9*x**3 + 98*x + (103*x**3 + 102*x)/(2*x**4 + 6*x**2 + 4) - 11*\operatorname{atan}(x)/2 - 118*\sqrt{2}*\operatorname{atan}(\sqrt{2}*x/2)$

$$3.82 \quad \int \frac{x^4(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} dx$$

Optimal. Leaf size=56

$$\frac{5x^3}{3} - \frac{(51x^2 + 50)x}{2(x^4 + 3x^2 + 2)} - 27x + \frac{13}{2} \tan^{-1}(x) + 33\sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Rubi [A] time = 0.07, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {1668, 1676, 1166, 203}

$$\frac{5x^3}{3} - \frac{(51x^2 + 50)x}{2(x^4 + 3x^2 + 2)} - 27x + \frac{13}{2} \tan^{-1}(x) + 33\sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

[In] Int[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]

[Out] -27*x + (5*x^3)/3 - (x*(50 + 51*x^2))/(2*(2 + 3*x^2 + x^4)) + (13*ArcTan[x])/2 + 33*sqrt[2]*ArcTan[x/sqrt[2]]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1668

Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]

Rule 1676

Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := Int[ExpandIntegrand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && Expon[Pq, x^2] > 1

Rubi steps

$$\begin{aligned}
\int \frac{x^4(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} dx &= -\frac{x(50+51x^2)}{2(2+3x^2+x^4)} - \frac{1}{4} \int \frac{-100-6x^2+48x^4-20x^6}{2+3x^2+x^4} dx \\
&= -\frac{x(50+51x^2)}{2(2+3x^2+x^4)} - \frac{1}{4} \int \left(108-20x^2 - \frac{2(158+145x^2)}{2+3x^2+x^4}\right) dx \\
&= -27x + \frac{5x^3}{3} - \frac{x(50+51x^2)}{2(2+3x^2+x^4)} + \frac{1}{2} \int \frac{158+145x^2}{2+3x^2+x^4} dx \\
&= -27x + \frac{5x^3}{3} - \frac{x(50+51x^2)}{2(2+3x^2+x^4)} + \frac{13}{2} \int \frac{1}{1+x^2} dx + 66 \int \frac{1}{2+x^2} dx \\
&= -27x + \frac{5x^3}{3} - \frac{x(50+51x^2)}{2(2+3x^2+x^4)} + \frac{13}{2} \tan^{-1}(x) + 33\sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 57, normalized size = 1.02

$$\frac{5x^3}{3} + \frac{-51x^3 - 50x}{2(x^4 + 3x^2 + 2)} - 27x + \frac{13}{2} \tan^{-1}(x) + 33\sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]

[Out] -27*x + (5*x^3)/3 + (-50*x - 51*x^3)/(2*(2 + 3*x^2 + x^4)) + (13*ArcTan[x])/2 + 33*sqrt(2)*ArcTan[x/sqrt(2)]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]

[Out] IntegrateAlgebraic[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2, x]

fricas [A] time = 1.10, size = 69, normalized size = 1.23

$$\frac{10x^7 - 132x^5 - 619x^3 + 198\sqrt{2}(x^4 + 3x^2 + 2)\arctan\left(\frac{1}{2}\sqrt{2}x\right) + 39(x^4 + 3x^2 + 2)\arctan(x) - 474x}{6(x^4 + 3x^2 + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")

[Out] 1/6*(10*x^7 - 132*x^5 - 619*x^3 + 198*sqrt(2)*(x^4 + 3*x^2 + 2)*arctan(1/2*sqrt(2)*x) + 39*(x^4 + 3*x^2 + 2)*arctan(x) - 474*x)/(x^4 + 3*x^2 + 2)

giac [A] time = 0.31, size = 48, normalized size = 0.86

$$\frac{5}{3}x^3 + 33\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - 27x - \frac{51x^3 + 50x}{2(x^4 + 3x^2 + 2)} + \frac{13}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")

[Out] $\frac{5}{3}x^3 + 33\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - 27x - \frac{1}{2}(51x^3 + 50x)/(x^4 + 3x^2 + 2) + 13/2\arctan(x)$

maple [A] time = 0.01, size = 46, normalized size = 0.82

$$\frac{5x^3}{3} - 27x + \frac{x}{2x^2 + 2} - \frac{26x}{x^2 + 2} + \frac{13\arctan(x)}{2} + 33\sqrt{2}\arctan\left(\frac{\sqrt{2}x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x)

[Out] $\frac{5}{3}x^3 - 27x + 1/2/(x^2+1)*x + 13/2*\arctan(x) - 26/(x^2+2)*x + 33*2^{(1/2)}*\arctan(1/2*2^{(1/2)}*x)$

maxima [A] time = 1.64, size = 48, normalized size = 0.86

$$\frac{5}{3}x^3 + 33\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - 27x - \frac{51x^3 + 50x}{2(x^4 + 3x^2 + 2)} + \frac{13}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")

[Out] $\frac{5}{3}x^3 + 33\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - 27x - \frac{1}{2}(51x^3 + 50x)/(x^4 + 3x^2 + 2) + 13/2\arctan(x)$

mupad [B] time = 0.92, size = 48, normalized size = 0.86

$$\frac{13\operatorname{atan}(x)}{2} - 27x + 33\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right) - \frac{\frac{51x^3}{2} + 25x}{x^4 + 3x^2 + 2} + \frac{5x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^2,x)

[Out] $\frac{(13*\operatorname{atan}(x))/2 - 27*x + 33*2^{(1/2)}*\operatorname{atan}((2^{(1/2)}*x)/2) - (25*x + (51*x^3)/2)/(3*x^2 + x^4 + 2) + (5*x^3)/3}$

sympy [A] time = 0.21, size = 54, normalized size = 0.96

$$\frac{5x^3}{3} - 27x + \frac{-51x^3 - 50x}{2x^4 + 6x^2 + 4} + \frac{13\operatorname{atan}(x)}{2} + 33\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)

[Out] $5*x**3/3 - 27*x + (-51*x**3 - 50*x)/(2*x**4 + 6*x**2 + 4) + 13*\operatorname{atan}(x)/2 + 33*\sqrt{2}*\operatorname{atan}(\sqrt{2}*x/2)$

$$3.83 \quad \int \frac{x^2(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} dx$$

Optimal. Leaf size=49

$$\frac{(25x^2 + 24)x}{2(x^4 + 3x^2 + 2)} + 5x - \frac{15}{2} \tan^{-1}(x) - \frac{7 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Rubi [A] time = 0.07, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {1668, 1676, 1166, 203}

$$\frac{(25x^2 + 24)x}{2(x^4 + 3x^2 + 2)} + 5x - \frac{15}{2} \tan^{-1}(x) - \frac{7 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]

[Out] 5*x + (x*(24 + 25*x^2))/(2*(2 + 3*x^2 + x^4)) - (15*ArcTan[x])/2 - (7*ArcTan[x/Sqrt[2]])/Sqrt[2]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1668

Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]

Rule 1676

Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := Int[ExpandIntegrand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && Expon[Pq, x^2] > 1

Rubi steps

$$\begin{aligned}
\int \frac{x^2(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} dx &= \frac{x(24+25x^2)}{2(2+3x^2+x^4)} - \frac{1}{4} \int \frac{48-2x^2-20x^4}{2+3x^2+x^4} dx \\
&= \frac{x(24+25x^2)}{2(2+3x^2+x^4)} - \frac{1}{4} \int \left(-20 + \frac{2(44+29x^2)}{2+3x^2+x^4} \right) dx \\
&= 5x + \frac{x(24+25x^2)}{2(2+3x^2+x^4)} - \frac{1}{2} \int \frac{44+29x^2}{2+3x^2+x^4} dx \\
&= 5x + \frac{x(24+25x^2)}{2(2+3x^2+x^4)} - 7 \int \frac{1}{2+x^2} dx - \frac{15}{2} \int \frac{1}{1+x^2} dx \\
&= 5x + \frac{x(24+25x^2)}{2(2+3x^2+x^4)} - \frac{15}{2} \tan^{-1}(x) - \frac{7 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 50, normalized size = 1.02

$$\frac{25x^3 + 24x}{2(x^4 + 3x^2 + 2)} + 5x - \frac{15}{2} \tan^{-1}(x) - \frac{7 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]

[Out] 5*x + (24*x + 25*x^3)/(2*(2 + 3*x^2 + x^4)) - (15*ArcTan[x])/2 - (7*ArcTan[x/Sqrt[2]])/Sqrt[2]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]

[Out] IntegrateAlgebraic[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2, x]

fricas [A] time = 1.31, size = 64, normalized size = 1.31

$$\frac{10x^5 + 55x^3 - 7\sqrt{2}(x^4 + 3x^2 + 2) \arctan\left(\frac{1}{2}\sqrt{2}x\right) - 15(x^4 + 3x^2 + 2) \arctan(x) + 44x}{2(x^4 + 3x^2 + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")

[Out] 1/2*(10*x^5 + 55*x^3 - 7*sqrt(2)*(x^4 + 3*x^2 + 2)*arctan(1/2*sqrt(2)*x) - 15*(x^4 + 3*x^2 + 2)*arctan(x) + 44*x)/(x^4 + 3*x^2 + 2)

giac [A] time = 0.34, size = 43, normalized size = 0.88

$$-\frac{7}{2}\sqrt{2} \arctan\left(\frac{1}{2}\sqrt{2}x\right) + 5x + \frac{25x^3 + 24x}{2(x^4 + 3x^2 + 2)} - \frac{15}{2} \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")

[Out] -7/2*sqrt(2)*arctan(1/2*sqrt(2)*x) + 5*x + 1/2*(25*x^3 + 24*x)/(x^4 + 3*x^2 + 2) - 15/2*arctan(x)

maple [A] time = 0.01, size = 41, normalized size = 0.84

$$5x - \frac{x}{2(x^2 + 1)} + \frac{13x}{x^2 + 2} - \frac{15 \arctan(x)}{2} - \frac{7\sqrt{2} \arctan\left(\frac{\sqrt{2}x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x)

[Out] 5*x-1/2/(x^2+1)*x-15/2*arctan(x)+13/(x^2+2)*x-7/2*2^(1/2)*arctan(1/2*2^(1/2)*x)

maxima [A] time = 1.63, size = 43, normalized size = 0.88

$$-\frac{7}{2}\sqrt{2} \arctan\left(\frac{1}{2}\sqrt{2}x\right) + 5x + \frac{25x^3 + 24x}{2(x^4 + 3x^2 + 2)} - \frac{15}{2} \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")

[Out] -7/2*sqrt(2)*arctan(1/2*sqrt(2)*x) + 5*x + 1/2*(25*x^3 + 24*x)/(x^4 + 3*x^2 + 2) - 15/2*arctan(x)

mupad [B] time = 0.07, size = 42, normalized size = 0.86

$$5x - \frac{15 \operatorname{atan}(x)}{2} - \frac{7\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{2} + \frac{\frac{25x^3}{2} + 12x}{x^4 + 3x^2 + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^2,x)

[Out] 5*x - (15*atan(x))/2 - (7*2^(1/2)*atan((2^(1/2)*x)/2))/2 + (12*x + (25*x^3)/2)/(3*x^2 + x^4 + 2)

sympy [A] time = 0.21, size = 48, normalized size = 0.98

$$5x + \frac{25x^3 + 24x}{2x^4 + 6x^2 + 4} - \frac{15 \operatorname{atan}(x)}{2} - \frac{7\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)

[Out] 5*x + (25*x**3 + 24*x)/(2*x**4 + 6*x**2 + 4) - 15*atan(x)/2 - 7*sqrt(2)*atan(sqrt(2)*x/2)/2

$$3.84 \quad \int \frac{4+x^2+3x^4+5x^6}{(2+3x^2+x^4)^2} dx$$

Optimal. Leaf size=48

$$-\frac{x(12x^2+11)}{2(x^4+3x^2+2)} + \frac{17}{2} \tan^{-1}(x) - \frac{19 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{2\sqrt{2}}$$

Rubi [A] time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {1678, 1166, 203}

$$-\frac{x(12x^2+11)}{2(x^4+3x^2+2)} + \frac{17}{2} \tan^{-1}(x) - \frac{19 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(2 + 3*x^2 + x^4)^2, x]

[Out] -(x*(11 + 12*x^2))/(2*(2 + 3*x^2 + x^4)) + (17*ArcTan[x])/2 - (19*ArcTan[x/Sqrt[2]])/(2*Sqrt[2])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[Rt[b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1678

Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d = Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p+1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p+1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p+1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p+1)*ExpandToSum[2*a*(p+1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p+3) - 2*a*c*d*(4*p+5) - a*b*e + c*(4*p+7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]

Rubi steps

$$\begin{aligned} \int \frac{4 + x^2 + 3x^4 + 5x^6}{(2 + 3x^2 + x^4)^2} dx &= -\frac{x(11 + 12x^2)}{2(2 + 3x^2 + x^4)} - \frac{1}{4} \int \frac{-30 + 4x^2}{2 + 3x^2 + x^4} dx \\ &= -\frac{x(11 + 12x^2)}{2(2 + 3x^2 + x^4)} + \frac{17}{2} \int \frac{1}{1 + x^2} dx - \frac{19}{2} \int \frac{1}{2 + x^2} dx \\ &= -\frac{x(11 + 12x^2)}{2(2 + 3x^2 + x^4)} + \frac{17}{2} \tan^{-1}(x) - \frac{19 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{2\sqrt{2}} \end{aligned}$$

Mathematica [A] time = 0.04, size = 46, normalized size = 0.96

$$\frac{1}{4} \left(-\frac{2x(12x^2 + 11)}{x^4 + 3x^2 + 2} + 34 \tan^{-1}(x) - 19\sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(2 + 3*x^2 + x^4)^2,x]

[Out] ((-2*x*(11 + 12*x^2))/(2 + 3*x^2 + x^4) + 34*ArcTan[x] - 19*Sqrt[2]*ArcTan[x/Sqrt[2]])/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4 + x^2 + 3x^4 + 5x^6}{(2 + 3x^2 + x^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(4 + x^2 + 3*x^4 + 5*x^6)/(2 + 3*x^2 + x^4)^2,x]

[Out] IntegrateAlgebraic[(4 + x^2 + 3*x^4 + 5*x^6)/(2 + 3*x^2 + x^4)^2, x]

fricas [A] time = 1.52, size = 59, normalized size = 1.23

$$\frac{24x^3 + 19\sqrt{2}(x^4 + 3x^2 + 2) \arctan\left(\frac{1}{2}\sqrt{2}x\right) - 34(x^4 + 3x^2 + 2) \arctan(x) + 22x}{4(x^4 + 3x^2 + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")

[Out] -1/4*(24*x^3 + 19*sqrt(2)*(x^4 + 3*x^2 + 2)*arctan(1/2*sqrt(2)*x) - 34*(x^4 + 3*x^2 + 2)*arctan(x) + 22*x)/(x^4 + 3*x^2 + 2)

giac [A] time = 0.34, size = 40, normalized size = 0.83

$$-\frac{19}{4} \sqrt{2} \arctan\left(\frac{1}{2}\sqrt{2}x\right) - \frac{12x^3 + 11x}{2(x^4 + 3x^2 + 2)} + \frac{17}{2} \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")

[Out] -19/4*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/2*(12*x^3 + 11*x)/(x^4 + 3*x^2 + 2) + 17/2*arctan(x)

maple [A] time = 0.01, size = 38, normalized size = 0.79

$$\frac{x}{2x^2 + 2} - \frac{13x}{2(x^2 + 2)} + \frac{17 \arctan(x)}{2} - \frac{19\sqrt{2} \arctan\left(\frac{\sqrt{2}x}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x)

[Out] 1/2/(x^2+1)*x+17/2*arctan(x)-13/2/(x^2+2)*x-19/4*2^(1/2)*arctan(1/2*2^(1/2)*x)

maxima [A] time = 1.53, size = 40, normalized size = 0.83

$$-\frac{19}{4}\sqrt{2} \arctan\left(\frac{1}{2}\sqrt{2}x\right) - \frac{12x^3 + 11x}{2(x^4 + 3x^2 + 2)} + \frac{17}{2} \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")

[Out] -19/4*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/2*(12*x^3 + 11*x)/(x^4 + 3*x^2 + 2) + 17/2*arctan(x)

mupad [B] time = 0.07, size = 40, normalized size = 0.83

$$\frac{17 \operatorname{atan}(x)}{2} - \frac{19\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{4} - \frac{6x^3 + \frac{11x}{2}}{x^4 + 3x^2 + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(3*x^2 + x^4 + 2)^2,x)

[Out] (17*atan(x))/2 - (19*2^(1/2)*atan((2^(1/2)*x)/2))/4 - ((11*x)/2 + 6*x^3)/(3*x^2 + x^4 + 2)

sympy [A] time = 0.20, size = 46, normalized size = 0.96

$$\frac{-12x^3 - 11x}{2x^4 + 6x^2 + 4} + \frac{17 \operatorname{atan}(x)}{2} - \frac{19\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)

[Out] (-12*x**3 - 11*x)/(2*x**4 + 6*x**2 + 4) + 17*atan(x)/2 - 19*sqrt(2)*atan(sqrt(2)*x/2)/4

$$3.85 \quad \int \frac{4+x^2+3x^4+5x^6}{x^2(2+3x^2+x^4)^2} dx$$

Optimal. Leaf size=53

$$\frac{x(11x^2+9)}{4(x^4+3x^2+2)} - \frac{1}{x} - \frac{19}{2} \tan^{-1}(x) + \frac{45 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Rubi [A] time = 0.07, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {1669, 1664, 203}

$$\frac{x(11x^2+9)}{4(x^4+3x^2+2)} - \frac{1}{x} - \frac{19}{2} \tan^{-1}(x) + \frac{45 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(2 + 3*x^2 + x^4)^2), x]

[Out] -x^(-1) + (x*(9 + 11*x^2))/(4*(2 + 3*x^2 + x^4)) - (19*ArcTan[x])/2 + (45*ArcTan[x/Sqrt[2]])/(4*Sqrt[2])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 1664

Int[(Pq_)*((d_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]

Rule 1669

Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]

Rubi steps

$$\begin{aligned}
\int \frac{4 + x^2 + 3x^4 + 5x^6}{x^2(2 + 3x^2 + x^4)^2} dx &= \frac{x(9 + 11x^2)}{4(2 + 3x^2 + x^4)} - \frac{1}{4} \int \frac{-8 + 19x^2 - 11x^4}{x^2(2 + 3x^2 + x^4)} dx \\
&= \frac{x(9 + 11x^2)}{4(2 + 3x^2 + x^4)} - \frac{1}{4} \int \left(-\frac{4}{x^2} + \frac{38}{1 + x^2} - \frac{45}{2 + x^2} \right) dx \\
&= -\frac{1}{x} + \frac{x(9 + 11x^2)}{4(2 + 3x^2 + x^4)} - \frac{19}{2} \int \frac{1}{1 + x^2} dx + \frac{45}{4} \int \frac{1}{2 + x^2} dx \\
&= -\frac{1}{x} + \frac{x(9 + 11x^2)}{4(2 + 3x^2 + x^4)} - \frac{19}{2} \tan^{-1}(x) + \frac{45 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 51, normalized size = 0.96

$$\frac{1}{8} \left(\frac{2x(11x^2 + 9)}{x^4 + 3x^2 + 2} - \frac{8}{x} - 76 \tan^{-1}(x) + 45\sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(2 + 3*x^2 + x^4)^2), x]

[Out] (-8/x + (2*x*(9 + 11*x^2))/(2 + 3*x^2 + x^4) - 76*ArcTan[x] + 45*Sqrt[2]*ArcTan[x/Sqrt[2]])/8

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4 + x^2 + 3x^4 + 5x^6}{x^2(2 + 3x^2 + x^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(2 + 3*x^2 + x^4)^2), x]

[Out] IntegrateAlgebraic[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(2 + 3*x^2 + x^4)^2), x]

fricas [A] time = 1.16, size = 68, normalized size = 1.28

$$\frac{14x^4 + 45\sqrt{2}(x^5 + 3x^3 + 2x) \arctan\left(\frac{1}{2}\sqrt{2}x\right) - 6x^2 - 76(x^5 + 3x^3 + 2x) \arctan(x) - 16}{8(x^5 + 3x^3 + 2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+3*x^2+2)^2,x, algorithm="fricas")

[Out] 1/8*(14*x^4 + 45*sqrt(2)*(x^5 + 3*x^3 + 2*x)*arctan(1/2*sqrt(2)*x) - 6*x^2 - 76*(x^5 + 3*x^3 + 2*x)*arctan(x) - 16)/(x^5 + 3*x^3 + 2*x)

giac [A] time = 0.38, size = 45, normalized size = 0.85

$$\frac{45}{8} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} x\right) + \frac{7x^4 - 3x^2 - 8}{4(x^5 + 3x^3 + 2x)} - \frac{19}{2} \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+3*x^2+2)^2,x, algorithm="giac")

[Out] $45/8*\sqrt{2}*\arctan(1/2*\sqrt{2}*x) + 1/4*(7*x^4 - 3*x^2 - 8)/(x^5 + 3*x^3 + 2*x) - 19/2*\arctan(x)$

maple [A] time = 0.01, size = 43, normalized size = 0.81

$$-\frac{x}{2(x^2+1)} + \frac{13x}{4(x^2+2)} - \frac{19\arctan(x)}{2} + \frac{45\sqrt{2}\arctan\left(\frac{\sqrt{2}x}{2}\right)}{8} - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((5*x^6+3*x^4+x^2+4)/x^2/(x^4+3*x^2+2)^2,x)`

[Out] $-1/x - 1/2/(x^2+1)*x - 19/2*\arctan(x) + 13/4/(x^2+2)*x + 45/8*2^{(1/2)}*\arctan(1/2*2^{(1/2)}*x)$

maxima [A] time = 1.55, size = 45, normalized size = 0.85

$$\frac{45}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + \frac{7x^4 - 3x^2 - 8}{4(x^5 + 3x^3 + 2x)} - \frac{19}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+3*x^2+2)^2,x, algorithm="maxima")`

[Out] $45/8*\sqrt{2}*\arctan(1/2*\sqrt{2}*x) + 1/4*(7*x^4 - 3*x^2 - 8)/(x^5 + 3*x^3 + 2*x) - 19/2*\arctan(x)$

mupad [B] time = 0.07, size = 45, normalized size = 0.85

$$\frac{45\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{8} - \frac{19\operatorname{atan}(x)}{2} - \frac{-\frac{7x^4}{4} + \frac{3x^2}{4} + 2}{x^5 + 3x^3 + 2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^2*(3*x^2 + x^4 + 2)^2),x)`

[Out] $(45*2^{(1/2)}*\operatorname{atan}((2^{(1/2)}*x)/2))/8 - (19*\operatorname{atan}(x))/2 - ((3*x^2)/4 - (7*x^4)/(4 + 2))/(2*x + 3*x^3 + x^5)$

sympy [A] time = 0.22, size = 49, normalized size = 0.92

$$\frac{7x^4 - 3x^2 - 8}{4x^5 + 12x^3 + 8x} - \frac{19\operatorname{atan}(x)}{2} + \frac{45\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x**6+3*x**4+x**2+4)/x**2/(x**4+3*x**2+2)**2,x)`

[Out] $(7*x**4 - 3*x**2 - 8)/(4*x**5 + 12*x**3 + 8*x) - 19*\operatorname{atan}(x)/2 + 45*\sqrt{2}*\operatorname{atan}(\sqrt{2}*x/2)/8$

$$3.86 \quad \int \frac{4+x^2+3x^4+5x^6}{x^4(2+3x^2+x^4)^2} dx$$

Optimal. Leaf size=62

$$-\frac{1}{3x^3} - \frac{x(9x^2+5)}{8(x^4+3x^2+2)} + \frac{11}{4x} + \frac{21}{2} \tan^{-1}(x) - \frac{71 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{8\sqrt{2}}$$

Rubi [A] time = 0.08, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {1669, 1664, 203}

$$-\frac{x(9x^2+5)}{8(x^4+3x^2+2)} - \frac{1}{3x^3} + \frac{11}{4x} + \frac{21}{2} \tan^{-1}(x) - \frac{71 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(2 + 3*x^2 + x^4)^2), x]

[Out] -1/(3*x^3) + 11/(4*x) - (x*(5 + 9*x^2))/(8*(2 + 3*x^2 + x^4)) + (21*ArcTan[x])/2 - (71*ArcTan[x/Sqrt[2]])/(8*Sqrt[2])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[Rt[b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 1664

Int[(Pq_)*((d_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]

Rule 1669

Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p+1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p+1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p+1)*(b^2 - 4*a*c)), Int[x^m*(a + b*x^2 + c*x^4)^(p+1)*ExpandToSum[(2*a*(p+1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p+3) - 2*a*c*d*(4*p+5) - a*b*e)/x^m + c*(4*p+7)*(b*d - 2*a*e)*x^(2-m), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]

Rubi steps

$$\begin{aligned}
\int \frac{4 + x^2 + 3x^4 + 5x^6}{x^4(2 + 3x^2 + x^4)^2} dx &= -\frac{x(5 + 9x^2)}{8(2 + 3x^2 + x^4)} - \frac{1}{4} \int \frac{-8 + 10x^2 - \frac{39x^4}{2} + \frac{9x^6}{2}}{x^4(2 + 3x^2 + x^4)} dx \\
&= -\frac{x(5 + 9x^2)}{8(2 + 3x^2 + x^4)} - \frac{1}{4} \int \left(-\frac{4}{x^4} + \frac{11}{x^2} - \frac{42}{1 + x^2} + \frac{71}{2(2 + x^2)} \right) dx \\
&= -\frac{1}{3x^3} + \frac{11}{4x} - \frac{x(5 + 9x^2)}{8(2 + 3x^2 + x^4)} - \frac{71}{8} \int \frac{1}{2 + x^2} dx + \frac{21}{2} \int \frac{1}{1 + x^2} dx \\
&= -\frac{1}{3x^3} + \frac{11}{4x} - \frac{x(5 + 9x^2)}{8(2 + 3x^2 + x^4)} + \frac{21}{2} \tan^{-1}(x) - \frac{71 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{8\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 56, normalized size = 0.90

$$\frac{1}{48} \left(-\frac{16}{x^3} - \frac{6x(9x^2 + 5)}{x^4 + 3x^2 + 2} + \frac{132}{x} + 504 \tan^{-1}(x) - 213\sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(2 + 3*x^2 + x^4)^2), x]

[Out] (-16/x^3 + 132/x - (6*x*(5 + 9*x^2))/(2 + 3*x^2 + x^4) + 504*ArcTan[x] - 213*sqrt[2]*ArcTan[x/sqrt[2]])/48

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4 + x^2 + 3x^4 + 5x^6}{x^4(2 + 3x^2 + x^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(2 + 3*x^2 + x^4)^2), x]

[Out] IntegrateAlgebraic[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(2 + 3*x^2 + x^4)^2), x]

fricas [A] time = 0.61, size = 79, normalized size = 1.27

$$\frac{78x^6 + 350x^4 - 213\sqrt{2}(x^7 + 3x^5 + 2x^3) \arctan\left(\frac{1}{2}\sqrt{2}x\right) + 216x^2 + 504(x^7 + 3x^5 + 2x^3) \arctan(x) - 32}{48(x^7 + 3x^5 + 2x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+3*x^2+2)^2,x, algorithm="fricas")

[Out] 1/48*(78*x^6 + 350*x^4 - 213*sqrt(2)*(x^7 + 3*x^5 + 2*x^3)*arctan(1/2*sqrt(2)*x) + 216*x^2 + 504*(x^7 + 3*x^5 + 2*x^3)*arctan(x) - 32)/(x^7 + 3*x^5 + 2*x^3)

giac [A] time = 0.39, size = 52, normalized size = 0.84

$$-\frac{71}{16} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} x\right) - \frac{9x^3 + 5x}{8(x^4 + 3x^2 + 2)} + \frac{33x^2 - 4}{12x^3} + \frac{21}{2} \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+3*x^2+2)^2,x, algorithm="giac")

[Out] $-71/16*\sqrt{2}*\arctan(1/2*\sqrt{2}*x) - 1/8*(9*x^3 + 5*x)/(x^4 + 3*x^2 + 2) + 1/12*(33*x^2 - 4)/x^3 + 21/2*\arctan(x)$

maple [A] time = 0.02, size = 48, normalized size = 0.77

$$\frac{x}{2x^2+2} - \frac{13x}{8(x^2+2)} + \frac{21 \arctan(x)}{2} - \frac{71\sqrt{2} \arctan\left(\frac{\sqrt{2}x}{2}\right)}{16} + \frac{11}{4x} - \frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^6+3*x^4+x^2+4)/x^4/(x^4+3*x^2+2)^2,x)

[Out] $-1/3/x^3+11/4/x+1/2/(x^2+1)*x+21/2*\arctan(x)-13/8/(x^2+2)*x-71/16*2^{(1/2)}*\arctan(1/2*2^{(1/2)}*x)$

maxima [A] time = 1.60, size = 52, normalized size = 0.84

$$-\frac{71}{16}\sqrt{2} \arctan\left(\frac{1}{2}\sqrt{2}x\right) + \frac{39x^6 + 175x^4 + 108x^2 - 16}{24(x^7 + 3x^5 + 2x^3)} + \frac{21}{2} \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+3*x^2+2)^2,x, algorithm="maxima")

[Out] $-71/16*\sqrt{2}*\arctan(1/2*\sqrt{2}*x) + 1/24*(39*x^6 + 175*x^4 + 108*x^2 - 16)/(x^7 + 3*x^5 + 2*x^3) + 21/2*\arctan(x)$

mupad [B] time = 0.92, size = 51, normalized size = 0.82

$$\frac{21 \operatorname{atan}(x)}{2} - \frac{71\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{16} + \frac{\frac{13x^6}{8} + \frac{175x^4}{24} + \frac{9x^2}{2} - \frac{2}{3}}{x^7 + 3x^5 + 2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^4*(3*x^2 + x^4 + 2)^2),x)

[Out] $(21*\operatorname{atan}(x))/2 - (71*2^{(1/2)}*\operatorname{atan}(2^{(1/2)}*x/2))/16 + ((9*x^2)/2 + (175*x^4)/24 + (13*x^6)/8 - 2/3)/(2*x^3 + 3*x^5 + x^7)$

sympy [A] time = 0.24, size = 56, normalized size = 0.90

$$\frac{21 \operatorname{atan}(x)}{2} - \frac{71\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{16} + \frac{39x^6 + 175x^4 + 108x^2 - 16}{24x^7 + 72x^5 + 48x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**6+3*x**4+x**2+4)/x**4/(x**4+3*x**2+2)**2,x)

[Out] $21*\operatorname{atan}(x)/2 - 71*\sqrt{2}*\operatorname{atan}(\sqrt{2}*x/2)/16 + (39*x**6 + 175*x**4 + 108*x**2 - 16)/(24*x**7 + 72*x**5 + 48*x**3)$

$$3.87 \quad \int \frac{4+x^2+3x^4+5x^6}{x^6(2+3x^2+x^4)^2} dx$$

Optimal. Leaf size=69

$$-\frac{1}{5x^5} + \frac{11}{12x^3} - \frac{x(3-5x^2)}{16(x^4+3x^2+2)} - \frac{23}{4x} - \frac{23}{2} \tan^{-1}(x) + \frac{97 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{16\sqrt{2}}$$

Rubi [A] time = 0.09, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {1669, 1664, 203}

$$-\frac{x(3-5x^2)}{16(x^4+3x^2+2)} + \frac{11}{12x^3} - \frac{1}{5x^5} - \frac{23}{4x} - \frac{23}{2} \tan^{-1}(x) + \frac{97 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{16\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^6*(2 + 3*x^2 + x^4)^2), x]

[Out] -1/(5*x^5) + 11/(12*x^3) - 23/(4*x) - (x*(3 - 5*x^2))/(16*(2 + 3*x^2 + x^4)) - (23*ArcTan[x])/2 + (97*ArcTan[x/Sqrt[2]])/(16*Sqrt[2])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 1664

Int[(Pq_)*((d_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]

Rule 1669

Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]

Rubi steps

$$\begin{aligned}
\int \frac{4 + x^2 + 3x^4 + 5x^6}{x^6(2 + 3x^2 + x^4)^2} dx &= -\frac{x(3 - 5x^2)}{16(2 + 3x^2 + x^4)} - \frac{1}{4} \int \frac{-8 + 10x^2 - 17x^4 + \frac{39x^6}{4} - \frac{5x^8}{4}}{x^6(2 + 3x^2 + x^4)} dx \\
&= -\frac{x(3 - 5x^2)}{16(2 + 3x^2 + x^4)} - \frac{1}{4} \int \left(-\frac{4}{x^6} + \frac{11}{x^4} - \frac{23}{x^2} + \frac{46}{1 + x^2} - \frac{97}{4(2 + x^2)} \right) dx \\
&= -\frac{1}{5x^5} + \frac{11}{12x^3} - \frac{23}{4x} - \frac{x(3 - 5x^2)}{16(2 + 3x^2 + x^4)} + \frac{97}{16} \int \frac{1}{2 + x^2} dx - \frac{23}{2} \int \frac{1}{1 + x^2} dx \\
&= -\frac{1}{5x^5} + \frac{11}{12x^3} - \frac{23}{4x} - \frac{x(3 - 5x^2)}{16(2 + 3x^2 + x^4)} - \frac{23}{2} \tan^{-1}(x) + \frac{97 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{16\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 61, normalized size = 0.88

$$\frac{1}{480} \left(-\frac{96}{x^5} + \frac{440}{x^3} + \frac{30x(5x^2 - 3)}{x^4 + 3x^2 + 2} - \frac{2760}{x} - 5520 \tan^{-1}(x) + 1455\sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^6*(2 + 3*x^2 + x^4)^2), x]

[Out] (-96/x^5 + 440/x^3 - 2760/x + (30*x*(-3 + 5*x^2))/(2 + 3*x^2 + x^4) - 5520*ArcTan[x] + 1455*sqrt[2]*ArcTan[x/sqrt[2]])/480

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4 + x^2 + 3x^4 + 5x^6}{x^6(2 + 3x^2 + x^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(4 + x^2 + 3*x^4 + 5*x^6)/(x^6*(2 + 3*x^2 + x^4)^2), x]

[Out] IntegrateAlgebraic[(4 + x^2 + 3*x^4 + 5*x^6)/(x^6*(2 + 3*x^2 + x^4)^2), x]

fricas [A] time = 1.02, size = 84, normalized size = 1.22

$$\frac{2610x^8 + 7930x^6 + 4296x^4 - 1455\sqrt{2}(x^9 + 3x^7 + 2x^5) \arctan\left(\frac{1}{2}\sqrt{2}x\right) - 592x^2 + 5520(x^9 + 3x^7 + 2x^5) \arctan(x) + 192}{480(x^9 + 3x^7 + 2x^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^6+3*x^4+x^2+4)/x^6/(x^4+3*x^2+2)^2,x, algorithm="fricas")

[Out] -1/480*(2610*x^8 + 7930*x^6 + 4296*x^4 - 1455*sqrt(2)*(x^9 + 3*x^7 + 2*x^5)*arctan(1/2*sqrt(2)*x) - 592*x^2 + 5520*(x^9 + 3*x^7 + 2*x^5)*arctan(x) + 192)/(x^9 + 3*x^7 + 2*x^5)

giac [A] time = 0.34, size = 57, normalized size = 0.83

$$\frac{97}{32} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} x\right) + \frac{5x^3 - 3x}{16(x^4 + 3x^2 + 2)} - \frac{345x^4 - 55x^2 + 12}{60x^5} - \frac{23}{2} \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^6+3*x^4+x^2+4)/x^6/(x^4+3*x^2+2)^2,x, algorithm="giac")

[Out] $97/32\sqrt{2}\arctan(1/2\sqrt{2}x) + 1/16(5x^3 - 3x)/(x^4 + 3x^2 + 2) - 1/60(345x^4 - 55x^2 + 12)/x^5 - 23/2\arctan(x)$

maple [A] time = 0.02, size = 53, normalized size = 0.77

$$-\frac{x}{2(x^2+1)} + \frac{13x}{16(x^2+2)} - \frac{23\arctan(x)}{2} + \frac{97\sqrt{2}\arctan\left(\frac{\sqrt{2}x}{2}\right)}{32} - \frac{23}{4x} + \frac{11}{12x^3} - \frac{1}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((5*x^6+3*x^4+x^2+4)/x^6/(x^4+3*x^2+2)^2,x)`

[Out] $-1/5/x^5 + 11/12/x^3 - 23/4/x - 1/2/(x^2+1)*x - 23/2*\arctan(x) + 13/16/(x^2+2)*x + 97/32*2^{1/2}*\arctan(1/2*2^{1/2}*x)$

maxima [A] time = 1.76, size = 57, normalized size = 0.83

$$\frac{97}{32}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - \frac{1305x^8 + 3965x^6 + 2148x^4 - 296x^2 + 96}{240(x^9 + 3x^7 + 2x^5)} - \frac{23}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^6+3*x^4+x^2+4)/x^6/(x^4+3*x^2+2)^2,x, algorithm="maxima")`

[Out] $97/32\sqrt{2}\arctan(1/2\sqrt{2}x) - 1/240*(1305x^8 + 3965x^6 + 2148x^4 - 296x^2 + 96)/(x^9 + 3x^7 + 2x^5) - 23/2*\arctan(x)$

mupad [B] time = 0.92, size = 57, normalized size = 0.83

$$\frac{97\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{32} - \frac{23\operatorname{atan}(x)}{2} - \frac{\frac{87x^8}{16} + \frac{793x^6}{48} + \frac{179x^4}{20} - \frac{37x^2}{30} + \frac{2}{5}}{x^9 + 3x^7 + 2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^6*(3*x^2 + x^4 + 2)^2),x)`

[Out] $(97*2^{1/2}*\operatorname{atan}((2^{1/2}*x)/2))/32 - (23*\operatorname{atan}(x))/2 - ((179*x^4)/20 - (37*x^2)/30 + (793*x^6)/48 + (87*x^8)/16 + 2/5)/(2*x^5 + 3*x^7 + x^9)$

sympy [A] time = 0.25, size = 61, normalized size = 0.88

$$-\frac{23\operatorname{atan}(x)}{2} + \frac{97\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{32} + \frac{-1305x^8 - 3965x^6 - 2148x^4 + 296x^2 - 96}{240x^9 + 720x^7 + 480x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x**6+3*x**4+x**2+4)/x**6/(x**4+3*x**2+2)**2,x)`

[Out] $-23*\operatorname{atan}(x)/2 + 97*\sqrt{2}*\operatorname{atan}(\sqrt{2}*x/2)/32 + (-1305*x**8 - 3965*x**6 - 2148*x**4 + 296*x**2 - 96)/(240*x**9 + 720*x**7 + 480*x**5)$

$$3.88 \quad \int \frac{4+x^2+3x^4+5x^6}{x^8(2+3x^2+x^4)^2} dx$$

Optimal. Leaf size=76

$$-\frac{1}{7x^7} + \frac{11}{20x^5} - \frac{23}{12x^3} + \frac{x(3x^2+19)}{32(x^4+3x^2+2)} + \frac{137}{16x} + \frac{25}{2} \tan^{-1}(x) - \frac{123 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{32\sqrt{2}}$$

Rubi [A] time = 0.10, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {1669, 1664, 203}

$$\frac{x(3x^2+19)}{32(x^4+3x^2+2)} - \frac{23}{12x^3} + \frac{11}{20x^5} - \frac{1}{7x^7} + \frac{137}{16x} + \frac{25}{2} \tan^{-1}(x) - \frac{123 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{32\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^8*(2 + 3*x^2 + x^4)^2), x]

[Out] -1/(7*x^7) + 11/(20*x^5) - 23/(12*x^3) + 137/(16*x) + (x*(19 + 3*x^2))/(32*(2 + 3*x^2 + x^4)) + (25*ArcTan[x])/2 - (123*ArcTan[x/Sqrt[2]])/(32*Sqrt[2])

Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 1664

Int[(Pq_)*((d_)*(x_)^(m_))*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]

Rule 1669

Int[(Pq_)*(x_)^(m_))*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p+1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p+1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p+1)*(b^2 - 4*a*c)), Int[x^m*(a + b*x^2 + c*x^4)^(p+1)*ExpandToSum[(2*a*(p+1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p+3) - 2*a*c*d*(4*p+5) - a*b*e)/x^m + c*(4*p+7)*(b*d - 2*a*e)*x^(2-m), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]

Rubi steps

$$\begin{aligned}
\int \frac{4 + x^2 + 3x^4 + 5x^6}{x^8(2 + 3x^2 + x^4)^2} dx &= \frac{x(19 + 3x^2)}{32(2 + 3x^2 + x^4)} - \frac{1}{4} \int \frac{-8 + 10x^2 - 17x^4 + \frac{21x^6}{2} - \frac{39x^8}{8} - \frac{3x^{10}}{8}}{x^8(2 + 3x^2 + x^4)} dx \\
&= \frac{x(19 + 3x^2)}{32(2 + 3x^2 + x^4)} - \frac{1}{4} \int \left(-\frac{4}{x^8} + \frac{11}{x^6} - \frac{23}{x^4} + \frac{137}{4x^2} - \frac{50}{1 + x^2} + \frac{123}{8(2 + x^2)} \right) dx \\
&= -\frac{1}{7x^7} + \frac{11}{20x^5} - \frac{23}{12x^3} + \frac{137}{16x} + \frac{x(19 + 3x^2)}{32(2 + 3x^2 + x^4)} - \frac{123}{32} \int \frac{1}{2 + x^2} dx + \frac{25}{2} \int \frac{1}{1 + x^2} dx \\
&= -\frac{1}{7x^7} + \frac{11}{20x^5} - \frac{23}{12x^3} + \frac{137}{16x} + \frac{x(19 + 3x^2)}{32(2 + 3x^2 + x^4)} + \frac{25}{2} \tan^{-1}(x) - \frac{123 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{32\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 77, normalized size = 1.01

$$-\frac{1}{7x^7} + \frac{11}{20x^5} - \frac{23}{12x^3} + \frac{3x^3 + 19x}{32(x^4 + 3x^2 + 2)} + \frac{137}{16x} + \frac{25}{2} \tan^{-1}(x) - \frac{123 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{32\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^8*(2 + 3*x^2 + x^4)^2), x]

[Out] -1/7*1/x^7 + 11/(20*x^5) - 23/(12*x^3) + 137/(16*x) + (19*x + 3*x^3)/(32*(2 + 3*x^2 + x^4)) + (25*ArcTan[x])/2 - (123*ArcTan[x/Sqrt[2]])/(32*Sqrt[2])

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4 + x^2 + 3x^4 + 5x^6}{x^8(2 + 3x^2 + x^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(4 + x^2 + 3*x^4 + 5*x^6)/(x^8*(2 + 3*x^2 + x^4)^2), x]

[Out] IntegrateAlgebraic[(4 + x^2 + 3*x^4 + 5*x^6)/(x^8*(2 + 3*x^2 + x^4)^2), x]

fricas [A] time = 1.30, size = 89, normalized size = 1.17

$$\frac{58170x^{10} + 163730x^8 + 80136x^6 - 15632x^4 - 12915\sqrt{2}(x^{11} + 3x^9 + 2x^7) \arctan\left(\frac{1}{2}\sqrt{2}x\right) + 4512x^2 + 84000(x^{11} + 3x^9 + 2x^7) \arctan(x) - 1920}{6720(x^{11} + 3x^9 + 2x^7)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^6+3*x^4+x^2+4)/x^8/(x^4+3*x^2+2)^2,x, algorithm="fricas")

[Out] 1/6720*(58170*x^10 + 163730*x^8 + 80136*x^6 - 15632*x^4 - 12915*sqrt(2)*(x^11 + 3*x^9 + 2*x^7)*arctan(1/2*sqrt(2)*x) + 4512*x^2 + 84000*(x^11 + 3*x^9 + 2*x^7)*arctan(x) - 1920)/(x^11 + 3*x^9 + 2*x^7)

giac [A] time = 0.45, size = 62, normalized size = 0.82

$$-\frac{123}{64} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} x\right) + \frac{3x^3 + 19x}{32(x^4 + 3x^2 + 2)} + \frac{14385x^6 - 3220x^4 + 924x^2 - 240}{1680x^7} + \frac{25}{2} \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^6+3*x^4+x^2+4)/x^8/(x^4+3*x^2+2)^2,x, algorithm="giac")

[Out] $-123/64*\sqrt{2}*\arctan(1/2*\sqrt{2}*x) + 1/32*(3*x^3 + 19*x)/(x^4 + 3*x^2 + 2) + 1/1680*(14385*x^6 - 3220*x^4 + 924*x^2 - 240)/x^7 + 25/2*\arctan(x)$

maple [A] time = 0.02, size = 58, normalized size = 0.76

$$\frac{x}{2x^2 + 2} - \frac{13x}{32(x^2 + 2)} + \frac{25 \arctan(x)}{2} - \frac{123\sqrt{2} \arctan\left(\frac{\sqrt{2}x}{2}\right)}{64} + \frac{137}{16x} - \frac{23}{12x^3} + \frac{11}{20x^5} - \frac{1}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((5*x^6+3*x^4+x^2+4)/x^8/(x^4+3*x^2+2)^2,x)`

[Out] $-1/7/x^7+11/20/x^5-23/12/x^3+137/16/x+1/2/(x^2+1)*x+25/2*\arctan(x)-13/32/(x^2+2)*x-123/64*2^{(1/2)}*\arctan(1/2*2^{(1/2)}*x)$

maxima [A] time = 1.53, size = 62, normalized size = 0.82

$$-\frac{123}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + \frac{29085x^{10} + 81865x^8 + 40068x^6 - 7816x^4 + 2256x^2 - 960}{3360(x^{11} + 3x^9 + 2x^7)} + \frac{25}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^6+3*x^4+x^2+4)/x^8/(x^4+3*x^2+2)^2,x, algorithm="maxima")`

[Out] $-123/64*\sqrt{2}*\arctan(1/2*\sqrt{2}*x) + 1/3360*(29085*x^{10} + 81865*x^8 + 40068*x^6 - 7816*x^4 + 2256*x^2 - 960)/(x^{11} + 3*x^9 + 2*x^7) + 25/2*\arctan(x)$

mupad [B] time = 0.07, size = 61, normalized size = 0.80

$$\frac{25 \operatorname{atan}(x)}{2} - \frac{123 \sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{64} + \frac{\frac{277x^{10}}{32} + \frac{2339x^8}{96} + \frac{477x^6}{40} - \frac{977x^4}{420} + \frac{47x^2}{70} - \frac{2}{7}}{x^{11} + 3x^9 + 2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^8*(3*x^2 + x^4 + 2)^2),x)`

[Out] $(25*\operatorname{atan}(x))/2 - (123*2^{(1/2)}*\operatorname{atan}((2^{(1/2)}*x)/2))/64 + ((47*x^2)/70 - (977*x^4)/420 + (477*x^6)/40 + (2339*x^8)/96 + (277*x^{10})/32 - 2/7)/(2*x^7 + 3*x^9 + x^{11})$

sympy [A] time = 0.28, size = 66, normalized size = 0.87

$$\frac{25 \operatorname{atan}(x)}{2} - \frac{123\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{64} + \frac{29085x^{10} + 81865x^8 + 40068x^6 - 7816x^4 + 2256x^2 - 960}{3360x^{11} + 10080x^9 + 6720x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x**6+3*x**4+x**2+4)/x**8/(x**4+3*x**2+2)**2,x)`

[Out] $25*\operatorname{atan}(x)/2 - 123*\sqrt{2}*\operatorname{atan}(\sqrt{2}*x/2)/64 + (29085*x^{10} + 81865*x^8 + 40068*x^6 - 7816*x^4 + 2256*x^2 - 960)/(3360*x^{11} + 10080*x^9 + 6720*x^7)$

$$3.89 \quad \int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3} dx$$

Optimal. Leaf size=81

$$x^5 - 14x^3 + \frac{(1669x^2 + 824)x}{8(x^4 + 3x^2 + 2)} + \frac{(415x^2 + 414)x}{4(x^4 + 3x^2 + 2)^2} + 214x + \frac{477}{8} \tan^{-1}(x) - 351\sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Rubi [A] time = 0.11, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {1668, 1678, 1676, 1166, 203}

$$x^5 - 14x^3 + \frac{(1669x^2 + 824)x}{8(x^4 + 3x^2 + 2)} + \frac{(415x^2 + 414)x}{4(x^4 + 3x^2 + 2)^2} + 214x + \frac{477}{8} \tan^{-1}(x) - 351\sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

[In] Int[(x^10*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]

[Out] 214*x - 14*x^3 + x^5 + (x*(414 + 415*x^2))/(4*(2 + 3*x^2 + x^4)^2) + (x*(824 + 1669*x^2))/(8*(2 + 3*x^2 + x^4)) + (477*ArcTan[x])/8 - 351*sqrt[2]*ArcTan[x/sqrt[2]]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1668

Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]

Rule 1676

Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := Int[ExpandIntegrand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && Expon[Pq, x^2] > 1

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := With[{d =
  Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{x^{10} (4 + x^2 + 3x^4 + 5x^6)}{(2 + 3x^2 + x^4)^3} dx &= \frac{x(414 + 415x^2)}{4(2 + 3x^2 + x^4)^2} - \frac{1}{8} \int \frac{828 - 2478x^2 - 840x^4 + 424x^6 - 216x^8 + 96x^{10} - 40x^{12}}{(2 + 3x^2 + x^4)^2} dx \\ &= \frac{x(414 + 415x^2)}{4(2 + 3x^2 + x^4)^2} + \frac{x(824 + 1669x^2)}{8(2 + 3x^2 + x^4)} + \frac{1}{32} \int \frac{-4952 - 2700x^2 + 3136x^4 - 864x^6 + 576x^8 - 40x^{10}}{2 + 3x^2 + x^4} dx \\ &= \frac{x(414 + 415x^2)}{4(2 + 3x^2 + x^4)^2} + \frac{x(824 + 1669x^2)}{8(2 + 3x^2 + x^4)} + \frac{1}{32} \int \left(6848 - 1344x^2 + 160x^4 - \frac{36(5x^6 - 4x^8 + 3x^{10} - 2x^{12})}{2 + 3x^2 + x^4} \right) dx \\ &= 214x - 14x^3 + x^5 + \frac{x(414 + 415x^2)}{4(2 + 3x^2 + x^4)^2} + \frac{x(824 + 1669x^2)}{8(2 + 3x^2 + x^4)} - \frac{9}{8} \int \frac{518 + 571x^2}{2 + 3x^2 + x^4} dx \\ &= 214x - 14x^3 + x^5 + \frac{x(414 + 415x^2)}{4(2 + 3x^2 + x^4)^2} + \frac{x(824 + 1669x^2)}{8(2 + 3x^2 + x^4)} + \frac{477}{8} \int \frac{1}{1 + x^2} dx \\ &= 214x - 14x^3 + x^5 + \frac{x(414 + 415x^2)}{4(2 + 3x^2 + x^4)^2} + \frac{x(824 + 1669x^2)}{8(2 + 3x^2 + x^4)} + \frac{477}{8} \tan^{-1}(x) - 351\sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) \end{aligned}$$

Mathematica [A] time = 0.06, size = 71, normalized size = 0.88

$$\frac{x(8x^{12} - 64x^{10} + 1144x^8 + 10581x^6 + 26775x^4 + 26736x^2 + 9324)}{8(x^4 + 3x^2 + 2)^2} + \frac{477}{8} \tan^{-1}(x) - 351\sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^10*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
```

```
[Out] (x*(9324 + 26736*x^2 + 26775*x^4 + 10581*x^6 + 1144*x^8 - 64*x^10 + 8*x^12)
)/(8*(2 + 3*x^2 + x^4)^2) + (477*ArcTan[x])/8 - 351*Sqrt[2]*ArcTan[x/Sqrt[2]]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10} (4 + x^2 + 3x^4 + 5x^6)}{(2 + 3x^2 + x^4)^3} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^10*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[(x^10*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3, x]
```

fricas [A] time = 1.23, size = 114, normalized size = 1.41

$$\frac{8x^{13} - 64x^{11} + 1144x^9 + 10581x^7 + 26775x^5 + 26736x^3 - 2808\sqrt{2}(x^8 + 6x^6 + 13x^4 + 12x^2 + 4)\arctan\left(\frac{1}{2}\sqrt{2}x\right) + 477(x^8 + 6x^6 + 13x^4 + 12x^2 + 4)\arctan(x) + 9324x}{8(x^8 + 6x^6 + 13x^4 + 12x^2 + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^10*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="fricas")

[Out] 1/8*(8*x^13 - 64*x^11 + 1144*x^9 + 10581*x^7 + 26775*x^5 + 26736*x^3 - 2808*sqrt(2)*(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)*arctan(1/2*sqrt(2)*x) + 477*(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)*arctan(x) + 9324*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)

giac [A] time = 0.32, size = 61, normalized size = 0.75

$$x^5 - 14x^3 - 351\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + 214x + \frac{1669x^7 + 5831x^5 + 6640x^3 + 2476x}{8(x^4 + 3x^2 + 2)^2} + \frac{477}{8}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^10*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="giac")

[Out] x^5 - 14*x^3 - 351*sqrt(2)*arctan(1/2*sqrt(2)*x) + 214*x + 1/8*(1669*x^7 + 5831*x^5 + 6640*x^3 + 2476*x)/(x^4 + 3*x^2 + 2)^2 + 477/8*arctan(x)

maple [A] time = 0.01, size = 64, normalized size = 0.79

$$x^5 - 14x^3 + 214x + \frac{477\arctan(x)}{8} - 351\sqrt{2}\arctan\left(\frac{\sqrt{2}x}{2}\right) + \frac{-\frac{11}{8}x^3 - \frac{13}{8}x}{(x^2 + 1)^2} - \frac{16\left(-\frac{105}{8}x^3 - \frac{79}{4}x\right)}{(x^2 + 2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^10*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x)

[Out] x^5-14*x^3+214*x+(-11/8*x^3-13/8*x)/(x^2+1)^2+477/8*arctan(x)-16*(-105/8*x^3-79/4*x)/(x^2+2)^2-351*2^(1/2)*arctan(1/2*2^(1/2)*x)

maxima [A] time = 1.58, size = 71, normalized size = 0.88

$$x^5 - 14x^3 - 351\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + 214x + \frac{1669x^7 + 5831x^5 + 6640x^3 + 2476x}{8(x^8 + 6x^6 + 13x^4 + 12x^2 + 4)} + \frac{477}{8}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^10*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="maxima")

[Out] x^5 - 14*x^3 - 351*sqrt(2)*arctan(1/2*sqrt(2)*x) + 214*x + 1/8*(1669*x^7 + 5831*x^5 + 6640*x^3 + 2476*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4) + 477/8*arctan(x)

mupad [B] time = 0.06, size = 70, normalized size = 0.86

$$214x + \frac{477\operatorname{atan}(x)}{8} - 351\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right) + \frac{\frac{1669x^7}{8} + \frac{5831x^5}{8} + 830x^3 + \frac{619x}{2}}{x^8 + 6x^6 + 13x^4 + 12x^2 + 4} - 14x^3 + x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^10*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^3,x)

[Out] $214x + (477 \operatorname{atan}(x))/8 - 351 \cdot 2^{(1/2)} \operatorname{atan}((2^{(1/2)}x)/2) + ((619x)/2 + 830x^3 + (5831x^5)/8 + (1669x^7)/8)/(12x^2 + 13x^4 + 6x^6 + x^8 + 4) - 14x^3 + x^5$

sympy [A] time = 0.26, size = 75, normalized size = 0.93

$$x^5 - 14x^3 + 214x + \frac{1669x^7 + 5831x^5 + 6640x^3 + 2476x}{8x^8 + 48x^6 + 104x^4 + 96x^2 + 32} + \frac{477 \operatorname{atan}(x)}{8} - 351\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**10*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**3,x)`

[Out] $x^5 - 14x^3 + 214x + (1669x^7 + 5831x^5 + 6640x^3 + 2476x)/(8x^8 + 48x^6 + 104x^4 + 96x^2 + 32) + 477 \operatorname{atan}(x)/8 - 351 \operatorname{sqrt}(2) \operatorname{atan}(\operatorname{sqrt}(2)x/2)$

$$3.90 \quad \int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3} dx$$

Optimal. Leaf size=80

$$\frac{5x^3}{3} + \frac{(24 - 409x^2)x}{8(x^4 + 3x^2 + 2)} - \frac{(207x^2 + 206)x}{4(x^4 + 3x^2 + 2)^2} - 42x - \frac{449}{8} \tan^{-1}(x) + \frac{219 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Rubi [A] time = 0.10, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {1668, 1678, 1676, 1166, 203}

$$\frac{5x^3}{3} + \frac{(24 - 409x^2)x}{8(x^4 + 3x^2 + 2)} - \frac{(207x^2 + 206)x}{4(x^4 + 3x^2 + 2)^2} - 42x - \frac{449}{8} \tan^{-1}(x) + \frac{219 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]

[Out] -42*x + (5*x^3)/3 - (x*(206 + 207*x^2))/(4*(2 + 3*x^2 + x^4)^2) + (x*(24 - 409*x^2))/(8*(2 + 3*x^2 + x^4)) - (449*ArcTan[x])/8 + (219*ArcTan[x/Sqrt[2]])/Sqrt[2]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1668

Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]

Rule 1676

Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := Int[ExpandIntegrand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && Expon[Pq, x^2] > 1

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := With[{d =
  Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{x^8 (4 + x^2 + 3x^4 + 5x^6)}{(2 + 3x^2 + x^4)^3} dx &= -\frac{x(206 + 207x^2)}{4(2 + 3x^2 + x^4)^2} - \frac{1}{8} \int \frac{-412 + 1230x^2 + 424x^4 - 216x^6 + 96x^8 - 40x^{10}}{(2 + 3x^2 + x^4)^2} dx \\ &= -\frac{x(206 + 207x^2)}{4(2 + 3x^2 + x^4)^2} + \frac{x(24 - 409x^2)}{8(2 + 3x^2 + x^4)} + \frac{1}{32} \int \frac{728 + 1500x^2 - 864x^4 + 160x^6}{2 + 3x^2 + x^4} dx \\ &= -\frac{x(206 + 207x^2)}{4(2 + 3x^2 + x^4)^2} + \frac{x(24 - 409x^2)}{8(2 + 3x^2 + x^4)} + \frac{1}{32} \int \left(-1344 + 160x^2 + \frac{4(854 + 1303x^2)}{2 + 3x^2 + x^4} \right) dx \\ &= -42x + \frac{5x^3}{3} - \frac{x(206 + 207x^2)}{4(2 + 3x^2 + x^4)^2} + \frac{x(24 - 409x^2)}{8(2 + 3x^2 + x^4)} + \frac{1}{8} \int \frac{854 + 1303x^2}{2 + 3x^2 + x^4} dx \\ &= -42x + \frac{5x^3}{3} - \frac{x(206 + 207x^2)}{4(2 + 3x^2 + x^4)^2} + \frac{x(24 - 409x^2)}{8(2 + 3x^2 + x^4)} - \frac{449}{8} \int \frac{1}{1 + x^2} dx + 219 \int \frac{1}{\sqrt{2 + 3x^2 + x^4}} dx \\ &= -42x + \frac{5x^3}{3} - \frac{x(206 + 207x^2)}{4(2 + 3x^2 + x^4)^2} + \frac{x(24 - 409x^2)}{8(2 + 3x^2 + x^4)} - \frac{449}{8} \tan^{-1}(x) + \frac{219 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}} \end{aligned}$$

Mathematica [A] time = 0.05, size = 66, normalized size = 0.82

$$\frac{x(40x^{10} - 768x^8 - 6755x^6 - 16233x^4 - 15416x^2 - 5124)}{24(x^4 + 3x^2 + 2)^2} - \frac{449}{8} \tan^{-1}(x) + \frac{219 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]

[Out] (x*(-5124 - 15416*x^2 - 16233*x^4 - 6755*x^6 - 768*x^8 + 40*x^10))/(24*(2 + 3*x^2 + x^4)^2) - (449*ArcTan[x])/8 + (219*ArcTan[x/Sqrt[2]])/Sqrt[2]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8 (4 + x^2 + 3x^4 + 5x^6)}{(2 + 3x^2 + x^4)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]

[Out] IntegrateAlgebraic[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3, x]

fricas [A] time = 1.32, size = 109, normalized size = 1.36

$$\frac{40x^{11} - 768x^9 - 6755x^7 - 16233x^5 - 15416x^3 + 2628\sqrt{2}(x^8 + 6x^6 + 13x^4 + 12x^2 + 4)\arctan\left(\frac{1}{2}\sqrt{2}x\right) - 1347(x^8 + 6x^6 + 13x^4 + 12x^2 + 4)\arctan(x) - 5124x}{24(x^8 + 6x^6 + 13x^4 + 12x^2 + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="fricas")

[Out] 1/24*(40*x^11 - 768*x^9 - 6755*x^7 - 16233*x^5 - 15416*x^3 + 2628*sqrt(2)*(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)*arctan(1/2*sqrt(2)*x) - 1347*(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)*arctan(x) - 5124*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)

giac [A] time = 0.41, size = 58, normalized size = 0.72

$$\frac{5}{3}x^3 + \frac{219}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - 42x - \frac{409x^7 + 1203x^5 + 1160x^3 + 364x}{8(x^4 + 3x^2 + 2)^2} - \frac{449}{8}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="giac")

[Out] 5/3*x^3 + 219/2*sqrt(2)*arctan(1/2*sqrt(2)*x) - 42*x - 1/8*(409*x^7 + 1203*x^5 + 1160*x^3 + 364*x)/(x^4 + 3*x^2 + 2)^2 - 449/8*arctan(x)

maple [A] time = 0.01, size = 62, normalized size = 0.78

$$\frac{5x^3}{3} - 42x - \frac{449\arctan(x)}{8} + \frac{219\sqrt{2}\arctan\left(\frac{\sqrt{2}x}{2}\right)}{2} - \frac{-\frac{15}{8}x^3 - \frac{17}{8}x}{(x^2 + 1)^2} + \frac{-53x^3 - 54x}{(x^2 + 2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x)

[Out] 5/3*x^3-42*x-(-15/8*x^3-17/8*x)/(x^2+1)^2-449/8*arctan(x)+16*(-53/16*x^3-27/8*x)/(x^2+2)^2+219/2*2^(1/2)*arctan(1/2*2^(1/2)*x)

maxima [A] time = 1.91, size = 68, normalized size = 0.85

$$\frac{5}{3}x^3 + \frac{219}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - 42x - \frac{409x^7 + 1203x^5 + 1160x^3 + 364x}{8(x^8 + 6x^6 + 13x^4 + 12x^2 + 4)} - \frac{449}{8}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="maxima")

[Out] 5/3*x^3 + 219/2*sqrt(2)*arctan(1/2*sqrt(2)*x) - 42*x - 1/8*(409*x^7 + 1203*x^5 + 1160*x^3 + 364*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4) - 449/8*arctan(x)

mupad [B] time = 0.05, size = 68, normalized size = 0.85

$$\frac{219\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{2} - \frac{449\operatorname{atan}(x)}{8} - 42x - \frac{\frac{409x^7}{8} + \frac{1203x^5}{8} + 145x^3 + \frac{91x}{2}}{x^8 + 6x^6 + 13x^4 + 12x^2 + 4} + \frac{5x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^8*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^3,x)

[Out] $(219 \cdot 2^{1/2} \cdot \operatorname{atan}((2^{1/2} \cdot x)/2))/2 - (449 \cdot \operatorname{atan}(x))/8 - 42x - ((91x)/2 + 145x^3 + (1203x^5)/8 + (409x^7)/8)/(12x^2 + 13x^4 + 6x^6 + x^8 + 4) + (5x^3)/3$

sympy [A] time = 0.26, size = 76, normalized size = 0.95

$$\frac{5x^3}{3} - 42x + \frac{-409x^7 - 1203x^5 - 1160x^3 - 364x}{8x^8 + 48x^6 + 104x^4 + 96x^2 + 32} - \frac{449 \operatorname{atan}(x)}{8} + \frac{219\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**8*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**3,x)`

[Out] $5x^3/3 - 42x + (-409x^7 - 1203x^5 - 1160x^3 - 364x)/(8x^8 + 48x^6 + 104x^4 + 96x^2 + 32) - 449 \cdot \operatorname{atan}(x)/8 + 219 \cdot \sqrt{2} \cdot \operatorname{atan}(\sqrt{2}x/2)/2$

$$3.91 \quad \int \frac{x^6(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3} dx$$

Optimal. Leaf size=75

$$-\frac{(15x^2 + 244)x}{8(x^4 + 3x^2 + 2)} + \frac{(103x^2 + 102)x}{4(x^4 + 3x^2 + 2)^2} + 5x + \frac{413}{8} \tan^{-1}(x) - \frac{191 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{2\sqrt{2}}$$

Rubi [A] time = 0.09, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {1668, 1678, 1676, 1166, 203}

$$-\frac{(15x^2 + 244)x}{8(x^4 + 3x^2 + 2)} + \frac{(103x^2 + 102)x}{4(x^4 + 3x^2 + 2)^2} + 5x + \frac{413}{8} \tan^{-1}(x) - \frac{191 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]

[Out] 5*x + (x*(102 + 103*x^2))/(4*(2 + 3*x^2 + x^4)^2) - (x*(244 + 15*x^2))/(8*(2 + 3*x^2 + x^4)) + (413*ArcTan[x])/8 - (191*ArcTan[x/Sqrt[2]])/(2*Sqrt[2])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1668

Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]

Rule 1676

Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := Int[ExpandIntegrand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && Expon[Pq, x^2] > 1

Rule 1678

```
Int[(Pq_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := With[{d =
  Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
 \int \frac{x^6 (4 + x^2 + 3x^4 + 5x^6)}{(2 + 3x^2 + x^4)^3} dx &= \frac{x(102 + 103x^2)}{4(2 + 3x^2 + x^4)^2} - \frac{1}{8} \int \frac{204 - 606x^2 - 216x^4 + 96x^6 - 40x^8}{(2 + 3x^2 + x^4)^2} dx \\
 &= \frac{x(102 + 103x^2)}{4(2 + 3x^2 + x^4)^2} - \frac{x(244 + 15x^2)}{8(2 + 3x^2 + x^4)} + \frac{1}{32} \int \frac{568 - 924x^2 + 160x^4}{2 + 3x^2 + x^4} dx \\
 &= \frac{x(102 + 103x^2)}{4(2 + 3x^2 + x^4)^2} - \frac{x(244 + 15x^2)}{8(2 + 3x^2 + x^4)} + \frac{1}{32} \int \left(160 + \frac{4(62 - 351x^2)}{2 + 3x^2 + x^4} \right) dx \\
 &= 5x + \frac{x(102 + 103x^2)}{4(2 + 3x^2 + x^4)^2} - \frac{x(244 + 15x^2)}{8(2 + 3x^2 + x^4)} + \frac{1}{8} \int \frac{62 - 351x^2}{2 + 3x^2 + x^4} dx \\
 &= 5x + \frac{x(102 + 103x^2)}{4(2 + 3x^2 + x^4)^2} - \frac{x(244 + 15x^2)}{8(2 + 3x^2 + x^4)} + \frac{413}{8} \int \frac{1}{1 + x^2} dx - \frac{191}{2} \int \frac{1}{2 + x^2} dx \\
 &= 5x + \frac{x(102 + 103x^2)}{4(2 + 3x^2 + x^4)^2} - \frac{x(244 + 15x^2)}{8(2 + 3x^2 + x^4)} + \frac{413}{8} \tan^{-1}(x) - \frac{191 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{2\sqrt{2}}
 \end{aligned}$$

Mathematica [A] time = 0.06, size = 60, normalized size = 0.80

$$\frac{1}{8} \left(\frac{x(40x^8 + 225x^6 + 231x^4 - 76x^2 - 124)}{(x^4 + 3x^2 + 2)^2} + 413 \tan^{-1}(x) - 382\sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]

[Out] ((x*(-124 - 76*x^2 + 231*x^4 + 225*x^6 + 40*x^8))/(2 + 3*x^2 + x^4)^2 + 413*ArcTan[x] - 382*Sqrt[2]*ArcTan[x/Sqrt[2]])/8

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6 (4 + x^2 + 3x^4 + 5x^6)}{(2 + 3x^2 + x^4)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]

[Out] IntegrateAlgebraic[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3, x]

fricas [A] time = 1.19, size = 104, normalized size = 1.39

$$\frac{40x^9 + 225x^7 + 231x^5 - 76x^3 - 382\sqrt{2}(x^8 + 6x^6 + 13x^4 + 12x^2 + 4)\arctan\left(\frac{1}{2}\sqrt{2}x\right) + 413(x^8 + 6x^6 + 13x^4 + 12x^2 + 4)\arctan(x) - 124x}{8(x^8 + 6x^6 + 13x^4 + 12x^2 + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="fricas")

[Out] 1/8*(40*x^9 + 225*x^7 + 231*x^5 - 76*x^3 - 382*sqrt(2)*(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)*arctan(1/2*sqrt(2)*x) + 413*(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)*arctan(x) - 124*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)

giac [A] time = 0.32, size = 53, normalized size = 0.71

$$-\frac{191}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + 5x - \frac{15x^7 + 289x^5 + 556x^3 + 284x}{8(x^4 + 3x^2 + 2)^2} + \frac{413}{8}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="giac")

[Out] -191/4*sqrt(2)*arctan(1/2*sqrt(2)*x) + 5*x - 1/8*(15*x^7 + 289*x^5 + 556*x^3 + 284*x)/(x^4 + 3*x^2 + 2)^2 + 413/8*arctan(x)

maple [A] time = 0.01, size = 56, normalized size = 0.75

$$5x + \frac{413\arctan(x)}{8} - \frac{191\sqrt{2}\arctan\left(\frac{\sqrt{2}x}{2}\right)}{4} + \frac{-\frac{19}{8}x^3 - \frac{21}{8}x}{(x^2 + 1)^2} - \frac{16\left(-\frac{1}{32}x^3 + \frac{25}{16}x\right)}{(x^2 + 2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x)

[Out] 5*x+(-19/8*x^3-21/8*x)/(x^2+1)^2+413/8*arctan(x)-16*(-1/32*x^3+25/16*x)/(x^2+2)^2-191/4*2^(1/2)*arctan(1/2*2^(1/2)*x)

maxima [A] time = 1.85, size = 63, normalized size = 0.84

$$-\frac{191}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + 5x - \frac{15x^7 + 289x^5 + 556x^3 + 284x}{8(x^8 + 6x^6 + 13x^4 + 12x^2 + 4)} + \frac{413}{8}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="maxima")

[Out] -191/4*sqrt(2)*arctan(1/2*sqrt(2)*x) + 5*x - 1/8*(15*x^7 + 289*x^5 + 556*x^3 + 284*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4) + 413/8*arctan(x)

mupad [B] time = 0.93, size = 63, normalized size = 0.84

$$5x + \frac{413\operatorname{atan}(x)}{8} - \frac{191\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{4} - \frac{\frac{15x^7}{8} + \frac{289x^5}{8} + \frac{139x^3}{2} + \frac{71x}{2}}{x^8 + 6x^6 + 13x^4 + 12x^2 + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^6*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^3,x)

[Out] 5*x + (413*atan(x))/8 - (191*2^(1/2)*atan((2^(1/2)*x)/2))/4 - ((71*x)/2 + (139*x^3)/2 + (289*x^5)/8 + (15*x^7)/8)/(12*x^2 + 13*x^4 + 6*x^6 + x^8 + 4)

sympy [A] time = 0.26, size = 70, normalized size = 0.93

$$5x + \frac{-15x^7 - 289x^5 - 556x^3 - 284x}{8x^8 + 48x^6 + 104x^4 + 96x^2 + 32} + \frac{413 \operatorname{atan}(x)}{8} - \frac{191\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**6*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**3,x)

[Out] 5*x + (-15*x**7 - 289*x**5 - 556*x**3 - 284*x)/(8*x**8 + 48*x**6 + 104*x**4 + 96*x**2 + 32) + 413*atan(x)/8 - 191*sqrt(2)*atan(sqrt(2)*x/2)/4

$$3.92 \quad \int \frac{x^4(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3} dx$$

Optimal. Leaf size=72

$$-\frac{x(51x^2+50)}{4(x^4+3x^2+2)^2} + \frac{x(125x^2+254)}{8(x^4+3x^2+2)} - \frac{369}{8} \tan^{-1}(x) + \frac{267 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Rubi [A] time = 0.07, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {1668, 1678, 1166, 203}

$$-\frac{x(51x^2+50)}{4(x^4+3x^2+2)^2} + \frac{x(125x^2+254)}{8(x^4+3x^2+2)} - \frac{369}{8} \tan^{-1}(x) + \frac{267 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]

[Out] -(x*(50 + 51*x^2))/(4*(2 + 3*x^2 + x^4)^2) + (x*(254 + 125*x^2))/(8*(2 + 3*x^2 + x^4)) - (369*ArcTan[x])/8 + (267*ArcTan[x/Sqrt[2]])/(4*Sqrt[2])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1668

Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :=> With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]

Rule 1678

Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :=> With[{d = Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a

+ b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]

Rubi steps

$$\begin{aligned} \int \frac{x^4(4 + x^2 + 3x^4 + 5x^6)}{(2 + 3x^2 + x^4)^3} dx &= -\frac{x(50 + 51x^2)}{4(2 + 3x^2 + x^4)^2} - \frac{1}{8} \int \frac{-100 + 294x^2 + 96x^4 - 40x^6}{(2 + 3x^2 + x^4)^2} dx \\ &= -\frac{x(50 + 51x^2)}{4(2 + 3x^2 + x^4)^2} + \frac{x(254 + 125x^2)}{8(2 + 3x^2 + x^4)} + \frac{1}{32} \int \frac{-816 + 660x^2}{2 + 3x^2 + x^4} dx \\ &= -\frac{x(50 + 51x^2)}{4(2 + 3x^2 + x^4)^2} + \frac{x(254 + 125x^2)}{8(2 + 3x^2 + x^4)} - \frac{369}{8} \int \frac{1}{1 + x^2} dx + \frac{267}{4} \int \frac{1}{2 + x^2} dx \\ &= -\frac{x(50 + 51x^2)}{4(2 + 3x^2 + x^4)^2} + \frac{x(254 + 125x^2)}{8(2 + 3x^2 + x^4)} - \frac{369}{8} \tan^{-1}(x) + \frac{267 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}} \end{aligned}$$

Mathematica [A] time = 0.06, size = 55, normalized size = 0.76

$$\frac{1}{8} \left(\frac{x(125x^6 + 629x^4 + 910x^2 + 408)}{(x^4 + 3x^2 + 2)^2} - 369 \tan^{-1}(x) + 267\sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]

[Out] ((x*(408 + 910*x^2 + 629*x^4 + 125*x^6))/(2 + 3*x^2 + x^4)^2 - 369*ArcTan[x] + 267*Sqrt[2]*ArcTan[x/Sqrt[2]])/8

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4(4 + x^2 + 3x^4 + 5x^6)}{(2 + 3x^2 + x^4)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]

[Out] IntegrateAlgebraic[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3, x]

fricas [A] time = 0.69, size = 99, normalized size = 1.38

$$\frac{125x^7 + 629x^5 + 910x^3 + 267\sqrt{2}(x^8 + 6x^6 + 13x^4 + 12x^2 + 4)\arctan\left(\frac{1}{2}\sqrt{2}x\right) - 369(x^8 + 6x^6 + 13x^4 + 12x^2 + 4)\arctan(x) + 408x}{8(x^8 + 6x^6 + 13x^4 + 12x^2 + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="fricas")

[Out] 1/8*(125*x^7 + 629*x^5 + 910*x^3 + 267*sqrt(2)*(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)*arctan(1/2*sqrt(2)*x) - 369*(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)*arc tan(x) + 408*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)

giac [A] time = 0.34, size = 50, normalized size = 0.69

$$\frac{267}{8} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} x\right) + \frac{125 x^7 + 629 x^5 + 910 x^3 + 408 x}{8 (x^4 + 3 x^2 + 2)^2} - \frac{369}{8} \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="giac")

[Out] 267/8*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/8*(125*x^7 + 629*x^5 + 910*x^3 + 408*x)/(x^4 + 3*x^2 + 2)^2 - 369/8*arctan(x)

maple [A] time = 0.01, size = 54, normalized size = 0.75

$$-\frac{369 \arctan(x)}{8} + \frac{267 \sqrt{2} \arctan\left(\frac{\sqrt{2} x}{2}\right)}{8} - \frac{-\frac{23}{8} x^3 - \frac{25}{8} x}{(x^2 + 1)^2} + \frac{\frac{51}{4} x^3 + \frac{77}{2} x}{(x^2 + 2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x)

[Out] -(-23/8*x^3-25/8*x)/(x^2+1)^2-369/8*arctan(x)+2*(51/8*x^3+77/4*x)/(x^2+2)^2+267/8*2^(1/2)*arctan(1/2*2^(1/2)*x)

maxima [A] time = 1.53, size = 60, normalized size = 0.83

$$\frac{267}{8} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} x\right) + \frac{125 x^7 + 629 x^5 + 910 x^3 + 408 x}{8 (x^8 + 6 x^6 + 13 x^4 + 12 x^2 + 4)} - \frac{369}{8} \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="maxima")

[Out] 267/8*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/8*(125*x^7 + 629*x^5 + 910*x^3 + 408*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4) - 369/8*arctan(x)

mupad [B] time = 0.93, size = 59, normalized size = 0.82

$$\frac{267 \sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2} x}{2}\right)}{8} - \frac{369 \operatorname{atan}(x)}{8} + \frac{\frac{125 x^7}{8} + \frac{629 x^5}{8} + \frac{455 x^3}{4} + 51 x}{x^8 + 6 x^6 + 13 x^4 + 12 x^2 + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^3,x)

[Out] (267*2^(1/2)*atan((2^(1/2)*x)/2))/8 - (369*atan(x))/8 + (51*x + (455*x^3)/4 + (629*x^5)/8 + (125*x^7)/8)/(12*x^2 + 13*x^4 + 6*x^6 + x^8 + 4)

sympy [A] time = 0.26, size = 65, normalized size = 0.90

$$\frac{125 x^7 + 629 x^5 + 910 x^3 + 408 x}{8 x^8 + 48 x^6 + 104 x^4 + 96 x^2 + 32} - \frac{369 \operatorname{atan}(x)}{8} + \frac{267 \sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2} x}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**3,x)

[Out] (125*x**7 + 629*x**5 + 910*x**3 + 408*x)/(8*x**8 + 48*x**6 + 104*x**4 + 96*x**2 + 32) - 369*atan(x)/8 + 267*sqrt(2)*atan(sqrt(2)*x/2)/8

$$3.93 \quad \int \frac{x^2(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3} dx$$

Optimal. Leaf size=72

$$\frac{x(25x^2 + 24)}{4(x^4 + 3x^2 + 2)^2} - \frac{x(130x^2 + 211)}{8(x^4 + 3x^2 + 2)} + \frac{317}{8} \tan^{-1}(x) - \frac{447 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{8\sqrt{2}}$$

Rubi [A] time = 0.07, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {1668, 1678, 1166, 203}

$$\frac{x(25x^2 + 24)}{4(x^4 + 3x^2 + 2)^2} - \frac{x(130x^2 + 211)}{8(x^4 + 3x^2 + 2)} + \frac{317}{8} \tan^{-1}(x) - \frac{447 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]

[Out] (x*(24 + 25*x^2))/(4*(2 + 3*x^2 + x^4)^2) - (x*(211 + 130*x^2))/(8*(2 + 3*x^2 + x^4)) + (317*ArcTan[x])/8 - (447*ArcTan[x/Sqrt[2]])/(8*Sqrt[2])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1668

Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]

Rule 1678

Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := With[{d = Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a

+ b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]

Rubi steps

$$\begin{aligned} \int \frac{x^2(4 + x^2 + 3x^4 + 5x^6)}{(2 + 3x^2 + x^4)^3} dx &= \frac{x(24 + 25x^2)}{4(2 + 3x^2 + x^4)^2} - \frac{1}{8} \int \frac{48 - 154x^2 - 40x^4}{(2 + 3x^2 + x^4)^2} dx \\ &= \frac{x(24 + 25x^2)}{4(2 + 3x^2 + x^4)^2} - \frac{x(211 + 130x^2)}{8(2 + 3x^2 + x^4)} + \frac{1}{32} \int \frac{748 - 520x^2}{2 + 3x^2 + x^4} dx \\ &= \frac{x(24 + 25x^2)}{4(2 + 3x^2 + x^4)^2} - \frac{x(211 + 130x^2)}{8(2 + 3x^2 + x^4)} + \frac{317}{8} \int \frac{1}{1 + x^2} dx - \frac{447}{8} \int \frac{1}{2 + x^2} dx \\ &= \frac{x(24 + 25x^2)}{4(2 + 3x^2 + x^4)^2} - \frac{x(211 + 130x^2)}{8(2 + 3x^2 + x^4)} + \frac{317}{8} \tan^{-1}(x) - \frac{447 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{8\sqrt{2}} \end{aligned}$$

Mathematica [A] time = 0.06, size = 56, normalized size = 0.78

$$\frac{1}{16} \left(-\frac{2x(130x^6 + 601x^4 + 843x^2 + 374)}{(x^4 + 3x^2 + 2)^2} + 634 \tan^{-1}(x) - 447\sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]

[Out] ((-2*x*(374 + 843*x^2 + 601*x^4 + 130*x^6))/(2 + 3*x^2 + x^4)^2 + 634*ArcTan[x] - 447*sqrt[2]*ArcTan[x/sqrt[2]])/16

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(4 + x^2 + 3x^4 + 5x^6)}{(2 + 3x^2 + x^4)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]

[Out] IntegrateAlgebraic[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3, x]

fricas [A] time = 1.40, size = 99, normalized size = 1.38

$$\frac{260x^7 + 1202x^5 + 1686x^3 + 447\sqrt{2}(x^8 + 6x^6 + 13x^4 + 12x^2 + 4)\arctan\left(\frac{1}{2}\sqrt{2}x\right) - 634(x^8 + 6x^6 + 13x^4 + 12x^2 + 4)\arctan(x) + 748x}{16(x^8 + 6x^6 + 13x^4 + 12x^2 + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="fricas")

[Out] -1/16*(260*x^7 + 1202*x^5 + 1686*x^3 + 447*sqrt(2)*(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)*arctan(1/2*sqrt(2)*x) - 634*(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)*arctan(x) + 748*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)

giac [A] time = 0.38, size = 50, normalized size = 0.69

$$-\frac{447}{16} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} x\right) - \frac{130 x^7 + 601 x^5 + 843 x^3 + 374 x}{8(x^4 + 3x^2 + 2)^2} + \frac{317}{8} \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="giac")

[Out] -447/16*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/8*(130*x^7 + 601*x^5 + 843*x^3 + 374*x)/(x^4 + 3*x^2 + 2)^2 + 317/8*arctan(x)

maple [A] time = 0.01, size = 53, normalized size = 0.74

$$\frac{317 \arctan(x)}{8} - \frac{447 \sqrt{2} \arctan\left(\frac{\sqrt{2} x}{2}\right)}{16} + \frac{-\frac{27}{8} x^3 - \frac{29}{8} x}{(x^2 + 1)^2} - \frac{\frac{103}{8} x^3 + \frac{129}{4} x}{(x^2 + 2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x)

[Out] (-27/8*x^3-29/8*x)/(x^2+1)^2+317/8*arctan(x)-(103/8*x^3+129/4*x)/(x^2+2)^2-447/16*2^(1/2)*arctan(1/2*2^(1/2)*x)

maxima [A] time = 1.74, size = 60, normalized size = 0.83

$$-\frac{447}{16} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} x\right) - \frac{130 x^7 + 601 x^5 + 843 x^3 + 374 x}{8(x^8 + 6x^6 + 13x^4 + 12x^2 + 4)} + \frac{317}{8} \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="maxima")

[Out] -447/16*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/8*(130*x^7 + 601*x^5 + 843*x^3 + 374*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4) + 317/8*arctan(x)

mupad [B] time = 0.07, size = 60, normalized size = 0.83

$$\frac{317 \operatorname{atan}(x)}{8} - \frac{447 \sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2} x}{2}\right)}{16} - \frac{\frac{65 x^7}{4} + \frac{601 x^5}{8} + \frac{843 x^3}{8} + \frac{187 x}{4}}{x^8 + 6 x^6 + 13 x^4 + 12 x^2 + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^3,x)

[Out] (317*atan(x))/8 - (447*2^(1/2)*atan((2^(1/2)*x)/2))/16 - ((187*x)/4 + (843*x^3)/8 + (601*x^5)/8 + (65*x^7)/4)/(12*x^2 + 13*x^4 + 6*x^6 + x^8 + 4)

sympy [A] time = 0.25, size = 66, normalized size = 0.92

$$\frac{-130x^7 - 601x^5 - 843x^3 - 374x}{8x^8 + 48x^6 + 104x^4 + 96x^2 + 32} + \frac{317 \operatorname{atan}(x)}{8} - \frac{447 \sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2} x}{2}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**3,x)

[Out] (-130*x**7 - 601*x**5 - 843*x**3 - 374*x)/(8*x**8 + 48*x**6 + 104*x**4 + 96*x**2 + 32) + 317*atan(x)/8 - 447*sqrt(2)*atan(sqrt(2)*x/2)/16

$$3.94 \quad \int \frac{4+x^2+3x^4+5x^6}{(2+3x^2+x^4)^3} dx$$

Optimal. Leaf size=72

$$-\frac{x(12x^2+11)}{4(x^4+3x^2+2)^2} + \frac{x(217x^2+335)}{16(x^4+3x^2+2)} - \frac{257}{8} \tan^{-1}(x) + \frac{731 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{16\sqrt{2}}$$

Rubi [A] time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1678, 1178, 1166, 203}

$$-\frac{x(12x^2+11)}{4(x^4+3x^2+2)^2} + \frac{x(217x^2+335)}{16(x^4+3x^2+2)} - \frac{257}{8} \tan^{-1}(x) + \frac{731 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{16\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(2 + 3*x^2 + x^4)^3, x]

[Out] -(x*(11 + 12*x^2))/(4*(2 + 3*x^2 + x^4)^2) + (x*(335 + 217*x^2))/(16*(2 + 3*x^2 + x^4)) - (257*ArcTan[x])/8 + (731*ArcTan[x/Sqrt[2]])/(16*Sqrt[2])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1178

Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2*p]

Rule 1678

Int[(Pq)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d = Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]

Rubi steps

$$\begin{aligned}
\int \frac{4 + x^2 + 3x^4 + 5x^6}{(2 + 3x^2 + x^4)^3} dx &= -\frac{x(11 + 12x^2)}{4(2 + 3x^2 + x^4)^2} - \frac{1}{8} \int \frac{-38 + 80x^2}{(2 + 3x^2 + x^4)^2} dx \\
&= -\frac{x(11 + 12x^2)}{4(2 + 3x^2 + x^4)^2} + \frac{x(335 + 217x^2)}{16(2 + 3x^2 + x^4)} + \frac{1}{32} \int \frac{-594 + 434x^2}{2 + 3x^2 + x^4} dx \\
&= -\frac{x(11 + 12x^2)}{4(2 + 3x^2 + x^4)^2} + \frac{x(335 + 217x^2)}{16(2 + 3x^2 + x^4)} - \frac{257}{8} \int \frac{1}{1 + x^2} dx + \frac{731}{16} \int \frac{1}{2 + x^2} dx \\
&= -\frac{x(11 + 12x^2)}{4(2 + 3x^2 + x^4)^2} + \frac{x(335 + 217x^2)}{16(2 + 3x^2 + x^4)} - \frac{257}{8} \tan^{-1}(x) + \frac{731 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{16\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 56, normalized size = 0.78

$$\frac{1}{32} \left(\frac{2x(217x^6 + 986x^4 + 1391x^2 + 626)}{(x^4 + 3x^2 + 2)^2} - 1028 \tan^{-1}(x) + 731\sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(2 + 3*x^2 + x^4)^3, x]

[Out] ((2*x*(626 + 1391*x^2 + 986*x^4 + 217*x^6))/(2 + 3*x^2 + x^4)^2 - 1028*ArcTan[x] + 731*Sqrt[2]*ArcTan[x/Sqrt[2]])/32

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4 + x^2 + 3x^4 + 5x^6}{(2 + 3x^2 + x^4)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(4 + x^2 + 3*x^4 + 5*x^6)/(2 + 3*x^2 + x^4)^3, x]

[Out] IntegrateAlgebraic[(4 + x^2 + 3*x^4 + 5*x^6)/(2 + 3*x^2 + x^4)^3, x]

fricas [A] time = 1.27, size = 99, normalized size = 1.38

$$\frac{434x^7 + 1972x^5 + 2782x^3 + 731\sqrt{2}(x^8 + 6x^6 + 13x^4 + 12x^2 + 4)\arctan\left(\frac{1}{2}\sqrt{2}x\right) - 1028(x^8 + 6x^6 + 13x^4 + 12x^2 + 4)\arctan(x) + 1252x}{32(x^8 + 6x^6 + 13x^4 + 12x^2 + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="fricas")

[Out] 1/32*(434*x^7 + 1972*x^5 + 2782*x^3 + 731*sqrt(2)*(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)*arctan(1/2*sqrt(2)*x) - 1028*(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)*arctan(x) + 1252*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)

giac [A] time = 0.33, size = 50, normalized size = 0.69

$$\frac{731}{32} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} x\right) + \frac{217x^7 + 986x^5 + 1391x^3 + 626x}{16(x^4 + 3x^2 + 2)^2} - \frac{257}{8} \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="giac")

[Out] 731/32*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/16*(217*x^7 + 986*x^5 + 1391*x^3 + 626*x)/(x^4 + 3*x^2 + 2)^2 - 257/8*arctan(x)

maple [A] time = 0.01, size = 53, normalized size = 0.74

$$-\frac{257 \arctan(x)}{8} + \frac{731\sqrt{2} \arctan\left(\frac{\sqrt{2}x}{2}\right)}{32} - \frac{-\frac{31}{8}x^3 - \frac{33}{8}x}{(x^2+1)^2} + \frac{\frac{155}{16}x^3 + \frac{181}{8}x}{(x^2+2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x)

[Out] -(-31/8*x^3-33/8*x)/(x^2+1)^2-257/8*arctan(x)+(155/16*x^3+181/8*x)/(x^2+2)^2+731/32*2^(1/2)*arctan(1/2*2^(1/2)*x)

maxima [A] time = 1.67, size = 60, normalized size = 0.83

$$\frac{731}{32} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} x\right) + \frac{217 x^7 + 986 x^5 + 1391 x^3 + 626 x}{16 (x^8 + 6 x^6 + 13 x^4 + 12 x^2 + 4)} - \frac{257}{8} \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="maxima")

[Out] 731/32*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/16*(217*x^7 + 986*x^5 + 1391*x^3 + 626*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4) - 257/8*arctan(x)

mupad [B] time = 0.07, size = 59, normalized size = 0.82

$$\frac{731 \sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{32} - \frac{257 \operatorname{atan}(x)}{8} + \frac{\frac{217x^7}{16} + \frac{493x^5}{8} + \frac{1391x^3}{16} + \frac{313x}{8}}{x^8 + 6x^6 + 13x^4 + 12x^2 + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(3*x^2 + x^4 + 2)^3,x)

[Out] (731*2^(1/2)*atan((2^(1/2)*x)/2))/32 - (257*atan(x))/8 + ((313*x)/8 + (1391*x^3)/16 + (493*x^5)/8 + (217*x^7)/16)/(12*x^2 + 13*x^4 + 6*x^6 + x^8 + 4)

sympy [A] time = 0.25, size = 65, normalized size = 0.90

$$\frac{217x^7 + 986x^5 + 1391x^3 + 626x}{16x^8 + 96x^6 + 208x^4 + 192x^2 + 64} - \frac{257 \operatorname{atan}(x)}{8} + \frac{731\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**3,x)

[Out] (217*x**7 + 986*x**5 + 1391*x**3 + 626*x)/(16*x**8 + 96*x**6 + 208*x**4 + 192*x**2 + 64) - 257*atan(x)/8 + 731*sqrt(2)*atan(sqrt(2)*x/2)/32

$$3.95 \quad \int \frac{4+x^2+3x^4+5x^6}{x^2(2+3x^2+x^4)^3} dx$$

Optimal. Leaf size=79

$$\frac{x(11x^2+9)}{8(x^4+3x^2+2)^2} - \frac{x(347x^2+547)}{32(x^4+3x^2+2)} - \frac{1}{2x} + \frac{189}{8} \tan^{-1}(x) - \frac{1119 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{32\sqrt{2}}$$

Rubi [A] time = 0.10, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {1669, 1664, 203}

$$\frac{x(11x^2+9)}{8(x^4+3x^2+2)^2} - \frac{x(347x^2+547)}{32(x^4+3x^2+2)} - \frac{1}{2x} + \frac{189}{8} \tan^{-1}(x) - \frac{1119 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{32\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(2 + 3*x^2 + x^4)^3), x]

[Out] -1/(2*x) + (x*(9 + 11*x^2))/(8*(2 + 3*x^2 + x^4)^2) - (x*(547 + 347*x^2))/(32*(2 + 3*x^2 + x^4)) + (189*ArcTan[x])/8 - (1119*ArcTan[x/Sqrt[2]])/(32*sqrt[2])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[Rt[b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 1664

Int[(Pq_)*((d_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]

Rule 1669

Int[(Pq_)*(x_)^(m_.))*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p+1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p+1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p+1)*(b^2 - 4*a*c)), Int[x^m*(a + b*x^2 + c*x^4)^(p+1)*ExpandToSum[(2*a*(p+1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p+3) - 2*a*c*d*(4*p+5) - a*b*e)/x^m + c*(4*p+7)*(b*d - 2*a*e)*x^(2-m), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]

Rubi steps

$$\begin{aligned}
\int \frac{4 + x^2 + 3x^4 + 5x^6}{x^2(2 + 3x^2 + x^4)^3} dx &= \frac{x(9 + 11x^2)}{8(2 + 3x^2 + x^4)^2} - \frac{1}{8} \int \frac{-16 + 29x^2 - 55x^4}{x^2(2 + 3x^2 + x^4)^2} dx \\
&= \frac{x(9 + 11x^2)}{8(2 + 3x^2 + x^4)^2} - \frac{x(547 + 347x^2)}{32(2 + 3x^2 + x^4)} + \frac{1}{32} \int \frac{32 + 441x^2 - 347x^4}{x^2(2 + 3x^2 + x^4)} dx \\
&= \frac{x(9 + 11x^2)}{8(2 + 3x^2 + x^4)^2} - \frac{x(547 + 347x^2)}{32(2 + 3x^2 + x^4)} + \frac{1}{32} \int \left(\frac{16}{x^2} + \frac{756}{1 + x^2} - \frac{1119}{2 + x^2} \right) dx \\
&= -\frac{1}{2x} + \frac{x(9 + 11x^2)}{8(2 + 3x^2 + x^4)^2} - \frac{x(547 + 347x^2)}{32(2 + 3x^2 + x^4)} + \frac{189}{8} \int \frac{1}{1 + x^2} dx - \frac{1119}{32} \int \frac{1}{2 + x^2} dx \\
&= -\frac{1}{2x} + \frac{x(9 + 11x^2)}{8(2 + 3x^2 + x^4)^2} - \frac{x(547 + 347x^2)}{32(2 + 3x^2 + x^4)} + \frac{189}{8} \tan^{-1}(x) - \frac{1119 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{32\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 63, normalized size = 0.80

$$\frac{1}{64} \left(-\frac{2(363x^8 + 1684x^6 + 2499x^4 + 1250x^2 + 64)}{x(x^4 + 3x^2 + 2)^2} + 1512 \tan^{-1}(x) - 1119\sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(2 + 3*x^2 + x^4)^3), x]

[Out] ((-2*(64 + 1250*x^2 + 2499*x^4 + 1684*x^6 + 363*x^8))/(x*(2 + 3*x^2 + x^4)^2) + 1512*ArcTan[x] - 1119*Sqrt[2]*ArcTan[x/Sqrt[2]])/64

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4 + x^2 + 3x^4 + 5x^6}{x^2(2 + 3x^2 + x^4)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(2 + 3*x^2 + x^4)^3), x]

[Out] IntegrateAlgebraic[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(2 + 3*x^2 + x^4)^3), x]

fricas [A] time = 1.27, size = 108, normalized size = 1.37

$$\frac{726x^8 + 3368x^6 + 4998x^4 + 1119\sqrt{2}(x^9 + 6x^7 + 13x^5 + 12x^3 + 4x) \arctan\left(\frac{1}{2}\sqrt{2}x\right) + 2500x^2 - 1512(x^9 + 6x^7 + 13x^5 + 12x^3 + 4x) \arctan(x) + 128}{64(x^9 + 6x^7 + 13x^5 + 12x^3 + 4x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+3*x^2+2)^3,x, algorithm="fricas")

[Out] -1/64*(726*x^8 + 3368*x^6 + 4998*x^4 + 1119*sqrt(2)*(x^9 + 6*x^7 + 13*x^5 + 12*x^3 + 4*x)*arctan(1/2*sqrt(2)*x) + 2500*x^2 - 1512*(x^9 + 6*x^7 + 13*x^5 + 12*x^3 + 4*x)*arctan(x) + 128)/(x^9 + 6*x^7 + 13*x^5 + 12*x^3 + 4*x)

giac [A] time = 0.34, size = 55, normalized size = 0.70

$$-\frac{1119}{64} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} x\right) - \frac{347x^7 + 1588x^5 + 2291x^3 + 1058x}{32(x^4 + 3x^2 + 2)^2} - \frac{1}{2x} + \frac{189}{8} \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+3*x^2+2)^3,x, algorithm="giac")

[Out] -1119/64*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/32*(347*x^7 + 1588*x^5 + 2291*x^3 + 1058*x)/(x^4 + 3*x^2 + 2)^2 - 1/2/x + 189/8*arctan(x)

maple [A] time = 0.01, size = 58, normalized size = 0.73

$$\frac{189 \arctan(x)}{8} - \frac{1119\sqrt{2} \arctan\left(\frac{\sqrt{2}x}{2}\right)}{64} - \frac{1}{2x} + \frac{-\frac{35}{8}x^3 - \frac{37}{8}x}{(x^2+1)^2} - \frac{\frac{207}{16}x^3 + \frac{233}{8}x}{2(x^2+2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^6+3*x^4+x^2+4)/x^2/(x^4+3*x^2+2)^3,x)

[Out] -1/2/x+(-35/8*x^3-37/8*x)/(x^2+1)^2+189/8*arctan(x)-1/2*(207/16*x^3+233/8*x)/(x^2+2)^2-1119/64*2^(1/2)*arctan(1/2*2^(1/2)*x)

maxima [A] time = 1.80, size = 65, normalized size = 0.82

$$-\frac{1119}{64} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} x\right) - \frac{363 x^8 + 1684 x^6 + 2499 x^4 + 1250 x^2 + 64}{32(x^9 + 6x^7 + 13x^5 + 12x^3 + 4x)} + \frac{189}{8} \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+3*x^2+2)^3,x, algorithm="maxima")

[Out] -1119/64*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/32*(363*x^8 + 1684*x^6 + 2499*x^4 + 1250*x^2 + 64)/(x^9 + 6*x^7 + 13*x^5 + 12*x^3 + 4*x) + 189/8*arctan(x)

mupad [B] time = 0.92, size = 65, normalized size = 0.82

$$\frac{189 \operatorname{atan}(x)}{8} - \frac{1119 \sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{64} - \frac{\frac{363x^8}{32} + \frac{421x^6}{8} + \frac{2499x^4}{32} + \frac{625x^2}{16} + 2}{x^9 + 6x^7 + 13x^5 + 12x^3 + 4x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^2*(3*x^2 + x^4 + 2)^3),x)

[Out] (189*atan(x))/8 - (1119*2^(1/2)*atan((2^(1/2)*x)/2))/64 - ((625*x^2)/16 + (2499*x^4)/32 + (421*x^6)/8 + (363*x^8)/32 + 2)/(4*x + 12*x^3 + 13*x^5 + 6*x^7 + x^9)

sympy [A] time = 0.28, size = 71, normalized size = 0.90

$$\frac{-363x^8 - 1684x^6 - 2499x^4 - 1250x^2 - 64}{32x^9 + 192x^7 + 416x^5 + 384x^3 + 128x} + \frac{189 \operatorname{atan}(x)}{8} - \frac{1119\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**6+3*x**4+x**2+4)/x**2/(x**4+3*x**2+2)**3,x)

[Out] (-363*x**8 - 1684*x**6 - 2499*x**4 - 1250*x**2 - 64)/(32*x**9 + 192*x**7 + 416*x**5 + 384*x**3 + 128*x) + 189*atan(x)/8 - 1119*sqrt(2)*atan(sqrt(2)*x/2)/64

$$3.96 \quad \int \frac{4+x^2+3x^4+5x^6}{x^4(2+3x^2+x^4)^3} dx$$

Optimal. Leaf size=86

$$-\frac{1}{6x^3} - \frac{x(9x^2+5)}{16(x^4+3x^2+2)^2} + \frac{x(571x^2+951)}{64(x^4+3x^2+2)} + \frac{17}{8x} - \frac{113}{8} \tan^{-1}(x) + \frac{1611 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{64\sqrt{2}}$$

Rubi [A] time = 0.12, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {1669, 1664, 203}

$$-\frac{x(9x^2+5)}{16(x^4+3x^2+2)^2} + \frac{x(571x^2+951)}{64(x^4+3x^2+2)} - \frac{1}{6x^3} + \frac{17}{8x} - \frac{113}{8} \tan^{-1}(x) + \frac{1611 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{64\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(2 + 3*x^2 + x^4)^3), x]

[Out] -1/(6*x^3) + 17/(8*x) - (x*(5 + 9*x^2))/(16*(2 + 3*x^2 + x^4)^2) + (x*(951 + 571*x^2))/(64*(2 + 3*x^2 + x^4)) - (113*ArcTan[x])/8 + (1611*ArcTan[x/Sqrt[2]])/(64*Sqrt[2])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 1664

Int[(Pq_)*((d_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]

Rule 1669

Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p+1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p+1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p+1)*(b^2 - 4*a*c)), Int[x^m*(a + b*x^2 + c*x^4)^(p+1)*ExpandToSum[(2*a*(p+1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p+3) - 2*a*c*d*(4*p+5) - a*b*e)/x^m + c*(4*p+7)*(b*d - 2*a*e)*x^(2-m), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]

Rubi steps

$$\begin{aligned}
\int \frac{4 + x^2 + 3x^4 + 5x^6}{x^4(2 + 3x^2 + x^4)^3} dx &= -\frac{x(5 + 9x^2)}{16(2 + 3x^2 + x^4)^2} - \frac{1}{8} \int \frac{-16 + 20x^2 - \frac{73x^4}{2} + \frac{45x^6}{2}}{x^4(2 + 3x^2 + x^4)^2} dx \\
&= -\frac{x(5 + 9x^2)}{16(2 + 3x^2 + x^4)^2} + \frac{x(951 + 571x^2)}{64(2 + 3x^2 + x^4)} + \frac{1}{32} \int \frac{32 - 88x^2 - \frac{573x^4}{2} + \frac{571x^6}{2}}{x^4(2 + 3x^2 + x^4)} dx \\
&= -\frac{x(5 + 9x^2)}{16(2 + 3x^2 + x^4)^2} + \frac{x(951 + 571x^2)}{64(2 + 3x^2 + x^4)} + \frac{1}{32} \int \left(\frac{16}{x^4} - \frac{68}{x^2} - \frac{452}{1 + x^2} + \frac{1611}{2(2 + x^2)} \right) dx \\
&= -\frac{1}{6x^3} + \frac{17}{8x} - \frac{x(5 + 9x^2)}{16(2 + 3x^2 + x^4)^2} + \frac{x(951 + 571x^2)}{64(2 + 3x^2 + x^4)} - \frac{113}{8} \int \frac{1}{1 + x^2} dx + \frac{1611}{64} \int \frac{1}{2 + x^2} dx \\
&= -\frac{1}{6x^3} + \frac{17}{8x} - \frac{x(5 + 9x^2)}{16(2 + 3x^2 + x^4)^2} + \frac{x(951 + 571x^2)}{64(2 + 3x^2 + x^4)} - \frac{113}{8} \tan^{-1}(x) + \frac{1611 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{64\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 78, normalized size = 0.91

$$\frac{1}{384} \left(-\frac{64}{x^3} - \frac{24x(9x^2 + 5)}{(x^4 + 3x^2 + 2)^2} + \frac{6x(571x^2 + 951)}{x^4 + 3x^2 + 2} + \frac{816}{x} - 5424 \tan^{-1}(x) + 4833\sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(2 + 3*x^2 + x^4)^3), x]

[Out] (-64/x^3 + 816/x - (24*x*(5 + 9*x^2))/(2 + 3*x^2 + x^4)^2 + (6*x*(951 + 571*x^2))/(2 + 3*x^2 + x^4) - 5424*ArcTan[x] + 4833*sqrt[2]*ArcTan[x/sqrt[2]])/384

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4 + x^2 + 3x^4 + 5x^6}{x^4(2 + 3x^2 + x^4)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(2 + 3*x^2 + x^4)^3), x]

[Out] IntegrateAlgebraic[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(2 + 3*x^2 + x^4)^3), x]

fricas [A] time = 0.96, size = 119, normalized size = 1.38

$$\frac{4242x^{10} + 20816x^8 + 33978x^6 + 20252x^4 + 4833\sqrt{2}(x^{11} + 6x^9 + 13x^7 + 12x^5 + 4x^3)\arctan\left(\frac{1}{2}\sqrt{2}x\right) + 2496x^2 - 5424(x^{11} + 6x^9 + 13x^7 + 12x^5 + 4x^3)\arctan(x) - 256}{384(x^{11} + 6x^9 + 13x^7 + 12x^5 + 4x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+3*x^2+2)^3,x, algorithm="fricas")

[Out] 1/384*(4242*x^10 + 20816*x^8 + 33978*x^6 + 20252*x^4 + 4833*sqrt(2)*(x^11 + 6*x^9 + 13*x^7 + 12*x^5 + 4*x^3)*arctan(1/2*sqrt(2)*x) + 2496*x^2 - 5424*(x^11 + 6*x^9 + 13*x^7 + 12*x^5 + 4*x^3)*arctan(x) - 256)/(x^11 + 6*x^9 + 13*x^7 + 12*x^5 + 4*x^3)

giac [A] time = 0.36, size = 62, normalized size = 0.72

$$\frac{1611}{128} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} x\right) + \frac{571x^7 + 2664x^5 + 3959x^3 + 1882x}{64(x^4 + 3x^2 + 2)^2} + \frac{51x^2 - 4}{24x^3} - \frac{113}{8} \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+3*x^2+2)^3,x, algorithm="giac")

[Out] 1611/128*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/64*(571*x^7 + 2664*x^5 + 3959*x^3 + 1882*x)/(x^4 + 3*x^2 + 2)^2 + 1/24*(51*x^2 - 4)/x^3 - 113/8*arctan(x)

maple [A] time = 0.02, size = 64, normalized size = 0.74

$$-\frac{113 \arctan(x)}{8} + \frac{1611\sqrt{2} \arctan\left(\frac{\sqrt{2}x}{2}\right)}{128} + \frac{17}{8x} - \frac{1}{6x^3} - \frac{-\frac{39}{8}x^3 - \frac{41}{8}x}{(x^2 + 1)^2} + \frac{\frac{259}{8}x^3 + \frac{285}{4}x}{8(x^2 + 2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^6+3*x^4+x^2+4)/x^4/(x^4+3*x^2+2)^3,x)

[Out] -1/6/x^3+17/8/x-(-39/8*x^3-41/8*x)/(x^2+1)^2-113/8*arctan(x)+1/8*(259/8*x^3+285/4*x)/(x^2+2)^2+1611/128*2^(1/2)*arctan(1/2*2^(1/2)*x)

maxima [A] time = 1.53, size = 72, normalized size = 0.84

$$\frac{1611}{128} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2}x\right) + \frac{2121x^{10} + 10408x^8 + 16989x^6 + 10126x^4 + 1248x^2 - 128}{192(x^{11} + 6x^9 + 13x^7 + 12x^5 + 4x^3)} - \frac{113}{8} \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+3*x^2+2)^3,x, algorithm="maxima")

[Out] 1611/128*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/192*(2121*x^10 + 10408*x^8 + 16989*x^6 + 10126*x^4 + 1248*x^2 - 128)/(x^11 + 6*x^9 + 13*x^7 + 12*x^5 + 4*x^3) - 113/8*arctan(x)

mupad [B] time = 0.92, size = 71, normalized size = 0.83

$$\frac{\frac{707x^{10}}{64} + \frac{1301x^8}{24} + \frac{5663x^6}{64} + \frac{5063x^4}{96} + \frac{13x^2}{2} - \frac{2}{3}}{x^{11} + 6x^9 + 13x^7 + 12x^5 + 4x^3} - \frac{113 \operatorname{atan}(x)}{8} + \frac{1611 \sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^4*(3*x^2 + x^4 + 2)^3),x)

[Out] ((13*x^2)/2 + (5063*x^4)/96 + (5663*x^6)/64 + (1301*x^8)/24 + (707*x^10)/64 - 2/3)/(4*x^3 + 12*x^5 + 13*x^7 + 6*x^9 + x^11) - (113*atan(x))/8 + (1611*2^(1/2)*atan(2^(1/2)*x/2))/128

sympy [A] time = 0.29, size = 76, normalized size = 0.88

$$-\frac{113 \operatorname{atan}(x)}{8} + \frac{1611\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{128} + \frac{2121x^{10} + 10408x^8 + 16989x^6 + 10126x^4 + 1248x^2 - 128}{192x^{11} + 1152x^9 + 2496x^7 + 2304x^5 + 768x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**6+3*x**4+x**2+4)/x**4/(x**4+3*x**2+2)**3,x)

[Out] -113*atan(x)/8 + 1611*sqrt(2)*atan(sqrt(2)*x/2)/128 + (2121*x**10 + 10408*x**8 + 16989*x**6 + 10126*x**4 + 1248*x**2 - 128)/(192*x**11 + 1152*x**9 + 2496*x**7 + 2304*x**5 + 768*x**3)

$$3.97 \quad \int \frac{4+x^2+3x^4+5x^6}{x^6(2+3x^2+x^4)^3} dx$$

Optimal. Leaf size=93

$$-\frac{1}{10x^5} + \frac{17}{24x^3} - \frac{x(3-5x^2)}{32(x^4+3x^2+2)^2} - \frac{x(999x^2+1771)}{128(x^4+3x^2+2)} - \frac{93}{16x} + \frac{29}{8} \tan^{-1}(x) - \frac{2207 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{128\sqrt{2}}$$

Rubi [A] time = 0.13, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {1669, 1664, 203}

$$-\frac{x(3-5x^2)}{32(x^4+3x^2+2)^2} - \frac{x(999x^2+1771)}{128(x^4+3x^2+2)} + \frac{17}{24x^3} - \frac{1}{10x^5} - \frac{93}{16x} + \frac{29}{8} \tan^{-1}(x) - \frac{2207 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{128\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^6*(2 + 3*x^2 + x^4)^3), x]

[Out] -1/(10*x^5) + 17/(24*x^3) - 93/(16*x) - (x*(3 - 5*x^2))/(32*(2 + 3*x^2 + x^4)^2) - (x*(1771 + 999*x^2))/(128*(2 + 3*x^2 + x^4)) + (29*ArcTan[x])/8 - (2207*ArcTan[x/Sqrt[2]])/(128*Sqrt[2])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 1664

Int[(Pq_)*((d_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]

Rule 1669

Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p+1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p+1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p+1)*(b^2 - 4*a*c)), Int[x^m*(a + b*x^2 + c*x^4)^(p+1)*ExpandToSum[(2*a*(p+1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p+3) - 2*a*c*d*(4*p+5) - a*b*e)/x^m + c*(4*p+7)*(b*d - 2*a*e)*x^(2-m), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]

Rubi steps

$$\begin{aligned}
\int \frac{4 + x^2 + 3x^4 + 5x^6}{x^6(2 + 3x^2 + x^4)^3} dx &= -\frac{x(3 - 5x^2)}{32(2 + 3x^2 + x^4)^2} - \frac{1}{8} \int \frac{-16 + 20x^2 - 34x^4 + \frac{81x^6}{4} - \frac{25x^8}{4}}{x^6(2 + 3x^2 + x^4)^2} dx \\
&= -\frac{x(3 - 5x^2)}{32(2 + 3x^2 + x^4)^2} - \frac{x(1771 + 999x^2)}{128(2 + 3x^2 + x^4)} + \frac{1}{32} \int \frac{32 - 88x^2 + 184x^4 + \frac{681x^6}{4} - \frac{9}{4}}{x^6(2 + 3x^2 + x^4)} dx \\
&= -\frac{x(3 - 5x^2)}{32(2 + 3x^2 + x^4)^2} - \frac{x(1771 + 999x^2)}{128(2 + 3x^2 + x^4)} + \frac{1}{32} \int \left(\frac{16}{x^6} - \frac{68}{x^4} + \frac{186}{x^2} + \frac{116}{1 + x^2} - \frac{9}{4} \right) dx \\
&= -\frac{1}{10x^5} + \frac{17}{24x^3} - \frac{93}{16x} - \frac{x(3 - 5x^2)}{32(2 + 3x^2 + x^4)^2} - \frac{x(1771 + 999x^2)}{128(2 + 3x^2 + x^4)} + \frac{29}{8} \int \frac{1}{1 + x^2} dx \\
&= -\frac{1}{10x^5} + \frac{17}{24x^3} - \frac{93}{16x} - \frac{x(3 - 5x^2)}{32(2 + 3x^2 + x^4)^2} - \frac{x(1771 + 999x^2)}{128(2 + 3x^2 + x^4)} + \frac{29}{8} \tan^{-1}(x) - \frac{9}{32} x
\end{aligned}$$

Mathematica [A] time = 0.08, size = 73, normalized size = 0.78

$$\frac{2(26145x^{12} + 137120x^{10} + 246477x^8 + 170702x^6 + 30816x^4 - 3136x^2 + 768)}{x^5(x^4 + 3x^2 + 2)^2} + 13920 \tan^{-1}(x) - 33105\sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

3840

Antiderivative was successfully verified.

[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^6*(2 + 3*x^2 + x^4)^3), x]

[Out] ((-2*(768 - 3136*x^2 + 30816*x^4 + 170702*x^6 + 246477*x^8 + 137120*x^10 + 26145*x^12))/(x^5*(2 + 3*x^2 + x^4)^2) + 13920*ArcTan[x] - 33105*Sqrt[2]*ArcTan[x/Sqrt[2]])/3840

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4 + x^2 + 3x^4 + 5x^6}{x^6(2 + 3x^2 + x^4)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(4 + x^2 + 3*x^4 + 5*x^6)/(x^6*(2 + 3*x^2 + x^4)^3), x]

[Out] IntegrateAlgebraic[(4 + x^2 + 3*x^4 + 5*x^6)/(x^6*(2 + 3*x^2 + x^4)^3), x]

fricas [A] time = 1.25, size = 124, normalized size = 1.33

$$\frac{52290x^{12} + 274240x^{10} + 492954x^8 + 341404x^6 + 61632x^4 + 33105\sqrt{2}(x^{13} + 6x^{11} + 13x^9 + 12x^7 + 4x^5)\arctan\left(\frac{1}{2}\sqrt{2}x\right) - 6272x^2 - 13920(x^{13} + 6x^{11} + 13x^9 + 12x^7 + 4x^5)\arctan(x) + 1536}{3840(x^{13} + 6x^{11} + 13x^9 + 12x^7 + 4x^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^6+3*x^4+x^2+4)/x^6/(x^4+3*x^2+2)^3,x, algorithm="fricas")

[Out] -1/3840*(52290*x^12 + 274240*x^10 + 492954*x^8 + 341404*x^6 + 61632*x^4 + 33105*sqrt(2)*(x^13 + 6*x^11 + 13*x^9 + 12*x^7 + 4*x^5)*arctan(1/2*sqrt(2)*x) - 6272*x^2 - 13920*(x^13 + 6*x^11 + 13*x^9 + 12*x^7 + 4*x^5)*arctan(x) + 1536)/(x^13 + 6*x^11 + 13*x^9 + 12*x^7 + 4*x^5)

giac [A] time = 0.36, size = 67, normalized size = 0.72

$$-\frac{2207}{256}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - \frac{999x^7 + 4768x^5 + 7291x^3 + 3554x}{128(x^4 + 3x^2 + 2)^2} - \frac{1395x^4 - 170x^2 + 24}{240x^5} + \frac{29}{8}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^6+3*x^4+x^2+4)/x^6/(x^4+3*x^2+2)^3,x, algorithm="giac")

[Out] $-2207/256*\sqrt{2}*\arctan(1/2*\sqrt{2}*x) - 1/128*(999*x^7 + 4768*x^5 + 7291*x^3 + 3554*x)/(x^4 + 3*x^2 + 2)^2 - 1/240*(1395*x^4 - 170*x^2 + 24)/x^5 + 29/8*\arctan(x)$

maple [A] time = 0.02, size = 68, normalized size = 0.73

$$\frac{29 \arctan(x)}{8} - \frac{2207\sqrt{2} \arctan\left(\frac{\sqrt{2}x}{2}\right)}{256} - \frac{93}{16x} + \frac{17}{24x^3} - \frac{1}{10x^5} + \frac{-\frac{43}{8}x^3 - \frac{45}{8}x}{(x^2 + 1)^2} - \frac{\frac{311}{8}x^3 + \frac{337}{4}x}{16(x^2 + 2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^6+3*x^4+x^2+4)/x^6/(x^4+3*x^2+2)^3,x)

[Out] $-1/10/x^5+17/24/x^3-93/16/x+(-43/8*x^3-45/8*x)/(x^2+1)^2+29/8*\arctan(x)-1/16*(311/8*x^3+337/4*x)/(x^2+2)^2-2207/256*2^{(1/2)}*\arctan(1/2*2^{(1/2)}*x)$

maxima [A] time = 1.74, size = 77, normalized size = 0.83

$$-\frac{2207}{256}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - \frac{26145x^{12} + 137120x^{10} + 246477x^8 + 170702x^6 + 30816x^4 - 3136x^2 + 768}{1920(x^{13} + 6x^{11} + 13x^9 + 12x^7 + 4x^5)} + \frac{29}{8}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^6+3*x^4+x^2+4)/x^6/(x^4+3*x^2+2)^3,x, algorithm="maxima")

[Out] $-2207/256*\sqrt{2}*\arctan(1/2*\sqrt{2}*x) - 1/1920*(26145*x^{12} + 137120*x^{10} + 246477*x^8 + 170702*x^6 + 30816*x^4 - 3136*x^2 + 768)/(x^{13} + 6*x^{11} + 13*x^9 + 12*x^7 + 4*x^5) + 29/8*\arctan(x)$

mupad [B] time = 0.93, size = 77, normalized size = 0.83

$$\frac{29 \operatorname{atan}(x)}{8} - \frac{2207\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{256} - \frac{\frac{1743x^{12}}{128} + \frac{857x^{10}}{12} + \frac{82159x^8}{640} + \frac{85351x^6}{960} + \frac{321x^4}{20} - \frac{49x^2}{30} + \frac{2}{5}}{x^{13} + 6x^{11} + 13x^9 + 12x^7 + 4x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^6*(3*x^2 + x^4 + 2)^3),x)

[Out] $(29*\operatorname{atan}(x))/8 - (2207*2^{(1/2)}*\operatorname{atan}(2^{(1/2)}*x/2))/256 - ((321*x^4)/20 - (49*x^2)/30 + (85351*x^6)/960 + (82159*x^8)/640 + (857*x^{10})/12 + (1743*x^{12})/128 + 2/5)/(4*x^5 + 12*x^7 + 13*x^9 + 6*x^{11} + x^{13})$

sympy [A] time = 0.31, size = 82, normalized size = 0.88

$$\frac{29 \operatorname{atan}(x)}{8} - \frac{2207\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{256} + \frac{-26145x^{12} - 137120x^{10} - 246477x^8 - 170702x^6 - 30816x^4 + 3136x^2 - 768}{1920x^{13} + 11520x^{11} + 24960x^9 + 23040x^7 + 7680x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**6+3*x**4+x**2+4)/x**6/(x**4+3*x**2+2)**3,x)

[Out] $29*\operatorname{atan}(x)/8 - 2207*\sqrt{2}*\operatorname{atan}(\sqrt{2}*x/2)/256 + (-26145*x^{12} - 137120*x^{10} - 246477*x^8 - 170702*x^6 - 30816*x^4 + 3136*x^2 - 768)/(1920*x^{13} + 11520*x^{11} + 24960*x^9 + 23040*x^7 + 7680*x^5)$

$$3.98 \quad \int \frac{x^9(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2} dx$$

Optimal. Leaf size=86

$$\frac{5x^8}{8} - \frac{17x^6}{6} + \frac{19x^4}{4} + 19x^2 + \frac{201 \tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}} - \frac{25(7x^2+15)}{8(x^4+2x^2+3)} - \frac{183}{4} \log(x^4+2x^2+3)$$

Rubi [A] time = 0.14, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {1663, 1660, 1657, 634, 618, 204, 628}

$$\frac{5x^8}{8} - \frac{17x^6}{6} + \frac{19x^4}{4} + 19x^2 - \frac{25(7x^2+15)}{8(x^4+2x^2+3)} - \frac{183}{4} \log(x^4+2x^2+3) + \frac{201 \tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(x^9*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]

[Out] 19*x^2 + (19*x^4)/4 - (17*x^6)/6 + (5*x^8)/8 - (25*(15 + 7*x^2))/(8*(3 + 2*x^2 + x^4)) + (201*ArcTan[(1 + x^2)/Sqrt[2]])/(8*Sqrt[2]) - (183*Log[3 + 2*x^2 + x^4])/4

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1657

Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[Expand[Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 1660

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P

```
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]], Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[p, -1]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\begin{aligned} \int \frac{x^9 (4 + x^2 + 3x^4 + 5x^6)}{(3 + 2x^2 + x^4)^2} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x^4 (4 + x + 3x^2 + 5x^3)}{(3 + 2x + x^2)^2} dx, x, x^2 \right) \\ &= -\frac{25(15 + 7x^2)}{8(3 + 2x^2 + x^4)} + \frac{1}{16} \text{Subst} \left(\int \frac{-150 - 400x + 200x^2 - 56x^4 + 40x^5}{3 + 2x + x^2} dx, x, x^2 \right) \\ &= -\frac{25(15 + 7x^2)}{8(3 + 2x^2 + x^4)} + \frac{1}{16} \text{Subst} \left(\int \left(304 + 152x - 136x^2 + 40x^3 - \frac{6(177 + 244x)}{3 + 2x + x^2} \right) dx, x, x^2 \right) \\ &= 19x^2 + \frac{19x^4}{4} - \frac{17x^6}{6} + \frac{5x^8}{8} - \frac{25(15 + 7x^2)}{8(3 + 2x^2 + x^4)} - \frac{3}{8} \text{Subst} \left(\int \frac{177 + 244x}{3 + 2x + x^2} dx, x, x^2 \right) \\ &= 19x^2 + \frac{19x^4}{4} - \frac{17x^6}{6} + \frac{5x^8}{8} - \frac{25(15 + 7x^2)}{8(3 + 2x^2 + x^4)} + \frac{201}{8} \text{Subst} \left(\int \frac{1}{3 + 2x + x^2} dx, x, x^2 \right) \\ &= 19x^2 + \frac{19x^4}{4} - \frac{17x^6}{6} + \frac{5x^8}{8} - \frac{25(15 + 7x^2)}{8(3 + 2x^2 + x^4)} - \frac{183}{4} \log(3 + 2x^2 + x^4) - \frac{201}{4} \text{Subst} \left(\int \frac{1}{3 + 2x + x^2} dx, x, x^2 \right) \\ &= 19x^2 + \frac{19x^4}{4} - \frac{17x^6}{6} + \frac{5x^8}{8} - \frac{25(15 + 7x^2)}{8(3 + 2x^2 + x^4)} + \frac{201 \tan^{-1} \left(\frac{1+x^2}{\sqrt{2}} \right)}{8\sqrt{2}} - \frac{183}{4} \log(3 + 2x^2 + x^4) \end{aligned}$$

Mathematica [A] time = 0.04, size = 78, normalized size = 0.91

$$\frac{1}{48} \left(30x^8 - 136x^6 + 228x^4 + 912x^2 + 603\sqrt{2} \tan^{-1} \left(\frac{x^2 + 1}{\sqrt{2}} \right) - \frac{150(7x^2 + 15)}{x^4 + 2x^2 + 3} - 2196 \log(x^4 + 2x^2 + 3) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^9*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] (912*x^2 + 228*x^4 - 136*x^6 + 30*x^8 - (150*(15 + 7*x^2))/(3 + 2*x^2 + x^4)
) + 603*sqrt[2]*ArcTan[(1 + x^2)/sqrt[2]] - 2196*Log[3 + 2*x^2 + x^4])/48
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9 (4 + x^2 + 3x^4 + 5x^6)}{(3 + 2x^2 + x^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^9*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]

[Out] IntegrateAlgebraic[(x^9*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2, x]

fricas [A] time = 1.14, size = 95, normalized size = 1.10

$$\frac{30x^{12} - 76x^{10} + 46x^8 + 960x^6 + 2508x^4 + 603\sqrt{2}(x^4 + 2x^2 + 3)\arctan\left(\frac{1}{2}\sqrt{2}(x^2 + 1)\right) + 1686x^2 - 2196(x^4 + 2x^2 + 3)\log(x^4 + 2x^2 + 3) - 2250}{48(x^4 + 2x^2 + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")

[Out] 1/48*(30*x^12 - 76*x^10 + 46*x^8 + 960*x^6 + 2508*x^4 + 603*sqrt(2)*(x^4 + 2*x^2 + 3)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 1686*x^2 - 2196*(x^4 + 2*x^2 + 3)*log(x^4 + 2*x^2 + 3) - 2250)/(x^4 + 2*x^2 + 3)

giac [A] time = 1.02, size = 76, normalized size = 0.88

$$\frac{5}{8}x^8 - \frac{17}{6}x^6 + \frac{19}{4}x^4 + 19x^2 + \frac{201}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x^2 + 1)\right) + \frac{366x^4 + 557x^2 + 723}{8(x^4 + 2x^2 + 3)} - \frac{183}{4}\log(x^4 + 2x^2 + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")

[Out] 5/8*x^8 - 17/6*x^6 + 19/4*x^4 + 19*x^2 + 201/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 1/8*(366*x^4 + 557*x^2 + 723)/(x^4 + 2*x^2 + 3) - 183/4*log(x^4 + 2*x^2 + 3)

maple [A] time = 0.01, size = 74, normalized size = 0.86

$$\frac{5x^8}{8} - \frac{17x^6}{6} + \frac{19x^4}{4} + 19x^2 + \frac{201\sqrt{2}\arctan\left(\frac{(2x^2+2)\sqrt{2}}{4}\right)}{16} - \frac{183\ln(x^4 + 2x^2 + 3)}{4} - \frac{\frac{175x^2}{4} + \frac{375}{4}}{2(x^4 + 2x^2 + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^9*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x)

[Out] 5/8*x^8-17/6*x^6+19/4*x^4+19*x^2-1/2*(175/4*x^2+375/4)/(x^4+2*x^2+3)-183/4*ln(x^4+2*x^2+3)+201/16*2^(1/2)*arctan(1/4*(2*x^2+2)*2^(1/2))

maxima [A] time = 1.33, size = 71, normalized size = 0.83

$$\frac{5}{8}x^8 - \frac{17}{6}x^6 + \frac{19}{4}x^4 + 19x^2 + \frac{201}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x^2 + 1)\right) - \frac{25(7x^2 + 15)}{8(x^4 + 2x^2 + 3)} - \frac{183}{4}\log(x^4 + 2x^2 + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")

[Out] 5/8*x^8 - 17/6*x^6 + 19/4*x^4 + 19*x^2 + 201/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 25/8*(7*x^2 + 15)/(x^4 + 2*x^2 + 3) - 183/4*log(x^4 + 2*x^2 + 3)

mupad [B] time = 0.90, size = 75, normalized size = 0.87

$$\frac{201\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x^2}{2} + \frac{\sqrt{2}}{2}\right)}{16} - \frac{\frac{175x^2}{8} + \frac{375}{8}}{x^4 + 2x^2 + 3} - \frac{183\ln(x^4 + 2x^2 + 3)}{4} + 19x^2 + \frac{19x^4}{4} - \frac{17x^6}{6} + \frac{5x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^9*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^2,x)`

[Out] $(201*2^{(1/2)}*\operatorname{atan}(2^{(1/2)}/2 + (2^{(1/2)}*x^2)/2))/16 - ((175*x^2)/8 + 375/8)/(2*x^2 + x^4 + 3) - (183*\log(2*x^2 + x^4 + 3))/4 + 19*x^2 + (19*x^4)/4 - (17*x^6)/6 + (5*x^8)/8$

sympy [A] time = 0.18, size = 87, normalized size = 1.01

$$\frac{5x^8}{8} - \frac{17x^6}{6} + \frac{19x^4}{4} + 19x^2 + \frac{-175x^2 - 375}{8x^4 + 16x^2 + 24} - \frac{183 \log(x^4 + 2x^2 + 3)}{4} + \frac{201\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}x^2}{2} + \frac{\sqrt{2}}{2}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**9*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)`

[Out] $5*x**8/8 - 17*x**6/6 + 19*x**4/4 + 19*x**2 + (-175*x**2 - 375)/(8*x**4 + 16*x**2 + 24) - 183*\log(x**4 + 2*x**2 + 3)/4 + 201*\sqrt{2}*\operatorname{atan}(\sqrt{2}*x**2/2 + \sqrt{2}/2)/16$

$$3.99 \quad \int \frac{x^7(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2} dx$$

Optimal. Leaf size=81

$$\frac{5x^6}{6} - \frac{17x^4}{4} + \frac{19x^2}{2} - \frac{455 \tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}} + \frac{25(5x^2+3)}{8(x^4+2x^2+3)} + \frac{19}{2} \log(x^4+2x^2+3)$$

Rubi [A] time = 0.13, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {1663, 1660, 1657, 634, 618, 204, 628}

$$\frac{5x^6}{6} - \frac{17x^4}{4} + \frac{19x^2}{2} + \frac{25(5x^2+3)}{8(x^4+2x^2+3)} + \frac{19}{2} \log(x^4+2x^2+3) - \frac{455 \tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(x^7*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]

[Out] (19*x^2)/2 - (17*x^4)/4 + (5*x^6)/6 + (25*(3 + 5*x^2))/(8*(3 + 2*x^2 + x^4)) - (455*ArcTan[(1 + x^2)/Sqrt[2]])/(8*Sqrt[2]) + (19*Log[3 + 2*x^2 + x^4])/2

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1657

Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 1660

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P

```

q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]], Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[p, -1]

```

Rule 1663

```

Int[(Pq_)*(x_)^(m_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]

```

Rubi steps

$$\begin{aligned}
\int \frac{x^7 (4 + x^2 + 3x^4 + 5x^6)}{(3 + 2x^2 + x^4)^2} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x^3 (4 + x + 3x^2 + 5x^3)}{(3 + 2x + x^2)^2} dx, x, x^2 \right) \\
&= \frac{25(3 + 5x^2)}{8(3 + 2x^2 + x^4)} + \frac{1}{16} \text{Subst} \left(\int \frac{-150 + 200x - 56x^3 + 40x^4}{3 + 2x + x^2} dx, x, x^2 \right) \\
&= \frac{25(3 + 5x^2)}{8(3 + 2x^2 + x^4)} + \frac{1}{16} \text{Subst} \left(\int \left(152 - 136x + 40x^2 - \frac{2(303 - 152x)}{3 + 2x + x^2} \right) dx, x, x^2 \right) \\
&= \frac{19x^2}{2} - \frac{17x^4}{4} + \frac{5x^6}{6} + \frac{25(3 + 5x^2)}{8(3 + 2x^2 + x^4)} - \frac{1}{8} \text{Subst} \left(\int \frac{303 - 152x}{3 + 2x + x^2} dx, x, x^2 \right) \\
&= \frac{19x^2}{2} - \frac{17x^4}{4} + \frac{5x^6}{6} + \frac{25(3 + 5x^2)}{8(3 + 2x^2 + x^4)} + \frac{19}{2} \text{Subst} \left(\int \frac{2 + 2x}{3 + 2x + x^2} dx, x, x^2 \right) - \\
&= \frac{19x^2}{2} - \frac{17x^4}{4} + \frac{5x^6}{6} + \frac{25(3 + 5x^2)}{8(3 + 2x^2 + x^4)} + \frac{19}{2} \log(3 + 2x^2 + x^4) + \frac{455}{4} \text{Subst} \left(\int \frac{1}{3 + 2x + x^2} dx, x, x^2 \right) \\
&= \frac{19x^2}{2} - \frac{17x^4}{4} + \frac{5x^6}{6} + \frac{25(3 + 5x^2)}{8(3 + 2x^2 + x^4)} - \frac{455 \tan^{-1} \left(\frac{1+x^2}{\sqrt{2}} \right)}{8\sqrt{2}} + \frac{19}{2} \log(3 + 2x^2 + x^4)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 73, normalized size = 0.90

$$\frac{1}{48} \left(40x^6 - 204x^4 + 456x^2 - 1365\sqrt{2} \tan^{-1} \left(\frac{x^2 + 1}{\sqrt{2}} \right) + \frac{150(5x^2 + 3)}{x^4 + 2x^2 + 3} + 456 \log(x^4 + 2x^2 + 3) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2, x]
```

```
[Out] (456*x^2 - 204*x^4 + 40*x^6 + (150*(3 + 5*x^2))/(3 + 2*x^2 + x^4) - 1365*sqrt[2]*ArcTan[(1 + x^2)/sqrt[2]] + 456*Log[3 + 2*x^2 + x^4])/48
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7 (4 + x^2 + 3x^4 + 5x^6)}{(3 + 2x^2 + x^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^7*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]

[Out] IntegrateAlgebraic[(x^7*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2, x]

fricas [A] time = 1.12, size = 90, normalized size = 1.11

$$\frac{40x^{10} - 124x^8 + 168x^6 + 300x^4 - 1365\sqrt{2}(x^4 + 2x^2 + 3)\arctan\left(\frac{1}{2}\sqrt{2}(x^2 + 1)\right) + 2118x^2 + 456(x^4 + 2x^2 + 3)\log(x^4 + 2x^2 + 3) + 450}{48(x^4 + 2x^2 + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")

[Out] 1/48*(40*x^10 - 124*x^8 + 168*x^6 + 300*x^4 - 1365*sqrt(2)*(x^4 + 2*x^2 + 3)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 2118*x^2 + 456*(x^4 + 2*x^2 + 3)*log(x^4 + 2*x^2 + 3) + 450)/(x^4 + 2*x^2 + 3)

giac [A] time = 1.17, size = 71, normalized size = 0.88

$$\frac{5}{6}x^6 - \frac{17}{4}x^4 + \frac{19}{2}x^2 - \frac{455}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x^2 + 1)\right) - \frac{76x^4 + 27x^2 + 153}{8(x^4 + 2x^2 + 3)} + \frac{19}{2}\log(x^4 + 2x^2 + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")

[Out] 5/6*x^6 - 17/4*x^4 + 19/2*x^2 - 455/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 1/8*(76*x^4 + 27*x^2 + 153)/(x^4 + 2*x^2 + 3) + 19/2*log(x^4 + 2*x^2 + 3)

maple [A] time = 0.01, size = 69, normalized size = 0.85

$$\frac{5x^6}{6} - \frac{17x^4}{4} + \frac{19x^2}{2} - \frac{455\sqrt{2}\arctan\left(\frac{(2x^2+2)\sqrt{2}}{4}\right)}{16} + \frac{19\ln(x^4 + 2x^2 + 3)}{2} + \frac{\frac{125x^2}{4} + \frac{75}{4}}{2x^4 + 4x^2 + 6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x)

[Out] 5/6*x^6-17/4*x^4+19/2*x^2+1/2*(125/4*x^2+75/4)/(x^4+2*x^2+3)+19/2*ln(x^4+2*x^2+3)-455/16*2^(1/2)*arctan(1/4*(2*x^2+2)*2^(1/2))

maxima [A] time = 1.82, size = 66, normalized size = 0.81

$$\frac{5}{6}x^6 - \frac{17}{4}x^4 + \frac{19}{2}x^2 - \frac{455}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x^2 + 1)\right) + \frac{25(5x^2 + 3)}{8(x^4 + 2x^2 + 3)} + \frac{19}{2}\log(x^4 + 2x^2 + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")

[Out] 5/6*x^6 - 17/4*x^4 + 19/2*x^2 - 455/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 25/8*(5*x^2 + 3)/(x^4 + 2*x^2 + 3) + 19/2*log(x^4 + 2*x^2 + 3)

mupad [B] time = 0.05, size = 69, normalized size = 0.85

$$\frac{19\ln(x^4 + 2x^2 + 3)}{2} + \frac{\frac{125x^2}{8} + \frac{75}{8}}{x^4 + 2x^2 + 3} - \frac{455\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x^2}{2} + \frac{\sqrt{2}}{2}\right)}{16} + \frac{19x^2}{2} - \frac{17x^4}{4} + \frac{5x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^7*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^2,x)

[Out] $(19 \log(2x^2 + x^4 + 3))/2 + ((125x^2)/8 + 75/8)/(2x^2 + x^4 + 3) - (455 \sqrt{2}) \operatorname{atan}(2^{1/2}/2 + (2^{1/2}x^2)/2)/16 + (19x^2)/2 - (17x^4)/4 + (5x^6)/6$

sympy [A] time = 0.18, size = 80, normalized size = 0.99

$$\frac{5x^6}{6} - \frac{17x^4}{4} + \frac{19x^2}{2} + \frac{125x^2 + 75}{8x^4 + 16x^2 + 24} + \frac{19 \log(x^4 + 2x^2 + 3)}{2} - \frac{455\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}x^2}{2} + \frac{\sqrt{2}}{2}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**7*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)`

[Out] $5x^{**6}/6 - 17x^{**4}/4 + 19x^{**2}/2 + (125x^{**2} + 75)/(8x^{**4} + 16x^{**2} + 24) + 19 \log(x^{**4} + 2x^{**2} + 3)/2 - 455 \sqrt{2} \operatorname{atan}(\sqrt{2}x^{**2}/2 + \sqrt{2}/2)/16$

$$3.100 \quad \int \frac{x^5(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2} dx$$

Optimal. Leaf size=74

$$\frac{5x^4}{4} - \frac{17x^2}{2} + \frac{203 \tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}} + \frac{25(3-x^2)}{8(x^4+2x^2+3)} + \frac{19}{4} \log(x^4+2x^2+3)$$

Rubi [A] time = 0.12, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {1663, 1660, 1657, 634, 618, 204, 628}

$$\frac{5x^4}{4} - \frac{17x^2}{2} + \frac{25(3-x^2)}{8(x^4+2x^2+3)} + \frac{19}{4} \log(x^4+2x^2+3) + \frac{203 \tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(x^5*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]

[Out] (-17*x^2)/2 + (5*x^4)/4 + (25*(3 - x^2))/(8*(3 + 2*x^2 + x^4)) + (203*ArcTan[(1 + x^2)/Sqrt[2]])/(8*Sqrt[2]) + (19*Log[3 + 2*x^2 + x^4])/4

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1657

Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 1660

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x +

```
c*x^2, x], x, 1]], Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[p, -1]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5(4 + x^2 + 3x^4 + 5x^6)}{(3 + 2x^2 + x^4)^2} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x^2(4 + x + 3x^2 + 5x^3)}{(3 + 2x + x^2)^2} dx, x, x^2 \right) \\
&= \frac{25(3 - x^2)}{8(3 + 2x^2 + x^4)} + \frac{1}{16} \text{Subst} \left(\int \frac{150 - 56x^2 + 40x^3}{3 + 2x + x^2} dx, x, x^2 \right) \\
&= \frac{25(3 - x^2)}{8(3 + 2x^2 + x^4)} + \frac{1}{16} \text{Subst} \left(\int \left(-136 + 40x + \frac{2(279 + 76x)}{3 + 2x + x^2} \right) dx, x, x^2 \right) \\
&= -\frac{17x^2}{2} + \frac{5x^4}{4} + \frac{25(3 - x^2)}{8(3 + 2x^2 + x^4)} + \frac{1}{8} \text{Subst} \left(\int \frac{279 + 76x}{3 + 2x + x^2} dx, x, x^2 \right) \\
&= -\frac{17x^2}{2} + \frac{5x^4}{4} + \frac{25(3 - x^2)}{8(3 + 2x^2 + x^4)} + \frac{19}{4} \text{Subst} \left(\int \frac{2 + 2x}{3 + 2x + x^2} dx, x, x^2 \right) + \frac{203}{8} \text{Subst} \left(\int \frac{1}{-8 - 2x} dx, x, x^2 \right) \\
&= -\frac{17x^2}{2} + \frac{5x^4}{4} + \frac{25(3 - x^2)}{8(3 + 2x^2 + x^4)} + \frac{19}{4} \log(3 + 2x^2 + x^4) - \frac{203}{4} \text{Subst} \left(\int \frac{1}{-8 - 2x} dx, x, x^2 \right) \\
&= -\frac{17x^2}{2} + \frac{5x^4}{4} + \frac{25(3 - x^2)}{8(3 + 2x^2 + x^4)} + \frac{203 \tan^{-1} \left(\frac{1+x^2}{\sqrt{2}} \right)}{8\sqrt{2}} + \frac{19}{4} \log(3 + 2x^2 + x^4)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 66, normalized size = 0.89

$$\frac{1}{16} \left(20x^4 - 136x^2 + 203\sqrt{2} \tan^{-1} \left(\frac{x^2 + 1}{\sqrt{2}} \right) - \frac{50(x^2 - 3)}{x^4 + 2x^2 + 3} + 76 \log(x^4 + 2x^2 + 3) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2, x]
```

```
[Out] (-136*x^2 + 20*x^4 - (50*(-3 + x^2))/(3 + 2*x^2 + x^4) + 203*sqrt[2]*ArcTan
[(1 + x^2)/sqrt[2]] + 76*Log[3 + 2*x^2 + x^4])/16
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5(4 + x^2 + 3x^4 + 5x^6)}{(3 + 2x^2 + x^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^5*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]

[Out] IntegrateAlgebraic[(x^5*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2, x]

fricas [A] time = 1.05, size = 85, normalized size = 1.15

$$\frac{20x^8 - 96x^6 - 212x^4 + 203\sqrt{2}(x^4 + 2x^2 + 3)\arctan\left(\frac{1}{2}\sqrt{2}(x^2 + 1)\right) - 458x^2 + 76(x^4 + 2x^2 + 3)\log(x^4 + 2x^2 + 3) + 150}{16(x^4 + 2x^2 + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")

[Out] 1/16*(20*x^8 - 96*x^6 - 212*x^4 + 203*sqrt(2)*(x^4 + 2*x^2 + 3)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 458*x^2 + 76*(x^4 + 2*x^2 + 3)*log(x^4 + 2*x^2 + 3) + 150)/(x^4 + 2*x^2 + 3)

giac [A] time = 1.09, size = 66, normalized size = 0.89

$$\frac{5}{4}x^4 - \frac{17}{2}x^2 + \frac{203}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x^2 + 1)\right) - \frac{38x^4 + 101x^2 + 39}{8(x^4 + 2x^2 + 3)} + \frac{19}{4}\log(x^4 + 2x^2 + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")

[Out] 5/4*x^4 - 17/2*x^2 + 203/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 1/8*(38*x^4 + 101*x^2 + 39)/(x^4 + 2*x^2 + 3) + 19/4*log(x^4 + 2*x^2 + 3)

maple [A] time = 0.01, size = 64, normalized size = 0.86

$$\frac{5x^4}{4} - \frac{17x^2}{2} + \frac{203\sqrt{2}\arctan\left(\frac{(2x^2+2)\sqrt{2}}{4}\right)}{16} + \frac{19\ln(x^4 + 2x^2 + 3)}{4} + \frac{-\frac{25x^2}{4} + \frac{75}{4}}{2x^4 + 4x^2 + 6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x)

[Out] 5/4*x^4-17/2*x^2+1/2*(-25/4*x^2+75/4)/(x^4+2*x^2+3)+19/4*ln(x^4+2*x^2+3)+203/16*2^(1/2)*arctan(1/4*(2*x^2+2)*2^(1/2))

maxima [A] time = 1.59, size = 59, normalized size = 0.80

$$\frac{5}{4}x^4 - \frac{17}{2}x^2 + \frac{203}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x^2 + 1)\right) - \frac{25(x^2 - 3)}{8(x^4 + 2x^2 + 3)} + \frac{19}{4}\log(x^4 + 2x^2 + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")

[Out] 5/4*x^4 - 17/2*x^2 + 203/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 25/8*(x^2 - 3)/(x^4 + 2*x^2 + 3) + 19/4*log(x^4 + 2*x^2 + 3)

mupad [B] time = 0.05, size = 65, normalized size = 0.88

$$\frac{19\ln(x^4 + 2x^2 + 3)}{4} - \frac{\frac{25x^2}{8} - \frac{75}{8}}{x^4 + 2x^2 + 3} + \frac{203\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x^2}{2} + \frac{\sqrt{2}}{2}\right)}{16} - \frac{17x^2}{2} + \frac{5x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^5*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^2,x)

[Out] $(19 \log(2x^2 + x^4 + 3))/4 - ((25x^2)/8 - 75/8)/(2x^2 + x^4 + 3) + (203 \cdot 2^{1/2} \operatorname{atan}(2^{1/2}/2 + (2^{1/2}x^2)/2))/16 - (17x^2)/2 + (5x^4)/4$

sympy [A] time = 0.18, size = 73, normalized size = 0.99

$$\frac{5x^4}{4} - \frac{17x^2}{2} + \frac{75 - 25x^2}{8x^4 + 16x^2 + 24} + \frac{19 \log(x^4 + 2x^2 + 3)}{4} + \frac{203\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}x^2}{2} + \frac{\sqrt{2}}{2}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)`

[Out] $5x^{**4}/4 - 17x^{**2}/2 + (75 - 25x^{**2})/(8x^{**4} + 16x^{**2} + 24) + 19 \log(x^{**4} + 2x^{**2} + 3)/4 + 203 \operatorname{sqrt}(2) \operatorname{atan}(\operatorname{sqrt}(2)x^{**2}/2 + \operatorname{sqrt}(2)/2)/16$

$$3.101 \quad \int \frac{x^3(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2} dx$$

Optimal. Leaf size=65

$$\frac{5x^2}{2} - \frac{17 \tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}} - \frac{25(x^2+3)}{8(x^4+2x^2+3)} - \frac{17}{4} \log(x^4+2x^2+3)$$

Rubi [A] time = 0.11, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {1663, 1660, 1657, 634, 618, 204, 628}

$$\frac{5x^2}{2} - \frac{25(x^2+3)}{8(x^4+2x^2+3)} - \frac{17}{4} \log(x^4+2x^2+3) - \frac{17 \tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(x^3*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]

[Out] (5*x^2)/2 - (25*(3 + x^2))/(8*(3 + 2*x^2 + x^4)) - (17*ArcTan[(1 + x^2)/Sqrt[2]])/(8*Sqrt[2]) - (17*Log[3 + 2*x^2 + x^4])/4

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[d*Log[RemoveContent[a + b*x + c*x^2, x]]/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1657

Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[Expand[Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 1660

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +

```
c*x^2, x], x, 1]], Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[p, -1]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3(4 + x^2 + 3x^4 + 5x^6)}{(3 + 2x^2 + x^4)^2} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x(4 + x + 3x^2 + 5x^3)}{(3 + 2x + x^2)^2} dx, x, x^2 \right) \\
&= -\frac{25(3 + x^2)}{8(3 + 2x^2 + x^4)} + \frac{1}{16} \text{Subst} \left(\int \frac{-50 - 56x + 40x^2}{3 + 2x + x^2} dx, x, x^2 \right) \\
&= -\frac{25(3 + x^2)}{8(3 + 2x^2 + x^4)} + \frac{1}{16} \text{Subst} \left(\int \left(40 - \frac{34(5 + 4x)}{3 + 2x + x^2} \right) dx, x, x^2 \right) \\
&= \frac{5x^2}{2} - \frac{25(3 + x^2)}{8(3 + 2x^2 + x^4)} - \frac{17}{8} \text{Subst} \left(\int \frac{5 + 4x}{3 + 2x + x^2} dx, x, x^2 \right) \\
&= \frac{5x^2}{2} - \frac{25(3 + x^2)}{8(3 + 2x^2 + x^4)} - \frac{17}{8} \text{Subst} \left(\int \frac{1}{3 + 2x + x^2} dx, x, x^2 \right) - \frac{17}{4} \text{Subst} \left(\int \frac{1}{3 + 2x + x^2} dx, x, x^2 \right) \\
&= \frac{5x^2}{2} - \frac{25(3 + x^2)}{8(3 + 2x^2 + x^4)} - \frac{17}{4} \log(3 + 2x^2 + x^4) + \frac{17}{4} \text{Subst} \left(\int \frac{1}{-8 - x^2} dx, x, x^2 \right) \\
&= \frac{5x^2}{2} - \frac{25(3 + x^2)}{8(3 + 2x^2 + x^4)} - \frac{17 \tan^{-1} \left(\frac{1+x^2}{\sqrt{2}} \right)}{8\sqrt{2}} - \frac{17}{4} \log(3 + 2x^2 + x^4)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 61, normalized size = 0.94

$$\frac{1}{16} \left(40x^2 - 17\sqrt{2} \tan^{-1} \left(\frac{x^2 + 1}{\sqrt{2}} \right) - \frac{50(x^2 + 3)}{x^4 + 2x^2 + 3} - 68 \log(x^4 + 2x^2 + 3) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] (40*x^2 - (50*(3 + x^2))/(3 + 2*x^2 + x^4) - 17*Sqrt[2]*ArcTan[(1 + x^2)/Sqrt[2]] - 68*Log[3 + 2*x^2 + x^4])/16
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(4 + x^2 + 3x^4 + 5x^6)}{(3 + 2x^2 + x^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^3*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]

[Out] IntegrateAlgebraic[(x^3*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2, x]

fricas [A] time = 1.00, size = 80, normalized size = 1.23

$$\frac{40x^6 + 80x^4 - 17\sqrt{2}(x^4 + 2x^2 + 3)\arctan\left(\frac{1}{2}\sqrt{2}(x^2 + 1)\right) + 70x^2 - 68(x^4 + 2x^2 + 3)\log(x^4 + 2x^2 + 3) - 150}{16(x^4 + 2x^2 + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")

[Out] 1/16*(40*x^6 + 80*x^4 - 17*sqrt(2)*(x^4 + 2*x^2 + 3)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 70*x^2 - 68*(x^4 + 2*x^2 + 3)*log(x^4 + 2*x^2 + 3) - 150)/(x^4 + 2*x^2 + 3)

giac [A] time = 1.18, size = 54, normalized size = 0.83

$$\frac{5}{2}x^2 - \frac{17}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x^2 + 1)\right) - \frac{25(x^2 + 3)}{8(x^4 + 2x^2 + 3)} - \frac{17}{4}\log(x^4 + 2x^2 + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")

[Out] 5/2*x^2 - 17/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 25/8*(x^2 + 3)/(x^4 + 2*x^2 + 3) - 17/4*log(x^4 + 2*x^2 + 3)

maple [A] time = 0.01, size = 59, normalized size = 0.91

$$\frac{5x^2}{2} - \frac{17\sqrt{2}\arctan\left(\frac{(2x^2+2)\sqrt{2}}{4}\right)}{16} - \frac{17\ln(x^4 + 2x^2 + 3)}{4} - \frac{\frac{25x^2}{4} + \frac{75}{4}}{2(x^4 + 2x^2 + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x)

[Out] 5/2*x^2-1/2*(25/4*x^2+75/4)/(x^4+2*x^2+3)-17/4*ln(x^4+2*x^2+3)-17/16*2^(1/2)*arctan(1/4*(2*x^2+2)*2^(1/2))

maxima [A] time = 1.72, size = 54, normalized size = 0.83

$$\frac{5}{2}x^2 - \frac{17}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x^2 + 1)\right) - \frac{25(x^2 + 3)}{8(x^4 + 2x^2 + 3)} - \frac{17}{4}\log(x^4 + 2x^2 + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")

[Out] 5/2*x^2 - 17/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 25/8*(x^2 + 3)/(x^4 + 2*x^2 + 3) - 17/4*log(x^4 + 2*x^2 + 3)

mupad [B] time = 0.92, size = 60, normalized size = 0.92

$$\frac{5x^2}{2} - \frac{\frac{25x^2}{8} + \frac{75}{8}}{x^4 + 2x^2 + 3} - \frac{17\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x^2}{2} + \frac{\sqrt{2}}{2}\right)}{16} - \frac{17\ln(x^4 + 2x^2 + 3)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^3*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^2,x)`

[Out] $(5x^2)/2 - ((25x^2)/8 + 75/8)/(2x^2 + x^4 + 3) - (17\sqrt{2} \operatorname{atan}(\sqrt{2}/2 + (\sqrt{2}x^2)/2))/16 - (17\log(2x^2 + x^4 + 3))/4$

sympy [A] time = 0.18, size = 68, normalized size = 1.05

$$\frac{5x^2}{2} + \frac{-25x^2 - 75}{8x^4 + 16x^2 + 24} - \frac{17\log(x^4 + 2x^2 + 3)}{4} - \frac{17\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}x^2}{2} + \frac{\sqrt{2}}{2}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)`

[Out] $5x^2/2 + (-25x^2 - 75)/(8x^4 + 16x^2 + 24) - 17\log(x^4 + 2x^2 + 3)/4 - 17\sqrt{2} \operatorname{atan}(\sqrt{2}x^2/2 + \sqrt{2}/2)/16$

$$3.102 \quad \int \frac{x(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2} dx$$

Optimal. Leaf size=58

$$-\frac{23 \tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}} + \frac{25(x^2+1)}{8(x^4+2x^2+3)} + \frac{5}{4} \log(x^4+2x^2+3)$$

Rubi [A] time = 0.07, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1663, 1660, 634, 618, 204, 628}

$$\frac{25(x^2+1)}{8(x^4+2x^2+3)} + \frac{5}{4} \log(x^4+2x^2+3) - \frac{23 \tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(x*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]

[Out] (25*(1 + x^2))/(8*(3 + 2*x^2 + x^4)) - (23*ArcTan[(1 + x^2)/Sqrt[2]])/(8*Sqrt[2]) + (5*Log[3 + 2*x^2 + x^4])/4

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[d*Log[RemoveContent[a + b*x + c*x^2, x]]/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1660

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(2*c*f - b*g), x], x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]

Rule 1663

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\begin{aligned} \int \frac{x(4 + x^2 + 3x^4 + 5x^6)}{(3 + 2x^2 + x^4)^2} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{4 + x + 3x^2 + 5x^3}{(3 + 2x + x^2)^2} dx, x, x^2 \right) \\ &= \frac{25(1 + x^2)}{8(3 + 2x^2 + x^4)} + \frac{1}{16} \text{Subst} \left(\int \frac{-6 + 40x}{3 + 2x + x^2} dx, x, x^2 \right) \\ &= \frac{25(1 + x^2)}{8(3 + 2x^2 + x^4)} + \frac{5}{4} \text{Subst} \left(\int \frac{2 + 2x}{3 + 2x + x^2} dx, x, x^2 \right) - \frac{23}{8} \text{Subst} \left(\int \frac{1}{3 + 2x + x^2} dx, x, x^2 \right) \\ &= \frac{25(1 + x^2)}{8(3 + 2x^2 + x^4)} + \frac{5}{4} \log(3 + 2x^2 + x^4) + \frac{23}{4} \text{Subst} \left(\int \frac{1}{-8 - x^2} dx, x, 2(1 + x^2) \right) \\ &= \frac{25(1 + x^2)}{8(3 + 2x^2 + x^4)} - \frac{23 \tan^{-1} \left(\frac{1+x^2}{\sqrt{2}} \right)}{8\sqrt{2}} + \frac{5}{4} \log(3 + 2x^2 + x^4) \end{aligned}$$

Mathematica [A] time = 0.02, size = 58, normalized size = 1.00

$$-\frac{23 \tan^{-1} \left(\frac{x^2+1}{\sqrt{2}} \right)}{8\sqrt{2}} + \frac{25(x^2+1)}{8(x^4+2x^2+3)} + \frac{5}{4} \log(x^4+2x^2+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] (25*(1 + x^2))/(8*(3 + 2*x^2 + x^4)) - (23*ArcTan[(1 + x^2)/Sqrt[2]])/(8*Sqrt[2]) + (5*Log[3 + 2*x^2 + x^4])/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(4 + x^2 + 3x^4 + 5x^6)}{(3 + 2x^2 + x^4)^2} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(x*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2, x]
```

fricas [A] time = 1.33, size = 70, normalized size = 1.21

$$\frac{23\sqrt{2}(x^4+2x^2+3)\arctan\left(\frac{1}{2}\sqrt{2}(x^2+1)\right)-50x^2-20(x^4+2x^2+3)\log(x^4+2x^2+3)-50}{16(x^4+2x^2+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```


[Out] $-1/16*(23*\sqrt{2}*(x^4 + 2*x^2 + 3)*\arctan(1/2*\sqrt{2}*(x^2 + 1)) - 50*x^2 - 20*(x^4 + 2*x^2 + 3)*\log(x^4 + 2*x^2 + 3) - 50)/(x^4 + 2*x^2 + 3)$

giac [A] time = 1.13, size = 49, normalized size = 0.84

$$-\frac{23}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x^2+1)\right)+\frac{25(x^2+1)}{8(x^4+2x^2+3)}+\frac{5}{4}\log(x^4+2x^2+3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")`

[Out] $-23/16*\sqrt{2}*\arctan(1/2*\sqrt{2}*(x^2 + 1)) + 25/8*(x^2 + 1)/(x^4 + 2*x^2 + 3) + 5/4*\log(x^4 + 2*x^2 + 3)$

maple [A] time = 0.01, size = 54, normalized size = 0.93

$$-\frac{23\sqrt{2}\arctan\left(\frac{(2x^2+2)\sqrt{2}}{4}\right)}{16}+\frac{5\ln(x^4+2x^2+3)}{4}+\frac{\frac{25x^2}{4}+\frac{25}{4}}{2x^4+4x^2+6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x)`

[Out] $1/2*(25/4*x^2+25/4)/(x^4+2*x^2+3)+5/4*\ln(x^4+2*x^2+3)-23/16*2^{(1/2)}*\arctan(1/4*(2*x^2+2)*2^{(1/2)})$

maxima [A] time = 1.41, size = 49, normalized size = 0.84

$$-\frac{23}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x^2+1)\right)+\frac{25(x^2+1)}{8(x^4+2x^2+3)}+\frac{5}{4}\log(x^4+2x^2+3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")`

[Out] $-23/16*\sqrt{2}*\arctan(1/2*\sqrt{2}*(x^2 + 1)) + 25/8*(x^2 + 1)/(x^4 + 2*x^2 + 3) + 5/4*\log(x^4 + 2*x^2 + 3)$

mupad [B] time = 0.05, size = 69, normalized size = 1.19

$$\frac{5\ln(x^4+2x^2+3)}{4}+\frac{25x^2}{8(x^4+2x^2+3)}+\frac{25}{8(x^4+2x^2+3)}-\frac{23\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^2,x)`

[Out] $(5*\log(2*x^2 + x^4 + 3))/4 + (25*x^2)/(8*(2*x^2 + x^4 + 3)) + 25/(8*(2*x^2 + x^4 + 3)) - (23*2^{(1/2)}*\operatorname{atan}(2^{(1/2)}/2 + (2^{(1/2)}*x^2)/2))/16$

sympy [A] time = 0.18, size = 60, normalized size = 1.03

$$\frac{25x^2+25}{8x^4+16x^2+24}+\frac{5\log(x^4+2x^2+3)}{4}-\frac{23\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)`

[Out] $(25*x**2 + 25)/(8*x**4 + 16*x**2 + 24) + 5*\log(x**4 + 2*x**2 + 3)/4 - 23*\sqrt{2}*\operatorname{atan}(\sqrt{2}*x**2/2 + \sqrt{2}/2)/16$

$$3.103 \quad \int \frac{4+x^2+3x^4+5x^6}{x(3+2x^2+x^4)^2} dx$$

Optimal. Leaf size=66

$$\frac{89 \tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{72\sqrt{2}} + \frac{25(1-x^2)}{24(x^4+2x^2+3)} - \frac{1}{9} \log(x^4+2x^2+3) + \frac{4 \log(x)}{9}$$

Rubi [A] time = 0.11, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {1663, 1646, 800, 634, 618, 204, 628}

$$\frac{25(1-x^2)}{24(x^4+2x^2+3)} - \frac{1}{9} \log(x^4+2x^2+3) + \frac{89 \tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{72\sqrt{2}} + \frac{4 \log(x)}{9}$$

Antiderivative was successfully verified.

[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x*(3 + 2*x^2 + x^4)^2), x]

[Out] (25*(1 - x^2))/(24*(3 + 2*x^2 + x^4)) + (89*ArcTan[(1 + x^2)/Sqrt[2]])/(72*Sqrt[2]) + (4*Log[x])/9 - Log[3 + 2*x^2 + x^4]/9

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 800

Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]

Rule 1646

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x

```

^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
+ 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
- ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x] /; FreeQ[{a, b, c, d,
e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]

```

Rule 1663

```

Int[(Pq_)*(x_)^(m_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]

```

Rubi steps

$$\begin{aligned}
\int \frac{4 + x^2 + 3x^4 + 5x^6}{x(3 + 2x^2 + x^4)^2} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{4 + x + 3x^2 + 5x^3}{x(3 + 2x + x^2)^2} dx, x, x^2 \right) \\
&= \frac{25(1 - x^2)}{24(3 + 2x^2 + x^4)} + \frac{1}{16} \text{Subst} \left(\int \frac{\frac{32}{3} + \frac{70x}{3}}{x(3 + 2x + x^2)} dx, x, x^2 \right) \\
&= \frac{25(1 - x^2)}{24(3 + 2x^2 + x^4)} + \frac{1}{16} \text{Subst} \left(\int \left(\frac{32}{9x} - \frac{2(-73 + 16x)}{9(3 + 2x + x^2)} \right) dx, x, x^2 \right) \\
&= \frac{25(1 - x^2)}{24(3 + 2x^2 + x^4)} + \frac{4 \log(x)}{9} - \frac{1}{72} \text{Subst} \left(\int \frac{-73 + 16x}{3 + 2x + x^2} dx, x, x^2 \right) \\
&= \frac{25(1 - x^2)}{24(3 + 2x^2 + x^4)} + \frac{4 \log(x)}{9} - \frac{1}{9} \text{Subst} \left(\int \frac{2 + 2x}{3 + 2x + x^2} dx, x, x^2 \right) + \frac{89}{72} \text{Subst} \left(\int \frac{1}{-8 - x^2} dx, x, x^2 \right) \\
&= \frac{25(1 - x^2)}{24(3 + 2x^2 + x^4)} + \frac{4 \log(x)}{9} - \frac{1}{9} \log(3 + 2x^2 + x^4) - \frac{89}{36} \text{Subst} \left(\int \frac{1}{-8 - x^2} dx, x, x^2 \right) \\
&= \frac{25(1 - x^2)}{24(3 + 2x^2 + x^4)} + \frac{89 \tan^{-1} \left(\frac{1+x^2}{\sqrt{2}} \right)}{72\sqrt{2}} + \frac{4 \log(x)}{9} - \frac{1}{9} \log(3 + 2x^2 + x^4)
\end{aligned}$$

Mathematica [C] time = 0.06, size = 93, normalized size = 1.41

$$\frac{1}{288} \left(-\sqrt{2} (16\sqrt{2} + 89i) \log(x^2 - i\sqrt{2} + 1) + \sqrt{2} (-16\sqrt{2} + 89i) \log(x^2 + i\sqrt{2} + 1) - \frac{300(x^2 - 1)}{x^4 + 2x^2 + 3} + 128 \log(x) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x*(3 + 2*x^2 + x^4)^2), x]
```

```
[Out] ((-300*(-1 + x^2))/(3 + 2*x^2 + x^4) + 128*Log[x] - Sqrt[2]*(89*I + 16*Sqrt[2])*Log[1 - I*Sqrt[2] + x^2] + Sqrt[2]*(89*I - 16*Sqrt[2])*Log[1 + I*Sqrt[2] + x^2])/288
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4 + x^2 + 3x^4 + 5x^6}{x(3 + 2x^2 + x^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(4 + x^2 + 3*x^4 + 5*x^6)/(x*(3 + 2*x^2 + x^4)^2), x]

[Out] IntegrateAlgebraic[(4 + x^2 + 3*x^4 + 5*x^6)/(x*(3 + 2*x^2 + x^4)^2), x]

fricas [A] time = 0.84, size = 84, normalized size = 1.27

$$\frac{89\sqrt{2}(x^4 + 2x^2 + 3)\arctan\left(\frac{1}{2}\sqrt{2}(x^2 + 1)\right) - 150x^2 - 16(x^4 + 2x^2 + 3)\log(x^4 + 2x^2 + 3) + 64(x^4 + 2x^2 + 3)\log(x) + 150}{144(x^4 + 2x^2 + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^6+3*x^4+x^2+4)/x/(x^4+2*x^2+3)^2,x, algorithm="fricas")

[Out] 1/144*(89*sqrt(2)*(x^4 + 2*x^2 + 3)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 150*x^2 - 16*(x^4 + 2*x^2 + 3)*log(x^4 + 2*x^2 + 3) + 64*(x^4 + 2*x^2 + 3)*log(x) + 150)/(x^4 + 2*x^2 + 3)

giac [A] time = 1.10, size = 62, normalized size = 0.94

$$\frac{89}{144}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x^2 + 1)\right) + \frac{8x^4 - 59x^2 + 99}{72(x^4 + 2x^2 + 3)} - \frac{1}{9}\log(x^4 + 2x^2 + 3) + \frac{2}{9}\log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^6+3*x^4+x^2+4)/x/(x^4+2*x^2+3)^2,x, algorithm="giac")

[Out] 89/144*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 1/72*(8*x^4 - 59*x^2 + 99)/(x^4 + 2*x^2 + 3) - 1/9*log(x^4 + 2*x^2 + 3) + 2/9*log(x^2)

maple [A] time = 0.01, size = 58, normalized size = 0.88

$$\frac{89\sqrt{2}\arctan\left(\frac{(2x^2+2)\sqrt{2}}{4}\right)}{144} + \frac{4\ln(x)}{9} - \frac{\ln(x^4 + 2x^2 + 3)}{9} - \frac{\frac{75x^2}{4} - \frac{75}{4}}{18(x^4 + 2x^2 + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^6+3*x^4+x^2+4)/x/(x^4+2*x^2+3)^2,x)

[Out] 4/9*ln(x)-1/18*(75/4*x^2-75/4)/(x^4+2*x^2+3)-1/9*ln(x^4+2*x^2+3)+89/144*2^(1/2)*arctan(1/4*(2*x^2+2)*2^(1/2))

maxima [A] time = 1.68, size = 55, normalized size = 0.83

$$\frac{89}{144}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x^2 + 1)\right) - \frac{25(x^2 - 1)}{24(x^4 + 2x^2 + 3)} - \frac{1}{9}\log(x^4 + 2x^2 + 3) + \frac{2}{9}\log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^6+3*x^4+x^2+4)/x/(x^4+2*x^2+3)^2,x, algorithm="maxima")

[Out] 89/144*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 25/24*(x^2 - 1)/(x^4 + 2*x^2 + 3) - 1/9*log(x^4 + 2*x^2 + 3) + 2/9*log(x^2)

mupad [B] time = 0.91, size = 59, normalized size = 0.89

$$\frac{4\ln(x)}{9} - \frac{\ln(x^4 + 2x^2 + 3)}{9} - \frac{\frac{25x^2}{24} - \frac{25}{24}}{x^4 + 2x^2 + 3} + \frac{89\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x^2}{2} + \frac{\sqrt{2}}{2}\right)}{144}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2 + 3*x^4 + 5*x^6 + 4)/(x*(2*x^2 + x^4 + 3)^2),x)`

[Out] $(4*\log(x))/9 - \log(2*x^2 + x^4 + 3)/9 - ((25*x^2)/24 - 25/24)/(2*x^2 + x^4 + 3) + (89*2^{(1/2)}*\operatorname{atan}(2^{(1/2)}/2 + (2^{(1/2)}*x^2)/2))/144$

sympy [A] time = 0.20, size = 65, normalized size = 0.98

$$\frac{25 - 25x^2}{24x^4 + 48x^2 + 72} + \frac{4\log(x)}{9} - \frac{\log(x^4 + 2x^2 + 3)}{9} + \frac{89\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}x^2}{2} + \frac{\sqrt{2}}{2}\right)}{144}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x**6+3*x**4+x**2+4)/x/(x**4+2*x**2+3)**2,x)`

[Out] $(25 - 25*x**2)/(24*x**4 + 48*x**2 + 72) + 4*\log(x)/9 - \log(x**4 + 2*x**2 + 3)/9 + 89*\operatorname{sqrt}(2)*\operatorname{atan}(\operatorname{sqrt}(2)*x**2/2 + \operatorname{sqrt}(2)/2)/144$

$$3.104 \quad \int \frac{4+x^2+3x^4+5x^6}{x^3(3+2x^2+x^4)^2} dx$$

Optimal. Leaf size=71

$$-\frac{2}{9x^2} - \frac{71 \tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{216\sqrt{2}} - \frac{25(x^2+5)}{72(x^4+2x^2+3)} + \frac{13}{108} \log(x^4+2x^2+3) - \frac{13 \log(x)}{27}$$

Rubi [A] time = 0.13, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {1663, 1646, 1628, 634, 618, 204, 628}

$$-\frac{25(x^2+5)}{72(x^4+2x^2+3)} - \frac{2}{9x^2} + \frac{13}{108} \log(x^4+2x^2+3) - \frac{71 \tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{216\sqrt{2}} - \frac{13 \log(x)}{27}$$

Antiderivative was successfully verified.

[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^3*(3 + 2*x^2 + x^4)^2), x]

[Out] -2/(9*x^2) - (25*(5 + x^2))/(72*(3 + 2*x^2 + x^4)) - (71*ArcTan[(1 + x^2)/Sqrt[2]])/(216*Sqrt[2]) - (13*Log[x])/27 + (13*Log[3 + 2*x^2 + x^4])/108

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1628

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 1646

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],

```
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]], Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
+ 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
- ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x] /; FreeQ[{a, b, c, d,
e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\begin{aligned} \int \frac{4 + x^2 + 3x^4 + 5x^6}{x^3(3 + 2x^2 + x^4)^2} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{4 + x + 3x^2 + 5x^3}{x^2(3 + 2x + x^2)^2} dx, x, x^2 \right) \\ &= -\frac{25(5 + x^2)}{72(3 + 2x^2 + x^4)} + \frac{1}{16} \text{Subst} \left(\int \frac{\frac{32}{3} - \frac{40x}{9} - \frac{50x^2}{9}}{x^2(3 + 2x + x^2)} dx, x, x^2 \right) \\ &= -\frac{25(5 + x^2)}{72(3 + 2x^2 + x^4)} + \frac{1}{16} \text{Subst} \left(\int \left(\frac{32}{9x^2} - \frac{104}{27x} + \frac{2(-19 + 52x)}{27(3 + 2x + x^2)} \right) dx, x, x^2 \right) \\ &= -\frac{2}{9x^2} - \frac{25(5 + x^2)}{72(3 + 2x^2 + x^4)} - \frac{13 \log(x)}{27} + \frac{1}{216} \text{Subst} \left(\int \frac{-19 + 52x}{3 + 2x + x^2} dx, x, x^2 \right) \\ &= -\frac{2}{9x^2} - \frac{25(5 + x^2)}{72(3 + 2x^2 + x^4)} - \frac{13 \log(x)}{27} + \frac{13}{108} \text{Subst} \left(\int \frac{2 + 2x}{3 + 2x + x^2} dx, x, x^2 \right) - \frac{71}{108} \log(3 + 2x + x^2) \\ &= -\frac{2}{9x^2} - \frac{25(5 + x^2)}{72(3 + 2x^2 + x^4)} - \frac{13 \log(x)}{27} + \frac{13}{108} \log(3 + 2x^2 + x^4) + \frac{71}{108} \text{Subst} \left(\int \frac{1}{3 + 2x + x^2} dx, x, x^2 \right) \\ &= -\frac{2}{9x^2} - \frac{25(5 + x^2)}{72(3 + 2x^2 + x^4)} - \frac{71 \tan^{-1} \left(\frac{1+x^2}{\sqrt{2}} \right)}{216\sqrt{2}} - \frac{13 \log(x)}{27} + \frac{13}{108} \log(3 + 2x^2 + x^4) \end{aligned}$$

Mathematica [C] time = 0.05, size = 97, normalized size = 1.37

$$\frac{1}{864} \left(-\frac{192}{x^2} + \sqrt{2} (52\sqrt{2} + 71i) \log(x^2 - i\sqrt{2} + 1) + \sqrt{2} (52\sqrt{2} - 71i) \log(x^2 + i\sqrt{2} + 1) - \frac{300(x^2 + 5)}{x^4 + 2x^2 + 3} - 416 \log(x) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^3*(3 + 2*x^2 + x^4)^2), x]
```

```
[Out] (-192/x^2 - (300*(5 + x^2))/(3 + 2*x^2 + x^4) - 416*Log[x] + Sqrt[2]*(71*I
+ 52*Sqrt[2])*Log[1 - I*Sqrt[2] + x^2] + Sqrt[2]*(-71*I + 52*Sqrt[2])*Log[1
+ I*Sqrt[2] + x^2])/864
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4 + x^2 + 3x^4 + 5x^6}{x^3(3 + 2x^2 + x^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(4 + x^2 + 3*x^4 + 5*x^6)/(x^3*(3 + 2*x^2 + x^4)^2), x]

[Out] IntegrateAlgebraic[(4 + x^2 + 3*x^4 + 5*x^6)/(x^3*(3 + 2*x^2 + x^4)^2), x]

fricas [A] time = 1.19, size = 105, normalized size = 1.48

$$\frac{246x^4 + 71\sqrt{2}(x^6 + 2x^4 + 3x^2)\arctan\left(\frac{1}{2}\sqrt{2}(x^2 + 1)\right) + 942x^2 - 52(x^6 + 2x^4 + 3x^2)\log(x^4 + 2x^2 + 3) + 208(x^6 + 2x^4 + 3x^2)\log(x) + 288}{432(x^6 + 2x^4 + 3x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^6+3*x^4+x^2+4)/x^3/(x^4+2*x^2+3)^2,x, algorithm="fricas")

[Out] -1/432*(246*x^4 + 71*sqrt(2)*(x^6 + 2*x^4 + 3*x^2)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 942*x^2 - 52*(x^6 + 2*x^4 + 3*x^2)*log(x^4 + 2*x^2 + 3) + 208*(x^6 + 2*x^4 + 3*x^2)*log(x) + 288)/(x^6 + 2*x^4 + 3*x^2)

giac [A] time = 1.07, size = 66, normalized size = 0.93

$$-\frac{71}{432}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x^2 + 1)\right) - \frac{41x^4 + 157x^2 + 48}{72(x^6 + 2x^4 + 3x^2)} + \frac{13}{108}\log(x^4 + 2x^2 + 3) - \frac{13}{54}\log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^6+3*x^4+x^2+4)/x^3/(x^4+2*x^2+3)^2,x, algorithm="giac")

[Out] -71/432*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 1/72*(41*x^4 + 157*x^2 + 48)/(x^6 + 2*x^4 + 3*x^2) + 13/108*log(x^4 + 2*x^2 + 3) - 13/54*log(x^2)

maple [A] time = 0.01, size = 63, normalized size = 0.89

$$-\frac{71\sqrt{2}\arctan\left(\frac{(2x^2+2)\sqrt{2}}{4}\right)}{432} - \frac{13\ln(x)}{27} + \frac{13\ln(x^4 + 2x^2 + 3)}{108} - \frac{2}{9x^2} + \frac{-\frac{75x^2}{4} - \frac{375}{4}}{54x^4 + 108x^2 + 162}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^6+3*x^4+x^2+4)/x^3/(x^4+2*x^2+3)^2,x)

[Out] -2/9/x^2-13/27*ln(x)+1/54*(-75/4*x^2-375/4)/(x^4+2*x^2+3)+13/108*ln(x^4+2*x^2+3)-71/432*2^(1/2)*arctan(1/4*(2*x^2+2)*2^(1/2))

maxima [A] time = 1.43, size = 66, normalized size = 0.93

$$-\frac{71}{432}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x^2 + 1)\right) - \frac{41x^4 + 157x^2 + 48}{72(x^6 + 2x^4 + 3x^2)} + \frac{13}{108}\log(x^4 + 2x^2 + 3) - \frac{13}{54}\log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^6+3*x^4+x^2+4)/x^3/(x^4+2*x^2+3)^2,x, algorithm="maxima")

[Out] -71/432*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 1/72*(41*x^4 + 157*x^2 + 48)/(x^6 + 2*x^4 + 3*x^2) + 13/108*log(x^4 + 2*x^2 + 3) - 13/54*log(x^2)

mpad [B] time = 0.06, size = 68, normalized size = 0.96

$$\frac{13\ln(x^4 + 2x^2 + 3)}{108} - \frac{13\ln(x)}{27} - \frac{\frac{41x^4}{72} + \frac{157x^2}{72} + \frac{2}{3}}{x^6 + 2x^4 + 3x^2} - \frac{71\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x^2}{2} + \frac{\sqrt{2}}{2}\right)}{432}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^3*(2*x^2 + x^4 + 3)^2),x)`

[Out] $(13 \log(2x^2 + x^4 + 3))/108 - (13 \log(x))/27 - ((157x^2)/72 + (41x^4)/72 + 2/3)/(3x^2 + 2x^4 + x^6) - (71 \cdot 2^{1/2} \cdot \operatorname{atan}(2^{1/2}/2 + (2^{1/2}x^2)/2))/432$

sympy [A] time = 0.21, size = 76, normalized size = 1.07

$$\frac{-41x^4 - 157x^2 - 48}{72x^6 + 144x^4 + 216x^2} - \frac{13 \log(x)}{27} + \frac{13 \log(x^4 + 2x^2 + 3)}{108} - \frac{71\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}x^2}{2} + \frac{\sqrt{2}}{2}\right)}{432}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x**6+3*x**4+x**2+4)/x**3/(x**4+2*x**2+3)**2,x)`

[Out] $(-41x^4 - 157x^2 - 48)/(72x^6 + 144x^4 + 216x^2) - 13 \log(x)/27 + 13 \log(x^4 + 2x^2 + 3)/108 - 71 \sqrt{2} \operatorname{atan}(\sqrt{2}x^2/2 + \sqrt{2}/2)/432$

$$3.105 \quad \int \frac{4+x^2+3x^4+5x^6}{x^5(3+2x^2+x^4)^2} dx$$

Optimal. Leaf size=80

$$-\frac{1}{9x^4} + \frac{13}{54x^2} + \frac{125 \tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{216\sqrt{2}} + \frac{25(5x^2+7)}{216(x^4+2x^2+3)} - \frac{13}{108} \log(x^4+2x^2+3) + \frac{13 \log(x)}{27}$$

Rubi [A] time = 0.14, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {1663, 1646, 1628, 634, 618, 204, 628}

$$\frac{25(5x^2+7)}{216(x^4+2x^2+3)} + \frac{13}{54x^2} - \frac{1}{9x^4} - \frac{13}{108} \log(x^4+2x^2+3) + \frac{125 \tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{216\sqrt{2}} + \frac{13 \log(x)}{27}$$

Antiderivative was successfully verified.

[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^5*(3 + 2*x^2 + x^4)^2), x]

[Out] -1/(9*x^4) + 13/(54*x^2) + (25*(7 + 5*x^2))/(216*(3 + 2*x^2 + x^4)) + (125*ArcTan[(1 + x^2)/Sqrt[2]])/(216*Sqrt[2]) + (13*Log[x])/27 - (13*Log[3 + 2*x^2 + x^4])/108

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1628

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 1646

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x

```

^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
+ 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
- ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x]] /; FreeQ[{a, b, c, d,
e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]

```

Rule 1663

```

Int[(Pq_)*(x_)^(m_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]

```

Rubi steps

$$\begin{aligned}
\int \frac{4 + x^2 + 3x^4 + 5x^6}{x^5(3 + 2x^2 + x^4)^2} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{4 + x + 3x^2 + 5x^3}{x^3(3 + 2x + x^2)^2} dx, x, x^2 \right) \\
&= \frac{25(7 + 5x^2)}{216(3 + 2x^2 + x^4)} + \frac{1}{16} \text{Subst} \left(\int \frac{\frac{32}{3} - \frac{40x}{9} + \frac{200x^2}{27} + \frac{250x^3}{27}}{x^3(3 + 2x + x^2)} dx, x, x^2 \right) \\
&= \frac{25(7 + 5x^2)}{216(3 + 2x^2 + x^4)} + \frac{1}{16} \text{Subst} \left(\int \left(\frac{32}{9x^3} - \frac{104}{27x^2} + \frac{104}{27x} - \frac{2(-73 + 52x)}{27(3 + 2x + x^2)} \right) dx, x, x^2 \right) \\
&= -\frac{1}{9x^4} + \frac{13}{54x^2} + \frac{25(7 + 5x^2)}{216(3 + 2x^2 + x^4)} + \frac{13 \log(x)}{27} - \frac{1}{216} \text{Subst} \left(\int \frac{-73 + 52x}{3 + 2x + x^2} dx, x, x^2 \right) \\
&= -\frac{1}{9x^4} + \frac{13}{54x^2} + \frac{25(7 + 5x^2)}{216(3 + 2x^2 + x^4)} + \frac{13 \log(x)}{27} - \frac{13}{108} \text{Subst} \left(\int \frac{2 + 2x}{3 + 2x + x^2} dx, x, x^2 \right) \\
&= -\frac{1}{9x^4} + \frac{13}{54x^2} + \frac{25(7 + 5x^2)}{216(3 + 2x^2 + x^4)} + \frac{13 \log(x)}{27} - \frac{13}{108} \log(3 + 2x^2 + x^4) - \frac{125}{108} \text{Subst} \left(\int \frac{1}{3 + 2x + x^2} dx, x, x^2 \right) \\
&= -\frac{1}{9x^4} + \frac{13}{54x^2} + \frac{25(7 + 5x^2)}{216(3 + 2x^2 + x^4)} + \frac{125 \tan^{-1} \left(\frac{1+x^2}{\sqrt{2}} \right)}{216\sqrt{2}} + \frac{13 \log(x)}{27} - \frac{13}{108} \log(3 + 2x^2 + x^4)
\end{aligned}$$

Mathematica [C] time = 0.06, size = 105, normalized size = 1.31

$$\frac{1}{864} \left(-\frac{96}{x^4} + \frac{208}{x^2} - \sqrt{2} (52\sqrt{2} + 125i) \log(x^2 - i\sqrt{2} + 1) + \sqrt{2} (-52\sqrt{2} + 125i) \log(x^2 + i\sqrt{2} + 1) + \frac{100(5x^2 + 7)}{x^4 + 2x^2 + 3} + 416 \log(x) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^5*(3 + 2*x^2 + x^4)^2), x]
```

```
[Out] (-96/x^4 + 208/x^2 + (100*(7 + 5*x^2))/(3 + 2*x^2 + x^4) + 416*Log[x] - Sqrt[2]*(125*I + 52*Sqrt[2])*Log[1 - I*Sqrt[2] + x^2] + Sqrt[2]*(125*I - 52*Sqrt[2])*Log[1 + I*Sqrt[2] + x^2])/864
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4 + x^2 + 3x^4 + 5x^6}{x^5(3 + 2x^2 + x^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(4 + x^2 + 3*x^4 + 5*x^6)/(x^5*(3 + 2*x^2 + x^4)^2), x]

[Out] IntegrateAlgebraic[(4 + x^2 + 3*x^4 + 5*x^6)/(x^5*(3 + 2*x^2 + x^4)^2), x]

fricas [A] time = 1.07, size = 110, normalized size = 1.38

$$\frac{354x^6 + 510x^4 + 125\sqrt{2}(x^8 + 2x^6 + 3x^4)\arctan\left(\frac{1}{2}\sqrt{2}(x^2 + 1)\right) + 216x^2 - 52(x^8 + 2x^6 + 3x^4)\log(x^4 + 2x^2 + 3) + 208(x^8 + 2x^6 + 3x^4)\log(x) - 144}{432(x^8 + 2x^6 + 3x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^6+3*x^4+x^2+4)/x^5/(x^4+2*x^2+3)^2,x, algorithm="fricas")

[Out] 1/432*(354*x^6 + 510*x^4 + 125*sqrt(2)*(x^8 + 2*x^6 + 3*x^4)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 216*x^2 - 52*(x^8 + 2*x^6 + 3*x^4)*log(x^4 + 2*x^2 + 3) + 208*(x^8 + 2*x^6 + 3*x^4)*log(x) - 144)/(x^8 + 2*x^6 + 3*x^4)

giac [A] time = 1.16, size = 79, normalized size = 0.99

$$\frac{125}{432}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x^2 + 1)\right) + \frac{26x^4 + 177x^2 + 253}{216(x^4 + 2x^2 + 3)} - \frac{39x^4 - 26x^2 + 12}{108x^4} - \frac{13}{108}\log(x^4 + 2x^2 + 3) + \frac{13}{54}\log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^6+3*x^4+x^2+4)/x^5/(x^4+2*x^2+3)^2,x, algorithm="giac")

[Out] 125/432*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 1/216*(26*x^4 + 177*x^2 + 253)/(x^4 + 2*x^2 + 3) - 1/108*(39*x^4 - 26*x^2 + 12)/x^4 - 13/108*log(x^4 + 2*x^2 + 3) + 13/54*log(x^2)

maple [A] time = 0.01, size = 68, normalized size = 0.85

$$\frac{125\sqrt{2}\arctan\left(\frac{(2x^2+2)\sqrt{2}}{4}\right)}{432} + \frac{13\ln(x)}{27} - \frac{13\ln(x^4 + 2x^2 + 3)}{108} + \frac{13}{54x^2} - \frac{1}{9x^4} - \frac{-\frac{125x^2}{4} - \frac{175}{4}}{54(x^4 + 2x^2 + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^6+3*x^4+x^2+4)/x^5/(x^4+2*x^2+3)^2,x)

[Out] -1/9/x^4+13/54/x^2+13/27*ln(x)-1/54*(-125/4*x^2-175/4)/(x^4+2*x^2+3)-13/108*ln(x^4+2*x^2+3)+125/432*2^(1/2)*arctan(1/4*(2*x^2+2)*2^(1/2))

maxima [A] time = 2.38, size = 71, normalized size = 0.89

$$\frac{125}{432}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x^2 + 1)\right) + \frac{59x^6 + 85x^4 + 36x^2 - 24}{72(x^8 + 2x^6 + 3x^4)} - \frac{13}{108}\log(x^4 + 2x^2 + 3) + \frac{13}{54}\log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^6+3*x^4+x^2+4)/x^5/(x^4+2*x^2+3)^2,x, algorithm="maxima")

[Out] 125/432*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 1/72*(59*x^6 + 85*x^4 + 36*x^2 - 24)/(x^8 + 2*x^6 + 3*x^4) - 13/108*log(x^4 + 2*x^2 + 3) + 13/54*log(x^2)

mupad [B] time = 0.06, size = 72, normalized size = 0.90

$$\frac{13\ln(x)}{27} - \frac{13\ln(x^4 + 2x^2 + 3)}{108} + \frac{\frac{59x^6}{72} + \frac{85x^4}{72} + \frac{x^2}{2} - \frac{1}{3}}{x^8 + 2x^6 + 3x^4} + \frac{125\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x^2}{2} + \frac{\sqrt{2}}{2}\right)}{432}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^5*(2*x^2 + x^4 + 3)^2),x)`

[Out] $(13*\log(x))/27 - (13*\log(2*x^2 + x^4 + 3))/108 + (x^2/2 + (85*x^4)/72 + (59*x^6)/72 - 1/3)/(3*x^4 + 2*x^6 + x^8) + (125*2^{(1/2)}*\operatorname{atan}(2^{(1/2)}/2 + (2^{(1/2)}*x^2)/2))/432$

sympy [A] time = 0.23, size = 80, normalized size = 1.00

$$\frac{13 \log(x)}{27} - \frac{13 \log(x^4 + 2x^2 + 3)}{108} + \frac{125\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}x^2}{2} + \frac{\sqrt{2}}{2}\right)}{432} + \frac{59x^6 + 85x^4 + 36x^2 - 24}{72x^8 + 144x^6 + 216x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x**6+3*x**4+x**2+4)/x**5/(x**4+2*x**2+3)**2,x)`

[Out] $13*\log(x)/27 - 13*\log(x**4 + 2*x**2 + 3)/108 + 125*\sqrt{2}*\operatorname{atan}(\sqrt{2}*x**2/2 + \sqrt{2}/2)/432 + (59*x**6 + 85*x**4 + 36*x**2 - 24)/(72*x**8 + 144*x**6 + 216*x**4)$

$$3.106 \quad \int \frac{4+x^2+3x^4+5x^6}{x^7(3+2x^2+x^4)^2} dx$$

Optimal. Leaf size=87

$$-\frac{2}{27x^6} + \frac{13}{108x^4} - \frac{13}{54x^2} - \frac{1237 \tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{1944\sqrt{2}} + \frac{25(1-7x^2)}{648(x^4+2x^2+3)} - \frac{61}{972} \log(x^4+2x^2+3) + \frac{61 \log(x)}{243}$$

Rubi [A] time = 0.15, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {1663, 1646, 1628, 634, 618, 204, 628}

$$\frac{25(1-7x^2)}{648(x^4+2x^2+3)} - \frac{13}{54x^2} + \frac{13}{108x^4} - \frac{2}{27x^6} - \frac{61}{972} \log(x^4+2x^2+3) - \frac{1237 \tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{1944\sqrt{2}} + \frac{61 \log(x)}{243}$$

Antiderivative was successfully verified.

[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^7*(3 + 2*x^2 + x^4)^2), x]

[Out] -2/(27*x^6) + 13/(108*x^4) - 13/(54*x^2) + (25*(1 - 7*x^2))/(648*(3 + 2*x^2 + x^4)) - (1237*ArcTan[(1 + x^2)/Sqrt[2]])/(1944*Sqrt[2]) + (61*Log[x])/243 - (61*Log[3 + 2*x^2 + x^4])/972

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1628

Int[(Pq_)*((d_.) + (e_.)*(x_)^m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^p_, x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 1646

Int[(Pq_)*((d_.) + (e_.)*(x_)^m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^p_, x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x

```

^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
+ 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m*
- ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x] /; FreeQ[{a, b, c, d,
e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]

```

Rule 1663

```

Int[(Pq_)*(x_)^(m_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]

```

Rubi steps

$$\begin{aligned}
\int \frac{4 + x^2 + 3x^4 + 5x^6}{x^7(3 + 2x^2 + x^4)^2} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{4 + x + 3x^2 + 5x^3}{x^4(3 + 2x + x^2)^2} dx, x, x^2 \right) \\
&= \frac{25(1 - 7x^2)}{648(3 + 2x^2 + x^4)} + \frac{1}{16} \text{Subst} \left(\int \frac{\frac{32}{3} - \frac{40x}{9} + \frac{200x^2}{27} + \frac{800x^3}{81} - \frac{350x^4}{81}}{x^4(3 + 2x + x^2)} dx, x, x^2 \right) \\
&= \frac{25(1 - 7x^2)}{648(3 + 2x^2 + x^4)} + \frac{1}{16} \text{Subst} \left(\int \left(\frac{32}{9x^4} - \frac{104}{27x^3} + \frac{104}{27x^2} + \frac{488}{243x} - \frac{2(1481 + 244x)}{243(3 + 2x + x^2)} \right) dx, x, x^2 \right) \\
&= -\frac{2}{27x^6} + \frac{13}{108x^4} - \frac{13}{54x^2} + \frac{25(1 - 7x^2)}{648(3 + 2x^2 + x^4)} + \frac{61 \log(x)}{243} - \frac{\text{Subst} \left(\int \frac{1481 + 244x}{3 + 2x + x^2} dx, x, x^2 \right)}{1944} \\
&= -\frac{2}{27x^6} + \frac{13}{108x^4} - \frac{13}{54x^2} + \frac{25(1 - 7x^2)}{648(3 + 2x^2 + x^4)} + \frac{61 \log(x)}{243} - \frac{61}{972} \text{Subst} \left(\int \frac{2 + x}{3 + 2x + x^2} dx, x, x^2 \right) \\
&= -\frac{2}{27x^6} + \frac{13}{108x^4} - \frac{13}{54x^2} + \frac{25(1 - 7x^2)}{648(3 + 2x^2 + x^4)} + \frac{61 \log(x)}{243} - \frac{61}{972} \log(3 + 2x^2 + x^4) \\
&= -\frac{2}{27x^6} + \frac{13}{108x^4} - \frac{13}{54x^2} + \frac{25(1 - 7x^2)}{648(3 + 2x^2 + x^4)} - \frac{1237 \tan^{-1} \left(\frac{1+x^2}{\sqrt{2}} \right)}{1944\sqrt{2}} + \frac{61 \log(x)}{243}
\end{aligned}$$

Mathematica [C] time = 0.07, size = 110, normalized size = 1.26

$$\frac{-\frac{576}{x^6} + \frac{936}{x^4} - \frac{1872}{x^2} + \sqrt{2}(-244\sqrt{2} + 1237i) \log(x^2 - i\sqrt{2} + 1) - \sqrt{2}(244\sqrt{2} + 1237i) \log(x^2 + i\sqrt{2} + 1) - \frac{300(7x^2 - 1)}{x^4 + 2x^2 + 3} + 1952 \log(x)}{7776}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^7*(3 + 2*x^2 + x^4)^2), x]
```

```
[Out] (-576/x^6 + 936/x^4 - 1872/x^2 - (300*(-1 + 7*x^2))/(3 + 2*x^2 + x^4) + 195
2*Log[x] + Sqrt[2]*(1237*I - 244*Sqrt[2])*Log[1 - I*Sqrt[2] + x^2] - Sqrt[2
]*(1237*I + 244*Sqrt[2])*Log[1 + I*Sqrt[2] + x^2])/7776
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4 + x^2 + 3x^4 + 5x^6}{x^7(3 + 2x^2 + x^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(4 + x^2 + 3*x^4 + 5*x^6)/(x^7*(3 + 2*x^2 + x^4)^2), x]

[Out] IntegrateAlgebraic[(4 + x^2 + 3*x^4 + 5*x^6)/(x^7*(3 + 2*x^2 + x^4)^2), x]

fricas [A] time = 1.21, size = 115, normalized size = 1.32

$$\frac{1986x^8 + 1254x^6 + 2160x^4 + 1237\sqrt{2}(x^{10} + 2x^8 + 3x^6)\arctan\left(\frac{1}{2}\sqrt{2}(x^2 + 1)\right) - 828x^2 + 244(x^{10} + 2x^8 + 3x^6)\log(x^4 + 2x^2 + 3) - 976(x^{10} + 2x^8 + 3x^6)\log(x) + 864}{3888(x^{10} + 2x^8 + 3x^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^6+3*x^4+x^2+4)/x^7/(x^4+2*x^2+3)^2,x, algorithm="fricas")

[Out] -1/3888*(1986*x^8 + 1254*x^6 + 2160*x^4 + 1237*sqrt(2)*(x^10 + 2*x^8 + 3*x^6)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 828*x^2 + 244*(x^10 + 2*x^8 + 3*x^6)*log(x^4 + 2*x^2 + 3) - 976*(x^10 + 2*x^8 + 3*x^6)*log(x) + 864)/(x^10 + 2*x^8 + 3*x^6)

giac [A] time = 1.17, size = 84, normalized size = 0.97

$$-\frac{1237}{3888}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x^2 + 1)\right) + \frac{122x^4 - 281x^2 + 441}{1944(x^4 + 2x^2 + 3)} - \frac{671x^6 + 702x^4 - 351x^2 + 216}{2916x^6} - \frac{61}{972}\log(x^4 + 2x^2 + 3) + \frac{61}{486}\log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^6+3*x^4+x^2+4)/x^7/(x^4+2*x^2+3)^2,x, algorithm="giac")

[Out] -1237/3888*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 1/1944*(122*x^4 - 281*x^2 + 441)/(x^4 + 2*x^2 + 3) - 1/2916*(671*x^6 + 702*x^4 - 351*x^2 + 216)/x^6 - 61/972*log(x^4 + 2*x^2 + 3) + 61/486*log(x^2)

maple [A] time = 0.01, size = 73, normalized size = 0.84

$$-\frac{1237\sqrt{2}\arctan\left(\frac{(2x^2+2)\sqrt{2}}{4}\right)}{3888} + \frac{61\ln(x)}{243} - \frac{61\ln(x^4 + 2x^2 + 3)}{972} - \frac{13}{54x^2} + \frac{13}{108x^4} - \frac{2}{27x^6} - \frac{\frac{525x^2}{4} - \frac{75}{4}}{486(x^4 + 2x^2 + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^6+3*x^4+x^2+4)/x^7/(x^4+2*x^2+3)^2,x)

[Out] -2/27/x^6+13/108/x^4-13/54/x^2+61/243*ln(x)-1/486*(525/4*x^2-75/4)/(x^4+2*x^2+3)-61/972*ln(x^4+2*x^2+3)-1237/3888*2^(1/2)*arctan(1/4*(2*x^2+2)*2^(1/2))

maxima [A] time = 2.47, size = 76, normalized size = 0.87

$$-\frac{1237}{3888}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x^2 + 1)\right) - \frac{331x^8 + 209x^6 + 360x^4 - 138x^2 + 144}{648(x^{10} + 2x^8 + 3x^6)} - \frac{61}{972}\log(x^4 + 2x^2 + 3) + \frac{61}{486}\log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^6+3*x^4+x^2+4)/x^7/(x^4+2*x^2+3)^2,x, algorithm="maxima")

[Out] -1237/3888*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 1/648*(331*x^8 + 209*x^6 + 360*x^4 - 138*x^2 + 144)/(x^10 + 2*x^8 + 3*x^6) - 61/972*log(x^4 + 2*x^2 + 3) + 61/486*log(x^2)

mupad [B] time = 0.07, size = 78, normalized size = 0.90

$$\frac{61\ln(x)}{243} - \frac{61\ln(x^4 + 2x^2 + 3)}{972} - \frac{\frac{331x^8}{648} + \frac{209x^6}{648} + \frac{5x^4}{9} - \frac{23x^2}{108} + \frac{2}{9}}{x^{10} + 2x^8 + 3x^6} - \frac{1237\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x^2}{2} + \frac{\sqrt{2}}{2}\right)}{3888}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^7*(2*x^2 + x^4 + 3)^2),x)`

[Out] $(61*\log(x))/243 - (61*\log(2*x^2 + x^4 + 3))/972 - ((5*x^4)/9 - (23*x^2)/108 + (209*x^6)/648 + (331*x^8)/648 + 2/9)/(3*x^6 + 2*x^8 + x^{10}) - (1237*2^{(1/2)}*atan(2^{(1/2)}/2 + (2^{(1/2)}*x^2)/2))/3888$

sympy [A] time = 0.24, size = 85, normalized size = 0.98

$$\frac{61 \log(x)}{243} - \frac{61 \log(x^4 + 2x^2 + 3)}{972} - \frac{1237\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}x^2}{2} + \frac{\sqrt{2}}{2}\right)}{3888} + \frac{-331x^8 - 209x^6 - 360x^4 + 138x^2 - 144}{648x^{10} + 1296x^8 + 1944x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x**6+3*x**4+x**2+4)/x**7/(x**4+2*x**2+3)**2,x)`

[Out] $61*\log(x)/243 - 61*\log(x**4 + 2*x**2 + 3)/972 - 1237*\sqrt{2}*atan(\sqrt{2}*x**2/2 + \sqrt{2}/2)/3888 + (-331*x**8 - 209*x**6 - 360*x**4 + 138*x**2 - 144)/(648*x**10 + 1296*x**8 + 1944*x**6)$

$$3.107 \quad \int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2} dx$$

Optimal. Leaf size=248

$$\frac{5x^7}{7} - \frac{17x^5}{5} + \frac{19x^3}{3} - \frac{1}{32} \sqrt{\frac{1}{2}(618291\sqrt{3} - 262771)} \log\left(x^2 - \sqrt{2(\sqrt{3}-1)}x + \sqrt{3}\right) + \frac{1}{32} \sqrt{\frac{1}{2}(618291\sqrt{3} - 262771)}$$

Rubi [A] time = 0.34, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {1668, 1676, 1169, 634, 618, 204, 628}

$$\frac{5x^7}{7} - \frac{17x^5}{5} + \frac{19x^3}{3} + \frac{25(5x^2+3)x}{8(x^2+2x^2+3)} - \frac{1}{32} \sqrt{\frac{1}{2}(618291\sqrt{3} - 262771)} \log(x^2 - \sqrt{2(\sqrt{3}-1)}x + \sqrt{3}) + \frac{1}{32} \sqrt{\frac{1}{2}(618291\sqrt{3} - 262771)} \log(x^2 + \sqrt{2(\sqrt{3}-1)}x + \sqrt{3}) + 38x + \frac{1}{16} \sqrt{\frac{1}{2}(262771 + 618291\sqrt{3})} \tan^{-1}\left(\frac{\sqrt{2(\sqrt{3}-1)} - 2x}{\sqrt{2(1+\sqrt{3})}}\right) - \frac{1}{16} \sqrt{\frac{1}{2}(262771 + 618291\sqrt{3})} \tan^{-1}\left(\frac{2x + \sqrt{2(\sqrt{3}-1)}}{\sqrt{2(1+\sqrt{3})}}\right)$$

Antiderivative was successfully verified.

[In] Int[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]

[Out] 38*x + (19*x^3)/3 - (17*x^5)/5 + (5*x^7)/7 + (25*x*(3 + 5*x^2))/(8*(3 + 2*x^2 + x^4)) + (Sqrt[(262771 + 618291*Sqrt[3])/2]*ArcTan[(Sqrt[2*(-1 + Sqrt[3]])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/16 - (Sqrt[(262771 + 618291*Sqrt[3])/2]*ArcTan[(Sqrt[2*(-1 + Sqrt[3]])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/16 - (Sqrt[(-262771 + 618291*Sqrt[3])/2]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]]*x + x^2))/32 + (Sqrt[(-262771 + 618291*Sqrt[3])/2]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]]*x + x^2))/32

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1169

Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]

Rule 1668

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :=
  With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
    e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
  x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
  2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
  nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
  mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
  + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x]] /; FreeQ[{a, b,
  c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &&
  & LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := Int[ExpandInte
  grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
  2] && Expon[Pq, x^2] > 1
```

Rubi steps

$$\begin{aligned}
 \int \frac{x^8(4 + x^2 + 3x^4 + 5x^6)}{(3 + 2x^2 + x^4)^2} dx &= \frac{25x(3 + 5x^2)}{8(3 + 2x^2 + x^4)} + \frac{1}{48} \int \frac{-450 - 1650x^2 + 1200x^4 - 336x^8 + 240x^{10}}{3 + 2x^2 + x^4} dx \\
 &= \frac{25x(3 + 5x^2)}{8(3 + 2x^2 + x^4)} + \frac{1}{48} \int \left(1824 + 912x^2 - 816x^4 + 240x^6 - \frac{6(987 + 1339x^2)}{3 + 2x^2 + x^4} \right) dx \\
 &= 38x + \frac{19x^3}{3} - \frac{17x^5}{5} + \frac{5x^7}{7} + \frac{25x(3 + 5x^2)}{8(3 + 2x^2 + x^4)} - \frac{1}{8} \int \frac{987 + 1339x^2}{3 + 2x^2 + x^4} dx \\
 &= 38x + \frac{19x^3}{3} - \frac{17x^5}{5} + \frac{5x^7}{7} + \frac{25x(3 + 5x^2)}{8(3 + 2x^2 + x^4)} - \frac{\int \frac{987\sqrt{2(-1+\sqrt{3})} - (987-1339\sqrt{3})}{\sqrt{3} - \sqrt{2(-1+\sqrt{3})}x + x^2}}{16\sqrt{6}(-1 + \sqrt{3})} dx \\
 &= 38x + \frac{19x^3}{3} - \frac{17x^5}{5} + \frac{5x^7}{7} + \frac{25x(3 + 5x^2)}{8(3 + 2x^2 + x^4)} - \frac{1}{32} (1339 + 329\sqrt{3}) \int \frac{1}{\sqrt{3} - \sqrt{2(-1+\sqrt{3})}x + x^2}} dx \\
 &= 38x + \frac{19x^3}{3} - \frac{17x^5}{5} + \frac{5x^7}{7} + \frac{25x(3 + 5x^2)}{8(3 + 2x^2 + x^4)} - \frac{1}{32} \sqrt{\frac{1}{2}(-262771 + 618291\sqrt{3})} \int \frac{1}{\sqrt{3} - \sqrt{2(-1+\sqrt{3})}x + x^2}} dx \\
 &= 38x + \frac{19x^3}{3} - \frac{17x^5}{5} + \frac{5x^7}{7} + \frac{25x(3 + 5x^2)}{8(3 + 2x^2 + x^4)} + \frac{1}{16} \sqrt{\frac{1}{2}(262771 + 618291\sqrt{3})} \int \frac{1}{\sqrt{3} - \sqrt{2(-1+\sqrt{3})}x + x^2}} dx
 \end{aligned}$$

Mathematica [C] time = 0.17, size = 145, normalized size = 0.58

$$\frac{5x^7}{7} - \frac{17x^5}{5} + \frac{19x^3}{3} + \frac{25(5x^2 + 3)x}{8(x^4 + 2x^2 + 3)} + 38x - \frac{(1339\sqrt{2} + 352i) \tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{16\sqrt{2-2i\sqrt{2}}} - \frac{(1339\sqrt{2} - 352i) \tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{16\sqrt{2+2i\sqrt{2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] 38*x + (19*x^3)/3 - (17*x^5)/5 + (5*x^7)/7 + (25*x*(3 + 5*x^2))/(8*(3 + 2*x
^2 + x^4)) - ((352*I + 1339*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/(16*Sqr
```

$t[2 - (2*I)*\text{Sqrt}[2]] - ((-352*I + 1339*\text{Sqrt}[2])* \text{ArcTan}[x/\text{Sqrt}[1 + I*\text{Sqrt}[2]]]) / (16*\text{Sqrt}[2 + (2*I)*\text{Sqrt}[2]])$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8(4 + x^2 + 3x^4 + 5x^6)}{(3 + 2x^2 + x^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]

[Out] IntegrateAlgebraic[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2, x]

fricas [B] time = 1.31, size = 519, normalized size = 2.09

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")

[Out] 1/338902147590720*(242072962564800*x^11 - 668121376678848*x^9 + 568064552152064*x^7 + 13714240239171136*x^5 - 102773860*14158657803^(1/4)*sqrt(68699)*sqrt(3)*sqrt(2)*(x^4 + 2*x^2 + 3)*sqrt(262771*sqrt(3) + 1854873)*arctan(1/3145089554732313026311937382*sqrt(50431867201)*14158657803^(3/4)*sqrt(68699)*sqrt(3*14158657803^(1/4)*sqrt(68699)*(1339*sqrt(3)*x - 987*x)*sqrt(262771*sqrt(3) + 1854873) + 453886804809*x^2 + 453886804809*sqrt(3))*(329*sqrt(3)*sqrt(2) - 1339*sqrt(2))*sqrt(262771*sqrt(3) + 1854873) - 1/20787713069048994*14158657803^(3/4)*sqrt(68699)*(329*sqrt(3)*sqrt(2)*x - 1339*sqrt(2)*x)*sqrt(262771*sqrt(3) + 1854873) + 1/2*sqrt(3)*sqrt(2) - 1/2*sqrt(2) - 102773860*14158657803^(1/4)*sqrt(68699)*sqrt(3)*sqrt(2)*(x^4 + 2*x^2 + 3)*sqrt(262771*sqrt(3) + 1854873)*arctan(1/3145089554732313026311937382*sqrt(50431867201)*14158657803^(3/4)*sqrt(68699)*sqrt(-3*14158657803^(1/4)*sqrt(68699)*(1339*sqrt(3)*x - 987*x)*sqrt(262771*sqrt(3) + 1854873) + 453886804809*x^2 + 453886804809*sqrt(3))*(329*sqrt(3)*sqrt(2) - 1339*sqrt(2))*sqrt(262771*sqrt(3) + 1854873) - 1/20787713069048994*14158657803^(3/4)*sqrt(68699)*(329*sqrt(3)*sqrt(2)*x - 1339*sqrt(2)*x)*sqrt(262771*sqrt(3) + 1854873) - 1/2*sqrt(3)*sqrt(2) + 1/2*sqrt(2) + 35*14158657803^(1/4)*sqrt(68699)*(1854873*x^4 + 3709746*x^2 - 262771*sqrt(3)*(x^4 + 2*x^2 + 3) + 5564619)*sqrt(262771*sqrt(3) + 1854873)*log(3*14158657803^(1/4)*sqrt(68699)*(1339*sqrt(3)*x - 987*x)*sqrt(262771*sqrt(3) + 1854873) + 453886804809*x^2 + 453886804809*sqrt(3)) - 35*14158657803^(1/4)*sqrt(68699)*(1854873*x^4 + 3709746*x^2 - 262771*sqrt(3)*(x^4 + 2*x^2 + 3) + 5564619)*sqrt(262771*sqrt(3) + 1854873)*log(-3*14158657803^(1/4)*sqrt(68699)*(1339*sqrt(3)*x - 987*x)*sqrt(262771*sqrt(3) + 1854873) + 453886804809*x^2 + 453886804809*sqrt(3)) + 37491050077223400*x^3 + 41812052459005080*x)/(x^4 + 2*x^2 + 3)

giac [B] time = 1.89, size = 585, normalized size = 2.36

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")

[Out] 5/7*x^7 - 17/5*x^5 + 19/3*x^3 + 1/20736*sqrt(2)*(1339*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 24102*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 24102*3^(3/4)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) + 1339*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 35532*3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) + 35532*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3)

) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) + 1/20736*sqrt(2)*(1339*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 24102*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 24102*3^(3/4)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) + 1339*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 35532*3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) + 35532*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3/4)*(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) + 1/41472*sqrt(2)*(24102*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 1339*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 1339*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 24102*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 35532*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) - 35532*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 + 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) - 1/41472*sqrt(2)*(24102*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 1339*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 1339*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 24102*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 35532*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) - 35532*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 - 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) + 38*x + 25/8*(5*x^3 + 3*x)/(x^4 + 2*x^2 + 3)

maple [B] time = 0.12, size = 427, normalized size = 1.72

$$\frac{5}{7}x^7 - \frac{17}{5}x^5 + \frac{19}{3}x^3 + 38x + \frac{25(5x^3 + 3x)}{8(x^4 + 2x^2 + 3)} - \frac{1}{8} \int \frac{1339x^2 + 987}{x^4 + 2x^2 + 3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x)

[Out] 5/7*x^7-17/5*x^5+19/3*x^3+38*x-(-125/8*x^3-75/8*x)/(x^4+2*x^2+3)-505/64*ln(x^2+3^(1/2))-x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)*3^(1/2)-11/4*ln(x^2+3^(1/2))-x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)-505/32/(2+2*3^(1/2))^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))*3^(1/2)-11/2/(2+2*3^(1/2))^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))-329/8/(2+2*3^(1/2))^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*3^(1/2)+505/64*ln(x^2+3^(1/2))+x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)*3^(1/2)+11/4*ln(x^2+3^(1/2))+x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)-505/32/(2+2*3^(1/2))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))*3^(1/2)-11/2/(2+2*3^(1/2))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))-329/8/(2+2*3^(1/2))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*3^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{5}{7}x^7 - \frac{17}{5}x^5 + \frac{19}{3}x^3 + 38x + \frac{25(5x^3 + 3x)}{8(x^4 + 2x^2 + 3)} - \frac{1}{8} \int \frac{1339x^2 + 987}{x^4 + 2x^2 + 3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")

[Out] 5/7*x^7 - 17/5*x^5 + 19/3*x^3 + 38*x + 25/8*(5*x^3 + 3*x)/(x^4 + 2*x^2 + 3) - 1/8*integrate((1339*x^2 + 987)/(x^4 + 2*x^2 + 3), x)

mupad [B] time = 0.11, size = 171, normalized size = 0.69

$$38x + \frac{125x^3 + 75x}{8x^4 + 2x^2 + 3} + \frac{19x^3}{3} - \frac{17x^5}{5} + \frac{5x^7}{7} + \frac{\operatorname{atan}\left(\frac{\sqrt{-262771 - \sqrt{2}734099i}734099i}{64\left(\frac{1112159985}{64} + \sqrt{2}72455713i\right)} + \frac{734099\sqrt{2} + \sqrt{-262771 - \sqrt{2}734099i}}{128\left(\frac{1112159985}{64} + \sqrt{2}72455713i\right)}\right)\sqrt{-262771 - \sqrt{2}734099i} + \operatorname{atan}\left(\frac{\sqrt{-262771 + \sqrt{2}734099i}734099i}{64\left(\frac{1112159985}{64} + \sqrt{2}72455713i\right)} - \frac{734099\sqrt{2} + \sqrt{-262771 + \sqrt{2}734099i}}{128\left(\frac{1112159985}{64} + \sqrt{2}72455713i\right)}\right)\sqrt{-262771 + \sqrt{2}734099i}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^8*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^2,x)

[Out] 38*x + (atan((x*(-2^(1/2)*734099i - 262771)^(1/2)*734099i)/(64*((2^(1/2)*72455713i)/128 - 1112159985/64)) + (734099*2^(1/2)*x*(-2^(1/2)*734099i - 2

$$\begin{aligned} & \frac{62771^{1/2}}{(128*((2^{1/2}*724555713i)/128 - 1112159985/64))} * (-2^{1/2} * \\ & 734099i - 262771^{1/2} * 1i) / 16 - (\operatorname{atan}((x*(2^{1/2}*734099i - 262771)^{1/2} * \\ & 734099i) / (64*((2^{1/2}*724555713i)/128 + 1112159985/64)) - (734099 * 2^{1/2} * \\ & x*(2^{1/2}*734099i - 262771)^{1/2})) / (128*((2^{1/2}*724555713i)/128 + 111215 \\ & 9985/64))) * (2^{1/2} * 734099i - 262771)^{1/2} * 1i) / 16 + ((75*x) / 8 + (125*x^3) / \\ & 8) / (2*x^2 + x^4 + 3) + (19*x^3) / 3 - (17*x^5) / 5 + (5*x^7) / 7 \end{aligned}$$

sympy [A] time = 0.61, size = 71, normalized size = 0.29

$$\frac{5x^7}{7} - \frac{17x^5}{5} + \frac{19x^3}{3} + 38x + \frac{125x^3 + 75x}{8x^4 + 16x^2 + 24} + \operatorname{RootSum}\left(1048576t^4 + 538155008t^2 + 1146851282043, \left(t \mapsto t \log\left(-\frac{16547840t^3}{453886804809} - \frac{11974973632t}{453886804809} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**8*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)

[Out] 5*x**7/7 - 17*x**5/5 + 19*x**3/3 + 38*x + (125*x**3 + 75*x)/(8*x**4 + 16*x**2 + 24) + RootSum(1048576*_t**4 + 538155008*_t**2 + 1146851282043, Lambda(_t, _t*log(-16547840*_t**3/453886804809 - 11974973632*_t/453886804809 + x))

$$3.108 \quad \int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2} dx$$

Optimal. Leaf size=237

$$x^5 - \frac{17x^3}{3} + \frac{3}{32} \sqrt{\frac{3}{2} (8669 + 5011\sqrt{3})} \log\left(x^2 - \sqrt{2(\sqrt{3}-1)}x + \sqrt{3}\right) - \frac{3}{32} \sqrt{\frac{3}{2} (8669 + 5011\sqrt{3})} \log\left(x^2 + \sqrt{2(\sqrt{3}-1)}x + \sqrt{3}\right)$$

Rubi [A] time = 0.29, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 31, number of rules / integrand size = 0.226, Rules used = {1668, 1676, 1169, 634, 618, 204, 628}

$$x^5 - \frac{17x^3}{3} + \frac{25(3-x^2)x}{8(3+2x^2+3)} + \frac{3}{32} \sqrt{\frac{3}{2} (8669 + 5011\sqrt{3})} \log\left(x^2 - \sqrt{2(\sqrt{3}-1)}x + \sqrt{3}\right) - \frac{3}{32} \sqrt{\frac{3}{2} (8669 + 5011\sqrt{3})} \log\left(x^2 + \sqrt{2(\sqrt{3}-1)}x + \sqrt{3}\right) + 19x + \frac{3}{16} \sqrt{\frac{3}{2} (5011\sqrt{3} - 8669)} \tan^{-1}\left(\frac{\sqrt{2(\sqrt{3}-1)} - 2x}{\sqrt{2(1+\sqrt{3})}}\right) - \frac{3}{16} \sqrt{\frac{3}{2} (5011\sqrt{3} - 8669)} \tan^{-1}\left(\frac{2x + \sqrt{2(\sqrt{3}-1)}}{\sqrt{2(1+\sqrt{3})}}\right)$$

Antiderivative was successfully verified.

[In] Int[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]

[Out] 19*x - (17*x^3)/3 + x^5 + (25*x*(3 - x^2))/(8*(3 + 2*x^2 + x^4)) + (3*sqrt[(3*(-8669 + 5011*sqrt(3)))/2]*ArcTan[(sqrt[2*(-1 + sqrt(3))]] - 2*x)/sqrt[2*(1 + sqrt(3))])/16 - (3*sqrt[(3*(-8669 + 5011*sqrt(3)))/2]*ArcTan[(sqrt[2*(-1 + sqrt(3))] + 2*x)/sqrt[2*(1 + sqrt(3))]])/16 + (3*sqrt[(3*(8669 + 5011*sqrt(3)))/2]*Log[sqrt(3) - sqrt[2*(-1 + sqrt(3))]*x + x^2])/32 - (3*sqrt[(3*(8669 + 5011*sqrt(3)))/2]*Log[sqrt(3) + sqrt[2*(-1 + sqrt(3))]*x + x^2])/32

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1169

Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]

Rule 1668

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :=
  With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
  e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
  x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
  2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
  nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
  mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
  + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x] /; FreeQ[{a, b,
  c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
  & LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := Int[ExpandInte
  grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
  2] && Expon[Pq, x^2] > 1
```

Rubi steps

$$\begin{aligned}
 \int \frac{x^6(4 + x^2 + 3x^4 + 5x^6)}{(3 + 2x^2 + x^4)^2} dx &= \frac{25x(3 - x^2)}{8(3 + 2x^2 + x^4)} + \frac{1}{48} \int \frac{-450 + 1050x^2 - 336x^6 + 240x^8}{3 + 2x^2 + x^4} dx \\
 &= \frac{25x(3 - x^2)}{8(3 + 2x^2 + x^4)} + \frac{1}{48} \int \left(912 - 816x^2 + 240x^4 - \frac{54(59 - 31x^2)}{3 + 2x^2 + x^4} \right) dx \\
 &= 19x - \frac{17x^3}{3} + x^5 + \frac{25x(3 - x^2)}{8(3 + 2x^2 + x^4)} - \frac{9}{8} \int \frac{59 - 31x^2}{3 + 2x^2 + x^4} dx \\
 &= 19x - \frac{17x^3}{3} + x^5 + \frac{25x(3 - x^2)}{8(3 + 2x^2 + x^4)} - \frac{1}{32} \left(3\sqrt{3(1 + \sqrt{3})} \right) \int \frac{59\sqrt{2(-1 + \sqrt{3})}}{\sqrt{3} - \sqrt{2(-1 + \sqrt{3})}} dx \\
 &= 19x - \frac{17x^3}{3} + x^5 + \frac{25x(3 - x^2)}{8(3 + 2x^2 + x^4)} - \frac{1}{16} \left(3\sqrt{\frac{3}{2}(3182 - 1829\sqrt{3})} \right) \int \frac{1}{\sqrt{3} - \sqrt{2(-1 + \sqrt{3})}} dx \\
 &= 19x - \frac{17x^3}{3} + x^5 + \frac{25x(3 - x^2)}{8(3 + 2x^2 + x^4)} + \frac{3}{32} \sqrt{\frac{3}{2}(8669 + 5011\sqrt{3})} \log \left(\sqrt{3} - \sqrt{2(-1 + \sqrt{3})} \right) \\
 &= 19x - \frac{17x^3}{3} + x^5 + \frac{25x(3 - x^2)}{8(3 + 2x^2 + x^4)} + \frac{3}{16} \sqrt{\frac{3}{2}(-8669 + 5011\sqrt{3})} \tan^{-1} \left(\frac{\sqrt{2(-1 + \sqrt{3})}}{\sqrt{2}} \right)
 \end{aligned}$$

Mathematica [C] time = 0.16, size = 132, normalized size = 0.56

$$x^5 - \frac{17x^3}{3} - \frac{25(x^2 - 3)x}{8(x^4 + 2x^2 + 3)} + 19x + \frac{9(31\sqrt{2} + 90i) \tan^{-1} \left(\frac{x}{\sqrt{1-i\sqrt{2}}} \right)}{16\sqrt{2 - 2i\sqrt{2}}} + \frac{9(31\sqrt{2} - 90i) \tan^{-1} \left(\frac{x}{\sqrt{1+i\sqrt{2}}} \right)}{16\sqrt{2 + 2i\sqrt{2}}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2, x]

[Out] 19*x - (17*x^3)/3 + x^5 - (25*x*(-3 + x^2))/(8*(3 + 2*x^2 + x^4)) + (9*(90*I + 31*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/(16*Sqrt[2 - (2*I)*Sqrt[2]])

+ (9*(-90*I + 31*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/(16*Sqrt[2 + (2*I)*Sqrt[2]])

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6(4 + x^2 + 3x^4 + 5x^6)}{(3 + 2x^2 + x^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]

[Out] IntegrateAlgebraic[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2, x]

fricas [B] time = 1.30, size = 476, normalized size = 2.01

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")

[Out] 1/287671488*(287671488*x^9 - 1054795456*x^7 + 3068495872*x^5 + 3588*677973267^(1/4)*sqrt(3)*sqrt(2)*(x^4 + 2*x^2 + 3)*sqrt(-43440359*sqrt(3) + 75330363)*arctan(1/1822344999502852422*677973267^(3/4)*sqrt(4494867)*sqrt(4494867*x^2 + 677973267^(1/4)*(31*sqrt(3)*x + 59*x)*sqrt(-43440359*sqrt(3) + 75330363) + 4494867*sqrt(3))*(59*sqrt(3)*sqrt(2) + 93*sqrt(2))*sqrt(-43440359*sqrt(3) + 75330363) - 1/405428013666*677973267^(3/4)*(59*sqrt(3)*sqrt(2)*x + 93*sqrt(2)*x)*sqrt(-43440359*sqrt(3) + 75330363) - 1/2*sqrt(3)*sqrt(2) + 1/2*sqrt(2) + 3588*677973267^(1/4)*sqrt(3)*sqrt(2)*(x^4 + 2*x^2 + 3)*sqrt(-43440359*sqrt(3) + 75330363)*arctan(1/1822344999502852422*677973267^(3/4)*sqrt(4494867)*sqrt(4494867*x^2 - 677973267^(1/4)*(31*sqrt(3)*x + 59*x)*sqrt(-43440359*sqrt(3) + 75330363) + 4494867*sqrt(3))*(59*sqrt(3)*sqrt(2) + 93*sqrt(2))*sqrt(-43440359*sqrt(3) + 75330363) - 1/405428013666*677973267^(3/4)*(59*sqrt(3)*sqrt(2)*x + 93*sqrt(2)*x)*sqrt(-43440359*sqrt(3) + 75330363) + 1/2*sqrt(3)*sqrt(2) - 1/2*sqrt(2) + 5142127848*x^3 - 3*677973267^(1/4)*(15033*x^4 + 30066*x^2 + 8669*sqrt(3)*(x^4 + 2*x^2 + 3) + 45099)*sqrt(-43440359*sqrt(3) + 75330363)*log(4494867*x^2 + 677973267^(1/4)*(31*sqrt(3)*x + 59*x)*sqrt(-43440359*sqrt(3) + 75330363) + 4494867*sqrt(3)) + 3*677973267^(1/4)*(15033*x^4 + 30066*x^2 + 8669*sqrt(3)*(x^4 + 2*x^2 + 3) + 45099)*sqrt(-43440359*sqrt(3) + 75330363)*log(4494867*x^2 - 677973267^(1/4)*(31*sqrt(3)*x + 59*x)*sqrt(-43440359*sqrt(3) + 75330363) + 4494867*sqrt(3)) + 19094195016*x)/(x^4 + 2*x^2 + 3)

giac [B] time = 1.85, size = 576, normalized size = 2.43

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")

[Out] x^5 - 17/3*x^3 - 1/2304*sqrt(2)*(31*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 558*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 558*3^(3/4)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) + 31*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 2124*3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) - 2124*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) - 1/2304*sqrt(2)*(31*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 558*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 558*3^(3/4)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) + 31*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 2124*3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) - 2124*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan

$n(1/3*3^{(3/4)}*(x - 3^{(1/4)}*\sqrt{-1/6*\sqrt{3} + 1/2}))/\sqrt{1/6*\sqrt{3} + 1/2}) - 1/4608*\sqrt{2}*(558*3^{(3/4)}*\sqrt{2}*(\sqrt{3} + 3)*\sqrt{-6*\sqrt{3} + 18}) - 31*3^{(3/4)}*\sqrt{2}*(-6*\sqrt{3} + 18)^{(3/2)} + 31*3^{(3/4)}*(6*\sqrt{3} + 18)^{(3/2)} + 558*3^{(3/4)}*\sqrt{2}*(6*\sqrt{3} + 18)*(\sqrt{3} - 3) + 2124*3^{(1/4)}*\sqrt{2}*\sqrt{-6*\sqrt{3} + 18} + 2124*3^{(1/4)}*\sqrt{2}*(6*\sqrt{3} + 18))*\log(x^2 + 2*3^{(1/4)}*x*\sqrt{-1/6*\sqrt{3} + 1/2} + \sqrt{3})) + 1/4608*\sqrt{2}*(558*3^{(3/4)}*\sqrt{2}*(\sqrt{3} + 3)*\sqrt{-6*\sqrt{3} + 18}) - 31*3^{(3/4)}*\sqrt{2}*(-6*\sqrt{3} + 18)^{(3/2)} + 31*3^{(3/4)}*(6*\sqrt{3} + 18)^{(3/2)} + 558*3^{(3/4)}*\sqrt{2}*(6*\sqrt{3} + 18)*(\sqrt{3} - 3) + 2124*3^{(1/4)}*\sqrt{2}*\sqrt{-6*\sqrt{3} + 18} + 2124*3^{(1/4)}*\sqrt{2}*(6*\sqrt{3} + 18))*\log(x^2 - 2*3^{(1/4)}*x*\sqrt{-1/6*\sqrt{3} + 1/2} + \sqrt{3})) + 19*x - 25/8*(x^3 - 3*x)/(x^4 + 2*x^2 + 3)$

maple [B] time = 0.03, size = 419, normalized size = 1.77

$$x^5 - \frac{17}{3}x^3 + 19x - \frac{25(x^3 - 3x)}{8(x^4 + 2x^2 + 3)} + \frac{9}{8} \int \frac{31x^2 - 59}{x^4 + 2x^2 + 3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x)

[Out] $x^5 - 17/3*x^3 + 19*x + (-25/8*x^3 + 75/8*x)/(x^4 + 2*x^2 + 3) + 57/16*(-2 + 2*3^{(1/2)})^{(1/2)}*3^{(1/2)}*\ln(x^2 - (-2 + 2*3^{(1/2)})^{(1/2)}*x + 3^{(1/2)}) + 405/64*(-2 + 2*3^{(1/2)})^{(1/2)}*3^{(1/2)}*\ln(x^2 - (-2 + 2*3^{(1/2)})^{(1/2)}*x + 3^{(1/2)}) + 57/8/(2 + 2*3^{(1/2)})^{(1/2)}*(-2 + 2*3^{(1/2)})^{(1/2)}*3^{(1/2)}*\arctan((2*x - (-2 + 2*3^{(1/2)})^{(1/2)})^{(1/2)})/(2 + 2*3^{(1/2)})^{(1/2)}) + 405/32/(2 + 2*3^{(1/2)})^{(1/2)}*(-2 + 2*3^{(1/2)})^{(1/2)}*3^{(1/2)}*\arctan((2*x - (-2 + 2*3^{(1/2)})^{(1/2)})^{(1/2)})/(2 + 2*3^{(1/2)})^{(1/2)}) - 177/8/(2 + 2*3^{(1/2)})^{(1/2)}*3^{(1/2)}*\arctan((2*x - (-2 + 2*3^{(1/2)})^{(1/2)})^{(1/2)})/(2 + 2*3^{(1/2)})^{(1/2)}) - 57/16*(-2 + 2*3^{(1/2)})^{(1/2)}*3^{(1/2)}*\ln(x^2 + (-2 + 2*3^{(1/2)})^{(1/2)}*x + 3^{(1/2)}) - 405/64*(-2 + 2*3^{(1/2)})^{(1/2)}*3^{(1/2)}*\ln(x^2 + (-2 + 2*3^{(1/2)})^{(1/2)}*x + 3^{(1/2)}) + 57/8/(2 + 2*3^{(1/2)})^{(1/2)}*(-2 + 2*3^{(1/2)})^{(1/2)}*3^{(1/2)}*\arctan((2*x + (-2 + 2*3^{(1/2)})^{(1/2)})^{(1/2)})/(2 + 2*3^{(1/2)})^{(1/2)}) + 405/32/(2 + 2*3^{(1/2)})^{(1/2)}*(-2 + 2*3^{(1/2)})^{(1/2)}*3^{(1/2)}*\arctan((2*x + (-2 + 2*3^{(1/2)})^{(1/2)})^{(1/2)})/(2 + 2*3^{(1/2)})^{(1/2)}) - 177/8/(2 + 2*3^{(1/2)})^{(1/2)}*3^{(1/2)}*\arctan((2*x + (-2 + 2*3^{(1/2)})^{(1/2)})^{(1/2)})/(2 + 2*3^{(1/2)})^{(1/2)})$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$x^5 - \frac{17}{3}x^3 + 19x - \frac{25(x^3 - 3x)}{8(x^4 + 2x^2 + 3)} + \frac{9}{8} \int \frac{31x^2 - 59}{x^4 + 2x^2 + 3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")

[Out] $x^5 - 17/3*x^3 + 19*x - 25/8*(x^3 - 3*x)/(x^4 + 2*x^2 + 3) + 9/8*\integrate((31*x^2 - 59)/(x^4 + 2*x^2 + 3), x)$

mupad [B] time = 0.94, size = 164, normalized size = 0.69

$$19x + \frac{75x}{x^4 + 2x^2 + 3} - \frac{25x^3}{8} - \frac{17x^3}{3} + x^5 - \frac{\operatorname{atan}\left(\frac{x\sqrt{26007 - \sqrt{2}897i}24219i}{64\left(\frac{1380483}{16} + \sqrt{2}4286763i\right)} - \frac{24219\sqrt{2}x\sqrt{26007 - \sqrt{2}897i}}{128\left(\frac{1380483}{16} + \sqrt{2}4286763i\right)}\right)\sqrt{26007 - \sqrt{2}897i}3i}{16} + \frac{\operatorname{atan}\left(\frac{x\sqrt{26007 + \sqrt{2}897i}24219i}{64\left(\frac{1380483}{16} + \sqrt{2}4286763i\right)} + \frac{24219\sqrt{2}x\sqrt{26007 + \sqrt{2}897i}}{128\left(\frac{1380483}{16} + \sqrt{2}4286763i\right)}\right)\sqrt{26007 + \sqrt{2}897i}3i}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^6*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^2,x)

[Out] $19*x + ((75*x)/8 - (25*x^3)/8)/(2*x^2 + x^4 + 3) - (\operatorname{atan}((x*(26007 - 2^{(1/2)})*897i)^{(1/2)}*24219i)/(64*((2^{(1/2)})*4286763i)/128 - 1380483/16)) - (24219*2^{(1/2)}*x*(26007 - 2^{(1/2)}*897i)^{(1/2)})/(128*((2^{(1/2)})*4286763i)/128 - 1380483/16))*(26007 - 2^{(1/2)}*897i)^{(1/2)}*3i/16 + (\operatorname{atan}((x*(2^{(1/2)})*897i + 26007)^{(1/2)}*24219i)/(64*((2^{(1/2)})*4286763i)/128 + 1380483/16)) + (24219*2^{(1/2)}*x*(2^{(1/2)}*897i + 26007)^{(1/2)})/(128*((2^{(1/2)})*4286763i)/128 + 1380483/16))*(2^{(1/2)}*897i + 26007)^{(1/2)}*3i/16 - (17*x^3)/3 + x^5$

sympy [B] time = 1.36, size = 1205, normalized size = 5.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**6*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)

[Out] $x^5 - 17x^3/3 + 19x + (-25x^3 + 75x)/(8x^4 + 16x^2 + 24) - 3\sqrt{26007/2048 + 15033\sqrt{3}/2048}\log(x^2 + x(-304\sqrt{2})\sqrt{8669 + 5011\sqrt{3}}/299 - 433349\sqrt{6}\sqrt{8669 + 5011\sqrt{3}}/1498289 + 152\sqrt{3}\sqrt{8669 + 5011\sqrt{3}})\sqrt{43440359\sqrt{3} + 75240962}/1498289) - 2882918249387\sqrt{2}\sqrt{43440359\sqrt{3} + 75240962}/2244869927521 - 993398584\sqrt{6}\sqrt{43440359\sqrt{3} + 75240962}/1343965233 + 49936376949404567/2244869927521 + 17261871038090\sqrt{3}/1343965233) + 3\sqrt{26007/2048 + 15033\sqrt{3}/2048}\log(x^2 + x(-152\sqrt{3})\sqrt{8669 + 5011\sqrt{3}})\sqrt{43440359\sqrt{3} + 75240962}/1498289 + 433349\sqrt{6}\sqrt{8669 + 5011\sqrt{3}}/1498289 + 304\sqrt{2}\sqrt{8669 + 5011\sqrt{3}}/299) - 2882918249387\sqrt{2}\sqrt{43440359\sqrt{3} + 75240962}/2244869927521 - 993398584\sqrt{6}\sqrt{43440359\sqrt{3} + 75240962}/1343965233 + 49936376949404567/2244869927521 + 17261871038090\sqrt{3}/1343965233) - 2\sqrt{-27}\sqrt{2}\sqrt{43440359\sqrt{3} + 75240962}/1024 + 234063/2048 + 405891\sqrt{3}/2048)\operatorname{atan}(2996578\sqrt{3}x/(17641\sqrt{2})\sqrt{-2}\sqrt{2}\sqrt{43440359\sqrt{3} + 75240962} + 8669 + 15033\sqrt{3})) + 152\sqrt{43440359\sqrt{3} + 75240962}\sqrt{-2}\sqrt{2}\sqrt{43440359\sqrt{3} + 75240962} + 8669 + 15033\sqrt{3})) - 1523344\sqrt{6}\sqrt{8669 + 5011\sqrt{3}}/(17641\sqrt{2})\sqrt{-2}\sqrt{2}\sqrt{43440359\sqrt{3} + 75240962} + 8669 + 15033\sqrt{3}) + 152\sqrt{43440359\sqrt{3} + 75240962}\sqrt{-2}\sqrt{2}\sqrt{43440359\sqrt{3} + 75240962} + 8669 + 15033\sqrt{3})) - 1300047\sqrt{2}\sqrt{8669 + 5011\sqrt{3}}/(17641\sqrt{2})\sqrt{-2}\sqrt{2}\sqrt{43440359\sqrt{3} + 75240962} + 8669 + 15033\sqrt{3}) + 152\sqrt{43440359\sqrt{3} + 75240962}\sqrt{-2}\sqrt{2}\sqrt{43440359\sqrt{3} + 75240962} + 8669 + 15033\sqrt{3})) + 456\sqrt{8669 + 5011\sqrt{3}})\sqrt{43440359\sqrt{3} + 75240962}/(17641\sqrt{2})\sqrt{-2}\sqrt{2}\sqrt{43440359\sqrt{3} + 75240962} + 8669 + 15033\sqrt{3}) + 152\sqrt{43440359\sqrt{3} + 75240962}\sqrt{-2}\sqrt{2}\sqrt{43440359\sqrt{3} + 75240962} + 8669 + 15033\sqrt{3})) - 2\sqrt{-27}\sqrt{2}\sqrt{43440359\sqrt{3} + 75240962}/1024 + 234063/2048 + 405891\sqrt{3}/2048)\operatorname{atan}(2996578\sqrt{3}x/(17641\sqrt{2})\sqrt{-2}\sqrt{2}\sqrt{43440359\sqrt{3} + 75240962} + 8669 + 15033\sqrt{3})) + 152\sqrt{43440359\sqrt{3} + 75240962}\sqrt{-2}\sqrt{2}\sqrt{43440359\sqrt{3} + 75240962} + 8669 + 15033\sqrt{3})) - 456\sqrt{8669 + 5011\sqrt{3}})\sqrt{43440359\sqrt{3} + 75240962}/(17641\sqrt{2})\sqrt{-2}\sqrt{2}\sqrt{43440359\sqrt{3} + 75240962} + 8669 + 15033\sqrt{3}) + 152\sqrt{43440359\sqrt{3} + 75240962}\sqrt{-2}\sqrt{2}\sqrt{43440359\sqrt{3} + 75240962} + 8669 + 15033\sqrt{3})) + 1300047\sqrt{2}\sqrt{8669 + 5011\sqrt{3}}/(17641\sqrt{2})\sqrt{-2}\sqrt{2}\sqrt{43440359\sqrt{3} + 75240962} + 8669 + 15033\sqrt{3})) + 152\sqrt{43440359\sqrt{3} + 75240962}\sqrt{-2}\sqrt{2}\sqrt{43440359\sqrt{3} + 75240962} + 8669 + 15033\sqrt{3})) + 1523344\sqrt{6}\sqrt{8669 + 5011\sqrt{3}}/(17641\sqrt{2})\sqrt{-2}\sqrt{2}\sqrt{43440359\sqrt{3} + 75240962} + 8669 + 15033\sqrt{3}) + 152\sqrt{43440359\sqrt{3} + 75240962}\sqrt{-2}\sqrt{2}\sqrt{43440359\sqrt{3} + 75240962} + 8669 + 15033\sqrt{3}))$

$$3.109 \quad \int \frac{x^4(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2} dx$$

Optimal. Leaf size=232

$$\frac{5x^3}{3} - \frac{1}{32} \sqrt{\frac{1}{2} (26499\sqrt{3} - 14395)} \log\left(x^2 - \sqrt{2(\sqrt{3}-1)}x + \sqrt{3}\right) + \frac{1}{32} \sqrt{\frac{1}{2} (26499\sqrt{3} - 14395)} \log\left(x^2 + \sqrt{2(\sqrt{3}-1)}x + \sqrt{3}\right)$$

Rubi [A] time = 0.29, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {1668, 1676, 1169, 634, 618, 204, 628}

$$\frac{5x^3}{3} - \frac{25(x^2+3)x}{8(x^2+2x+3)} - \frac{1}{32} \sqrt{\frac{1}{2} (26499\sqrt{3} - 14395)} \log\left(x^2 - \sqrt{2(\sqrt{3}-1)}x + \sqrt{3}\right) + \frac{1}{32} \sqrt{\frac{1}{2} (26499\sqrt{3} - 14395)} \log\left(x^2 + \sqrt{2(\sqrt{3}-1)}x + \sqrt{3}\right) - 17x - \frac{1}{16} \sqrt{\frac{1}{2} (14395 + 26499\sqrt{3})} \tan^{-1}\left(\frac{\sqrt{2(\sqrt{3}-1)} - 2x}{\sqrt{2(x+\sqrt{3})}}\right) + \frac{1}{16} \sqrt{\frac{1}{2} (14395 + 26499\sqrt{3})} \tan^{-1}\left(\frac{2x + \sqrt{2(\sqrt{3}-1)}}{\sqrt{2(x+\sqrt{3})}}\right)$$

Antiderivative was successfully verified.

[In] Int[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]

[Out] -17*x + (5*x^3)/3 - (25*x*(3 + x^2))/(8*(3 + 2*x^2 + x^4)) - (Sqrt[(14395 + 26499*Sqrt[3])/2]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])]) - 2*x]/Sqrt[2*(1 + Sqrt[3])])]/16 + (Sqrt[(14395 + 26499*Sqrt[3])/2]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])]) + 2*x]/Sqrt[2*(1 + Sqrt[3])])]/16 - (Sqrt[(-14395 + 26499*Sqrt[3])/2]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/32 + (Sqrt[(-14395 + 26499*Sqrt[3])/2]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/32

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1169

Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]

Rule 1668

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :=
  With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
    e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
  x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
  2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
  nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
  mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
  + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x] /; FreeQ[{a, b,
  c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &&
  & LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := Int[ExpandInte
  grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
  2] && Expon[Pq, x^2] > 1
```

Rubi steps

$$\begin{aligned}
 \int \frac{x^4(4 + x^2 + 3x^4 + 5x^6)}{(3 + 2x^2 + x^4)^2} dx &= -\frac{25x(3 + x^2)}{8(3 + 2x^2 + x^4)} + \frac{1}{48} \int \frac{450 - 150x^2 - 336x^4 + 240x^6}{3 + 2x^2 + x^4} dx \\
 &= -\frac{25x(3 + x^2)}{8(3 + 2x^2 + x^4)} + \frac{1}{48} \int \left(-816 + 240x^2 + \frac{6(483 + 127x^2)}{3 + 2x^2 + x^4} \right) dx \\
 &= -17x + \frac{5x^3}{3} - \frac{25x(3 + x^2)}{8(3 + 2x^2 + x^4)} + \frac{1}{8} \int \frac{483 + 127x^2}{3 + 2x^2 + x^4} dx \\
 &= -17x + \frac{5x^3}{3} - \frac{25x(3 + x^2)}{8(3 + 2x^2 + x^4)} + \frac{\int \frac{483\sqrt{2(-1+\sqrt{3})} - (483-127\sqrt{3})x}{\sqrt{3}-\sqrt{2(-1+\sqrt{3})}x+x^2} dx}{16\sqrt{6(-1+\sqrt{3})}} + \frac{\int \frac{483\sqrt{2(-1-\sqrt{3})} - (483-127\sqrt{3})x}{\sqrt{3}+\sqrt{2(-1-\sqrt{3})}x+x^2} dx}{16\sqrt{6(-1-\sqrt{3})}} \\
 &= -17x + \frac{5x^3}{3} - \frac{25x(3 + x^2)}{8(3 + 2x^2 + x^4)} + \frac{1}{32} (127 + 161\sqrt{3}) \int \frac{1}{\sqrt{3} - \sqrt{2(-1 + \sqrt{3})}x} dx \\
 &= -17x + \frac{5x^3}{3} - \frac{25x(3 + x^2)}{8(3 + 2x^2 + x^4)} - \frac{1}{32} \sqrt{\frac{1}{2}(-14395 + 26499\sqrt{3})} \log\left(\sqrt{3} - \sqrt{2(-1 + \sqrt{3})}x\right) \\
 &= -17x + \frac{5x^3}{3} - \frac{25x(3 + x^2)}{8(3 + 2x^2 + x^4)} - \frac{1}{16} \sqrt{\frac{1}{2}(14395 + 26499\sqrt{3})} \tan^{-1}\left(\frac{\sqrt{2(-1 - \sqrt{3})}x}{\sqrt{2(-1 - \sqrt{3})}}\right)
 \end{aligned}$$

Mathematica [C] time = 0.16, size = 129, normalized size = 0.56

$$\frac{5x^3}{3} - \frac{25(x^2 + 3)x}{8(x^4 + 2x^2 + 3)} - 17x + \frac{(127\sqrt{2} - 356i) \tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{16\sqrt{2-2i\sqrt{2}}} + \frac{(127\sqrt{2} + 356i) \tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{16\sqrt{2+2i\sqrt{2}}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]

```
[Out] -17*x + (5*x^3)/3 - (25*x*(3 + x^2))/(8*(3 + 2*x^2 + x^4)) + ((-356*I + 127
*sqrt(2))*ArcTan[x/Sqrt[1 - I*sqrt(2)]])/(16*sqrt(2 - (2*I)*sqrt(2))) + ((3
56*I + 127*sqrt(2))*ArcTan[x/Sqrt[1 + I*sqrt(2)]])/(16*sqrt(2 + (2*I)*sqrt(
2]))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4(4 + x^2 + 3x^4 + 5x^6)}{(3 + 2x^2 + x^4)^2} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2, x]
```

fricas [B] time = 1.38, size = 508, normalized size = 2.19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] 1/1295793216*(2159655360*x^7 - 17709173952*x^5 - 123268*143883^(1/4)*sqrt(2
19)*sqrt(3)*sqrt(2)*(x^4 + 2*x^2 + 3)*sqrt(14395*sqrt(3) + 79497)*arctan(1/
658350237832613766*sqrt(24746051)*143883^(3/4)*sqrt(219)*sqrt(11*143883^(1/
4)*sqrt(219)*(127*sqrt(3)*x - 483*x)*sqrt(14395*sqrt(3) + 79497) + 22271445
9*x^2 + 222714459*sqrt(3))*(161*sqrt(3)*sqrt(2) - 127*sqrt(2))*sqrt(14395*s
qrt(3) + 79497) - 1/8868084822*143883^(3/4)*sqrt(219)*(161*sqrt(3)*sqrt(2)*
x - 127*sqrt(2)*x)*sqrt(14395*sqrt(3) + 79497) + 1/2*sqrt(3)*sqrt(2) - 1/2*
sqrt(2) - 123268*143883^(1/4)*sqrt(219)*sqrt(3)*sqrt(2)*(x^4 + 2*x^2 + 3)*
sqrt(14395*sqrt(3) + 79497)*arctan(1/658350237832613766*sqrt(24746051)*1438
83^(3/4)*sqrt(219)*sqrt(-11*143883^(1/4)*sqrt(219)*(127*sqrt(3)*x - 483*x)*
sqrt(14395*sqrt(3) + 79497) + 222714459*x^2 + 222714459*sqrt(3))*(161*sqrt(
3)*sqrt(2) - 127*sqrt(2))*sqrt(14395*sqrt(3) + 79497) - 1/8868084822*143883
^(3/4)*sqrt(219)*(161*sqrt(3)*sqrt(2)*x - 127*sqrt(2)*x)*sqrt(14395*sqrt(3)
+ 79497) - 1/2*sqrt(3)*sqrt(2) + 1/2*sqrt(2) - 143883^(1/4)*sqrt(219)*(79
497*x^4 + 158994*x^2 - 14395*sqrt(3)*(x^4 + 2*x^2 + 3) + 238491)*sqrt(14395
*sqrt(3) + 79497)*log(11*143883^(1/4)*sqrt(219)*(127*sqrt(3)*x - 483*x)*sqr
t(14395*sqrt(3) + 79497) + 222714459*x^2 + 222714459*sqrt(3)) + 143883^(1/4
)*sqrt(219)*(79497*x^4 + 158994*x^2 - 14395*sqrt(3)*(x^4 + 2*x^2 + 3) + 238
491)*sqrt(14395*sqrt(3) + 79497)*log(-11*143883^(1/4)*sqrt(219)*(127*sqrt(3
)*x - 483*x)*sqrt(14395*sqrt(3) + 79497) + 222714459*x^2 + 222714459*sqrt(3
)) - 41627357064*x^3 - 78233515416*x)/(x^4 + 2*x^2 + 3)
```

giac [B] time = 1.85, size = 573, normalized size = 2.47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] 5/3*x^3 - 1/20736*sqrt(2)*(127*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 228
6*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 2286*3^(3/4)*(sqrt(3
) + 3)*sqrt(-6*sqrt(3) + 18) + 127*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 17388*
3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) + 17388*3^(1/4)*sqrt(-6*sqrt(3) + 18))
*arctan(1/3*3^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3)
+ 1/2)) - 1/20736*sqrt(2)*(127*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 22
86*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 2286*3^(3/4)*(sqrt(
```

$3) + 3) \sqrt{-6\sqrt{3} + 18} + 127 \cdot 3^{3/4} \cdot (-6\sqrt{3} + 18)^{3/2} - 17388 \cdot 3^{1/4} \sqrt{2} \sqrt{6\sqrt{3} + 18} + 17388 \cdot 3^{1/4} \sqrt{-6\sqrt{3} + 18}$
 $) \arctan(1/3 \cdot 3^{3/4} \cdot (x - 3^{1/4}) \sqrt{-1/6\sqrt{3} + 1/2}) / \sqrt{1/6\sqrt{3} + 1/2}) - 1/41472 \sqrt{2} \cdot (2286 \cdot 3^{3/4} \sqrt{2} \cdot (\sqrt{3} + 3) \sqrt{-6\sqrt{3} + 18} - 127 \cdot 3^{3/4} \sqrt{2} \cdot (-6\sqrt{3} + 18)^{3/2} + 127 \cdot 3^{3/4} \cdot (6\sqrt{3} + 18)^{3/2} + 2286 \cdot 3^{3/4} \sqrt{2} \cdot (\sqrt{3} + 3) \sqrt{-6\sqrt{3} + 18} - 17388 \cdot 3^{1/4} \sqrt{2} \sqrt{6\sqrt{3} + 18} - 17388 \cdot 3^{1/4} \sqrt{-6\sqrt{3} + 18}) \cdot \log(x^2 + 2 \cdot 3^{1/4} \cdot x \sqrt{-1/6\sqrt{3} + 1/2} + \sqrt{3}) + 1/41472 \sqrt{2} \cdot (2286 \cdot 3^{3/4} \sqrt{2} \cdot (\sqrt{3} + 3) \sqrt{-6\sqrt{3} + 18} - 127 \cdot 3^{3/4} \sqrt{2} \cdot (-6\sqrt{3} + 18)^{3/2} + 127 \cdot 3^{3/4} \cdot (6\sqrt{3} + 18)^{3/2} + 2286 \cdot 3^{3/4} \sqrt{2} \cdot (\sqrt{3} + 3) \sqrt{-6\sqrt{3} + 18} \cdot (\sqrt{3} - 3) - 17388 \cdot 3^{1/4} \sqrt{2} \sqrt{-6\sqrt{3} + 18} - 17388 \cdot 3^{1/4} \sqrt{6\sqrt{3} + 18}) \cdot \log(x^2 - 2 \cdot 3^{1/4} \cdot x \sqrt{-1/6\sqrt{3} + 1/2} + \sqrt{3}) - 17 \cdot x - 25/8 \cdot (x^3 + 3x) / (x^4 + 2x^2 + 3)$

maple [B] time = 0.03, size = 416, normalized size = 1.79

$\frac{5x^3}{3} - 17x - \frac{25(x^3 + 3x)}{8(x^4 + 2x^2 + 3)} + \frac{1}{8} \int \frac{127x^2 + 483}{x^4 + 2x^2 + 3} dx$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x)

[Out] $5/3 \cdot x^3 - 17x + (-25/8 \cdot x^3 - 75/8 \cdot x) / (x^4 + 2x^2 + 3) - 17/64 \cdot (-2 + 2 \cdot 3^{1/2})^{1/2} \cdot 3^{1/2} \cdot \ln(x^2 - (-2 + 2 \cdot 3^{1/2})^{1/2} \cdot x + 3^{1/2}) - 89/32 \cdot (-2 + 2 \cdot 3^{1/2})^{1/2} \cdot \ln(x^2 - (-2 + 2 \cdot 3^{1/2})^{1/2} \cdot x + 3^{1/2}) - 17/32 \cdot (2 + 2 \cdot 3^{1/2})^{1/2} \cdot (-2 + 2 \cdot 3^{1/2}) \cdot 3^{1/2} \cdot \arctan((2x - (-2 + 2 \cdot 3^{1/2})^{1/2}) / (2 + 2 \cdot 3^{1/2})^{1/2}) - 89/16 \cdot (2 + 2 \cdot 3^{1/2})^{1/2} \cdot (-2 + 2 \cdot 3^{1/2}) \cdot \arctan((2x - (-2 + 2 \cdot 3^{1/2})^{1/2}) / (2 + 2 \cdot 3^{1/2})^{1/2}) + 161/8 \cdot (2 + 2 \cdot 3^{1/2})^{1/2} \cdot 3^{1/2} \cdot \arctan((2x - (-2 + 2 \cdot 3^{1/2})^{1/2}) / (2 + 2 \cdot 3^{1/2})^{1/2}) + 17/64 \cdot (-2 + 2 \cdot 3^{1/2})^{1/2} \cdot 3^{1/2} \cdot \ln(x^2 + (-2 + 2 \cdot 3^{1/2})^{1/2} \cdot x + 3^{1/2}) + 89/32 \cdot (-2 + 2 \cdot 3^{1/2})^{1/2} \cdot \ln(x^2 + (-2 + 2 \cdot 3^{1/2})^{1/2} \cdot x + 3^{1/2}) - 17/32 \cdot (2 + 2 \cdot 3^{1/2})^{1/2} \cdot (-2 + 2 \cdot 3^{1/2}) \cdot 3^{1/2} \cdot \arctan((2x + (-2 + 2 \cdot 3^{1/2})^{1/2}) / (2 + 2 \cdot 3^{1/2})^{1/2}) - 89/16 \cdot (2 + 2 \cdot 3^{1/2})^{1/2} \cdot (-2 + 2 \cdot 3^{1/2}) \cdot \arctan((2x + (-2 + 2 \cdot 3^{1/2})^{1/2}) / (2 + 2 \cdot 3^{1/2})^{1/2}) + 161/8 \cdot (2 + 2 \cdot 3^{1/2})^{1/2} \cdot 3^{1/2} \cdot \arctan((2x + (-2 + 2 \cdot 3^{1/2})^{1/2}) / (2 + 2 \cdot 3^{1/2})^{1/2})$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{5}{3} x^3 - 17x - \frac{25(x^3 + 3x)}{8(x^4 + 2x^2 + 3)} + \frac{1}{8} \int \frac{127x^2 + 483}{x^4 + 2x^2 + 3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")

[Out] $5/3 \cdot x^3 - 17x - 25/8 \cdot (x^3 + 3x) / (x^4 + 2x^2 + 3) + 1/8 \cdot \text{integrate}((127 \cdot x^2 + 483) / (x^4 + 2x^2 + 3), x)$

mupad [B] time = 0.09, size = 162, normalized size = 0.70

$$\frac{5x^3}{3} - \frac{25x^3 + 75x}{8(x^4 + 2x^2 + 3)} - 17x + \frac{\operatorname{atan}\left(\frac{x\sqrt{-14395 - \sqrt{2} \cdot 30817i} - 30817\sqrt{2}x\sqrt{-14395 - \sqrt{2} \cdot 30817i}}{64\left(\frac{1571667}{64} + \frac{\sqrt{2} \cdot 14884611i}{128}\right)} - \frac{30817\sqrt{2}x\sqrt{-14395 - \sqrt{2} \cdot 30817i}}{128\left(\frac{1571667}{64} + \frac{\sqrt{2} \cdot 14884611i}{128}\right)}\right)\sqrt{-14395 - \sqrt{2} \cdot 30817i} + \operatorname{atan}\left(\frac{x\sqrt{-14395 + \sqrt{2} \cdot 30817i} + 30817\sqrt{2}x\sqrt{-14395 + \sqrt{2} \cdot 30817i}}{64\left(\frac{1571667}{64} + \frac{\sqrt{2} \cdot 14884611i}{128}\right)} + \frac{30817\sqrt{2}x\sqrt{-14395 + \sqrt{2} \cdot 30817i}}{128\left(\frac{1571667}{64} + \frac{\sqrt{2} \cdot 14884611i}{128}\right)}\right)\sqrt{-14395 + \sqrt{2} \cdot 30817i}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^2,x)

[Out] $(\operatorname{atan}((x \cdot (-2^{1/2}) \cdot 30817i - 14395)^{1/2} \cdot 30817i) / (64 \cdot ((2^{1/2}) \cdot 14884611i) / 128 - 1571667/64)) - (30817 \cdot 2^{1/2} \cdot x \cdot (-2^{1/2}) \cdot 30817i - 14395)^{1/2} / (128 \cdot ((2^{1/2}) \cdot 14884611i) / 128 - 1571667/64)) \cdot (-2^{1/2}) \cdot 30817i - 14395)^{1/2}$

$\frac{i}{16} - \frac{(75x)/8 + (25x^3)/8}{(2x^2 + x^4 + 3)} - 17x - \frac{\operatorname{atan}\left(\frac{x(2^{1/2} \cdot 30817i - 14395)^{1/2} \cdot 30817i}{64 \cdot ((2^{1/2} \cdot 14884611i)/128 + 1571667/64)}\right) + (30817 \cdot 2^{1/2} \cdot x \cdot (2^{1/2} \cdot 30817i - 14395)^{1/2})}{128 \cdot ((2^{1/2} \cdot 14884611i)/128 + 1571667/64)} \cdot (2^{1/2} \cdot 30817i - 14395)^{1/2} \cdot \frac{i}{16} + (5x^3)/3$

sympy [A] time = 0.61, size = 60, normalized size = 0.26

$$\frac{5x^3}{3} - 17x + \frac{-25x^3 - 75x}{8x^4 + 16x^2 + 24} + \operatorname{RootSum}\left(1048576t^4 + 29480960t^2 + 2106591003, \left(t \mapsto t \log\left(\frac{557056t^3}{816619683} + \frac{166600064t}{816619683} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)

[Out] $5x^3/3 - 17x + (-25x^3 - 75x)/(8x^4 + 16x^2 + 24) + \operatorname{RootSum}(1048576*_t^4 + 29480960*_t^2 + 2106591003, \operatorname{Lambda}(_t, _t \cdot \log(557056*_t^3/816619683 + 166600064*_t/816619683 + x)))$

$$3.110 \quad \int \frac{x^2(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2} dx$$

Optimal. Leaf size=225

$$-\frac{1}{32}\sqrt{\frac{1}{6}(12899\sqrt{3}-19291)} \log\left(x^2 - \sqrt{2(\sqrt{3}-1)}x + \sqrt{3}\right) + \frac{1}{32}\sqrt{\frac{1}{6}(12899\sqrt{3}-19291)} \log\left(x^2 + \sqrt{2(\sqrt{3}-1)}x + \sqrt{3}\right)$$

Rubi [A] time = 0.30, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {1668, 1676, 1169, 634, 618, 204, 628}

$$\frac{25(x^2+1)x}{8(x^2+2x^2+3)} - \frac{1}{32}\sqrt{\frac{1}{6}(12899\sqrt{3}-19291)} \log\left(x^2 - \sqrt{2(\sqrt{3}-1)}x + \sqrt{3}\right) + \frac{1}{32}\sqrt{\frac{1}{6}(12899\sqrt{3}-19291)} \log\left(x^2 + \sqrt{2(\sqrt{3}-1)}x + \sqrt{3}\right) + 5x + \frac{1}{16}\sqrt{\frac{1}{6}(19291+12899\sqrt{3})} \tan^{-1}\left(\frac{\sqrt{2(\sqrt{3}-1)}-2x}{\sqrt{2(1+\sqrt{3})}}\right) - \frac{1}{16}\sqrt{\frac{1}{6}(19291+12899\sqrt{3})} \tan^{-1}\left(\frac{2x+\sqrt{2(\sqrt{3}-1)}}{\sqrt{2(1+\sqrt{3})}}\right)$$

Antiderivative was successfully verified.

[In] Int[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]

[Out] 5*x + (25*x*(1 + x^2))/(8*(3 + 2*x^2 + x^4)) + (Sqrt[(19291 + 12899*Sqrt[3])/6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])]) - 2*x]/Sqrt[2*(1 + Sqrt[3])])]/16 - (Sqrt[(19291 + 12899*Sqrt[3])/6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])]) + 2*x]/Sqrt[2*(1 + Sqrt[3])])]/16 - (Sqrt[(-19291 + 12899*Sqrt[3])/6]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]]*x + x^2)/32 + (Sqrt[(-19291 + 12899*Sqrt[3])/6]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]]*x + x^2)/32

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1169

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]

Rule 1668

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :=
  With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
    e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
  x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
  2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
  nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
  mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
  + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x] /; FreeQ[{a, b,
  c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
  & LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := Int[ExpandInte
  grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
  2] && Expon[Pq, x^2] > 1
```

Rubi steps

$$\begin{aligned}
 \int \frac{x^2(4 + x^2 + 3x^4 + 5x^6)}{(3 + 2x^2 + x^4)^2} dx &= \frac{25x(1 + x^2)}{8(3 + 2x^2 + x^4)} + \frac{1}{48} \int \frac{-150 - 186x^2 + 240x^4}{3 + 2x^2 + x^4} dx \\
 &= \frac{25x(1 + x^2)}{8(3 + 2x^2 + x^4)} + \frac{1}{48} \int \left(240 - \frac{6(145 + 111x^2)}{3 + 2x^2 + x^4} \right) dx \\
 &= 5x + \frac{25x(1 + x^2)}{8(3 + 2x^2 + x^4)} - \frac{1}{8} \int \frac{145 + 111x^2}{3 + 2x^2 + x^4} dx \\
 &= 5x + \frac{25x(1 + x^2)}{8(3 + 2x^2 + x^4)} - \frac{\int \frac{145\sqrt{2(-1+\sqrt{3})} - (145-111\sqrt{3})x}{\sqrt{3} - \sqrt{2(-1+\sqrt{3})}x + x^2} dx}{16\sqrt{6(-1+\sqrt{3})}} - \frac{\int \frac{145\sqrt{2(-1+\sqrt{3})} + (145-111\sqrt{3})x}{\sqrt{3} + \sqrt{2(-1+\sqrt{3})}x + x^2} dx}{16\sqrt{6(-1+\sqrt{3})}} \\
 &= 5x + \frac{25x(1 + x^2)}{8(3 + 2x^2 + x^4)} - \frac{1}{96} (333 + 145\sqrt{3}) \int \frac{1}{\sqrt{3} - \sqrt{2(-1+\sqrt{3})}x + x^2} dx \\
 &= 5x + \frac{25x(1 + x^2)}{8(3 + 2x^2 + x^4)} - \frac{1}{32} \sqrt{\frac{1}{6}(-19291 + 12899\sqrt{3})} \log\left(\sqrt{3} - \sqrt{2(-1+\sqrt{3})}x + x^2\right) \\
 &= 5x + \frac{25x(1 + x^2)}{8(3 + 2x^2 + x^4)} + \frac{1}{16} \sqrt{\frac{1}{6}(19291 + 12899\sqrt{3})} \tan^{-1}\left(\frac{\sqrt{2(-1+\sqrt{3})} - 2}{\sqrt{2(1+\sqrt{3})}}\right)
 \end{aligned}$$

Mathematica [C] time = 0.16, size = 121, normalized size = 0.54

$$\frac{25(x^3 + x)}{8(x^4 + 2x^2 + 3)} + 5x - \frac{(111\sqrt{2} - 34i) \tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{16\sqrt{2-2i\sqrt{2}}} - \frac{(111\sqrt{2} + 34i) \tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{16\sqrt{2+2i\sqrt{2}}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2, x]

```
[Out] 5*x + (25*(x + x^3))/(8*(3 + 2*x^2 + x^4)) - ((-34*I + 111*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/(16*Sqrt[2 - (2*I)*Sqrt[2]]) - ((34*I + 111*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/(16*Sqrt[2 + (2*I)*Sqrt[2]])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(4 + x^2 + 3x^4 + 5x^6)}{(3 + 2x^2 + x^4)^2} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2, x]
```

fricas [B] time = 1.22, size = 459, normalized size = 2.04

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] 1/19736089152*(98680445760*x^5 + 31876*499152603^(1/4)*sqrt(2)*(x^4 + 2*x^2 + 3)*sqrt(248834609*sqrt(3) + 499152603)*arctan(1/2453286601800494203302*499152603^(3/4)*sqrt(308376393)*sqrt(308376393*x^2 + 499152603^(1/4)*(145*sqrt(3)*x - 333*x)*sqrt(248834609*sqrt(3) + 499152603) + 308376393*sqrt(3))*(111*sqrt(3)*sqrt(2) - 145*sqrt(2))*sqrt(248834609*sqrt(3) + 499152603) - 1/7955494186614*499152603^(3/4)*(111*sqrt(3)*sqrt(2)*x - 145*sqrt(2)*x)*sqrt(248834609*sqrt(3) + 499152603) + 1/2*sqrt(3)*sqrt(2) - 1/2*sqrt(2)) + 31876*499152603^(1/4)*sqrt(2)*(x^4 + 2*x^2 + 3)*sqrt(248834609*sqrt(3) + 499152603)*arctan(1/2453286601800494203302*499152603^(3/4)*sqrt(308376393)*sqrt(308376393*x^2 - 499152603^(1/4)*(145*sqrt(3)*x - 333*x)*sqrt(248834609*sqrt(3) + 499152603) + 308376393*sqrt(3))*(111*sqrt(3)*sqrt(2) - 145*sqrt(2))*sqrt(248834609*sqrt(3) + 499152603) - 1/7955494186614*499152603^(3/4)*(111*sqrt(3)*sqrt(2)*x - 145*sqrt(2)*x)*sqrt(248834609*sqrt(3) + 499152603) - 1/2*sqrt(3)*sqrt(2) + 1/2*sqrt(2)) + 259036170120*x^3 + 499152603^(1/4)*(19291*x^4 + 38582*x^2 - 12899*sqrt(3)*(x^4 + 2*x^2 + 3) + 57873)*sqrt(248834609*sqrt(3) + 499152603)*log(308376393*x^2 + 499152603^(1/4)*(145*sqrt(3)*x - 333*x)*sqrt(248834609*sqrt(3) + 499152603) + 308376393*sqrt(3)) - 499152603^(1/4)*(19291*x^4 + 38582*x^2 - 12899*sqrt(3)*(x^4 + 2*x^2 + 3) + 57873)*sqrt(248834609*sqrt(3) + 499152603)*log(308376393*x^2 - 499152603^(1/4)*(145*sqrt(3)*x - 333*x)*sqrt(248834609*sqrt(3) + 499152603) + 308376393*sqrt(3)) + 357716615880*x)/(x^4 + 2*x^2 + 3)
```

giac [B] time = 1.82, size = 566, normalized size = 2.52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] 1/6912*sqrt(2)*(37*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 666*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 666*3^(3/4)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) + 37*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 1740*3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) + 1740*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) + 1/6912*sqrt(2)*(37*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 666*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 666*3^(3/4)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) + 37*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 1740*3^(1/4)*sqrt(2)*sqrt(6
```

```
*sqrt(3) + 18) + 1740*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3/4)*(x
- 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) + 1/13824*sqrt
(2)*(666*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 37*3^(3/4)*s
qrt(2)*(-6*sqrt(3) + 18)^(3/2) + 37*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 666*3^
(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 1740*3^(1/4)*sqrt(2)*sqrt(-6*sq
rt(3) + 18) - 1740*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 + 2*3^(1/4)*x*sqrt(
-1/6*sqrt(3) + 1/2) + sqrt(3)) - 1/13824*sqrt(2)*(666*3^(3/4)*sqrt(2)*(sqrt
(3) + 3)*sqrt(-6*sqrt(3) + 18) - 37*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2)
+ 37*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 666*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sq
rt(3) - 3) - 1740*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) - 1740*3^(1/4)*sqrt
(6*sqrt(3) + 18))*log(x^2 - 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3))
+ 5*x + 25/8*(x^3 + x)/(x^4 + 2*x^2 + 3)
```

maple [B] time = 0.03, size = 412, normalized size = 1.83

$$\frac{\arctan\left(\frac{x\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right) + \frac{17(2+2\sqrt{3})\sqrt{3}\arctan\left(\frac{x\sqrt{2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right) + 14\sqrt{3}\arctan\left(\frac{x\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right) + \frac{17(2+2\sqrt{3})\sqrt{3}\arctan\left(\frac{x\sqrt{2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right) + 14\sqrt{3}\arctan\left(\frac{x\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)}{24\sqrt{2+2\sqrt{3}}}}{24\sqrt{2+2\sqrt{3}}} + \frac{\arctan\left(\frac{x\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right) + \frac{17(2+2\sqrt{3})\sqrt{3}\arctan\left(\frac{x\sqrt{2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right) + 14\sqrt{3}\arctan\left(\frac{x\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right) + \frac{17(2+2\sqrt{3})\sqrt{3}\arctan\left(\frac{x\sqrt{2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right) + 14\sqrt{3}\arctan\left(\frac{x\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)}{24\sqrt{2+2\sqrt{3}}}}{24\sqrt{2+2\sqrt{3}}}}{24\sqrt{2+2\sqrt{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x)
[Out] 5*x*(-25/8*x^3-25/8*x)/(x^4+2*x^2+3)-47/96*(-2+2*3^(1/2))^(1/2)*3^(1/2)*ln(
x^2-(-2+2*3^(1/2))^(1/2)*x+3^(1/2))+17/64*(-2+2*3^(1/2))^(1/2)*ln(x^2-(-2+2
*3^(1/2))^(1/2)*x+3^(1/2))-47/48/(2+2*3^(1/2))^(1/2)*(-2+2*3^(1/2))*3^(1/2)
*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))+17/32/(2+2*3^(1/2))
^(1/2)*(-2+2*3^(1/2))*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2)
)-145/24/(2+2*3^(1/2))^(1/2)*3^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2
*3^(1/2))^(1/2))+47/96*(-2+2*3^(1/2))^(1/2)*3^(1/2)*ln(x^2+(-2+2*3^(1/2))^(
1/2)*x+3^(1/2))-17/64*(-2+2*3^(1/2))^(1/2)*ln(x^2+(-2+2*3^(1/2))^(1/2)*x+3^
(1/2))-47/48/(2+2*3^(1/2))^(1/2)*(-2+2*3^(1/2))*3^(1/2)*arctan((2*x+(-2+2*3
^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))+17/32/(2+2*3^(1/2))^(1/2)*(-2+2*3^(1/2)
)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))-145/24/(2+2*3^(1/2)
)^(1/2)*3^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$5x + \frac{25(x^3 + x)}{8(x^4 + 2x^2 + 3)} - \frac{1}{8} \int \frac{111x^2 + 145}{x^4 + 2x^2 + 3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
[Out] 5*x + 25/8*(x^3 + x)/(x^4 + 2*x^2 + 3) - 1/8*integrate((111*x^2 + 145)/(x^4
+ 2*x^2 + 3), x)
```

mupad [B] time = 0.96, size = 156, normalized size = 0.69

$$5x + \frac{\frac{25x^3}{8} + \frac{25x}{8}}{x^4 + 2x^2 + 3} + \frac{\operatorname{atan}\left(\frac{x\sqrt{-57873-\sqrt{2}23907i}-7969i}{576\left(\frac{374543}{96} + \frac{\sqrt{2}1155505i}{384}\right)} + \frac{7969\sqrt{2}x\sqrt{-57873-\sqrt{2}23907i}}{1152\left(\frac{374543}{96} + \frac{\sqrt{2}1155505i}{384}\right)}\right)\sqrt{-57873-\sqrt{2}23907i} + \operatorname{atan}\left(\frac{x\sqrt{-57873+\sqrt{2}23907i}-7969i}{576\left(\frac{374543}{96} + \frac{\sqrt{2}1155505i}{384}\right)} - \frac{7969\sqrt{2}x\sqrt{-57873+\sqrt{2}23907i}}{1152\left(\frac{374543}{96} + \frac{\sqrt{2}1155505i}{384}\right)}\right)\sqrt{-57873+\sqrt{2}23907i}}{48}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^2,x)
[Out] 5*x + ((25*x)/8 + (25*x^3)/8)/(2*x^2 + x^4 + 3) + (atan((x*(- 2^(1/2)*23907
i - 57873)^(1/2)*7969i)/(576*((2^(1/2)*1155505i)/384 - 374543/96)) + (7969*
2^(1/2)*x*(- 2^(1/2)*23907i - 57873)^(1/2))/(1152*((2^(1/2)*1155505i)/384 -
374543/96)))*(- 2^(1/2)*23907i - 57873)^(1/2)*1i)/48 - (atan((x*(2^(1/2)*2
3907i - 57873)^(1/2)*7969i)/(576*((2^(1/2)*1155505i)/384 + 374543/96)) - (7
969*2^(1/2)*x*(2^(1/2)*23907i - 57873)^(1/2))/(1152*((2^(1/2)*1155505i)/384
+ 374543/96)))*(2^(1/2)*23907i - 57873)^(1/2)*1i)/48
```

sympy [A] time = 0.60, size = 51, normalized size = 0.23

$$5x + \frac{25x^3 + 25x}{8x^4 + 16x^2 + 24} + \text{RootSum}\left(3145728t^4 + 39507968t^2 + 166384201, \left(t \mapsto t \log\left(-\frac{9240576t^3}{102792131} - \frac{95003488t}{102792131} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)

[Out] 5*x + (25*x**3 + 25*x)/(8*x**4 + 16*x**2 + 24) + RootSum(3145728*_t**4 + 39507968*_t**2 + 166384201, Lambda(_t, _t*log(-9240576*_t**3/102792131 - 95003488*_t/102792131 + x)))

$$3.111 \quad \int \frac{4+x^2+3x^4+5x^6}{(3+2x^2+x^4)^2} dx$$

Optimal. Leaf size=224

$$\frac{1}{96} \sqrt{\frac{1}{6} (11567 + 12897\sqrt{3})} \log\left(x^2 - \sqrt{2(\sqrt{3}-1)}x + \sqrt{3}\right) - \frac{1}{96} \sqrt{\frac{1}{6} (11567 + 12897\sqrt{3})} \log\left(x^2 + \sqrt{2(\sqrt{3}-1)}x + \sqrt{3}\right)$$

Rubi [A] time = 0.22, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1678, 1169, 634, 618, 204, 628}

$$\frac{25x(1-x^2)}{24(x^2+2x^2+3)} + \frac{1}{96} \sqrt{\frac{1}{6} (11567 + 12897\sqrt{3})} \log\left(x^2 - \sqrt{2(\sqrt{3}-1)}x + \sqrt{3}\right) - \frac{1}{96} \sqrt{\frac{1}{6} (11567 + 12897\sqrt{3})} \log\left(x^2 + \sqrt{2(\sqrt{3}-1)}x + \sqrt{3}\right) - \frac{1}{48} \sqrt{\frac{1}{6} (12897\sqrt{3} - 11567)} \tan^{-1}\left(\frac{\sqrt{2(\sqrt{3}-1)} - 2x}{\sqrt{2(1+\sqrt{3})}}\right) + \frac{1}{48} \sqrt{\frac{1}{6} (12897\sqrt{3} - 11567)} \tan^{-1}\left(\frac{2x + \sqrt{2(\sqrt{3}-1)}}{\sqrt{2(1+\sqrt{3})}}\right)$$

Antiderivative was successfully verified.

[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(3 + 2*x^2 + x^4)^2,x]

[Out] (25*x*(1 - x^2))/(24*(3 + 2*x^2 + x^4)) - (Sqrt[(-11567 + 12897*Sqrt[3])/6] * ArcTan[(Sqrt[2*(-1 + Sqrt[3])]) - 2*x]/Sqrt[2*(1 + Sqrt[3])])]/48 + (Sqrt[(-11567 + 12897*Sqrt[3])/6] * ArcTan[(Sqrt[2*(-1 + Sqrt[3])]) + 2*x]/Sqrt[2*(1 + Sqrt[3])])]/48 + (Sqrt[(11567 + 12897*Sqrt[3])/6] * Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/96 - (Sqrt[(11567 + 12897*Sqrt[3])/6] * Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/96

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1169

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]

Rule 1678

Int[(Pq_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := With[{d = Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]

Rubi steps

$$\begin{aligned} \int \frac{4 + x^2 + 3x^4 + 5x^6}{(3 + 2x^2 + x^4)^2} dx &= \frac{25x(1 - x^2)}{24(3 + 2x^2 + x^4)} + \frac{1}{48} \int \frac{14 + 190x^2}{3 + 2x^2 + x^4} dx \\ &= \frac{25x(1 - x^2)}{24(3 + 2x^2 + x^4)} + \frac{\int \frac{14\sqrt{2(-1+\sqrt{3})} - (14-190\sqrt{3})x}{\sqrt{3} - \sqrt{2(-1+\sqrt{3})}x + x^2} dx}{96\sqrt{6(-1+\sqrt{3})}} + \frac{\int \frac{14\sqrt{2(-1+\sqrt{3})} + (14-190\sqrt{3})x}{\sqrt{3} + \sqrt{2(-1+\sqrt{3})}x + x^2} dx}{96\sqrt{6(-1+\sqrt{3})}} \\ &= \frac{25x(1 - x^2)}{24(3 + 2x^2 + x^4)} + \frac{(7 - 95\sqrt{3}) \int \frac{\sqrt{2(-1+\sqrt{3})} + 2x}{\sqrt{3} + \sqrt{2(-1+\sqrt{3})}x + x^2} dx}{96\sqrt{6(-1+\sqrt{3})}} + \frac{1}{288} (285 + 7\sqrt{3}) \int \frac{1}{\sqrt{3} + \sqrt{2(-1+\sqrt{3})}x + x^2} dx \\ &= \frac{25x(1 - x^2)}{24(3 + 2x^2 + x^4)} + \frac{1}{96} \sqrt{\frac{11567}{6} + \frac{4299\sqrt{3}}{2}} \log\left(\sqrt{3} - \sqrt{2(-1+\sqrt{3})}x + x^2\right) - \frac{1}{96} \sqrt{\frac{11567}{6} + \frac{4299\sqrt{3}}{2}} \log\left(\sqrt{3} + \sqrt{2(-1+\sqrt{3})}x + x^2\right) \\ &= \frac{25x(1 - x^2)}{24(3 + 2x^2 + x^4)} - \frac{1}{48} \sqrt{\frac{1}{6}(-11567 + 12897\sqrt{3})} \tan^{-1}\left(\frac{\sqrt{2(-1+\sqrt{3})} - 2x}{\sqrt{2(1+\sqrt{3})}}\right) + \frac{1}{288} (285 + 7\sqrt{3}) \int \frac{1}{\sqrt{3} + \sqrt{2(-1+\sqrt{3})}x + x^2} dx \end{aligned}$$

Mathematica [C] time = 0.26, size = 115, normalized size = 0.51

$$\frac{1}{48} \left(\frac{50x(x^2 - 1)}{x^4 + 2x^2 + 3} + \frac{(95 + 44i\sqrt{2}) \tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}} + \frac{(95 - 44i\sqrt{2}) \tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(3 + 2*x^2 + x^4)^2, x]

[Out] ((-50*x*(-1 + x^2))/(3 + 2*x^2 + x^4) + ((95 + (44*I)*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/Sqrt[1 - I*Sqrt[2]] + ((95 - (44*I)*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/Sqrt[1 + I*Sqrt[2]])/48

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4 + x^2 + 3x^4 + 5x^6}{(3 + 2x^2 + x^4)^2} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(4 + x^2 + 3*x^4 + 5*x^6)/(3 + 2*x^2 + x^4)^2,x]
[Out] IntegrateAlgebraic[(4 + x^2 + 3*x^4 + 5*x^6)/(3 + 2*x^2 + x^4)^2, x]
fricas [B]   time = 1.41, size = 454, normalized size = 2.03
```

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
[Out] -1/33461214912*(54052*6160467^(1/4)*sqrt(2)*(x^4 + 2*x^2 + 3)*sqrt(-1491795
99*sqrt(3) + 498997827)*arctan(1/29015889224422097862*sqrt(19364129)*616046
7^(3/4)*sqrt(174277161*x^2 + 6160467^(1/4)*(7*sqrt(3)*x - 285*x)*sqrt(-1491
79599*sqrt(3) + 498997827) + 174277161*sqrt(3))*(95*sqrt(3)*sqrt(2) - 7*sq
rt(2))*sqrt(-149179599*sqrt(3) + 498997827) - 1/499478343426*6160467^(3/4)*(
95*sqrt(3)*sqrt(2)*x - 7*sqrt(2)*x)*sqrt(-149179599*sqrt(3) + 498997827) +
1/2*sqrt(3)*sqrt(2) - 1/2*sqrt(2)) + 54052*6160467^(1/4)*sqrt(2)*(x^4 + 2*x
^2 + 3)*sqrt(-149179599*sqrt(3) + 498997827)*arctan(1/29015889224422097862*
sqrt(19364129)*6160467^(3/4)*sqrt(174277161*x^2 - 6160467^(1/4)*(7*sqrt(3)*
x - 285*x)*sqrt(-149179599*sqrt(3) + 498997827) + 174277161*sqrt(3))*(95*sq
rt(3)*sqrt(2) - 7*sqrt(2))*sqrt(-149179599*sqrt(3) + 498997827) - 1/4994783
43426*6160467^(3/4)*(95*sqrt(3)*sqrt(2)*x - 7*sqrt(2)*x)*sqrt(-149179599*sq
rt(3) + 498997827) - 1/2*sqrt(3)*sqrt(2) + 1/2*sqrt(2)) + 34855432200*x^3 -
6160467^(1/4)*(11567*x^4 + 23134*x^2 + 12897*sqrt(3))*(x^4 + 2*x^2 + 3) + 3
4701)*sqrt(-149179599*sqrt(3) + 498997827)*log(174277161*x^2 + 6160467^(1/4
))*(7*sqrt(3)*x - 285*x)*sqrt(-149179599*sqrt(3) + 498997827) + 174277161*sq
rt(3)) + 6160467^(1/4)*(11567*x^4 + 23134*x^2 + 12897*sqrt(3))*(x^4 + 2*x^2
+ 3) + 34701)*sqrt(-149179599*sqrt(3) + 498997827)*log(174277161*x^2 - 6160
467^(1/4)*(7*sqrt(3)*x - 285*x)*sqrt(-149179599*sqrt(3) + 498997827) + 1742
77161*sqrt(3)) - 34855432200*x)/(x^4 + 2*x^2 + 3)
giac [B]   time = 1.82, size = 565, normalized size = 2.52
```

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")
[Out] -1/62208*sqrt(2)*(95*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 1710*3^(3/4)*
sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 1710*3^(3/4)*(sqrt(3) + 3)*sq
rt(-6*sqrt(3) + 18) + 95*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 252*3^(1/4)*sqrt(
2)*sqrt(6*sqrt(3) + 18) + 252*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(
3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) - 1/62
208*sqrt(2)*(95*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 1710*3^(3/4)*sqrt(
2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 1710*3^(3/4)*(sqrt(3) + 3)*sqrt(-6*
sqrt(3) + 18) + 95*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 252*3^(1/4)*sqrt(2)*sq
rt(6*sqrt(3) + 18) + 252*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3/4)*
(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) - 1/124416*
sqrt(2)*(1710*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 95*3^(3
/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 95*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 1
710*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 252*3^(1/4)*sqrt(2)*sqrt(-
6*sqrt(3) + 18) - 252*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 + 2*3^(1/4)*x*s
qrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) + 1/124416*sqrt(2)*(1710*3^(3/4)*sqrt(2)
*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 95*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)
^(3/2) + 95*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 1710*3^(3/4)*sqrt(6*sqrt(3) +
18)*(sqrt(3) - 3) - 252*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) - 252*3^(1/4)
*sqrt(6*sqrt(3) + 18))*log(x^2 - 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqr
t(3)) - 25/24*(x^3 - x)/(x^4 + 2*x^2 + 3)
```


maple [B] time = 0.03, size = 408, normalized size = 1.82

$$\frac{19(-2+2\sqrt{3})\sqrt{3}\arctan\left(\frac{2x-2\sqrt{3}}{3\sqrt{2+3}}\right) + 11(-2+2\sqrt{3})\arctan\left(\frac{2x-2\sqrt{3}}{3\sqrt{2+3}}\right) + 7\sqrt{3}\arctan\left(\frac{2x-2\sqrt{3}}{3\sqrt{2+3}}\right) + 19(-2+2\sqrt{3})\sqrt{3}\arctan\left(\frac{2x-2\sqrt{3}}{3\sqrt{2+3}}\right) + 11(-2+2\sqrt{3})\arctan\left(\frac{2x-2\sqrt{3}}{3\sqrt{2+3}}\right) + 7\sqrt{3}\arctan\left(\frac{2x-2\sqrt{3}}{3\sqrt{2+3}}\right) + 19\sqrt{-2+2\sqrt{3}}\sqrt{3}\ln\left(\frac{x-\sqrt{-2+2\sqrt{3}}+x\sqrt{3}}{3}\right) + 11\sqrt{-2+2\sqrt{3}}\sqrt{3}\ln\left(\frac{x-\sqrt{-2+2\sqrt{3}}+x\sqrt{3}}{3}\right) + 19\sqrt{-2+2\sqrt{3}}\sqrt{3}\ln\left(\frac{x-\sqrt{-2+2\sqrt{3}}+x\sqrt{3}}{3}\right) + 11\sqrt{-2+2\sqrt{3}}\sqrt{3}\ln\left(\frac{x-\sqrt{-2+2\sqrt{3}}+x\sqrt{3}}{3}\right) + \frac{25x}{24} - \frac{25x^3}{24}}{288\sqrt{2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x)
[Out] (-25/24*x^3+25/24*x)/(x^4+2*x^2+3)+139/576*(-2+2*3^(1/2))^(1/2)*3^(1/2)*ln(x^2-(-2+2*3^(1/2))^(1/2)*x+3^(1/2))+11/48*(-2+2*3^(1/2))^(1/2)*ln(x^2-(-2+2*3^(1/2))^(1/2)*x+3^(1/2))+139/288/(2+2*3^(1/2))^(1/2)*(-2+2*3^(1/2))*3^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))+11/24/(2+2*3^(1/2))^(1/2)*(-2+2*3^(1/2))*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))+7/72/(2+2*3^(1/2))^(1/2)*3^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))-139/576*(-2+2*3^(1/2))^(1/2)*3^(1/2)*ln(x^2+(-2+2*3^(1/2))^(1/2)*x+3^(1/2))-11/48*(-2+2*3^(1/2))^(1/2)*ln(x^2+(-2+2*3^(1/2))^(1/2)*x+3^(1/2))+139/288/(2+2*3^(1/2))^(1/2)*(-2+2*3^(1/2))*3^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))+11/24/(2+2*3^(1/2))^(1/2)*(-2+2*3^(1/2))*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))+7/72/(2+2*3^(1/2))^(1/2)*3^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{25(x^3 - x)}{24(x^4 + 2x^2 + 3)} + \frac{1}{24} \int \frac{95x^2 + 7}{x^4 + 2x^2 + 3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
[Out] -25/24*(x^3 - x)/(x^4 + 2*x^2 + 3) + 1/24*integrate((95*x^2 + 7)/(x^4 + 2*x^2 + 3), x)
```

mupad [B] time = 0.13, size = 153, normalized size = 0.68

$$\frac{25x}{24} - \frac{25x^3}{24} - \frac{\operatorname{atan}\left(\frac{x\sqrt{34701-\sqrt{2}40539i}-13513i}{15552\left(\frac{1878307}{5184}+\frac{\sqrt{2}94591i}{10368}\right)} + \frac{13513\sqrt{2}x\sqrt{34701-\sqrt{2}40539i}}{31104\left(\frac{1878307}{5184}+\frac{\sqrt{2}94591i}{10368}\right)}\right)\sqrt{34701-\sqrt{2}40539i}}{144} + \frac{\operatorname{atan}\left(\frac{x\sqrt{34701+\sqrt{2}40539i}-13513i}{15552\left(\frac{1878307}{5184}+\frac{\sqrt{2}94591i}{10368}\right)} - \frac{13513\sqrt{2}x\sqrt{34701+\sqrt{2}40539i}}{31104\left(\frac{1878307}{5184}+\frac{\sqrt{2}94591i}{10368}\right)}\right)\sqrt{34701+\sqrt{2}40539i}}{144}}{x^4 + 2x^2 + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(2*x^2 + x^4 + 3)^2,x)
[Out] ((25*x)/24 - (25*x^3)/24)/(2*x^2 + x^4 + 3) - (atan((x*(34701 - 2^(1/2)*40539i)^(1/2)*13513i)/(15552*((2^(1/2)*94591i)/10368 - 1878307/5184))) + (13513*2^(1/2)*x*(34701 - 2^(1/2)*40539i)^(1/2))/(31104*((2^(1/2)*94591i)/10368 - 1878307/5184)))*(34701 - 2^(1/2)*40539i)^(1/2)*1i/144 + (atan((x*(2^(1/2)*40539i + 34701)^(1/2)*13513i)/(15552*((2^(1/2)*94591i)/10368 + 1878307/5184))) - (13513*2^(1/2)*x*(2^(1/2)*40539i + 34701)^(1/2))/(31104*((2^(1/2)*94591i)/10368 + 1878307/5184)))*(2^(1/2)*40539i + 34701)^(1/2)*1i/144
```

sympy [B] time = 1.29, size = 1185, normalized size = 5.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
[Out] (-25*x**3 + 25*x)/(24*x**4 + 48*x**2 + 72) + sqrt(11567/55296 + 1433*sqrt(3)/6144)*log(x**2 + x*(-556*sqrt(2)*sqrt(11567 + 12897*sqrt(3)))/13513 - 1040345*sqrt(6)*sqrt(11567 + 12897*sqrt(3))/174277161 + 278*sqrt(3)*sqrt(11567 + 12897*sqrt(3))*sqrt(149179599*sqrt(3) + 316396658)/174277161) - 47610276200401*sqrt(2)*sqrt(149179599*sqrt(3) + 316396658)/30372528846219921 - 43908
```

$$\begin{aligned}
& 31246\sqrt{6}\sqrt{149179599\sqrt{3} + 316396658}/7065021829779 + 128104648 \\
& 1635939181/30372528846219921 + 200684595453464\sqrt{3}/7065021829779) - \sqrt{3} \\
& \sqrt{11567/55296 + 1433\sqrt{3}/6144} \log(x^2 + x(-278\sqrt{3}\sqrt{11567 + \\
& 12897\sqrt{3}))\sqrt{149179599\sqrt{3} + 316396658}/174277161 + 1040345\sqrt{6} \\
& \sqrt{11567 + 12897\sqrt{3}}/174277161 + 556\sqrt{2}\sqrt{11567 + 12897\sqrt{3}}/13513) - \\
& 47610276200401\sqrt{2}\sqrt{149179599\sqrt{3} + 316396658})/30372528846219921 - \\
& 4390831246\sqrt{6}\sqrt{149179599\sqrt{3} + 316396658})/7065021829779 + 128104648 \\
& 1635939181/30372528846219921 + 200684595453464\sqrt{3}/7065021829779) + \\
& 2\sqrt{-\sqrt{2}\sqrt{149179599\sqrt{3} + 316396658})/27648 + 11567/55296 + 1433\sqrt{3}/2048} \\
& \operatorname{atan}(348554322\sqrt{3}x/(94591\sqrt{2}\sqrt{-2\sqrt{2}\sqrt{149179599\sqrt{3} + 316396658} + \\
& 11567 + 38691\sqrt{3}}) + 278\sqrt{149179599\sqrt{3} + 316396658}\sqrt{-2\sqrt{2}\sqrt{149179599\sqrt{3} + 316396658} + \\
& 11567 + 38691\sqrt{3}}) - 7170732\sqrt{6}\sqrt{11567 + 12897\sqrt{3}}/(94591\sqrt{2}\sqrt{-2\sqrt{2}\sqrt{149179599\sqrt{3} + 316396658} + \\
& 11567 + 38691\sqrt{3}}) + 278\sqrt{149179599\sqrt{3} + 316396658}\sqrt{-2\sqrt{2}\sqrt{149179599\sqrt{3} + 316396658} + \\
& 11567 + 38691\sqrt{3}}) - 3121035\sqrt{2}\sqrt{11567 + 12897\sqrt{3}}/(94591\sqrt{2}\sqrt{-2\sqrt{2}\sqrt{149179599\sqrt{3} + 316396658} + \\
& 11567 + 38691\sqrt{3}}) + 278\sqrt{149179599\sqrt{3} + 316396658}\sqrt{-2\sqrt{2}\sqrt{149179599\sqrt{3} + 316396658} + \\
& 11567 + 38691\sqrt{3}}) + 834\sqrt{11567 + 12897\sqrt{3}}\sqrt{149179599\sqrt{3} + 316396658}/(94591\sqrt{2}\sqrt{-2\sqrt{2}\sqrt{149179599\sqrt{3} + 316396658} + \\
& 11567 + 38691\sqrt{3}}) + 278\sqrt{149179599\sqrt{3} + 316396658}\sqrt{-2\sqrt{2}\sqrt{149179599\sqrt{3} + 316396658} + \\
& 11567 + 38691\sqrt{3}}) + 2\sqrt{-\sqrt{2}\sqrt{149179599\sqrt{3} + 316396658})/27648 + 11567/55296 + 1433\sqrt{3}/2048} \\
& \operatorname{atan}(348554322\sqrt{3}x/(94591\sqrt{2}\sqrt{-2\sqrt{2}\sqrt{149179599\sqrt{3} + 316396658} + 11567 + \\
& 38691\sqrt{3}}) + 278\sqrt{149179599\sqrt{3} + 316396658}\sqrt{-2\sqrt{2}\sqrt{149179599\sqrt{3} + 316396658} + \\
& 11567 + 38691\sqrt{3}}) - 834\sqrt{11567 + 12897\sqrt{3}}\sqrt{149179599\sqrt{3} + 316396658}/(94591\sqrt{2}\sqrt{-2\sqrt{2}\sqrt{149179599\sqrt{3} + 316396658} + \\
& 11567 + 38691\sqrt{3}}) + 278\sqrt{149179599\sqrt{3} + 316396658}\sqrt{-2\sqrt{2}\sqrt{149179599\sqrt{3} + 316396658} + \\
& 11567 + 38691\sqrt{3}}) + 3121035\sqrt{2}\sqrt{11567 + 12897\sqrt{3}}/(94591\sqrt{2}\sqrt{-2\sqrt{2}\sqrt{149179599\sqrt{3} + 316396658} + \\
& 11567 + 38691\sqrt{3}}) + 278\sqrt{149179599\sqrt{3} + 316396658}\sqrt{-2\sqrt{2}\sqrt{149179599\sqrt{3} + 316396658} + \\
& 11567 + 38691\sqrt{3}}) + 7170732\sqrt{6}\sqrt{11567 + 12897\sqrt{3}}/(94591\sqrt{2}\sqrt{-2\sqrt{2}\sqrt{149179599\sqrt{3} + 316396658} + \\
& 11567 + 38691\sqrt{3}}) + 278\sqrt{149179599\sqrt{3} + 316396658}\sqrt{-2\sqrt{2}\sqrt{149179599\sqrt{3} + 316396658} + \\
& 11567 + 38691\sqrt{3}})
\end{aligned}$$

$$3.112 \quad \int \frac{4+x^2+3x^4+5x^6}{x^2(3+2x^2+x^4)^2} dx$$

Optimal. Leaf size=229

$$-\frac{1}{96}\sqrt{\frac{1}{6}(965+699\sqrt{3})} \log\left(x^2 - \sqrt{2(\sqrt{3}-1)}x + \sqrt{3}\right) + \frac{1}{96}\sqrt{\frac{1}{6}(965+699\sqrt{3})} \log\left(x^2 + \sqrt{2(\sqrt{3}-1)}x\right)$$

Rubi [A] time = 0.31, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {1669, 1664, 1169, 634, 618, 204, 628}

$$-\frac{25x(x^2+5)}{72(x^4+2x^2+3)} - \frac{1}{96}\sqrt{\frac{1}{6}(965+699\sqrt{3})} \log\left(x^2 - \sqrt{2(\sqrt{3}-1)}x + \sqrt{3}\right) + \frac{1}{96}\sqrt{\frac{1}{6}(965+699\sqrt{3})} \log\left(x^2 + \sqrt{2(\sqrt{3}-1)}x + \sqrt{3}\right) - \frac{4}{9x} + \frac{1}{48}\sqrt{\frac{1}{6}(699\sqrt{3}-965)} \tan^{-1}\left(\frac{\sqrt{2(\sqrt{3}-1)}-2x}{\sqrt{2(1+\sqrt{3})}}\right) - \frac{1}{48}\sqrt{\frac{1}{6}(699\sqrt{3}-965)} \tan^{-1}\left(\frac{2x+\sqrt{2(\sqrt{3}-1)}}{\sqrt{2(1+\sqrt{3})}}\right)$$

Antiderivative was successfully verified.

[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(3 + 2*x^2 + x^4)^2), x]

[Out] -4/(9*x) - (25*x*(5 + x^2))/(72*(3 + 2*x^2 + x^4)) + (Sqrt[(-965 + 699*Sqrt[3])/6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])]) - 2*x]/Sqrt[2*(1 + Sqrt[3])])]/48 - (Sqrt[(-965 + 699*Sqrt[3])/6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])]) + 2*x]/Sqrt[2*(1 + Sqrt[3])])]/48 - (Sqrt[(965 + 699*Sqrt[3])/6]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/96 + (Sqrt[(965 + 699*Sqrt[3])/6]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/96

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1169

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]

Rule 1664

```
Int[(Pq_)*((d_)*(x_)^(m_))*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_
Symbol] := Int[ExpandIntegrand[(d*x)^(m)*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1669

```
Int[(Pq_)*(x_)^(m_))*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :=
With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{4 + x^2 + 3x^4 + 5x^6}{x^2(3 + 2x^2 + x^4)^2} dx &= -\frac{25x(5 + x^2)}{72(3 + 2x^2 + x^4)} + \frac{1}{48} \int \frac{64 + \frac{170x^2}{3} - \frac{50x^4}{3}}{x^2(3 + 2x^2 + x^4)} dx \\
&= -\frac{25x(5 + x^2)}{72(3 + 2x^2 + x^4)} + \frac{1}{48} \int \left(\frac{64}{3x^2} - \frac{2(-7 + 19x^2)}{3 + 2x^2 + x^4} \right) dx \\
&= -\frac{4}{9x} - \frac{25x(5 + x^2)}{72(3 + 2x^2 + x^4)} - \frac{1}{24} \int \frac{-7 + 19x^2}{3 + 2x^2 + x^4} dx \\
&= -\frac{4}{9x} - \frac{25x(5 + x^2)}{72(3 + 2x^2 + x^4)} - \frac{\int \frac{-7\sqrt{2(-1+\sqrt{3})} - (-7-19\sqrt{3})x}{\sqrt{3} - \sqrt{2(-1+\sqrt{3})}x + x^2} dx}{48\sqrt{6(-1+\sqrt{3})}} - \frac{\int \frac{-7\sqrt{2(-1+\sqrt{3})} + (-7-19\sqrt{3})x}{\sqrt{3} + \sqrt{2(-1+\sqrt{3})}x + x^2} dx}{48\sqrt{6(-1+\sqrt{3})}} \\
&= -\frac{4}{9x} - \frac{25x(5 + x^2)}{72(3 + 2x^2 + x^4)} - \frac{1}{48} \sqrt{\frac{1}{6}(566 - 133\sqrt{3})} \int \frac{1}{\sqrt{3} - \sqrt{2(-1+\sqrt{3})}x + x^2} dx \\
&= -\frac{4}{9x} - \frac{25x(5 + x^2)}{72(3 + 2x^2 + x^4)} - \frac{1}{96} \sqrt{\frac{1}{6}(965 + 699\sqrt{3})} \log\left(\sqrt{3} - \sqrt{2(-1+\sqrt{3})}x + x^2\right) \\
&= -\frac{4}{9x} - \frac{25x(5 + x^2)}{72(3 + 2x^2 + x^4)} + \frac{1}{48} \sqrt{\frac{1}{6}(-965 + 699\sqrt{3})} \tan^{-1}\left(\frac{\sqrt{2(-1+\sqrt{3})} - 2x}{\sqrt{2(1+\sqrt{3})}}\right)
\end{aligned}$$

Mathematica [C] time = 0.18, size = 126, normalized size = 0.55

$$-\frac{25x(x^2 + 5)}{72(x^4 + 2x^2 + 3)} - \frac{4}{9x} - \frac{(19\sqrt{2} + 26i) \tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{48\sqrt{2-2i\sqrt{2}}} - \frac{(19\sqrt{2} - 26i) \tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{48\sqrt{2+2i\sqrt{2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(3 + 2*x^2 + x^4)^2), x]
```

```
[Out] -4/(9*x) - (25*x*(5 + x^2))/(72*(3 + 2*x^2 + x^4)) - ((26*I + 19*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/(48*Sqrt[2 - (2*I)*Sqrt[2]]) - ((-26*I + 19*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/(48*Sqrt[2 + (2*I)*Sqrt[2]])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4 + x^2 + 3x^4 + 5x^6}{x^2(3 + 2x^2 + x^4)^2} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(3 + 2*x^2 + x^4)^2), x]
```

```
[Out] IntegrateAlgebraic[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(3 + 2*x^2 + x^4)^2), x]
```

fricas [B] time = 1.35, size = 471, normalized size = 2.06

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] -1/208156608*(164790648*x^4 - 2068*1465803^(1/4)*sqrt(2)*(x^5 + 2*x^3 + 3*x)*sqrt(-674535*sqrt(3) + 1465803)*arctan(1/547726639257666*1465803^(3/4)*sqrt(120461)*sqrt(1084149*x^2 + 1465803^(1/4)*(7*sqrt(3)*x + 57*x))*sqrt(-674535*sqrt(3) + 1465803) + 1084149*sqrt(3))*(19*sqrt(3)*sqrt(2) + 7*sqrt(2))*sqrt(-674535*sqrt(3) + 1465803) - 1/1515640302*1465803^(3/4)*(19*sqrt(3)*sqrt(2)*x + 7*sqrt(2)*x)*sqrt(-674535*sqrt(3) + 1465803) - 1/2*sqrt(3)*sqrt(2) + 1/2*sqrt(2)) - 2068*1465803^(1/4)*sqrt(2)*(x^5 + 2*x^3 + 3*x)*sqrt(-674535*sqrt(3) + 1465803)*arctan(1/547726639257666*1465803^(3/4)*sqrt(120461)*sqrt(1084149*x^2 - 1465803^(1/4)*(7*sqrt(3)*x + 57*x))*sqrt(-674535*sqrt(3) + 1465803) + 1084149*sqrt(3))*(19*sqrt(3)*sqrt(2) + 7*sqrt(2))*sqrt(-674535*sqrt(3) + 1465803) - 1/1515640302*1465803^(3/4)*(19*sqrt(3)*sqrt(2)*x + 7*sqrt(2)*x)*sqrt(-674535*sqrt(3) + 1465803) + 1/2*sqrt(3)*sqrt(2) - 1/2*sqrt(2)) - 1465803^(1/4)*(965*x^5 + 1930*x^3 + 699*sqrt(3)*(x^5 + 2*x^3 + 3*x) + 2895*x)*sqrt(-674535*sqrt(3) + 1465803)*log(1084149*x^2 + 1465803^(1/4)*(7*sqrt(3)*x + 57*x))*sqrt(-674535*sqrt(3) + 1465803) + 1084149*sqrt(3)) + 1465803^(1/4)*(965*x^5 + 1930*x^3 + 699*sqrt(3)*(x^5 + 2*x^3 + 3*x) + 2895*x)*sqrt(-674535*sqrt(3) + 1465803)*log(1084149*x^2 - 1465803^(1/4)*(7*sqrt(3)*x + 57*x))*sqrt(-674535*sqrt(3) + 1465803) + 1084149*sqrt(3)) + 546411096*x^2 + 277542144)/(x^5 + 2*x^3 + 3*x)
```

giac [B] time = 1.94, size = 572, normalized size = 2.50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] 1/62208*sqrt(2)*(19*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 342*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 342*3^(3/4)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) + 19*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 252*3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) - 252*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) + 1/62208*sqrt(2)*(19*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 342*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 342*3^(3/4)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) + 19*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 252*3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) - 252*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3/4)*(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) + 1/124416*sqrt(
```

2)*(342*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 19*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 19*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 342*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) + 252*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) + 252*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 + 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) - 1/124416*sqrt(2)*(342*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 19*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 19*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 342*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) + 252*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) + 252*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 - 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) - 1/24*(19*x^4 + 63*x^2 + 32)/(x^5 + 2*x^3 + 3*x)

maple [B] time = 0.03, size = 414, normalized size = 1.81

$$\frac{(-2+2\sqrt{3})\sqrt{3}\arctan\left(\frac{x-\sqrt{2+2\sqrt{3}}}{2+2\sqrt{3}}\right) - 13(-2+2\sqrt{3})\arctan\left(\frac{x-\sqrt{2+2\sqrt{3}}}{2+2\sqrt{3}}\right) - 7\sqrt{3}\arctan\left(\frac{x-\sqrt{2+2\sqrt{3}}}{2+2\sqrt{3}}\right) - (-2+2\sqrt{3})\sqrt{3}\arctan\left(\frac{x-\sqrt{2+2\sqrt{3}}}{2+2\sqrt{3}}\right) - 13(-2+2\sqrt{3})\arctan\left(\frac{x-\sqrt{2+2\sqrt{3}}}{2+2\sqrt{3}}\right) - 7\sqrt{3}\arctan\left(\frac{x-\sqrt{2+2\sqrt{3}}}{2+2\sqrt{3}}\right) - \sqrt{-2+2\sqrt{3}}\sqrt{3}\ln\left(\frac{x^2-\sqrt{-2+2\sqrt{3}}x+\sqrt{3}}{18}\right) - 13\sqrt{-2+2\sqrt{3}}\sqrt{3}\ln\left(\frac{x^2-\sqrt{-2+2\sqrt{3}}x+\sqrt{3}}{18}\right) - \sqrt{-2+2\sqrt{3}}\sqrt{3}\ln\left(\frac{x^2-\sqrt{-2+2\sqrt{3}}x+\sqrt{3}}{18}\right) - 13\sqrt{-2+2\sqrt{3}}\sqrt{3}\ln\left(\frac{x^2-\sqrt{-2+2\sqrt{3}}x+\sqrt{3}}{18}\right) - \frac{4}{15}\frac{2x^2+3}{x^5+2x^3+3x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^6+3*x^4+x^2+4)/x^2/(x^4+2*x^2+3)^2,x)
 [Out] -4/9/x-1/9*(25/8*x^3+125/8*x)/(x^4+2*x^2+3)-1/18*(-2+2*3^(1/2))^(1/2)*3^(1/2)*ln(x^2-(-2+2*3^(1/2))^(1/2)*x+3^(1/2))-13/192*(-2+2*3^(1/2))^(1/2)*ln(x^2-(-2+2*3^(1/2))^(1/2)*x+3^(1/2))-1/9/(2+2*3^(1/2))^(1/2)*(-2+2*3^(1/2))*3^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))-13/96/(2+2*3^(1/2))^(1/2)*(-2+2*3^(1/2))*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))+7/72/(2+2*3^(1/2))^(1/2)*3^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))+1/18*(-2+2*3^(1/2))^(1/2)*3^(1/2)*ln(x^2+(-2+2*3^(1/2))^(1/2)*x+3^(1/2))+13/192*(-2+2*3^(1/2))^(1/2)*ln(x^2+(-2+2*3^(1/2))^(1/2)*x+3^(1/2))-1/9/(2+2*3^(1/2))^(1/2)*(-2+2*3^(1/2))*3^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))-13/96/(2+2*3^(1/2))^(1/2)*(-2+2*3^(1/2))*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))+7/72/(2+2*3^(1/2))^(1/2)*3^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{19x^4 + 63x^2 + 32}{24(x^5 + 2x^3 + 3x)} - \frac{1}{24} \int \frac{19x^2 - 7}{x^4 + 2x^2 + 3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+2*x^2+3)^2,x, algorithm="maxima")
 [Out] -1/24*(19*x^4 + 63*x^2 + 32)/(x^5 + 2*x^3 + 3*x) - 1/24*integrate((19*x^2 - 7)/(x^4 + 2*x^2 + 3), x)

mupad [B] time = 0.14, size = 159, normalized size = 0.69

$$-\frac{\frac{19x^4}{24} + \frac{21x^2}{8} + \frac{4}{3}}{x^5 + 2x^3 + 3x} - \frac{\operatorname{atan}\left(\frac{x\sqrt{2895-\sqrt{2}1551i}517i}{15552\left(\frac{517}{162} + \frac{\sqrt{2}3619i}{10368}\right)} + \frac{517\sqrt{2}x\sqrt{2895-\sqrt{2}1551i}}{31104\left(\frac{517}{162} + \frac{\sqrt{2}3619i}{10368}\right)}\right)\sqrt{2895-\sqrt{2}1551i}i}{144} + \frac{\operatorname{atan}\left(\frac{x\sqrt{2895+\sqrt{2}1551i}517i}{15552\left(-\frac{517}{162} + \frac{\sqrt{2}3619i}{10368}\right)} - \frac{517\sqrt{2}x\sqrt{2895+\sqrt{2}1551i}}{31104\left(-\frac{517}{162} + \frac{\sqrt{2}3619i}{10368}\right)}\right)\sqrt{2895+\sqrt{2}1551i}i}{144}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^2*(2*x^2 + x^4 + 3)^2),x)
 [Out] (atan((x*(2^(1/2)*1551i + 2895)^(1/2)*517i)/(15552*((2^(1/2)*3619i)/10368 - 517/162))) - (517*2^(1/2)*x*(2^(1/2)*1551i + 2895)^(1/2))/(31104*((2^(1/2)*3619i)/10368 - 517/162)))*(2^(1/2)*1551i + 2895)^(1/2)*1i/144 - (atan((x*(2895 - 2^(1/2)*1551i)^(1/2)*517i)/(15552*((2^(1/2)*3619i)/10368 + 517/162))) + (517*2^(1/2)*x*(2895 - 2^(1/2)*1551i)^(1/2))/(31104*((2^(1/2)*3619i)/10368 + 517/162)))*(2895 - 2^(1/2)*1551i)^(1/2)*1i/144 - ((21*x^2)/8 + (19*x^4)/24 + 4/3)/(3*x + 2*x^3 + x^5)

sympy [B] time = 1.32, size = 1192, normalized size = 5.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**2/(x**4+2*x**2+3)**2,x)
```

```
[Out] (-19*x**4 - 63*x**2 - 32)/(24*x**5 + 48*x**3 + 72*x) - sqrt(965/55296 + 233
*sqrt(3)/18432)*log(x**2 + x*(-128*sqrt(2)*sqrt(965 + 699*sqrt(3)))/517 - 21
793*sqrt(6)*sqrt(965 + 699*sqrt(3))/361383 + 64*sqrt(3)*sqrt(965 + 699*sqrt
(3))*sqrt(674535*sqrt(3) + 1198514)/361383) - 8882635459*sqrt(2)*sqrt(67453
5*sqrt(3) + 1198514)/130597672689 - 20458048*sqrt(6)*sqrt(674535*sqrt(3) +
1198514)/560505033 + 18567565928783/130597672689 + 46950427730*sqrt(3)/5605
05033) + sqrt(965/55296 + 233*sqrt(3)/18432)*log(x**2 + x*(-64*sqrt(3)*sqrt
(965 + 699*sqrt(3))*sqrt(674535*sqrt(3) + 1198514)/361383 + 21793*sqrt(6)*s
qrt(965 + 699*sqrt(3))/361383 + 128*sqrt(2)*sqrt(965 + 699*sqrt(3))/517) -
8882635459*sqrt(2)*sqrt(674535*sqrt(3) + 1198514)/130597672689 - 20458048*s
qrt(6)*sqrt(674535*sqrt(3) + 1198514)/560505033 + 18567565928783/1305976726
89 + 46950427730*sqrt(3)/560505033) + 2*sqrt(-sqrt(2)*sqrt(674535*sqrt(3) +
1198514)/27648 + 965/55296 + 233*sqrt(3)/6144)*atan(722766*sqrt(3)*x/(-64*
sqrt(674535*sqrt(3) + 1198514)*sqrt(-2*sqrt(2)*sqrt(674535*sqrt(3) + 119851
4) + 965 + 2097*sqrt(3)) + 3619*sqrt(2)*sqrt(-2*sqrt(2)*sqrt(674535*sqrt(3)
+ 1198514) + 965 + 2097*sqrt(3))) + 89472*sqrt(6)*sqrt(965 + 699*sqrt(3))/
(-64*sqrt(674535*sqrt(3) + 1198514)*sqrt(-2*sqrt(2)*sqrt(674535*sqrt(3) + 1
198514) + 965 + 2097*sqrt(3)) + 3619*sqrt(2)*sqrt(-2*sqrt(2)*sqrt(674535*s
qrt(3) + 1198514) + 965 + 2097*sqrt(3))) + 65379*sqrt(2)*sqrt(965 + 699*sqrt
(3))/(-64*sqrt(674535*sqrt(3) + 1198514)*sqrt(-2*sqrt(2)*sqrt(674535*sqrt(3
) + 1198514) + 965 + 2097*sqrt(3)) + 3619*sqrt(2)*sqrt(-2*sqrt(2)*sqrt(6745
35*sqrt(3) + 1198514) + 965 + 2097*sqrt(3))) - 192*sqrt(965 + 699*sqrt(3))*
sqrt(674535*sqrt(3) + 1198514)/(-64*sqrt(674535*sqrt(3) + 1198514)*sqrt(-2*
sqrt(2)*sqrt(674535*sqrt(3) + 1198514) + 965 + 2097*sqrt(3)) + 3619*sqrt(2)
*sqrt(-2*sqrt(2)*sqrt(674535*sqrt(3) + 1198514) + 965 + 2097*sqrt(3)))) + 2
*sqrt(-sqrt(2)*sqrt(674535*sqrt(3) + 1198514)/27648 + 965/55296 + 233*sqrt(
3)/6144)*atan(722766*sqrt(3)*x/(-64*sqrt(674535*sqrt(3) + 1198514)*sqrt(-2*
sqrt(2)*sqrt(674535*sqrt(3) + 1198514) + 965 + 2097*sqrt(3)) + 3619*sqrt(2)
*sqrt(-2*sqrt(2)*sqrt(674535*sqrt(3) + 1198514) + 965 + 2097*sqrt(3))) + 19
2*sqrt(965 + 699*sqrt(3))*sqrt(674535*sqrt(3) + 1198514)/(-64*sqrt(674535*s
qrt(3) + 1198514)*sqrt(-2*sqrt(2)*sqrt(674535*sqrt(3) + 1198514) + 965 + 20
97*sqrt(3)) + 3619*sqrt(2)*sqrt(-2*sqrt(2)*sqrt(674535*sqrt(3) + 1198514) +
965 + 2097*sqrt(3))) - 65379*sqrt(2)*sqrt(965 + 699*sqrt(3))/(-64*sqrt(674
535*sqrt(3) + 1198514)*sqrt(-2*sqrt(2)*sqrt(674535*sqrt(3) + 1198514) + 965
+ 2097*sqrt(3)) + 3619*sqrt(2)*sqrt(-2*sqrt(2)*sqrt(674535*sqrt(3) + 11985
14) + 965 + 2097*sqrt(3))) - 89472*sqrt(6)*sqrt(965 + 699*sqrt(3))/(-64*sq
rt(674535*sqrt(3) + 1198514)*sqrt(-2*sqrt(2)*sqrt(674535*sqrt(3) + 1198514)
+ 965 + 2097*sqrt(3)) + 3619*sqrt(2)*sqrt(-2*sqrt(2)*sqrt(674535*sqrt(3) +
1198514) + 965 + 2097*sqrt(3))))
```

$$3.113 \quad \int \frac{4+x^2+3x^4+5x^6}{x^4(3+2x^2+x^4)^2} dx$$

Optimal. Leaf size=238

$$-\frac{4}{27x^3} + \frac{1}{864} \sqrt{\frac{1}{6}(56673\sqrt{3} - 6073)} \log\left(x^2 - \sqrt{2(\sqrt{3}-1)}x + \sqrt{3}\right) - \frac{1}{864} \sqrt{\frac{1}{6}(56673\sqrt{3} - 6073)} \log\left(x^2 + \sqrt{2(\sqrt{3}-1)}x + \sqrt{3}\right)$$

Rubi [A] time = 0.34, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {1669, 1664, 1169, 634, 618, 204, 628}

$$\frac{25x(5x^2+7)}{216(x^4+2x^2+3)} - \frac{4}{27x^3} + \frac{1}{864} \sqrt{\frac{1}{6}(56673\sqrt{3}-6073)} \log(x^2 - \sqrt{2(\sqrt{3}-1)}x + \sqrt{3}) - \frac{1}{864} \sqrt{\frac{1}{6}(56673\sqrt{3}-6073)} \log(x^2 + \sqrt{2(\sqrt{3}-1)}x + \sqrt{3}) + \frac{13}{27x} - \frac{1}{432} \sqrt{\frac{1}{6}(6073+56673\sqrt{3})} \tan^{-1}\left(\frac{\sqrt{2(\sqrt{3}-1)}-2x}{\sqrt{2(1+\sqrt{3})}}\right) + \frac{1}{432} \sqrt{\frac{1}{6}(6073+56673\sqrt{3})} \tan^{-1}\left(\frac{2x+\sqrt{2(\sqrt{3}-1)}}{\sqrt{2(1+\sqrt{3})}}\right)$$

Antiderivative was successfully verified.

[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(3 + 2*x^2 + x^4)^2), x]

[Out] -4/(27*x^3) + 13/(27*x) + (25*x*(7 + 5*x^2))/(216*(3 + 2*x^2 + x^4)) - (Sqrt[(6073 + 56673*Sqrt[3])/6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3]]) - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/432 + (Sqrt[(6073 + 56673*Sqrt[3])/6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3]]) + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/432 + (Sqrt[(-6073 + 56673*Sqrt[3])/6]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/864 - (Sqrt[(-6073 + 56673*Sqrt[3])/6]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/864

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1169

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]

Rule 1664

```
Int[(Pq_)*((d_)*(x_)^(m_))*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1669

```
Int[(Pq_)*(x_)^(m_))*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :>
With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{4 + x^2 + 3x^4 + 5x^6}{x^4(3 + 2x^2 + x^4)^2} dx &= \frac{25x(7 + 5x^2)}{216(3 + 2x^2 + x^4)} + \frac{1}{48} \int \frac{64 - \frac{80x^2}{3} + \frac{50x^4}{9} + \frac{250x^6}{9}}{x^4(3 + 2x^2 + x^4)} dx \\
&= \frac{25x(7 + 5x^2)}{216(3 + 2x^2 + x^4)} + \frac{1}{48} \int \left(\frac{64}{3x^4} - \frac{208}{9x^2} + \frac{2(137 + 229x^2)}{9(3 + 2x^2 + x^4)} \right) dx \\
&= -\frac{4}{27x^3} + \frac{13}{27x} + \frac{25x(7 + 5x^2)}{216(3 + 2x^2 + x^4)} + \frac{1}{216} \int \frac{137 + 229x^2}{3 + 2x^2 + x^4} dx \\
&= -\frac{4}{27x^3} + \frac{13}{27x} + \frac{25x(7 + 5x^2)}{216(3 + 2x^2 + x^4)} + \frac{\int \frac{137\sqrt{2(-1+\sqrt{3})} - (137-229\sqrt{3})x}{\sqrt{3} - \sqrt{2(-1+\sqrt{3})x+x^2}} dx}{432\sqrt{6(-1+\sqrt{3})}} + \frac{\int \frac{137\sqrt{2}}{\sqrt{3}}}{432} \\
&= -\frac{4}{27x^3} + \frac{13}{27x} + \frac{25x(7 + 5x^2)}{216(3 + 2x^2 + x^4)} + \frac{1}{432} \sqrt{\frac{1}{6}(88046 + 31373\sqrt{3})} \int \frac{1}{\sqrt{3} - \sqrt{2(-1+\sqrt{3})x+x^2}} \\
&= -\frac{4}{27x^3} + \frac{13}{27x} + \frac{25x(7 + 5x^2)}{216(3 + 2x^2 + x^4)} + \frac{1}{864} \sqrt{\frac{1}{6}(-6073 + 56673\sqrt{3})} \log\left(\sqrt{3} - \sqrt{2(-1+\sqrt{3})x+x^2}\right) \\
&= -\frac{4}{27x^3} + \frac{13}{27x} + \frac{25x(7 + 5x^2)}{216(3 + 2x^2 + x^4)} - \frac{1}{432} \sqrt{\frac{1}{6}(6073 + 56673\sqrt{3})} \tan^{-1}\left(\frac{\sqrt{2(-1+\sqrt{3})x+x^2}}{\sqrt{2(-1+\sqrt{3})x+x^2}}\right)
\end{aligned}$$

Mathematica [C] time = 0.29, size = 131, normalized size = 0.55

$$\frac{1}{864} \left(\frac{4(229x^6 + 351x^4 + 248x^2 - 96)}{x^3(x^4 + 2x^2 + 3)} + \frac{2(229 + 46i\sqrt{2}) \tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}} + \frac{2(229 - 46i\sqrt{2}) \tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(3 + 2*x^2 + x^4)^2), x]

```
[Out] ((4*(-96 + 248*x^2 + 351*x^4 + 229*x^6))/(x^3*(3 + 2*x^2 + x^4)) + (2*(229 + (46*I)*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/Sqrt[1 - I*Sqrt[2]] + (2*(229 - (46*I)*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/Sqrt[1 + I*Sqrt[2]])/64
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4 + x^2 + 3x^4 + 5x^6}{x^4(3 + 2x^2 + x^4)^2} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(3 + 2*x^2 + x^4)^2), x]
```

```
[Out] IntegrateAlgebraic[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(3 + 2*x^2 + x^4)^2), x]
```

fricas [B] time = 1.36, size = 528, normalized size = 2.22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] 1/2261454002496*(2397560030424*x^6 + 3674862754056*x^4 - 277108*118956627^(1/4)*sqrt(6297)*sqrt(2)*(x^7 + 2*x^5 + 3*x^3)*sqrt(6073*sqrt(3) + 170019)*arctan(1/295480530439458889122*118956627^(3/4)*sqrt(81861)*sqrt(6297)*sqrt(3*118956627^(1/4)*sqrt(6297)*(137*sqrt(3)*x - 687*x)*sqrt(6073*sqrt(3) + 170019) + 3926135421*x^2 + 3926135421*sqrt(3))*(229*sqrt(3)*sqrt(2) - 137*sqrt(2))*sqrt(6073*sqrt(3) + 170019) - 1/16481916497358*118956627^(3/4)*sqrt(6297)*(229*sqrt(3)*sqrt(2)*x - 137*sqrt(2)*x)*sqrt(6073*sqrt(3) + 170019) + 1/2*sqrt(3)*sqrt(2) - 1/2*sqrt(2)) - 277108*118956627^(1/4)*sqrt(6297)*sqrt(2)*(x^7 + 2*x^5 + 3*x^3)*sqrt(6073*sqrt(3) + 170019)*arctan(1/295480530439458889122*118956627^(3/4)*sqrt(81861)*sqrt(6297)*sqrt(-3*118956627^(1/4)*sqrt(6297)*(137*sqrt(3)*x - 687*x)*sqrt(6073*sqrt(3) + 170019) + 3926135421*x^2 + 3926135421*sqrt(3))*(229*sqrt(3)*sqrt(2) - 137*sqrt(2))*sqrt(6073*sqrt(3) + 170019) - 1/16481916497358*118956627^(3/4)*sqrt(6297)*(229*sqrt(3)*sqrt(2)*x - 137*sqrt(2)*x)*sqrt(6073*sqrt(3) + 170019) - 1/2*sqrt(3)*sqrt(2) + 1/2*sqrt(2)) - 118956627^(1/4)*sqrt(6297)*(6073*x^7 + 12146*x^5 + 18219*x^3 - 56673*sqrt(3)*(x^7 + 2*x^5 + 3*x^3))*sqrt(6073*sqrt(3) + 170019)*log(3*118956627^(1/4)*sqrt(6297)*(137*sqrt(3)*x - 687*x)*sqrt(6073*sqrt(3) + 170019) + 3926135421*x^2 + 3926135421*sqrt(3)) + 118956627^(1/4)*sqrt(6297)*(6073*x^7 + 12146*x^5 + 18219*x^3 - 56673*sqrt(3)*(x^7 + 2*x^5 + 3*x^3))*sqrt(6073*sqrt(3) + 170019)*log(-3*118956627^(1/4)*sqrt(6297)*(137*sqrt(3)*x - 687*x)*sqrt(6073*sqrt(3) + 170019) + 3926135421*x^2 + 3926135421*sqrt(3)) + 2596484225088*x^2 - 1005090667776)/(x^7 + 2*x^5 + 3*x^3)
```

giac [B] time = 1.85, size = 579, normalized size = 2.43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] -1/559872*sqrt(2)*(229*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 4122*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 4122*3^(3/4)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) + 229*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 4932*3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) + 4932*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) - 1/559872*sqrt(2)*(229*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 4122*3^(3/4)
```

) * sqrt(2) * sqrt(6 * sqrt(3) + 18) * (sqrt(3) - 3) - 4122 * 3^(3/4) * (sqrt(3) + 3) * sqrt(-6 * sqrt(3) + 18) + 229 * 3^(3/4) * (-6 * sqrt(3) + 18)^(3/2) - 4932 * 3^(1/4) * sqrt(2) * sqrt(6 * sqrt(3) + 18) + 4932 * 3^(1/4) * sqrt(-6 * sqrt(3) + 18) * arctan(1/3 * 3^(3/4) * (x - 3^(1/4) * sqrt(-1/6 * sqrt(3) + 1/2)) / sqrt(1/6 * sqrt(3) + 1/2)) - 1/1119744 * sqrt(2) * (4122 * 3^(3/4) * sqrt(2) * (sqrt(3) + 3) * sqrt(-6 * sqrt(3) + 18) - 229 * 3^(3/4) * sqrt(2) * (-6 * sqrt(3) + 18)^(3/2) + 229 * 3^(3/4) * (6 * sqrt(3) + 18)^(3/2) + 4122 * 3^(3/4) * sqrt(6 * sqrt(3) + 18) * (sqrt(3) - 3) - 4932 * 3^(1/4) * sqrt(2) * sqrt(-6 * sqrt(3) + 18) - 4932 * 3^(1/4) * sqrt(6 * sqrt(3) + 18)) * log(x^2 + 2 * 3^(1/4) * x * sqrt(-1/6 * sqrt(3) + 1/2) + sqrt(3)) + 1/1119744 * sqrt(2) * (4122 * 3^(3/4) * sqrt(2) * (sqrt(3) + 3) * sqrt(-6 * sqrt(3) + 18) - 229 * 3^(3/4) * sqrt(2) * (-6 * sqrt(3) + 18)^(3/2) + 229 * 3^(3/4) * (6 * sqrt(3) + 18)^(3/2) + 4122 * 3^(3/4) * sqrt(6 * sqrt(3) + 18) * (sqrt(3) - 3) - 4932 * 3^(1/4) * sqrt(2) * sqrt(-6 * sqrt(3) + 18) - 4932 * 3^(1/4) * sqrt(6 * sqrt(3) + 18)) * log(x^2 - 2 * 3^(1/4) * x * sqrt(-1/6 * sqrt(3) + 1/2) + sqrt(3)) + 25/216 * (5 * x^3 + 7 * x) / (x^4 + 2 * x^2 + 3) + 1/27 * (13 * x^2 - 4) / x^3

maple [B] time = 0.04, size = 419, normalized size = 1.76

$$\frac{229(-2+2\sqrt{3})\sqrt{2}\arctan\left(\frac{x\sqrt{-18219-\sqrt{2}2078311}692271}{3779136+\sqrt{2}9490949}\right)+23(-2+2\sqrt{3})\arctan\left(\frac{x\sqrt{-18219-\sqrt{2}2078311}692271}{3779136+\sqrt{2}9490949}\right)+137\sqrt{2}\arctan\left(\frac{x\sqrt{-18219-\sqrt{2}2078311}692271}{3779136+\sqrt{2}9490949}\right)+229(-2+2\sqrt{3})\sqrt{2}\arctan\left(\frac{x\sqrt{-18219-\sqrt{2}2078311}692271}{3779136+\sqrt{2}9490949}\right)+23(-2+2\sqrt{3})\arctan\left(\frac{x\sqrt{-18219-\sqrt{2}2078311}692271}{3779136+\sqrt{2}9490949}\right)+137\sqrt{2}\arctan\left(\frac{x\sqrt{-18219-\sqrt{2}2078311}692271}{3779136+\sqrt{2}9490949}\right)+229\sqrt{-2+2\sqrt{3}}\sqrt{2}\ln\left(\frac{x\sqrt{-18219-\sqrt{2}2078311}692271}{3779136+\sqrt{2}9490949}\right)+23\sqrt{-2+2\sqrt{3}}\ln\left(\frac{x\sqrt{-18219-\sqrt{2}2078311}692271}{3779136+\sqrt{2}9490949}\right)+25\sqrt{-2+2\sqrt{3}}\sqrt{2}\ln\left(\frac{x\sqrt{-18219-\sqrt{2}2078311}692271}{3779136+\sqrt{2}9490949}\right)+23\sqrt{-2+2\sqrt{3}}\ln\left(\frac{x\sqrt{-18219-\sqrt{2}2078311}692271}{3779136+\sqrt{2}9490949}\right)+\frac{11}{27}\frac{4}{27}\frac{13}{27}\frac{1}{27}}{216\sqrt{2}+2\sqrt{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^6+3*x^4+x^2+4)/x^4/(x^4+2*x^2+3)^2,x)

[Out] -4/27/x^3+13/27/x+1/27*(125/8*x^3+175/8*x)/(x^4+2*x^2+3)+275/5184*(-2+2*3^(1/2))^(1/2)*3^(1/2)*ln(x^2-(-2+2*3^(1/2))^(1/2)*x+3^(1/2))+23/864*(-2+2*3^(1/2))^(1/2)*ln(x^2-(-2+2*3^(1/2))^(1/2)*x+3^(1/2))+275/2592/(2+2*3^(1/2))^(1/2)*(-2+2*3^(1/2))*3^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))+23/432/(2+2*3^(1/2))^(1/2)*(-2+2*3^(1/2))*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))+137/648/(2+2*3^(1/2))^(1/2)*3^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))-275/5184*(-2+2*3^(1/2))^(1/2)*3^(1/2)*ln(x^2+(-2+2*3^(1/2))^(1/2)*x+3^(1/2))-23/864*(-2+2*3^(1/2))^(1/2)*ln(x^2+(-2+2*3^(1/2))^(1/2)*x+3^(1/2))+275/2592/(2+2*3^(1/2))^(1/2)*(-2+2*3^(1/2))*3^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))+23/432/(2+2*3^(1/2))^(1/2)*(-2+2*3^(1/2))*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))+137/648/(2+2*3^(1/2))^(1/2)*3^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{229x^6 + 351x^4 + 248x^2 - 96}{216(x^7 + 2x^5 + 3x^3)} + \frac{1}{216} \int \frac{229x^2 + 137}{x^4 + 2x^2 + 3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+2*x^2+3)^2,x, algorithm="maxima")

[Out] 1/216*(229*x^6 + 351*x^4 + 248*x^2 - 96)/(x^7 + 2*x^5 + 3*x^3) + 1/216*integrate((229*x^2 + 137)/(x^4 + 2*x^2 + 3), x)

mupad [B] time = 0.14, size = 165, normalized size = 0.69

$$\frac{229x^6 + 351x^4 + 248x^2 - 96}{216(x^7 + 2x^5 + 3x^3)} - \frac{4}{27} \operatorname{atan}\left(\frac{x\sqrt{-18219-\sqrt{2}2078311}692271}{11337408\left(\frac{19051175\sqrt{2}9490949}{3779136+\sqrt{2}9490949}\right)} + \frac{69227\sqrt{2}x\sqrt{-18219-\sqrt{2}2078311}}{22674816\left(\frac{19051175\sqrt{2}9490949}{3779136+\sqrt{2}9490949}\right)}\right) \sqrt{-18219-\sqrt{2}2078311} \operatorname{li} \operatorname{atan}\left(\frac{x\sqrt{-18219+\sqrt{2}2078311}692271}{11337408\left(\frac{19051175\sqrt{2}9490949}{3779136+\sqrt{2}9490949}\right)} - \frac{69227\sqrt{2}x\sqrt{-18219+\sqrt{2}2078311}}{22674816\left(\frac{19051175\sqrt{2}9490949}{3779136+\sqrt{2}9490949}\right)}\right) \sqrt{-18219+\sqrt{2}2078311} \operatorname{li} + \frac{1296}{1296}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^4*(2*x^2 + x^4 + 3)^2),x)

[Out] ((31*x^2)/27 + (13*x^4)/8 + (229*x^6)/216 - 4/9)/(3*x^3 + 2*x^5 + x^7) - (atan((x*(-2^(1/2)*2078311i - 18219)^(1/2)*69277i)/(11337408*((2^(1/2)*9490949i)/7558272 - 19051175/3779136))) + (69277*2^(1/2)*x*(-2^(1/2)*2078311i - 18

$$\frac{219^{1/2}}{(22674816 * ((2^{1/2} * 9490949i) / 7558272 - 19051175 / 3779136))} * (-2^{1/2} * 207831i - 18219)^{1/2} * 1i / 1296 + (\operatorname{atan}((x * (2^{1/2} * 207831i - 18219)^{1/2} * 69277i) / (11337408 * ((2^{1/2} * 9490949i) / 7558272 + 19051175 / 3779136)) - (69277 * 2^{1/2} * x * (2^{1/2} * 207831i - 18219)^{1/2})) / (22674816 * ((2^{1/2} * 9490949i) / 7558272 + 19051175 / 3779136)))) * (2^{1/2} * 207831i - 18219)^{1/2} * 1i) / 1296$$

sympy [A] time = 0.65, size = 60, normalized size = 0.25

$$\operatorname{RootSum}\left(2293235712t^4 + 12437504t^2 + 4405801, \left(t \mapsto t \log\left(\frac{19707494400t^3}{145412423} + \frac{357152768t}{145412423} + x\right)\right)\right) + \frac{229x^6 + 351x^4 + 248x^2 - 96}{216x^7 + 432x^5 + 648x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**6+3*x**4+x**2+4)/x**4/(x**4+2*x**2+3)**2,x)

[Out] RootSum(2293235712*_t**4 + 12437504*_t**2 + 4405801, Lambda(_t, _t*log(19707494400*_t**3/145412423 + 357152768*_t/145412423 + x))) + (229*x**6 + 351*x**4 + 248*x**2 - 96)/(216*x**7 + 432*x**5 + 648*x**3)

$$3.114 \quad \int \frac{4+x^2+3x^4+5x^6}{x^6(3+2x^2+x^4)^2} dx$$

Optimal. Leaf size=245

$$-\frac{4}{45x^5} + \frac{13}{81x^3} - \frac{\sqrt{\frac{1}{6}(1139381 + 688419\sqrt{3})} \log\left(x^2 - \sqrt{2(\sqrt{3}-1)}x + \sqrt{3}\right)}{2592} + \frac{\sqrt{\frac{1}{6}(1139381 + 688419\sqrt{3})} \log\left(x^2 + \sqrt{2(\sqrt{3}-1)}x + \sqrt{3}\right)}{2592}$$

Rubi [A] time = 0.33, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 31, number of rules / integrand size = 0.226, Rules used = {1669, 1664, 1169, 634, 618, 204, 628}

$$\frac{25x(1-7x^2)}{648(x^2+2x^2+3)^2} + \frac{13}{81x^3} - \frac{4}{45x^5} - \frac{\sqrt{\frac{1}{6}(1139381 + 688419\sqrt{3})} \log(x^2 - \sqrt{2(\sqrt{3}-1)}x + \sqrt{3})}{2592} + \frac{\sqrt{\frac{1}{6}(1139381 + 688419\sqrt{3})} \log(x^2 + \sqrt{2(\sqrt{3}-1)}x + \sqrt{3})}{2592} - \frac{13}{27x} + \frac{\sqrt{\frac{1}{6}(688419\sqrt{3} - 1139381)} \tan^{-1}\left(\frac{\sqrt{2(\sqrt{3}-1)}x}{\sqrt{2(1+\sqrt{3})}}\right)}{1296} - \frac{\sqrt{\frac{1}{6}(688419\sqrt{3} - 1139381)} \tan^{-1}\left(\frac{2x + \sqrt{2(\sqrt{3}-1)}}{\sqrt{2(1+\sqrt{3})}}\right)}{1296}$$

Antiderivative was successfully verified.

[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^6*(3 + 2*x^2 + x^4)^2), x]

[Out] -4/(45*x^5) + 13/(81*x^3) - 13/(27*x) + (25*x*(1 - 7*x^2))/(648*(3 + 2*x^2 + x^4)) + (Sqrt[(-1139381 + 688419*Sqrt[3])/6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3]]) - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/1296 - (Sqrt[(-1139381 + 688419*Sqrt[3])/6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3]]) + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/1296 - (Sqrt[(1139381 + 688419*Sqrt[3])/6]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/2592 + (Sqrt[(1139381 + 688419*Sqrt[3])/6]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/2592

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1169

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ

$[b^2 - 4ac, 0] \ \&\& \ \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \ \&\& \ \text{NegQ}[b^2 - 4ac]$

Rule 1664

$\text{Int}[(Pq_)*((d_)*(x_))^{(m_)}*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^{(p_)}, x_Symbol] \ :> \ \text{Int}[\text{ExpandIntegrand}[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] \ /;$
 $\text{FreeQ}\{a, b, c, d, m\}, x] \ \&\& \ \text{PolyQ}[Pq, x^2] \ \&\& \ \text{IGtQ}[p, -2]$

Rule 1669

$\text{Int}[(Pq_)*(x_)^{(m_)}*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^{(p_)}, x_Symbol] \ :>$
 $\text{With}\{d = \text{Coeff}[\text{PolynomialRemainder}[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],$
 $e = \text{Coeff}[\text{PolynomialRemainder}[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]\}, \text{Simp}[($
 $x*(a + b*x^2 + c*x^4)^{(p+1)}*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)$
 $)/(2*a*(p+1)*(b^2 - 4*a*c)), x] + \text{Dist}[1/(2*a*(p+1)*(b^2 - 4*a*c)), \text{I}$
 $\text{nt}[x^m*(a + b*x^2 + c*x^4)^{(p+1)}*\text{ExpandToSum}[(2*a*(p+1)*(b^2 - 4*a*c)*\text{P}$
 $\text{olynomialQuotient}[x^m*Pq, a + b*x^2 + c*x^4, x)]/x^m + (b^2*d*(2*p+3) - 2$
 $*a*c*d*(4*p+5) - a*b*e)/x^m + c*(4*p+7)*(b*d - 2*a*e)*x^{(2-m)}, x], x]$
 $/;$ $\text{FreeQ}\{a, b, c\}, x] \ \&\& \ \text{PolyQ}[Pq, x^2] \ \&\& \ \text{GtQ}[\text{Expon}[Pq, x^2], 1] \ \&\&$
 $\text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ \text{ILtQ}[m/2, 0]$

Rubi steps

$$\int \frac{4 + x^2 + 3x^4 + 5x^6}{x^6(3 + 2x^2 + x^4)^2} dx = \frac{25x(1 - 7x^2)}{648(3 + 2x^2 + x^4)} + \frac{1}{48} \int \frac{64 - \frac{80x^2}{3} + \frac{400x^4}{9} + \frac{1550x^6}{27} - \frac{350x^8}{27}}{x^6(3 + 2x^2 + x^4)} dx$$

$$= \frac{25x(1 - 7x^2)}{648(3 + 2x^2 + x^4)} + \frac{1}{48} \int \left(\frac{64}{3x^6} - \frac{208}{9x^4} + \frac{208}{9x^2} - \frac{2(-463 + 487x^2)}{27(3 + 2x^2 + x^4)} \right) dx$$

$$= -\frac{4}{45x^5} + \frac{13}{81x^3} - \frac{13}{27x} + \frac{25x(1 - 7x^2)}{648(3 + 2x^2 + x^4)} - \frac{1}{648} \int \frac{-463 + 487x^2}{3 + 2x^2 + x^4} dx$$

$$= -\frac{4}{45x^5} + \frac{13}{81x^3} - \frac{13}{27x} + \frac{25x(1 - 7x^2)}{648(3 + 2x^2 + x^4)} - \frac{\int \frac{-463\sqrt{2(-1+\sqrt{3})} - (-463-487\sqrt{3})x}{\sqrt{3} - \sqrt{2(-1+\sqrt{3})}x + x^2} dx}{1296\sqrt{6(-1+\sqrt{3})}}$$

$$= -\frac{4}{45x^5} + \frac{13}{81x^3} - \frac{13}{27x} + \frac{25x(1 - 7x^2)}{648(3 + 2x^2 + x^4)} - \frac{(1461 - 463\sqrt{3}) \int \frac{1}{\sqrt{3} - \sqrt{2(-1+\sqrt{3})}x + x^2}}{7776}$$

$$= -\frac{4}{45x^5} + \frac{13}{81x^3} - \frac{13}{27x} + \frac{25x(1 - 7x^2)}{648(3 + 2x^2 + x^4)} - \frac{\sqrt{\frac{1}{6}(1139381 + 688419\sqrt{3})} \log(\sqrt{3} - \sqrt{2(-1+\sqrt{3})}x + x^2)}{2592}$$

$$= -\frac{4}{45x^5} + \frac{13}{81x^3} - \frac{13}{27x} + \frac{25x(1 - 7x^2)}{648(3 + 2x^2 + x^4)} + \frac{\sqrt{\frac{1}{6}(-1139381 + 688419\sqrt{3})} \tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{1296}$$

Mathematica [C] time = 0.29, size = 140, normalized size = 0.57

$$\frac{4(2435x^8 + 2475x^6 + 3928x^4 - 984x^2 + 864)}{x^5(x^4 + 2x^2 + 3)} - \frac{10i(475\sqrt{2} - 487i) \tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}} + \frac{10i(475\sqrt{2} + 487i) \tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}}$$

12960

Antiderivative was successfully verified.

[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^6*(3 + 2*x^2 + x^4)^2), x]

[Out] ((-4*(864 - 984*x^2 + 3928*x^4 + 2475*x^6 + 2435*x^8))/(x^5*(3 + 2*x^2 + x^4)) - ((10*I)*(-487*I + 475*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/Sqrt[1 - I*Sqrt[2]] + ((10*I)*(487*I + 475*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/Sqrt[1 + I*Sqrt[2]])/12960

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4 + x^2 + 3x^4 + 5x^6}{x^6(3 + 2x^2 + x^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(4 + x^2 + 3*x^4 + 5*x^6)/(x^6*(3 + 2*x^2 + x^4)^2), x]

[Out] IntegrateAlgebraic[(4 + x^2 + 3*x^4 + 5*x^6)/(x^6*(3 + 2*x^2 + x^4)^2), x]

fricas [B] time = 1.44, size = 496, normalized size = 2.02

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^6+3*x^4+x^2+4)/x^6/(x^4+2*x^2+3)^2,x, algorithm="fricas")

[Out] -1/1478473537631040*(1111136748188760*x^8 + 1129389507912600*x^6 + 1792421004881088*x^4 - 4971380*216699003^(1/4)*sqrt(2)*(x^9 + 2*x^7 + 3*x^5)*sqrt(-784371528639*sqrt(3) + 1421762158683)*arctan(1/6144866223568721756453718*sqrt(704195977)*216699003^(3/4)*sqrt(57039874137*x^2 + 216699003^(1/4)*(463*sqrt(3)*x + 1461*x)*sqrt(-784371528639*sqrt(3) + 1421762158683) + 57039874137*sqrt(3))*(487*sqrt(3)*sqrt(2) + 463*sqrt(2))*sqrt(-784371528639*sqrt(3) + 1421762158683) - 1/969563780580726*216699003^(3/4)*(487*sqrt(3)*sqrt(2)*x + 463*sqrt(2)*x)*sqrt(-784371528639*sqrt(3) + 1421762158683) - 1/2*sqrt(3)*sqrt(2) + 1/2*sqrt(2)) - 4971380*216699003^(1/4)*sqrt(2)*(x^9 + 2*x^7 + 3*x^5)*sqrt(-784371528639*sqrt(3) + 1421762158683)*arctan(1/6144866223568721756453718*sqrt(704195977)*216699003^(3/4)*sqrt(57039874137*x^2 - 216699003^(1/4)*(463*sqrt(3)*x + 1461*x)*sqrt(-784371528639*sqrt(3) + 1421762158683) + 57039874137*sqrt(3))*(487*sqrt(3)*sqrt(2) + 463*sqrt(2))*sqrt(-784371528639*sqrt(3) + 1421762158683) - 1/969563780580726*216699003^(3/4)*(487*sqrt(3)*sqrt(2)*x + 463*sqrt(2)*x)*sqrt(-784371528639*sqrt(3) + 1421762158683) + 1/2*sqrt(3)*sqrt(2) - 1/2*sqrt(2)) - 5*216699003^(1/4)*(1139381*x^9 + 2278762*x^7 + 3418143*x^5 + 688419*sqrt(3)*(x^9 + 2*x^7 + 3*x^5))*sqrt(-784371528639*sqrt(3) + 1421762158683)*log(57039874137*x^2 + 216699003^(1/4)*(463*sqrt(3)*x + 1461*x)*sqrt(-784371528639*sqrt(3) + 1421762158683) + 57039874137*sqrt(3)) + 5*216699003^(1/4)*(1139381*x^9 + 2278762*x^7 + 3418143*x^5 + 688419*sqrt(3)*(x^9 + 2*x^7 + 3*x^5))*sqrt(-784371528639*sqrt(3) + 1421762158683)*log(57039874137*x^2 - 216699003^(1/4)*(463*sqrt(3)*x + 1461*x)*sqrt(-784371528639*sqrt(3) + 1421762158683) + 57039874137*sqrt(3)) - 449017889206464*x^2 + 394259610034944)/(x^9 + 2*x^7 + 3*x^5)

giac [B] time = 1.78, size = 584, normalized size = 2.38

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^6+3*x^4+x^2+4)/x^6/(x^4+2*x^2+3)^2,x, algorithm="giac")

[Out] 1/1679616*sqrt(2)*(487*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 8766*3^(3/4))*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 8766*3^(3/4)*(sqrt(3) + 3)*s

$$\begin{aligned} & \text{qrt}(-6*\text{sqrt}(3) + 18) + 487*3^{(3/4)}*(-6*\text{sqrt}(3) + 18)^{(3/2)} + 16668*3^{(1/4)}* \\ & \text{sqrt}(2)*\text{sqrt}(6*\text{sqrt}(3) + 18) - 16668*3^{(1/4)}*\text{sqrt}(-6*\text{sqrt}(3) + 18)*\text{arctan}(\\ & 1/3*3^{(3/4)}*(x + 3^{(1/4)}*\text{sqrt}(-1/6*\text{sqrt}(3) + 1/2))/\text{sqrt}(1/6*\text{sqrt}(3) + 1/2)) \\ & + 1/1679616*\text{sqrt}(2)*(487*3^{(3/4)}*\text{sqrt}(2)*(6*\text{sqrt}(3) + 18)^{(3/2)} + 8766*3^{(3/4)}* \\ & \text{sqrt}(2)*\text{sqrt}(6*\text{sqrt}(3) + 18)*(\text{sqrt}(3) - 3) - 8766*3^{(3/4)}*(\text{sqrt}(3) + 3) \\ &)*\text{sqrt}(-6*\text{sqrt}(3) + 18) + 487*3^{(3/4)}*(-6*\text{sqrt}(3) + 18)^{(3/2)} + 16668*3^{(1/4)}* \\ & \text{sqrt}(2)*\text{sqrt}(6*\text{sqrt}(3) + 18) - 16668*3^{(1/4)}*\text{sqrt}(-6*\text{sqrt}(3) + 18))*\text{arctan}(\\ & 1/3*3^{(3/4)}*(x - 3^{(1/4)}*\text{sqrt}(-1/6*\text{sqrt}(3) + 1/2))/\text{sqrt}(1/6*\text{sqrt}(3) + 1/2)) \\ & + 1/3359232*\text{sqrt}(2)*(8766*3^{(3/4)}*\text{sqrt}(2)*(\text{sqrt}(3) + 3)*\text{sqrt}(-6*\text{sqrt}(3) \\ & + 18) - 487*3^{(3/4)}*\text{sqrt}(2)*(-6*\text{sqrt}(3) + 18)^{(3/2)} + 487*3^{(3/4)}*(6*\text{sqrt}(3) \\ & + 18)^{(3/2)} + 8766*3^{(3/4)}*\text{sqrt}(6*\text{sqrt}(3) + 18)*(\text{sqrt}(3) - 3) + 16668*3^{(1/4)}* \\ & \text{sqrt}(2)*\text{sqrt}(-6*\text{sqrt}(3) + 18) + 16668*3^{(1/4)}*\text{sqrt}(6*\text{sqrt}(3) + 18))*\log(x^2 + 2*3^{(1/4)}* \\ & x*\text{sqrt}(-1/6*\text{sqrt}(3) + 1/2) + \text{sqrt}(3)) - 1/3359232*\text{sqrt}(2)*(8766*3^{(3/4)}*\text{sqrt}(2) \\ & *(\text{sqrt}(3) + 3)*\text{sqrt}(-6*\text{sqrt}(3) + 18) - 487*3^{(3/4)}*\text{sqrt}(2)*(-6*\text{sqrt}(3) + 18)^{(3/2)} \\ & + 487*3^{(3/4)}*(6*\text{sqrt}(3) + 18)^{(3/2)} + 8766*3^{(3/4)}*\text{sqrt}(6*\text{sqrt}(3) + 18)*(\text{sqrt}(3) - 3) \\ & + 16668*3^{(1/4)}*\text{sqrt}(2)*\text{sqrt}(-6*\text{sqrt}(3) + 18) + 16668*3^{(1/4)}*\text{sqrt}(6*\text{sqrt}(3) + 18))* \\ & \log(x^2 - 2*3^{(1/4)}*x*\text{sqrt}(-1/6*\text{sqrt}(3) + 1/2) + \text{sqrt}(3)) - 25/648*(7*x^3 - x)/(x^4 + 2*x^2 + 3) - \\ & 1/405*(195*x^4 - 65*x^2 + 36)/x^5 \end{aligned}$$

maple [B] time = 0.03, size = 424, normalized size = 1.73

$\frac{487(-2+2\sqrt{3})\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}\sqrt{3+2\sqrt{3}}}{3\sqrt{3+2\sqrt{3}}}\right) - 487(-2+2\sqrt{3})\operatorname{atan}\left(\frac{\sqrt{3}\sqrt{3-2\sqrt{3}}}{3\sqrt{3-2\sqrt{3}}}\right) - 487\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}\sqrt{3+2\sqrt{3}}}{3\sqrt{3+2\sqrt{3}}}\right) - 487(-2+2\sqrt{3})\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}\sqrt{3-2\sqrt{3}}}{3\sqrt{3-2\sqrt{3}}}\right) - 487(-2+2\sqrt{3})\operatorname{atan}\left(\frac{\sqrt{3}\sqrt{3+2\sqrt{3}}}{3\sqrt{3+2\sqrt{3}}}\right) - 487\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}\sqrt{3-2\sqrt{3}}}{3\sqrt{3-2\sqrt{3}}}\right) - 487\sqrt{3+2\sqrt{3}}\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}\sqrt{3+2\sqrt{3}}}{3\sqrt{3+2\sqrt{3}}}\right) - 487\sqrt{3+2\sqrt{3}}\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}\sqrt{3-2\sqrt{3}}}{3\sqrt{3-2\sqrt{3}}}\right) - 487\sqrt{3-2\sqrt{3}}\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}\sqrt{3+2\sqrt{3}}}{3\sqrt{3+2\sqrt{3}}}\right) - 487\sqrt{3-2\sqrt{3}}\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}\sqrt{3-2\sqrt{3}}}{3\sqrt{3-2\sqrt{3}}}\right) - \frac{487}{3\sqrt{3}} - \frac{487}{3\sqrt{3-2\sqrt{3}}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^6+3*x^4+x^2+4)/x^6/(x^4+2*x^2+3)^2,x)

[Out]
$$\begin{aligned} & -4/45/x^5+13/81/x^3-13/27/x-1/27*(175/24*x^3-25/24*x)/(x^4+2*x^2+3)-481/777 \\ & 6*(-2+2*3^{(1/2)})^{(1/2)}*3^{(1/2)}*\ln(x^2-(-2+2*3^{(1/2)})^{(1/2)}*x+3^{(1/2)})-475/5 \\ & 184*(-2+2*3^{(1/2)})^{(1/2)}*\ln(x^2-(-2+2*3^{(1/2)})^{(1/2)}*x+3^{(1/2)})-481/3888/(2 \\ & +2*3^{(1/2)})^{(1/2)}*(-2+2*3^{(1/2)})*3^{(1/2)}*\arctan((2*x-(-2+2*3^{(1/2)})^{(1/2)})/ \\ & (2+2*3^{(1/2)})^{(1/2)})-475/2592/(2+2*3^{(1/2)})^{(1/2)}*(-2+2*3^{(1/2)})*\arctan((2* \\ & x-(-2+2*3^{(1/2)})^{(1/2)})/(2+2*3^{(1/2)})^{(1/2)})+463/1944/(2+2*3^{(1/2)})^{(1/2)}*3 \\ & ^{(1/2)}*\arctan((2*x-(-2+2*3^{(1/2)})^{(1/2)})/(2+2*3^{(1/2)})^{(1/2)})+481/7776*(-2+ \\ & 2*3^{(1/2)})^{(1/2)}*3^{(1/2)}*\ln(x^2+(-2+2*3^{(1/2)})^{(1/2)}*x+3^{(1/2)})+475/5184*(- \\ & 2+2*3^{(1/2)})^{(1/2)}*\ln(x^2+(-2+2*3^{(1/2)})^{(1/2)}*x+3^{(1/2)})-481/3888/(2+2*3^{(1/2)} \\ & ^{(1/2)})^{(1/2)}*(-2+2*3^{(1/2)})*3^{(1/2)}*\arctan((2*x+(-2+2*3^{(1/2)})^{(1/2)})/(2+2*3 \\ & ^{(1/2)})^{(1/2)})-475/2592/(2+2*3^{(1/2)})^{(1/2)}*(-2+2*3^{(1/2)})*\arctan((2*x+(-2+ \\ & 2*3^{(1/2)})^{(1/2)})/(2+2*3^{(1/2)})^{(1/2)})+463/1944/(2+2*3^{(1/2)})^{(1/2)}*3^{(1/2)} \\ & *\arctan((2*x+(-2+2*3^{(1/2)})^{(1/2)})/(2+2*3^{(1/2)})^{(1/2)}) \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2435x^8 + 2475x^6 + 3928x^4 - 984x^2 + 864}{3240(x^9 + 2x^7 + 3x^5)} - \frac{1}{648} \int \frac{487x^2 - 463}{x^4 + 2x^2 + 3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^6+3*x^4+x^2+4)/x^6/(x^4+2*x^2+3)^2,x, algorithm="maxima")

[Out]
$$-1/3240*(2435*x^8 + 2475*x^6 + 3928*x^4 - 984*x^2 + 864)/(x^9 + 2*x^7 + 3*x^5) - 1/648*\text{integrate}((487*x^2 - 463)/(x^4 + 2*x^2 + 3), x)$$

mapad [B] time = 0.14, size = 171, normalized size = 0.70

$\frac{487x^8}{648} + \frac{55x^6}{72} + \frac{491x^4}{405} - \frac{41x^2}{135} + \frac{4}{15} \operatorname{atan}\left(\frac{x\sqrt{3418143-\sqrt{2}7457071}248569}{306110016} + \frac{248569\sqrt{2}x\sqrt{3418143-\sqrt{2}7457071}}{612220032} + \frac{119561689\sqrt{2}1150874471}{5018336} + \frac{\sqrt{2}1150874471}{20407344}\right) \sqrt{3418143-\sqrt{2}7457071} \operatorname{li}\left(\frac{x\sqrt{3418143+\sqrt{2}7457071}248569}{306110016} - \frac{248569\sqrt{2}x\sqrt{3418143+\sqrt{2}7457071}}{612220032} - \frac{119561689\sqrt{2}1150874471}{5018336} + \frac{\sqrt{2}1150874471}{20407344}\right) \sqrt{3418143+\sqrt{2}7457071} \operatorname{li}\left(\frac{x\sqrt{3418143-\sqrt{2}7457071}248569}{306110016} + \frac{248569\sqrt{2}x\sqrt{3418143-\sqrt{2}7457071}}{612220032} + \frac{119561689\sqrt{2}1150874471}{5018336} + \frac{\sqrt{2}1150874471}{20407344}\right) - \frac{119561689\sqrt{2}1150874471}{5018336} + \frac{\sqrt{2}1150874471}{20407344}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^6*(2*x^2 + x^4 + 3)^2),x)


```
[Out] (atan((x*(2^(1/2)*745707i + 3418143)^(1/2)*248569i)/(306110016*((2^(1/2)*115087447i)/204073344 - 119561689/51018336)) - (248569*2^(1/2)*x*(2^(1/2)*745707i + 3418143)^(1/2))/(612220032*((2^(1/2)*115087447i)/204073344 - 119561689/51018336)))*(2^(1/2)*745707i + 3418143)^(1/2)*i)/3888 - (atan((x*(3418143 - 2^(1/2)*745707i)^(1/2)*248569i)/(306110016*((2^(1/2)*115087447i)/204073344 + 119561689/51018336)) + (248569*2^(1/2)*x*(3418143 - 2^(1/2)*745707i)^(1/2))/(612220032*((2^(1/2)*115087447i)/204073344 + 119561689/51018336)))*(3418143 - 2^(1/2)*745707i)^(1/2)*i)/3888 - ((491*x^4)/405 - (41*x^2)/135 + (55*x^6)/72 + (487*x^8)/648 + 4/15)/(3*x^5 + 2*x^7 + x^9)
```

sympy [B] time = 1.33, size = 1202, normalized size = 4.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**6/(x**4+2*x**2+3)**2,x)
```

```
[Out] -sqrt(1139381/40310784 + 2833*sqrt(3)/165888)*log(x**2 + x*(-3848*sqrt(2)*sqrt(1139381 + 688419*sqrt(3)))/248569 - 769085497*sqrt(6)*sqrt(1139381 + 688419*sqrt(3))/171119622411 + 1924*sqrt(3)*sqrt(1139381 + 688419*sqrt(3))*sqrt(784371528639*sqrt(3) + 1359975610922)/171119622411) - 8677510907569510603*sqrt(2)*sqrt(784371528639*sqrt(3) + 1359975610922)/29281925174083213452921 - 21752950947364*sqrt(6)*sqrt(784371528639*sqrt(3) + 1359975610922)/127605100269239577 + 20196165220927340076543947/29281925174083213452921 + 50945036826336313070*sqrt(3)/127605100269239577) + sqrt(1139381/40310784 + 2833*sqrt(3)/165888)*log(x**2 + x*(-1924*sqrt(3)*sqrt(1139381 + 688419*sqrt(3))*sqrt(784371528639*sqrt(3) + 1359975610922)/171119622411 + 769085497*sqrt(6)*sqrt(1139381 + 688419*sqrt(3))/171119622411 + 3848*sqrt(2)*sqrt(1139381 + 688419*sqrt(3))/248569) - 8677510907569510603*sqrt(2)*sqrt(784371528639*sqrt(3) + 1359975610922)/29281925174083213452921 - 21752950947364*sqrt(6)*sqrt(784371528639*sqrt(3) + 1359975610922)/127605100269239577 + 20196165220927340076543947/29281925174083213452921 + 50945036826336313070*sqrt(3)/127605100269239577) + 2*sqrt(-sqrt(2)*sqrt(784371528639*sqrt(3) + 1359975610922)/2015392 + 1139381/40310784 + 2833*sqrt(3)/55296)*atan(342239244822*sqrt(3)*x/(-1924*sqrt(784371528639*sqrt(3) + 1359975610922)*sqrt(-2*sqrt(2)*sqrt(784371528639*sqrt(3) + 1359975610922) + 1139381 + 2065257*sqrt(3)) + 115087447*sqrt(2)*sqrt(-2*sqrt(2)*sqrt(784371528639*sqrt(3) + 1359975610922) + 1139381 + 2065257*sqrt(3))) + 2649036312*sqrt(6)*sqrt(1139381 + 688419*sqrt(3))/(-1924*sqrt(784371528639*sqrt(3) + 1359975610922)*sqrt(-2*sqrt(2)*sqrt(784371528639*sqrt(3) + 1359975610922) + 1139381 + 2065257*sqrt(3)) + 115087447*sqrt(2)*sqrt(-2*sqrt(2)*sqrt(784371528639*sqrt(3) + 1359975610922) + 1139381 + 2065257*sqrt(3))) + 2307256491*sqrt(2)*sqrt(1139381 + 688419*sqrt(3))/(-1924*sqrt(784371528639*sqrt(3) + 1359975610922)*sqrt(-2*sqrt(2)*sqrt(784371528639*sqrt(3) + 1359975610922) + 1139381 + 2065257*sqrt(3)) + 115087447*sqrt(2)*sqrt(-2*sqrt(2)*sqrt(784371528639*sqrt(3) + 1359975610922) + 1139381 + 2065257*sqrt(3))) - 5772*sqrt(1139381 + 688419*sqrt(3))*sqrt(784371528639*sqrt(3) + 1359975610922)/(-1924*sqrt(784371528639*sqrt(3) + 1359975610922)*sqrt(-2*sqrt(2)*sqrt(784371528639*sqrt(3) + 1359975610922) + 1139381 + 2065257*sqrt(3)) + 115087447*sqrt(2)*sqrt(-2*sqrt(2)*sqrt(784371528639*sqrt(3) + 1359975610922) + 1139381 + 2065257*sqrt(3))) + 2*sqrt(-sqrt(2)*sqrt(784371528639*sqrt(3) + 1359975610922)/2015392 + 1139381/40310784 + 2833*sqrt(3)/55296)*atan(342239244822*sqrt(3)*x/(-1924*sqrt(784371528639*sqrt(3) + 1359975610922)*sqrt(-2*sqrt(2)*sqrt(784371528639*sqrt(3) + 1359975610922) + 1139381 + 2065257*sqrt(3)) + 115087447*sqrt(2)*sqrt(-2*sqrt(2)*sqrt(784371528639*sqrt(3) + 1359975610922) + 1139381 + 2065257*sqrt(3))) + 5772*sqrt(1139381 + 688419*sqrt(3))*sqrt(784371528639*sqrt(3) + 1359975610922)/(-1924*sqrt(784371528639*sqrt(3) + 1359975610922)*sqrt(-2*sqrt(2)*sqrt(784371528639*sqrt(3) + 1359975610922) + 1139381 + 2065257*sqrt(3)) + 115087447*sqrt(2)*sqrt(-2*sqrt(2)*sqrt(784371528639*sqrt(3) + 1359975610922) + 1139381 + 2065257*sqrt(3))) - 2307256491*sqrt(2)*sqrt(1139381 + 688419*sqrt(3))/(-1924*sqrt
```

$$\begin{aligned}
& (784371528639\sqrt{3} + 1359975610922)\sqrt{-2\sqrt{2}\sqrt{784371528639\sqrt{3} + 1359975610922} + 1139381 + 2065257\sqrt{3}} + 115087447\sqrt{2}\sqrt{-2\sqrt{2}\sqrt{784371528639\sqrt{3} + 1359975610922} + 1139381 + 2065257\sqrt{3}} \\
& - 2649036312\sqrt{6}\sqrt{1139381 + 688419\sqrt{3}} / (-1924\sqrt{784371528639\sqrt{3} + 1359975610922}\sqrt{-2\sqrt{2}\sqrt{784371528639\sqrt{3} + 1359975610922} + 1139381 + 2065257\sqrt{3}} + 115087447\sqrt{2}\sqrt{-2\sqrt{2}\sqrt{784371528639\sqrt{3} + 1359975610922} + 1139381 + 2065257\sqrt{3}})) \\
& + (-2435x^8 - 2475x^6 - 3928x^4 + 984x^2 - 864) / (3240x^9 + 6480x^7 + 9720x^5)
\end{aligned}$$

$$3.115 \quad \int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3} dx$$

Optimal. Leaf size=243

$$x^5 - 9x^3 + \frac{3}{512} \sqrt{8595619 + 7678611\sqrt{3}} \log\left(x^2 - \sqrt{2(\sqrt{3}-1)}x + \sqrt{3}\right) - \frac{3}{512} \sqrt{8595619 + 7678611\sqrt{3}} \log\left(x^2 + \sqrt{2(\sqrt{3}-1)}x + \sqrt{3}\right)$$

Rubi [A] time = 0.36, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {1668, 1678, 1676, 1169, 634, 618, 204, 628}

$$x^5 - 9x^3 + \frac{(252x^2 + 3305)x}{64(x^4 + 2x^2 + 3)} - \frac{25(7x^2 + 15)x}{16(x^4 + 2x^2 + 3)} + \frac{3}{512} \sqrt{8595619 + 7678611\sqrt{3}} \log\left(x^2 - \sqrt{2(\sqrt{3}-1)}x + \sqrt{3}\right) - \frac{3}{512} \sqrt{8595619 + 7678611\sqrt{3}} \log\left(x^2 + \sqrt{2(\sqrt{3}-1)}x + \sqrt{3}\right) + 58x + \frac{3}{256} \sqrt{7678611\sqrt{3} - 8595619} \tan^{-1}\left(\frac{\sqrt{2(\sqrt{3}-1)} - 2x}{\sqrt{2(1+\sqrt{3})}}\right) - \frac{3}{256} \sqrt{7678611\sqrt{3} - 8595619} \tan^{-1}\left(\frac{2x + \sqrt{2(\sqrt{3}-1)}}{\sqrt{2(1+\sqrt{3})}}\right)$$

Antiderivative was successfully verified.

[In] Int[(x^10*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]

[Out] 58*x - 9*x^3 + x^5 - (25*x*(15 + 7*x^2))/(16*(3 + 2*x^2 + x^4)^2) + (x*(3305 + 252*x^2))/(64*(3 + 2*x^2 + x^4)) + (3*Sqrt[-8595619 + 7678611*Sqrt[3]]*ArcTan[(Sqrt[2*(-1 + Sqrt[3]])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/256 - (3*Sqrt[-8595619 + 7678611*Sqrt[3]]*ArcTan[(Sqrt[2*(-1 + Sqrt[3]])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/256 + (3*Sqrt[8595619 + 7678611*Sqrt[3]]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/512 - (3*Sqrt[8595619 + 7678611*Sqrt[3]]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/512

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1169

Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]

Rule 1668

```

Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :=
  With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
    e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
  x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
  2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
  nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
  mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
  + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x]] /; FreeQ[{a, b,
  c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
  & LtQ[p, -1] && IGtQ[m/2, 0]

```

Rule 1676

```

Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := Int[ExpandInte
  grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
  2] && Expon[Pq, x^2] > 1

```

Rule 1678

```

Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := With[{d =
  Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
  nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
  ^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
  b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
  x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
  + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
  + 7)*(b*d - 2*a*e)*x^2, x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
  2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]

```

Rubi steps

$$\begin{aligned}
\int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3} dx &= -\frac{25x(15+7x^2)}{16(3+2x^2+x^4)^2} + \frac{1}{96} \int \frac{2250-2850x^2-4800x^4+2400x^6-672x^{10}+}{(3+2x^2+x^4)^2} \\
&= -\frac{25x(15+7x^2)}{16(3+2x^2+x^4)^2} + \frac{x(3305+252x^2)}{64(3+2x^2+x^4)} + \frac{\int \frac{-201960+193248x^2+87552x^4-78336x^6}{3+2x^2+x^4}}{4608} \\
&= -\frac{25x(15+7x^2)}{16(3+2x^2+x^4)^2} + \frac{x(3305+252x^2)}{64(3+2x^2+x^4)} + \frac{\int (267264-124416x^2+23040)}{4608} \\
&= 58x-9x^3+x^5 - \frac{25x(15+7x^2)}{16(3+2x^2+x^4)^2} + \frac{x(3305+252x^2)}{64(3+2x^2+x^4)} - \frac{3}{64} \int \frac{4647-14}{3+2x^2+} \\
&= 58x-9x^3+x^5 - \frac{25x(15+7x^2)}{16(3+2x^2+x^4)^2} + \frac{x(3305+252x^2)}{64(3+2x^2+x^4)} - \frac{1}{256} \sqrt{3(1+\sqrt{3})} \\
&= 58x-9x^3+x^5 - \frac{25x(15+7x^2)}{16(3+2x^2+x^4)^2} + \frac{x(3305+252x^2)}{64(3+2x^2+x^4)} - \frac{1}{256} \left(3\sqrt{722010}\right) \\
&= 58x-9x^3+x^5 - \frac{25x(15+7x^2)}{16(3+2x^2+x^4)^2} + \frac{x(3305+252x^2)}{64(3+2x^2+x^4)} + \frac{3}{512} \sqrt{8595619} - \\
&= 58x-9x^3+x^5 - \frac{25x(15+7x^2)}{16(3+2x^2+x^4)^2} + \frac{x(3305+252x^2)}{64(3+2x^2+x^4)} + \frac{3}{256} \sqrt{-8595619}
\end{aligned}$$

Mathematica [C] time = 0.22, size = 156, normalized size = 0.64

$$x^5 - 9x^3 + \frac{(252x^2 + 3305)x}{64(x^4 + 2x^2 + 3)} - \frac{25(7x^2 + 15)x}{16(x^4 + 2x^2 + 3)^2} + 58x + \frac{3(148\sqrt{2} + 4795i) \tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{128\sqrt{2-2i\sqrt{2}}} + \frac{3(148\sqrt{2} - 4795i) \tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{128\sqrt{2+2i\sqrt{2}}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^10*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]

[Out] 58*x - 9*x^3 + x^5 - (25*x*(15 + 7*x^2))/(16*(3 + 2*x^2 + x^4)^2) + (x*(3305 + 252*x^2))/(64*(3 + 2*x^2 + x^4)) + (3*(4795*I + 148*sqrt(2))*ArcTan[x/Sqrt[1 - I*sqrt(2)]])/(128*sqrt(2 - (2*I)*sqrt(2))) + (3*(-4795*I + 148*sqrt(2))*ArcTan[x/Sqrt[1 + I*sqrt(2)]])/(128*sqrt(2 + (2*I)*sqrt(2)))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^10*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]

[Out] IntegrateAlgebraic[(x^10*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3, x]

fricas [B] time = 1.36, size = 561, normalized size = 2.31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="fricas")
[Out] 1/18808834881088512*(18808834881088512*x^13 - 94044174405442560*x^11 + 6018
82716194832384*x^9 + 2970620359031916864*x^7 + 10166469141273357744*x^5 + 5
7410392*2183743218123^(1/4)*sqrt(3)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sq
t(-66002414605209*sqrt(3) + 176883200667963)*arctan(1/863545621466021963404
537403089353*sqrt(6122667604521)*2183743218123^(3/4)*sqrt(55104008440689*x^
2 + 2183743218123^(1/4)*(148*sqrt(3)*sqrt(2)*x + 4647*sqrt(2)*x)*sqrt(-6600
2414605209*sqrt(3) + 176883200667963) + 55104008440689*sqrt(3))*(1549*sqrt(
3) + 148)*sqrt(-66002414605209*sqrt(3) + 176883200667963) - 1/4701358281741
8600331*2183743218123^(3/4)*(1549*sqrt(3)*x + 148*x)*sqrt(-66002414605209*s
qrt(3) + 176883200667963) - 1/2*sqrt(3)*sqrt(2) + 1/2*sqrt(2)) + 57410392*2
183743218123^(1/4)*sqrt(3)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqrt(-660024
14605209*sqrt(3) + 176883200667963)*arctan(1/863545621466021963404537403089
353*sqrt(6122667604521)*2183743218123^(3/4)*sqrt(55104008440689*x^2 - 21837
43218123^(1/4)*(148*sqrt(3)*sqrt(2)*x + 4647*sqrt(2)*x)*sqrt(-6600241460520
9*sqrt(3) + 176883200667963) + 55104008440689*sqrt(3))*(1549*sqrt(3) + 148)
*sqrt(-66002414605209*sqrt(3) + 176883200667963) - 1/47013582817418600331*2
183743218123^(3/4)*(1549*sqrt(3)*x + 148*x)*sqrt(-66002414605209*sqrt(3) +
176883200667963) + 1/2*sqrt(3)*sqrt(2) - 1/2*sqrt(2)) + 1352649115995281020
8*x^3 - 2183743218123^(1/4)*(8595619*sqrt(3)*sqrt(2)*(x^8 + 4*x^6 + 10*x^4
+ 12*x^2 + 9) + 23035833*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9))*sqrt(
-66002414605209*sqrt(3) + 176883200667963)*log(55104008440689*x^2 + 2183743
218123^(1/4)*(148*sqrt(3)*sqrt(2)*x + 4647*sqrt(2)*x)*sqrt(-66002414605209*
sqrt(3) + 176883200667963) + 55104008440689*sqrt(3)) + 2183743218123^(1/4)*
(8595619*sqrt(3)*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9) + 23035833*sq
rt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9))*sqrt(-66002414605209*sqrt(3) + 17
6883200667963)*log(55104008440689*x^2 - 2183743218123^(1/4)*(148*sqrt(3)*sq
rt(2)*x + 4647*sqrt(2)*x)*sqrt(-66002414605209*sqrt(3) + 176883200667963) +
55104008440689*sqrt(3)) + 12291279706746325584*x)/(x^8 + 4*x^6 + 10*x^4 +
12*x^2 + 9)
```

giac [B] time = 2.69, size = 588, normalized size = 2.42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="giac")
[Out] x^5 - 9*x^3 - 1/13824*sqrt(2)*(37*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) +
666*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 666*3^(3/4)*(sqrt(
3) + 3)*sqrt(-6*sqrt(3) + 18) + 37*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 41823*
3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) - 41823*3^(1/4)*sqrt(-6*sqrt(3) + 18))
*arctan(1/3*3^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3)
+ 1/2)) - 1/13824*sqrt(2)*(37*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 666
*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 666*3^(3/4)*(sqrt(3)
+ 3)*sqrt(-6*sqrt(3) + 18) + 37*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 41823*3^(
1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) - 41823*3^(1/4)*sqrt(-6*sqrt(3) + 18))*ar
ctan(1/3*3^(3/4)*(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) +
1/2)) - 1/27648*sqrt(2)*(666*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3)
+ 18) - 37*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 37*3^(3/4)*(6*sqrt(3)
+ 18)^(3/2) + 666*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) + 41823*3^(1/4
)*sqrt(2)*sqrt(-6*sqrt(3) + 18) + 41823*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x
^2 + 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) + 1/27648*sqrt(2)*(666
*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 37*3^(3/4)*sqrt(2)*(-
6*sqrt(3) + 18)^(3/2) + 37*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 666*3^(3/4)*sq
rt(6*sqrt(3) + 18)*(sqrt(3) - 3) + 41823*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) +
18) + 41823*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 - 2*3^(1/4)*x*sqrt(-1/6*s
```

$\text{qrt}(3) + 1/2) + \text{sqrt}(3)) + 58*x + 1/64*(252*x^7 + 3809*x^5 + 6666*x^3 + 8415*x)/(x^4 + 2*x^2 + 3)^2$

maple [B] time = 0.04, size = 429, normalized size = 1.77

$$\frac{x^5 - 9x^3 + 58x + \frac{1}{64} \left(\frac{252x^7 + 3809x^5 + 6666x^3 + 8415x}{x^4 + 2x^2 + 3} \right)}{(x^4 + 2x^2 + 3)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^{10}*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3, x)$

[Out] $x^5 - 9x^3 + 58x + \frac{63}{16}x^7 + \frac{3809}{64}x^5 + \frac{3333}{32}x^3 + \frac{8415}{64}x$ / $(x^4 + 2x^2 + 3)^2 + \frac{5091}{1024}(-2 + 2*3^{1/2})^{1/2} * 3^{1/2} * \ln(x^2 - (-2 + 2*3^{1/2})^{1/2} * x + 3^{1/2}) + \frac{14385}{1024}(-2 + 2*3^{1/2})^{1/2} * \ln(x^2 - (-2 + 2*3^{1/2})^{1/2} * x + 3^{1/2}) + \frac{5091}{512}((2 + 2*3^{1/2})^{1/2} * (-2 + 2*3^{1/2}) * 3^{1/2} * \arctan((2*x - (-2 + 2*3^{1/2})^{1/2})^{1/2}) / ((2 + 2*3^{1/2})^{1/2}) + 14385/512 / ((2 + 2*3^{1/2})^{1/2}) * (-2 + 2*3^{1/2}) * \arctan((2*x - (-2 + 2*3^{1/2})^{1/2})^{1/2}) / ((2 + 2*3^{1/2})^{1/2}) - 4647/64 / ((2 + 2*3^{1/2})^{1/2}) * 3^{1/2} * \arctan((2*x - (-2 + 2*3^{1/2})^{1/2})^{1/2}) / ((2 + 2*3^{1/2})^{1/2}) - 5091/1024 * (-2 + 2*3^{1/2})^{1/2} * 3^{1/2} * \ln(x^2 + (-2 + 2*3^{1/2})^{1/2} * x + 3^{1/2}) - 14385/1024 * (-2 + 2*3^{1/2})^{1/2} * \ln(x^2 + (-2 + 2*3^{1/2})^{1/2} * x + 3^{1/2}) + 5091/512 / ((2 + 2*3^{1/2})^{1/2}) * (-2 + 2*3^{1/2}) * 3^{1/2} * \arctan((2*x + (-2 + 2*3^{1/2})^{1/2})^{1/2}) / ((2 + 2*3^{1/2})^{1/2}) + 14385/512 / ((2 + 2*3^{1/2})^{1/2}) * (-2 + 2*3^{1/2}) * \arctan((2*x + (-2 + 2*3^{1/2})^{1/2})^{1/2}) / ((2 + 2*3^{1/2})^{1/2}) - 4647/64 / ((2 + 2*3^{1/2})^{1/2}) * 3^{1/2} * \arctan((2*x + (-2 + 2*3^{1/2})^{1/2})^{1/2}) / ((2 + 2*3^{1/2})^{1/2})$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$x^5 - 9x^3 + 58x + \frac{252x^7 + 3809x^5 + 6666x^3 + 8415x}{64(x^8 + 4x^6 + 10x^4 + 12x^2 + 9)} + \frac{3}{64} \int \frac{148x^2 - 4647}{x^4 + 2x^2 + 3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^{10}*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3, x, \text{algorithm}="maxima")$

[Out] $x^5 - 9x^3 + 58x + 1/64*(252*x^7 + 3809*x^5 + 6666*x^3 + 8415*x)/(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9) + 3/64*\text{integrate}((148*x^2 - 4647)/(x^4 + 2*x^2 + 3), x)$

mupad [B] time = 0.11, size = 184, normalized size = 0.76

$$58x + \frac{15}{x^3 + 4x^5 + 10x^4 + 12x^2 + 9} + \frac{\text{atan}\left(\frac{\sqrt{17191238 - \sqrt{2} \cdot 14352598i} - 193760073i}{131072}, \frac{\sqrt{17191238 - \sqrt{2} \cdot 14352598i}}{262144} + \frac{193760073i}{131072}\right) \sqrt{17191238 - \sqrt{2} \cdot 14352598i}}{256} + \frac{\text{atan}\left(\frac{\sqrt{17191238 + \sqrt{2} \cdot 14352598i} - 193760073i}{131072}, \frac{\sqrt{17191238 + \sqrt{2} \cdot 14352598i}}{262144} + \frac{193760073i}{131072}\right) \sqrt{17191238 + \sqrt{2} \cdot 14352598i}}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((x^{10}*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^3, x)$

[Out] $58x - \frac{\text{atan}((x*(17191238 - 2^{1/2}*14352598i))^{1/2}*193760073i)/(131072*((2^{1/2}*900403059231i)/131072 - 986432531643/131072)) - (193760073*2^{1/2}*x*(17191238 - 2^{1/2}*14352598i))^{1/2})/(262144*((2^{1/2}*900403059231i)/131072 - 986432531643/131072)) * (17191238 - 2^{1/2}*14352598i)^{1/2} * 3i}{256} + \frac{\text{atan}((x*(2^{1/2}*14352598i + 17191238))^{1/2}*193760073i)/(131072*((2^{1/2}*900403059231i)/131072 + 986432531643/131072)) + (193760073*2^{1/2}*x*(2^{1/2}*14352598i + 17191238))^{1/2})/(262144*((2^{1/2}*900403059231i)/131072 + 986432531643/131072)) * (2^{1/2}*14352598i + 17191238)^{1/2} * 3i}{256} + ((8415*x)/64 + (3333*x^3)/32 + (3809*x^5)/64 + (63*x^7)/16)/(12*x^2 + 10*x^4 + 4*x^6 + x^8 + 9) - 9*x^3 + x^5$

sympy [B] time = 1.35, size = 1204, normalized size = 4.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**10*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**3,x)

[Out] $x^{10} - 9x^8 + 58x^6 + (252x^7 + 3809x^5 + 6666x^3 + 8415x)/(64x^8 + 256x^6 + 640x^4 + 768x^2 + 576) - 3\sqrt{8595619/262144 + 7678611\sqrt{3}}/262144 \cdot \log(x^2 + x(-6788\sqrt{3})\sqrt{8595619 + 7678611\sqrt{3}})/7176299 - 2313785528\sqrt{8595619 + 7678611\sqrt{3}}/18368002813563 + 1697\sqrt{2}\sqrt{8595619 + 7678611\sqrt{3}}\sqrt{66002414605209\sqrt{3} + 125383933330562}/18368002813563 - 1218095240252468879279\sqrt{2}\sqrt{66002414605209\sqrt{3} + 125383933330562}/1012150582077174852410264907 - 134353410196228\sqrt{6}\sqrt{66002414605209\sqrt{3} + 125383933330562}/395442840668908030011 + 18391902996311867463806959889/1012150582077174852410264907 + 5204579286823805792980\sqrt{3}/395442840668908030011 + 3\sqrt{8595619/262144 + 7678611\sqrt{3}}/262144 \cdot \log(x^2 + x(-1697\sqrt{2})\sqrt{8595619 + 7678611\sqrt{3}})\sqrt{66002414605209\sqrt{3} + 125383933330562}/18368002813563 + 2313785528\sqrt{8595619 + 7678611\sqrt{3}}/18368002813563 + 6788\sqrt{3}\sqrt{8595619 + 7678611\sqrt{3}}/7176299 - 1218095240252468879279\sqrt{2}\sqrt{66002414605209\sqrt{3} + 125383933330562}/1012150582077174852410264907 - 134353410196228\sqrt{6}\sqrt{66002414605209\sqrt{3} + 125383933330562}/395442840668908030011 + 18391902996311867463806959889/1012150582077174852410264907 + 5204579286823805792980\sqrt{3}/395442840668908030011 - 2\sqrt{-9\sqrt{2}}\sqrt{66002414605209\sqrt{3} + 125383933330562}/131072 + 77360571/262144 + 207322497\sqrt{3}/262144 \cdot \operatorname{atan}(110208016881378x/(22232174302\sqrt{-2\sqrt{2}}\sqrt{66002414605209\sqrt{3} + 125383933330562} + 8595619 + 23035833\sqrt{3})) + 1697\sqrt{2}\sqrt{66002414605209\sqrt{3} + 125383933330562}\sqrt{-2\sqrt{2}}\sqrt{66002414605209\sqrt{3} + 125383933330562} + 8595619 + 23035833\sqrt{3})) - 52122411468\sqrt{3}\sqrt{8595619 + 7678611\sqrt{3}}/(22232174302\sqrt{-2\sqrt{2}}\sqrt{66002414605209\sqrt{3} + 125383933330562} + 8595619 + 23035833\sqrt{3}) + 1697\sqrt{2}\sqrt{66002414605209\sqrt{3} + 125383933330562}\sqrt{-2\sqrt{2}}\sqrt{66002414605209\sqrt{3} + 125383933330562} + 8595619 + 23035833\sqrt{3})) - 6941356584\sqrt{8595619 + 7678611\sqrt{3}}/(22232174302\sqrt{-2\sqrt{2}}\sqrt{66002414605209\sqrt{3} + 125383933330562} + 8595619 + 23035833\sqrt{3}) + 1697\sqrt{2}\sqrt{66002414605209\sqrt{3} + 125383933330562}\sqrt{-2\sqrt{2}}\sqrt{66002414605209\sqrt{3} + 125383933330562} + 8595619 + 23035833\sqrt{3})) + 5091\sqrt{2}\sqrt{8595619 + 7678611\sqrt{3}}\sqrt{66002414605209\sqrt{3} + 125383933330562}/(22232174302\sqrt{-2\sqrt{2}}\sqrt{66002414605209\sqrt{3} + 125383933330562} + 8595619 + 23035833\sqrt{3}) + 1697\sqrt{2}\sqrt{66002414605209\sqrt{3} + 125383933330562}\sqrt{-2\sqrt{2}}\sqrt{66002414605209\sqrt{3} + 125383933330562} + 8595619 + 23035833\sqrt{3})) - 2\sqrt{-9\sqrt{2}}\sqrt{66002414605209\sqrt{3} + 125383933330562}/131072 + 77360571/262144 + 207322497\sqrt{3}/262144 \cdot \operatorname{atan}(110208016881378x/(22232174302\sqrt{-2\sqrt{2}}\sqrt{66002414605209\sqrt{3} + 125383933330562} + 8595619 + 23035833\sqrt{3}) + 1697\sqrt{2}\sqrt{66002414605209\sqrt{3} + 125383933330562}\sqrt{-2\sqrt{2}}\sqrt{66002414605209\sqrt{3} + 125383933330562} + 8595619 + 23035833\sqrt{3})) - 5091\sqrt{2}\sqrt{8595619 + 7678611\sqrt{3}}\sqrt{66002414605209\sqrt{3} + 125383933330562}/(22232174302\sqrt{-2\sqrt{2}}\sqrt{66002414605209\sqrt{3} + 125383933330562} + 8595619 + 23035833\sqrt{3}) + 1697\sqrt{2}\sqrt{66002414605209\sqrt{3} + 125383933330562}\sqrt{-2\sqrt{2}}\sqrt{66002414605209\sqrt{3} + 125383933330562} + 8595619 + 23035833\sqrt{3})) + 6941356584\sqrt{8595619 + 7678611\sqrt{3}}/(22232174302\sqrt{-2\sqrt{2}}\sqrt{66002414605209\sqrt{3} + 125383933330562} + 8595619 + 23035833\sqrt{3}) + 1697\sqrt{2}\sqrt{66002414605209\sqrt{3} + 125383933330562}\sqrt{-2\sqrt{2}}\sqrt{66002414605209\sqrt{3} + 125383933330562} + 8595619 + 23035833\sqrt{3})) + 52122411468\sqrt{3}\sqrt{8595619 + 7678611\sqrt{3}}/(22232174302\sqrt{-2\sqrt{2}}\sqrt{66002414605209\sqrt{3} + 125383933330562} + 8595619 + 23035833\sqrt{3}) + 1697\sqrt{2}\sqrt{66002414605209\sqrt{3} + 125383933330562}\sqrt{-2\sqrt{2}}\sqrt{66002414605209\sqrt{3} + 125383933330562} + 8595619 + 23035833\sqrt{3}))$

$$3.116 \quad \int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3} dx$$

Optimal. Leaf size=242

$$\frac{5x^3}{3} - \frac{21}{512} \sqrt{22721\sqrt{3} - 34271} \log\left(x^2 - \sqrt{2(\sqrt{3}-1)}x + \sqrt{3}\right) + \frac{21}{512} \sqrt{22721\sqrt{3} - 34271} \log\left(x^2 + \sqrt{2(\sqrt{3}-1)}x + \sqrt{3}\right)$$

Rubi [A] time = 0.31, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {1668, 1678, 1676, 1169, 634, 618, 204, 628}

$$\frac{5x^3}{3} - \frac{(835x^2 + 1468)x}{64(x^2 + 2x^2 + 3)} + \frac{25(5x^2 + 3)x}{16(x^2 + 2x^2 + 3)^2} - \frac{21}{512} \sqrt{22721\sqrt{3} - 34271} \log(x^2 - \sqrt{2(\sqrt{3}-1)}x + \sqrt{3}) + \frac{21}{512} \sqrt{22721\sqrt{3} - 34271} \log(x^2 + \sqrt{2(\sqrt{3}-1)}x + \sqrt{3}) - \frac{27x}{256} \sqrt{34271 + 22721\sqrt{3}} \tan^{-1}\left(\frac{\sqrt{2(\sqrt{3}-1)} - 2x}{\sqrt{2(1+\sqrt{3})}}\right) + \frac{21}{256} \sqrt{34271 + 22721\sqrt{3}} \tan^{-1}\left(\frac{2x + \sqrt{2(\sqrt{3}-1)}}{\sqrt{2(1+\sqrt{3})}}\right)$$

Antiderivative was successfully verified.

[In] Int[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]

[Out] -27*x + (5*x^3)/3 + (25*x*(3 + 5*x^2))/(16*(3 + 2*x^2 + x^4)^2) - (x*(1468 + 835*x^2))/(64*(3 + 2*x^2 + x^4)) - (21*sqrt[34271 + 22721*sqrt[3]]*ArcTan[(sqrt[2*(-1 + sqrt[3])]) - 2*x]/sqrt[2*(1 + sqrt[3])])]/256 + (21*sqrt[34271 + 22721*sqrt[3]]*ArcTan[(sqrt[2*(-1 + sqrt[3])]) + 2*x]/sqrt[2*(1 + sqrt[3])])]/256 - (21*sqrt[-34271 + 22721*sqrt[3]]*Log[sqrt[3] - sqrt[2*(-1 + sqrt[3])]]*x + x^2)/512 + (21*sqrt[-34271 + 22721*sqrt[3]]*Log[sqrt[3] + sqrt[2*(-1 + sqrt[3])]]*x + x^2)/512

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1169

Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]

Rule 1668

```

Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :=
  With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
    e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
  x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
  2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
  nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
  mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
  + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x]] /; FreeQ[{a, b,
  c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
  & LtQ[p, -1] && IGtQ[m/2, 0]

```

Rule 1676

```

Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := Int[ExpandInte
  grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
  2] && Expon[Pq, x^2] > 1

```

Rule 1678

```

Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := With[{d =
  Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
  nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
  ^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
  b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
  x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
  + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
  + 7)*(b*d - 2*a*e)*x^2, x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
  2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]

```

Rubi steps

$$\begin{aligned}
\int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3} dx &= \frac{25x(3+5x^2)}{16(3+2x^2+x^4)^2} + \frac{1}{96} \int \frac{-450-1050x^2+2400x^4-672x^8+480x^{10}}{(3+2x^2+x^4)^2} dx \\
&= \frac{25x(3+5x^2)}{16(3+2x^2+x^4)^2} - \frac{x(1468+835x^2)}{64(3+2x^2+x^4)} + \frac{\int \frac{98496+27432x^2-78336x^4+23040x^6}{3+2x^2+x^4} dx}{4608} \\
&= \frac{25x(3+5x^2)}{16(3+2x^2+x^4)^2} - \frac{x(1468+835x^2)}{64(3+2x^2+x^4)} + \frac{\int \left(-124416+23040x^2+\frac{1512(312+3x^2)}{3+2x^2+x^4}\right) dx}{4608} \\
&= -27x + \frac{5x^3}{3} + \frac{25x(3+5x^2)}{16(3+2x^2+x^4)^2} - \frac{x(1468+835x^2)}{64(3+2x^2+x^4)} + \frac{21}{64} \int \frac{312+137x^2}{3+2x^2+x^4} dx \\
&= -27x + \frac{5x^3}{3} + \frac{25x(3+5x^2)}{16(3+2x^2+x^4)^2} - \frac{x(1468+835x^2)}{64(3+2x^2+x^4)} + \frac{1}{256} \left(7\sqrt{3(1+\sqrt{3})}\right) \\
&= -27x + \frac{5x^3}{3} + \frac{25x(3+5x^2)}{16(3+2x^2+x^4)^2} - \frac{x(1468+835x^2)}{64(3+2x^2+x^4)} - \frac{1}{512} \left(21\sqrt{-34271+2272x}\right) \\
&= -27x + \frac{5x^3}{3} + \frac{25x(3+5x^2)}{16(3+2x^2+x^4)^2} - \frac{x(1468+835x^2)}{64(3+2x^2+x^4)} - \frac{21}{512} \sqrt{-34271+2272x} \\
&= -27x + \frac{5x^3}{3} + \frac{25x(3+5x^2)}{16(3+2x^2+x^4)^2} - \frac{x(1468+835x^2)}{64(3+2x^2+x^4)} - \frac{21}{256} \sqrt{34271+2272x}
\end{aligned}$$

Mathematica [C] time = 0.20, size = 155, normalized size = 0.64

$$\frac{5x^3}{3} - \frac{(835x^2+1468)x}{64(x^4+2x^2+3)} + \frac{25(5x^2+3)x}{16(x^4+2x^2+3)^2} - 27x + \frac{21(137\sqrt{2}-175i)\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{128\sqrt{2-2i\sqrt{2}}} + \frac{21(137\sqrt{2}+175i)\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{128\sqrt{2+2i\sqrt{2}}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]

[Out] -27*x + (5*x^3)/3 + (25*x*(3 + 5*x^2))/(16*(3 + 2*x^2 + x^4)^2) - (x*(1468 + 835*x^2))/(64*(3 + 2*x^2 + x^4)) + (21*(-175*I + 137*sqrt(2))*ArcTan[x/Sqrt[1 - I*sqrt(2)]])/(128*sqrt(2 - (2*I)*sqrt(2))) + (21*(175*I + 137*sqrt(2))*ArcTan[x/Sqrt[1 + I*sqrt(2)]])/(128*sqrt(2 + (2*I)*sqrt(2)))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]

[Out] IntegrateAlgebraic[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3, x]

fricas [B] time = 1.38, size = 557, normalized size = 2.30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="fricas")
```

```
[Out] 1/954779317248*(1591298862080*x^11 - 19413846117376*x^9 - 99660064046704*x^7 - 285508852710816*x^5 - 2298072*1548731523^(1/4)*sqrt(3)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqrt(778671391*sqrt(3) + 1548731523)*arctan(1/19753021371716480527209*1548731523^(3/4)*sqrt(932401677)*sqrt(932401677*x^2 + 1548731523^(1/4)*(137*sqrt(3)*sqrt(2)*x - 312*sqrt(2)*x)*sqrt(778671391*sqrt(3) + 1548731523) + 932401677*sqrt(3))*sqrt(778671391*sqrt(3) + 1548731523)*(104*sqrt(3) - 137) - 1/21185098503117*1548731523^(3/4)*(104*sqrt(3)*x - 137*x)*sqrt(778671391*sqrt(3) + 1548731523) + 1/2*sqrt(3)*sqrt(2) - 1/2*sqrt(2)) - 2298072*1548731523^(1/4)*sqrt(3)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqrt(778671391*sqrt(3) + 1548731523)*arctan(1/19753021371716480527209*1548731523^(3/4)*sqrt(932401677)*sqrt(932401677*x^2 - 1548731523^(1/4)*(137*sqrt(3)*sqrt(2)*x - 312*sqrt(2)*x)*sqrt(778671391*sqrt(3) + 1548731523) + 932401677*sqrt(3))*sqrt(778671391*sqrt(3) + 1548731523)*(104*sqrt(3) - 137) - 1/21185098503117*1548731523^(3/4)*(104*sqrt(3)*x - 137*x)*sqrt(778671391*sqrt(3) + 1548731523) - 1/2*sqrt(3)*sqrt(2) + 1/2*sqrt(2)) - 368738756006544*x^3 + 21*1548731523^(1/4)*(34271*sqrt(3)*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9) - 68163*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9))*sqrt(778671391*sqrt(3) + 1548731523)*log(932401677*x^2 + 1548731523^(1/4)*(137*sqrt(3)*sqrt(2)*x - 312*sqrt(2)*x)*sqrt(778671391*sqrt(3) + 1548731523) + 932401677*sqrt(3)) - 21*1548731523^(1/4)*(34271*sqrt(3)*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9) - 68163*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9))*sqrt(778671391*sqrt(3) + 1548731523)*log(932401677*x^2 - 1548731523^(1/4)*(137*sqrt(3)*sqrt(2)*x - 312*sqrt(2)*x)*sqrt(778671391*sqrt(3) + 1548731523) + 932401677*sqrt(3)) - 293236597809792*x)/(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)
```

giac [B] time = 2.59, size = 585, normalized size = 2.42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="giac")
```

```
[Out] 5/3*x^3 - 7/55296*sqrt(2)*(137*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 2466*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 2466*3^(3/4)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) + 137*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 11232*3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) + 11232*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) - 7/55296*sqrt(2)*(137*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 2466*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 2466*3^(3/4)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) + 137*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 11232*3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) + 11232*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3/4)*(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) - 7/110592*sqrt(2)*(2466*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 137*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 137*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 2466*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 11232*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) - 11232*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 + 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) + 7/110592*sqrt(2)*(2466*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 137*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 137*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 2466*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 11232*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) - 11232*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 - 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) - 27*x - 1/64*(835*x^7 + 3138*x^5 + 4941*x^3 + 4104*x)/(x^4 + 2*x^2 + 3)^2
```

maple [B] time = 0.03, size = 426, normalized size = 1.76

1/27 * (1101*(2+2*sqrt(3))^(1/4)*sqrt(3)*arctan(1/3*3^(3/4)*(x+3^(1/4)*sqrt(-1/6*sqrt(3)+1/2))/sqrt(1/6*sqrt(3)+1/2)) - 7/55296*sqrt(2)*(137*3^(3/4)*sqrt(2)*(6*sqrt(3)+18)^(3/2) + 2466*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3)+18)*(sqrt(3)-3) - 2466*3^(3/4)*(sqrt(3)+3)*sqrt(-6*sqrt(3)+18) + 137*3^(3/4)*(-6*sqrt(3)+18)^(3/2) - 11232*3^(1/4)*sqrt(2)*sqrt(6*sqrt(3)+18) + 11232*3^(1/4)*sqrt(-6*sqrt(3)+18))*arctan(1/3*3^(3/4)*(x+3^(1/4)*sqrt(-1/6*sqrt(3)+1/2))/sqrt(1/6*sqrt(3)+1/2)) - 7/55296*sqrt(2)*(137*3^(3/4)*sqrt(2)*(6*sqrt(3)+18)^(3/2) + 2466*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3)+18)*(sqrt(3)-3) - 2466*3^(3/4)*(sqrt(3)+3)*sqrt(-6*sqrt(3)+18) + 137*3^(3/4)*(-6*sqrt(3)+18)^(3/2) - 11232*3^(1/4)*sqrt(2)*sqrt(6*sqrt(3)+18) + 11232*3^(1/4)*sqrt(-6*sqrt(3)+18))*arctan(1/3*3^(3/4)*(x-3^(1/4)*sqrt(-1/6*sqrt(3)+1/2))/sqrt(1/6*sqrt(3)+1/2)) - 7/110592*sqrt(2)*(2466*3^(3/4)*sqrt(2)*(sqrt(3)+3)*sqrt(-6*sqrt(3)+18) - 137*3^(3/4)*sqrt(2)*(-6*sqrt(3)+18)^(3/2) + 137*3^(3/4)*(6*sqrt(3)+18)^(3/2) + 2466*3^(3/4)*sqrt(6*sqrt(3)+18)*(sqrt(3)-3) - 11232*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3)+18) - 11232*3^(1/4)*sqrt(6*sqrt(3)+18))*log(x^2+2*3^(1/4)*x*sqrt(-1/6*sqrt(3)+1/2)+sqrt(3)) + 7/110592*sqrt(2)*(2466*3^(3/4)*sqrt(2)*(sqrt(3)+3)*sqrt(-6*sqrt(3)+18) - 137*3^(3/4)*sqrt(2)*(-6*sqrt(3)+18)^(3/2) + 137*3^(3/4)*(6*sqrt(3)+18)^(3/2) + 2466*3^(3/4)*sqrt(6*sqrt(3)+18)*(sqrt(3)-3) - 11232*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3)+18) - 11232*3^(1/4)*sqrt(6*sqrt(3)+18))*log(x^2-2*3^(1/4)*x*sqrt(-1/6*sqrt(3)+1/2)+sqrt(3)) - 27*x - 1/64*(835*x^7+3138*x^5+4941*x^3+4104*x)/(x^4+2*x^2+3)^2

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3, x)$

[Out] $\frac{5}{3}x^3 - 27x + (-835/64*x^7 - 1569/32*x^5 - 4941/64*x^3 - 513/8*x)/(x^4+2*x^2+3)^2 + 693/1024*(-2+2*3^{(1/2)})^{(1/2)}*3^{(1/2)}*\ln(x^2 - (-2+2*3^{(1/2)})^{(1/2)}*x+3^{(1/2)}) - 3675/1024*(-2+2*3^{(1/2)})^{(1/2)}*\ln(x^2 - (-2+2*3^{(1/2)})^{(1/2)}*x+3^{(1/2)}) + 693/512/(2+2*3^{(1/2)})^{(1/2)}*(-2+2*3^{(1/2)})*3^{(1/2)}*\arctan((2*x - (-2+2*3^{(1/2)})^{(1/2)})^{(1/2)})/(2+2*3^{(1/2)})^{(1/2)}) - 3675/512/(2+2*3^{(1/2)})^{(1/2)}*(-2+2*3^{(1/2)})*\arctan((2*x - (-2+2*3^{(1/2)})^{(1/2)})^{(1/2)})/(2+2*3^{(1/2)})^{(1/2)}) + 273/8/(2+2*3^{(1/2)})^{(1/2)}*3^{(1/2)}*\arctan((2*x - (-2+2*3^{(1/2)})^{(1/2)})^{(1/2)})/(2+2*3^{(1/2)})^{(1/2)}) - 693/1024*(-2+2*3^{(1/2)})^{(1/2)}*3^{(1/2)}*\ln(x^2 + (-2+2*3^{(1/2)})^{(1/2)}*x+3^{(1/2)}) + 3675/1024*(-2+2*3^{(1/2)})^{(1/2)}*\ln(x^2 + (-2+2*3^{(1/2)})^{(1/2)}*x+3^{(1/2)}) + 693/512/(2+2*3^{(1/2)})^{(1/2)}*(-2+2*3^{(1/2)})*3^{(1/2)}*\arctan((2*x + (-2+2*3^{(1/2)})^{(1/2)})^{(1/2)})/(2+2*3^{(1/2)})^{(1/2)}) - 3675/512/(2+2*3^{(1/2)})^{(1/2)}*(-2+2*3^{(1/2)})*\arctan((2*x + (-2+2*3^{(1/2)})^{(1/2)})^{(1/2)})/(2+2*3^{(1/2)})^{(1/2)}) + 273/8/(2+2*3^{(1/2)})^{(1/2)}*3^{(1/2)}*\arctan((2*x + (-2+2*3^{(1/2)})^{(1/2)})^{(1/2)})/(2+2*3^{(1/2)})^{(1/2)})$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{5}{3}x^3 - 27x - \frac{835x^7 + 3138x^5 + 4941x^3 + 4104x}{64(x^8 + 4x^6 + 10x^4 + 12x^2 + 9)} + \frac{21}{64} \int \frac{137x^2 + 312}{x^4 + 2x^2 + 3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3, x, \text{algorithm}="maxima")$

[Out] $\frac{5}{3}x^3 - 27x - \frac{1}{64}(835x^7 + 3138x^5 + 4941x^3 + 4104x)/(x^8 + 4x^6 + 10x^4 + 12x^2 + 9) + \frac{21}{64}\text{integrate}((137x^2 + 312)/(x^4 + 2x^2 + 3), x)$

mupad [B] time = 0.94, size = 182, normalized size = 0.75

$$\frac{5x^3}{3} - \frac{835x^7}{64} + \frac{1569x^5}{32} + \frac{4941x^3}{64} + \frac{513x}{8} - 27x + \frac{\text{atan}\left(\frac{x\sqrt{-68542-\sqrt{2}27358i}-126681219i}{131072\left(\frac{12541440681}{131072} + \frac{\sqrt{2}4940567541i}{16384}\right)} + \frac{126681219\sqrt{2}x\sqrt{-68542-\sqrt{2}27358i}}{256}\right) - \text{atan}\left(\frac{x\sqrt{-68542+\sqrt{2}27358i}-126681219i}{131072\left(\frac{12541440681}{131072} + \frac{\sqrt{2}4940567541i}{16384}\right)} + \frac{126681219\sqrt{2}x\sqrt{-68542+\sqrt{2}27358i}}{256}\right)}{\sqrt{-68542+\sqrt{2}27358i}-21i}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((x^8*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^3, x)$

[Out] $(\text{atan}((x*(-2^{(1/2)}*27358i - 68542)^{(1/2)}*126681219i)/(131072*((2^{(1/2)}*4940567541i)/16384 + 12541440681/131072))) - (126681219*2^{(1/2)}*x*(-2^{(1/2)}*27358i - 68542)^{(1/2)})/(262144*((2^{(1/2)}*4940567541i)/16384 + 12541440681/131072))) * (-2^{(1/2)}*27358i - 68542)^{(1/2)}*21i)/256 - ((513*x)/8 + (4941*x^3)/64 + (1569*x^5)/32 + (835*x^7)/64)/(12*x^2 + 10*x^4 + 4*x^6 + x^8 + 9) - 27*x - (\text{atan}((x*(2^{(1/2)}*27358i - 68542)^{(1/2)}*126681219i)/(131072*((2^{(1/2)}*4940567541i)/16384 - 12541440681/131072))) + (126681219*2^{(1/2)}*x*(2^{(1/2)}*27358i - 68542)^{(1/2)})/(262144*((2^{(1/2)}*4940567541i)/16384 - 12541440681/131072))) * (2^{(1/2)}*27358i - 68542)^{(1/2)}*21i)/256 + (5*x^3)/3$

sympy [A] time = 0.68, size = 82, normalized size = 0.34

$$\frac{5x^3}{3} - 27x + \frac{-835x^7 - 3138x^5 - 4941x^3 - 4104x}{64x^8 + 256x^6 + 640x^4 + 768x^2 + 576} + 21 \text{RootSum}\left(17179869184t^4 + 8983937024t^2 + 1548731523, \left(t \mapsto t \log\left(-\frac{1107296256t^3}{310800559} + \frac{438857984t}{310800559} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x**8*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**3, x)$

[Out] $5*x**3/3 - 27*x + (-835*x**7 - 3138*x**5 - 4941*x**3 - 4104*x)/(64*x**8 + 256*x**6 + 640*x**4 + 768*x**2 + 576) + 21*\text{RootSum}(17179869184*_t**4 + 8983937024*_t**2 + 1548731523, \text{Lambda}(_t, _t*\log(-1107296256*_t**3/310800559 + 438857984*_t/310800559 + x)))$

$$3.117 \quad \int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3} dx$$

Optimal. Leaf size=235

$$-\frac{1}{512}\sqrt{1176531\sqrt{3}-827621}\log\left(x^2-\sqrt{2(\sqrt{3}-1)}x+\sqrt{3}\right)+\frac{1}{512}\sqrt{1176531\sqrt{3}-827621}\log\left(x^2+\sqrt{2(\sqrt{3}-1)}x+\sqrt{3}\right)$$

Rubi [A] time = 0.30, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {1668, 1678, 1676, 1169, 634, 618, 204, 628}

$$\frac{7(58x^2+11)x}{64(x^4+2x^2+3)} + \frac{25(3-x^2)x}{16(x^4+2x^2+3)} - \frac{1}{512}\sqrt{1176531\sqrt{3}-827621}\log\left(x^2-\sqrt{2(\sqrt{3}-1)}x+\sqrt{3}\right) + \frac{1}{512}\sqrt{1176531\sqrt{3}-827621}\log\left(x^2+\sqrt{2(\sqrt{3}-1)}x+\sqrt{3}\right) + \frac{5x}{256}\sqrt{827621+1176531\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{2(\sqrt{3}-1)}-2x}{\sqrt{2(1+\sqrt{3})}}\right) - \frac{1}{256}\sqrt{827621+1176531\sqrt{3}}\tan^{-1}\left(\frac{2x+\sqrt{2(\sqrt{3}-1)}}{\sqrt{2(1+\sqrt{3})}}\right)$$

Antiderivative was successfully verified.

[In] Int[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]

[Out] 5*x + (25*x*(3 - x^2))/(16*(3 + 2*x^2 + x^4)^2) + (7*x*(11 + 58*x^2))/(64*(3 + 2*x^2 + x^4)) + (Sqrt[827621 + 1176531*Sqrt[3]]*ArcTan[(Sqrt[2*(-1 + Sqrt[3]]) - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/256 - (Sqrt[827621 + 1176531*Sqrt[3]]*ArcTan[(Sqrt[2*(-1 + Sqrt[3]]) + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/256 - (Sqrt[-827621 + 1176531*Sqrt[3]]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]]*x + x^2))/512 + (Sqrt[-827621 + 1176531*Sqrt[3]]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]]*x + x^2))/512

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1169

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]

Rule 1668

```

Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :=
  With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
    e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
  x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
  2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
  nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
  mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
  + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x]] /; FreeQ[{a, b,
  c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
  & LtQ[p, -1] && IGtQ[m/2, 0]

```

Rule 1676

```

Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := Int[ExpandInte
  grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
  2] && Expon[Pq, x^2] > 1

```

Rule 1678

```

Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := With[{d =
  Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
  nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
  ^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
  b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
  x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
  + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
  + 7)*(b*d - 2*a*e)*x^2, x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
  2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]

```

Rubi steps

$$\begin{aligned}
\int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3} dx &= \frac{25x(3-x^2)}{16(3+2x^2+x^4)^2} + \frac{1}{96} \int \frac{-450+1650x^2-672x^6+480x^8}{(3+2x^2+x^4)^2} dx \\
&= \frac{25x(3-x^2)}{16(3+2x^2+x^4)^2} + \frac{7x(11+58x^2)}{64(3+2x^2+x^4)} + \frac{\int \frac{-12744-49104x^2+23040x^4}{3+2x^2+x^4} dx}{4608} \\
&= \frac{25x(3-x^2)}{16(3+2x^2+x^4)^2} + \frac{7x(11+58x^2)}{64(3+2x^2+x^4)} + \frac{\int \left(23040 - \frac{72(1137+1322x^2)}{3+2x^2+x^4}\right) dx}{4608} \\
&= 5x + \frac{25x(3-x^2)}{16(3+2x^2+x^4)^2} + \frac{7x(11+58x^2)}{64(3+2x^2+x^4)} - \frac{1}{64} \int \frac{1137+1322x^2}{3+2x^2+x^4} dx \\
&= 5x + \frac{25x(3-x^2)}{16(3+2x^2+x^4)^2} + \frac{7x(11+58x^2)}{64(3+2x^2+x^4)} - \frac{\int \frac{1137\sqrt{2(-1+\sqrt{3})} - (1137-1322\sqrt{3})x}{\sqrt{3}-\sqrt{2(-1+\sqrt{3})}x+x^2} dx}{128\sqrt{6(-1+\sqrt{3})}} \\
&= 5x + \frac{25x(3-x^2)}{16(3+2x^2+x^4)^2} + \frac{7x(11+58x^2)}{64(3+2x^2+x^4)} - \frac{1}{256} (1322+379\sqrt{3}) \int \frac{1}{\sqrt{3}-\sqrt{2(-1+\sqrt{3})}x+x^2} dx \\
&= 5x + \frac{25x(3-x^2)}{16(3+2x^2+x^4)^2} + \frac{7x(11+58x^2)}{64(3+2x^2+x^4)} - \frac{1}{512} \sqrt{-827621+1176531\sqrt{3}} \operatorname{arctan} \left(\frac{\sqrt{3}-\sqrt{2(-1+\sqrt{3})}x}{\sqrt{-827621+1176531\sqrt{3}}} \right) \\
&= 5x + \frac{25x(3-x^2)}{16(3+2x^2+x^4)^2} + \frac{7x(11+58x^2)}{64(3+2x^2+x^4)} + \frac{1}{256} \sqrt{827621+1176531\sqrt{3}} \operatorname{arctan} \left(\frac{\sqrt{3}-\sqrt{2(-1+\sqrt{3})}x}{\sqrt{827621+1176531\sqrt{3}}} \right)
\end{aligned}$$

Mathematica [C] time = 0.32, size = 138, normalized size = 0.59

$$\frac{1}{256} \left(\frac{4x(320x^8+1686x^6+4089x^4+5112x^2+3411)}{(x^4+2x^2+3)^2} - \frac{i(185\sqrt{2}-2644i)\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}} + \frac{i(185\sqrt{2}+2644i)\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]

[Out] ((4*x*(3411 + 5112*x^2 + 4089*x^4 + 1686*x^6 + 320*x^8))/(3 + 2*x^2 + x^4)^2 - (I*(-2644*I + 185*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/Sqrt[1 - I*Sqrt[2]] + (I*(2644*I + 185*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/Sqrt[1 + I*Sqrt[2]])/256

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]

[Out] IntegrateAlgebraic[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]

fricas [B] time = 1.39, size = 551, normalized size = 2.34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="fricas")
[Out] 1/4759173538071552*(23795867690357760*x^9 + 125374477893572448*x^7 + 304066
571830852752*x^5 - 10534088*4152675581883^(1/4)*sqrt(3)*(x^8 + 4*x^6 + 10*x
^4 + 12*x^2 + 9)*sqrt(973721762751*sqrt(3) + 4152675581883)*arctan(1/847120
6900375217227324302495633*4152675581883^(3/4)*sqrt(516403378697)*sqrt(46476
30408273*x^2 + 4152675581883^(1/4)*(1322*sqrt(3)*sqrt(2)*x - 1137*sqrt(2)*x
)*sqrt(973721762751*sqrt(3) + 4152675581883) + 4647630408273*sqrt(3))*sqrt(
973721762751*sqrt(3) + 4152675581883)*(379*sqrt(3) - 1322) - 1/546808125187
5840963*4152675581883^(3/4)*(379*sqrt(3)*x - 1322*x)*sqrt(973721762751*sqrt
(3) + 4152675581883) + 1/2*sqrt(3)*sqrt(2) - 1/2*sqrt(2)) - 10534088*415267
5581883^(1/4)*sqrt(3)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqrt(973721762751
*sqrt(3) + 4152675581883)*arctan(1/8471206900375217227324302495633*41526755
81883^(3/4)*sqrt(516403378697)*sqrt(4647630408273*x^2 - 4152675581883^(1/4)
*(1322*sqrt(3)*sqrt(2)*x - 1137*sqrt(2)*x)*sqrt(973721762751*sqrt(3) + 4152
675581883) + 4647630408273*sqrt(3))*sqrt(973721762751*sqrt(3) + 41526755818
83)*(379*sqrt(3) - 1322) - 1/5468081251875840963*4152675581883^(3/4)*(379*s
qrt(3)*x - 1322*x)*sqrt(973721762751*sqrt(3) + 4152675581883) - 1/2*sqrt(3)
*sqrt(2) + 1/2*sqrt(2)) + 380138986353465216*x^3 - 4152675581883^(1/4)*(827
621*sqrt(3)*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9) - 3529593*sqrt(2)*(
x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9))*sqrt(973721762751*sqrt(3) + 41526755818
83)*log(4647630408273*x^2 + 4152675581883^(1/4)*(1322*sqrt(3)*sqrt(2)*x - 1
137*sqrt(2)*x)*sqrt(973721762751*sqrt(3) + 4152675581883) + 4647630408273*s
qrt(3) + 4152675581883^(1/4)*(827621*sqrt(3)*sqrt(2)*(x^8 + 4*x^6 + 10*x^4
+ 12*x^2 + 9) - 3529593*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9))*sqrt(
973721762751*sqrt(3) + 4152675581883)*log(4647630408273*x^2 - 4152675581883
^(1/4)*(1322*sqrt(3)*sqrt(2)*x - 1137*sqrt(2)*x)*sqrt(973721762751*sqrt(3)
+ 4152675581883) + 4647630408273*sqrt(3)) + 253649077161907248*x)/(x^8 + 4*
x^6 + 10*x^4 + 12*x^2 + 9)
```

giac [B] time = 2.61, size = 580, normalized size = 2.47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="giac")
[Out] 1/82944*sqrt(2)*(661*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 11898*3^(3/4)
*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 11898*3^(3/4)*(sqrt(3) + 3)*s
qrt(-6*sqrt(3) + 18) + 661*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 20466*3^(1/4)*
sqrt(2)*sqrt(6*sqrt(3) + 18) + 20466*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(
1/3*3^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2))
+ 1/82944*sqrt(2)*(661*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 11898*3^(3
/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 11898*3^(3/4)*(sqrt(3) + 3
)*sqrt(-6*sqrt(3) + 18) + 661*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 20466*3^(1/
4)*sqrt(2)*sqrt(6*sqrt(3) + 18) + 20466*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arct
an(1/3*3^(3/4)*(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/
2)) + 1/165888*sqrt(2)*(11898*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3)
+ 18) - 661*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 661*3^(3/4)*(6*sqrt(
3) + 18)^(3/2) + 11898*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 20466*3
^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) - 20466*3^(1/4)*sqrt(6*sqrt(3) + 18))*
log(x^2 + 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) - 1/165888*sqrt(2)
*(11898*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 661*3^(3/4)*
sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 661*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 1189
```

$$8 \cdot 3^{3/4} \cdot \sqrt{6 \cdot \sqrt{3} + 18} \cdot (\sqrt{3} - 3) - 20466 \cdot 3^{1/4} \cdot \sqrt{2} \cdot \sqrt{-6 \cdot \sqrt{3} + 18} - 20466 \cdot 3^{1/4} \cdot \sqrt{6 \cdot \sqrt{3} + 18} \cdot \log(x^2 - 2 \cdot 3^{1/4} \cdot x \cdot \sqrt{-1/6 \cdot \sqrt{3} + 1/2} + \sqrt{3}) + 5 \cdot x + 1/64 \cdot (406 \cdot x^7 + 889 \cdot x^5 + 1272 \cdot x^3 + 531 \cdot x) / (x^4 + 2 \cdot x^2 + 3)^2$$

maple [B] time = 0.04, size = 422, normalized size = 1.80

$$\frac{8 \cdot \sqrt{3} \cdot \sqrt[4]{3} \cdot \sqrt{6 \cdot \sqrt{3} + 18} \cdot (\sqrt{3} - 3) - 20466 \cdot \sqrt{2} \cdot \sqrt{3} \cdot \sqrt{-6 \cdot \sqrt{3} + 18} - 20466 \cdot \sqrt{3} \cdot \sqrt{6 \cdot \sqrt{3} + 18} \cdot \log(x^2 - 2 \cdot \sqrt{3} \cdot x \cdot \sqrt{-1/6 \cdot \sqrt{3} + 1/2} + \sqrt{3}) + 5 \cdot x + \frac{1}{64} (406 x^7 + 889 x^5 + 1272 x^3 + 531 x)}{(x^4 + 2x^2 + 3)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x)

[Out] $5x - (-203/32 x^7 - 889/64 x^5 - 159/8 x^3 - 531/64 x) / (x^4 + 2x^2 + 3)^2 - 943/1024 \cdot (-2 + 2 \cdot 3^{1/2})^{1/2} \cdot 3^{1/2} \cdot \ln(x^2 - (-2 + 2 \cdot 3^{1/2})^{1/2} \cdot x + 3^{1/2}) - 185/1024 \cdot (-2 + 2 \cdot 3^{1/2})^{1/2} \cdot \ln(x^2 - (-2 + 2 \cdot 3^{1/2})^{1/2} \cdot x + 3^{1/2}) - 943/512 \cdot (2 + 2 \cdot 3^{1/2})^{1/2} \cdot (-2 + 2 \cdot 3^{1/2}) \cdot 3^{1/2} \cdot \arctan((2x - (-2 + 2 \cdot 3^{1/2})^{1/2})^{1/2}) / (2 + 2 \cdot 3^{1/2})^{1/2} - 185/512 \cdot (2 + 2 \cdot 3^{1/2})^{1/2} \cdot (-2 + 2 \cdot 3^{1/2}) \cdot \arctan((2x - (-2 + 2 \cdot 3^{1/2})^{1/2})^{1/2}) / (2 + 2 \cdot 3^{1/2})^{1/2} - 379/64 \cdot (2 + 2 \cdot 3^{1/2})^{1/2} \cdot 3^{1/2} \cdot \arctan((2x - (-2 + 2 \cdot 3^{1/2})^{1/2})^{1/2}) / (2 + 2 \cdot 3^{1/2})^{1/2} + 943/1024 \cdot (-2 + 2 \cdot 3^{1/2})^{1/2} \cdot 3^{1/2} \cdot \ln(x^2 + (-2 + 2 \cdot 3^{1/2})^{1/2} \cdot x + 3^{1/2}) + 185/1024 \cdot (-2 + 2 \cdot 3^{1/2})^{1/2} \cdot \ln(x^2 + (-2 + 2 \cdot 3^{1/2})^{1/2} \cdot x + 3^{1/2}) - 943/512 \cdot (2 + 2 \cdot 3^{1/2})^{1/2} \cdot (-2 + 2 \cdot 3^{1/2}) \cdot 3^{1/2} \cdot \arctan((2x + (-2 + 2 \cdot 3^{1/2})^{1/2})^{1/2}) / (2 + 2 \cdot 3^{1/2})^{1/2} - 185/512 \cdot (2 + 2 \cdot 3^{1/2})^{1/2} \cdot (-2 + 2 \cdot 3^{1/2}) \cdot \arctan((2x + (-2 + 2 \cdot 3^{1/2})^{1/2})^{1/2}) / (2 + 2 \cdot 3^{1/2})^{1/2} - 379/64 \cdot (2 + 2 \cdot 3^{1/2})^{1/2} \cdot 3^{1/2} \cdot \arctan((2x + (-2 + 2 \cdot 3^{1/2})^{1/2})^{1/2}) / (2 + 2 \cdot 3^{1/2})^{1/2}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$5x + \frac{406x^7 + 889x^5 + 1272x^3 + 531x}{64(x^8 + 4x^6 + 10x^4 + 12x^2 + 9)} - \frac{1}{64} \int \frac{1322x^2 + 1137}{x^4 + 2x^2 + 3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="maxima")

[Out] $5x + 1/64 \cdot (406 \cdot x^7 + 889 \cdot x^5 + 1272 \cdot x^3 + 531 \cdot x) / (x^8 + 4 \cdot x^6 + 10 \cdot x^4 + 12 \cdot x^2 + 9) - 1/64 \cdot \text{integrate}((1322 \cdot x^2 + 1137) / (x^4 + 2 \cdot x^2 + 3), x)$

mupad [B] time = 0.99, size = 176, normalized size = 0.75

$$5x + \frac{406x^7 + 889x^5 + 1272x^3 + 531x}{64x^8 + 256x^6 + 640x^4 + 768x^2 + 576} + \frac{\text{atan}\left(\frac{x\sqrt{-1655242 - \sqrt{2} 2633522i - 1316761i}}{131072(-3725116869 + \sqrt{2} 1497157257i) - 262144(3725116869 + \sqrt{2} 1497157257i)}\right) + \frac{1316761\sqrt{2}x\sqrt{-1655242 - \sqrt{2} 2633522i - 1316761i}}{256} \sqrt{-1655242 - \sqrt{2} 2633522i} + \text{atan}\left(\frac{x\sqrt{-1655242 + \sqrt{2} 2633522i - 1316761i}}{131072(-3725116869 + \sqrt{2} 1497157257i) - 262144(3725116869 + \sqrt{2} 1497157257i)}\right) - \frac{1316761\sqrt{2}x\sqrt{-1655242 + \sqrt{2} 2633522i - 1316761i}}{256} \sqrt{-1655242 + \sqrt{2} 2633522i}}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^6*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^3,x)

[Out] $5x + \frac{\text{atan}((x \cdot (-2^{1/2}) \cdot 2633522i - 1655242)^{1/2} \cdot 1316761i) / (131072 \cdot ((2^{1/2}) \cdot 1497157257i) / 131072 - 3725116869 / 131072)) + (1316761 \cdot 2^{1/2} \cdot x \cdot (-2^{1/2}) \cdot 2633522i - 1655242)^{1/2} / (262144 \cdot ((2^{1/2}) \cdot 1497157257i) / 131072 - 3725116869 / 131072)) \cdot (-2^{1/2}) \cdot 2633522i - 1655242)^{1/2} \cdot i)}{256} - \frac{\text{atan}((x \cdot (2^{1/2}) \cdot 2633522i - 1655242)^{1/2} \cdot 1316761i) / (131072 \cdot ((2^{1/2}) \cdot 1497157257i) / 131072 + 3725116869 / 131072)) - (1316761 \cdot 2^{1/2} \cdot x \cdot (2^{1/2}) \cdot 2633522i - 1655242)^{1/2} / (262144 \cdot ((2^{1/2}) \cdot 1497157257i) / 131072 + 3725116869 / 131072)) \cdot (2^{1/2}) \cdot 2633522i - 1655242)^{1/2} \cdot i)}{256} + \frac{(531 \cdot x)}{64} + \frac{(159 \cdot x^3)}{8} + \frac{(889 \cdot x^5)}{64} + \frac{(203 \cdot x^7)}{32} / (12 \cdot x^2 + 10 \cdot x^4 + 4 \cdot x^6 + x^8 + 9)$

sympy [A] time = 0.66, size = 71, normalized size = 0.30

$$5x + \frac{406x^7 + 889x^5 + 1272x^3 + 531x}{64x^8 + 256x^6 + 640x^4 + 768x^2 + 576} + \text{RootSum}\left(17179869184t^4 + 216955879424t^2 + 4152675581883, \left(t \mapsto t \log\left(-\frac{31641829376t^3}{1549210136091} - \frac{455309168896t}{1549210136091} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**3,x)
```

```
[Out] 5*x + (406*x**7 + 889*x**5 + 1272*x**3 + 531*x)/(64*x**8 + 256*x**6 + 640*x**4 + 768*x**2 + 576) + RootSum(17179869184*_t**4 + 216955879424*_t**2 + 4152675581883, Lambda(_t, _t*log(-31641829376*_t**3/1549210136091 - 455309168896*_t/1549210136091 + x)))
```

$$3.118 \quad \int \frac{x^4(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3} dx$$

Optimal. Leaf size=238

$$\frac{1}{512} \sqrt{3(48835 + 32827\sqrt{3})} \log\left(x^2 - \sqrt{2(\sqrt{3}-1)}x + \sqrt{3}\right) - \frac{1}{512} \sqrt{3(48835 + 32827\sqrt{3})} \log\left(x^2 + \sqrt{2(\sqrt{3}-1)}x + \sqrt{3}\right)$$

Rubi [A] time = 0.29, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {1668, 1678, 1169, 634, 618, 204, 628}

$$\frac{x(238-59x^2)}{64(x^4+2x^2+3)} - \frac{25x(x^2+3)}{16(x^4+2x^2+3)} + \frac{1}{512} \sqrt{3(48835+32827\sqrt{3})} \log(x^2 - \sqrt{2(\sqrt{3}-1)}x + \sqrt{3}) - \frac{1}{512} \sqrt{3(48835+32827\sqrt{3})} \log(x^2 + \sqrt{2(\sqrt{3}-1)}x + \sqrt{3}) - \frac{1}{256} \sqrt{3(32827\sqrt{3}-48835)} \tan^{-1}\left(\frac{\sqrt{2(\sqrt{3}-1)}-2x}{\sqrt{2(1+\sqrt{3})}}\right) + \frac{1}{256} \sqrt{3(32827\sqrt{3}-48835)} \tan^{-1}\left(\frac{2x+\sqrt{2(\sqrt{3}-1)}}{\sqrt{2(1+\sqrt{3})}}\right)$$

Antiderivative was successfully verified.

[In] Int[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]

[Out] (-25*x*(3 + x^2))/(16*(3 + 2*x^2 + x^4)^2) + (x*(238 - 59*x^2))/(64*(3 + 2*x^2 + x^4)) - (Sqrt[3*(-48835 + 32827*Sqrt[3])]*ArcTan[(Sqrt[2*(-1 + Sqrt[3]]) - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/256 + (Sqrt[3*(-48835 + 32827*Sqrt[3])]*ArcTan[(Sqrt[2*(-1 + Sqrt[3]]) + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/256 + (Sqrt[3*(48835 + 32827*Sqrt[3])]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/512 - (Sqrt[3*(48835 + 32827*Sqrt[3])]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/512

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1169

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]

Rule 1668

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :=
  With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
    e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
  x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
  2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
  nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
  mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
  + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x]] /; FreeQ[{a, b,
  c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &&
  & LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := With[{d =
  Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
  nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
  ^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
  b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
  x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
  + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
  + 7)*(b*d - 2*a*e)*x^2, x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
  2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
 \int \frac{x^4(4 + x^2 + 3x^4 + 5x^6)}{(3 + 2x^2 + x^4)^3} dx &= -\frac{25x(3 + x^2)}{16(3 + 2x^2 + x^4)^2} + \frac{1}{96} \int \frac{450 - 750x^2 - 672x^4 + 480x^6}{(3 + 2x^2 + x^4)^2} dx \\
 &= -\frac{25x(3 + x^2)}{16(3 + 2x^2 + x^4)^2} + \frac{x(238 - 59x^2)}{64(3 + 2x^2 + x^4)} + \frac{\int \frac{-9936 + 18792x^2}{3 + 2x^2 + x^4} dx}{4608} \\
 &= -\frac{25x(3 + x^2)}{16(3 + 2x^2 + x^4)^2} + \frac{x(238 - 59x^2)}{64(3 + 2x^2 + x^4)} + \frac{\int \frac{-9936\sqrt{2(-1+\sqrt{3})} - (-9936-18792\sqrt{3})}{\sqrt{3}-\sqrt{2(-1+\sqrt{3})}x+x^2} dx}{9216\sqrt{6(-1+\sqrt{3})}} \\
 &= -\frac{25x(3 + x^2)}{16(3 + 2x^2 + x^4)^2} + \frac{x(238 - 59x^2)}{64(3 + 2x^2 + x^4)} + \frac{1}{256} (261 - 46\sqrt{3}) \int \frac{1}{\sqrt{3} - \sqrt{2(-1+\sqrt{3})}x+x^2} dx \\
 &= -\frac{25x(3 + x^2)}{16(3 + 2x^2 + x^4)^2} + \frac{x(238 - 59x^2)}{64(3 + 2x^2 + x^4)} + \frac{1}{512} \sqrt{146505 + 98481\sqrt{3}} \log\left(\frac{\sqrt{3} - \sqrt{2(-1+\sqrt{3})}x+x^2}{\sqrt{3} - \sqrt{2(-1+\sqrt{3})}x+x^2}\right) \\
 &= -\frac{25x(3 + x^2)}{16(3 + 2x^2 + x^4)^2} + \frac{x(238 - 59x^2)}{64(3 + 2x^2 + x^4)} - \frac{1}{256} \sqrt{3(-48835 + 32827\sqrt{3})} \tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right) + \frac{1}{256} \sqrt{3(-48835 + 32827\sqrt{3})} \tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)
 \end{aligned}$$

Mathematica [C] time = 0.30, size = 129, normalized size = 0.54

$$\frac{1}{256} \left(\frac{4x(-59x^6 + 120x^4 + 199x^2 + 414)}{(x^4 + 2x^2 + 3)^2} + \frac{3(174 + 133i\sqrt{2}) \tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}} + \frac{3(174 - 133i\sqrt{2}) \tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]

[Out] ((4*x*(414 + 199*x^2 + 120*x^4 - 59*x^6))/(3 + 2*x^2 + x^4)^2 + (3*(174 + (133*I)*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/Sqrt[1 - I*Sqrt[2]] + (3*(174 - (133*I)*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/Sqrt[1 + I*Sqrt[2]])/256

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4(4 + x^2 + 3x^4 + 5x^6)}{(3 + 2x^2 + x^4)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]

[Out] IntegrateAlgebraic[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3, x]

fricas [B] time = 1.40, size = 546, normalized size = 2.29

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="fricas")

[Out] -1/2076490005504*(1914264223824*x^7 - 3893418760320*x^5 + 164728*29095522083^(1/4)*sqrt(3)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqrt(-1603106545*sqrt(3) + 3232835787)*arctan(1/1214880276996365518761363*29095522083^(3/4)*sqrt(2027822271)*sqrt(2027822271*x^2 + 29095522083^(1/4)*(87*sqrt(3)*sqrt(2)*x + 46*sqrt(2)*x)*sqrt(-1603106545*sqrt(3) + 3232835787) + 2027822271*sqrt(3))*(46*sqrt(3) + 261)*sqrt(-1603106545*sqrt(3) + 3232835787) - 1/599105895211053*29095522083^(3/4)*(46*sqrt(3)*x + 261*x)*sqrt(-1603106545*sqrt(3) + 3232835787) - 1/2*sqrt(3)*sqrt(2) + 1/2*sqrt(2)) + 164728*29095522083^(1/4)*sqrt(3)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqrt(-1603106545*sqrt(3) + 3232835787)*arctan(1/1214880276996365518761363*29095522083^(3/4)*sqrt(2027822271)*sqrt(2027822271*x^2 - 29095522083^(1/4)*(87*sqrt(3)*sqrt(2)*x + 46*sqrt(2)*x)*sqrt(-1603106545*sqrt(3) + 3232835787) + 2027822271*sqrt(3))*(46*sqrt(3) + 261)*sqrt(-1603106545*sqrt(3) + 3232835787) - 1/599105895211053*29095522083^(3/4)*(46*sqrt(3)*x + 261*x)*sqrt(-1603106545*sqrt(3) + 3232835787) + 1/2*sqrt(3)*sqrt(2) - 1/2*sqrt(2)) - 6456586110864*x^3 + 29095522083^(1/4)*(48835*sqrt(3)*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9) + 98481*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9))*sqrt(-1603106545*sqrt(3) + 3232835787)*log(2027822271*x^2 + 29095522083^(1/4)*(87*sqrt(3)*sqrt(2)*x + 46*sqrt(2)*x)*sqrt(-1603106545*sqrt(3) + 3232835787) + 2027822271*sqrt(3)) - 29095522083^(1/4)*(48835*sqrt(3)*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9) + 98481*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9))*sqrt(-1603106545*sqrt(3) + 3232835787)*log(2027822271*x^2 - 29095522083^(1/4)*(87*sqrt(3)*sqrt(2)*x + 46*sqrt(2)*x)*sqrt(-1603106545*sqrt(3) + 3232835787) + 2027822271*sqrt(3)) - 13432294723104*x)/(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)

giac [B] time = 2.60, size = 577, normalized size = 2.42

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="giac")

[Out] -1/18432*sqrt(2)*(29*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 522*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 522*3^(3/4)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) + 29*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 552*3^(1/4)*sqrt(2)

*sqrt(6*sqrt(3) + 18) - 552*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) - 1/18432*sqrt(2)*(29*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 522*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 522*3^(3/4)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) + 29*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 552*3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) - 552*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3/4)*(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) - 1/36864*sqrt(2)*(522*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 29*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 29*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 522*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) + 552*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) + 552*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 + 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) + 1/36864*sqrt(2)*(522*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 29*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 29*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 522*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) + 552*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) + 552*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 - 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) - 1/64*(59*x^7 - 120*x^5 - 199*x^3 - 414*x)/(x^4 + 2*x^2 + 3)^2

maple [B] time = 0.03, size = 418, normalized size = 1.76

$$\frac{30(-2+2\sqrt{3})\sqrt{\arctan\left(\frac{x-\sqrt{2+2\sqrt{3}}}{2\sqrt{3}}\right)}}{32\sqrt{2+2\sqrt{3}}} - \frac{30(-2+2\sqrt{3})\arctan\left(\frac{x-\sqrt{2+2\sqrt{3}}}{2\sqrt{3}}\right)}{32\sqrt{2+2\sqrt{3}}} + \frac{21\sqrt{3}\arctan\left(\frac{x-\sqrt{2+2\sqrt{3}}}{2\sqrt{3}}\right)}{32\sqrt{2+2\sqrt{3}}} - \frac{30(-2+2\sqrt{3})\sqrt{\arctan\left(\frac{x-\sqrt{2+2\sqrt{3}}}{2\sqrt{3}}\right)}}{32\sqrt{2+2\sqrt{3}}} - \frac{30(-2+2\sqrt{3})\arctan\left(\frac{x-\sqrt{2+2\sqrt{3}}}{2\sqrt{3}}\right)}{32\sqrt{2+2\sqrt{3}}} + \frac{21\sqrt{3}\arctan\left(\frac{x-\sqrt{2+2\sqrt{3}}}{2\sqrt{3}}\right)}{32\sqrt{2+2\sqrt{3}}} - \frac{30\sqrt{-2+2\sqrt{3}}\sqrt{\ln\left(\frac{x-\sqrt{-2+2\sqrt{3}}+x\sqrt{3}}{x-\sqrt{-2+2\sqrt{3}}+x\sqrt{3}}\right)}}{32\sqrt{2+2\sqrt{3}}} - \frac{30\sqrt{-2+2\sqrt{3}}\sqrt{\ln\left(\frac{x-\sqrt{-2+2\sqrt{3}}+x\sqrt{3}}{x-\sqrt{-2+2\sqrt{3}}+x\sqrt{3}}\right)}}{32\sqrt{2+2\sqrt{3}}} - \frac{30\sqrt{-2+2\sqrt{3}}\sqrt{\ln\left(\frac{x-\sqrt{-2+2\sqrt{3}}+x\sqrt{3}}{x-\sqrt{-2+2\sqrt{3}}+x\sqrt{3}}\right)}}{32\sqrt{2+2\sqrt{3}}} - \frac{30\sqrt{-2+2\sqrt{3}}\sqrt{\ln\left(\frac{x-\sqrt{-2+2\sqrt{3}}+x\sqrt{3}}{x-\sqrt{-2+2\sqrt{3}}+x\sqrt{3}}\right)}}{32\sqrt{2+2\sqrt{3}}} - \frac{30\sqrt{-2+2\sqrt{3}}\sqrt{\ln\left(\frac{x-\sqrt{-2+2\sqrt{3}}+x\sqrt{3}}{x-\sqrt{-2+2\sqrt{3}}+x\sqrt{3}}\right)}}{32\sqrt{2+2\sqrt{3}}} - \frac{30\sqrt{-2+2\sqrt{3}}\sqrt{\ln\left(\frac{x-\sqrt{-2+2\sqrt{3}}+x\sqrt{3}}{x-\sqrt{-2+2\sqrt{3}}+x\sqrt{3}}\right)}}{32\sqrt{2+2\sqrt{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x)

[Out] (-59/64*x^7+15/8*x^5+199/64*x^3+207/32*x)/(x^4+2*x^2+3)^2+307/1024*(-2+2*3^(1/2))^(1/2)*3^(1/2)*ln(x^2-(-2+2*3^(1/2))^(1/2)*x+3^(1/2))+399/1024*(-2+2*3^(1/2))^(1/2)*ln(x^2-(-2+2*3^(1/2))^(1/2)*x+3^(1/2))+307/512/(2+2*3^(1/2))^(1/2)*(-2+2*3^(1/2))*3^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))+399/512/(2+2*3^(1/2))^(1/2)*(-2+2*3^(1/2))*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))-23/32/(2+2*3^(1/2))^(1/2)*3^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))-307/1024*(-2+2*3^(1/2))^(1/2)*3^(1/2)*ln(x^2+(-2+2*3^(1/2))^(1/2)*x+3^(1/2))-399/1024*(-2+2*3^(1/2))^(1/2)*ln(x^2+(-2+2*3^(1/2))^(1/2)*x+3^(1/2))+307/512/(2+2*3^(1/2))^(1/2)*(-2+2*3^(1/2))*3^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))+399/512/(2+2*3^(1/2))^(1/2)*(-2+2*3^(1/2))*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))-23/32/(2+2*3^(1/2))^(1/2)*3^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{59x^7 - 120x^5 - 199x^3 - 414x}{64(x^8 + 4x^6 + 10x^4 + 12x^2 + 9)} + \frac{3}{64} \int \frac{87x^2 - 46}{x^4 + 2x^2 + 3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="maxima")

[Out] -1/64*(59*x^7 - 120*x^5 - 199*x^3 - 414*x)/(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9) + 3/64*integrate((87*x^2 - 46)/(x^4 + 2*x^2 + 3), x)

mupad [B] time = 0.15, size = 173, normalized size = 0.73

$$\frac{-\frac{59x^7}{64} + \frac{15x^5}{8} + \frac{199x^3}{64} + \frac{207x}{32}}{x^8 + 4x^6 + 10x^4 + 12x^2 + 9} + \frac{\operatorname{atan}\left(\frac{x\sqrt{293010-\sqrt{2}1235461}617731}{131072\left(\frac{5886923}{131072} + \sqrt{2}4262351\right)} + \frac{61773\sqrt{2}x\sqrt{293010-\sqrt{2}1235461}}{262144\left(\frac{5886923}{131072} + \sqrt{2}4262351\right)}\right)\sqrt{293010-\sqrt{2}1235461}11}{256} - \frac{\operatorname{atan}\left(\frac{x\sqrt{293010+\sqrt{2}1235461}617731}{131072\left(\frac{5886923}{131072} + \sqrt{2}4262351\right)} - \frac{61773\sqrt{2}x\sqrt{293010+\sqrt{2}1235461}}{262144\left(\frac{5886923}{131072} + \sqrt{2}4262351\right)}\right)\sqrt{293010+\sqrt{2}1235461}11}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^3,x)

```
[Out] ((207*x)/32 + (199*x^3)/64 + (15*x^5)/8 - (59*x^7)/64)/(12*x^2 + 10*x^4 + 4
*x^6 + x^8 + 9) + (atan((x*(293010 - 2^(1/2)*123546i)^(1/2)*61773i)/(131072
*((2^(1/2)*4262337i)/65536 + 56892933/131072))) + (61773*2^(1/2)*x*(293010 -
2^(1/2)*123546i)^(1/2))/(262144*((2^(1/2)*4262337i)/65536 + 56892933/13107
2))))*(293010 - 2^(1/2)*123546i)^(1/2)*1i)/256 - (atan((x*(2^(1/2)*123546i +
293010)^(1/2)*61773i)/(131072*((2^(1/2)*4262337i)/65536 - 56892933/131072)
) - (61773*2^(1/2)*x*(2^(1/2)*123546i + 293010)^(1/2))/(262144*((2^(1/2)*42
62337i)/65536 - 56892933/131072))))*(2^(1/2)*123546i + 293010)^(1/2)*1i)/256
```

sympy [B] time = 1.31, size = 1198, normalized size = 5.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**3,x)
```

```
[Out] (-59*x**7 + 120*x**5 + 199*x**3 + 414*x)/(64*x**8 + 256*x**6 + 640*x**4 + 7
68*x**2 + 576) - sqrt(146505/262144 + 98481*sqrt(3)/262144)*log(x**2 + x*(-
307*sqrt(6)*sqrt(48835 + 32827*sqrt(3))*sqrt(1603106545*sqrt(3) + 280884650
6)/675940757 + 10626354*sqrt(3)*sqrt(48835 + 32827*sqrt(3))/675940757 + 122
8*sqrt(48835 + 32827*sqrt(3))/20591) - 941929306825573*sqrt(2)*sqrt(1603106
545*sqrt(3) + 2808846506)/456895906973733049 - 47771215762*sqrt(6)*sqrt(160
3106545*sqrt(3) + 2808846506)/41754888382161 + 97477949666790882353/4568959
06973733049 + 5200450130596150*sqrt(3)/41754888382161 + sqrt(146505/262144
+ 98481*sqrt(3)/262144)*log(x**2 + x*(-1228*sqrt(48835 + 32827*sqrt(3))/20
591 - 10626354*sqrt(3)*sqrt(48835 + 32827*sqrt(3))/675940757 + 307*sqrt(6)*
sqrt(48835 + 32827*sqrt(3))*sqrt(1603106545*sqrt(3) + 2808846506)/675940757
) - 941929306825573*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506)/456895906
973733049 - 47771215762*sqrt(6)*sqrt(1603106545*sqrt(3) + 2808846506)/41754
888382161 + 97477949666790882353/456895906973733049 + 5200450130596150*sqrt
(3)/41754888382161 + 2*sqrt(-3*sqrt(2)*sqrt(1603106545*sqrt(3) + 280884650
6)/131072 + 146505/262144 + 295443*sqrt(3)/262144)*atan(1351881514*sqrt(3)*
x/(-1894372*sqrt(-2*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506) + 48835 +
98481*sqrt(3)) + 307*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506)*sqrt(-2
*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506) + 48835 + 98481*sqrt(3))) -
40311556*sqrt(3)*sqrt(48835 + 32827*sqrt(3))/(-1894372*sqrt(-2*sqrt(2)*sqrt
(1603106545*sqrt(3) + 2808846506) + 48835 + 98481*sqrt(3)) + 307*sqrt(2)*sq
rt(1603106545*sqrt(3) + 2808846506)*sqrt(-2*sqrt(2)*sqrt(1603106545*sqrt(3)
+ 2808846506) + 48835 + 98481*sqrt(3))) - 31879062*sqrt(48835 + 32827*sqrt
(3))/(-1894372*sqrt(-2*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506) + 4883
5 + 98481*sqrt(3)) + 307*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506)*sqrt
(-2*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506) + 48835 + 98481*sqrt(3)))
+ 921*sqrt(2)*sqrt(48835 + 32827*sqrt(3))*sqrt(1603106545*sqrt(3) + 280884
6506)/(-1894372*sqrt(-2*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506) + 488
35 + 98481*sqrt(3)) + 307*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506)*sq
rt(-2*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506) + 48835 + 98481*sqrt(3)
)) + 2*sqrt(-3*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506)/131072 + 14650
5/262144 + 295443*sqrt(3)/262144)*atan(1351881514*sqrt(3)*x/(-1894372*sqrt(
-2*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506) + 48835 + 98481*sqrt(3)) +
307*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506)*sqrt(-2*sqrt(2)*sqrt(160
3106545*sqrt(3) + 2808846506) + 48835 + 98481*sqrt(3))) - 921*sqrt(2)*sqrt(
48835 + 32827*sqrt(3))*sqrt(1603106545*sqrt(3) + 2808846506)/(-1894372*sqrt
(-2*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506) + 48835 + 98481*sqrt(3))
+ 307*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506)*sqrt(-2*sqrt(2)*sqrt(16
03106545*sqrt(3) + 2808846506) + 48835 + 98481*sqrt(3))) + 31879062*sqrt(48
835 + 32827*sqrt(3))/(-1894372*sqrt(-2*sqrt(2)*sqrt(1603106545*sqrt(3) + 28
08846506) + 48835 + 98481*sqrt(3)) + 307*sqrt(2)*sqrt(1603106545*sqrt(3) +
2808846506)*sqrt(-2*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506) + 48835 +
98481*sqrt(3))) + 40311556*sqrt(3)*sqrt(48835 + 32827*sqrt(3))/(-1894372*s
qrt(-2*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506) + 48835 + 98481*sqrt(3
```


)) + 307*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506)*sqrt(-2*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506) + 48835 + 98481*sqrt(3)))

$$3.119 \quad \int \frac{x^2(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3} dx$$

Optimal. Leaf size=246

$$\frac{11\sqrt{\frac{1}{3}(1825+1089\sqrt{3})} \log\left(x^2 - \sqrt{2(\sqrt{3}-1)}x + \sqrt{3}\right)}{1536} + \frac{11\sqrt{\frac{1}{3}(1825+1089\sqrt{3})} \log\left(x^2 + \sqrt{2(\sqrt{3}-1)}x + \sqrt{3}\right)}{1536}$$

Rubi [A] time = 0.28, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {1668, 1678, 1169, 634, 618, 204, 628}

$$\frac{25x(x^2+1)}{16(x^4+2x^2+3)^2} - \frac{x(88x^2+353)}{192(x^4+2x^2+3)} - \frac{11\sqrt{\frac{1}{3}(1825+1089\sqrt{3})} \log\left(x^2 - \sqrt{2(\sqrt{3}-1)}x + \sqrt{3}\right)}{1536} + \frac{11\sqrt{\frac{1}{3}(1825+1089\sqrt{3})} \log\left(x^2 + \sqrt{2(\sqrt{3}-1)}x + \sqrt{3}\right)}{1536} - \frac{11}{768} \sqrt{\frac{1}{3}(1089\sqrt{3}-1825)} \tan^{-1}\left(\frac{\sqrt{2(\sqrt{3}-1)}-2x}{\sqrt{2(1+\sqrt{3})}}\right) + \frac{11}{768} \sqrt{\frac{1}{3}(1089\sqrt{3}-1825)} \tan^{-1}\left(\frac{2x+\sqrt{2(\sqrt{3}-1)}}{\sqrt{2(1+\sqrt{3})}}\right)$$

Antiderivative was successfully verified.

[In] Int[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]

[Out] (25*x*(1 + x^2))/(16*(3 + 2*x^2 + x^4)^2) - (x*(353 + 88*x^2))/(192*(3 + 2*x^2 + x^4)) - (11*sqrt[(-1825 + 1089*sqrt[3])/3]*ArcTan[(sqrt[2*(-1 + sqrt[3])]) - 2*x]/sqrt[2*(1 + sqrt[3])]])/768 + (11*sqrt[(-1825 + 1089*sqrt[3])/3]*ArcTan[(sqrt[2*(-1 + sqrt[3])]) + 2*x]/sqrt[2*(1 + sqrt[3])]])/768 - (11*sqrt[(1825 + 1089*sqrt[3])/3]*Log[sqrt[3] - sqrt[2*(-1 + sqrt[3])]*x + x^2])/1536 + (11*sqrt[(1825 + 1089*sqrt[3])/3]*Log[sqrt[3] + sqrt[2*(-1 + sqrt[3])]*x + x^2])/1536

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1169

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]

Rule 1668

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :=
  With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
    e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
  x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
  2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
  nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
  mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
  + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x]] /; FreeQ[{a, b,
  c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
  & LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := With[{d =
  Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
  nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
  ^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
  b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
  x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
  + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
  + 7)*(b*d - 2*a*e)*x^2, x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
  2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3} dx &= \frac{25x(1+x^2)}{16(3+2x^2+x^4)^2} + \frac{1}{96} \int \frac{-150+78x^2+480x^4}{(3+2x^2+x^4)^2} dx \\
&= \frac{25x(1+x^2)}{16(3+2x^2+x^4)^2} - \frac{x(353+88x^2)}{192(3+2x^2+x^4)} + \frac{\int \frac{6072-2112x^2}{3+2x^2+x^4} dx}{4608} \\
&= \frac{25x(1+x^2)}{16(3+2x^2+x^4)^2} - \frac{x(353+88x^2)}{192(3+2x^2+x^4)} + \frac{\int \frac{6072\sqrt{2(-1+\sqrt{3})}-(6072+2112\sqrt{3})x}{\sqrt{3}-\sqrt{2(-1+\sqrt{3})}x+x^2} dx}{9216\sqrt{6(-1+\sqrt{3})}} \\
&= \frac{25x(1+x^2)}{16(3+2x^2+x^4)^2} - \frac{x(353+88x^2)}{192(3+2x^2+x^4)} - \frac{(11(24-23\sqrt{3})) \int \frac{1}{\sqrt{3}-\sqrt{2(-1+\sqrt{3})}x+x^2}}{2304} \\
&= \frac{25x(1+x^2)}{16(3+2x^2+x^4)^2} - \frac{x(353+88x^2)}{192(3+2x^2+x^4)} - \frac{11}{768} \sqrt{\frac{1825}{12} + \frac{363\sqrt{3}}{4}} \log\left(\sqrt{3} - \frac{x}{\sqrt{1-i\sqrt{2}}}\right) \\
&= \frac{25x(1+x^2)}{16(3+2x^2+x^4)^2} - \frac{x(353+88x^2)}{192(3+2x^2+x^4)} - \frac{11}{768} \sqrt{\frac{1}{3}(-1825+1089\sqrt{3})} \tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)
\end{aligned}$$

Mathematica [C] time = 0.30, size = 133, normalized size = 0.54

$$\frac{1}{768} \left(\frac{4x(88x^6+529x^4+670x^2+759)}{(x^4+2x^2+3)^2} - \frac{11i(31\sqrt{2}-16i)\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}} + \frac{11i(31\sqrt{2}+16i)\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]

[Out] ((-4*x*(759 + 670*x^2 + 529*x^4 + 88*x^6))/(3 + 2*x^2 + x^4)^2 - ((11*I)*(-16*I + 31*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/Sqrt[1 - I*Sqrt[2]] + ((11*I)*(16*I + 31*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/Sqrt[1 + I*Sqrt[2]])/768

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(4 + x^2 + 3x^4 + 5x^6)}{(3 + 2x^2 + x^4)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]

[Out] IntegrateAlgebraic[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3, x]

fricas [B] time = 1.35, size = 570, normalized size = 2.32

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="fricas")

[Out] -1/27952128*(12811392*x^7 + 77013936*x^5 + 1348*sqrt(6)*3^(3/4)*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqrt(-1987425*sqrt(3) + 3557763)*arctan(1/2226179538*sqrt(3707)*sqrt(6)*3^(3/4)*sqrt(sqrt(6)*3^(1/4)*(8*sqrt(3)*x + 23*x)*sqrt(-1987425*sqrt(3) + 3557763) + 33363*x^2 + 33363*sqrt(3))*(23*sqrt(3)*sqrt(2) + 24*sqrt(2))*sqrt(-1987425*sqrt(3) + 3557763) - 1/200178*sqrt(6)*3^(3/4)*(23*sqrt(3)*sqrt(2)*x + 24*sqrt(2)*x)*sqrt(-1987425*sqrt(3) + 3557763) - 1/2*sqrt(3)*sqrt(2) + 1/2*sqrt(2)) + 1348*sqrt(6)*3^(3/4)*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqrt(-1987425*sqrt(3) + 3557763)*arctan(1/2226179538*sqrt(3707)*sqrt(6)*3^(3/4)*sqrt(-sqrt(6)*3^(1/4)*(8*sqrt(3)*x + 23*x)*sqrt(-1987425*sqrt(3) + 3557763) + 33363*x^2 + 33363*sqrt(3))*(23*sqrt(3)*sqrt(2) + 24*sqrt(2))*sqrt(-1987425*sqrt(3) + 3557763) - 1/200178*sqrt(6)*3^(3/4)*(23*sqrt(3)*sqrt(2)*x + 24*sqrt(2)*x)*sqrt(-1987425*sqrt(3) + 3557763) + 1/2*sqrt(3)*sqrt(2) - 1/2*sqrt(2)) - sqrt(6)*3^(1/4)*(3267*x^8 + 13068*x^6 + 32670*x^4 + 39204*x^2 + 1825*sqrt(3)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9) + 29403)*sqrt(-1987425*sqrt(3) + 3557763)*log(sqrt(6)*3^(1/4)*(8*sqrt(3)*x + 23*x)*sqrt(-1987425*sqrt(3) + 3557763) + 33363*x^2 + 33363*sqrt(3)) + sqrt(6)*3^(1/4)*(3267*x^8 + 13068*x^6 + 32670*x^4 + 39204*x^2 + 1825*sqrt(3)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9) + 29403)*sqrt(-1987425*sqrt(3) + 3557763)*log(-sqrt(6)*3^(1/4)*(8*sqrt(3)*x + 23*x)*sqrt(-1987425*sqrt(3) + 3557763) + 33363*x^2 + 33363*sqrt(3)) + 97541280*x^3 + 110498256*x)/(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)

giac [B] time = 2.69, size = 577, normalized size = 2.35

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="giac")

[Out] 11/124416*sqrt(2)*(2*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 36*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 36*3^(3/4)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) + 2*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 207*3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) - 207*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3/4)*

$(x + 3^{1/4} \sqrt{-1/6 \sqrt{3} + 1/2}) / \sqrt{1/6 \sqrt{3} + 1/2} + 11/124416 \sqrt{2} * (2 * 3^{3/4} \sqrt{2} * (6 \sqrt{3} + 18)^{3/2} + 36 * 3^{3/4} \sqrt{2} \sqrt{6 \sqrt{3} + 18} * (\sqrt{3} - 3) - 36 * 3^{3/4} * (\sqrt{3} + 3) \sqrt{-6 \sqrt{3} + 18} + 2 * 3^{3/4} * (-6 \sqrt{3} + 18)^{3/2} + 207 * 3^{1/4} \sqrt{2} \sqrt{6 \sqrt{3} + 18} - 207 * 3^{1/4} \sqrt{-6 \sqrt{3} + 18}) * \arctan(1/3 * 3^{3/4} * (x - 3^{1/4} \sqrt{-1/6 \sqrt{3} + 1/2}) / \sqrt{1/6 \sqrt{3} + 1/2}) + 11/248832 \sqrt{2} * (36 * 3^{3/4} \sqrt{2} * (\sqrt{3} + 3) \sqrt{-6 \sqrt{3} + 18} - 2 * 3^{3/4} \sqrt{2} * (-6 \sqrt{3} + 18)^{3/2} + 2 * 3^{3/4} * (6 \sqrt{3} + 18)^{3/2} + 36 * 3^{3/4} \sqrt{6 \sqrt{3} + 18} * (\sqrt{3} - 3) + 207 * 3^{1/4} \sqrt{2} \sqrt{6 \sqrt{3} + 18} + 207 * 3^{1/4} \sqrt{-6 \sqrt{3} + 18}) * \log(x^2 + 2 * 3^{1/4} * x \sqrt{-1/6 \sqrt{3} + 1/2} + \sqrt{3}) - 11/248832 \sqrt{2} * (36 * 3^{3/4} \sqrt{2} * (\sqrt{3} + 3) \sqrt{-6 \sqrt{3} + 18} - 2 * 3^{3/4} \sqrt{2} * (-6 \sqrt{3} + 18)^{3/2} + 2 * 3^{3/4} * (6 \sqrt{3} + 18)^{3/2} + 36 * 3^{3/4} \sqrt{6 \sqrt{3} + 18} * (\sqrt{3} - 3) + 207 * 3^{1/4} \sqrt{2} \sqrt{6 \sqrt{3} + 18} + 207 * 3^{1/4} \sqrt{-6 \sqrt{3} + 18}) * \log(x^2 - 2 * 3^{1/4} * x \sqrt{-1/6 \sqrt{3} + 1/2} + \sqrt{3}) - 1/192 * (88 * x^7 + 529 * x^5 + 670 * x^3 + 759 * x) / (x^4 + 2 * x^2 + 3)^2$

maple [B] time = 0.03, size = 418, normalized size = 1.70

$$\frac{88x^7 + 529x^5 + 670x^3 + 759x}{(x^4 + 2x^2 + 3)^2} - \frac{11}{192} \int \frac{8x^2 - 23}{x^4 + 2x^2 + 3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x)

[Out] $(-11/24 * x^7 - 529/192 * x^5 - 335/96 * x^3 - 253/64 * x) / (x^4 + 2 * x^2 + 3)^2 - 517/9216 * (-2 + 2 * 3^{1/2})^{1/2} * 3^{1/2} * \ln(x^2 - (-2 + 2 * 3^{1/2})^{1/2} * x + 3^{1/2}) - 341/3072 * (-2 + 2 * 3^{1/2})^{1/2} * \ln(x^2 - (-2 + 2 * 3^{1/2})^{1/2} * x + 3^{1/2}) - 517/4608 / (2 + 2 * 3^{1/2})^{1/2} * (-2 + 2 * 3^{1/2})^{1/2} * 3^{1/2} * \arctan((2 * x - (-2 + 2 * 3^{1/2})^{1/2}) / (2 + 2 * 3^{1/2})^{1/2}) - 341/1536 / (2 + 2 * 3^{1/2})^{1/2} * (-2 + 2 * 3^{1/2})^{1/2} * \arctan((2 * x - (-2 + 2 * 3^{1/2})^{1/2}) / (2 + 2 * 3^{1/2})^{1/2}) + 253/576 / (2 + 2 * 3^{1/2})^{1/2} * 3^{1/2} * \arctan((2 * x - (-2 + 2 * 3^{1/2})^{1/2}) / (2 + 2 * 3^{1/2})^{1/2}) + 517/9216 * (-2 + 2 * 3^{1/2})^{1/2} * 3^{1/2} * \ln(x^2 + (-2 + 2 * 3^{1/2})^{1/2} * x + 3^{1/2}) + 341/3072 * (-2 + 2 * 3^{1/2})^{1/2} * \ln(x^2 + (-2 + 2 * 3^{1/2})^{1/2} * x + 3^{1/2}) - 517/4608 / (2 + 2 * 3^{1/2})^{1/2} * (-2 + 2 * 3^{1/2})^{1/2} * 3^{1/2} * \arctan((2 * x + (-2 + 2 * 3^{1/2})^{1/2}) / (2 + 2 * 3^{1/2})^{1/2}) - 341/1536 / (2 + 2 * 3^{1/2})^{1/2} * (-2 + 2 * 3^{1/2})^{1/2} * \arctan((2 * x + (-2 + 2 * 3^{1/2})^{1/2}) / (2 + 2 * 3^{1/2})^{1/2}) + 253/576 / (2 + 2 * 3^{1/2})^{1/2} * 3^{1/2} * \arctan((2 * x + (-2 + 2 * 3^{1/2})^{1/2}) / (2 + 2 * 3^{1/2})^{1/2})$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{88x^7 + 529x^5 + 670x^3 + 759x}{192(x^8 + 4x^6 + 10x^4 + 12x^2 + 9)} - \frac{11}{192} \int \frac{8x^2 - 23}{x^4 + 2x^2 + 3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="maxima")

[Out] $-1/192 * (88 * x^7 + 529 * x^5 + 670 * x^3 + 759 * x) / (x^8 + 4 * x^6 + 10 * x^4 + 12 * x^2 + 9) - 11/192 * \text{integrate}((8 * x^2 - 23) / (x^4 + 2 * x^2 + 3), x)$

mupad [B] time = 1.01, size = 174, normalized size = 0.71

$$\frac{11x^7 + 529x^5 + 670x^3 + 759x}{192(x^8 + 4x^6 + 10x^4 + 12x^2 + 9)} + \frac{\text{atan}\left(\frac{x\sqrt{10950 - \sqrt{2}2022i} - 448547i}{31850496\sqrt{10616832} + \sqrt{2}10616832i} - \frac{448547\sqrt{2}x\sqrt{10950 - \sqrt{2}2022i}}{63700992\sqrt{10616832} + \sqrt{2}10616832i}\right)\sqrt{10950 - \sqrt{2}2022i} + \text{atan}\left(\frac{x\sqrt{10950 + \sqrt{2}2022i} + 448547i}{31850496\sqrt{10616832} + \sqrt{2}10616832i} + \frac{448547\sqrt{2}x\sqrt{10950 + \sqrt{2}2022i}}{63700992\sqrt{10616832} + \sqrt{2}10616832i}\right)\sqrt{10950 + \sqrt{2}2022i}}{2304}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^3,x)

[Out] $(\text{atan}((x * (10950 - 2^{1/2} * 2022i))^{1/2} * 448547i) / (31850496 * ((2^{1/2} * 10616832i) / 10616832 - 21081709 / 10616832)) - (448547 * 2^{1/2} * x * (10950 - 2^{1/2} * 2022i) / (31850496 * ((2^{1/2} * 10616832i) / 10616832 + 21081709 / 10616832)))) * \sqrt{10950 - \sqrt{2}2022i} + (\text{atan}((x * (10950 + 2^{1/2} * 2022i))^{1/2} * 448547i) / (31850496 * ((2^{1/2} * 10616832i) / 10616832 + 21081709 / 10616832)) + (448547 * 2^{1/2} * x * (10950 + 2^{1/2} * 2022i) / (31850496 * ((2^{1/2} * 10616832i) / 10616832 + 21081709 / 10616832)))) * \sqrt{10950 + \sqrt{2}2022i} - 11/192 * \text{integrate}((8 * x^2 - 23) / (x^4 + 2 * x^2 + 3), x)$

$$2i)^{(1/2)})/(63700992*((2^{(1/2)}*10316581i)/10616832 - 21081709/10616832))* (10950 - 2^{(1/2)}*2022i)^{(1/2)}*11i)/2304 - ((253*x)/64 + (335*x^3)/96 + (529*x^5)/192 + (11*x^7)/24)/(12*x^2 + 10*x^4 + 4*x^6 + x^8 + 9) - (\text{atan}((x*(2^{(1/2)}*2022i + 10950)^{(1/2)}*448547i)/(31850496*((2^{(1/2)}*10316581i)/10616832 + 21081709/10616832)) + (448547*2^{(1/2)}*x*(2^{(1/2)}*2022i + 10950)^{(1/2)}))/(63700992*((2^{(1/2)}*10316581i)/10616832 + 21081709/10616832)))*(2^{(1/2)}*2022i + 10950)^{(1/2)}*11i)/2304$$

sympy [B] time = 1.33, size = 1200, normalized size = 4.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**3,x)

[Out] $(-88*x**7 - 529*x**5 - 670*x**3 - 759*x)/(192*x**8 + 768*x**6 + 1920*x**4 + 2304*x**2 + 1728) - \sqrt{220825/7077888 + 14641*\sqrt{3}/786432}*\log(x**2 + x*(-47*\sqrt{6}*\sqrt{1825 + 1089*\sqrt{3}})*\sqrt{1987425*\sqrt{3} + 3444194})/366993 + 52016*\sqrt{3}*\sqrt{1825 + 1089*\sqrt{3}}/366993 + 188*\sqrt{1825 + 1089*\sqrt{3}}/337 - 24765218375*\sqrt{2}*\sqrt{1987425*\sqrt{3} + 3444194}/134683862049 - 38128468*\sqrt{6}*\sqrt{1987425*\sqrt{3} + 3444194}/371029923 + 90413874433403/134683862049 + 144251139148*\sqrt{3}/371029923 + \sqrt{220825/7077888 + 14641*\sqrt{3}/786432}*\log(x**2 + x*(-188*\sqrt{1825 + 1089*\sqrt{3}})/337 - 52016*\sqrt{3}*\sqrt{1825 + 1089*\sqrt{3}}/366993 + 47*\sqrt{6}*\sqrt{1825 + 1089*\sqrt{3}})*\sqrt{1987425*\sqrt{3} + 3444194}/366993) - 24765218375*\sqrt{2}*\sqrt{1987425*\sqrt{3} + 3444194}/134683862049 - 38128468*\sqrt{6}*\sqrt{1987425*\sqrt{3} + 3444194}/371029923 + 90413874433403/134683862049 + 144251139148*\sqrt{3}/371029923 + 2*\sqrt{-121*\sqrt{2}*\sqrt{1987425*\sqrt{3} + 3444194}}/3538944 + 220825/7077888 + 14641*\sqrt{3}/262144)*\text{atan}(733986*\sqrt{3}*x/(15502*\sqrt{-2*\sqrt{2}*\sqrt{1987425*\sqrt{3} + 3444194}} + 1825 + 3267*\sqrt{3}) + 47*\sqrt{2}*\sqrt{1987425*\sqrt{3} + 3444194}*\sqrt{-2*\sqrt{2}*\sqrt{1987425*\sqrt{3} + 3444194}} + 1825 + 3267*\sqrt{3})) - 204732*\sqrt{3}*\sqrt{1825 + 1089*\sqrt{3}}/(15502*\sqrt{-2*\sqrt{2}*\sqrt{1987425*\sqrt{3} + 3444194}} + 1825 + 3267*\sqrt{3}) + 47*\sqrt{2}*\sqrt{1987425*\sqrt{3} + 3444194}*\sqrt{-2*\sqrt{2}*\sqrt{1987425*\sqrt{3} + 3444194}} + 1825 + 3267*\sqrt{3})) - 156048*\sqrt{1825 + 1089*\sqrt{3}}/(15502*\sqrt{-2*\sqrt{2}*\sqrt{1987425*\sqrt{3} + 3444194}} + 1825 + 3267*\sqrt{3}) + 47*\sqrt{2}*\sqrt{1987425*\sqrt{3} + 3444194}*\sqrt{-2*\sqrt{2}*\sqrt{1987425*\sqrt{3} + 3444194}} + 1825 + 3267*\sqrt{3})) + 141*\sqrt{2}*\sqrt{1825 + 1089*\sqrt{3}}*\sqrt{1987425*\sqrt{3} + 3444194}/(15502*\sqrt{-2*\sqrt{2}*\sqrt{1987425*\sqrt{3} + 3444194}} + 1825 + 3267*\sqrt{3}) + 47*\sqrt{2}*\sqrt{1987425*\sqrt{3} + 3444194}*\sqrt{-2*\sqrt{2}*\sqrt{1987425*\sqrt{3} + 3444194}} + 1825 + 3267*\sqrt{3})) + 2*\sqrt{-121*\sqrt{2}*\sqrt{1987425*\sqrt{3} + 3444194}}/3538944 + 220825/7077888 + 14641*\sqrt{3}/262144)*\text{atan}(733986*\sqrt{3}*x/(15502*\sqrt{-2*\sqrt{2}*\sqrt{1987425*\sqrt{3} + 3444194}} + 1825 + 3267*\sqrt{3}) + 47*\sqrt{2}*\sqrt{1987425*\sqrt{3} + 3444194}*\sqrt{-2*\sqrt{2}*\sqrt{1987425*\sqrt{3} + 3444194}} + 1825 + 3267*\sqrt{3})) - 141*\sqrt{2}*\sqrt{1825 + 1089*\sqrt{3}}*\sqrt{1987425*\sqrt{3} + 3444194}/(15502*\sqrt{-2*\sqrt{2}*\sqrt{1987425*\sqrt{3} + 3444194}} + 1825 + 3267*\sqrt{3}) + 47*\sqrt{2}*\sqrt{1987425*\sqrt{3} + 3444194}*\sqrt{-2*\sqrt{2}*\sqrt{1987425*\sqrt{3} + 3444194}} + 1825 + 3267*\sqrt{3})) + 156048*\sqrt{1825 + 1089*\sqrt{3}}/(15502*\sqrt{-2*\sqrt{2}*\sqrt{1987425*\sqrt{3} + 3444194}} + 1825 + 3267*\sqrt{3}) + 47*\sqrt{2}*\sqrt{1987425*\sqrt{3} + 3444194}*\sqrt{-2*\sqrt{2}*\sqrt{1987425*\sqrt{3} + 3444194}} + 1825 + 3267*\sqrt{3})) + 204732*\sqrt{3}*\sqrt{1825 + 1089*\sqrt{3}}/(15502*\sqrt{-2*\sqrt{2}*\sqrt{1987425*\sqrt{3} + 3444194}} + 1825 + 3267*\sqrt{3}) + 47*\sqrt{2}*\sqrt{1987425*\sqrt{3} + 3444194}*\sqrt{-2*\sqrt{2}*\sqrt{1987425*\sqrt{3} + 3444194}} + 1825 + 3267*\sqrt{3}))$

$$3.120 \quad \int \frac{4+x^2+3x^4+5x^6}{(3+2x^2+x^4)^3} dx$$

Optimal. Leaf size=248

$$\frac{1}{512} \sqrt{\frac{1}{3} (1291 + 1019\sqrt{3})} \log\left(x^2 - \sqrt{2(\sqrt{3}-1)}x + \sqrt{3}\right) - \frac{1}{512} \sqrt{\frac{1}{3} (1291 + 1019\sqrt{3})} \log\left(x^2 + \sqrt{2(\sqrt{3}-1)}x + \sqrt{3}\right)$$

Rubi [A] time = 0.25, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 28, number of rules / integrand size = 0.250, Rules used = {1678, 1178, 1169, 634, 618, 204, 628}

$$\frac{25x(1-x^2)}{48(x^4+2x^2+3)^3} + \frac{x(51x^2+64)}{192(x^4+2x^2+3)^2} + \frac{1}{512} \sqrt{\frac{1}{3} (1291 + 1019\sqrt{3})} \log\left(x^2 - \sqrt{2(\sqrt{3}-1)}x + \sqrt{3}\right) - \frac{1}{512} \sqrt{\frac{1}{3} (1291 + 1019\sqrt{3})} \log\left(x^2 + \sqrt{2(\sqrt{3}-1)}x + \sqrt{3}\right) - \frac{1}{256} \sqrt{\frac{1}{3} (1019\sqrt{3} - 1291)} \tan^{-1}\left(\frac{\sqrt{2(\sqrt{3}-1)} - 2x}{\sqrt{2(1+\sqrt{3})}}\right) + \frac{1}{256} \sqrt{\frac{1}{3} (1019\sqrt{3} - 1291)} \tan^{-1}\left(\frac{2x + \sqrt{2(\sqrt{3}-1)}}{\sqrt{2(1+\sqrt{3})}}\right)$$

Antiderivative was successfully verified.

[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(3 + 2*x^2 + x^4)^3, x]

[Out] (25*x*(1 - x^2))/(48*(3 + 2*x^2 + x^4)^2) + (x*(64 + 51*x^2))/(192*(3 + 2*x^2 + x^4)) - (Sqrt[(-1291 + 1019*Sqrt[3])/3]*ArcTan[(Sqrt[2*(-1 + Sqrt[3]]) - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/256 + (Sqrt[(-1291 + 1019*Sqrt[3])/3]*ArcTan[(Sqrt[2*(-1 + Sqrt[3]]) + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/256 + (Sqrt[(1291 + 1019*Sqrt[3])/3]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/512 - (Sqrt[(1291 + 1019*Sqrt[3])/3]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/512

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1169

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]

Rule 1178

```
Int[((d_) + (e_)*(x_)^2)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol]
:= Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := With[{d = Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{4 + x^2 + 3x^4 + 5x^6}{(3 + 2x^2 + x^4)^3} dx &= \frac{25x(1 - x^2)}{48(3 + 2x^2 + x^4)^2} + \frac{1}{96} \int \frac{78 + 230x^2}{(3 + 2x^2 + x^4)^2} dx \\ &= \frac{25x(1 - x^2)}{48(3 + 2x^2 + x^4)^2} + \frac{x(64 + 51x^2)}{192(3 + 2x^2 + x^4)} + \frac{\int \frac{-288 + 1224x^2}{3 + 2x^2 + x^4} dx}{4608} \\ &= \frac{25x(1 - x^2)}{48(3 + 2x^2 + x^4)^2} + \frac{x(64 + 51x^2)}{192(3 + 2x^2 + x^4)} + \frac{\int \frac{-288\sqrt{2(-1 + \sqrt{3})} - (-288 - 1224\sqrt{3})x}{\sqrt{3} - \sqrt{2(-1 + \sqrt{3})}x + x^2} dx}{9216\sqrt{6(-1 + \sqrt{3})}} + \dots \\ &= \frac{25x(1 - x^2)}{48(3 + 2x^2 + x^4)^2} + \frac{x(64 + 51x^2)}{192(3 + 2x^2 + x^4)} + \frac{1}{768}(51 - 4\sqrt{3}) \int \frac{1}{\sqrt{3} - \sqrt{2(-1 + \sqrt{3})}} \\ &= \frac{25x(1 - x^2)}{48(3 + 2x^2 + x^4)^2} + \frac{x(64 + 51x^2)}{192(3 + 2x^2 + x^4)} + \frac{1}{512} \sqrt{\frac{1}{3}(1291 + 1019\sqrt{3})} \log\left(\sqrt{3} - \sqrt{\dots}\right) \\ &= \frac{25x(1 - x^2)}{48(3 + 2x^2 + x^4)^2} + \frac{x(64 + 51x^2)}{192(3 + 2x^2 + x^4)} - \frac{1}{256} \sqrt{\frac{1}{3}(-1291 + 1019\sqrt{3})} \tan^{-1}\left(\frac{\sqrt{2}(\dots)}{\dots}\right) \end{aligned}$$

Mathematica [C] time = 0.29, size = 129, normalized size = 0.52

$$\frac{1}{768} \left(\frac{4x(51x^6 + 166x^4 + 181x^2 + 292)}{(x^4 + 2x^2 + 3)^2} + \frac{3(34 + 21i\sqrt{2}) \tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}} + \frac{3(34 - 21i\sqrt{2}) \tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(3 + 2*x^2 + x^4)^3, x]


```
[Out] ((4*x*(292 + 181*x^2 + 166*x^4 + 51*x^6))/(3 + 2*x^2 + x^4)^2 + (3*(34 + (2
1*I)*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/Sqrt[1 - I*Sqrt[2]] + (3*(34 -
(21*I)*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/Sqrt[1 + I*Sqrt[2]])/768
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4 + x^2 + 3x^4 + 5x^6}{(3 + 2x^2 + x^4)^3} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(4 + x^2 + 3*x^4 + 5*x^6)/(3 + 2*x^2 + x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[(4 + x^2 + 3*x^4 + 5*x^6)/(3 + 2*x^2 + x^4)^3, x]
```

fricas [B] time = 1.34, size = 576, normalized size = 2.32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="fricas")
```

```
[Out] 1/7991829504*(2122829712*x^7 + 6909602592*x^5 - 3404*3115083^(1/4)*sqrt(6)*
sqrt(3)*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqrt(-1315529*sqrt(3) +
3115083)*arctan(1/41378565634793586*3115083^(3/4)*sqrt(2601507)*sqrt(6)*sq
rt(3115083^(1/4)*sqrt(6)*(17*sqrt(3)*x + 4*x)*sqrt(-1315529*sqrt(3) + 31150
83) + 2601507*x^2 + 2601507*sqrt(3))*(4*sqrt(3)*sqrt(2) + 51*sqrt(2))*sqrt(
-1315529*sqrt(3) + 3115083) - 1/15905613798*3115083^(3/4)*sqrt(6)*(4*sqrt(3
)*sqrt(2)*x + 51*sqrt(2)*x)*sqrt(-1315529*sqrt(3) + 3115083) - 1/2*sqrt(3)*
sqrt(2) + 1/2*sqrt(2)) - 3404*3115083^(1/4)*sqrt(6)*sqrt(3)*sqrt(2)*(x^8 +
4*x^6 + 10*x^4 + 12*x^2 + 9)*sqrt(-1315529*sqrt(3) + 3115083)*arctan(1/4137
8565634793586*3115083^(3/4)*sqrt(2601507)*sqrt(6)*sqrt(-3115083^(1/4)*sqrt(
6)*(17*sqrt(3)*x + 4*x)*sqrt(-1315529*sqrt(3) + 3115083) + 2601507*x^2 + 26
01507*sqrt(3))*(4*sqrt(3)*sqrt(2) + 51*sqrt(2))*sqrt(-1315529*sqrt(3) + 311
5083) - 1/15905613798*3115083^(3/4)*sqrt(6)*(4*sqrt(3)*sqrt(2)*x + 51*sqrt(
2)*x)*sqrt(-1315529*sqrt(3) + 3115083) + 1/2*sqrt(3)*sqrt(2) - 1/2*sqrt(2))
- 3115083^(1/4)*sqrt(6)*(3057*x^8 + 12228*x^6 + 30570*x^4 + 36684*x^2 + 12
91*sqrt(3)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9) + 27513)*sqrt(-1315529*sqrt(
3) + 3115083)*log(3115083^(1/4)*sqrt(6)*(17*sqrt(3)*x + 4*x)*sqrt(-1315529*
sqrt(3) + 3115083) + 2601507*x^2 + 2601507*sqrt(3)) + 3115083^(1/4)*sqrt(6)
*(3057*x^8 + 12228*x^6 + 30570*x^4 + 36684*x^2 + 1291*sqrt(3)*(x^8 + 4*x^6
+ 10*x^4 + 12*x^2 + 9) + 27513)*sqrt(-1315529*sqrt(3) + 3115083)*log(-31150
83^(1/4)*sqrt(6)*(17*sqrt(3)*x + 4*x)*sqrt(-1315529*sqrt(3) + 3115083) + 26
01507*x^2 + 2601507*sqrt(3)) + 7533964272*x^3 + 12154240704*x)/(x^8 + 4*x^6
+ 10*x^4 + 12*x^2 + 9)
```

giac [B] time = 2.62, size = 577, normalized size = 2.33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="giac")
```

```
[Out] -1/165888*sqrt(2)*(17*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 306*3^(3/4)*
sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 306*3^(3/4)*(sqrt(3) + 3)*sqrt
(-6*sqrt(3) + 18) + 17*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 144*3^(1/4)*sqrt(2
)*sqrt(6*sqrt(3) + 18) - 144*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3
/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) - 1/165
888*sqrt(2)*(17*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 306*3^(3/4)*sqrt(2
```

```
) * sqrt(6 * sqrt(3) + 18) * (sqrt(3) - 3) - 306 * 3^(3/4) * (sqrt(3) + 3) * sqrt(-6 * sqrt(3) + 18) + 17 * 3^(3/4) * (-6 * sqrt(3) + 18)^(3/2) + 144 * 3^(1/4) * sqrt(2) * sqrt(6 * sqrt(3) + 18) - 144 * 3^(1/4) * sqrt(-6 * sqrt(3) + 18) * arctan(1/3 * 3^(3/4) * (x - 3^(1/4) * sqrt(-1/6 * sqrt(3) + 1/2)) / sqrt(1/6 * sqrt(3) + 1/2)) - 1/331776 * sqrt(2) * (306 * 3^(3/4) * sqrt(2) * (sqrt(3) + 3) * sqrt(-6 * sqrt(3) + 18) - 17 * 3^(3/4) * sqrt(2) * (-6 * sqrt(3) + 18)^(3/2) + 17 * 3^(3/4) * (6 * sqrt(3) + 18)^(3/2) + 306 * 3^(3/4) * sqrt(6 * sqrt(3) + 18) * (sqrt(3) - 3) + 144 * 3^(1/4) * sqrt(2) * sqrt(-6 * sqrt(3) + 18) + 144 * 3^(1/4) * sqrt(6 * sqrt(3) + 18)) * log(x^2 + 2 * 3^(1/4) * x * sqrt(-1/6 * sqrt(3) + 1/2) + sqrt(3)) + 1/331776 * sqrt(2) * (306 * 3^(3/4) * sqrt(2) * (sqrt(3) + 3) * sqrt(-6 * sqrt(3) + 18) - 17 * 3^(3/4) * sqrt(2) * (-6 * sqrt(3) + 18)^(3/2) + 17 * 3^(3/4) * (6 * sqrt(3) + 18)^(3/2) + 306 * 3^(3/4) * sqrt(6 * sqrt(3) + 18) * (sqrt(3) - 3) + 144 * 3^(1/4) * sqrt(2) * sqrt(-6 * sqrt(3) + 18) + 144 * 3^(1/4) * sqrt(6 * sqrt(3) + 18)) * log(x^2 - 2 * 3^(1/4) * x * sqrt(-1/6 * sqrt(3) + 1/2) + sqrt(3)) + 1/192 * (51 * x^7 + 166 * x^5 + 181 * x^3 + 292 * x) / (x^4 + 2 * x^2 + 3)^2
```

maple [B] time = 0.03, size = 418, normalized size = 1.69

$$\frac{51(-2+2\sqrt{3})\sqrt{3}\operatorname{atan}\left(\frac{x-\sqrt[4]{3}}{3\sqrt{3}}\right) - 2(-2+2\sqrt{3})\operatorname{atan}\left(\frac{x-\sqrt[4]{3}}{3\sqrt{3}}\right) - \sqrt{3}\operatorname{atan}\left(\frac{x-\sqrt[4]{3}}{3\sqrt{3}}\right) - 51(-2+2\sqrt{3})\sqrt{3}\operatorname{atan}\left(\frac{x-\sqrt[4]{3}}{3\sqrt{3}}\right) - 2(-2+2\sqrt{3})\operatorname{atan}\left(\frac{x-\sqrt[4]{3}}{3\sqrt{3}}\right) - \sqrt{3}\operatorname{atan}\left(\frac{x-\sqrt[4]{3}}{3\sqrt{3}}\right) - 51\sqrt{-2+2\sqrt{3}}\sqrt{3}\ln\left(\frac{x-\sqrt{-2+2\sqrt{3}}+i}{x-\sqrt{-2+2\sqrt{3}}-i}\right) - 2\sqrt{-2+2\sqrt{3}}\ln\left(\frac{x-\sqrt{-2+2\sqrt{3}}+i}{x-\sqrt{-2+2\sqrt{3}}-i}\right) - \sqrt{-2+2\sqrt{3}}\ln\left(\frac{x-\sqrt{-2+2\sqrt{3}}+i}{x-\sqrt{-2+2\sqrt{3}}-i}\right) - 2\sqrt{-2+2\sqrt{3}}\ln\left(\frac{x-\sqrt{-2+2\sqrt{3}}+i}{x-\sqrt{-2+2\sqrt{3}}-i}\right) - \frac{51x^7+166x^5+181x^3+292x}{(x^4+2x^2+3)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x)
[Out] (17/64*x^7+83/96*x^5+181/192*x^3+73/48*x)/(x^4+2*x^2+3)^2+55/3072*(-2+2*3^(1/2))^(1/2)*3^(1/2)*ln(x^2-(-2+2*3^(1/2))^(1/2)*x+3^(1/2))+21/1024*(-2+2*3^(1/2))^(1/2)*ln(x^2-(-2+2*3^(1/2))^(1/2)*x+3^(1/2))+55/1536/(2+2*3^(1/2))^(1/2)*(-2+2*3^(1/2))*3^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))+21/512/(2+2*3^(1/2))^(1/2)*(-2+2*3^(1/2))*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))-1/48/(2+2*3^(1/2))^(1/2)*3^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))-55/3072*(-2+2*3^(1/2))^(1/2)*3^(1/2)*ln(x^2+(-2+2*3^(1/2))^(1/2)*x+3^(1/2))-21/1024*(-2+2*3^(1/2))^(1/2)*ln(x^2+(-2+2*3^(1/2))^(1/2)*x+3^(1/2))+55/1536/(2+2*3^(1/2))^(1/2)*(-2+2*3^(1/2))*3^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))+21/512/(2+2*3^(1/2))^(1/2)*(-2+2*3^(1/2))*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))-1/48/(2+2*3^(1/2))^(1/2)*3^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{51x^7 + 166x^5 + 181x^3 + 292x}{192(x^8 + 4x^6 + 10x^4 + 12x^2 + 9)} + \frac{1}{64} \int \frac{17x^2 - 4}{x^4 + 2x^2 + 3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="maxima")
[Out] 1/192*(51*x^7 + 166*x^5 + 181*x^3 + 292*x)/(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9) + 1/64*integrate((17*x^2 - 4)/(x^4 + 2*x^2 + 3), x)
```

mupad [B] time = 1.01, size = 173, normalized size = 0.70

$$\frac{\frac{17x^7}{64} + \frac{83x^5}{96} + \frac{181x^3}{192} + \frac{73x}{48}}{x^8 + 4x^6 + 10x^4 + 12x^2 + 9} + \frac{\operatorname{atan}\left(\frac{x\sqrt{7746-\sqrt{2}5106i}851i}{1179648\left(\frac{46805}{393216} + \frac{\sqrt{2}851i}{98304}\right)} + \frac{851\sqrt{2}x\sqrt{7746-\sqrt{2}5106i}}{2359296\left(\frac{46805}{393216} + \frac{\sqrt{2}851i}{98304}\right)}\right)\sqrt{7746-\sqrt{2}5106i} \operatorname{li} + \operatorname{atan}\left(\frac{x\sqrt{7746+\sqrt{2}5106i}851i}{1179648\left(\frac{46805}{393216} + \frac{\sqrt{2}851i}{98304}\right)} - \frac{851\sqrt{2}x\sqrt{7746+\sqrt{2}5106i}}{2359296\left(\frac{46805}{393216} + \frac{\sqrt{2}851i}{98304}\right)}\right)\sqrt{7746+\sqrt{2}5106i} \operatorname{li}}{768}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(2*x^2 + x^4 + 3)^3,x)
[Out] ((73*x)/48 + (181*x^3)/192 + (83*x^5)/96 + (17*x^7)/64)/(12*x^2 + 10*x^4 + 4*x^6 + x^8 + 9) + (atan(x*(7746 - 2^(1/2)*5106i)^(1/2)*851i)/(1179648*((2^(1/2)*851i)/98304 + 46805/393216)) + (851*2^(1/2)*x*(7746 - 2^(1/2)*5106i)
```

$$\begin{aligned} & \sqrt[3]{(2^{1/2})} / (2359296 * ((2^{1/2}) * 851i) / 98304 + 46805 / 393216)) * (7746 - 2^{1/2} * 5106i)^{1/2} * i / 768 - (\operatorname{atan}((x * (2^{1/2}) * 5106i + 7746)^{1/2} * 851i) / (1179648 * ((2^{1/2}) * 851i) / 98304 - 46805 / 393216))) - (851 * 2^{1/2} * x * (2^{1/2}) * 5106i + 7746)^{1/2} / (2359296 * ((2^{1/2}) * 851i) / 98304 - 46805 / 393216)) * (2^{1/2} * 5106i + 7746)^{1/2} * i / 768 \end{aligned}$$

sympy [B] time = 1.34, size = 1195, normalized size = 4.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**3,x)

[Out]
$$\begin{aligned} & (51*x**7 + 166*x**5 + 181*x**3 + 292*x) / (192*x**8 + 768*x**6 + 1920*x**4 + 2304*x**2 + 1728) - \sqrt{1291/786432 + 1019*\sqrt{3}} / 786432 * \log(x**2 + x * (-55*\sqrt{6}) * \sqrt{1291 + 1019*\sqrt{3}}) * \sqrt{1315529*\sqrt{3} + 2390882} / 867169 \\ & + 49606*\sqrt{3} * \sqrt{1291 + 1019*\sqrt{3}} / 867169 + 220*\sqrt{1291 + 1019*\sqrt{3}} / 851 - 26628761029*\sqrt{2} * \sqrt{1315529*\sqrt{3} + 2390882} / 751982074561 \\ & - 40176070*\sqrt{6} * \sqrt{1315529*\sqrt{3} + 2390882} / 2213882457 + 76094994709709 / 751982074561 + 133967471914*\sqrt{3} / 2213882457 + \sqrt{1291/786432 + 1019*\sqrt{3}} / 786432 * \log(x**2 + x * (-220*\sqrt{1291 + 1019*\sqrt{3}}) / 851 - 49606*\sqrt{3} * \sqrt{1291 + 1019*\sqrt{3}}) / 867169 + 55*\sqrt{6} * \sqrt{1291 + 1019*\sqrt{3}} * \sqrt{1315529*\sqrt{3} + 2390882} / 867169 \\ & - 26628761029*\sqrt{2} * \sqrt{1315529*\sqrt{3} + 2390882} / 751982074561 - 40176070*\sqrt{6} * \sqrt{1315529*\sqrt{3} + 2390882} / 2213882457 + 76094994709709 / 751982074561 + 133967471914*\sqrt{3} / 2213882457 + 2*\sqrt{-\sqrt{2}} * \sqrt{1315529*\sqrt{3} + 2390882} / 393216 + 1291/786432 + 1019*\sqrt{3} / 262144 * \operatorname{atan}(1734338*\sqrt{3} * x / (-6808*\sqrt{-2*\sqrt{2}} * \sqrt{1315529*\sqrt{3} + 2390882} + 1291 + 3057*\sqrt{3}) + 55*\sqrt{2} * \sqrt{1315529*\sqrt{3} + 2390882} * \sqrt{-2*\sqrt{2}} * \sqrt{1315529*\sqrt{3} + 2390882} + 1291 + 3057*\sqrt{3})) - 224180*\sqrt{3} * \sqrt{1291 + 1019*\sqrt{3}} / (-6808*\sqrt{-2*\sqrt{2}} * \sqrt{1315529*\sqrt{3} + 2390882} + 1291 + 3057*\sqrt{3}) + 55*\sqrt{2} * \sqrt{1315529*\sqrt{3} + 2390882} * \sqrt{-2*\sqrt{2}} * \sqrt{1315529*\sqrt{3} + 2390882} + 1291 + 3057*\sqrt{3})) - 148818*\sqrt{1291 + 1019*\sqrt{3}} / (-6808*\sqrt{-2*\sqrt{2}} * \sqrt{1315529*\sqrt{3} + 2390882} + 1291 + 3057*\sqrt{3}) + 55*\sqrt{2} * \sqrt{1315529*\sqrt{3} + 2390882} * \sqrt{-2*\sqrt{2}} * \sqrt{1315529*\sqrt{3} + 2390882} + 1291 + 3057*\sqrt{3})) + 165*\sqrt{2} * \sqrt{1291 + 1019*\sqrt{3}} * \sqrt{1315529*\sqrt{3} + 2390882} / (-6808*\sqrt{-2*\sqrt{2}} * \sqrt{1315529*\sqrt{3} + 2390882} + 1291 + 3057*\sqrt{3}) + 55*\sqrt{2} * \sqrt{1315529*\sqrt{3} + 2390882} * \sqrt{-2*\sqrt{2}} * \sqrt{1315529*\sqrt{3} + 2390882} + 1291 + 3057*\sqrt{3})) + 2*\sqrt{-\sqrt{2}} * \sqrt{1315529*\sqrt{3} + 2390882} / 393216 + 1291/786432 + 1019*\sqrt{3} / 262144 * \operatorname{atan}(1734338*\sqrt{3} * x / (-6808*\sqrt{-2*\sqrt{2}} * \sqrt{1315529*\sqrt{3} + 2390882} + 1291 + 3057*\sqrt{3}) + 55*\sqrt{2} * \sqrt{1315529*\sqrt{3} + 2390882} * \sqrt{-2*\sqrt{2}} * \sqrt{1315529*\sqrt{3} + 2390882} + 1291 + 3057*\sqrt{3})) - 165*\sqrt{2} * \sqrt{1291 + 1019*\sqrt{3}} * \sqrt{1315529*\sqrt{3} + 2390882} / (-6808*\sqrt{-2*\sqrt{2}} * \sqrt{1315529*\sqrt{3} + 2390882} + 1291 + 3057*\sqrt{3}) + 55*\sqrt{2} * \sqrt{1315529*\sqrt{3} + 2390882} * \sqrt{-2*\sqrt{2}} * \sqrt{1315529*\sqrt{3} + 2390882} + 1291 + 3057*\sqrt{3})) + 148818*\sqrt{1291 + 1019*\sqrt{3}} / (-6808*\sqrt{-2*\sqrt{2}} * \sqrt{1315529*\sqrt{3} + 2390882} + 1291 + 3057*\sqrt{3}) + 55*\sqrt{2} * \sqrt{1315529*\sqrt{3} + 2390882} * \sqrt{-2*\sqrt{2}} * \sqrt{1315529*\sqrt{3} + 2390882} + 1291 + 3057*\sqrt{3})) + 224180*\sqrt{3} * \sqrt{1291 + 1019*\sqrt{3}} / (-6808*\sqrt{-2*\sqrt{2}} * \sqrt{1315529*\sqrt{3} + 2390882} + 1291 + 3057*\sqrt{3}) + 55*\sqrt{2} * \sqrt{1315529*\sqrt{3} + 2390882} * \sqrt{-2*\sqrt{2}} * \sqrt{1315529*\sqrt{3} + 2390882} + 1291 + 3057*\sqrt{3})) \end{aligned}$$

$$3.121 \quad \int \frac{4+x^2+3x^4+5x^6}{x^2(3+2x^2+x^4)^3} dx$$

Optimal. Leaf size=253

$$\frac{\sqrt{\frac{1}{3}(55161\sqrt{3}-59711)} \log\left(x^2 - \sqrt{2(\sqrt{3}-1)}x + \sqrt{3}\right)}{4608} + \frac{\sqrt{\frac{1}{3}(55161\sqrt{3}-59711)} \log\left(x^2 + \sqrt{2(\sqrt{3}-1)}x + \sqrt{3}\right)}{4608}$$

Rubi [A] time = 0.34, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {1669, 1664, 1169, 634, 618, 204, 628}

$$\frac{25x(x^2+5)}{144(x^4+2x^2+3)^2} - \frac{x(242x^2+325)}{1728(x^4+2x^2+3)} - \frac{\sqrt{\frac{1}{3}(55161\sqrt{3}-59711)} \log\left(x^2 - \sqrt{2(\sqrt{3}-1)}x + \sqrt{3}\right)}{4608} + \frac{\sqrt{\frac{1}{3}(55161\sqrt{3}-59711)} \log\left(x^2 + \sqrt{2(\sqrt{3}-1)}x + \sqrt{3}\right)}{4608} - \frac{4}{27x} + \frac{\sqrt{\frac{1}{3}(59711+55161\sqrt{3})} \tan^{-1}\left(\frac{\sqrt{2(\sqrt{3}-1)-2x}}{\sqrt{2(1+\sqrt{3})}}\right)}{2304} - \frac{\sqrt{\frac{1}{3}(59711+55161\sqrt{3})} \tan^{-1}\left(\frac{2x+\sqrt{2(\sqrt{3}-1)}}{\sqrt{2(1+\sqrt{3})}}\right)}{2304}$$

Antiderivative was successfully verified.

[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(3 + 2*x^2 + x^4)^3), x]

[Out] -4/(27*x) - (25*x*(5 + x^2))/(144*(3 + 2*x^2 + x^4)^2) - (x*(325 + 242*x^2))/(1728*(3 + 2*x^2 + x^4)) + (Sqrt[(59711 + 55161*Sqrt[3])/3]*ArcTan[(Sqrt[2*(-1 + Sqrt[3]]) - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/2304 - (Sqrt[(59711 + 55161*Sqrt[3])/3]*ArcTan[(Sqrt[2*(-1 + Sqrt[3]]) + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/2304 - (Sqrt[(-59711 + 55161*Sqrt[3])/3]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]]*x + x^2)/4608 + (Sqrt[(-59711 + 55161*Sqrt[3])/3]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]]*x + x^2)/4608

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1169

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ

$[b^2 - 4ac, 0] \&\& \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \&\& \text{NegQ}[b^2 - 4ac]$

Rule 1664

$\text{Int}[(Pq_)*((d_)*(x_))^{(m_)}*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^{(p_)}, x_Symbol] :> \text{Int}[\text{ExpandIntegrand}[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;$
 $\text{FreeQ}\{a, b, c, d, m\}, x] \&\& \text{PolyQ}[Pq, x^2] \&\& \text{IGtQ}[p, -2]$

Rule 1669

$\text{Int}[(Pq_)*(x_)^{(m_)}*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^{(p_)}, x_Symbol] :>$
 $\text{With}\{d = \text{Coeff}[\text{PolynomialRemainder}[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],$
 $e = \text{Coeff}[\text{PolynomialRemainder}[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]\}, \text{Simp}[($
 $x*(a + b*x^2 + c*x^4)^{(p + 1)}*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)$
 $)/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + \text{Dist}[1/(2*a*(p + 1)*(b^2 - 4*a*c)), \text{I}$
 $\text{nt}[x^m*(a + b*x^2 + c*x^4)^{(p + 1)}*\text{ExpandToSum}[(2*a*(p + 1)*(b^2 - 4*a*c)*\text{P}$
 $\text{olynomialQuotient}[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2$
 $*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^{(2 - m)}, x], x]$
 $]; \text{FreeQ}\{a, b, c\}, x] \&\& \text{PolyQ}[Pq, x^2] \&\& \text{GtQ}[\text{Expon}[Pq, x^2], 1] \&\&$
 $\text{NeQ}[b^2 - 4ac, 0] \&\& \text{LtQ}[p, -1] \&\& \text{ILtQ}[m/2, 0]$

Rubi steps

$$\int \frac{4 + x^2 + 3x^4 + 5x^6}{x^2(3 + 2x^2 + x^4)^3} dx = -\frac{25x(5 + x^2)}{144(3 + 2x^2 + x^4)^2} + \frac{1}{96} \int \frac{128 + 30x^2 - \frac{250x^4}{3}}{x^2(3 + 2x^2 + x^4)^2} dx$$

$$= -\frac{25x(5 + x^2)}{144(3 + 2x^2 + x^4)^2} - \frac{x(325 + 242x^2)}{1728(3 + 2x^2 + x^4)} + \frac{\int \frac{2048 - \frac{56x^2}{3} - \frac{1936x^4}{3}}{x^2(3 + 2x^2 + x^4)} dx}{4608}$$

$$= -\frac{25x(5 + x^2)}{144(3 + 2x^2 + x^4)^2} - \frac{x(325 + 242x^2)}{1728(3 + 2x^2 + x^4)} + \frac{\int \left(\frac{2048}{3x^2} - \frac{8(173 + 166x^2)}{3 + 2x^2 + x^4} \right) dx}{4608}$$

$$= -\frac{4}{27x} - \frac{25x(5 + x^2)}{144(3 + 2x^2 + x^4)^2} - \frac{x(325 + 242x^2)}{1728(3 + 2x^2 + x^4)} - \frac{1}{576} \int \frac{173 + 166x^2}{3 + 2x^2 + x^4} dx$$

$$= -\frac{4}{27x} - \frac{25x(5 + x^2)}{144(3 + 2x^2 + x^4)^2} - \frac{x(325 + 242x^2)}{1728(3 + 2x^2 + x^4)} - \frac{\int \frac{173\sqrt{2(-1 + \sqrt{3})} - (173 - 166\sqrt{3})}{\sqrt{3} - \sqrt{2(-1 + \sqrt{3})}x + x^2} dx}{1152\sqrt{6(-1 + \sqrt{3})}}$$

$$= -\frac{4}{27x} - \frac{25x(5 + x^2)}{144(3 + 2x^2 + x^4)^2} - \frac{x(325 + 242x^2)}{1728(3 + 2x^2 + x^4)} - \frac{\sqrt{\frac{1}{3}(-59711 + 55161\sqrt{3})}}{4608}$$

$$= -\frac{4}{27x} - \frac{25x(5 + x^2)}{144(3 + 2x^2 + x^4)^2} - \frac{x(325 + 242x^2)}{1728(3 + 2x^2 + x^4)} - \frac{\sqrt{\frac{1}{3}(-59711 + 55161\sqrt{3})}}{4608}$$

$$= -\frac{4}{27x} - \frac{25x(5 + x^2)}{144(3 + 2x^2 + x^4)^2} - \frac{x(325 + 242x^2)}{1728(3 + 2x^2 + x^4)} + \frac{\sqrt{\frac{1}{3}(59711 + 55161\sqrt{3})}}{2304}$$

Mathematica [C] time = 0.37, size = 140, normalized size = 0.55

$$\frac{\frac{12(166x^8+611x^6+1412x^4+1849x^2+768)}{x(x^4+2x^2+3)^2} + \frac{3i(7\sqrt{2}+332i)\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}} - \frac{3i(7\sqrt{2}-332i)\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}}}{6912}$$

Antiderivative was successfully verified.

[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(3 + 2*x^2 + x^4)^3), x]

[Out] ((-12*(768 + 1849*x^2 + 1412*x^4 + 611*x^6 + 166*x^8))/(x*(3 + 2*x^2 + x^4)^2) + ((3*I)*(332*I + 7*sqrt[2])*ArcTan[x/Sqrt[1 - I*sqrt[2]]])/sqrt[1 - I*sqrt[2]] - ((3*I)*(-332*I + 7*sqrt[2])*ArcTan[x/Sqrt[1 + I*sqrt[2]]])/sqrt[1 + I*sqrt[2]])/6912

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4 + x^2 + 3x^4 + 5x^6}{x^2(3 + 2x^2 + x^4)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(3 + 2*x^2 + x^4)^3), x]

[Out] IntegrateAlgebraic[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(3 + 2*x^2 + x^4)^3), x]

fricas [B] time = 1.39, size = 630, normalized size = 2.49

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+2*x^2+3)^3,x, algorithm="fricas")

[Out] -1/2978955242496*(858518351136*x^8 + 3159968147856*x^6 + 210956*1391283^(1/4)*sqrt(681)*sqrt(6)*sqrt(3)*sqrt(2)*(x^9 + 4*x^7 + 10*x^5 + 12*x^3 + 9*x)*sqrt(59711*sqrt(3) + 165483)*arctan(1/15811665652336538898*sqrt(11971753)*1391283^(3/4)*sqrt(681)*sqrt(6)*sqrt(1391283^(1/4)*sqrt(681)*sqrt(6)*(166*sqrt(3)*x - 173*x)*sqrt(59711*sqrt(3) + 165483) + 107745777*x^2 + 107745777*sqrt(3))*(173*sqrt(3)*sqrt(2) - 498*sqrt(2))*sqrt(59711*sqrt(3) + 165483) - 1/440249244822*1391283^(3/4)*sqrt(681)*sqrt(6)*(173*sqrt(3)*sqrt(2)*x - 498*sqrt(2)*x)*sqrt(59711*sqrt(3) + 165483) + 1/2*sqrt(3)*sqrt(2) - 1/2*sqrt(2)) + 210956*1391283^(1/4)*sqrt(681)*sqrt(6)*sqrt(3)*sqrt(2)*(x^9 + 4*x^7 + 10*x^5 + 12*x^3 + 9*x)*sqrt(59711*sqrt(3) + 165483)*arctan(1/47434996957009616694*sqrt(11971753)*1391283^(3/4)*sqrt(681)*sqrt(6)*sqrt(-9*1391283^(1/4)*sqrt(681)*sqrt(6)*(166*sqrt(3)*x - 173*x)*sqrt(59711*sqrt(3) + 165483) + 969711993*x^2 + 969711993*sqrt(3))*(173*sqrt(3)*sqrt(2) - 498*sqrt(2))*sqrt(59711*sqrt(3) + 165483) - 1/440249244822*1391283^(3/4)*sqrt(681)*sqrt(6)*(173*sqrt(3)*sqrt(2)*x - 498*sqrt(2)*x)*sqrt(59711*sqrt(3) + 165483) - 1/2*sqrt(3)*sqrt(2) + 1/2*sqrt(2)) + 7302577781952*x^4 - 1391283^(1/4)*sqrt(681)*sqrt(6)*(165483*x^9 + 661932*x^7 + 1654830*x^5 + 1985796*x^3 - 59711*sqrt(3)*(x^9 + 4*x^7 + 10*x^5 + 12*x^3 + 9*x) + 1489347*x)*sqrt(59711*sqrt(3) + 165483)*log(9*1391283^(1/4)*sqrt(681)*sqrt(6)*(166*sqrt(3)*x - 173*x)*sqrt(59711*sqrt(3) + 165483) + 969711993*x^2 + 969711993*sqrt(3)) + 1391283^(1/4)*sqrt(681)*sqrt(6)*(165483*x^9 + 661932*x^7 + 1654830*x^5 + 1985796*x^3 - 59711*sqrt(3)*(x^9 + 4*x^7 + 10*x^5 + 12*x^3 + 9*x) + 1489347*x)*sqrt(59711*sqrt(3) + 165483)*log(-9*1391283^(1/4)*sqrt(681)*sqrt(6)*(166*sqrt(3)*x - 173*x)*sqrt(59711*sqrt(3) + 165483) + 969711993*x^2 + 969711993*sqrt(3)) + 9562653200304*x^2 + 3971940323328)/(x^9 + 4*x^7 + 10*x^5 + 12*x^3 + 9*x)

giac [B] time = 3.27, size = 582, normalized size = 2.30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+2*x^2+3)^3,x, algorithm="giac")
[Out] 1/746496*sqrt(2)*(83*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 1494*3^(3/4)*
sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 1494*3^(3/4)*(sqrt(3) + 3)*sqrt
(-6*sqrt(3) + 18) + 83*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 3114*3^(1/4)*sqrt
(2)*sqrt(6*sqrt(3) + 18) + 3114*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3
^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) + 1/
746496*sqrt(2)*(83*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 1494*3^(3/4)*sq
rt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 1494*3^(3/4)*(sqrt(3) + 3)*sqrt(
-6*sqrt(3) + 18) + 83*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 3114*3^(1/4)*sqrt(2)
)*sqrt(6*sqrt(3) + 18) + 3114*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(
3/4)*(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) + 1/14
92992*sqrt(2)*(1494*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 8
3*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 83*3^(3/4)*(6*sqrt(3) + 18)^(3/
2) + 1494*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 3114*3^(1/4)*sqrt(2)
)*sqrt(-6*sqrt(3) + 18) - 3114*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 + 2*3^(
1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) - 1/1492992*sqrt(2)*(1494*3^(3/4)
)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 83*3^(3/4)*sqrt(2)*(-6*sqrt
(3) + 18)^(3/2) + 83*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 1494*3^(3/4)*sqrt(6*s
qrt(3) + 18)*(sqrt(3) - 3) - 3114*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) - 3
114*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 - 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) +
1/2) + sqrt(3)) - 1/1728*(242*x^7 + 809*x^5 + 1676*x^3 + 2475*x)/(x^4 + 2*
x^2 + 3)^2 - 4/27/x
```

maple [B] time = 0.03, size = 424, normalized size = 1.68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/x^2/(x^4+2*x^2+3)^3,x)
[Out] -4/27/x-1/27*(121/32*x^7+809/64*x^5+419/16*x^3+2475/64*x)/(x^4+2*x^2+3)^2-3
25/27648*(-2+2*3^(1/2))^(1/2)*3^(1/2)*ln(x^2-(-2+2*3^(1/2))^(1/2)*x+3^(1/2)
)+7/9216*(-2+2*3^(1/2))^(1/2)*ln(x^2-(-2+2*3^(1/2))^(1/2)*x+3^(1/2))-325/13
824/(2+2*3^(1/2))^(1/2)*(-2+2*3^(1/2))*3^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(
1/2))/(2+2*3^(1/2))^(1/2))+7/4608/(2+2*3^(1/2))^(1/2)*(-2+2*3^(1/2))*arctan
((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))-173/1728/(2+2*3^(1/2))^(1/
2)*3^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))+325/27648
*(-2+2*3^(1/2))^(1/2)*3^(1/2)*ln(x^2+(-2+2*3^(1/2))^(1/2)*x+3^(1/2))-7/9216
*(-2+2*3^(1/2))^(1/2)*ln(x^2+(-2+2*3^(1/2))^(1/2)*x+3^(1/2))-325/13824/(2+2
*3^(1/2))^(1/2)*(-2+2*3^(1/2))*3^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2
+2*3^(1/2))^(1/2))+7/4608/(2+2*3^(1/2))^(1/2)*(-2+2*3^(1/2))*arctan((2*x+(-
2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))-173/1728/(2+2*3^(1/2))^(1/2)*3^(1/
2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{166x^8 + 611x^6 + 1412x^4 + 1849x^2 + 768}{576(x^9 + 4x^7 + 10x^5 + 12x^3 + 9x)} - \frac{1}{576} \int \frac{166x^2 + 173}{x^4 + 2x^2 + 3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+2*x^2+3)^3,x, algorithm="maxima")
```

[Out] $-1/576*(166*x^8 + 611*x^6 + 1412*x^4 + 1849*x^2 + 768)/(x^9 + 4*x^7 + 10*x^5 + 12*x^3 + 9*x) - 1/576*\text{integrate}((166*x^2 + 173)/(x^4 + 2*x^2 + 3), x)$

mupad [B] time = 0.99, size = 179, normalized size = 0.71

$$\frac{\frac{83}{288}x^8 + \frac{611}{576}x^6 + \frac{353}{144}x^4 + \frac{1849}{576}x^2 + \frac{4}{3}}{x^9 + 4x^7 + 10x^5 + 12x^3 + 9x} + \frac{\text{atan}\left(\frac{x\sqrt{-358266 - \sqrt{2}316434i} - 52739i}{859963392\left(\frac{17140175}{286654464} + \frac{\sqrt{2}9123847i}{286654464}\right)} + \frac{52739\sqrt{2}x\sqrt{-358266 - \sqrt{2}316434i}}{1719926784\left(\frac{17140175}{286654464} + \frac{\sqrt{2}9123847i}{286654464}\right)}\right)\sqrt{-358266 - \sqrt{2}316434i}i}{6912} - \frac{\text{atan}\left(\frac{x\sqrt{-358266 + \sqrt{2}316434i} - 52739i}{859963392\left(\frac{17140175}{286654464} + \frac{\sqrt{2}9123847i}{286654464}\right)} + \frac{52739\sqrt{2}x\sqrt{-358266 + \sqrt{2}316434i}}{1719926784\left(\frac{17140175}{286654464} + \frac{\sqrt{2}9123847i}{286654464}\right)}\right)\sqrt{-358266 + \sqrt{2}316434i}i}{6912}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((x^2 + 3*x^4 + 5*x^6 + 4)/(x^2*(2*x^2 + x^4 + 3)^3), x)$

[Out] $(\text{atan}((x*(-2^{(1/2)}*316434i - 358266)^{(1/2)}*52739i)/(859963392*((2^{(1/2)}*9123847i)/286654464 - 17140175/286654464))) + (52739*2^{(1/2)}*x*(-2^{(1/2)}*316434i - 358266)^{(1/2)})/(1719926784*((2^{(1/2)}*9123847i)/286654464 - 17140175/286654464))) * (-2^{(1/2)}*316434i - 358266)^{(1/2)}*i)/6912 - (\text{atan}((x*(2^{(1/2)}*316434i - 358266)^{(1/2)}*52739i)/(859963392*((2^{(1/2)}*9123847i)/286654464 + 17140175/286654464))) - (52739*2^{(1/2)}*x*(2^{(1/2)}*316434i - 358266)^{(1/2)})/(1719926784*((2^{(1/2)}*9123847i)/286654464 + 17140175/286654464))) * (2^{(1/2)}*316434i - 358266)^{(1/2)}*i)/6912 - ((1849*x^2)/576 + (353*x^4)/144 + (611*x^6)/576 + (83*x^8)/288 + 4/3)/(9*x + 12*x^3 + 10*x^5 + 4*x^7 + x^9)$

sympy [A] time = 0.67, size = 75, normalized size = 0.30

$$\frac{-166x^8 - 611x^6 - 1412x^4 - 1849x^2 - 768}{576x^9 + 2304x^7 + 5760x^5 + 6912x^3 + 5184x} + \text{RootSum}\left(4174708211712t^4 + 15652880384t^2 + 37564641, \left(t \mapsto t \log\left(-\frac{98146713600t^3}{11971753} - \frac{9639364864t}{323237331} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((5*x**6+3*x**4+x**2+4)/x**2/(x**4+2*x**2+3)**3, x)$

[Out] $(-166*x**8 - 611*x**6 - 1412*x**4 - 1849*x**2 - 768)/(576*x**9 + 2304*x**7 + 5760*x**5 + 6912*x**3 + 5184*x) + \text{RootSum}(4174708211712*_t**4 + 15652880384*_t**2 + 37564641, \text{Lambda}(_t, _t*\log(-98146713600*_t**3/11971753 - 9639364864*_t/323237331 + x)))$

$$3.122 \quad \int \frac{4+x^2+3x^4+5x^6}{x^4(3+2x^2+x^4)^3} dx$$

Optimal. Leaf size=262

$$-\frac{4}{81x^3} + \frac{\sqrt{\frac{1}{3}(11240451\sqrt{3} - 10004741)} \log\left(x^2 - \sqrt{2(\sqrt{3} - 1)}x + \sqrt{3}\right)}{41472} - \frac{\sqrt{\frac{1}{3}(11240451\sqrt{3} - 10004741)} \log\left(x^2 + \sqrt{2(\sqrt{3} - 1)}x + \sqrt{3}\right)}{41472}$$

Rubi [A] time = 0.37, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 31, number of rules used = 0.226, Rules used = {1669, 1664, 1169, 634, 618, 204, 628}

$$\frac{25x(5x^2+7)}{432(x^2+2x+3)^2} + \frac{x(1025x^2+1474)}{5184(x^2+2x+3)} - \frac{4}{81x^3} + \frac{\sqrt{\frac{1}{3}(11240451\sqrt{3}-10004741)} \log(x^2-\sqrt{2(\sqrt{3}-1)}x+\sqrt{3})}{41472} + \frac{\sqrt{\frac{1}{3}(11240451\sqrt{3}-10004741)} \log(x^2+\sqrt{2(\sqrt{3}-1)}x+\sqrt{3})}{41472} + \frac{7}{27x} - \frac{\sqrt{\frac{1}{3}(10004741+11240451\sqrt{3})} \tan^{-1}\left(\frac{\sqrt{3(\sqrt{3}-1)-2x}}{\sqrt{3(\sqrt{3}-1)}}\right)}{20736} + \frac{\sqrt{\frac{1}{3}(10004741+11240451\sqrt{3})} \tan^{-1}\left(\frac{2x+\sqrt{3(\sqrt{3}-1)}}{\sqrt{3(\sqrt{3}-1)}}\right)}{20736}$$

Antiderivative was successfully verified.

[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(3 + 2*x^2 + x^4)^3), x]

[Out] -4/(81*x^3) + 7/(27*x) + (25*x*(7 + 5*x^2))/(432*(3 + 2*x^2 + x^4)^2) + (x*(1474 + 1025*x^2))/(5184*(3 + 2*x^2 + x^4)) - (Sqrt[(10004741 + 11240451*Sqrt[3])/3]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])]) - 2*x]/Sqrt[2*(1 + Sqrt[3])])]/20736 + (Sqrt[(10004741 + 11240451*Sqrt[3])/3]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])]) + 2*x]/Sqrt[2*(1 + Sqrt[3])])]/20736 + (Sqrt[(-10004741 + 11240451*Sqrt[3])/3]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/41472 - (Sqrt[(-10004741 + 11240451*Sqrt[3])/3]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/41472

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1169

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ

$[b^2 - 4ac, 0] \ \&\& \ \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \ \&\& \ \text{NegQ}[b^2 - 4ac]$

Rule 1664

$\text{Int}[(Pq_)*((d_)*(x_))^{(m_)}*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^{(p_)}, x_Symbol] \ :> \ \text{Int}[\text{ExpandIntegrand}[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] \ /;$
 $\text{FreeQ}\{a, b, c, d, m\}, x] \ \&\& \ \text{PolyQ}[Pq, x^2] \ \&\& \ \text{IGtQ}[p, -2]$

Rule 1669

$\text{Int}[(Pq_)*(x_)^{(m_)}*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^{(p_)}, x_Symbol] \ :>$
 $\text{With}\{d = \text{Coeff}[\text{PolynomialRemainder}[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],$
 $e = \text{Coeff}[\text{PolynomialRemainder}[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]\}, \text{Simp}[($
 $x*(a + b*x^2 + c*x^4)^{(p+1)}*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)$
 $)/(2*a*(p+1)*(b^2 - 4*a*c)), x] + \text{Dist}[1/(2*a*(p+1)*(b^2 - 4*a*c)), \text{Int}$
 $[x^m*(a + b*x^2 + c*x^4)^{(p+1)}*\text{ExpandToSum}[(2*a*(p+1)*(b^2 - 4*a*c)*\text{P}$
 $\text{olynomialQuotient}[x^m*Pq, a + b*x^2 + c*x^4, x)]/x^m + (b^2*d*(2*p+3) - 2$
 $*a*c*d*(4*p+5) - a*b*e)/x^m + c*(4*p+7)*(b*d - 2*a*e)*x^{(2-m)}, x], x]$
 $/; \text{FreeQ}\{a, b, c\}, x] \ \&\& \ \text{PolyQ}[Pq, x^2] \ \&\& \ \text{GtQ}[\text{Expon}[Pq, x^2], 1] \ \&\&$
 $\text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ \text{ILtQ}[m/2, 0]$

Rubi steps

$$\int \frac{4 + x^2 + 3x^4 + 5x^6}{x^4(3 + 2x^2 + x^4)^3} dx = \frac{25x(7 + 5x^2)}{432(3 + 2x^2 + x^4)^2} + \frac{1}{96} \int \frac{128 - \frac{160x^2}{3} + 50x^4 + \frac{1250x^6}{9}}{x^4(3 + 2x^2 + x^4)^2} dx$$

$$= \frac{25x(7 + 5x^2)}{432(3 + 2x^2 + x^4)^2} + \frac{x(1474 + 1025x^2)}{5184(3 + 2x^2 + x^4)} + \frac{\int \frac{2048 - \frac{6656x^2}{3} + \frac{2576x^4}{9} + \frac{8200x^6}{9}}{x^4(3 + 2x^2 + x^4)} dx}{4608}$$

$$= \frac{25x(7 + 5x^2)}{432(3 + 2x^2 + x^4)^2} + \frac{x(1474 + 1025x^2)}{5184(3 + 2x^2 + x^4)} + \frac{\int \left(\frac{2048}{3x^4} - \frac{3584}{3x^2} + \frac{8(2242 + 2369x^2)}{9(3 + 2x^2 + x^4)} \right) dx}{4608}$$

$$= -\frac{4}{81x^3} + \frac{7}{27x} + \frac{25x(7 + 5x^2)}{432(3 + 2x^2 + x^4)^2} + \frac{x(1474 + 1025x^2)}{5184(3 + 2x^2 + x^4)} + \frac{\int \frac{2242 + 2369x^2}{3 + 2x^2 + x^4} dx}{5184}$$

$$= -\frac{4}{81x^3} + \frac{7}{27x} + \frac{25x(7 + 5x^2)}{432(3 + 2x^2 + x^4)^2} + \frac{x(1474 + 1025x^2)}{5184(3 + 2x^2 + x^4)} + \frac{\int \frac{2242\sqrt{2(-1+\sqrt{3})} - (2242 - 2369\sqrt{3})}{\sqrt{3} - \sqrt{2(-1+\sqrt{3})}} dx}{10368\sqrt{6(-1+\sqrt{3})}}$$

$$= -\frac{4}{81x^3} + \frac{7}{27x} + \frac{25x(7 + 5x^2)}{432(3 + 2x^2 + x^4)^2} + \frac{x(1474 + 1025x^2)}{5184(3 + 2x^2 + x^4)} + \frac{(2242 - 2369\sqrt{3}) \int \frac{1}{\sqrt{3} - \sqrt{2(-1+\sqrt{3})}} dx}{20736\sqrt{6(-1+\sqrt{3})}}$$

$$= -\frac{4}{81x^3} + \frac{7}{27x} + \frac{25x(7 + 5x^2)}{432(3 + 2x^2 + x^4)^2} + \frac{x(1474 + 1025x^2)}{5184(3 + 2x^2 + x^4)} + \frac{\sqrt{-\frac{10004741}{12} + \frac{3746817}{4}}}{20736\sqrt{6(-1+\sqrt{3})}}$$

$$= -\frac{4}{81x^3} + \frac{7}{27x} + \frac{25x(7 + 5x^2)}{432(3 + 2x^2 + x^4)^2} + \frac{x(1474 + 1025x^2)}{5184(3 + 2x^2 + x^4)} - \frac{\sqrt{\frac{1}{3}(10004741 + 1124\sqrt{3})}}{20736\sqrt{6(-1+\sqrt{3})}}$$

Mathematica [C] time = 0.32, size = 139, normalized size = 0.53

$$\frac{4(2369x^{10}+8644x^8+19939x^6+20090x^4+9024x^2-2304)}{x^3(x^4+2x^2+3)^2} + \frac{(4738+127i\sqrt{2})\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}} + \frac{(4738-127i\sqrt{2})\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}}$$

20736

Antiderivative was successfully verified.

[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(3 + 2*x^2 + x^4)^3), x]

[Out] ((4*(-2304 + 9024*x^2 + 20090*x^4 + 19939*x^6 + 8644*x^8 + 2369*x^10))/(x^3*(3 + 2*x^2 + x^4)^2) + ((4738 + (127*I)*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/Sqrt[1 - I*Sqrt[2]] + ((4738 - (127*I)*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/Sqrt[1 + I*Sqrt[2]])/20736

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4 + x^2 + 3x^4 + 5x^6}{x^4(3 + 2x^2 + x^4)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(3 + 2*x^2 + x^4)^3), x]

[Out] IntegrateAlgebraic[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(3 + 2*x^2 + x^4)^3), x]

fricas [B] time = 1.04, size = 652, normalized size = 2.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+2*x^2+3)^3,x, algorithm="fricas")

[Out] 1/135934787413472256*(62119890312985296*x^10 + 226662866975704896*x^8 + 522840224968600176*x^6 + 47239676*713236683^(1/4)*sqrt(15419)*sqrt(6)*sqrt(3)*sqrt(2)*(x^11 + 4*x^9 + 10*x^7 + 12*x^5 + 9*x^3)*sqrt(10004741*sqrt(3) + 33721353)*arctan(1/27609352591972558367520653346*sqrt(182097141061)*713236683^(3/4)*sqrt(15419)*sqrt(6)*sqrt(3)*sqrt(713236683^(1/4)*sqrt(15419)*sqrt(6)*(2369*sqrt(3)*x - 2242*x)*sqrt(10004741*sqrt(3) + 33721353) + 546291423183*x^2 + 546291423183*sqrt(3))*(2242*sqrt(3)*sqrt(2) - 7107*sqrt(2))*sqrt(10004741*sqrt(3) + 33721353) - 1/50539604724352062*713236683^(3/4)*sqrt(15419)*sqrt(6)*(2242*sqrt(3)*sqrt(2)*x - 7107*sqrt(2)*x)*sqrt(10004741*sqrt(3) + 33721353) + 1/2*sqrt(3)*sqrt(2) - 1/2*sqrt(2)) + 47239676*713236683^(1/4)*sqrt(15419)*sqrt(6)*sqrt(3)*sqrt(2)*(x^11 + 4*x^9 + 10*x^7 + 12*x^5 + 9*x^3)*sqrt(10004741*sqrt(3) + 33721353)*arctan(1/82828057775917675102561960038*sqrt(182097141061)*713236683^(3/4)*sqrt(15419)*sqrt(6)*sqrt(-27*713236683^(1/4)*sqrt(15419)*sqrt(6)*(2369*sqrt(3)*x - 2242*x)*sqrt(10004741*sqrt(3) + 33721353) + 14749868425941*x^2 + 14749868425941*sqrt(3))*(2242*sqrt(3)*sqrt(2) - 7107*sqrt(2))*sqrt(10004741*sqrt(3) + 33721353) - 1/50539604724352062*713236683^(3/4)*sqrt(15419)*sqrt(6)*(2242*sqrt(3)*sqrt(2)*x - 7107*sqrt(2)*x)*sqrt(10004741*sqrt(3) + 33721353) - 1/2*sqrt(3)*sqrt(2) + 1/2*sqrt(2)) + 526799745203830560*x^4 - 713236683^(1/4)*sqrt(15419)*sqrt(6)*(33721353*x^11 + 134885412*x^9 + 337213530*x^7 + 404656236*x^5 + 303492177*x^3 - 10004741*sqrt(3)*(x^11 + 4*x^9 + 10*x^7 + 12*x^5 + 9*x^3))*sqrt(10004741*sqrt(3) + 33721353)*log(27*713236683^(1/4)*sqrt(15419)*sqrt(6)*(2369*sqrt(3)*x - 2242*x)*sqrt(10004741*sqrt(3) + 33721353) + 14749868425941*x^2 + 14749868425941*sqrt(3)) + 713236683^(1/4)*sqrt(15419)*sqrt(6)*(33721353*x^11 + 134885412*x^9 + 337213530*x^7 + 404656236*x^5 + 303492177*x^3 - 10004741*sqrt(3)*(x^11 + 4*x^9 + 10*x^7 + 12*x^5 + 9*x^3))*sqrt(10004741*sqrt(3) + 33721353)*log(-27*713236683^(1/4)*sqrt(15419)*sqrt(6)*(2369*sqrt(3)*x - 2242*x)*sqrt(10004741*sqrt(3) + 33721353) + 14749868425941*x^2 + 14749868425941*sqrt(3))

$$004741*\sqrt{3} + 33721353) + 14749868425941*x^2 + 14749868425941*\sqrt{3}) + 236627222534562816*x^2 - 60415461072654336)/(x^{11} + 4*x^9 + 10*x^7 + 12*x^5 + 9*x^3)$$

giac [B] time = 2.99, size = 589, normalized size = 2.25

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+2*x^2+3)^3,x, algorithm="giac")

[Out] -1/13436928*sqrt(2)*(2369*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 42642*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 42642*3^(3/4)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) + 2369*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 80712*3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) + 80712*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) - 1/13436928*sqrt(2)*(2369*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 42642*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 42642*3^(3/4)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) + 2369*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 80712*3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) + 80712*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3/4)*(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) - 1/26873856*sqrt(2)*(42642*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 2369*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 2369*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 42642*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 80712*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) - 80712*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 + 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) + 1/26873856*sqrt(2)*(42642*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 2369*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 2369*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 42642*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 80712*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) - 80712*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 - 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) + 1/5184*(1025*x^7 + 3524*x^5 + 7523*x^3 + 6522*x)/(x^4 + 2*x^2 + 3)^2 + 1/81*(21*x^2 - 4)/x^3

maple [B] time = 0.04, size = 429, normalized size = 1.64

$$\frac{888(-2+2\sqrt{3})\sqrt{3}\operatorname{arctan}\left(\frac{\sqrt{2}\sqrt{3}}{2\sqrt{3}+2}\right) + 127(-2+2\sqrt{3})\operatorname{arctan}\left(\frac{\sqrt{2}\sqrt{3}}{2\sqrt{3}+2}\right) + 1121\sqrt{3}\operatorname{arctan}\left(\frac{\sqrt{2}\sqrt{3}}{2\sqrt{3}+2}\right) + 888(-2+2\sqrt{3})\sqrt{3}\operatorname{arctan}\left(\frac{\sqrt{2}\sqrt{3}}{2\sqrt{3}+2}\right) + 127(-2+2\sqrt{3})\operatorname{arctan}\left(\frac{\sqrt{2}\sqrt{3}}{2\sqrt{3}+2}\right) + 1121\sqrt{3}\operatorname{arctan}\left(\frac{\sqrt{2}\sqrt{3}}{2\sqrt{3}+2}\right) + 888\sqrt{2+2\sqrt{3}}\sqrt{3}\ln\left(\frac{\sqrt{2}\sqrt{3}}{2\sqrt{3}+2}\right) + 127\sqrt{2+2\sqrt{3}}\ln\left(\frac{\sqrt{2}\sqrt{3}}{2\sqrt{3}+2}\right) + 1121\sqrt{2+2\sqrt{3}}\ln\left(\frac{\sqrt{2}\sqrt{3}}{2\sqrt{3}+2}\right) + 888\sqrt{2+2\sqrt{3}}\sqrt{3}\ln\left(\frac{\sqrt{2}\sqrt{3}}{2\sqrt{3}+2}\right) + 127\sqrt{2+2\sqrt{3}}\ln\left(\frac{\sqrt{2}\sqrt{3}}{2\sqrt{3}+2}\right) + 1121\sqrt{2+2\sqrt{3}}\ln\left(\frac{\sqrt{2}\sqrt{3}}{2\sqrt{3}+2}\right) + \frac{2}{3\sqrt{3}} + \frac{1}{3\sqrt{3}}}{2(3+2\sqrt{3})}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^6+3*x^4+x^2+4)/x^4/(x^4+2*x^2+3)^3,x)

[Out] -4/81/x^3+7/27/x+1/27*(1025/192*x^7+881/48*x^5+7523/192*x^3+1087/32*x)/(x^4+2*x^2+3)^2+4865/248832*(-2+2*3^(1/2))^(1/2)*3^(1/2)*ln(x^2-(-2+2*3^(1/2))^(1/2)*x+3^(1/2))+127/82944*(-2+2*3^(1/2))^(1/2)*ln(x^2-(-2+2*3^(1/2))^(1/2)*x+3^(1/2))+4865/124416/(2+2*3^(1/2))^(1/2)*(-2+2*3^(1/2))*3^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))+127/41472/(2+2*3^(1/2))^(1/2)*(-2+2*3^(1/2))*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))+1121/7776/(2+2*3^(1/2))^(1/2)*3^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))-4865/248832*(-2+2*3^(1/2))^(1/2)*3^(1/2)*ln(x^2+(-2+2*3^(1/2))^(1/2)*x+3^(1/2))-127/82944*(-2+2*3^(1/2))^(1/2)*ln(x^2+(-2+2*3^(1/2))^(1/2)*x+3^(1/2))+4865/124416/(2+2*3^(1/2))^(1/2)*(-2+2*3^(1/2))*3^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))+127/41472/(2+2*3^(1/2))^(1/2)*(-2+2*3^(1/2))*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))+1121/7776/(2+2*3^(1/2))^(1/2)*3^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2369 x^{10} + 8644 x^8 + 19939 x^6 + 20090 x^4 + 9024 x^2 - 2304}{5184 (x^{11} + 4 x^9 + 10 x^7 + 12 x^5 + 9 x^3)} + \frac{1}{5184} \int \frac{2369 x^2 + 2242}{x^4 + 2 x^2 + 3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+2*x^2+3)^3,x, algorithm="maxima")

[Out] 1/5184*(2369*x^10 + 8644*x^8 + 19939*x^6 + 20090*x^4 + 9024*x^2 - 2304)/(x^11 + 4*x^9 + 10*x^7 + 12*x^5 + 9*x^3) + 1/5184*integrate((2369*x^2 + 2242)/(x^4 + 2*x^2 + 3), x)

mupad [B] time = 1.02, size = 185, normalized size = 0.71

$$\frac{\frac{2369x^{10} + 8644x^8 + 19939x^6 + 20090x^4 + 9024x^2 - 2304}{x^{11} + 4x^9 + 10x^7 + 12x^5 + 9x^3} \operatorname{atan}\left(\frac{x\sqrt{-60028446 - \sqrt{2}70859514i}11809919i}{626913312768\left(\frac{294825935}{20897104256} + \frac{\sqrt{2}13238919199i}{104485552128}\right)} + \frac{11809919\sqrt{2}x\sqrt{-60028446 - \sqrt{2}70859514i}}{1253826625536\left(\frac{294825935}{20897104256} + \frac{\sqrt{2}13238919199i}{104485552128}\right)}\right) + \frac{\sqrt{-60028446 - \sqrt{2}70859514i}i}{62208} \operatorname{atan}\left(\frac{x\sqrt{-60028446 + \sqrt{2}70859514i}11809919i}{626913312768\left(\frac{294825935}{20897104256} + \frac{\sqrt{2}13238919199i}{104485552128}\right)} - \frac{11809919\sqrt{2}x\sqrt{-60028446 + \sqrt{2}70859514i}}{1253826625536\left(\frac{294825935}{20897104256} + \frac{\sqrt{2}13238919199i}{104485552128}\right)}\right)}{62208} + \frac{\sqrt{-60028446 + \sqrt{2}70859514i}i}{62208}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^4*(2*x^2 + x^4 + 3)^3),x)

[Out] ((47*x^2)/27 + (10045*x^4)/2592 + (19939*x^6)/5184 + (2161*x^8)/1296 + (2369*x^10)/5184 - 4/9)/(9*x^3 + 12*x^5 + 10*x^7 + 4*x^9 + x^11) - (atan((x*(-2^(1/2)*70859514i - 60028446)^(1/2)*11809919i)/(626913312768*((2^(1/2)*13238919199i)/104485552128 - 57455255935/208971104256))) + (11809919*2^(1/2)*x*(-2^(1/2)*70859514i - 60028446)^(1/2))/(1253826625536*((2^(1/2)*13238919199i)/104485552128 - 57455255935/208971104256))))*(-2^(1/2)*70859514i - 60028446)^(1/2)*i)/62208 + (atan((x*(2^(1/2)*70859514i - 60028446)^(1/2)*11809919i)/(626913312768*((2^(1/2)*13238919199i)/104485552128 + 57455255935/208971104256))) - (11809919*2^(1/2)*x*(2^(1/2)*70859514i - 60028446)^(1/2))/(1253826625536*((2^(1/2)*13238919199i)/104485552128 + 57455255935/208971104256))))*(2^(1/2)*70859514i - 60028446)^(1/2)*i)/62208

sympy [A] time = 0.69, size = 80, normalized size = 0.31

$$\operatorname{RootSum}\left(338151365148672t^4 + 2622682824704t^2 + 19257390441, \left(t \mapsto t \log\left(\frac{357010935644160t^3}{182097141061} + \frac{26016957890816t}{1638874269549} + x\right)\right) + \frac{2369x^{10} + 8644x^8 + 19939x^6 + 20090x^4 + 9024x^2 - 2304}{5184x^{11} + 20736x^9 + 51840x^7 + 62208x^5 + 46656x^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**6+3*x**4+x**2+4)/x**4/(x**4+2*x**2+3)**3,x)

[Out] RootSum(338151365148672*_t**4 + 2622682824704*_t**2 + 19257390441, Lambda(_t, _t*log(357010935644160*_t**3/182097141061 + 26016957890816*_t/1638874269549 + x))) + (2369*x**10 + 8644*x**8 + 19939*x**6 + 20090*x**4 + 9024*x**2 - 2304)/(5184*x**11 + 20736*x**9 + 51840*x**7 + 62208*x**5 + 46656*x**3)

$$3.123 \quad \int \frac{x(d+ex^2+fx^4+gx^6)}{a+bx^2+cx^4} dx$$

Optimal. Leaf size=149

$$\frac{\log(a+bx^2+cx^4)(-c(ag+bf)+b^2g+c^2e)}{4c^3} - \frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(-c^2(2af+be)+bc(3ag+bf)+b^3(-g)+2c^3d)}{2c^3\sqrt{b^2-4ac}}$$

Rubi [A] time = 0.29, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1663, 1657, 634, 618, 206, 628}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(-c^2(2af+be)+bc(3ag+bf)+b^3(-g)+2c^3d)}{2c^3\sqrt{b^2-4ac}} + \frac{\log(a+bx^2+cx^4)(-c(ag+bf)+b^2g+c^2e)}{4c^3} + \frac{x^2(cf-bg)}{2c^2} + \frac{gx^4}{4c}$$

Antiderivative was successfully verified.

[In] Int[(x*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4), x]

[Out] ((c*f - b*g)*x^2)/(2*c^2) + (g*x^4)/(4*c) - ((2*c^3*d - c^2*(b*e + 2*a*f) - b^3*g + b*c*(b*f + 3*a*g))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^3*Sqrt[b^2 - 4*a*c]) + ((c^2*e + b^2*g - c*(b*f + a*g))*Log[a + b*x^2 + c*x^4])/(4*c^3)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1657

Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 1663

Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m-1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[

(m - 1)/2]

Rubi steps

$$\begin{aligned}
\int \frac{x(d + ex^2 + fx^4 + gx^6)}{a + bx^2 + cx^4} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{d + ex + fx^2 + gx^3}{a + bx + cx^2} dx, x, x^2 \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \left(\frac{cf - bg}{c^2} + \frac{gx}{c} + \frac{c^2d - acf + abg + (c^2e + b^2g - c(bf + ag))x}{c^2(a + bx + cx^2)} \right) dx \right) \\
&= \frac{(cf - bg)x^2}{2c^2} + \frac{gx^4}{4c} + \frac{\text{Subst} \left(\int \frac{c^2d - acf + abg + (c^2e + b^2g - c(bf + ag))x}{a + bx + cx^2} dx, x, x^2 \right)}{2c^2} \\
&= \frac{(cf - bg)x^2}{2c^2} + \frac{gx^4}{4c} + \frac{(c^2e + b^2g - c(bf + ag)) \text{Subst} \left(\int \frac{b + 2cx}{a + bx + cx^2} dx, x, x^2 \right)}{4c^3} + \\
&= \frac{(cf - bg)x^2}{2c^2} + \frac{gx^4}{4c} + \frac{(c^2e + b^2g - c(bf + ag)) \log(a + bx^2 + cx^4)}{4c^3} - \frac{(2c^3d -}{4c^3} \\
&= \frac{(cf - bg)x^2}{2c^2} + \frac{gx^4}{4c} - \frac{(2c^3d - c^2(be + 2af) - b^3g + bc(bf + 3ag)) \tanh^{-1} \left(\frac{b +}{\sqrt{b^2}} \right)}{2c^3 \sqrt{b^2 - 4ac}}
\end{aligned}$$

Mathematica [A] time = 0.12, size = 142, normalized size = 0.95

$$\frac{\log(a + bx^2 + cx^4)(-c(ag + bf) + b^2g + c^2e) + \frac{2 \tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)(-c^2(2af+be)+bc(3ag+bf)+b^3(-g)+2c^3d)}{\sqrt{4ac-b^2}} + 2c^2(cf - bg) + c^2gx^4}{4c^3}$$

Antiderivative was successfully verified.

[In] Integrate[(x*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4), x]

[Out] (2*c*(c*f - b*g)*x^2 + c^2*g*x^4 + (2*(2*c^3*d - c^2*(b*e + 2*a*f) - b^3*g + b*c*(b*f + 3*a*g))*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (c^2*e + b^2*g - c*(b*f + a*g))*Log[a + b*x^2 + c*x^4]/(4*c^3)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(d + ex^2 + fx^4 + gx^6)}{a + bx^2 + cx^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4), x]**[Out]** IntegrateAlgebraic[(x*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4), x]**fricas [A]** time = 1.59, size = 486, normalized size = 3.26

$$\frac{\left((b^2 - 4ac)^2 \log\left(\frac{b + 2cx}{\sqrt{4ac - b^2}} \right) + (b^2 - 4ac) \log\left(\frac{c^2d - acf + abg + (c^2e + b^2g - c(bf + ag))x}{a + bx + cx^2} \right) + (b^2 - 4ac) \log(a + bx^2 + cx^4) \right) + (b^2 - 4ac) \log(a + bx^2 + cx^4)}{4(b^2 - 4ac)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a), x, algorithm="fricas")

[Out] [1/4*((b^2*c^2 - 4*a*c^3)*g*x^4 + 2*((b^2*c^2 - 4*a*c^3)*f - (b^3*c - 4*a*b*c^2)*g)*x^2 + (2*c^3*d - b*c^2*e + (b^2*c - 2*a*c^2)*f - (b^3 - 3*a*b*c)*g)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b

) $\sqrt{b^2 - 4ac}$)/($c^2x^4 + b^2x^2 + a$) + (($b^2c^2 - 4ac^3$) $e - (b^3c - 4ab^2c^2)*f + (b^4 - 5ab^2c + 4a^2c^2)*g$)* $\log(c^2x^4 + b^2x^2 + a)$)/($b^2c^3 - 4ac^4$), $1/4*((b^2c^2 - 4ac^3)*g*x^4 + 2*((b^2c^2 - 4ac^3)*f - (b^3c - 4ab^2c^2)*g)*x^2 - 2*(2c^3d - b^3c^2e + (b^2c - 2ac^2)*f - (b^3 - 3ab^2c)*g)*\sqrt{-b^2 + 4ac}*\arctan(-(2c^2x^2 + b)*\sqrt{-b^2 + 4ac})/(b^2 - 4ac)) + ((b^2c^2 - 4ac^3)*e - (b^3c - 4ab^2c^2)*f + (b^4 - 5ab^2c + 4a^2c^2)*g)*\log(c^2x^4 + b^2x^2 + a)/(b^2c^3 - 4ac^4)$]

giac [A] time = 1.90, size = 146, normalized size = 0.98

$$\frac{cgx^4 + 2cfx^2 - 2bgx^2}{4c^2} - \frac{(bcf - b^2g + acg - c^2e)\log(cx^4 + bx^2 + a)}{4c^3} + \frac{(2c^3d + b^2cf - 2ac^2f - b^3g + 3abcg - bc^2e)\arctan\left(\frac{2cx^2 + b}{\sqrt{-b^2 + 4ac}}\right)}{2\sqrt{-b^2 + 4ac}c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate($x*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)$), x, algorithm="giac")

[Out] $1/4*(c*g*x^4 + 2*c*f*x^2 - 2*b*g*x^2)/c^2 - 1/4*(b*c*f - b^2*g + a*c*g - c^2*e)*\log(c*x^4 + b*x^2 + a)/c^3 + 1/2*(2*c^3*d + b^2*c*f - 2*a*c^2*f - b^3*g + 3*a*b*c*g - b*c^2*e)*\arctan((2*c*x^2 + b)/\sqrt{-b^2 + 4*a*c})/(\sqrt{-b^2 + 4*a*c})*c^3$

maple [B] time = 0.01, size = 357, normalized size = 2.40

$$\frac{gx^4}{4c} + \frac{3abg\arctan\left(\frac{2x^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}c^2} - \frac{af\arctan\left(\frac{2x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}c} - \frac{b^2g\arctan\left(\frac{2x^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}c^3} + \frac{b^2f\arctan\left(\frac{2x^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}c^2} - \frac{be\arctan\left(\frac{2x^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}c} - \frac{bgx^2}{2c^2} + \frac{fx^2}{2c} + \frac{d\arctan\left(\frac{2x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} - \frac{ag\ln(cx^4+bx^2+a)}{4c^2} + \frac{b^2g\ln(cx^4+bx^2+a)}{4c^3} - \frac{bf\ln(cx^4+bx^2+a)}{4c^2} + \frac{e\ln(cx^4+bx^2+a)}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int($x*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)$), x)

[Out] $1/4*g*x^4/c - 1/2/c^2*x^2*b*g + 1/2/c*f*x^2 - 1/4/c^2*\ln(c*x^4+b*x^2+a)*a*g + 1/4/c^3*\ln(c*x^4+b*x^2+a)*b^2*g - 1/4*b/c^2*f*\ln(c*x^4+b*x^2+a) + 1/4/c*e*\ln(c*x^4+b*x^2+a) + 3/2/c^2/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x^2+b)/(4*a*c-b^2)^{(1/2)})*a*b*g - 1/(4*a*c-b^2)^{(1/2)}*a/c*f*\arctan((2*c*x^2+b)/(4*a*c-b^2)^{(1/2)}) + 1/(4*a*c-b^2)^{(1/2)}*d*\arctan((2*c*x^2+b)/(4*a*c-b^2)^{(1/2)}) - 1/2/c^3/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x^2+b)/(4*a*c-b^2)^{(1/2)})*b^3*g + 1/2/(4*a*c-b^2)^{(1/2)}*b^2/c^2*f*\arctan((2*c*x^2+b)/(4*a*c-b^2)^{(1/2)}) - 1/2/(4*a*c-b^2)^{(1/2)}*b/c*e*\arctan((2*c*x^2+b)/(4*a*c-b^2)^{(1/2)})$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate($x*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)$), x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 1.68, size = 1834, normalized size = 12.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(($x*(d + e*x^2 + f*x^4 + g*x^6)$)/($a + b*x^2 + c*x^4$)), x)

[Out] $x^2*(f/(2*c) - (b*g)/(2*c^2)) + (g*x^4)/(4*c) - (\log(a + b*x^2 + c*x^4))*(2*b^4*g + 2*b^2*c^2*e + 8*a^2*c^2*g - 8*a*c^3*e - 2*b^3*c*f + 8*a*b*c^2*f - 10*a*b^2*c*g)/(2*(16*a*c^4 - 4*b^2*c^3)) + (\operatorname{atan}((2*c^4*(4*a*c - b^2)*(x^2*$

$$\begin{aligned} & \left(\frac{(((4c^6d + 6b^2c^4f - 6b^3c^3g - 4ac^5f - 6b^5c^5e + 10ab^4c^4g)/c^4 - (4b^2c^2(2b^4g + 2b^2c^2e + 8a^2c^2g - 8a^3c^3e - 2b^3c^3f + 8ab^2c^2f - 10ab^2c^2g)))/(16a^4c^4 - 4b^2c^3)) * (2c^3d - b^3g - 2ac^2f - b^2c^2e + b^2c^2f + 3ab^2c^2g)) / (8c^3(4ac - b^2)^{(1/2)}) - (b(2c^3d - b^3g - 2ac^2f - b^2c^2e + b^2c^2f + 3ab^2c^2g)) * (2b^4g + 2b^2c^2e + 8a^2c^2g - 8a^3c^3e - 2b^3c^3f + 8ab^2c^2f - 10ab^2c^2g)) / (2c(4ac - b^2)^{(1/2)} * (16a^4c^4 - 4b^2c^3)) \right) / a + (b * \left(\frac{(((4c^6d + 6b^2c^4f - 6b^3c^3g - 4ac^5f - 6b^5c^5e + 10ab^4c^4g)/c^4 - (4b^2c^2(2b^4g + 2b^2c^2e + 8a^2c^2g - 8a^3c^3e - 2b^3c^3f + 8ab^2c^2f - 10ab^2c^2g)))/(16a^4c^4 - 4b^2c^3)) * (2b^4g + 2b^2c^2e + 8a^2c^2g - 8a^3c^3e - 2b^3c^3f + 8ab^2c^2f - 10ab^2c^2g)) / (2 * (16a^4c^4 - 4b^2c^3)) - (b^5g^2 + b^4c^4e^2 + b^3c^2f^2 - c^5d^2e + 2a^2b^2c^2g^2 + a^4c^4d^2g + a^4c^4e^2f + b^4c^4d^2f - 2b^4c^4f^2g - ab^3c^3f^2 - 3ab^3c^3g^2 - b^2c^3d^2g - 2b^2c^3e^2f - a^2c^3f^2g + 2b^3c^2e^2g + 4ab^2c^2f^2g - 3ab^2c^3e^2g) / c^4 + (b(2c^3d - b^3g - 2ac^2f - b^2c^2e + b^2c^2f + 3ab^2c^2g))^2 / (2c^4(4ac - b^2))) \right) / (2a(4ac - b^2)^{(1/2)}) + \left(\frac{(((8a^2c^4g - 8a^5c^5e + 8ab^4c^4f - 8ab^2c^3g) / c^4 - (8a^2c^2(2b^4g + 2b^2c^2e + 8a^2c^2g - 8a^3c^3e - 2b^3c^3f + 8ab^2c^2f - 10ab^2c^2g)))/(16a^4c^4 - 4b^2c^3)) * (2c^3d - b^3g - 2ac^2f - b^2c^2e + b^2c^2f + 3ab^2c^2g)) / (8c^3(4ac - b^2)^{(1/2)}) - (a(2c^3d - b^3g - 2ac^2f - b^2c^2e + b^2c^2f + 3ab^2c^2g)) * (2b^4g + 2b^2c^2e + 8a^2c^2g - 8a^3c^3e - 2b^3c^3f + 8ab^2c^2f - 10ab^2c^2g)) / (c(4ac - b^2)^{(1/2)} * (16a^4c^4 - 4b^2c^3)) \right) / a + (b * \left(\frac{(((8a^2c^4g - 8a^5c^5e + 8ab^4c^4f - 8ab^2c^3g) / c^4 - (8a^2c^2(2b^4g + 2b^2c^2e + 8a^2c^2g - 8a^3c^3e - 2b^3c^3f + 8ab^2c^2f - 10ab^2c^2g)))/(16a^4c^4 - 4b^2c^3)) * (2b^4g + 2b^2c^2e + 8a^2c^2g - 8a^3c^3e - 2b^3c^3f + 8ab^2c^2f - 10ab^2c^2g)) / (2 * (16a^4c^4 - 4b^2c^3)) - (ac^4e^2 + ab^4g^2 + a^3c^2g^2 + ab^2c^2f^2 - 2a^2b^2c^2g^2 - 2a^2c^3e^2g + 2ab^2c^2e^2g + 2a^2b^2c^2f^2g - 2ab^2c^3e^2f - 2ab^3c^3f^2g) / c^4 + (a(2c^3d - b^3g - 2ac^2f - b^2c^2e + b^2c^2f + 3ab^2c^2g))^2 / (c^4(4ac - b^2))) \right) / (2a(4ac - b^2)^{(1/2)}) \right) / (4c^6d^2 + b^6g^2 + 4a^2c^4f^2 + b^2c^4e^2 + b^4c^2f^2 - 4ab^2c^3f^2 - 8a^5c^5d^2f - 4b^5c^5d^2e - 2b^5c^5f^2g + 9a^2b^2c^2g^2 - 6ab^4c^2g^2 + 4b^2c^4d^2f - 4b^3c^3d^2g - 2b^3c^3e^2f + 2b^4c^2e^2g - 6ab^2c^3e^2g + 10ab^3c^2f^2g - 12a^2b^2c^3f^2g + 12ab^2c^4d^2g + 4ab^2c^4e^2f) * (2c^3d - b^3g - 2ac^2f - b^2c^2e + b^2c^2f + 3ab^2c^2g)) / (2c^3(4ac - b^2)^{(1/2)}) \end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(g*x**6+f*x**4+e*x**2+d)/(c*x**4+b*x**2+a), x)

[Out] Timed out

3.124 $\int \frac{x^4(d+ex^2+fx^4+gx^6)}{(a+bx^2+cx^4)^2} dx$

Optimal. Leaf size=594

$$\frac{x \left(a \left(-c^2(2af + be) + bc(3ag + bf) + b^3(-g) + 2c^3d \right) + x^2 \left(-b^2c(ce - 4ag) + bc^2(cd - 3af) + 2ac^2(ce - ag) + b^4(-g) \right) \right)}{2c^3 \left(b^2 - 4ac \right) \left(a + bx^2 + cx^4 \right)}$$

Rubi [A] time = 14.11, antiderivative size = 594, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {1668, 1676, 1166, 205}

$$\frac{x \left(\frac{c^2(d+ex^2+fx^4+gx^6)}{2c^3(b^2-4ac)} + \frac{c^2(d+ex^2+fx^4+gx^6)}{2c^3(b^2-4ac)} \right)}{2c^3(b^2-4ac)\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4)^2,x]
[Out] ((c*f - 2*b*g)*x)/c^3 + (g*x^3)/(3*c^2) + (x*(a*(2*c^3*d - c^2*(b*e + 2*a*f) - b^3*g + b*c*(b*f + 3*a*g)) + (b^3*c*f + b*c^2*(c*d - 3*a*f) - b^4*g - b^2*c*(c*e - 4*a*g) + 2*a*c^2*(c*e - a*g))*x^2))/(2*c^3*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((3*b^3*c*f - b*c^2*(c*d + 13*a*f) - 5*b^4*g - b^2*c*(c*e - 24*a*g) + 2*a*c^2*(3*c*e - 7*a*g) - (3*b^4*c*f - 4*a*c^3*(c*d - 5*a*f) - b^2*c^2*(c*d + 19*a*f) - 5*b^5*g - b^3*c*(c*e - 34*a*g) + 4*a*b*c^2*(2*c*e - 13*a*g))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/(2*Sqrt[2]*c^(7/2)*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((3*b^3*c*f - b*c^2*(c*d + 13*a*f) - 5*b^4*g - b^2*c*(c*e - 24*a*g) + 2*a*c^2*(3*c*e - 7*a*g) + (3*b^4*c*f - 4*a*c^3*(c*d - 5*a*f) - b^2*c^2*(c*d + 19*a*f) - 5*b^5*g - b^3*c*(c*e - 34*a*g) + 4*a*b*c^2*(2*c*e - 13*a*g))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]/(2*Sqrt[2]*c^(7/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1668

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1676

`Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1`

Rubi steps

$$\begin{aligned} \int \frac{x^4 (d + ex^2 + fx^4 + gx^6)}{(a + bx^2 + cx^4)^2} dx &= \frac{x (a (2c^3 d - c^2 (be + 2af) - b^3 g + bc (bf + 3ag)) + (b^3 cf + bc^2 (cd - 3af) - b^3 c^2 e + b^2 c^2 f - b^3 cg) x^2 + a^2 c (3b^3 g - 2c (f + gx^2)) + a (-b^3 g + 2c^3 (d + ex^2) - b c^2 (e + 3f x^2) + b^2 c (f + 4g x^2)))}{2c^3 (b^2 - 4ac) (a + bx^2 + cx^4)} \\ &= \frac{x (a (2c^3 d - c^2 (be + 2af) - b^3 g + bc (bf + 3ag)) + (b^3 cf + bc^2 (cd - 3af) - b^3 c^2 e + b^2 c^2 f - b^3 cg) x^2 + a^2 c (3b^3 g - 2c (f + gx^2)) + a (-b^3 g + 2c^3 (d + ex^2) - b c^2 (e + 3f x^2) + b^2 c (f + 4g x^2)))}{2c^3 (b^2 - 4ac) (a + bx^2 + cx^4)} \\ &= \frac{(cf - 2bg)x}{c^3} + \frac{gx^3}{3c^2} + \frac{x (a (2c^3 d - c^2 (be + 2af) - b^3 g + bc (bf + 3ag)) + (b^3 cf + bc^2 (cd - 3af) - b^3 c^2 e + b^2 c^2 f - b^3 cg) x^2 + a^2 c (3b^3 g - 2c (f + gx^2)) + a (-b^3 g + 2c^3 (d + ex^2) - b c^2 (e + 3f x^2) + b^2 c (f + 4g x^2)))}{2c^3 (b^2 - 4ac)} \\ &= \frac{(cf - 2bg)x}{c^3} + \frac{gx^3}{3c^2} + \frac{x (a (2c^3 d - c^2 (be + 2af) - b^3 g + bc (bf + 3ag)) + (b^3 cf + bc^2 (cd - 3af) - b^3 c^2 e + b^2 c^2 f - b^3 cg) x^2 + a^2 c (3b^3 g - 2c (f + gx^2)) + a (-b^3 g + 2c^3 (d + ex^2) - b c^2 (e + 3f x^2) + b^2 c (f + 4g x^2)))}{2c^3 (b^2 - 4ac)} \\ &= \frac{(cf - 2bg)x}{c^3} + \frac{gx^3}{3c^2} + \frac{x (a (2c^3 d - c^2 (be + 2af) - b^3 g + bc (bf + 3ag)) + (b^3 cf + bc^2 (cd - 3af) - b^3 c^2 e + b^2 c^2 f - b^3 cg) x^2 + a^2 c (3b^3 g - 2c (f + gx^2)) + a (-b^3 g + 2c^3 (d + ex^2) - b c^2 (e + 3f x^2) + b^2 c (f + 4g x^2)))}{2c^3 (b^2 - 4ac)} \end{aligned}$$

Mathematica [A] time = 2.64, size = 721, normalized size = 1.21

Integrate[(x^4*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4)^2, x]

Antiderivative was successfully verified.

[In] Integrate[(x^4*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4)^2, x]

[Out] (12* \sqrt{c} *(c*f - 2*b*g)*x + 4*c^(3/2)*g*x^3 + (6* \sqrt{c} *x*(b*(c^3*d - b*c^2*e + b^2*c*f - b^3*g)*x^2 + a^2*c*(3*b*g - 2*c*(f + g*x^2)) + a*(-b^3*g + 2*c^3*(d + e*x^2) - b*c^2*(e + 3*f*x^2) + b^2*c*(f + 4*g*x^2))))/(b^2 - 4*a*c)*(a + b*x^2 + c*x^4) + (3* $\sqrt{2}$ *(-5*b^5*g - b^3*c*(c*e + 3* $\sqrt{2}$ *b^2 - 4*a*c)*f - 34*a*g) + b^4*(3*c*f + 5* $\sqrt{2}$ *b^2 - 4*a*c)*g) + 2*a*c^2*(-2*c^2*d - 3*c* $\sqrt{2}$ *b^2 - 4*a*c)*e + 10*a*c*f + 7*a* $\sqrt{2}$ *b^2 - 4*a*c)*g) - b^2*c*(c^2*d - c* $\sqrt{2}$ *b^2 - 4*a*c)*e + 19*a*c*f + 24*a* $\sqrt{2}$ *b^2 - 4*a*c)*g) + b*c^2*(c*($\sqrt{2}$ *b^2 - 4*a*c)*d + 8*a*e) + 13*a*($\sqrt{2}$ *b^2 - 4*a*c)*f - 4*a*g))*ArcTan[($\sqrt{2}$ * \sqrt{c} *x)/ $\sqrt{b - \sqrt{b^2 - 4*a*c}}$]]/(b^2 - 4*a*c)^(3/2)* $\sqrt{b - \sqrt{b^2 - 4*a*c}}$) + (3* $\sqrt{2}$ *(5*b^5*g + b^3*c*(c*e - 3* $\sqrt{2}$ *b^2 - 4*a*c)*f - 34*a*g) + b^4*(-3*c*f + 5* $\sqrt{2}$ *b^2 - 4*a*c)*g) + b^2*c*(c^2*d + c* $\sqrt{2}$ *b^2 - 4*a*c)*e + 19*a*c*f - 24*a* $\sqrt{2}$ *b^2 - 4*a*c)*g) + 2*a*c^2*(2*c^2*d - 3*c* $\sqrt{2}$ *b^2 - 4*a*c)*e - 10*a*c*f + 7*a* $\sqrt{2}$ *b^2 - 4*a*c)*g) + b*c^2*(c*($\sqrt{2}$ *b^2 - 4*a*c)*d - 8*a*e) + 13*a*($\sqrt{2}$ *b^2 - 4*a*c)*f + 4*a*g))*ArcTan[($\sqrt{2}$ * \sqrt{c} *x)/ $\sqrt{b + \sqrt{b^2 - 4*a*c}}$]]/(b^2 - 4*a*c)^(3/2)* $\sqrt{b + \sqrt{b^2 - 4*a*c}}$)]/(12*c^(7/2))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 (d + ex^2 + fx^4 + gx^6)}{(a + bx^2 + cx^4)^2} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^4*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4)^2,
x]
```

```
[Out] IntegrateAlgebraic[(x^4*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4)^2,
x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [B] time = 9.95, size = 10761, normalized size = 18.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*c^3*d*x^3 + b^3*c*f*x^3 - 3*a*b*c^2*f*x^3 - b^4*g*x^3 + 4*a*b^2*c*g*
x^3 - 2*a^2*c^2*g*x^3 - b^2*c^2*x^3*e + 2*a*c^3*x^3*e + 2*a*c^3*d*x + a*b^2
*c*f*x - 2*a^2*c^2*f*x - a*b^3*g*x + 3*a^2*b*c*g*x - a*b*c^2*x*e)/((b^2*c^3
- 4*a*c^4)*(c*x^4 + b*x^2 + a)) + 1/16*((2*b^3*c^5 - 8*a*b*c^6 - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 4*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 + 2*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^5 - 2*(b^2 - 4*a*c)*b*c^5*(b^2*c^3 - 4
*a*c^4)^2*d - (6*b^5*c^3 - 50*a*b^3*c^4 + 104*a^2*b*c^5 - 3*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c + 25*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 52*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 26*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 13*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 6*(b^2 - 4*a*c)*b^3*c^3 + 26*(b^2 - 4*a*c)*a
*b*c^4*(b^2*c^3 - 4*a*c^4)^2*f + (10*b^6*c^2 - 88*a*b^4*c^3 + 220*a^2*b^2*
c^4 - 112*a^3*c^5 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*b^6 + 44*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b
^4*c + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c -
110*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2
- 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 -
5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 56*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 28*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 24*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 14*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 10*(b^2 - 4*a*c)*
b^4*c^2 + 48*(b^2 - 4*a*c)*a*b^2*c^3 - 28*(b^2 - 4*a*c)*a^2*c^4*(b^2*c^3 -
4*a*c^4)^2*g + (2*b^4*c^4 - 20*a*b^2*c^5 + 48*a^2*c^6 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 10*sqrt(2)*sqrt(b^2 - 4*a
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*b^2*c^4 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*c^5 - 2*(b^2 - 4*a*c)*b^2*c^4 + 12*(b^2 - 4*a*c)*a*c^5*(b^2
```

$$\begin{aligned}
& *c^3 - 4*a*c^4)^2*e - 4*(\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a*b^4*c^7 \\
& - 8*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^2*b^2*c^8 - 2*\text{sqrt}(2)*\text{sqrt}(b* \\
& c + \text{sqrt}(b^2 - 4*a*c))*c)*a*b^3*c^8 - 2*a*b^4*c^8 + 16*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sq} \\
& \text{rt}(b^2 - 4*a*c))*c)*a^3*c^9 + 8*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^2* \\
& b*c^9 + \text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a*b^2*c^9 + 16*a^2*b^2*c^9 \\
& - 4*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^2*c^10 - 32*a^3*c^10 + 2*(b^2 \\
& - 4*a*c)*a*b^2*c^8 - 8*(b^2 - 4*a*c)*a^2*c^9)*d*\text{abs}(b^2*c^3 - 4*a*c^4) - 2 \\
& *(3*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a*b^6*c^5 - 34*\text{sqrt}(2)*\text{sqrt}(b*c \\
& + \text{sqrt}(b^2 - 4*a*c))*c)*a^2*b^4*c^6 - 6*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c) \\
&)*c)*a*b^5*c^6 - 6*a*b^6*c^6 + 128*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)* \\
& a^3*b^2*c^7 + 44*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^2*b^3*c^7 + 3*\text{sq} \\
& \text{rt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a*b^4*c^7 + 68*a^2*b^4*c^7 - 160*\text{sqrt} \\
& (2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^4*c^8 - 80*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^ \\
& 2 - 4*a*c))*c)*a^3*b*c^8 - 22*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^2*b^ \\
& 2*c^8 - 256*a^3*b^2*c^8 + 40*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^3*c^ \\
& 9 + 320*a^4*c^9 + 6*(b^2 - 4*a*c)*a*b^4*c^6 - 44*(b^2 - 4*a*c)*a^2*b^2*c^7 \\
& + 80*(b^2 - 4*a*c)*a^3*c^8)*f*\text{abs}(b^2*c^3 - 4*a*c^4) + 2*(5*\text{sqrt}(2)*\text{sqrt}(b* \\
& c + \text{sqrt}(b^2 - 4*a*c))*c)*a*b^7*c^4 - 59*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c) \\
&)*c)*a^2*b^5*c^5 - 10*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a*b^6*c^5 - 1 \\
& 0*a*b^7*c^5 + 232*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^3*b^3*c^6 + 78* \\
& \text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^2*b^4*c^6 + 5*\text{sqrt}(2)*\text{sqrt}(b*c + \\
& \text{sqrt}(b^2 - 4*a*c))*c)*a*b^5*c^6 + 118*a^2*b^5*c^6 - 304*\text{sqrt}(2)*\text{sqrt}(b*c + \text{s} \\
& \text{qrt}(b^2 - 4*a*c))*c)*a^4*b*c^7 - 152*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c) \\
& *a^3*b^2*c^7 - 39*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^2*b^3*c^7 - 464 \\
& *a^3*b^3*c^7 + 76*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^3*b*c^8 + 608*a \\
& ^4*b*c^8 + 10*(b^2 - 4*a*c)*a*b^5*c^5 - 78*(b^2 - 4*a*c)*a^2*b^3*c^6 + 152* \\
& (b^2 - 4*a*c)*a^3*b*c^7)*g*\text{abs}(b^2*c^3 - 4*a*c^4) + 2*(\text{sqrt}(2)*\text{sqrt}(b*c + \text{s} \\
& \text{qrt}(b^2 - 4*a*c))*c)*a*b^5*c^6 - 8*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a \\
& ^2*b^3*c^7 - 2*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a*b^4*c^7 - 2*a*b^5* \\
& c^7 + 16*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^3*b*c^8 + 8*\text{sqrt}(2)*\text{sqrt} \\
& (b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^2*b^2*c^8 + \text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a \\
& *c))*c)*a*b^3*c^8 + 16*a^2*b^3*c^8 - 4*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))* \\
& c)*a^2*b*c^9 - 32*a^3*b*c^9 + 2*(b^2 - 4*a*c)*a*b^3*c^7 - 8*(b^2 - 4*a*c)*a \\
& ^2*b*c^8)*\text{abs}(b^2*c^3 - 4*a*c^4)*e - (2*b^7*c^11 - 8*a*b^5*c^12 - 32*a^2*b^ \\
& 3*c^13 + 128*a^3*b*c^14 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4 \\
& *a*c))*c)*b^7*c^9 + 4*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c) \\
&)*c)*a*b^5*c^10 + 2*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c) \\
& *b^6*c^10 + 16*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a \\
& ^2*b^3*c^11 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*b^5 \\
& *c^11 - 64*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^3*b* \\
& c^12 - 32*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^2*b^2 \\
& *c^12 + 16*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^2*b* \\
& c^13 - 2*(b^2 - 4*a*c)*b^5*c^11 + 32*(b^2 - 4*a*c)*a^2*b*c^13)*d + (6*b^9*c \\
& ^9 - 86*a*b^7*c^10 + 440*a^2*b^5*c^11 - 928*a^3*b^3*c^12 + 640*a^4*b*c^13 - \\
& 3*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*b^9*c^7 + 43*s \\
& \text{qrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a*b^7*c^8 + 6*\text{sqrt} \\
& (2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*b^8*c^8 - 220*\text{sqrt}(2) \\
& *\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^2*b^5*c^9 - 62*\text{sqrt}(2) \\
& *\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a*b^6*c^9 - 3*\text{sqrt}(2)*\text{sq} \\
& \text{rt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*b^7*c^9 + 464*\text{sqrt}(2)*\text{sqrt}(\\
& b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^3*b^3*c^10 + 192*\text{sqrt}(2)*\text{sq} \\
& \text{rt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^2*b^4*c^10 + 31*\text{sqrt}(2)*\text{sq} \\
& \text{rt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a*b^5*c^10 - 320*\text{sqrt}(2)*\text{sq} \\
& \text{rt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^4*b*c^11 - 160*\text{sqrt}(2)*\text{sq} \\
& \text{rt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^3*b^2*c^11 - 96*\text{sqrt}(2)*\text{s} \\
& \text{qrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^2*b^3*c^11 + 80*\text{sqrt}(2)* \\
& \text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^3*b*c^12 - 6*(b^2 - 4*a \\
& *c)*b^7*c^9 + 62*(b^2 - 4*a*c)*a*b^5*c^10 - 192*(b^2 - 4*a*c)*a^2*b^3*c^11 \\
& + 160*(b^2 - 4*a*c)*a^3*b*c^12)*f - (10*b^10*c^8 - 148*a*b^8*c^9 + 808*a^2*
\end{aligned}$$

$$\begin{aligned}
& b^6c^{10} - 1920a^3b^4c^{11} + 1664a^4b^2c^{12} - 5\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}\sqrt{b^2 - 4ac} \\
& \sqrt{bc + \sqrt{b^2 - 4ac}}\sqrt{b^2 - 4ac}b^{10}c^6 + 74\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}\sqrt{b^2 - 4ac} \\
& \sqrt{bc + \sqrt{b^2 - 4ac}}\sqrt{b^2 - 4ac}a^2b^8c^7 + 10\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}\sqrt{b^2 - 4ac} \\
& \sqrt{bc + \sqrt{b^2 - 4ac}}\sqrt{b^2 - 4ac}a^2b^6c^8 - 108\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}\sqrt{b^2 - 4ac} \\
& \sqrt{bc + \sqrt{b^2 - 4ac}}\sqrt{b^2 - 4ac}a^2b^7c^8 - 5\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}\sqrt{b^2 - 4ac} \\
& \sqrt{bc + \sqrt{b^2 - 4ac}}\sqrt{b^2 - 4ac}b^8c^8 + 960\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}\sqrt{b^2 - 4ac} \\
& \sqrt{bc + \sqrt{b^2 - 4ac}}\sqrt{b^2 - 4ac}a^3b^4c^9 + 376\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}\sqrt{b^2 - 4ac} \\
& \sqrt{bc + \sqrt{b^2 - 4ac}}\sqrt{b^2 - 4ac}a^2b^5c^9 + 54\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}\sqrt{b^2 - 4ac} \\
& \sqrt{bc + \sqrt{b^2 - 4ac}}\sqrt{b^2 - 4ac}a^2b^6c^9 - 832\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}\sqrt{b^2 - 4ac} \\
& \sqrt{bc + \sqrt{b^2 - 4ac}}\sqrt{b^2 - 4ac}a^4b^2c^{10} - 416\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}\sqrt{b^2 - 4ac} \\
& \sqrt{bc + \sqrt{b^2 - 4ac}}\sqrt{b^2 - 4ac}a^3b^3c^{10} - 188\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}\sqrt{b^2 - 4ac} \\
& \sqrt{bc + \sqrt{b^2 - 4ac}}\sqrt{b^2 - 4ac}a^2b^4c^{10} + 208\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}\sqrt{b^2 - 4ac} \\
& \sqrt{bc + \sqrt{b^2 - 4ac}}\sqrt{b^2 - 4ac}a^3b^2c^{11} - 10(b^2 - 4ac)b^8c^8 + 108(b^2 - 4ac)a^2b^6c^9 - 376(b^2 - 4ac)a^2b^4c^{10} \\
& + 416(b^2 - 4ac)a^3b^2c^{11})g - (2b^8c^{10} - 32a^2b^6c^{11} + 160a^2b^4c^{12} - 256a^3b^2c^{13} - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}\sqrt{b^2 - 4ac}) \\
& b^8c^8 + 16\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}\sqrt{b^2 - 4ac}a^2b^6c^9 + 2\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}\sqrt{b^2 - 4ac} \\
& \sqrt{bc + \sqrt{b^2 - 4ac}}\sqrt{b^2 - 4ac}a^2b^4c^{10} - 24\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}\sqrt{b^2 - 4ac} \\
& \sqrt{bc + \sqrt{b^2 - 4ac}}\sqrt{b^2 - 4ac}a^2b^5c^{10} - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}\sqrt{b^2 - 4ac} \\
& \sqrt{bc + \sqrt{b^2 - 4ac}}\sqrt{b^2 - 4ac}b^6c^{10} + 128\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}\sqrt{b^2 - 4ac} \\
& \sqrt{bc + \sqrt{b^2 - 4ac}}\sqrt{b^2 - 4ac}a^3b^2c^{11} + 64\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}\sqrt{b^2 - 4ac} \\
& \sqrt{bc + \sqrt{b^2 - 4ac}}\sqrt{b^2 - 4ac}a^2b^3c^{11} + 12\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}\sqrt{b^2 - 4ac} \\
& \sqrt{bc + \sqrt{b^2 - 4ac}}\sqrt{b^2 - 4ac}a^2b^4c^{11} - 32\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}\sqrt{b^2 - 4ac} \\
& \sqrt{bc + \sqrt{b^2 - 4ac}}\sqrt{b^2 - 4ac}a^2b^2c^{12} - 2(b^2 - 4ac)b^6c^{10} + 24(b^2 - 4ac)a^2b^4c^{11} - 64(b^2 - 4ac)a^2b^2c^{12})e \\
& \arctan(2\sqrt{1/2}x/\sqrt{(b^3c^3 - 4aab^2c^4 + \sqrt{(b^3c^3 - 4aab^2c^4)^2 - 4(a^2b^2c^3 - 4a^2c^4)(b^2c^4 - 4aac^5))})/(b^2c^4 - 4aac^5)) \\
& /((a^2b^6c^7 - 12a^2b^4c^8 - 2a^2b^5c^8 + 48a^3b^2c^9 + 16a^2b^3c^9 + a^2b^4c^9 - 64a^4c^{10} - 32a^3b^2c^{10} - 8a^2b^2c^{10} + 16a^3c^{11}) \\
& \text{abs}(b^2c^3 - 4aac^4)\text{abs}(c)) + 1/16((2b^3c^5 - 8aab^2c^6 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}\sqrt{b^2 - 4ac}) \\
& b^3c^3 + 4\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}\sqrt{b^2 - 4ac}a^2b^2c^4 + 2\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}\sqrt{b^2 - 4ac}) \\
& b^2c^4 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}\sqrt{b^2 - 4ac}b^2c^5 - 2(b^2 - 4ac)b^2c^5)(b^2c^3 - 4aac^4)^2d - (6b^5c^3 - 50a^2b^3c^4 + 104a^2b^2c^5 - 3\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}\sqrt{b^2 - 4ac}) \\
& b^5c + 25\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}\sqrt{b^2 - 4ac}a^2b^3c^2 + 6\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}\sqrt{b^2 - 4ac}) \\
& b^4c^2 - 52\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}\sqrt{b^2 - 4ac}a^2b^2c^3 - 26\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}\sqrt{b^2 - 4ac}) \\
& b^2c^3 - 3\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}\sqrt{b^2 - 4ac}) \\
& \sqrt{bc - \sqrt{b^2 - 4ac}}\sqrt{b^2 - 4ac}b^3c^3 + 13\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}\sqrt{b^2 - 4ac}) \\
& \sqrt{bc - \sqrt{b^2 - 4ac}}\sqrt{b^2 - 4ac}a^2b^2c^3 - 6(b^2 - 4ac)b^3c^3 + 26(b^2 - 4ac)a^2b^2c^3)(b^2c^3 - 4aac^4)^2f + (10b^6c^2 - 88a^2b^4c^3 + 2 \\
& 20a^2b^2c^4 - 112a^3c^5 - 5\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}\sqrt{b^2 - 4ac}) \\
& b^6 + 44\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}\sqrt{b^2 - 4ac}a^2b^4c + 10\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}\sqrt{b^2 - 4ac}) \\
& b^5c - 110\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}\sqrt{b^2 - 4ac}a^2b^2c^2 - 48\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}\sqrt{b^2 - 4ac}) \\
& a^2b^2c^2 - 5\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}\sqrt{b^2 - 4ac}b^4c^2 + 56\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}\sqrt{b^2 - 4ac}) \\
& a^3c^3 + 28\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}\sqrt{b^2 - 4ac}a^2b^2c^3 + 24\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}\sqrt{b^2 - 4ac}) \\
& a^2b^2c^3 - 14\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}\sqrt{b^2 - 4ac}a^2c^4 - 10(b^2 - 4ac)b^4c^2 + 48(b^2 - 4ac)a^2b^2c^3 - 28(b^2 - 4ac)a^2c^4) \\
& (b^2c^3 - 4aac^4)^2g + (2b^4c^4 - 20a^2b^2c^5 + 48a^2c^6 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}\sqrt{b^2 - 4ac}) \\
& b^4c^2 + 10\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}\sqrt{b^2 - 4ac}) \\
& \sqrt{bc - \sqrt{b^2 - 4ac}}\sqrt{b^2 - 4ac}a^2b^2c^3 + 2\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}\sqrt{b^2 - 4ac})
\end{aligned}$$

$$\begin{aligned}
&^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*b^3*c^3 - 24*\text{sqrt}(2)*\text{sqrt}(b^2 - \\
&4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*a^2*c^4 - 12*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a \\
&*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*a*b*c^4 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sq} \\
&\text{rt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*b^2*c^4 + 6*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c \\
&- \text{sqrt}(b^2 - 4*a*c)*c)*a*c^5 - 2*(b^2 - 4*a*c)*b^2*c^4 + 12*(b^2 - 4*a*c)* \\
&a*c^5)*(b^2*c^3 - 4*a*c^4)^2*e - 4*(\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c) \\
&)*a*b^4*c^7 - 8*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*a^2*b^2*c^8 - 2*\text{sqrt} \\
&(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*a*b^3*c^8 + 2*a*b^4*c^8 + 16*\text{sqrt}(2)*\text{sq} \\
&\text{rt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*a^3*c^9 + 8*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a \\
&*c)*c)*a^2*b*c^9 + \text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*a*b^2*c^9 - 16*a \\
&^2*b^2*c^9 - 4*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*a^2*c^10 + 32*a^3*c^ \\
&10 - 2*(b^2 - 4*a*c)*a*b^2*c^8 + 8*(b^2 - 4*a*c)*a^2*c^9)*d*\text{abs}(b^2*c^3 - 4 \\
&*a*c^4) - 2*(3*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*a*b^6*c^5 - 34*\text{sqrt}(\\
&2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*a^2*b^4*c^6 - 6*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(\\
&b^2 - 4*a*c)*c)*a*b^5*c^6 + 6*a*b^6*c^6 + 128*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - \\
&4*a*c)*c)*a^3*b^2*c^7 + 44*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*a^2*b^3 \\
&*c^7 + 3*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*a*b^4*c^7 - 68*a^2*b^4*c^7 \\
&- 160*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*a^4*c^8 - 80*\text{sqrt}(2)*\text{sqrt}(b* \\
&c - \text{sqrt}(b^2 - 4*a*c)*c)*a^3*b*c^8 - 22*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c \\
&)*c)*a^2*b^2*c^8 + 256*a^3*b^2*c^8 + 40*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c \\
&)*c)*a^3*c^9 - 320*a^4*c^9 - 6*(b^2 - 4*a*c)*a*b^4*c^6 + 44*(b^2 - 4*a*c)*a \\
&^2*b^2*c^7 - 80*(b^2 - 4*a*c)*a^3*c^8)*f*\text{abs}(b^2*c^3 - 4*a*c^4) + 2*(5*\text{sqrt} \\
&(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*a*b^7*c^4 - 59*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(\\
&b^2 - 4*a*c)*c)*a^2*b^5*c^5 - 10*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*a* \\
&b^6*c^5 + 10*a*b^7*c^5 + 232*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*a^3*b^ \\
&3*c^6 + 78*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*a^2*b^4*c^6 + 5*\text{sqrt}(2)* \\
&\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*a*b^5*c^6 - 118*a^2*b^5*c^6 - 304*\text{sqrt}(2)*\text{s} \\
&\text{qrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*a^4*b*c^7 - 152*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 \\
&- 4*a*c)*c)*a^3*b^2*c^7 - 39*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*a^2*b^ \\
&3*c^7 + 464*a^3*b^3*c^7 + 76*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*a^3*b* \\
&c^8 - 608*a^4*b*c^8 - 10*(b^2 - 4*a*c)*a*b^5*c^5 + 78*(b^2 - 4*a*c)*a^2*b^3 \\
&*c^6 - 152*(b^2 - 4*a*c)*a^3*b*c^7)*g*\text{abs}(b^2*c^3 - 4*a*c^4) + 2*(\text{sqrt}(2)*\text{s} \\
&\text{qrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*a*b^5*c^6 - 8*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - \\
&4*a*c)*c)*a^2*b^3*c^7 - 2*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*a*b^4*c^7 \\
&+ 2*a*b^5*c^7 + 16*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*a^3*b*c^8 + 8*s \\
&\text{qrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*a^2*b^2*c^8 + \text{sqrt}(2)*\text{sqrt}(b*c - \text{sq} \\
&\text{rt}(b^2 - 4*a*c)*c)*a*b^3*c^8 - 16*a^2*b^3*c^8 - 4*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^ \\
&2 - 4*a*c)*c)*a^2*b*c^9 + 32*a^3*b*c^9 - 2*(b^2 - 4*a*c)*a*b^3*c^7 + 8*(b^2 \\
&- 4*a*c)*a^2*b*c^8)*\text{abs}(b^2*c^3 - 4*a*c^4)*e - (2*b^7*c^11 - 8*a*b^5*c^12 \\
&- 32*a^2*b^3*c^13 + 128*a^3*b*c^14 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - s \\
&\text{qrt}(b^2 - 4*a*c)*c)*b^7*c^9 + 4*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b \\
&^2 - 4*a*c)*c)*a*b^5*c^10 + 2*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 \\
&- 4*a*c)*c)*b^6*c^10 + 16*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - \\
&4*a*c)*c)*a^2*b^3*c^11 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4* \\
&a*c)*c)*b^5*c^11 - 64*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c \\
&)*c)*a^3*b*c^12 - 32*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c) \\
&)*c)*a^2*b^2*c^12 + 16*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c \\
&)*c)*a^2*b*c^13 - 2*(b^2 - 4*a*c)*b^5*c^11 + 32*(b^2 - 4*a*c)*a^2*b*c^13)*d \\
&+ (6*b^9*c^9 - 86*a*b^7*c^10 + 440*a^2*b^5*c^11 - 928*a^3*b^3*c^12 + 640*a \\
&^4*b*c^13 - 3*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*b^9 \\
&*c^7 + 43*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*a*b^7*c \\
&^8 + 6*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*b^8*c^8 - \\
&220*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*a^2*b^5*c^9 - \\
&62*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*a*b^6*c^9 - 3 \\
&*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*b^7*c^9 + 464*\text{sq} \\
&\text{rt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*a^3*b^3*c^10 + 192* \\
&\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*a^2*b^4*c^10 + 31 \\
&*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*a*b^5*c^10 - 320 \\
&*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*a^4*b*c^11 - 160
\end{aligned}$$

```

*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^11 - 9
6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^11 +
80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^12 - 6
*(b^2 - 4*a*c)*b^7*c^9 + 62*(b^2 - 4*a*c)*a*b^5*c^10 - 192*(b^2 - 4*a*c)*a^
2*b^3*c^11 + 160*(b^2 - 4*a*c)*a^3*b*c^12)*f - (10*b^10*c^8 - 148*a*b^8*c^9
+ 808*a^2*b^6*c^10 - 1920*a^3*b^4*c^11 + 1664*a^4*b^2*c^12 - 5*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^10*c^6 + 74*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^8*c^7 + 10*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^9*c^7 - 404*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^8 - 108*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^7*c^8 - 5*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^8*c^8 + 960*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^9 + 376*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^9 + 54*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^9 - 832*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^10 - 416*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^10 - 188*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^10 + 208*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^11 - 10*(b^2 - 4*a*c)*b^8
*c^8 + 108*(b^2 - 4*a*c)*a*b^6*c^9 - 376*(b^2 - 4*a*c)*a^2*b^4*c^10 + 416*(
b^2 - 4*a*c)*a^3*b^2*c^11)*g - (2*b^8*c^10 - 32*a*b^6*c^11 + 160*a^2*b^4*c^
12 - 256*a^3*b^2*c^13 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*b^8*c^8 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a*b^6*c^9 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
b^7*c^9 - 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*
b^4*c^10 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b
^5*c^10 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^1
0 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c
^11 + 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*
c^11 + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c
^11 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*
c^12 - 2*(b^2 - 4*a*c)*b^6*c^10 + 24*(b^2 - 4*a*c)*a*b^4*c^11 - 64*(b^2 - 4
*a*c)*a^2*b^2*c^12)*e)*arctan(2*sqrt(1/2)*x/sqrt((b^3*c^3 - 4*a*b*c^4 - sqr
t((b^3*c^3 - 4*a*b*c^4)^2 - 4*(a*b^2*c^3 - 4*a^2*c^4)*(b^2*c^4 - 4*a*c^5)))
/(b^2*c^4 - 4*a*c^5)))/((a*b^6*c^7 - 12*a^2*b^4*c^8 - 2*a*b^5*c^8 + 48*a^3*
b^2*c^9 + 16*a^2*b^3*c^9 + a*b^4*c^9 - 64*a^4*c^10 - 32*a^3*b*c^10 - 8*a^2*
b^2*c^10 + 16*a^3*c^11)*abs(b^2*c^3 - 4*a*c^4)*abs(c)) + 1/3*(c^4*g*x^3 + 3
*c^4*f*x - 6*b*c^3*g*x)/c^6

```

maple [B] time = 0.06, size = 3028, normalized size = 5.10

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^4*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2, x)$

[Out]
$$-1/4/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*b^3/c*e*arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)+13/4/c/(4*a*c-b^2)*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*a*b*f-13/4/(4*a*c-b^2)*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*a*b/c*f*arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)+1/3*g*x^3/c^2-5/4/c^3/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^5*g+6/c^2/(4*a*c-b^2)*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*a*b^2*g-5/4/c^3/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^5*g-6/c^2/(4*a*c-b^2)*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*a*b^2*g-2/c^3*x*b*g+1/c^2*f*x-2/c^2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^3*a*b^2*g-13/c/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}$$

$$\begin{aligned}
& 2)) * c)^{(1/2)} * \operatorname{arctanh}(2^{(1/2)} / ((-b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * cx) * a^2 * b * g \\
& + 17/2 / c^2 / (4ac - b^2) / (-4ac + b^2)^{(1/2)} * 2^{(1/2)} / ((b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctan}(2^{(1/2)} / ((b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * cx) * a^2 * b^3 * g - 13 / c / (4 \\
& * ac - b^2) / (-4ac + b^2)^{(1/2)} * 2^{(1/2)} / ((b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctan}(2^{(1/2)} / ((b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * cx) * a^2 * b * g + 17/2 / c^2 / (4ac - b^2 \\
&) / (-4ac + b^2)^{(1/2)} * 2^{(1/2)} / ((-b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctanh}(2^{(1/2)} / ((-b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * cx) * a * b^3 * g + 1/2 / (cx^4 + b * x^2 + a) / (4ac \\
& * c - b^2) * a * b / c * e * x + 3/2 / (cx^4 + b * x^2 + a) / (4ac - b^2) * a * b / c * f * x^3 - 1/2 / (cx^4 + b * \\
& x^2 + a) / (4ac - b^2) * a * b^2 / c^2 * f * x - 3/4 / c^2 / (4ac - b^2) * 2^{(1/2)} / ((-b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctanh}(2^{(1/2)} / ((-b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * cx) * \\
& b^3 * f + 1/4 / c / (4ac - b^2) * 2^{(1/2)} / ((-b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctanh}(2^{(1/2)} / ((-b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * cx) * b^2 * e + 3/4 / (4ac - b^2) * 2^{(1/2)} \\
& / ((b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * b^3 / c^2 * f * \operatorname{arctan}(2^{(1/2)} / ((b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * cx) - 1/4 / (4ac - b^2) * 2^{(1/2)} / ((b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * b^2 / c * e * \operatorname{arctan}(2^{(1/2)} / ((b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * cx) - 1/4 / (4ac \\
& - b^2) / (-4ac + b^2)^{(1/2)} * 2^{(1/2)} / ((-b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctanh}(2^{(1/2)} / ((-b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * cx) * b^2 * d - 1/4 / (4ac - b^2) / (-4ac \\
& + b^2)^{(1/2)} * 2^{(1/2)} / ((b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * b^2 * d * \operatorname{arctan}(2^{(1/2)} / \\
& ((b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * cx) + 5 / (4ac - b^2) / (-4ac + b^2)^{(1/2)} * 2^{(1/2)} / ((-b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctanh}(2^{(1/2)} / ((-b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * cx) * a^2 * f + 3/4 / c^2 / (4ac - b^2) / (-4ac + b^2)^{(1/2)} * 2^{(1/2)} / ((-b \\
& + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctanh}(2^{(1/2)} / ((-b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * cx) * b^4 * f - 1/4 / c / (4ac - b^2) / (-4ac + b^2)^{(1/2)} * 2^{(1/2)} / ((-b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctanh}(2^{(1/2)} / ((-b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * cx) * \\
& b^3 * e + 2 / (4ac - b^2) / (-4ac + b^2)^{(1/2)} * 2^{(1/2)} / ((b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * a * b * e * \operatorname{arctan}(2^{(1/2)} / ((b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * cx) - 1 / (cx^4 + b * \\
& x^2 + a) / (4ac - b^2) * a * d * x - 1 / (cx^4 + b * x^2 + a) / (4ac - b^2) * a * e * x^3 - 1/2 / (cx^4 + b \\
& * x^2 + a) / (4ac - b^2) * b * d * x^3 - 3/2 / c^2 / (cx^4 + b * x^2 + a) * a^2 / (4ac - b^2) * x * b * g + 1 \\
& / 2 / c^3 / (cx^4 + b * x^2 + a) * a / (4ac - b^2) * x * b^3 * g - 5/4 / c^3 / (4ac - b^2) * 2^{(1/2)} / ((\\
& b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctan}(2^{(1/2)} / ((b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * cx) * b^4 * g + 7/2 / c / (4ac - b^2) * 2^{(1/2)} / ((-b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * \\
& \operatorname{arctanh}(2^{(1/2)} / ((-b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * cx) * a^2 * g + 5/4 / c^3 / (4ac \\
& - b^2) * 2^{(1/2)} / ((-b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctanh}(2^{(1/2)} / ((-b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * cx) * b^4 * g - 7/2 / c / (4ac - b^2) * 2^{(1/2)} / ((b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctan}(2^{(1/2)} / ((b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * cx) * a^2 * \\
& g - 1 / (4ac - b^2) / (-4ac + b^2)^{(1/2)} * 2^{(1/2)} / ((b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * \\
& a * c * d * \operatorname{arctan}(2^{(1/2)} / ((b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * cx) + 3/4 / (4ac - b^2) \\
& / (-4ac + b^2)^{(1/2)} * 2^{(1/2)} / ((b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * b^4 / c^2 * f * \operatorname{arctan}(2^{(1/2)} / ((b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * cx) + 2 / (4ac - b^2) / (-4ac + b^2)^{(1/2)} * 2^{(1/2)} / ((-b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctanh}(2^{(1/2)} / ((-b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * cx) * a * b * e - c / (4ac - b^2) / (-4ac + b^2)^{(1/2)} * 2^{(1/2)} / ((-b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctanh}(2^{(1/2)} / ((-b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * cx) * a * d - 19/4 / c / (4ac - b^2) / (-4ac + b^2)^{(1/2)} * 2^{(1/2)} / ((-b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctanh}(2^{(1/2)} / ((-b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * cx) * a * b^2 * f - 19/4 / (4ac - b^2) / (-4ac + b^2)^{(1/2)} * 2^{(1/2)} / ((b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * a * b^2 / c * f * \operatorname{arctan}(2^{(1/2)} / ((b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * cx) - 1/2 / (cx^4 + b * x^2 + a) / (4ac - b^2) * b^3 / c^2 * f * x^3 + 1/2 / (cx^4 + b * x^2 + a) / (4ac - b^2) * b^2 / c * e * x^3 + 1 / (cx^4 + b * x^2 + a) / (4ac - b^2) * a^2 / c * f * x - 3/2 / (4ac - b^2) * 2^{(1/2)} / ((-b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctanh}(2^{(1/2)} / ((-b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * cx) * a * e + 1/4 / (4ac - b^2) * 2^{(1/2)} / ((-b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctanh}(2^{(1/2)} / ((-b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * cx) * b * d + 3/2 / (4ac - b^2) * 2^{(1/2)} / ((b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * a * e * \operatorname{arctan}(2^{(1/2)} / ((b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * cx) - 1/4 / (4ac - b^2) * 2^{(1/2)} / ((b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * b * d * \operatorname{arctan}(2^{(1/2)} / ((b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * cx) + 5 / (4ac - b^2) / (-4ac + b^2)^{(1/2)} * 2^{(1/2)} / ((b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * a^2 * f * \operatorname{arctan}(2^{(1/2)} / ((b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * cx) + 1 / c / (cx^4 + b * x^2 + a) / (4ac - b^2) * x^3 * a^2 * g + 1/2 / c^3 / (cx^4 + b * x^2 + a) / (4ac - b^2) * x^3 * b^4 * g
\end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(bc^3d - (b^2c^2 - 2ac^3)e + (b^3c - 3abc^2)f - (b^4 - 4ab^2c + 2a^2c^2)g)x^3 + (2ac^3d - abc^2e + (ab^2c - 2a^2c^2)f - (ab^3 - 3a^2bc)g)x + \int \frac{2ac^3d - abc^2e - (bc^3d + (b^2c^2 - 6ac^3)e - (3b^3c - 13ab^2c^2)f + (5b^4 - 24ab^2c + 14a^2c^2)g)^2 + (3ab^2c - 10a^2c^2)f - (5ab^3 - 19a^2bc)g}{c^4 + bx^2 + a} dx + cgx^3 + 3(cf - 2bg)x}{2(ab^2c^3 - 4a^2c^4 + (b^2c^4 - 4ac^5)x^4 + (b^3c^3 - 4abc^4)x^2)} + \frac{c^2gx^3 + 3(cf - 2bg)x}{3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")

[Out] 1/2*((b*c^3*d - (b^2*c^2 - 2*a*c^3)*e + (b^3*c - 3*a*b*c^2)*f - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*g)*x^3 + (2*a*c^3*d - a*b*c^2*e + (a*b^2*c - 2*a^2*c^2)*f - (a*b^3 - 3*a^2*b*c)*g)*x)/(a*b^2*c^3 - 4*a^2*c^4 + (b^2*c^4 - 4*a*c^5)*x^4 + (b^3*c^3 - 4*a*b*c^4)*x^2) + 1/2*integrate(-(2*a*c^3*d - a*b*c^2*e - (b*c^3*d + (b^2*c^2 - 6*a*c^3)*e - (3*b^3*c - 13*a*b*c^2)*f + (5*b^4 - 24*a*b^2*c + 14*a^2*c^2)*g)*x^2 + (3*a*b^2*c - 10*a^2*c^2)*f - (5*a*b^3 - 19*a^2*b*c)*g)/(c*x^4 + b*x^2 + a), x)/(b^2*c^3 - 4*a*c^4) + 1/3*(c*g*x^3 + 3*(c*f - 2*b*g)*x)/c^3

mupad [B] time = 4.73, size = 47339, normalized size = 79.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4)^2,x)

[Out] ((x^3*(b^4*g + b^2*c^2*e + 2*a^2*c^2*g - 2*a*c^3*e - b*c^3*d - b^3*c*f + 3*a*b*c^2*f - 4*a*b^2*c*g))/(2*(4*a*c - b^2)) + (x*(2*a^2*c^2*f - 2*a*c^3*d + a*b^3*g + a*b*c^2*e - a*b^2*c*f - 3*a^2*b*c*g))/(2*(4*a*c - b^2)))/(a*c^3 + c^4*x^4 + b*c^3*x^2) + x*(f/c^2 - (2*b*g)/c^3) + atan((((2048*a^4*c^10*d - 10240*a^5*c^9*f + 384*a^2*b^4*c^8*d - 1536*a^3*b^2*c^9*d - 192*a^2*b^5*c^7*e + 768*a^3*b^3*c^8*e + 736*a^2*b^6*c^6*f - 4224*a^3*b^4*c^7*f + 10752*a^4*b^2*c^8*f - 1264*a^2*b^7*c^5*g + 7488*a^3*b^5*c^6*g - 19712*a^4*b^3*c^7*g - 32*a*b^6*c^7*d + 16*a*b^7*c^6*e - 1024*a^4*b*c^9*e - 48*a*b^8*c^5*f + 80*a*b^9*c^4*g + 19456*a^5*b*c^8*g)/(8*(64*a^3*c^8 - b^6*c^5 + 12*a*b^4*c^6 - 48*a^2*b^2*c^7)) - (x*(-(25*b^15*g^2 + b^9*c^6*d^2 + c^6*d^2*(-(4*a*c - b^2)^9)^(1/2) + b^11*c^4*e^2 + 9*b^13*c^2*f^2 + 25*b^6*g^2*(-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^10*d^2 - 27*a*b^9*c^5*e^2 - 3840*a^5*b*c^9*e^2 - 9*a*c^5*e^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c^3*f^2 + 26880*a^6*b*c^8*f^2 - 80640*a^7*b*c^7*g^2 - 30*b^14*c*f*g - 96*a^2*b^5*c^8*d^2 + 512*a^3*b^3*c^9*d^2 + 288*a^2*b^7*c^6*e^2 - 1504*a^3*b^5*c^7*e^2 + 3840*a^4*b^3*c^8*e^2 + 2077*a^2*b^9*c^4*f^2 - 10656*a^3*b^7*c^5*f^2 + 30240*a^4*b^5*c^6*f^2 - 44800*a^5*b^3*c^7*f^2 + 25*a^2*c^4*f^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 6366*a^2*b^11*c^2*g^2 - 35767*a^3*b^9*c^3*g^2 + 116928*a^4*b^7*c^4*g^2 - 219744*a^5*b^5*c^5*g^2 + 215040*a^6*b^3*c^6*g^2 - 49*a^3*c^3*g^2*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c*g^2 + 3072*a^5*c^10*d*e + 2*b^10*c^5*d*e - 7168*a^6*c^9*d*g - 15360*a^6*c^9*e*f - 6*b^11*c^4*d*f + 10*b^12*c^3*d*g - 6*b^12*c^3*e*f + 35840*a^7*c^8*f*g + 10*b^13*c^2*e*g - 36*a*b^8*c^6*d*e + 98*a*b^9*c^5*d*f - 1536*a^5*b*c^9*d*f - 10*a*c^5*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2*b*c^5*d*e*(-(4*a*c - b^2)^9)^(1/2) - 168*a*b^10*c^4*d*g + 152*a*b^10*c^4*e*f - 258*a*b^11*c^3*e*g + 43520*a^6*b*c^8*e*g + 724*a*b^12*c^2*f*g - 30*b^5*c*f*g*(-(4*a*c - b^2)^9)^(1/2) + 246*a^2*b^2*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) + 192*a^2*b^6*c^7*d*e - 128*a^3*b^4*c^8*d*e - 1536*a^4*b^2*c^9*d*e - 165*a*b^4*c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 576*a^2*b^7*c^6*d*f + 1344*a^3*b^5*c^7*d*f - 512*a^4*b^3*c^8*d*f + 1044*a^2*b^8*c^5*d*g - 1548*a^2*b^8*c^5*e*f - 2688*a^3*b^6*c^6*d*g + 8064*a^3*b^6*c^6*e*f + 1152*a^4*b^4*c^7*d*g - 22400*a^4*b^4*c^7*e*f + 6144*a^5*b^2*c^8*d*g + 30720*a^5*b^2*c^8*e*f - 6*b^2*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2706*a^2*b^9*c^4*e*g - 14784*a^3*b^7*c^5*e*g + 44352*a^4*b^5*c^6*e*g - 69120*a^5*b^3*c^7*e*g + 42*a^2*c^4*e*g*(-(4*a*c - b^2)^9)^(1/2) + 10*b^3*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*b^3*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^2*b^10*c^3*f*g + 39132*a^3*b^8*c^4*f*g - 119

$$\begin{aligned} & 616a^4b^6c^5f^*g + 201600a^5b^4c^6f^*g - 161280a^6b^2c^7f^*g + 10* \\ & b^4c^2e^*g*(-(4a*c - b^2)^9)^{(1/2)} - 51a*b^2c^3f^*g*(-(4a*c - b^2)^9)^{(1/2)} \\ & + 12a*b*c^4d^*g*(-(4a*c - b^2)^9)^{(1/2)} + 44a*b*c^4e^*f*(-(4a*c - \\ & b^2)^9)^{(1/2)} - 78a*b^2c^3e^*g*(-(4a*c - b^2)^9)^{(1/2)} + 184a*b^3c^2* \\ & f^*g*(-(4a*c - b^2)^9)^{(1/2)} - 186a^2b*c^3f^*g*(-(4a*c - b^2)^9)^{(1/2))/ \\ & (32*(4096a^6c^13 + b^12c^7 - 24a*b^10c^8 + 240a^2b^8c^9 - 1280a^3* \\ & b^6c^10 + 3840a^4b^4c^11 - 6144a^5b^2c^12))^{(1/2)}*(16b^7c^7 - 192 \\ & *a*b^5c^8 - 1024a^3b*c^10 + 768a^2b^3c^9))/(2*(16a^2c^7 + b^4c^5 - \\ & 8a*b^2c^6)))*(-(25b^15g^2 + b^9c^6d^2 + c^6d^2*(-(4a*c - b^2)^9)^{(1/2)} \\ & + b^11c^4e^2 + 9b^13c^2f^2 + 25b^6g^2*(-(4a*c - b^2)^9)^{(1/2)} \\ & - 768a^4b*c^10d^2 - 27a*b^9c^5e^2 - 3840a^5b*c^9e^2 - 9a*c^5e^2* \\ & (-(4a*c - b^2)^9)^{(1/2)} - 213a*b^11c^3f^2 + 26880a^6b*c^8f^2 - 80640 \\ & *a^7b*c^7g^2 - 30b^14c*f^g - 96a^2b^5c^8d^2 + 512a^3b^3c^9d^2 + \\ & 288a^2b^7c^6e^2 - 1504a^3b^5c^7e^2 + 3840a^4b^3c^8e^2 + 2077a \\ & ^2b^9c^4f^2 - 10656a^3b^7c^5f^2 + 30240a^4b^5c^6f^2 - 44800a^5* \\ & b^3c^7f^2 + 25a^2c^4f^2*(-(4a*c - b^2)^9)^{(1/2)} + b^2c^4e^2*(-(4a* \\ & c - b^2)^9)^{(1/2)} + 6366a^2b^11c^2g^2 - 35767a^3b^9c^3g^2 + 116928* \\ & a^4b^7c^4g^2 - 219744a^5b^5c^5g^2 + 215040a^6b^3c^6g^2 - 49a^3* \\ & c^3g^2*(-(4a*c - b^2)^9)^{(1/2)} + 9b^4c^2f^2*(-(4a*c - b^2)^9)^{(1/2)} - \\ & 615a*b^13c*g^2 + 3072a^5c^10d*e + 2b^10c^5d*e - 7168a^6c^9d*g - \\ & 15360a^6c^9e*f - 6b^11c^4d*f + 10b^12c^3d*g - 6b^12c^3e*f + 35 \\ & 840a^7c^8f^*g + 10b^13c^2e^*g - 36a*b^8c^6d*e + 98a*b^9c^5d*f - 1 \\ & 536a^5b*c^9d*f - 10a*c^5d*f*(-(4a*c - b^2)^9)^{(1/2)} + 2b*c^5d*e*(-(\\ & 4a*c - b^2)^9)^{(1/2)} - 168a*b^10c^4d*g + 152a*b^10c^4e*f - 258a*b^1 \\ & 1c^3e^*g + 43520a^6b*c^8e^*g + 724a*b^12c^2f^*g - 30b^5c*f^*g*(-(4a* \\ & c - b^2)^9)^{(1/2)} + 246a^2b^2c^2g^2*(-(4a*c - b^2)^9)^{(1/2)} + 192a^2* \\ & b^6c^7d*e - 128a^3b^4c^8d*e - 1536a^4b^2c^9d*e - 165a*b^4c*g^2* \\ & (- (4a*c - b^2)^9)^{(1/2)} - 576a^2b^7c^6d*f + 1344a^3b^5c^7d*f - 512 \\ & *a^4b^3c^8d*f + 1044a^2b^8c^5d*g - 1548a^2b^8c^5e*f - 2688a^3b \\ & ^6c^6d*g + 8064a^3b^6c^6e*f + 1152a^4b^4c^7d*g - 22400a^4b^4c^ \\ & 7e*f + 6144a^5b^2c^8d*g + 30720a^5b^2c^8e*f - 6b^2c^4d*f*(-(4a* \\ & *c - b^2)^9)^{(1/2)} + 2706a^2b^9c^4e^*g - 14784a^3b^7c^5e^*g + 44352a \\ & ^4b^5c^6e^*g - 69120a^5b^3c^7e^*g + 42a^2c^4e^*g*(-(4a*c - b^2)^9)^{(1/2)} \\ & + 10b^3c^3d*g*(-(4a*c - b^2)^9)^{(1/2)} - 6b^3c^3e*f*(-(4a*c - \\ & b^2)^9)^{(1/2)} - 7278a^2b^10c^3f^*g + 39132a^3b^8c^4f^*g - 119616a^4* \\ & b^6c^5f^*g + 201600a^5b^4c^6f^*g - 161280a^6b^2c^7f^*g + 10*b^4c^2* \\ & e^*g*(-(4a*c - b^2)^9)^{(1/2)} - 51a*b^2c^3f^*g*(-(4a*c - b^2)^9)^{(1/2)} + \\ & 12a*b*c^4d^*g*(-(4a*c - b^2)^9)^{(1/2)} + 44a*b*c^4e^*f*(-(4a*c - b^2)^9) \\ & ^{(1/2)} - 78a*b^2c^3e^*g*(-(4a*c - b^2)^9)^{(1/2)} + 184a*b^3c^2*f^*g*(-(4 \\ & *a*c - b^2)^9)^{(1/2)} - 186a^2b*c^3f^*g*(-(4a*c - b^2)^9)^{(1/2))/(32*(409 \\ & 6a^6c^13 + b^12c^7 - 24a*b^10c^8 + 240a^2b^8c^9 - 1280a^3b^6c^10 \\ & + 3840a^4b^4c^11 - 6144a^5b^2c^12))^{(1/2)} - (x*(25b^10g^2 + 8a^2 \\ & *c^8d^2 - 72a^3c^7e^2 + b^4c^6d^2 + 200a^4c^6f^2 + b^6c^4e^2 - 3 \\ & 92a^5c^5g^2 + 9b^8c^2f^2 + 2a*b^2c^7d^2 - 16a*b^4c^5e^2 - 114a \\ & *b^6c^3f^2 - 30b^9c*f^g + 74a^2b^2c^6e^2 + 481a^2b^4c^4f^2 - 71 \\ & 8a^3b^2c^5f^2 + 1676a^2b^6c^2g^2 - 3536a^3b^4c^3g^2 + 2794a^4* \\ & b^2c^4g^2 - 340a*b^8c*g^2 - 80a^3c^7d*f + 2b^5c^5d*e - 6b^6c^4* \\ & d*f + 336a^4c^6e^*g + 10b^7c^3d*g - 6b^7c^3e*f + 10b^8c^2e^*g - 1 \\ & 4a*b^3c^6d*e - 8a^2b*c^7d*e + 32a*b^4c^5d*f - 58a*b^5c^4d*g + 8 \\ & 6a*b^5c^4e*f + 152a^3b*c^6d*g + 472a^3b*c^6e*f - 148a*b^6c^3e^*g \\ & + 394a*b^7c^2f^*g - 1768a^4b*c^5f^*g + 4a^2b^2c^6d*f + 26a^2b^3* \\ & c^5d*g - 374a^2b^3c^5e*f + 698a^2b^4c^4e^*g - 1132a^3b^2c^5e^*g \\ & - 1804a^2b^5c^3f^*g + 3266a^3b^3c^4f^*g))/(2*(16a^2c^7 + b^4c^5 - \\ & 8a*b^2c^6)))*(-(25b^15g^2 + b^9c^6d^2 + c^6d^2*(-(4a*c - b^2)^9)^{(1/2)} \\ & + b^11c^4e^2 + 9b^13c^2f^2 + 25b^6g^2*(-(4a*c - b^2)^9)^{(1/2)} - \\ & 768a^4b*c^10d^2 - 27a*b^9c^5e^2 - 3840a^5b*c^9e^2 - 9a*c^5e^2*(\\ & -(4a*c - b^2)^9)^{(1/2)} - 213a*b^11c^3f^2 + 26880a^6b*c^8f^2 - 80640 \\ & *a^7b*c^7g^2 - 30b^14c*f^*g - 96a^2b^5c^8d^2 + 512a^3b^3c^9d^2 + \\ & 288a^2b^7c^6e^2 - 1504a^3b^5c^7e^2 + 3840a^4b^3c^8e^2 + 2077a^ \\ & \end{aligned}$$

$$\begin{aligned}
& 2*b^9*c^4*f^2 - 10656*a^3*b^7*c^5*f^2 + 30240*a^4*b^5*c^6*f^2 - 44800*a^5*b^3*c^7*f^2 + 25*a^2*c^4*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + b^2*c^4*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 6366*a^2*b^11*c^2*g^2 - 35767*a^3*b^9*c^3*g^2 + 116928*a^4*b^7*c^4*g^2 - 219744*a^5*b^5*c^5*g^2 + 215040*a^6*b^3*c^6*g^2 - 49*a^3*c^3*g^2*(-(4*a*c - b^2)^9)^{(1/2)} + 9*b^4*c^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 615*a*b^13*c*g^2 + 3072*a^5*c^10*d*e + 2*b^10*c^5*d*e - 7168*a^6*c^9*d*g - 15360*a^6*c^9*e*f - 6*b^11*c^4*d*f + 10*b^12*c^3*d*g - 6*b^12*c^3*e*f + 35840*a^7*c^8*f*g + 10*b^13*c^2*e*g - 36*a*b^8*c^6*d*e + 98*a*b^9*c^5*d*f - 1536*a^5*b*c^9*d*f - 10*a*c^5*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 2*b*c^5*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 168*a*b^10*c^4*d*g + 152*a*b^10*c^4*e*f - 258*a*b^11*c^3*e*g + 43520*a^6*b*c^8*e*g + 724*a*b^12*c^2*f*g - 30*b^5*c*f*g*(-(4*a*c - b^2)^9)^{(1/2)} + 246*a^2*b^2*c^2*g^2*(-(4*a*c - b^2)^9)^{(1/2)} + 192*a^2*b^6*c^7*d*e - 128*a^3*b^4*c^8*d*e - 1536*a^4*b^2*c^9*d*e - 165*a*b^4*c*g^2*(-(4*a*c - b^2)^9)^{(1/2)} - 576*a^2*b^7*c^6*d*f + 1344*a^3*b^5*c^7*d*f - 512*a^4*b^3*c^8*d*f + 1044*a^2*b^8*c^5*d*g - 1548*a^2*b^8*c^5*e*f - 2688*a^3*b^6*c^6*d*g + 8064*a^3*b^6*c^6*e*f + 1152*a^4*b^4*c^7*d*g - 22400*a^4*b^4*c^7*e*f + 6144*a^5*b^2*c^8*d*g + 30720*a^5*b^2*c^8*e*f - 6*b^2*c^4*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 2706*a^2*b^9*c^4*e*g - 14784*a^3*b^7*c^5*e*g + 44352*a^4*b^5*c^6*e*g - 69120*a^5*b^3*c^7*e*g + 42*a^2*c^4*e*g*(-(4*a*c - b^2)^9)^{(1/2)} + 10*b^3*c^3*d*g*(-(4*a*c - b^2)^9)^{(1/2)} - 6*b^3*c^3*e*f*(-(4*a*c - b^2)^9)^{(1/2)} - 7278*a^2*b^10*c^3*f*g + 39132*a^3*b^8*c^4*f*g - 119616*a^4*b^6*c^5*f*g + 201600*a^5*b^4*c^6*f*g - 161280*a^6*b^2*c^7*f*g + 10*b^4*c^2*e*g*(-(4*a*c - b^2)^9)^{(1/2)} - 51*a*b^2*c^3*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + 12*a*b*c^4*d*g*(-(4*a*c - b^2)^9)^{(1/2)} + 44*a*b*c^4*e*f*(-(4*a*c - b^2)^9)^{(1/2)} - 78*a*b^2*c^3*e*g*(-(4*a*c - b^2)^9)^{(1/2)} + 184*a*b^3*c^2*f*g*(-(4*a*c - b^2)^9)^{(1/2)} - 186*a^2*b*c^3*f*g*(-(4*a*c - b^2)^9)^{(1/2)}/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^{(1/2)}*i - (((2048*a^4*c^10*d - 10240*a^5*c^9*f + 384*a^2*b^4*c^8*d - 1536*a^3*b^2*c^9*d - 192*a^2*b^5*c^7*e + 768*a^3*b^3*c^8*e + 736*a^2*b^6*c^6*f - 4224*a^3*b^4*c^7*f + 10752*a^4*b^2*c^8*f - 1264*a^2*b^7*c^5*g + 7488*a^3*b^5*c^6*g - 19712*a^4*b^3*c^7*g - 32*a*b^6*c^7*d + 16*a*b^7*c^6*e - 1024*a^4*b*c^9*e - 48*a*b^8*c^5*f + 80*a*b^9*c^4*g + 19456*a^5*b*c^8*g)/(8*(64*a^3*c^8 - b^6*c^5 + 12*a*b^4*c^6 - 48*a^2*b^2*c^7)) + (x*(-(25*b^15*g^2 + b^9*c^6*d^2 + c^6*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + b^11*c^4*e^2 + 9*b^13*c^2*f^2 + 25*b^6*g^2*(-(4*a*c - b^2)^9)^{(1/2)} - 768*a^4*b*c^10*d^2 - 27*a*b^9*c^5*e^2 - 3840*a^5*b*c^9*e^2 - 9*a*c^5*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 213*a*b^11*c^3*f^2 + 26880*a^6*b*c^8*f^2 - 80640*a^7*b*c^7*g^2 - 30*b^14*c*f*g - 96*a^2*b^5*c^8*d^2 + 512*a^3*b^3*c^9*d^2 + 288*a^2*b^7*c^6*e^2 - 1504*a^3*b^5*c^7*e^2 + 3840*a^4*b^3*c^8*e^2 + 2077*a^2*b^9*c^4*f^2 - 10656*a^3*b^7*c^5*f^2 + 30240*a^4*b^5*c^6*f^2 - 44800*a^5*b^3*c^7*f^2 + 25*a^2*c^4*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + b^2*c^4*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 6366*a^2*b^11*c^2*g^2 - 35767*a^3*b^9*c^3*g^2 + 116928*a^4*b^7*c^4*g^2 - 219744*a^5*b^5*c^5*g^2 + 215040*a^6*b^3*c^6*g^2 - 49*a^3*c^3*g^2*(-(4*a*c - b^2)^9)^{(1/2)} + 9*b^4*c^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 615*a*b^13*c*g^2 + 3072*a^5*c^10*d*e + 2*b^10*c^5*d*e - 7168*a^6*c^9*d*g - 15360*a^6*c^9*e*f - 6*b^11*c^4*d*f + 10*b^12*c^3*d*g - 6*b^12*c^3*e*f + 35840*a^7*c^8*f*g + 10*b^13*c^2*e*g - 36*a*b^8*c^6*d*e + 98*a*b^9*c^5*d*f - 1536*a^5*b*c^9*d*f - 10*a*c^5*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 2*b*c^5*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 168*a*b^10*c^4*d*g + 152*a*b^10*c^4*e*f - 258*a*b^11*c^3*e*g + 43520*a^6*b*c^8*e*g + 724*a*b^12*c^2*f*g - 30*b^5*c*f*g*(-(4*a*c - b^2)^9)^{(1/2)} + 246*a^2*b^2*c^2*g^2*(-(4*a*c - b^2)^9)^{(1/2)} + 192*a^2*b^6*c^7*d*e - 128*a^3*b^4*c^8*d*e - 1536*a^4*b^2*c^9*d*e - 165*a*b^4*c*g^2*(-(4*a*c - b^2)^9)^{(1/2)} - 576*a^2*b^7*c^6*d*f + 1344*a^3*b^5*c^7*d*f - 512*a^4*b^3*c^8*d*f + 1044*a^2*b^8*c^5*d*g - 1548*a^2*b^8*c^5*e*f - 2688*a^3*b^6*c^6*d*g + 8064*a^3*b^6*c^6*e*f + 1152*a^4*b^4*c^7*d*g - 22400*a^4*b^4*c^7*e*f + 6144*a^5*b^2*c^8*d*g + 30720*a^5*b^2*c^8*e*f - 6*b^2*c^4*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 2706*a^2*b^9*c^4*e*g - 14784*a^3*b^7*c^5*e*g + 44352*a^4*b^5*c^6*e*g - 69120*a^5*b^3*c^7*e*g + 42*a^2*c^4*e*g*(-(4*a*c - b^2)^9)^{(1/2)} + 10*b^3*c^3*d*g*(-(4*a*c - b^2)^9)^{(1/2)} - 6*b^3*c^3*e*f*(-(4*a*c - b^2)^9)^{(1/2)}
\end{aligned}$$

$$\begin{aligned}
& *c - b^2)^9)^{(1/2)} - 7278*a^2*b^{10}*c^3*f*g + 39132*a^3*b^8*c^4*f*g - 119616 \\
& *a^4*b^6*c^5*f*g + 201600*a^5*b^4*c^6*f*g - 161280*a^6*b^2*c^7*f*g + 10*b^4 \\
& *c^2*e*g*(-(4*a*c - b^2)^9)^{(1/2)} - 51*a*b^2*c^3*f^2*(-(4*a*c - b^2)^9)^{(1/2)} \\
& + 12*a*b*c^4*d*g*(-(4*a*c - b^2)^9)^{(1/2)} + 44*a*b*c^4*e*f*(-(4*a*c - b^2)^9)^{(1/2)} \\
& - 78*a*b^2*c^3*e*g*(-(4*a*c - b^2)^9)^{(1/2)} + 184*a*b^3*c^2*f*g \\
& *(-4*a*c - b^2)^9)^{(1/2)} - 186*a^2*b*c^3*f*g*(-(4*a*c - b^2)^9)^{(1/2)} / (32 \\
& *(4096*a^6*c^{13} + b^{12}*c^7 - 24*a*b^{10}*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6 \\
& *c^{10} + 3840*a^4*b^4*c^{11} - 6144*a^5*b^2*c^{12}))^{(1/2)} * (16*b^7*c^7 - 192*a* \\
& b^5*c^8 - 1024*a^3*b*c^{10} + 768*a^2*b^3*c^9) / (2*(16*a^2*c^7 + b^4*c^5 - 8* \\
& a*b^2*c^6)) * (-25*b^{15}*g^2 + b^9*c^6*d^2 + c^6*d^2*(-(4*a*c - b^2)^9)^{(1/2)} \\
& + b^{11}*c^4*e^2 + 9*b^{13}*c^2*f^2 + 25*b^6*g^2*(-(4*a*c - b^2)^9)^{(1/2)} - 7 \\
& 68*a^4*b*c^{10}*d^2 - 27*a*b^9*c^5*e^2 - 3840*a^5*b*c^9*e^2 - 9*a*c^5*e^2*(-(4 \\
& 4*a*c - b^2)^9)^{(1/2)} - 213*a*b^{11}*c^3*f^2 + 26880*a^6*b*c^8*f^2 - 80640*a^7 \\
& *b*c^7*g^2 - 30*b^{14}*c*f*g - 96*a^2*b^5*c^8*d^2 + 512*a^3*b^3*c^9*d^2 + 28 \\
& 8*a^2*b^7*c^6*e^2 - 1504*a^3*b^5*c^7*e^2 + 3840*a^4*b^3*c^8*e^2 + 2077*a^2* \\
& b^9*c^4*f^2 - 10656*a^3*b^7*c^5*f^2 + 30240*a^4*b^5*c^6*f^2 - 44800*a^5*b^3 \\
& *c^7*f^2 + 25*a^2*c^4*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + b^2*c^4*e^2*(-(4*a*c - \\
& b^2)^9)^{(1/2)} + 6366*a^2*b^{11}*c^2*g^2 - 35767*a^3*b^9*c^3*g^2 + 116928*a^4 \\
& *b^7*c^4*g^2 - 219744*a^5*b^5*c^5*g^2 + 215040*a^6*b^3*c^6*g^2 - 49*a^3*c^3 \\
& *g^2*(-(4*a*c - b^2)^9)^{(1/2)} + 9*b^4*c^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 61 \\
& 5*a*b^{13}*c*g^2 + 3072*a^5*c^{10}*d*e + 2*b^{10}*c^5*d*e - 7168*a^6*c^9*d*g - 15 \\
& 360*a^6*c^9*e*f - 6*b^{11}*c^4*d*f + 10*b^{12}*c^3*d*g - 6*b^{12}*c^3*e*f + 35840 \\
& *a^7*c^8*f*g + 10*b^{13}*c^2*e*g - 36*a*b^8*c^6*d*e + 98*a*b^9*c^5*d*f - 1536 \\
& *a^5*b*c^9*d*f - 10*a*c^5*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 2*b*c^5*d*e*(-(4*a \\
& *c - b^2)^9)^{(1/2)} - 168*a*b^{10}*c^4*d*g + 152*a*b^{10}*c^4*e*f - 258*a*b^{11}*c \\
& ^3*e*g + 43520*a^6*b*c^8*e*g + 724*a*b^{12}*c^2*f*g - 30*b^5*c*f*g*(-(4*a*c - \\
& b^2)^9)^{(1/2)} + 246*a^2*b^2*c^2*g^2*(-(4*a*c - b^2)^9)^{(1/2)} + 192*a^2*b^6 \\
& *c^7*d*e - 128*a^3*b^4*c^8*d*e - 1536*a^4*b^2*c^9*d*e - 165*a*b^4*c*g^2*(-(4 \\
& 4*a*c - b^2)^9)^{(1/2)} - 576*a^2*b^7*c^6*d*f + 1344*a^3*b^5*c^7*d*f - 512*a^4 \\
& *b^3*c^8*d*f + 1044*a^2*b^8*c^5*d*g - 1548*a^2*b^8*c^5*e*f - 2688*a^3*b^6* \\
& c^6*d*g + 8064*a^3*b^6*c^6*e*f + 1152*a^4*b^4*c^7*d*g - 22400*a^4*b^4*c^7*e \\
& *f + 6144*a^5*b^2*c^8*d*g + 30720*a^5*b^2*c^8*e*f - 6*b^2*c^4*d*f*(-(4*a*c \\
& - b^2)^9)^{(1/2)} + 2706*a^2*b^9*c^4*e*g - 14784*a^3*b^7*c^5*e*g + 44352*a^4* \\
& b^5*c^6*e*g - 69120*a^5*b^3*c^7*e*g + 42*a^2*c^4*e*g*(-(4*a*c - b^2)^9)^{(1/2)} \\
& + 10*b^3*c^3*d*g*(-(4*a*c - b^2)^9)^{(1/2)} - 6*b^3*c^3*e*f*(-(4*a*c - b^2 \\
&)^9)^{(1/2)} - 7278*a^2*b^{10}*c^3*f*g + 39132*a^3*b^8*c^4*f*g - 119616*a^4*b^6 \\
& *c^5*f*g + 201600*a^5*b^4*c^6*f*g - 161280*a^6*b^2*c^7*f*g + 10*b^4*c^2*e*g \\
& *(-4*a*c - b^2)^9)^{(1/2)} - 51*a*b^2*c^3*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + 12* \\
& a*b*c^4*d*g*(-(4*a*c - b^2)^9)^{(1/2)} + 44*a*b*c^4*e*f*(-(4*a*c - b^2)^9)^{(1 \\
& /2)} - 78*a*b^2*c^3*e*g*(-(4*a*c - b^2)^9)^{(1/2)} + 184*a*b^3*c^2*f*g*(-(4*a* \\
& c - b^2)^9)^{(1/2)} - 186*a^2*b*c^3*f*g*(-(4*a*c - b^2)^9)^{(1/2)} / (32*(4096*a \\
& ^6*c^{13} + b^{12}*c^7 - 24*a*b^{10}*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^{10} + \\
& 3840*a^4*b^4*c^{11} - 6144*a^5*b^2*c^{12}))^{(1/2)} + (x*(25*b^{10}*g^2 + 8*a^2*c^ \\
& 8*d^2 - 72*a^3*c^7*e^2 + b^4*c^6*d^2 + 200*a^4*c^6*f^2 + b^6*c^4*e^2 - 392* \\
& a^5*c^5*g^2 + 9*b^8*c^2*f^2 + 2*a*b^2*c^7*d^2 - 16*a*b^4*c^5*e^2 - 114*a*b^ \\
& 6*c^3*f^2 - 30*b^9*c*f*g + 74*a^2*b^2*c^6*e^2 + 481*a^2*b^4*c^4*f^2 - 718*a \\
& ^3*b^2*c^5*f^2 + 1676*a^2*b^6*c^2*g^2 - 3536*a^3*b^4*c^3*g^2 + 2794*a^4*b^2 \\
& *c^4*g^2 - 340*a*b^8*c*g^2 - 80*a^3*c^7*d*f + 2*b^5*c^5*d*e - 6*b^6*c^4*d*f \\
& + 336*a^4*c^6*e*g + 10*b^7*c^3*d*g - 6*b^7*c^3*e*f + 10*b^8*c^2*e*g - 14*a \\
& *b^3*c^6*d*e - 8*a^2*b*c^7*d*e + 32*a*b^4*c^5*d*f - 58*a*b^5*c^4*d*g + 86*a \\
& *b^5*c^4*e*f + 152*a^3*b*c^6*d*g + 472*a^3*b*c^6*e*f - 148*a*b^6*c^3*e*g + \\
& 394*a*b^7*c^2*f*g - 1768*a^4*b*c^5*f*g + 4*a^2*b^2*c^6*d*f + 26*a^2*b^3*c^5 \\
& *d*g - 374*a^2*b^3*c^5*e*f + 698*a^2*b^4*c^4*e*g - 1132*a^3*b^2*c^5*e*g - 1 \\
& 804*a^2*b^5*c^3*f*g + 3266*a^3*b^3*c^4*f*g) / (2*(16*a^2*c^7 + b^4*c^5 - 8*a \\
& *b^2*c^6)) * (-25*b^{15}*g^2 + b^9*c^6*d^2 + c^6*d^2*(-(4*a*c - b^2)^9)^{(1/2)} \\
& + b^{11}*c^4*e^2 + 9*b^{13}*c^2*f^2 + 25*b^6*g^2*(-(4*a*c - b^2)^9)^{(1/2)} - 76 \\
& 8*a^4*b*c^{10}*d^2 - 27*a*b^9*c^5*e^2 - 3840*a^5*b*c^9*e^2 - 9*a*c^5*e^2*(-(4 \\
& *a*c - b^2)^9)^{(1/2)} - 213*a*b^{11}*c^3*f^2 + 26880*a^6*b*c^8*f^2 - 80640*a^7 \\
& *b*c^7*g^2 - 30*b^{14}*c*f*g - 96*a^2*b^5*c^8*d^2 + 512*a^3*b^3*c^9*d^2 + 288
\end{aligned}$$

$$\begin{aligned}
& *a^2*b^7*c^6*e^2 - 1504*a^3*b^5*c^7*e^2 + 3840*a^4*b^3*c^8*e^2 + 2077*a^2*b^9*c^4*f^2 - 10656*a^3*b^7*c^5*f^2 + 30240*a^4*b^5*c^6*f^2 - 44800*a^5*b^3*c^7*f^2 + 25*a^2*c^4*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + b^2*c^4*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 6366*a^2*b^11*c^2*g^2 - 35767*a^3*b^9*c^3*g^2 + 116928*a^4*b^7*c^4*g^2 - 219744*a^5*b^5*c^5*g^2 + 215040*a^6*b^3*c^6*g^2 - 49*a^3*c^3*g^2*(-(4*a*c - b^2)^9)^{(1/2)} + 9*b^4*c^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 615*a*b^13*c*g^2 + 3072*a^5*c^10*d*e + 2*b^10*c^5*d*e - 7168*a^6*c^9*d*g - 15360*a^6*c^9*e*f - 6*b^11*c^4*d*f + 10*b^12*c^3*d*g - 6*b^12*c^3*e*f + 35840*a^7*c^8*f*g + 10*b^13*c^2*e*g - 36*a*b^8*c^6*d*e + 98*a*b^9*c^5*d*f - 1536*a^5*b*c^9*d*f - 10*a*c^5*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 2*b*c^5*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 168*a*b^10*c^4*d*g + 152*a*b^10*c^4*e*f - 258*a*b^11*c^3*e*g + 43520*a^6*b*c^8*e*g + 724*a*b^12*c^2*f*g - 30*b^5*c*f*g*(-(4*a*c - b^2)^9)^{(1/2)} + 246*a^2*b^2*c^2*g^2*(-(4*a*c - b^2)^9)^{(1/2)} + 192*a^2*b^6*c^7*d*e - 128*a^3*b^4*c^8*d*e - 1536*a^4*b^2*c^9*d*e - 165*a*b^4*c*g^2*(-(4*a*c - b^2)^9)^{(1/2)} - 576*a^2*b^7*c^6*d*f + 1344*a^3*b^5*c^7*d*f - 512*a^4*b^3*c^8*d*f + 1044*a^2*b^8*c^5*d*g - 1548*a^2*b^8*c^5*e*f - 2688*a^3*b^6*c^6*d*g + 8064*a^3*b^6*c^6*e*f + 1152*a^4*b^4*c^7*d*g - 22400*a^4*b^4*c^7*e*f + 6144*a^5*b^2*c^8*d*g + 30720*a^5*b^2*c^8*e*f - 6*b^2*c^4*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 2706*a^2*b^9*c^4*e*g - 14784*a^3*b^7*c^5*e*g + 44352*a^4*b^5*c^6*e*g - 69120*a^5*b^3*c^7*e*g + 42*a^2*c^4*e*g*(-(4*a*c - b^2)^9)^{(1/2)} + 10*b^3*c^3*d*g*(-(4*a*c - b^2)^9)^{(1/2)} - 6*b^3*c^3*e*f*(-(4*a*c - b^2)^9)^{(1/2)} - 7278*a^2*b^10*c^3*f*g + 39132*a^3*b^8*c^4*f*g - 119616*a^4*b^6*c^5*f*g + 201600*a^5*b^4*c^6*f*g - 161280*a^6*b^2*c^7*f*g + 10*b^4*c^2*e*g*(-(4*a*c - b^2)^9)^{(1/2)} - 51*a*b^2*c^3*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + 12*a*b*c^4*d*g*(-(4*a*c - b^2)^9)^{(1/2)} + 44*a*b*c^4*e*f*(-(4*a*c - b^2)^9)^{(1/2)} - 78*a*b^2*c^3*e*g*(-(4*a*c - b^2)^9)^{(1/2)} + 184*a*b^3*c^2*f*g*(-(4*a*c - b^2)^9)^{(1/2)} - 186*a^2*b*c^3*f*g*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12))^(1/2)*i)/((((2048*a^4*c^10*d - 10240*a^5*c^9*f + 384*a^2*b^4*c^8*d - 1536*a^3*b^2*c^9*d - 192*a^2*b^5*c^7*e + 768*a^3*b^3*c^8*e + 736*a^2*b^6*c^6*f - 4224*a^3*b^4*c^7*f + 10752*a^4*b^2*c^8*f - 1264*a^2*b^7*c^5*g + 7488*a^3*b^5*c^6*g - 19712*a^4*b^3*c^7*g - 32*a*b^6*c^7*d + 16*a*b^7*c^6*e - 1024*a^4*b*c^9*e - 48*a*b^8*c^5*f + 80*a*b^9*c^4*g + 19456*a^5*b*c^8*g)/(8*(64*a^3*c^8 - b^6*c^5 + 12*a*b^4*c^6 - 48*a^2*b^2*c^7)) - (x*(-(25*b^15*g^2 + b^9*c^6*d^2 + c^6*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + b^11*c^4*e^2 + 9*b^13*c^2*f^2 + 25*b^6*g^2*(-(4*a*c - b^2)^9)^{(1/2)} - 768*a^4*b*c^10*d^2 - 27*a*b^9*c^5*e^2 - 3840*a^5*b*c^9*e^2 - 9*a*c^5*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 213*a*b^11*c^3*f^2 + 26880*a^6*b*c^8*f^2 - 80640*a^7*b*c^7*g^2 - 30*b^14*c*f*g - 96*a^2*b^5*c^8*d^2 + 512*a^3*b^3*c^9*d^2 + 288*a^2*b^7*c^6*e^2 - 1504*a^3*b^5*c^7*e^2 + 3840*a^4*b^3*c^8*e^2 + 2077*a^2*b^9*c^4*f^2 - 10656*a^3*b^7*c^5*f^2 + 30240*a^4*b^5*c^6*f^2 - 44800*a^5*b^3*c^7*f^2 + 25*a^2*c^4*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + b^2*c^4*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 6366*a^2*b^11*c^2*g^2 - 35767*a^3*b^9*c^3*g^2 + 116928*a^4*b^7*c^4*g^2 - 219744*a^5*b^5*c^5*g^2 + 215040*a^6*b^3*c^6*g^2 - 49*a^3*c^3*g^2*(-(4*a*c - b^2)^9)^{(1/2)} + 9*b^4*c^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 615*a*b^13*c*g^2 + 3072*a^5*c^10*d*e + 2*b^10*c^5*d*e - 7168*a^6*c^9*d*g - 15360*a^6*c^9*e*f - 6*b^11*c^4*d*f + 10*b^12*c^3*d*g - 6*b^12*c^3*e*f + 35840*a^7*c^8*f*g + 10*b^13*c^2*e*g - 36*a*b^8*c^6*d*e + 98*a*b^9*c^5*d*f - 1536*a^5*b*c^9*d*f - 10*a*c^5*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 2*b*c^5*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 168*a*b^10*c^4*d*g + 152*a*b^10*c^4*e*f - 258*a*b^11*c^3*e*g + 43520*a^6*b*c^8*e*g + 724*a*b^12*c^2*f*g - 30*b^5*c*f*g*(-(4*a*c - b^2)^9)^{(1/2)} + 246*a^2*b^2*c^2*g^2*(-(4*a*c - b^2)^9)^{(1/2)} + 192*a^2*b^6*c^7*d*e - 128*a^3*b^4*c^8*d*e - 1536*a^4*b^2*c^9*d*e - 165*a*b^4*c*g^2*(-(4*a*c - b^2)^9)^{(1/2)} - 576*a^2*b^7*c^6*d*f + 1344*a^3*b^5*c^7*d*f - 512*a^4*b^3*c^8*d*f + 1044*a^2*b^8*c^5*d*g - 1548*a^2*b^8*c^5*e*f - 2688*a^3*b^6*c^6*d*g + 8064*a^3*b^6*c^6*e*f + 1152*a^4*b^4*c^7*d*g - 22400*a^4*b^4*c^7*e*f + 6144*a^5*b^2*c^8*d*g + 30720*a^5*b^2*c^8*e*f - 6*b^2*c^4*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 2706*a^2*b^9*c^4*e*g - 14784*a^3*b^7*c^5*e*g + 44352*a^4*b^5*c^6*e*g - 69120*a^5*b^3*c^7*e*g + 42*a^2*c^4*e*g*(-(4*a*c - b^2)^9)^{(1/2)}
\end{aligned}$$

$$\begin{aligned}
&)^{(1/2)} + 10*b^3*c^3*d*g*(-(4*a*c - b^2)^9)^{(1/2)} - 6*b^3*c^3*e*f*(-(4*a*c - b^2)^9)^{(1/2)} - 7278*a^2*b^10*c^3*f*g + 39132*a^3*b^8*c^4*f*g - 119616*a^4*b^6*c^5*f*g + 201600*a^5*b^4*c^6*f*g - 161280*a^6*b^2*c^7*f*g + 10*b^4*c^2*e*g*(-(4*a*c - b^2)^9)^{(1/2)} - 51*a*b^2*c^3*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + 12*a*b*c^4*d*g*(-(4*a*c - b^2)^9)^{(1/2)} + 44*a*b*c^4*e*f*(-(4*a*c - b^2)^9)^{(1/2)} - 78*a*b^2*c^3*e*g*(-(4*a*c - b^2)^9)^{(1/2)} + 184*a*b^3*c^2*f*g*(-(4*a*c - b^2)^9)^{(1/2)} - 186*a^2*b*c^3*f*g*(-(4*a*c - b^2)^9)^{(1/2))/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12))^{(1/2)}*(16*b^7*c^7 - 192*a*b^5*c^8 - 1024*a^3*b*c^10 + 768*a^2*b^3*c^9))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))*(-(25*b^15*g^2 + b^9*c^6*d^2 + c^6*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + b^11*c^4*e^2 + 9*b^13*c^2*f^2 + 25*b^6*g^2*(-(4*a*c - b^2)^9)^{(1/2)} - 768*a^4*b*c^10*d^2 - 27*a*b^9*c^5*e^2 - 3840*a^5*b*c^9*e^2 - 9*a*c^5*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 213*a*b^11*c^3*f^2 + 26880*a^6*b*c^8*f^2 - 80640*a^7*b*c^7*g^2 - 30*b^14*c*f*g - 96*a^2*b^5*c^8*d^2 + 512*a^3*b^3*c^9*d^2 + 288*a^2*b^7*c^6*e^2 - 1504*a^3*b^5*c^7*e^2 + 3840*a^4*b^3*c^8*e^2 + 2077*a^2*b^9*c^4*f^2 - 10656*a^3*b^7*c^5*f^2 + 30240*a^4*b^5*c^6*f^2 - 44800*a^5*b^3*c^7*f^2 + 25*a^2*c^4*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + b^2*c^4*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 6366*a^2*b^11*c^2*g^2 - 35767*a^3*b^9*c^3*g^2 + 116928*a^4*b^7*c^4*g^2 - 219744*a^5*b^5*c^5*g^2 + 215040*a^6*b^3*c^6*g^2 - 49*a^3*c^3*g^2*(-(4*a*c - b^2)^9)^{(1/2)} + 9*b^4*c^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 615*a*b^13*c*g^2 + 3072*a^5*c^10*d*e + 2*b^10*c^5*d*e - 7168*a^6*c^9*d*g - 15360*a^6*c^9*e*f - 6*b^11*c^4*d*f + 10*b^12*c^3*d*g - 6*b^12*c^3*e*f + 35840*a^7*c^8*f*g + 10*b^13*c^2*e*g - 36*a*b^8*c^6*d*e + 98*a*b^9*c^5*d*f - 1536*a^5*b*c^9*d*f - 10*a*c^5*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 2*b*c^5*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 168*a*b^10*c^4*d*g + 152*a*b^10*c^4*e*f - 258*a*b^11*c^3*e*g + 43520*a^6*b*c^8*e*g + 724*a*b^12*c^2*f*g - 30*b^5*c*f*g*(-(4*a*c - b^2)^9)^{(1/2)} + 246*a^2*b^2*c^2*g^2*(-(4*a*c - b^2)^9)^{(1/2)} + 192*a^2*b^6*c^7*d*e - 128*a^3*b^4*c^8*d*e - 1536*a^4*b^2*c^9*d*e - 165*a*b^4*c*g^2*(-(4*a*c - b^2)^9)^{(1/2)} - 576*a^2*b^7*c^6*d*f + 1344*a^3*b^5*c^7*d*f - 512*a^4*b^3*c^8*d*f + 1044*a^2*b^8*c^5*d*g - 1548*a^2*b^8*c^5*e*f - 2688*a^3*b^6*c^6*d*g + 8064*a^3*b^6*c^6*e*f + 1152*a^4*b^4*c^7*d*g - 22400*a^4*b^4*c^7*e*f + 6144*a^5*b^2*c^8*d*g + 30720*a^5*b^2*c^8*e*f - 6*b^2*c^4*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 2706*a^2*b^9*c^4*e*g - 14784*a^3*b^7*c^5*e*g + 44352*a^4*b^5*c^6*e*g - 69120*a^5*b^3*c^7*e*g + 42*a^2*c^4*e*g*(-(4*a*c - b^2)^9)^{(1/2)} + 10*b^3*c^3*d*g*(-(4*a*c - b^2)^9)^{(1/2)} - 6*b^3*c^3*e*f*(-(4*a*c - b^2)^9)^{(1/2)} - 7278*a^2*b^10*c^3*f*g + 39132*a^3*b^8*c^4*f*g - 119616*a^4*b^6*c^5*f*g + 201600*a^5*b^4*c^6*f*g - 161280*a^6*b^2*c^7*f*g + 10*b^4*c^2*e*g*(-(4*a*c - b^2)^9)^{(1/2)} - 51*a*b^2*c^3*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + 12*a*b*c^4*d*g*(-(4*a*c - b^2)^9)^{(1/2)} + 44*a*b*c^4*e*f*(-(4*a*c - b^2)^9)^{(1/2)} - 78*a*b^2*c^3*e*g*(-(4*a*c - b^2)^9)^{(1/2)} + 184*a*b^3*c^2*f*g*(-(4*a*c - b^2)^9)^{(1/2)} - 186*a^2*b*c^3*f*g*(-(4*a*c - b^2)^9)^{(1/2))/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12))^{(1/2)} - (x*(25*b^10*g^2 + 8*a^2*c^8*d^2 - 72*a^3*c^7*e^2 + b^4*c^6*d^2 + 200*a^4*c^6*f^2 + b^6*c^4*e^2 - 392*a^5*c^5*g^2 + 9*b^8*c^2*f^2 + 2*a*b^2*c^7*d^2 - 16*a*b^4*c^5*e^2 - 114*a*b^6*c^3*f^2 - 30*b^9*c*f*g + 74*a^2*b^2*c^6*e^2 + 481*a^2*b^4*c^4*f^2 - 718*a^3*b^2*c^5*f^2 + 1676*a^2*b^6*c^2*g^2 - 3536*a^3*b^4*c^3*g^2 + 2794*a^4*b^2*c^4*g^2 - 340*a*b^8*c*g^2 - 80*a^3*c^7*d*f + 2*b^5*c^5*d*e - 6*b^6*c^4*d*f + 336*a^4*c^6*e*g + 10*b^7*c^3*d*g - 6*b^7*c^3*e*f + 10*b^8*c^2*e*g - 14*a*b^3*c^6*d*e - 8*a^2*b*c^7*d*e + 32*a*b^4*c^5*d*f - 58*a*b^5*c^4*d*g + 86*a*b^5*c^4*e*f + 152*a^3*b*c^6*d*g + 472*a^3*b*c^6*e*f - 148*a*b^6*c^3*e*g + 394*a*b^7*c^2*f*g - 1768*a^4*b*c^5*f*g + 4*a^2*b^2*c^6*d*f + 26*a^2*b^3*c^5*d*g - 374*a^2*b^3*c^5*e*f + 698*a^2*b^4*c^4*e*g - 1132*a^3*b^2*c^5*e*g - 1804*a^2*b^5*c^3*f*g + 3266*a^3*b^3*c^4*f*g))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))*(-(25*b^15*g^2 + b^9*c^6*d^2 + c^6*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + b^11*c^4*e^2 + 9*b^13*c^2*f^2 + 25*b^6*g^2*(-(4*a*c - b^2)^9)^{(1/2)} - 768*a^4*b*c^10*d^2 - 27*a*b^9*c^5*e^2 - 3840*a^5*b*c^9*e^2 - 9*a*c^5*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 213*a*b^11*c^3*f^2 + 26880*a^6*b*c^8*f^2 - 80640*a^7*b*
\end{aligned}$$

$$\begin{aligned}
& c^7g^2 - 30b^{14}cf^2g - 96a^2b^5c^8d^2 + 512a^3b^3c^9d^2 + 288a^2b^7c^6e^2 - 1504a^3b^5c^7e^2 + 3840a^4b^3c^8e^2 + 2077a^2b^9c^4f^2 - 10656a^3b^7c^5f^2 + 30240a^4b^5c^6f^2 - 44800a^5b^3c^7f^2 + 25a^2c^4f^2 * (-4ac - b^2)^9)^{(1/2)} + b^2c^4e^2 * (-4ac - b^2)^9)^{(1/2)} + 6366a^2b^{11}c^2g^2 - 35767a^3b^9c^3g^2 + 116928a^4b^7c^4g^2 - 219744a^5b^5c^5g^2 + 215040a^6b^3c^6g^2 - 49a^3c^3g^2 * (-4ac - b^2)^9)^{(1/2)} + 9b^4c^2f^2 * (-4ac - b^2)^9)^{(1/2)} - 615ab^{13}c^3g^2 + 3072a^5c^{10}d^2e + 2b^{10}c^5d^2e - 7168a^6c^9d^2g - 15360a^6c^9e^2f - 6b^{11}c^4d^2f + 10b^{12}c^3d^2g - 6b^{12}c^3e^2f + 35840a^7c^8f^2g + 10b^{13}c^2e^2g - 36ab^8c^6d^2e + 98ab^9c^5d^2f - 1536a^5b^3c^9d^2f - 10ac^5d^2f * (-4ac - b^2)^9)^{(1/2)} + 2b^5c^5d^2e * (-4ac - b^2)^9)^{(1/2)} - 168ab^{10}c^4d^2g + 152ab^{10}c^4e^2f - 258ab^{11}c^3e^2g + 43520a^6b^3c^8e^2g + 724ab^{12}c^2f^2g - 30b^5c^5f^2g * (-4ac - b^2)^9)^{(1/2)} + 246a^2b^2c^2g^2 * (-4ac - b^2)^9)^{(1/2)} + 192a^2b^6c^7d^2e - 128a^3b^4c^8d^2e - 1536a^4b^2c^9d^2e - 165ab^4c^6g^2 * (-4ac - b^2)^9)^{(1/2)} - 576a^2b^7c^6d^2f + 1344a^3b^5c^7d^2f - 512a^4b^3c^8d^2f + 1044a^2b^8c^5d^2g - 1548a^2b^8c^5e^2f - 2688a^3b^6c^6d^2g + 8064a^3b^6c^6e^2f + 1152a^4b^4c^7d^2g - 22400a^4b^4c^7e^2f + 6144a^5b^2c^8d^2g + 30720a^5b^2c^8e^2f - 6b^2c^4d^2f * (-4ac - b^2)^9)^{(1/2)} + 2706a^2b^9c^4e^2g - 14784a^3b^7c^5e^2g + 44352a^4b^5c^6e^2g - 69120a^5b^3c^7e^2g + 42a^2c^4e^2g * (-4ac - b^2)^9)^{(1/2)} + 10b^3c^3d^2g * (-4ac - b^2)^9)^{(1/2)} - 6b^3c^3e^2f * (-4ac - b^2)^9)^{(1/2)} - 7278a^2b^{10}c^3f^2g + 39132a^3b^8c^4f^2g - 119616a^4b^6c^5f^2g + 201600a^5b^4c^6f^2g - 161280a^6b^2c^7f^2g + 10b^4c^2e^2g * (-4ac - b^2)^9)^{(1/2)} - 51ab^2c^3f^2 * (-4ac - b^2)^9)^{(1/2)} + 12ab^2c^4d^2g * (-4ac - b^2)^9)^{(1/2)} + 44ab^2c^4e^2f * (-4ac - b^2)^9)^{(1/2)} - 78ab^2c^3e^2g * (-4ac - b^2)^9)^{(1/2)} + 184ab^3c^2f^2g * (-4ac - b^2)^9)^{(1/2)} - 186a^2b^3c^3f^2g * (-4ac - b^2)^9)^{(1/2))} / (32(4096a^6c^{13} + b^{12}c^7 - 24ab^{10}c^8 + 240a^2b^8c^9 - 1280a^3b^6c^{10} + 3840a^4b^4c^{11} - 6144a^5b^2c^{12}))^{(1/2)} + (((2048a^4c^{10}d - 10240a^5c^9f + 384a^2b^4c^8d - 1536a^3b^2c^9d - 192a^2b^5c^7e + 768a^3b^3c^8e + 736a^2b^6c^6f - 4224a^3b^4c^7f + 10752a^4b^2c^8f - 1264a^2b^7c^5g + 7488a^3b^5c^6g - 19712a^4b^3c^7g - 32ab^6c^7d + 16ab^7c^6e - 1024a^4b^3c^9e - 48ab^8c^5f + 80ab^9c^4g + 19456a^5b^3c^8g) / (8(64a^3c^8 - b^6c^5 + 12ab^4c^6 - 48a^2b^2c^7))) + (x * (-25b^{15}g^2 + b^9c^6d^2 + c^6d^2 * (-4ac - b^2)^9)^{(1/2)} + b^{11}c^4e^2 + 9b^{13}c^2f^2 + 25b^6g^2 * (-4ac - b^2)^9)^{(1/2)} - 768a^4b^3c^{10}d^2 - 27ab^9c^5e^2 - 3840a^5b^3c^9e^2 - 9ac^5e^2 * (-4ac - b^2)^9)^{(1/2)} - 213ab^{11}c^3f^2 + 26880a^6b^3c^8f^2 - 80640a^7b^3c^7g^2 - 30b^{14}cf^2g - 96a^2b^5c^8d^2 + 512a^3b^3c^9d^2 + 288a^2b^7c^6e^2 - 1504a^3b^5c^7e^2 + 3840a^4b^3c^8e^2 + 2077a^2b^9c^4f^2 - 10656a^3b^7c^5f^2 + 30240a^4b^5c^6f^2 - 44800a^5b^3c^7f^2 + 25a^2c^4f^2 * (-4ac - b^2)^9)^{(1/2)} + b^2c^4e^2 * (-4ac - b^2)^9)^{(1/2)} + 6366a^2b^{11}c^2g^2 - 35767a^3b^9c^3g^2 + 116928a^4b^7c^4g^2 - 219744a^5b^5c^5g^2 + 215040a^6b^3c^6g^2 - 49a^3c^3g^2 * (-4ac - b^2)^9)^{(1/2)} + 9b^4c^2f^2 * (-4ac - b^2)^9)^{(1/2)} - 615ab^{13}c^3g^2 + 3072a^5c^{10}d^2e + 2b^{10}c^5d^2e - 7168a^6c^9d^2g - 15360a^6c^9e^2f - 6b^{11}c^4d^2f + 10b^{12}c^3d^2g - 6b^{12}c^3e^2f + 35840a^7c^8f^2g + 10b^{13}c^2e^2g - 36ab^8c^6d^2e + 98ab^9c^5d^2f - 1536a^5b^3c^9d^2f - 10ac^5d^2f * (-4ac - b^2)^9)^{(1/2)} + 2b^5c^5d^2e * (-4ac - b^2)^9)^{(1/2)} - 168ab^{10}c^4d^2g + 152ab^{10}c^4e^2f - 258ab^{11}c^3e^2g + 43520a^6b^3c^8e^2g + 724ab^{12}c^2f^2g - 30b^5c^5f^2g * (-4ac - b^2)^9)^{(1/2)} + 246a^2b^2c^2g^2 * (-4ac - b^2)^9)^{(1/2)} + 192a^2b^6c^7d^2e - 128a^3b^4c^8d^2e - 1536a^4b^2c^9d^2e - 165ab^4c^6g^2 * (-4ac - b^2)^9)^{(1/2)} - 576a^2b^7c^6d^2f + 1344a^3b^5c^7d^2f - 512a^4b^3c^8d^2f + 1044a^2b^8c^5d^2g - 1548a^2b^8c^5e^2f - 2688a^3b^6c^6d^2g + 8064a^3b^6c^6e^2f + 1152a^4b^4c^7d^2g - 22400a^4b^4c^7e^2f + 6144a^5b^2c^8d^2g + 30720a^5b^2c^8e^2f - 6b^2c^4d^2f * (-4ac - b^2)^9)^{(1/2)} + 2706a^2b^9c^4e^2g - 14784a^3b^7c^5e^2g + 44352a^4b^5c^6e^2g
\end{aligned}$$

$$\begin{aligned}
& ^5c^6e^g - 69120a^5b^3c^7e^g + 42a^2c^4e^g*(-(4ac - b^2)^9)^{(1/2)} \\
& + 10b^3c^3d^g*(-(4ac - b^2)^9)^{(1/2)} - 6b^3c^3e^f*(-(4ac - b^2)^9)^{(1/2)} \\
& - 7278a^2b^{10}c^3f^g + 39132a^3b^8c^4f^g - 119616a^4b^6c^5f^g \\
& + 201600a^5b^4c^6f^g - 161280a^6b^2c^7f^g + 10b^4c^2e^g*(-(4ac - b^2)^9)^{(1/2)} \\
& - 51ab^2c^3f^2*(-(4ac - b^2)^9)^{(1/2)} + 12ab^2c^4d^g*(-(4ac - b^2)^9)^{(1/2)} \\
& + 44ab^2c^4e^f*(-(4ac - b^2)^9)^{(1/2)} - 78ab^2c^3e^g*(-(4ac - b^2)^9)^{(1/2)} \\
& + 184ab^3c^2f^g*(-(4ac - b^2)^9)^{(1/2)} - 186a^2b^3c^3f^g*(-(4ac - b^2)^9)^{(1/2)}) \\
& / (32(4096a^6c^{13} + b^{12}c^7 - 24ab^{10}c^8 + 240a^2b^8c^9 - 1280a^3b^6c^{10} + 3840a^4b^4c^{11} \\
& - 6144a^5b^2c^{12}))^{(1/2)} * (16b^7c^7 - 192ab^5c^8 - 1024a^3b^6c^{10} + 768a^2b^3c^9) \\
& / (2(16a^2c^7 + b^4c^5 - 8ab^2c^6)) * (-25b^{15}g^2 + b^9c^6d^2 + c^6d^2*(-(4ac - b^2)^9)^{(1/2)} \\
& + b^{11}c^4e^2 + 9b^{13}c^2f^2 + 25b^6g^2*(-(4ac - b^2)^9)^{(1/2)} - 768a^4b^3c^{10}d^2 \\
& - 27ab^9c^5e^2 - 3840a^5b^3c^9e^2 - 9a^5c^5e^2*(-(4ac - b^2)^9)^{(1/2)} \\
& - 213ab^{11}c^3f^2 + 26880a^6b^3c^8f^2 - 80640a^7b^3c^7g^2 - 30b^{14}c^3f^g \\
& - 96a^2b^5c^8d^2 + 512a^3b^3c^9d^2 + 288a^2b^7c^6e^2 - 1504a^3b^5c^7e^2 \\
& + 3840a^4b^3c^8e^2 + 2077a^2b^9c^4f^2 - 10656a^3b^7c^5f^2 + 30240a^4b^5c^6f^2 \\
& - 44800a^5b^3c^7f^2 + 25a^2c^4f^2*(-(4ac - b^2)^9)^{(1/2)} + b^2c^4e^2*(-(4ac - b^2)^9)^{(1/2)} \\
& + 6366a^2b^{11}c^2g^2 - 35767a^3b^9c^3g^2 + 116928a^4b^7c^4g^2 - 219744a^5b^5c^5g^2 \\
& + 215040a^6b^3c^6g^2 - 49a^3c^3g^2*(-(4ac - b^2)^9)^{(1/2)} + 9b^4c^2f^2*(-(4ac - b^2)^9)^{(1/2)} \\
& - 615ab^{13}c^3g^2 + 3072a^5c^{10}d^e + 2b^{10}c^5d^e - 7168a^6c^9d^g - 15360a^6c^9e^f \\
& - 6b^{11}c^4d^f + 10b^{12}c^3d^g - 6b^{12}c^3e^f + 35840a^7c^8f^g + 10b^{13}c^2e^g \\
& - 36ab^8c^6d^e + 98ab^9c^5d^f - 1536a^5b^3c^9d^f - 10a^5c^5d^f*(-(4ac - b^2)^9)^{(1/2)} \\
& + 2b^2c^5d^e*(-(4ac - b^2)^9)^{(1/2)} - 168ab^{10}c^4d^g + 152ab^{10}c^4e^f \\
& - 258ab^{11}c^3e^g + 43520a^6b^3c^8e^g + 724ab^{12}c^2f^g - 30b^5c^3f^g*(-(4ac - b^2)^9)^{(1/2)} \\
& + 246a^2b^2c^2g^2*(-(4ac - b^2)^9)^{(1/2)} + 192a^2b^6c^7d^e - 128a^3b^4c^8d^e \\
& - 1536a^4b^2c^9d^e - 165ab^4c^6g^2*(-(4ac - b^2)^9)^{(1/2)} - 576a^2b^7c^6d^f \\
& + 1344a^3b^5c^7d^f - 512a^4b^3c^8d^f + 1044a^2b^8c^5d^g - 1548a^2b^8c^5e^f \\
& - 2688a^3b^6c^6d^g + 8064a^3b^6c^6e^f + 1152a^4b^4c^7d^g - 22400a^4b^4c^7e^f \\
& + 6144a^5b^2c^8d^g + 30720a^5b^2c^8e^f - 6b^2c^4d^f*(-(4ac - b^2)^9)^{(1/2)} \\
& + 2706a^2b^9c^4e^g - 14784a^3b^7c^5e^g + 44352a^4b^5c^6e^g - 69120a^5b^3c^7e^g \\
& + 42a^2c^4e^g*(-(4ac - b^2)^9)^{(1/2)} + 10b^3c^3d^g*(-(4ac - b^2)^9)^{(1/2)} \\
& - 6b^3c^3e^f*(-(4ac - b^2)^9)^{(1/2)} - 7278a^2b^{10}c^3f^g + 39132a^3b^8c^4f^g \\
& - 119616a^4b^6c^5f^g + 201600a^5b^4c^6f^g - 161280a^6b^2c^7f^g + 10b^4c^2e^g*(-(4ac - b^2)^9)^{(1/2)} \\
& - 51ab^2c^3f^2*(-(4ac - b^2)^9)^{(1/2)} + 12ab^2c^4d^g*(-(4ac - b^2)^9)^{(1/2)} \\
& + 44ab^2c^4e^f*(-(4ac - b^2)^9)^{(1/2)} - 78ab^2c^3e^g*(-(4ac - b^2)^9)^{(1/2)} \\
& + 184ab^3c^2f^g*(-(4ac - b^2)^9)^{(1/2)} - 186a^2b^3c^3f^g*(-(4ac - b^2)^9)^{(1/2)}) \\
& / (32(4096a^6c^{13} + b^{12}c^7 - 24ab^{10}c^8 + 240a^2b^8c^9 - 1280a^3b^6c^{10} + 3840a^4b^4c^{11} \\
& - 6144a^5b^2c^{12}))^{(1/2)} + (x(25b^{10}g^2 + 8a^2c^8d^2 - 72a^3c^7e^2 + b^4c^6d^2 \\
& + 200a^4c^6f^2 + b^6c^4e^2 - 392a^5c^5g^2 + 9b^8c^2f^2 + 2ab^2c^7d^2 \\
& - 16ab^4c^5e^2 - 114ab^6c^3f^2 - 30b^9c^3f^g + 74a^2b^2c^6e^2 \\
& + 481a^2b^4c^4f^2 - 718a^3b^2c^5f^2 + 1676a^2b^6c^2g^2 - 3536a^3b^4c^3g^2 \\
& + 2794a^4b^2c^4g^2 - 340ab^8c^3g^2 - 80a^3c^7d^f + 2b^5c^5d^e - 6b^6c^4d^f \\
& + 336a^4c^6e^g + 10b^7c^3d^g - 6b^7c^3e^f + 10b^8c^2e^g - 14ab^3c^6d^e \\
& - 8a^2b^3c^7d^e + 32ab^4c^5d^f - 58ab^5c^4d^g + 86ab^5c^4e^f \\
& + 152a^3b^3c^6d^g + 472a^3b^3c^6e^f - 148ab^6c^3e^g + 394ab^7c^2f^g \\
& - 1768a^4b^3c^5f^g + 4a^2b^2c^6d^f + 26a^2b^3c^5d^g - 374a^2b^3c^5e^f \\
& + 698a^2b^4c^4e^g - 1132a^3b^2c^5e^g - 1804a^2b^5c^3f^g \\
& + 3266a^3b^3c^4f^g) / (2(16a^2c^7 + b^4c^5 - 8ab^2c^6)) * (-25b^{15}g^2 \\
& + b^9c^6d^2 + c^6d^2*(-(4ac - b^2)^9)^{(1/2)} + b^{11}c^4e^2 + 9b^{13}c^2f^2 \\
& + 25b^6g^2*(-(4ac - b^2)^9)^{(1/2)} - 768a^4b^3c^{10}d^2 - 27ab^9c^5e^2 \\
& - 3840a^5b^3c^9e^2 - 9a^5c^5e^2*(-(4ac - b^2)^9)^{(1/2)}
\end{aligned}$$

$$\begin{aligned}
& 2)^9)^{(1/2)} - 213*a*b^{11}*c^3*f^2 + 26880*a^6*b*c^8*f^2 - 80640*a^7*b*c^7*g^2 - 30*b^{14}*c*f*g - 96*a^2*b^5*c^8*d^2 + 512*a^3*b^3*c^9*d^2 + 288*a^2*b^7*c^6*e^2 - 1504*a^3*b^5*c^7*e^2 + 3840*a^4*b^3*c^8*e^2 + 2077*a^2*b^9*c^4*f^2 - 10656*a^3*b^7*c^5*f^2 + 30240*a^4*b^5*c^6*f^2 - 44800*a^5*b^3*c^7*f^2 + 25*a^2*c^4*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + b^2*c^4*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 6366*a^2*b^{11}*c^2*g^2 - 35767*a^3*b^9*c^3*g^2 + 116928*a^4*b^7*c^4*g^2 - 219744*a^5*b^5*c^5*g^2 + 215040*a^6*b^3*c^6*g^2 - 49*a^3*c^3*g^2*(-(4*a*c - b^2)^9)^{(1/2)} + 9*b^4*c^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 615*a*b^{13}*c*g^2 + 3072*a^5*c^{10}*d*e + 2*b^{10}*c^5*d*e - 7168*a^6*c^9*d*g - 15360*a^6*c^9*e*f - 6*b^{11}*c^4*d*f + 10*b^{12}*c^3*d*g - 6*b^{12}*c^3*e*f + 35840*a^7*c^8*f*g + 10*b^{13}*c^2*e*g - 36*a*b^8*c^6*d*e + 98*a*b^9*c^5*d*f - 1536*a^5*b*c^9*d*f - 10*a*c^5*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 2*b*c^5*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 168*a*b^{10}*c^4*d*g + 152*a*b^{10}*c^4*e*f - 258*a*b^{11}*c^3*e*g + 43520*a^6*b*c^8*e*g + 724*a*b^{12}*c^2*f*g - 30*b^5*c*f*g*(-(4*a*c - b^2)^9)^{(1/2)} + 246*a^2*b^2*c^2*g^2*(-(4*a*c - b^2)^9)^{(1/2)} + 192*a^2*b^6*c^7*d*e - 128*a^3*b^4*c^8*d*e - 1536*a^4*b^2*c^9*d*e - 165*a*b^4*c*g^2*(-(4*a*c - b^2)^9)^{(1/2)} - 576*a^2*b^7*c^6*d*f + 1344*a^3*b^5*c^7*d*f - 512*a^4*b^3*c^8*d*f + 1044*a^2*b^8*c^5*d*g - 1548*a^2*b^8*c^5*e*f - 2688*a^3*b^6*c^6*d*g + 8064*a^3*b^6*c^6*e*f + 1152*a^4*b^4*c^7*d*g - 22400*a^4*b^4*c^7*e*f + 6144*a^5*b^2*c^8*d*g + 30720*a^5*b^2*c^8*e*f - 6*b^2*c^4*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 2706*a^2*b^9*c^4*e*g - 14784*a^3*b^7*c^5*e*g + 44352*a^4*b^5*c^6*e*g - 69120*a^5*b^3*c^7*e*g + 42*a^2*c^4*e*g*(-(4*a*c - b^2)^9)^{(1/2)} + 10*b^3*c^3*d*g*(-(4*a*c - b^2)^9)^{(1/2)} - 6*b^3*c^3*e*f*(-(4*a*c - b^2)^9)^{(1/2)} - 7278*a^2*b^{10}*c^3*f*g + 39132*a^3*b^8*c^4*f*g - 119616*a^4*b^6*c^5*f*g + 201600*a^5*b^4*c^6*f*g - 161280*a^6*b^2*c^7*f*g + 10*b^4*c^2*e*g*(-(4*a*c - b^2)^9)^{(1/2)} - 51*a*b^2*c^3*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + 12*a*b*c^4*d*g*(-(4*a*c - b^2)^9)^{(1/2)} + 44*a*b*c^4*e*f*(-(4*a*c - b^2)^9)^{(1/2)} - 78*a*b^2*c^3*e*g*(-(4*a*c - b^2)^9)^{(1/2)} + 184*a*b^3*c^2*f*g*(-(4*a*c - b^2)^9)^{(1/2)} - 186*a^2*b*c^3*f*g*(-(4*a*c - b^2)^9)^{(1/2))/(32*(4096*a^6*c^{13} + b^{12}*c^7 - 24*a*b^{10}*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^{10} + 3840*a^4*b^4*c^{11} - 6144*a^5*b^2*c^{12}))^{(1/2)} - (2744*a^7*c^3*g^3 - 225*a^4*b^6*g^3 - 216*a^4*c^6*e^3 + 3*a*b^3*c^6*d^3 + 4*a^2*b*c^7*d^3 + 1300*a^5*b*c^4*f^3 - 24*a^3*c^7*d^2*e + 2060*a^5*b^4*c*g^3 - 125*a^2*b^8*e*g^2 + 56*a^4*c^6*d^2*g - 600*a^5*c^5*e*f^2 + 175*a^3*b^7*f*g^2 + 1512*a^5*c^5*e^2*g - 3528*a^6*c^4*e*g^2 + 1400*a^6*c^4*f^2*g - 5*a^2*b^4*c^4*e^3 + 66*a^3*b^2*c^5*e^3 + 63*a^3*b^5*c^2*f^3 - 573*a^4*b^3*c^3*f^3 - 5334*a^6*b^2*c^2*g^3 + 75*a*b^9*d*g^2 + 240*a^4*c^6*d*e*f - 560*a^5*c^5*d*f*g + 6*a*b^4*c^5*d^2*e + 3*a*b^5*c^4*d*e^2 + 204*a^3*b*c^6*d*e^2 - 18*a*b^5*c^4*d^2*f + 27*a*b^7*c^2*d*f^2 + 12*a^3*b*c^6*d^2*f - 420*a^4*b*c^5*d*f^2 + 30*a*b^6*c^3*d^2*g - 845*a^2*b^7*c*d*g^2 + 924*a^4*b*c^5*e^2*f + 2044*a^5*b*c^4*d*g^2 + 1350*a^3*b^6*c*e*g^2 - 210*a^3*b^6*c*f^2*g - 1485*a^4*b^5*c*f*g^2 + 364*a^6*b*c^3*f*g^2 - 42*a^2*b^2*c^6*d^2*e - 51*a^2*b^3*c^5*d*e^2 + 81*a^2*b^3*c^5*d^2*f - 279*a^2*b^5*c^3*d*f^2 + 801*a^3*b^3*c^4*d*f^2 - 149*a^2*b^4*c^4*d^2*g + 30*a^2*b^5*c^3*e^2*f - 45*a^2*b^6*c^2*e*f^2 + 78*a^3*b^2*c^5*d^2*g - 339*a^3*b^3*c^4*e^2*f + 402*a^3*b^4*c^3*e*f^2 + 3198*a^3*b^5*c^2*d*g^2 - 762*a^4*b^2*c^4*e*f^2 - 4571*a^4*b^3*c^3*d*g^2 - 50*a^2*b^6*c^2*e^2*g + 600*a^3*b^4*c^3*e^2*g - 2002*a^4*b^2*c^4*e^2*g - 4835*a^4*b^4*c^2*e*g^2 + 6598*a^5*b^2*c^3*e*g^2 + 1927*a^4*b^4*c^2*f^2*g - 4722*a^5*b^2*c^3*f^2*g + 3061*a^5*b^3*c^2*f*g^2 - 90*a*b^8*c*d*f*g - 18*a*b^6*c^3*d*e*f + 30*a*b^7*c^2*d*e*g - 1352*a^4*b*c^5*d*e*g + 150*a^2*b^7*c*e*f*g - 2312*a^5*b*c^4*e*f*g + 246*a^2*b^4*c^4*d*e*f - 804*a^3*b^2*c^5*d*e*f - 424*a^2*b^5*c^3*d*e*g + 1578*a^3*b^3*c^4*d*e*g + 972*a^2*b^6*c^2*d*f*g - 3244*a^3*b^4*c^3*d*f*g + 3276*a^4*b^2*c^4*d*f*g - 1480*a^3*b^5*c^2*e*f*g + 4122*a^4*b^3*c^3*e*f*g)/(4*(64*a^3*c^8 - b^6*c^5 + 12*a*b^4*c^6 - 48*a^2*b^2*c^7)))*(-(25*b^{15}*g^2 + b^9*c^6*d^2 + c^6*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + b^{11}*c^4*e^2 + 9*b^{13}*c^2*f^2 + 25*b^6*g^2*(-(4*a*c - b^2)^9)^{(1/2)} - 768*a^4*b*c^{10}*d^2 - 27*a*b^9*c^5*e^2 - 3840*a^5*b*c^9*e^2 - 9*a*c^5*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 213*a*b^{11}*c^3*f^2 + 26880*a^6*b*c^8*f^2 - 80640*a^7*b*c^7*g^2 - 30*b^{14}*c*f*g - 96*a^2*b^5*c^8*d^2 + 512*a^3*b^3*c^9*d^2 + 288*a^2*b^7*c^6*e^2 - 1504*a^3*b^5*c^7*e^2 + 3840*a^4
\end{aligned}$$

$$\begin{aligned}
& *b^3*c^8*e^2 + 2077*a^2*b^9*c^4*f^2 - 10656*a^3*b^7*c^5*f^2 + 30240*a^4*b^5 \\
& *c^6*f^2 - 44800*a^5*b^3*c^7*f^2 + 25*a^2*c^4*f^2*(-(4*a*c - b^2)^9)^{(1/2)} \\
& + b^2*c^4*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 6366*a^2*b^11*c^2*g^2 - 35767*a^3* \\
& b^9*c^3*g^2 + 116928*a^4*b^7*c^4*g^2 - 219744*a^5*b^5*c^5*g^2 + 215040*a^6* \\
& b^3*c^6*g^2 - 49*a^3*c^3*g^2*(-(4*a*c - b^2)^9)^{(1/2)} + 9*b^4*c^2*f^2*(-(4* \\
& a*c - b^2)^9)^{(1/2)} - 615*a*b^13*c*g^2 + 3072*a^5*c^10*d*e + 2*b^10*c^5*d*e \\
& - 7168*a^6*c^9*d*g - 15360*a^6*c^9*e*f - 6*b^11*c^4*d*f + 10*b^12*c^3*d*g \\
& - 6*b^12*c^3*e*f + 35840*a^7*c^8*f*g + 10*b^13*c^2*e*g - 36*a*b^8*c^6*d*e + \\
& 98*a*b^9*c^5*d*f - 1536*a^5*b*c^9*d*f - 10*a*c^5*d*f*(-(4*a*c - b^2)^9)^{(1 \\
& /2)} + 2*b*c^5*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 168*a*b^10*c^4*d*g + 152*a*b^1 \\
& 0*c^4*e*f - 258*a*b^11*c^3*e*g + 43520*a^6*b*c^8*e*g + 724*a*b^12*c^2*f*g - \\
& 30*b^5*c*f*g*(-(4*a*c - b^2)^9)^{(1/2)} + 246*a^2*b^2*c^2*g^2*(-(4*a*c - b^2 \\
&)^9)^{(1/2)} + 192*a^2*b^6*c^7*d*e - 128*a^3*b^4*c^8*d*e - 1536*a^4*b^2*c^9*d \\
& *e - 165*a*b^4*c*g^2*(-(4*a*c - b^2)^9)^{(1/2)} - 576*a^2*b^7*c^6*d*f + 1344* \\
& a^3*b^5*c^7*d*f - 512*a^4*b^3*c^8*d*f + 1044*a^2*b^8*c^5*d*g - 1548*a^2*b^8 \\
& *c^5*e*f - 2688*a^3*b^6*c^6*d*g + 8064*a^3*b^6*c^6*e*f + 1152*a^4*b^4*c^7*d \\
& *g - 22400*a^4*b^4*c^7*e*f + 6144*a^5*b^2*c^8*d*g + 30720*a^5*b^2*c^8*e*f - \\
& 6*b^2*c^4*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 2706*a^2*b^9*c^4*e*g - 14784*a^3* \\
& b^7*c^5*e*g + 44352*a^4*b^5*c^6*e*g - 69120*a^5*b^3*c^7*e*g + 42*a^2*c^4*e* \\
& g*(-(4*a*c - b^2)^9)^{(1/2)} + 10*b^3*c^3*d*g*(-(4*a*c - b^2)^9)^{(1/2)} - 6*b^ \\
& 3*c^3*e*f*(-(4*a*c - b^2)^9)^{(1/2)} - 7278*a^2*b^10*c^3*f*g + 39132*a^3*b^8* \\
& c^4*f*g - 119616*a^4*b^6*c^5*f*g + 201600*a^5*b^4*c^6*f*g - 161280*a^6*b^2* \\
& c^7*f*g + 10*b^4*c^2*e*g*(-(4*a*c - b^2)^9)^{(1/2)} - 51*a*b^2*c^3*f^2*(-(4*a \\
& *c - b^2)^9)^{(1/2)} + 12*a*b*c^4*d*g*(-(4*a*c - b^2)^9)^{(1/2)} + 44*a*b*c^4*e \\
& *f*(-(4*a*c - b^2)^9)^{(1/2)} - 78*a*b^2*c^3*e*g*(-(4*a*c - b^2)^9)^{(1/2)} + 1 \\
& 84*a*b^3*c^2*f*g*(-(4*a*c - b^2)^9)^{(1/2)} - 186*a^2*b*c^3*f*g*(-(4*a*c - b^ \\
& 2)^9)^{(1/2)})/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^ \\
& 9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^{(1/2)}*2i + \\
& \operatorname{atan}((((2048*a^4*c^10*d - 10240*a^5*c^9*f + 384*a^2*b^4*c^8*d - 1536*a^3* \\
& b^2*c^9*d - 192*a^2*b^5*c^7*e + 768*a^3*b^3*c^8*e + 736*a^2*b^6*c^6*f - 422 \\
& 4*a^3*b^4*c^7*f + 10752*a^4*b^2*c^8*f - 1264*a^2*b^7*c^5*g + 7488*a^3*b^5*c \\
& ^6*g - 19712*a^4*b^3*c^7*g - 32*a*b^6*c^7*d + 16*a*b^7*c^6*e - 1024*a^4*b*c \\
& ^9*e - 48*a*b^8*c^5*f + 80*a*b^9*c^4*g + 19456*a^5*b*c^8*g)/(8*(64*a^3*c^8 \\
& - b^6*c^5 + 12*a*b^4*c^6 - 48*a^2*b^2*c^7)) - (x*((c^6*d^2*(-(4*a*c - b^2)^ \\
& 9)^{(1/2)} - b^9*c^6*d^2 - 25*b^15*g^2 - b^11*c^4*e^2 - 9*b^13*c^2*f^2 + 25*b \\
& ^6*g^2*(-(4*a*c - b^2)^9)^{(1/2)} + 768*a^4*b*c^10*d^2 + 27*a*b^9*c^5*e^2 + 3 \\
& 840*a^5*b*c^9*e^2 - 9*a*c^5*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 213*a*b^11*c^3*f \\
& ^2 - 26880*a^6*b*c^8*f^2 + 80640*a^7*b*c^7*g^2 + 30*b^14*c*f*g + 96*a^2*b^5 \\
& *c^8*d^2 - 512*a^3*b^3*c^9*d^2 - 288*a^2*b^7*c^6*e^2 + 1504*a^3*b^5*c^7*e^2 \\
& - 3840*a^4*b^3*c^8*e^2 - 2077*a^2*b^9*c^4*f^2 + 10656*a^3*b^7*c^5*f^2 - 30 \\
& 240*a^4*b^5*c^6*f^2 + 44800*a^5*b^3*c^7*f^2 + 25*a^2*c^4*f^2*(-(4*a*c - b^2 \\
&)^9)^{(1/2)} + b^2*c^4*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 6366*a^2*b^11*c^2*g^2 + \\
& 35767*a^3*b^9*c^3*g^2 - 116928*a^4*b^7*c^4*g^2 + 219744*a^5*b^5*c^5*g^2 - \\
& 215040*a^6*b^3*c^6*g^2 - 49*a^3*c^3*g^2*(-(4*a*c - b^2)^9)^{(1/2)} + 9*b^4*c^ \\
& 2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + 615*a*b^13*c*g^2 - 3072*a^5*c^10*d*e - 2*b \\
& ^10*c^5*d*e + 7168*a^6*c^9*d*g + 15360*a^6*c^9*e*f + 6*b^11*c^4*d*f - 10*b^ \\
& 12*c^3*d*g + 6*b^12*c^3*e*f - 35840*a^7*c^8*f*g - 10*b^13*c^2*e*g + 36*a*b^ \\
& 8*c^6*d*e - 98*a*b^9*c^5*d*f + 1536*a^5*b*c^9*d*f - 10*a*c^5*d*f*(-(4*a*c - \\
& b^2)^9)^{(1/2)} + 2*b*c^5*d*e*(-(4*a*c - b^2)^9)^{(1/2)} + 168*a*b^10*c^4*d*g \\
& - 152*a*b^10*c^4*e*f + 258*a*b^11*c^3*e*g - 43520*a^6*b*c^8*e*g - 724*a*b^1 \\
& 2*c^2*f*g - 30*b^5*c*f*g*(-(4*a*c - b^2)^9)^{(1/2)} + 246*a^2*b^2*c^2*g^2*(-(\\
& 4*a*c - b^2)^9)^{(1/2)} - 192*a^2*b^6*c^7*d*e + 128*a^3*b^4*c^8*d*e + 1536*a^ \\
& 4*b^2*c^9*d*e - 165*a*b^4*c*g^2*(-(4*a*c - b^2)^9)^{(1/2)} + 576*a^2*b^7*c^6* \\
& d*f - 1344*a^3*b^5*c^7*d*f + 512*a^4*b^3*c^8*d*f - 1044*a^2*b^8*c^5*d*g + 1 \\
& 548*a^2*b^8*c^5*e*f + 2688*a^3*b^6*c^6*d*g - 8064*a^3*b^6*c^6*e*f - 1152*a^ \\
& 4*b^4*c^7*d*g + 22400*a^4*b^4*c^7*e*f - 6144*a^5*b^2*c^8*d*g - 30720*a^5*b^ \\
& 2*c^8*e*f - 6*b^2*c^4*d*f*(-(4*a*c - b^2)^9)^{(1/2)} - 2706*a^2*b^9*c^4*e*g + \\
& 14784*a^3*b^7*c^5*e*g - 44352*a^4*b^5*c^6*e*g + 69120*a^5*b^3*c^7*e*g + 42 \\
& *a^2*c^4*e*g*(-(4*a*c - b^2)^9)^{(1/2)} + 10*b^3*c^3*d*g*(-(4*a*c - b^2)^9)^{(
\end{aligned}$$

$$\begin{aligned}
& 1/2) - 6*b^3*c^3*e*f*(-(4*a*c - b^2)^9)^{(1/2)} + 7278*a^2*b^{10}*c^3*f*g - 391 \\
& 32*a^3*b^8*c^4*f*g + 119616*a^4*b^6*c^5*f*g - 201600*a^5*b^4*c^6*f*g + 1612 \\
& 80*a^6*b^2*c^7*f*g + 10*b^4*c^2*e*g*(-(4*a*c - b^2)^9)^{(1/2)} - 51*a*b^2*c^3 \\
& *f^2*(-(4*a*c - b^2)^9)^{(1/2)} + 12*a*b*c^4*d*g*(-(4*a*c - b^2)^9)^{(1/2)} + 4 \\
& 4*a*b*c^4*e*f*(-(4*a*c - b^2)^9)^{(1/2)} - 78*a*b^2*c^3*e*g*(-(4*a*c - b^2)^9 \\
&)^{(1/2)} + 184*a*b^3*c^2*f*g*(-(4*a*c - b^2)^9)^{(1/2)} - 186*a^2*b*c^3*f*g*(- \\
& (4*a*c - b^2)^9)^{(1/2))/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240 \\
& *a^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12))) \\
& ^{(1/2)}*(16*b^7*c^7 - 192*a*b^5*c^8 - 1024*a^3*b*c^10 + 768*a^2*b^3*c^9))/(2 \\
& *(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))*((c^6*d^2*(-(4*a*c - b^2)^9)^{(1/2)} \\
& - b^9*c^6*d^2 - 25*b^15*g^2 - b^11*c^4*e^2 - 9*b^13*c^2*f^2 + 25*b^6*g^2*(- \\
& (4*a*c - b^2)^9)^{(1/2)} + 768*a^4*b*c^10*d^2 + 27*a*b^9*c^5*e^2 + 3840*a^5*b \\
& *c^9*e^2 - 9*a*c^5*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 213*a*b^11*c^3*f^2 - 2688 \\
& 0*a^6*b*c^8*f^2 + 80640*a^7*b*c^7*g^2 + 30*b^14*c*f*g + 96*a^2*b^5*c^8*d^2 \\
& - 512*a^3*b^3*c^9*d^2 - 288*a^2*b^7*c^6*e^2 + 1504*a^3*b^5*c^7*e^2 - 3840*a \\
& ^4*b^3*c^8*e^2 - 2077*a^2*b^9*c^4*f^2 + 10656*a^3*b^7*c^5*f^2 - 30240*a^4*b \\
& ^5*c^6*f^2 + 44800*a^5*b^3*c^7*f^2 + 25*a^2*c^4*f^2*(-(4*a*c - b^2)^9)^{(1/2)} \\
&) + b^2*c^4*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 6366*a^2*b^11*c^2*g^2 + 35767*a^ \\
& 3*b^9*c^3*g^2 - 116928*a^4*b^7*c^4*g^2 + 219744*a^5*b^5*c^5*g^2 - 215040*a^ \\
& 6*b^3*c^6*g^2 - 49*a^3*c^3*g^2*(-(4*a*c - b^2)^9)^{(1/2)} + 9*b^4*c^2*f^2*(- \\
& (4*a*c - b^2)^9)^{(1/2)} + 615*a*b^13*c*g^2 - 3072*a^5*c^10*d*e - 2*b^10*c^5*d \\
& *e + 7168*a^6*c^9*d*g + 15360*a^6*c^9*e*f + 6*b^11*c^4*d*f - 10*b^12*c^3*d* \\
& g + 6*b^12*c^3*e*f - 35840*a^7*c^8*f*g - 10*b^13*c^2*e*g + 36*a*b^8*c^6*d*e \\
& - 98*a*b^9*c^5*d*f + 1536*a^5*b*c^9*d*f - 10*a*c^5*d*f*(-(4*a*c - b^2)^9)^ \\
& (1/2) + 2*b*c^5*d*e*(-(4*a*c - b^2)^9)^{(1/2)} + 168*a*b^10*c^4*d*g - 152*a*b \\
& ^10*c^4*e*f + 258*a*b^11*c^3*e*g - 43520*a^6*b*c^8*e*g - 724*a*b^12*c^2*f*g \\
& - 30*b^5*c*f*g*(-(4*a*c - b^2)^9)^{(1/2)} + 246*a^2*b^2*c^2*g^2*(-(4*a*c - b \\
& ^2)^9)^{(1/2)} - 192*a^2*b^6*c^7*d*e + 128*a^3*b^4*c^8*d*e + 1536*a^4*b^2*c^9 \\
& *d*e - 165*a*b^4*c*g^2*(-(4*a*c - b^2)^9)^{(1/2)} + 576*a^2*b^7*c^6*d*f - 134 \\
& 4*a^3*b^5*c^7*d*f + 512*a^4*b^3*c^8*d*f - 1044*a^2*b^8*c^5*d*g + 1548*a^2*b \\
& ^8*c^5*e*f + 2688*a^3*b^6*c^6*d*g - 8064*a^3*b^6*c^6*e*f - 1152*a^4*b^4*c^7 \\
& *d*g + 22400*a^4*b^4*c^7*e*f - 6144*a^5*b^2*c^8*d*g - 30720*a^5*b^2*c^8*e*f \\
& - 6*b^2*c^4*d*f*(-(4*a*c - b^2)^9)^{(1/2)} - 2706*a^2*b^9*c^4*e*g + 14784*a^ \\
& 3*b^7*c^5*e*g - 44352*a^4*b^5*c^6*e*g + 69120*a^5*b^3*c^7*e*g + 42*a^2*c^4* \\
& e*g*(-(4*a*c - b^2)^9)^{(1/2)} + 10*b^3*c^3*d*g*(-(4*a*c - b^2)^9)^{(1/2)} - 6* \\
& b^3*c^3*e*f*(-(4*a*c - b^2)^9)^{(1/2)} + 7278*a^2*b^{10}*c^3*f*g - 39132*a^3*b^ \\
& 8*c^4*f*g + 119616*a^4*b^6*c^5*f*g - 201600*a^5*b^4*c^6*f*g + 161280*a^6*b^ \\
& 2*c^7*f*g + 10*b^4*c^2*e*g*(-(4*a*c - b^2)^9)^{(1/2)} - 51*a*b^2*c^3*f^2*(-(4 \\
& *a*c - b^2)^9)^{(1/2)} + 12*a*b*c^4*d*g*(-(4*a*c - b^2)^9)^{(1/2)} + 44*a*b*c^4 \\
& *e*f*(-(4*a*c - b^2)^9)^{(1/2)} - 78*a*b^2*c^3*e*g*(-(4*a*c - b^2)^9)^{(1/2)} + \\
& 184*a*b^3*c^2*f*g*(-(4*a*c - b^2)^9)^{(1/2)} - 186*a^2*b*c^3*f*g*(-(4*a*c - \\
& b^2)^9)^{(1/2))/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8* \\
& c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^{(1/2)} - \\
& (x*(25*b^10*g^2 + 8*a^2*c^8*d^2 - 72*a^3*c^7*e^2 + b^4*c^6*d^2 + 200*a^4*c^ \\
& 6*f^2 + b^6*c^4*e^2 - 392*a^5*c^5*g^2 + 9*b^8*c^2*f^2 + 2*a*b^2*c^7*d^2 - 1 \\
& 6*a*b^4*c^5*e^2 - 114*a*b^6*c^3*f^2 - 30*b^9*c*f*g + 74*a^2*b^2*c^6*e^2 + 4 \\
& 81*a^2*b^4*c^4*f^2 - 718*a^3*b^2*c^5*f^2 + 1676*a^2*b^6*c^2*g^2 - 3536*a^3* \\
& b^4*c^3*g^2 + 2794*a^4*b^2*c^4*g^2 - 340*a*b^8*c*g^2 - 80*a^3*c^7*d*f + 2*b \\
& ^5*c^5*d*e - 6*b^6*c^4*d*f + 336*a^4*c^6*e*g + 10*b^7*c^3*d*g - 6*b^7*c^3*e \\
& *f + 10*b^8*c^2*e*g - 14*a*b^3*c^6*d*e - 8*a^2*b*c^7*d*e + 32*a*b^4*c^5*d*f \\
& - 58*a*b^5*c^4*d*g + 86*a*b^5*c^4*e*f + 152*a^3*b*c^6*d*g + 472*a^3*b*c^6* \\
& e*f - 148*a*b^6*c^3*e*g + 394*a*b^7*c^2*f*g - 1768*a^4*b*c^5*f*g + 4*a^2*b^ \\
& 2*c^6*d*f + 26*a^2*b^3*c^5*d*g - 374*a^2*b^3*c^5*e*f + 698*a^2*b^4*c^4*e*g \\
& - 1132*a^3*b^2*c^5*e*g - 1804*a^2*b^5*c^3*f*g + 3266*a^3*b^3*c^4*f*g))/(2*(\\
& 16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))*((c^6*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - \\
& b^9*c^6*d^2 - 25*b^15*g^2 - b^11*c^4*e^2 - 9*b^13*c^2*f^2 + 25*b^6*g^2*(-(4 \\
& *a*c - b^2)^9)^{(1/2)} + 768*a^4*b*c^10*d^2 + 27*a*b^9*c^5*e^2 + 3840*a^5*b*c \\
& ^9*e^2 - 9*a*c^5*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 213*a*b^11*c^3*f^2 - 26880* \\
& a^6*b*c^8*f^2 + 80640*a^7*b*c^7*g^2 + 30*b^14*c*f*g + 96*a^2*b^5*c^8*d^2 -
\end{aligned}$$

$$\begin{aligned}
& 512a^3b^3c^9d^2 - 288a^2b^7c^6e^2 + 1504a^3b^5c^7e^2 - 3840a^4 \\
& *b^3c^8e^2 - 2077a^2b^9c^4f^2 + 10656a^3b^7c^5f^2 - 30240a^4b^5 \\
& *c^6f^2 + 44800a^5b^3c^7f^2 + 25a^2c^4f^2 * (-4ac - b^2)^9)^{(1/2)} \\
& + b^2c^4e^2 * (-4ac - b^2)^9)^{(1/2)} - 6366a^2b^{11}c^2g^2 + 35767a^3 \\
& b^9c^3g^2 - 116928a^4b^7c^4g^2 + 219744a^5b^5c^5g^2 - 215040a^6 \\
& b^3c^6g^2 - 49a^3c^3g^2 * (-4ac - b^2)^9)^{(1/2)} + 9b^4c^2f^2 * (-4 \\
& ac - b^2)^9)^{(1/2)} + 615a*b^{13}c*g^2 - 3072a^5c^{10}d*e - 2b^{10}c^5d*e \\
& + 7168a^6c^9d*g + 15360a^6c^9e*f + 6b^{11}c^4d*f - 10b^{12}c^3d*g \\
& + 6b^{12}c^3e*f - 35840a^7c^8f*g - 10b^{13}c^2e*g + 36a*b^8c^6d*e - \\
& 98a*b^9c^5d*f + 1536a^5b*c^9d*f - 10a*c^5d*f * (-4ac - b^2)^9)^{(1/2)} \\
& + 2b*c^5d*e * (-4ac - b^2)^9)^{(1/2)} + 168a*b^{10}c^4d*g - 152a*b^{10} \\
& c^4e*f + 258a*b^{11}c^3e*g - 43520a^6b*c^8e*g - 724a*b^{12}c^2f*g - \\
& 30b^5c*f*g * (-4ac - b^2)^9)^{(1/2)} + 246a^2b^2c^2g^2 * (-4ac - b^2 \\
&)^9)^{(1/2)} - 192a^2b^6c^7d*e + 128a^3b^4c^8d*e + 1536a^4b^2c^9d \\
& *e - 165a*b^4c*g^2 * (-4ac - b^2)^9)^{(1/2)} + 576a^2b^7c^6d*f - 1344 \\
& a^3b^5c^7d*f + 512a^4b^3c^8d*f - 1044a^2b^8c^5d*g + 1548a^2b^8 \\
& *c^5e*f + 2688a^3b^6c^6d*g - 8064a^3b^6c^6e*f - 1152a^4b^4c^7d \\
& *g + 22400a^4b^4c^7e*f - 6144a^5b^2c^8d*g - 30720a^5b^2c^8e*f - \\
& 6b^2c^4d*f * (-4ac - b^2)^9)^{(1/2)} - 2706a^2b^9c^4e*g + 14784a^3 \\
& b^7c^5e*g - 44352a^4b^5c^6e*g + 69120a^5b^3c^7e*g + 42a^2c^4e \\
& *g * (-4ac - b^2)^9)^{(1/2)} + 10b^3c^3d*g * (-4ac - b^2)^9)^{(1/2)} - 6b^ \\
& 3c^3e*f * (-4ac - b^2)^9)^{(1/2)} + 7278a^2b^{10}c^3f*g - 39132a^3b^8 \\
& c^4f*g + 119616a^4b^6c^5f*g - 201600a^5b^4c^6f*g + 161280a^6b^2 \\
& c^7f*g + 10b^4c^2e*g * (-4ac - b^2)^9)^{(1/2)} - 51a*b^2c^3f^2 * (-4a \\
& c - b^2)^9)^{(1/2)} + 12a*b*c^4d*g * (-4ac - b^2)^9)^{(1/2)} + 44a*b*c^4e \\
& *f * (-4ac - b^2)^9)^{(1/2)} - 78a*b^2c^3e*g * (-4ac - b^2)^9)^{(1/2)} + 1 \\
& 84a*b^3c^2f*g * (-4ac - b^2)^9)^{(1/2)} - 186a^2b*c^3f*g * (-4ac - b^ \\
& 2)^9)^{(1/2))} / (32(4096a^6c^{13} + b^{12}c^7 - 24a*b^{10}c^8 + 240a^2b^8c^ \\
& 9 - 1280a^3b^6c^{10} + 3840a^4b^4c^{11} - 6144a^5b^2c^{12}))^{(1/2)} * i - \\
& (((2048a^4c^{10}d - 10240a^5c^9f + 384a^2b^4c^8d - 1536a^3b^2c^ \\
& 9d - 192a^2b^5c^7e + 768a^3b^3c^8e + 736a^2b^6c^6f - 4224a^3 \\
& b^4c^7f + 10752a^4b^2c^8f - 1264a^2b^7c^5g + 7488a^3b^5c^6g - \\
& 19712a^4b^3c^7g - 32a*b^6c^7d + 16a*b^7c^6e - 1024a^4b*c^9e - \\
& 48a*b^8c^5f + 80a*b^9c^4g + 19456a^5b*c^8g) / (8(64a^3c^8 - b^6 \\
& c^5 + 12a*b^4c^6 - 48a^2b^2c^7)) + (x((c^6d^2 * (-4ac - b^2)^9)^{(1/2)} \\
& - b^9c^6d^2 - 25b^{15}g^2 - b^{11}c^4e^2 - 9b^{13}c^2f^2 + 25b^6g^2 \\
& * (-4ac - b^2)^9)^{(1/2)} + 768a^4b*c^{10}d^2 + 27a*b^9c^5e^2 + 3840a^ \\
& 5b*c^9e^2 - 9a*c^5e^2 * (-4ac - b^2)^9)^{(1/2)} + 213a*b^{11}c^3f^2 - 2 \\
& 6880a^6b*c^8f^2 + 80640a^7b*c^7g^2 + 30b^{14}c*f*g + 96a^2b^5c^8d \\
& ^2 - 512a^3b^3c^9d^2 - 288a^2b^7c^6e^2 + 1504a^3b^5c^7e^2 - 384 \\
& 0a^4b^3c^8e^2 - 2077a^2b^9c^4f^2 + 10656a^3b^7c^5f^2 - 30240a^4 \\
& b^5c^6f^2 + 44800a^5b^3c^7f^2 + 25a^2c^4f^2 * (-4ac - b^2)^9)^{(1/2)} \\
& + b^2c^4e^2 * (-4ac - b^2)^9)^{(1/2)} - 6366a^2b^{11}c^2g^2 + 35767 \\
& a^3b^9c^3g^2 - 116928a^4b^7c^4g^2 + 219744a^5b^5c^5g^2 - 215040 \\
& a^6b^3c^6g^2 - 49a^3c^3g^2 * (-4ac - b^2)^9)^{(1/2)} + 9b^4c^2f^2 * \\
& (-4ac - b^2)^9)^{(1/2)} + 615a*b^{13}c*g^2 - 3072a^5c^{10}d*e - 2b^{10}c^ \\
& 5d*e + 7168a^6c^9d*g + 15360a^6c^9e*f + 6b^{11}c^4d*f - 10b^{12}c^3 \\
& *d*g + 6b^{12}c^3e*f - 35840a^7c^8f*g - 10b^{13}c^2e*g + 36a*b^8c^6 \\
& d*e - 98a*b^9c^5d*f + 1536a^5b*c^9d*f - 10a*c^5d*f * (-4ac - b^2)^ \\
& 9)^{(1/2)} + 2b*c^5d*e * (-4ac - b^2)^9)^{(1/2)} + 168a*b^{10}c^4d*g - 152 \\
& a*b^{10}c^4e*f + 258a*b^{11}c^3e*g - 43520a^6b*c^8e*g - 724a*b^{12}c^2 \\
& f*g - 30b^5c*f*g * (-4ac - b^2)^9)^{(1/2)} + 246a^2b^2c^2g^2 * (-4ac \\
& - b^2)^9)^{(1/2)} - 192a^2b^6c^7d*e + 128a^3b^4c^8d*e + 1536a^4b^2 \\
& c^9d*e - 165a*b^4c*g^2 * (-4ac - b^2)^9)^{(1/2)} + 576a^2b^7c^6d*f - \\
& 1344a^3b^5c^7d*f + 512a^4b^3c^8d*f - 1044a^2b^8c^5d*g + 1548a^ \\
& 2b^8c^5e*f + 2688a^3b^6c^6d*g - 8064a^3b^6c^6e*f - 1152a^4b^4 \\
& c^7d*g + 22400a^4b^4c^7e*f - 6144a^5b^2c^8d*g - 30720a^5b^2c^8 \\
& e*f - 6b^2c^4d*f * (-4ac - b^2)^9)^{(1/2)} - 2706a^2b^9c^4e*g + 14784 \\
& a^3b^7c^5e*g - 44352a^4b^5c^6e*g + 69120a^5b^3c^7e*g + 42a^2c^
\end{aligned}$$

$$\begin{aligned}
& ^4e*g*(-(4*a*c - b^2)^9)^{(1/2)} + 10*b^3*c^3*d*g*(-(4*a*c - b^2)^9)^{(1/2)} - \\
& 6*b^3*c^3*e*f*(-(4*a*c - b^2)^9)^{(1/2)} + 7278*a^2*b^10*c^3*f*g - 39132*a^3 \\
& *b^8*c^4*f*g + 119616*a^4*b^6*c^5*f*g - 201600*a^5*b^4*c^6*f*g + 161280*a^6 \\
& *b^2*c^7*f*g + 10*b^4*c^2*e*g*(-(4*a*c - b^2)^9)^{(1/2)} - 51*a*b^2*c^3*f^2*(\\
& -(4*a*c - b^2)^9)^{(1/2)} + 12*a*b*c^4*d*g*(-(4*a*c - b^2)^9)^{(1/2)} + 44*a*b* \\
& c^4*e*f*(-(4*a*c - b^2)^9)^{(1/2)} - 78*a*b^2*c^3*e*g*(-(4*a*c - b^2)^9)^{(1/2)} \\
&) + 184*a*b^3*c^2*f*g*(-(4*a*c - b^2)^9)^{(1/2)} - 186*a^2*b*c^3*f*g*(-(4*a*c \\
& - b^2)^9)^{(1/2)})/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b \\
& ^8*c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^{(1/2)} \\
& *(16*b^7*c^7 - 192*a*b^5*c^8 - 1024*a^3*b*c^10 + 768*a^2*b^3*c^9))/(2*(16*a \\
& ^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))*((c^6*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - b^9*c \\
& ^6*d^2 - 25*b^15*g^2 - b^11*c^4*e^2 - 9*b^13*c^2*f^2 + 25*b^6*g^2*(-(4*a*c \\
& - b^2)^9)^{(1/2)} + 768*a^4*b*c^10*d^2 + 27*a*b^9*c^5*e^2 + 3840*a^5*b*c^9*e \\
& ^2 - 9*a*c^5*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 213*a*b^11*c^3*f^2 - 26880*a^6* \\
& b*c^8*f^2 + 80640*a^7*b*c^7*g^2 + 30*b^14*c*f*g + 96*a^2*b^5*c^8*d^2 - 512* \\
& a^3*b^3*c^9*d^2 - 288*a^2*b^7*c^6*e^2 + 1504*a^3*b^5*c^7*e^2 - 3840*a^4*b^3 \\
& *c^8*e^2 - 2077*a^2*b^9*c^4*f^2 + 10656*a^3*b^7*c^5*f^2 - 30240*a^4*b^5*c^6 \\
& *f^2 + 44800*a^5*b^3*c^7*f^2 + 25*a^2*c^4*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + b^ \\
& 2*c^4*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 6366*a^2*b^11*c^2*g^2 + 35767*a^3*b^9* \\
& c^3*g^2 - 116928*a^4*b^7*c^4*g^2 + 219744*a^5*b^5*c^5*g^2 - 215040*a^6*b^3* \\
& c^6*g^2 - 49*a^3*c^3*g^2*(-(4*a*c - b^2)^9)^{(1/2)} + 9*b^4*c^2*f^2*(-(4*a*c \\
& - b^2)^9)^{(1/2)} + 615*a*b^13*c*g^2 - 3072*a^5*c^10*d*e - 2*b^10*c^5*d*e + 7 \\
& 168*a^6*c^9*d*g + 15360*a^6*c^9*e*f + 6*b^11*c^4*d*f - 10*b^12*c^3*d*g + 6* \\
& b^12*c^3*e*f - 35840*a^7*c^8*f*g - 10*b^13*c^2*e*g + 36*a*b^8*c^6*d*e - 98* \\
& a*b^9*c^5*d*f + 1536*a^5*b*c^9*d*f - 10*a*c^5*d*f*(-(4*a*c - b^2)^9)^{(1/2)} \\
& + 2*b*c^5*d*e*(-(4*a*c - b^2)^9)^{(1/2)} + 168*a*b^10*c^4*d*g - 152*a*b^10*c^ \\
& 4*e*f + 258*a*b^11*c^3*e*g - 43520*a^6*b*c^8*e*g - 724*a*b^12*c^2*f*g - 30* \\
& b^5*c*f*g*(-(4*a*c - b^2)^9)^{(1/2)} + 246*a^2*b^2*c^2*g^2*(-(4*a*c - b^2)^9) \\
& ^{(1/2)} - 192*a^2*b^6*c^7*d*e + 128*a^3*b^4*c^8*d*e + 1536*a^4*b^2*c^9*d*e - \\
& 165*a*b^4*c*g^2*(-(4*a*c - b^2)^9)^{(1/2)} + 576*a^2*b^7*c^6*d*f - 1344*a^3* \\
& b^5*c^7*d*f + 512*a^4*b^3*c^8*d*f - 1044*a^2*b^8*c^5*d*g + 1548*a^2*b^8*c^5 \\
& *e*f + 2688*a^3*b^6*c^6*d*g - 8064*a^3*b^6*c^6*e*f - 1152*a^4*b^4*c^7*d*g + \\
& 22400*a^4*b^4*c^7*e*f - 6144*a^5*b^2*c^8*d*g - 30720*a^5*b^2*c^8*e*f - 6*b \\
& ^2*c^4*d*f*(-(4*a*c - b^2)^9)^{(1/2)} - 2706*a^2*b^9*c^4*e*g + 14784*a^3*b^7* \\
& c^5*e*g - 44352*a^4*b^5*c^6*e*g + 69120*a^5*b^3*c^7*e*g + 42*a^2*c^4*e*g*(- \\
& (4*a*c - b^2)^9)^{(1/2)} + 10*b^3*c^3*d*g*(-(4*a*c - b^2)^9)^{(1/2)} - 6*b^3*c^ \\
& 3*e*f*(-(4*a*c - b^2)^9)^{(1/2)} + 7278*a^2*b^10*c^3*f*g - 39132*a^3*b^8*c^4* \\
& f*g + 119616*a^4*b^6*c^5*f*g - 201600*a^5*b^4*c^6*f*g + 161280*a^6*b^2*c^7* \\
& f*g + 10*b^4*c^2*e*g*(-(4*a*c - b^2)^9)^{(1/2)} - 51*a*b^2*c^3*f^2*(-(4*a*c - \\
& b^2)^9)^{(1/2)} + 12*a*b*c^4*d*g*(-(4*a*c - b^2)^9)^{(1/2)} + 44*a*b*c^4*e*f*(\\
& -(4*a*c - b^2)^9)^{(1/2)} - 78*a*b^2*c^3*e*g*(-(4*a*c - b^2)^9)^{(1/2)} + 184*a \\
& *b^3*c^2*f*g*(-(4*a*c - b^2)^9)^{(1/2)} - 186*a^2*b*c^3*f*g*(-(4*a*c - b^2)^9) \\
&)^{(1/2)})/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - \\
& 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^{(1/2)} + (x*(25 \\
& *b^10*g^2 + 8*a^2*c^8*d^2 - 72*a^3*c^7*e^2 + b^4*c^6*d^2 + 200*a^4*c^6*f^2 \\
& + b^6*c^4*e^2 - 392*a^5*c^5*g^2 + 9*b^8*c^2*f^2 + 2*a*b^2*c^7*d^2 - 16*a*b^ \\
& 4*c^5*e^2 - 114*a*b^6*c^3*f^2 - 30*b^9*c*f*g + 74*a^2*b^2*c^6*e^2 + 481*a^2 \\
& *b^4*c^4*f^2 - 718*a^3*b^2*c^5*f^2 + 1676*a^2*b^6*c^2*g^2 - 3536*a^3*b^4*c^ \\
& 3*g^2 + 2794*a^4*b^2*c^4*g^2 - 340*a*b^8*c*g^2 - 80*a^3*c^7*d*f + 2*b^5*c^5 \\
& *d*e - 6*b^6*c^4*d*f + 336*a^4*c^6*e*g + 10*b^7*c^3*d*g - 6*b^7*c^3*e*f + 1 \\
& 0*b^8*c^2*e*g - 14*a*b^3*c^6*d*e - 8*a^2*b*c^7*d*e + 32*a*b^4*c^5*d*f - 58* \\
& a*b^5*c^4*d*g + 86*a*b^5*c^4*e*f + 152*a^3*b*c^6*d*g + 472*a^3*b*c^6*e*f - \\
& 148*a*b^6*c^3*e*g + 394*a*b^7*c^2*f*g - 1768*a^4*b*c^5*f*g + 4*a^2*b^2*c^6* \\
& d*f + 26*a^2*b^3*c^5*d*g - 374*a^2*b^3*c^5*e*f + 698*a^2*b^4*c^4*e*g - 1132 \\
& *a^3*b^2*c^5*e*g - 1804*a^2*b^5*c^3*f*g + 3266*a^3*b^3*c^4*f*g))/(2*(16*a^2 \\
& *c^7 + b^4*c^5 - 8*a*b^2*c^6)))*((c^6*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - b^9*c^ \\
& 6*d^2 - 25*b^15*g^2 - b^11*c^4*e^2 - 9*b^13*c^2*f^2 + 25*b^6*g^2*(-(4*a*c - \\
& b^2)^9)^{(1/2)} + 768*a^4*b*c^10*d^2 + 27*a*b^9*c^5*e^2 + 3840*a^5*b*c^9*e^2 \\
& - 9*a*c^5*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 213*a*b^11*c^3*f^2 - 26880*a^6*b*
\end{aligned}$$

$$\begin{aligned}
& c^8 f^2 + 80640 a^7 b^3 c^7 g^2 + 30 b^{14} c^4 f g + 96 a^2 b^5 c^8 d^2 - 512 a^3 b^3 c^9 d^2 - 288 a^2 b^7 c^6 e^2 + 1504 a^3 b^5 c^7 e^2 - 3840 a^4 b^3 c^8 e^2 - 2077 a^2 b^9 c^4 f^2 + 10656 a^3 b^7 c^5 f^2 - 30240 a^4 b^5 c^6 f^2 + 44800 a^5 b^3 c^7 f^2 + 25 a^2 c^4 f^2 (-4 a c - b^2)^9)^{(1/2)} + b^2 c^4 e^2 (-4 a c - b^2)^9)^{(1/2)} - 6366 a^2 b^{11} c^2 g^2 + 35767 a^3 b^9 c^3 g^2 - 116928 a^4 b^7 c^4 g^2 + 219744 a^5 b^5 c^5 g^2 - 215040 a^6 b^3 c^6 g^2 - 49 a^3 c^3 g^2 (-4 a c - b^2)^9)^{(1/2)} + 9 b^4 c^2 f^2 (-4 a c - b^2)^9)^{(1/2)} + 615 a^2 b^{13} c^3 g^2 - 3072 a^5 c^{10} d e - 2 b^{10} c^5 d e + 7168 a^6 c^9 d g + 15360 a^6 c^9 e f + 6 b^{11} c^4 d f - 10 b^{12} c^3 d g + 6 b^{12} c^3 e f - 35840 a^7 c^8 f g - 10 b^{13} c^2 e g + 36 a^2 b^8 c^6 d e - 98 a^2 b^9 c^5 d f + 1536 a^5 b^3 c^9 d f - 10 a^2 c^5 d f (-4 a c - b^2)^9)^{(1/2)} + 2 b^2 c^5 d e (-4 a c - b^2)^9)^{(1/2)} + 168 a^2 b^{10} c^4 d g - 152 a^2 b^{10} c^4 e f + 258 a^2 b^{11} c^3 e g - 43520 a^6 b^3 c^8 e g - 724 a^2 b^{12} c^2 f g - 30 b^5 c^4 f g (-4 a c - b^2)^9)^{(1/2)} + 246 a^2 b^2 c^2 g^2 (-4 a c - b^2)^9)^{(1/2)} - 192 a^2 b^6 c^7 d e + 128 a^3 b^4 c^8 d e + 1536 a^4 b^2 c^9 d e - 165 a^2 b^4 c^3 g^2 (-4 a c - b^2)^9)^{(1/2)} + 576 a^2 b^7 c^6 d f - 1344 a^3 b^5 c^7 d f + 512 a^4 b^3 c^8 d f - 1044 a^2 b^8 c^5 d g + 1548 a^2 b^8 c^5 e f + 2688 a^3 b^6 c^6 d g - 8064 a^3 b^6 c^6 e f - 1152 a^4 b^4 c^7 d g + 22400 a^4 b^4 c^7 e f - 6144 a^5 b^2 c^8 d g - 30720 a^5 b^2 c^8 e f - 6 b^2 c^4 d f (-4 a c - b^2)^9)^{(1/2)} - 2706 a^2 b^9 c^4 e g + 14784 a^3 b^7 c^5 e g - 44352 a^4 b^5 c^6 e g + 69120 a^5 b^3 c^7 e g + 42 a^2 c^4 e g (-4 a c - b^2)^9)^{(1/2)} + 10 b^3 c^3 d g (-4 a c - b^2)^9)^{(1/2)} - 6 b^3 c^3 e f (-4 a c - b^2)^9)^{(1/2)} + 7278 a^2 b^{10} c^3 f g - 39132 a^3 b^8 c^4 f g + 119616 a^4 b^6 c^5 f g - 201600 a^5 b^4 c^6 f g + 161280 a^6 b^2 c^7 f g + 10 b^4 c^2 e g (-4 a c - b^2)^9)^{(1/2)} - 51 a^2 b^2 c^3 f^2 (-4 a c - b^2)^9)^{(1/2)} + 12 a^2 b^4 c^4 d g (-4 a c - b^2)^9)^{(1/2)} + 44 a^2 b^4 c^4 e f (-4 a c - b^2)^9)^{(1/2)} - 78 a^2 b^2 c^3 e g (-4 a c - b^2)^9)^{(1/2)} + 184 a^2 b^3 c^2 f g (-4 a c - b^2)^9)^{(1/2)} - 186 a^2 b^3 c^3 f g (-4 a c - b^2)^9)^{(1/2))} / (32 (4096 a^6 c^{13} + b^{12} c^7 - 24 a^2 b^{10} c^8 + 240 a^2 b^8 c^9 - 1280 a^3 b^6 c^{10} + 3840 a^4 b^4 c^{11} - 6144 a^5 b^2 c^{12}))^{(1/2)} * i) / (((2048 a^4 c^{10} d - 10240 a^5 c^9 f + 384 a^2 b^4 c^8 d - 1536 a^3 b^2 c^9 d - 192 a^2 b^5 c^7 e + 768 a^3 b^3 c^8 e + 736 a^2 b^6 c^6 f - 4224 a^3 b^4 c^7 f + 10752 a^4 b^2 c^8 f - 1264 a^2 b^7 c^5 g + 7488 a^3 b^5 c^6 g - 19712 a^4 b^3 c^7 g - 32 a^2 b^6 c^7 d + 16 a^2 b^7 c^6 e - 1024 a^4 b^3 c^9 e - 48 a^2 b^8 c^5 f + 80 a^2 b^9 c^4 g + 19456 a^5 b^3 c^8 g) / (8 (64 a^3 c^8 - b^6 c^5 + 12 a^2 b^4 c^6 - 48 a^2 b^2 c^7)) - (x ((c^6 d^2 (-4 a c - b^2)^9)^{(1/2)} - b^9 c^6 d^2 - 25 b^{15} g^2 - b^{11} c^4 e^2 - 9 b^{13} c^2 f^2 + 25 b^6 g^2 (-4 a c - b^2)^9)^{(1/2)} + 768 a^4 b^3 c^{10} d^2 + 27 a^2 b^9 c^5 e^2 + 3840 a^5 b^3 c^9 e^2 - 9 a^2 c^5 e^2 (-4 a c - b^2)^9)^{(1/2)} + 213 a^2 b^{11} c^3 f^2 - 26880 a^6 b^3 c^8 f^2 + 80640 a^7 b^3 c^7 g^2 + 30 b^{14} c^4 f g + 96 a^2 b^5 c^8 d^2 - 512 a^3 b^3 c^9 d^2 - 288 a^2 b^7 c^6 e^2 + 1504 a^3 b^5 c^7 e^2 - 3840 a^4 b^3 c^8 e^2 - 2077 a^2 b^9 c^4 f^2 + 10656 a^3 b^7 c^5 f^2 - 30240 a^4 b^5 c^6 f^2 + 44800 a^5 b^3 c^7 f^2 + 25 a^2 c^4 f^2 (-4 a c - b^2)^9)^{(1/2)} + b^2 c^4 e^2 (-4 a c - b^2)^9)^{(1/2)} - 6366 a^2 b^{11} c^2 g^2 + 35767 a^3 b^9 c^3 g^2 - 116928 a^4 b^7 c^4 g^2 + 219744 a^5 b^5 c^5 g^2 - 215040 a^6 b^3 c^6 g^2 - 49 a^3 c^3 g^2 (-4 a c - b^2)^9)^{(1/2)} + 9 b^4 c^2 f^2 (-4 a c - b^2)^9)^{(1/2)} + 615 a^2 b^{13} c^3 g^2 - 3072 a^5 c^{10} d e - 2 b^{10} c^5 d e + 7168 a^6 c^9 d g + 15360 a^6 c^9 e f + 6 b^{11} c^4 d f - 10 b^{12} c^3 d g + 6 b^{12} c^3 e f - 35840 a^7 c^8 f g - 10 b^{13} c^2 e g + 36 a^2 b^8 c^6 d e - 98 a^2 b^9 c^5 d f + 1536 a^5 b^3 c^9 d f - 10 a^2 c^5 d f (-4 a c - b^2)^9)^{(1/2)} + 2 b^2 c^5 d e (-4 a c - b^2)^9)^{(1/2)} + 168 a^2 b^{10} c^4 d g - 152 a^2 b^{10} c^4 e f + 258 a^2 b^{11} c^3 e g - 43520 a^6 b^3 c^8 e g - 724 a^2 b^{12} c^2 f g - 30 b^5 c^4 f g (-4 a c - b^2)^9)^{(1/2)} + 246 a^2 b^2 c^2 g^2 (-4 a c - b^2)^9)^{(1/2)} - 192 a^2 b^6 c^7 d e + 128 a^3 b^4 c^8 d e + 1536 a^4 b^2 c^9 d e - 165 a^2 b^4 c^3 g^2 (-4 a c - b^2)^9)^{(1/2)} + 576 a^2 b^7 c^6 d f - 1344 a^3 b^5 c^7 d f + 512 a^4 b^3 c^8 d f - 1044 a^2 b^8 c^5 d g + 1548 a^2 b^8 c^5 e f + 2688 a^3 b^6 c^6 d g - 8064 a^3 b^6 c^6 e f - 1152 a^4 b^4 c^7 d g + 22400 a^4 b^4 c^7 e f - 6144 a^5 b^2 c^8 d g - 30720 a^5 b^2 c^8 e f - 6 b^2 c^4 d f (-4 a c - b^2)^9)^{(1/2)} - 2706 a^2 b^9 c^4 e g + 14784 a^3 b^7 c^5 e g
\end{aligned}$$

$$\begin{aligned}
& ^7c^5e^*g - 44352a^4b^5c^6e^*g + 69120a^5b^3c^7e^*g + 42a^2c^4e^*g \\
& *(-4a^*c - b^2)^9)^{(1/2)} + 10b^3c^3d^*g*(-(4a^*c - b^2)^9)^{(1/2)} - 6b^3 \\
& *c^3e^*f*(-(4a^*c - b^2)^9)^{(1/2)} + 7278a^2b^10c^3f^*g - 39132a^3b^8c \\
& ^4f^*g + 119616a^4b^6c^5f^*g - 201600a^5b^4c^6f^*g + 161280a^6b^2c \\
& ^7f^*g + 10b^4c^2e^*g*(-(4a^*c - b^2)^9)^{(1/2)} - 51a^*b^2c^3f^2*(-(4a^* \\
& c - b^2)^9)^{(1/2)} + 12a^*b^*c^4d^*g*(-(4a^*c - b^2)^9)^{(1/2)} + 44a^*b^*c^4e^* \\
& f*(-(4a^*c - b^2)^9)^{(1/2)} - 78a^*b^2c^3e^*g*(-(4a^*c - b^2)^9)^{(1/2)} + 18 \\
& 4a^*b^3c^2f^*g*(-(4a^*c - b^2)^9)^{(1/2)} - 186a^2b^*c^3f^*g*(-(4a^*c - b^2 \\
&)^9)^{(1/2)))/(32*(4096a^6c^13 + b^12c^7 - 24a^*b^10c^8 + 240a^2b^8c^9 \\
& - 1280a^3b^6c^10 + 3840a^4b^4c^11 - 6144a^5b^2c^12))^{(1/2)}*(16b \\
& ^7c^7 - 192a^*b^5c^8 - 1024a^3b^*c^10 + 768a^2b^3c^9))/(2*(16a^2c^7 \\
& + b^4c^5 - 8a^*b^2c^6)))*((c^6d^2*(-(4a^*c - b^2)^9)^{(1/2)} - b^9c^6d^ \\
& 2 - 25b^15g^2 - b^11c^4e^2 - 9b^13c^2f^2 + 25b^6g^2*(-(4a^*c - b^2 \\
&)^9)^{(1/2)} + 768a^4b^*c^10d^2 + 27a^*b^9c^5e^2 + 3840a^5b^*c^9e^2 - 9 \\
& *a^*c^5e^2*(-(4a^*c - b^2)^9)^{(1/2)} + 213a^*b^11c^3f^2 - 26880a^6b^*c^8 \\
& f^2 + 80640a^7b^*c^7g^2 + 30b^14c^*f^*g + 96a^2b^5c^8d^2 - 512a^3b^ \\
& 3c^9d^2 - 288a^2b^7c^6e^2 + 1504a^3b^5c^7e^2 - 3840a^4b^3c^8e \\
& ^2 - 2077a^2b^9c^4f^2 + 10656a^3b^7c^5f^2 - 30240a^4b^5c^6f^2 + \\
& 44800a^5b^3c^7f^2 + 25a^2c^4f^2*(-(4a^*c - b^2)^9)^{(1/2)} + b^2c^4* \\
& e^2*(-(4a^*c - b^2)^9)^{(1/2)} - 6366a^2b^11c^2g^2 + 35767a^3b^9c^3g^ \\
& 2 - 116928a^4b^7c^4g^2 + 219744a^5b^5c^5g^2 - 215040a^6b^3c^6g^ \\
& 2 - 49a^3c^3g^2*(-(4a^*c - b^2)^9)^{(1/2)} + 9b^4c^2f^2*(-(4a^*c - b^2) \\
& ^9)^{(1/2)} + 615a^*b^13c^*g^2 - 3072a^5c^10d^*e - 2b^10c^5d^*e + 7168a^ \\
& 6c^9d^*g + 15360a^6c^9e^*f + 6b^11c^4d^*f - 10b^12c^3d^*g + 6b^12c \\
& ^3e^*f - 35840a^7c^8f^*g - 10b^13c^2e^*g + 36a^*b^8c^6d^*e - 98a^*b^9* \\
& c^5d^*f + 1536a^5b^*c^9d^*f - 10a^*c^5d^*f*(-(4a^*c - b^2)^9)^{(1/2)} + 2b^* \\
& c^5d^*e*(-(4a^*c - b^2)^9)^{(1/2)} + 168a^*b^10c^4d^*g - 152a^*b^10c^4e^*f \\
& + 258a^*b^11c^3e^*g - 43520a^6b^*c^8e^*g - 724a^*b^12c^2f^*g - 30b^5c^* \\
& f^*g*(-(4a^*c - b^2)^9)^{(1/2)} + 246a^2b^2c^2g^2*(-(4a^*c - b^2)^9)^{(1/2)} \\
& - 192a^2b^6c^7d^*e + 128a^3b^4c^8d^*e + 1536a^4b^2c^9d^*e - 165a^ \\
& *b^4c^*g^2*(-(4a^*c - b^2)^9)^{(1/2)} + 576a^2b^7c^6d^*f - 1344a^3b^5c^ \\
& 7d^*f + 512a^4b^3c^8d^*f - 1044a^2b^8c^5d^*g + 1548a^2b^8c^5e^*f + \\
& 2688a^3b^6c^6d^*g - 8064a^3b^6c^6e^*f - 1152a^4b^4c^7d^*g + 22400 \\
& *a^4b^4c^7e^*f - 6144a^5b^2c^8d^*g - 30720a^5b^2c^8e^*f - 6b^2c^4 \\
& *d^*f*(-(4a^*c - b^2)^9)^{(1/2)} - 2706a^2b^9c^4e^*g + 14784a^3b^7c^5e^* \\
& g - 44352a^4b^5c^6e^*g + 69120a^5b^3c^7e^*g + 42a^2c^4e^*g*(-(4a^*c \\
& - b^2)^9)^{(1/2)} + 10b^3c^3d^*g*(-(4a^*c - b^2)^9)^{(1/2)} - 6b^3c^3e^*f* \\
& (-4a^*c - b^2)^9)^{(1/2)} + 7278a^2b^10c^3f^*g - 39132a^3b^8c^4f^*g + \\
& 119616a^4b^6c^5f^*g - 201600a^5b^4c^6f^*g + 161280a^6b^2c^7f^*g + \\
& 10b^4c^2e^*g*(-(4a^*c - b^2)^9)^{(1/2)} - 51a^*b^2c^3f^2*(-(4a^*c - b^2) \\
& ^9)^{(1/2)} + 12a^*b^*c^4d^*g*(-(4a^*c - b^2)^9)^{(1/2)} + 44a^*b^*c^4e^*f*(-(4a^* \\
& c - b^2)^9)^{(1/2)} - 78a^*b^2c^3e^*g*(-(4a^*c - b^2)^9)^{(1/2)} + 184a^*b^3c \\
& ^2f^*g*(-(4a^*c - b^2)^9)^{(1/2)} - 186a^2b^*c^3f^*g*(-(4a^*c - b^2)^9)^{(1/2} \\
&))/(32*(4096a^6c^13 + b^12c^7 - 24a^*b^10c^8 + 240a^2b^8c^9 - 1280a \\
& ^3b^6c^10 + 3840a^4b^4c^11 - 6144a^5b^2c^12))^{(1/2)} - (x*(25b^10* \\
& g^2 + 8a^2c^8d^2 - 72a^3c^7e^2 + b^4c^6d^2 + 200a^4c^6f^2 + b^6* \\
& c^4e^2 - 392a^5c^5g^2 + 9b^8c^2f^2 + 2a^*b^2c^7d^2 - 16a^*b^4c^5* \\
& e^2 - 114a^*b^6c^3f^2 - 30b^9c^*f^*g + 74a^2b^2c^6e^2 + 481a^2b^4c \\
& ^4f^2 - 718a^3b^2c^5f^2 + 1676a^2b^6c^2g^2 - 3536a^3b^4c^3g^2 \\
& + 2794a^4b^2c^4g^2 - 340a^*b^8c^*g^2 - 80a^3c^7d^*f + 2b^5c^5d^*e - \\
& 6b^6c^4d^*f + 336a^4c^6e^*g + 10b^7c^3d^*g - 6b^7c^3e^*f + 10b^8* \\
& c^2e^*g - 14a^*b^3c^6d^*e - 8a^2b^*c^7d^*e + 32a^*b^4c^5d^*f - 58a^*b^5* \\
& c^4d^*g + 86a^*b^5c^4e^*f + 152a^3b^*c^6d^*g + 472a^3b^*c^6e^*f - 148a^* \\
& b^6c^3e^*g + 394a^*b^7c^2f^*g - 1768a^4b^*c^5f^*g + 4a^2b^2c^6d^*f + \\
& 26a^2b^3c^5d^*g - 374a^2b^3c^5e^*f + 698a^2b^4c^4e^*g - 1132a^3b \\
& ^2c^5e^*g - 1804a^2b^5c^3f^*g + 3266a^3b^3c^4f^*g))/(2*(16a^2c^7 + \\
& b^4c^5 - 8a^*b^2c^6)))*((c^6d^2*(-(4a^*c - b^2)^9)^{(1/2)} - b^9c^6d^2 \\
& - 25b^15g^2 - b^11c^4e^2 - 9b^13c^2f^2 + 25b^6g^2*(-(4a^*c - b^2) \\
& ^9)^{(1/2)} + 768a^4b^*c^10d^2 + 27a^*b^9c^5e^2 + 3840a^5b^*c^9e^2 - 9a
\end{aligned}$$

$$\begin{aligned}
& *c^5 * e^2 * (- (4 * a * c - b^2)^9)^{(1/2)} + 213 * a * b^{11} * c^3 * f^2 - 26880 * a^6 * b * c^8 * f^2 \\
& + 80640 * a^7 * b * c^7 * g^2 + 30 * b^{14} * c * f * g + 96 * a^2 * b^5 * c^8 * d^2 - 512 * a^3 * b^3 * \\
& c^9 * d^2 - 288 * a^2 * b^7 * c^6 * e^2 + 1504 * a^3 * b^5 * c^7 * e^2 - 3840 * a^4 * b^3 * c^8 * e^2 \\
& - 2077 * a^2 * b^9 * c^4 * f^2 + 10656 * a^3 * b^7 * c^5 * f^2 - 30240 * a^4 * b^5 * c^6 * f^2 + 4 \\
& 4800 * a^5 * b^3 * c^7 * f^2 + 25 * a^2 * c^4 * f^2 * (- (4 * a * c - b^2)^9)^{(1/2)} + b^2 * c^4 * e^2 \\
& * (- (4 * a * c - b^2)^9)^{(1/2)} - 6366 * a^2 * b^{11} * c^2 * g^2 + 35767 * a^3 * b^9 * c^3 * g^2 \\
& - 116928 * a^4 * b^7 * c^4 * g^2 + 219744 * a^5 * b^5 * c^5 * g^2 - 215040 * a^6 * b^3 * c^6 * g^2 \\
& - 49 * a^3 * c^3 * g^2 * (- (4 * a * c - b^2)^9)^{(1/2)} + 9 * b^4 * c^2 * f^2 * (- (4 * a * c - b^2)^9)^{(1/2)} \\
& + 615 * a * b^{13} * c * g^2 - 3072 * a^5 * c^{10} * d * e - 2 * b^{10} * c^5 * d * e + 7168 * a^6 * \\
& c^9 * d * g + 15360 * a^6 * c^9 * e * f + 6 * b^{11} * c^4 * d * f - 10 * b^{12} * c^3 * d * g + 6 * b^{12} * c^3 \\
& * e * f - 35840 * a^7 * c^8 * f * g - 10 * b^{13} * c^2 * e * g + 36 * a * b^8 * c^6 * d * e - 98 * a * b^9 * c^5 \\
& * d * f + 1536 * a^5 * b * c^9 * d * f - 10 * a * c^5 * d * f * (- (4 * a * c - b^2)^9)^{(1/2)} + 2 * b * c^5 \\
& * d * e * (- (4 * a * c - b^2)^9)^{(1/2)} + 168 * a * b^{10} * c^4 * d * g - 152 * a * b^{10} * c^4 * e * f + \\
& 258 * a * b^{11} * c^3 * e * g - 43520 * a^6 * b * c^8 * e * g - 724 * a * b^{12} * c^2 * f * g - 30 * b^5 * c * f * \\
& g * (- (4 * a * c - b^2)^9)^{(1/2)} + 246 * a^2 * b^2 * c^2 * g^2 * (- (4 * a * c - b^2)^9)^{(1/2)} - \\
& 192 * a^2 * b^6 * c^7 * d * e + 128 * a^3 * b^4 * c^8 * d * e + 1536 * a^4 * b^2 * c^9 * d * e - 165 * a * b \\
& ^4 * c * g^2 * (- (4 * a * c - b^2)^9)^{(1/2)} + 576 * a^2 * b^7 * c^6 * d * f - 1344 * a^3 * b^5 * c^7 * \\
& d * f + 512 * a^4 * b^3 * c^8 * d * f - 1044 * a^2 * b^8 * c^5 * d * g + 1548 * a^2 * b^8 * c^5 * e * f + 2 \\
& 688 * a^3 * b^6 * c^6 * d * g - 8064 * a^3 * b^6 * c^6 * e * f - 1152 * a^4 * b^4 * c^7 * d * g + 22400 * a \\
& ^4 * b^4 * c^7 * e * f - 6144 * a^5 * b^2 * c^8 * d * g - 30720 * a^5 * b^2 * c^8 * e * f - 6 * b^2 * c^4 * d \\
& * f * (- (4 * a * c - b^2)^9)^{(1/2)} - 2706 * a^2 * b^9 * c^4 * e * g + 14784 * a^3 * b^7 * c^5 * e * g \\
& - 44352 * a^4 * b^5 * c^6 * e * g + 69120 * a^5 * b^3 * c^7 * e * g + 42 * a^2 * c^4 * e * g * (- (4 * a * c - \\
& b^2)^9)^{(1/2)} + 10 * b^3 * c^3 * d * g * (- (4 * a * c - b^2)^9)^{(1/2)} - 6 * b^3 * c^3 * e * f * (- \\
& (4 * a * c - b^2)^9)^{(1/2)} + 7278 * a^2 * b^{10} * c^3 * f * g - 39132 * a^3 * b^8 * c^4 * f * g + 11 \\
& 9616 * a^4 * b^6 * c^5 * f * g - 201600 * a^5 * b^4 * c^6 * f * g + 161280 * a^6 * b^2 * c^7 * f * g + 10 \\
& * b^4 * c^2 * e * g * (- (4 * a * c - b^2)^9)^{(1/2)} - 51 * a * b^2 * c^3 * f^2 * (- (4 * a * c - b^2)^9)^{(1/2)} \\
& + 12 * a * b * c^4 * d * g * (- (4 * a * c - b^2)^9)^{(1/2)} + 44 * a * b * c^4 * e * f * (- (4 * a * c \\
& - b^2)^9)^{(1/2)} - 78 * a * b^2 * c^3 * e * g * (- (4 * a * c - b^2)^9)^{(1/2)} + 184 * a * b^3 * c^2 \\
& * f * g * (- (4 * a * c - b^2)^9)^{(1/2)} - 186 * a^2 * b * c^3 * f * g * (- (4 * a * c - b^2)^9)^{(1/2)} \\
& / (32 * (4096 * a^6 * c^{13} + b^{12} * c^7 - 24 * a * b^{10} * c^8 + 240 * a^2 * b^8 * c^9 - 1280 * a^3 \\
& * b^6 * c^{10} + 3840 * a^4 * b^4 * c^{11} - 6144 * a^5 * b^2 * c^{12}))^{(1/2)} + (((2048 * a^4 * c^ \\
& 10 * d - 10240 * a^5 * c^9 * f + 384 * a^2 * b^4 * c^8 * d - 1536 * a^3 * b^2 * c^9 * d - 192 * a^2 * b \\
& ^5 * c^7 * e + 768 * a^3 * b^3 * c^8 * e + 736 * a^2 * b^6 * c^6 * f - 4224 * a^3 * b^4 * c^7 * f + 107 \\
& 52 * a^4 * b^2 * c^8 * f - 1264 * a^2 * b^7 * c^5 * g + 7488 * a^3 * b^5 * c^6 * g - 19712 * a^4 * b^3 * \\
& c^7 * g - 32 * a * b^6 * c^7 * d + 16 * a * b^7 * c^6 * e - 1024 * a^4 * b * c^9 * e - 48 * a * b^8 * c^5 * f \\
& + 80 * a * b^9 * c^4 * g + 19456 * a^5 * b * c^8 * g) / (8 * (64 * a^3 * c^8 - b^6 * c^5 + 12 * a * b^4 * \\
& c^6 - 48 * a^2 * b^2 * c^7)) + (x * ((c^6 * d^2 * (- (4 * a * c - b^2)^9)^{(1/2)} - b^9 * c^6 * d^2 \\
& - 25 * b^{15} * g^2 - b^{11} * c^4 * e^2 - 9 * b^{13} * c^2 * f^2 + 25 * b^6 * g^2 * (- (4 * a * c - b^2)^9)^{(1/2)} \\
& + 768 * a^4 * b * c^{10} * d^2 + 27 * a * b^9 * c^5 * e^2 + 3840 * a^5 * b * c^9 * e^2 - 9 \\
& * a * c^5 * e^2 * (- (4 * a * c - b^2)^9)^{(1/2)} + 213 * a * b^{11} * c^3 * f^2 - 26880 * a^6 * b * c^8 * \\
& f^2 + 80640 * a^7 * b * c^7 * g^2 + 30 * b^{14} * c * f * g + 96 * a^2 * b^5 * c^8 * d^2 - 512 * a^3 * b^3 * \\
& c^9 * d^2 - 288 * a^2 * b^7 * c^6 * e^2 + 1504 * a^3 * b^5 * c^7 * e^2 - 3840 * a^4 * b^3 * c^8 * e^2 \\
& - 2077 * a^2 * b^9 * c^4 * f^2 + 10656 * a^3 * b^7 * c^5 * f^2 - 30240 * a^4 * b^5 * c^6 * f^2 + 4 \\
& 4800 * a^5 * b^3 * c^7 * f^2 + 25 * a^2 * c^4 * f^2 * (- (4 * a * c - b^2)^9)^{(1/2)} + b^2 * c^4 * e^2 \\
& * (- (4 * a * c - b^2)^9)^{(1/2)} - 6366 * a^2 * b^{11} * c^2 * g^2 + 35767 * a^3 * b^9 * c^3 * g^2 \\
& - 116928 * a^4 * b^7 * c^4 * g^2 + 219744 * a^5 * b^5 * c^5 * g^2 - 215040 * a^6 * b^3 * c^6 * g^2 \\
& - 49 * a^3 * c^3 * g^2 * (- (4 * a * c - b^2)^9)^{(1/2)} + 9 * b^4 * c^2 * f^2 * (- (4 * a * c - b^2)^9)^{(1/2)} \\
& + 615 * a * b^{13} * c * g^2 - 3072 * a^5 * c^{10} * d * e - 2 * b^{10} * c^5 * d * e + 7168 * a^6 * \\
& c^9 * d * g + 15360 * a^6 * c^9 * e * f + 6 * b^{11} * c^4 * d * f - 10 * b^{12} * c^3 * d * g + 6 * b^{12} * c^3 \\
& * e * f - 35840 * a^7 * c^8 * f * g - 10 * b^{13} * c^2 * e * g + 36 * a * b^8 * c^6 * d * e - 98 * a * b^9 * \\
& c^5 * d * f + 1536 * a^5 * b * c^9 * d * f - 10 * a * c^5 * d * f * (- (4 * a * c - b^2)^9)^{(1/2)} + 2 * b * \\
& c^5 * d * e * (- (4 * a * c - b^2)^9)^{(1/2)} + 168 * a * b^{10} * c^4 * d * g - 152 * a * b^{10} * c^4 * e * f \\
& + 258 * a * b^{11} * c^3 * e * g - 43520 * a^6 * b * c^8 * e * g - 724 * a * b^{12} * c^2 * f * g - 30 * b^5 * c * \\
& f * g * (- (4 * a * c - b^2)^9)^{(1/2)} + 246 * a^2 * b^2 * c^2 * g^2 * (- (4 * a * c - b^2)^9)^{(1/2)} \\
& - 192 * a^2 * b^6 * c^7 * d * e + 128 * a^3 * b^4 * c^8 * d * e + 1536 * a^4 * b^2 * c^9 * d * e - 165 * a \\
& * b^4 * c * g^2 * (- (4 * a * c - b^2)^9)^{(1/2)} + 576 * a^2 * b^7 * c^6 * d * f - 1344 * a^3 * b^5 * c^7 * \\
& d * f + 512 * a^4 * b^3 * c^8 * d * f - 1044 * a^2 * b^8 * c^5 * d * g + 1548 * a^2 * b^8 * c^5 * e * f + \\
& 2688 * a^3 * b^6 * c^6 * d * g - 8064 * a^3 * b^6 * c^6 * e * f - 1152 * a^4 * b^4 * c^7 * d * g + 22400 \\
& * a^4 * b^4 * c^7 * e * f - 6144 * a^5 * b^2 * c^8 * d * g - 30720 * a^5 * b^2 * c^8 * e * f - 6 * b^2 * c^4
\end{aligned}$$

$$\begin{aligned}
& *d*f*(-(4*a*c - b^2)^9)^{(1/2)} - 2706*a^2*b^9*c^4*e*g + 14784*a^3*b^7*c^5*e*g \\
& g - 44352*a^4*b^5*c^6*e*g + 69120*a^5*b^3*c^7*e*g + 42*a^2*c^4*e*g*(-(4*a*c \\
& - b^2)^9)^{(1/2)} + 10*b^3*c^3*d*g*(-(4*a*c - b^2)^9)^{(1/2)} - 6*b^3*c^3*e*f* \\
& (- (4*a*c - b^2)^9)^{(1/2)} + 7278*a^2*b^10*c^3*f*g - 39132*a^3*b^8*c^4*f*g + \\
& 119616*a^4*b^6*c^5*f*g - 201600*a^5*b^4*c^6*f*g + 161280*a^6*b^2*c^7*f*g + \\
& 10*b^4*c^2*e*g*(-(4*a*c - b^2)^9)^{(1/2)} - 51*a*b^2*c^3*f^2*(-(4*a*c - b^2)^ \\
& 9)^{(1/2)} + 12*a*b*c^4*d*g*(-(4*a*c - b^2)^9)^{(1/2)} + 44*a*b*c^4*e*f*(-(4*a* \\
& c - b^2)^9)^{(1/2)} - 78*a*b^2*c^3*e*g*(-(4*a*c - b^2)^9)^{(1/2)} + 184*a*b^3*c \\
& ^2*f*g*(-(4*a*c - b^2)^9)^{(1/2)} - 186*a^2*b*c^3*f*g*(-(4*a*c - b^2)^9)^{(1/2)} \\
&))/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a \\
& ^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^{(1/2)}*(16*b^7*c^7 - \\
& 192*a*b^5*c^8 - 1024*a^3*b*c^10 + 768*a^2*b^3*c^9))/(2*(16*a^2*c^7 + b^4*c^ \\
& 5 - 8*a*b^2*c^6)))*((c^6*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - b^9*c^6*d^2 - 25*b^ \\
& 15*g^2 - b^11*c^4*e^2 - 9*b^13*c^2*f^2 + 25*b^6*g^2*(-(4*a*c - b^2)^9)^{(1/2)} \\
&) + 768*a^4*b*c^10*d^2 + 27*a*b^9*c^5*e^2 + 3840*a^5*b*c^9*e^2 - 9*a*c^5*e^ \\
& 2*(-(4*a*c - b^2)^9)^{(1/2)} + 213*a*b^11*c^3*f^2 - 26880*a^6*b*c^8*f^2 + 806 \\
& 40*a^7*b*c^7*g^2 + 30*b^14*c*f*g + 96*a^2*b^5*c^8*d^2 - 512*a^3*b^3*c^9*d^2 \\
& - 288*a^2*b^7*c^6*e^2 + 1504*a^3*b^5*c^7*e^2 - 3840*a^4*b^3*c^8*e^2 - 2077 \\
& *a^2*b^9*c^4*f^2 + 10656*a^3*b^7*c^5*f^2 - 30240*a^4*b^5*c^6*f^2 + 44800*a^ \\
& 5*b^3*c^7*f^2 + 25*a^2*c^4*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + b^2*c^4*e^2*(-(4* \\
& a*c - b^2)^9)^{(1/2)} - 6366*a^2*b^11*c^2*g^2 + 35767*a^3*b^9*c^3*g^2 - 11692 \\
& 8*a^4*b^7*c^4*g^2 + 219744*a^5*b^5*c^5*g^2 - 215040*a^6*b^3*c^6*g^2 - 49*a^ \\
& 3*c^3*g^2*(-(4*a*c - b^2)^9)^{(1/2)} + 9*b^4*c^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} \\
& + 615*a*b^13*c*g^2 - 3072*a^5*c^10*d*e - 2*b^10*c^5*d*e + 7168*a^6*c^9*d*g \\
& + 15360*a^6*c^9*e*f + 6*b^11*c^4*d*f - 10*b^12*c^3*d*g + 6*b^12*c^3*e*f - \\
& 35840*a^7*c^8*f*g - 10*b^13*c^2*e*g + 36*a*b^8*c^6*d*e - 98*a*b^9*c^5*d*f + \\
& 1536*a^5*b*c^9*d*f - 10*a*c^5*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 2*b*c^5*d*e*(\\
& -(4*a*c - b^2)^9)^{(1/2)} + 168*a*b^10*c^4*d*g - 152*a*b^10*c^4*e*f + 258*a*b \\
& ^11*c^3*e*g - 43520*a^6*b*c^8*e*g - 724*a*b^12*c^2*f*g - 30*b^5*c*f*g*(-(4* \\
& a*c - b^2)^9)^{(1/2)} + 246*a^2*b^2*c^2*g^2*(-(4*a*c - b^2)^9)^{(1/2)} - 192*a^ \\
& 2*b^6*c^7*d*e + 128*a^3*b^4*c^8*d*e + 1536*a^4*b^2*c^9*d*e - 165*a*b^4*c*g^ \\
& 2*(-(4*a*c - b^2)^9)^{(1/2)} + 576*a^2*b^7*c^6*d*f - 1344*a^3*b^5*c^7*d*f + 5 \\
& 12*a^4*b^3*c^8*d*f - 1044*a^2*b^8*c^5*d*g + 1548*a^2*b^8*c^5*e*f + 2688*a^3 \\
& *b^6*c^6*d*g - 8064*a^3*b^6*c^6*e*f - 1152*a^4*b^4*c^7*d*g + 22400*a^4*b^4* \\
& c^7*e*f - 6144*a^5*b^2*c^8*d*g - 30720*a^5*b^2*c^8*e*f - 6*b^2*c^4*d*f*(-(4 \\
& *a*c - b^2)^9)^{(1/2)} - 2706*a^2*b^9*c^4*e*g + 14784*a^3*b^7*c^5*e*g - 44352 \\
& *a^4*b^5*c^6*e*g + 69120*a^5*b^3*c^7*e*g + 42*a^2*c^4*e*g*(-(4*a*c - b^2)^9 \\
&)^{(1/2)} + 10*b^3*c^3*d*g*(-(4*a*c - b^2)^9)^{(1/2)} - 6*b^3*c^3*e*f*(-(4*a*c \\
& - b^2)^9)^{(1/2)} + 7278*a^2*b^10*c^3*f*g - 39132*a^3*b^8*c^4*f*g + 119616*a^ \\
& 4*b^6*c^5*f*g - 201600*a^5*b^4*c^6*f*g + 161280*a^6*b^2*c^7*f*g + 10*b^4*c^ \\
& 2*e*g*(-(4*a*c - b^2)^9)^{(1/2)} - 51*a*b^2*c^3*f^2*(-(4*a*c - b^2)^9)^{(1/2)} \\
& + 12*a*b*c^4*d*g*(-(4*a*c - b^2)^9)^{(1/2)} + 44*a*b*c^4*e*f*(-(4*a*c - b^2)^ \\
& 9)^{(1/2)} - 78*a*b^2*c^3*e*g*(-(4*a*c - b^2)^9)^{(1/2)} + 184*a*b^3*c^2*f*g*(- \\
& (4*a*c - b^2)^9)^{(1/2)} - 186*a^2*b*c^3*f*g*(-(4*a*c - b^2)^9)^{(1/2)))/(32*(4 \\
& 096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^ \\
& 10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^{(1/2)} + (x*(25*b^10*g^2 + 8*a \\
& ^2*c^8*d^2 - 72*a^3*c^7*e^2 + b^4*c^6*d^2 + 200*a^4*c^6*f^2 + b^6*c^4*e^2 - \\
& 392*a^5*c^5*g^2 + 9*b^8*c^2*f^2 + 2*a*b^2*c^7*d^2 - 16*a*b^4*c^5*e^2 - 114 \\
& *a*b^6*c^3*f^2 - 30*b^9*c*f*g + 74*a^2*b^2*c^6*e^2 + 481*a^2*b^4*c^4*f^2 - \\
& 718*a^3*b^2*c^5*f^2 + 1676*a^2*b^6*c^2*g^2 - 3536*a^3*b^4*c^3*g^2 + 2794*a^ \\
& 4*b^2*c^4*g^2 - 340*a*b^8*c*g^2 - 80*a^3*c^7*d*f + 2*b^5*c^5*d*e - 6*b^6*c^ \\
& 4*d*f + 336*a^4*c^6*e*g + 10*b^7*c^3*d*g - 6*b^7*c^3*e*f + 10*b^8*c^2*e*g - \\
& 14*a*b^3*c^6*d*e - 8*a^2*b*c^7*d*e + 32*a*b^4*c^5*d*f - 58*a*b^5*c^4*d*g + \\
& 86*a*b^5*c^4*e*f + 152*a^3*b*c^6*d*g + 472*a^3*b*c^6*e*f - 148*a*b^6*c^3*e \\
& *g + 394*a*b^7*c^2*f*g - 1768*a^4*b*c^5*f*g + 4*a^2*b^2*c^6*d*f + 26*a^2*b^ \\
& 3*c^5*d*g - 374*a^2*b^3*c^5*e*f + 698*a^2*b^4*c^4*e*g - 1132*a^3*b^2*c^5*e* \\
& g - 1804*a^2*b^5*c^3*f*g + 3266*a^3*b^3*c^4*f*g))/(2*(16*a^2*c^7 + b^4*c^5 \\
& - 8*a*b^2*c^6)))*((c^6*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - b^9*c^6*d^2 - 25*b^15 \\
& *g^2 - b^11*c^4*e^2 - 9*b^13*c^2*f^2 + 25*b^6*g^2*(-(4*a*c - b^2)^9)^{(1/2)}
\end{aligned}$$

$$\begin{aligned}
& + 768a^4b^2c^{10}d^2 + 27a^2b^9c^5e^2 + 3840a^5b^2c^9e^2 - 9a^2c^5e^2 \\
& (-4ac - b^2)^9)^{(1/2)} + 213a^2b^{11}c^3f^2 - 26880a^6b^2c^8f^2 + 80640 \\
& a^7b^2c^7g^2 + 30b^{14}c^2fg + 96a^2b^5c^8d^2 - 512a^3b^3c^9d^2 - \\
& 288a^2b^7c^6e^2 + 1504a^3b^5c^7e^2 - 3840a^4b^3c^8e^2 - 2077a^2 \\
& b^9c^4f^2 + 10656a^3b^7c^5f^2 - 30240a^4b^5c^6f^2 + 44800a^5b^3 \\
& c^7f^2 + 25a^2c^4f^2(-4ac - b^2)^9)^{(1/2)} + b^2c^4e^2(-4ac \\
& c - b^2)^9)^{(1/2)} - 6366a^2b^{11}c^2g^2 + 35767a^3b^9c^3g^2 - 116928a^4 \\
& b^7c^4g^2 + 219744a^5b^5c^5g^2 - 215040a^6b^3c^6g^2 - 49a^3c^3 \\
& g^2(-4ac - b^2)^9)^{(1/2)} + 9b^4c^2f^2(-4ac - b^2)^9)^{(1/2)} + \\
& 615a^2b^{13}c^2g^2 - 3072a^5c^{10}d^2e - 2b^{10}c^5d^2e + 7168a^6c^9d^2g + \\
& 15360a^6c^9e^2f + 6b^{11}c^4d^2f - 10b^{12}c^3d^2g + 6b^{12}c^3e^2f - 35 \\
& 840a^7c^8f^2g - 10b^{13}c^2e^2g + 36a^2b^8c^6d^2e - 98a^2b^9c^5d^2f + 1 \\
& 536a^5b^2c^9d^2f - 10a^2c^5d^2f(-4ac - b^2)^9)^{(1/2)} + 2b^2c^5d^2e(- \\
& 4ac - b^2)^9)^{(1/2)} + 168a^2b^{10}c^4d^2g - 152a^2b^{10}c^4e^2f + 258a^2b^{11} \\
& c^3e^2g - 43520a^6b^2c^8e^2g - 724a^2b^{12}c^2f^2g - 30b^5c^2fg(-4ac \\
& c - b^2)^9)^{(1/2)} + 246a^2b^2c^2g^2(-4ac - b^2)^9)^{(1/2)} - 192a^2b^6 \\
& c^7d^2e + 128a^3b^4c^8d^2e + 1536a^4b^2c^9d^2e - 165a^2b^4c^2g^2(- \\
& 4ac - b^2)^9)^{(1/2)} + 576a^2b^7c^6d^2f - 1344a^3b^5c^7d^2f + 512 \\
& a^4b^3c^8d^2f - 1044a^2b^8c^5d^2g + 1548a^2b^8c^5e^2f + 2688a^3b^6 \\
& c^6d^2g - 8064a^3b^6c^6e^2f - 1152a^4b^4c^7d^2g + 22400a^4b^4c^7 \\
& e^2f - 6144a^5b^2c^8d^2g - 30720a^5b^2c^8e^2f - 6b^2c^4d^2fg(-4ac \\
& c - b^2)^9)^{(1/2)} - 2706a^2b^9c^4e^2g + 14784a^3b^7c^5e^2g - 44352a^4 \\
& b^5c^6e^2g + 69120a^5b^3c^7e^2g + 42a^2c^4e^2fg(-4ac - b^2)^9)^{(1/2)} \\
& + 10b^3c^3d^2fg(-4ac - b^2)^9)^{(1/2)} - 6b^3c^3e^2fg(-4ac - \\
& b^2)^9)^{(1/2)} + 7278a^2b^{10}c^3fg - 39132a^3b^8c^4fg + 119616a^4b^6 \\
& c^5fg - 201600a^5b^4c^6fg + 161280a^6b^2c^7fg + 10b^4c^2e^2 \\
& fg(-4ac - b^2)^9)^{(1/2)} - 51a^2b^2c^3f^2(-4ac - b^2)^9)^{(1/2)} + \\
& 12a^2b^2c^4d^2fg(-4ac - b^2)^9)^{(1/2)} + 44a^2b^2c^4e^2fg(-4ac - b^2)^9 \\
&)^{(1/2)} - 78a^2b^2c^3e^2fg(-4ac - b^2)^9)^{(1/2)} + 184a^2b^3c^2fg(-4 \\
& ac - b^2)^9)^{(1/2)} - 186a^2b^3c^3fg(-4ac - b^2)^9)^{(1/2)} / (32(409 \\
& 6a^6c^{13} + b^{12}c^7 - 24a^2b^{10}c^8 + 240a^2b^8c^9 - 1280a^3b^6c^{10} \\
& + 3840a^4b^4c^{11} - 6144a^5b^2c^{12}))^{(1/2)} - (2744a^7c^3g^3 - 225 \\
& a^4b^6g^3 - 216a^4c^6e^3 + 3a^2b^3c^6d^3 + 4a^2b^2c^7d^3 + 1300a^5 \\
& b^2c^4f^3 - 24a^3c^7d^2e + 2060a^5b^4c^2g^3 - 125a^2b^8e^2g^2 + \\
& 56a^4c^6d^2g - 600a^5c^5e^2f^2 + 175a^3b^7fg^2 + 1512a^5c^5e^2 \\
& fg - 3528a^6c^4e^2fg^2 + 1400a^6c^4f^2g - 5a^2b^4c^4e^3 + 66a^3b^2 \\
& c^5e^3 + 63a^3b^5c^2f^3 - 573a^4b^3c^3f^3 - 5334a^6b^2c^2g^3 \\
& + 75a^2b^9d^2g^2 + 240a^4c^6d^2ef - 560a^5c^5d^2fg + 6a^2b^4c^5d^2 \\
& e + 3a^2b^5c^4d^2e + 204a^3b^2c^6d^2e - 18a^2b^5c^4d^2f + 27a^2b^7 \\
& c^2d^2f + 12a^3b^2c^6d^2f - 420a^4b^2c^5d^2f + 30a^2b^6c^3d^2g \\
& - 845a^2b^7c^2d^2g + 924a^4b^2c^5e^2f + 2044a^5b^2c^4d^2g^2 + 1350 \\
& a^3b^6c^2e^2g^2 - 210a^3b^6c^2f^2g - 1485a^4b^5c^2fg^2 + 364a^6b^2c^3 \\
& fg^2 - 42a^2b^2c^6d^2e - 51a^2b^3c^5d^2e + 81a^2b^3c^5d^2e^2 \\
& f - 279a^2b^5c^3d^2f + 801a^3b^3c^4d^2f - 149a^2b^4c^4d^2g \\
& + 30a^2b^5c^3e^2f - 45a^2b^6c^2e^2f + 78a^3b^2c^5d^2g - 339a^3 \\
& b^3c^4e^2f + 402a^3b^4c^3e^2f + 3198a^3b^5c^2d^2g^2 - 762a^4 \\
& b^2c^4e^2f - 4571a^4b^3c^3d^2g^2 - 50a^2b^6c^2e^2g + 600a^3b^4 \\
& c^3e^2g - 2002a^4b^2c^4e^2g - 4835a^4b^4c^2e^2g^2 + 6598a^5b^2 \\
& c^3e^2g^2 + 1927a^4b^4c^2f^2g - 4722a^5b^2c^3f^2g + 3061a^5b^3 \\
& c^2fg^2 - 90a^2b^8c^2d^2fg - 18a^2b^6c^3d^2ef + 30a^2b^7c^2d^2ef - \\
& 1352a^4b^2c^5d^2ef + 150a^2b^7c^2efg - 2312a^5b^2c^4efg + 246a^2 \\
& b^4c^4d^2ef - 804a^3b^2c^5d^2ef - 424a^2b^5c^3d^2ef + 1578a^3b^3 \\
& c^4d^2ef + 972a^2b^6c^2d^2ef - 3244a^3b^4c^3d^2ef + 3276a^4b^2 \\
& c^4d^2ef - 1480a^3b^5c^2e^2fg + 4122a^4b^3c^3e^2fg) / (4(64a^3c^8 \\
& - b^6c^5 + 12a^2b^4c^6 - 48a^2b^2c^7))) * ((c^6d^2(-4ac - b^2)^9)^{(1/2)} \\
& - b^9c^6d^2 - 25b^{15}g^2 - b^{11}c^4e^2 - 9b^{13}c^2f^2 + 25b^6 \\
& g^2(-4ac - b^2)^9)^{(1/2)} + 768a^4b^2c^{10}d^2 + 27a^2b^9c^5e^2 + \\
& 3840a^5b^2c^9e^2 - 9a^2c^5e^2(-4ac - b^2)^9)^{(1/2)} + 213a^2b^{11}c^3 \\
& f^2 - 26880a^6b^2c^8f^2 + 80640a^7b^2c^7g^2 + 30b^{14}c^2fg + 96a^2b^
\end{aligned}$$

$$\begin{aligned}
& 5c^8d^2 - 512a^3b^3c^9d^2 - 288a^2b^7c^6e^2 + 1504a^3b^5c^7e^2 \\
& - 3840a^4b^3c^8e^2 - 2077a^2b^9c^4f^2 + 10656a^3b^7c^5f^2 - 3 \\
& 0240a^4b^5c^6f^2 + 44800a^5b^3c^7f^2 + 25a^2c^4f^2(-4ac - b^2)^9)^{(1/2)} + b^2c^4e^2(-4ac - b^2)^9)^{(1/2)} - 6366a^2b^{11}c^2g^2 \\
& + 35767a^3b^9c^3g^2 - 116928a^4b^7c^4g^2 + 219744a^5b^5c^5g^2 - \\
& 215040a^6b^3c^6g^2 - 49a^3c^3g^2(-4ac - b^2)^9)^{(1/2)} + 9b^4c^2f^2(-4ac - b^2)^9)^{(1/2)} + 615a^2b^{13}c^2g^2 - 3072a^5c^{10}d^2e - 2 \\
& b^{10}c^5d^2e + 7168a^6c^9d^2g + 15360a^6c^9e^2f + 6b^{11}c^4d^2f - 10b^{12}c^3d^2g + 6b^{12}c^3e^2f - 35840a^7c^8f^2g - 10b^{13}c^2e^2g + 36a^2b^8c^6d^2e \\
& - 98a^2b^9c^5d^2f + 1536a^5b^3c^9d^2f - 10a^2c^5d^2f(-4ac - b^2)^9)^{(1/2)} + 2b^2c^5d^2e(-4ac - b^2)^9)^{(1/2)} + 168a^2b^{10}c^4d^2g \\
& - 152a^2b^{10}c^4e^2f + 258a^2b^{11}c^3e^2g - 43520a^6b^2c^8e^2g - 724a^2b^{12}c^2f^2g - 30b^5c^5f^2g(-4ac - b^2)^9)^{(1/2)} + 246a^2b^2c^2g^2(-4ac - b^2)^9)^{(1/2)} \\
& - 192a^2b^6c^7d^2e + 128a^3b^4c^8d^2e + 1536a^4b^2c^9d^2e - 165a^2b^4c^2g^2(-4ac - b^2)^9)^{(1/2)} + 576a^2b^7c^6d^2f - 1344a^3b^5c^7d^2f + 512a^4b^3c^8d^2f - 1044a^2b^8c^5d^2g + \\
& 1548a^2b^8c^5e^2f + 2688a^3b^6c^6d^2g - 8064a^3b^6c^6e^2f - 1152a^4b^4c^7d^2g + 22400a^4b^4c^7e^2f - 6144a^5b^2c^8d^2g - 30720a^5b^2c^8e^2f - 6b^2c^4d^2f(-4ac - b^2)^9)^{(1/2)} - 2706a^2b^9c^4e^2g \\
& + 14784a^3b^7c^5e^2g - 44352a^4b^5c^6e^2g + 69120a^5b^3c^7e^2g + 4 \\
& 2a^2c^4e^2g(-4ac - b^2)^9)^{(1/2)} + 10b^3c^3d^2g(-4ac - b^2)^9)^{(1/2)} - 6b^3c^3e^2f(-4ac - b^2)^9)^{(1/2)} + 7278a^2b^{10}c^3f^2g - 39 \\
& 132a^3b^8c^4f^2g + 119616a^4b^6c^5f^2g - 201600a^5b^4c^6f^2g + 161 \\
& 280a^6b^2c^7f^2g + 10b^4c^2e^2g(-4ac - b^2)^9)^{(1/2)} - 51a^2b^2c^3f^2(-4ac - b^2)^9)^{(1/2)} + 12a^2b^2c^4d^2g(-4ac - b^2)^9)^{(1/2)} + \\
& 44a^2b^2c^4e^2f(-4ac - b^2)^9)^{(1/2)} - 78a^2b^2c^3e^2g(-4ac - b^2)^9)^{(1/2)} + 184a^2b^3c^2f^2g(-4ac - b^2)^9)^{(1/2)} - 186a^2b^2c^3f^2g(-4ac - b^2)^9)^{(1/2)} \\
& - 186a^2b^2c^3f^2g(-4ac - b^2)^9)^{(1/2)} / (32(4096a^6c^{13} + b^{12}c^7 - 24a^2b^{10}c^8 + 24 \\
& 0a^2b^8c^9 - 1280a^3b^6c^{10} + 3840a^4b^4c^{11} - 6144a^5b^2c^{12})) \\
&)^{(1/2)} * 2i + (g^2x^3)/(3c^2)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4*(g*x**6+f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)

[Out] Timed out

$$3.125 \quad \int \frac{x^2(d+ex^2+fx^4+gx^6)}{(a+bx^2+cx^4)^2} dx$$

Optimal. Leaf size=471

$$\frac{x \left(x^2 \left(-c^2(2af + be) + bc(3ag + bf) + b^3(-g) + 2c^3d \right) - ab^2g + bc(af + cd) - 2ac(ce - ag) \right)}{2c^2 \left(b^2 - 4ac \right) \left(a + bx^2 + cx^4 \right)} \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c}}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)$$

Rubi [A] time = 6.66, antiderivative size = 471, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 35, number of rules / integrand size = 0.114, Rules used = {1668, 1676, 1166, 205}

$$\frac{x \left(c^2 \left(-c^2(2af + be) + bc(3ag + bf) + b^3(-g) + 2c^3d \right) - ab^2g + bc(af + cd) - 2ac(ce - ag) \right)}{2c^2 \left(b^2 - 4ac \right) \left(a + bx^2 + cx^4 \right)} \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c}}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right) + \frac{c^2}{2c^2}$$

Antiderivative was successfully verified.

[In] Int[(x^2*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4)^2,x]

[Out] (g*x)/c^2 - (x*(b*c*(c*d + a*f) - a*b^2*g - 2*a*c*(c*e - a*g) + (2*c^3*d - c^2*(b*e + 2*a*f) - b^3*g + b*c*(b*f + 3*a*g))*x^2))/(2*c^2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((2*c^3*d - c^2*(b*e - 6*a*f) + 3*b^3*g - b*c*(b*f + 13*a*g) + (b^3*c*f - 4*b*c^2*(c*d + 2*a*f) - 3*b^4*g + 4*a*c^2*(c*e - 5*a*g) + b^2*c*(c*e + 19*a*g))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(2*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((2*c^3*d - c^2*(b*e - 6*a*f) + 3*b^3*g - b*c*(b*f + 13*a*g) - (b^3*c*f - 4*b*c^2*(c*d + 2*a*f) - 3*b^4*g + 4*a*c^2*(c*e - 5*a*g) + b^2*c*(c*e + 19*a*g))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(2*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1668

Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :=> With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]

Rule 1676

```
Int[(Pq_)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] := Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rubi steps

$$\int \frac{x^2 (d + ex^2 + fx^4 + gx^6)}{(a + bx^2 + cx^4)^2} dx = -\frac{x (bc(cd + af) - ab^2g - 2ac(ce - ag) + (2c^3d - c^2(be + 2af) - b^3g + bc(bf + 3ag)))}{2c^2 (b^2 - 4ac) (a + bx^2 + cx^4)}$$

$$= -\frac{x (bc(cd + af) - ab^2g - 2ac(ce - ag) + (2c^3d - c^2(be + 2af) - b^3g + bc(bf + 3ag)))}{2c^2 (b^2 - 4ac) (a + bx^2 + cx^4)}$$

$$= \frac{gx}{c^2} - \frac{x (bc(cd + af) - ab^2g - 2ac(ce - ag) + (2c^3d - c^2(be + 2af) - b^3g + bc(bf + 3ag)))}{2c^2 (b^2 - 4ac) (a + bx^2 + cx^4)}$$

$$= \frac{gx}{c^2} - \frac{x (bc(cd + af) - ab^2g - 2ac(ce - ag) + (2c^3d - c^2(be + 2af) - b^3g + bc(bf + 3ag)))}{2c^2 (b^2 - 4ac) (a + bx^2 + cx^4)}$$

$$= \frac{gx}{c^2} - \frac{x (bc(cd + af) - ab^2g - 2ac(ce - ag) + (2c^3d - c^2(be + 2af) - b^3g + bc(bf + 3ag)))}{2c^2 (b^2 - 4ac) (a + bx^2 + cx^4)}$$

Mathematica [A] time = 1.95, size = 575, normalized size = 1.22

$$\frac{\sqrt{c} \operatorname{arctan}\left(\frac{x \sqrt{c}}{\sqrt{a+bx^2+cx^4}}\right) \left[2x^2(-3b^2g+bx^2c^2e+2ac^2d+2a^2c^2f+19a^2cg)+b^3c^2(-c^2e+3c^2f+3cg)+b^2c^2(-c^2e+3c^2f+3cg)+b^2c^2(-c^2e+3c^2f+3cg)+b^2c^2(-c^2e+3c^2f+3cg)\right] + 2c^2 \operatorname{arctan}\left(\frac{x \sqrt{c}}{\sqrt{a+bx^2+cx^4}}\right) \left[2x^2(-3b^2g+bx^2c^2e+2ac^2d+2a^2c^2f+19a^2cg)+b^3c^2(-c^2e+3c^2f+3cg)+b^2c^2(-c^2e+3c^2f+3cg)+b^2c^2(-c^2e+3c^2f+3cg)\right] + 4\sqrt{c}x}{(b^2-4ac)^2 \sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (4*Sqrt[c]*g*x - (2*Sqrt[c]*x*(-(b^3*g*x^2) + b^2*(-(a*g) + c*f*x^2) + 2*c*(a^2*g + c^2*d*x^2 - a*c*(e + f*x^2)) + b*c*(c*(d - e*x^2) + a*(f + 3*g*x^2))))/(b^2 - 4*a*c)*(a + b*x^2 + c*x^4) - (Sqrt[2]*(-3*b^4*g + b^2*c*(c*e - Sqrt[b^2 - 4*a*c]*f + 19*a*g) + 2*c^2*(c*Sqrt[b^2 - 4*a*c]*d + 2*a*c*e + 3*a*Sqrt[b^2 - 4*a*c]*f - 10*a^2*g) + b^3*(c*f + 3*Sqrt[b^2 - 4*a*c]*g) - b*c*(4*c^2*d + c*Sqrt[b^2 - 4*a*c]*e + 8*a*c*f + 13*a*Sqrt[b^2 - 4*a*c]*g))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*(3*b^4*g - b^2*c*(c*e + Sqrt[b^2 - 4*a*c]*f + 19*a*g) + 2*c^2*(c*Sqrt[b^2 - 4*a*c]*d - 2*a*c*e + 3*a*Sqrt[b^2 - 4*a*c]*f + 10*a^2*g) + b^3*(-(c*f) + 3*Sqrt[b^2 - 4*a*c]*g) + b*c*(4*c^2*d - c*Sqrt[b^2 - 4*a*c]*e + 8*a*c*f - 13*a*Sqrt[b^2 - 4*a*c]*g))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])/(4*c^(5/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 (d + ex^2 + fx^4 + gx^6)}{(a + bx^2 + cx^4)^2} dx$$

Verification is not applicable to the result.

$$\begin{aligned}
& (2) \sqrt{b^2 - 4ac} \sqrt{bc - \sqrt{b^2 - 4ac}c} a^2 b^3 c^8 - \sqrt{2} \\
& \sqrt{b^2 - 4ac} \sqrt{bc - \sqrt{b^2 - 4ac}c} b^5 c^8 - 64 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc - \sqrt{b^2 - 4ac}c} a^3 b^3 c^9 - 32 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc - \sqrt{b^2 - 4ac}c} a^2 b^2 c^9 + 16 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc - \sqrt{b^2 - 4ac}c} a^2 b^3 c^{10} - 2(b^2 - 4ac) b^5 c^8 + 32(b^2 - 4ac) a^2 b^3 c^{10} e) \arctan(2\sqrt{1/2} x / \sqrt{(b^3 c^2 - 4ab^2 c^3 + \sqrt{(b^3 c^2 - 4ab^2 c^3)^2 - 4(a^2 b^2 c^2 - 4a^2 c^3)(b^2 c^3 - 4ac^4))} / (b^2 c^3 - 4ac^4)) / ((ab^6 c^5 - 12a^2 b^4 c^6 - 2ab^5 c^6 + 48a^3 b^2 c^7 + 16a^2 b^3 c^7 + ab^4 c^7 - 64a^4 c^8 - 32a^3 b^2 c^8 - 8a^2 b^2 c^8 + 16a^3 c^9) \operatorname{abs}(-b^2 c^2 + 4ac^3) \operatorname{abs}(c)) + 1/16(2(2b^2 c^5 - 8ac^6 - \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}c}) b^2 c^3 + 4\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}c}) a^2 c^4 + 2\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}c}) b^2 c^4 - \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}c}) c^5 - 2(b^2 - 4ac) c^5) (b^2 c^2 - 4ac^3)^2 d - (2b^4 c^3 - 20ab^2 c^4 + 48a^2 c^5 - \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}c}) b^4 c + 10\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}c}) a^2 b^2 c^2 + 2\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}c}) b^3 c^2 - 24\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}c}) a^2 c^3 - 12\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}c}) ab^2 c^3 - \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}c}) b^2 c^3 + 6\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}c}) a^2 c^4 - 2(b^2 - 4ac) b^2 c^3 + 12(b^2 - 4ac) a^2 c^4) (b^2 c^2 - 4ac^3)^2 f + (6b^5 c^2 - 50ab^3 c^3 + 104a^2 b^2 c^4 - 3\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}c}) b^5 + 25\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}c}) a^2 b^3 c^3 + 6\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}c}) b^4 c - 52\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}c}) a^2 b^2 c^2 - 26\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}c}) ab^2 c^2 - 3\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}c}) b^3 c^2 + 13\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}c}) ab^2 c^3 - 6(b^2 - 4ac) b^3 c^2 + 26(b^2 - 4ac) ab^2 c^3) (b^2 c^2 - 4ac^3)^2 g - (2b^3 c^4 - 8ab^2 c^5 - \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}c}) b^3 c^2 + 4\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}c}) ab^2 c^3 + 2\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}c}) b^2 c^3 - \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}c}) b^2 c^4 - 2(b^2 - 4ac) b^2 c^4) (b^2 c^2 - 4ac^3)^2 e + 2(\sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}c}) b^5 c^5 - 8\sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}c}) ab^3 c^6 - 2\sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}c}) b^4 c^6 - 2b^5 c^6 + 16\sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}c}) a^2 b^2 c^7 + 8\sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}c}) ab^2 c^7 + \sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}c}) b^3 c^7 + 16ab^3 c^7 - 4\sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}c}) ab^2 c^8 - 32a^2 b^2 c^8 + 2(b^2 - 4ac) b^3 c^6 - 8(b^2 - 4ac) ab^2 c^7) d \operatorname{abs}(-b^2 c^2 + 4ac^3) + 2(\sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}c}) ab^5 c^4 - 8\sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}c}) a^2 b^3 c^5 - 2\sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}c}) ab^4 c^5 - 2ab^5 c^5 + 16\sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}c}) a^3 b^2 c^6 + 8\sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}c}) a^2 b^2 c^6 + \sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}c}) ab^3 c^6 + 16a^2 b^3 c^6 - 4\sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}c}) a^2 b^2 c^7 - 32a^3 b^2 c^7 + 2(b^2 - 4ac) ab^3 c^5 - 8(b^2 - 4ac) a^2 b^2 c^6) f \operatorname{abs}(-b^2 c^2 + 4ac^3) - 2(3\sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}c}) ab^6 c^3 - 34\sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}c}) a^2 b^4 c^4 - 6\sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}c}) ab^5 c^4 - 6ab^6 c^4 + 128\sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}c}) a^3 b^2 c^5 + 44\sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}c}) a^2 b^3 c^5 + 3\sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}c}) ab^4 c^5 + 68a^2 b^4 c^5 - 160\sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}c}) a^3 b^2 c^6 - 22\sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}c}) a^2 b^2 c^6 - 256a^3 b^2 c^6 + 40\sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}c}) a^3 c^7 + 320a^4 c^7 + 6(b^2 - 4ac) ab^4 c^4 - 44(b^2 - 4ac) a^2 b^2 c^5 + 80(b^2 - 4ac) a^3 c^6) g \operatorname{abs}(-b^2 c^2 + 4ac^3) - 4(\sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}c})
\end{aligned}$$

$$\begin{aligned}
& 4*a*c)*c)*a*b^4*c^5 - 8*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^2*b^2*c^6 \\
& - 2*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a*b^3*c^6 - 2*a*b^4*c^6 + 16* \\
& \text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^3*c^7 + 8*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(\\
& (b^2 - 4*a*c))*c)*a^2*b*c^7 + \text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a*b^2* \\
& c^7 + 16*a^2*b^2*c^7 - 4*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^2*c^8 - \\
& 32*a^3*c^8 + 2*(b^2 - 4*a*c)*a*b^2*c^6 - 8*(b^2 - 4*a*c)*a^2*c^7)*\text{abs}(-b^2*c^2 \\
& + 4*a*c^3)*e - 4*(2*b^6*c^9 - 16*a*b^4*c^10 + 32*a^2*b^2*c^11 - \text{sqrt}(2) \\
& *\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*b^6*c^7 + 8*\text{sqrt}(2)*\text{sqrt}(\\
& (b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a*b^4*c^8 + 2*\text{sqrt}(2)*\text{sqrt}(b^ \\
& 2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*b^5*c^8 - 16*\text{sqrt}(2)*\text{sqrt}(b^2 - \\
& 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^2*b^2*c^9 - 8*\text{sqrt}(2)*\text{sqrt}(b^2 - 4 \\
& *a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a*b^3*c^9 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c) \\
& *\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*b^4*c^9 + 4*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(\\
& (b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a*b^2*c^10 - 2*(b^2 - 4*a*c)*b^4*c^9 + 8*(b^2 - \\
& 4*a*c)*a*b^2*c^10)*d + (2*b^8*c^7 - 32*a*b^6*c^8 + 160*a^2*b^4*c^9 - 256*a \\
& ^3*b^2*c^10 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*b^8 \\
& *c^5 + 16*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a*b^6*c \\
& ^6 + 2*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*b^7*c^6 - \\
& 80*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^2*b^4*c^7 - \\
& 24*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a*b^5*c^7 - \text{sq} \\
& \text{rt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*b^6*c^7 + 128*\text{sqrt}(\\
& 2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^3*b^2*c^8 + 64*\text{sqrt}(\\
& 2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^2*b^3*c^8 + 12*\text{sqrt}(\\
& 2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a*b^4*c^8 - 32*\text{sqrt}(2) \\
& *\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^2*b^2*c^9 - 2*(b^2 - 4 \\
& *a*c)*b^6*c^7 + 24*(b^2 - 4*a*c)*a*b^4*c^8 - 64*(b^2 - 4*a*c)*a^2*b^2*c^9)* \\
& f - (6*b^9*c^6 - 86*a*b^7*c^7 + 440*a^2*b^5*c^8 - 928*a^3*b^3*c^9 + 640*a^4 \\
& *b*c^10 - 3*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*b^9*c \\
& ^4 + 43*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a*b^7*c^5 \\
& + 6*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*b^8*c^5 - 22 \\
& 0*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^2*b^5*c^6 - 6 \\
& 2*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a*b^6*c^6 - 3*s \\
& \text{qrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*b^7*c^6 + 464*\text{sqrt} \\
& (2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^3*b^3*c^7 + 192*\text{sq} \\
& \text{rt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^2*b^4*c^7 + 31*\text{sq} \\
& \text{rt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a*b^5*c^7 - 320*\text{sqrt} \\
& (2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^4*b*c^8 - 160*\text{sqrt}(\\
& 2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^3*b^2*c^8 - 96*\text{sqrt}(\\
& 2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^2*b^3*c^8 + 80*\text{sqrt}(\\
& 2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^3*b*c^9 - 6*(b^2 - 4 \\
& *a*c)*b^7*c^6 + 62*(b^2 - 4*a*c)*a*b^5*c^7 - 192*(b^2 - 4*a*c)*a^2*b^3*c^8 \\
& + 160*(b^2 - 4*a*c)*a^3*b*c^9)*g + (2*b^7*c^8 - 8*a*b^5*c^9 - 32*a^2*b^3*c^ \\
& 10 + 128*a^3*b*c^11 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c) \\
&))*c)*b^7*c^6 + 4*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)* \\
& a*b^5*c^7 + 2*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*b^6 \\
& *c^7 + 16*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^2*b^3 \\
& *c^8 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*b^5*c^8 - \\
& 64*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^3*b*c^9 - 32 \\
& *\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^2*b^2*c^9 + 16 \\
& *\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^2*b*c^10 - 2*(\\
& b^2 - 4*a*c)*b^5*c^8 + 32*(b^2 - 4*a*c)*a^2*b*c^10)*e)*\text{arctan}(2*\text{sqrt}(1/2)*x \\
& / \text{sqrt}((b^3*c^2 - 4*a*b*c^3 - \text{sqrt}((b^3*c^2 - 4*a*b*c^3)^2 - 4*(a*b^2*c^2 - \\
& 4*a^2*c^3))*(b^2*c^3 - 4*a*c^4)))/(b^2*c^3 - 4*a*c^4)))/((a*b^6*c^5 - 12*a^2 \\
& *b^4*c^6 - 2*a*b^5*c^6 + 48*a^3*b^2*c^7 + 16*a^2*b^3*c^7 + a*b^4*c^7 - 64*a \\
& ^4*c^8 - 32*a^3*b*c^8 - 8*a^2*b^2*c^8 + 16*a^3*c^9)*\text{abs}(-b^2*c^2 + 4*a*c^3) \\
& *\text{abs}(c))
\end{aligned}$$

maple [B] time = 0.05, size = 2300, normalized size = 4.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\int (x^2*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2, x)$

[Out]
$$\begin{aligned} & -19/4/c/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} \\ & *arctanh(2^{(1/2)/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*a*b^2*g-19/4/c \\ & / (4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*ar \\ & ctan(2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*a*b^2*g+g*x/c^2+3/4/c^2/ \\ & (4*a*c-b^2)*2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*arctan(2^{(1/2)/((b+(-4 \\ & *a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^3*g+5/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/ \\ & 2)/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*arctan(2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2)})* \\ & c)^{(1/2)}*c*x)*a^2*g-3/4/c^2/(4*a*c-b^2)*2^{(1/2)/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} \\ & *arctanh(2^{(1/2)/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^3*g+5/(4*a*c \\ & -b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*arctanh \\ & (2^{(1/2)/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*a^2*g+1/4/(4*a*c-b^2)/c*2^{(\\ & 1/2)/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*arctanh(2^{(1/2)/((-b+(-4*a*c+b^2)^{(1 \\ & /2)})*c)^{(1/2)}*c*x)*b^2*f-1/4/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)/((-b+(- \\ & 4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*arctanh(2^{(1/2)/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} \\ &)*c*x)*b^2*e-1/4/(4*a*c-b^2)*2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*b^2/c \\ & *f*arctan(2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)-1/4/(4*a*c-b^2)/(-4 \\ & *a*c+b^2)^{(1/2)}*2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*b^2*e*arctan(2^{(1/ \\ & 2)/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)-1/2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^ \\ & 3*b*e-1/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*a*e+1/2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x \\ & *b*d+1/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1 \\ & /2)}*b*c*d*arctan(2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)+13/4/c/(4*a* \\ & c-b^2)*2^{(1/2)/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*arctanh(2^{(1/2)/((-b+(-4*a \\ & *c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*a*b*g+3/4/c^2/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2 \\ & ^{(1/2)/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*arctanh(2^{(1/2)/((-b+(-4*a*c+b^2)^{(1/2)} \\ &)*c)^{(1/2)}*c*x)*b^4*g+3/4/c^2/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)/((\\ & b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*arctan(2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1 \\ & /2)}*c*x)*b^4*g-13/4/c/(4*a*c-b^2)*2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} \\ & *arctan(2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*a*b*g-1/2/c^2/(c*x^4+ \\ & b*x^2+a)/(4*a*c-b^2)*x*a*b^2*g+2/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)/((- \\ & b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*arctanh(2^{(1/2)/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} \\ &)*c*x)*a*b*f-1/(4*a*c-b^2)*c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)/((-b+(-4*a*c+b^2)^{(1/2)} \\ &)*c)^{(1/2)}*arctanh(2^{(1/2)/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*a \\ & *e-1/4/(4*a*c-b^2)/c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} \\ & *arctanh(2^{(1/2)/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^3*f+1/(4*a* \\ & c-b^2)*c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*arcta \\ & nh(2^{(1/2)/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b*d+2/(4*a*c-b^2)/(-4*a*c \\ & +b^2)^{(1/2)}*2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*a*b*f*arctan(2^{(1/2)/((\\ & b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)-1/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/ \\ & 2)/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*a*c*e*arctan(2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2)} \\ &)*c)^{(1/2)}*c*x)-1/2/c^2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^3*b^3*g+1/2/c/(c* \\ & x^4+b*x^2+a)/(4*a*c-b^2)*x^3*b^2*f+1/c/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*a^2*g+ \\ & 3/2/c/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^3*a*b*g+1/2/c/(c*x^4+b*x^2+a)/(4*a*c-b^ \\ & 2)*x*a*b*f-1/4/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2)} \\ &))*c)^{(1/2)}*b^3/c*f*arctan(2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)+c/ \\ & (c*x^4+b*x^2+a)/(4*a*c-b^2)*x^3*d-1/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^3*a*f-3/2 \\ & / (4*a*c-b^2)*2^{(1/2)/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*arctanh(2^{(1/2)/((-b \\ & +(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*a*f+1/4/(4*a*c-b^2)*2^{(1/2)/((-b+(-4*a*c \\ & +b^2)^{(1/2)})*c)^{(1/2)}*arctanh(2^{(1/2)/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x \\ &)*b*e-1/2/(4*a*c-b^2)*c*2^{(1/2)/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*arctanh(2 \\ & ^{(1/2)/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*d+3/2/(4*a*c-b^2)*2^{(1/2)/((b \\ & +(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*a*f*arctan(2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} \\ &)*c*x)-1/4/(4*a*c-b^2)*2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*b*e*ar \\ & ctan(2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)+1/2/(4*a*c-b^2)*2^{(1/2)/ \\ & ((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*d*arctan(2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2)} \\ &)*c)^{(1/2)}*c*x) \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")

[Out]
$$-1/2*((2*c^3*d - b*c^2*e + (b^2*c - 2*a*c^2)*f - (b^3 - 3*a*b*c)*g)*x^3 + (b*c^2*d - 2*a*c^2*e + a*b*c*f - (a*b^2 - 2*a^2*c)*g)*x)/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2) + g*x/c^2 + 1/2*integrate((b*c^2*d - 2*a*c^2*e + a*b*c*f - (2*c^3*d - b*c^2*e - (b^2*c - 6*a*c^2)*f + (3*b^3 - 13*a*b*c)*g)*x^2 - (3*a*b^2 - 10*a^2*c)*g)/(c*x^4 + b*x^2 + a), x)/(b^2*c^2 - 4*a*c^3)$$

mupad [B] time = 4.22, size = 36589, normalized size = 77.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4)^2,x)

[Out]
$$\frac{(x^3(2c^3d - b^3g - 2a^2c^2f - b^2c^2e + b^2c^2f + 3a^2b^2cg)) + (x(b^2c^2d - 2a^2c^2e - a^2b^2g + 2a^2c^2g + a^2b^2cf))}{(2(4ac - b^2)) + (x(b^2c^2d - 2a^2c^2e - a^2b^2g + 2a^2c^2g + a^2b^2cf))} / (2(4ac - b^2)) / (a^2c^2 + c^3x^4 + b^2c^2x^2) - \operatorname{atan}\left(\frac{(10240a^5c^7g - 16b^7c^5d - 2048a^4c^8e - 768a^2b^3c^7d - 384a^2b^4c^6e + 1536a^3b^2c^7e + 192a^2b^5c^5f - 768a^3b^3c^6f - 736a^2b^6c^4g + 4224a^3b^4c^5g - 10752a^4b^2c^6g + 192a^2b^5c^6d + 1024a^3b^2c^8d + 32a^2b^6c^5e - 16a^2b^7c^4f + 1024a^4b^2c^7f + 48a^2b^8c^3g)}{(8(64a^3c^6 - b^6c^3 + 12a^2b^4c^4 - 48a^2b^2c^5)) - (x((c^5d^2 - (-4ac - b^2)^9)^{1/2} - b^9c^5d^2 - 9a^2b^13g^2 + 768a^4b^2c^9d^2 - a^2b^9c^4e^2 + 768a^5b^2c^8e^2 - a^2c^4e^2(-4ac - b^2)^9)^{1/2} - a^2b^11c^2f^2 + 3840a^6b^2c^7f^2 - 9a^2b^4g^2(-4ac - b^2)^9)^{1/2} + 213a^2b^11c^2g^2 - 26880a^7b^2c^6g^2 + 96a^2b^5c^7d^2 - 512a^3b^3c^8d^2 + 96a^3b^5c^6e^2 - 512a^4b^3c^7e^2 + 27a^2b^9c^3f^2 - 288a^3b^7c^4f^2 + 1504a^4b^5c^5f^2 - 3840a^5b^3c^6f^2 + 9a^2c^3f^2(-4ac - b^2)^9)^{1/2} - 2077a^3b^9c^2g^2 + 10656a^4b^7c^3g^2 - 30240a^5b^5c^4g^2 + 44800a^6b^3c^5g^2 - 25a^3c^2g^2(-4ac - b^2)^9)^{1/2} - 1024a^5c^9d^2e + 5120a^6c^8d^2g - 3072a^6c^8e^2f + 15360a^7c^7f^2g + 12a^2b^8c^5d^2e + 6a^2b^9c^4d^2f + 3584a^5b^2c^8d^2f + 6a^2c^4d^2f(-4ac - b^2)^9)^{1/2} - 18a^2b^10c^3d^2g - 2a^2b^10c^3e^2f + 6a^2b^11c^2e^2g + 1536a^6b^2c^7e^2g - 128a^2b^6c^6d^2e + 384a^3b^4c^7d^2e - 128a^2b^7c^5d^2f + 960a^3b^5c^6d^2f - 3072a^4b^3c^7d^2f + 324a^2b^8c^4d^2g + 36a^2b^8c^4e^2f - 2240a^3b^6c^5d^2g - 192a^3b^6c^5e^2f + 7296a^4b^4c^6d^2g + 128a^4b^4c^6e^2f - 10752a^5b^2c^7d^2g + 1536a^5b^2c^7e^2f - 98a^2b^9c^3e^2g + 576a^3b^7c^4e^2g - 1344a^4b^5c^5e^2g + 512a^5b^3c^6e^2g + 10a^2c^3e^2g(-4ac - b^2)^9)^{1/2} - 152a^2b^10c^2f^2g + 1548a^3b^8c^3f^2g - 8064a^4b^6c^4f^2g + 22400a^5b^4c^5f^2g - 30720a^6b^2c^6f^2g + 6a^2b^12c^2f^2g - a^2b^2c^2f^2(-4ac - b^2)^9)^{1/2} + 51a^2b^2c^2g^2(-4ac - b^2)^9)^{1/2} - 18a^2b^2c^2g^2(-4ac - b^2)^9)^{1/2} - 2a^2b^2c^2e^2f(-4ac - b^2)^9)^{1/2} + 6a^2b^3c^2f^2g(-4ac - b^2)^9)^{1/2} + 6a^2b^2c^2e^2g(-4ac - b^2)^9)^{1/2} - 44a^2b^2c^2f^2g(-4ac - b^2)^9)^{1/2}) / (32(4096a^7c^11 + a^2b^12c^5 - 24a^2b^10c^6 + 240a^3b^8c^7 - 1280a^4b^6c^8 + 3840a^5b^4c^9 - 6144a^6b^2c^10))^{1/2} * (16b^7c^5 - 192a^2b^5c^6 - 1024a^3b^3c^8 + 768a^2b^3c^7)) / (2(16a^2c^5 + b^4c^3 - 8a^2b^2c^4)) * ((c^5d^2(-4ac - b^2)^9)^{1/2} - b^9c^5d^2 - 9a^2b^13g^2 + 768a^4b^2c^9d^2 - a^2b^9c^4e^2 + 768a^5b^2c^8e^2 - a^2c^4e^2(-4ac - b^2)^9)^{1/2} - a^2b^11c^2f^2 + 3840a^6b^2c^7f^2 - 9a^2b^4g^2(-4ac - b^2)^9)^{1/2} + 213a^2b^11c^2g^2 - 26880a^7b^2c^6g^2 + 96a^2b^5c^7d^2 - 512a^3b^3c^8d^2 + 96a^3b^5c^6e^2 - 512a^4b^3c^8$$

$$\begin{aligned}
& 7e^2 + 27a^2b^9c^3f^2 - 288a^3b^7c^4f^2 + 1504a^4b^5c^5f^2 - 3 \\
& 840a^5b^3c^6f^2 + 9a^2c^3f^2(-4ac - b^2)^9)^{(1/2)} - 2077a^3b^9 \\
& c^2g^2 + 10656a^4b^7c^3g^2 - 30240a^5b^5c^4g^2 + 44800a^6b^3c^5 \\
& g^2 - 25a^3c^2g^2(-4ac - b^2)^9)^{(1/2)} - 1024a^5c^9d^2e + 5120a \\
& ^6c^8d^2g - 3072a^6c^8e^2f + 15360a^7c^7f^2g + 12a^8b^8c^5d^2e + 6a^8 \\
& b^9c^4d^2f + 3584a^5b^8c^8d^2f + 6a^8c^4d^2f(-4ac - b^2)^9)^{(1/2)} - 1 \\
& 8a^8b^10c^3d^2g - 2a^8b^10c^3e^2f + 6a^8b^11c^2e^2g + 1536a^6b^8c^7e^2g \\
& - 128a^2b^6c^6d^2e + 384a^3b^4c^7d^2e - 128a^2b^7c^5d^2f + 960a^3 \\
& b^5c^6d^2f - 3072a^4b^3c^7d^2f + 324a^2b^8c^4d^2g + 36a^2b^8c^4 \\
& e^2f - 2240a^3b^6c^5d^2g - 192a^3b^6c^5e^2f + 7296a^4b^4c^6d^2g + \\
& 128a^4b^4c^6e^2f - 10752a^5b^2c^7d^2g + 1536a^5b^2c^7e^2f - 98a^2 \\
& b^9c^3e^2g + 576a^3b^7c^4e^2g - 1344a^4b^5c^5e^2g + 512a^5b^3c^6 \\
& e^2g + 10a^2c^3e^2g(-4ac - b^2)^9)^{(1/2)} - 152a^2b^10c^2f^2g + 154 \\
& 8a^3b^8c^3f^2g - 8064a^4b^6c^4f^2g + 22400a^5b^4c^5f^2g - 30720a^6 \\
& b^2c^6f^2g + 6a^8b^12c^2f^2g - a^8b^2c^2f^2(-4ac - b^2)^9)^{(1/2)} + 5 \\
& 1a^2b^2c^2g^2(-4ac - b^2)^9)^{(1/2)} - 18a^8b^8c^3d^2g(-4ac - b^2)^9 \\
&)^{(1/2)} - 2a^8b^8c^3e^2f(-4ac - b^2)^9)^{(1/2)} + 6a^8b^3c^2f^2g(-4ac - \\
& b^2)^9)^{(1/2)} + 6a^8b^2c^2e^2g(-4ac - b^2)^9)^{(1/2)} - 44a^2b^8c^2f^2 \\
& g(-4ac - b^2)^9)^{(1/2)}/(32(4096a^7c^11 + a^8b^12c^5 - 24a^2b^10c^6 \\
& + 240a^3b^8c^7 - 1280a^4b^6c^8 + 3840a^5b^4c^9 - 6144a^6b^2c^{10}))^{(1/2)} - (x(9b^8g^2 - 8a^8c^7d^2 \\
& + 8a^2c^6e^2 + 10b^2c^6d^2 \\
& - 72a^3c^5f^2 + b^4c^4e^2 + 200a^4c^4g^2 + b^6c^2f^2 + 2a^8b^2c^5 \\
& e^2 - 16a^8b^4c^3f^2 - 6b^7c^2f^2g + 74a^2b^2c^4f^2 + 481a^2b^4c^2 \\
& g^2 - 718a^3b^2c^3g^2 - 114a^8b^6c^2g^2 - 48a^2c^6d^2f - 6b^3c^5 \\
& d^2e - 6b^4c^4d^2f - 80a^3c^5e^2g + 18b^5c^3d^2g + 2b^5c^3e^2f - 6 \\
& b^6c^2e^2g + 52a^8b^2c^5d^2f - 126a^8b^3c^4d^2g - 14a^8b^3c^4e^2f + 18 \\
& 4a^2b^8c^5d^2g - 8a^2b^8c^5e^2f + 32a^8b^4c^3e^2g + 86a^8b^5c^2f^2g + 4 \\
& 72a^3b^8c^4f^2g + 4a^2b^2c^4e^2g - 374a^2b^3c^3f^2g - 8a^8b^8c^6d^2e) \\
&)/(2(16a^2c^5 + b^4c^3 - 8a^8b^2c^4))((c^5d^2(-4ac - b^2)^9)^{(1/2)} - b^9 \\
& c^5d^2 - 9a^8b^13g^2 + 768a^4b^8c^9d^2 - a^8b^9c^4e^2 + 768a^5 \\
& b^8c^8e^2 - a^8c^4e^2(-4ac - b^2)^9)^{(1/2)} - a^8b^11c^2f^2 + 3840a^6 \\
& b^8c^7f^2 - 9a^8b^4g^2(-4ac - b^2)^9)^{(1/2)} + 213a^2b^11c^2g^2 - \\
& 26880a^7b^8c^6g^2 + 96a^2b^5c^7d^2 - 512a^3b^3c^8d^2 + 96a^3b^5 \\
& c^6e^2 - 512a^4b^3c^7e^2 + 27a^2b^9c^3f^2 - 288a^3b^7c^4f^2 \\
& + 1504a^4b^5c^5f^2 - 3840a^5b^3c^6f^2 + 9a^2c^3f^2(-4ac - b^2)^9)^{(1/2)} - \\
& 2077a^3b^9c^2g^2 + 10656a^4b^7c^3g^2 - 30240a^5b^5c^4g^2 + 44800a^6b^3c^5 \\
& g^2 - 25a^3c^2g^2(-4ac - b^2)^9)^{(1/2)} - 1024a^5c^9d^2e + 5120a^6c^8d^2g \\
& - 3072a^6c^8e^2f + 15360a^7c^7f^2g + 12a^8b^8c^5d^2e + 6a^8b^9c^4d^2f \\
& + 3584a^5b^8c^8d^2f + 6a^8c^4d^2f(-4ac - b^2)^9)^{(1/2)} - 18a^8b^10c^3d^2g \\
& - 2a^8b^10c^3e^2f + 6a^8b^11c^2e^2g + 1536a^6b^8c^7e^2g - 128a^2b^6c^6d^2e \\
& + 384a^3b^4c^7d^2e - 128a^2b^7c^5d^2f + 960a^3b^5c^6d^2f - 3072a^4b^3c^7d^2f \\
& + 324a^2b^8c^4d^2g + 36a^2b^8c^4e^2f - 2240a^3b^6c^5d^2g - 192a^3b^6c^5e^2f \\
& + 7296a^4b^4c^6d^2g + 128a^4b^4c^6e^2f - 10752a^5b^2c^7d^2g + 153 \\
& 6a^5b^2c^7e^2f - 98a^2b^9c^3e^2g + 576a^3b^7c^4e^2g - 1344a^4b^5 \\
& c^5e^2g + 512a^5b^3c^6e^2g + 10a^2c^3e^2g(-4ac - b^2)^9)^{(1/2)} - \\
& 152a^2b^10c^2f^2g + 1548a^3b^8c^3f^2g - 8064a^4b^6c^4f^2g + 22400a^5 \\
& b^4c^5f^2g - 30720a^6b^2c^6f^2g + 6a^8b^12c^2f^2g - a^8b^2c^2f^2(-4ac - \\
& b^2)^9)^{(1/2)} + 51a^2b^2c^2g^2(-4ac - b^2)^9)^{(1/2)} - 18a^8b^8 \\
& c^3d^2g(-4ac - b^2)^9)^{(1/2)} - 2a^8b^8c^3e^2f(-4ac - b^2)^9)^{(1/2)} \\
& + 6a^8b^3c^2f^2g(-4ac - b^2)^9)^{(1/2)} + 6a^8b^2c^2e^2g(-4ac - b^2)^9 \\
&)^{(1/2)} - 44a^2b^8c^2f^2g(-4ac - b^2)^9)^{(1/2)}/(32(4096a^7c^11 + \\
& a^8b^12c^5 - 24a^2b^10c^6 + 240a^3b^8c^7 - 1280a^4b^6c^8 + 3840a^5 \\
& b^4c^9 - 6144a^6b^2c^{10}))^{(1/2)}*1i - (((10240a^5c^7g - 16b^7c^5 \\
& d - 2048a^4c^8e - 768a^2b^3c^7d - 384a^2b^4c^6e + 1536a^3b^2c^7 \\
& e + 192a^2b^5c^5f - 768a^3b^3c^6f - 736a^2b^6c^4g + 4224a^3 \\
& b^4c^5g - 10752a^4b^2c^6g + 192a^8b^5c^6d + 1024a^3b^8c^8d + 32 \\
& a^8b^6c^5e - 16a^8b^7c^4f + 1024a^4b^8c^7f + 48a^8b^8c^3g)/(8(64a^3 \\
& c^6 - b^6c^3 + 12a^8b^4c^4 - 48a^2b^2c^5)) + (x((c^5d^2(-4ac
\end{aligned}$$

$$\begin{aligned}
& - b^2)^9)^{(1/2)} - b^9*c^5*d^2 - 9*a*b^{13}*g^2 + 768*a^4*b*c^9*d^2 - a*b^9*c^4*e^2 + 768*a^5*b*c^8*e^2 - a*c^4*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - a*b^{11}*c^2*f^2 + 3840*a^6*b*c^7*f^2 - 9*a*b^4*g^2*(-(4*a*c - b^2)^9)^{(1/2)} + 213*a^2*b^{11}*c*g^2 - 26880*a^7*b*c^6*g^2 + 96*a^2*b^5*c^7*d^2 - 512*a^3*b^3*c^8*d^2 + 96*a^3*b^5*c^6*e^2 - 512*a^4*b^3*c^7*e^2 + 27*a^2*b^9*c^3*f^2 - 288*a^3*b^7*c^4*f^2 + 1504*a^4*b^5*c^5*f^2 - 3840*a^5*b^3*c^6*f^2 + 9*a^2*c^3*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 2077*a^3*b^9*c^2*g^2 + 10656*a^4*b^7*c^3*g^2 - 30240*a^5*b^5*c^4*g^2 + 44800*a^6*b^3*c^5*g^2 - 25*a^3*c^2*g^2*(-(4*a*c - b^2)^9)^{(1/2)} - 1024*a^5*c^9*d*e + 5120*a^6*c^8*d*g - 3072*a^6*c^8*e*f + 15360*a^7*c^7*f*g + 12*a*b^8*c^5*d*e + 6*a*b^9*c^4*d*f + 3584*a^5*b*c^8*d*f + 6*a*c^4*d*f*(-(4*a*c - b^2)^9)^{(1/2)} - 18*a*b^{10}*c^3*d*g - 2*a*b^{10}*c^3*e*f + 6*a*b^{11}*c^2*e*g + 1536*a^6*b*c^7*e*g - 128*a^2*b^6*c^6*d*e + 384*a^3*b^4*c^7*d*e - 128*a^2*b^7*c^5*d*f + 960*a^3*b^5*c^6*d*f - 3072*a^4*b^3*c^7*d*f + 324*a^2*b^8*c^4*d*g + 36*a^2*b^8*c^4*e*f - 2240*a^3*b^6*c^5*d*g - 192*a^3*b^6*c^5*e*f + 7296*a^4*b^4*c^6*d*g + 128*a^4*b^4*c^6*e*f - 10752*a^5*b^2*c^7*d*g + 1536*a^5*b^2*c^7*e*f - 98*a^2*b^9*c^3*e*g + 576*a^3*b^7*c^4*e*g - 1344*a^4*b^5*c^5*e*g + 512*a^5*b^3*c^6*e*g + 10*a^2*c^3*e*g*(-(4*a*c - b^2)^9)^{(1/2)} - 152*a^2*b^{10}*c^2*f*g + 1548*a^3*b^8*c^3*f*g - 8064*a^4*b^6*c^4*f*g + 22400*a^5*b^4*c^5*f*g - 30720*a^6*b^2*c^6*f*g + 6*a*b^{12}*c*f*g - a*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + 51*a^2*b^2*c*g^2*(-(4*a*c - b^2)^9)^{(1/2)} - 18*a*b*c^3*d*g*(-(4*a*c - b^2)^9)^{(1/2)} - 2*a*b*c^3*e*f*(-(4*a*c - b^2)^9)^{(1/2)} + 6*a*b^3*c*f*g*(-(4*a*c - b^2)^9)^{(1/2)} + 6*a*b^2*c^2*e*g*(-(4*a*c - b^2)^9)^{(1/2)} - 44*a^2*b*c^2*f*g*(-(4*a*c - b^2)^9)^{(1/2)))/(32*(4096*a^7*c^{11} + a*b^{12}*c^5 - 24*a^2*b^{10}*c^6 + 240*a^3*b^8*c^7 - 1280*a^4*b^6*c^8 + 3840*a^5*b^4*c^9 - 6144*a^6*b^2*c^{10}))^{(1/2)}*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((c^5*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - b^9*c^5*d^2 - 9*a*b^{13}*g^2 + 768*a^4*b*c^9*d^2 - a*b^9*c^4*e^2 + 768*a^5*b*c^8*e^2 - a*c^4*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - a*b^{11}*c^2*f^2 + 3840*a^6*b*c^7*f^2 - 9*a*b^4*g^2*(-(4*a*c - b^2)^9)^{(1/2)} + 213*a^2*b^{11}*c*g^2 - 26880*a^7*b*c^6*g^2 + 96*a^2*b^5*c^7*d^2 - 512*a^3*b^3*c^8*d^2 + 96*a^3*b^5*c^6*e^2 - 512*a^4*b^3*c^7*e^2 + 27*a^2*b^9*c^3*f^2 - 288*a^3*b^7*c^4*f^2 + 1504*a^4*b^5*c^5*f^2 - 3840*a^5*b^3*c^6*f^2 + 9*a^2*c^3*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 2077*a^3*b^9*c^2*g^2 + 10656*a^4*b^7*c^3*g^2 - 30240*a^5*b^5*c^4*g^2 + 44800*a^6*b^3*c^5*g^2 - 25*a^3*c^2*g^2*(-(4*a*c - b^2)^9)^{(1/2)} - 1024*a^5*c^9*d*e + 5120*a^6*c^8*d*g - 3072*a^6*c^8*e*f + 15360*a^7*c^7*f*g + 12*a*b^8*c^5*d*e + 6*a*b^9*c^4*d*f + 3584*a^5*b*c^8*d*f + 6*a*c^4*d*f*(-(4*a*c - b^2)^9)^{(1/2)} - 18*a*b^{10}*c^3*d*g - 2*a*b^{10}*c^3*e*f + 6*a*b^{11}*c^2*e*g + 1536*a^6*b*c^7*e*g - 128*a^2*b^6*c^6*d*e + 384*a^3*b^4*c^7*d*e - 128*a^2*b^7*c^5*d*f + 960*a^3*b^5*c^6*d*f - 3072*a^4*b^3*c^7*d*f + 324*a^2*b^8*c^4*d*g + 36*a^2*b^8*c^4*e*f - 2240*a^3*b^6*c^5*d*g - 192*a^3*b^6*c^5*e*f + 7296*a^4*b^4*c^6*d*g + 128*a^4*b^4*c^6*e*f - 10752*a^5*b^2*c^7*d*g + 1536*a^5*b^2*c^7*e*f - 98*a^2*b^9*c^3*e*g + 576*a^3*b^7*c^4*e*g - 1344*a^4*b^5*c^5*e*g + 512*a^5*b^3*c^6*e*g + 10*a^2*c^3*e*g*(-(4*a*c - b^2)^9)^{(1/2)} - 152*a^2*b^{10}*c^2*f*g + 1548*a^3*b^8*c^3*f*g - 8064*a^4*b^6*c^4*f*g + 22400*a^5*b^4*c^5*f*g - 30720*a^6*b^2*c^6*f*g + 6*a*b^{12}*c*f*g - a*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + 51*a^2*b^2*c*g^2*(-(4*a*c - b^2)^9)^{(1/2)} - 18*a*b*c^3*d*g*(-(4*a*c - b^2)^9)^{(1/2)} - 2*a*b*c^3*e*f*(-(4*a*c - b^2)^9)^{(1/2)} + 6*a*b^3*c*f*g*(-(4*a*c - b^2)^9)^{(1/2)} + 6*a*b^2*c^2*e*g*(-(4*a*c - b^2)^9)^{(1/2)} - 44*a^2*b*c^2*f*g*(-(4*a*c - b^2)^9)^{(1/2)))/(32*(4096*a^7*c^{11} + a*b^{12}*c^5 - 24*a^2*b^{10}*c^6 + 240*a^3*b^8*c^7 - 1280*a^4*b^6*c^8 + 3840*a^5*b^4*c^9 - 6144*a^6*b^2*c^{10}))^{(1/2)} + (x*(9*b^8*g^2 - 8*a*c^7*d^2 + 8*a^2*c^6*e^2 + 10*b^2*c^6*d^2 - 72*a^3*c^5*f^2 + b^4*c^4*e^2 + 200*a^4*c^4*g^2 + b^6*c^2*f^2 + 2*a*b^2*c^5*e^2 - 16*a*b^4*c^3*f^2 - 6*b^7*c*f*g + 74*a^2*b^2*c^4*f^2 + 481*a^2*b^4*c^2*g^2 - 718*a^3*b^2*c^3*g^2 - 114*a*b^6*c*g^2 - 48*a^2*c^6*d*f - 6*b^3*c^5*d*e - 6*b^4*c^4*d*f - 80*a^3*c^5*e*g + 18*b^5*c^3*d*g + 2*b^5*c^3*e*f - 6*b^6*c^2*e*g + 52*a*b^2*c^5*d*f - 126*a*b^3*c^4*d*g - 14*a*b^3*c^4*e*f + 184*a^2*b*c^5*d*g - 8*a^2*b*c^5*e*f + 32*a*b^4*c^3*e*g + 86*a*b^5*c^2*f*g + 472*a^3*b*c^4*f*g + 4*a^2*b^2*c^4*e*g - 374*a^2*b^3*c^3*f*g - 8*a*b*c^6*d*e))/(2*(16*a
\end{aligned}$$

$$\begin{aligned}
& \left(c^5 d^2 + b^4 c^3 - 8 a^2 b^2 c^4 \right) \left((c^5 d^2 (-4 a^2 c - b^2)^9)^{1/2} - b^9 c^5 d^2 - 9 a^2 b^3 c^7 g^2 + 768 a^4 b^2 c^9 d^2 - a^2 b^9 c^4 e^2 + 768 a^5 b^2 c^8 e^2 - a^2 c^4 e^2 (-4 a^2 c - b^2)^9)^{1/2} - a^2 b^{11} c^2 f^2 + 3840 a^6 b^2 c^7 f^2 - 9 a^2 b^4 g^2 (-4 a^2 c - b^2)^9)^{1/2} + 213 a^2 b^{11} c^2 g^2 - 26880 a^7 b^2 c^6 g^2 + 96 a^2 b^5 c^7 d^2 - 512 a^3 b^3 c^8 d^2 + 96 a^3 b^5 c^6 e^2 - 512 a^4 b^3 c^7 e^2 + 27 a^2 b^9 c^3 f^2 - 288 a^3 b^7 c^4 f^2 + 1504 a^4 b^5 c^5 f^2 - 3840 a^5 b^3 c^6 f^2 + 9 a^2 c^3 f^2 (-4 a^2 c - b^2)^9)^{1/2} - 2077 a^3 b^9 c^2 g^2 + 10656 a^4 b^7 c^3 g^2 - 30240 a^5 b^5 c^4 g^2 + 44800 a^6 b^3 c^5 g^2 - 25 a^3 c^2 g^2 (-4 a^2 c - b^2)^9)^{1/2} - 1024 a^5 c^9 d e + 5120 a^6 c^8 d g - 3072 a^6 c^8 e f + 15360 a^7 c^7 f g + 12 a^2 b^8 c^5 d e + 6 a^2 b^9 c^4 d f + 3584 a^5 b^2 c^8 d f + 6 a^2 c^4 d f (-4 a^2 c - b^2)^9)^{1/2} - 18 a^2 b^{10} c^3 d g - 2 a^2 b^{10} c^3 e f + 6 a^2 b^{11} c^2 e g + 1536 a^6 b^2 c^7 e g - 128 a^2 b^6 c^6 d e + 384 a^3 b^4 c^7 d e - 128 a^2 b^7 c^5 d f + 960 a^3 b^5 c^6 d f - 3072 a^4 b^3 c^7 d f + 324 a^2 b^8 c^4 d g + 36 a^2 b^8 c^4 e f - 2240 a^3 b^6 c^5 d g - 192 a^3 b^6 c^5 e f + 7296 a^4 b^4 c^6 d g + 128 a^4 b^4 c^6 e f - 10752 a^5 b^2 c^7 d g + 1536 a^5 b^2 c^7 e f - 98 a^2 b^9 c^3 e g + 576 a^3 b^7 c^4 e g - 1344 a^4 b^5 c^5 e g + 512 a^5 b^3 c^6 e g + 10 a^2 c^3 e g (-4 a^2 c - b^2)^9)^{1/2} - 152 a^2 b^{10} c^2 f g + 1548 a^3 b^8 c^3 f g - 8064 a^4 b^6 c^4 f g + 22400 a^5 b^4 c^5 f g - 30720 a^6 b^2 c^6 f g + 6 a^2 b^{12} c^2 f g - a^2 b^2 c^2 f^2 (-4 a^2 c - b^2)^9)^{1/2} + 51 a^2 b^2 c^2 g^2 (-4 a^2 c - b^2)^9)^{1/2} - 18 a^2 b^3 c^3 d g (-4 a^2 c - b^2)^9)^{1/2} - 2 a^2 b^3 c^3 e f (-4 a^2 c - b^2)^9)^{1/2} + 6 a^2 b^3 c^3 f g (-4 a^2 c - b^2)^9)^{1/2} + 6 a^2 b^2 c^2 e g (-4 a^2 c - b^2)^9)^{1/2} - 44 a^2 b^2 c^2 f g (-4 a^2 c - b^2)^9)^{1/2} / (32 (4096 a^7 c^{11} + a^2 b^{12} c^5 - 24 a^2 b^{10} c^6 + 240 a^3 b^8 c^7 - 1280 a^4 b^6 c^8 + 3840 a^5 b^4 c^9 - 6144 a^6 b^2 c^{10}))^{1/2} * i) / (((10240 a^5 c^7 g - 16 b^7 c^5 d - 2048 a^4 c^8 e - 768 a^2 b^3 c^7 d - 384 a^2 b^4 c^6 e + 1536 a^3 b^2 c^7 e + 192 a^2 b^5 c^5 f - 768 a^3 b^3 c^6 f - 736 a^2 b^6 c^4 g + 4224 a^3 b^4 c^5 g - 10752 a^4 b^2 c^6 g + 192 a^2 b^5 c^6 d + 1024 a^3 b^2 c^8 d + 32 a^2 b^6 c^5 e - 16 a^2 b^7 c^4 f + 1024 a^4 b^2 c^7 f + 48 a^2 b^8 c^3 g) / (8 (64 a^3 c^6 - b^6 c^3 + 12 a^2 b^4 c^4 - 48 a^2 b^2 c^5)) - (x ((c^5 d^2 (-4 a^2 c - b^2)^9)^{1/2} - b^9 c^5 d^2 - 9 a^2 b^3 c^7 g^2 + 768 a^4 b^2 c^9 d^2 - a^2 b^9 c^4 e^2 + 768 a^5 b^2 c^8 e^2 - a^2 c^4 e^2 (-4 a^2 c - b^2)^9)^{1/2} - a^2 b^{11} c^2 f^2 + 3840 a^6 b^2 c^7 f^2 - 9 a^2 b^4 g^2 (-4 a^2 c - b^2)^9)^{1/2} + 213 a^2 b^{11} c^2 g^2 - 26880 a^7 b^2 c^6 g^2 + 96 a^2 b^5 c^7 d^2 - 512 a^3 b^3 c^8 d^2 + 96 a^3 b^5 c^6 e^2 - 512 a^4 b^3 c^7 e^2 + 27 a^2 b^9 c^3 f^2 - 288 a^3 b^7 c^4 f^2 + 1504 a^4 b^5 c^5 f^2 - 3840 a^5 b^3 c^6 f^2 + 9 a^2 c^3 f^2 (-4 a^2 c - b^2)^9)^{1/2} - 2077 a^3 b^9 c^2 g^2 + 10656 a^4 b^7 c^3 g^2 - 30240 a^5 b^5 c^4 g^2 + 44800 a^6 b^3 c^5 g^2 - 25 a^3 c^2 g^2 (-4 a^2 c - b^2)^9)^{1/2} - 1024 a^5 c^9 d e + 5120 a^6 c^8 d g - 3072 a^6 c^8 e f + 15360 a^7 c^7 f g + 12 a^2 b^8 c^5 d e + 6 a^2 b^9 c^4 d f + 3584 a^5 b^2 c^8 d f + 6 a^2 c^4 d f (-4 a^2 c - b^2)^9)^{1/2} - 18 a^2 b^{10} c^3 d g - 2 a^2 b^{10} c^3 e f + 6 a^2 b^{11} c^2 e g + 1536 a^6 b^2 c^7 e g - 128 a^2 b^6 c^6 d e + 384 a^3 b^4 c^7 d e - 128 a^2 b^7 c^5 d f + 960 a^3 b^5 c^6 d f - 3072 a^4 b^3 c^7 d f + 324 a^2 b^8 c^4 d g + 36 a^2 b^8 c^4 e f - 2240 a^3 b^6 c^5 d g - 192 a^3 b^6 c^5 e f + 7296 a^4 b^4 c^6 d g + 128 a^4 b^4 c^6 e f - 10752 a^5 b^2 c^7 d g + 1536 a^5 b^2 c^7 e f - 98 a^2 b^9 c^3 e g + 576 a^3 b^7 c^4 e g - 1344 a^4 b^5 c^5 e g + 512 a^5 b^3 c^6 e g + 10 a^2 c^3 e g (-4 a^2 c - b^2)^9)^{1/2} - 152 a^2 b^{10} c^2 f g + 1548 a^3 b^8 c^3 f g - 8064 a^4 b^6 c^4 f g + 22400 a^5 b^4 c^5 f g - 30720 a^6 b^2 c^6 f g + 6 a^2 b^{12} c^2 f g - a^2 b^2 c^2 f^2 (-4 a^2 c - b^2)^9)^{1/2} + 51 a^2 b^2 c^2 g^2 (-4 a^2 c - b^2)^9)^{1/2} - 18 a^2 b^3 c^3 d g (-4 a^2 c - b^2)^9)^{1/2} - 2 a^2 b^3 c^3 e f (-4 a^2 c - b^2)^9)^{1/2} + 6 a^2 b^3 c^3 f g (-4 a^2 c - b^2)^9)^{1/2} + 6 a^2 b^2 c^2 e g (-4 a^2 c - b^2)^9)^{1/2} - 44 a^2 b^2 c^2 f g (-4 a^2 c - b^2)^9)^{1/2} / (32 (4096 a^7 c^{11} + a^2 b^{12} c^5 - 24 a^2 b^{10} c^6 + 240 a^3 b^8 c^7 - 1280 a^4 b^6 c^8 + 3840 a^5 b^4 c^9 - 6144 a^6 b^2 c^{10}))^{1/2} * (16 b^7 c^5 - 192 a^2 b^5 c^6 - 1024 a^3 b^2 c^8 + 768 a^2 b^3 c^7) / (2 (16 a^2 c^5 + b^4 c^3 - 8 a^2 b^2 c^4))) * ((c^5 d^2 (-4 a^2 c - b^2)^9)^{1/2} - b^9 c^5 d^2 - 9 a^2 b^3 c^7 g^2 + 768 a^4 b^2 c^9 d^2 - a^2 b^9 c^4 e^2 + 768 a^5 b^2 c^8 e^2 - a^2 c^4 e^2 (-4 a^2 c - b^2)^9)^{1/2}
\end{aligned}$$

$$\begin{aligned}
& 1/2) - a*b^{11}*c^2*f^2 + 3840*a^6*b*c^7*f^2 - 9*a*b^4*g^2*(-(4*a*c - b^2)^9)^{1/2} + 213*a^2*b^{11}*c*g^2 - 26880*a^7*b*c^6*g^2 + 96*a^2*b^5*c^7*d^2 - 51 \\
& 2*a^3*b^3*c^8*d^2 + 96*a^3*b^5*c^6*e^2 - 512*a^4*b^3*c^7*e^2 + 27*a^2*b^9*c^3*f^2 - 288*a^3*b^7*c^4*f^2 + 1504*a^4*b^5*c^5*f^2 - 3840*a^5*b^3*c^6*f^2 \\
& + 9*a^2*c^3*f^2*(-(4*a*c - b^2)^9)^{1/2} - 2077*a^3*b^9*c^2*g^2 + 10656*a^4*b^7*c^3*g^2 - 30240*a^5*b^5*c^4*g^2 + 44800*a^6*b^3*c^5*g^2 - 25*a^3*c^2*g \\
& ^2*(-(4*a*c - b^2)^9)^{1/2} - 1024*a^5*c^9*d*e + 5120*a^6*c^8*d*g - 3072*a^6*c^8*e*f + 15360*a^7*c^7*f*g + 12*a*b^8*c^5*d*e + 6*a*b^9*c^4*d*f + 3584*a \\
& ^5*b*c^8*d*f + 6*a*c^4*d*f*(-(4*a*c - b^2)^9)^{1/2} - 18*a*b^10*c^3*d*g - 2*a*b^10*c^3*e*f + 6*a*b^11*c^2*e*g + 1536*a^6*b*c^7*e*g - 128*a^2*b^6*c^6*d \\
& *e + 384*a^3*b^4*c^7*d*e - 128*a^2*b^7*c^5*d*f + 960*a^3*b^5*c^6*d*f - 3072*a^4*b^3*c^7*d*f + 324*a^2*b^8*c^4*d*g + 36*a^2*b^8*c^4*e*f - 2240*a^3*b^6* \\
& c^5*d*g - 192*a^3*b^6*c^5*e*f + 7296*a^4*b^4*c^6*d*g + 128*a^4*b^4*c^6*e*f - 10752*a^5*b^2*c^7*d*g + 1536*a^5*b^2*c^7*e*f - 98*a^2*b^9*c^3*e*g + 576*a \\
& ^3*b^7*c^4*e*g - 1344*a^4*b^5*c^5*e*g + 512*a^5*b^3*c^6*e*g + 10*a^2*c^3*e*g*(-(4*a*c - b^2)^9)^{1/2} - 152*a^2*b^10*c^2*f*g + 1548*a^3*b^8*c^3*f*g - \\
& 8064*a^4*b^6*c^4*f*g + 22400*a^5*b^4*c^5*f*g - 30720*a^6*b^2*c^6*f*g + 6*a*b^12*c*f*g - a*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^{1/2} + 51*a^2*b^2*c*g^2*(-(4 \\
& *a*c - b^2)^9)^{1/2} - 18*a*b*c^3*d*g*(-(4*a*c - b^2)^9)^{1/2} - 2*a*b*c^3*e*f*(-(4*a*c - b^2)^9)^{1/2} + 6*a*b^3*c*f*g*(-(4*a*c - b^2)^9)^{1/2} + 6*a \\
& *b^2*c^2*e*g*(-(4*a*c - b^2)^9)^{1/2} - 44*a^2*b*c^2*f*g*(-(4*a*c - b^2)^9)^{1/2})/(32*(4096*a^7*c^11 + a*b^12*c^5 - 24*a^2*b^10*c^6 + 240*a^3*b^8*c^7 \\
& - 1280*a^4*b^6*c^8 + 3840*a^5*b^4*c^9 - 6144*a^6*b^2*c^10)))^{1/2} - (x*(9 \\
& *b^8*g^2 - 8*a*c^7*d^2 + 8*a^2*c^6*e^2 + 10*b^2*c^6*d^2 - 72*a^3*c^5*f^2 + \\
& b^4*c^4*e^2 + 200*a^4*c^4*g^2 + b^6*c^2*f^2 + 2*a*b^2*c^5*e^2 - 16*a*b^4*c^3 \\
& *f^2 - 6*b^7*c*f*g + 74*a^2*b^2*c^4*f^2 + 481*a^2*b^4*c^2*g^2 - 718*a^3*b^2 \\
& *c^3*g^2 - 114*a*b^6*c*g^2 - 48*a^2*c^6*d*f - 6*b^3*c^5*d*e - 6*b^4*c^4*d* \\
& f - 80*a^3*c^5*e*g + 18*b^5*c^3*d*g + 2*b^5*c^3*e*f - 6*b^6*c^2*e*g + 52*a* \\
& b^2*c^5*d*f - 126*a*b^3*c^4*d*g - 14*a*b^3*c^4*e*f + 184*a^2*b*c^5*d*g - 8* \\
& a^2*b*c^5*e*f + 32*a*b^4*c^3*e*g + 86*a*b^5*c^2*f*g + 472*a^3*b*c^4*f*g + 4 \\
& *a^2*b^2*c^4*e*g - 374*a^2*b^3*c^3*f*g - 8*a*b*c^6*d*e))/(2*(16*a^2*c^5 + b \\
& ^4*c^3 - 8*a*b^2*c^4)))*((c^5*d^2*(-(4*a*c - b^2)^9)^{1/2} - b^9*c^5*d^2 - \\
& 9*a*b^13*g^2 + 768*a^4*b*c^9*d^2 - a*b^9*c^4*e^2 + 768*a^5*b*c^8*e^2 - a*c^4 \\
& *e^2*(-(4*a*c - b^2)^9)^{1/2} - a*b^11*c^2*f^2 + 3840*a^6*b*c^7*f^2 - 9*a* \\
& b^4*g^2*(-(4*a*c - b^2)^9)^{1/2} + 213*a^2*b^11*c*g^2 - 26880*a^7*b*c^6*g^2 \\
& + 96*a^2*b^5*c^7*d^2 - 512*a^3*b^3*c^8*d^2 + 96*a^3*b^5*c^6*e^2 - 512*a^4* \\
& b^3*c^7*e^2 + 27*a^2*b^9*c^3*f^2 - 288*a^3*b^7*c^4*f^2 + 1504*a^4*b^5*c^5*f \\
& ^2 - 3840*a^5*b^3*c^6*f^2 + 9*a^2*c^3*f^2*(-(4*a*c - b^2)^9)^{1/2} - 2077*a \\
& ^3*b^9*c^2*g^2 + 10656*a^4*b^7*c^3*g^2 - 30240*a^5*b^5*c^4*g^2 + 44800*a^6* \\
& b^3*c^5*g^2 - 25*a^3*c^2*g^2*(-(4*a*c - b^2)^9)^{1/2} - 1024*a^5*c^9*d*e + \\
& 5120*a^6*c^8*d*g - 3072*a^6*c^8*e*f + 15360*a^7*c^7*f*g + 12*a*b^8*c^5*d*e \\
& + 6*a*b^9*c^4*d*f + 3584*a^5*b*c^8*d*f + 6*a*c^4*d*f*(-(4*a*c - b^2)^9)^{1/2} \\
& - 18*a*b^10*c^3*d*g - 2*a*b^10*c^3*e*f + 6*a*b^11*c^2*e*g + 1536*a^6*b*c^7 \\
& *e*g - 128*a^2*b^6*c^6*d*e + 384*a^3*b^4*c^7*d*e - 128*a^2*b^7*c^5*d*f + \\
& 960*a^3*b^5*c^6*d*f - 3072*a^4*b^3*c^7*d*f + 324*a^2*b^8*c^4*d*g + 36*a^2*b \\
& ^8*c^4*e*f - 2240*a^3*b^6*c^5*d*g - 192*a^3*b^6*c^5*e*f + 7296*a^4*b^4*c^6* \\
& d*g + 128*a^4*b^4*c^6*e*f - 10752*a^5*b^2*c^7*d*g + 1536*a^5*b^2*c^7*e*f - \\
& 98*a^2*b^9*c^3*e*g + 576*a^3*b^7*c^4*e*g - 1344*a^4*b^5*c^5*e*g + 512*a^5*b \\
& ^3*c^6*e*g + 10*a^2*c^3*e*g*(-(4*a*c - b^2)^9)^{1/2} - 152*a^2*b^10*c^2*f*g \\
& + 1548*a^3*b^8*c^3*f*g - 8064*a^4*b^6*c^4*f*g + 22400*a^5*b^4*c^5*f*g - 30 \\
& 720*a^6*b^2*c^6*f*g + 6*a*b^12*c*f*g - a*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^{1/2} \\
& + 51*a^2*b^2*c*g^2*(-(4*a*c - b^2)^9)^{1/2} - 18*a*b*c^3*d*g*(-(4*a*c - \\
& b^2)^9)^{1/2} - 2*a*b*c^3*e*f*(-(4*a*c - b^2)^9)^{1/2} + 6*a*b^3*c*f*g*(-(4 \\
& *a*c - b^2)^9)^{1/2} + 6*a*b^2*c^2*e*g*(-(4*a*c - b^2)^9)^{1/2} - 44*a^2*b* \\
& c^2*f*g*(-(4*a*c - b^2)^9)^{1/2})/(32*(4096*a^7*c^11 + a*b^12*c^5 - 24*a^2* \\
& b^10*c^6 + 240*a^3*b^8*c^7 - 1280*a^4*b^6*c^8 + 3840*a^5*b^4*c^9 - 6144*a^6 \\
& *b^2*c^10)))^{1/2} + (((10240*a^5*c^7*g - 16*b^7*c^5*d - 2048*a^4*c^8*e - 7 \\
& 68*a^2*b^3*c^7*d - 384*a^2*b^4*c^6*e + 1536*a^3*b^2*c^7*e + 192*a^2*b^5*c^5 \\
& *f - 768*a^3*b^3*c^6*f - 736*a^2*b^6*c^4*g + 4224*a^3*b^4*c^5*g - 10752*a^4
\end{aligned}$$

$$\begin{aligned}
& *b^2*c^6*g + 192*a*b^5*c^6*d + 1024*a^3*b*c^8*d + 32*a*b^6*c^5*e - 16*a*b^7 \\
& *c^4*f + 1024*a^4*b*c^7*f + 48*a*b^8*c^3*g)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a \\
& *b^4*c^4 - 48*a^2*b^2*c^5)) + (x*((c^5*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - b^9*c \\
& ^5*d^2 - 9*a*b^13*g^2 + 768*a^4*b*c^9*d^2 - a*b^9*c^4*e^2 + 768*a^5*b*c^8*e \\
& ^2 - a*c^4*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - a*b^11*c^2*f^2 + 3840*a^6*b*c^7*f \\
& ^2 - 9*a*b^4*g^2*(-(4*a*c - b^2)^9)^{(1/2)} + 213*a^2*b^11*c*g^2 - 26880*a^7* \\
& b*c^6*g^2 + 96*a^2*b^5*c^7*d^2 - 512*a^3*b^3*c^8*d^2 + 96*a^3*b^5*c^6*e^2 - \\
& 512*a^4*b^3*c^7*e^2 + 27*a^2*b^9*c^3*f^2 - 288*a^3*b^7*c^4*f^2 + 1504*a^4* \\
& b^5*c^5*f^2 - 3840*a^5*b^3*c^6*f^2 + 9*a^2*c^3*f^2*(-(4*a*c - b^2)^9)^{(1/2)} \\
& - 2077*a^3*b^9*c^2*g^2 + 10656*a^4*b^7*c^3*g^2 - 30240*a^5*b^5*c^4*g^2 + 4 \\
& 4800*a^6*b^3*c^5*g^2 - 25*a^3*c^2*g^2*(-(4*a*c - b^2)^9)^{(1/2)} - 1024*a^5*c \\
& ^9*d*e + 5120*a^6*c^8*d*g - 3072*a^6*c^8*e*f + 15360*a^7*c^7*f*g + 12*a*b^8 \\
& *c^5*d*e + 6*a*b^9*c^4*d*f + 3584*a^5*b*c^8*d*f + 6*a*c^4*d*f*(-(4*a*c - b^ \\
& 2)^9)^{(1/2)} - 18*a*b^10*c^3*d*g - 2*a*b^10*c^3*e*f + 6*a*b^11*c^2*e*g + 153 \\
& 6*a^6*b*c^7*e*g - 128*a^2*b^6*c^6*d*e + 384*a^3*b^4*c^7*d*e - 128*a^2*b^7*c \\
& ^5*d*f + 960*a^3*b^5*c^6*d*f - 3072*a^4*b^3*c^7*d*f + 324*a^2*b^8*c^4*d*g + \\
& 36*a^2*b^8*c^4*e*f - 2240*a^3*b^6*c^5*d*g - 192*a^3*b^6*c^5*e*f + 7296*a^4 \\
& *b^4*c^6*d*g + 128*a^4*b^4*c^6*e*f - 10752*a^5*b^2*c^7*d*g + 1536*a^5*b^2*c \\
& ^7*e*f - 98*a^2*b^9*c^3*e*g + 576*a^3*b^7*c^4*e*g - 1344*a^4*b^5*c^5*e*g + \\
& 512*a^5*b^3*c^6*e*g + 10*a^2*c^3*e*g*(-(4*a*c - b^2)^9)^{(1/2)} - 152*a^2*b^1 \\
& 0*c^2*f*g + 1548*a^3*b^8*c^3*f*g - 8064*a^4*b^6*c^4*f*g + 22400*a^5*b^4*c^5 \\
& *f*g - 30720*a^6*b^2*c^6*f*g + 6*a*b^12*c*f*g - a*b^2*c^2*f^2*(-(4*a*c - b^ \\
& 2)^9)^{(1/2)} + 51*a^2*b^2*c*g^2*(-(4*a*c - b^2)^9)^{(1/2)} - 18*a*b*c^3*d*g*(- \\
& (4*a*c - b^2)^9)^{(1/2)} - 2*a*b*c^3*e*f*(-(4*a*c - b^2)^9)^{(1/2)} + 6*a*b^3*c \\
& *f*g*(-(4*a*c - b^2)^9)^{(1/2)} + 6*a*b^2*c^2*e*g*(-(4*a*c - b^2)^9)^{(1/2)} - \\
& 44*a^2*b*c^2*f*g*(-(4*a*c - b^2)^9)^{(1/2)))/(32*(4096*a^7*c^11 + a*b^12*c^5 \\
& - 24*a^2*b^10*c^6 + 240*a^3*b^8*c^7 - 1280*a^4*b^6*c^8 + 3840*a^5*b^4*c^9 - \\
& 6144*a^6*b^2*c^10)))^{(1/2)}*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^3*b*c^8 + \\
& 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((c^5*d^2*(-(4* \\
& a*c - b^2)^9)^{(1/2)} - b^9*c^5*d^2 - 9*a*b^13*g^2 + 768*a^4*b*c^9*d^2 - a*b^ \\
& 9*c^4*e^2 + 768*a^5*b*c^8*e^2 - a*c^4*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - a*b^11 \\
& *c^2*f^2 + 3840*a^6*b*c^7*f^2 - 9*a*b^4*g^2*(-(4*a*c - b^2)^9)^{(1/2)} + 213* \\
& a^2*b^11*c*g^2 - 26880*a^7*b*c^6*g^2 + 96*a^2*b^5*c^7*d^2 - 512*a^3*b^3*c^8 \\
& *d^2 + 96*a^3*b^5*c^6*e^2 - 512*a^4*b^3*c^7*e^2 + 27*a^2*b^9*c^3*f^2 - 288* \\
& a^3*b^7*c^4*f^2 + 1504*a^4*b^5*c^5*f^2 - 3840*a^5*b^3*c^6*f^2 + 9*a^2*c^3*f \\
& ^2*(-(4*a*c - b^2)^9)^{(1/2)} - 2077*a^3*b^9*c^2*g^2 + 10656*a^4*b^7*c^3*g^2 \\
& - 30240*a^5*b^5*c^4*g^2 + 44800*a^6*b^3*c^5*g^2 - 25*a^3*c^2*g^2*(-(4*a*c - \\
& b^2)^9)^{(1/2)} - 1024*a^5*c^9*d*e + 5120*a^6*c^8*d*g - 3072*a^6*c^8*e*f + 1 \\
& 5360*a^7*c^7*f*g + 12*a*b^8*c^5*d*e + 6*a*b^9*c^4*d*f + 3584*a^5*b*c^8*d*f \\
& + 6*a*c^4*d*f*(-(4*a*c - b^2)^9)^{(1/2)} - 18*a*b^10*c^3*d*g - 2*a*b^10*c^3*e \\
& *f + 6*a*b^11*c^2*e*g + 1536*a^6*b*c^7*e*g - 128*a^2*b^6*c^6*d*e + 384*a^3* \\
& b^4*c^7*d*e - 128*a^2*b^7*c^5*d*f + 960*a^3*b^5*c^6*d*f - 3072*a^4*b^3*c^7* \\
& d*f + 324*a^2*b^8*c^4*d*g + 36*a^2*b^8*c^4*e*f - 2240*a^3*b^6*c^5*d*g - 192 \\
& *a^3*b^6*c^5*e*f + 7296*a^4*b^4*c^6*d*g + 128*a^4*b^4*c^6*e*f - 10752*a^5*b \\
& ^2*c^7*d*g + 1536*a^5*b^2*c^7*e*f - 98*a^2*b^9*c^3*e*g + 576*a^3*b^7*c^4*e* \\
& g - 1344*a^4*b^5*c^5*e*g + 512*a^5*b^3*c^6*e*g + 10*a^2*c^3*e*g*(-(4*a*c - \\
& b^2)^9)^{(1/2)} - 152*a^2*b^10*c^2*f*g + 1548*a^3*b^8*c^3*f*g - 8064*a^4*b^6* \\
& c^4*f*g + 22400*a^5*b^4*c^5*f*g - 30720*a^6*b^2*c^6*f*g + 6*a*b^12*c*f*g - \\
& a*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + 51*a^2*b^2*c*g^2*(-(4*a*c - b^2)^9 \\
&)^{(1/2)} - 18*a*b*c^3*d*g*(-(4*a*c - b^2)^9)^{(1/2)} - 2*a*b*c^3*e*f*(-(4*a*c \\
& - b^2)^9)^{(1/2)} + 6*a*b^3*c*f*g*(-(4*a*c - b^2)^9)^{(1/2)} + 6*a*b^2*c^2*e*g* \\
& (- (4*a*c - b^2)^9)^{(1/2)} - 44*a^2*b*c^2*f*g*(-(4*a*c - b^2)^9)^{(1/2)))/(32*(\\
& 4096*a^7*c^11 + a*b^12*c^5 - 24*a^2*b^10*c^6 + 240*a^3*b^8*c^7 - 1280*a^4*b \\
& ^6*c^8 + 3840*a^5*b^4*c^9 - 6144*a^6*b^2*c^10)))^{(1/2)} + (x*(9*b^8*g^2 - 8* \\
& a*c^7*d^2 + 8*a^2*c^6*e^2 + 10*b^2*c^6*d^2 - 72*a^3*c^5*f^2 + b^4*c^4*e^2 + \\
& 200*a^4*c^4*g^2 + b^6*c^2*f^2 + 2*a*b^2*c^5*e^2 - 16*a*b^4*c^3*f^2 - 6*b^7 \\
& *c*f*g + 74*a^2*b^2*c^4*f^2 + 481*a^2*b^4*c^2*g^2 - 718*a^3*b^2*c^3*g^2 - 1 \\
& 14*a*b^6*c*g^2 - 48*a^2*c^6*d*f - 6*b^3*c^5*d*e - 6*b^4*c^4*d*f - 80*a^3*c^ \\
& 5*e*g + 18*b^5*c^3*d*g + 2*b^5*c^3*e*f - 6*b^6*c^2*e*g + 52*a*b^2*c^5*d*f -
\end{aligned}$$

$$\begin{aligned}
& 126*a*b^3*c^4*d*g - 14*a*b^3*c^4*e*f + 184*a^2*b*c^5*d*g - 8*a^2*b*c^5*e*f \\
& + 32*a*b^4*c^3*e*g + 86*a*b^5*c^2*f*g + 472*a^3*b*c^4*f*g + 4*a^2*b^2*c^4* \\
& e*g - 374*a^2*b^3*c^3*f*g - 8*a*b*c^6*d*e)) / (2*(16*a^2*c^5 + b^4*c^3 - 8*a* \\
& b^2*c^4))) * ((c^5*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - b^9*c^5*d^2 - 9*a*b^13*g^2 \\
& + 768*a^4*b*c^9*d^2 - a*b^9*c^4*e^2 + 768*a^5*b*c^8*e^2 - a*c^4*e^2*(-(4*a* \\
& c - b^2)^9)^{(1/2)} - a*b^11*c^2*f^2 + 3840*a^6*b*c^7*f^2 - 9*a*b^4*g^2*(-(4* \\
& a*c - b^2)^9)^{(1/2)} + 213*a^2*b^11*c*g^2 - 26880*a^7*b*c^6*g^2 + 96*a^2*b^5* \\
& c^7*d^2 - 512*a^3*b^3*c^8*d^2 + 96*a^3*b^5*c^6*e^2 - 512*a^4*b^3*c^7*e^2 + \\
& 27*a^2*b^9*c^3*f^2 - 288*a^3*b^7*c^4*f^2 + 1504*a^4*b^5*c^5*f^2 - 3840*a^5* \\
& b^3*c^6*f^2 + 9*a^2*c^3*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 2077*a^3*b^9*c^2*g^2 \\
& + 10656*a^4*b^7*c^3*g^2 - 30240*a^5*b^5*c^4*g^2 + 44800*a^6*b^3*c^5*g^2 - \\
& 25*a^3*c^2*g^2*(-(4*a*c - b^2)^9)^{(1/2)} - 1024*a^5*c^9*d*e + 5120*a^6*c^8* \\
& d*g - 3072*a^6*c^8*e*f + 15360*a^7*c^7*f*g + 12*a*b^8*c^5*d*e + 6*a*b^9*c^4* \\
& d*f + 3584*a^5*b*c^8*d*f + 6*a*c^4*d*f*(-(4*a*c - b^2)^9)^{(1/2)} - 18*a*b^1 \\
& 0*c^3*d*g - 2*a*b^10*c^3*e*f + 6*a*b^11*c^2*e*g + 1536*a^6*b*c^7*e*g - 128* \\
& a^2*b^6*c^6*d*e + 384*a^3*b^4*c^7*d*e - 128*a^2*b^7*c^5*d*f + 960*a^3*b^5*c^ \\
& 6*d*f - 3072*a^4*b^3*c^7*d*f + 324*a^2*b^8*c^4*d*g + 36*a^2*b^8*c^4*e*f - \\
& 2240*a^3*b^6*c^5*d*g - 192*a^3*b^6*c^5*e*f + 7296*a^4*b^4*c^6*d*g + 128*a^4* \\
& b^4*c^6*e*f - 10752*a^5*b^2*c^7*d*g + 1536*a^5*b^2*c^7*e*f - 98*a^2*b^9*c^ \\
& 3*e*g + 576*a^3*b^7*c^4*e*g - 1344*a^4*b^5*c^5*e*g + 512*a^5*b^3*c^6*e*g + \\
& 10*a^2*c^3*e*g*(-(4*a*c - b^2)^9)^{(1/2)} - 152*a^2*b^10*c^2*f*g + 1548*a^3*b^ \\
& 8*c^3*f*g - 8064*a^4*b^6*c^4*f*g + 22400*a^5*b^4*c^5*f*g - 30720*a^6*b^2*c^ \\
& 6*f*g + 6*a*b^12*c*f*g - a*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + 51*a^2*b^ \\
& 2*c*g^2*(-(4*a*c - b^2)^9)^{(1/2)} - 18*a*b*c^3*d*g*(-(4*a*c - b^2)^9)^{(1/2)} \\
& - 2*a*b*c^3*e*f*(-(4*a*c - b^2)^9)^{(1/2)} + 6*a*b^3*c*f*g*(-(4*a*c - b^2)^9 \\
&)^{(1/2)} + 6*a*b^2*c^2*e*g*(-(4*a*c - b^2)^9)^{(1/2)} - 44*a^2*b*c^2*f*g*(-(4* \\
& a*c - b^2)^9)^{(1/2))} / (32*(4096*a^7*c^11 + a*b^12*c^5 - 24*a^2*b^10*c^6 + 24 \\
& 0*a^3*b^8*c^7 - 1280*a^4*b^6*c^8 + 3840*a^5*b^4*c^9 - 6144*a^6*b^2*c^10)))^ \\
& (1/2) - (8*a*c^7*d^3 + 9*b^8*d*g^2 + 6*b^2*c^6*d^3 - 63*a^3*b^5*g^3 + 216*a^ \\
& 4*c^4*f^3 - 3*a*b^3*c^4*e^3 - 4*a^2*b*c^5*e^3 + 8*a^2*c^6*d*e^2 + 573*a^4* \\
& b^3*c*g^3 - 1300*a^5*b*c^2*g^3 + 72*a^2*c^6*d^2*f + 216*a^3*c^5*d*f^2 - 5*b^ \\
& 3*c^5*d^2*e + b^4*c^4*d*e^2 + 24*a^3*c^5*e^2*f + 200*a^4*c^4*d*g^2 - 5*b^4* \\
& c^4*d^2*f + b^6*c^2*d*f^2 + 45*a^2*b^6*f*f*g^2 + 15*b^5*c^3*d^2*g + 600*a^5* \\
& c^3*f*g^2 + 5*a^2*b^4*c^2*f^3 - 66*a^3*b^2*c^3*f^3 - 27*a*b^7*e*g^2 - 28*a* \\
& b*c^6*d^2*e - 78*a*b^6*c*d*g^2 - 80*a^3*c^5*d*e*g + 2*b^5*c^3*d*e*f - 6*b^6* \\
& c^2*d*e*g - 240*a^4*c^4*e*f*g + 18*a*b^2*c^5*d*e^2 + 26*a*b^2*c^5*d^2*f - \\
& 12*a*b^4*c^3*d*f^2 - 53*a*b^3*c^4*d^2*g - 6*a*b^4*c^3*e^2*f - 3*a*b^5*c^2*e* \\
& f^2 - 76*a^2*b*c^5*d^2*g - 204*a^3*b*c^4*e*f^2 + 18*a*b^5*c^2*e^2*g + 279* \\
& a^2*b^5*c*e*g^2 - 12*a^3*b*c^4*e^2*g + 420*a^4*b*c^3*e*g^2 - 30*a^2*b^5*c*f^ \\
& 2*g - 402*a^3*b^4*c*f*g^2 - 924*a^4*b*c^3*f^2*g - 6*b^7*c*d*f*f*g + 2*a^2*b^ \\
& 2*c^4*d*f^2 + 42*a^2*b^2*c^4*e^2*f + 51*a^2*b^3*c^3*e*f^2 + 133*a^2*b^4*c^2* \\
& d*g^2 + 114*a^3*b^2*c^3*d*g^2 - 81*a^2*b^3*c^3*e^2*g - 801*a^3*b^3*c^2*e*g^ \\
& 2 + 339*a^3*b^3*c^2*f^2*g + 762*a^4*b^2*c^2*f*f*g^2 + 18*a*b^6*c*e*f*f*g + 6*a* \\
& b^3*c^4*d*e*f - 152*a^2*b*c^5*d*e*f - 28*a*b^4*c^3*d*e*g + 62*a*b^5*c^2*d* \\
& f*g - 536*a^3*b*c^4*d*f*f*g + 276*a^2*b^2*c^4*d*e*g - 42*a^2*b^3*c^3*d*f*f*g - \\
& 246*a^2*b^4*c^2*e*f*f*g + 804*a^3*b^2*c^3*e*f*f*g) / (4*(64*a^3*c^6 - b^6*c^3 + 1 \\
& 2*a*b^4*c^4 - 48*a^2*b^2*c^5))) * ((c^5*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - b^9*c^ \\
& 5*d^2 - 9*a*b^13*g^2 + 768*a^4*b*c^9*d^2 - a*b^9*c^4*e^2 + 768*a^5*b*c^8*e^ \\
& 2 - a*c^4*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - a*b^11*c^2*f^2 + 3840*a^6*b*c^7*f^ \\
& 2 - 9*a*b^4*g^2*(-(4*a*c - b^2)^9)^{(1/2)} + 213*a^2*b^11*c*g^2 - 26880*a^7* \\
& b*c^6*g^2 + 96*a^2*b^5*c^7*d^2 - 512*a^3*b^3*c^8*d^2 + 96*a^3*b^5*c^6*e^2 - \\
& 512*a^4*b^3*c^7*e^2 + 27*a^2*b^9*c^3*f^2 - 288*a^3*b^7*c^4*f^2 + 1504*a^4* \\
& b^5*c^5*f^2 - 3840*a^5*b^3*c^6*f^2 + 9*a^2*c^3*f^2*(-(4*a*c - b^2)^9)^{(1/2)} \\
& - 2077*a^3*b^9*c^2*g^2 + 10656*a^4*b^7*c^3*g^2 - 30240*a^5*b^5*c^4*g^2 + 4 \\
& 4800*a^6*b^3*c^5*g^2 - 25*a^3*c^2*g^2*(-(4*a*c - b^2)^9)^{(1/2)} - 1024*a^5*c^ \\
& 9*d*e + 5120*a^6*c^8*d*g - 3072*a^6*c^8*e*f + 15360*a^7*c^7*f*g + 12*a*b^8* \\
& c^5*d*e + 6*a*b^9*c^4*d*f + 3584*a^5*b*c^8*d*f + 6*a*c^4*d*f*(-(4*a*c - b^ \\
& 2)^9)^{(1/2)} - 18*a*b^10*c^3*d*g - 2*a*b^10*c^3*e*f + 6*a*b^11*c^2*e*g + 153 \\
& 6*a^6*b*c^7*e*g - 128*a^2*b^6*c^6*d*e + 384*a^3*b^4*c^7*d*e - 128*a^2*b^7*c
\end{aligned}$$

$$\begin{aligned}
& ^5d*f + 960*a^3*b^5*c^6*d*f - 3072*a^4*b^3*c^7*d*f + 324*a^2*b^8*c^4*d*g + \\
& 36*a^2*b^8*c^4*e*f - 2240*a^3*b^6*c^5*d*g - 192*a^3*b^6*c^5*e*f + 7296*a^4 \\
& *b^4*c^6*d*g + 128*a^4*b^4*c^6*e*f - 10752*a^5*b^2*c^7*d*g + 1536*a^5*b^2*c \\
& ^7*e*f - 98*a^2*b^9*c^3*e*g + 576*a^3*b^7*c^4*e*g - 1344*a^4*b^5*c^5*e*g + \\
& 512*a^5*b^3*c^6*e*g + 10*a^2*c^3*e*g*(-(4*a*c - b^2)^9)^{(1/2)} - 152*a^2*b^1 \\
& 0*c^2*f*g + 1548*a^3*b^8*c^3*f*g - 8064*a^4*b^6*c^4*f*g + 22400*a^5*b^4*c^5 \\
& *f*g - 30720*a^6*b^2*c^6*f*g + 6*a*b^12*c*f*g - a*b^2*c^2*f^2*(-(4*a*c - b^ \\
& 2)^9)^{(1/2)} + 51*a^2*b^2*c*g^2*(-(4*a*c - b^2)^9)^{(1/2)} - 18*a*b*c^3*d*g*(- \\
& (4*a*c - b^2)^9)^{(1/2)} - 2*a*b*c^3*e*f*(-(4*a*c - b^2)^9)^{(1/2)} + 6*a*b^3*c \\
& *f*g*(-(4*a*c - b^2)^9)^{(1/2)} + 6*a*b^2*c^2*e*g*(-(4*a*c - b^2)^9)^{(1/2)} - \\
& 44*a^2*b*c^2*f*g*(-(4*a*c - b^2)^9)^{(1/2)}/(32*(4096*a^7*c^11 + a*b^12*c^5 \\
& - 24*a^2*b^10*c^6 + 240*a^3*b^8*c^7 - 1280*a^4*b^6*c^8 + 3840*a^5*b^4*c^9 - \\
& 6144*a^6*b^2*c^10)))^{(1/2)}*i - \operatorname{atan}((((10240*a^5*c^7*g - 16*b^7*c^5*d - \\
& 2048*a^4*c^8*e - 768*a^2*b^3*c^7*d - 384*a^2*b^4*c^6*e + 1536*a^3*b^2*c^7*e \\
& + 192*a^2*b^5*c^5*f - 768*a^3*b^3*c^6*f - 736*a^2*b^6*c^4*g + 4224*a^3*b^4 \\
& *c^5*g - 10752*a^4*b^2*c^6*g + 192*a*b^5*c^6*d + 1024*a^3*b*c^8*d + 32*a*b^6 \\
& *c^5*e - 16*a*b^7*c^4*f + 1024*a^4*b*c^7*f + 48*a*b^8*c^3*g)/(8*(64*a^3*c^6 \\
& - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) - (x*((768*a^4*b*c^9*d^2 - b^ \\
& 9*c^5*d^2 - c^5*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - 9*a*b^13*g^2 - a*b^9*c^4*e^2 \\
& + 768*a^5*b*c^8*e^2 + a*c^4*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - a*b^11*c^2*f^2 \\
& + 3840*a^6*b*c^7*f^2 + 9*a*b^4*g^2*(-(4*a*c - b^2)^9)^{(1/2)} + 213*a^2*b^11* \\
& c*g^2 - 26880*a^7*b*c^6*g^2 + 96*a^2*b^5*c^7*d^2 - 512*a^3*b^3*c^8*d^2 + 96 \\
& *a^3*b^5*c^6*e^2 - 512*a^4*b^3*c^7*e^2 + 27*a^2*b^9*c^3*f^2 - 288*a^3*b^7*c \\
& ^4*f^2 + 1504*a^4*b^5*c^5*f^2 - 3840*a^5*b^3*c^6*f^2 - 9*a^2*c^3*f^2*(-(4*a \\
& *c - b^2)^9)^{(1/2)} - 2077*a^3*b^9*c^2*g^2 + 10656*a^4*b^7*c^3*g^2 - 30240*a \\
& ^5*b^5*c^4*g^2 + 44800*a^6*b^3*c^5*g^2 + 25*a^3*c^2*g^2*(-(4*a*c - b^2)^9)^ \\
& (1/2) - 1024*a^5*c^9*d*e + 5120*a^6*c^8*d*g - 3072*a^6*c^8*e*f + 15360*a^7* \\
& c^7*f*g + 12*a*b^8*c^5*d*e + 6*a*b^9*c^4*d*f + 3584*a^5*b*c^8*d*f - 6*a*c^4 \\
& *d*f*(-(4*a*c - b^2)^9)^{(1/2)} - 18*a*b^10*c^3*d*g - 2*a*b^10*c^3*e*f + 6*a* \\
& b^11*c^2*e*g + 1536*a^6*b*c^7*e*g - 128*a^2*b^6*c^6*d*e + 384*a^3*b^4*c^7*d \\
& *e - 128*a^2*b^7*c^5*d*f + 960*a^3*b^5*c^6*d*f - 3072*a^4*b^3*c^7*d*f + 324 \\
& *a^2*b^8*c^4*d*g + 36*a^2*b^8*c^4*e*f - 2240*a^3*b^6*c^5*d*g - 192*a^3*b^6* \\
& c^5*e*f + 7296*a^4*b^4*c^6*d*g + 128*a^4*b^4*c^6*e*f - 10752*a^5*b^2*c^7*d* \\
& g + 1536*a^5*b^2*c^7*e*f - 98*a^2*b^9*c^3*e*g + 576*a^3*b^7*c^4*e*g - 1344* \\
& a^4*b^5*c^5*e*g + 512*a^5*b^3*c^6*e*g - 10*a^2*c^3*e*g*(-(4*a*c - b^2)^9)^{(\\
& 1/2)} - 152*a^2*b^10*c^2*f*g + 1548*a^3*b^8*c^3*f*g - 8064*a^4*b^6*c^4*f*g + \\
& 22400*a^5*b^4*c^5*f*g - 30720*a^6*b^2*c^6*f*g + 6*a*b^12*c*f*g + a*b^2*c^2 \\
& *f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 51*a^2*b^2*c*g^2*(-(4*a*c - b^2)^9)^{(1/2)} + \\
& 18*a*b*c^3*d*g*(-(4*a*c - b^2)^9)^{(1/2)} + 2*a*b*c^3*e*f*(-(4*a*c - b^2)^9) \\
& ^{(1/2)} - 6*a*b^3*c*f*g*(-(4*a*c - b^2)^9)^{(1/2)} - 6*a*b^2*c^2*e*g*(-(4*a*c \\
& - b^2)^9)^{(1/2)} + 44*a^2*b*c^2*f*g*(-(4*a*c - b^2)^9)^{(1/2)}/(32*(4096*a^7* \\
& c^11 + a*b^12*c^5 - 24*a^2*b^10*c^6 + 240*a^3*b^8*c^7 - 1280*a^4*b^6*c^8 + \\
& 3840*a^5*b^4*c^9 - 6144*a^6*b^2*c^10)))^{(1/2)}*(16*b^7*c^5 - 192*a*b^5*c^6 - \\
& 1024*a^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4) \\
&))*((768*a^4*b*c^9*d^2 - b^9*c^5*d^2 - c^5*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - 9 \\
& *a*b^13*g^2 - a*b^9*c^4*e^2 + 768*a^5*b*c^8*e^2 + a*c^4*e^2*(-(4*a*c - b^2) \\
& ^9)^{(1/2)} - a*b^11*c^2*f^2 + 3840*a^6*b*c^7*f^2 + 9*a*b^4*g^2*(-(4*a*c - b^ \\
& 2)^9)^{(1/2)} + 213*a^2*b^11*c*g^2 - 26880*a^7*b*c^6*g^2 + 96*a^2*b^5*c^7*d^2 \\
& - 512*a^3*b^3*c^8*d^2 + 96*a^3*b^5*c^6*e^2 - 512*a^4*b^3*c^7*e^2 + 27*a^2*b \\
& ^9*c^3*f^2 - 288*a^3*b^7*c^4*f^2 + 1504*a^4*b^5*c^5*f^2 - 3840*a^5*b^3*c^6 \\
& *f^2 - 9*a^2*c^3*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 2077*a^3*b^9*c^2*g^2 + 1065 \\
& 6*a^4*b^7*c^3*g^2 - 30240*a^5*b^5*c^4*g^2 + 44800*a^6*b^3*c^5*g^2 + 25*a^3*c^2 \\
& *g^2*(-(4*a*c - b^2)^9)^{(1/2)} - 1024*a^5*c^9*d*e + 5120*a^6*c^8*d*g - 30 \\
& 72*a^6*c^8*e*f + 15360*a^7*c^7*f*g + 12*a*b^8*c^5*d*e + 6*a*b^9*c^4*d*f + 3 \\
& 584*a^5*b*c^8*d*f - 6*a*c^4*d*f*(-(4*a*c - b^2)^9)^{(1/2)} - 18*a*b^10*c^3*d* \\
& g - 2*a*b^10*c^3*e*f + 6*a*b^11*c^2*e*g + 1536*a^6*b*c^7*e*g - 128*a^2*b^6* \\
& c^6*d*e + 384*a^3*b^4*c^7*d*e - 128*a^2*b^7*c^5*d*f + 960*a^3*b^5*c^6*d*f - \\
& 3072*a^4*b^3*c^7*d*f + 324*a^2*b^8*c^4*d*g + 36*a^2*b^8*c^4*e*f - 2240*a^3 \\
& *b^6*c^5*d*g - 192*a^3*b^6*c^5*e*f + 7296*a^4*b^4*c^6*d*g + 128*a^4*b^4*c^6
\end{aligned}$$

$$\begin{aligned}
& *e*f - 10752*a^5*b^2*c^7*d*g + 1536*a^5*b^2*c^7*e*f - 98*a^2*b^9*c^3*e*g + \\
& 576*a^3*b^7*c^4*e*g - 1344*a^4*b^5*c^5*e*g + 512*a^5*b^3*c^6*e*g - 10*a^2*c^3*e*g*(-(4*a*c - b^2)^9)^{(1/2)} - 152*a^2*b^10*c^2*f*g + 1548*a^3*b^8*c^3*f \\
& *g - 8064*a^4*b^6*c^4*f*g + 22400*a^5*b^4*c^5*f*g - 30720*a^6*b^2*c^6*f*g + \\
& 6*a*b^12*c*f*g + a*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 51*a^2*b^2*c*g^2 \\
& *(- (4*a*c - b^2)^9)^{(1/2)} + 18*a*b*c^3*d*g*(-(4*a*c - b^2)^9)^{(1/2)} + 2*a*b \\
& *c^3*e*f*(-(4*a*c - b^2)^9)^{(1/2)} - 6*a*b^3*c*f*g*(-(4*a*c - b^2)^9)^{(1/2)} \\
& - 6*a*b^2*c^2*e*g*(-(4*a*c - b^2)^9)^{(1/2)} + 44*a^2*b*c^2*f*g*(-(4*a*c - b^2)^9)^{(1/2))}/(32*(4096*a^7*c^11 + a*b^12*c^5 - 24*a^2*b^10*c^6 + 240*a^3*b^8 \\
& *c^7 - 1280*a^4*b^6*c^8 + 3840*a^5*b^4*c^9 - 6144*a^6*b^2*c^10)))^{(1/2)} - \\
& (x*(9*b^8*g^2 - 8*a*c^7*d^2 + 8*a^2*c^6*e^2 + 10*b^2*c^6*d^2 - 72*a^3*c^5*f^2 + b^4*c^4*e^2 + 200*a^4*c^4*g^2 + b^6*c^2*f^2 + 2*a*b^2*c^5*e^2 - 16*a*b^4 \\
& *c^3*f^2 - 6*b^7*c*f*g + 74*a^2*b^2*c^4*f^2 + 481*a^2*b^4*c^2*g^2 - 718*a^3*b^2*c^3*g^2 - 114*a*b^6*c*g^2 - 48*a^2*c^6*d*f - 6*b^3*c^5*d*e - 6*b^4*c^4 \\
& *d*f - 80*a^3*c^5*e*g + 18*b^5*c^3*d*g + 2*b^5*c^3*e*f - 6*b^6*c^2*e*g + \\
& 52*a*b^2*c^5*d*f - 126*a*b^3*c^4*d*g - 14*a*b^3*c^4*e*f + 184*a^2*b*c^5*d*g \\
& - 8*a^2*b*c^5*e*f + 32*a*b^4*c^3*e*g + 86*a*b^5*c^2*f*g + 472*a^3*b*c^4*f* \\
& g + 4*a^2*b^2*c^4*e*g - 374*a^2*b^3*c^3*f*g - 8*a*b*c^6*d*e))/ (2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((768*a^4*b*c^9*d^2 - b^9*c^5*d^2 - c^5*d^2*(- \\
& (4*a*c - b^2)^9)^{(1/2)} - 9*a*b^13*g^2 - a*b^9*c^4*e^2 + 768*a^5*b*c^8*e^2 + \\
& a*c^4*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - a*b^11*c^2*f^2 + 3840*a^6*b*c^7*f^2 + \\
& 9*a*b^4*g^2*(-(4*a*c - b^2)^9)^{(1/2)} + 213*a^2*b^11*c*g^2 - 26880*a^7*b*c^6 \\
& *g^2 + 96*a^2*b^5*c^7*d^2 - 512*a^3*b^3*c^8*d^2 + 96*a^3*b^5*c^6*e^2 - 512 \\
& *a^4*b^3*c^7*e^2 + 27*a^2*b^9*c^3*f^2 - 288*a^3*b^7*c^4*f^2 + 1504*a^4*b^5*c^5 \\
& *f^2 - 3840*a^5*b^3*c^6*f^2 - 9*a^2*c^3*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 2 \\
& 077*a^3*b^9*c^2*g^2 + 10656*a^4*b^7*c^3*g^2 - 30240*a^5*b^5*c^4*g^2 + 44800 \\
& *a^6*b^3*c^5*g^2 + 25*a^3*c^2*g^2*(-(4*a*c - b^2)^9)^{(1/2)} - 1024*a^5*c^9*d \\
& *e + 5120*a^6*c^8*d*g - 3072*a^6*c^8*e*f + 15360*a^7*c^7*f*g + 12*a*b^8*c^5 \\
& *d*e + 6*a*b^9*c^4*d*f + 3584*a^5*b*c^8*d*f - 6*a*c^4*d*f*(-(4*a*c - b^2)^9 \\
&)^{(1/2)} - 18*a*b^10*c^3*d*g - 2*a*b^10*c^3*e*f + 6*a*b^11*c^2*e*g + 1536*a^6 \\
& *b*c^7*e*g - 128*a^2*b^6*c^6*d*e + 384*a^3*b^4*c^7*d*e - 128*a^2*b^7*c^5*d \\
& *f + 960*a^3*b^5*c^6*d*f - 3072*a^4*b^3*c^7*d*f + 324*a^2*b^8*c^4*d*g + 36* \\
& a^2*b^8*c^4*e*f - 2240*a^3*b^6*c^5*d*g - 192*a^3*b^6*c^5*e*f + 7296*a^4*b^4 \\
& *c^6*d*g + 128*a^4*b^4*c^6*e*f - 10752*a^5*b^2*c^7*d*g + 1536*a^5*b^2*c^7*e \\
& *f - 98*a^2*b^9*c^3*e*g + 576*a^3*b^7*c^4*e*g - 1344*a^4*b^5*c^5*e*g + 512* \\
& a^5*b^3*c^6*e*g - 10*a^2*c^3*e*g*(-(4*a*c - b^2)^9)^{(1/2)} - 152*a^2*b^10*c^2 \\
& *f*g + 1548*a^3*b^8*c^3*f*g - 8064*a^4*b^6*c^4*f*g + 22400*a^5*b^4*c^5*f*g \\
& - 30720*a^6*b^2*c^6*f*g + 6*a*b^12*c*f*g + a*b^2*c^2*f^2*(-(4*a*c - b^2)^9 \\
&)^{(1/2)} - 51*a^2*b^2*c*g^2*(-(4*a*c - b^2)^9)^{(1/2)} + 18*a*b*c^3*d*g*(-(4*a \\
& *c - b^2)^9)^{(1/2)} + 2*a*b*c^3*e*f*(-(4*a*c - b^2)^9)^{(1/2)} - 6*a*b^3*c*f*g \\
& *(- (4*a*c - b^2)^9)^{(1/2)} - 6*a*b^2*c^2*e*g*(-(4*a*c - b^2)^9)^{(1/2)} + 44*a \\
& ^2*b*c^2*f*g*(-(4*a*c - b^2)^9)^{(1/2))}/(32*(4096*a^7*c^11 + a*b^12*c^5 - 24 \\
& *a^2*b^10*c^6 + 240*a^3*b^8*c^7 - 1280*a^4*b^6*c^8 + 3840*a^5*b^4*c^9 - 614 \\
& 4*a^6*b^2*c^10)))^{(1/2)}*i - (((10240*a^5*c^7*g - 16*b^7*c^5*d - 2048*a^4*c^8 \\
& *e - 768*a^2*b^3*c^7*d - 384*a^2*b^4*c^6*e + 1536*a^3*b^2*c^7*e + 192*a^2 \\
& *b^5*c^5*f - 768*a^3*b^3*c^6*f - 736*a^2*b^6*c^4*g + 4224*a^3*b^4*c^5*g - 1 \\
& 0752*a^4*b^2*c^6*g + 192*a*b^5*c^6*d + 1024*a^3*b*c^8*d + 32*a*b^6*c^5*e - \\
& 16*a*b^7*c^4*f + 1024*a^4*b*c^7*f + 48*a*b^8*c^3*g)/(8*(64*a^3*c^6 - b^6*c^3 \\
& + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) + (x*((768*a^4*b*c^9*d^2 - b^9*c^5*d^2 \\
& - c^5*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - 9*a*b^13*g^2 - a*b^9*c^4*e^2 + 768*a^5 \\
& *b*c^8*e^2 + a*c^4*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - a*b^11*c^2*f^2 + 3840*a^6 \\
& *b*c^7*f^2 + 9*a*b^4*g^2*(-(4*a*c - b^2)^9)^{(1/2)} + 213*a^2*b^11*c*g^2 - 26 \\
& 880*a^7*b*c^6*g^2 + 96*a^2*b^5*c^7*d^2 - 512*a^3*b^3*c^8*d^2 + 96*a^3*b^5*c^6 \\
& *e^2 - 512*a^4*b^3*c^7*e^2 + 27*a^2*b^9*c^3*f^2 - 288*a^3*b^7*c^4*f^2 + 1 \\
& 504*a^4*b^5*c^5*f^2 - 3840*a^5*b^3*c^6*f^2 - 9*a^2*c^3*f^2*(-(4*a*c - b^2)^9 \\
&)^{(1/2)} - 2077*a^3*b^9*c^2*g^2 + 10656*a^4*b^7*c^3*g^2 - 30240*a^5*b^5*c^4 \\
& *g^2 + 44800*a^6*b^3*c^5*g^2 + 25*a^3*c^2*g^2*(-(4*a*c - b^2)^9)^{(1/2)} - 10 \\
& 24*a^5*c^9*d*e + 5120*a^6*c^8*d*g - 3072*a^6*c^8*e*f + 15360*a^7*c^7*f*g + \\
& 12*a*b^8*c^5*d*e + 6*a*b^9*c^4*d*f + 3584*a^5*b*c^8*d*f - 6*a*c^4*d*f*(-(4*
\end{aligned}$$

$$\begin{aligned}
& a^2c - b^2)^9)^{(1/2)} - 18*a*b^{10}*c^3*d*g - 2*a*b^{10}*c^3*e*f + 6*a*b^{11}*c^2*e \\
& *g + 1536*a^6*b*c^7*e*g - 128*a^2*b^6*c^6*d*e + 384*a^3*b^4*c^7*d*e - 128*a \\
& ^2*b^7*c^5*d*f + 960*a^3*b^5*c^6*d*f - 3072*a^4*b^3*c^7*d*f + 324*a^2*b^8*c \\
& ^4*d*g + 36*a^2*b^8*c^4*e*f - 2240*a^3*b^6*c^5*d*g - 192*a^3*b^6*c^5*e*f + \\
& 7296*a^4*b^4*c^6*d*g + 128*a^4*b^4*c^6*e*f - 10752*a^5*b^2*c^7*d*g + 1536*a \\
& ^5*b^2*c^7*e*f - 98*a^2*b^9*c^3*e*g + 576*a^3*b^7*c^4*e*g - 1344*a^4*b^5*c^ \\
& 5*e*g + 512*a^5*b^3*c^6*e*g - 10*a^2*c^3*e*g*(-(4*a*c - b^2)^9)^{(1/2)} - 152 \\
& *a^2*b^{10}*c^2*f*g + 1548*a^3*b^8*c^3*f*g - 8064*a^4*b^6*c^4*f*g + 22400*a^5 \\
& *b^4*c^5*f*g - 30720*a^6*b^2*c^6*f*g + 6*a*b^{12}*c*f*g + a*b^2*c^2*f^2*(-(4* \\
& a*c - b^2)^9)^{(1/2)} - 51*a^2*b^2*c*g^2*(-(4*a*c - b^2)^9)^{(1/2)} + 18*a*b*c^ \\
& 3*d*g*(-(4*a*c - b^2)^9)^{(1/2)} + 2*a*b*c^3*e*f*(-(4*a*c - b^2)^9)^{(1/2)} - 6 \\
& *a*b^3*c*f*g*(-(4*a*c - b^2)^9)^{(1/2)} - 6*a*b^2*c^2*e*g*(-(4*a*c - b^2)^9)^{ \\
& (1/2)} + 44*a^2*b*c^2*f*g*(-(4*a*c - b^2)^9)^{(1/2))}/(32*(4096*a^7*c^11 + a*b \\
& ^12*c^5 - 24*a^2*b^{10}*c^6 + 240*a^3*b^8*c^7 - 1280*a^4*b^6*c^8 + 3840*a^5*b \\
& ^4*c^9 - 6144*a^6*b^2*c^10)))^{(1/2)}*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^3* \\
& b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((768*a \\
& ^4*b*c^9*d^2 - b^9*c^5*d^2 - c^5*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - 9*a*b^{13}*g^ \\
& 2 - a*b^9*c^4*e^2 + 768*a^5*b*c^8*e^2 + a*c^4*e^2*(-(4*a*c - b^2)^9)^{(1/2)} \\
& - a*b^{11}*c^2*f^2 + 3840*a^6*b*c^7*f^2 + 9*a*b^4*g^2*(-(4*a*c - b^2)^9)^{(1/2)} \\
&) + 213*a^2*b^{11}*c*g^2 - 26880*a^7*b*c^6*g^2 + 96*a^2*b^5*c^7*d^2 - 512*a^3 \\
& *b^3*c^8*d^2 + 96*a^3*b^5*c^6*e^2 - 512*a^4*b^3*c^7*e^2 + 27*a^2*b^9*c^3*f^ \\
& 2 - 288*a^3*b^7*c^4*f^2 + 1504*a^4*b^5*c^5*f^2 - 3840*a^5*b^3*c^6*f^2 - 9*a \\
& ^2*c^3*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 2077*a^3*b^9*c^2*g^2 + 10656*a^4*b^7* \\
& c^3*g^2 - 30240*a^5*b^5*c^4*g^2 + 44800*a^6*b^3*c^5*g^2 + 25*a^3*c^2*g^2*(- \\
& (4*a*c - b^2)^9)^{(1/2)} - 1024*a^5*c^9*d*e + 5120*a^6*c^8*d*g - 3072*a^6*c^8 \\
& *e*f + 15360*a^7*c^7*f*g + 12*a*b^8*c^5*d*e + 6*a*b^9*c^4*d*f + 3584*a^5*b* \\
& c^8*d*f - 6*a*c^4*d*f*(-(4*a*c - b^2)^9)^{(1/2)} - 18*a*b^{10}*c^3*d*g - 2*a*b^ \\
& 10*c^3*e*f + 6*a*b^{11}*c^2*e*g + 1536*a^6*b*c^7*e*g - 128*a^2*b^6*c^6*d*e + \\
& 384*a^3*b^4*c^7*d*e - 128*a^2*b^7*c^5*d*f + 960*a^3*b^5*c^6*d*f - 3072*a^4* \\
& b^3*c^7*d*f + 324*a^2*b^8*c^4*d*g + 36*a^2*b^8*c^4*e*f - 2240*a^3*b^6*c^5*d \\
& *g - 192*a^3*b^6*c^5*e*f + 7296*a^4*b^4*c^6*d*g + 128*a^4*b^4*c^6*e*f - 107 \\
& 52*a^5*b^2*c^7*d*g + 1536*a^5*b^2*c^7*e*f - 98*a^2*b^9*c^3*e*g + 576*a^3*b^ \\
& 7*c^4*e*g - 1344*a^4*b^5*c^5*e*g + 512*a^5*b^3*c^6*e*g - 10*a^2*c^3*e*g*(-(\\
& 4*a*c - b^2)^9)^{(1/2)} - 152*a^2*b^{10}*c^2*f*g + 1548*a^3*b^8*c^3*f*g - 8064* \\
& a^4*b^6*c^4*f*g + 22400*a^5*b^4*c^5*f*g - 30720*a^6*b^2*c^6*f*g + 6*a*b^{12}* \\
& c*f*g + a*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 51*a^2*b^2*c*g^2*(-(4*a*c \\
& - b^2)^9)^{(1/2)} + 18*a*b*c^3*d*g*(-(4*a*c - b^2)^9)^{(1/2)} + 2*a*b*c^3*e*f*(- \\
& (4*a*c - b^2)^9)^{(1/2)} - 6*a*b^3*c*f*g*(-(4*a*c - b^2)^9)^{(1/2)} - 6*a*b^2* \\
& c^2*e*g*(-(4*a*c - b^2)^9)^{(1/2)} + 44*a^2*b*c^2*f*g*(-(4*a*c - b^2)^9)^{(1/2)} \\
&))/(32*(4096*a^7*c^11 + a*b^12*c^5 - 24*a^2*b^{10}*c^6 + 240*a^3*b^8*c^7 - 12 \\
& 80*a^4*b^6*c^8 + 3840*a^5*b^4*c^9 - 6144*a^6*b^2*c^10)))^{(1/2)} + (x*(9*b^8* \\
& g^2 - 8*a*c^7*d^2 + 8*a^2*c^6*e^2 + 10*b^2*c^6*d^2 - 72*a^3*c^5*f^2 + b^4*c \\
& ^4*e^2 + 200*a^4*c^4*g^2 + b^6*c^2*f^2 + 2*a*b^2*c^5*e^2 - 16*a*b^4*c^3*f^2 \\
& - 6*b^7*c*f*g + 74*a^2*b^2*c^4*f^2 + 481*a^2*b^4*c^2*g^2 - 718*a^3*b^2*c^3 \\
& *g^2 - 114*a*b^6*c*g^2 - 48*a^2*c^6*d*f - 6*b^3*c^5*d*e - 6*b^4*c^4*d*f - 8 \\
& 0*a^3*c^5*e*g + 18*b^5*c^3*d*g + 2*b^5*c^3*e*f - 6*b^6*c^2*e*g + 52*a*b^2*c \\
& ^5*d*f - 126*a*b^3*c^4*d*g - 14*a*b^3*c^4*e*f + 184*a^2*b*c^5*d*g - 8*a^2*b \\
& *c^5*e*f + 32*a*b^4*c^3*e*g + 86*a*b^5*c^2*f*g + 472*a^3*b*c^4*f*g + 4*a^2* \\
& b^2*c^4*e*g - 374*a^2*b^3*c^3*f*g - 8*a*b*c^6*d*e))/(2*(16*a^2*c^5 + b^4*c^ \\
& 3 - 8*a*b^2*c^4)))*((768*a^4*b*c^9*d^2 - b^9*c^5*d^2 - c^5*d^2*(-(4*a*c - b \\
& ^2)^9)^{(1/2)} - 9*a*b^{13}*g^2 - a*b^9*c^4*e^2 + 768*a^5*b*c^8*e^2 + a*c^4*e^2 \\
& *(- (4*a*c - b^2)^9)^{(1/2)} - a*b^{11}*c^2*f^2 + 3840*a^6*b*c^7*f^2 + 9*a*b^4*g \\
& ^2*(-(4*a*c - b^2)^9)^{(1/2)} + 213*a^2*b^{11}*c*g^2 - 26880*a^7*b*c^6*g^2 + 96 \\
& *a^2*b^5*c^7*d^2 - 512*a^3*b^3*c^8*d^2 + 96*a^3*b^5*c^6*e^2 - 512*a^4*b^3*c \\
& ^7*e^2 + 27*a^2*b^9*c^3*f^2 - 288*a^3*b^7*c^4*f^2 + 1504*a^4*b^5*c^5*f^2 - \\
& 3840*a^5*b^3*c^6*f^2 - 9*a^2*c^3*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 2077*a^3*b^ \\
& 9*c^2*g^2 + 10656*a^4*b^7*c^3*g^2 - 30240*a^5*b^5*c^4*g^2 + 44800*a^6*b^3*c \\
& ^5*g^2 + 25*a^3*c^2*g^2*(-(4*a*c - b^2)^9)^{(1/2)} - 1024*a^5*c^9*d*e + 5120* \\
& a^6*c^8*d*g - 3072*a^6*c^8*e*f + 15360*a^7*c^7*f*g + 12*a*b^8*c^5*d*e + 6*a
\end{aligned}$$

$$\begin{aligned}
& *b^9c^4d*f + 3584a^5b^8c^8d*f - 6a^4c^4d*f*(-(4a^2c - b^2)^9)^{(1/2)} - \\
& 18a^2b^10c^3d*g - 2a^2b^10c^3e*f + 6a^2b^11c^2e*g + 1536a^6b^7c^7e* \\
& g - 128a^2b^6c^6d*e + 384a^3b^4c^7d*e - 128a^2b^7c^5d*f + 960a^ \\
& ^3b^5c^6d*f - 3072a^4b^3c^7d*f + 324a^2b^8c^4d*g + 36a^2b^8c^ \\
& 4e*f - 2240a^3b^6c^5d*g - 192a^3b^6c^5e*f + 7296a^4b^4c^6d*g + \\
& 128a^4b^4c^6e*f - 10752a^5b^2c^7d*g + 1536a^5b^2c^7e*f - 98a^ \\
& 2b^9c^3e*g + 576a^3b^7c^4e*g - 1344a^4b^5c^5e*g + 512a^5b^3c^ \\
& 6e*g - 10a^2c^3e*g*(-(4a^2c - b^2)^9)^{(1/2)} - 152a^2b^10c^2f*g + 15 \\
& 48a^3b^8c^3f*g - 8064a^4b^6c^4f*g + 22400a^5b^4c^5f*g - 30720a^ \\
& ^6b^2c^6f*g + 6a^2b^12c^2f*g + a^2b^2c^2f^2*(-(4a^2c - b^2)^9)^{(1/2)} - \\
& 51a^2b^2c^2g^2*(-(4a^2c - b^2)^9)^{(1/2)} + 18a^2b^2c^2d*g*(-(4a^2c - b^2)^ \\
& ^9)^{(1/2)} + 2a^2b^2c^2e*f*(-(4a^2c - b^2)^9)^{(1/2)} - 6a^2b^2c^2f*g*(-(4a^2c \\
& - b^2)^9)^{(1/2)} - 6a^2b^2c^2e*g*(-(4a^2c - b^2)^9)^{(1/2)} + 44a^2b^2c^2f \\
& *g*(-(4a^2c - b^2)^9)^{(1/2)}/(32*(4096a^7c^11 + a^2b^12c^5 - 24a^2b^10c^ \\
& ^6 + 240a^3b^8c^7 - 1280a^4b^6c^8 + 3840a^5b^4c^9 - 6144a^6b^2c^ \\
& ^10)))^{(1/2)}*1i)/((((10240a^5c^7g - 16b^7c^5d - 2048a^4c^8e - 768 \\
& a^2b^3c^7d - 384a^2b^4c^6e + 1536a^3b^2c^7e + 192a^2b^5c^5f \\
& - 768a^3b^3c^6f - 736a^2b^6c^4g + 4224a^3b^4c^5g - 10752a^4b^ \\
& ^2c^6g + 192a^2b^5c^6d + 1024a^3b^8c^8d + 32a^2b^6c^5e - 16a^2b^7c^ \\
& ^4f + 1024a^4b^7c^7f + 48a^2b^8c^3g)/(8*(64a^3c^6 - b^6c^3 + 12a^2b^ \\
& ^4c^4 - 48a^2b^2c^5)) - (x*((768a^4b^9c^9d^2 - b^9c^5d^2 - c^5d^2* \\
& ^(-(4a^2c - b^2)^9)^{(1/2)} - 9a^2b^13g^2 - a^2b^9c^4e^2 + 768a^5b^8c^8e^2 \\
& + a^2c^4e^2*(-(4a^2c - b^2)^9)^{(1/2)} - a^2b^11c^2f^2 + 3840a^6b^7c^7f^2 \\
& + 9a^2b^4g^2*(-(4a^2c - b^2)^9)^{(1/2)} + 213a^2b^11c^2g^2 - 26880a^7b^6c^6g^2 \\
& + 96a^2b^5c^7d^2 - 512a^3b^3c^8d^2 + 96a^3b^5c^6e^2 - 5 \\
& 12a^4b^3c^7e^2 + 27a^2b^9c^3f^2 - 288a^3b^7c^4f^2 + 1504a^4b^5c^5f^2 - 3840a^5b^3c^6f^2 \\
& - 9a^2c^3f^2*(-(4a^2c - b^2)^9)^{(1/2)} - 2077a^3b^9c^2g^2 + 10656a^4b^7c^3g^2 - 30240a^5b^5c^4g^2 \\
& + 44800a^6b^3c^5g^2 + 25a^3c^2g^2*(-(4a^2c - b^2)^9)^{(1/2)} - 1024a^5c^9 \\
& *d*e + 5120a^6c^8d*g - 3072a^6c^8e*f + 15360a^7c^7f*g + 12a^2b^8c^ \\
& ^5d*e + 6a^2b^9c^4d*f + 3584a^5b^8c^8d*f - 6a^4c^4d*f*(-(4a^2c - b^2) \\
& ^9)^{(1/2)} - 18a^2b^10c^3d*g - 2a^2b^10c^3e*f + 6a^2b^11c^2e*g + 1536a^ \\
& ^6b^7c^7e*g - 128a^2b^6c^6d*e + 384a^3b^4c^7d*e - 128a^2b^7c^5 \\
& *d*f + 960a^3b^5c^6d*f - 3072a^4b^3c^7d*f + 324a^2b^8c^4d*g + 3 \\
& 6a^2b^8c^4e*f - 2240a^3b^6c^5d*g - 192a^3b^6c^5e*f + 7296a^4b^4c^6d*g + \\
& 128a^4b^4c^6e*f - 10752a^5b^2c^7d*g + 1536a^5b^2c^7e*f - 98a^2b^9c^3e*g \\
& + 576a^3b^7c^4e*g - 1344a^4b^5c^5e*g + 512a^5b^3c^6e*g - 10a^2c^3e*g*(-(4a^2c - b^2)^9)^{(1/2)} \\
& - 152a^2b^10c^2f*g + 1548a^3b^8c^3f*g - 8064a^4b^6c^4f*g + 22400a^5b^4c^5f \\
& *g - 30720a^6b^2c^6f*g + 6a^2b^12c^2f*g + a^2b^2c^2f^2*(-(4a^2c - b^2) \\
& ^9)^{(1/2)} - 51a^2b^2c^2g^2*(-(4a^2c - b^2)^9)^{(1/2)} + 18a^2b^2c^2d*g*(-(4 \\
& a^2c - b^2)^9)^{(1/2)} + 2a^2b^2c^2e*f*(-(4a^2c - b^2)^9)^{(1/2)} - 6a^2b^2c^2f \\
& *g*(-(4a^2c - b^2)^9)^{(1/2)} - 6a^2b^2c^2e*g*(-(4a^2c - b^2)^9)^{(1/2)} + 44 \\
& a^2b^2c^2f*g*(-(4a^2c - b^2)^9)^{(1/2)}/(32*(4096a^7c^11 + a^2b^12c^5 - \\
& 24a^2b^10c^6 + 240a^3b^8c^7 - 1280a^4b^6c^8 + 3840a^5b^4c^9 - 6 \\
& 144a^6b^2c^10)))^{(1/2)}*(16b^7c^5 - 192a^2b^5c^6 - 1024a^3b^8c^8 + 76 \\
& 8a^2b^3c^7)/(2*(16a^2c^5 + b^4c^3 - 8a^2b^2c^4))*((768a^4b^9c^9d^2 - b^9c^5d^2 - c^5d^2*(-(4a^2c - b^2)^9)^{(1/2)} \\
& - 9a^2b^13g^2 - a^2b^9c^4e^2 + 768a^5b^8c^8e^2 + a^2c^4e^2*(-(4a^2c - b^2)^9)^{(1/2)} - a^2b^11c^2f^2 \\
& + 3840a^6b^7c^7f^2 + 9a^2b^4g^2*(-(4a^2c - b^2)^9)^{(1/2)} + 213a^2b^11c^2g^2 - 26880a^7b^6c^6g^2 \\
& + 96a^2b^5c^7d^2 - 512a^3b^3c^8d^2 + 27a^2b^9c^3f^2 - 288a^3b^7c^4f^2 + 1504a^4b^5c^5f^2 - 3840a^5b^3c^6f^2 \\
& - 9a^2c^3f^2*(-(4a^2c - b^2)^9)^{(1/2)} - 2077a^3b^9c^2g^2 + 10656a^4b^7c^3g^2 - 30240a^5b^5c^4g^2 \\
& + 44800a^6b^3c^5g^2 + 25a^3c^2g^2*(-(4a^2c - b^2)^9)^{(1/2)} - 1024a^5c^9d*e + 5120a^6c^8d*g - 3072a^6c^8e*f \\
& + 15360a^7c^7f*g + 12a^2b^8c^5d*e + 6a^2b^9c^4d*f + 3584a^5b^8c^8d*f - 6a^4c^4d*f*(-(4a^2c - b^2)^9)^{(1/2)} \\
& - 18a^2b^10c^3d*g - 2a^2b^10c^3e*f + 6a^2b^11c^2e*g + 1536a^6b^7c^7e*g - 128a^2b^6c^6d*e + 384a^3b^
\end{aligned}$$

$$\begin{aligned}
& 4c^7d^2e - 128a^2b^7c^5d^2f + 960a^3b^5c^6d^2f - 3072a^4b^3c^7d^2f \\
& + 324a^2b^8c^4d^2g + 36a^2b^8c^4e^2f - 2240a^3b^6c^5d^2g - 192a^3b^6c^5e^2f \\
& + 7296a^4b^4c^6d^2g + 128a^4b^4c^6e^2f - 10752a^5b^2c^7d^2g + 1536a^5b^2c^7e^2f \\
& - 98a^2b^9c^3e^2g + 576a^3b^7c^4e^2g - 1344a^4b^5c^5e^2g + 512a^5b^3c^6e^2g \\
& - 10a^2c^3e^2g * (-4ac - b^2)^9)^{(1/2)} - 152a^2b^{10}c^2f^2g + 1548a^3b^8c^3f^2g - 8064a^4b^6c^4f^2g \\
& + 22400a^5b^4c^5f^2g - 30720a^6b^2c^6f^2g + 6a^2b^{12}c^2f^2g + ab^2c^2f^2 * (-4ac - b^2)^9)^{(1/2)} \\
& - 51a^2b^2c^2g^2 * (-4ac - b^2)^9)^{(1/2)} + 18a^2b^2c^2d^2g * (-4ac - b^2)^9)^{(1/2)} + 2a^2b^2c^2e^2f * (-4ac - b^2)^9)^{(1/2)} \\
& - 6a^2b^3c^2f^2g * (-4ac - b^2)^9)^{(1/2)} - 6a^2b^2c^2e^2g * (-4ac - b^2)^9)^{(1/2)} + 44a^2b^2c^2f^2g * (-4ac - b^2)^9)^{(1/2)} \\
& / (32 * (4096a^7c^{11} + ab^{12}c^5 - 24a^2b^{10}c^6 + 240a^3b^8c^7 - 1280a^4b^6c^8 + 3840a^5b^4c^9 - 6144a^6b^2c^{10}))^{(1/2)} \\
& - (x * (9b^8g^2 - 8ac^7d^2 + 8a^2c^6e^2 + 10b^2c^6d^2 - 72a^3c^5f^2 + b^4c^4e^2 + 200a^4c^4g^2 \\
& + b^6c^2f^2 + 2a^2b^2c^5e^2 - 16a^2b^4c^3f^2 - 6b^7c^2f^2g + 74a^2b^2c^4f^2 + 481a^2b^4c^2g^2 - 718a^3b^2c^3g^2 - 114a^2b^6c^2g^2 \\
& - 48a^2c^6d^2f - 6b^3c^5d^2e - 6b^4c^4d^2f - 80a^3c^5e^2g + 18b^5c^3d^2g + 2b^5c^3e^2f - 6b^6c^2e^2g + 52a^2b^2c^5d^2f - 126a^2b^3c^4d^2g \\
& - 14a^2b^3c^4e^2f + 184a^2b^2c^5d^2g - 8a^2b^2c^5e^2f + 32a^2b^4c^3e^2g + 86a^2b^5c^2f^2g + 472a^3b^2c^4f^2g + 4a^2b^2c^4e^2g \\
& - 374a^2b^3c^3f^2g - 8a^2b^3c^6d^2e)) / (2 * (16a^2c^5 + b^4c^3 - 8a^2b^2c^4)) * ((768a^4b^2c^9d^2 - b^9c^5d^2 - c^5d^2 * (-4ac - b^2)^9)^{(1/2)} \\
& - 9a^2b^{13}g^2 - ab^9c^4e^2 + 768a^5b^2c^8e^2 + ac^4e^2 * (-4ac - b^2)^9)^{(1/2)} - ab^{11}c^2f^2 + 3840a^6b^2c^7f^2 + 9a^2b^4g^2 * (-4ac - b^2)^9)^{(1/2)} \\
& + 213a^2b^{11}c^2g^2 - 26880a^7b^2c^6g^2 + 96a^2b^5c^7d^2 - 512a^3b^3c^8d^2 + 96a^3b^5c^6e^2 - 512a^4b^3c^7e^2 + 27a^2b^9c^3f^2 - 288a^3b^7c^4f^2 \\
& + 1504a^4b^5c^5f^2 - 3840a^5b^3c^6f^2 - 9a^2c^3f^2 * (-4ac - b^2)^9)^{(1/2)} - 2077a^3b^9c^2g^2 + 10656a^4b^7c^3g^2 - 30240a^5b^5c^4g^2 \\
& + 44800a^6b^3c^5g^2 + 25a^3c^2g^2 * (-4ac - b^2)^9)^{(1/2)} - 1024a^5c^9d^2e + 5120a^6c^8d^2g - 3072a^6c^8e^2f + 15360a^7c^7f^2g \\
& + 12a^2b^8c^5d^2e + 6a^2b^9c^4d^2f + 3584a^5b^2c^8d^2f - 6a^2c^4d^2f * (-4ac - b^2)^9)^{(1/2)} - 18a^2b^{10}c^3d^2g - 2a^2b^{10}c^3e^2f \\
& + 6a^2b^{11}c^2e^2g + 1536a^6b^2c^7e^2g - 128a^2b^6c^6d^2e + 384a^3b^4c^7d^2e - 128a^2b^7c^5d^2f + 960a^3b^5c^6d^2f - 3072a^4b^3c^7d^2f \\
& + 324a^2b^8c^4d^2g + 36a^2b^8c^4e^2f - 2240a^3b^6c^5d^2g - 192a^3b^6c^5e^2f + 7296a^4b^4c^6d^2g + 128a^4b^4c^6e^2f - 10752a^5b^2c^7d^2g \\
& + 1536a^5b^2c^7e^2f - 98a^2b^9c^3e^2g + 576a^3b^7c^4e^2g - 1344a^4b^5c^5e^2g + 512a^5b^3c^6e^2g - 10a^2c^3e^2g * (-4ac - b^2)^9)^{(1/2)} \\
& - 152a^2b^{10}c^2f^2g + 1548a^3b^8c^3f^2g - 8064a^4b^6c^4f^2g + 22400a^5b^4c^5f^2g - 30720a^6b^2c^6f^2g + 6a^2b^{12}c^2f^2g + ab^2c^2f^2 * (-4ac - b^2)^9)^{(1/2)} \\
& - 51a^2b^2c^2g^2 * (-4ac - b^2)^9)^{(1/2)} + 18a^2b^2c^2d^2g * (-4ac - b^2)^9)^{(1/2)} + 2a^2b^2c^2e^2f * (-4ac - b^2)^9)^{(1/2)} - 6a^2b^3c^2f^2g * (-4ac - b^2)^9)^{(1/2)} \\
& - 6a^2b^2c^2e^2g * (-4ac - b^2)^9)^{(1/2)} + 44a^2b^2c^2f^2g * (-4ac - b^2)^9)^{(1/2)} / (32 * (4096a^7c^{11} + ab^{12}c^5 - 24a^2b^{10}c^6 + 240a^3b^8c^7 \\
& - 1280a^4b^6c^8 + 3840a^5b^4c^9 - 6144a^6b^2c^{10}))^{(1/2)} + (((10240a^5c^7g - 16b^7c^5d - 2048a^4c^8e - 768a^2b^3c^7d - 384a^2b^4c^6e \\
& + 1536a^3b^2c^7e + 192a^2b^5c^5f - 768a^3b^3c^6f - 736a^2b^6c^4g + 4224a^3b^4c^5g - 10752a^4b^2c^6g + 192a^2b^5c^6d + 1024a^3b^2c^8d \\
& + 32a^2b^6c^5e - 16a^2b^7c^4f + 1024a^4b^2c^7f + 48a^2b^8c^3g) / (8 * (64a^3c^6 - b^6c^3 + 12a^2b^4c^4 - 48a^2b^2c^5)) + (x * ((768a^4b^2c^9d^2 - b^9c^5d^2 - c^5d^2 * (-4ac - b^2)^9)^{(1/2)} \\
& - 9a^2b^{13}g^2 - ab^9c^4e^2 + 768a^5b^2c^8e^2 + ac^4e^2 * (-4ac - b^2)^9)^{(1/2)} - ab^{11}c^2f^2 + 3840a^6b^2c^7f^2 + 9a^2b^4g^2 * (-4ac - b^2)^9)^{(1/2)} \\
& + 213a^2b^{11}c^2g^2 - 26880a^7b^2c^6g^2 + 96a^2b^5c^7d^2 - 512a^3b^3c^8d^2 + 96a^3b^5c^6e^2 - 512a^4b^3c^7e^2 + 27a^2b^9c^3f^2 - 288a^3b^7c^4f^2 \\
& + 1504a^4b^5c^5f^2 - 3840a^5b^3c^6f^2 - 9a^2c^3f^2 * (-4ac - b^2)^9)^{(1/2)} - 2077a^3b^9c^2g^2 + 10656a^4b^7c^3g^2 - 30240a^5b^5c^4g^2 + 44800a^6b^3c^5g^2
\end{aligned}$$

$$\begin{aligned}
& 5g^2 + 25a^3c^2g^2(-4ac - b^2)^9)^{(1/2)} - 1024a^5c^9d^2e + 5120a^6c^8d^2g - 3072a^6c^8e^2f + 15360a^7c^7f^2g + 12ab^8c^5d^2e + 6ab^9c^4d^2f + 3584a^5b^3c^8d^2f - 6a^3c^4d^2f(-4ac - b^2)^9)^{(1/2)} - 18ab^10c^3d^2g - 2ab^10c^3e^2f + 6ab^11c^2e^2g + 1536a^6b^3c^7e^2g - 128a^2b^6c^6d^2e + 384a^3b^4c^7d^2e - 128a^2b^7c^5d^2f + 960a^3b^5c^6d^2f - 3072a^4b^3c^7d^2f + 324a^2b^8c^4d^2g + 36a^2b^8c^4e^2f - 2240a^3b^6c^5d^2g - 192a^3b^6c^5e^2f + 7296a^4b^4c^6d^2g + 128a^4b^4c^6e^2f - 10752a^5b^2c^7d^2g + 1536a^5b^2c^7e^2f - 98a^2b^9c^3e^2g + 576a^3b^7c^4e^2g - 1344a^4b^5c^5e^2g + 512a^5b^3c^6e^2g - 10a^2c^3e^2g(-4ac - b^2)^9)^{(1/2)} - 152a^2b^10c^2f^2g + 1548a^3b^8c^3f^2g - 8064a^4b^6c^4f^2g + 22400a^5b^4c^5f^2g - 30720a^6b^2c^6f^2g + 6ab^12c^2f^2(-4ac - b^2)^9)^{(1/2)} - 51a^2b^2c^2g^2(-4ac - b^2)^9)^{(1/2)} + 18ab^3c^3d^2g(-4ac - b^2)^9)^{(1/2)} + 2ab^3c^3e^2f(-4ac - b^2)^9)^{(1/2)} - 6ab^3c^3f^2g(-4ac - b^2)^9)^{(1/2)} - 6ab^2c^2e^2g(-4ac - b^2)^9)^{(1/2)} + 44a^2b^2c^2f^2g(-4ac - b^2)^9)^{(1/2)}/(32(4096a^7c^11 + ab^12c^5 - 24a^2b^10c^6 + 240a^3b^8c^7 - 1280a^4b^6c^8 + 3840a^5b^4c^9 - 6144a^6b^2c^10)))^{(1/2)}(16b^7c^5 - 192ab^5c^6 - 1024a^3b^3c^8 + 768a^2b^3c^7))^{(1/2)}(2(16a^2c^5 + b^4c^3 - 8ab^2c^4))^{(1/2)}((768a^4b^3c^9d^2 - b^9c^5d^2 - c^5d^2(-4ac - b^2)^9)^{(1/2)} - 9ab^13g^2 - ab^9c^4e^2 + 768a^5b^3c^8e^2 + ac^4e^2(-4ac - b^2)^9)^{(1/2)} - ab^11c^2f^2 + 3840a^6b^3c^7f^2 + 9ab^4g^2(-4ac - b^2)^9)^{(1/2)} + 213a^2b^11c^2g^2 - 26880a^7b^3c^6g^2 + 96a^2b^5c^7d^2 - 512a^3b^3c^8d^2 + 96a^3b^5c^6e^2 - 512a^4b^3c^7e^2 + 27a^2b^9c^3f^2 - 288a^3b^7c^4f^2 + 1504a^4b^5c^5f^2 - 3840a^5b^3c^6f^2 - 9a^2c^3f^2(-4ac - b^2)^9)^{(1/2)} - 2077a^3b^9c^2g^2 + 10656a^4b^7c^3g^2 - 30240a^5b^5c^4g^2 + 44800a^6b^3c^5g^2 + 25a^3c^2g^2(-4ac - b^2)^9)^{(1/2)} - 1024a^5c^9d^2e + 5120a^6c^8d^2g - 3072a^6c^8e^2f + 15360a^7c^7f^2g + 12ab^8c^5d^2e + 6ab^9c^4d^2f + 3584a^5b^3c^8d^2f - 6a^3c^4d^2f(-4ac - b^2)^9)^{(1/2)} - 18ab^10c^3d^2g - 2ab^10c^3e^2f + 6ab^11c^2e^2g + 1536a^6b^3c^7e^2g - 128a^2b^6c^6d^2e + 384a^3b^4c^7d^2e - 128a^2b^7c^5d^2f + 960a^3b^5c^6d^2f - 3072a^4b^3c^7d^2f + 324a^2b^8c^4d^2g + 36a^2b^8c^4e^2f - 2240a^3b^6c^5d^2g - 192a^3b^6c^5e^2f + 7296a^4b^4c^6d^2g + 128a^4b^4c^6e^2f - 10752a^5b^2c^7d^2g + 1536a^5b^2c^7e^2f - 98a^2b^9c^3e^2g + 576a^3b^7c^4e^2g - 1344a^4b^5c^5e^2g + 512a^5b^3c^6e^2g - 10a^2c^3e^2g(-4ac - b^2)^9)^{(1/2)} - 152a^2b^10c^2f^2g + 1548a^3b^8c^3f^2g - 8064a^4b^6c^4f^2g + 22400a^5b^4c^5f^2g - 30720a^6b^2c^6f^2g + 6ab^12c^2f^2(-4ac - b^2)^9)^{(1/2)} - 51a^2b^2c^2g^2(-4ac - b^2)^9)^{(1/2)} + 18ab^3c^3d^2g(-4ac - b^2)^9)^{(1/2)} + 2ab^3c^3e^2f(-4ac - b^2)^9)^{(1/2)} - 6ab^3c^3f^2g(-4ac - b^2)^9)^{(1/2)} - 6ab^2c^2e^2g(-4ac - b^2)^9)^{(1/2)} + 44a^2b^2c^2f^2g(-4ac - b^2)^9)^{(1/2)}/(32(4096a^7c^11 + ab^12c^5 - 24a^2b^10c^6 + 240a^3b^8c^7 - 1280a^4b^6c^8 + 3840a^5b^4c^9 - 6144a^6b^2c^10)))^{(1/2)} + (x(9b^8g^2 - 8a^3c^7d^2 + 8a^2c^6e^2 + 10b^2c^6d^2 - 72a^3c^5f^2 + b^4c^4e^2 + 200a^4c^4g^2 + b^6c^2f^2 + 2ab^2c^5e^2 - 16ab^4c^3f^2 - 6b^7c^2f^2g + 74a^2b^2c^4f^2 + 481a^2b^4c^2g^2 - 718a^3b^2c^3g^2 - 114ab^6c^2g^2 - 48a^2c^6d^2f - 6b^3c^5d^2e - 6b^4c^4d^2f - 80a^3c^5e^2g + 18b^5c^3d^2g + 2b^5c^3e^2f - 6b^6c^2e^2g + 52ab^2c^5d^2f - 126ab^3c^4d^2g - 14ab^3c^4e^2f + 184a^2b^3c^5d^2g - 8a^2b^3c^5e^2f + 32ab^4c^3e^2g + 86ab^5c^2f^2g + 472a^3b^3c^4f^2g + 4a^2b^2c^4e^2g - 374a^2b^3c^3f^2g - 8ab^3c^6d^2e))/2(16a^2c^5 + b^4c^3 - 8ab^2c^4))^{(1/2)}((768a^4b^3c^9d^2 - b^9c^5d^2 - c^5d^2(-4ac - b^2)^9)^{(1/2)} - 9ab^13g^2 - ab^9c^4e^2 + 768a^5b^3c^8e^2 + ac^4e^2(-4ac - b^2)^9)^{(1/2)} - ab^11c^2f^2 + 3840a^6b^3c^7f^2 + 9ab^4g^2(-4ac - b^2)^9)^{(1/2)} + 213a^2b^11c^2g^2 - 26880a^7b^3c^6g^2 + 96a^2b^5c^7d^2 - 512a^3b^3c^8d^2 + 96a^3b^5c^6e^2 - 512a^4b^3c^7e^2 + 27a^2b^9c^3f^2 - 288a^3b^7c^4f^2 + 1504a^4b^5c^5f^2 - 3840a^5b^3c^6f^2 - 9a^2c^3f^2(-4ac - b^2)^9)^{(1/2)} - 2077a^3b^9c^2g^2 + 10656a^4b^
\end{aligned}$$

$$\begin{aligned}
& ^7c^3g^2 - 30240a^5b^5c^4g^2 + 44800a^6b^3c^5g^2 + 25a^3c^2g^2 \\
& *(-(4ac - b^2)^9)^{(1/2)} - 1024a^5c^9d^2e + 5120a^6c^8d^2g - 3072a^6c^8e^2f + 15360a^7c^7f^2g + 12a^8b^8c^5d^2e + 6a^8b^9c^4d^2f + 3584a^5 \\
& *b^8c^8d^2f - 6a^8c^4d^2f*(-(4ac - b^2)^9)^{(1/2)} - 18a^8b^10c^3d^2g - 2a^8 \\
& *b^10c^3e^2f + 6a^8b^11c^2e^2g + 1536a^6b^7c^7e^2g - 128a^2b^6c^6d^2e + 384a^3b^4c^7d^2e - 128a^2b^7c^5d^2f + 960a^3b^5c^6d^2f - 3072a^4 \\
& *b^3c^7d^2f + 324a^2b^8c^4d^2g + 36a^2b^8c^4e^2f - 2240a^3b^6c^5d^2g - 192a^3b^6c^5e^2f + 7296a^4b^4c^6d^2g + 128a^4b^4c^6e^2f - \\
& 10752a^5b^2c^7d^2g + 1536a^5b^2c^7e^2f - 98a^2b^9c^3e^2g + 576a^3b^7c^4e^2g - 1344a^4b^5c^5e^2g + 512a^5b^3c^6e^2g - 10a^2c^3e^2g * \\
& (-(4ac - b^2)^9)^{(1/2)} - 152a^2b^10c^2f^2g + 1548a^3b^8c^3f^2g - 8064a^4b^6c^4f^2g + 22400a^5b^4c^5f^2g - 30720a^6b^2c^6f^2g + 6a^8b^12 \\
& *c^2f^2g + a^8b^2c^2f^2*(-(4ac - b^2)^9)^{(1/2)} - 51a^2b^2c^2g^2*(-(4ac - b^2)^9)^{(1/2)} + 18a^8b^3c^3d^2g*(-(4ac - b^2)^9)^{(1/2)} + 2a^8b^3c^3e^2 \\
& *f*(-(4ac - b^2)^9)^{(1/2)} - 6a^8b^3c^3f^2g*(-(4ac - b^2)^9)^{(1/2)} - 6a^8b^2c^2e^2g*(-(4ac - b^2)^9)^{(1/2)} + 44a^2b^2c^2f^2g*(-(4ac - b^2)^9)^{(1/2)} \\
&)/(32*(4096a^7c^11 + a^8b^12c^5 - 24a^2b^10c^6 + 240a^3b^8c^7 - 1280a^4b^6c^8 + 3840a^5b^4c^9 - 6144a^6b^2c^10)))^{(1/2)} - (8a^8c^7d^3 + 9b^8d^2g^2 + 6b^2c^6d^3 - 63a^3b^5g^3 + 216a^4c^4f^3 - 3a^8b^3c^4e^3 - 4a^2b^2c^5e^3 + 8a^2c^6d^2e^2 + 573a^4b^3c^3g^3 - 1300a^5b^2c^2g^3 + 72a^2c^6d^2f + 216a^3c^5d^2f^2 - 5b^3c^5d^2e + b^4c^4d^2e^2 + 24a^3c^5e^2f + 200a^4c^4d^2g^2 - 5b^4c^4d^2f + b^6c^2d^2f^2 + 45a^2b^6f^2g^2 + 15b^5c^3d^2g + 600a^5c^3f^2g^2 + 5a^2b^4c^2f^3 - 66a^3b^2c^3f^3 - 27a^8b^7e^2g^2 - 28a^8b^6c^6d^2e - 78a^8b^6c^6d^2g^2 - 80a^3c^5d^2e^2g + 2b^5c^3d^2e^2f - 6b^6c^2d^2e^2g - 240a^4c^4e^2f^2g + 18a^8b^2c^5d^2e^2 + 26a^8b^2c^5d^2f^2 - 12a^8b^4c^3d^2f^2 - 53a^8b^3c^4d^2g - 6a^8b^4c^3e^2f - 3a^8b^5c^2e^2f^2 - 76a^2b^8c^5d^2g - 204a^3b^8c^4e^2f^2 + 18a^8b^5c^2e^2g + 279a^2b^5c^2e^2g^2 - 12a^3b^8c^4e^2g + 420a^4b^8c^3e^2g^2 - 30a^2b^5c^2f^2g - 402a^3b^4c^4f^2g^2 - 924a^4b^8c^3f^2g - 6b^7c^4d^2f^2g + 2a^2b^2c^4d^2f^2 + 42a^2b^2c^4e^2f + 51a^2b^3c^3e^2f^2 + 133a^2b^4c^2d^2g^2 + 114a^3b^2c^3d^2g^2 - 81a^2b^3c^3e^2g - 801a^3b^3c^2e^2g^2 + 339a^3b^3c^2f^2g + 762a^4b^2c^2f^2g^2 + 18a^8b^6c^2e^2f^2g + 6a^8b^3c^4d^2e^2f - 152a^2b^2c^5d^2e^2f - 28a^8b^4c^3d^2e^2g + 62a^8b^5c^2d^2f^2g - 536a^3b^4c^4d^2f^2g + 276a^2b^2c^4d^2e^2g - 42a^2b^3c^3d^2f^2g - 246a^2b^4c^2e^2f^2g + 804a^3b^2c^3e^2f^2g)/(4*(64a^3c^6 - b^6c^3 + 12a^8b^4c^4 - 48a^2b^2c^5))) * ((768a^4b^8c^9d^2 - b^9c^5d^2 - c^5d^2*(-(4ac - b^2)^9)^{(1/2)} - 9a^8b^13g^2 - a^8b^9c^4e^2 + 768a^5b^8c^8e^2 + a^8c^4e^2*(-(4ac - b^2)^9)^{(1/2)} - a^8b^11c^2f^2 + 3840a^6b^7c^7f^2 + 9a^8b^4g^2*(-(4ac - b^2)^9)^{(1/2)} + 213a^2b^11c^2g^2 - 26880a^7b^8c^6g^2 + 96a^2b^5c^7d^2 - 512a^3b^3c^8d^2 + 96a^3b^5c^6e^2 - 512a^4b^3c^7e^2 + 27a^2b^9c^3f^2 - 288a^3b^7c^4f^2 + 1504a^4b^5c^5f^2 - 3840a^5b^3c^6f^2 - 9a^2c^3f^2*(-(4ac - b^2)^9)^{(1/2)} - 2077a^3b^9c^2g^2 + 10656a^4b^7c^3g^2 - 30240a^5b^5c^4g^2 + 44800a^6b^3c^5g^2 + 25a^3c^2g^2*(-(4ac - b^2)^9)^{(1/2)} - 1024a^5c^9d^2e + 5120a^6c^8d^2g - 3072a^6c^8e^2f + 15360a^7c^7f^2g + 12a^8b^8c^5d^2e + 6a^8b^9c^4d^2f + 3584a^5b^8c^8d^2f - 6a^8c^4d^2f*(-(4ac - b^2)^9)^{(1/2)} - 18a^8b^10c^3d^2g - 2a^8b^10c^3e^2f + 6a^8b^11c^2e^2g + 1536a^6b^7c^7e^2g - 128a^2b^6c^6d^2e + 384a^3b^4c^7d^2e - 128a^2b^7c^5d^2f + 960a^3b^5c^6d^2f - 3072a^4b^3c^7d^2f + 324a^2b^8c^4d^2g + 36a^2b^8c^4e^2f - 2240a^3b^6c^5d^2g - 192a^3b^6c^5e^2f + 7296a^4b^4c^6d^2g + 128a^4b^4c^6e^2f - 10752a^5b^2c^7d^2g + 1536a^5b^2c^7e^2f - 98a^2b^9c^3e^2g + 576a^3b^7c^4e^2g - 1344a^4b^5c^5e^2g + 512a^5b^3c^6e^2g - 10a^2c^3e^2g * (-(4ac - b^2)^9)^{(1/2)} - 152a^2b^10c^2f^2g + 1548a^3b^8c^3f^2g - 8064a^4b^6c^4f^2g + 22400a^5b^4c^5f^2g - 30720a^6b^2c^6f^2g + 6a^8b^12c^2f^2g + a^8b^2c^2f^2*(-(4ac - b^2)^9)^{(1/2)} - 51a^2b^2c^2g^2*(-(4ac - b^2)^9)^{(1/2)} + 18a^8b^3c^3d^2g*(-(4ac - b^2)^9)^{(1/2)} + 2a^8b^3c^3e^2f*(-(4ac - b^2)^9)^{(1/2)} - 6a^8b^3c^3f^2g*(-(4ac - b^2)^9)^{(1/2)} - 6a^8b^2c^2e^2g*(-(4ac - b^2)^9)^{(1/2)} + 44a^2b^2c^2f^2g*(-(4ac - b^2)^9)^{(1/2)}
\end{aligned}$$

$$g \cdot (-4ac - b^2)^{9/2} / (32(4096a^7c^{11} + ab^{12}c^5 - 24a^2b^{10}c^6 + 240a^3b^8c^7 - 1280a^4b^6c^8 + 3840a^5b^4c^9 - 6144a^6b^2c^{10}))^{1/2} + (gx)/c^2$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(g*x**6+f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)

[Out] Timed out

$$3.126 \quad \int \frac{d+ex^2+fx^4+gx^6}{(a+bx^2+cx^4)^2} dx$$

Optimal. Leaf size=449

$$\frac{x \left(x^2 (-ab^2g + bc(af + cd) - 2ac(ce - ag)) + c \left(-\frac{ab(ag+ce)}{c} - 2a(cd - af) + b^2d \right) \right)}{2ac(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right) \left(\frac{ab^2g}{c} + \dots \right)}{2ac(b^2 - 4ac)(a + bx^2 + cx^4)}$$

Rubi [A] time = 2.87, antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 32, number of rules / integrand size = 0.094, Rules used = {1678, 1166, 205}

$$\frac{\tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right) \left(\frac{b^2(ad - af) - ab^3g + 4bc(2ac + c^2) - 4ac^2(ef + ckd)}{c\sqrt{b^2 - 4ac}} + \frac{ab^2g}{c} + b(af + cd) - 2a(3ag + ce) \right) + \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b + \sqrt{b^2 - 4ac}}} \right) \left(\frac{b^2(ad - af) - ab^3g + 4bc(2ac + c^2) - 4ac^2(ef + ckd)}{c\sqrt{b^2 - 4ac}} + \frac{ab^2g}{c} + b(af + cd) - 2a(3ag + ce) \right) + x^2 \left(-ab^2g + bc(af + cd) - 2ac(ce - ag) \right) + c \left(-\frac{ab(ag+ce)}{c} - 2a(cd - af) + b^2d \right)}{2\sqrt{2}a\sqrt{c} \sqrt{b^2 - 4ac} \sqrt{b - \sqrt{b^2 - 4ac}} + 2\sqrt{2}a\sqrt{c} \sqrt{b^2 - 4ac} \sqrt{b + \sqrt{b^2 - 4ac}} + 2ac(b^2 - 4ac)(a + bx^2 + cx^4)}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2 + f*x^4 + g*x^6)/(a + b*x^2 + c*x^4)^2,x]

[Out] (x*(c*(b^2*d - 2*a*(c*d - a*f) - (a*b*(c*e + a*g))/c) + (b*c*(c*d + a*f) - a*b^2*g - 2*a*c*(c*e - a*g))*x^2)/(2*a*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4) + ((b*(c*d + a*f) + (a*b^2*g)/c - 2*a*(c*e + 3*a*g) + (b^2*c*(c*d - a*f) - 4*a*c^2*(3*c*d + a*f) - a*b^3*g + 4*a*b*c*(c*e + 2*a*g))/(c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(2*Sqrt[2]*a*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b*(c*d + a*f) + (a*b^2*g)/c - 2*a*(c*e + 3*a*g) - (b^2*c*(c*d - a*f) - 4*a*c^2*(3*c*d + a*f) - a*b^3*g + 4*a*b*c*(c*e + 2*a*g))/(c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(2*Sqrt[2]*a*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1678

Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := With[{d = Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]

Rubi steps

$$\int \frac{d + ex^2 + fx^4 + gx^6}{(a + bx^2 + cx^4)^2} dx = \frac{x \left(c \left(b^2d - 2a(cd - af) - \frac{ab(ce+ag)}{c} \right) + (bc(cd + af) - ab^2g - 2ac(ce - ag)) x^2 \right)}{2ac(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{\int \dots}{\dots}$$

$$= \frac{x \left(c \left(b^2d - 2a(cd - af) - \frac{ab(ce+ag)}{c} \right) + (bc(cd + af) - ab^2g - 2ac(ce - ag)) x^2 \right)}{2ac(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{(b(\dots))}{\dots}$$

$$= \frac{x \left(c \left(b^2d - 2a(cd - af) - \frac{ab(ce+ag)}{c} \right) + (bc(cd + af) - ab^2g - 2ac(ce - ag)) x^2 \right)}{2ac(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{(b(\dots))}{\dots}$$

Mathematica [A] time = 1.65, size = 512, normalized size = 1.14

$$\frac{2\sqrt{c}((b^2-cg)^2+ac^2+2a^2f^2+2cd-2ag^2)+2a(c(f+g^2)-c(f+g^2))}{(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}cx}{\sqrt{b^2-4ac}}\right)\left(b\left(\frac{ab^2g+cd\sqrt{b^2-4ac}+af\sqrt{b^2-4ac}+4ac}{\sqrt{b^2-4ac}}\right)-ab^2g-2a\left(\frac{c\sqrt{b^2-4ac}+3ag\sqrt{b^2-4ac}+2af+4c^2}{\sqrt{b^2-4ac}}\right)+b^2\left(\frac{ag\sqrt{b^2-4ac}-af+g^2}{\sqrt{b^2-4ac}}\right)\right)}{(b^2-4ac)^{3/2}\sqrt{b^2-4ac}} + \frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}cx}{\sqrt{b^2-4ac}}\right)\left(b\left(\frac{-ab^2g+cd\sqrt{b^2-4ac}+af\sqrt{b^2-4ac}-4ac}{\sqrt{b^2-4ac}}\right)+ab^2g+2a\left(\frac{-c\sqrt{b^2-4ac}-3ag\sqrt{b^2-4ac}+2af+4c^2}{\sqrt{b^2-4ac}}\right)+b^2\left(\frac{ag\sqrt{b^2-4ac}+af+g^2-cd}{\sqrt{b^2-4ac}}\right)\right)}{(b^2-4ac)^{3/2}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2 + f*x^4 + g*x^6)/(a + b*x^2 + c*x^4)^2,x]
[Out] ((2*Sqrt[c]*x*(b*(-(a*c*e) - a^2*g + c^2*d*x^2 + a*c*f*x^2) + b^2*(c*d - a*g*x^2) + 2*a*c*(-(c*(d + e*x^2)) + a*(f + g*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(-(a*b^3*g) + b*c*(c*Sqrt[b^2 - 4*a*c]*d + 4*a*c*e + a*Sqrt[b^2 - 4*a*c]*f + 8*a^2*g) + b^2*(c^2*d - a*c*f + a*Sqrt[b^2 - 4*a*c]*g) - 2*a*c*(6*c^2*d + c*Sqrt[b^2 - 4*a*c]*e + 2*a*c*f + 3*a*Sqrt[b^2 - 4*a*c]*g))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(a*b^3*g + b*c*(c*Sqrt[b^2 - 4*a*c]*d - 4*a*c*e + a*Sqrt[b^2 - 4*a*c]*f - 8*a^2*g) + 2*a*c*(6*c^2*d - c*Sqrt[b^2 - 4*a*c]*e + 2*a*c*f - 3*a*Sqrt[b^2 - 4*a*c]*g) + b^2*(-(c^2*d) + a*c*f + a*Sqrt[b^2 - 4*a*c]*g))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])/(4*a*c^(3/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex^2 + fx^4 + gx^6}{(a + bx^2 + cx^4)^2} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x^2 + f*x^4 + g*x^6)/(a + b*x^2 + c*x^4)^2,x]
[Out] IntegrateAlgebraic[(d + e*x^2 + f*x^4 + g*x^6)/(a + b*x^2 + c*x^4)^2, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
[Out] Timed out
```

giac [B] time = 8.51, size = 8913, normalized size = 19.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
[Out] 1/2*(b*c^2*d*x^3 + a*b*c*f*x^3 - a*b^2*g*x^3 + 2*a^2*c*g*x^3 - 2*a*c^2*x^3*
e + b^2*c*d*x - 2*a*c^2*d*x + 2*a^2*c*f*x - a^2*b*g*x - a*b*c*x*e)/((c*x^4
+ b*x^2 + a)*(a*b^2*c - 4*a^2*c^2)) + 1/16*((2*b^3*c^4 - 8*a*b*c^5 - sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + 2*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^4 - 2*(b^2 - 4*a*c)*b*c^4)*(a*b^2*c
- 4*a^2*c^2)^2*d + (2*a*b^3*c^3 - 8*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*a*b*c^3)*(a*b^2*c - 4*a^2*c^2)^
2*f + (2*a*b^4*c^2 - 20*a^2*b^2*c^3 + 48*a^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^3*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b^2*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^2*c^3 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + 12*(b^2 - 4*a*c)*a^2*c^3)*(a
*b^2*c - 4*a^2*c^2)^2*g - 2*(2*a*b^2*c^4 - 8*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*a*c^4)*(a*b^2*c - 4*a^2*c^2)
^2*e + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3 - 14*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*b^5*c^4 - 2*a*b^6*c^4 + 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^3*b^2*c^5 + 20*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 +
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 + 28*a^2*b^4*c^5 - 96*sq
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^6 - 48*sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^3*b*c^6 - 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*
b^2*c^6 - 128*a^3*b^2*c^6 + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*
c^7 + 192*a^4*c^7 + 2*(b^2 - 4*a*c)*a*b^4*c^4 - 20*(b^2 - 4*a*c)*a^2*b^2*c^
5 + 48*(b^2 - 4*a*c)*a^3*c^6)*d*abs(a*b^2*c - 4*a^2*c^2) - 4*(sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^4*b^2*c^4 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^4
- 2*a^3*b^4*c^4 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*c^5 + 8*s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^5 + sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^3*b^2*c^5 + 16*a^4*b^2*c^5 - 4*sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^4*c^6 - 32*a^5*c^6 + 2*(b^2 - 4*a*c)*a^3*b^2*c^4 - 8*(b^2 -
4*a*c)*a^4*c^5)*f*abs(a*b^2*c - 4*a^2*c^2) + 2*(sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^3*b^5*c^2 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b
^3*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 - 2*a^3*b^5*
c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^4 + 8*sqrt(2)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^3*b^3*c^4 + 16*a^4*b^3*c^4 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^4*b*c^5 - 32*a^5*b*c^5 + 2*(b^2 - 4*a*c)*a^3*b^3*c^3 - 8*(b^2 - 4*a*
c)*a^4*b*c^4)*g*abs(a*b^2*c - 4*a^2*c^2) + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^2*b^5*c^3 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*
c^4 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 - 2*a^2*b^5*c^4
+ 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^5 + 8*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^5 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^2*b^3*c^5 + 16*a^3*b^3*c^5 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^3*b*c^6 - 32*a^4*b*c^6 + 2*(b^2 - 4*a*c)*a^2*b^3*c^4 - 8*(b^2 - 4*a*c)*
a^3*b*c^5)*abs(a*b^2*c - 4*a^2*c^2)*e + (2*a^2*b^7*c^6 - 40*a^3*b^5*c^7 + 2
```

$$\begin{aligned}
& 24a^4b^3c^8 - 384a^5b^4c^9 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}} \\
& \sqrt{b^2 - 4ac} a^2b^7c^4 + 20\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}} \\
& \sqrt{b^2 - 4ac} a^3b^5c^5 + 2\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}} \\
& \sqrt{b^2 - 4ac} a^2b^6c^5 - 112\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}} \\
& \sqrt{b^2 - 4ac} a^4b^3c^6 - 32\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}} \\
& \sqrt{b^2 - 4ac} a^3b^4c^6 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}} \\
& \sqrt{b^2 - 4ac} a^2b^5c^6 + 192\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}} \\
& \sqrt{b^2 - 4ac} a^5b^2c^7 + 96\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}} \\
& \sqrt{b^2 - 4ac} a^4b^2c^7 + 16\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}} \\
& \sqrt{b^2 - 4ac} a^3b^3c^7 - 48\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}} \\
& \sqrt{b^2 - 4ac} a^4b^4c^8 - 2(b^2 - 4ac)a^2b^5c^6 + 32(b^2 - 4ac)a^3b^3c^7 \\
& - 96(b^2 - 4ac)a^4b^4c^8)d - (2a^3b^7c^5 - 8a^4b^5c^6 - 32a^5b^3c^7 \\
& + 128a^6b^4c^8 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}} \\
& \sqrt{b^2 - 4ac} a^3b^7c^3 + 4\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}} \\
& \sqrt{b^2 - 4ac} a^4b^5c^4 + 2\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}} \\
& \sqrt{b^2 - 4ac} a^3b^6c^4 + 16\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}} \\
& \sqrt{b^2 - 4ac} a^5b^3c^5 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}} \\
& \sqrt{b^2 - 4ac} a^3b^5c^5 - 64\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}} \\
& \sqrt{b^2 - 4ac} a^6b^2c^6 - 32\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}} \\
& \sqrt{b^2 - 4ac} a^5b^2c^6 + 16\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}} \\
& \sqrt{b^2 - 4ac} a^5b^4c^7 - 2(b^2 - 4ac)a^3b^5c^5 + 32(b^2 - 4ac)a^5b^4c^7 \\
&)f - (2a^3b^8c^4 - 32a^4b^6c^5 + 160a^5b^4c^6 - 256a^6b^2c^7 \\
& - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}} \\
& \sqrt{b^2 - 4ac} a^3b^8c^2 + 16\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}} \\
& \sqrt{b^2 - 4ac} a^4b^6c^3 + 2\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}} \\
& \sqrt{b^2 - 4ac} a^3b^7c^3 - 80\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}} \\
& \sqrt{b^2 - 4ac} a^5b^4c^4 - 24\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}} \\
& \sqrt{b^2 - 4ac} a^4b^5c^4 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}} \\
& \sqrt{b^2 - 4ac} a^3b^6c^4 + 128\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}} \\
& \sqrt{b^2 - 4ac} a^6b^2c^5 + 64\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}} \\
& \sqrt{b^2 - 4ac} a^5b^3c^5 + 12\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}} \\
& \sqrt{b^2 - 4ac} a^4b^4c^5 - 32\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}} \\
& \sqrt{b^2 - 4ac} a^5b^2c^6 - 2(b^2 - 4ac)a^3b^6c^4 + 24(b^2 - 4ac)a^4b^4c^5 \\
& - 64(b^2 - 4ac)a^5b^2c^6)g + 4(2a^3b^6c^6 - 16a^4b^4c^7 + 32a^5b^2c^8 \\
& - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}} \\
& \sqrt{b^2 - 4ac} a^3b^6c^4 + 8\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}} \\
& \sqrt{b^2 - 4ac} a^4b^4c^5 + 2\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}} \\
& \sqrt{b^2 - 4ac} a^3b^5c^5 - 16\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}} \\
& \sqrt{b^2 - 4ac} a^5b^2c^6 - 8\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}} \\
& \sqrt{b^2 - 4ac} a^4b^3c^6 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}} \\
& \sqrt{b^2 - 4ac} a^3b^4c^6 + 4\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}} \\
& \sqrt{b^2 - 4ac} a^4b^2c^7 - 2(b^2 - 4ac)a^3b^4c^6 + 8(b^2 - 4ac)a^4b^2c^7 \\
&)e) \arctan(2\sqrt{1/2}x/\sqrt{(ab^3c - 4a^2b^2c^2 + \sqrt{(ab^3c - 4a^2b^2c^2)^2 - 4(a^2b^2c - 4a^3c^2)(ab^2c^2 - 4a^2c^3)})}) \\
& /(\sqrt{(ab^2c^2 - 4a^2c^3)}) / ((a^3b^6c^3 - 12a^4b^4c^4 - 2a^3b^5c^4 + 48a^5b^2c^5 + 16a^4b^3c^5 + a^3b^4c^5 \\
& - 64a^6c^6 - 32a^5b^4c^6 - 8a^4b^2c^6 + 16a^5c^7) \operatorname{abs}(ab^2c - 4a^2c^2) \operatorname{abs}(c)) - 1/16((2b^3c^4 - 8ab^3c^5 - \sqrt{2}\sqrt{b^2 - 4ac}) \\
& \sqrt{bc - \sqrt{b^2 - 4ac}} \\
& \sqrt{b^2 - 4ac} a^3b^3c^2 + 4\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}} \\
& \sqrt{bc - \sqrt{b^2 - 4ac}} \\
& \sqrt{b^2 - 4ac} a^2b^2c^3 + 2\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}} \\
& \sqrt{bc - \sqrt{b^2 - 4ac}} \\
& \sqrt{b^2 - 4ac} a^2b^2c^3 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}} \\
& \sqrt{bc - \sqrt{b^2 - 4ac}} \\
& \sqrt{b^2 - 4ac} a^2b^2c^3 + 2\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}} \\
& \sqrt{bc - \sqrt{b^2 - 4ac}} \\
& \sqrt{b^2 - 4ac} a^2b^2c^3 - 2(b^2 - 4ac)ab^2c^3 + 4\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}} \\
& \sqrt{bc - \sqrt{b^2 - 4ac}} \\
& \sqrt{b^2 - 4ac} a^2b^2c^3 + 2\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}} \\
& \sqrt{bc - \sqrt{b^2 - 4ac}} \\
& \sqrt{b^2 - 4ac} a^2b^2c^3 - 2(b^2 - 4ac)ab^2c^3 + 4\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}} \\
& \sqrt{bc - \sqrt{b^2 - 4ac}} \\
& \sqrt{b^2 - 4ac} a^2b^2c^3 - 20a^2b^2c^3 + 48a^3c^4 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}} \\
& \sqrt{bc - \sqrt{b^2 - 4ac}} \\
& \sqrt{b^2 - 4ac} a^2b^2c^3 + 10\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}} \\
& \sqrt{bc - \sqrt{b^2 - 4ac}} \\
& \sqrt{b^2 - 4ac} a^2b^2c^3 + 2\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}} \\
& \sqrt{bc - \sqrt{b^2 - 4ac}}
\end{aligned}$$

$$\begin{aligned}
& *a*c)*c)*a*b^3*c - 24*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}} \\
&)*c)*a^3*c^2 - 12*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c) \\
& *a^2*b*c^2 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^2 \\
& *c^2 + 6*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*c^3 \\
& - 2*(b^2 - 4*a*c)*a*b^2*c^2 + 12*(b^2 - 4*a*c)*a^2*c^3)*(a*b^2*c - 4*a^2*c \\
& ^2)^2*g - 2*(2*a*b^2*c^4 - 8*a^2*c^5 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \\
& \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c^2 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}} \\
& *c)*a^2*c^3 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b*c^3 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}} \\
& *c)*a*c^4 - 2*(b^2 - 4*a*c)*a*c^4)*(a*b^2*c - 4*a^2*c^2)^2*e - 2*(\sqrt{2} \\
&)*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^6*c^3 - 14*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b^4*c^4 - 2*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^5 \\
& *c^4 + 2*a*b^6*c^4 + 64*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*b^2*c^5 \\
& + 20*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b^3*c^5 + \sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^4*c^5 - 28*a^2*b^4*c^5 - 96*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^4*c^6 - 48*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)* \\
& a^3*b*c^6 - 10*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b^2*c^6 + 128*a^3 \\
& *b^2*c^6 + 24*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*c^7 - 192*a^4*c^7 \\
& - 2*(b^2 - 4*a*c)*a*b^4*c^4 + 20*(b^2 - 4*a*c)*a^2*b^2*c^5 - 48*(b^2 - 4* \\
& a*c)*a^3*c^6)*d*abs(a*b^2*c - 4*a^2*c^2) + 4*(\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}} \\
& *c)*a^3*b^4*c^3 - 8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^4*b^2* \\
& c^4 - 2*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*b^3*c^4 + 2*a^3*b^4*c^4 \\
& + 16*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^5*c^5 + 8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^4*b*c^5 + \sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)* \\
& a^3*b^2*c^5 - 16*a^4*b^2*c^5 - 4*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^4 \\
& *c^6 + 32*a^5*c^6 - 2*(b^2 - 4*a*c)*a^3*b^2*c^4 + 8*(b^2 - 4*a*c)*a^4*c^5) \\
& *f*abs(a*b^2*c - 4*a^2*c^2) - 2*(\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3 \\
& *b^5*c^2 - 8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^4*b^3*c^3 - 2*\sqrt{2} \\
& *\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*b^4*c^3 + 2*a^3*b^5*c^3 + 16*\sqrt{2} \\
& *\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^5*b*c^4 + 8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4 \\
& *a*c}}*c)*a^4*b^2*c^4 + \sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*b^3*c^4 \\
& ^4 - 16*a^4*b^3*c^4 - 4*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^4*b*c^5 + \\
& 32*a^5*b*c^5 - 2*(b^2 - 4*a*c)*a^3*b^3*c^3 + 8*(b^2 - 4*a*c)*a^4*b*c^4)*g* \\
& abs(a*b^2*c - 4*a^2*c^2) - 2*(\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b \\
& ^5*c^3 - 8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*b^3*c^4 - 2*\sqrt{2} \\
& *\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b^4*c^4 + 2*a^2*b^5*c^4 + 16*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4 \\
& *a*c}}*c)*a^4*b*c^5 + 8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*b^2*c^5 + \sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b^3*c^5 \\
& - 16*a^3*b^3*c^5 - 4*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*b*c^6 + 32 \\
& *a^4*b*c^6 - 2*(b^2 - 4*a*c)*a^2*b^3*c^4 + 8*(b^2 - 4*a*c)*a^3*b*c^5)*abs(a \\
& *b^2*c - 4*a^2*c^2)*e + (2*a^2*b^7*c^6 - 40*a^3*b^5*c^7 + 224*a^4*b^3*c^8 - \\
& 384*a^5*b*c^9 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)* \\
& a^2*b^7*c^4 + 20*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)* \\
& a^3*b^5*c^5 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2 \\
& *b^6*c^5 - 112*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)* \\
& a^4*b^3*c^6 - 32*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)* \\
& a^3*b^4*c^6 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2 \\
& *b^5*c^6 + 192*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^5 \\
& *b*c^7 + 96*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^4* \\
& b^2*c^7 + 16*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3* \\
& b^3*c^7 - 48*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^4* \\
& b*c^8 - 2*(b^2 - 4*a*c)*a^2*b^5*c^6 + 32*(b^2 - 4*a*c)*a^3*b^3*c^7 - 96*(b^2 \\
& - 4*a*c)*a^4*b*c^8)*d - (2*a^3*b^7*c^5 - 8*a^4*b^5*c^6 - 32*a^5*b^3*c^7 + \\
& 128*a^6*b*c^8 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)* \\
& a^3*b^7*c^3 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^4 \\
& *b^5*c^4 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3 \\
& *b^6*c^4 + 16*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^5 \\
& *b^3*c^5 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*b \\
& ^5*c^5 - 64*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^6*b
\end{aligned}$$

```

*c^6 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^2
*c^6 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c
^7 - 2*(b^2 - 4*a*c)*a^3*b^5*c^5 + 32*(b^2 - 4*a*c)*a^5*b*c^7)*f - (2*a^3*b
^8*c^4 - 32*a^4*b^6*c^5 + 160*a^5*b^4*c^6 - 256*a^6*b^2*c^7 - sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^8*c^2 + 16*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^3 + 2*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^7*c^3 - 80*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^4 - 24*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^4 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c^4 + 128*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^5 + 64*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^5 + 12*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^5 - 32*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^6 - 2*(b^2 - 4*a*c)*a^3
*b^6*c^4 + 24*(b^2 - 4*a*c)*a^4*b^4*c^5 - 64*(b^2 - 4*a*c)*a^5*b^2*c^6)*g +
4*(2*a^3*b^6*c^6 - 16*a^4*b^4*c^7 + 32*a^5*b^2*c^8 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c^4 + 8*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^5 - 16*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^6 - 8*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^6 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^7 - 2*(b^2 - 4*a*c)*a^3*b^4*c^6 + 8
*(b^2 - 4*a*c)*a^4*b^2*c^7)*e)*arctan(2*sqrt(1/2)*x/sqrt((a*b^3*c - 4*a^2*b
*c^2 - sqrt((a*b^3*c - 4*a^2*b*c^2)^2 - 4*(a^2*b^2*c - 4*a^3*c^2)*(a*b^2*c^
2 - 4*a^2*c^3)))/(a*b^2*c^2 - 4*a^2*c^3)))/(a^3*b^6*c^3 - 12*a^4*b^4*c^4 -
2*a^3*b^5*c^4 + 48*a^5*b^2*c^5 + 16*a^4*b^3*c^5 + a^3*b^4*c^5 - 64*a^6*c^6
- 32*a^5*b*c^6 - 8*a^4*b^2*c^6 + 16*a^5*c^7)*abs(a*b^2*c - 4*a^2*c^2)*abs(
c))

```

maple [B] time = 0.05, size = 1760, normalized size = 3.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x)$

[Out]
$$-3/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c^2*d*arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)+(-1/2/a*(2*a^2*c*g-a*b^2*g+a*b*c*f-2*a*c^2*e+b*c^2*d)/(4*a*c-b^2)/c*x^3+1/2*(a^2*b*g-2*a^2*c*f+a*b*c*e+2*a*c^2*d-b^2*c*d)/a/c/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+1/4/a/(4*a*c-b^2)*c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^2*d+1/4/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}/a*b^2*c*d*arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)-1/4/(4*a*c-b^2)/c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^3*g+2*a/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^3*g+1/4/(4*a*c-b^2)/c*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^2*g-1/4/(4*a*c-b^2)/c*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^2*g-3/2*a/(4*a*c-b^2)*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*g+3/2*a/(4*a*c-b^2)*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*g-1/4/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^2*f-3/(4*a*c-b^2)*c^2/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x$$

$$\begin{aligned} &) * d - 1/4 / (4 * a * c - b^2) / (-4 * a * c + b^2)^{(1/2)} * 2^{(1/2)} / ((b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * b^2 * f * \arctan(2^{(1/2)} / ((b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) + 2 * a / (4 * a * c - b^2) / (-4 * a * c + b^2)^{(1/2)} * 2^{(1/2)} / ((b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * \arctan(2^{(1/2)} / ((b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * b * g - 1/2 / (4 * a * c - b^2) * c * 2^{(1/2)} / ((-b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctanh}(2^{(1/2)} / ((-b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * e - 1/4 / (4 * a * c - b^2) * 2^{(1/2)} / ((b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * b * f * \arctan(2^{(1/2)} / ((b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) + 1/4 * a / (4 * a * c - b^2) * c * 2^{(1/2)} / ((-b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctanh}(2^{(1/2)} / ((-b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * b * d - a / (4 * a * c - b^2) * c / (-4 * a * c + b^2)^{(1/2)} * 2^{(1/2)} / ((-b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctanh}(2^{(1/2)} / ((-b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * f + 1 / (4 * a * c - b^2) * c / (-4 * a * c + b^2)^{(1/2)} * 2^{(1/2)} / ((-b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctanh}(2^{(1/2)} / ((-b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * b * e - 1/4 / (4 * a * c - b^2) * 2^{(1/2)} / ((b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} / a * b * c * d * \arctan(2^{(1/2)} / ((b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) - 1 / (4 * a * c - b^2) / (-4 * a * c + b^2)^{(1/2)} * 2^{(1/2)} / ((b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * a * c * f * \arctan(2^{(1/2)} / ((b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) + 1 / (4 * a * c - b^2) / (-4 * a * c + b^2)^{(1/2)} * 2^{(1/2)} / ((b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * b * c * e * \arctan(2^{(1/2)} / ((b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) + 1/4 / (4 * a * c - b^2) * 2^{(1/2)} / ((-b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctanh}(2^{(1/2)} / ((-b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * b * f + 1/2 / (4 * a * c - b^2) * 2^{(1/2)} / ((b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * c * e * \arctan(2^{(1/2)} / ((b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(bc^2d - 2ac^2e + abcf - (ab^2 - 2a^2c)g)x^3 - (abce - 2a^2cf + a^2bg - (b^2c - 2ac^2)d)x - \int \frac{abce - 2a^2cf + a^2bg + (bc^2d - 2ac^2e + abcf + (ab^2 - 6a^2c)g)x^2 + (b^2c - 6ac^2)d}{c^4 + bx^2 + a} dx}{2(a^2b^2c - 4a^3c^2 + (ab^2c^2 - 4a^2c^3)x^4 + (ab^3c - 4a^2bc^2)x^2)} - \frac{1}{2(ab^2c - 4a^2c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")

[Out] $1/2 * ((b * c^2 * d - 2 * a * c^2 * e + a * b * c * f - (a * b^2 - 2 * a^2 * c) * g) * x^3 - (a * b * c * e - 2 * a^2 * c * f + a^2 * b * g - (b^2 * c - 2 * a * c^2) * d) * x) / (a^2 * b^2 * c - 4 * a^3 * c^2 + (a * b^2 * c^2 - 4 * a^2 * c^3) * x^4 + (a * b^3 * c - 4 * a^2 * b * c^2) * x^2) - 1/2 * \operatorname{integrate}(- (a * b * c * e - 2 * a^2 * c * f + a^2 * b * g + (b * c^2 * d - 2 * a * c^2 * e + a * b * c * f + (a * b^2 - 6 * a^2 * c) * g) * x^2 + (b^2 * c - 6 * a * c^2) * d) / (c * x^4 + b * x^2 + a), x) / (a * b^2 * c - 4 * a^2 * c^2)$

mupad [B] time = 5.82, size = 32587, normalized size = 72.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x^2 + f*x^4 + g*x^6)/(a + b*x^2 + c*x^4)^2,x)

[Out] $((x * (2 * a * c^2 * d - b^2 * c * d + a^2 * b * g - 2 * a^2 * c * f + a * b * c * e)) / (2 * a * c * (4 * a * c - b^2)) - (x^3 * (b * c^2 * d - 2 * a * c^2 * e - a * b^2 * g + 2 * a^2 * c * g + a * b * c * f)) / (2 * a * c * (4 * a * c - b^2))) / (a + b * x^2 + c * x^4) - \operatorname{atan}((((6144 * a^5 * c^7 * d + 2048 * a^6 * c^6 * f - 288 * a^2 * b^6 * c^4 * d + 1920 * a^3 * b^4 * c^5 * d - 5632 * a^4 * b^2 * c^6 * d + 16 * a^2 * b^7 * c^3 * e - 192 * a^3 * b^5 * c^4 * e + 768 * a^4 * b^3 * c^5 * e - 32 * a^3 * b^6 * c^3 * f + 384 * a^4 * b^4 * c^4 * f - 1536 * a^5 * b^2 * c^5 * f + 16 * a^3 * b^7 * c^2 * g - 192 * a^4 * b^5 * c^3 * g + 768 * a^5 * b^3 * c^4 * g + 16 * a * b^8 * c^3 * d - 1024 * a^5 * b * c^6 * e - 1024 * a^6 * b * c^5 * g) / (8 * (64 * a^5 * c^4 - a^2 * b^6 * c + 12 * a^3 * b^4 * c^2 - 48 * a^4 * b^2 * c^3)) - (x * ((27 * a * b^9 * c^4 * d^2 - a^3 * b^11 * g^2 - b^11 * c^3 * d^2 + 3840 * a^5 * b * c^8 * d^2 - 9 * a * c^4 * d^2 * (-4 * a * c - b^2)^9)^{(1/2)} + 768 * a^6 * b * c^7 * e^2 + 768 * a^7 * b * c^6 * f^2 + 27 * a^4 * b^9 * c * g^2 + 3840 * a^8 * b * c^5 * g^2 + 9 * a^4 * c * g^2 * (-4 * a * c - b^2)^9)^{(1/2)} - 288 * a^2 * b^7 * c^5 * d^2 + 1504 * a^3 * b^5 * c^6 * d^2 - 3840 * a^4 * b^3 * c^7 * d^2 - a^2 * b^9 * c^3 * e^2 + 96 * a^4 * b^5 * c^5 * e^2 - 512 * a^5 * b^3 * c^6 * e^2 + a^2 * c^3 * e^2 * (-4 * a * c - b^2)^9)^{(1/2)} + b^2 * c^3 * d^2 * (-4 * a * c - b^2)^9)^{(1/2)} - a^3 * b^9 * c^2 * f^2 + 96 * a^5 * b^5 * c^4 * f^2 - 512 * a^6 * b^3 * c^5 * f^2 - a^3 * b^2 * g^2 * (-4 * a * c - b^2)^9)^{(1/2)} - a^3 * c^2 * f^2 * (-4 * a * c - b^2)^9)^{(1/2)} - 288 * a^5 * b^7 * c^2 * g^2 + 1504 * a^6 * b^5 * c^3 * g^2 - 3840 * a^7 * b^3 * c^4 * g^2 - 3072 * a^6 * c^8 * d * e - 9216 * a^7 * c^7 * d * g -$

$$\begin{aligned}
& 1024a^7c^7e^f - 3072a^8c^6f^*g - 2a^*b^{10}c^3d^*e + 3584a^6b^*c^7d^*f \\
& + 3584a^7b^*c^6e^*g - 2a^3b^{10}c^*f^*g + 36a^2b^8c^4d^*e - 192a^3b^6 \\
& c^5d^*e + 128a^4b^4c^6d^*e + 1536a^5b^2c^7d^*e + 6a^2b^9c^3d^*f - \\
& 128a^3b^7c^4d^*f + 960a^4b^5c^5d^*f - 3072a^5b^3c^6d^*f - 6a^2c^3d^*f \\
& (-4a^*c - b^2)^9)^{(1/2)} - 20a^3b^8c^3d^*g + 12a^3b^8c^3e^*f + \\
& 384a^4b^6c^4d^*g - 128a^4b^6c^4e^*f - 2688a^5b^4c^5d^*g + 384a^5 \\
& b^4c^5e^*f + 8192a^6b^2c^6d^*g + 6a^3b^9c^2e^*g - 128a^4b^7c^3e^* \\
& g + 960a^5b^5c^4e^*g - 3072a^6b^3c^5e^*g + 6a^3c^2e^*g(-4a^*c - \\
& b^2)^9)^{(1/2)} + 36a^4b^8c^2f^*g - 192a^5b^6c^3f^*g + 128a^6b^4c^4 \\
& f^*g + 1536a^7b^2c^5f^*g + 2a^*b^*c^3d^*e^*(-4a^*c - b^2)^9)^{(1/2)} - 2a^3 \\
& b^*c^*f^*g(-4a^*c - b^2)^9)^{(1/2)} / (32(4096a^9c^9 + a^3b^{12}c^3 - 24a^4 \\
& b^{10}c^4 + 240a^5b^8c^5 - 1280a^6b^6c^6 + 3840a^7b^4c^7 - 6144a^8b^2 \\
& c^8))^{(1/2)} * (1024a^5b^*c^6 - 16a^2b^7c^3 + 192a^3b^5c^4 - 76 \\
& 8a^4b^3c^5) / (2(16a^4c^3 + a^2b^4c - 8a^3b^2c^2)) * ((27a^*b^9c^ \\
& 4d^2 - a^3b^{11}g^2 - b^{11}c^3d^2 + 3840a^5b^*c^8d^2 - 9a^*c^4d^2(-4 \\
& a^*c - b^2)^9)^{(1/2)} + 768a^6b^*c^7e^2 + 768a^7b^*c^6f^2 + 27a^4b^9c \\
& g^2 + 3840a^8b^*c^5g^2 + 9a^4c^*g^2(-4a^*c - b^2)^9)^{(1/2)} - 288a^2b^7 \\
& c^5d^2 + 1504a^3b^5c^6d^2 - 3840a^4b^3c^7d^2 - a^2b^9c^3e^2 \\
& + 96a^4b^5c^5e^2 - 512a^5b^3c^6e^2 + a^2c^3e^2(-4a^*c - b^2)^9 \\
&)^{(1/2)} + b^2c^3d^2(-4a^*c - b^2)^9)^{(1/2)} - a^3b^9c^2f^2 + 96a^5b \\
& ^5c^4f^2 - 512a^6b^3c^5f^2 - a^3b^2g^2(-4a^*c - b^2)^9)^{(1/2)} - a \\
& ^3c^2f^2(-4a^*c - b^2)^9)^{(1/2)} - 288a^5b^7c^2g^2 + 1504a^6b^5c^3 \\
& g^2 - 3840a^7b^3c^4g^2 - 3072a^6c^8d^*e - 9216a^7c^7d^*g - 1024a^ \\
& ^7c^7e^*f - 3072a^8c^6f^*g - 2a^*b^{10}c^3d^*e + 3584a^6b^*c^7d^*f + 358 \\
& 4a^7b^*c^6e^*g - 2a^3b^{10}c^*f^*g + 36a^2b^8c^4d^*e - 192a^3b^6c^5d^* \\
& e + 128a^4b^4c^6d^*e + 1536a^5b^2c^7d^*e + 6a^2b^9c^3d^*f - 128a^ \\
& ^3b^7c^4d^*f + 960a^4b^5c^5d^*f - 3072a^5b^3c^6d^*f - 6a^2c^3d^*f \\
& (-4a^*c - b^2)^9)^{(1/2)} - 20a^3b^8c^3d^*g + 12a^3b^8c^3e^*f + 384a^ \\
& ^4b^6c^4d^*g - 128a^4b^6c^4e^*f - 2688a^5b^4c^5d^*g + 384a^5b^4c^ \\
& ^5e^*f + 8192a^6b^2c^6d^*g + 6a^3b^9c^2e^*g - 128a^4b^7c^3e^*g + 9 \\
& 60a^5b^5c^4e^*g - 3072a^6b^3c^5e^*g + 6a^3c^2e^*g(-4a^*c - b^2)^9 \\
&)^{(1/2)} + 36a^4b^8c^2f^*g - 192a^5b^6c^3f^*g + 128a^6b^4c^4f^*g + \\
& 1536a^7b^2c^5f^*g + 2a^*b^*c^3d^*e^*(-4a^*c - b^2)^9)^{(1/2)} - 2a^3b^*c^*f^* \\
& g^*(-4a^*c - b^2)^9)^{(1/2)} / (32(4096a^9c^9 + a^3b^{12}c^3 - 24a^4b^{10} \\
& c^4 + 240a^5b^8c^5 - 1280a^6b^6c^6 + 3840a^7b^4c^7 - 6144a^8b^2 \\
& c^8))^{(1/2)} + (x(72a^2c^6d^2 - 8a^3c^5e^2 + b^4c^4d^2 + a^2b^6g^2 \\
& + 8a^4c^4f^2 - 72a^5c^3g^2 - 14a^*b^2c^5d^2 - 16a^3b^4c^*g^2 \\
& + 10a^2b^2c^4e^2 + a^2b^4c^2f^2 + 2a^3b^2c^3f^2 + 74a^4b^2c^2 \\
& g^2 + 48a^3c^5d^*f - 48a^4c^4e^*g + 2a^*b^3c^4d^*e - 40a^2b^*c^5d^*e \\
& - 72a^3b^*c^4d^*g - 8a^3b^*c^4e^*f + 2a^2b^5c^*f^*g - 8a^4b^*c^3f^*g + \\
& 4a^2b^2c^4d^*f + 10a^2b^3c^3d^*g - 6a^2b^3c^3e^*f - 6a^2b^4c^2 \\
& e^*g + 52a^3b^2c^3e^*g - 14a^3b^3c^2f^*g) / (2(16a^4c^3 + a^2b^4c \\
& - 8a^3b^2c^2)) * ((27a^*b^9c^4d^2 - a^3b^{11}g^2 - b^{11}c^3d^2 + 3840 \\
& a^5b^*c^8d^2 - 9a^*c^4d^2(-4a^*c - b^2)^9)^{(1/2)} + 768a^6b^*c^7e^2 + \\
& 768a^7b^*c^6f^2 + 27a^4b^9c^*g^2 + 3840a^8b^*c^5g^2 + 9a^4c^*g^2(- \\
& (4a^*c - b^2)^9)^{(1/2)} - 288a^2b^7c^5d^2 + 1504a^3b^5c^6d^2 - 3840 \\
& a^4b^3c^7d^2 - a^2b^9c^3e^2 + 96a^4b^5c^5e^2 - 512a^5b^3c^6e^2 \\
& + a^2c^3e^2(-4a^*c - b^2)^9)^{(1/2)} + b^2c^3d^2(-4a^*c - b^2)^9)^{(\\
& 1/2)} - a^3b^9c^2f^2 + 96a^5b^5c^4f^2 - 512a^6b^3c^5f^2 - a^3b^2 \\
& g^2(-4a^*c - b^2)^9)^{(1/2)} - a^3c^2f^2(-4a^*c - b^2)^9)^{(1/2)} - 288 \\
& a^5b^7c^2g^2 + 1504a^6b^5c^3g^2 - 3840a^7b^3c^4g^2 - 3072a^6c^8 \\
& d^*e - 9216a^7c^7d^*g - 1024a^7c^7e^*f - 3072a^8c^6f^*g - 2a^*b^{10}c^3 \\
& d^*e + 3584a^6b^*c^7d^*f + 3584a^7b^*c^6e^*g - 2a^3b^{10}c^*f^*g + 36a^2 \\
& b^8c^4d^*e - 192a^3b^6c^5d^*e + 128a^4b^4c^6d^*e + 1536a^5b^2c^7 \\
& d^*e + 6a^2b^9c^3d^*f - 128a^3b^7c^4d^*f + 960a^4b^5c^5d^*f - 307 \\
& 2a^5b^3c^6d^*f - 6a^2c^3d^*f(-4a^*c - b^2)^9)^{(1/2)} - 20a^3b^8c^3 \\
& d^*g + 12a^3b^8c^3e^*f + 384a^4b^6c^4d^*g - 128a^4b^6c^4e^*f - 268 \\
& 8a^5b^4c^5d^*g + 384a^5b^4c^5e^*f + 8192a^6b^2c^6d^*g + 6a^3b^9c^ \\
& ^2e^*g - 128a^4b^7c^3e^*g + 960a^5b^5c^4e^*g - 3072a^6b^3c^5e^*g
\end{aligned}$$

$$\begin{aligned}
& + 6a^3c^2e*g*(-(4ac - b^2)^9)^{(1/2)} + 36a^4b^8c^2f*g - 192a^5b^6c^3f*g + 128a^6b^4c^4f*g + 1536a^7b^2c^5f*g + 2a*b*c^3d*e*(-(4ac - b^2)^9)^{(1/2)} - 2a^3b*c*f*g*(-(4ac - b^2)^9)^{(1/2)}/(32*(4096a^9c^9 + a^3b^12c^3 - 24a^4b^10c^4 + 240a^5b^8c^5 - 1280a^6b^6c^6 + 3840a^7b^4c^7 - 6144a^8b^2c^8)))^{(1/2)}*1i - (((6144a^5c^7d + 2048a^6c^6f - 288a^2b^6c^4d + 1920a^3b^4c^5d - 5632a^4b^2c^6d + 16a^2b^7c^3e - 192a^3b^5c^4e + 768a^4b^3c^5e - 32a^3b^6c^3f + 384a^4b^4c^4f - 1536a^5b^2c^5f + 16a^3b^7c^2g - 192a^4b^5c^3g + 768a^5b^3c^4g + 16a*b^8c^3d - 1024a^5b*c^6e - 1024a^6b*c^5g)/(8*(64a^5c^4 - a^2b^6c + 12a^3b^4c^2 - 48a^4b^2c^3))) + (x*((27a*b^9c^4d^2 - a^3b^11g^2 - b^11c^3d^2 + 3840a^5b*c^8d^2 - 9a*c^4d^2*(-(4ac - b^2)^9)^{(1/2)} + 768a^6b*c^7e^2 + 768a^7b*c^6f^2 + 27a^4b^9c*g^2 + 3840a^8b*c^5g^2 + 9a^4c*g^2*(-(4ac - b^2)^9)^{(1/2)} - 288a^2b^7c^5d^2 + 1504a^3b^5c^6d^2 - 3840a^4b^3c^7d^2 - a^2b^9c^3e^2 + 96a^4b^5c^5e^2 - 512a^5b^3c^6e^2 + a^2c^3e^2*(-(4ac - b^2)^9)^{(1/2)} + b^2c^3d^2*(-(4ac - b^2)^9)^{(1/2)} - a^3b^9c^2f^2 + 96a^5b^5c^4f^2 - 512a^6b^3c^5f^2 - a^3b^2g^2*(-(4ac - b^2)^9)^{(1/2)} - a^3c^2f^2*(-(4ac - b^2)^9)^{(1/2)} - 288a^5b^7c^2g^2 + 1504a^6b^5c^3g^2 - 3840a^7b^3c^4g^2 - 3072a^6c^8d*e - 9216a^7c^7d*g - 1024a^7c^7e*f - 3072a^8c^6f*g - 2a*b^10c^3d*e + 3584a^6b*c^7d*f + 3584a^7b*c^6e*g - 2a^3b^10c*f*g + 36a^2b^8c^4d*e - 192a^3b^6c^5d*e + 128a^4b^4c^6d*e + 1536a^5b^2c^7d*e + 6a^2b^9c^3d*f - 128a^3b^7c^4d*f + 960a^4b^5c^5d*f - 3072a^5b^3c^6d*f - 6a^2c^3d*f*(-(4ac - b^2)^9)^{(1/2)} - 20a^3b^8c^3d*g + 12a^3b^8c^3e*f + 384a^4b^6c^4d*g - 128a^4b^6c^4e*f - 2688a^5b^4c^5d*g + 384a^5b^4c^5e*f + 8192a^6b^2c^6d*g + 6a^3b^9c^2e*g - 128a^4b^7c^3e*g + 960a^5b^5c^4e*g - 3072a^6b^3c^5e*g + 6a^3c^2e*g*(-(4ac - b^2)^9)^{(1/2)} + 36a^4b^8c^2f*g - 192a^5b^6c^3f*g + 128a^6b^4c^4f*g + 1536a^7b^2c^5f*g + 2a*b*c^3d*e*(-(4ac - b^2)^9)^{(1/2)} - 2a^3b*c*f*g*(-(4ac - b^2)^9)^{(1/2)}/(32*(4096a^9c^9 + a^3b^12c^3 - 24a^4b^10c^4 + 240a^5b^8c^5 - 1280a^6b^6c^6 + 3840a^7b^4c^7 - 6144a^8b^2c^8)))^{(1/2)}*(1024a^5b*c^6 - 16a^2b^7c^3 + 192a^3b^5c^4 - 768a^4b^3c^5))/(2*(16a^4c^3 + a^2b^4c - 8a^3b^2c^2)))*((27a*b^9c^4d^2 - a^3b^11g^2 - b^11c^3d^2 + 3840a^5b*c^8d^2 - 9a*c^4d^2*(-(4ac - b^2)^9)^{(1/2)} + 768a^6b*c^7e^2 + 768a^7b*c^6f^2 + 27a^4b^9c*g^2 + 3840a^8b*c^5g^2 + 9a^4c*g^2*(-(4ac - b^2)^9)^{(1/2)} - 288a^2b^7c^5d^2 + 1504a^3b^5c^6d^2 - 3840a^4b^3c^7d^2 - a^2b^9c^3e^2 + 96a^4b^5c^5e^2 - 512a^5b^3c^6e^2 + a^2c^3e^2*(-(4ac - b^2)^9)^{(1/2)} + b^2c^3d^2*(-(4ac - b^2)^9)^{(1/2)} - a^3b^9c^2f^2 + 96a^5b^5c^4f^2 - 512a^6b^3c^5f^2 - a^3b^2g^2*(-(4ac - b^2)^9)^{(1/2)} - a^3c^2f^2*(-(4ac - b^2)^9)^{(1/2)} - 288a^5b^7c^2g^2 + 1504a^6b^5c^3g^2 - 3840a^7b^3c^4g^2 - 3072a^6c^8d*e - 9216a^7c^7d*g - 1024a^7c^7e*f - 3072a^8c^6f*g - 2a*b^10c^3d*e + 3584a^6b*c^7d*f + 3584a^7b*c^6e*g - 2a^3b^10c*f*g + 36a^2b^8c^4d*e - 192a^3b^6c^5d*e + 128a^4b^4c^6d*e + 1536a^5b^2c^7d*e + 6a^2b^9c^3d*f - 128a^3b^7c^4d*f + 960a^4b^5c^5d*f - 3072a^5b^3c^6d*f - 6a^2c^3d*f*(-(4ac - b^2)^9)^{(1/2)} - 20a^3b^8c^3d*g + 12a^3b^8c^3e*f + 384a^4b^6c^4d*g - 128a^4b^6c^4e*f - 2688a^5b^4c^5d*g + 384a^5b^4c^5e*f + 8192a^6b^2c^6d*g + 6a^3b^9c^2e*g - 128a^4b^7c^3e*g + 960a^5b^5c^4e*g - 3072a^6b^3c^5e*g + 6a^3c^2e*g*(-(4ac - b^2)^9)^{(1/2)} + 36a^4b^8c^2f*g - 192a^5b^6c^3f*g + 128a^6b^4c^4f*g + 1536a^7b^2c^5f*g + 2a*b*c^3d*e*(-(4ac - b^2)^9)^{(1/2)} - 2a^3b*c*f*g*(-(4ac - b^2)^9)^{(1/2)}/(32*(4096a^9c^9 + a^3b^12c^3 - 24a^4b^10c^4 + 240a^5b^8c^5 - 1280a^6b^6c^6 + 3840a^7b^4c^7 - 6144a^8b^2c^8)))^{(1/2)} - (x*(72a^2c^6d^2 - 8a^3c^5e^2 + b^4c^4d^2 + a^2b^6g^2 + 8a^4c^4f^2 - 72a^5c^3g^2 - 14a*b^2c^5d^2 - 16a^3b^4c*g^2 + 10a^2b^2c^4e^2 + a^2b^4c^2f^2 + 2a^3b^2c^3f^2 + 74a^4b^2c^2g^2 + 48a^3c^5d*f - 48a^4c^4e*g + 2a*b^3c^4d*e - 40a^2b*c^5d*e - 72a^3b*c^4d*g - 8a^3b*c^4e*f + 2a^2b^5c*f*g - 8a^4b*c
\end{aligned}$$

$$\begin{aligned}
& ^3f*g + 4*a^2*b^2*c^4*d*f + 10*a^2*b^3*c^3*d*g - 6*a^2*b^3*c^3*e*f - 6*a^2 \\
& *b^4*c^2*e*g + 52*a^3*b^2*c^3*e*g - 14*a^3*b^3*c^2*f*g)/(2*(16*a^4*c^3 + a \\
& ^2*b^4*c - 8*a^3*b^2*c^2)))*((27*a*b^9*c^4*d^2 - a^3*b^11*g^2 - b^11*c^3*d^ \\
& 2 + 3840*a^5*b*c^8*d^2 - 9*a*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^6*b*c \\
& ^7*e^2 + 768*a^7*b*c^6*f^2 + 27*a^4*b^9*c*g^2 + 3840*a^8*b*c^5*g^2 + 9*a^4* \\
& c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^2*b^7*c^5*d^2 + 1504*a^3*b^5*c^6*d^2 \\
& - 3840*a^4*b^3*c^7*d^2 - a^2*b^9*c^3*e^2 + 96*a^4*b^5*c^5*e^2 - 512*a^5*b^ \\
& 3*c^6*e^2 + a^2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^3*d^2*(-(4*a*c - b \\
& ^2)^9)^(1/2) - a^3*b^9*c^2*f^2 + 96*a^5*b^5*c^4*f^2 - 512*a^6*b^3*c^5*f^2 - \\
& a^3*b^2*g^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) \\
&) - 288*a^5*b^7*c^2*g^2 + 1504*a^6*b^5*c^3*g^2 - 3840*a^7*b^3*c^4*g^2 - 307 \\
& 2*a^6*c^8*d*e - 9216*a^7*c^7*d*g - 1024*a^7*c^7*e*f - 3072*a^8*c^6*f*g - 2* \\
& a*b^10*c^3*d*e + 3584*a^6*b*c^7*d*f + 3584*a^7*b*c^6*e*g - 2*a^3*b^10*c*f*g \\
& + 36*a^2*b^8*c^4*d*e - 192*a^3*b^6*c^5*d*e + 128*a^4*b^4*c^6*d*e + 1536*a^ \\
& 5*b^2*c^7*d*e + 6*a^2*b^9*c^3*d*f - 128*a^3*b^7*c^4*d*f + 960*a^4*b^5*c^5*d \\
& *f - 3072*a^5*b^3*c^6*d*f - 6*a^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 20*a^3 \\
& *b^8*c^3*d*g + 12*a^3*b^8*c^3*e*f + 384*a^4*b^6*c^4*d*g - 128*a^4*b^6*c^4*e \\
& *f - 2688*a^5*b^4*c^5*d*g + 384*a^5*b^4*c^5*e*f + 8192*a^6*b^2*c^6*d*g + 6* \\
& a^3*b^9*c^2*e*g - 128*a^4*b^7*c^3*e*g + 960*a^5*b^5*c^4*e*g - 3072*a^6*b^3* \\
& c^5*e*g + 6*a^3*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c^2*f*g - 192 \\
& *a^5*b^6*c^3*f*g + 128*a^6*b^4*c^4*f*g + 1536*a^7*b^2*c^5*f*g + 2*a*b*c^3*d \\
& *e*(-(4*a*c - b^2)^9)^(1/2) - 2*a^3*b*c*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(\\
& 4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6* \\
& b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2)*1i)/((((6144*a^5*c^7 \\
& *d + 2048*a^6*c^6*f - 288*a^2*b^6*c^4*d + 1920*a^3*b^4*c^5*d - 5632*a^4*b^2 \\
& *c^6*d + 16*a^2*b^7*c^3*e - 192*a^3*b^5*c^4*e + 768*a^4*b^3*c^5*e - 32*a^3* \\
& b^6*c^3*f + 384*a^4*b^4*c^4*f - 1536*a^5*b^2*c^5*f + 16*a^3*b^7*c^2*g - 192 \\
& *a^4*b^5*c^3*g + 768*a^5*b^3*c^4*g + 16*a*b^8*c^3*d - 1024*a^5*b*c^6*e - 10 \\
& 24*a^6*b*c^5*g)/(8*(64*a^5*c^4 - a^2*b^6*c + 12*a^3*b^4*c^2 - 48*a^4*b^2*c^ \\
& 3)) - (x*((27*a*b^9*c^4*d^2 - a^3*b^11*g^2 - b^11*c^3*d^2 + 3840*a^5*b*c^8* \\
& d^2 - 9*a*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^6*b*c^7*e^2 + 768*a^7*b* \\
& c^6*f^2 + 27*a^4*b^9*c*g^2 + 3840*a^8*b*c^5*g^2 + 9*a^4*c*g^2*(-(4*a*c - b^ \\
& 2)^9)^(1/2) - 288*a^2*b^7*c^5*d^2 + 1504*a^3*b^5*c^6*d^2 - 3840*a^4*b^3*c^7 \\
& *d^2 - a^2*b^9*c^3*e^2 + 96*a^4*b^5*c^5*e^2 - 512*a^5*b^3*c^6*e^2 + a^2*c^3 \\
& *e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^3* \\
& b^9*c^2*f^2 + 96*a^5*b^5*c^4*f^2 - 512*a^6*b^3*c^5*f^2 - a^3*b^2*g^2*(-(4*a \\
& *c - b^2)^9)^(1/2) - a^3*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2 \\
& *g^2 + 1504*a^6*b^5*c^3*g^2 - 3840*a^7*b^3*c^4*g^2 - 3072*a^6*c^8*d*e - 921 \\
& 6*a^7*c^7*d*g - 1024*a^7*c^7*e*f - 3072*a^8*c^6*f*g - 2*a*b^10*c^3*d*e + 35 \\
& 84*a^6*b*c^7*d*f + 3584*a^7*b*c^6*e*g - 2*a^3*b^10*c*f*g + 36*a^2*b^8*c^4*d \\
& *e - 192*a^3*b^6*c^5*d*e + 128*a^4*b^4*c^6*d*e + 1536*a^5*b^2*c^7*d*e + 6*a \\
& ^2*b^9*c^3*d*f - 128*a^3*b^7*c^4*d*f + 960*a^4*b^5*c^5*d*f - 3072*a^5*b^3*c \\
& ^6*d*f - 6*a^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 20*a^3*b^8*c^3*d*g + 12*a \\
& ^3*b^8*c^3*e*f + 384*a^4*b^6*c^4*d*g - 128*a^4*b^6*c^4*e*f - 2688*a^5*b^4*c \\
& ^5*d*g + 384*a^5*b^4*c^5*e*f + 8192*a^6*b^2*c^6*d*g + 6*a^3*b^9*c^2*e*g - 1 \\
& 28*a^4*b^7*c^3*e*g + 960*a^5*b^5*c^4*e*g - 3072*a^6*b^3*c^5*e*g + 6*a^3*c^2 \\
& *e*g*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c^2*f*g - 192*a^5*b^6*c^3*f*g + \\
& 128*a^6*b^4*c^4*f*g + 1536*a^7*b^2*c^5*f*g + 2*a*b*c^3*d*e*(-(4*a*c - b^2)^ \\
& 9)^(1/2) - 2*a^3*b*c*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^9 + a^3* \\
& b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7* \\
& b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2)*(1024*a^5*b*c^6 - 16*a^2*b^7*c^3 + 192* \\
& a^3*b^5*c^4 - 768*a^4*b^3*c^5))/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2) \\
&))*((27*a*b^9*c^4*d^2 - a^3*b^11*g^2 - b^11*c^3*d^2 + 3840*a^5*b*c^8*d^2 - \\
& 9*a*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^6*b*c^7*e^2 + 768*a^7*b*c^6*f^ \\
& 2 + 27*a^4*b^9*c*g^2 + 3840*a^8*b*c^5*g^2 + 9*a^4*c*g^2*(-(4*a*c - b^2)^9) \\
& ^2 + 27*a^4*b^9*c*g^2 + 3840*a^8*b*c^5*g^2 + 9*a^4*c*g^2*(-(4*a*c - b^2)^9) \\
& ^2 - 288*a^2*b^7*c^5*d^2 + 1504*a^3*b^5*c^6*d^2 - 3840*a^4*b^3*c^7*d^2 - \\
& a^2*b^9*c^3*e^2 + 96*a^4*b^5*c^5*e^2 - 512*a^5*b^3*c^6*e^2 + a^2*c^3*e^2*(- \\
& (4*a*c - b^2)^9)^(1/2) + b^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*c^ \\
& 2*f^2 + 96*a^5*b^5*c^4*f^2 - 512*a^6*b^3*c^5*f^2 - a^3*b^2*g^2*(-(4*a*c - b
\end{aligned}$$

$$\begin{aligned}
& ^2)^9)^{(1/2)} - a^3c^2f^2*(-(4ac - b^2)^9)^{(1/2)} - 288a^5b^7c^2g^2 + \\
& 1504a^6b^5c^3g^2 - 3840a^7b^3c^4g^2 - 3072a^6c^8d^2e - 9216a^7c^7d^2g - 1024a^7c^7e^2f - 3072a^8c^6f^2g - 2ab^{10}c^3d^2e + 3584a^6 \\
& b^7c^2d^2f + 3584a^7b^7c^6e^2g - 2a^3b^{10}c^2f^2g + 36a^2b^8c^4d^2e - 1 \\
& 92a^3b^6c^5d^2e + 128a^4b^4c^6d^2e + 1536a^5b^2c^7d^2e + 6a^2b^9 \\
& c^3d^2f - 128a^3b^7c^4d^2f + 960a^4b^5c^5d^2f - 3072a^5b^3c^6d^2f \\
& - 6a^2c^3d^2f*(-(4ac - b^2)^9)^{(1/2)} - 20a^3b^8c^3d^2g + 12a^3b^8 \\
& c^3e^2f + 384a^4b^6c^4d^2g - 128a^4b^6c^4e^2f - 2688a^5b^4c^5d^2g \\
& + 384a^5b^4c^5e^2f + 8192a^6b^2c^6d^2g + 6a^3b^9c^2e^2g - 128a^4 \\
& b^7c^3e^2g + 960a^5b^5c^4e^2g - 3072a^6b^3c^5e^2g + 6a^3c^2e^2g*(\\
& -(4ac - b^2)^9)^{(1/2)} + 36a^4b^8c^2f^2g - 192a^5b^6c^3f^2g + 128a^6 \\
& b^4c^4f^2g + 1536a^7b^2c^5f^2g + 2ab^7c^3d^2e*(-(4ac - b^2)^9)^{(1/ \\
& 2)} - 2a^3b^7c^3d^2e*(-(4ac - b^2)^9)^{(1/2)})/(32*(4096a^9c^9 + a^3b^{12}c \\
& ^3 - 24a^4b^{10}c^4 + 240a^5b^8c^5 - 1280a^6b^6c^6 + 3840a^7b^4c^7 - 6144a^8b^2c^8)))^{(1/2)} + (x*(72a^2c^6d^2 - 8a^3c^5e^2 + b^4c^4 \\
& d^2 + a^2b^6g^2 + 8a^4c^4f^2 - 72a^5c^3g^2 - 14ab^2c^5d^2 - 1 \\
& 6a^3b^4c^2g^2 + 10a^2b^2c^4e^2 + a^2b^4c^2f^2 + 2a^3b^2c^3f^2 \\
& + 74a^4b^2c^2g^2 + 48a^3c^5d^2f - 48a^4c^4e^2g + 2ab^3c^4d^2e - \\
& 40a^2b^7c^5d^2e - 72a^3b^6c^4d^2g - 8a^3b^6c^4e^2f + 2a^2b^5c^3f^2g - 8 \\
& a^4b^5c^3f^2g + 4a^2b^2c^4d^2f + 10a^2b^3c^3d^2g - 6a^2b^3c^3e^2f \\
& - 6a^2b^4c^2e^2g + 52a^3b^2c^3e^2g - 14a^3b^3c^2f^2g))/(2*(16a^4 \\
& c^3 + a^2b^4c - 8a^3b^2c^2)))*((27ab^9c^4d^2 - a^3b^{11}g^2 - b^{11} \\
& c^3d^2 + 3840a^5b^8c^8d^2 - 9a^4c^4d^2*(-(4ac - b^2)^9)^{(1/2)} + 768 \\
& a^6b^7c^7e^2 + 768a^7b^6c^6f^2 + 27a^4b^9c^2g^2 + 3840a^8b^5c^5g^2 \\
& + 9a^4c^2g^2*(-(4ac - b^2)^9)^{(1/2)} - 288a^2b^7c^5d^2 + 1504a^3b^5 \\
& c^6d^2 - 3840a^4b^3c^7d^2 - a^2b^9c^3e^2 + 96a^4b^5c^5e^2 - 51 \\
& 2a^5b^3c^6e^2 + a^2c^3e^2*(-(4ac - b^2)^9)^{(1/2)} + b^2c^3d^2*(-(4 \\
& ac - b^2)^9)^{(1/2)} - a^3b^9c^2f^2 + 96a^5b^5c^4f^2 - 512a^6b^3c^5 \\
& f^2 - a^3b^2g^2*(-(4ac - b^2)^9)^{(1/2)} - a^3c^2f^2*(-(4ac - b^2) \\
& ^9)^{(1/2)} - 288a^5b^7c^2g^2 + 1504a^6b^5c^3g^2 - 3840a^7b^3c^4g^ \\
& ^2 - 3072a^6c^8d^2e - 9216a^7c^7d^2g - 1024a^7c^7e^2f - 3072a^8c^6 \\
& f^2g - 2ab^{10}c^3d^2e + 3584a^6b^7c^2d^2f + 3584a^7b^7c^6e^2g - 2a^3b^ \\
& ^{10}c^2f^2g + 36a^2b^8c^4d^2e - 192a^3b^6c^5d^2e + 128a^4b^4c^6d^2e + \\
& 1536a^5b^2c^7d^2e + 6a^2b^9c^3d^2f - 128a^3b^7c^4d^2f + 960a^4b^5 \\
& c^5d^2f - 3072a^5b^3c^6d^2f - 6a^2c^3d^2f*(-(4ac - b^2)^9)^{(1/2)} \\
& - 20a^3b^8c^3d^2g + 12a^3b^8c^3e^2f + 384a^4b^6c^4d^2g - 128a^4b^ \\
& ^6c^4e^2f - 2688a^5b^4c^5d^2g + 384a^5b^4c^5e^2f + 8192a^6b^2c^6d^2 \\
& g + 6a^3b^9c^2e^2g - 128a^4b^7c^3e^2g + 960a^5b^5c^4e^2g - 3072a^ \\
& ^6b^3c^5e^2g + 6a^3c^2e^2g*(-(4ac - b^2)^9)^{(1/2)} + 36a^4b^8c^2f^2 \\
& *g - 192a^5b^6c^3f^2g + 128a^6b^4c^4f^2g + 1536a^7b^2c^5f^2g + 2a \\
& *b^7c^3d^2e*(-(4ac - b^2)^9)^{(1/2)} - 2a^3b^7c^3d^2e*(-(4ac - b^2)^9)^{(1/2)} \\
&))/(32*(4096a^9c^9 + a^3b^{12}c^3 - 24a^4b^{10}c^4 + 240a^5b^8c^5 - 1 \\
& 280a^6b^6c^6 + 3840a^7b^4c^7 - 6144a^8b^2c^8)))^{(1/2)} + (((6144a^ \\
& 5c^7d^2 + 2048a^6c^6f^2 - 288a^2b^6c^4d^2 + 1920a^3b^4c^5d^2 - 5632a^ \\
& 4b^2c^6d^2 + 16a^2b^7c^3e^2 - 192a^3b^5c^4e^2 + 768a^4b^3c^5e^2 - 32 \\
& a^3b^6c^3f^2 + 384a^4b^4c^4f^2 - 1536a^5b^2c^5f^2 + 16a^3b^7c^2g^2 \\
& - 192a^4b^5c^3g^2 + 768a^5b^3c^4g^2 + 16ab^8c^3d^2 - 1024a^5b^8c^6e^2 \\
& - 1024a^6b^7c^5g^2)/(8*(64a^5c^4 - a^2b^6c + 12a^3b^4c^2 - 48a^4b^ \\
& ^2c^3)) + (x*((27ab^9c^4d^2 - a^3b^{11}g^2 - b^{11}c^3d^2 + 3840a^5b^ \\
& ^8c^8d^2 - 9a^4c^4d^2*(-(4ac - b^2)^9)^{(1/2)} + 768a^6b^7c^7e^2 + 768a^ \\
& ^7b^6c^6f^2 + 27a^4b^9c^2g^2 + 3840a^8b^5c^5g^2 + 9a^4c^2g^2*(-(4ac \\
& - b^2)^9)^{(1/2)} - 288a^2b^7c^5d^2 + 1504a^3b^5c^6d^2 - 3840a^4b^ \\
& ^3c^7d^2 - a^2b^9c^3e^2 + 96a^4b^5c^5e^2 - 512a^5b^3c^6e^2 + a^ \\
& ^2c^3e^2*(-(4ac - b^2)^9)^{(1/2)} + b^2c^3d^2*(-(4ac - b^2)^9)^{(1/2)} - \\
& a^3b^9c^2f^2 + 96a^5b^5c^4f^2 - 512a^6b^3c^5f^2 - a^3b^2g^2*(- \\
& -(4ac - b^2)^9)^{(1/2)} - a^3c^2f^2*(-(4ac - b^2)^9)^{(1/2)} - 288a^5b^ \\
& ^7c^2g^2 + 1504a^6b^5c^3g^2 - 3840a^7b^3c^4g^2 - 3072a^6c^8d^2e \\
& - 9216a^7c^7d^2g - 1024a^7c^7e^2f - 3072a^8c^6f^2g - 2ab^{10}c^3d^2e \\
& + 3584a^6b^7c^2d^2f + 3584a^7b^7c^6e^2g - 2a^3b^{10}c^2f^2g + 36a^2b^8
\end{aligned}$$

$$\begin{aligned}
& c^4*d*e - 192*a^3*b^6*c^5*d*e + 128*a^4*b^4*c^6*d*e + 1536*a^5*b^2*c^7*d*e \\
& + 6*a^2*b^9*c^3*d*f - 128*a^3*b^7*c^4*d*f + 960*a^4*b^5*c^5*d*f - 3072*a^5*b^3*c^6*d*f - 6*a^2*c^3*d*f*(-(4*a*c - b^2)^9)^{(1/2)} - 20*a^3*b^8*c^3*d*g + \\
& 12*a^3*b^8*c^3*e*f + 384*a^4*b^6*c^4*d*g - 128*a^4*b^6*c^4*e*f - 2688*a^5*b^4*c^5*d*g + 384*a^5*b^4*c^5*e*f + 8192*a^6*b^2*c^6*d*g + 6*a^3*b^9*c^2*e* \\
& g - 128*a^4*b^7*c^3*e*g + 960*a^5*b^5*c^4*e*g - 3072*a^6*b^3*c^5*e*g + 6*a^3*c^2*e*g*(-(4*a*c - b^2)^9)^{(1/2)} + 36*a^4*b^8*c^2*f*g - 192*a^5*b^6*c^3*f* \\
& *g + 128*a^6*b^4*c^4*f*g + 1536*a^7*b^2*c^5*f*g + 2*a*b*c^3*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 2*a^3*b*c*f*g*(-(4*a*c - b^2)^9)^{(1/2))}/(32*(4096*a^9*c^9 + \\
& a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^{(1/2)}*(1024*a^5*b*c^6 - 16*a^2*b^7*c^3 + \\
& 192*a^3*b^5*c^4 - 768*a^4*b^3*c^5)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))*((27*a*b^9*c^4*d^2 - a^3*b^11*g^2 - b^11*c^3*d^2 + 3840*a^5*b*c^8*d^2 - 9*a*c^4*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + 768*a^6*b*c^7*e^2 + 768*a^7*b*c^6*f^2 + 27*a^4*b^9*c*g^2 + 3840*a^8*b*c^5*g^2 + 9*a^4*c*g^2*(-(4*a*c - b^2)^9)^{(1/2)} - 288*a^2*b^7*c^5*d^2 + 1504*a^3*b^5*c^6*d^2 - 3840*a^4*b^3*c^7*d^2 - a^2*b^9*c^3*e^2 + 96*a^4*b^5*c^5*e^2 - 512*a^5*b^3*c^6*e^2 + a^2*c^3*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + b^2*c^3*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - a^3*b^9*c^2*f^2 + 96*a^5*b^5*c^4*f^2 - 512*a^6*b^3*c^5*f^2 - a^3*b^2*g^2*(-(4*a*c - b^2)^9)^{(1/2)} - a^3*c^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 288*a^5*b^7*c^2*g^2 + 1504*a^6*b^5*c^3*g^2 - 3840*a^7*b^3*c^4*g^2 - 3072*a^6*c^8*d*e - 9216*a^7*c^7*d*g - 1024*a^7*c^7*e*f - 3072*a^8*c^6*f*g - 2*a*b^10*c^3*d*e + 3584*a^6*b*c^7*d*f + 3584*a^7*b*c^6*e*g - 2*a^3*b^10*c*f*g + 36*a^2*b^8*c^4*d*e - 192*a^3*b^6*c^5*d*e + 128*a^4*b^4*c^6*d*e + 1536*a^5*b^2*c^7*d*e + 6*a^2*b^9*c^3*d*f - 128*a^3*b^7*c^4*d*f + 960*a^4*b^5*c^5*d*f - 3072*a^5*b^3*c^6*d*f - 6*a^2*c^3*d*f*(-(4*a*c - b^2)^9)^{(1/2)} - 20*a^3*b^8*c^3*d*g + 12*a^3*b^8*c^3*e*f + 384*a^4*b^6*c^4*d*g - 128*a^4*b^6*c^4*e*f - 2688*a^5*b^4*c^5*d*g + 384*a^5*b^4*c^5*e*f + 8192*a^6*b^2*c^6*d*g + 6*a^3*b^9*c^2*e*g - 128*a^4*b^7*c^3*e*g + 960*a^5*b^5*c^4*e*g - 3072*a^6*b^3*c^5*e*g + 6*a^3*c^2*e*g*(-(4*a*c - b^2)^9)^{(1/2)} + 36*a^4*b^8*c^2*f*g - 192*a^5*b^6*c^3*f*g + 128*a^6*b^4*c^4*f*g + 1536*a^7*b^2*c^5*f*g + 2*a*b*c^3*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 2*a^3*b*c*f*g*(-(4*a*c - b^2)^9)^{(1/2))}/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^{(1/2)} - (x*(72*a^2*c^6*d^2 - 8*a^3*c^5*e^2 + b^4*c^4*d^2 + a^2*b^6*g^2 + 8*a^4*c^4*f^2 - 72*a^5*c^3*g^2 - 14*a*b^2*c^5*d^2 - 16*a^3*b^4*c*g^2 + 10*a^2*b^2*c^4*e^2 + a^2*b^4*c^2*f^2 + 2*a^3*b^2*c^3*f^2 + 74*a^4*b^2*c^2*g^2 + 48*a^3*c^5*d*f - 48*a^4*c^4*e*g + 2*a*b^3*c^4*d*e - 40*a^2*b*c^5*d*e - 72*a^3*b*c^4*d*g - 8*a^3*b*c^4*e*f + 2*a^2*b^5*c*f*g - 8*a^4*b*c^3*f*g + 4*a^2*b^2*c^4*d*f + 10*a^2*b^3*c^3*d*g - 6*a^2*b^3*c^3*e*f - 6*a^2*b^4*c^2*e*g + 52*a^3*b^2*c^3*e*g - 14*a^3*b^3*c^2*f*g))/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))*((27*a*b^9*c^4*d^2 - a^3*b^11*g^2 - b^11*c^3*d^2 + 3840*a^5*b*c^8*d^2 - 9*a*c^4*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + 768*a^6*b*c^7*e^2 + 768*a^7*b*c^6*f^2 + 27*a^4*b^9*c*g^2 + 3840*a^8*b*c^5*g^2 + 9*a^4*c*g^2*(-(4*a*c - b^2)^9)^{(1/2)} - 288*a^2*b^7*c^5*d^2 + 1504*a^3*b^5*c^6*d^2 - 3840*a^4*b^3*c^7*d^2 - a^2*b^9*c^3*e^2 + 96*a^4*b^5*c^5*e^2 - 512*a^5*b^3*c^6*e^2 + a^2*c^3*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + b^2*c^3*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - a^3*b^9*c^2*f^2 + 96*a^5*b^5*c^4*f^2 - 512*a^6*b^3*c^5*f^2 - a^3*b^2*g^2*(-(4*a*c - b^2)^9)^{(1/2)} - a^3*c^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 288*a^5*b^7*c^2*g^2 + 1504*a^6*b^5*c^3*g^2 - 3840*a^7*b^3*c^4*g^2 - 3072*a^6*c^8*d*e - 9216*a^7*c^7*d*g - 1024*a^7*c^7*e*f - 3072*a^8*c^6*f*g - 2*a*b^10*c^3*d*e + 3584*a^6*b*c^7*d*f + 3584*a^7*b*c^6*e*g - 2*a^3*b^10*c*f*g + 36*a^2*b^8*c^4*d*e - 192*a^3*b^6*c^5*d*e + 128*a^4*b^4*c^6*d*e + 1536*a^5*b^2*c^7*d*e + 6*a^2*b^9*c^3*d*f - 128*a^3*b^7*c^4*d*f + 960*a^4*b^5*c^5*d*f - 3072*a^5*b^3*c^6*d*f - 6*a^2*c^3*d*f*(-(4*a*c - b^2)^9)^{(1/2)} - 20*a^3*b^8*c^3*d*g + 12*a^3*b^8*c^3*e*f + 384*a^4*b^6*c^4*d*g - 128*a^4*b^6*c^4*e*f - 2688*a^5*b^4*c^5*d*g + 384*a^5*b^4*c^5*e*f + 8192*a^6*b^2*c^6*d*g + 6*a^3*b^9*c^2*e*g - 128*a^4*b^7*c^3*e*g + 960*a^5*b^5*c^4*e*g - 3072*a^6*b^3*c^5*e*g + 6*a^3*c^2*e*g*(-(4*a*c - b^2)^9)^{(1/2)} + 36*a^4*b^8*c^2*f*g - 192*a^5*b^6*c^3*f*g + 128*a^6*b^4*c^4*f*g + 1536*a^7*b^2*c^5*f*g
\end{aligned}$$

$$\begin{aligned}
& + 2*a*b*c^3*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 2*a^3*b*c*f*g*(-(4*a*c - b^2)^9)^{(1/2)} \\
& / (32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8))^{(1/2)} + (8*a^3*c^5*e^3 + 5*b^3*c^5*d^3 + 5*a^4*b^4*g^3 + 216*a^6*c^2*g^3 - 4*a^4*b*c^3*f^3 + 72*a^2*c^6*d^2*e - 66*a^5*b^2*c*g^3 - 3*b^4*c^4*d^2*e + a^2*b^6*e*g^2 + 216*a^3*c^5*d^2*g + 8*a^4*c^4*e*f^2 + b^5*c^3*d^2*f - 3*a^3*b^5*f*g^2 + 72*a^4*c^4*e^2*g + 216*a^5*c^3*e*g^2 + b^6*c^2*d^2*g + 24*a^5*c^3*f^2*g + 6*a^2*b^2*c^4*e^3 - 3*a^3*b^3*c^2*f^3 - 36*a*b*c^6*d^3 + a*b^7*d*g^2 + 48*a^3*c^5*d*e*f + 144*a^4*c^4*d*f*g + 18*a*b^2*c^5*d^2*e + 3*a*b^3*c^4*d*e^2 - 60*a^2*b*c^5*d^2*f - 28*a^3*b*c^4*d*f^2 - 10*a*b^4*c^3*d^2*g - 21*a^2*b^5*c*d*g^2 - 28*a^3*b*c^4*e^2*f - 396*a^4*b*c^3*d*g^2 - 12*a^3*b^4*c*e*g^2 - 6*a^3*b^4*c*f^2*g + 51*a^4*b^3*c*f*g^2 - 204*a^5*b*c^2*f*g^2 - 9*a^2*b^3*c^3*d*f^2 - 6*a^2*b^2*c^4*d^2*g - 5*a^2*b^3*c^3*e^2*f + a^2*b^4*c^2*e*f^2 + 18*a^3*b^2*c^3*e*f^2 + 155*a^3*b^3*c^2*d*g^2 - 5*a^2*b^4*c^2*e^2*g + 26*a^3*b^2*c^3*e^2*g + 2*a^4*b^2*c^2*e*g^2 + 42*a^4*b^2*c^2*f^2*g + 2*a*b^6*c*d*f*g - 4*a*b^4*c^3*d*e*f - 4*a*b^5*c^2*d*e*g - 312*a^3*b*c^4*d*e*g + 2*a^2*b^5*c*e*f*g - 152*a^4*b*c^3*e*f*g + 52*a^2*b^2*c^4*d*e*f + 70*a^2*b^3*c^3*d*e*g - 30*a^2*b^4*c^2*d*f*g + 100*a^3*b^2*c^3*d*f*g + 6*a^3*b^3*c^2*e*f*g) / (4*(64*a^5*c^4 - a^2*b^6*c + 12*a^3*b^4*c^2 - 48*a^4*b^2*c^3))) * ((27*a*b^9*c^4*d^2 - a^3*b^11*g^2 - b^11*c^3*d^2 + 3840*a^5*b*c^8*d^2 - 9*a*c^4*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + 768*a^6*b*c^7*e^2 + 768*a^7*b*c^6*f^2 + 27*a^4*b^9*c*g^2 + 3840*a^8*b*c^5*g^2 + 9*a^4*c*g^2*(-(4*a*c - b^2)^9)^{(1/2)} - 288*a^2*b^7*c^5*d^2 + 1504*a^3*b^5*c^6*d^2 - 3840*a^4*b^3*c^7*d^2 - a^2*b^9*c^3*e^2 + 96*a^4*b^5*c^5*e^2 - 512*a^5*b^3*c^6*e^2 + a^2*c^3*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + b^2*c^3*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - a^3*b^9*c^2*f^2 + 96*a^5*b^5*c^4*f^2 - 512*a^6*b^3*c^5*f^2 - a^3*b^2*g^2*(-(4*a*c - b^2)^9)^{(1/2)} - a^3*c^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 288*a^5*b^7*c^2*g^2 + 1504*a^6*b^5*c^3*g^2 - 3840*a^7*b^3*c^4*g^2 - 3072*a^6*c^8*d*e - 9216*a^7*c^7*d*g - 1024*a^7*c^7*e*f - 3072*a^8*c^6*f*g - 2*a*b^10*c^3*d*e + 3584*a^6*b*c^7*d*f + 3584*a^7*b*c^6*e*g - 2*a^3*b^10*c*f*g + 36*a^2*b^8*c^4*d*e - 192*a^3*b^6*c^5*d*e + 128*a^4*b^4*c^6*d*e + 1536*a^5*b^2*c^7*d*e + 6*a^2*b^9*c^3*d*f - 128*a^3*b^7*c^4*d*f + 960*a^4*b^5*c^5*d*f - 3072*a^5*b^3*c^6*d*f - 6*a^2*c^3*d*f*(-(4*a*c - b^2)^9)^{(1/2)} - 20*a^3*b^8*c^3*d*g + 12*a^3*b^8*c^3*e*f + 384*a^4*b^6*c^4*d*g - 128*a^4*b^6*c^4*e*f - 2688*a^5*b^4*c^5*d*g + 384*a^5*b^4*c^5*e*f + 8192*a^6*b^2*c^6*d*g + 6*a^3*b^9*c^2*e*g - 128*a^4*b^7*c^3*e*g + 960*a^5*b^5*c^4*e*g - 3072*a^6*b^3*c^5*e*g + 6*a^3*c^2*e*g*(-(4*a*c - b^2)^9)^{(1/2)} + 36*a^4*b^8*c^2*f*g - 192*a^5*b^6*c^3*f*g + 128*a^6*b^4*c^4*f*g + 1536*a^7*b^2*c^5*f*g + 2*a*b*c^3*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 2*a^3*b*c*f*g*(-(4*a*c - b^2)^9)^{(1/2)} / (32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8))^{(1/2)} * 2i - \text{atan}((((6144*a^5*c^7*d + 2048*a^6*c^6*f - 288*a^2*b^6*c^4*d + 1920*a^3*b^4*c^5*d - 5632*a^4*b^2*c^6*d + 16*a^2*b^7*c^3*e - 192*a^3*b^5*c^4*e + 768*a^4*b^3*c^5*e - 32*a^3*b^6*c^3*f + 384*a^4*b^4*c^4*f - 1536*a^5*b^2*c^5*f + 16*a^3*b^7*c^2*g - 192*a^4*b^5*c^3*g + 768*a^5*b^3*c^4*g + 16*a*b^8*c^3*d - 1024*a^5*b*c^6*e - 1024*a^6*b*c^5*g) / (8*(64*a^5*c^4 - a^2*b^6*c + 12*a^3*b^4*c^2 - 48*a^4*b^2*c^3)) - (x*((27*a*b^9*c^4*d^2 - a^3*b^11*g^2 - b^11*c^3*d^2 + 3840*a^5*b*c^8*d^2 + 9*a*c^4*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + 768*a^6*b*c^7*e^2 + 768*a^7*b*c^6*f^2 + 27*a^4*b^9*c*g^2 + 3840*a^8*b*c^5*g^2 - 9*a^4*c*g^2*(-(4*a*c - b^2)^9)^{(1/2)} - 288*a^2*b^7*c^5*d^2 + 1504*a^3*b^5*c^6*d^2 - 3840*a^4*b^3*c^7*d^2 - a^2*b^9*c^3*e^2 + 96*a^4*b^5*c^5*e^2 - 512*a^5*b^3*c^6*e^2 - a^2*c^3*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - b^2*c^3*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - a^3*b^9*c^2*f^2 + 96*a^5*b^5*c^4*f^2 - 512*a^6*b^3*c^5*f^2 + a^3*b^2*g^2*(-(4*a*c - b^2)^9)^{(1/2)} + a^3*c^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 288*a^5*b^7*c^2*g^2 + 1504*a^6*b^5*c^3*g^2 - 3840*a^7*b^3*c^4*g^2 - 3072*a^6*c^8*d*e - 9216*a^7*c^7*d*g - 1024*a^7*c^7*e*f - 3072*a^8*c^6*f*g - 2*a*b^10*c^3*d*e + 3584*a^6*b*c^7*d*f + 3584*a^7*b*c^6*e*g - 2*a^3*b^10*c*f*g + 36*a^2*b^8*c^4*d*e - 192*a^3*b^6*c^5*d*e + 128*a^4*b^4*c^6*d*e + 1536*a^5*b^2*c^7*d*e + 6*a^2*b^9*c^3*d*f - 128*a^3*b^7*c^4*d*f + 960*a^4*b^5*c^5*d
\end{aligned}$$

$$\begin{aligned}
& *f - 3072*a^5*b^3*c^6*d*f + 6*a^2*c^3*d*f*(-(4*a*c - b^2)^9)^{(1/2)} - 20*a^3 \\
& *b^8*c^3*d*g + 12*a^3*b^8*c^3*e*f + 384*a^4*b^6*c^4*d*g - 128*a^4*b^6*c^4*e \\
& *f - 2688*a^5*b^4*c^5*d*g + 384*a^5*b^4*c^5*e*f + 8192*a^6*b^2*c^6*d*g + 6* \\
& a^3*b^9*c^2*e*g - 128*a^4*b^7*c^3*e*g + 960*a^5*b^5*c^4*e*g - 3072*a^6*b^3* \\
& c^5*e*g - 6*a^3*c^2*e*g*(-(4*a*c - b^2)^9)^{(1/2)} + 36*a^4*b^8*c^2*f*g - 192 \\
& *a^5*b^6*c^3*f*g + 128*a^6*b^4*c^4*f*g + 1536*a^7*b^2*c^5*f*g - 2*a*b*c^3*d \\
& *e*(-(4*a*c - b^2)^9)^{(1/2)} + 2*a^3*b*c*f*g*(-(4*a*c - b^2)^9)^{(1/2)}/(32*(\\
& 4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6* \\
& b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8))^{(1/2)}*(1024*a^5*b*c^6 - 16 \\
& *a^2*b^7*c^3 + 192*a^3*b^5*c^4 - 768*a^4*b^3*c^5))/(2*(16*a^4*c^3 + a^2*b^4 \\
& *c - 8*a^3*b^2*c^2)))*((27*a*b^9*c^4*d^2 - a^3*b^11*g^2 - b^11*c^3*d^2 + 38 \\
& 40*a^5*b*c^8*d^2 + 9*a*c^4*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + 768*a^6*b*c^7*e^2 \\
& + 768*a^7*b*c^6*f^2 + 27*a^4*b^9*c*g^2 + 3840*a^8*b*c^5*g^2 - 9*a^4*c*g^2* \\
& (- (4*a*c - b^2)^9)^{(1/2)} - 288*a^2*b^7*c^5*d^2 + 1504*a^3*b^5*c^6*d^2 - 384 \\
& 0*a^4*b^3*c^7*d^2 - a^2*b^9*c^3*e^2 + 96*a^4*b^5*c^5*e^2 - 512*a^5*b^3*c^6* \\
& e^2 - a^2*c^3*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - b^2*c^3*d^2*(-(4*a*c - b^2)^9) \\
& ^{(1/2)} - a^3*b^9*c^2*f^2 + 96*a^5*b^5*c^4*f^2 - 512*a^6*b^3*c^5*f^2 + a^3*b \\
& ^2*g^2*(-(4*a*c - b^2)^9)^{(1/2)} + a^3*c^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 28 \\
& 8*a^5*b^7*c^2*g^2 + 1504*a^6*b^5*c^3*g^2 - 3840*a^7*b^3*c^4*g^2 - 3072*a^6* \\
& c^8*d*e - 9216*a^7*c^7*d*g - 1024*a^7*c^7*e*f - 3072*a^8*c^6*f*g - 2*a*b^10 \\
& *c^3*d*e + 3584*a^6*b*c^7*d*f + 3584*a^7*b*c^6*e*g - 2*a^3*b^10*c*f*g + 36* \\
& a^2*b^8*c^4*d*e - 192*a^3*b^6*c^5*d*e + 128*a^4*b^4*c^6*d*e + 1536*a^5*b^2* \\
& c^7*d*e + 6*a^2*b^9*c^3*d*f - 128*a^3*b^7*c^4*d*f + 960*a^4*b^5*c^5*d*f - 3 \\
& 072*a^5*b^3*c^6*d*f + 6*a^2*c^3*d*f*(-(4*a*c - b^2)^9)^{(1/2)} - 20*a^3*b^8*c \\
& ^3*d*g + 12*a^3*b^8*c^3*e*f + 384*a^4*b^6*c^4*d*g - 128*a^4*b^6*c^4*e*f - 2 \\
& 688*a^5*b^4*c^5*d*g + 384*a^5*b^4*c^5*e*f + 8192*a^6*b^2*c^6*d*g + 6*a^3*b^ \\
& 9*c^2*e*g - 128*a^4*b^7*c^3*e*g + 960*a^5*b^5*c^4*e*g - 3072*a^6*b^3*c^5*e* \\
& g - 6*a^3*c^2*e*g*(-(4*a*c - b^2)^9)^{(1/2)} + 36*a^4*b^8*c^2*f*g - 192*a^5*b \\
& ^6*c^3*f*g + 128*a^6*b^4*c^4*f*g + 1536*a^7*b^2*c^5*f*g - 2*a*b*c^3*d*e*(-(\\
& 4*a*c - b^2)^9)^{(1/2)} + 2*a^3*b*c*f*g*(-(4*a*c - b^2)^9)^{(1/2)}/(32*(4096*a \\
& ^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^ \\
& 6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8))^{(1/2)} + (x*(72*a^2*c^6*d^2 - 8*a \\
& ^3*c^5*e^2 + b^4*c^4*d^2 + a^2*b^6*g^2 + 8*a^4*c^4*f^2 - 72*a^5*c^3*g^2 - 1 \\
& 4*a*b^2*c^5*d^2 - 16*a^3*b^4*c*g^2 + 10*a^2*b^2*c^4*e^2 + a^2*b^4*c^2*f^2 + \\
& 2*a^3*b^2*c^3*f^2 + 74*a^4*b^2*c^2*g^2 + 48*a^3*c^5*d*f - 48*a^4*c^4*e*g + \\
& 2*a*b^3*c^4*d*e - 40*a^2*b*c^5*d*e - 72*a^3*b*c^4*d*g - 8*a^3*b*c^4*e*f + \\
& 2*a^2*b^5*c*f*g - 8*a^4*b*c^3*f*g + 4*a^2*b^2*c^4*d*f + 10*a^2*b^3*c^3*d*g \\
& - 6*a^2*b^3*c^3*e*f - 6*a^2*b^4*c^2*e*g + 52*a^3*b^2*c^3*e*g - 14*a^3*b^3*c \\
& ^2*f*g))/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))*((27*a*b^9*c^4*d^2 - \\
& a^3*b^11*g^2 - b^11*c^3*d^2 + 3840*a^5*b*c^8*d^2 + 9*a*c^4*d^2*(-(4*a*c - \\
& b^2)^9)^{(1/2)} + 768*a^6*b*c^7*e^2 + 768*a^7*b*c^6*f^2 + 27*a^4*b^9*c*g^2 + \\
& 3840*a^8*b*c^5*g^2 - 9*a^4*c*g^2*(-(4*a*c - b^2)^9)^{(1/2)} - 288*a^2*b^7*c^5 \\
& *d^2 + 1504*a^3*b^5*c^6*d^2 - 3840*a^4*b^3*c^7*d^2 - a^2*b^9*c^3*e^2 + 96*a \\
& ^4*b^5*c^5*e^2 - 512*a^5*b^3*c^6*e^2 - a^2*c^3*e^2*(-(4*a*c - b^2)^9)^{(1/2)} \\
& - b^2*c^3*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - a^3*b^9*c^2*f^2 + 96*a^5*b^5*c^4* \\
& f^2 - 512*a^6*b^3*c^5*f^2 + a^3*b^2*g^2*(-(4*a*c - b^2)^9)^{(1/2)} + a^3*c^2* \\
& f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 288*a^5*b^7*c^2*g^2 + 1504*a^6*b^5*c^3*g^2 - \\
& 3840*a^7*b^3*c^4*g^2 - 3072*a^6*c^8*d*e - 9216*a^7*c^7*d*g - 1024*a^7*c^7* \\
& e*f - 3072*a^8*c^6*f*g - 2*a*b^10*c^3*d*e + 3584*a^6*b*c^7*d*f + 3584*a^7*b \\
& *c^6*e*g - 2*a^3*b^10*c*f*g + 36*a^2*b^8*c^4*d*e - 192*a^3*b^6*c^5*d*e + 12 \\
& 8*a^4*b^4*c^6*d*e + 1536*a^5*b^2*c^7*d*e + 6*a^2*b^9*c^3*d*f - 128*a^3*b^7* \\
& c^4*d*f + 960*a^4*b^5*c^5*d*f - 3072*a^5*b^3*c^6*d*f + 6*a^2*c^3*d*f*(-(4*a \\
& *c - b^2)^9)^{(1/2)} - 20*a^3*b^8*c^3*d*g + 12*a^3*b^8*c^3*e*f + 384*a^4*b^6* \\
& c^4*d*g - 128*a^4*b^6*c^4*e*f - 2688*a^5*b^4*c^5*d*g + 384*a^5*b^4*c^5*e*f \\
& + 8192*a^6*b^2*c^6*d*g + 6*a^3*b^9*c^2*e*g - 128*a^4*b^7*c^3*e*g + 960*a^5* \\
& b^5*c^4*e*g - 3072*a^6*b^3*c^5*e*g - 6*a^3*c^2*e*g*(-(4*a*c - b^2)^9)^{(1/2)} \\
& + 36*a^4*b^8*c^2*f*g - 192*a^5*b^6*c^3*f*g + 128*a^6*b^4*c^4*f*g + 1536*a^ \\
& 7*b^2*c^5*f*g - 2*a*b*c^3*d*e*(-(4*a*c - b^2)^9)^{(1/2)} + 2*a^3*b*c*f*g*(-(4 \\
& *a*c - b^2)^9)^{(1/2)}/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 +
\end{aligned}$$

$$\begin{aligned}
& 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8))) \\
& ^{(1/2)*i - (((6144*a^5*c^7*d + 2048*a^6*c^6*f - 288*a^2*b^6*c^4*d + 1920*a \\
& ^3*b^4*c^5*d - 5632*a^4*b^2*c^6*d + 16*a^2*b^7*c^3*e - 192*a^3*b^5*c^4*e + \\
& 768*a^4*b^3*c^5*e - 32*a^3*b^6*c^3*f + 384*a^4*b^4*c^4*f - 1536*a^5*b^2*c^5 \\
& *f + 16*a^3*b^7*c^2*g - 192*a^4*b^5*c^3*g + 768*a^5*b^3*c^4*g + 16*a*b^8*c^ \\
& 3*d - 1024*a^5*b*c^6*e - 1024*a^6*b*c^5*g)/(8*(64*a^5*c^4 - a^2*b^6*c + 12* \\
& a^3*b^4*c^2 - 48*a^4*b^2*c^3)) + (x*((27*a*b^9*c^4*d^2 - a^3*b^11*g^2 - b^1 \\
& 1*c^3*d^2 + 3840*a^5*b*c^8*d^2 + 9*a*c^4*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + 768 \\
& *a^6*b*c^7*e^2 + 768*a^7*b*c^6*f^2 + 27*a^4*b^9*c*g^2 + 3840*a^8*b*c^5*g^2 \\
& - 9*a^4*c*g^2*(-(4*a*c - b^2)^9)^{(1/2)} - 288*a^2*b^7*c^5*d^2 + 1504*a^3*b^5 \\
& *c^6*d^2 - 3840*a^4*b^3*c^7*d^2 - a^2*b^9*c^3*e^2 + 96*a^4*b^5*c^5*e^2 - 51 \\
& 2*a^5*b^3*c^6*e^2 - a^2*c^3*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - b^2*c^3*d^2*(-(4 \\
& *a*c - b^2)^9)^{(1/2)} - a^3*b^9*c^2*f^2 + 96*a^5*b^5*c^4*f^2 - 512*a^6*b^3*c \\
& ^5*f^2 + a^3*b^2*g^2*(-(4*a*c - b^2)^9)^{(1/2)} + a^3*c^2*f^2*(-(4*a*c - b^2) \\
& ^9)^{(1/2)} - 288*a^5*b^7*c^2*g^2 + 1504*a^6*b^5*c^3*g^2 - 3840*a^7*b^3*c^4*g \\
& ^2 - 3072*a^6*c^8*d*e - 9216*a^7*c^7*d*g - 1024*a^7*c^7*e*f - 3072*a^8*c^6* \\
& f*g - 2*a*b^10*c^3*d*e + 3584*a^6*b*c^7*d*f + 3584*a^7*b*c^6*e*g - 2*a^3*b^ \\
& 10*c*f*g + 36*a^2*b^8*c^4*d*e - 192*a^3*b^6*c^5*d*e + 128*a^4*b^4*c^6*d*e + \\
& 1536*a^5*b^2*c^7*d*e + 6*a^2*b^9*c^3*d*f - 128*a^3*b^7*c^4*d*f + 960*a^4*b \\
& ^5*c^5*d*f - 3072*a^5*b^3*c^6*d*f + 6*a^2*c^3*d*f*(-(4*a*c - b^2)^9)^{(1/2)} \\
& - 20*a^3*b^8*c^3*d*g + 12*a^3*b^8*c^3*e*f + 384*a^4*b^6*c^4*d*g - 128*a^4*b \\
& ^6*c^4*e*f - 2688*a^5*b^4*c^5*d*g + 384*a^5*b^4*c^5*e*f + 8192*a^6*b^2*c^6* \\
& d*g + 6*a^3*b^9*c^2*e*g - 128*a^4*b^7*c^3*e*g + 960*a^5*b^5*c^4*e*g - 3072* \\
& a^6*b^3*c^5*e*g - 6*a^3*c^2*e*g*(-(4*a*c - b^2)^9)^{(1/2)} + 36*a^4*b^8*c^2*f \\
& *g - 192*a^5*b^6*c^3*f*g + 128*a^6*b^4*c^4*f*g + 1536*a^7*b^2*c^5*f*g - 2*a \\
& *b*c^3*d*e*(-(4*a*c - b^2)^9)^{(1/2)} + 2*a^3*b*c*f*g*(-(4*a*c - b^2)^9)^{(1/2)} \\
&))/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1 \\
& 280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^{(1/2)}*(1024*a^5*b* \\
& c^6 - 16*a^2*b^7*c^3 + 192*a^3*b^5*c^4 - 768*a^4*b^3*c^5))/(2*(16*a^4*c^3 + \\
& a^2*b^4*c - 8*a^3*b^2*c^2))))*((27*a*b^9*c^4*d^2 - a^3*b^11*g^2 - b^11*c^3* \\
& d^2 + 3840*a^5*b*c^8*d^2 + 9*a*c^4*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + 768*a^6*b \\
& *c^7*e^2 + 768*a^7*b*c^6*f^2 + 27*a^4*b^9*c*g^2 + 3840*a^8*b*c^5*g^2 - 9*a^ \\
& 4*c*g^2*(-(4*a*c - b^2)^9)^{(1/2)} - 288*a^2*b^7*c^5*d^2 + 1504*a^3*b^5*c^6*d \\
& ^2 - 3840*a^4*b^3*c^7*d^2 - a^2*b^9*c^3*e^2 + 96*a^4*b^5*c^5*e^2 - 512*a^5* \\
& b^3*c^6*e^2 - a^2*c^3*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - b^2*c^3*d^2*(-(4*a*c - \\
& b^2)^9)^{(1/2)} - a^3*b^9*c^2*f^2 + 96*a^5*b^5*c^4*f^2 - 512*a^6*b^3*c^5*f^2 \\
& + a^3*b^2*g^2*(-(4*a*c - b^2)^9)^{(1/2)} + a^3*c^2*f^2*(-(4*a*c - b^2)^9)^{(1 \\
& /2)} - 288*a^5*b^7*c^2*g^2 + 1504*a^6*b^5*c^3*g^2 - 3840*a^7*b^3*c^4*g^2 - 3 \\
& 072*a^6*c^8*d*e - 9216*a^7*c^7*d*g - 1024*a^7*c^7*e*f - 3072*a^8*c^6*f*g - \\
& 2*a*b^10*c^3*d*e + 3584*a^6*b*c^7*d*f + 3584*a^7*b*c^6*e*g - 2*a^3*b^10*c*f \\
& *g + 36*a^2*b^8*c^4*d*e - 192*a^3*b^6*c^5*d*e + 128*a^4*b^4*c^6*d*e + 1536* \\
& a^5*b^2*c^7*d*e + 6*a^2*b^9*c^3*d*f - 128*a^3*b^7*c^4*d*f + 960*a^4*b^5*c^5 \\
& *d*f - 3072*a^5*b^3*c^6*d*f + 6*a^2*c^3*d*f*(-(4*a*c - b^2)^9)^{(1/2)} - 20*a \\
& ^3*b^8*c^3*d*g + 12*a^3*b^8*c^3*e*f + 384*a^4*b^6*c^4*d*g - 128*a^4*b^6*c^4 \\
& *e*f - 2688*a^5*b^4*c^5*d*g + 384*a^5*b^4*c^5*e*f + 8192*a^6*b^2*c^6*d*g + \\
& 6*a^3*b^9*c^2*e*g - 128*a^4*b^7*c^3*e*g + 960*a^5*b^5*c^4*e*g - 3072*a^6*b^ \\
& 3*c^5*e*g - 6*a^3*c^2*e*g*(-(4*a*c - b^2)^9)^{(1/2)} + 36*a^4*b^8*c^2*f*g - 1 \\
& 92*a^5*b^6*c^3*f*g + 128*a^6*b^4*c^4*f*g + 1536*a^7*b^2*c^5*f*g - 2*a*b*c^3 \\
& *d*e*(-(4*a*c - b^2)^9)^{(1/2)} + 2*a^3*b*c*f*g*(-(4*a*c - b^2)^9)^{(1/2)))/(32 \\
& *(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^ \\
& 6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^{(1/2)} - (x*(72*a^2*c^6*d \\
& ^2 - 8*a^3*c^5*e^2 + b^4*c^4*d^2 + a^2*b^6*g^2 + 8*a^4*c^4*f^2 - 72*a^5*c^3 \\
& *g^2 - 14*a*b^2*c^5*d^2 - 16*a^3*b^4*c*g^2 + 10*a^2*b^2*c^4*e^2 + a^2*b^4*c \\
& ^2*f^2 + 2*a^3*b^2*c^3*f^2 + 74*a^4*b^2*c^2*g^2 + 48*a^3*c^5*d*f - 48*a^4*c \\
& ^4*e*g + 2*a*b^3*c^4*d*e - 40*a^2*b*c^5*d*e - 72*a^3*b*c^4*d*g - 8*a^3*b*c^ \\
& 4*e*f + 2*a^2*b^5*c*f*g - 8*a^4*b*c^3*f*g + 4*a^2*b^2*c^4*d*f + 10*a^2*b^3* \\
& c^3*d*g - 6*a^2*b^3*c^3*e*f - 6*a^2*b^4*c^2*e*g + 52*a^3*b^2*c^3*e*g - 14*a \\
& ^3*b^3*c^2*f*g))/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2))))*((27*a*b^9*c \\
& ^4*d^2 - a^3*b^11*g^2 - b^11*c^3*d^2 + 3840*a^5*b*c^8*d^2 + 9*a*c^4*d^2*(-(
\end{aligned}$$

$$\begin{aligned}
& 4*a*c - b^2)^9)^{(1/2)} + 768*a^6*b*c^7*e^2 + 768*a^7*b*c^6*f^2 + 27*a^4*b^9* \\
& c*g^2 + 3840*a^8*b*c^5*g^2 - 9*a^4*c*g^2*(-(4*a*c - b^2)^9)^{(1/2)} - 288*a^2 \\
& *b^7*c^5*d^2 + 1504*a^3*b^5*c^6*d^2 - 3840*a^4*b^3*c^7*d^2 - a^2*b^9*c^3*e^ \\
& 2 + 96*a^4*b^5*c^5*e^2 - 512*a^5*b^3*c^6*e^2 - a^2*c^3*e^2*(-(4*a*c - b^2)^ \\
& 9)^{(1/2)} - b^2*c^3*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - a^3*b^9*c^2*f^2 + 96*a^5* \\
& b^5*c^4*f^2 - 512*a^6*b^3*c^5*f^2 + a^3*b^2*g^2*(-(4*a*c - b^2)^9)^{(1/2)} + \\
& a^3*c^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 288*a^5*b^7*c^2*g^2 + 1504*a^6*b^5*c \\
& ^3*g^2 - 3840*a^7*b^3*c^4*g^2 - 3072*a^6*c^8*d*e - 9216*a^7*c^7*d*g - 1024* \\
& a^7*c^7*e*f - 3072*a^8*c^6*f*g - 2*a*b^10*c^3*d*e + 3584*a^6*b*c^7*d*f + 35 \\
& 84*a^7*b*c^6*e*g - 2*a^3*b^10*c*f*g + 36*a^2*b^8*c^4*d*e - 192*a^3*b^6*c^5* \\
& d*e + 128*a^4*b^4*c^6*d*e + 1536*a^5*b^2*c^7*d*e + 6*a^2*b^9*c^3*d*f - 128* \\
& a^3*b^7*c^4*d*f + 960*a^4*b^5*c^5*d*f - 3072*a^5*b^3*c^6*d*f + 6*a^2*c^3*d* \\
& f*(-(4*a*c - b^2)^9)^{(1/2)} - 20*a^3*b^8*c^3*d*g + 12*a^3*b^8*c^3*e*f + 384* \\
& a^4*b^6*c^4*d*g - 128*a^4*b^6*c^4*e*f - 2688*a^5*b^4*c^5*d*g + 384*a^5*b^4* \\
& c^5*e*f + 8192*a^6*b^2*c^6*d*g + 6*a^3*b^9*c^2*e*g - 128*a^4*b^7*c^3*e*g + \\
& 960*a^5*b^5*c^4*e*g - 3072*a^6*b^3*c^5*e*g - 6*a^3*c^2*e*g*(-(4*a*c - b^2)^ \\
& 9)^{(1/2)} + 36*a^4*b^8*c^2*f*g - 192*a^5*b^6*c^3*f*g + 128*a^6*b^4*c^4*f*g + \\
& 1536*a^7*b^2*c^5*f*g - 2*a*b*c^3*d*e*(-(4*a*c - b^2)^9)^{(1/2)} + 2*a^3*b*c* \\
& f*g*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^1 \\
& 0*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^ \\
& 2*c^8)))^{(1/2)}*i)/((((6144*a^5*c^7*d + 2048*a^6*c^6*f - 288*a^2*b^6*c^4*d \\
& + 1920*a^3*b^4*c^5*d - 5632*a^4*b^2*c^6*d + 16*a^2*b^7*c^3*e - 192*a^3*b^5* \\
& c^4*e + 768*a^4*b^3*c^5*e - 32*a^3*b^6*c^3*f + 384*a^4*b^4*c^4*f - 1536*a^5 \\
& *b^2*c^5*f + 16*a^3*b^7*c^2*g - 192*a^4*b^5*c^3*g + 768*a^5*b^3*c^4*g + 16* \\
& a*b^8*c^3*d - 1024*a^5*b*c^6*e - 1024*a^6*b*c^5*g)/(8*(64*a^5*c^4 - a^2*b^6 \\
& *c + 12*a^3*b^4*c^2 - 48*a^4*b^2*c^3)) - (x*((27*a*b^9*c^4*d^2 - a^3*b^11*g \\
& ^2 - b^11*c^3*d^2 + 3840*a^5*b*c^8*d^2 + 9*a*c^4*d^2*(-(4*a*c - b^2)^9)^{(1/ \\
& 2)} + 768*a^6*b*c^7*e^2 + 768*a^7*b*c^6*f^2 + 27*a^4*b^9*c*g^2 + 3840*a^8*b* \\
& c^5*g^2 - 9*a^4*c*g^2*(-(4*a*c - b^2)^9)^{(1/2)} - 288*a^2*b^7*c^5*d^2 + 1504 \\
& *a^3*b^5*c^6*d^2 - 3840*a^4*b^3*c^7*d^2 - a^2*b^9*c^3*e^2 + 96*a^4*b^5*c^5* \\
& e^2 - 512*a^5*b^3*c^6*e^2 - a^2*c^3*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - b^2*c^3* \\
& d^2*(-(4*a*c - b^2)^9)^{(1/2)} - a^3*b^9*c^2*f^2 + 96*a^5*b^5*c^4*f^2 - 512*a \\
& ^6*b^3*c^5*f^2 + a^3*b^2*g^2*(-(4*a*c - b^2)^9)^{(1/2)} + a^3*c^2*f^2*(-(4*a* \\
& c - b^2)^9)^{(1/2)} - 288*a^5*b^7*c^2*g^2 + 1504*a^6*b^5*c^3*g^2 - 3840*a^7*b \\
& ^3*c^4*g^2 - 3072*a^6*c^8*d*e - 9216*a^7*c^7*d*g - 1024*a^7*c^7*e*f - 3072* \\
& a^8*c^6*f*g - 2*a*b^10*c^3*d*e + 3584*a^6*b*c^7*d*f + 3584*a^7*b*c^6*e*g - \\
& 2*a^3*b^10*c*f*g + 36*a^2*b^8*c^4*d*e - 192*a^3*b^6*c^5*d*e + 128*a^4*b^4*c \\
& ^6*d*e + 1536*a^5*b^2*c^7*d*e + 6*a^2*b^9*c^3*d*f - 128*a^3*b^7*c^4*d*f + 9 \\
& 60*a^4*b^5*c^5*d*f - 3072*a^5*b^3*c^6*d*f + 6*a^2*c^3*d*f*(-(4*a*c - b^2)^9 \\
&)^{(1/2)} - 20*a^3*b^8*c^3*d*g + 12*a^3*b^8*c^3*e*f + 384*a^4*b^6*c^4*d*g - 1 \\
& 28*a^4*b^6*c^4*e*f - 2688*a^5*b^4*c^5*d*g + 384*a^5*b^4*c^5*e*f + 8192*a^6* \\
& b^2*c^6*d*g + 6*a^3*b^9*c^2*e*g - 128*a^4*b^7*c^3*e*g + 960*a^5*b^5*c^4*e*g \\
& - 3072*a^6*b^3*c^5*e*g - 6*a^3*c^2*e*g*(-(4*a*c - b^2)^9)^{(1/2)} + 36*a^4*b \\
& ^8*c^2*f*g - 192*a^5*b^6*c^3*f*g + 128*a^6*b^4*c^4*f*g + 1536*a^7*b^2*c^5*f \\
& *g - 2*a*b*c^3*d*e*(-(4*a*c - b^2)^9)^{(1/2)} + 2*a^3*b*c*f*g*(-(4*a*c - b^2) \\
& ^9)^{(1/2)})/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8 \\
& *c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^{(1/2)}*(102 \\
& 4*a^5*b*c^6 - 16*a^2*b^7*c^3 + 192*a^3*b^5*c^4 - 768*a^4*b^3*c^5))/(2*(16*a \\
& ^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2))*((27*a*b^9*c^4*d^2 - a^3*b^11*g^2 - b \\
& ^11*c^3*d^2 + 3840*a^5*b*c^8*d^2 + 9*a*c^4*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + 7 \\
& 68*a^6*b*c^7*e^2 + 768*a^7*b*c^6*f^2 + 27*a^4*b^9*c*g^2 + 3840*a^8*b*c^5*g^ \\
& 2 - 9*a^4*c*g^2*(-(4*a*c - b^2)^9)^{(1/2)} - 288*a^2*b^7*c^5*d^2 + 1504*a^3*b \\
& ^5*c^6*d^2 - 3840*a^4*b^3*c^7*d^2 - a^2*b^9*c^3*e^2 + 96*a^4*b^5*c^5*e^2 - \\
& 512*a^5*b^3*c^6*e^2 - a^2*c^3*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - b^2*c^3*d^2*(- \\
& (4*a*c - b^2)^9)^{(1/2)} - a^3*b^9*c^2*f^2 + 96*a^5*b^5*c^4*f^2 - 512*a^6*b^3 \\
& *c^5*f^2 + a^3*b^2*g^2*(-(4*a*c - b^2)^9)^{(1/2)} + a^3*c^2*f^2*(-(4*a*c - b^ \\
& 2)^9)^{(1/2)} - 288*a^5*b^7*c^2*g^2 + 1504*a^6*b^5*c^3*g^2 - 3840*a^7*b^3*c^4 \\
& *g^2 - 3072*a^6*c^8*d*e - 9216*a^7*c^7*d*g - 1024*a^7*c^7*e*f - 3072*a^8*c^ \\
& 6*f*g - 2*a*b^10*c^3*d*e + 3584*a^6*b*c^7*d*f + 3584*a^7*b*c^6*e*g - 2*a^3*
\end{aligned}$$

$$\begin{aligned}
& b^{10}c^*fg + 36a^2b^8c^4d^*e - 192a^3b^6c^5d^*e + 128a^4b^4c^6d^*e \\
& + 1536a^5b^2c^7d^*e + 6a^2b^9c^3d^*f - 128a^3b^7c^4d^*f + 960a^4 \\
& *b^5c^5d^*f - 3072a^5b^3c^6d^*f + 6a^2c^3d^*f*(-(4a*c - b^2)^9)^{(1/2)} \\
&) - 20a^3b^8c^3d^*g + 12a^3b^8c^3e^*f + 384a^4b^6c^4d^*g - 128a^4 \\
& *b^6c^4e^*f - 2688a^5b^4c^5d^*g + 384a^5b^4c^5e^*f + 8192a^6b^2c^6 \\
& d^*g + 6a^3b^9c^2e^*g - 128a^4b^7c^3e^*g + 960a^5b^5c^4e^*g - 307 \\
& 2a^6b^3c^5e^*g - 6a^3c^2e^*g*(-(4a*c - b^2)^9)^{(1/2)} + 36a^4b^8c^2 \\
& *f^*g - 192a^5b^6c^3f^*g + 128a^6b^4c^4f^*g + 1536a^7b^2c^5f^*g - 2 \\
& *a*b*c^3d^*e*(-(4a*c - b^2)^9)^{(1/2)} + 2a^3b*c^3f^*g*(-(4a*c - b^2)^9)^{(1 \\
& /2)))/(32*(4096a^9c^9 + a^3b^12c^3 - 24a^4b^10c^4 + 240a^5b^8c^5 - \\
& 1280a^6b^6c^6 + 3840a^7b^4c^7 - 6144a^8b^2c^8)))^{(1/2)} + (x*(72a \\
& ^2c^6d^2 - 8a^3c^5e^2 + b^4c^4d^2 + a^2b^6g^2 + 8a^4c^4f^2 - 72 \\
& *a^5c^3g^2 - 14a*b^2c^5d^2 - 16a^3b^4c^*g^2 + 10a^2b^2c^4e^2 + a \\
& ^2b^4c^2f^2 + 2a^3b^2c^3f^2 + 74a^4b^2c^2g^2 + 48a^3c^5d^*f - \\
& 48a^4c^4e^*g + 2a*b^3c^4d^*e - 40a^2b*c^5d^*e - 72a^3b*c^4d^*g - 8* \\
& a^3b*c^4e^*f + 2a^2b^5c^*f^*g - 8a^4b*c^3f^*g + 4a^2b^2c^4d^*f + 10* \\
& a^2b^3c^3d^*g - 6a^2b^3c^3e^*f - 6a^2b^4c^2e^*g + 52a^3b^2c^3e^* \\
& g - 14a^3b^3c^2f^*g))/(2*(16a^4c^3 + a^2b^4c - 8a^3b^2c^2)))*((27 \\
& *a*b^9c^4d^2 - a^3b^11g^2 - b^11c^3d^2 + 3840a^5b*c^8d^2 + 9a*c^4 \\
& *d^2*(-(4a*c - b^2)^9)^{(1/2)} + 768a^6b*c^7e^2 + 768a^7b*c^6f^2 + 27* \\
& a^4b^9c^*g^2 + 3840a^8b*c^5g^2 - 9a^4c^*g^2*(-(4a*c - b^2)^9)^{(1/2)} - \\
& 288a^2b^7c^5d^2 + 1504a^3b^5c^6d^2 - 3840a^4b^3c^7d^2 - a^2b^ \\
& 9c^3e^2 + 96a^4b^5c^5e^2 - 512a^5b^3c^6e^2 - a^2c^3e^2*(-(4a*c \\
& - b^2)^9)^{(1/2)} - b^2c^3d^2*(-(4a*c - b^2)^9)^{(1/2)} - a^3b^9c^2f^2 + \\
& 96a^5b^5c^4f^2 - 512a^6b^3c^5f^2 + a^3b^2g^2*(-(4a*c - b^2)^9)^{ \\
& (1/2)} + a^3c^2f^2*(-(4a*c - b^2)^9)^{(1/2)} - 288a^5b^7c^2g^2 + 1504a \\
& ^6b^5c^3g^2 - 3840a^7b^3c^4g^2 - 3072a^6c^8d^*e - 9216a^7c^7d^*g \\
& - 1024a^7c^7e^*f - 3072a^8c^6f^*g - 2a*b^10c^3d^*e + 3584a^6b*c^7* \\
& d^*f + 3584a^7b*c^6e^*g - 2a^3b^10c^*f^*g + 36a^2b^8c^4d^*e - 192a^3* \\
& b^6c^5d^*e + 128a^4b^4c^6d^*e + 1536a^5b^2c^7d^*e + 6a^2b^9c^3d^* \\
& f - 128a^3b^7c^4d^*f + 960a^4b^5c^5d^*f - 3072a^5b^3c^6d^*f + 6a^ \\
& 2c^3d^*f*(-(4a*c - b^2)^9)^{(1/2)} - 20a^3b^8c^3d^*g + 12a^3b^8c^3e^* \\
& f + 384a^4b^6c^4d^*g - 128a^4b^6c^4e^*f - 2688a^5b^4c^5d^*g + 384* \\
& a^5b^4c^5e^*f + 8192a^6b^2c^6d^*g + 6a^3b^9c^2e^*g - 128a^4b^7c^ \\
& 3e^*g + 960a^5b^5c^4e^*g - 3072a^6b^3c^5e^*g - 6a^3c^2e^*g*(-(4a*c \\
& - b^2)^9)^{(1/2)} + 36a^4b^8c^2f^*g - 192a^5b^6c^3f^*g + 128a^6b^4c^ \\
& ^4f^*g + 1536a^7b^2c^5f^*g - 2a*b*c^3d^*e*(-(4a*c - b^2)^9)^{(1/2)} + 2* \\
& a^3b*c^3f^*g*(-(4a*c - b^2)^9)^{(1/2)))/(32*(4096a^9c^9 + a^3b^12c^3 - 24 \\
& *a^4b^10c^4 + 240a^5b^8c^5 - 1280a^6b^6c^6 + 3840a^7b^4c^7 - 614 \\
& 4a^8b^2c^8)))^{(1/2)} + (((6144a^5c^7*d + 2048a^6c^6*f - 288a^2b^6c^ \\
& ^4*d + 1920a^3b^4c^5*d - 5632a^4b^2c^6*d + 16a^2b^7c^3*e - 192a^3 \\
& *b^5c^4*e + 768a^4b^3c^5*e - 32a^3b^6c^3*f + 384a^4b^4c^4*f - 153 \\
& 6a^5b^2c^5*f + 16a^3b^7c^2*g - 192a^4b^5c^3*g + 768a^5b^3c^4*g \\
& + 16a*b^8c^3*d - 1024a^5b*c^6*e - 1024a^6b*c^5*g)/(8*(64a^5c^4 - a^ \\
& 2b^6c + 12a^3b^4c^2 - 48a^4b^2c^3)) + (x*((27a*b^9c^4d^2 - a^3b \\
& ^11g^2 - b^11c^3d^2 + 3840a^5b*c^8d^2 + 9a*c^4d^2*(-(4a*c - b^2)^9 \\
&)^{(1/2)} + 768a^6b*c^7e^2 + 768a^7b*c^6f^2 + 27a^4b^9c^*g^2 + 3840a \\
& ^8b*c^5g^2 - 9a^4c^*g^2*(-(4a*c - b^2)^9)^{(1/2)} - 288a^2b^7c^5d^2 + \\
& 1504a^3b^5c^6d^2 - 3840a^4b^3c^7d^2 - a^2b^9c^3e^2 + 96a^4b^5 \\
& *c^5e^2 - 512a^5b^3c^6e^2 - a^2c^3e^2*(-(4a*c - b^2)^9)^{(1/2)} - b^2 \\
& *c^3d^2*(-(4a*c - b^2)^9)^{(1/2)} - a^3b^9c^2f^2 + 96a^5b^5c^4f^2 - \\
& 512a^6b^3c^5f^2 + a^3b^2g^2*(-(4a*c - b^2)^9)^{(1/2)} + a^3c^2f^2*(- \\
& (4a*c - b^2)^9)^{(1/2)} - 288a^5b^7c^2g^2 + 1504a^6b^5c^3g^2 - 3840* \\
& a^7b^3c^4g^2 - 3072a^6c^8d^*e - 9216a^7c^7d^*g - 1024a^7c^7e^*f - \\
& 3072a^8c^6f^*g - 2a*b^10c^3d^*e + 3584a^6b*c^7d^*f + 3584a^7b*c^6e \\
& *g - 2a^3b^10c^*f^*g + 36a^2b^8c^4d^*e - 192a^3b^6c^5d^*e + 128a^4* \\
& b^4c^6d^*e + 1536a^5b^2c^7d^*e + 6a^2b^9c^3d^*f - 128a^3b^7c^4d^* \\
& f + 960a^4b^5c^5d^*f - 3072a^5b^3c^6d^*f + 6a^2c^3d^*f*(-(4a*c - b \\
& ^2)^9)^{(1/2)} - 20a^3b^8c^3d^*g + 12a^3b^8c^3e^*f + 384a^4b^6c^4d^*
\end{aligned}$$

$$\begin{aligned}
&g - 128a^4b^6c^4ef - 2688a^5b^4c^5d^2g + 384a^5b^4c^5ef + 8192 \\
&a^6b^2c^6d^2g + 6a^3b^9c^2e^2g - 128a^4b^7c^3e^2g + 960a^5b^5c^4 \\
&4e^2g - 3072a^6b^3c^5e^2g - 6a^3c^2e^2g(-4ac - b^2)^9)^{(1/2)} + 36 \\
&a^4b^8c^2f^2g - 192a^5b^6c^3f^2g + 128a^6b^4c^4f^2g + 1536a^7b^2c^5 \\
&c^5f^2g - 2a^3b^2c^3d^2e^2(-4ac - b^2)^9)^{(1/2)} + 2a^3b^2c^3d^2e^2(-4ac - \\
&b^2)^9)^{(1/2)} / (32(4096a^9c^9 + a^3b^12c^3 - 24a^4b^10c^4 + 240a^5 \\
&5b^8c^5 - 1280a^6b^6c^6 + 3840a^7b^4c^7 - 6144a^8b^2c^8))^{(1/2)} \\
&*(1024a^5b^2c^6 - 16a^2b^7c^3 + 192a^3b^5c^4 - 768a^4b^3c^5) / (2 \\
&(16a^4c^3 + a^2b^4c - 8a^3b^2c^2)) * ((27a^3b^9c^4d^2 - a^3b^11g^2 \\
&2 - b^11c^3d^2 + 3840a^5b^2c^8d^2 + 9a^3c^4d^2(-4ac - b^2)^9)^{(1/2)} \\
&+ 768a^6b^2c^7e^2 + 768a^7b^2c^6f^2 + 27a^4b^9c^2g^2 + 3840a^8b^2c^5 \\
&5g^2 - 9a^4c^2g^2(-4ac - b^2)^9)^{(1/2)} - 288a^2b^7c^5d^2 + 1504a^3 \\
&a^3b^5c^6d^2 - 3840a^4b^3c^7d^2 - a^2b^9c^3e^2 + 96a^4b^5c^5e^2 - 512a^5 \\
&a^5b^3c^6e^2 - a^2c^3e^2(-4ac - b^2)^9)^{(1/2)} - b^2c^3d^2(-4ac - b^2)^9)^{(1/2)} \\
&- a^3b^9c^2f^2 + 96a^5b^5c^4f^2 - 512a^6b^3c^5f^2 + a^3b^2g^2(-4ac - b^2)^9)^{(1/2)} \\
&+ a^3c^2f^2(-4ac - b^2)^9)^{(1/2)} - 288a^5b^7c^2g^2 + 1504a^6b^5c^3g^2 - 3840a^7b^3 \\
&3c^4g^2 - 3072a^6c^8d^2e - 9216a^7c^7d^2g - 1024a^7c^7e^2f - 3072a^8 \\
&a^8c^6f^2g - 2a^3b^10c^3d^2e + 3584a^6b^2c^7d^2f + 3584a^7b^2c^6e^2g - 2 \\
&a^3b^10c^3f^2g + 36a^2b^8c^4d^2e - 192a^3b^6c^5d^2e + 128a^4b^4c^6 \\
&6d^2e + 1536a^5b^2c^7d^2e + 6a^2b^9c^3d^2f - 128a^3b^7c^4d^2f + 96 \\
&0a^4b^5c^5d^2f - 3072a^5b^3c^6d^2f + 6a^2c^3d^2f(-4ac - b^2)^9)^{(1/2)} \\
&- 20a^3b^8c^3d^2g + 12a^3b^8c^3e^2f + 384a^4b^6c^4d^2g - 12 \\
&8a^4b^6c^4e^2f - 2688a^5b^4c^5d^2g + 384a^5b^4c^5e^2f + 8192a^6b^2c^6 \\
&a^6b^2c^6d^2g + 6a^3b^9c^2e^2g - 128a^4b^7c^3e^2g + 960a^5b^5c^4e^2g \\
&- 3072a^6b^3c^5e^2g - 6a^3c^2e^2g(-4ac - b^2)^9)^{(1/2)} + 36a^4b^8 \\
&8c^2f^2g - 192a^5b^6c^3f^2g + 128a^6b^4c^4f^2g + 1536a^7b^2c^5f^2g \\
&g - 2a^3b^2c^3d^2e^2(-4ac - b^2)^9)^{(1/2)} + 2a^3b^2c^3d^2e^2(-4ac - b^2)^9 \\
&9)^{(1/2)} / (32(4096a^9c^9 + a^3b^12c^3 - 24a^4b^10c^4 + 240a^5b^8c^5 - 1280a^6 \\
&b^6c^6 + 3840a^7b^4c^7 - 6144a^8b^2c^8))^{(1/2)} - (x \\
&(72a^2c^6d^2 - 8a^3c^5e^2 + b^4c^4d^2 + a^2b^6g^2 + 8a^4c^4f^2 \\
&- 72a^5c^3g^2 - 14a^2b^2c^5d^2 - 16a^3b^4c^2g^2 + 10a^2b^2c^4e^2 \\
&2 + a^2b^4c^2f^2 + 2a^3b^2c^3f^2 + 74a^4b^2c^2g^2 + 48a^3c^5d \\
&>f - 48a^4c^4e^2g + 2a^3b^3c^4d^2e - 40a^2b^2c^5d^2e - 72a^3b^2c^4d^2g \\
&- 8a^3b^2c^4e^2f + 2a^2b^5c^2f^2g - 8a^4b^2c^3f^2g + 4a^2b^2c^4d^2f \\
&+ 10a^2b^3c^3d^2g - 6a^2b^3c^3e^2f - 6a^2b^4c^2e^2g + 52a^3b^2c^3 \\
&>e^2g - 14a^3b^3c^2f^2g) / (2(16a^4c^3 + a^2b^4c - 8a^3b^2c^2)) \\
&*((27a^3b^9c^4d^2 - a^3b^11g^2 - b^11c^3d^2 + 3840a^5b^2c^8d^2 + 9 \\
&>a^3c^4d^2(-4ac - b^2)^9)^{(1/2)} + 768a^6b^2c^7e^2 + 768a^7b^2c^6f^2 \\
&+ 27a^4b^9c^2g^2 + 3840a^8b^2c^5g^2 - 9a^4c^2g^2(-4ac - b^2)^9)^{(1/2)} \\
&- 288a^2b^7c^5d^2 + 1504a^3b^5c^6d^2 - 3840a^4b^3c^7d^2 - a^2b^9c^3e^2 \\
&+ 96a^4b^5c^5e^2 - 512a^5b^3c^6e^2 - a^2c^3e^2(-4ac - b^2)^9)^{(1/2)} - b^2c^3 \\
&>d^2(-4ac - b^2)^9)^{(1/2)} - a^3b^9c^2f^2 + 96a^5b^5c^4f^2 - 512a^6b^3c^5f^2 \\
&+ a^3b^2g^2(-4ac - b^2)^9)^{(1/2)} + a^3c^2f^2(-4ac - b^2)^9)^{(1/2)} - 288a^5 \\
&b^7c^2g^2 + 1504a^6b^5c^3g^2 - 3840a^7b^3c^4g^2 - 3072a^6c^8d^2e - 9216a^7 \\
&>c^7d^2g - 1024a^7c^7e^2f - 3072a^8c^6f^2g - 2a^3b^10c^3d^2e + 3584a^6b^2 \\
&>c^7d^2f + 3584a^7b^2c^6e^2g - 2a^3b^10c^3f^2g + 36a^2b^8c^4d^2e - 192 \\
&>a^3b^6c^5d^2e + 128a^4b^4c^6d^2e + 1536a^5b^2c^7d^2e + 6a^2b^9c^3d^2f \\
&- 128a^3b^7c^4d^2f + 960a^4b^5c^5d^2f - 3072a^5b^3c^6d^2f + 6a^2c^3d^2f \\
&>f(-4ac - b^2)^9)^{(1/2)} - 20a^3b^8c^3d^2g + 12a^3b^8c^3e^2f + 384a^4b^6c^4 \\
&>d^2g - 128a^4b^6c^4e^2f - 2688a^5b^4c^5d^2g + 384a^5b^4c^5e^2f + 8192a^6b^2c^6 \\
&>d^2g + 6a^3b^9c^2e^2g - 128a^4b^7c^3e^2g + 960a^5b^5c^4e^2g - 3072a^6b^3c^5 \\
&>e^2g - 6a^3c^2e^2g(-4ac - b^2)^9)^{(1/2)} + 36a^4b^8c^2f^2g - 192a^5b^6c^3f^2g \\
&+ 128a^6b^4c^4f^2g + 1536a^7b^2c^5f^2g - 2a^3b^2c^3d^2e^2(-4ac - b^2)^9)^{(1/2)} \\
&+ 2a^3b^2c^3d^2e^2(-4ac - b^2)^9)^{(1/2)} / (32(4096a^9c^9 + a^3b^12c^3 - 24a^4b^10c^4 \\
&+ 240a^5b^8c^5 - 1280a^6b^6c^6 + 3840a^7b^4c^7 - 6144a^8b^2c^8))^{(1/2)} + (8a^3c^5e^3 \\
&+ 5b^3c^5d^3 + 5a^4b^4g^2
\end{aligned}$$

$$\begin{aligned}
& 3 + 216a^6c^2g^3 - 4a^4b^3c^3f^3 + 72a^2c^6d^2e - 66a^5b^2c^3g^3 \\
& - 3b^4c^4d^2e + a^2b^6e^2g^2 + 216a^3c^5d^2g + 8a^4c^4e^2f^2 + \\
& b^5c^3d^2f - 3a^3b^5f^2g^2 + 72a^4c^4e^2g + 216a^5c^3e^2g^2 + b^6 \\
& c^2d^2g + 24a^5c^3f^2g + 6a^2b^2c^4e^3 - 3a^3b^3c^2f^3 - 36 \\
& a^2b^3c^6d^3 + a^2b^7d^2g^2 + 48a^3c^5d^2e^2f + 144a^4c^4d^2f^2g + 18a^2b^2 \\
& c^5d^2e + 3a^2b^3c^4d^2e^2 - 60a^2b^3c^5d^2e^2 - a^2b^3c^4d^2e^2f + a^2 \\
& b^5c^2d^2f^2 - 60a^2b^3c^5d^2e^2f - 28a^3b^3c^4d^2f^2 - 10a^2b^4c^3d^2 \\
& g - 21a^2b^5c^3d^2g^2 - 28a^3b^3c^4e^2f^2 - 396a^4b^3c^3d^2g^2 - 12a^3b^4 \\
& c^2e^2g^2 - 6a^3b^4c^2f^2g + 51a^4b^3c^2f^2g^2 - 204a^5b^3c^2f^2g^2 \\
& - 9a^2b^3c^3d^2f^2 - 6a^2b^2c^4d^2g - 5a^2b^3c^3e^2f + a^2b^4 \\
& c^2e^2f^2 + 18a^3b^2c^3e^2f^2 + 155a^3b^3c^2d^2g^2 - 5a^2b^4c^2e^2 \\
& ^2g + 26a^3b^2c^3e^2g + 2a^4b^2c^2e^2g^2 + 42a^4b^2c^2f^2g + \\
& 2a^2b^6c^2d^2f^2g - 4a^2b^4c^3d^2e^2f - 4a^2b^5c^2d^2e^2g - 312a^3b^3c^4d^2e \\
& ^2g + 2a^2b^5c^2e^2f^2g - 152a^4b^3c^3e^2f^2g + 52a^2b^2c^4d^2e^2f + 70a^2 \\
& b^3c^3d^2e^2g - 30a^2b^4c^2d^2f^2g + 100a^3b^2c^3d^2f^2g + 6a^3b^3c^2 \\
& c^2e^2f^2g)/(4*(64a^5c^4 - a^2b^6c + 12a^3b^4c^2 - 48a^4b^2c^3))) \\
& *((27a^9b^9c^4d^2 - a^3b^11g^2 - b^11c^3d^2 + 3840a^5b^8c^8d^2 + 9a \\
& a^4c^4d^2*(-(4a^2c - b^2)^9)^{(1/2)} + 768a^6b^7c^7e^2 + 768a^7b^6c^6f^2 \\
& + 27a^4b^9c^3g^2 + 3840a^8b^5c^5g^2 - 9a^4c^3g^2*(-(4a^2c - b^2)^9)^{(1/2)} \\
& - 288a^2b^7c^5d^2 + 1504a^3b^5c^6d^2 - 3840a^4b^3c^7d^2 - a^2b^9c^3e^2 \\
& + 96a^4b^5c^5e^2 - 512a^5b^3c^6e^2 - a^2c^3e^2*(-(4a^2c - b^2)^9)^{(1/2)} \\
& - b^2c^3d^2*(-(4a^2c - b^2)^9)^{(1/2)} - a^3b^9c^2f^2 + 96a^5b^5c^4f^2 \\
& - 512a^6b^3c^5f^2 + a^3b^2g^2*(-(4a^2c - b^2)^9)^{(1/2)} + a^3c^2f^2 \\
& *(-(4a^2c - b^2)^9)^{(1/2)} - 288a^5b^7c^2g^2 + 1504a^6b^5c^3g^2 - 3840a^7b^3c^4g^2 \\
& - 3072a^6c^8d^2e - 9216a^7c^7d^2g - 1024a^7c^7e^2f - 3072a^8c^6f^2g - 2a^2b^10c^3d^2e \\
& + 3584a^6b^7c^7d^2f + 3584a^7b^6c^6e^2g - 2a^3b^10c^3f^2g + 36a^2b^8c^4d^2e - 192 \\
& a^3b^6c^5d^2e + 128a^4b^4c^6d^2e + 1536a^5b^2c^7d^2e + 6a^2b^9c^3d^2f \\
& - 128a^3b^7c^4d^2f + 960a^4b^5c^5d^2f - 3072a^5b^3c^6d^2f + 6a^2c^3d^2f^2 \\
& *(-(4a^2c - b^2)^9)^{(1/2)} - 20a^3b^8c^3d^2g + 12a^3b^8c^3e^2f \\
& + 384a^4b^6c^4d^2g - 128a^4b^6c^4e^2f - 2688a^5b^4c^5d^2g + 384a^5b^4c^5e^2f \\
& + 8192a^6b^2c^6d^2g + 6a^3b^9c^2e^2g - 128a^4b^7c^3e^2g + 960a^5b^5c^4e^2g \\
& - 3072a^6b^3c^5e^2g - 6a^3c^2e^2g*(-(4a^2c - b^2)^9)^{(1/2)} + 36a^4b^8c^2f^2g \\
& - 192a^5b^6c^3f^2g + 128a^6b^4c^4f^2g + 1536a^7b^2c^5f^2g - 2a^2b^3c^3d^2e \\
& *(-(4a^2c - b^2)^9)^{(1/2)} + 2a^3b^3c^3f^2g*(-(4a^2c - b^2)^9)^{(1/2)})/(32*(4096a^9c^9 \\
& + a^3b^12c^3 - 24a^4b^10c^4 + 240a^5b^8c^5 - 1280a^6b^6c^6 + 3840a^7b^4c^7 \\
& - 6144a^8b^2c^8)))^{(1/2)}*2i
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x**6+f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)

[Out] Timed out

$$3.127 \quad \int \frac{d+ex^2+fx^4+gx^6}{x^2(a+bx^2+cx^4)^2} dx$$

Optimal. Leaf size=460

$$\frac{x \left(a \left(-2a^2g + \frac{b^3d}{a} + a(bf + 2ce) - b(be + 3cd) \right) + x^2 \left(-ab(ag + ce) - 2ac(cd - af) + b^2cd \right) \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{2a^2 (b^2 - 4ac) (a + bx^2 + cx^4)}$$

Rubi [A] time = 2.79, antiderivative size = 460, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {1669, 1664, 1166, 205}

$$\frac{x \left(a \left(-2a^2g + \frac{b^3d}{a} + a(bf + 2ce) - b(be + 3cd) \right) + x^2 \left(-ab(ag + ce) - 2ac(cd - af) + b^2cd \right) \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{2a^2 (b^2 - 4ac) (a + bx^2 + cx^4)}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2 + f*x^4 + g*x^6)/(x^2*(a + b*x^2 + c*x^4)^2), x]

[Out] -(d/(a^2*x)) - (x*(a*((b^3*d)/a - b*(3*c*d + b*e) + a*(2*c*e + b*f) - 2*a^2*g) + (b^2*c*d - 2*a*c*(c*d - a*f) - a*b*(c*e + a*g))*x^2)/(2*a^2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((3*b^2*c*d - 2*a*c*(5*c*d - a*f) - a*b*(c*e + a*g) + (3*b^3*c*d - 4*a*b*c*(4*c*d + a*f) - a*b^2*(c*e - a*g) + 4*a^2*c*(3*c*e + a*g))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/(2*Sqrt[2]*a^2*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((3*b^2*c*d - 2*a*c*(5*c*d - a*f) - a*b*(c*e + a*g) - (3*b^3*c*d - 4*a*b*c*(4*c*d + a*f) - a*b^2*(c*e - a*g) + 4*a^2*c*(3*c*e + a*g))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]/(2*Sqrt[2]*a^2*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1664

Int[(Pq_)*((d_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]

Rule 1669

Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2

```
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rubi steps

$$\int \frac{d + ex^2 + fx^4 + gx^6}{x^2(a + bx^2 + cx^4)^2} dx = -\frac{x \left(a \left(\frac{b^3 d}{a} - b(3cd + be) + a(2ce + bf) - 2a^2 g \right) + (b^2 cd - 2ac(cd - af) - ab(ce + af) - a^2 g) \right)}{2a^2(b^2 - 4ac)(a + bx^2 + cx^4)}$$

$$= -\frac{x \left(a \left(\frac{b^3 d}{a} - b(3cd + be) + a(2ce + bf) - 2a^2 g \right) + (b^2 cd - 2ac(cd - af) - ab(ce + af) - a^2 g) \right)}{2a^2(b^2 - 4ac)(a + bx^2 + cx^4)}$$

$$= -\frac{d}{a^2 x} - \frac{x \left(a \left(\frac{b^3 d}{a} - b(3cd + be) + a(2ce + bf) - 2a^2 g \right) + (b^2 cd - 2ac(cd - af) - ab(ce + af) - a^2 g) \right)}{2a^2(b^2 - 4ac)(a + bx^2 + cx^4)}$$

$$= -\frac{d}{a^2 x} - \frac{x \left(a \left(\frac{b^3 d}{a} - b(3cd + be) + a(2ce + bf) - 2a^2 g \right) + (b^2 cd - 2ac(cd - af) - ab(ce + af) - a^2 g) \right)}{2a^2(b^2 - 4ac)(a + bx^2 + cx^4)}$$

$$= -\frac{d}{a^2 x} - \frac{x \left(a \left(\frac{b^3 d}{a} - b(3cd + be) + a(2ce + bf) - 2a^2 g \right) + (b^2 cd - 2ac(cd - af) - ab(ce + af) - a^2 g) \right)}{2a^2(b^2 - 4ac)(a + bx^2 + cx^4)}$$

Mathematica [A] time = 2.43, size = 529, normalized size = 1.15

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2 + f*x^4 + g*x^6)/(x^2*(a + b*x^2 + c*x^4)^2), x]
[Out] -1/4*((4*d)/x - (2*x*(-(b^3*d) + b^2*(a*e - c*d*x^2) + a*b*(3*c*d - a*f + c
*e*x^2 + a*g*x^2) + 2*a*(a^2*g + c^2*d*x^2 - a*c*(e + f*x^2))))/(b^2 - 4*a
*c)*(a + b*x^2 + c*x^4) + (Sqrt[2]*(3*b^3*c*d + b^2*(3*c*Sqrt[b^2 - 4*a*c]
*d - a*c*e + a^2*g) + 2*a*c*(-5*c*Sqrt[b^2 - 4*a*c]*d + 6*a*c*e + a*Sqrt[b^
2 - 4*a*c]*f + 2*a^2*g) - a*b*(16*c^2*d + c*Sqrt[b^2 - 4*a*c]*e + 4*a*c*f +
a*Sqrt[b^2 - 4*a*c]*g))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a
*c]]]/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]
*(-3*b^3*c*d + b^2*(3*c*Sqrt[b^2 - 4*a*c]*d + a*c*e - a^2*g) - 2*a*c*(5*c*S
qrt[b^2 - 4*a*c]*d + 6*a*c*e - a*Sqrt[b^2 - 4*a*c]*f + 2*a^2*g) + a*b*(16*c
^2*d - c*Sqrt[b^2 - 4*a*c]*e + 4*a*c*f - a*Sqrt[b^2 - 4*a*c]*g))*ArcTan[(Sq
rt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)
*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/a^2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex^2 + fx^4 + gx^6}{x^2(a + bx^2 + cx^4)^2} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x^2 + f*x^4 + g*x^6)/(x^2*(a + b*x^2 + c*x^4)^2), x]
```

[Out] IntegrateAlgebraic[(d + e*x^2 + f*x^4 + g*x^6)/(x^2*(a + b*x^2 + c*x^4)^2), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^6+f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")

[Out] Timed out

giac [B] time = 8.30, size = 9176, normalized size = 19.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^6+f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="giac")

[Out]
$$-1/2*(3*b^2*c*d*x^4 - 10*a*c^2*d*x^4 + 2*a^2*c*f*x^4 - a^2*b*g*x^4 - a*b*c*x^4*e + 3*b^3*d*x^2 - 11*a*b*c*d*x^2 + a^2*b*f*x^2 - 2*a^3*g*x^2 - a*b^2*x^2*e + 2*a^2*c*x^2*e + 2*a*b^2*d - 8*a^2*c*d)/(c*x^5 + b*x^3 + a*x)*(a^2*b^2 - 4*a^3*c) - 1/16*((6*b^4*c^3 - 44*a*b^2*c^4 + 80*a^2*c^5 - 3*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^4*c + 22*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*a*b^2*c^2 + 6*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*b^3*c^2 - 40*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*a^2*c^3 - 20*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*a*b*c^3 - 3*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*b^2*c^3 + 10*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*a*c^4 - 6*(b^2 - 4*a*c)*b^2*c^3 + 20*(b^2 - 4*a*c)*a*c^4)*(a^2*b^2 - 4*a^3*c)^2*d + 2*(2*a^2*b^2*c^3 - 8*a^3*c^4 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*a^2*b^2*c + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*a^3*c^2 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*a^2*b*c^2 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*a^2*c^3 - 2*(b^2 - 4*a*c)*a^2*c^3)*(a^2*b^2 - 4*a^3*c)^2*f - (2*a^2*b^3*c^2 - 8*a^3*b*c^3 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*a^2*b^3 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*a^3*b*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*a^2*b^2*c - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*a^2*b*c^2 - 2*(b^2 - 4*a*c)*a^2*b*c^2)*(a^2*b^2 - 4*a^3*c)^2*g - (2*a*b^3*c^3 - 8*a^2*b*c^4 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*a*b^3*c + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*a^2*b*c^2 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*a*b^2*c^2 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*a*b*c^3 - 2*(b^2 - 4*a*c)*a*b*c^3)*(a^2*b^2 - 4*a^3*c)^2*e + 2*(3*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*a^2*b^7*c - 37*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*a^3*b^5*c^2 - 6*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*a^2*b^6*c^2 - 6*a^2*b^7*c^2 + 152*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*a^4*b^3*c^3 + 50*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*a^3*b^4*c^3 + 3*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*a^2*b^5*c^3 + 74*a^3*b^5*c^3 - 208*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*a^5*b*c^4 - 104*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*a^4*b^2*c^4 - 25*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*a^3*b^3*c^4 - 304*a^4*b^3*c^4 + 52*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*a^4*b*c^5 + 416*a^5*b*c^5 + 6*(b^2 - 4*a*c)*a^2*b^5*c^2 - 50*(b^2 - 4*a*c)*a^3*b^3*c^3 + 104*(b^2 - 4*a*c)*a^4*b*c^4)*d*abs(a^2*b^2 - 4*a^3*c) - 2*(\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*a^4*b^5*c - 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*a^5*b^3*c^2 - 2*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*a^4*b^4*c^2 - 2*a^4*b^5*c^2 + 16*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*a^6*b*c^3 + 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*a^5*b^2*c^3 + \sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*a^4*b^3*c^3 + 16*a^5*b^3*c^3 - 4*\sqrt{2}*\sqrt{b$$

$$\begin{aligned}
& c + \sqrt{b^2 - 4ac} * c * a^5 * b^4 * c^4 - 32 * a^6 * b^4 * c^4 + 2 * (b^2 - 4ac) * a^4 * b^3 * c^2 - 8 * (b^2 - 4ac) * a^5 * b^3 * c^3 * f * \text{abs}(a^2 * b^2 - 4a^3 * c) + 4 * (\sqrt{2} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c) * a^5 * b^4 * c - 8 * \sqrt{2} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c * a^6 * b^2 * c^2 - 2 * \sqrt{2} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c * a^5 * b^3 * c^2 - 2 * a^5 * b^4 * c^2 + 16 * \sqrt{2} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c * a^7 * c^3 + 8 * \sqrt{2} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c * a^6 * b * c^3 + \sqrt{2} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c * a^5 * b^2 * c^3 + 16 * a^6 * b^2 * c^3 - 4 * \sqrt{2} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c * a^6 * c^4 - 32 * a^7 * c^4 + 2 * (b^2 - 4ac) * a^5 * b^2 * c^2 - 8 * (b^2 - 4ac) * a^6 * c^3 * g * \text{abs}(a^2 * b^2 - 4a^3 * c) - 2 * (\sqrt{2} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c) * a^3 * b^6 * c - 14 * \sqrt{2} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c * a^4 * b^4 * c^2 - 2 * \sqrt{2} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c * a^3 * b^5 * c^2 - 2 * a^3 * b^6 * c^2 + 64 * \sqrt{2} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c * a^5 * b^2 * c^3 + 20 * \sqrt{2} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c * a^4 * b^3 * c^3 + \sqrt{2} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c * a^3 * b^4 * c^3 + 28 * a^4 * b^4 * c^3 - 96 * \sqrt{2} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c * a^6 * c^4 - 48 * \sqrt{2} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c * a^5 * b * c^4 - 10 * \sqrt{2} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c * a^4 * b^2 * c^4 - 128 * a^5 * b^2 * c^4 + 24 * \sqrt{2} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c * a^5 * c^5 + 192 * a^6 * c^5 + 2 * (b^2 - 4ac) * a^3 * b^4 * c^2 - 20 * (b^2 - 4ac) * a^4 * b^2 * c^3 + 48 * (b^2 - 4ac) * a^5 * c^4 * \text{abs}(a^2 * b^2 - 4a^3 * c) * e + (6 * a^4 * b^8 * c^3 - 80 * a^5 * b^6 * c^4 + 352 * a^6 * b^4 * c^5 - 512 * a^7 * b^2 * c^6 - 3 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c) * a^4 * b^8 * c + 40 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c * a^5 * b^6 * c^2 + 6 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c * a^4 * b^7 * c^2 - 176 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c * a^6 * b^4 * c^3 - 56 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c * a^5 * b^5 * c^3 - 3 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c * a^4 * b^6 * c^3 + 256 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c * a^7 * b^2 * c^4 + 128 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c * a^6 * b^3 * c^4 + 28 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c * a^5 * b^4 * c^4 - 64 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c * a^6 * b^2 * c^5 - 6 * (b^2 - 4ac) * a^4 * b^6 * c^3 + 56 * (b^2 - 4ac) * a^5 * b^4 * c^4 - 128 * (b^2 - 4ac) * a^6 * b^2 * c^5 * d - 4 * (2 * a^6 * b^6 * c^3 - 16 * a^7 * b^4 * c^4 + 32 * a^8 * b^2 * c^5 - \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c) * a^6 * b^6 * c + 8 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c * a^7 * b^4 * c^2 + 2 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c * a^6 * b^5 * c^2 - 16 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c * a^8 * b^2 * c^3 - 8 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c * a^7 * b^3 * c^3 - \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c * a^6 * b^4 * c^3 + 4 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c * a^7 * b^2 * c^4 - 2 * (b^2 - 4ac) * a^6 * b^4 * c^3 + 8 * (b^2 - 4ac) * a^7 * b^2 * c^4 * f + (2 * a^6 * b^7 * c^2 - 8 * a^7 * b^5 * c^3 - 32 * a^8 * b^3 * c^4 + 128 * a^9 * b * c^5 - \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c) * a^6 * b^7 + 4 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c * a^7 * b^5 * c + 2 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c * a^6 * b^6 * c + 16 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c * a^8 * b^3 * c^2 - \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c * a^6 * b^5 * c^2 - 64 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c * a^9 * b * c^3 - 32 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c * a^8 * b^2 * c^3 + 16 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c * a^8 * b * c^4 - 2 * (b^2 - 4ac) * a^6 * b^5 * c^2 + 32 * (b^2 - 4ac) * a^8 * b * c^4 * g - (2 * a^5 * b^7 * c^3 - 40 * a^6 * b^5 * c^4 + 224 * a^7 * b^3 * c^5 - 384 * a^8 * b * c^6 - \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c) * a^5 * b^7 * c + 20 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c * a^6 * b^5 * c^2 + 2 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c * a^5 * b^6 * c^2 - 112 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c * a^7 * b^3 * c^3 - 32 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c * a^6 * b^4 * c^3 - \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c * a^5 * b^5 * c^3 + 192 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c * a^8 * b * c^4 + 96 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c * a^7 * b^2 * c^4 + 16 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c * a^6 * b^3 * c^4 - 48 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c + \sqrt{b^2 - 4ac}} * c * a^
\end{aligned}$$

$$\begin{aligned}
& 7*b*c^5 - 2*(b^2 - 4*a*c)*a^5*b^5*c^3 + 32*(b^2 - 4*a*c)*a^6*b^3*c^4 - 96*(\\
& b^2 - 4*a*c)*a^7*b*c^5)*e)*\arctan(2*\sqrt{1/2}*x/\sqrt{(a^2*b^3 - 4*a^3*b*c + \\
& \sqrt{(a^2*b^3 - 4*a^3*b*c)^2 - 4*(a^3*b^2 - 4*a^4*c)*(a^2*b^2*c - 4*a^3*c^2)} \\
& 2)))/(a^2*b^2*c - 4*a^3*c^2)))/((a^5*b^6*c - 12*a^6*b^4*c^2 - 2*a^5*b^5*c^2 \\
& + 48*a^7*b^2*c^3 + 16*a^6*b^3*c^3 + a^5*b^4*c^3 - 64*a^8*c^4 - 32*a^7*b*c^4 \\
& - 8*a^6*b^2*c^4 + 16*a^7*c^5)*\text{abs}(a^2*b^2 - 4*a^3*c)*\text{abs}(c)) + 1/16*((6*b \\
& ^4*c^3 - 44*a*b^2*c^4 + 80*a^2*c^5 - 3*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \\
& \sqrt{b^2 - 4*a*c}}*c)*b^4*c + 22*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{ \\
& b^2 - 4*a*c}}*c)*a*b^2*c^2 + 6*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 \\
& - 4*a*c}}*c)*b^3*c^2 - 40*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4 \\
& *a*c}}*c)*a^2*c^3 - 20*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c} \\
& }*c)*a*b*c^3 - 3*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)* \\
& b^2*c^3 + 10*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*c^4 \\
& - 6*(b^2 - 4*a*c)*b^2*c^3 + 20*(b^2 - 4*a*c)*a*c^4)*(a^2*b^2 - 4*a^3*c)^2 \\
& *d + 2*(2*a^2*b^2*c^3 - 8*a^3*c^4 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{ \\
& b^2 - 4*a*c}}*c)*a^2*b^2*c + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{ \\
& b^2 - 4*a*c}}*c)*a^3*c^2 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4 \\
& *a*c}}*c)*a^2*b*c^2 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c} \\
& }*c)*a^2*c^3 - 2*(b^2 - 4*a*c)*a^2*c^3)*(a^2*b^2 - 4*a^3*c)^2*f - (2*a^2*b \\
& ^3*c^2 - 8*a^3*b*c^3 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c} \\
& }*c)*a^2*b^3 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c) \\
& *a^3*b*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2* \\
& b^2*c - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b*c^2 \\
& - 2*(b^2 - 4*a*c)*a^2*b*c^2)*(a^2*b^2 - 4*a^3*c)^2*g - (2*a*b^3*c^3 - 8*a^ \\
& 2*b*c^4 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^3*c \\
& + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b*c^2 + \\
& 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c^2 - \sqrt{ \\
& 2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b*c^3 - 2*(b^2 - 4 \\
& *a*c)*a*b*c^3)*(a^2*b^2 - 4*a^3*c)^2*e - 2*(3*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - \\
& 4*a*c}}*c)*a^2*b^7*c - 37*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*b^5*c \\
& ^2 - 6*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b^6*c^2 + 6*a^2*b^7*c^2 \\
& + 152*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^4*b^3*c^3 + 50*\sqrt{2}*\sqrt{ \\
& b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*b^4*c^3 + 3*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4 \\
& *a*c}}*c)*a^2*b^5*c^3 - 74*a^3*b^5*c^3 - 208*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4 \\
& *a*c}}*c)*a^5*b*c^4 - 104*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^4*b^2*c^ \\
& 4 - 25*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*b^3*c^4 + 304*a^4*b^3*c^ \\
& 4 + 52*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^4*b*c^5 - 416*a^5*b*c^5 - \\
& 6*(b^2 - 4*a*c)*a^2*b^5*c^2 + 50*(b^2 - 4*a*c)*a^3*b^3*c^3 - 104*(b^2 - 4*a \\
& *c)*a^4*b*c^4)*d*\text{abs}(a^2*b^2 - 4*a^3*c) + 2*(\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - \\
& 4*a*c}}*c)*a^4*b^5*c - 8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^5*b^3*c^2 \\
& - 2*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^4*b^4*c^2 + 2*a^4*b^5*c^2 + \\
& 16*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^6*b*c^3 + 8*\sqrt{2}*\sqrt{b*c - \\
& \sqrt{b^2 - 4*a*c}}*c)*a^5*b^2*c^3 + \sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c) \\
& *a^4*b^3*c^3 - 16*a^5*b^3*c^3 - 4*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a \\
& ^5*b*c^4 + 32*a^6*b*c^4 - 2*(b^2 - 4*a*c)*a^4*b^3*c^2 + 8*(b^2 - 4*a*c)*a^5 \\
& *b*c^3)*f*\text{abs}(a^2*b^2 - 4*a^3*c) - 4*(\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}* \\
& c)*a^5*b^4*c - 8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^6*b^2*c^2 - 2*\sqrt{ \\
& 2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^5*b^3*c^2 + 2*a^5*b^4*c^2 + 16*\sqrt{ \\
& 2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^7*c^3 + 8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - \\
& 4*a*c}}*c)*a^6*b*c^3 + \sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^5*b^2*c^ \\
& 3 - 16*a^6*b^2*c^3 - 4*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^6*c^4 + 32 \\
& *a^7*c^4 - 2*(b^2 - 4*a*c)*a^5*b^2*c^2 + 8*(b^2 - 4*a*c)*a^6*c^3)*g*\text{abs}(a^2 \\
& *b^2 - 4*a^3*c) + 2*(\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*b^6*c - 14 \\
& *\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^4*b^4*c^2 - 2*\sqrt{2}*\sqrt{b*c - \\
& \sqrt{b^2 - 4*a*c}}*c)*a^3*b^5*c^2 + 2*a^3*b^6*c^2 + 64*\sqrt{2}*\sqrt{b*c - \sqrt{ \\
& b^2 - 4*a*c}}*c)*a^5*b^2*c^3 + 20*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c) \\
& *a^4*b^3*c^3 + \sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*b^4*c^3 - 28*a^ \\
& 4*b^4*c^3 - 96*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^6*c^4 - 48*\sqrt{2} \\
& *\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^5*b*c^4 - 10*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4
\end{aligned}$$

$$\begin{aligned}
& - 4*a*c)*c)*a^4*b^2*c^4 + 128*a^5*b^2*c^4 + 24*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^5*c^5 - 192*a^6*c^5 - 2*(b^2 - 4*a*c)*a^3*b^4*c^2 + 20*(b^2 \\
& - 4*a*c)*a^4*b^2*c^3 - 48*(b^2 - 4*a*c)*a^5*c^4)*\text{abs}(a^2*b^2 - 4*a^3*c)*e + \\
& (6*a^4*b^8*c^3 - 80*a^5*b^6*c^4 + 352*a^6*b^4*c^5 - 512*a^7*b^2*c^6 - 3*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^4*b^8*c + 40*\sqrt{2} \\
& *\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^5*b^6*c^2 + 6*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^4*b^7*c^2 - 176*\sqrt{2} \\
& *\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^6*b^4*c^3 - 56*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^5*b^5*c^3 - 3*\sqrt{2} \\
& *\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^4*b^6*c^3 + 256*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^7*b^2*c^4 + 128*\sqrt{2} \\
& *\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^6*b^3*c^4 + 28*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^5*b^4*c^4 - 64*\sqrt{2} \\
& *\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^6*b^2*c^5 - 6*(b^2 - 4*a*c)*a^4*b^6*c^3 + 56*(b^2 - 4*a*c)*a^5*b^4*c^4 - 128*(b^2 - 4*a*c)*a^6 \\
& *b^2*c^5)*d - 4*(2*a^6*b^6*c^3 - 16*a^7*b^4*c^4 + 32*a^8*b^2*c^5 - \sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^6*b^6*c + 8*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^7*b^4*c^2 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^6*b^5*c^2 - 16*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^8*b^2*c^3 - 8*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^7*b^3*c^3 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^6*b^4*c^3 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^7*b^2*c^4 - 2*(b^2 - 4*a*c)*a^6*b^4*c^3 + 8*(b^2 - 4*a*c)*a^7*b^2*c^4)*f + (2*a^6*b^7*c^2 - 8*a^7*b^5*c^3 - 32*a^8*b^3*c^4 + 128*a^9*b*c^5 - \sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^6*b^7 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^7*b^5*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^6*b^6*c + 16*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^8*b^3*c^2 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^6*b^5*c^2 - 64*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^9*b*c^3 - 32*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^8*b^2*c^3 + 16*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^8*b*c^4 - 2*(b^2 - 4*a*c)*a^6*b^5*c^2 + 32*(b^2 - 4*a*c)*a^8*b*c^4)*g - (2*a^5*b^7*c^3 - 40*a^6*b^5*c^4 + 224*a^7*b^3*c^5 - 384*a^8*b*c^6 - \sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^5*b^7*c + 20*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^6*b^5*c^2 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^5*b^6*c^2 - 112*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^7*b^3*c^3 - 32*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^6*b^4*c^3 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^5*b^5*c^3 + 192*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^8*b*c^4 + 96*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^7*b^2*c^4 + 16*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^6*b^3*c^4 - 48*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^7*b*c^5 - 2*(b^2 - 4*a*c)*a^5*b^5*c^3 + 32*(b^2 - 4*a*c)*a^6*b^3*c^4 - 96*(b^2 - 4*a*c)*a^7*b*c^5)*e) *arctan(2*\sqrt{1/2}*x/\sqrt{((a^2*b^3 - 4*a^3*b*c - \sqrt{((a^2*b^3 - 4*a^3*b*c)^2 - 4*(a^3*b^2 - 4*a^4*c)*(a^2*b^2*c - 4*a^3*c^2)))/(a^2*b^2*c - 4*a^3*c^2)))/(a^5*b^6*c - 12*a^6*b^4*c^2 - 2*a^5*b^5*c^2 + 48*a^7*b^2*c^3 + 16*a^6*b^3*c^3 + a^5*b^4*c^3 - 64*a^8*c^4 - 32*a^7*b*c^4 - 8*a^6*b^2*c^4 + 16*a^7*c^5)*\text{abs}(a^2*b^2 - 4*a^3*c)*\text{abs}(c))
\end{aligned}$$

maple [B] time = 0.06, size = 2045, normalized size = 4.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((g*x^6+f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a)^2, x)$

[Out] $4/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}/a*b*c^2*d*\text{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)+4/a*c^2/(4*a*$

$$\begin{aligned} & c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh} \\ & (2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b*d-3/4/a^2*c/(4*a*c-b^2)/ \\ & (-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)} \\ & /((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^3*d-3/4/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)} \\ & *2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}/a^2*b^3*c*d*\operatorname{arctan}(2^{(1/2)}/ \\ & (b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)+1/4/a*c/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)} \\ & *2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2) \\ &)^{(1/2)})*c)^{(1/2)}*c*x)*b^2*e+1/4/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b \\ & +(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}/a*b^2*c*e*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)} \\ &))*c)^{(1/2)}*c*x)-1/a^2*d/x-1/4/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+ \\ & (-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} \\ &)*c*x)*b^2*g+1/4/a*c/(4*a*c-b^2)*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}* \\ & \operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b*e-a*c/(4*a*c-b^2)/ \\ & (-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctan}(2^{(1/2)}/ \\ & ((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*g-a*c/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}* \\ & 2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2) \\ &)^{(1/2)})*c)^{(1/2)}*c*x)*g-1/(c*x^4+b*x^2+a)/(4*a*c-b^2)/a*c^2*d*x^3-1/2/(c*x^ \\ & 4+b*x^2+a)/(4*a*c-b^2)/a*b^2*e*x+1/2/(c*x^4+b*x^2+a)/(4*a*c-b^2)/a^2*b^3*d* \\ & x-1/2*c/(4*a*c-b^2)*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)} \\ & /((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*f+1/2/(4*a*c-b^2)*2^{(1/2)}/((b+(-4 \\ & *a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*f*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} \\ &)*c*x)-1/4/(4*a*c-b^2)*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}/a*b*c*e*\operatorname{ar} \\ & \operatorname{ctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)+c/(4*a*c-b^2)/(-4*a*c+b^ \\ & 2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4 \\ & *a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b*f+1/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)} \\ & /((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*b*c*f*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)} \\ &))*c)^{(1/2)}*c*x)-3/4/a^2*c/(4*a*c-b^2)*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c) \\ & ^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^2*d+3/4/(4* \\ & a*c-b^2)*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}/a^2*b^2*c*d*\operatorname{arctan}(2^{(1/2)} \\ &)/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)-1/4/(4*a*c-b^2)*2^{(1/2)}/((b+(-4*a*c \\ & +b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)* \\ & b*g+1/4/(4*a*c-b^2)*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)} \\ & /((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b*g+1/(c*x^4+b*x^2+a)/(4*a*c-b^2) \\ & *c*e*x+1/2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*b*f*x-1/2/(c*x^4+b*x^2+a)/(4*a*c-b^2) \\ &)*x^3*b*g-a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*g+1/(c*x^4+b*x^2+a)/(4*a*c-b^2)*c \\ & *f*x^3-1/4/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})* \\ & c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^2*g+1/2/(\\ & c*x^4+b*x^2+a)/(4*a*c-b^2)/a^2*b^2*c*d*x^3+5/2/a*c^2/(4*a*c-b^2)*2^{(1/2)}/((\\ & -b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c) \\ & ^{(1/2)}*c*x)*d-5/2/(4*a*c-b^2)*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}/a*c^ \\ & 2*d*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)-3/(4*a*c-b^2)/(-4* \\ & a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c^2*e*\operatorname{arctan}(2^{(1/2)} \\ &)/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)-3*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)} \\ &)*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^ \\ & 2)^{(1/2)})*c)^{(1/2)}*c*x)*e-1/2/(c*x^4+b*x^2+a)/(4*a*c-b^2)/a*b*c*e*x^3-3/2/(\\ & c*x^4+b*x^2+a)/(4*a*c-b^2)/a*b*c*d*x \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^6+f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")

[Out] $\frac{1}{2}*((a*b*c*e - 2*a^2*c*f + a^2*b*g - (3*b^2*c - 10*a*c^2)*d)*x^4 - (a^2*b*f - 2*a^3*g + (3*b^3 - 11*a*b*c)*d - (a*b^2 - 2*a^2*c)*e)*x^2 - 2*(a*b^2 - 4*a^2*c)*d)/((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x) + \frac{1}{2}*\operatorname{integrate}((a^2*b*f - 2*a^3*g + (a*b*c*e - 2*a^2*c*f + a^2*b*g - (3*b^2*c - 10*a*c^2)*d)*x^2 - (3*b^3 - 13*a*b*c)*d + (a*b^2 -$

$$6a^2c)e)/(cx^4 + bx^2 + a), x)/(a^2b^2 - 4a^3c)$$

mupad [B] time = 7.76, size = 40860, normalized size = 88.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((d + ex^2 + fx^4 + gx^6)/(x^2(a + bx^2 + cx^4)^2), x)$

[Out] $\text{atan}\left(\frac{\left(\left(\left(213ab^{11}c^2d^2 - a^5b^9g^2 - a^5g^2(-4ac - b^2)^9\right)^{1/2} - 9b^{13}cd^2 - 26880a^6b^7c^7d^2 - a^2b^{11}c^7e^2 + 3840a^7b^6c^6e^2 + 9b^4cd^2(-4ac - b^2)^9\right)^{1/2} - a^4b^9c^7f^2 + 768a^8b^5c^5f^2 + a^4c^7f^2(-4ac - b^2)^9\right)^{1/2} + 768a^9b^4c^4g^2 - 2077a^2b^9c^3d^2 + 10656a^3b^7c^4d^2 - 30240a^4b^5c^5d^2 + 44800a^5b^3c^6d^2 + 25a^2c^3d^2(-4ac - b^2)^9\right)^{1/2} + 27a^3b^9c^2e^2 - 288a^4b^7c^3e^2 + 1504a^5b^5c^4e^2 - 3840a^6b^3c^5e^2 - 9a^3c^2e^2(-4ac - b^2)^9\right)^{1/2} + 96a^6b^5c^3f^2 - 512a^7b^3c^4f^2 + 96a^7b^5c^2g^2 - 512a^8b^3c^3g^2 + 15360a^7c^7d^2e + 5120a^8c^6d^2g - 3072a^8c^6e^2f - 1024a^9c^5f^2g + 6a^2b^{11}cd^2f + 1536a^7b^6cd^2f - 18a^3b^{10}cd^2g - 2a^3b^{10}c^2e^2f + 6a^4b^9c^2e^2g + 3584a^8b^5c^2e^2g - 6a^4c^2e^2g(-4ac - b^2)^9\right)^{1/2} + 12a^5b^8c^2f^2g - 152a^2b^{10}c^2d^2e + 1548a^3b^8c^3d^2e - 8064a^4b^6c^4d^2e + 22400a^5b^4c^5d^2e - 30720a^6b^2c^6d^2e - 98a^3b^9c^2d^2f + 576a^4b^7c^3d^2f - 1344a^5b^5c^4d^2f + 512a^6b^3c^5d^2f - 10a^3c^2d^2f(-4ac - b^2)^9\right)^{1/2} + 324a^4b^8c^2d^2g + 36a^4b^8c^2e^2f - 2240a^5b^6c^3d^2g - 192a^5b^6c^3e^2f + 7296a^6b^4c^4d^2g + 128a^6b^4c^4e^2f - 10752a^7b^2c^5d^2g + 1536a^7b^2c^5e^2f - 128a^5b^7c^2e^2g + 960a^6b^5c^3e^2g - 3072a^7b^3c^4e^2g - 128a^6b^6c^2f^2g + 384a^7b^4c^3f^2g + 6ab^{12}cd^2e - 51ab^2c^2d^2(-4ac - b^2)^9\right)^{1/2} + a^2b^2c^2e^2(-4ac - b^2)^9\right)^{1/2} - 6ab^3cd^2e(-4ac - b^2)^9\right)^{1/2} + 18a^3b^3cd^2g(-4ac - b^2)^9\right)^{1/2} + 2a^3b^3c^2e^2f(-4ac - b^2)^9\right)^{1/2} + 44a^2b^2c^2d^2e(-4ac - b^2)^9\right)^{1/2} - 6a^2b^2cd^2f(-4ac - b^2)^9\right)^{1/2})/(32(4096a^{11}c^7 + a^5b^{12}c - 24a^6b^{10}c^2 + 240a^7b^8c^3 - 1280a^8b^6c^4 + 3840a^9b^4c^5 - 6144a^{10}b^2c^6)))^{1/2} * (x((213ab^{11}c^2d^2 - a^5b^9g^2 - a^5g^2(-4ac - b^2)^9)^{1/2} - 9b^{13}cd^2 - 26880a^6b^7c^7d^2 - a^2b^{11}c^7e^2 + 3840a^7b^6c^6e^2 + 9b^4cd^2(-4ac - b^2)^9)^{1/2} - a^4b^9c^7f^2 + 768a^8b^5c^5f^2 + a^4c^7f^2(-4ac - b^2)^9)^{1/2} + 768a^9b^4c^4g^2 - 2077a^2b^9c^3d^2 + 10656a^3b^7c^4d^2 - 30240a^4b^5c^5d^2 + 44800a^5b^3c^6d^2 + 25a^2c^3d^2(-4ac - b^2)^9)^{1/2} + 27a^3b^9c^2e^2 - 288a^4b^7c^3e^2 + 1504a^5b^5c^4e^2 - 3840a^6b^3c^5e^2 - 9a^3c^2e^2(-4ac - b^2)^9)^{1/2} + 96a^6b^5c^3f^2 - 512a^7b^3c^4f^2 + 96a^7b^5c^2g^2 - 512a^8b^3c^3g^2 + 15360a^7c^7d^2e + 5120a^8c^6d^2g - 3072a^8c^6e^2f - 1024a^9c^5f^2g + 6a^2b^{11}cd^2f + 1536a^7b^6cd^2f - 18a^3b^{10}cd^2g - 2a^3b^{10}c^2e^2f + 6a^4b^9c^2e^2g + 3584a^8b^5c^2e^2g - 6a^4c^2e^2g(-4ac - b^2)^9)^{1/2} + 12a^5b^8c^2f^2g - 152a^2b^{10}c^2d^2e + 1548a^3b^8c^3d^2e - 8064a^4b^6c^4d^2e + 22400a^5b^4c^5d^2e - 30720a^6b^2c^6d^2e - 98a^3b^9c^2d^2f + 576a^4b^7c^3d^2f - 1344a^5b^5c^4d^2f + 512a^6b^3c^5d^2f - 10a^3c^2d^2f(-4ac - b^2)^9)^{1/2} + 324a^4b^8c^2d^2g + 36a^4b^8c^2e^2f - 2240a^5b^6c^3d^2g - 192a^5b^6c^3e^2f + 7296a^6b^4c^4d^2g + 128a^6b^4c^4e^2f - 10752a^7b^2c^5d^2g + 1536a^7b^2c^5e^2f - 128a^5b^7c^2e^2g + 960a^6b^5c^3e^2g - 3072a^7b^3c^4e^2g - 128a^6b^6c^2f^2g + 384a^7b^4c^3f^2g + 6ab^{12}cd^2e - 51ab^2c^2d^2(-4ac - b^2)^9)^{1/2} + a^2b^2c^2e^2(-4ac - b^2)^9\right)^{1/2} - 6ab^3cd^2e(-4ac - b^2)^9\right)^{1/2} + 18a^3b^3cd^2g(-4ac - b^2)^9\right)^{1/2} + 2a^3b^3c^2e^2f(-4ac - b^2)^9\right)^{1/2} + 44a^2b^2c^2d^2e(-4ac - b^2)^9\right)^{1/2} - 6a^2b^2cd^2f(-4ac - b^2)^9\right)^{1/2})/(32(4096a^{11}c^7 + a^5b^{12}c - 24a^6b^{10}c^2 + 240a^7b^8c^3 - 1280a^8b^6c^4 + 3840a^9b^4c^5 - 6144a^{10}b^2c^6)))^{1/2} * (1048576a^{16}b^8c^8 + 256a^{10}b^{13}c^2 - 6144a^{11}b^{11}c^3 + 61440a^{12}b$

$$\begin{aligned}
& ^9c^4 - 327680a^{13}b^7c^5 + 983040a^{14}b^5c^6 - 1572864a^{15}b^3c^7) \\
& - 131072a^{16}c^7g - 393216a^{15}c^8e + 192a^8b^{13}c^2d - 4672a^9b^{11}c^3d + 47360a^{10}b^9c^4d - 256000a^{11}b^7c^5d + 778240a^{12}b^5c^6d - 1261568a^{13}b^3c^7d - 64a^9b^{12}c^2e + 1664a^{10}b^{10}c^3e - 17920a^{11}b^8c^4e + 102400a^{12}b^6c^5e - 327680a^{13}b^4c^6e + 557056a^{14}b^2c^7e - 64a^{10}b^{11}c^2f + 1280a^{11}b^9c^3f - 10240a^{12}b^7c^4f + 40960a^{13}b^5c^5f - 81920a^{14}b^3c^6f + 128a^{11}b^{10}c^2g - 2560a^{12}b^8c^3g + 20480a^{13}b^6c^4g - 81920a^{14}b^4c^5g + 163840a^{15}b^2c^6g + 851968a^{14}b^3c^8d + 65536a^{15}b^4c^7f) + x*(204800a^{12}c^9d^2 - 73728a^{13}c^8e^2 + 8192a^{14}c^7f^2 - 8192a^{15}c^6g^2 + 16a^{10}b^{10}c^2g^2 + 144a^6b^{12}c^3d^2 - 3264a^7b^{10}c^4d^2 + 30112a^8b^8c^5d^2 - 143360a^9b^6c^6d^2 + 365568a^{10}b^4c^7d^2 - 458752a^{11}b^2c^8d^2 + 16a^8b^{10}c^3e^2 - 416a^9b^8c^4e^2 + 4608a^{10}b^6c^5e^2 - 25600a^{11}b^4c^6e^2 + 69632a^{12}b^2c^7e^2 + 160a^{10}b^8c^3f^2 - 2048a^{11}b^6c^4f^2 + 9216a^{12}b^4c^5f^2 - 16384a^{13}b^2c^6f^2 - 160a^{11}b^8c^2g^2 + 512a^{12}b^6c^3g^2 - 1024a^{13}b^4c^4g^2 + 4096a^{14}b^2c^5g^2 - 81920a^{13}c^8d^2f - 49152a^{14}c^7e^2g + 237568a^{12}b^3c^8d^2e + 106496a^{13}b^2c^7d^2g + 40960a^{13}b^3c^7e^2f + 8192a^{14}b^2c^6f^2g - 96a^7b^{11}c^3d^2e + 2336a^8b^9c^4d^2e - 22528a^9b^7c^5d^2e + 107520a^{10}b^5c^6d^2e - 253952a^{11}b^3c^7d^2e - 96a^8b^{10}c^3d^2f + 1472a^9b^8c^4d^2f - 7168a^{10}b^6c^5d^2f + 6144a^{11}b^4c^6d^2f + 40960a^{12}b^2c^7d^2f + 288a^9b^9c^3d^2g + 32a^9b^9c^3e^2f - 5120a^{10}b^7c^4d^2g - 1024a^{10}b^7c^4e^2f + 33792a^{11}b^5c^5d^2g + 9216a^{11}b^5c^5e^2f - 98304a^{12}b^3c^6d^2g - 32768a^{12}b^3c^6e^2f + 64a^{10}b^8c^3e^2g - 6144a^{12}b^4c^5e^2g + 32768a^{13}b^2c^6e^2g - 96a^{10}b^9c^2f^2g + 1024a^{11}b^7c^3f^2g - 3072a^{12}b^5c^4f^2g) * ((213a^2b^{11}c^2d^2 - a^5b^9g^2 - a^5g^2 * (-4a^2c - b^2)^9)^{1/2} - 9b^{13}c^2d^2 - 26880a^6b^3c^7d^2 - a^2b^{11}c^2e^2 + 3840a^7b^3c^6e^2 + 9b^4c^2d^2 * (-4a^2c - b^2)^9)^{1/2} - a^4b^9c^2f^2 + 768a^8b^3c^5f^2 + a^4c^2f^2 * (-4a^2c - b^2)^9)^{1/2} + 768a^9b^3c^4g^2 - 2077a^2b^9c^3d^2 + 10656a^3b^7c^4d^2 - 30240a^4b^5c^5d^2 + 44800a^5b^3c^6d^2 + 25a^2c^3d^2 * (-4a^2c - b^2)^9)^{1/2} + 27a^3b^9c^2e^2 - 288a^4b^7c^3e^2 + 1504a^5b^5c^4e^2 - 3840a^6b^3c^5e^2 - 9a^3c^2e^2 * (-4a^2c - b^2)^9)^{1/2} + 96a^6b^5c^3f^2 - 512a^7b^3c^4f^2 + 96a^7b^5c^2g^2 - 512a^8b^3c^3g^2 + 15360a^7c^7d^2e + 5120a^8c^6d^2g - 3072a^8c^6e^2f - 1024a^9c^5f^2g + 6a^2b^{11}c^2d^2f + 1536a^7b^3c^6d^2f - 18a^3b^{10}c^2d^2g - 2a^3b^{10}c^2e^2f + 6a^4b^9c^2e^2g + 3584a^8b^3c^5e^2g - 6a^4c^2e^2g * (-4a^2c - b^2)^9)^{1/2} + 12a^5b^8c^2f^2g - 152a^2b^{10}c^2d^2e + 1548a^3b^8c^3d^2e - 8064a^4b^6c^4d^2e + 22400a^5b^4c^5d^2e - 30720a^6b^2c^6d^2e - 98a^3b^9c^2d^2f + 576a^4b^7c^3d^2f - 1344a^5b^5c^4d^2f + 512a^6b^3c^5d^2f - 10a^3c^2d^2f * (-4a^2c - b^2)^9)^{1/2} + 324a^4b^8c^2d^2g + 36a^4b^8c^2e^2f - 2240a^5b^6c^3d^2g - 192a^5b^6c^3e^2f + 7296a^6b^4c^4d^2g + 128a^6b^4c^4e^2f - 10752a^7b^2c^5d^2g + 1536a^7b^2c^5e^2f - 128a^5b^7c^2e^2g + 960a^6b^5c^3e^2g - 3072a^7b^3c^4e^2g - 128a^6b^6c^2f^2g + 384a^7b^4c^3f^2g + 6a^2b^{12}c^2d^2e - 51a^2b^2c^2d^2 * (-4a^2c - b^2)^9)^{1/2} + a^2b^2c^2e^2 * (-4a^2c - b^2)^9)^{1/2} - 6a^2b^3c^2d^2e * (-4a^2c - b^2)^9)^{1/2} + 18a^3b^3c^2d^2g * (-4a^2c - b^2)^9)^{1/2} + 2a^3b^3c^2e^2f * (-4a^2c - b^2)^9)^{1/2} + 44a^2b^3c^2d^2e * (-4a^2c - b^2)^9)^{1/2} - 6a^2b^2c^2d^2f * (-4a^2c - b^2)^9)^{1/2} / (32 * (4096a^{11}c^7 + a^5b^{12}c - 24a^6b^{10}c^2 + 240a^7b^8c^3 - 1280a^8b^6c^4 + 3840a^9b^4c^5 - 6144a^{10}b^2c^6)))^{1/2} * i + (((213a^2b^{11}c^2d^2 - a^5b^9g^2 - a^5g^2 * (-4a^2c - b^2)^9)^{1/2} - 9b^{13}c^2d^2 - 26880a^6b^3c^7d^2 - a^2b^{11}c^2e^2 + 3840a^7b^3c^6e^2 + 9b^4c^2d^2 * (-4a^2c - b^2)^9)^{1/2} - a^4b^9c^2f^2 + 768a^8b^3c^5f^2 + a^4c^2f^2 * (-4a^2c - b^2)^9)^{1/2} + 768a^9b^3c^4g^2 - 2077a^2b^9c^3d^2 + 10656a^3b^7c^4d^2 - 30240a^4b^5c^5d^2 + 44800a^5b^3c^6d^2 + 25a^2c^3d^2 * (-4a^2c - b^2)^9)^{1/2} + 27a^3b^9c^2e^2 - 288a^4b^7c^3e^2 + 1504a^5b^5c^4e^2 - 3840a^6b^3c^5e^2 - 9a^3c^2e^2 * (-4a^2c - b^2)^9)^{1/2} + 96a^6b^5c^3f^2 - 512a^7b^3c^4f^2 + 96a^7b^5c^2g^2 - 512a^8
\end{aligned}$$

$$\begin{aligned}
& *b^3*c^3*g^2 + 15360*a^7*c^7*d*e + 5120*a^8*c^6*d*g - 3072*a^8*c^6*e*f - 10 \\
& 24*a^9*c^5*f*g + 6*a^2*b^{11}*c*d*f + 1536*a^7*b*c^6*d*f - 18*a^3*b^{10}*c*d*g \\
& - 2*a^3*b^{10}*c*e*f + 6*a^4*b^9*c*e*g + 3584*a^8*b*c^5*e*g - 6*a^4*c*e*g*(- \\
& (4*a*c - b^2)^9)^{(1/2)} + 12*a^5*b^8*c*f*g - 152*a^2*b^{10}*c^2*d*e + 1548*a^3* \\
& b^8*c^3*d*e - 8064*a^4*b^6*c^4*d*e + 22400*a^5*b^4*c^5*d*e - 30720*a^6*b^2* \\
& c^6*d*e - 98*a^3*b^9*c^2*d*f + 576*a^4*b^7*c^3*d*f - 1344*a^5*b^5*c^4*d*f + \\
& 512*a^6*b^3*c^5*d*f - 10*a^3*c^2*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 324*a^4*b^ \\
& 8*c^2*d*g + 36*a^4*b^8*c^2*e*f - 2240*a^5*b^6*c^3*d*g - 192*a^5*b^6*c^3*e*f \\
& + 7296*a^6*b^4*c^4*d*g + 128*a^6*b^4*c^4*e*f - 10752*a^7*b^2*c^5*d*g + 153 \\
& 6*a^7*b^2*c^5*e*f - 128*a^5*b^7*c^2*e*g + 960*a^6*b^5*c^3*e*g - 3072*a^7*b^ \\
& 3*c^4*e*g - 128*a^6*b^6*c^2*f*g + 384*a^7*b^4*c^3*f*g + 6*a*b^{12}*c*d*e - 51 \\
& *a*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + a^2*b^2*c*e^2*(-(4*a*c - b^2)^9)^{ \\
& (1/2)} - 6*a*b^3*c*d*e*(-(4*a*c - b^2)^9)^{(1/2)} + 18*a^3*b*c*d*g*(-(4*a*c - \\
& b^2)^9)^{(1/2)} + 2*a^3*b*c*e*f*(-(4*a*c - b^2)^9)^{(1/2)} + 44*a^2*b*c^2*d*e*(- \\
& -(4*a*c - b^2)^9)^{(1/2)} - 6*a^2*b^2*c*d*f*(-(4*a*c - b^2)^9)^{(1/2)}/(32*(40 \\
& 96*a^{11}*c^7 + a^5*b^{12}*c - 24*a^6*b^{10}*c^2 + 240*a^7*b^8*c^3 - 1280*a^8*b^6 \\
& *c^4 + 3840*a^9*b^4*c^5 - 6144*a^{10}*b^2*c^6)))^{(1/2)}*(393216*a^{15}*c^8*e + 1 \\
& 31072*a^{16}*c^7*g + x*((213*a*b^{11}*c^2*d^2 - a^5*b^9*g^2 - a^5*g^2*(-(4*a*c \\
& - b^2)^9)^{(1/2)} - 9*b^{13}*c*d^2 - 26880*a^6*b*c^7*d^2 - a^2*b^{11}*c*e^2 + 384 \\
& 0*a^7*b*c^6*e^2 + 9*b^4*c*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - a^4*b^9*c*f^2 + 76 \\
& 8*a^8*b*c^5*f^2 + a^4*c*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + 768*a^9*b*c^4*g^2 - \\
& 2077*a^2*b^9*c^3*d^2 + 10656*a^3*b^7*c^4*d^2 - 30240*a^4*b^5*c^5*d^2 + 4480 \\
& 0*a^5*b^3*c^6*d^2 + 25*a^2*c^3*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + 27*a^3*b^9*c^ \\
& 2*e^2 - 288*a^4*b^7*c^3*e^2 + 1504*a^5*b^5*c^4*e^2 - 3840*a^6*b^3*c^5*e^2 - \\
& 9*a^3*c^2*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 96*a^6*b^5*c^3*f^2 - 512*a^7*b^3* \\
& c^4*f^2 + 96*a^7*b^5*c^2*g^2 - 512*a^8*b^3*c^3*g^2 + 15360*a^7*c^7*d*e + 51 \\
& 20*a^8*c^6*d*g - 3072*a^8*c^6*e*f - 1024*a^9*c^5*f*g + 6*a^2*b^{11}*c*d*f + 1 \\
& 536*a^7*b*c^6*d*f - 18*a^3*b^{10}*c*d*g - 2*a^3*b^{10}*c*e*f + 6*a^4*b^9*c*e*g \\
& + 3584*a^8*b*c^5*e*g - 6*a^4*c*e*g*(-(4*a*c - b^2)^9)^{(1/2)} + 12*a^5*b^8*c* \\
& f*g - 152*a^2*b^{10}*c^2*d*e + 1548*a^3*b^8*c^3*d*e - 8064*a^4*b^6*c^4*d*e + \\
& 22400*a^5*b^4*c^5*d*e - 30720*a^6*b^2*c^6*d*e - 98*a^3*b^9*c^2*d*f + 576*a^ \\
& 4*b^7*c^3*d*f - 1344*a^5*b^5*c^4*d*f + 512*a^6*b^3*c^5*d*f - 10*a^3*c^2*d*f \\
& *(-(4*a*c - b^2)^9)^{(1/2)} + 324*a^4*b^8*c^2*d*g + 36*a^4*b^8*c^2*e*f - 2240 \\
& *a^5*b^6*c^3*d*g - 192*a^5*b^6*c^3*e*f + 7296*a^6*b^4*c^4*d*g + 128*a^6*b^4 \\
& *c^4*e*f - 10752*a^7*b^2*c^5*d*g + 1536*a^7*b^2*c^5*e*f - 128*a^5*b^7*c^2*e \\
& *g + 960*a^6*b^5*c^3*e*g - 3072*a^7*b^3*c^4*e*g - 128*a^6*b^6*c^2*f*g + 384 \\
& *a^7*b^4*c^3*f*g + 6*a*b^{12}*c*d*e - 51*a*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^{(1/ \\
& 2)} + a^2*b^2*c*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 6*a*b^3*c*d*e*(-(4*a*c - b^2) \\
& ^9)^{(1/2)} + 18*a^3*b*c*d*g*(-(4*a*c - b^2)^9)^{(1/2)} + 2*a^3*b*c*e*f*(-(4*a* \\
& c - b^2)^9)^{(1/2)} + 44*a^2*b*c^2*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 6*a^2*b^2*c \\
& *d*f*(-(4*a*c - b^2)^9)^{(1/2)}/(32*(4096*a^{11}*c^7 + a^5*b^{12}*c - 24*a^6*b^ \\
& 10*c^2 + 240*a^7*b^8*c^3 - 1280*a^8*b^6*c^4 + 3840*a^9*b^4*c^5 - 6144*a^{10}*b \\
& ^2*c^6)))^{(1/2)}*(1048576*a^{16}*b*c^8 + 256*a^{10}*b^{13}*c^2 - 6144*a^{11}*b^{11}*c^ \\
& 3 + 61440*a^{12}*b^9*c^4 - 327680*a^{13}*b^7*c^5 + 983040*a^{14}*b^5*c^6 - 157286 \\
& 4*a^{15}*b^3*c^7) - 192*a^8*b^{13}*c^2*d + 4672*a^9*b^{11}*c^3*d - 47360*a^{10}*b^9 \\
& *c^4*d + 256000*a^{11}*b^7*c^5*d - 778240*a^{12}*b^5*c^6*d + 1261568*a^{13}*b^3*c \\
& ^7*d + 64*a^9*b^{12}*c^2*e - 1664*a^{10}*b^{10}*c^3*e + 17920*a^{11}*b^8*c^4*e - 10 \\
& 2400*a^{12}*b^6*c^5*e + 327680*a^{13}*b^4*c^6*e - 557056*a^{14}*b^2*c^7*e + 64*a^ \\
& 10*b^{11}*c^2*f - 1280*a^{11}*b^9*c^3*f + 10240*a^{12}*b^7*c^4*f - 40960*a^{13}*b^5 \\
& *c^5*f + 81920*a^{14}*b^3*c^6*f - 128*a^{11}*b^{10}*c^2*g + 2560*a^{12}*b^8*c^3*g - \\
& 20480*a^{13}*b^6*c^4*g + 81920*a^{14}*b^4*c^5*g - 163840*a^{15}*b^2*c^6*g - 8519 \\
& 68*a^{14}*b*c^8*d - 65536*a^{15}*b*c^7*f) + x*(204800*a^{12}*c^9*d^2 - 73728*a^{13} \\
& *c^8*e^2 + 8192*a^{14}*c^7*f^2 - 8192*a^{15}*c^6*g^2 + 16*a^{10}*b^{10}*c*g^2 + 144 \\
& *a^6*b^{12}*c^3*d^2 - 3264*a^7*b^{10}*c^4*d^2 + 30112*a^8*b^8*c^5*d^2 - 143360* \\
& a^9*b^6*c^6*d^2 + 365568*a^{10}*b^4*c^7*d^2 - 458752*a^{11}*b^2*c^8*d^2 + 16*a^ \\
& 8*b^{10}*c^3*e^2 - 416*a^9*b^8*c^4*e^2 + 4608*a^{10}*b^6*c^5*e^2 - 25600*a^{11}*b \\
& ^4*c^6*e^2 + 69632*a^{12}*b^2*c^7*e^2 + 160*a^{10}*b^8*c^3*f^2 - 2048*a^{11}*b^6* \\
& c^4*f^2 + 9216*a^{12}*b^4*c^5*f^2 - 16384*a^{13}*b^2*c^6*f^2 - 160*a^{11}*b^8*c^2 \\
& *g^2 + 512*a^{12}*b^6*c^3*g^2 - 1024*a^{13}*b^4*c^4*g^2 + 4096*a^{14}*b^2*c^5*g^2
\end{aligned}$$

$$\begin{aligned}
& - 81920a^{13}c^8d^2f - 49152a^{14}c^7e^2g + 237568a^{12}b^2c^8d^2e + 106496 \\
& a^{13}b^2c^7d^2g + 40960a^{13}b^2c^7e^2f + 8192a^{14}b^2c^6f^2g - 96a^7b^{11}c^3d^2e + 2336a^8b^9c^4d^2e - 22528a^9b^7c^5d^2e + 107520a^{10}b^5c^6d^2e - 253952a^{11}b^3c^7d^2e - 96a^8b^{10}c^3d^2f + 1472a^9b^8c^4d^2f \\
& - 7168a^{10}b^6c^5d^2f + 6144a^{11}b^4c^6d^2f + 40960a^{12}b^2c^7d^2f + 288a^9b^9c^3d^2g + 32a^9b^9c^3e^2f - 5120a^{10}b^7c^4d^2g - 1024a^{10}b^7c^4e^2f + 33792a^{11}b^5c^5d^2g + 9216a^{11}b^5c^5e^2f - 98304a^{12}b^3c^6d^2g - 32768a^{12}b^3c^6e^2f + 64a^{10}b^8c^3e^2g - 6144a^{12}b^4c^5e^2g + 32768a^{13}b^2c^6e^2g - 96a^{10}b^9c^2f^2g + 1024a^{11}b^7c^3f^2g - 3072a^{12}b^5c^4f^2g) \cdot ((213a^2b^{11}c^2d^2 - a^5b^9g^2 - a^5g^2 \cdot (-4ac - b^2)^9)^{1/2} - 9b^{13}cd^2 - 26880a^6b^2c^7d^2 - a^2b^{11}c^2e^2 + 3840a^7b^2c^6e^2 + 9b^4c^2d^2 \cdot (-4ac - b^2)^9)^{1/2} - a^4b^9c^2f^2 + 768a^8b^2c^5f^2 + a^4c^2f^2 \cdot (-4ac - b^2)^9)^{1/2} + 768a^9b^2c^4g^2 - 2077a^2b^9c^3d^2 + 10656a^3b^7c^4d^2 - 30240a^4b^5c^5d^2 + 44800a^5b^3c^6d^2 + 25a^2c^3d^2 \cdot (-4ac - b^2)^9)^{1/2} + 27a^3b^9c^2e^2 - 288a^4b^7c^3e^2 + 1504a^5b^5c^4e^2 - 3840a^6b^3c^5e^2 - 9a^3c^2e^2 \cdot (-4ac - b^2)^9)^{1/2} + 96a^6b^5c^3f^2 - 512a^7b^3c^4f^2 + 96a^7b^5c^2g^2 - 512a^8b^3c^3g^2 + 15360a^7c^7d^2e + 5120a^8c^6d^2g - 3072a^8c^6e^2f - 1024a^9c^5f^2g + 6a^2b^{11}c^2d^2f + 1536a^7b^2c^6d^2f - 18a^3b^{10}c^2d^2g - 2a^3b^{10}c^2e^2f + 6a^4b^9c^2e^2g + 3584a^8b^2c^5e^2g - 6a^4c^2e^2g \cdot (-4ac - b^2)^9)^{1/2} + 12a^5b^8c^2f^2g - 152a^2b^{10}c^2d^2e + 1548a^3b^8c^3d^2e - 8064a^4b^6c^4d^2e + 22400a^5b^4c^5d^2e - 30720a^6b^2c^6d^2e - 98a^3b^9c^2d^2f + 576a^4b^7c^3d^2f - 1344a^5b^5c^4d^2f + 512a^6b^3c^5d^2f - 10a^3c^2d^2f \cdot (-4ac - b^2)^9)^{1/2} + 324a^4b^8c^2d^2g + 36a^4b^8c^2e^2f - 2240a^5b^6c^3d^2g - 192a^5b^6c^3e^2f + 7296a^6b^4c^4d^2g + 128a^6b^4c^4e^2f - 10752a^7b^2c^5d^2g + 1536a^7b^2c^5e^2f - 128a^5b^7c^2e^2g + 960a^6b^5c^3e^2g - 3072a^7b^3c^4e^2g - 128a^6b^6c^2f^2g + 384a^7b^4c^3f^2g + 6a^2b^{12}c^2d^2e - 51a^2b^2c^2d^2e \cdot (-4ac - b^2)^9)^{1/2} + a^2b^2c^2e^2 \cdot (-4ac - b^2)^9)^{1/2} - 6a^2b^3c^2d^2e \cdot (-4ac - b^2)^9)^{1/2} + 18a^3b^2c^2d^2e \cdot (-4ac - b^2)^9)^{1/2} + 2a^3b^2c^2d^2e \cdot (-4ac - b^2)^9)^{1/2} + 44a^2b^2c^2d^2e \cdot (-4ac - b^2)^9)^{1/2} - 6a^2b^2c^2d^2e \cdot (-4ac - b^2)^9)^{1/2}) / (32 \cdot (4096a^{11}c^7 + a^5b^{12}c - 24a^6b^{10}c^2 + 240a^7b^8c^3 - 1280a^8b^6c^4 + 3840a^9b^4c^5 - 6144a^{10}b^2c^6))^{1/2} \cdot i) / (((213a^2b^{11}c^2d^2 - a^5b^9g^2 - a^5g^2 \cdot (-4ac - b^2)^9)^{1/2} - 9b^{13}cd^2 - 26880a^6b^2c^7d^2 - a^2b^{11}c^2e^2 + 3840a^7b^2c^6e^2 + 9b^4c^2d^2 \cdot (-4ac - b^2)^9)^{1/2} - a^4b^9c^2f^2 + 768a^8b^2c^5f^2 + a^4c^2f^2 \cdot (-4ac - b^2)^9)^{1/2} + 768a^9b^2c^4g^2 - 2077a^2b^9c^3d^2 + 10656a^3b^7c^4d^2 - 30240a^4b^5c^5d^2 + 44800a^5b^3c^6d^2 + 25a^2c^3d^2 \cdot (-4ac - b^2)^9)^{1/2} + 27a^3b^9c^2e^2 - 288a^4b^7c^3e^2 + 1504a^5b^5c^4e^2 - 3840a^6b^3c^5e^2 - 9a^3c^2e^2 \cdot (-4ac - b^2)^9)^{1/2} + 96a^6b^5c^3f^2 - 512a^7b^3c^4f^2 + 96a^7b^5c^2g^2 - 512a^8b^3c^3g^2 + 15360a^7c^7d^2e + 5120a^8c^6d^2g - 3072a^8c^6e^2f - 1024a^9c^5f^2g + 6a^2b^{11}c^2d^2f + 1536a^7b^2c^6d^2f - 18a^3b^{10}c^2d^2g - 2a^3b^{10}c^2e^2f + 6a^4b^9c^2e^2g + 3584a^8b^2c^5e^2g - 6a^4c^2e^2g \cdot (-4ac - b^2)^9)^{1/2} + 12a^5b^8c^2f^2g - 152a^2b^{10}c^2d^2e + 1548a^3b^8c^3d^2e - 8064a^4b^6c^4d^2e + 22400a^5b^4c^5d^2e - 30720a^6b^2c^6d^2e - 98a^3b^9c^2d^2f + 576a^4b^7c^3d^2f - 1344a^5b^5c^4d^2f + 512a^6b^3c^5d^2f - 10a^3c^2d^2f \cdot (-4ac - b^2)^9)^{1/2} + 324a^4b^8c^2d^2g + 36a^4b^8c^2e^2f - 2240a^5b^6c^3d^2g - 192a^5b^6c^3e^2f + 7296a^6b^4c^4d^2g + 128a^6b^4c^4e^2f - 10752a^7b^2c^5d^2g + 1536a^7b^2c^5e^2f - 128a^5b^7c^2e^2g + 960a^6b^5c^3e^2g - 3072a^7b^3c^4e^2g - 128a^6b^6c^2f^2g + 384a^7b^4c^3f^2g + 6a^2b^{12}c^2d^2e - 51a^2b^2c^2d^2e \cdot (-4ac - b^2)^9)^{1/2} + a^2b^2c^2e^2 \cdot (-4ac - b^2)^9)^{1/2} - 6a^2b^3c^2d^2e \cdot (-4ac - b^2)^9)^{1/2} + 18a^3b^2c^2d^2e \cdot (-4ac - b^2)^9)^{1/2} + 2a^3b^2c^2d^2e \cdot (-4ac - b^2)^9)^{1/2} + 44a^2b^2c^2d^2e \cdot (-4ac - b^2)^9)^{1/2} - 6a^2b^2c^2d^2e \cdot (-4ac - b^2)^9)^{1/2}) / (32 \cdot (4096a^{11}c^7 + a^5b^{12}c - 24a^6b^{10}c^2 + 240a^7b^8c^3 - 1280a^8b^6c^4 + 3840a^9b^4c^5
\end{aligned}$$

$$\begin{aligned}
& - 6144a^{10}b^2c^6))^{(1/2)} * (393216a^{15}c^8e + 131072a^{16}c^7g + x((2 \\
& 13a^*b^{11}c^2d^2 - a^5b^9g^2 - a^5g^2*(-(4a*c - b^2)^9)^{(1/2)} - 9b^{13} \\
& *c*d^2 - 26880a^6b^7c^7d^2 - a^2b^{11}c^7e^2 + 3840a^7b^6c^6e^2 + 9b^4* \\
& c*d^2*(-(4a*c - b^2)^9)^{(1/2)} - a^4b^9c^7f^2 + 768a^8b^5c^5f^2 + a^4c* \\
& f^2*(-(4a*c - b^2)^9)^{(1/2)} + 768a^9b^4c^4g^2 - 2077a^2b^9c^3d^2 + 1 \\
& 0656a^3b^7c^4d^2 - 30240a^4b^5c^5d^2 + 44800a^5b^3c^6d^2 + 25a \\
& ^2c^3d^2*(-(4a*c - b^2)^9)^{(1/2)} + 27a^3b^9c^2e^2 - 288a^4b^7c^3 \\
& e^2 + 1504a^5b^5c^4e^2 - 3840a^6b^3c^5e^2 - 9a^3c^2e^2*(-(4a*c \\
& - b^2)^9)^{(1/2)} + 96a^6b^5c^3f^2 - 512a^7b^3c^4f^2 + 96a^7b^5c^2 \\
& *g^2 - 512a^8b^3c^3g^2 + 15360a^7c^7d^2e + 5120a^8c^6d^2g - 3072a^ \\
& 8c^6e^2f - 1024a^9c^5f^2g + 6a^2b^{11}c^7d^2f + 1536a^7b^6c^6d^2f - 18a \\
& ^3b^{10}c^7d^2g - 2a^3b^{10}c^7e^2f + 6a^4b^9c^6e^2g + 3584a^8b^5c^5e^2g - 6 \\
& *a^4c^7e^2g*(-(4a*c - b^2)^9)^{(1/2)} + 12a^5b^8c^6f^2g - 152a^2b^{10}c^2d \\
& *e + 1548a^3b^8c^3d^2e - 8064a^4b^6c^4d^2e + 22400a^5b^4c^5d^2e - \\
& 30720a^6b^2c^6d^2e - 98a^3b^9c^2d^2f + 576a^4b^7c^3d^2f - 1344a^5 \\
& *b^5c^4d^2f + 512a^6b^3c^5d^2f - 10a^3c^2d^2f*(-(4a*c - b^2)^9)^{(1/2)} \\
&) + 324a^4b^8c^2d^2g + 36a^4b^8c^2e^2f - 2240a^5b^6c^3d^2g - 192a \\
& ^5b^6c^3e^2f + 7296a^6b^4c^4d^2g + 128a^6b^4c^4e^2f - 10752a^7b^2 \\
& *c^5d^2g + 1536a^7b^2c^5e^2f - 128a^5b^7c^2e^2g + 960a^6b^5c^3e^2g \\
& - 3072a^7b^3c^4e^2g - 128a^6b^6c^2f^2g + 384a^7b^4c^3f^2g + 6a^*b \\
& ^{12}c^7d^2e - 51a^*b^2c^2d^2*(-(4a*c - b^2)^9)^{(1/2)} + a^2b^2c^2e^2*(-(4* \\
& a*c - b^2)^9)^{(1/2)} - 6a^*b^3c^7d^2e*(-(4a*c - b^2)^9)^{(1/2)} + 18a^3b^3c^7d \\
& *e*(-(4a*c - b^2)^9)^{(1/2)} + 2a^3b^3c^7e^2f*(-(4a*c - b^2)^9)^{(1/2)} + 44a \\
& ^2b^3c^7d^2e*(-(4a*c - b^2)^9)^{(1/2)} - 6a^2b^2c^7d^2e*(-(4a*c - b^2)^9)^{(1/2)} \\
& (1/2))/((32*(4096a^{11}c^7 + a^5b^{12}c - 24a^6b^{10}c^2 + 240a^7b^8c^3 \\
& - 1280a^8b^6c^4 + 3840a^9b^4c^5 - 6144a^{10}b^2c^6)))^{(1/2)} * (1048576 \\
& *a^{16}b^8c^8 + 256a^{10}b^{13}c^2 - 6144a^{11}b^{11}c^3 + 61440a^{12}b^9c^4 - \\
& 327680a^{13}b^7c^5 + 983040a^{14}b^5c^6 - 1572864a^{15}b^3c^7) - 192a^ \\
& 8b^{13}c^2d + 4672a^9b^{11}c^3d - 47360a^{10}b^9c^4d + 256000a^{11}b^7 \\
& *c^5d - 778240a^{12}b^5c^6d + 1261568a^{13}b^3c^7d + 64a^9b^{12}c^2e \\
& - 1664a^{10}b^{10}c^3e + 17920a^{11}b^8c^4e - 102400a^{12}b^6c^5e + 32 \\
& 7680a^{13}b^4c^6e - 557056a^{14}b^2c^7e + 64a^{10}b^{11}c^2f - 1280a^{11} \\
& b^9c^3f + 10240a^{12}b^7c^4f - 40960a^{13}b^5c^5f + 81920a^{14}b^3c \\
& ^6f - 128a^{11}b^{10}c^2g + 2560a^{12}b^8c^3g - 20480a^{13}b^6c^4g + \\
& 81920a^{14}b^4c^5g - 163840a^{15}b^2c^6g - 851968a^{14}b^6c^8d - 65536* \\
& a^{15}b^7c^7f) + x*(204800a^{12}c^9d^2 - 73728a^{13}c^8e^2 + 8192a^{14}c^7 \\
& *f^2 - 8192a^{15}c^6g^2 + 16a^{10}b^{10}c^7g^2 + 144a^6b^{12}c^3d^2 - 3264 \\
& *a^7b^{10}c^4d^2 + 30112a^8b^8c^5d^2 - 143360a^9b^6c^6d^2 + 365568 \\
& *a^{10}b^4c^7d^2 - 458752a^{11}b^2c^8d^2 + 16a^8b^{10}c^3e^2 - 416a^9 \\
& *b^8c^4e^2 + 4608a^{10}b^6c^5e^2 - 25600a^{11}b^4c^6e^2 + 69632a^{12} \\
& b^2c^7e^2 + 160a^{10}b^8c^3f^2 - 2048a^{11}b^6c^4f^2 + 9216a^{12}b^4c \\
& ^5f^2 - 16384a^{13}b^2c^6f^2 - 160a^{11}b^8c^2g^2 + 512a^{12}b^6c^3g \\
& ^2 - 1024a^{13}b^4c^4g^2 + 4096a^{14}b^2c^5g^2 - 81920a^{13}c^8d^2f - \\
& 49152a^{14}c^7e^2g + 237568a^{12}b^8c^8d^2e + 106496a^{13}b^6c^7d^2g + 40960* \\
& a^{13}b^7c^7e^2f + 8192a^{14}b^5c^6f^2g - 96a^7b^{11}c^3d^2e + 2336a^8b^9c \\
& ^4d^2e - 22528a^9b^7c^5d^2e + 107520a^{10}b^5c^6d^2e - 253952a^{11}b^3c \\
& ^7d^2e - 96a^8b^{10}c^3d^2f + 1472a^9b^8c^4d^2f - 7168a^{10}b^6c^5d^2f \\
& + 6144a^{11}b^4c^6d^2f + 40960a^{12}b^2c^7d^2f + 288a^9b^9c^3d^2g + \\
& 32a^9b^9c^3e^2f - 5120a^{10}b^7c^4d^2g - 1024a^{10}b^7c^4e^2f + 33792* \\
& a^{11}b^5c^5d^2g + 9216a^{11}b^5c^5e^2f - 98304a^{12}b^3c^6d^2g - 32768a \\
& ^{12}b^3c^6e^2f + 64a^{10}b^8c^3e^2g - 6144a^{12}b^4c^5e^2g + 32768a^{13} \\
& b^2c^6e^2g - 96a^{10}b^9c^2f^2g + 1024a^{11}b^7c^3f^2g - 3072a^{12}b^5c \\
& ^4f^2g)) * ((213a^*b^{11}c^2d^2 - a^5b^9g^2 - a^5g^2*(-(4a*c - b^2)^9)^{(1/2)} - 9b^{13} \\
& *c*d^2 - 26880a^6b^7c^7d^2 - a^2b^{11}c^7e^2 + 3840a^7b^6c^6e^2 + 9b^4* \\
& c*d^2*(-(4a*c - b^2)^9)^{(1/2)} - a^4b^9c^7f^2 + 768a^8b^5c^5f^2 + a^4c* \\
& f^2*(-(4a*c - b^2)^9)^{(1/2)} + 768a^9b^4c^4g^2 - 2077a^2b^9c^3d^2 + 1 \\
& 0656a^3b^7c^4d^2 - 30240a^4b^5c^5d^2 + 44800a^5b^3c^6d^2 + 25a \\
& ^2c^3d^2*(-(4a*c - b^2)^9)^{(1/2)} + 27a^3b^9c^2e^2 - 288a^4b^7c^3 \\
& e^2 + 1504a^5b^5c^4e^2 - 3840a^6b^3c^5e^2 - 9a^3c^2e^2)
\end{aligned}$$

$$\begin{aligned}
& ^2*(-(4*a*c - b^2)^9)^{(1/2)} + 96*a^6*b^5*c^3*f^2 - 512*a^7*b^3*c^4*f^2 + 96 \\
& *a^7*b^5*c^2*g^2 - 512*a^8*b^3*c^3*g^2 + 15360*a^7*c^7*d*e + 5120*a^8*c^6*d \\
& *g - 3072*a^8*c^6*e*f - 1024*a^9*c^5*f*g + 6*a^2*b^11*c*d*f + 1536*a^7*b*c^6 \\
& *d*f - 18*a^3*b^10*c*d*g - 2*a^3*b^10*c*e*f + 6*a^4*b^9*c*e*g + 3584*a^8*b \\
& *c^5*e*g - 6*a^4*c*e*g*(-(4*a*c - b^2)^9)^{(1/2)} + 12*a^5*b^8*c*f*g - 152*a^2 \\
& *b^10*c^2*d*e + 1548*a^3*b^8*c^3*d*e - 8064*a^4*b^6*c^4*d*e + 22400*a^5*b^4 \\
& *c^5*d*e - 30720*a^6*b^2*c^6*d*e - 98*a^3*b^9*c^2*d*f + 576*a^4*b^7*c^3*d* \\
& f - 1344*a^5*b^5*c^4*d*f + 512*a^6*b^3*c^5*d*f - 10*a^3*c^2*d*f*(-(4*a*c - \\
& b^2)^9)^{(1/2)} + 324*a^4*b^8*c^2*d*g + 36*a^4*b^8*c^2*e*f - 2240*a^5*b^6*c^3 \\
& *d*g - 192*a^5*b^6*c^3*e*f + 7296*a^6*b^4*c^4*d*g + 128*a^6*b^4*c^4*e*f - 1 \\
& 0752*a^7*b^2*c^5*d*g + 1536*a^7*b^2*c^5*e*f - 128*a^5*b^7*c^2*e*g + 960*a^6 \\
& *b^5*c^3*e*g - 3072*a^7*b^3*c^4*e*g - 128*a^6*b^6*c^2*f*g + 384*a^7*b^4*c^3 \\
& *f*g + 6*a*b^12*c*d*e - 51*a*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + a^2*b^2 \\
& *c*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 6*a*b^3*c*d*e*(-(4*a*c - b^2)^9)^{(1/2)} + \\
& 18*a^3*b*c*d*g*(-(4*a*c - b^2)^9)^{(1/2)} + 2*a^3*b*c*e*f*(-(4*a*c - b^2)^9)^{(1 \\
& /2)} + 44*a^2*b*c^2*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 6*a^2*b^2*c*d*f*(-(4*a* \\
& c - b^2)^9)^{(1/2)))/(32*(4096*a^11*c^7 + a^5*b^12*c - 24*a^6*b^10*c^2 + 240* \\
& a^7*b^8*c^3 - 1280*a^8*b^6*c^4 + 3840*a^9*b^4*c^5 - 6144*a^10*b^2*c^6)))^{(1 \\
& /2)} - (((213*a*b^11*c^2*d^2 - a^5*b^9*g^2 - a^5*g^2*(-(4*a*c - b^2)^9)^{(1/2)} \\
&) - 9*b^13*c*d^2 - 26880*a^6*b*c^7*d^2 - a^2*b^11*c*e^2 + 3840*a^7*b*c^6*e^2 \\
& + 9*b^4*c*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - a^4*b^9*c*f^2 + 768*a^8*b*c^5*f^2 \\
& + a^4*c*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + 768*a^9*b*c^4*g^2 - 2077*a^2*b^9*c^3 \\
& *d^2 + 10656*a^3*b^7*c^4*d^2 - 30240*a^4*b^5*c^5*d^2 + 44800*a^5*b^3*c^6*d^2 \\
& + 25*a^2*c^3*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + 27*a^3*b^9*c^2*e^2 - 288*a^4 \\
& *b^7*c^3*e^2 + 1504*a^5*b^5*c^4*e^2 - 3840*a^6*b^3*c^5*e^2 - 9*a^3*c^2*e^2 \\
& *(-(4*a*c - b^2)^9)^{(1/2)} + 96*a^6*b^5*c^3*f^2 - 512*a^7*b^3*c^4*f^2 + 96*a^7 \\
& *b^5*c^2*g^2 - 512*a^8*b^3*c^3*g^2 + 15360*a^7*c^7*d*e + 5120*a^8*c^6*d*g - \\
& 3072*a^8*c^6*e*f - 1024*a^9*c^5*f*g + 6*a^2*b^11*c*d*f + 1536*a^7*b*c^6*d* \\
& f - 18*a^3*b^10*c*d*g - 2*a^3*b^10*c*e*f + 6*a^4*b^9*c*e*g + 3584*a^8*b*c^5 \\
& *e*g - 6*a^4*c*e*g*(-(4*a*c - b^2)^9)^{(1/2)} + 12*a^5*b^8*c*f*g - 152*a^2*b^ \\
& 10*c^2*d*e + 1548*a^3*b^8*c^3*d*e - 8064*a^4*b^6*c^4*d*e + 22400*a^5*b^4*c^ \\
& 5*d*e - 30720*a^6*b^2*c^6*d*e - 98*a^3*b^9*c^2*d*f + 576*a^4*b^7*c^3*d*f - \\
& 1344*a^5*b^5*c^4*d*f + 512*a^6*b^3*c^5*d*f - 10*a^3*c^2*d*f*(-(4*a*c - b^2) \\
& ^9)^{(1/2)} + 324*a^4*b^8*c^2*d*g + 36*a^4*b^8*c^2*e*f - 2240*a^5*b^6*c^3*d \\
& *g - 192*a^5*b^6*c^3*e*f + 7296*a^6*b^4*c^4*d*g + 128*a^6*b^4*c^4*e*f - 107 \\
& 52*a^7*b^2*c^5*d*g + 1536*a^7*b^2*c^5*e*f - 128*a^5*b^7*c^2*e*g + 960*a^6*b^ \\
& ^5*c^3*e*g - 3072*a^7*b^3*c^4*e*g - 128*a^6*b^6*c^2*f*g + 384*a^7*b^4*c^3*f \\
& *g + 6*a*b^12*c*d*e - 51*a*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + a^2*b^2*c \\
& *e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 6*a*b^3*c*d*e*(-(4*a*c - b^2)^9)^{(1/2)} + 18 \\
& *a^3*b*c*d*g*(-(4*a*c - b^2)^9)^{(1/2)} + 2*a^3*b*c*e*f*(-(4*a*c - b^2)^9)^{(1 \\
& /2)} + 44*a^2*b*c^2*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 6*a^2*b^2*c*d*f*(-(4*a*c \\
& - b^2)^9)^{(1/2)))/(32*(4096*a^11*c^7 + a^5*b^12*c - 24*a^6*b^10*c^2 + 240*a^ \\
& 7*b^8*c^3 - 1280*a^8*b^6*c^4 + 3840*a^9*b^4*c^5 - 6144*a^10*b^2*c^6)))^{(1/2)} \\
&)*(x*((213*a*b^11*c^2*d^2 - a^5*b^9*g^2 - a^5*g^2*(-(4*a*c - b^2)^9)^{(1/2)} \\
& - 9*b^13*c*d^2 - 26880*a^6*b*c^7*d^2 - a^2*b^11*c*e^2 + 3840*a^7*b*c^6*e^2 \\
& + 9*b^4*c*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - a^4*b^9*c*f^2 + 768*a^8*b*c^5*f^2 \\
& + a^4*c*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + 768*a^9*b*c^4*g^2 - 2077*a^2*b^9*c^3 \\
& *d^2 + 10656*a^3*b^7*c^4*d^2 - 30240*a^4*b^5*c^5*d^2 + 44800*a^5*b^3*c^6*d^2 \\
& + 25*a^2*c^3*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + 27*a^3*b^9*c^2*e^2 - 288*a^4*b^ \\
& 7*c^3*e^2 + 1504*a^5*b^5*c^4*e^2 - 3840*a^6*b^3*c^5*e^2 - 9*a^3*c^2*e^2*(- \\
& -(4*a*c - b^2)^9)^{(1/2)} + 96*a^6*b^5*c^3*f^2 - 512*a^7*b^3*c^4*f^2 + 96*a^7 \\
& *b^5*c^2*g^2 - 512*a^8*b^3*c^3*g^2 + 15360*a^7*c^7*d*e + 5120*a^8*c^6*d*g - \\
& 3072*a^8*c^6*e*f - 1024*a^9*c^5*f*g + 6*a^2*b^11*c*d*f + 1536*a^7*b*c^6*d* \\
& f - 18*a^3*b^10*c*d*g - 2*a^3*b^10*c*e*f + 6*a^4*b^9*c*e*g + 3584*a^8*b*c^5 \\
& *e*g - 6*a^4*c*e*g*(-(4*a*c - b^2)^9)^{(1/2)} + 12*a^5*b^8*c*f*g - 152*a^2*b^ \\
& 10*c^2*d*e + 1548*a^3*b^8*c^3*d*e - 8064*a^4*b^6*c^4*d*e + 22400*a^5*b^4*c^ \\
& 5*d*e - 30720*a^6*b^2*c^6*d*e - 98*a^3*b^9*c^2*d*f + 576*a^4*b^7*c^3*d*f - \\
& 1344*a^5*b^5*c^4*d*f + 512*a^6*b^3*c^5*d*f - 10*a^3*c^2*d*f*(-(4*a*c - b^2) \\
& ^9)^{(1/2)} + 324*a^4*b^8*c^2*d*g + 36*a^4*b^8*c^2*e*f - 2240*a^5*b^6*c^3*d*g
\end{aligned}$$

$$\begin{aligned}
& - 192a^5b^6c^3ef + 7296a^6b^4c^4d^2g + 128a^6b^4c^4ef - 10752 \\
& a^7b^2c^5d^2g + 1536a^7b^2c^5ef - 128a^5b^7c^2efg + 960a^6b^5 \\
& c^3efg - 3072a^7b^3c^4efg - 128a^6b^6c^2f^2g + 384a^7b^4c^3f^2g \\
& + 6a^6b^12c^2d^2e - 51a^6b^2c^2d^2e(-4ac - b^2)^9)^{(1/2)} + a^2b^2c^2e \\
& ^2(-4ac - b^2)^9)^{(1/2)} - 6a^6b^3c^2d^2e(-4ac - b^2)^9)^{(1/2)} + 18a^ \\
& ^3b^3c^2d^2g(-4ac - b^2)^9)^{(1/2)} + 2a^3b^3c^2ef(-4ac - b^2)^9)^{(1/2)} \\
&) + 44a^2b^3c^2d^2e(-4ac - b^2)^9)^{(1/2)} - 6a^2b^2c^2d^2f(-4ac - \\
& b^2)^9)^{(1/2)))/(32(4096a^{11}c^7 + a^5b^{12}c - 24a^6b^{10}c^2 + 240a^7 \\
& b^8c^3 - 1280a^8b^6c^4 + 3840a^9b^4c^5 - 6144a^{10}b^2c^6)))^{(1/2)} * \\
& (1048576a^{16}b^8c^8 + 256a^{10}b^{13}c^2 - 6144a^{11}b^{11}c^3 + 61440a^{12}b^ \\
& ^9c^4 - 327680a^{13}b^7c^5 + 983040a^{14}b^5c^6 - 1572864a^{15}b^3c^7) \\
& - 131072a^{16}c^7g - 393216a^{15}c^8e + 192a^8b^{13}c^2d - 4672a^9b^{11} \\
& c^3d + 47360a^{10}b^9c^4d - 256000a^{11}b^7c^5d + 778240a^{12}b^5c^ \\
& 6d - 1261568a^{13}b^3c^7d - 64a^9b^{12}c^2e + 1664a^{10}b^{10}c^3e - 1 \\
& 7920a^{11}b^8c^4e + 102400a^{12}b^6c^5e - 327680a^{13}b^4c^6e + 55705 \\
& 6a^{14}b^2c^7e - 64a^{10}b^{11}c^2f + 1280a^{11}b^9c^3f - 10240a^{12}b^ \\
& 7c^4f + 40960a^{13}b^5c^5f - 81920a^{14}b^3c^6f + 128a^{11}b^{10}c^2g \\
& - 2560a^{12}b^8c^3g + 20480a^{13}b^6c^4g - 81920a^{14}b^4c^5g + 1638 \\
& 40a^{15}b^2c^6g + 851968a^{14}b^3c^8d + 65536a^{15}b^3c^7f) + x(204800a^ \\
& ^{12}c^9d^2 - 73728a^{13}c^8e^2 + 8192a^{14}c^7f^2 - 8192a^{15}c^6g^2 + \\
& 16a^{10}b^{10}c^5d^2 + 144a^6b^{12}c^3d^2 - 3264a^7b^{10}c^4d^2 + 30112a^ \\
& ^8b^8c^5d^2 - 143360a^9b^6c^6d^2 + 365568a^{10}b^4c^7d^2 - 458752a^ \\
& ^{11}b^2c^8d^2 + 16a^8b^{10}c^3e^2 - 416a^9b^8c^4e^2 + 4608a^{10}b^ \\
& 6c^5e^2 - 25600a^{11}b^4c^6e^2 + 69632a^{12}b^2c^7e^2 + 160a^{10}b^8c^ \\
& ^3f^2 - 2048a^{11}b^6c^4f^2 + 9216a^{12}b^4c^5f^2 - 16384a^{13}b^2c^ \\
& 6f^2 - 160a^{11}b^8c^2g^2 + 512a^{12}b^6c^3g^2 - 1024a^{13}b^4c^4g^2 \\
& + 4096a^{14}b^2c^5g^2 - 81920a^{13}c^8d^2f - 49152a^{14}c^7e^2g + 237568 \\
& a^{12}b^3c^8d^2e + 106496a^{13}b^3c^7d^2g + 40960a^{13}b^3c^7e^2f + 8192a^{14} \\
& b^3c^6f^2g - 96a^7b^{11}c^3d^2e + 2336a^8b^9c^4d^2e - 22528a^9b^7c^5 \\
& d^2e + 107520a^{10}b^5c^6d^2e - 253952a^{11}b^3c^7d^2e - 96a^8b^{10}c^3d \\
& ^2f + 1472a^9b^8c^4d^2f - 7168a^{10}b^6c^5d^2f + 6144a^{11}b^4c^6d^2f + \\
& 40960a^{12}b^2c^7d^2f + 288a^9b^9c^3d^2g + 32a^9b^9c^3e^2f - 5120a^ \\
& ^{10}b^7c^4d^2g - 1024a^{10}b^7c^4e^2f + 33792a^{11}b^5c^5d^2g + 9216a^{11} \\
& b^5c^5e^2f - 98304a^{12}b^3c^6d^2g - 32768a^{12}b^3c^6e^2f + 64a^{10}b^ \\
& ^8c^3e^2g - 6144a^{12}b^4c^5e^2g + 32768a^{13}b^2c^6e^2g - 96a^{10}b^9c^ \\
& ^2f^2g + 1024a^{11}b^7c^3f^2g - 3072a^{12}b^5c^4f^2g) * ((213a^6b^{11}c^2d \\
& ^2 - a^5b^9g^2 - a^5g^2(-4ac - b^2)^9)^{(1/2)} - 9b^{13}c^2d^2 - 26880a^ \\
& ^6b^3c^7d^2 - a^2b^{11}c^2e^2 + 3840a^7b^3c^6e^2 + 9b^4c^2d^2(-4ac - \\
& b^2)^9)^{(1/2)} - a^4b^9c^2f^2 + 768a^8b^3c^5f^2 + a^4c^2f^2(-4ac - \\
& b^2)^9)^{(1/2)} + 768a^9b^3c^4g^2 - 2077a^2b^9c^3d^2 + 10656a^3b^7c^ \\
& 4d^2 - 30240a^4b^5c^5d^2 + 44800a^5b^3c^6d^2 + 25a^2c^3d^2(-4 \\
& ac - b^2)^9)^{(1/2)} + 27a^3b^9c^2e^2 - 288a^4b^7c^3e^2 + 1504a^5b^ \\
& ^5c^4e^2 - 3840a^6b^3c^5e^2 - 9a^3c^2e^2(-4ac - b^2)^9)^{(1/2)} \\
& + 96a^6b^5c^3f^2 - 512a^7b^3c^4f^2 + 96a^7b^5c^2g^2 - 512a^8b^ \\
& ^3c^3g^2 + 15360a^7c^7d^2e + 5120a^8c^6d^2g - 3072a^8c^6e^2f - 102 \\
& 4a^9c^5f^2g + 6a^2b^{11}c^2d^2f + 1536a^7b^3c^6d^2f - 18a^3b^{10}c^2d^2g - \\
& 2a^3b^{10}c^2ef + 6a^4b^9c^2ef + 3584a^8b^3c^5ef - 6a^4c^2ef(-4 \\
& ac - b^2)^9)^{(1/2)} + 12a^5b^8c^2f^2g - 152a^2b^{10}c^2d^2e + 1548a^3b^ \\
& ^8c^3d^2e - 8064a^4b^6c^4d^2e + 22400a^5b^4c^5d^2e - 30720a^6b^2c^ \\
& ^6d^2e - 98a^3b^9c^2d^2f + 576a^4b^7c^3d^2f - 1344a^5b^5c^4d^2f + \\
& 512a^6b^3c^5d^2f - 10a^3c^2d^2f(-4ac - b^2)^9)^{(1/2)} + 324a^4b^8 \\
& c^2d^2g + 36a^4b^8c^2e^2f - 2240a^5b^6c^3d^2g - 192a^5b^6c^3e^2f \\
& + 7296a^6b^4c^4d^2g + 128a^6b^4c^4ef - 10752a^7b^2c^5d^2g + 1536 \\
& a^7b^2c^5ef - 128a^5b^7c^2efg + 960a^6b^5c^3efg - 3072a^7b^3 \\
& c^4efg - 128a^6b^6c^2f^2g + 384a^7b^4c^3f^2g + 6a^6b^{12}c^2d^2e - 51 \\
& a^6b^2c^2d^2(-4ac - b^2)^9)^{(1/2)} + a^2b^2c^2e^2(-4ac - b^2)^9)^{(\\
& 1/2)} - 6a^6b^3c^2d^2e(-4ac - b^2)^9)^{(1/2)} + 18a^3b^3c^2d^2g(-4ac - b \\
& ^2)^9)^{(1/2)} + 2a^3b^3c^2ef(-4ac - b^2)^9)^{(1/2)} + 44a^2b^3c^2d^2e(- \\
& 4ac - b^2)^9)^{(1/2)} - 6a^2b^2c^2d^2f(-4ac - b^2)^9)^{(1/2)))/(32(409
\end{aligned}$$

$$\begin{aligned}
& 6*a^{11}*c^7 + a^5*b^{12}*c - 24*a^6*b^{10}*c^2 + 240*a^7*b^8*c^3 - 1280*a^8*b^6*c^4 + 3840*a^9*b^4*c^5 - 6144*a^{10}*b^2*c^6))^{(1/2)} - 128000*a^{10}*c^9*d^3 + \\
& 1024*a^{13}*c^6*f^3 - 4608*a^{11}*b*c^7*e^3 - 24*a^{11}*b^7*c*g^3 - 46080*a^{11}*c^8*d*e^2 - 512*a^{14}*b*c^4*g^3 + 76800*a^{11}*c^8*d^2*f - 15360*a^{12}*c^7*d*f^2 \\
& + 9216*a^{12}*c^7*e^2*f - 5120*a^{13}*c^6*d*g^2 + 1024*a^{14}*c^5*f*g^2 - 504*a^6*b^8*c^5*d^3 + 8112*a^7*b^6*c^6*d^3 - 48704*a^8*b^4*c^7*d^3 + 129280*a^9*b^2*c^8*d^3 + 40*a^8*b^7*c^4*e^3 - 608*a^9*b^5*c^5*e^3 + 2944*a^{10}*b^3*c^6*e^3 + 48*a^{10}*b^6*c^3*f^3 - 320*a^{11}*b^4*c^4*f^3 + 256*a^{12}*b^2*c^5*f^3 + 160*a^{12}*b^5*c^2*g^3 - 128*a^{13}*b^3*c^3*g^3 - 30720*a^{12}*c^7*d*e*g + 6144*a^{13}*c^6*e*f*g + 84480*a^{10}*b*c^8*d^2*e - 24*a^8*b^{10}*c*d*g^2 + 2560*a^{11}*b*c^7*d^2*g - 7680*a^{12}*b*c^6*e*f^2 + 8*a^9*b^9*c*e*g^2 - 7680*a^{12}*b*c^6*e^2*g - 3584*a^{13}*b*c^5*e*g^2 + 8*a^{10}*b^8*c*f*g^2 - 3584*a^{13}*b*c^5*f^2*g + 360*a^6*b^9*c^4*d^2*e - 5736*a^7*b^7*c^5*d^2*e - 240*a^7*b^8*c^4*d*e^2 + 33888*a^8*b^5*c^6*d^2*e + 3792*a^8*b^6*c^5*d*e^2 - 87936*a^9*b^3*c^7*d^2*e - 21696*a^9*b^4*c^6*d*e^2 + 52992*a^{10}*b^2*c^7*d*e^2 - 216*a^6*b^{10}*c^3*d^2*f + 3744*a^7*b^8*c^4*d^2*f - 25200*a^8*b^6*c^5*d^2*f - 72*a^8*b^8*c^3*d*f^2 + 81984*a^9*b^4*c^6*d^2*f + 1296*a^9*b^6*c^4*d*f^2 - 128256*a^{10}*b^2*c^7*d^2*f - 7872*a^{10}*b^4*c^5*d*f^2 + 19200*a^{11}*b^2*c^6*d*f^2 + 72*a^6*b^{11}*c^2*d^2*g - 1128*a^7*b^9*c^3*d^2*g + 6488*a^8*b^7*c^4*d^2*g - 24*a^8*b^8*c^3*e^2*f - 16032*a^9*b^5*c^5*d^2*g + 336*a^9*b^6*c^4*e^2*f + 24*a^9*b^7*c^3*e*f^2 + 368*a^9*b^8*c^2*d*g^2 + 13440*a^{10}*b^3*c^6*d^2*g - 960*a^{10}*b^4*c^5*e^2*f - 672*a^{10}*b^5*c^4*e*f^2 - 1840*a^{10}*b^6*c^3*d*g^2 - 2304*a^{11}*b^2*c^6*e^2*f + 4224*a^{11}*b^3*c^5*e*f^2 + 2880*a^{11}*b^4*c^4*d*g^2 + 1792*a^{12}*b^2*c^5*d*g^2 + 8*a^8*b^9*c^2*e^2*g - 72*a^9*b^7*c^3*e^2*g - 288*a^{10}*b^5*c^4*e^2*g - 136*a^{10}*b^7*c^2*e*g^2 + 3712*a^{11}*b^3*c^5*e^2*g + 480*a^{11}*b^5*c^3*e*g^2 + 640*a^{12}*b^3*c^4*e*g^2 - 40*a^{10}*b^7*c^2*f^2*g + 96*a^{11}*b^5*c^3*f^2*g + 80*a^{11}*b^6*c^2*f*g^2 + 1152*a^{12}*b^3*c^4*f^2*g - 960*a^{12}*b^4*c^3*f*g^2 + 1792*a^{13}*b^2*c^4*f*g^2 + 21504*a^{11}*b*c^7*d*e*f + 17408*a^{12}*b*c^6*d*f*g + 144*a^7*b^9*c^3*d*e*f - 2256*a^8*b^7*c^4*d*e*f + 12480*a^9*b^5*c^5*d*e*f - 28416*a^{10}*b^3*c^6*d*e*f - 48*a^7*b^{10}*c^2*d*e*g + 592*a^8*b^8*c^3*d*e*g - 1632*a^9*b^6*c^4*d*e*g - 4992*a^{10}*b^4*c^5*d*e*g + 28160*a^{11}*b^2*c^6*d*e*g + 96*a^8*b^9*c^2*d*f*g - 1616*a^9*b^7*c^3*d*f*g + 9408*a^{10}*b^5*c^4*d*f*g - 22272*a^{11}*b^3*c^5*d*f*g - 32*a^9*b^8*c^2*e*f*g + 672*a^{10}*b^6*c^3*e*f*g - 3456*a^{11}*b^4*c^4*e*f*g + 3584*a^{12}*b^2*c^5*e*f*g))*((213*a*b^{11}*c^2*d^2 - a^5*b^9*g^2 - a^5*g^2*(-(4*a*c - b^2)^9)^{(1/2)} - 9*b^{13}*c*d^2 - 26880*a^6*b*c^7*d^2 - a^2*b^{11}*c*e^2 + 3840*a^7*b*c^6*e^2 + 9*b^4*c*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - a^4*b^9*c*f^2 + 768*a^8*b*c^5*f^2 + a^4*c*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + 768*a^9*b*c^4*g^2 - 2077*a^2*b^9*c^3*d^2 + 10656*a^3*b^7*c^4*d^2 - 30240*a^4*b^5*c^5*d^2 + 44800*a^5*b^3*c^6*d^2 + 25*a^2*c^3*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + 27*a^3*b^9*c^2*e^2 - 288*a^4*b^7*c^3*e^2 + 1504*a^5*b^5*c^4*e^2 - 3840*a^6*b^3*c^5*e^2 - 9*a^3*c^2*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 96*a^6*b^5*c^3*f^2 - 512*a^7*b^3*c^4*f^2 + 96*a^7*b^5*c^2*g^2 - 512*a^8*b^3*c^3*g^2 + 15360*a^7*c^7*d*e + 5120*a^8*c^6*d*g - 3072*a^8*c^6*e*f - 1024*a^9*c^5*f*g + 6*a^2*b^{11}*c*d*f + 1536*a^7*b*c^6*d*f - 18*a^3*b^{10}*c*d*g - 2*a^3*b^{10}*c*e*f + 6*a^4*b^9*c*e*g + 3584*a^8*b*c^5*e*g - 6*a^4*c*e*g*(-(4*a*c - b^2)^9)^{(1/2)} + 12*a^5*b^8*c*f*g - 152*a^2*b^{10}*c^2*d*e + 1548*a^3*b^8*c^3*d*e - 8064*a^4*b^6*c^4*d*e + 22400*a^5*b^4*c^5*d*e - 30720*a^6*b^2*c^6*d*e - 98*a^3*b^9*c^2*d*f + 576*a^4*b^7*c^3*d*f - 1344*a^5*b^5*c^4*d*f + 512*a^6*b^3*c^5*d*f - 10*a^3*c^2*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 324*a^4*b^8*c^2*d*g + 36*a^4*b^8*c^2*e*f - 2240*a^5*b^6*c^3*d*g - 192*a^5*b^6*c^3*e*f + 7296*a^6*b^4*c^4*d*g + 128*a^6*b^4*c^4*e*f - 10752*a^7*b^2*c^5*d*g + 1536*a^7*b^2*c^5*e*f - 128*a^5*b^7*c^2*e*g + 960*a^6*b^5*c^3*e*g - 3072*a^7*b^3*c^4*e*g - 128*a^6*b^6*c^2*f*g + 384*a^7*b^4*c^3*f*g + 6*a*b^{12}*c*d*e - 51*a*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + a^2*b^2*c*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 6*a*b^3*c*d*e*(-(4*a*c - b^2)^9)^{(1/2)} + 18*a^3*b*c*d*g*(-(4*a*c - b^2)^9)^{(1/2)} + 2*a^3*b*c*e*f*(-(4*a*c - b^2)^9)^{(1/2)} + 44*a^2*b*c^2*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 6*a^2*b^2*c*d*f*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(4096*a^{11}*c^7 + a^5*b^{12}*c - 24*a^6*b^{10}*c^2 + 240*a^7*b^8*c^3 - 1280*a^8*b^6*c^4 + 3840*a^9*b^4*c^5 - 6144*a^{10}*b^2*c^6))^{(1/2)}*2i - (d/a - (x^2*(3*b^3*
\end{aligned}$$

$$\begin{aligned}
& d - 2a^3g - ab^2e + a^2b^2f + 2a^2c^2e - 11ab^2cd) / (2a^2(4ac - b^2)) + (x^4(10a^2c^2d - 3b^2cd + a^2b^2g - 2a^2c^2f + ab^2e)) / (2a^2(4ac - b^2)) / (ax + bx^3 + cx^5) + \operatorname{atan}\left(\frac{(a^5g^2 - (4ac - b^2)^9)^{1/2} - a^5b^9g^2 - 9b^{13}cd^2 + 213ab^{11}c^2d^2 - 26880a^6b^7c^7d^2 - a^2b^{11}c^2e^2 + 3840a^7b^6c^6e^2 - 9b^4cd^2(-4ac - b^2)^9)^{1/2} - a^4b^9c^2f^2 + 768a^8b^5c^5f^2 - a^4c^2f^2(-4ac - b^2)^9)^{1/2} + 768a^9b^4c^4g^2 - 2077a^2b^9c^3d^2 + 10656a^3b^7c^4d^2 - 30240a^4b^5c^5d^2 + 44800a^5b^3c^6d^2 - 25a^2c^3d^2(-4ac - b^2)^9)^{1/2} + 27a^3b^9c^2e^2 - 288a^4b^7c^3e^2 + 1504a^5b^5c^4e^2 - 3840a^6b^3c^5e^2 + 9a^3c^2e^2(-4ac - b^2)^9)^{1/2} + 96a^6b^5c^3f^2 - 512a^7b^3c^4f^2 + 96a^7b^5c^2g^2 - 512a^8b^3c^3g^2 + 15360a^7c^7d^2e + 5120a^8c^6d^2g - 3072a^8c^6e^2f - 1024a^9c^5f^2g + 6a^2b^{11}c^2d^2f + 1536a^7b^6c^6d^2f - 18a^3b^{10}c^2d^2g - 2a^3b^{10}c^2e^2f + 6a^4b^9c^2e^2g + 3584a^8b^5c^5e^2g + 6a^4c^2e^2g(-4ac - b^2)^9)^{1/2} + 12a^5b^8c^2f^2g - 152a^2b^{10}c^2d^2e + 1548a^3b^8c^3d^2e - 8064a^4b^6c^4d^2e + 22400a^5b^4c^5d^2e - 30720a^6b^2c^6d^2e - 98a^3b^9c^2d^2f + 576a^4b^7c^3d^2f - 1344a^5b^5c^4d^2f + 512a^6b^3c^5d^2f + 10a^3c^2d^2f(-4ac - b^2)^9)^{1/2} + 324a^4b^8c^2d^2g + 36a^4b^8c^2e^2f - 2240a^5b^6c^3d^2g - 192a^5b^6c^3e^2f + 7296a^6b^4c^4d^2g + 128a^6b^4c^4e^2f - 10752a^7b^2c^5d^2g + 1536a^7b^2c^5e^2f - 128a^5b^7c^2e^2g + 960a^6b^5c^3e^2g - 3072a^7b^3c^4e^2g - 128a^6b^6c^2f^2g + 384a^7b^4c^3f^2g + 6a^2b^{12}c^2d^2e + 51a^2b^2c^2d^2(-4ac - b^2)^9)^{1/2} - a^2b^2c^2e^2(-4ac - b^2)^9)^{1/2} + 6a^2b^3c^2d^2e(-4ac - b^2)^9)^{1/2} - 18a^3b^3c^2d^2g(-4ac - b^2)^9)^{1/2} - 2a^3b^3c^2e^2f(-4ac - b^2)^9)^{1/2} - 44a^2b^3c^2d^2e(-4ac - b^2)^9)^{1/2} + 6a^2b^2c^2d^2f(-4ac - b^2)^9)^{1/2} / (32(4096a^{11}c^7 + a^5b^{12}c - 24a^6b^{10}c^2 + 240a^7b^8c^3 - 1280a^8b^6c^4 + 3840a^9b^4c^5 - 6144a^{10}b^2c^6))^{1/2} * (x((a^5g^2 - (4ac - b^2)^9)^{1/2} - a^5b^9g^2 - 9b^{13}cd^2 + 213ab^{11}c^2d^2 - 26880a^6b^7c^7d^2 - a^2b^{11}c^2e^2 + 3840a^7b^6c^6e^2 - 9b^4cd^2(-4ac - b^2)^9)^{1/2} - a^4b^9c^2f^2 + 768a^8b^5c^5f^2 - a^4c^2f^2(-4ac - b^2)^9)^{1/2} + 768a^9b^4c^4g^2 - 2077a^2b^9c^3d^2 + 10656a^3b^7c^4d^2 - 30240a^4b^5c^5d^2 + 44800a^5b^3c^6d^2 - 25a^2c^3d^2(-4ac - b^2)^9)^{1/2} + 27a^3b^9c^2e^2 - 288a^4b^7c^3e^2 + 1504a^5b^5c^4e^2 - 3840a^6b^3c^5e^2 + 9a^3c^2e^2(-4ac - b^2)^9)^{1/2} + 96a^6b^5c^3f^2 - 512a^7b^3c^4f^2 + 96a^7b^5c^2g^2 - 512a^8b^3c^3g^2 + 15360a^7c^7d^2e + 5120a^8c^6d^2g - 3072a^8c^6e^2f - 1024a^9c^5f^2g + 6a^2b^{11}c^2d^2f + 1536a^7b^6c^6d^2f - 18a^3b^{10}c^2d^2g - 2a^3b^{10}c^2e^2f + 6a^4b^9c^2e^2g + 3584a^8b^5c^5e^2g + 6a^4c^2e^2g(-4ac - b^2)^9)^{1/2} + 12a^5b^8c^2f^2g - 152a^2b^{10}c^2d^2e + 1548a^3b^8c^3d^2e - 8064a^4b^6c^4d^2e + 22400a^5b^4c^5d^2e - 30720a^6b^2c^6d^2e - 98a^3b^9c^2d^2f + 576a^4b^7c^3d^2f - 1344a^5b^5c^4d^2f + 512a^6b^3c^5d^2f + 10a^3c^2d^2f(-4ac - b^2)^9)^{1/2} + 324a^4b^8c^2d^2g + 36a^4b^8c^2e^2f - 2240a^5b^6c^3d^2g - 192a^5b^6c^3e^2f + 7296a^6b^4c^4d^2g + 128a^6b^4c^4e^2f - 10752a^7b^2c^5d^2g + 1536a^7b^2c^5e^2f - 128a^5b^7c^2e^2g + 960a^6b^5c^3e^2g - 3072a^7b^3c^4e^2g - 128a^6b^6c^2f^2g + 384a^7b^4c^3f^2g + 6a^2b^{12}c^2d^2e + 51a^2b^2c^2d^2(-4ac - b^2)^9)^{1/2} - a^2b^2c^2e^2(-4ac - b^2)^9)^{1/2} + 6a^2b^3c^2d^2e(-4ac - b^2)^9)^{1/2} - 18a^3b^3c^2d^2g(-4ac - b^2)^9)^{1/2} - 2a^3b^3c^2e^2f(-4ac - b^2)^9)^{1/2} - 44a^2b^3c^2d^2e(-4ac - b^2)^9)^{1/2} + 6a^2b^2c^2d^2f(-4ac - b^2)^9)^{1/2} / (32(4096a^{11}c^7 + a^5b^{12}c - 24a^6b^{10}c^2 + 240a^7b^8c^3 - 1280a^8b^6c^4 + 3840a^9b^4c^5 - 6144a^{10}b^2c^6))^{1/2} * (1048576a^{16}b^8c^8 + 256a^{10}b^{13}c^2 - 6144a^{11}b^{11}c^3 + 61440a^{12}b^9c^4 - 327680a^{13}b^7c^5 + 983040a^{14}b^5c^6 - 1572864a^{15}b^3c^7) - 131072a^{16}c^7g - 393216a^{15}c^8e + 192a^8b^{13}c^2d - 4672a^9b^{11}c^3d + 47360a^{10}b^9c^4d - 256000a^{11}b^7c^5d + 778240a^{12}b^5c^6d - 1261568a^{13}b^3c^7d - 64a^9b^{12}c^2e + 1664a^{10}b^{10}c^3e - 17920a^{11}b^8c^4e + 102400a^{12}b^6c^5e - 327680a^{13}b^4c^6e + 557056a^{14}b^2c^7e - 64a^{10}b^{11}
\end{aligned}$$

$$\begin{aligned}
& c^2f + 1280a^{11}b^9c^3f - 10240a^{12}b^7c^4f + 40960a^{13}b^5c^5f - \\
& 81920a^{14}b^3c^6f + 128a^{11}b^{10}c^2g - 2560a^{12}b^8c^3g + 20480a^{13}b^6c^4g - 81920a^{14}b^4c^5g + 163840a^{15}b^2c^6g + 851968a^{14}b^2c^8d + 65536a^{15}b^2c^7f) + x(204800a^{12}c^9d^2 - 73728a^{13}c^8e^2 \\
& + 8192a^{14}c^7f^2 - 8192a^{15}c^6g^2 + 16a^{10}b^{10}c^2g^2 + 144a^6b^{11}2c^3d^2 - 3264a^7b^{10}c^4d^2 + 30112a^8b^8c^5d^2 - 143360a^9b^6c^6d^2 + 365568a^{10}b^4c^7d^2 - 458752a^{11}b^2c^8d^2 + 16a^8b^{10}c^3e^2 - 416a^9b^8c^4e^2 + 4608a^{10}b^6c^5e^2 - 25600a^{11}b^4c^6e^2 + 69632a^{12}b^2c^7e^2 + 160a^{10}b^8c^3f^2 - 2048a^{11}b^6c^4f^2 \\
& + 9216a^{12}b^4c^5f^2 - 16384a^{13}b^2c^6f^2 - 160a^{11}b^8c^2g^2 + 512a^{12}b^6c^3g^2 - 1024a^{13}b^4c^4g^2 + 4096a^{14}b^2c^5g^2 - 81920a^{13}c^8d^2f - 49152a^{14}c^7e^2g + 237568a^{12}b^2c^8d^2e + 106496a^{13}b^2c^7d^2g + 40960a^{13}b^2c^7e^2f + 8192a^{14}b^2c^6f^2g - 96a^7b^{11}c^3d^2e + 2336a^8b^9c^4d^2e - 22528a^9b^7c^5d^2e + 107520a^{10}b^5c^6d^2e - 253952a^{11}b^3c^7d^2e - 96a^8b^{10}c^3d^2f + 1472a^9b^8c^4d^2f - 7168a^{10}b^6c^5d^2f + 6144a^{11}b^4c^6d^2f + 40960a^{12}b^2c^7d^2f + 288a^9b^9c^3d^2g + 32a^9b^9c^3e^2f - 5120a^{10}b^7c^4d^2g - 1024a^{10}b^7c^4e^2f + 33792a^{11}b^5c^5d^2g + 9216a^{11}b^5c^5e^2f - 98304a^{12}b^3c^6d^2g - 32768a^{12}b^3c^6e^2f + 64a^{10}b^8c^3e^2g - 6144a^{12}b^4c^5e^2g + 32768a^{13}b^2c^6e^2g - 96a^{10}b^9c^2f^2g + 1024a^{11}b^7c^3f^2g - 3072a^{12}b^5c^4f^2g) * ((a^5g^2 * (-4ac - b^2)^9)^{(1/2)} - a^5b^9g^2 - 9b^{13}c^2d^2 + 213a^2b^{11}c^2d^2 - 26880a^6b^2c^7d^2 - a^2b^{11}c^2e^2 + 3840a^7b^2c^6e^2 - 9b^4c^2d^2 * (-4ac - b^2)^9)^{(1/2)} - a^4b^9c^2f^2 + 768a^8b^2c^5f^2 - a^4c^2f^2 * (-4ac - b^2)^9)^{(1/2)} + 768a^9b^2c^4g^2 - 2077a^2b^9c^3d^2 + 10656a^3b^7c^4d^2 - 30240a^4b^5c^5d^2 + 44800a^5b^3c^6d^2 - 25a^2c^3d^2 * (-4ac - b^2)^9)^{(1/2)} + 27a^3b^9c^2e^2 - 288a^4b^7c^3e^2 + 1504a^5b^5c^4e^2 - 3840a^6b^3c^5e^2 + 9a^3c^2e^2 * (-4ac - b^2)^9)^{(1/2)} + 96a^6b^5c^3f^2 - 512a^7b^3c^4f^2 + 96a^7b^5c^2g^2 - 512a^8b^3c^3g^2 + 15360a^7c^7d^2e + 5120a^8c^6d^2g - 3072a^8c^6e^2f - 1024a^9c^5f^2g + 6a^2b^{11}c^2d^2f + 1536a^7b^2c^6d^2f - 18a^3b^{10}c^2d^2g - 2a^3b^{10}c^2e^2f + 6a^4b^9c^2e^2g + 3584a^8b^2c^5e^2g + 6a^4c^2e^2g * (-4ac - b^2)^9)^{(1/2)} + 12a^5b^8c^2f^2g - 152a^2b^{10}c^2d^2e + 1548a^3b^8c^3d^2e - 8064a^4b^6c^4d^2e + 22400a^5b^4c^5d^2e - 30720a^6b^2c^6d^2e - 98a^3b^9c^2d^2f + 576a^4b^7c^3d^2f - 1344a^5b^5c^4d^2f + 512a^6b^3c^5d^2f + 10a^3c^2d^2f * (-4ac - b^2)^9)^{(1/2)} + 324a^4b^8c^2d^2g + 36a^4b^8c^2e^2f - 2240a^5b^6c^3d^2g - 192a^5b^6c^3e^2f + 7296a^6b^4c^4d^2g + 128a^6b^4c^4e^2f - 10752a^7b^2c^5d^2g + 1536a^7b^2c^5e^2f - 128a^5b^7c^2e^2g + 960a^6b^5c^3e^2g - 3072a^7b^3c^4e^2g - 128a^6b^6c^2f^2g + 384a^7b^4c^3f^2g + 6a^2b^{12}c^2d^2 * (-4ac - b^2)^9)^{(1/2)} - a^2b^2c^2e^2 * (-4ac - b^2)^9)^{(1/2)} + 6a^2b^3c^2d^2e * (-4ac - b^2)^9)^{(1/2)} - 18a^3b^2c^2d^2e * (-4ac - b^2)^9)^{(1/2)} - 2a^3b^2c^2e^2f * (-4ac - b^2)^9)^{(1/2)} - 44a^2b^2c^2d^2e * (-4ac - b^2)^9)^{(1/2)} + 6a^2b^2c^2d^2f * (-4ac - b^2)^9)^{(1/2)}) / (32 * (4096a^{11}c^7 + a^5b^{12}c - 24a^6b^{10}c^2 + 240a^7b^8c^3 - 1280a^8b^6c^4 + 3840a^9b^4c^5 - 6144a^{10}b^2c^6)))^{(1/2)} * i + (((a^5g^2 * (-4ac - b^2)^9)^{(1/2)} - a^5b^9g^2 - 9b^{13}c^2d^2 + 213a^2b^{11}c^2d^2 - 26880a^6b^2c^7d^2 - a^2b^{11}c^2e^2 + 3840a^7b^2c^6e^2 - 9b^4c^2d^2 * (-4ac - b^2)^9)^{(1/2)} - a^4b^9c^2f^2 + 768a^8b^2c^5f^2 - a^4c^2f^2 * (-4ac - b^2)^9)^{(1/2)} + 768a^9b^2c^4g^2 - 2077a^2b^9c^3d^2 + 10656a^3b^7c^4d^2 - 30240a^4b^5c^5d^2 + 44800a^5b^3c^6d^2 - 25a^2c^3d^2 * (-4ac - b^2)^9)^{(1/2)} + 27a^3b^9c^2e^2 - 288a^4b^7c^3e^2 + 1504a^5b^5c^4e^2 - 3840a^6b^3c^5e^2 + 9a^3c^2e^2 * (-4ac - b^2)^9)^{(1/2)} + 96a^6b^5c^3f^2 - 512a^7b^3c^4f^2 + 96a^7b^5c^2g^2 - 512a^8b^3c^3g^2 + 15360a^7c^7d^2e + 5120a^8c^6d^2g - 3072a^8c^6e^2f - 1024a^9c^5f^2g + 6a^2b^{11}c^2d^2f + 1536a^7b^2c^6d^2f - 18a^3b^{10}c^2d^2g - 2a^3b^{10}c^2e^2f + 6a^4b^9c^2e^2g + 3584a^8b^2c^5e^2g + 6a^4c^2e^2g * (-4ac - b^2)^9)^{(1/2)} + 12a^5b^8c^2f^2g - 152a^2b^{10}c^2d^2e + 1548a^3b^8c^3d^2e - 8064a^4b^6c^4d^2e + 22400a^5b^4c^5d^2e - 30720a^6b^2c^6d^2e - 98a^3b^9c^2d^2f + 5
\end{aligned}$$

$$\begin{aligned}
& 76a^4b^7c^3d^2f - 1344a^5b^5c^4d^2f + 512a^6b^3c^5d^2f + 10a^3c^2d^2f(-4ac - b^2)^9)^{(1/2)} + 324a^4b^8c^2d^2g + 36a^4b^8c^2e^2f - \\
& 2240a^5b^6c^3d^2g - 192a^5b^6c^3e^2f + 7296a^6b^4c^4d^2g + 128a^6b^4c^4e^2f - 10752a^7b^2c^5d^2g + 1536a^7b^2c^5e^2f - 128a^5b^7c^2e^2g + 960a^6b^5c^3e^2g - \\
& 3072a^7b^3c^4e^2g - 128a^6b^6c^2f^2g + 384a^7b^4c^3f^2g + 6a^2b^12c^2d^2e + 51a^2b^2c^2d^2e(-4ac - b^2)^9)^{(1/2)} - a^2b^2c^2e^2(-4ac - b^2)^9)^{(1/2)} + \\
& 6a^2b^3c^2d^2e(-4ac - b^2)^9)^{(1/2)} - 18a^3b^3c^2d^2g(-4ac - b^2)^9)^{(1/2)} - 2a^3b^3c^2e^2f(-4ac - b^2)^9)^{(1/2)} - 44a^2b^3c^2d^2e(-4ac - b^2)^9)^{(1/2)} + \\
& 6a^2b^2c^2d^2f(-4ac - b^2)^9)^{(1/2)} / (32(4096a^{11}c^7 + a^5b^{12}c - 24a^6b^{10}c^2 + 240a^7b^8c^3 - 1280a^8b^6c^4 + 3840a^9b^4c^5 - 6144a^{10}b^2c^6))^{(1/2)} * \\
& (393216a^{15}c^8e + 131072a^{16}c^7g + x((a^5g^2(-4ac - b^2)^9)^{(1/2)} - a^5b^9g^2 - 9b^{13}c^2d^2 + 213a^2b^{11}c^2d^2 - 26880a^6b^3c^7d^2 - \\
& a^2b^{11}c^2e^2 + 3840a^7b^3c^6e^2 - 9b^4c^2d^2(-4ac - b^2)^9)^{(1/2)} - a^4b^9c^2f^2 + 768a^8b^3c^5f^2 - a^4c^2f^2(-4ac - b^2)^9)^{(1/2)} + \\
& 768a^9b^3c^4g^2 - 2077a^2b^9c^3d^2 + 10656a^3b^7c^4d^2 - 30240a^4b^5c^5d^2 + 44800a^5b^3c^6d^2 - 25a^2c^3d^2(-4ac - b^2)^9)^{(1/2)} + 27a^3b^9c^2e^2 - \\
& 288a^4b^7c^3e^2 + 1504a^5b^5c^4e^2 - 3840a^6b^3c^5e^2 + 9a^3c^2e^2(-4ac - b^2)^9)^{(1/2)} + 96a^6b^5c^3f^2 - 512a^7b^3c^4f^2 + 96a^7b^5c^2g^2 - \\
& 512a^8b^3c^3g^2 + 15360a^7c^7d^2e + 5120a^8c^6d^2g - 3072a^8c^6e^2f - 1024a^9c^5f^2g + 6a^2b^{11}c^2d^2f + 1536a^7b^3c^6d^2f - \\
& 18a^3b^{10}c^2d^2g - 2a^3b^{10}c^2e^2f + 6a^4b^9c^2e^2g + 3584a^8b^3c^5e^2g + 6a^4c^2e^2g(-4ac - b^2)^9)^{(1/2)} + 12a^5b^8c^2f^2g - \\
& 152a^2b^{10}c^2d^2e + 1548a^3b^8c^3d^2e - 8064a^4b^6c^4d^2e + 22400a^5b^4c^5d^2e - 30720a^6b^2c^6d^2e - 98a^3b^9c^2d^2f + \\
& 576a^4b^7c^3d^2f - 1344a^5b^5c^4d^2f + 512a^6b^3c^5d^2f + 10a^3c^2d^2f(-4ac - b^2)^9)^{(1/2)} + 324a^4b^8c^2d^2g + \\
& 36a^4b^8c^2e^2f - 2240a^5b^6c^3d^2g - 192a^5b^6c^3e^2f + 7296a^6b^4c^4d^2g + 128a^6b^4c^4e^2f - 10752a^7b^2c^5d^2g + \\
& 1536a^7b^2c^5e^2f - 128a^5b^7c^2e^2g + 960a^6b^5c^3e^2g - 3072a^7b^3c^4e^2g - 128a^6b^6c^2f^2g + 384a^7b^4c^3f^2g + \\
& 6a^2b^{12}c^2d^2e + 51a^2b^2c^2d^2e(-4ac - b^2)^9)^{(1/2)} - a^2b^2c^2e^2(-4ac - b^2)^9)^{(1/2)} + 6a^2b^3c^2d^2e(-4ac - b^2)^9)^{(1/2)} - \\
& 18a^3b^3c^2d^2g(-4ac - b^2)^9)^{(1/2)} - 2a^3b^3c^2e^2f(-4ac - b^2)^9)^{(1/2)} - 44a^2b^3c^2d^2e(-4ac - b^2)^9)^{(1/2)} + \\
& 6a^2b^2c^2d^2f(-4ac - b^2)^9)^{(1/2)} / (32(4096a^{11}c^7 + a^5b^{12}c - 24a^6b^{10}c^2 + 240a^7b^8c^3 - 1280a^8b^6c^4 + 3840a^9b^4c^5 - \\
& 6144a^{10}b^2c^6))^{(1/2)} * (1048576a^{16}b^8c^8 + 256a^{10}b^{13}c^2 - 6144a^{11}b^{11}c^3 + 61440a^{12}b^9c^4 - 327680a^{13}b^7c^5 + \\
& 983040a^{14}b^5c^6 - 1572864a^{15}b^3c^7) - 192a^8b^{13}c^2d + 4672a^9b^{11}c^3d - 47360a^{10}b^9c^4d + 256000a^{11}b^7c^5d - \\
& 778240a^{12}b^5c^6d + 1261568a^{13}b^3c^7d + 64a^9b^{12}c^2e - 1664a^{10}b^{10}c^3e + 17920a^{11}b^8c^4e - 102400a^{12}b^6c^5e + \\
& 327680a^{13}b^4c^6e - 557056a^{14}b^2c^7e + 64a^{10}b^{11}c^2f - 1280a^{11}b^9c^3f + 10240a^{12}b^7c^4f - 40960a^{13}b^5c^5f + \\
& 81920a^{14}b^3c^6f - 128a^{11}b^{10}c^2g + 2560a^{12}b^8c^3g - 20480a^{13}b^6c^4g + 81920a^{14}b^4c^5g - 163840a^{15}b^2c^6g - \\
& 851968a^{14}b^3c^8d - 65536a^{15}b^4c^7f) + x(204800a^{12}c^9d^2 - 73728a^{13}c^8e^2 + 8192a^{14}c^7f^2 - 8192a^{15}c^6g^2 + \\
& 16a^{10}b^{10}c^4d^2 + 30112a^8b^8c^5d^2 - 143360a^9b^6c^6d^2 + 365568a^{10}b^4c^7d^2 - 458752a^{11}b^2c^8d^2 + \\
& 16a^8b^{10}c^3e^2 - 416a^9b^8c^4e^2 + 4608a^{10}b^6c^5e^2 - 25600a^{11}b^4c^6e^2 + 69632a^{12}b^2c^7e^2 + 160a^{10}b^8c^3f^2 - \\
& 2048a^{11}b^6c^4f^2 + 9216a^{12}b^4c^5f^2 - 16384a^{13}b^2c^6f^2 - 160a^{11}b^8c^2g^2 + 512a^{12}b^6c^3g^2 - 1024a^{13}b^4c^4g^2 + \\
& 4096a^{14}b^2c^5g^2 - 81920a^{13}c^8d^2f - 49152a^{14}c^7e^2g + 237568a^{12}b^3c^8d^2e + 106496a^{13}b^2c^7d^2g + 40960a^{13}b^3c^7e^2f + \\
& 8192a^{14}b^2c^6f^2g - 96a^7b^{11}c^3d^2e + 2336a^8b^9c^4d^2e - 22528a^9b^7c^5d^2e + 107520a^{10}b^5c^6d^2e - \\
& 253952a^{11}b^3c^7d^2e - 96a^8b^{10}c^3d^2f + 1472a^9b^8c^4d^2f - 7168a^{10}b^6c^5d^2f + 6144a^{11}b^4c^6d^2f + \\
& 40960a^{12}b^2c^7d^2f + 288a^9b^9c^3d^2g + 32a^9b
\end{aligned}$$

$$\begin{aligned}
& ^9c^3*ef - 5120*a^{10}*b^7*c^4*d*g - 1024*a^{10}*b^7*c^4*ef + 33792*a^{11}*b^5 \\
& *c^5*d*g + 9216*a^{11}*b^5*c^5*ef - 98304*a^{12}*b^3*c^6*d*g - 32768*a^{12}*b^3* \\
& c^6*ef + 64*a^{10}*b^8*c^3*eg - 6144*a^{12}*b^4*c^5*eg + 32768*a^{13}*b^2*c^6* \\
& eg - 96*a^{10}*b^9*c^2*f*g + 1024*a^{11}*b^7*c^3*f*g - 3072*a^{12}*b^5*c^4*f*g) \\
& *((a^5*g^2*(-(4*a*c - b^2)^9)^{(1/2)} - a^5*b^9*g^2 - 9*b^{13}*c*d^2 + 213*a*b^{11}* \\
& c^2*d^2 - 26880*a^6*b*c^7*d^2 - a^2*b^{11}*c*e^2 + 3840*a^7*b*c^6*e^2 - 9* \\
& b^4*c*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - a^4*b^9*c*f^2 + 768*a^8*b*c^5*f^2 - a^4* \\
& c*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + 768*a^9*b*c^4*g^2 - 2077*a^2*b^9*c^3*d^2 \\
& + 10656*a^3*b^7*c^4*d^2 - 30240*a^4*b^5*c^5*d^2 + 44800*a^5*b^3*c^6*d^2 - \\
& 25*a^2*c^3*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + 27*a^3*b^9*c^2*e^2 - 288*a^4*b^7* \\
& c^3*e^2 + 1504*a^5*b^5*c^4*e^2 - 3840*a^6*b^3*c^5*e^2 + 9*a^3*c^2*e^2*(-(4* \\
& a*c - b^2)^9)^{(1/2)} + 96*a^6*b^5*c^3*f^2 - 512*a^7*b^3*c^4*f^2 + 96*a^7*b^5* \\
& c^2*g^2 - 512*a^8*b^3*c^3*g^2 + 15360*a^7*c^7*d*e + 5120*a^8*c^6*d*g - 307 \\
& 2*a^8*c^6*ef - 1024*a^9*c^5*f*g + 6*a^2*b^{11}*c*d*f + 1536*a^7*b*c^6*d*f - \\
& 18*a^3*b^{10}*c*d*g - 2*a^3*b^{10}*c*ef + 6*a^4*b^9*c*eg + 3584*a^8*b*c^5*eg \\
& + 6*a^4*c*eg*(-(4*a*c - b^2)^9)^{(1/2)} + 12*a^5*b^8*c*f*g - 152*a^2*b^{10}*c \\
& ^2*d*e + 1548*a^3*b^8*c^3*d*e - 8064*a^4*b^6*c^4*d*e + 22400*a^5*b^4*c^5*d* \\
& e - 30720*a^6*b^2*c^6*d*e - 98*a^3*b^9*c^2*d*f + 576*a^4*b^7*c^3*d*f - 1344 \\
& *a^5*b^5*c^4*d*f + 512*a^6*b^3*c^5*d*f + 10*a^3*c^2*d*f*(-(4*a*c - b^2)^9)^{(1/2)} \\
& + 324*a^4*b^8*c^2*d*g + 36*a^4*b^8*c^2*ef - 2240*a^5*b^6*c^3*d*g - 1 \\
& 92*a^5*b^6*c^3*ef + 7296*a^6*b^4*c^4*d*g + 128*a^6*b^4*c^4*ef - 10752*a^7 \\
& *b^2*c^5*d*g + 1536*a^7*b^2*c^5*ef - 128*a^5*b^7*c^2*eg + 960*a^6*b^5*c^3 \\
& *eg - 3072*a^7*b^3*c^4*eg - 128*a^6*b^6*c^2*f*g + 384*a^7*b^4*c^3*f*g + 6 \\
& *a*b^{12}*c*d*e + 51*a*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - a^2*b^2*c*e^2* \\
& (- (4*a*c - b^2)^9)^{(1/2)} + 6*a*b^3*c*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 18*a^3*b \\
& *c*d*g*(-(4*a*c - b^2)^9)^{(1/2)} - 2*a^3*b*c*ef*(-(4*a*c - b^2)^9)^{(1/2)} - \\
& 44*a^2*b*c^2*d*e*(-(4*a*c - b^2)^9)^{(1/2)} + 6*a^2*b^2*c*d*f*(-(4*a*c - b^2) \\
& ^9)^{(1/2)))/(32*(4096*a^{11}*c^7 + a^5*b^{12}*c - 24*a^6*b^{10}*c^2 + 240*a^7*b^8* \\
& c^3 - 1280*a^8*b^6*c^4 + 3840*a^9*b^4*c^5 - 6144*a^{10}*b^2*c^6))^{(1/2)}*i)/ \\
& (((a^5*g^2*(-(4*a*c - b^2)^9)^{(1/2)} - a^5*b^9*g^2 - 9*b^{13}*c*d^2 + 213*a*b^{11}* \\
& c^2*d^2 - 26880*a^6*b*c^7*d^2 - a^2*b^{11}*c*e^2 + 3840*a^7*b*c^6*e^2 - 9* \\
& b^4*c*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - a^4*b^9*c*f^2 + 768*a^8*b*c^5*f^2 - a^4* \\
& c*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + 768*a^9*b*c^4*g^2 - 2077*a^2*b^9*c^3*d^2 \\
& + 10656*a^3*b^7*c^4*d^2 - 30240*a^4*b^5*c^5*d^2 + 44800*a^5*b^3*c^6*d^2 - \\
& 25*a^2*c^3*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + 27*a^3*b^9*c^2*e^2 - 288*a^4*b^7* \\
& c^3*e^2 + 1504*a^5*b^5*c^4*e^2 - 3840*a^6*b^3*c^5*e^2 + 9*a^3*c^2*e^2*(-(4* \\
& a*c - b^2)^9)^{(1/2)} + 96*a^6*b^5*c^3*f^2 - 512*a^7*b^3*c^4*f^2 + 96*a^7*b^5* \\
& c^2*g^2 - 512*a^8*b^3*c^3*g^2 + 15360*a^7*c^7*d*e + 5120*a^8*c^6*d*g - 30 \\
& 72*a^8*c^6*ef - 1024*a^9*c^5*f*g + 6*a^2*b^{11}*c*d*f + 1536*a^7*b*c^6*d*f - \\
& 18*a^3*b^{10}*c*d*g - 2*a^3*b^{10}*c*ef + 6*a^4*b^9*c*eg + 3584*a^8*b*c^5*eg \\
& g + 6*a^4*c*eg*(-(4*a*c - b^2)^9)^{(1/2)} + 12*a^5*b^8*c*f*g - 152*a^2*b^{10}* \\
& c^2*d*e + 1548*a^3*b^8*c^3*d*e - 8064*a^4*b^6*c^4*d*e + 22400*a^5*b^4*c^5*d \\
& *e - 30720*a^6*b^2*c^6*d*e - 98*a^3*b^9*c^2*d*f + 576*a^4*b^7*c^3*d*f - 134 \\
& 4*a^5*b^5*c^4*d*f + 512*a^6*b^3*c^5*d*f + 10*a^3*c^2*d*f*(-(4*a*c - b^2)^9) \\
& ^{(1/2)} + 324*a^4*b^8*c^2*d*g + 36*a^4*b^8*c^2*ef - 2240*a^5*b^6*c^3*d*g - \\
& 192*a^5*b^6*c^3*ef + 7296*a^6*b^4*c^4*d*g + 128*a^6*b^4*c^4*ef - 10752*a^7 \\
& *b^2*c^5*d*g + 1536*a^7*b^2*c^5*ef - 128*a^5*b^7*c^2*eg + 960*a^6*b^5*c^3 \\
& *eg - 3072*a^7*b^3*c^4*eg - 128*a^6*b^6*c^2*f*g + 384*a^7*b^4*c^3*f*g + \\
& 6*a*b^{12}*c*d*e + 51*a*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - a^2*b^2*c*e^2* \\
& (- (4*a*c - b^2)^9)^{(1/2)} + 6*a*b^3*c*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 18*a^3*b \\
& *c*d*g*(-(4*a*c - b^2)^9)^{(1/2)} - 2*a^3*b*c*ef*(-(4*a*c - b^2)^9)^{(1/2)} - \\
& 44*a^2*b*c^2*d*e*(-(4*a*c - b^2)^9)^{(1/2)} + 6*a^2*b^2*c*d*f*(-(4*a*c - b^2) \\
& ^9)^{(1/2)))/(32*(4096*a^{11}*c^7 + a^5*b^{12}*c - 24*a^6*b^{10}*c^2 + 240*a^7*b^8* \\
& c^3 - 1280*a^8*b^6*c^4 + 3840*a^9*b^4*c^5 - 6144*a^{10}*b^2*c^6))^{(1/2)}*(39 \\
& 3216*a^{15}*c^8*e + 131072*a^{16}*c^7*g + x*((a^5*g^2*(-(4*a*c - b^2)^9)^{(1/2)} \\
& - a^5*b^9*g^2 - 9*b^{13}*c*d^2 + 213*a*b^{11}*c^2*d^2 - 26880*a^6*b*c^7*d^2 - a^2* \\
& b^{11}*c*e^2 + 3840*a^7*b*c^6*e^2 - 9*b^4*c*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - \\
& a^4*b^9*c*f^2 + 768*a^8*b*c^5*f^2 - a^4*c*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + 7 \\
& 68*a^9*b*c^4*g^2 - 2077*a^2*b^9*c^3*d^2 + 10656*a^3*b^7*c^4*d^2 - 30240*a^4
\end{aligned}$$

$$\begin{aligned}
& *b^5*c^5*d^2 + 44800*a^5*b^3*c^6*d^2 - 25*a^2*c^3*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + 27*a^3*b^9*c^2*e^2 - 288*a^4*b^7*c^3*e^2 + 1504*a^5*b^5*c^4*e^2 - 384 \\
& 0*a^6*b^3*c^5*e^2 + 9*a^3*c^2*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 96*a^6*b^5*c^3 \\
& *f^2 - 512*a^7*b^3*c^4*f^2 + 96*a^7*b^5*c^2*g^2 - 512*a^8*b^3*c^3*g^2 + 153 \\
& 60*a^7*c^7*d*e + 5120*a^8*c^6*d*g - 3072*a^8*c^6*e*f - 1024*a^9*c^5*f*g + 6 \\
& *a^2*b^11*c*d*f + 1536*a^7*b*c^6*d*f - 18*a^3*b^10*c*d*g - 2*a^3*b^10*c*e*f \\
& + 6*a^4*b^9*c*e*g + 3584*a^8*b*c^5*e*g + 6*a^4*c*e*g*(-(4*a*c - b^2)^9)^{(1/2)} + 12*a^5*b^8*c*f*g - 152*a^2*b^10*c^2*d*e + 1548*a^3*b^8*c^3*d*e - 8064 \\
& *a^4*b^6*c^4*d*e + 22400*a^5*b^4*c^5*d*e - 30720*a^6*b^2*c^6*d*e - 98*a^3*b \\
& ^9*c^2*d*f + 576*a^4*b^7*c^3*d*f - 1344*a^5*b^5*c^4*d*f + 512*a^6*b^3*c^5*d \\
& *f + 10*a^3*c^2*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 324*a^4*b^8*c^2*d*g + 36*a^4 \\
& *b^8*c^2*e*f - 2240*a^5*b^6*c^3*d*g - 192*a^5*b^6*c^3*e*f + 7296*a^6*b^4*c^ \\
& 4*d*g + 128*a^6*b^4*c^4*e*f - 10752*a^7*b^2*c^5*d*g + 1536*a^7*b^2*c^5*e*f \\
& - 128*a^5*b^7*c^2*e*g + 960*a^6*b^5*c^3*e*g - 3072*a^7*b^3*c^4*e*g - 128*a^ \\
& 6*b^6*c^2*f*g + 384*a^7*b^4*c^3*f*g + 6*a*b^12*c*d*e + 51*a*b^2*c^2*d^2*(-(\\
& 4*a*c - b^2)^9)^{(1/2)} - a^2*b^2*c*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 6*a*b^3*c* \\
& d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 18*a^3*b*c*d*g*(-(4*a*c - b^2)^9)^{(1/2)} - 2* \\
& a^3*b*c*e*f*(-(4*a*c - b^2)^9)^{(1/2)} - 44*a^2*b*c^2*d*e*(-(4*a*c - b^2)^9)^{ \\
& (1/2)} + 6*a^2*b^2*c*d*f*(-(4*a*c - b^2)^9)^{(1/2)}/(32*(4096*a^11*c^7 + a^5* \\
& b^12*c - 24*a^6*b^10*c^2 + 240*a^7*b^8*c^3 - 1280*a^8*b^6*c^4 + 3840*a^9*b^ \\
& 4*c^5 - 6144*a^10*b^2*c^6)))^{(1/2)}*(1048576*a^16*b*c^8 + 256*a^10*b^13*c^2 \\
& - 6144*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c^5 + 983040*a^ \\
& 14*b^5*c^6 - 1572864*a^15*b^3*c^7) - 192*a^8*b^13*c^2*d + 4672*a^9*b^11*c^3 \\
& *d - 47360*a^10*b^9*c^4*d + 256000*a^11*b^7*c^5*d - 778240*a^12*b^5*c^6*d + \\
& 1261568*a^13*b^3*c^7*d + 64*a^9*b^12*c^2*e - 1664*a^10*b^10*c^3*e + 17920* \\
& a^11*b^8*c^4*e - 102400*a^12*b^6*c^5*e + 327680*a^13*b^4*c^6*e - 557056*a^1 \\
& 4*b^2*c^7*e + 64*a^10*b^11*c^2*f - 1280*a^11*b^9*c^3*f + 10240*a^12*b^7*c^4 \\
& *f - 40960*a^13*b^5*c^5*f + 81920*a^14*b^3*c^6*f - 128*a^11*b^10*c^2*g + 25 \\
& 60*a^12*b^8*c^3*g - 20480*a^13*b^6*c^4*g + 81920*a^14*b^4*c^5*g - 163840*a^ \\
& 15*b^2*c^6*g - 851968*a^14*b*c^8*d - 65536*a^15*b*c^7*f) + x*(204800*a^12*c \\
& ^9*d^2 - 73728*a^13*c^8*e^2 + 8192*a^14*c^7*f^2 - 8192*a^15*c^6*g^2 + 16*a^ \\
& 10*b^10*c*g^2 + 144*a^6*b^12*c^3*d^2 - 3264*a^7*b^10*c^4*d^2 + 30112*a^8*b^ \\
& 8*c^5*d^2 - 143360*a^9*b^6*c^6*d^2 + 365568*a^10*b^4*c^7*d^2 - 458752*a^11* \\
& b^2*c^8*d^2 + 16*a^8*b^10*c^3*e^2 - 416*a^9*b^8*c^4*e^2 + 4608*a^10*b^6*c^5 \\
& *e^2 - 25600*a^11*b^4*c^6*e^2 + 69632*a^12*b^2*c^7*e^2 + 160*a^10*b^8*c^3*f \\
& ^2 - 2048*a^11*b^6*c^4*f^2 + 9216*a^12*b^4*c^5*f^2 - 16384*a^13*b^2*c^6*f^2 \\
& - 160*a^11*b^8*c^2*g^2 + 512*a^12*b^6*c^3*g^2 - 1024*a^13*b^4*c^4*g^2 + 40 \\
& 96*a^14*b^2*c^5*g^2 - 81920*a^13*c^8*d*f - 49152*a^14*c^7*e*g + 237568*a^12 \\
& *b*c^8*d*e + 106496*a^13*b*c^7*d*g + 40960*a^13*b*c^7*e*f + 8192*a^14*b*c^6 \\
& *f*g - 96*a^7*b^11*c^3*d*e + 2336*a^8*b^9*c^4*d*e - 22528*a^9*b^7*c^5*d*e + \\
& 107520*a^10*b^5*c^6*d*e - 253952*a^11*b^3*c^7*d*e - 96*a^8*b^10*c^3*d*f + \\
& 1472*a^9*b^8*c^4*d*f - 7168*a^10*b^6*c^5*d*f + 6144*a^11*b^4*c^6*d*f + 4096 \\
& 0*a^12*b^2*c^7*d*f + 288*a^9*b^9*c^3*d*g + 32*a^9*b^9*c^3*e*f - 5120*a^10*b \\
& ^7*c^4*d*g - 1024*a^10*b^7*c^4*e*f + 33792*a^11*b^5*c^5*d*g + 9216*a^11*b^5 \\
& *c^5*e*f - 98304*a^12*b^3*c^6*d*g - 32768*a^12*b^3*c^6*e*f + 64*a^10*b^8*c^ \\
& 3*e*g - 6144*a^12*b^4*c^5*e*g + 32768*a^13*b^2*c^6*e*g - 96*a^10*b^9*c^2*f* \\
& g + 1024*a^11*b^7*c^3*f*g - 3072*a^12*b^5*c^4*f*g))*((a^5*g^2*(-(4*a*c - b^ \\
& 2)^9)^{(1/2)} - a^5*b^9*g^2 - 9*b^13*c*d^2 + 213*a*b^11*c^2*d^2 - 26880*a^6*b \\
& *c^7*d^2 - a^2*b^11*c*e^2 + 3840*a^7*b*c^6*e^2 - 9*b^4*c*d^2*(-(4*a*c - b^2 \\
&)^9)^{(1/2)} - a^4*b^9*c*f^2 + 768*a^8*b*c^5*f^2 - a^4*c*f^2*(-(4*a*c - b^2)^ \\
& 9)^{(1/2)} + 768*a^9*b*c^4*g^2 - 2077*a^2*b^9*c^3*d^2 + 10656*a^3*b^7*c^4*d^2 \\
& - 30240*a^4*b^5*c^5*d^2 + 44800*a^5*b^3*c^6*d^2 - 25*a^2*c^3*d^2*(-(4*a*c \\
& - b^2)^9)^{(1/2)} + 27*a^3*b^9*c^2*e^2 - 288*a^4*b^7*c^3*e^2 + 1504*a^5*b^5*c \\
& ^4*e^2 - 3840*a^6*b^3*c^5*e^2 + 9*a^3*c^2*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 96 \\
& *a^6*b^5*c^3*f^2 - 512*a^7*b^3*c^4*f^2 + 96*a^7*b^5*c^2*g^2 - 512*a^8*b^3*c \\
& ^3*g^2 + 15360*a^7*c^7*d*e + 5120*a^8*c^6*d*g - 3072*a^8*c^6*e*f - 1024*a^9 \\
& *c^5*f*g + 6*a^2*b^11*c*d*f + 1536*a^7*b*c^6*d*f - 18*a^3*b^10*c*d*g - 2*a^ \\
& 3*b^10*c*e*f + 6*a^4*b^9*c*e*g + 3584*a^8*b*c^5*e*g + 6*a^4*c*e*g*(-(4*a*c \\
& - b^2)^9)^{(1/2)} + 12*a^5*b^8*c*f*g - 152*a^2*b^10*c^2*d*e + 1548*a^3*b^8*c^
\end{aligned}$$

$$\begin{aligned}
& 3*d*e - 8064*a^4*b^6*c^4*d*e + 22400*a^5*b^4*c^5*d*e - 30720*a^6*b^2*c^6*d* \\
& e - 98*a^3*b^9*c^2*d*f + 576*a^4*b^7*c^3*d*f - 1344*a^5*b^5*c^4*d*f + 512*a \\
& ^6*b^3*c^5*d*f + 10*a^3*c^2*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 324*a^4*b^8*c^2* \\
& d*g + 36*a^4*b^8*c^2*e*f - 2240*a^5*b^6*c^3*d*g - 192*a^5*b^6*c^3*e*f + 729 \\
& 6*a^6*b^4*c^4*d*g + 128*a^6*b^4*c^4*e*f - 10752*a^7*b^2*c^5*d*g + 1536*a^7* \\
& b^2*c^5*e*f - 128*a^5*b^7*c^2*e*g + 960*a^6*b^5*c^3*e*g - 3072*a^7*b^3*c^4* \\
& e*g - 128*a^6*b^6*c^2*f*g + 384*a^7*b^4*c^3*f*g + 6*a*b^12*c*d*e + 51*a*b^2 \\
& *c^2*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - a^2*b^2*c*e^2*(-(4*a*c - b^2)^9)^{(1/2)} \\
& + 6*a*b^3*c*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 18*a^3*b*c*d*g*(-(4*a*c - b^2)^9 \\
&)^{(1/2)} - 2*a^3*b*c*e*f*(-(4*a*c - b^2)^9)^{(1/2)} - 44*a^2*b*c^2*d*e*(-(4*a* \\
& c - b^2)^9)^{(1/2)} + 6*a^2*b^2*c*d*f*(-(4*a*c - b^2)^9)^{(1/2)}/(32*(4096*a^1 \\
& 1*c^7 + a^5*b^12*c - 24*a^6*b^10*c^2 + 240*a^7*b^8*c^3 - 1280*a^8*b^6*c^4 + \\
& 3840*a^9*b^4*c^5 - 6144*a^10*b^2*c^6))^{(1/2)} - (((a^5*g^2*(-(4*a*c - b^2) \\
& ^9)^{(1/2)} - a^5*b^9*g^2 - 9*b^13*c*d^2 + 213*a*b^11*c^2*d^2 - 26880*a^6*b*c \\
& ^7*d^2 - a^2*b^11*c*e^2 + 3840*a^7*b*c^6*e^2 - 9*b^4*c*d^2*(-(4*a*c - b^2)^ \\
& 9)^{(1/2)} - a^4*b^9*c*f^2 + 768*a^8*b*c^5*f^2 - a^4*c*f^2*(-(4*a*c - b^2)^9) \\
& ^{(1/2)} + 768*a^9*b*c^4*g^2 - 2077*a^2*b^9*c^3*d^2 + 10656*a^3*b^7*c^4*d^2 - \\
& 30240*a^4*b^5*c^5*d^2 + 44800*a^5*b^3*c^6*d^2 - 25*a^2*c^3*d^2*(-(4*a*c - \\
& b^2)^9)^{(1/2)} + 27*a^3*b^9*c^2*e^2 - 288*a^4*b^7*c^3*e^2 + 1504*a^5*b^5*c^4 \\
& *e^2 - 3840*a^6*b^3*c^5*e^2 + 9*a^3*c^2*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 96*a \\
& ^6*b^5*c^3*f^2 - 512*a^7*b^3*c^4*f^2 + 96*a^7*b^5*c^2*g^2 - 512*a^8*b^3*c^3 \\
& *g^2 + 15360*a^7*c^7*d*e + 5120*a^8*c^6*d*g - 3072*a^8*c^6*e*f - 1024*a^9*c \\
& ^5*f*g + 6*a^2*b^11*c*d*f + 1536*a^7*b*c^6*d*f - 18*a^3*b^10*c*d*g - 2*a^3* \\
& b^10*c*e*f + 6*a^4*b^9*c*e*g + 3584*a^8*b*c^5*e*g + 6*a^4*c*e*g*(-(4*a*c - \\
& b^2)^9)^{(1/2)} + 12*a^5*b^8*c*f*g - 152*a^2*b^10*c^2*d*e + 1548*a^3*b^8*c^3* \\
& d*e - 8064*a^4*b^6*c^4*d*e + 22400*a^5*b^4*c^5*d*e - 30720*a^6*b^2*c^6*d*e \\
& - 98*a^3*b^9*c^2*d*f + 576*a^4*b^7*c^3*d*f - 1344*a^5*b^5*c^4*d*f + 512*a^6 \\
& *b^3*c^5*d*f + 10*a^3*c^2*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 324*a^4*b^8*c^2*d* \\
& g + 36*a^4*b^8*c^2*e*f - 2240*a^5*b^6*c^3*d*g - 192*a^5*b^6*c^3*e*f + 7296* \\
& a^6*b^4*c^4*d*g + 128*a^6*b^4*c^4*e*f - 10752*a^7*b^2*c^5*d*g + 1536*a^7*b^ \\
& 2*c^5*e*f - 128*a^5*b^7*c^2*e*g + 960*a^6*b^5*c^3*e*g - 3072*a^7*b^3*c^4*e* \\
& g - 128*a^6*b^6*c^2*f*g + 384*a^7*b^4*c^3*f*g + 6*a*b^12*c*d*e + 51*a*b^2*c \\
& ^2*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - a^2*b^2*c*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + \\
& 6*a*b^3*c*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 18*a^3*b*c*d*g*(-(4*a*c - b^2)^9) \\
& ^{(1/2)} - 2*a^3*b*c*e*f*(-(4*a*c - b^2)^9)^{(1/2)} - 44*a^2*b*c^2*d*e*(-(4*a*c \\
& - b^2)^9)^{(1/2)} + 6*a^2*b^2*c*d*f*(-(4*a*c - b^2)^9)^{(1/2)}/(32*(4096*a^11* \\
& c^7 + a^5*b^12*c - 24*a^6*b^10*c^2 + 240*a^7*b^8*c^3 - 1280*a^8*b^6*c^4 + 3 \\
& 840*a^9*b^4*c^5 - 6144*a^10*b^2*c^6))^{(1/2)}*(x*((a^5*g^2*(-(4*a*c - b^2)^9 \\
&)^{(1/2)} - a^5*b^9*g^2 - 9*b^13*c*d^2 + 213*a*b^11*c^2*d^2 - 26880*a^6*b*c^7 \\
& *d^2 - a^2*b^11*c*e^2 + 3840*a^7*b*c^6*e^2 - 9*b^4*c*d^2*(-(4*a*c - b^2)^9) \\
& ^{(1/2)} - a^4*b^9*c*f^2 + 768*a^8*b*c^5*f^2 - a^4*c*f^2*(-(4*a*c - b^2)^9)^{(\\
& 1/2)} + 768*a^9*b*c^4*g^2 - 2077*a^2*b^9*c^3*d^2 + 10656*a^3*b^7*c^4*d^2 - 3 \\
& 0240*a^4*b^5*c^5*d^2 + 44800*a^5*b^3*c^6*d^2 - 25*a^2*c^3*d^2*(-(4*a*c - b^ \\
& 2)^9)^{(1/2)} + 27*a^3*b^9*c^2*e^2 - 288*a^4*b^7*c^3*e^2 + 1504*a^5*b^5*c^4*e \\
& ^2 - 3840*a^6*b^3*c^5*e^2 + 9*a^3*c^2*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 96*a^6 \\
& *b^5*c^3*f^2 - 512*a^7*b^3*c^4*f^2 + 96*a^7*b^5*c^2*g^2 - 512*a^8*b^3*c^3*g \\
& ^2 + 15360*a^7*c^7*d*e + 5120*a^8*c^6*d*g - 3072*a^8*c^6*e*f - 1024*a^9*c^5 \\
& *f*g + 6*a^2*b^11*c*d*f + 1536*a^7*b*c^6*d*f - 18*a^3*b^10*c*d*g - 2*a^3*b^ \\
& 10*c*e*f + 6*a^4*b^9*c*e*g + 3584*a^8*b*c^5*e*g + 6*a^4*c*e*g*(-(4*a*c - b^ \\
& 2)^9)^{(1/2)} + 12*a^5*b^8*c*f*g - 152*a^2*b^10*c^2*d*e + 1548*a^3*b^8*c^3*d* \\
& e - 8064*a^4*b^6*c^4*d*e + 22400*a^5*b^4*c^5*d*e - 30720*a^6*b^2*c^6*d*e - \\
& 98*a^3*b^9*c^2*d*f + 576*a^4*b^7*c^3*d*f - 1344*a^5*b^5*c^4*d*f + 512*a^6*b \\
& ^3*c^5*d*f + 10*a^3*c^2*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 324*a^4*b^8*c^2*d*g \\
& + 36*a^4*b^8*c^2*e*f - 2240*a^5*b^6*c^3*d*g - 192*a^5*b^6*c^3*e*f + 7296*a^ \\
& 6*b^4*c^4*d*g + 128*a^6*b^4*c^4*e*f - 10752*a^7*b^2*c^5*d*g + 1536*a^7*b^2* \\
& c^5*e*f - 128*a^5*b^7*c^2*e*g + 960*a^6*b^5*c^3*e*g - 3072*a^7*b^3*c^4*e*g \\
& - 128*a^6*b^6*c^2*f*g + 384*a^7*b^4*c^3*f*g + 6*a*b^12*c*d*e + 51*a*b^2*c^2 \\
& *d^2*(-(4*a*c - b^2)^9)^{(1/2)} - a^2*b^2*c*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 6* \\
& a*b^3*c*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 18*a^3*b*c*d*g*(-(4*a*c - b^2)^9)^{(1
\end{aligned}$$

$$\begin{aligned}
& /2) - 2*a^3*b*c*e*f*(-(4*a*c - b^2)^9)^{(1/2)} - 44*a^2*b*c^2*d*e*(-(4*a*c - \\
& b^2)^9)^{(1/2)} + 6*a^2*b^2*c*d*f*(-(4*a*c - b^2)^9)^{(1/2)}/(32*(4096*a^{11}*c^7 \\
& + a^5*b^{12}*c - 24*a^6*b^{10}*c^2 + 240*a^7*b^8*c^3 - 1280*a^8*b^6*c^4 + 384 \\
& 0*a^9*b^4*c^5 - 6144*a^{10}*b^2*c^6)))^{(1/2)}*(1048576*a^{16}*b*c^8 + 256*a^{10}*b \\
& ^{13}*c^2 - 6144*a^{11}*b^{11}*c^3 + 61440*a^{12}*b^9*c^4 - 327680*a^{13}*b^7*c^5 + 9 \\
& 83040*a^{14}*b^5*c^6 - 1572864*a^{15}*b^3*c^7) - 131072*a^{16}*c^7*g - 393216*a^{15} \\
& *c^8*e + 192*a^8*b^{13}*c^2*d - 4672*a^9*b^{11}*c^3*d + 47360*a^{10}*b^9*c^4*d - \\
& 256000*a^{11}*b^7*c^5*d + 778240*a^{12}*b^5*c^6*d - 1261568*a^{13}*b^3*c^7*d - 6 \\
& 4*a^9*b^{12}*c^2*e + 1664*a^{10}*b^{10}*c^3*e - 17920*a^{11}*b^8*c^4*e + 102400*a^{12} \\
& *b^6*c^5*e - 327680*a^{13}*b^4*c^6*e + 557056*a^{14}*b^2*c^7*e - 64*a^{10}*b^{11} \\
& *c^2*f + 1280*a^{11}*b^9*c^3*f - 10240*a^{12}*b^7*c^4*f + 40960*a^{13}*b^5*c^5*f - \\
& 81920*a^{14}*b^3*c^6*f + 128*a^{11}*b^{10}*c^2*g - 2560*a^{12}*b^8*c^3*g + 20480*a^{13} \\
& *b^6*c^4*g - 81920*a^{14}*b^4*c^5*g + 163840*a^{15}*b^2*c^6*g + 851968*a^{14} \\
& *b*c^8*d + 65536*a^{15}*b*c^7*f) + x*(204800*a^{12}*c^9*d^2 - 73728*a^{13}*c^8*e^2 \\
& + 8192*a^{14}*c^7*f^2 - 8192*a^{15}*c^6*g^2 + 16*a^{10}*b^{10}*c*g^2 + 144*a^6*b^{11} \\
& *c^3*d^2 - 3264*a^7*b^{10}*c^4*d^2 + 30112*a^8*b^8*c^5*d^2 - 143360*a^9*b^6 \\
& *c^6*d^2 + 365568*a^{10}*b^4*c^7*d^2 - 458752*a^{11}*b^2*c^8*d^2 + 16*a^8*b^{10}*c \\
& ^3*e^2 - 416*a^9*b^8*c^4*e^2 + 4608*a^{10}*b^6*c^5*e^2 - 25600*a^{11}*b^4*c^6*e \\
& ^2 + 69632*a^{12}*b^2*c^7*e^2 + 160*a^{10}*b^8*c^3*f^2 - 2048*a^{11}*b^6*c^4*f^2 \\
& + 9216*a^{12}*b^4*c^5*f^2 - 16384*a^{13}*b^2*c^6*f^2 - 160*a^{11}*b^8*c^2*g^2 + 5 \\
& 12*a^{12}*b^6*c^3*g^2 - 1024*a^{13}*b^4*c^4*g^2 + 4096*a^{14}*b^2*c^5*g^2 - 81920 \\
& *a^{13}*c^8*d*f - 49152*a^{14}*c^7*e*g + 237568*a^{12}*b*c^8*d*e + 106496*a^{13}*b \\
& *c^7*d*g + 40960*a^{13}*b*c^7*e*f + 8192*a^{14}*b*c^6*f*g - 96*a^7*b^{11}*c^3*d*e \\
& + 2336*a^8*b^9*c^4*d*e - 22528*a^9*b^7*c^5*d*e + 107520*a^{10}*b^5*c^6*d*e - \\
& 253952*a^{11}*b^3*c^7*d*e - 96*a^8*b^{10}*c^3*d*f + 1472*a^9*b^8*c^4*d*f - 7168 \\
& *a^{10}*b^6*c^5*d*f + 6144*a^{11}*b^4*c^6*d*f + 40960*a^{12}*b^2*c^7*d*f + 288*a^9 \\
& *b^9*c^3*d*g + 32*a^9*b^9*c^3*e*f - 5120*a^{10}*b^7*c^4*d*g - 1024*a^{10}*b^7 \\
& *c^4*e*f + 33792*a^{11}*b^5*c^5*d*g + 9216*a^{11}*b^5*c^5*e*f - 98304*a^{12}*b^3*c \\
& ^6*d*g - 32768*a^{12}*b^3*c^6*e*f + 64*a^{10}*b^8*c^3*e*g - 6144*a^{12}*b^4*c^5*e \\
& *g + 32768*a^{13}*b^2*c^6*e*g - 96*a^{10}*b^9*c^2*f*g + 1024*a^{11}*b^7*c^3*f*g - \\
& 3072*a^{12}*b^5*c^4*f*g)*((a^5*g^2*(-(4*a*c - b^2)^9)^{(1/2)} - a^5*b^9*g^2 - \\
& 9*b^{13}*c*d^2 + 213*a*b^{11}*c^2*d^2 - 26880*a^6*b*c^7*d^2 - a^2*b^{11}*c*e^2 + \\
& 3840*a^7*b*c^6*e^2 - 9*b^4*c*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - a^4*b^9*c*f^2 \\
& + 768*a^8*b*c^5*f^2 - a^4*c*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + 768*a^9*b*c^4*g^2 \\
& - 2077*a^2*b^9*c^3*d^2 + 10656*a^3*b^7*c^4*d^2 - 30240*a^4*b^5*c^5*d^2 + \\
& 44800*a^5*b^3*c^6*d^2 - 25*a^2*c^3*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + 27*a^3*b^9 \\
& *c^2*e^2 - 288*a^4*b^7*c^3*e^2 + 1504*a^5*b^5*c^4*e^2 - 3840*a^6*b^3*c^5*e \\
& ^2 + 9*a^3*c^2*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 96*a^6*b^5*c^3*f^2 - 512*a^7*b^3 \\
& *c^4*f^2 + 96*a^7*b^5*c^2*g^2 - 512*a^8*b^3*c^3*g^2 + 15360*a^7*c^7*d*e \\
& + 5120*a^8*c^6*d*g - 3072*a^8*c^6*e*f - 1024*a^9*c^5*f*g + 6*a^2*b^{11}*c*d*f \\
& + 1536*a^7*b*c^6*d*f - 18*a^3*b^{10}*c*d*g - 2*a^3*b^{10}*c*e*f + 6*a^4*b^9*c \\
& *e*g + 3584*a^8*b*c^5*e*g + 6*a^4*c*e*g*(-(4*a*c - b^2)^9)^{(1/2)} + 12*a^5*b^8 \\
& *c*f*g - 152*a^2*b^{10}*c^2*d*e + 1548*a^3*b^8*c^3*d*e - 8064*a^4*b^6*c^4*d \\
& *e + 22400*a^5*b^4*c^5*d*e - 30720*a^6*b^2*c^6*d*e - 98*a^3*b^9*c^2*d*f + 57 \\
& 6*a^4*b^7*c^3*d*f - 1344*a^5*b^5*c^4*d*f + 512*a^6*b^3*c^5*d*f + 10*a^3*c^2 \\
& *d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 324*a^4*b^8*c^2*d*g + 36*a^4*b^8*c^2*e*f - \\
& 2240*a^5*b^6*c^3*d*g - 192*a^5*b^6*c^3*e*f + 7296*a^6*b^4*c^4*d*g + 128*a^6 \\
& *b^4*c^4*e*f - 10752*a^7*b^2*c^5*d*g + 1536*a^7*b^2*c^5*e*f - 128*a^5*b^7*c \\
& ^2*e*g + 960*a^6*b^5*c^3*e*g - 3072*a^7*b^3*c^4*e*g - 128*a^6*b^6*c^2*f*g + \\
& 384*a^7*b^4*c^3*f*g + 6*a*b^{12}*c*d*e + 51*a*b^2*c^2*d^2*(-(4*a*c - b^2)^9) \\
& ^{(1/2)} - a^2*b^2*c*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 6*a*b^3*c*d*e*(-(4*a*c - \\
& b^2)^9)^{(1/2)} - 18*a^3*b*c*d*g*(-(4*a*c - b^2)^9)^{(1/2)} - 2*a^3*b*c*e*f*(-(\\
& 4*a*c - b^2)^9)^{(1/2)} - 44*a^2*b*c^2*d*e*(-(4*a*c - b^2)^9)^{(1/2)} + 6*a^2*b \\
& ^2*c*d*f*(-(4*a*c - b^2)^9)^{(1/2)}/(32*(4096*a^{11}*c^7 + a^5*b^{12}*c - 24*a^6 \\
& *b^{10}*c^2 + 240*a^7*b^8*c^3 - 1280*a^8*b^6*c^4 + 3840*a^9*b^4*c^5 - 6144*a^ \\
& 10*b^2*c^6)))^{(1/2)} - 128000*a^{10}*c^9*d^3 + 1024*a^{13}*c^6*f^3 - 4608*a^{11}*b \\
& *c^7*e^3 - 24*a^{11}*b^7*c*g^3 - 46080*a^{11}*c^8*d*e^2 - 512*a^{14}*b*c^4*g^3 + \\
& 76800*a^{11}*c^8*d^2*f - 15360*a^{12}*c^7*d*f^2 + 9216*a^{12}*c^7*e^2*f - 5120*a^ \\
& 13*c^6*d*g^2 + 1024*a^{14}*c^5*f*g^2 - 504*a^6*b^8*c^5*d^3 + 8112*a^7*b^6*c^6
\end{aligned}$$

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*d^3 - 48704*a^8*b^4*c^7*d^3 + 129280*a^9*b^2*c^8*d^3 + 40*a^8*b^7*c^4*e^3
- 608*a^9*b^5*c^5*e^3 + 2944*a^10*b^3*c^6*e^3 + 48*a^10*b^6*c^3*f^3 - 320*a
^11*b^4*c^4*f^3 + 256*a^12*b^2*c^5*f^3 + 160*a^12*b^5*c^2*g^3 - 128*a^13*b^
3*c^3*g^3 - 30720*a^12*c^7*d*e*g + 6144*a^13*c^6*e*f*g + 84480*a^10*b*c^8*d
^2*e - 24*a^8*b^10*c*d*g^2 + 2560*a^11*b*c^7*d^2*g - 7680*a^12*b*c^6*e*f^2
+ 8*a^9*b^9*c*e*g^2 - 7680*a^12*b*c^6*e^2*g - 3584*a^13*b*c^5*e*g^2 + 8*a^1
0*b^8*c*f*g^2 - 3584*a^13*b*c^5*f^2*g + 360*a^6*b^9*c^4*d^2*e - 5736*a^7*b^
7*c^5*d^2*e - 240*a^7*b^8*c^4*d*e^2 + 33888*a^8*b^5*c^6*d^2*e + 3792*a^8*b^
6*c^5*d*e^2 - 87936*a^9*b^3*c^7*d^2*e - 21696*a^9*b^4*c^6*d*e^2 + 52992*a^1
0*b^2*c^7*d*e^2 - 216*a^6*b^10*c^3*d^2*f + 3744*a^7*b^8*c^4*d^2*f - 25200*a
^8*b^6*c^5*d^2*f - 72*a^8*b^8*c^3*d*f^2 + 81984*a^9*b^4*c^6*d^2*f + 1296*a^
9*b^6*c^4*d*f^2 - 128256*a^10*b^2*c^7*d^2*f - 7872*a^10*b^4*c^5*d*f^2 + 192
00*a^11*b^2*c^6*d*f^2 + 72*a^6*b^11*c^2*d^2*g - 1128*a^7*b^9*c^3*d^2*g + 64
88*a^8*b^7*c^4*d^2*g - 24*a^8*b^8*c^3*e^2*f - 16032*a^9*b^5*c^5*d^2*g + 336
*a^9*b^6*c^4*e^2*f + 24*a^9*b^7*c^3*e*f^2 + 368*a^9*b^8*c^2*d*g^2 + 13440*a
^10*b^3*c^6*d^2*g - 960*a^10*b^4*c^5*e^2*f - 672*a^10*b^5*c^4*e*f^2 - 1840*
a^10*b^6*c^3*d*g^2 - 2304*a^11*b^2*c^6*e^2*f + 4224*a^11*b^3*c^5*e*f^2 + 28
80*a^11*b^4*c^4*d*g^2 + 1792*a^12*b^2*c^5*d*g^2 + 8*a^8*b^9*c^2*e^2*g - 72*
a^9*b^7*c^3*e^2*g - 288*a^10*b^5*c^4*e^2*g - 136*a^10*b^7*c^2*e*g^2 + 3712*
a^11*b^3*c^5*e^2*g + 480*a^11*b^5*c^3*e*g^2 + 640*a^12*b^3*c^4*e*g^2 - 40*a
^10*b^7*c^2*f^2*g + 96*a^11*b^5*c^3*f^2*g + 80*a^11*b^6*c^2*f*g^2 + 1152*a^
12*b^3*c^4*f^2*g - 960*a^12*b^4*c^3*f*g^2 + 1792*a^13*b^2*c^4*f*g^2 + 21504
*a^11*b*c^7*d*e*f + 17408*a^12*b*c^6*d*f*g + 144*a^7*b^9*c^3*d*e*f - 2256*a
^8*b^7*c^4*d*e*f + 12480*a^9*b^5*c^5*d*e*f - 28416*a^10*b^3*c^6*d*e*f - 48*
a^7*b^10*c^2*d*e*g + 592*a^8*b^8*c^3*d*e*g - 1632*a^9*b^6*c^4*d*e*g - 4992*
a^10*b^4*c^5*d*e*g + 28160*a^11*b^2*c^6*d*e*g + 96*a^8*b^9*c^2*d*f*g - 1616
*a^9*b^7*c^3*d*f*g + 9408*a^10*b^5*c^4*d*f*g - 22272*a^11*b^3*c^5*d*f*g - 3
2*a^9*b^8*c^2*e*f*g + 672*a^10*b^6*c^3*e*f*g - 3456*a^11*b^4*c^4*e*f*g + 35
84*a^12*b^2*c^5*e*f*g))((a^5*g^2*(-(4*a*c - b^2)^9)^(1/2) - a^5*b^9*g^2 -
9*b^13*c*d^2 + 213*a*b^11*c^2*d^2 - 26880*a^6*b*c^7*d^2 - a^2*b^11*c*e^2 +
3840*a^7*b*c^6*e^2 - 9*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^4*b^9*c*f^2 +
768*a^8*b*c^5*f^2 - a^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^9*b*c^4*g^2
- 2077*a^2*b^9*c^3*d^2 + 10656*a^3*b^7*c^4*d^2 - 30240*a^4*b^5*c^5*d^2 + 4
4800*a^5*b^3*c^6*d^2 - 25*a^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^3*b^9
*c^2*e^2 - 288*a^4*b^7*c^3*e^2 + 1504*a^5*b^5*c^4*e^2 - 3840*a^6*b^3*c^5*e^
2 + 9*a^3*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 96*a^6*b^5*c^3*f^2 - 512*a^7*b
^3*c^4*f^2 + 96*a^7*b^5*c^2*g^2 - 512*a^8*b^3*c^3*g^2 + 15360*a^7*c^7*d*e +
5120*a^8*c^6*d*g - 3072*a^8*c^6*e*f - 1024*a^9*c^5*f*g + 6*a^2*b^11*c*d*f
+ 1536*a^7*b*c^6*d*f - 18*a^3*b^10*c*d*g - 2*a^3*b^10*c*e*f + 6*a^4*b^9*c*e
*g + 3584*a^8*b*c^5*e*g + 6*a^4*c*e*g*(-(4*a*c - b^2)^9)^(1/2) + 12*a^5*b^8
*c*f*g - 152*a^2*b^10*c^2*d*e + 1548*a^3*b^8*c^3*d*e - 8064*a^4*b^6*c^4*d*e
+ 22400*a^5*b^4*c^5*d*e - 30720*a^6*b^2*c^6*d*e - 98*a^3*b^9*c^2*d*f + 576
*a^4*b^7*c^3*d*f - 1344*a^5*b^5*c^4*d*f + 512*a^6*b^3*c^5*d*f + 10*a^3*c^2*
d*f*(-(4*a*c - b^2)^9)^(1/2) + 324*a^4*b^8*c^2*d*g + 36*a^4*b^8*c^2*e*f - 2
240*a^5*b^6*c^3*d*g - 192*a^5*b^6*c^3*e*f + 7296*a^6*b^4*c^4*d*g + 128*a^6*
b^4*c^4*e*f - 10752*a^7*b^2*c^5*d*g + 1536*a^7*b^2*c^5*e*f - 128*a^5*b^7*c^
2*e*g + 960*a^6*b^5*c^3*e*g - 3072*a^7*b^3*c^4*e*g - 128*a^6*b^6*c^2*f*g +
384*a^7*b^4*c^3*f*g + 6*a*b^12*c*d*e + 51*a*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(
1/2) - a^2*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^3*c*d*e*(-(4*a*c - b
^2)^9)^(1/2) - 18*a^3*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) - 2*a^3*b*c*e*f*(-(4
*a*c - b^2)^9)^(1/2) - 44*a^2*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) + 6*a^2*b^
2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^11*c^7 + a^5*b^12*c - 24*a^6*
b^10*c^2 + 240*a^7*b^8*c^3 - 1280*a^8*b^6*c^4 + 3840*a^9*b^4*c^5 - 6144*a^1
0*b^2*c^6)))^(1/2)*2i

```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**6+f*x**4+e*x**2+d)/x**2/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.128 $\int \frac{d+ex^2+fx^4+gx^6}{x^4(a+bx^2+cx^4)^2} dx$

Optimal. Leaf size=542

$$\frac{2bd - ae}{a^3x} - \frac{d}{3a^2x^3} + \frac{x \left(a^2 \left(\frac{b^4d}{a^2} - \frac{b^2(be+4cd)}{a} - a(bg + 2cf) + b^2f + 3bce + 2c^2d \right) + cx^2 (2a^2(ce - ag) - ab^2e - ab(3cd - 2ae) - ab^2c) \right)}{2a^3(b^2 - 4ac)(a + bx^2 + cx^4)}$$

Rubi [A] time = 7.26, antiderivative size = 542, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {1669, 1664, 1166, 205}

$$\frac{x^2 \left(\frac{d^2(b^2+4ac)}{a^2} - \frac{d(bg+2cf)}{a} + \frac{b^2f+3bce+2c^2d}{a} \right) + \frac{2d^2(b^2+4ac)}{a^2} \sqrt{\frac{b^2+4ac}{b^2-4ac}}}{2a^3(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{b^2-4ac}}{2\sqrt{2}a^2(b^2-4ac)} \left(\frac{2b^2d+4cd}{\sqrt{b^2-4ac}} + \frac{2b^2d+4cd}{\sqrt{b^2-4ac}} \right) + \frac{\sqrt{b^2-4ac}}{2\sqrt{2}a^2(b^2-4ac)} \left(\frac{2b^2d+4cd}{\sqrt{b^2-4ac}} + \frac{2b^2d+4cd}{\sqrt{b^2-4ac}} \right) + \frac{2bd-ae}{a^3x} - \frac{d}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2 + f*x^4 + g*x^6)/(x^4*(a + b*x^2 + c*x^4)^2), x]
[Out] -d/(3*a^2*x^3) + (2*b*d - a*e)/(a^3*x) + (x*(a^2*((b^4*d)/a^2 + 2*c^2*d + 3*b*c*e - (b^2*(4*c*d + b*e))/a + b^2*f - a*(2*c*f + b*g)) + c*(b^3*d - a*b^2*e - a*b*(3*c*d - a*f) + 2*a^2*(c*e - a*g))*x^2)/(2*a^3*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[c]*(5*b^3*d - 3*a*b^2*e - a*b*(19*c*d - a*f) + 2*a^2*(5*c*e - a*g) + (5*b^4*d - 3*a*b^3*e + 4*a^2*c*(7*c*d - 3*a*f) - a*b^2*(29*c*d - a*f) + 4*a^2*b*(4*c*e + a*g))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/(2*Sqrt[2]*a^3*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(5*b^3*d - 3*a*b^2*e - a*b*(19*c*d - a*f) + 2*a^2*(5*c*e - a*g) - (5*b^4*d - 3*a*b^3*e + 4*a^2*c*(7*c*d - 3*a*f) - a*b^2*(29*c*d - a*f) + 4*a^2*b*(4*c*e + a*g))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]/(2*Sqrt[2]*a^3*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1664

```
Int[(Pq_)*((d_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1669

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
```

```

olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]

```

Rubi steps

$$\begin{aligned}
 \int \frac{d + ex^2 + fx^4 + gx^6}{x^4(a + bx^2 + cx^4)^2} dx &= \frac{x \left(a^2 \left(\frac{b^4 d}{a^2} + 2c^2 d + 3bce - \frac{b^2(4cd+be)}{a} + b^2 f - a(2cf + bg) \right) + c(b^3 d - ab^2 e - ab(3d + e)) \right)}{2a^3(b^2 - 4ac)(a + bx^2 + cx^4)} \\
 &= \frac{x \left(a^2 \left(\frac{b^4 d}{a^2} + 2c^2 d + 3bce - \frac{b^2(4cd+be)}{a} + b^2 f - a(2cf + bg) \right) + c(b^3 d - ab^2 e - ab(3d + e)) \right)}{2a^3(b^2 - 4ac)(a + bx^2 + cx^4)} \\
 &= -\frac{d}{3a^2 x^3} + \frac{2bd - ae}{a^3 x} + \frac{x \left(a^2 \left(\frac{b^4 d}{a^2} + 2c^2 d + 3bce - \frac{b^2(4cd+be)}{a} + b^2 f - a(2cf + bg) \right) + c(b^3 d - ab^2 e - ab(3d + e)) \right)}{2a^3(b^2 - 4ac)(a + bx^2 + cx^4)} \\
 &= -\frac{d}{3a^2 x^3} + \frac{2bd - ae}{a^3 x} + \frac{x \left(a^2 \left(\frac{b^4 d}{a^2} + 2c^2 d + 3bce - \frac{b^2(4cd+be)}{a} + b^2 f - a(2cf + bg) \right) + c(b^3 d - ab^2 e - ab(3d + e)) \right)}{2a^3(b^2 - 4ac)(a + bx^2 + cx^4)} \\
 &= -\frac{d}{3a^2 x^3} + \frac{2bd - ae}{a^3 x} + \frac{x \left(a^2 \left(\frac{b^4 d}{a^2} + 2c^2 d + 3bce - \frac{b^2(4cd+be)}{a} + b^2 f - a(2cf + bg) \right) + c(b^3 d - ab^2 e - ab(3d + e)) \right)}{2a^3(b^2 - 4ac)(a + bx^2 + cx^4)}
 \end{aligned}$$

Mathematica [A] time = 2.15, size = 612, normalized size = 1.13

$$\frac{(-4ad)/x^3 + (24bd - 12ae)/x + (6x(b^4d + b^3(-ae) + cd*x^2) + a*b^2*(af - c*(4d + e*x^2)) + a*b*(-a^2*g) - 3*c^2*d*x^2 + a*c*(3e + f*x^2)) + 2*a^2*c*(c*(d + e*x^2) - a*(f + g*x^2)))/((b^2 - 4ac)*(a + b*x^2 + c*x^4)) + (3*sqrt(2)*sqrt(c)*(5*b^4*d + b^3*(5*sqrt(b^2 - 4ac)*d - 3*ae) + a*b^2*(-29*c*d - 3*sqrt(b^2 - 4ac)*e + af) + a*b*(-19*c*sqrt(b^2 - 4ac)*d + 16*a*c*e + a*sqrt(b^2 - 4ac)*f + 4*a^2*g) - 2*a^2*(-14*c^2*d - 5*c*sqrt(b^2 - 4ac)*e + 6*a*c*f + a*sqrt(b^2 - 4ac)*g))*ArcTan[(sqrt(2)*sqrt(c)*x)/sqrt(b - sqrt(b^2 - 4ac))]/((b^2 - 4ac)^(3/2)*sqrt(b - sqrt(b^2 - 4ac))) - (3*sqrt(2)*sqrt(c)*(5*b^4*d - b^3*(5*sqrt(b^2 - 4ac)*d + 3*ae) + a*b^2*(-29*c*d + 3*sqrt(b^2 - 4ac)*e + af) + a*b*(19*c*sqrt(b^2 - 4ac)*d + 16*a*c*e - a*sqrt(b^2 - 4ac)*f + 4*a^2*g) + 2*a^2*(14*c^2*d - 5*c*sqrt(b^2 - 4ac)*e - 6*a*c*f + a*sqrt(b^2 - 4ac)*g))*ArcTan[(sqrt(2)*sqrt(c)*x)/sqrt(b + sqrt(b^2 - 4ac))]/((b^2 - 4ac)^(3/2)*sqrt(b + sqrt(b^2 - 4ac)))/(12*a^3)$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2 + f*x^4 + g*x^6)/(x^4*(a + b*x^2 + c*x^4)^2), x]

[Out] ((-4*a*d)/x^3 + (24*b*d - 12*a*e)/x + (6*x*(b^4*d + b^3*(-a*e) + c*d*x^2) + a*b^2*(a*f - c*(4*d + e*x^2)) + a*b*(-a^2*g) - 3*c^2*d*x^2 + a*c*(3*e + f*x^2)) + 2*a^2*c*(c*(d + e*x^2) - a*(f + g*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (3*sqrt(2)*sqrt(c)*(5*b^4*d + b^3*(5*sqrt(b^2 - 4*a*c)*d - 3*a*e) + a*b^2*(-29*c*d - 3*sqrt(b^2 - 4*a*c)*e + a*f) + a*b*(-19*c*sqrt(b^2 - 4*a*c)*d + 16*a*c*e + a*sqrt(b^2 - 4*a*c)*f + 4*a^2*g) - 2*a^2*(-14*c^2*d - 5*c*sqrt(b^2 - 4*a*c)*e + 6*a*c*f + a*sqrt(b^2 - 4*a*c)*g))*ArcTan[(sqrt(2)*sqrt(c)*x)/sqrt(b - sqrt(b^2 - 4*a*c))]/((b^2 - 4*a*c)^(3/2)*sqrt(b - sqrt(b^2 - 4*a*c))) - (3*sqrt(2)*sqrt(c)*(5*b^4*d - b^3*(5*sqrt(b^2 - 4*a*c)*d + 3*a*e) + a*b^2*(-29*c*d + 3*sqrt(b^2 - 4*a*c)*e + a*f) + a*b*(19*c*sqrt(b^2 - 4*a*c)*d + 16*a*c*e - a*sqrt(b^2 - 4*a*c)*f + 4*a^2*g) + 2*a^2*(14*c^2*d - 5*c*sqrt(b^2 - 4*a*c)*e - 6*a*c*f + a*sqrt(b^2 - 4*a*c)*g))*ArcTan[(sqrt(2)*sqrt(c)*x)/sqrt(b + sqrt(b^2 - 4*a*c))]/((b^2 - 4*a*c)^(3/2)*sqrt(b + sqrt(b^2 - 4*a*c)))/(12*a^3)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex^2 + fx^4 + gx^6}{x^4(a + bx^2 + cx^4)^2} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x^2 + f*x^4 + g*x^6)/(x^4*(a + b*x^2 + c*x^4)^2),
x]
```

```
[Out] IntegrateAlgebraic[(d + e*x^2 + f*x^4 + g*x^6)/(x^4*(a + b*x^2 + c*x^4)^2),
x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^6+f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a)^2,x, algorithm="fricas"
)
```

[Out] Timed out

giac [B] time = 9.06, size = 10422, normalized size = 19.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^6+f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^3*c*d*x^3 - 3*a*b*c^2*d*x^3 + a^2*b*c*f*x^3 - 2*a^3*c*g*x^3 - a*b^2*c*x^3*e + 2*a^2*c^2*x^3*e + b^4*d*x - 4*a*b^2*c*d*x + 2*a^2*c^2*d*x + a^2*b^2*f*x - 2*a^3*c*f*x - a^3*b*g*x - a*b^3*x*e + 3*a^2*b*c*x*e)/((a^3*b^2 - 4*a^4*c)*(c*x^4 + b*x^2 + a)) + 1/16*((10*b^5*c^2 - 78*a*b^3*c^3 + 152*a^2*b*c^4 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 39*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 76*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 38*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 19*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 10*(b^2 - 4*a*c)*b^3*c^2 + 38*(b^2 - 4*a*c)*a*b*c^3)*(a^3*b^2 - 4*a^4*c)^2*d + (2*a^2*b^3*c^2 - 8*a^3*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 2*(b^2 - 4*a*c)*a^2*b*c^2)*(a^3*b^2 - 4*a^4*c)^2*f - 2*(2*a^3*b^2*c^2 - 8*a^4*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^2 - 2*(b^2 - 4*a*c)*a^3*c^2)*(a^3*b^2 - 4*a^4*c)^2*g - (6*a*b^4*c^2 - 44*a^2*b^2*c^3 + 80*a^3*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4 + 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^2 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 6*(b^2 - 4*a*c)*a*b^2*c^2 + 20*(b^2 - 4*a*c)*a^2*c^3)*(a^3*b^2 - 4*a^4*c)^2*e + 2*(5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^8 - 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c - 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^7*c - 10*a^3*b^8*c + 286*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^2 + 88*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^2 + 5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c^2 + 128*a^4*b^6*c^2 - 496*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^3 - 220*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^3 - 44*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^3 - 572*a^5*b^4*c^3 + 224*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*c^4 + 112*sqrt(2)*sq
```


$$\begin{aligned}
& \text{rt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^6*b*c^4 + 110*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - \\
& 4*a*c))*c)*a^5*b^2*c^4 + 992*a^6*b^2*c^4 - 56*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - \\
& 4*a*c))*c)*a^6*c^5 - 448*a^7*c^5 + 10*(b^2 - 4*a*c)*a^3*b^6*c - 88*(b^2 - 4 \\
& *a*c)*a^4*b^4*c^2 + 220*(b^2 - 4*a*c)*a^5*b^2*c^3 - 112*(b^2 - 4*a*c)*a^6*c \\
& ^4)*d*\text{abs}(a^3*b^2 - 4*a^4*c) + 2*(\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a \\
& ^5*b^6 - 14*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^6*b^4*c - 2*\text{sqrt}(2)*s \\
& \text{qrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^5*b^5*c - 2*a^5*b^6*c + 64*\text{sqrt}(2)*\text{sqrt}(b* \\
& c + \text{sqrt}(b^2 - 4*a*c))*c)*a^7*b^2*c^2 + 20*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a \\
& *c))*c)*a^6*b^3*c^2 + \text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^5*b^4*c^2 + \\
& 28*a^6*b^4*c^2 - 96*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^8*c^3 - 48*\text{sq} \\
& \text{rt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^7*b*c^3 - 10*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqr} \\
& \text{t}(b^2 - 4*a*c))*c)*a^6*b^2*c^3 - 128*a^7*b^2*c^3 + 24*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqr} \\
& \text{t}(b^2 - 4*a*c))*c)*a^7*c^4 + 192*a^8*c^4 + 2*(b^2 - 4*a*c)*a^5*b^4*c - 20*(b \\
& ^2 - 4*a*c)*a^6*b^2*c^2 + 48*(b^2 - 4*a*c)*a^7*c^3)*f*\text{abs}(a^3*b^2 - 4*a^4*c \\
&) + 2*(\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^6*b^5 - 8*\text{sqrt}(2)*\text{sqrt}(b*c \\
& + \text{sqrt}(b^2 - 4*a*c))*c)*a^7*b^3*c - 2*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))* \\
& c)*a^6*b^4*c - 2*a^6*b^5*c + 16*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^8 \\
& *b*c^2 + 8*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^7*b^2*c^2 + \text{sqrt}(2)*\text{sq} \\
& \text{rt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^6*b^3*c^2 + 16*a^7*b^3*c^2 - 4*\text{sqrt}(2)*\text{sqrt} \\
& (b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^7*b*c^3 - 32*a^8*b*c^3 + 2*(b^2 - 4*a*c)*a^6* \\
& b^3*c - 8*(b^2 - 4*a*c)*a^7*b*c^2)*g*\text{abs}(a^3*b^2 - 4*a^4*c) - 2*(3*\text{sqrt}(2)* \\
& \text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^4*b^7 - 37*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - \\
& 4*a*c))*c)*a^5*b^5*c - 6*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^4*b^6*c - \\
& 6*a^4*b^7*c + 152*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^6*b^3*c^2 + 50 \\
& *\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^5*b^4*c^2 + 3*\text{sqrt}(2)*\text{sqrt}(b*c + \\
& \text{sqrt}(b^2 - 4*a*c))*c)*a^4*b^5*c^2 + 74*a^5*b^5*c^2 - 208*\text{sqrt}(2)*\text{sqrt}(b*c + \\
& \text{sqrt}(b^2 - 4*a*c))*c)*a^7*b*c^3 - 104*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))* \\
& c)*a^6*b^2*c^3 - 25*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^5*b^3*c^3 - 3 \\
& 04*a^6*b^3*c^3 + 52*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^6*b*c^4 + 416 \\
& *a^7*b*c^4 + 6*(b^2 - 4*a*c)*a^4*b^5*c - 50*(b^2 - 4*a*c)*a^5*b^3*c^2 + 104 \\
& *(b^2 - 4*a*c)*a^6*b*c^3)*\text{abs}(a^3*b^2 - 4*a^4*c)*e + (10*a^6*b^9*c^2 - 138* \\
& a^7*b^7*c^3 + 680*a^8*b^5*c^4 - 1376*a^9*b^3*c^5 + 896*a^10*b*c^6 - 5*\text{sqrt}(\\
& 2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^6*b^9 + 69*\text{sqrt}(2)*s \\
& \text{qrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^7*b^7*c + 10*\text{sqrt}(2)*\text{sq} \\
& \text{rt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^6*b^8*c - 340*\text{sqrt}(2)*\text{sqrt} \\
& (b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^8*b^5*c^2 - 98*\text{sqrt}(2)*\text{sqrt} \\
& (b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^7*b^6*c^2 - 5*\text{sqrt}(2)*\text{sqrt}(\\
& b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^6*b^7*c^2 + 688*\text{sqrt}(2)*\text{sqrt} \\
& (b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^9*b^3*c^3 + 288*\text{sqrt}(2)*\text{sq} \\
& \text{rt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^8*b^4*c^3 + 49*\text{sqrt}(2)*\text{sq} \\
& \text{rt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^7*b^5*c^3 - 448*\text{sqrt}(2)*\text{sq} \\
& \text{rt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^10*b*c^4 - 224*\text{sqrt}(2)*\text{sq} \\
& \text{rt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^9*b^2*c^4 - 144*\text{sqrt}(2)*s \\
& \text{qrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^8*b^3*c^4 + 112*\text{sqrt}(2)* \\
& \text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^9*b*c^5 - 10*(b^2 - 4*a \\
& *c)*a^6*b^7*c^2 + 98*(b^2 - 4*a*c)*a^7*b^5*c^3 - 288*(b^2 - 4*a*c)*a^8*b^3* \\
& c^4 + 224*(b^2 - 4*a*c)*a^9*b*c^5)*d + (2*a^8*b^7*c^2 - 40*a^9*b^5*c^3 + 22 \\
& 4*a^10*b^3*c^4 - 384*a^11*b*c^5 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt} \\
& (b^2 - 4*a*c))*c)*a^8*b^7 + 20*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 \\
& - 4*a*c))*c)*a^9*b^5*c + 2*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - \\
& 4*a*c))*c)*a^8*b^6*c - 112*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4 \\
& *a*c))*c)*a^10*b^3*c^2 - 32*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - \\
& 4*a*c))*c)*a^9*b^4*c^2 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a \\
& *c))*c)*a^8*b^5*c^2 + 192*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4* \\
& a*c))*c)*a^11*b*c^3 + 96*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a \\
& *c))*c)*a^10*b^2*c^3 + 16*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4* \\
& a*c))*c)*a^9*b^3*c^3 - 48*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4* \\
& a*c))*c)*a^10*b*c^4 - 2*(b^2 - 4*a*c)*a^8*b^5*c^2 + 32*(b^2 - 4*a*c)*a^9*b^3 \\
& *c^3 - 96*(b^2 - 4*a*c)*a^10*b*c^4)*f + 4*(2*a^9*b^6*c^2 - 16*a^10*b^4*c^3
\end{aligned}$$

$$\begin{aligned}
& + 32*a^{11}*b^2*c^4 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}* \\
& c)*a^9*b^6 + 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^ \\
& 10*b^4*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^9* \\
& b^5*c - 16*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^{11}*b \\
& ^2*c^2 - 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^{10}*b \\
& ^3*c^2 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^9*b^4* \\
& c^2 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^{10}*b^2* \\
& c^3 - 2*(b^2 - 4*a*c)*a^9*b^4*c^2 + 8*(b^2 - 4*a*c)*a^{10}*b^2*c^3)*g - (6*a^ \\
& 7*b^8*c^2 - 80*a^8*b^6*c^3 + 352*a^9*b^4*c^4 - 512*a^{10}*b^2*c^5 - 3*\sqrt{2} \\
& *\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^7*b^8 + 40*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^8*b^6*c + 6*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^7*b^7*c - 176*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^9*b^4*c^2 - 56*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^8*b^5*c^2 - 3*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^7*b^6*c^2 + 256*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^{10}*b^2*c^3 + 128*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^9*b^3*c^3 + 28*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^8*b^4*c^3 - 64*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^9*b^2*c^4 - 6*(b^2 - 4*a*c)*a^7*b^6*c^2 + 56*(b^2 - 4*a*c)*a^8*b^4*c^3 - 128*(b^2 - 4*a*c)*a^9*b^2*c^4)*e)*\arctan(2*\sqrt{1/2}*x/\sqrt{(a^3*b^3 - 4*a^4*b*c + \sqrt{(a^3*b^3 - 4*a^4*b*c)^2 - 4*(a^4*b^2 - 4*a^5*c)*(a^3*b^2*c - 4*a^4*c^2)}))/((a^3*b^2*c - 4*a^4*c^2)))/((a^7*b^6 - 12*a^8*b^4*c - 2*a^7*b^5*c + 48*a^9*b^2*c^2 + 16*a^8*b^3*c^2 + a^7*b^4*c^2 - 64*a^{10}*c^3 - 32*a^9*b*c^3 - 8*a^8*b^2*c^3 + 16*a^9*c^4)*\text{abs}(a^3*b^2 - 4*a^4*c)*\text{abs}(c)) - 1/16*((10*b^5*c^2 - 78*a*b^3*c^3 + 152*a^2*b*c^4 - 5*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^5 + 39*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^3*c + 10*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^4*c - 76*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b*c^2 - 38*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c^2 - 5*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^3*c^2 + 19*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b*c^3 - 10*(b^2 - 4*a*c)*b^3*c^2 + 38*(b^2 - 4*a*c)*a*b*c^3)*(a^3*b^2 - 4*a^4*c)^2*d + (2*a^2*b^3*c^2 - 8*a^3*b*c^3 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b^3 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*b*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b^2*c - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b*c^2 - 2*(b^2 - 4*a*c)*a^2*b*c^2)*(a^3*b^2 - 4*a^4*c)^2*f - 2*(2*a^3*b^2*c^2 - 8*a^4*c^3 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*b^2 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^4*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*b*c - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*c^2 - 2*(b^2 - 4*a*c)*a^3*c^2)*(a^3*b^2 - 4*a^4*c)^2*g - (6*a*b^4*c^2 - 44*a^2*b^2*c^3 + 80*a^3*c^4 - 3*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^4 + 22*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b^2*c + 6*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^3*c - 40*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*c^2 - 20*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b*c^2 - 3*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c^2 + 10*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*c^3 - 6*(b^2 - 4*a*c)*a*b^2*c^2 + 20*(b^2 - 4*a*c)*a^2*c^3)*(a^3*b^2 - 4*a^4*c)^2*e - 2*(5*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*b^8 - 64*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^4*b^6*c - 10*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*b^7*c + 10*a^3*b^8*c + 286*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^5*b^4*c^2 + 88*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^4*b^5*c^2 + 5*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*b^6*c^2 - 128*a^4*b^6*c^2 - 496*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^6*b^2*c^3 - 220*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^5*b^3*c^3 - 44*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^4*b^4*c^3 + 572*a^5*b^4*c^3 + 224*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^7*c^4 + 112*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^6*c^4 + 112*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^5*c^4 + 112*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^4*c^4 + 112*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*c^4 + 112*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*c^4 + 112*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*c^4 + 112*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*c^4
\end{aligned}$$

$$\begin{aligned}
& (2) \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} a^6 b^2 c^4 + 110 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} a^5 b^2 c^4 - 992 a^6 b^2 c^4 - 56 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} a^6 c^5 + 448 a^7 c^5 - 10 (b^2 - 4ac) a^3 b^6 c + 88 (b^2 - 4ac) a^4 b^4 c^2 - 220 (b^2 - 4ac) a^5 b^2 c^3 + 112 (b^2 - 4ac) a^6 c^4) d \operatorname{abs}(a^3 b^2 - 4a^4 c) - 2 (\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} a^5 b^6 - 14 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} a^6 b^4 c - 2 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} a^5 b^5 c + 2 a^5 b^6 c + 64 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} a^7 b^2 c^2 + 20 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} a^6 b^3 c^2 + \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} a^5 b^4 c^2 - 28 a^6 b^4 c^2 - 96 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} a^8 c^3 - 48 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} a^7 b^2 c^3 - 10 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} a^6 b^2 c^3 + 128 a^7 b^2 c^3 + 24 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} a^7 c^4 - 192 a^8 c^4 - 2 (b^2 - 4ac) a^5 b^4 c + 20 (b^2 - 4ac) a^6 b^2 c^2 - 48 (b^2 - 4ac) a^7 c^3) f \operatorname{abs}(a^3 b^2 - 4a^4 c) - 2 (\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} a^6 b^5 - 8 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} a^7 b^3 c - 2 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} a^6 b^4 c + 2 a^6 b^5 c + 16 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} a^8 b^2 c^2 + 8 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} a^7 b^2 c^2 + \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} a^6 b^3 c^2 - 16 a^7 b^3 c^2 - 4 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} a^7 b^2 c^3 + 32 a^8 b^2 c^3 - 2 (b^2 - 4ac) a^6 b^3 c + 8 (b^2 - 4ac) a^7 b^2 c^2) g \operatorname{abs}(a^3 b^2 - 4a^4 c) + 2 (3 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} a^4 b^7 - 37 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} a^5 b^5 c - 6 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} a^4 b^6 c + 6 a^4 b^7 c + 152 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} a^6 b^3 c^2 + 50 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} a^5 b^4 c^2 + 3 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} a^4 b^5 c^2 - 74 a^5 b^5 c^2 - 208 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} a^7 b^2 c^3 - 104 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} a^6 b^2 c^3 - 25 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} a^5 b^3 c^3 + 304 a^6 b^3 c^3 + 52 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} a^6 b^2 c^4 - 416 a^7 b^2 c^4 - 6 (b^2 - 4ac) a^4 b^5 c + 50 (b^2 - 4ac) a^5 b^3 c^2 - 104 (b^2 - 4ac) a^6 b^2 c^3) \operatorname{abs}(a^3 b^2 - 4a^4 c) e + (10 a^6 b^9 c^2 - 138 a^7 b^7 c^3 + 680 a^8 b^5 c^4 - 1376 a^9 b^3 c^5 + 896 a^{10} b^2 c^6 - 5 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} a^6 b^9 + 69 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} a^7 b^7 c + 10 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} a^6 b^8 c - 340 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} a^8 b^5 c^2 - 98 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} a^7 b^6 c^2 - 5 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} a^6 b^7 c^2 + 688 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} a^9 b^3 c^3 + 288 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} a^8 b^4 c^3 + 49 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} a^7 b^5 c^3 - 448 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} a^{10} b^2 c^4 - 224 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} a^9 b^2 c^4 - 144 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} a^8 b^3 c^4 + 112 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} a^9 b^2 c^5 - 10 (b^2 - 4ac) a^6 b^7 c^2 + 98 (b^2 - 4ac) a^7 b^5 c^3 - 288 (b^2 - 4ac) a^8 b^3 c^4 + 224 (b^2 - 4ac) a^9 b^2 c^5) d + (2 a^8 b^7 c^2 - 40 a^9 b^5 c^3 + 224 a^{10} b^3 c^4 - 384 a^{11} b^2 c^5 - \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} a^8 b^7 + 20 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} a^9 b^5 c + 2 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} a^8 b^6 c - 112 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} a^{10} b^3 c^2 - 32 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} a^9 b^4 c^2 - \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} a^8 b^5 c^2 + 192 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} a^{11} b^2 c^3 + 96 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} a^{10} b^2 c^3 + 16 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} a^9 b^3 c^3 - 48 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} a^{10} b^2 c^4 - 2 (b^2 - 4ac) a^8 b^5 c^2 + 32 (b^2 - 4ac) a^9 b^3 c^3 - 96 (b^2 - 4ac) a^{10} b^2 c^4) f + 4 (2 a^9 b^6 c^2 - 16 a^{10} b^5 c^3 - 96 (b^2 - 4ac) a^{10} b^2 c^4) g + 4 (2 a^9 b^6 c^2 - 16 a^{10} b^5 c^3 - 96 (b^2 - 4ac) a^{10} b^2 c^4) h + 4 (2 a^9 b^6 c^2 - 16 a^{10} b^5 c^3 - 96 (b^2 - 4ac) a^{10} b^2 c^4) i + 4 (2 a^9 b^6 c^2 - 16 a^{10} b^5 c^3 - 96 (b^2 - 4ac) a^{10} b^2 c^4) j + 4 (2 a^9 b^6 c^2 - 16 a^{10} b^5 c^3 - 96 (b^2 - 4ac) a^{10} b^2 c^4) k + 4 (2 a^9 b^6 c^2 - 16 a^{10} b^5 c^3 - 96 (b^2 - 4ac) a^{10} b^2 c^4) l + 4 (2 a^9 b^6 c^2 - 16 a^{10} b^5 c^3 - 96 (b^2 - 4ac) a^{10} b^2 c^4) m + 4 (2 a^9 b^6 c^2 - 16 a^{10} b^5 c^3 - 96 (b^2 - 4ac) a^{10} b^2 c^4) n + 4 (2 a^9 b^6 c^2 - 16 a^{10} b^5 c^3 - 96 (b^2 - 4ac) a^{10} b^2 c^4) o + 4 (2 a^9 b^6 c^2 - 16 a^{10} b^5 c^3 - 96 (b^2 - 4ac) a^{10} b^2 c^4) p + 4 (2 a^9 b^6 c^2 - 16 a^{10} b^5 c^3 - 96 (b^2 - 4ac) a^{10} b^2 c^4) q + 4 (2 a^9 b^6 c^2 - 16 a^{10} b^5 c^3 - 96 (b^2 - 4ac) a^{10} b^2 c^4) r + 4 (2 a^9 b^6 c^2 - 16 a^{10} b^5 c^3 - 96 (b^2 - 4ac) a^{10} b^2 c^4) s + 4 (2 a^9 b^6 c^2 - 16 a^{10} b^5 c^3 - 96 (b^2 - 4ac) a^{10} b^2 c^4) t + 4 (2 a^9 b^6 c^2 - 16 a^{10} b^5 c^3 - 96 (b^2 - 4ac) a^{10} b^2 c^4) u + 4 (2 a^9 b^6 c^2 - 16 a^{10} b^5 c^3 - 96 (b^2 - 4ac) a^{10} b^2 c^4) v + 4 (2 a^9 b^6 c^2 - 16 a^{10} b^5 c^3 - 96 (b^2 - 4ac) a^{10} b^2 c^4) w + 4 (2 a^9 b^6 c^2 - 16 a^{10} b^5 c^3 - 96 (b^2 - 4ac) a^{10} b^2 c^4) x + 4 (2 a^9 b^6 c^2 - 16 a^{10} b^5 c^3 - 96 (b^2 - 4ac) a^{10} b^2 c^4) y + 4 (2 a^9 b^6 c^2 - 16 a^{10} b^5 c^3 - 96 (b^2 - 4ac) a^{10} b^2 c^4) z
\end{aligned}$$

$$4c^3 + 32a^{11}b^2c^4 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}} \\
 *a^9b^6 + 8\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}} \\
 *c)a^{10}b^4c + 2\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}} \\
)a^9b^5c - 16\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}} \\
 *c)a^{11}b^2c^2 - 8\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}} \\
 *c)a^{10}b^3c^2 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}} \\
 *c)a^9b^4c^2 + 4\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}} \\
 *c)a^{10}b^2c^3 - 2(b^2 - 4ac)a^9b^4c^2 + 8(b^2 - 4ac)a^{10}b^2c^3 \\
 *g - (6a^7b^8c^2 - 80a^8b^6c^3 + 352a^9b^4c^4 - 512a^{10}b^2c^5 - 3 \\
 \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}})a^7b^8 + 40\sqrt{2} \\
 \sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}})a^8b^6c + 6\sqrt{2} \\
 \sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}})a^7b^7c - 176\sqrt{2} \\
 \sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}})a^9b^4c^2 - 56\sqrt{2} \\
 \sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}})a^8b^5c^2 - 3\sqrt{2} \\
 \sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}})a^7b^6c^2 + 256\sqrt{2} \\
 \sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}})a^{10}b^2c^3 + 128\sqrt{2} \\
 \sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}})a^9b^3c^3 + 28\sqrt{2} \\
 \sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}})a^8b^4c^3 - 64\sqrt{2} \\
 \sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}})a^9b^2c^4 - 6(b^2 - 4 \\
 ac)a^7b^6c^2 + 56(b^2 - 4ac)a^8b^4c^3 - 128(b^2 - 4ac)a^9b^2 \\
 c^4)e) \arctan(2\sqrt{1/2}x/\sqrt{(a^3b^3 - 4a^4bc - \sqrt{(a^3b^3 - 4a^4bc)^2 - 4(a^4b^2 - 4a^5c)(a^3b^2c - 4a^4c^2)})) / (a^3b^2c - 4a^4c^2)) / ((a^7b^6 - 12a^8b^4c - 2a^7b^5c + 48a^9b^2c^2 + 16a^8b^3c^2 + a^7b^4c^2 - 64a^{10}c^3 - 32a^9bc^3 - 8a^8b^2c^3 + 16a^9c^4) \operatorname{abs}(a^3b^2 - 4a^4c) \operatorname{abs}(c)) + 1/3(6bdx^2 - 3ax^2e - ad) / (a^3x^3)$$

maple [B] time = 0.07, size = 2503, normalized size = 4.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{int}((gx^6+fx^4+ex^2+d)/x^4/(cx^4+bx^2+a)^2, x)$

[Out]
$$-1/3/a^2d/x^3+c/(4ac-b^2)/(-4ac+b^2)^{(1/2)}2^{(1/2)}/((-b+(-4ac+b^2)^{(1/2)})c)^{(1/2)}\operatorname{arctanh}(2^{(1/2)}/((-b+(-4ac+b^2)^{(1/2)})c)^{(1/2)}cx)bg+c/(4ac-b^2)/(-4ac+b^2)^{(1/2)}2^{(1/2)}/((b+(-4ac+b^2)^{(1/2)})c)^{(1/2)}\operatorname{arctan}(2^{(1/2)}/((b+(-4ac+b^2)^{(1/2)})c)^{(1/2)}cx)bg+2/a^3bd/x-1/2c/(4ac-b^2)2^{(1/2)}/((-b+(-4ac+b^2)^{(1/2)})c)^{(1/2)}\operatorname{arctanh}(2^{(1/2)}/((-b+(-4ac+b^2)^{(1/2)})c)^{(1/2)}cx)g+1/2c/(4ac-b^2)2^{(1/2)}/((b+(-4ac+b^2)^{(1/2)})c)^{(1/2)}\operatorname{arctan}(2^{(1/2)}/((b+(-4ac+b^2)^{(1/2)})c)^{(1/2)}cx)g-5/4/(4ac-b^2)2^{(1/2)}/((b+(-4ac+b^2)^{(1/2)})c)^{(1/2)}/a^3b^3cd\operatorname{arctan}(2^{(1/2)}/((b+(-4ac+b^2)^{(1/2)})c)^{(1/2)}cx)+7/a^3/(4ac-b^2)/(-4ac+b^2)^{(1/2)}2^{(1/2)}/((-b+(-4ac+b^2)^{(1/2)})c)^{(1/2)}\operatorname{arctanh}(2^{(1/2)}/((-b+(-4ac+b^2)^{(1/2)})c)^{(1/2)}cx)d+1/4ac/(4ac-b^2)2^{(1/2)}/((-b+(-4ac+b^2)^{(1/2)})c)^{(1/2)}\operatorname{arctanh}(2^{(1/2)}/((-b+(-4ac+b^2)^{(1/2)})c)^{(1/2)}cx)bf-3/4/a^2c/(4ac-b^2)2^{(1/2)}/((-b+(-4ac+b^2)^{(1/2)})c)^{(1/2)}\operatorname{arctanh}(2^{(1/2)}/((-b+(-4ac+b^2)^{(1/2)})c)^{(1/2)}cx)b^2e-1/4/(4ac-b^2)2^{(1/2)}/((b+(-4ac+b^2)^{(1/2)})c)^{(1/2)}/abcf\operatorname{arctan}(2^{(1/2)}/((b+(-4ac+b^2)^{(1/2)})c)^{(1/2)}cx)+3/4/(4ac-b^2)2^{(1/2)}/((b+(-4ac+b^2)^{(1/2)})c)^{(1/2)}/a^2b^2ce\operatorname{arctan}(2^{(1/2)}/((b+(-4ac+b^2)^{(1/2)})c)^{(1/2)}cx)+19/4/(4ac-b^2)2^{(1/2)}/((b+(-4ac+b^2)^{(1/2)})c)^{(1/2)}/a^2b^2cd\operatorname{arctan}(2^{(1/2)}/((b+(-4ac+b^2)^{(1/2)})c)^{(1/2)}cx)+7/(4ac-b^2)/(-4ac+b^2)^{(1/2)}2^{(1/2)}/((b+(-4ac+b^2)^{(1/2)})c)^{(1/2)}/a^3d\operatorname{arctan}(2^{(1/2)}/((b+(-4ac+b^2)^{(1/2)})c)^{(1/2)}cx)-19/4/a^2c^2/(4ac-b^2)2^{(1/2)}/((-b+(-4ac+b^2)^{(1/2)})c)^{(1/2)}\operatorname{arctanh}(2^{(1/2)}/((-b+(-4ac+b^2)^{(1/2)})c)^{(1/2)}cx)bd+5/4/a^3c/(4ac-b^2)2^{(1/2)}/((-b+(-4ac+b^2)^{(1/2)})c)^{(1/2)}\operatorname{arctanh}(2^{(1/2)}/((-b+(-4ac+b^2)^{(1/2)})c)^{(1/2)}cx)b^3d+1/(cx^4+bx^2+a)c/(4ac-b^2)x^3g+1/2/(cx^4+bx^2+a)/(4ac-b^2)xbg-29/4/a^2c^2/(4ac-b^2)/(-4ac+b^2)^{(1/2)}2^{(1/2)}/((-b+(-4ac+b^2)^{(1/2)})c)^{(1/2)}\operatorname{arctanh}(2^{(1/2)}/((-b+(-4ac+b^2)^{(1/2)})c)^{(1/2)}cx)$$

$$\begin{aligned} & (1/2)/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^2*d+1/4/(4*a*c-b^2)/(-4*a*c+ \\ & b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}/a*b^2*c*f*\arctan(2^{(1/2)} \\ &)/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)+4/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)} \\ &)/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}/a*b*c^2*e*\arctan(2^{(1/2)}/((b+(-4*a*c \\ & +b^2)^{(1/2)})*c)^{(1/2)}*c*x)-3/4/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+ \\ & -4*a*c+b^2)^{(1/2)})*c)^{(1/2)}/a^2*b^3*c*e*\arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)} \\ &)*c)^{(1/2)}*c*x)+4/a*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a \\ & *c+b^2)^{(1/2)})*c)^{(1/2)}*\arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c \\ & *x)*b*e-3/4/a^2*c/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)} \\ &)*c)^{(1/2)}*\arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^3* \\ & e+1/4/a*c/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c \\ &)^{(1/2)}*\arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^2*f+5/4/(4 \\ & *a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}/a^3*b \\ & ^4*c*d*\arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)+5/4/a^3*c/(4*a* \\ & c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctanh \\ & (2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^4*d-29/4/(4*a*c-b^2)/(-4* \\ & a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}/a^2*b^2*c^2*d*\arctan \\ & (2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)-1/2/(c*x^4+b*x^2+a)/(4*a*c- \\ & b^2)/a*b*c*f*x^3+1/2/(c*x^4+b*x^2+a)/(4*a*c-b^2)/a^2*b^2*c*e*x^3+3/2/(c*x^4 \\ & +b*x^2+a)/(4*a*c-b^2)/a^2*b*c^2*d*x^3-3/2/(c*x^4+b*x^2+a)/(4*a*c-b^2)/a*b*c \\ & *e*x+2/(c*x^4+b*x^2+a)/(4*a*c-b^2)/a^2*b^2*c*d*x-1/2/(c*x^4+b*x^2+a)/(4*a*c \\ & -b^2)/a^3*b^3*c*d*x^3+5/2/a*c^2/(4*a*c-b^2)*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)} \\ &)*c)^{(1/2)}*\arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*e-5/2/(4* \\ & a*c-b^2)*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}/a*c^2*e*\arctan(2^{(1/2)}/((\\ & b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)-3/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)} \\ &)/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c^2*f*\arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)} \\ &)*c)^{(1/2)}*c*x)-3*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a* \\ & c+b^2)^{(1/2)})*c)^{(1/2)}*\arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c* \\ & x)*f+1/(c*x^4+b*x^2+a)/(4*a*c-b^2)*c*f*x-1/(c*x^4+b*x^2+a)/(4*a*c-b^2)/a*c^ \\ & 2*e*x^3-1/2/(c*x^4+b*x^2+a)/(4*a*c-b^2)/a^3*b^4*d*x-1/2/(c*x^4+b*x^2+a)/(4* \\ & a*c-b^2)/a*b^2*f*x-1/(c*x^4+b*x^2+a)/(4*a*c-b^2)/a*c^2*d*x+1/2/(c*x^4+b*x^2 \\ & +a)/(4*a*c-b^2)/a^2*b^3*e*x-1/a^2*e/x \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3(a^2bcf - 2a^3cg + (5b^2c - 19abc^2)d - (3ab^2c - 10a^2c^2)e)^2 - (3a^2bg - (15b^4 - 62ab^2c + 14a^2c^2)d + 3(3ab^3 - 11a^2bc)e - 3(a^2b^2 - 2a^3c)f)^2 + 2(5(ab^3 - 4a^2bc)d - 3(a^2b^2 - 4a^3c)e)^2 - 2(a^2b^2 - 4a^3c)d - \int \frac{a^2bc(a^2bcf - 2a^3cg + (5b^2c - 19abc^2)d - (3ab^2c - 10a^2c^2)e)^2 - (3a^2bg - (15b^4 - 62ab^2c + 14a^2c^2)d + 3(3ab^3 - 11a^2bc)e - 3(a^2b^2 - 2a^3c)f)^2 + 2(5(ab^3 - 4a^2bc)d - 3(a^2b^2 - 4a^3c)e)^2 - 2(a^2b^2 - 4a^3c)d}{6((a^2b^2c - 4a^3c)^2 + (a^2b^3 - 4a^4c)e^2 + (a^2b^2 - 4a^3c)f^2)} dx}{2(a^2b^2 - 4a^3c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^6+f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")

[Out] 1/6*(3*(a^2*b*c*f - 2*a^3*c*g + (5*b^3*c - 19*a*b*c^2)*d - (3*a*b^2*c - 10*a^2*c^2)*e)*x^6 - (3*a^3*b*g - (15*b^4 - 62*a*b^2*c + 14*a^2*c^2)*d + 3*(3*a*b^3 - 11*a^2*b*c)*e - 3*(a^2*b^2 - 2*a^3*c)*f)*x^4 + 2*(5*(a*b^3 - 4*a^2*b*c)*d - 3*(a^2*b^2 - 4*a^3*c)*e)*x^2 - 2*(a^2*b^2 - 4*a^3*c)*d)/((a^3*b^2*c - 4*a^4*c^2)*x^7 + (a^3*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3) - 1/2*integrate(-(a^3*b*g + (a^2*b*c*f - 2*a^3*c*g + (5*b^3*c - 19*a*b*c^2)*d - (3*a*b^2*c - 10*a^2*c^2)*e)*x^2 + (5*b^4 - 24*a*b^2*c + 14*a^2*c^2)*d - (3*a*b^3 - 13*a^2*b*c)*e + (a^2*b^2 - 6*a^3*c)*f)/(c*x^4 + b*x^2 + a), x)/(a^3*b^2 - 4*a^4*c)

mupad [B] time = 8.47, size = 51386, normalized size = 94.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x^2 + f*x^4 + g*x^6)/(x^4*(a + b*x^2 + c*x^4)^2), x)

[Out] atan((((-(25*b^15*d^2 + 9*a^2*b^13*e^2 + 25*b^6*d^2*(-(4*a*c - b^2)^9))^(1/2) + a^4*b^11*f^2 + a^6*b^9*g^2 + a^6*g^2*(-(4*a*c - b^2)^9))^(1/2) - 80640*a^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^2

$$\begin{aligned}
& - 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 768*a^{10}*b*c^4*g^2 - 30*a*b^{14}*d*e + 6366*a^2*b^{11}*c^2*d^2 - 35767*a^3*b^9*c^3*d^2 + 116 \\
& 928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^{(1/2)} \\
& + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^2 - 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + 25*a^4*c^2* \\
& e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^2 + 3840*a^8*b^3*c^4*f^2 - 96*a^8*b^5*c^2*g^2 + 512*a^9*b^3*c^3*g^2 - 615*a*b^13*c*d^2 \\
& + 10*a^2*b^13*d*f + 35840*a^8*c^7*d*e + 10*a^3*b^12*d*g - 6*a^3*b^12*e*f - 6*a^4*b^11*e*g - 7168*a^9*c^6*d*g - 15360*a^9*c^6*e*f + 2*a^5*b^10 \\
& *f*g + 3072*a^10*c^5*f*g - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^{(1/2)} + 724*a^2*b^12*c*d*e - 258*a^3*b^11*c*d*f + 43520*a^8*b*c^6*d*f - 168*a^4*b^10*c*d*g \\
& + 152*a^4*b^10*c*e*f + 98*a^5*b^9*c*e*g - 1536*a^9*b*c^5*e*g + 2*a^5*b*f*g*(-(4*a*c - b^2)^9)^{(1/2)} - 10*a^5*c*e*g*(-(4*a*c - b^2)^9)^{(1/2)} - 36*a^6*b^8*c*f*g \\
& + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - 7278*a^3*b^10*c^2*d*e + 39132*a^4*b^8*c^3*d*e - 1 \\
& 19616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 2706*a^4*b^9*c^2*d*f - 14784*a^5*b^7*c^3*d*f \\
& + 44352*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f + 10*a^3*b^3*d*g*(-(4*a*c - b^2)^9)^{(1/2)} - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^{(1/2)} + 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^{(1/2)} \\
& + 1044*a^5*b^8*c^2*d*g - 1548*a^5*b^8*c^2*e*f - 2688*a^6*b^6*c^3*d*g + 8064*a^6*b^6*c^3*e*f + 1152*a^7*b^4*c^4*d*g - 22400*a^7*b^4*c^4*e*f + 6144*a^8*b^2*c^5*d*g \\
& + 30720*a^8*b^2*c^5*e*f - 6*a^4*b^2*e*g*(-(4*a*c - b^2)^9)^{(1/2)} - 576*a^6*b^7*c^2*e*g + 1344*a^7*b^5*c^3*e*g - 512*a^8*b^3*c^4*e*g + 192*a^7*b^6*c^2*f*g - 128*a^8*b^4*c^3*f*g - 1 \\
& 536*a^9*b^2*c^4*f*g - 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 12*a^4*b*c*d*g*(-(4*a*c - b^2)^9)^{(1/2)} + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^{(1/2)} + \\
& 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^7*b^12 + 4 \\
& 096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5))^{(1/2)}*(393216*a^20*c^8*f - 917504*a^19*c^9*d \\
& + x*(-(25*b^15*d^2 + 9*a^2*b^13*e^2 + 25*b^6*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + a^4*b^11*f^2 + a^6*b^9*g^2 + a^6*g^2*(-(4*a*c - b^2)^9)^{(1/2)} - 80640 \\
& *a^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^2 - 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 768*a^{10}*b*c^4*g^2 - 30*a*b^{14}*d*e \\
& + 6366*a^2*b^{11}*c^2*d^2 - 35767*a^3*b^9*c^3*d^2 + 116928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^{(1/2)} \\
& + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^2 - 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + 25*a^4*c^2* \\
& e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^2 + 3840*a^8*b^3*c^4*f^2 - 96*a^8*b^5*c^2*g^2 + 512*a^9*b^3*c^3*g^2 - 615*a*b^13*c*d^2 \\
& + 10*a^2*b^13*d*f + 35840*a^8*c^7*d*e + 10*a^3*b^12*d*g - 6*a^3*b^12*e*f - 6*a^4*b^11*e*g - 7168*a^9*c^6*d*g - 15360*a^9*c^6*e*f + 2*a^5*b^10*f*g + 3072*a^10*c^5*f*g \\
& - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^{(1/2)} + 724*a^2*b^12*c*d*e - 258*a^3*b^11*c*d*f + 43520*a^8*b*c^6*d*f - 168*a^4*b^10*c*d*g + 152*a^4*b^10*c*e*f + 98*a^5*b^9*c*e*g \\
& - 1536*a^9*b*c^5*e*g + 2*a^5*b*f*g*(-(4*a*c - b^2)^9)^{(1/2)} - 10*a^5*c*e*g*(-(4*a*c - b^2)^9)^{(1/2)} - 36*a^6*b^8*c*f*g + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - 165*a*b^4*c*d^2 \\
& *(-4*a*c - b^2)^9)^{(1/2)} - 7278*a^3*b^10*c^2*d*e + 39132*a^4*b^8*c^3*d*e - 119616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^{(1/2)} \\
& + 2706*a^4*b^9*c^2*d*f - 14784*a^5*b^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f + 10*a^3*b^3*d*g*(-(4*a*c - b^2)^9)^{(1/2)} - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^{(1/2)} + 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^{(1/2)} \\
& + 1044*a^5*b^8*c^2*d*g - 1548*a^5*b^8*c^2*e*f - 2688*a^6*b^6*c^3*d*g + 8064*a^6*b^6*c^3*e*f + 1152*a^7*b^4*c^4*d*g - 22400*a^7*b^4*c^4*e*f + 6144*a^8*b^2*c^5*d*g + 30720*a^8*b^2*c^5*e*f - 6 \\
& *a^4*b^2*e*g*(-(4*a*c - b^2)^9)^{(1/2)} - 576*a^6*b^7*c^2*e*g + 1344*a^7*b^5*c^3*e*g - 512*a^8*b^3*c^4*e*g + 192*a^7*b^6*c^2*f*g - 128*a^8*b^4*c^3*f*g -
\end{aligned}$$

$$\begin{aligned}
& 1536*a^9*b^2*c^4*f*g - 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 12*a^4* \\
& b*c*d*g*(-(4*a*c - b^2)^9)^{(1/2)} + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^{(1/2)} \\
& + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 186*a^3*b*c^2*d*e*(-(4*a*c - \\
& b^2)^9)^{(1/2)} - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^{(1/2)}/(32*(a^7*b^12 + \\
& 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840 \\
& *a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^{(1/2)}*(1048576*a^21*b*c^8 + 256*a^15*b \\
& ^13*c^2 - 6144*a^16*b^11*c^3 + 61440*a^17*b^9*c^4 - 327680*a^18*b^7*c^5 + 9 \\
& 83040*a^19*b^5*c^6 - 1572864*a^20*b^3*c^7) + 320*a^12*b^14*c^2*d - 7936*a^1 \\
& 3*b^12*c^3*d + 82816*a^14*b^10*c^4*d - 468480*a^15*b^8*c^5*d + 1536000*a^16 \\
& *b^6*c^6*d - 2867200*a^17*b^4*c^7*d + 2719744*a^18*b^2*c^8*d - 192*a^13*b^1 \\
& 3*c^2*e + 4672*a^14*b^11*c^3*e - 47360*a^15*b^9*c^4*e + 256000*a^16*b^7*c^5 \\
& *e - 778240*a^17*b^5*c^6*e + 1261568*a^18*b^3*c^7*e + 64*a^14*b^12*c^2*f - \\
& 1664*a^15*b^10*c^3*f + 17920*a^16*b^8*c^4*f - 102400*a^17*b^6*c^5*f + 32768 \\
& 0*a^18*b^4*c^6*f - 557056*a^19*b^2*c^7*f + 64*a^15*b^11*c^2*g - 1280*a^16*b \\
& ^9*c^3*g + 10240*a^17*b^7*c^4*g - 40960*a^18*b^5*c^5*g + 81920*a^19*b^3*c^6 \\
& *g - 851968*a^19*b*c^8*e - 65536*a^20*b*c^7*g) + x*(204800*a^17*c^9*e^2 - 4 \\
& 01408*a^16*c^10*d^2 - 73728*a^18*c^8*f^2 + 8192*a^19*c^7*g^2 + 400*a^9*b^14 \\
& *c^3*d^2 - 9440*a^10*b^12*c^4*d^2 + 92816*a^11*b^10*c^5*d^2 - 488096*a^12*b \\
& ^8*c^6*d^2 + 1458688*a^13*b^6*c^7*d^2 - 2401280*a^14*b^4*c^8*d^2 + 1871872* \\
& a^15*b^2*c^9*d^2 + 144*a^11*b^12*c^3*e^2 - 3264*a^12*b^10*c^4*e^2 + 30112*a \\
& ^13*b^8*c^5*e^2 - 143360*a^14*b^6*c^6*e^2 + 365568*a^15*b^4*c^7*e^2 - 45875 \\
& 2*a^16*b^2*c^8*e^2 + 16*a^13*b^10*c^3*f^2 - 416*a^14*b^8*c^4*f^2 + 4608*a^1 \\
& 5*b^6*c^5*f^2 - 25600*a^16*b^4*c^6*f^2 + 69632*a^17*b^2*c^7*f^2 + 160*a^15* \\
& b^8*c^3*g^2 - 2048*a^16*b^6*c^4*g^2 + 9216*a^17*b^4*c^5*g^2 - 16384*a^18*b^ \\
& 2*c^6*g^2 + 344064*a^17*c^9*d*f - 81920*a^18*c^8*e*g - 1236992*a^16*b*c^9*d \\
& *e + 40960*a^17*b*c^8*d*g + 237568*a^17*b*c^8*e*f + 40960*a^18*b*c^7*f*g - \\
& 480*a^10*b^13*c^3*d*e + 11104*a^11*b^11*c^4*d*e - 105824*a^12*b^9*c^5*d*e + \\
& 530432*a^13*b^7*c^6*d*e - 1469440*a^14*b^5*c^7*d*e + 2121728*a^15*b^3*c^8* \\
& d*e + 160*a^11*b^12*c^3*d*f - 3968*a^12*b^10*c^4*d*f + 39488*a^13*b^8*c^5*d \\
& *f - 200704*a^14*b^6*c^6*d*f + 542720*a^15*b^4*c^7*d*f - 720896*a^16*b^2*c^ \\
& 8*d*f + 160*a^12*b^11*c^3*d*g - 96*a^12*b^11*c^3*e*f - 2528*a^13*b^9*c^4*d* \\
& g + 2336*a^13*b^9*c^4*e*f + 14336*a^14*b^7*c^5*d*g - 22528*a^14*b^7*c^5*e*f \\
& - 31744*a^15*b^5*c^6*d*g + 107520*a^15*b^5*c^6*e*f + 8192*a^16*b^3*c^7*d*g \\
& - 253952*a^16*b^3*c^7*e*f - 96*a^13*b^10*c^3*e*g + 1472*a^14*b^8*c^4*e*g - \\
& 7168*a^15*b^6*c^5*e*g + 6144*a^16*b^4*c^6*e*g + 40960*a^17*b^2*c^7*e*g + 3 \\
& 2*a^14*b^9*c^3*f*g - 1024*a^15*b^7*c^4*f*g + 9216*a^16*b^5*c^5*f*g - 32768* \\
& a^17*b^3*c^6*f*g)*(-(25*b^15*d^2 + 9*a^2*b^13*e^2 + 25*b^6*d^2*(-(4*a*c - \\
& b^2)^9)^{(1/2)} + a^4*b^11*f^2 + a^6*b^9*g^2 + a^6*g^2*(-(4*a*c - b^2)^9)^{(1/ \\
& 2)} - 80640*a^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^ \\
& 5*b^9*c*f^2 - 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 7 \\
& 68*a^10*b*c^4*g^2 - 30*a*b^14*d*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c \\
& ^3*d^2 + 116928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c \\
& ^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 49*a^3*c^3*d^2*(-(4*a*c - \\
& b^2)^9)^{(1/2)} + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b \\
& ^5*c^4*e^2 - 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + \\
& 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b \\
& ^5*c^3*f^2 + 3840*a^8*b^3*c^4*f^2 - 96*a^8*b^5*c^2*g^2 + 512*a^9*b^3*c^3*g^ \\
& 2 - 615*a*b^13*c*d^2 + 10*a^2*b^13*d*f + 35840*a^8*c^7*d*e + 10*a^3*b^12*d* \\
& g - 6*a^3*b^12*e*f - 6*a^4*b^11*e*g - 7168*a^9*c^6*d*g - 15360*a^9*c^6*e*f \\
& + 2*a^5*b^10*f*g + 3072*a^10*c^5*f*g - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^{(1/2 \\
&)} + 724*a^2*b^12*c*d*e - 258*a^3*b^11*c*d*f + 43520*a^8*b*c^6*d*f - 168*a^4 \\
& *b^10*c*d*g + 152*a^4*b^10*c*e*f + 98*a^5*b^9*c*e*g - 1536*a^9*b*c^5*e*g + \\
& 2*a^5*b*f*g*(-(4*a*c - b^2)^9)^{(1/2)} - 10*a^5*c*e*g*(-(4*a*c - b^2)^9)^{(1/2 \\
&)} - 36*a^6*b^8*c*f*g + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - 165*a \\
& *b^4*c*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - 7278*a^3*b^10*c^2*d*e + 39132*a^4*b^8 \\
& *c^3*d*e - 119616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5*d*e - 161280*a^7*b^2 \\
& *c^6*d*e + 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 2706*a^4*b^9*c^2*d*f - \\
& 14784*a^5*b^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f + 10 \\
& *a^3*b^3*d*g*(-(4*a*c - b^2)^9)^{(1/2)} - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^{(1
\end{aligned}$$

$$\begin{aligned}
& /2) + 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 1044*a^5*b^8*c^2*d*g - 1548 \\
& *a^5*b^8*c^2*e*f - 2688*a^6*b^6*c^3*d*g + 8064*a^6*b^6*c^3*e*f + 1152*a^7*b \\
& ^4*c^4*d*g - 22400*a^7*b^4*c^4*e*f + 6144*a^8*b^2*c^5*d*g + 30720*a^8*b^2*c \\
& ^5*e*f - 6*a^4*b^2*e*g*(-(4*a*c - b^2)^9)^{(1/2)} - 576*a^6*b^7*c^2*e*g + 134 \\
& 4*a^7*b^5*c^3*e*g - 512*a^8*b^3*c^4*e*g + 192*a^7*b^6*c^2*f*g - 128*a^8*b^4 \\
& *c^3*f*g - 1536*a^9*b^2*c^4*f*g - 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^{(1/2)} \\
& + 12*a^4*b*c*d*g*(-(4*a*c - b^2)^9)^{(1/2)} + 44*a^4*b*c*e*f*(-(4*a*c - b^2) \\
& ^9)^{(1/2)} + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 186*a^3*b*c^2*d*e* \\
& (- (4*a*c - b^2)^9)^{(1/2)} - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^{(1/2)}/(32*(\\
& a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6* \\
& c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^{(1/2)}*1i + ((-(25*b^15*d^2 + \\
& 9*a^2*b^13*e^2 + 25*b^6*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + a^4*b^11*f^2 + a^6* \\
& b^9*g^2 + a^6*g^2*(-(4*a*c - b^2)^9)^{(1/2)} - 80640*a^7*b*c^7*d^2 - 213*a^3* \\
& b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^2 - 3840*a^9*b*c^5*f^2 - \\
& 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 768*a^10*b*c^4*g^2 - 30*a*b^14*d*e + \\
& 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^2 + 116928*a^4*b^7*c^4*d^2 - 2 \\
& 19744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c - b \\
& ^2)^9)^{(1/2)} - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + 2077*a^4*b^9*c^2*e \\
& ^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^2 - 44800*a^7*b^3*c^5*e^2 \\
& + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9) \\
& ^{(1/2)} + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^2 + 3840*a^8*b^3*c^4*f^2 - \\
& 96*a^8*b^5*c^2*g^2 + 512*a^9*b^3*c^3*g^2 - 615*a*b^13*c*d^2 + 10*a^2*b^13* \\
& d*f + 35840*a^8*c^7*d*e + 10*a^3*b^12*d*g - 6*a^3*b^12*e*f - 6*a^4*b^11*e*g \\
& - 7168*a^9*c^6*d*g - 15360*a^9*c^6*e*f + 2*a^5*b^10*f*g + 3072*a^10*c^5*f* \\
& g - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^{(1/2)} + 724*a^2*b^12*c*d*e - 258*a^3*b^ \\
& 11*c*d*f + 43520*a^8*b*c^6*d*f - 168*a^4*b^10*c*d*g + 152*a^4*b^10*c*e*f + \\
& 98*a^5*b^9*c*e*g - 1536*a^9*b*c^5*e*g + 2*a^5*b*f*g*(-(4*a*c - b^2)^9)^{(1/2)} \\
&) - 10*a^5*c*e*g*(-(4*a*c - b^2)^9)^{(1/2)} - 36*a^6*b^8*c*f*g + 246*a^2*b^2* \\
& c^2*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^{(1/2)} \\
& - 7278*a^3*b^10*c^2*d*e + 39132*a^4*b^8*c^3*d*e - 119616*a^5*b^6*c^4*d*e + \\
& 201600*a^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*f*(-(4*a*c \\
& - b^2)^9)^{(1/2)} + 2706*a^4*b^9*c^2*d*f - 14784*a^5*b^7*c^3*d*f + 44352*a^6* \\
& b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f + 10*a^3*b^3*d*g*(-(4*a*c - b^2)^9)^{(1/ \\
& 2)} - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^{(1/2)} + 42*a^4*c^2*d*f*(-(4*a*c - b^2 \\
&)^9)^{(1/2)} + 1044*a^5*b^8*c^2*d*g - 1548*a^5*b^8*c^2*e*f - 2688*a^6*b^6*c^3 \\
& *d*g + 8064*a^6*b^6*c^3*e*f + 1152*a^7*b^4*c^4*d*g - 22400*a^7*b^4*c^4*e*f \\
& + 6144*a^8*b^2*c^5*d*g + 30720*a^8*b^2*c^5*e*f - 6*a^4*b^2*e*g*(-(4*a*c - b \\
& ^2)^9)^{(1/2)} - 576*a^6*b^7*c^2*e*g + 1344*a^7*b^5*c^3*e*g - 512*a^8*b^3*c^4 \\
& *e*g + 192*a^7*b^6*c^2*f*g - 128*a^8*b^4*c^3*f*g - 1536*a^9*b^2*c^4*f*g - 5 \\
& 1*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 12*a^4*b*c*d*g*(-(4*a*c - b^2)^9 \\
&)^{(1/2)} + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^{(1/2)} + 184*a^2*b^3*c*d*e*(-(4* \\
& a*c - b^2)^9)^{(1/2)} - 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 78*a^3*b \\
& ^2*c*d*f*(-(4*a*c - b^2)^9)^{(1/2)}/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b \\
& ^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12 \\
& *b^2*c^5)))^{(1/2)}*(917504*a^19*c^9*d - 393216*a^20*c^8*f + x*(-(25*b^15*d^2 \\
& + 9*a^2*b^13*e^2 + 25*b^6*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + a^4*b^11*f^2 + a^ \\
& 6*b^9*g^2 + a^6*g^2*(-(4*a*c - b^2)^9)^{(1/2)} - 80640*a^7*b*c^7*d^2 - 213*a^ \\
& 3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^2 - 3840*a^9*b*c^5*f^2 \\
& - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 768*a^10*b*c^4*g^2 - 30*a*b^14*d*e \\
& + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^2 + 116928*a^4*b^7*c^4*d^2 - \\
& 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c - \\
& b^2)^9)^{(1/2)} - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + 2077*a^4*b^9*c^2 \\
& *e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^2 - 44800*a^7*b^3*c^5*e^ \\
& 2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9 \\
&)^{(1/2)} + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^2 + 3840*a^8*b^3*c^4*f^2 \\
& - 96*a^8*b^5*c^2*g^2 + 512*a^9*b^3*c^3*g^2 - 615*a*b^13*c*d^2 + 10*a^2*b^1 \\
& 3*d*f + 35840*a^8*c^7*d*e + 10*a^3*b^12*d*g - 6*a^3*b^12*e*f - 6*a^4*b^11*e \\
& *g - 7168*a^9*c^6*d*g - 15360*a^9*c^6*e*f + 2*a^5*b^10*f*g + 3072*a^10*c^5* \\
& f*g - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^{(1/2)} + 724*a^2*b^12*c*d*e - 258*a^3*
\end{aligned}$$

$$\begin{aligned}
& b^{11}c^d f + 43520a^8b^6c^d f - 168a^4b^{10}c^d g + 152a^4b^{10}c^e f \\
& + 98a^5b^9c^e g - 1536a^9b^6c^5e g + 2a^5b^6f g^2 - (4a^2c - b^2)^9)^{(1/2)} \\
& - 10a^5c^e g^2 - (4a^2c - b^2)^9)^{(1/2)} - 36a^6b^8c^f g + 246a^2b^2c^2d^2 \\
& - (4a^2c - b^2)^9)^{(1/2)} - 165a^6b^4c^d^2 - (4a^2c - b^2)^9)^{(1/2)} - 7278a^3b^{10}c^2d^2e \\
& + 39132a^4b^8c^3d^2e - 119616a^5b^6c^4d^2e + 201600a^6b^4c^5d^2e \\
& - 161280a^7b^2c^6d^2e + 10a^2b^4d^2f - (4a^2c - b^2)^9)^{(1/2)} + 2706a^4b^9c^2d^2f \\
& - 14784a^5b^7c^3d^2f + 44352a^6b^5c^4d^2f - 69120a^7b^3c^5d^2f + 10a^3b^3d^2g^2 \\
& - (4a^2c - b^2)^9)^{(1/2)} - 6a^3b^3c^e f^2 - (4a^2c - b^2)^9)^{(1/2)} + 42a^4c^2d^2f^2 \\
& - (4a^2c - b^2)^9)^{(1/2)} + 1044a^5b^8c^2d^2g - 1548a^5b^8c^2e f^2 - 2688a^6b^6c^3d^2g \\
& + 8064a^6b^6c^3e f^2 + 1152a^7b^4c^4d^2g - 22400a^7b^4c^4e f^2 + 6144a^8b^2c^5d^2g \\
& + 30720a^8b^2c^5e f^2 - 6a^4b^2e^2g^2 - (4a^2c - b^2)^9)^{(1/2)} - 576a^6b^7c^2e^2g \\
& + 1344a^7b^5c^3e^2g - 512a^8b^3c^4e^2g + 192a^7b^6c^2f^2g - 128a^8b^4c^3f^2g \\
& - 1536a^9b^2c^4f^2g - 51a^3b^2c^e^2 - (4a^2c - b^2)^9)^{(1/2)} + 12a^4b^6c^d g^2 \\
& - (4a^2c - b^2)^9)^{(1/2)} + 44a^4b^6c^e f^2 - (4a^2c - b^2)^9)^{(1/2)} + 184a^2b^3c^d e^2 \\
& - (4a^2c - b^2)^9)^{(1/2)} - 186a^3b^6c^2d^2e^2 - (4a^2c - b^2)^9)^{(1/2)} - 78a^3b^2c^d f^2 \\
& - (4a^2c - b^2)^9)^{(1/2)} / (32(a^7b^{12} + 4096a^{13}c^6 - 24a^8b^{10}c + 240a^9b^8c^2 \\
& - 1280a^{10}b^6c^3 + 3840a^{11}b^4c^4 - 6144a^{12}b^2c^5))^{(1/2)} * (1048576a^{21}b^8c^8 \\
& + 256a^{15}b^{13}c^2 - 6144a^{16}b^{11}c^3 + 61440a^{17}b^9c^4 - 327680a^{18}b^7c^5 \\
& + 983040a^{19}b^5c^6 - 1572864a^{20}b^3c^7) - 320a^{12}b^{14}c^2d + 7936a^{13}b^{12}c^3d \\
& - 82816a^{14}b^{10}c^4d + 468480a^{15}b^8c^5d - 1536000a^{16}b^6c^6d + 2867200a^{17}b^4c^7d \\
& - 2719744a^{18}b^2c^8d + 192a^{13}b^{13}c^2e - 4672a^{14}b^{11}c^3e + 47360a^{15}b^9c^4e \\
& - 256000a^{16}b^7c^5e + 778240a^{17}b^5c^6e - 1261568a^{18}b^3c^7e - 64a^{14}b^{12}c^2f \\
& + 1664a^{15}b^{10}c^3f - 17920a^{16}b^8c^4f + 102400a^{17}b^6c^5f - 327680a^{18}b^4c^6f \\
& + 557056a^{19}b^2c^7f - 64a^{15}b^{11}c^2g + 1280a^{16}b^9c^3g - 10240a^{17}b^7c^4g \\
& + 40960a^{18}b^5c^5g - 81920a^{19}b^3c^6g + 851968a^{19}b^6c^8e + 65536a^{20}b^6c^7g \\
& + x(204800a^{17}c^9e^2 - 401408a^{16}c^{10}d^2 - 73728a^{18}c^8f^2 + 8192a^{19}c^7g^2 \\
& + 400a^9b^{14}c^3d^2 - 9440a^{10}b^{12}c^4d^2 + 92816a^{11}b^{10}c^5d^2 - 488096a^{12}b^8c^6d^2 \\
& + 1458688a^{13}b^6c^7d^2 - 2401280a^{14}b^4c^8d^2 + 1871872a^{15}b^2c^9d^2 + 144a^{11}b^{12}c^3e^2 \\
& - 3264a^{12}b^{10}c^4e^2 + 30112a^{13}b^8c^5e^2 - 143360a^{14}b^6c^6e^2 + 365568a^{15}b^4c^7e^2 \\
& - 458752a^{16}b^2c^8e^2 + 16a^{13}b^{10}c^3f^2 - 416a^{14}b^8c^4f^2 + 4608a^{15}b^6c^5f^2 \\
& - 25600a^{16}b^4c^6f^2 + 69632a^{17}b^2c^7f^2 + 160a^{15}b^8c^3g^2 - 2048a^{16}b^6c^4g^2 \\
& + 9216a^{17}b^4c^5g^2 - 16384a^{18}b^2c^6g^2 + 344064a^{17}c^9d^2f - 81920a^{18}c^8e^2g \\
& - 1236992a^{16}b^6c^9d^2e + 40960a^{17}b^5c^8d^2g + 237568a^{17}b^5c^8e^2f + 40960a^{18}b^4c^7f^2g \\
& - 480a^{10}b^{13}c^3d^2e + 11104a^{11}b^{11}c^4d^2e - 105824a^{12}b^9c^5d^2e + 530432a^{13}b^7c^6d^2e \\
& - 1469440a^{14}b^5c^7d^2e + 2121728a^{15}b^3c^8d^2e + 160a^{11}b^{12}c^3d^2f - 3968a^{12}b^{10}c^4d^2f \\
& + 39488a^{13}b^8c^5d^2f - 200704a^{14}b^6c^6d^2f + 542720a^{15}b^4c^7d^2f - 720896a^{16}b^2c^8d^2f \\
& + 160a^{12}b^{11}c^3d^2g - 96a^{12}b^{11}c^3e^2f - 2528a^{13}b^9c^4d^2g + 2336a^{13}b^9c^4e^2f \\
& + 14336a^{14}b^7c^5d^2g - 22528a^{14}b^7c^5e^2f - 31744a^{15}b^5c^6d^2g + 107520a^{15}b^5c^6e^2f \\
& + 8192a^{16}b^3c^7d^2g - 253952a^{16}b^3c^7e^2f - 96a^{13}b^{10}c^3e^2g + 1472a^{14}b^8c^4e^2g \\
& - 7168a^{15}b^6c^5e^2g + 6144a^{16}b^4c^6e^2g + 40960a^{17}b^2c^7e^2g + 32a^{14}b^9c^3f^2g - 1024a^{15}b^7c^4f^2g \\
& + 9216a^{16}b^5c^5f^2g - 32768a^{17}b^3c^6f^2g)) * (-25b^{15}d^2 + 9a^2b^{13}e^2 + 25b^6d^2 \\
& - (4a^2c - b^2)^9)^{(1/2)} + a^4b^{11}f^2 + a^6b^9g^2 + a^6g^2 - (4a^2c - b^2)^9)^{(1/2)} - 80640a^7b^6c^7d^2 \\
& - 213a^3b^{11}c^e^2 + 26880a^8b^6c^6e^2 - 27a^5b^9c^f^2 - 3840a^9b^6c^5f^2 - 9a^5c^f^2 \\
& - (4a^2c - b^2)^9)^{(1/2)} - 768a^{10}b^6c^4g^2 - 30a^8b^{14}d^2e + 6366a^2b^{11}c^2d^2 \\
& - 35767a^3b^9c^3d^2 + 116928a^4b^7c^4d^2 - 219744a^5b^5c^5d^2 + 215040a^6b^3c^6d^2 \\
& + 9a^2b^4e^2 - (4a^2c - b^2)^9)^{(1/2)} - 49a^3c^3d^2 - (4a^2c - b^2)^9)^{(1/2)} + 2077a^4b^9c^2e^2 \\
& - 10656a^5b^7c^3e^2 + 30240a^6b^5c^4e^2 - 44800a^7b^3c^5e^2 + a^4b^2f^2 - (4a^2c - b^2)^9)^{(1/2)} \\
& + 25a^4c^2e^2 - (4a^2c - b^2)^9)^{(1/2)}
\end{aligned}$$

$$\begin{aligned}
& c - b^2)^9)^{(1/2)} + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^2 + 3840*a^8*b^3*c^4*f^2 - 96*a^8*b^5*c^2*g^2 + 512*a^9*b^3*c^3*g^2 - 615*a*b^13*c*d^2 + \\
& 10*a^2*b^13*d*f + 35840*a^8*c^7*d*e + 10*a^3*b^12*d*g - 6*a^3*b^12*e*f - 6*a^4*b^11*e*g - 7168*a^9*c^6*d*g - 15360*a^9*c^6*e*f + 2*a^5*b^10*f*g + 3072 \\
& *a^10*c^5*f*g - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^{(1/2)} + 724*a^2*b^12*c*d*e - 258*a^3*b^11*c*d*f + 43520*a^8*b*c^6*d*f - 168*a^4*b^10*c*d*g + 152*a^4*b^10*c*e*f + 98*a^5*b^9*c*e*g - 1536*a^9*b*c^5*e*g + 2*a^5*b*f*g*(-(4*a*c - \\
& b^2)^9)^{(1/2)} - 10*a^5*c*e*g*(-(4*a*c - b^2)^9)^{(1/2)} - 36*a^6*b^8*c*f*g + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - 7278*a^3*b^10*c^2*d*e + 39132*a^4*b^8*c^3*d*e - 119616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d \\
& *f*(-(4*a*c - b^2)^9)^{(1/2)} + 2706*a^4*b^9*c^2*d*f - 14784*a^5*b^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f + 10*a^3*b^3*d*g*(-(4*a*c - b^2)^9)^{(1/2)} - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^{(1/2)} + 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 1044*a^5*b^8*c^2*d*g - 1548*a^5*b^8*c^2*e*f - 2688 \\
& *a^6*b^6*c^3*d*g + 8064*a^6*b^6*c^3*e*f + 1152*a^7*b^4*c^4*d*g - 22400*a^7*b^4*c^4*e*f + 6144*a^8*b^2*c^5*d*g + 30720*a^8*b^2*c^5*e*f - 6*a^4*b^2*e*g*(-(4*a*c - b^2)^9)^{(1/2)} - 576*a^6*b^7*c^2*e*g + 1344*a^7*b^5*c^3*e*g - 512 \\
& *a^8*b^3*c^4*e*g + 192*a^7*b^6*c^2*f*g - 128*a^8*b^4*c^3*f*g - 1536*a^9*b^2*c^4*f*g - 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 12*a^4*b*c*d*g*(-(4*a*c - b^2)^9)^{(1/2)} + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^{(1/2)} + 184*a^2*b^3 \\
& *c*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^{(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^{(1/2)}*i)/(((-(25*b^15*d^2 + 9*a^2*b^13*e^2 + 25*b^6 \\
& *d^2*(-(4*a*c - b^2)^9)^{(1/2)} + a^4*b^11*f^2 + a^6*b^9*g^2 + a^6*g^2*(-(4*a*c - b^2)^9)^{(1/2)} - 80640*a^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^2 - 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 768*a^10*b*c^4*g^2 - 30*a*b^14*d*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^2 + 116928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^2 - 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^2 + 3840*a^8*b^3*c^4*f^2 - 96*a^8*b^5*c^2*g^2 + 512*a^9*b^3*c^3*g^2 - 615*a*b^13*c*d^2 + 10*a^2*b^13*d*f + 35840*a^8*c^7*d*e + 10*a^3*b^12*d*g - 6*a^3*b^12*e*f - 6*a^4*b^11*e*g - 7168*a^9*c^6*d*g - 15360*a^9*c^6*e*f + 2*a^5*b^10*f*g + 3072*a^10*c^5*f*g - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^{(1/2)} + 724*a^2*b^12*c*d*e - 258*a^3*b^11*c*d*f + 43520*a^8*b*c^6*d*f - 168*a^4*b^10*c*d*g + 152*a^4*b^10*c*e*f + 98*a^5*b^9*c*e*g - 1536*a^9*b*c^5*e*g + 2*a^5*b*f*g*(-(4*a*c - b^2)^9)^{(1/2)} - 10*a^5*c*e*g*(-(4*a*c - b^2)^9)^{(1/2)} - 36*a^6*b^8*c*f*g + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - 7278*a^3*b^10*c^2*d*e + 39132*a^4*b^8*c^3*d*e - 119616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 2706*a^4*b^9*c^2*d*f - 14784*a^5*b^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f + 10*a^3*b^3*d*g*(-(4*a*c - b^2)^9)^{(1/2)} - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^{(1/2)} + 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 1044*a^5*b^8*c^2*d*g - 1548*a^5*b^8*c^2*e*f - 2688*a^6*b^6*c^3*d*g + 8064*a^6*b^6*c^3*e*f + 1152*a^7*b^4*c^4*d*g - 22400*a^7*b^4*c^4*e*f + 6144*a^8*b^2*c^5*d*g + 30720*a^8*b^2*c^5*e*f - 6*a^4*b^2*e*g*(-(4*a*c - b^2)^9)^{(1/2)} - 576*a^6*b^7*c^2*e*g + 1344*a^7*b^5*c^3*e*g - 512*a^8*b^3*c^4*e*g + 192*a^7*b^6*c^2*f*g - 128*a^8*b^4*c^3*f*g - 1536*a^9*b^2*c^4*f*g - 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 12*a^4*b*c*d*g*(-(4*a*c - b^2)^9)^{(1/2)} + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^{(1/2)} + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^{(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^{(1/2)}*(393216*a^20*c^8*f - 917504*a^19*c^9*d + x*(-(25*b^15*d^2 + 9*a^2*b^13*e^2 + 25*
\end{aligned}$$

$$\begin{aligned}
& b^6 d^2 (-4ac - b^2)^9)^{1/2} + a^4 b^{11} f^2 + a^6 b^9 g^2 + a^6 g^2 (- \\
& (4ac - b^2)^9)^{1/2} - 80640 a^7 b^7 c^7 d^2 - 213 a^3 b^{11} c^6 e^2 + 26880 a^8 \\
& b^7 c^6 e^2 - 27 a^5 b^9 c^3 f^2 - 3840 a^9 b^7 c^5 f^2 - 9 a^5 c^3 f^2 (-4ac \\
& - b^2)^9)^{1/2} - 768 a^{10} b^4 c^4 g^2 - 30 a^2 b^{14} d^2 e + 6366 a^2 b^{11} c^2 d^2 \\
& - 35767 a^3 b^9 c^3 d^2 + 116928 a^4 b^7 c^4 d^2 - 219744 a^5 b^5 c^5 d^2 \\
& + 215040 a^6 b^3 c^6 d^2 + 9 a^2 b^4 e^2 (-4ac - b^2)^9)^{1/2} - 49 a^3 \\
& c^3 d^2 (-4ac - b^2)^9)^{1/2} + 2077 a^4 b^9 c^2 e^2 - 10656 a^5 b^7 c^3 \\
& e^2 + 30240 a^6 b^5 c^4 e^2 - 44800 a^7 b^3 c^5 e^2 + a^4 b^2 f^2 (-4ac \\
& - b^2)^9)^{1/2} + 25 a^4 c^2 e^2 (-4ac - b^2)^9)^{1/2} + 288 a^6 b^7 c^2 \\
& f^2 - 1504 a^7 b^5 c^3 f^2 + 3840 a^8 b^3 c^4 f^2 - 96 a^8 b^5 c^2 g^2 + \\
& 512 a^9 b^3 c^3 g^2 - 615 a^2 b^{13} c^4 d^2 + 10 a^2 b^{13} d^2 f + 35840 a^8 c^7 d \\
& e + 10 a^3 b^{12} d^2 g - 6 a^3 b^{12} e^2 f - 6 a^4 b^{11} e^2 g - 7168 a^9 c^6 d^2 g - \\
& 15360 a^9 c^6 e^2 f + 2 a^5 b^{10} f^2 g + 3072 a^{10} c^5 f^2 g - 30 a^2 b^5 d^2 e (-4 \\
& ac - b^2)^9)^{1/2} + 724 a^2 b^{12} c^4 d^2 e - 258 a^3 b^{11} c^4 d^2 f + 43520 a^8 b^7 \\
& c^6 d^2 f - 168 a^4 b^{10} c^4 d^2 g + 152 a^4 b^{10} c^4 e^2 f + 98 a^5 b^9 c^4 e^2 g - 15 \\
& 36 a^9 b^7 c^5 e^2 g + 2 a^5 b^7 f^2 g (-4ac - b^2)^9)^{1/2} - 10 a^5 c^4 e^2 g (-4 \\
& ac - b^2)^9)^{1/2} - 36 a^6 b^8 c^4 f^2 g + 246 a^2 b^2 c^2 d^2 (-4ac - b^2)^9)^{1/2} \\
& - 165 a^2 b^4 c^4 d^2 (-4ac - b^2)^9)^{1/2} - 7278 a^3 b^{10} c^2 d^2 \\
& e + 39132 a^4 b^8 c^3 d^2 e - 119616 a^5 b^6 c^4 d^2 e + 201600 a^6 b^4 c^5 d^2 \\
& e - 161280 a^7 b^2 c^6 d^2 e + 10 a^2 b^4 d^2 f (-4ac - b^2)^9)^{1/2} + 270 \\
& 6 a^4 b^9 c^2 d^2 f - 14784 a^5 b^7 c^3 d^2 f + 44352 a^6 b^5 c^4 d^2 f - 69120 a^7 \\
& b^3 c^5 d^2 f + 10 a^3 b^3 d^2 g (-4ac - b^2)^9)^{1/2} - 6 a^3 b^3 e^2 f (-4 \\
& ac - b^2)^9)^{1/2} + 42 a^4 c^2 d^2 f (-4ac - b^2)^9)^{1/2} + 1044 a^5 \\
& b^8 c^2 d^2 g - 1548 a^5 b^8 c^2 e^2 f - 2688 a^6 b^6 c^3 d^2 g + 8064 a^6 b^6 c^3 \\
& e^2 f + 1152 a^7 b^4 c^4 d^2 g - 22400 a^7 b^4 c^4 e^2 f + 6144 a^8 b^2 c^5 d^2 g \\
& + 30720 a^8 b^2 c^5 e^2 f - 6 a^4 b^2 e^2 g (-4ac - b^2)^9)^{1/2} - 576 a^6 \\
& b^7 c^2 e^2 g + 1344 a^7 b^5 c^3 e^2 g - 512 a^8 b^3 c^4 e^2 g + 192 a^7 b^6 c^2 \\
& f^2 g - 128 a^8 b^4 c^3 f^2 g - 1536 a^9 b^2 c^4 f^2 g - 51 a^3 b^2 c^2 e^2 (-4 \\
& ac - b^2)^9)^{1/2} + 12 a^4 b^3 c^2 d^2 g (-4ac - b^2)^9)^{1/2} + 44 a^4 b^3 c^2 \\
& e^2 f (-4ac - b^2)^9)^{1/2} + 184 a^2 b^3 c^2 d^2 e (-4ac - b^2)^9)^{1/2} - \\
& 186 a^3 b^3 c^2 d^2 e (-4ac - b^2)^9)^{1/2} - 78 a^3 b^2 c^2 d^2 f (-4ac - b^2)^9)^{1/2} \\
& - 78 a^3 b^2 c^2 d^2 f (-4ac - b^2)^9)^{1/2} / (32 (a^7 b^{12} + 4096 a^{13} c^6 - 24 a^8 b^{10} c \\
& + 240 a^9 b^8 c^2 - 1280 a^{10} b^6 c^3 + 3840 a^{11} b^4 c^4 - 6144 a^{12} b^2 c^5))^{1/2} * (10 \\
& 48576 a^{21} b^8 c^8 + 256 a^{15} b^{13} c^2 - 6144 a^{16} b^{11} c^3 + 61440 a^{17} b^9 c^4 \\
& - 327680 a^{18} b^7 c^5 + 983040 a^{19} b^5 c^6 - 1572864 a^{20} b^3 c^7) + 3 \\
& 20 a^{12} b^{14} c^2 d - 7936 a^{13} b^{12} c^3 d + 82816 a^{14} b^{10} c^4 d - 468480 a^{15} \\
& b^8 c^5 d + 1536000 a^{16} b^6 c^6 d - 2867200 a^{17} b^4 c^7 d + 2719744 a^{18} b^2 \\
& c^8 d - 192 a^{13} b^{13} c^2 e + 4672 a^{14} b^{11} c^3 e - 47360 a^{15} b^9 c^4 e + \\
& 256000 a^{16} b^7 c^5 e - 778240 a^{17} b^5 c^6 e + 1261568 a^{18} b^3 c^7 e + 64 a^{14} \\
& b^{12} c^2 f - 1664 a^{15} b^{10} c^3 f + 17920 a^{16} b^8 c^4 f - 102400 a^{17} b^6 c^5 f + \\
& 327680 a^{18} b^4 c^6 f - 557056 a^{19} b^2 c^7 f + 64 a^{15} b^{11} c^2 g - 1280 a^{16} b^9 \\
& c^3 g + 10240 a^{17} b^7 c^4 g - 40960 a^{18} b^5 c^5 g + 81920 a^{19} b^3 c^6 g - 851968 \\
& a^{19} b^3 c^8 e - 65536 a^{20} b^3 c^7 g) + x (204800 a^{17} c^9 e^2 - 401408 a^{16} c^{10} d^2 \\
& - 73728 a^{18} c^8 f^2 + 8192 a^{19} c^7 g^2 + 400 a^9 b^{14} c^3 d^2 - 9440 a^{10} b^{12} c^4 d^2 \\
& + 92816 a^{11} b^{10} c^5 d^2 - 488096 a^{12} b^8 c^6 d^2 + 1458688 a^{13} b^6 c^7 d^2 - 240128 \\
& 0 a^{14} b^4 c^8 d^2 + 1871872 a^{15} b^2 c^9 d^2 + 144 a^{11} b^{12} c^3 e^2 - 3264 a^{12} b^{10} \\
& c^4 e^2 + 30112 a^{13} b^8 c^5 e^2 - 143360 a^{14} b^6 c^6 e^2 + 365568 a^{15} b^4 c^7 e^2 \\
& - 458752 a^{16} b^2 c^8 e^2 + 16 a^{13} b^{10} c^3 f^2 - 416 a^{14} b^8 c^4 f^2 + 4608 a^{15} \\
& b^6 c^5 f^2 - 25600 a^{16} b^4 c^6 f^2 + 69632 a^{17} b^2 c^7 f^2 + 160 a^{15} b^8 c^3 g^2 \\
& - 2048 a^{16} b^6 c^4 g^2 + 9216 a^{17} b^4 c^5 g^2 - 16384 a^{18} b^2 c^6 g^2 + 344064 a^{17} \\
& c^9 d^2 f - 81920 a^{18} c^8 e^2 g - 1236992 a^{16} b^3 c^9 d^2 e + 40960 a^{17} b^3 c^8 d^2 g \\
& + 237568 a^{17} b^3 c^8 e^2 f + 40960 a^{18} b^3 c^7 f^2 g - 480 a^{10} b^{13} c^3 d^2 e + 11104 \\
& a^{11} b^{11} c^4 d^2 e - 105824 a^{12} b^9 c^5 d^2 e + 530432 a^{13} b^7 c^6 d^2 e - 1469440 \\
& a^{14} b^5 c^7 d^2 e + 2121728 a^{15} b^3 c^8 d^2 e + 160 a^{11} b^{12} c^3 d^2 f - 3968 a^{12} b^{10} \\
& c^4 d^2 f + 39488 a^{13} b^8 c^5 d^2 f - 200704 a^{14} b^6 c^6 d^2 f + 542720 a^{15} b^4 \\
& c^7 d^2 f - 720896 a^{16} b^2 c^8 d^2 f + 160 a^{12} b^{11} c^3 d^2 g - 96 a^{12} b^{11} c^3 \\
& e^2 f - 2528 a^{13} b^9 c^4 d^2 g + 2336 a^{13} b^9 c^4 e^2 f + 14336 a^{14} b^7 c^5
\end{aligned}$$

$$\begin{aligned}
& *d*g - 22528*a^{14}*b^7*c^5*e*f - 31744*a^{15}*b^5*c^6*d*g + 107520*a^{15}*b^5*c^6*e*f + 8192*a^{16}*b^3*c^7*d*g - 253952*a^{16}*b^3*c^7*e*f - 96*a^{13}*b^{10}*c^3*e*g + 1472*a^{14}*b^8*c^4*e*g - 7168*a^{15}*b^6*c^5*e*g + 6144*a^{16}*b^4*c^6*e*g + 40960*a^{17}*b^2*c^7*e*g + 32*a^{14}*b^9*c^3*f*g - 1024*a^{15}*b^7*c^4*f*g + 9216*a^{16}*b^5*c^5*f*g - 32768*a^{17}*b^3*c^6*f*g) * (- (25*b^{15}*d^2 + 9*a^2*b^{13}*e^2 + 25*b^6*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + a^4*b^{11}*f^2 + a^6*b^9*g^2 + a^6*g^2*(-(4*a*c - b^2)^9)^{(1/2)} - 80640*a^7*b*c^7*d^2 - 213*a^3*b^{11}*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^2 - 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 768*a^{10}*b*c^4*g^2 - 30*a*b^{14}*d*e + 6366*a^2*b^{11}*c^2*d^2 - 35767*a^3*b^9*c^3*d^2 + 116928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^2 - 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^2 + 3840*a^8*b^3*c^4*f^2 - 96*a^8*b^5*c^2*g^2 + 512*a^9*b^3*c^3*g^2 - 615*a*b^{13}*c*d^2 + 10*a^2*b^{13}*d*f + 35840*a^8*c^7*d*e + 10*a^3*b^{12}*d*g - 6*a^3*b^{12}*e*f - 6*a^4*b^{11}*e*g - 7168*a^9*c^6*d*g - 15360*a^9*c^6*e*f + 2*a^5*b^{10}*f*g + 3072*a^{10}*c^5*f*g - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^{(1/2)} + 724*a^2*b^{12}*c*d*e - 258*a^3*b^{11}*c*d*f + 43520*a^8*b*c^6*d*f - 168*a^4*b^{10}*c*d*g + 152*a^4*b^{10}*c*e*f + 98*a^5*b^9*c*e*g - 1536*a^9*b*c^5*e*g + 2*a^5*b*f*g*(-(4*a*c - b^2)^9)^{(1/2)} - 10*a^5*c*e*g*(-(4*a*c - b^2)^9)^{(1/2)} - 36*a^6*b^8*c*f*g + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - 7278*a^3*b^{10}*c^2*d*e + 39132*a^4*b^8*c^3*d*e - 119616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 2706*a^4*b^9*c^2*d*f - 14784*a^5*b^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f + 10*a^3*b^3*d*g*(-(4*a*c - b^2)^9)^{(1/2)} - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^{(1/2)} + 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 1044*a^5*b^8*c^2*d*g - 1548*a^5*b^8*c^2*e*f - 2688*a^6*b^6*c^3*d*g + 8064*a^6*b^6*c^3*e*f + 1152*a^7*b^4*c^4*d*g - 22400*a^7*b^4*c^4*e*f + 6144*a^8*b^2*c^5*d*g + 30720*a^8*b^2*c^5*e*f - 6*a^4*b^2*e*g*(-(4*a*c - b^2)^9)^{(1/2)} - 576*a^6*b^7*c^2*e*g + 1344*a^7*b^5*c^3*e*g - 512*a^8*b^3*c^4*e*g + 192*a^7*b^6*c^2*f*g - 128*a^8*b^4*c^3*f*g - 1536*a^9*b^2*c^4*f*g - 51*a^3*b^2*c^5*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 12*a^4*b*c*d*g*(-(4*a*c - b^2)^9)^{(1/2)} + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^{(1/2)} + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^{(1/2)}/(32*(a^7*b^{12} + 4096*a^{13}*c^6 - 24*a^8*b^{10}*c + 240*a^9*b^8*c^2 - 1280*a^{10}*b^6*c^3 + 3840*a^{11}*b^4*c^4 - 6144*a^{12}*b^2*c^5)))^{(1/2)} - ((-(25*b^{15}*d^2 + 9*a^2*b^{13}*e^2 + 25*b^6*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + a^4*b^{11}*f^2 + a^6*b^9*g^2 + a^6*g^2*(-(4*a*c - b^2)^9)^{(1/2)} - 80640*a^7*b*c^7*d^2 - 213*a^3*b^{11}*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^2 - 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 768*a^{10}*b*c^4*g^2 - 30*a*b^{14}*d*e + 6366*a^2*b^{11}*c^2*d^2 - 35767*a^3*b^9*c^3*d^2 + 116928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^2 - 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^2 + 3840*a^8*b^3*c^4*f^2 - 96*a^8*b^5*c^2*g^2 + 512*a^9*b^3*c^3*g^2 - 615*a*b^{13}*c*d^2 + 10*a^2*b^{13}*d*f + 35840*a^8*c^7*d*e + 10*a^3*b^{12}*d*g - 6*a^3*b^{12}*e*f - 6*a^4*b^{11}*e*g - 7168*a^9*c^6*d*g - 15360*a^9*c^6*e*f + 2*a^5*b^{10}*f*g + 3072*a^{10}*c^5*f*g - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^{(1/2)} + 724*a^2*b^{12}*c*d*e - 258*a^3*b^{11}*c*d*f + 43520*a^8*b*c^6*d*f - 168*a^4*b^{10}*c*d*g + 152*a^4*b^{10}*c*e*f + 98*a^5*b^9*c*e*g - 1536*a^9*b*c^5*e*g + 2*a^5*b*f*g*(-(4*a*c - b^2)^9)^{(1/2)} - 10*a^5*c*e*g*(-(4*a*c - b^2)^9)^{(1/2)} - 36*a^6*b^8*c*f*g + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - 7278*a^3*b^{10}*c^2*d*e + 39132*a^4*b^8*c^3*d*e - 119616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 2706*a^4*b^9*c^2*d*f - 14784*a^
\end{aligned}$$

$$\begin{aligned}
& 5*b^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f + 10*a^3*b^3* \\
& d*g*(-(4*a*c - b^2)^9)^{(1/2)} - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^{(1/2)} + 42* \\
& a^4*c^2*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 1044*a^5*b^8*c^2*d*g - 1548*a^5*b^8* \\
& c^2*e*f - 2688*a^6*b^6*c^3*d*g + 8064*a^6*b^6*c^3*e*f + 1152*a^7*b^4*c^4*d* \\
& g - 22400*a^7*b^4*c^4*e*f + 6144*a^8*b^2*c^5*d*g + 30720*a^8*b^2*c^5*e*f - \\
& 6*a^4*b^2*e*g*(-(4*a*c - b^2)^9)^{(1/2)} - 576*a^6*b^7*c^2*e*g + 1344*a^7*b^5* \\
& c^3*e*g - 512*a^8*b^3*c^4*e*g + 192*a^7*b^6*c^2*f*g - 128*a^8*b^4*c^3*f*g \\
& - 1536*a^9*b^2*c^4*f*g - 51*a^3*b^2*c^e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 12*a^4 \\
& *b*c*d*g*(-(4*a*c - b^2)^9)^{(1/2)} + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^{(1/2)} \\
& + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 186*a^3*b*c^2*d*e*(-(4*a*c \\
& - b^2)^9)^{(1/2)} - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^{(1/2))}/(32*(a^7*b^12 \\
& + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 384 \\
& 0*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^{(1/2)}*(917504*a^19*c^9*d - 393216*a^2 \\
& 0*c^8*f + x*(-(25*b^15*d^2 + 9*a^2*b^13*e^2 + 25*b^6*d^2*(-(4*a*c - b^2)^9) \\
& ^{(1/2)} + a^4*b^11*f^2 + a^6*b^9*g^2 + a^6*g^2*(-(4*a*c - b^2)^9)^{(1/2)} - 80 \\
& 640*a^7*b*c^7*d^2 - 213*a^3*b^11*c^e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c \\
& *f^2 - 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^{(1/2)} - 768*a^10 \\
& *b*c^4*g^2 - 30*a*b^14*d*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^2 \\
& + 116928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2 \\
& + 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9 \\
&)^{(1/2)} + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^4* \\
& e^2 - 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + 25*a^4 \\
& *c^2*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^3* \\
& f^2 + 3840*a^8*b^3*c^4*f^2 - 96*a^8*b^5*c^2*g^2 + 512*a^9*b^3*c^3*g^2 - 615 \\
& *a*b^13*c*d^2 + 10*a^2*b^13*d*f + 35840*a^8*c^7*d*e + 10*a^3*b^12*d*g - 6*a \\
& ^3*b^12*e*f - 6*a^4*b^11*e*g - 7168*a^9*c^6*d*g - 15360*a^9*c^6*e*f + 2*a^5 \\
& *b^10*f*g + 3072*a^10*c^5*f*g - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^{(1/2)} + 724 \\
& *a^2*b^12*c*d*e - 258*a^3*b^11*c*d*f + 43520*a^8*b*c^6*d*f - 168*a^4*b^10*c \\
& *d*g + 152*a^4*b^10*c*e*f + 98*a^5*b^9*c*e*g - 1536*a^9*b*c^5*e*g + 2*a^5*b \\
& *f*g*(-(4*a*c - b^2)^9)^{(1/2)} - 10*a^5*c*e*g*(-(4*a*c - b^2)^9)^{(1/2)} - 36* \\
& a^6*b^8*c*f*g + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - 165*a*b^4*c* \\
& d^2*(-(4*a*c - b^2)^9)^{(1/2)} - 7278*a^3*b^10*c^2*d*e + 39132*a^4*b^8*c^3*d* \\
& e - 119616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d* \\
& e + 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 2706*a^4*b^9*c^2*d*f - 14784* \\
& a^5*b^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f + 10*a^3*b^ \\
& 3*d*g*(-(4*a*c - b^2)^9)^{(1/2)} - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^{(1/2)} + 4 \\
& 2*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^{(1/2)} + 1044*a^5*b^8*c^2*d*g - 1548*a^5*b^ \\
& 8*c^2*e*f - 2688*a^6*b^6*c^3*d*g + 8064*a^6*b^6*c^3*e*f + 1152*a^7*b^4*c^4* \\
& d*g - 22400*a^7*b^4*c^4*e*f + 6144*a^8*b^2*c^5*d*g + 30720*a^8*b^2*c^5*e*f \\
& - 6*a^4*b^2*e*g*(-(4*a*c - b^2)^9)^{(1/2)} - 576*a^6*b^7*c^2*e*g + 1344*a^7*b \\
& ^5*c^3*e*g - 512*a^8*b^3*c^4*e*g + 192*a^7*b^6*c^2*f*g - 128*a^8*b^4*c^3*f* \\
& g - 1536*a^9*b^2*c^4*f*g - 51*a^3*b^2*c^e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 12*a \\
& ^4*b*c*d*g*(-(4*a*c - b^2)^9)^{(1/2)} + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^{(1/ \\
& 2)} + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 186*a^3*b*c^2*d*e*(-(4*a* \\
& c - b^2)^9)^{(1/2)} - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^{(1/2))}/(32*(a^7*b^1 \\
& 2 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3 \\
& 840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^{(1/2)}*(1048576*a^21*b*c^8 + 256*a^1 \\
& 5*b^13*c^2 - 6144*a^16*b^11*c^3 + 61440*a^17*b^9*c^4 - 327680*a^18*b^7*c^5 \\
& + 983040*a^19*b^5*c^6 - 1572864*a^20*b^3*c^7) - 320*a^12*b^14*c^2*d + 7936* \\
& a^13*b^12*c^3*d - 82816*a^14*b^10*c^4*d + 468480*a^15*b^8*c^5*d - 1536000*a \\
& ^16*b^6*c^6*d + 2867200*a^17*b^4*c^7*d - 2719744*a^18*b^2*c^8*d + 192*a^13* \\
& b^13*c^2*e - 4672*a^14*b^11*c^3*e + 47360*a^15*b^9*c^4*e - 256000*a^16*b^7* \\
& c^5*e + 778240*a^17*b^5*c^6*e - 1261568*a^18*b^3*c^7*e - 64*a^14*b^12*c^2*f \\
& + 1664*a^15*b^10*c^3*f - 17920*a^16*b^8*c^4*f + 102400*a^17*b^6*c^5*f - 32 \\
& 7680*a^18*b^4*c^6*f + 557056*a^19*b^2*c^7*f - 64*a^15*b^11*c^2*g + 1280*a^1 \\
& 6*b^9*c^3*g - 10240*a^17*b^7*c^4*g + 40960*a^18*b^5*c^5*g - 81920*a^19*b^3* \\
& c^6*g + 851968*a^19*b*c^8*e + 65536*a^20*b*c^7*g) + x*(204800*a^17*c^9*e^2 \\
& - 401408*a^16*c^10*d^2 - 73728*a^18*c^8*f^2 + 8192*a^19*c^7*g^2 + 400*a^9*b \\
& ^14*c^3*d^2 - 9440*a^10*b^12*c^4*d^2 + 92816*a^11*b^10*c^5*d^2 - 488096*a^1
\end{aligned}$$

$$\begin{aligned}
& 2*b^8*c^6*d^2 + 1458688*a^13*b^6*c^7*d^2 - 2401280*a^14*b^4*c^8*d^2 + 18718 \\
& 72*a^15*b^2*c^9*d^2 + 144*a^11*b^12*c^3*e^2 - 3264*a^12*b^10*c^4*e^2 + 3011 \\
& 2*a^13*b^8*c^5*e^2 - 143360*a^14*b^6*c^6*e^2 + 365568*a^15*b^4*c^7*e^2 - 45 \\
& 8752*a^16*b^2*c^8*e^2 + 16*a^13*b^10*c^3*f^2 - 416*a^14*b^8*c^4*f^2 + 4608* \\
& a^15*b^6*c^5*f^2 - 25600*a^16*b^4*c^6*f^2 + 69632*a^17*b^2*c^7*f^2 + 160*a^ \\
& 15*b^8*c^3*g^2 - 2048*a^16*b^6*c^4*g^2 + 9216*a^17*b^4*c^5*g^2 - 16384*a^18 \\
& *b^2*c^6*g^2 + 344064*a^17*c^9*d*f - 81920*a^18*c^8*e*g - 1236992*a^16*b*c^ \\
& 9*d*e + 40960*a^17*b*c^8*d*g + 237568*a^17*b*c^8*e*f + 40960*a^18*b*c^7*f*g \\
& - 480*a^10*b^13*c^3*d*e + 11104*a^11*b^11*c^4*d*e - 105824*a^12*b^9*c^5*d* \\
& e + 530432*a^13*b^7*c^6*d*e - 1469440*a^14*b^5*c^7*d*e + 2121728*a^15*b^3*c \\
& ^8*d*e + 160*a^11*b^12*c^3*d*f - 3968*a^12*b^10*c^4*d*f + 39488*a^13*b^8*c^ \\
& 5*d*f - 200704*a^14*b^6*c^6*d*f + 542720*a^15*b^4*c^7*d*f - 720896*a^16*b^2 \\
& *c^8*d*f + 160*a^12*b^11*c^3*d*g - 96*a^12*b^11*c^3*e*f - 2528*a^13*b^9*c^4 \\
& *d*g + 2336*a^13*b^9*c^4*e*f + 14336*a^14*b^7*c^5*d*g - 22528*a^14*b^7*c^5* \\
& e*f - 31744*a^15*b^5*c^6*d*g + 107520*a^15*b^5*c^6*e*f + 8192*a^16*b^3*c^7* \\
& d*g - 253952*a^16*b^3*c^7*e*f - 96*a^13*b^10*c^3*e*g + 1472*a^14*b^8*c^4*e* \\
& g - 7168*a^15*b^6*c^5*e*g + 6144*a^16*b^4*c^6*e*g + 40960*a^17*b^2*c^7*e*g \\
& + 32*a^14*b^9*c^3*f*g - 1024*a^15*b^7*c^4*f*g + 9216*a^16*b^5*c^5*f*g - 327 \\
& 68*a^17*b^3*c^6*f*g) * (- (25*b^15*d^2 + 9*a^2*b^13*e^2 + 25*b^6*d^2 * (- (4*a*c \\
& - b^2)^9)^(1/2) + a^4*b^11*f^2 + a^6*b^9*g^2 + a^6*g^2 * (- (4*a*c - b^2)^9)^(\\
& 1/2) - 80640*a^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27 \\
& *a^5*b^9*c*f^2 - 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2 * (- (4*a*c - b^2)^9)^(1/2) \\
& - 768*a^10*b*c^4*g^2 - 30*a*b^14*d*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^ \\
& 9*c^3*d^2 + 116928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^ \\
& 3*c^6*d^2 + 9*a^2*b^4*e^2 * (- (4*a*c - b^2)^9)^(1/2) - 49*a^3*c^3*d^2 * (- (4*a* \\
& c - b^2)^9)^(1/2) + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^ \\
& 6*b^5*c^4*e^2 - 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2 * (- (4*a*c - b^2)^9)^(1/2 \\
&) + 25*a^4*c^2*e^2 * (- (4*a*c - b^2)^9)^(1/2) + 288*a^6*b^7*c^2*f^2 - 1504*a^ \\
& 7*b^5*c^3*f^2 + 3840*a^8*b^3*c^4*f^2 - 96*a^8*b^5*c^2*g^2 + 512*a^9*b^3*c^3 \\
& *g^2 - 615*a*b^13*c*d^2 + 10*a^2*b^13*d*f + 35840*a^8*c^7*d*e + 10*a^3*b^12 \\
& *d*g - 6*a^3*b^12*e*f - 6*a^4*b^11*e*g - 7168*a^9*c^6*d*g - 15360*a^9*c^6*e \\
& *f + 2*a^5*b^10*f*g + 3072*a^10*c^5*f*g - 30*a*b^5*d*e * (- (4*a*c - b^2)^9)^(\\
& 1/2) + 724*a^2*b^12*c*d*e - 258*a^3*b^11*c*d*f + 43520*a^8*b*c^6*d*f - 168* \\
& a^4*b^10*c*d*g + 152*a^4*b^10*c*e*f + 98*a^5*b^9*c*e*g - 1536*a^9*b*c^5*e*g \\
& + 2*a^5*b*f*g * (- (4*a*c - b^2)^9)^(1/2) - 10*a^5*c*e*g * (- (4*a*c - b^2)^9)^(\\
& 1/2) - 36*a^6*b^8*c*f*g + 246*a^2*b^2*c^2*d^2 * (- (4*a*c - b^2)^9)^(1/2) - 16 \\
& 5*a*b^4*c*d^2 * (- (4*a*c - b^2)^9)^(1/2) - 7278*a^3*b^10*c^2*d*e + 39132*a^4* \\
& b^8*c^3*d*e - 119616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5*d*e - 161280*a^7* \\
& b^2*c^6*d*e + 10*a^2*b^4*d*f * (- (4*a*c - b^2)^9)^(1/2) + 2706*a^4*b^9*c^2*d* \\
& f - 14784*a^5*b^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f + \\
& 10*a^3*b^3*d*g * (- (4*a*c - b^2)^9)^(1/2) - 6*a^3*b^3*e*f * (- (4*a*c - b^2)^9) \\
& ^ (1/2) + 42*a^4*c^2*d*f * (- (4*a*c - b^2)^9)^(1/2) + 1044*a^5*b^8*c^2*d*g - 1 \\
& 548*a^5*b^8*c^2*e*f - 2688*a^6*b^6*c^3*d*g + 8064*a^6*b^6*c^3*e*f + 1152*a^ \\
& 7*b^4*c^4*d*g - 22400*a^7*b^4*c^4*e*f + 6144*a^8*b^2*c^5*d*g + 30720*a^8*b^ \\
& 2*c^5*e*f - 6*a^4*b^2*e*g * (- (4*a*c - b^2)^9)^(1/2) - 576*a^6*b^7*c^2*e*g + \\
& 1344*a^7*b^5*c^3*e*g - 512*a^8*b^3*c^4*e*g + 192*a^7*b^6*c^2*f*g - 128*a^8* \\
& b^4*c^3*f*g - 1536*a^9*b^2*c^4*f*g - 51*a^3*b^2*c*e^2 * (- (4*a*c - b^2)^9)^(1 \\
& /2) + 12*a^4*b*c*d*g * (- (4*a*c - b^2)^9)^(1/2) + 44*a^4*b*c*e*f * (- (4*a*c - b \\
& ^2)^9)^(1/2) + 184*a^2*b^3*c*d*e * (- (4*a*c - b^2)^9)^(1/2) - 186*a^3*b*c^2*d \\
& *e * (- (4*a*c - b^2)^9)^(1/2) - 78*a^3*b^2*c*d*f * (- (4*a*c - b^2)^9)^(1/2)) / (3 \\
& 2*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b \\
& ^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2) - 128000*a^15*c^9*e \\
& ^3 + 1024*a^18*c^6*g^3 + 476672*a^13*b*c^10*d^3 - 4608*a^16*b*c^7*f^3 - 250 \\
& 880*a^14*c^10*d^2*e + 50176*a^15*c^9*d^2*g - 46080*a^16*c^8*e*f^2 + 76800*a \\
& ^16*c^8*e^2*g - 15360*a^17*c^7*e*g^2 + 9216*a^17*c^7*f^2*g + 1800*a^9*b^9*c \\
& ^6*d^3 - 29080*a^10*b^7*c^7*d^3 + 176032*a^11*b^5*c^8*d^3 - 473216*a^12*b^3 \\
& *c^9*d^3 - 504*a^11*b^8*c^5*e^3 + 8112*a^12*b^6*c^6*e^3 - 48704*a^13*b^4*c^ \\
& 7*e^3 + 129280*a^14*b^2*c^8*e^3 + 40*a^13*b^7*c^4*f^3 - 608*a^14*b^5*c^5*f^ \\
& 3 + 2944*a^15*b^3*c^6*f^3 + 48*a^15*b^6*c^3*g^3 - 320*a^16*b^4*c^4*g^3 + 25
\end{aligned}$$

$$\begin{aligned}
& 6a^{17}b^2c^5g^3 + 215040a^{15}c^9d^2ef - 43008a^{16}c^8d^2fg + 442880a^{14}b^2c^9d^2ef - 433664a^{14}b^2c^9d^2ef + 109056a^{15}b^2c^8d^2fg^2 + 84480a^{15}b^2c^8e^2f + 43520a^{16}b^2c^7d^2fg^2 - 7680a^{17}b^2c^6fg^2 - 1400a^9b^{10}c^5d^2e + 21680a^{10}b^8c^6d^2e + 1680a^{10}b^9c^5d^2e - 121648a^{11}b^6c^7d^2e - 27176a^{11}b^7c^6d^2e + 275264a^{12}b^4c^8d^2e + 164448a^{12}b^5c^7d^2e - 121088a^{13}b^2c^9d^2e - 441216a^{13}b^3c^8d^2e + 1000a^9b^{11}c^4d^2f - 17800a^{10}b^9c^5d^2f + 124280a^{11}b^7c^6d^2f + 400a^{11}b^9c^4d^2f - 422944a^{12}b^5c^7d^2f - 6600a^{12}b^7c^5d^2f + 694912a^{13}b^3c^8d^2f + 40416a^{13}b^5c^6d^2f - 108928a^{14}b^3c^7d^2f - 600a^9b^{12}c^3d^2g + 10960a^{10}b^{10}c^4d^2g - 78904a^{11}b^8c^5d^2g + 360a^{11}b^9c^4e^2f + 278096a^{12}b^6c^6d^2g - 5736a^{12}b^7c^5e^2f - 240a^{12}b^8c^4e^2f + 120a^{12}b^9c^3d^2g - 472000a^{13}b^4c^7d^2g + 33888a^{13}b^5c^6e^2f + 3792a^{13}b^6c^5e^2f - 2216a^{13}b^7c^4d^2g + 284416a^{14}b^2c^8d^2g - 87936a^{14}b^3c^7e^2f - 21696a^{14}b^4c^6e^2f + 14688a^{14}b^5c^5d^2g + 52992a^{15}b^2c^7e^2f - 41856a^{15}b^3c^6d^2g - 216a^{11}b^{10}c^3e^2g + 3744a^{12}b^8c^4e^2g - 25200a^{13}b^6c^5e^2g - 72a^{13}b^8c^3e^2g + 81984a^{14}b^4c^6e^2g + 1296a^{14}b^6c^4e^2g - 128256a^{15}b^2c^7e^2g - 7872a^{15}b^4c^5e^2g + 19200a^{16}b^2c^6e^2g - 24a^{13}b^8c^3f^2g + 336a^{14}b^6c^4f^2g + 24a^{14}b^7c^3f^2g - 960a^{15}b^4c^5f^2g - 672a^{15}b^5c^4f^2g - 2304a^{16}b^2c^6f^2g + 4224a^{16}b^3c^5f^2g - 306176a^{15}b^2c^8d^2ef + 21504a^{16}b^2c^7d^2ef - 1200a^{10}b^{10}c^4d^2ef + 20240a^{11}b^8c^5d^2ef - 130656a^{12}b^6c^6d^2ef + 394368a^{13}b^4c^7d^2ef - 528896a^{14}b^2c^8d^2ef + 720a^{10}b^{11}c^3d^2ef - 12816a^{11}b^9c^4d^2ef + 89264a^{12}b^7c^5d^2ef - 302400a^{13}b^5c^6d^2ef + 493824a^{14}b^3c^7d^2ef - 240a^{11}b^{10}c^3d^2ef + 3872a^{12}b^8c^4d^2ef - 22368a^{13}b^6c^5d^2ef + 51840a^{14}b^4c^6d^2ef - 25088a^{15}b^2c^7d^2ef + 144a^{12}b^9c^3d^2ef - 2256a^{13}b^7c^4d^2ef + 12480a^{14}b^5c^5d^2ef - 28416a^{15}b^3c^6d^2ef) * (- (25b^{15}d^2 + 9a^2b^{13}e^2 + 25b^6d^2 * (- (4ac - b^2)^9)^{1/2} + a^4b^{11}f^2 + a^6b^9g^2 + a^6g^2 * (- (4ac - b^2)^9)^{1/2} - 80640a^7b^2c^7d^2 - 213a^3b^{11}c^2e^2 + 26880a^8b^2c^6e^2 - 27a^5b^9c^2f^2 - 3840a^9b^2c^5f^2 - 9a^5c^2f^2 * (- (4ac - b^2)^9)^{1/2} - 768a^{10}b^2c^4g^2 - 30a^2b^{14}d^2e + 6366a^2b^{11}c^2d^2 - 35767a^3b^9c^3d^2 + 116928a^4b^7c^4d^2 - 219744a^5b^5c^5d^2 + 215040a^6b^3c^6d^2 + 9a^2b^4e^2 * (- (4ac - b^2)^9)^{1/2} - 49a^3c^3d^2 * (- (4ac - b^2)^9)^{1/2} + 2077a^4b^9c^2e^2 - 10656a^5b^7c^3e^2 + 30240a^6b^5c^4e^2 - 44800a^7b^3c^5e^2 + a^4b^2f^2 * (- (4ac - b^2)^9)^{1/2} + 25a^4c^2e^2 * (- (4ac - b^2)^9)^{1/2} + 288a^6b^7c^2f^2 - 1504a^7b^5c^3f^2 + 3840a^8b^3c^4f^2 - 96a^8b^5c^2g^2 + 512a^9b^3c^3g^2 - 615a^2b^{13}c^2d^2 + 10a^2b^{13}d^2f + 35840a^8c^7d^2e + 10a^3b^{12}d^2g - 6a^3b^{12}e^2f - 6a^4b^{11}e^2g - 7168a^9c^6d^2g - 15360a^9c^6e^2f + 2a^5b^{10}f^2g + 3072a^{10}c^5f^2g - 30a^2b^5d^2e * (- (4ac - b^2)^9)^{1/2} + 724a^2b^{12}c^2d^2e - 258a^3b^{11}c^2d^2f + 43520a^8b^2c^6d^2f - 168a^4b^{10}c^2d^2g + 152a^4b^{10}c^2e^2f + 98a^5b^9c^2e^2g - 1536a^9b^2c^5e^2g + 2a^5b^2f^2g * (- (4ac - b^2)^9)^{1/2} - 10a^5c^2e^2g * (- (4ac - b^2)^9)^{1/2} - 36a^6b^8c^2f^2g + 246a^2b^2c^2d^2 * (- (4ac - b^2)^9)^{1/2} - 165a^2b^4c^2d^2 * (- (4ac - b^2)^9)^{1/2} - 7278a^3b^{10}c^2d^2e + 39132a^4b^8c^3d^2e - 119616a^5b^6c^4d^2e + 201600a^6b^4c^5d^2e - 161280a^7b^2c^6d^2e + 10a^2b^4d^2f * (- (4ac - b^2)^9)^{1/2} + 2706a^4b^9c^2d^2f - 14784a^5b^7c^3d^2f + 44352a^6b^5c^4d^2f - 69120a^7b^3c^5d^2f + 10a^3b^3d^2g * (- (4ac - b^2)^9)^{1/2} - 6a^3b^3e^2f * (- (4ac - b^2)^9)^{1/2} + 42a^4c^2d^2f * (- (4ac - b^2)^9)^{1/2} + 1044a^5b^8c^2d^2g - 1548a^5b^8c^2e^2f - 2688a^6b^6c^3d^2g + 8064a^6b^6c^3e^2f + 1152a^7b^4c^4d^2g - 22400a^7b^4c^4e^2f + 6144a^8b^2c^5d^2g + 30720a^8b^2c^5e^2f - 6a^4b^2e^2g * (- (4ac - b^2)^9)^{1/2} - 576a^6b^7c^2e^2g + 1344a^7b^5c^3e^2g - 512a^8b^3c^4e^2g + 192a^7b^6c^2f^2g - 128a^8b^4c^3f^2g - 1536a^9b^2c^4f^2g - 51a^3b^2c^2e^2 * (- (4ac - b^2)^9)^{1/2} + 12a^4b^2c^2d^2g * (- (4ac - b^2)^9)^{1/2} + 44a^4b^2c^2e^2f * (- (4ac - b^2)^9)^{1/2} + 184a^2b^3c^2d^2e * (- (4ac - b^2)^9)^{1/2}
\end{aligned}$$

$$\begin{aligned}
& - (4ac - b^2)^9)^{(1/2)} - 186a^3b^2c^2d^2e^2(-4ac - b^2)^9)^{(1/2)} - 78a^3b^2c^2d^2e^2(-4ac - b^2)^9)^{(1/2)} / (32(a^7b^{12} + 4096a^{13}c^6 - 24a^8b^{10}c + 240a^9b^8c^2 - 1280a^{10}b^6c^3 + 3840a^{11}b^4c^4 - 6144a^{12}b^2c^5))^{(1/2)} * 2i - (d/(3a) + (x^2(3ae - 5bd))/(3a^2) + (x^4(15b^4d + 14a^2c^2d + 3a^2b^2f - 9ab^3e - 3a^3b^3g - 6a^3c^3f - 62ab^2cd + 33a^2b^2ce)) / (6a^3(4ac - b^2)) + (cx^6(5b^3d - 2a^3g - 3ab^2e + a^2bf + 10a^2ce - 19abc^2d)) / (2a^3(4ac - b^2))) / (ax^3 + bx^5 + cx^7) + \operatorname{atan}(\frac{((25b^6d^2(-4ac - b^2)^9)^{(1/2)} - 9a^2b^{13}e^2 - 25b^{15}d^2 - a^4b^{11}f^2 - a^6b^9g^2 + a^6g^2(-4ac - b^2)^9)^{(1/2)} + 80640a^7b^7d^2 + 213a^3b^{11}c^2e^2 - 26880a^8b^6e^2 + 27a^5b^9c^2f^2 + 3840a^9b^5c^2f^2 - 9a^5c^2f^2(-4ac - b^2)^9)^{(1/2)} + 768a^{10}b^4g^2 + 30ab^{14}de - 6366a^2b^{11}c^2d^2 + 35767a^3b^9c^3d^2 - 116928a^4b^7c^4d^2 + 219744a^5b^5c^5d^2 - 215040a^6b^3c^6d^2 + 9a^2b^4e^2(-4ac - b^2)^9)^{(1/2)} - 49a^3c^3d^2(-4ac - b^2)^9)^{(1/2)} - 2077a^4b^9c^2e^2 + 10656a^5b^7c^3e^2 - 30240a^6b^5c^4e^2 + 44800a^7b^3c^5e^2 + a^4b^2f^2(-4ac - b^2)^9)^{(1/2)} + 25a^4c^2e^2(-4ac - b^2)^9)^{(1/2)} - 288a^6b^7c^2f^2 + 1504a^7b^5c^3f^2 - 3840a^8b^3c^4f^2 + 96a^8b^5c^2g^2 - 512a^9b^3c^3g^2 + 615ab^{13}cd^2 - 10a^2b^{13}df - 35840a^8c^7de - 10a^3b^{12}dg + 6a^3b^{12}ef + 6a^4b^{11}eg + 7168a^9c^6d^2g + 15360a^9c^6ef - 2a^5b^{10}fg - 3072a^{10}c^5fg - 30ab^5de(-4ac - b^2)^9)^{(1/2)} - 724a^2b^{12}cde + 258a^3b^{11}cdf - 43520a^8b^6cdf + 168a^4b^{10}cdg - 152a^4b^{10}cef - 98a^5b^9ceg + 1536a^9b^5ceg + 2a^5b^9ceg(-4ac - b^2)^9)^{(1/2)} - 10a^5ceg(-4ac - b^2)^9)^{(1/2)} + 36a^6b^8c^2fg + 246a^2b^2c^2d^2(-4ac - b^2)^9)^{(1/2)} - 165ab^4cd^2(-4ac - b^2)^9)^{(1/2)} + 7278a^3b^{10}c^2de - 39132a^4b^8c^3de + 119616a^5b^6c^4de - 201600a^6b^4c^5de + 161280a^7b^2c^6de + 10a^2b^4d^2e(-4ac - b^2)^9)^{(1/2)} - 2706a^4b^9c^2d^2e + 14784a^5b^7c^3d^2e - 44352a^6b^5c^4d^2e + 69120a^7b^3c^5d^2e + 10a^3b^3d^2eg(-4ac - b^2)^9)^{(1/2)} - 6a^3b^3e^2f(-4ac - b^2)^9)^{(1/2)} + 42a^4c^2d^2e(-4ac - b^2)^9)^{(1/2)} - 1044a^5b^8c^2d^2g + 1548a^5b^8c^2ef + 2688a^6b^6c^3d^2g - 8064a^6b^6c^3ef - 1152a^7b^4c^4d^2g + 22400a^7b^4c^4ef - 6144a^8b^2c^5d^2g - 30720a^8b^2c^5ef - 6a^4b^2e^2g(-4ac - b^2)^9)^{(1/2)} + 576a^6b^7c^2e^2g - 1344a^7b^5c^3e^2g + 512a^8b^3c^4e^2g - 192a^7b^6c^2f^2g + 128a^8b^4c^3f^2g + 1536a^9b^2c^4f^2g - 51a^3b^2c^2e^2(-4ac - b^2)^9)^{(1/2)} + 12a^4b^2cd^2g(-4ac - b^2)^9)^{(1/2)} + 44a^4b^2c^2ef(-4ac - b^2)^9)^{(1/2)} + 184a^2b^3c^2de(-4ac - b^2)^9)^{(1/2)} - 186a^3b^2c^2d^2e(-4ac - b^2)^9)^{(1/2)} - 78a^3b^2c^2d^2e(-4ac - b^2)^9)^{(1/2)} / (32(a^7b^{12} + 4096a^{13}c^6 - 24a^8b^{10}c + 240a^9b^8c^2 - 1280a^{10}b^6c^3 + 3840a^{11}b^4c^4 - 6144a^{12}b^2c^5))^{(1/2)} * (393216a^{20}c^8f - 917504a^{19}c^9d + x((25b^6d^2(-4ac - b^2)^9)^{(1/2)} - 9a^2b^{13}e^2 - 25b^{15}d^2 - a^4b^{11}f^2 - a^6b^9g^2 + a^6g^2(-4ac - b^2)^9)^{(1/2)} + 80640a^7b^7d^2 + 213a^3b^{11}c^2e^2 - 26880a^8b^6e^2 + 27a^5b^9c^2f^2 + 3840a^9b^5c^2f^2 - 9a^5c^2f^2(-4ac - b^2)^9)^{(1/2)} + 768a^{10}b^4g^2 + 30ab^{14}de - 6366a^2b^{11}c^2d^2 + 35767a^3b^9c^3d^2 - 116928a^4b^7c^4d^2 + 219744a^5b^5c^5d^2 - 215040a^6b^3c^6d^2 + 9a^2b^4e^2(-4ac - b^2)^9)^{(1/2)} - 49a^3c^3d^2(-4ac - b^2)^9)^{(1/2)} - 2077a^4b^9c^2e^2 + 10656a^5b^7c^3e^2 - 30240a^6b^5c^4e^2 + 44800a^7b^3c^5e^2 + a^4b^2f^2(-4ac - b^2)^9)^{(1/2)} + 25a^4c^2e^2(-4ac - b^2)^9)^{(1/2)} - 288a^6b^7c^2f^2 + 1504a^7b^5c^3f^2 - 3840a^8b^3c^4f^2 + 96a^8b^5c^2g^2 - 512a^9b^3c^3g^2 + 615ab^{13}cd^2 - 10a^2b^{13}df - 35840a^8c^7de - 10a^3b^{12}dg + 6a^3b^{12}ef + 6a^4b^{11}eg + 7168a^9c^6d^2g + 15360a^9c^6ef - 2a^5b^{10}fg - 3072a^{10}c^5fg - 30ab^5de(-4ac - b^2)^9)^{(1/2)} - 724a^2b^{12}cde + 258a^3b^{11}cdf - 43520a^8b^6cdf + 168a^4b^{10}cdg - 152a^4b^{10}cef - 98a^5b^9ceg + 1536a^9b^5ceg + 2a^5b^9ceg(-4ac - b^2)^9)^{(1/2)} - 10a^5ceg(-4ac - b^2)^9)^{(1/2)} + 36a^6b^8c^2fg + 246a^2b^2c^2d^2(-4ac - b^
\end{aligned}$$

$$\begin{aligned}
& 2)^9)^{(1/2)} - 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + 7278*a^3*b^{10}*c^2*d*e - 39132*a^4*b^8*c^3*d*e + 119616*a^5*b^6*c^4*d*e - 201600*a^6*b^4*c^5*d*e + 161280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^{(1/2)} - 2706*a^4*b^9*c^2*d*f + 14784*a^5*b^7*c^3*d*f - 44352*a^6*b^5*c^4*d*f + 69120*a^7*b^3*c^5*d*f + 10*a^3*b^3*d*g*(-(4*a*c - b^2)^9)^{(1/2)} - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^{(1/2)} + 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^{(1/2)} - 1044*a^5*b^8*c^2*d*g + 1548*a^5*b^8*c^2*e*f + 2688*a^6*b^6*c^3*d*g - 8064*a^6*b^6*c^3*e*f - 1152*a^7*b^4*c^4*d*g + 22400*a^7*b^4*c^4*e*f - 6144*a^8*b^2*c^5*d*g - 30720*a^8*b^2*c^5*e*f - 6*a^4*b^2*e*g*(-(4*a*c - b^2)^9)^{(1/2)} + 576*a^6*b^7*c^2*e*g - 1344*a^7*b^5*c^3*e*g + 512*a^8*b^3*c^4*e*g - 192*a^7*b^6*c^2*f*g + 128*a^8*b^4*c^3*f*g + 1536*a^9*b^2*c^4*f*g - 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 12*a^4*b*c*d*g*(-(4*a*c - b^2)^9)^{(1/2)} + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^{(1/2)} + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^{(1/2)))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5))^(1/2)*(1048576*a^21*b*c^8 + 256*a^15*b^13*c^2 - 6144*a^16*b^11*c^3 + 61440*a^17*b^9*c^4 - 327680*a^18*b^7*c^5 + 983040*a^19*b^5*c^6 - 1572864*a^20*b^3*c^7) + 320*a^12*b^14*c^2*d - 7936*a^13*b^12*c^3*d + 82816*a^14*b^10*c^4*d - 468480*a^15*b^8*c^5*d + 1536000*a^16*b^6*c^6*d - 2867200*a^17*b^4*c^7*d + 2719744*a^18*b^2*c^8*d - 192*a^13*b^13*c^2*e + 4672*a^14*b^11*c^3*e - 47360*a^15*b^9*c^4*e + 256000*a^16*b^7*c^5*e - 778240*a^17*b^5*c^6*e + 1261568*a^18*b^3*c^7*e + 64*a^14*b^12*c^2*f - 1664*a^15*b^10*c^3*f + 17920*a^16*b^8*c^4*f - 102400*a^17*b^6*c^5*f + 327680*a^18*b^4*c^6*f - 557056*a^19*b^2*c^7*f + 64*a^15*b^11*c^2*g - 1280*a^16*b^9*c^3*g + 10240*a^17*b^7*c^4*g - 40960*a^18*b^5*c^5*g + 81920*a^19*b^3*c^6*g - 851968*a^19*b*c^8*e - 65536*a^20*b*c^7*g) + x*(204800*a^17*c^9*e^2 - 401408*a^16*c^10*d^2 - 73728*a^18*c^8*f^2 + 8192*a^19*c^7*g^2 + 400*a^9*b^14*c^3*d^2 - 9440*a^10*b^12*c^4*d^2 + 92816*a^11*b^10*c^5*d^2 - 488096*a^12*b^8*c^6*d^2 + 1458688*a^13*b^6*c^7*d^2 - 2401280*a^14*b^4*c^8*d^2 + 1871872*a^15*b^2*c^9*d^2 + 144*a^11*b^12*c^3*e^2 - 3264*a^12*b^10*c^4*e^2 + 30112*a^13*b^8*c^5*e^2 - 143360*a^14*b^6*c^6*e^2 + 365568*a^15*b^4*c^7*e^2 - 458752*a^16*b^2*c^8*e^2 + 16*a^13*b^10*c^3*f^2 - 416*a^14*b^8*c^4*f^2 + 4608*a^15*b^6*c^5*f^2 - 25600*a^16*b^4*c^6*f^2 + 69632*a^17*b^2*c^7*f^2 + 160*a^15*b^8*c^3*g^2 - 2048*a^16*b^6*c^4*g^2 + 9216*a^17*b^4*c^5*g^2 - 16384*a^18*b^2*c^6*g^2 + 344064*a^17*c^9*d*f - 81920*a^18*c^8*e*g - 1236992*a^16*b*c^9*d*e + 40960*a^17*b*c^8*d*g + 237568*a^17*b*c^8*e*f + 40960*a^18*b*c^7*f*g - 480*a^10*b^13*c^3*d*e + 11104*a^11*b^11*c^4*d*e - 105824*a^12*b^9*c^5*d*e + 530432*a^13*b^7*c^6*d*e - 1469440*a^14*b^5*c^7*d*e + 2121728*a^15*b^3*c^8*d*e + 160*a^11*b^12*c^3*d*f - 3968*a^12*b^10*c^4*d*f + 39488*a^13*b^8*c^5*d*f - 200704*a^14*b^6*c^6*d*f + 542720*a^15*b^4*c^7*d*f - 720896*a^16*b^2*c^8*d*f + 160*a^12*b^11*c^3*d*g - 96*a^12*b^11*c^3*e*f - 2528*a^13*b^9*c^4*d*g + 2336*a^13*b^9*c^4*e*f + 14336*a^14*b^7*c^5*d*g - 22528*a^14*b^7*c^5*e*f - 31744*a^15*b^5*c^6*d*g + 107520*a^15*b^5*c^6*e*f + 8192*a^16*b^3*c^7*d*g - 253952*a^16*b^3*c^7*e*f - 96*a^13*b^10*c^3*e*g + 1472*a^14*b^8*c^4*e*g - 7168*a^15*b^6*c^5*e*g + 6144*a^16*b^4*c^6*e*g + 40960*a^17*b^2*c^7*e*g + 32*a^14*b^9*c^3*f*g - 1024*a^15*b^7*c^4*f*g + 9216*a^16*b^5*c^5*f*g - 32768*a^17*b^3*c^6*f*g))*((25*b^6*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - 9*a^2*b^13*e^2 - 25*b^15*d^2 - a^4*b^11*f^2 - a^6*b^9*g^2 + a^6*g^2*(-(4*a*c - b^2)^9)^{(1/2)} + 80640*a^7*b*c^7*d^2 + 213*a^3*b^11*c*e^2 - 26880*a^8*b*c^6*e^2 + 27*a^5*b^9*c*f^2 + 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + 768*a^10*b*c^4*g^2 + 30*a*b^14*d*e - 6366*a^2*b^11*c^2*d^2 + 35767*a^3*b^9*c^3*d^2 - 116928*a^4*b^7*c^4*d^2 + 219744*a^5*b^5*c^5*d^2 - 215040*a^6*b^3*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - 2077*a^4*b^9*c^2*e^2 + 10656*a^5*b^7*c^3*e^2 - 30240*a^6*b^5*c^4*e^2 + 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 288*a^6*b^7*c^2*f^2 + 1504*a^7*b^5*c^3*f^2 - 3840*a^8*b^3*c^4*f^2 + 96*a^8*b^5*c^2*g^2 - 512*a^9*b^3*c^3*g^2 + 615*a*b^13*c*d^2 - 10*a^2*b^13*d*f - 35840*a^8*c^7*d*e - 10*a^3*b^12*d*g + 6*a^3*b^12*e*f + 6*a^4*b^11*e*g + 7168*a^9
\end{aligned}$$

$$\begin{aligned}
& c^6*d*g + 15360*a^9*c^6*e*f - 2*a^5*b^{10}*f*g - 3072*a^{10}*c^5*f*g - 30*a*b^5 \\
& *d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 724*a^2*b^{12}*c*d*e + 258*a^3*b^{11}*c*d*f - 4 \\
& 3520*a^8*b*c^6*d*f + 168*a^4*b^{10}*c*d*g - 152*a^4*b^{10}*c*e*f - 98*a^5*b^9*c \\
& *e*g + 1536*a^9*b*c^5*e*g + 2*a^5*b*f*g*(-(4*a*c - b^2)^9)^{(1/2)} - 10*a^5*c \\
& *e*g*(-(4*a*c - b^2)^9)^{(1/2)} + 36*a^6*b^8*c*f*g + 246*a^2*b^2*c^2*d^2*(-(4 \\
& *a*c - b^2)^9)^{(1/2)} - 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + 7278*a^3* \\
& b^{10}*c^2*d*e - 39132*a^4*b^8*c^3*d*e + 119616*a^5*b^6*c^4*d*e - 201600*a^6* \\
& b^4*c^5*d*e + 161280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^{(1 \\
& /2)} - 2706*a^4*b^9*c^2*d*f + 14784*a^5*b^7*c^3*d*f - 44352*a^6*b^5*c^4*d*f \\
& + 69120*a^7*b^3*c^5*d*f + 10*a^3*b^3*d*g*(-(4*a*c - b^2)^9)^{(1/2)} - 6*a^3*b \\
& ^3*e*f*(-(4*a*c - b^2)^9)^{(1/2)} + 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^{(1/2)} - \\
& 1044*a^5*b^8*c^2*d*g + 1548*a^5*b^8*c^2*e*f + 2688*a^6*b^6*c^3*d*g - 8064* \\
& a^6*b^6*c^3*e*f - 1152*a^7*b^4*c^4*d*g + 22400*a^7*b^4*c^4*e*f - 6144*a^8*b \\
& ^2*c^5*d*g - 30720*a^8*b^2*c^5*e*f - 6*a^4*b^2*e*g*(-(4*a*c - b^2)^9)^{(1/2)} \\
& + 576*a^6*b^7*c^2*e*g - 1344*a^7*b^5*c^3*e*g + 512*a^8*b^3*c^4*e*g - 192*a \\
& ^7*b^6*c^2*f*g + 128*a^8*b^4*c^3*f*g + 1536*a^9*b^2*c^4*f*g - 51*a^3*b^2*c* \\
& e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 12*a^4*b*c*d*g*(-(4*a*c - b^2)^9)^{(1/2)} + 44 \\
& *a^4*b*c*e*f*(-(4*a*c - b^2)^9)^{(1/2)} + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9 \\
&)^{(1/2)} - 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 78*a^3*b^2*c*d*f*(-(\\
& 4*a*c - b^2)^9)^{(1/2)}/(32*(a^7*b^{12} + 4096*a^{13}*c^6 - 24*a^8*b^{10}*c + 240* \\
& a^9*b^8*c^2 - 1280*a^{10}*b^6*c^3 + 3840*a^{11}*b^4*c^4 - 6144*a^{12}*b^2*c^5))^{(\\
& 1/2)}*1i + (((25*b^6*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - 9*a^2*b^{13}*e^2 - 25*b^1 \\
& 5*d^2 - a^4*b^{11}*f^2 - a^6*b^9*g^2 + a^6*g^2*(-(4*a*c - b^2)^9)^{(1/2)} + 806 \\
& 40*a^7*b*c^7*d^2 + 213*a^3*b^{11}*c*e^2 - 26880*a^8*b*c^6*e^2 + 27*a^5*b^9*c* \\
& f^2 + 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + 768*a^{10} \\
& b*c^4*g^2 + 30*a*b^{14}*d*e - 6366*a^2*b^{11}*c^2*d^2 + 35767*a^3*b^9*c^3*d^2 - \\
& 116928*a^4*b^7*c^4*d^2 + 219744*a^5*b^5*c^5*d^2 - 215040*a^6*b^3*c^6*d^2 + \\
& 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9) \\
& ^{(1/2)} - 2077*a^4*b^9*c^2*e^2 + 10656*a^5*b^7*c^3*e^2 - 30240*a^6*b^5*c^4*e \\
& ^2 + 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + 25*a^4* \\
& c^2*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 288*a^6*b^7*c^2*f^2 + 1504*a^7*b^5*c^3*f \\
& ^2 - 3840*a^8*b^3*c^4*f^2 + 96*a^8*b^5*c^2*g^2 - 512*a^9*b^3*c^3*g^2 + 615* \\
& a*b^{13}*c*d^2 - 10*a^2*b^{13}*d*f - 35840*a^8*c^7*d*e - 10*a^3*b^{12}*d*g + 6*a^ \\
& 3*b^{12}*e*f + 6*a^4*b^{11}*e*g + 7168*a^9*c^6*d*g + 15360*a^9*c^6*e*f - 2*a^5* \\
& b^{10}*f*g - 3072*a^{10}*c^5*f*g - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 724* \\
& a^2*b^{12}*c*d*e + 258*a^3*b^{11}*c*d*f - 43520*a^8*b*c^6*d*f + 168*a^4*b^{10}*c* \\
& d*g - 152*a^4*b^{10}*c*e*f - 98*a^5*b^9*c*e*g + 1536*a^9*b*c^5*e*g + 2*a^5*b* \\
& f*g*(-(4*a*c - b^2)^9)^{(1/2)} - 10*a^5*c*e*g*(-(4*a*c - b^2)^9)^{(1/2)} + 36*a \\
& ^6*b^8*c*f*g + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - 165*a*b^4*c*d \\
& ^2*(-(4*a*c - b^2)^9)^{(1/2)} + 7278*a^3*b^{10}*c^2*d*e - 39132*a^4*b^8*c^3*d*e \\
& + 119616*a^5*b^6*c^4*d*e - 201600*a^6*b^4*c^5*d*e + 161280*a^7*b^2*c^6*d*e \\
& + 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^{(1/2)} - 2706*a^4*b^9*c^2*d*f + 14784*a \\
& ^5*b^7*c^3*d*f - 44352*a^6*b^5*c^4*d*f + 69120*a^7*b^3*c^5*d*f + 10*a^3*b^3 \\
& *d*g*(-(4*a*c - b^2)^9)^{(1/2)} - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^{(1/2)} + 42 \\
& *a^4*c^2*d*f*(-(4*a*c - b^2)^9)^{(1/2)} - 1044*a^5*b^8*c^2*d*g + 1548*a^5*b^8 \\
& *c^2*e*f + 2688*a^6*b^6*c^3*d*g - 8064*a^6*b^6*c^3*e*f - 1152*a^7*b^4*c^4*d \\
& *g + 22400*a^7*b^4*c^4*e*f - 6144*a^8*b^2*c^5*d*g - 30720*a^8*b^2*c^5*e*f - \\
& 6*a^4*b^2*e*g*(-(4*a*c - b^2)^9)^{(1/2)} + 576*a^6*b^7*c^2*e*g - 1344*a^7*b^ \\
& 5*c^3*e*g + 512*a^8*b^3*c^4*e*g - 192*a^7*b^6*c^2*f*g + 128*a^8*b^4*c^3*f*g \\
& + 1536*a^9*b^2*c^4*f*g - 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 12*a^ \\
& 4*b*c*d*g*(-(4*a*c - b^2)^9)^{(1/2)} + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^{(1/2 \\
&)} + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 186*a^3*b*c^2*d*e*(-(4*a*c \\
& - b^2)^9)^{(1/2)} - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^{(1/2)}/(32*(a^7*b^{12} \\
& + 4096*a^{13}*c^6 - 24*a^8*b^{10}*c + 240*a^9*b^8*c^2 - 1280*a^{10}*b^6*c^3 + 38 \\
& 40*a^{11}*b^4*c^4 - 6144*a^{12}*b^2*c^5))^{(1/2)}*(917504*a^{19}*c^9*d - 393216*a^ \\
& 20*c^8*f + x*((25*b^6*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - 9*a^2*b^{13}*e^2 - 25*b^ \\
& 15*d^2 - a^4*b^{11}*f^2 - a^6*b^9*g^2 + a^6*g^2*(-(4*a*c - b^2)^9)^{(1/2)} + 80 \\
& 640*a^7*b*c^7*d^2 + 213*a^3*b^{11}*c*e^2 - 26880*a^8*b*c^6*e^2 + 27*a^5*b^9*c \\
& *f^2 + 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + 768*a^{10}
\end{aligned}$$

$$\begin{aligned}
& *b^4c^4g^2 + 30a^2b^{14}d^2e - 6366a^2b^{11}c^2d^2 + 35767a^3b^9c^3d^2 \\
& - 116928a^4b^7c^4d^2 + 219744a^5b^5c^5d^2 - 215040a^6b^3c^6d^2 \\
& + 9a^2b^4e^2(-4a^2c - b^2)^9)^{(1/2)} - 49a^3c^3d^2(-4a^2c - b^2)^9 \\
&)^{(1/2)} - 2077a^4b^9c^2e^2 + 10656a^5b^7c^3e^2 - 30240a^6b^5c^4e \\
& e^2 + 44800a^7b^3c^5e^2 + a^4b^2f^2(-4a^2c - b^2)^9)^{(1/2)} + 25a^4 \\
& *c^2e^2(-4a^2c - b^2)^9)^{(1/2)} - 288a^6b^7c^2f^2 + 1504a^7b^5c^3 \\
& f^2 - 3840a^8b^3c^4f^2 + 96a^8b^5c^2g^2 - 512a^9b^3c^3g^2 + 615 \\
& *a^2b^{13}c^4d^2 - 10a^2b^{13}d^2f - 35840a^8c^7d^2e - 10a^3b^{12}d^2g + 6a \\
& ^3b^{12}e^2f + 6a^4b^{11}e^2g + 7168a^9c^6d^2g + 15360a^9c^6e^2f - 2a^5 \\
& *b^{10}f^2g - 3072a^{10}c^5f^2g - 30a^2b^5d^2e(-4a^2c - b^2)^9)^{(1/2)} - 724 \\
& *a^2b^{12}c^4d^2e + 258a^3b^{11}c^4d^2f - 43520a^8b^6c^6d^2f + 168a^4b^{10}c \\
& *d^2g - 152a^4b^{10}c^2e^2f - 98a^5b^9c^2e^2g + 1536a^9b^3c^5e^2g + 2a^5b \\
& *f^2g(-4a^2c - b^2)^9)^{(1/2)} - 10a^5c^2e^2g(-4a^2c - b^2)^9)^{(1/2)} + 36 \\
& a^6b^8c^2f^2g + 246a^2b^2c^2d^2(-4a^2c - b^2)^9)^{(1/2)} - 165a^2b^4c^2 \\
& d^2(-4a^2c - b^2)^9)^{(1/2)} + 7278a^3b^{10}c^2d^2e - 39132a^4b^8c^3d^2e \\
& e + 119616a^5b^6c^4d^2e - 201600a^6b^4c^5d^2e + 161280a^7b^2c^6d^2e \\
& e + 10a^2b^4d^2f(-4a^2c - b^2)^9)^{(1/2)} - 2706a^4b^9c^2d^2f + 14784a \\
& ^5b^7c^3d^2f - 44352a^6b^5c^4d^2f + 69120a^7b^3c^5d^2f + 10a^3b^3 \\
& *d^2g(-4a^2c - b^2)^9)^{(1/2)} - 6a^3b^3e^2f(-4a^2c - b^2)^9)^{(1/2)} + 4 \\
& 2a^4c^2d^2f(-4a^2c - b^2)^9)^{(1/2)} - 1044a^5b^8c^2d^2g + 1548a^5b^8 \\
& *c^2e^2f + 2688a^6b^6c^3d^2g - 8064a^6b^6c^3e^2f - 1152a^7b^4c^4 \\
& d^2g + 22400a^7b^4c^4e^2f - 6144a^8b^2c^5d^2g - 30720a^8b^2c^5e^2f \\
& - 6a^4b^2e^2g(-4a^2c - b^2)^9)^{(1/2)} + 576a^6b^7c^2e^2g - 1344a^7b^5 \\
& *c^3e^2g + 512a^8b^3c^4e^2g - 192a^7b^6c^2f^2g + 128a^8b^4c^3f^2 \\
& g + 1536a^9b^2c^4f^2g - 51a^3b^2c^2e^2(-4a^2c - b^2)^9)^{(1/2)} + 12a \\
& ^4b^2c^2d^2g(-4a^2c - b^2)^9)^{(1/2)} + 44a^4b^2c^2e^2f(-4a^2c - b^2)^9)^{(1/ \\
& 2)} + 184a^2b^3c^2d^2e(-4a^2c - b^2)^9)^{(1/2)} - 186a^3b^3c^2d^2e(-4a^2 \\
& c - b^2)^9)^{(1/2)} - 78a^3b^2c^2d^2f(-4a^2c - b^2)^9)^{(1/2)))/(32(a^7b^1 \\
& 2 + 4096a^{13}c^6 - 24a^8b^{10}c + 240a^9b^8c^2 - 1280a^{10}b^6c^3 + 3 \\
& 840a^{11}b^4c^4 - 6144a^{12}b^2c^5)))^{(1/2)}(1048576a^{21}b^8c^8 + 256a^{1 \\
& 5}b^{13}c^2 - 6144a^{16}b^{11}c^3 + 61440a^{17}b^9c^4 - 327680a^{18}b^7c^5 \\
& + 983040a^{19}b^5c^6 - 1572864a^{20}b^3c^7) - 320a^{12}b^{14}c^2d + 7936a \\
& ^{13}b^{12}c^3d - 82816a^{14}b^{10}c^4d + 468480a^{15}b^8c^5d - 1536000a \\
& ^{16}b^6c^6d + 2867200a^{17}b^4c^7d - 2719744a^{18}b^2c^8d + 192a^{13} \\
& b^{13}c^2e - 4672a^{14}b^{11}c^3e + 47360a^{15}b^9c^4e - 256000a^{16}b^7 \\
& c^5e + 778240a^{17}b^5c^6e - 1261568a^{18}b^3c^7e - 64a^{14}b^{12}c^2f \\
& + 1664a^{15}b^{10}c^3f - 17920a^{16}b^8c^4f + 102400a^{17}b^6c^5f - 32 \\
& 7680a^{18}b^4c^6f + 557056a^{19}b^2c^7f - 64a^{15}b^{11}c^2g + 1280a^{1 \\
& 6}b^9c^3g - 10240a^{17}b^7c^4g + 40960a^{18}b^5c^5g - 81920a^{19}b^3 \\
& c^6g + 851968a^{19}b^3c^8e + 65536a^{20}b^3c^7g) + x(204800a^{17}c^9e^2 \\
& - 401408a^{16}c^{10}d^2 - 73728a^{18}c^8f^2 + 8192a^{19}c^7g^2 + 400a^9b \\
& ^{14}c^3d^2 - 9440a^{10}b^{12}c^4d^2 + 92816a^{11}b^{10}c^5d^2 - 488096a^{1 \\
& 2}b^8c^6d^2 + 1458688a^{13}b^6c^7d^2 - 2401280a^{14}b^4c^8d^2 + 18718 \\
& 72a^{15}b^2c^9d^2 + 144a^{11}b^{12}c^3e^2 - 3264a^{12}b^{10}c^4e^2 + 3011 \\
& 2a^{13}b^8c^5e^2 - 143360a^{14}b^6c^6e^2 + 365568a^{15}b^4c^7e^2 - 45 \\
& 8752a^{16}b^2c^8e^2 + 16a^{13}b^{10}c^3f^2 - 416a^{14}b^8c^4f^2 + 4608a \\
& ^{15}b^6c^5f^2 - 25600a^{16}b^4c^6f^2 + 69632a^{17}b^2c^7f^2 + 160a^{1 \\
& 5}b^8c^3g^2 - 2048a^{16}b^6c^4g^2 + 9216a^{17}b^4c^5g^2 - 16384a^{18} \\
& *b^2c^6g^2 + 344064a^{17}c^9d^2f - 81920a^{18}c^8e^2g - 1236992a^{16}b^3c^ \\
& 9d^2e + 40960a^{17}b^3c^8d^2g + 237568a^{17}b^3c^8e^2f + 40960a^{18}b^3c^7 \\
& f^2g - 480a^{10}b^{13}c^3d^2e + 11104a^{11}b^{11}c^4d^2e - 105824a^{12}b^9c^5d^2 \\
& e + 530432a^{13}b^7c^6d^2e - 1469440a^{14}b^5c^7d^2e + 2121728a^{15}b^3c^ \\
& 8d^2e + 160a^{11}b^{12}c^3d^2f - 3968a^{12}b^{10}c^4d^2f + 39488a^{13}b^8c^ \\
& 5d^2f - 200704a^{14}b^6c^6d^2f + 542720a^{15}b^4c^7d^2f - 720896a^{16}b^2 \\
& *c^8d^2f + 160a^{12}b^{11}c^3d^2g - 96a^{12}b^{11}c^3e^2f - 2528a^{13}b^9c^4 \\
& *d^2g + 2336a^{13}b^9c^4e^2f + 14336a^{14}b^7c^5d^2g - 22528a^{14}b^7c^5 \\
& e^2f - 31744a^{15}b^5c^6d^2g + 107520a^{15}b^5c^6e^2f + 8192a^{16}b^3c^7 \\
& *d^2g - 253952a^{16}b^3c^7e^2f - 96a^{13}b^{10}c^3e^2g + 1472a^{14}b^8c^4e^2 \\
& g - 7168a^{15}b^6c^5e^2g + 6144a^{16}b^4c^6e^2g + 40960a^{17}b^2c^7e^2g
\end{aligned}$$

$$\begin{aligned}
& + 32*a^{14}*b^9*c^3*f*g - 1024*a^{15}*b^7*c^4*f*g + 9216*a^{16}*b^5*c^5*f*g - 327 \\
& 68*a^{17}*b^3*c^6*f*g) * ((25*b^6*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - 9*a^2*b^{13}*e^2 \\
& - 25*b^{15}*d^2 - a^4*b^{11}*f^2 - a^6*b^9*g^2 + a^6*g^2*(-(4*a*c - b^2)^9)^{(1/2)} \\
& + 80640*a^7*b*c^7*d^2 + 213*a^3*b^{11}*c*e^2 - 26880*a^8*b*c^6*e^2 + 27* \\
& a^5*b^9*c*f^2 + 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + \\
& 768*a^{10}*b*c^4*g^2 + 30*a*b^{14}*d*e - 6366*a^2*b^{11}*c^2*d^2 + 35767*a^3*b^9 \\
& *c^3*d^2 - 116928*a^4*b^7*c^4*d^2 + 219744*a^5*b^5*c^5*d^2 - 215040*a^6*b^3 \\
& *c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 49*a^3*c^3*d^2*(-(4*a*c \\
& - b^2)^9)^{(1/2)} - 2077*a^4*b^9*c^2*e^2 + 10656*a^5*b^7*c^3*e^2 - 30240*a^6 \\
& *b^5*c^4*e^2 + 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} \\
& + 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 288*a^6*b^7*c^2*f^2 + 1504*a^7 \\
& *b^5*c^3*f^2 - 3840*a^8*b^3*c^4*f^2 + 96*a^8*b^5*c^2*g^2 - 512*a^9*b^3*c^3* \\
& g^2 + 615*a*b^{13}*c*d^2 - 10*a^2*b^{13}*d*f - 35840*a^8*c^7*d*e - 10*a^3*b^{12}* \\
& d*g + 6*a^3*b^{12}*e*f + 6*a^4*b^{11}*e*g + 7168*a^9*c^6*d*g + 15360*a^9*c^6*e* \\
& f - 2*a^5*b^{10}*f*g - 3072*a^{10}*c^5*f*g - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^{(1 \\
& /2)} - 724*a^2*b^{12}*c*d*e + 258*a^3*b^{11}*c*d*f - 43520*a^8*b*c^6*d*f + 168*a \\
& ^4*b^{10}*c*d*g - 152*a^4*b^{10}*c*e*f - 98*a^5*b^9*c*e*g + 1536*a^9*b*c^5*e*g \\
& + 2*a^5*b*f*g*(-(4*a*c - b^2)^9)^{(1/2)} - 10*a^5*c*e*g*(-(4*a*c - b^2)^9)^{(1 \\
& /2)} + 36*a^6*b^8*c*f*g + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - 165 \\
& *a*b^4*c*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + 7278*a^3*b^{10}*c^2*d*e - 39132*a^4*b \\
& ^8*c^3*d*e + 119616*a^5*b^6*c^4*d*e - 201600*a^6*b^4*c^5*d*e + 161280*a^7*b \\
& ^2*c^6*d*e + 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^{(1/2)} - 2706*a^4*b^9*c^2*d*f \\
& + 14784*a^5*b^7*c^3*d*f - 44352*a^6*b^5*c^4*d*f + 69120*a^7*b^3*c^5*d*f + \\
& 10*a^3*b^3*d*g*(-(4*a*c - b^2)^9)^{(1/2)} - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^{(1 \\
& /2)} + 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^{(1/2)} - 1044*a^5*b^8*c^2*d*g + 15 \\
& 48*a^5*b^8*c^2*e*f + 2688*a^6*b^6*c^3*d*g - 8064*a^6*b^6*c^3*e*f - 1152*a^7 \\
& *b^4*c^4*d*g + 22400*a^7*b^4*c^4*e*f - 6144*a^8*b^2*c^5*d*g - 30720*a^8*b^2 \\
& *c^5*e*f - 6*a^4*b^2*e*g*(-(4*a*c - b^2)^9)^{(1/2)} + 576*a^6*b^7*c^2*e*g - 1 \\
& 344*a^7*b^5*c^3*e*g + 512*a^8*b^3*c^4*e*g - 192*a^7*b^6*c^2*f*g + 128*a^8*b \\
& ^4*c^3*f*g + 1536*a^9*b^2*c^4*f*g - 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^{(1/ \\
& 2)} + 12*a^4*b*c*d*g*(-(4*a*c - b^2)^9)^{(1/2)} + 44*a^4*b*c*e*f*(-(4*a*c - b^ \\
& 2)^9)^{(1/2)} + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 186*a^3*b*c^2*d* \\
& e*(-(4*a*c - b^2)^9)^{(1/2)} - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^{(1/2)}) / (32 \\
& *(a^7*b^{12} + 4096*a^{13}*c^6 - 24*a^8*b^{10}*c + 240*a^9*b^8*c^2 - 1280*a^{10}*b^ \\
& 6*c^3 + 3840*a^{11}*b^4*c^4 - 6144*a^{12}*b^2*c^5))^{(1/2)} * i) / (((((25*b^6*d^2*(\\
& -(4*a*c - b^2)^9)^{(1/2)} - 9*a^2*b^{13}*e^2 - 25*b^{15}*d^2 - a^4*b^{11}*f^2 - a^6 \\
& *b^9*g^2 + a^6*g^2*(-(4*a*c - b^2)^9)^{(1/2)} + 80640*a^7*b*c^7*d^2 + 213*a^3 \\
& *b^{11}*c*e^2 - 26880*a^8*b*c^6*e^2 + 27*a^5*b^9*c*f^2 + 3840*a^9*b*c^5*f^2 - \\
& 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + 768*a^{10}*b*c^4*g^2 + 30*a*b^{14}*d*e \\
& - 6366*a^2*b^{11}*c^2*d^2 + 35767*a^3*b^9*c^3*d^2 - 116928*a^4*b^7*c^4*d^2 + \\
& 219744*a^5*b^5*c^5*d^2 - 215040*a^6*b^3*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c - \\
& b^2)^9)^{(1/2)} - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - 2077*a^4*b^9*c^2* \\
& e^2 + 10656*a^5*b^7*c^3*e^2 - 30240*a^6*b^5*c^4*e^2 + 44800*a^7*b^3*c^5*e^2 \\
& + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9) \\
& ^{(1/2)} - 288*a^6*b^7*c^2*f^2 + 1504*a^7*b^5*c^3*f^2 - 3840*a^8*b^3*c^4*f^2 \\
& + 96*a^8*b^5*c^2*g^2 - 512*a^9*b^3*c^3*g^2 + 615*a*b^{13}*c*d^2 - 10*a^2*b^{13} \\
& *d*f - 35840*a^8*c^7*d*e - 10*a^3*b^{12}*d*g + 6*a^3*b^{12}*e*f + 6*a^4*b^{11}*e* \\
& g + 7168*a^9*c^6*d*g + 15360*a^9*c^6*e*f - 2*a^5*b^{10}*f*g - 3072*a^{10}*c^5*f \\
& *g - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 724*a^2*b^{12}*c*d*e + 258*a^3*b \\
& ^{11}*c*d*f - 43520*a^8*b*c^6*d*f + 168*a^4*b^{10}*c*d*g - 152*a^4*b^{10}*c*e*f - \\
& 98*a^5*b^9*c*e*g + 1536*a^9*b*c^5*e*g + 2*a^5*b*f*g*(-(4*a*c - b^2)^9)^{(1/ \\
& 2)} - 10*a^5*c*e*g*(-(4*a*c - b^2)^9)^{(1/2)} + 36*a^6*b^8*c*f*g + 246*a^2*b^2 \\
& *c^2*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^{(1/2) \\
&) + 7278*a^3*b^{10}*c^2*d*e - 39132*a^4*b^8*c^3*d*e + 119616*a^5*b^6*c^4*d*e \\
& - 201600*a^6*b^4*c^5*d*e + 161280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*f*(-(4*a*c \\
& - b^2)^9)^{(1/2)} - 2706*a^4*b^9*c^2*d*f + 14784*a^5*b^7*c^3*d*f - 44352*a^6 \\
& *b^5*c^4*d*f + 69120*a^7*b^3*c^5*d*f + 10*a^3*b^3*d*g*(-(4*a*c - b^2)^9)^{(1 \\
& /2)} - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^{(1/2)} + 42*a^4*c^2*d*f*(-(4*a*c - b^ \\
& 2)^9)^{(1/2)} - 1044*a^5*b^8*c^2*d*g + 1548*a^5*b^8*c^2*e*f + 2688*a^6*b^6*c^
\end{aligned}$$

$$\begin{aligned}
& 3*d*g - 8064*a^6*b^6*c^3*e*f - 1152*a^7*b^4*c^4*d*g + 22400*a^7*b^4*c^4*e*f \\
& - 6144*a^8*b^2*c^5*d*g - 30720*a^8*b^2*c^5*e*f - 6*a^4*b^2*e*g*(-(4*a*c - \\
& b^2)^9)^{(1/2)} + 576*a^6*b^7*c^2*e*g - 1344*a^7*b^5*c^3*e*g + 512*a^8*b^3*c^4 \\
& *e*g - 192*a^7*b^6*c^2*f*g + 128*a^8*b^4*c^3*f*g + 1536*a^9*b^2*c^4*f*g - \\
& 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 12*a^4*b*c*d*g*(-(4*a*c - b^2)^9)^{(1/2)} \\
& + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^{(1/2)} + 184*a^2*b^3*c*d*e*(-(4 \\
& *a*c - b^2)^9)^{(1/2)} - 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 78*a^3* \\
& b^2*c*d*f*(-(4*a*c - b^2)^9)^{(1/2)}/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8* \\
& b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^1 \\
& 2*b^2*c^5)))^{(1/2)}*(393216*a^20*c^8*f - 917504*a^19*c^9*d + x*((25*b^6*d^2* \\
& -(4*a*c - b^2)^9)^{(1/2)} - 9*a^2*b^13*e^2 - 25*b^15*d^2 - a^4*b^11*f^2 - a^ \\
& 6*b^9*g^2 + a^6*g^2*(-(4*a*c - b^2)^9)^{(1/2)} + 80640*a^7*b*c^7*d^2 + 213*a^ \\
& 3*b^11*c*e^2 - 26880*a^8*b*c^6*e^2 + 27*a^5*b^9*c*f^2 + 3840*a^9*b*c^5*f^2 \\
& - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + 768*a^10*b*c^4*g^2 + 30*a*b^14*d*e \\
& - 6366*a^2*b^11*c^2*d^2 + 35767*a^3*b^9*c^3*d^2 - 116928*a^4*b^7*c^4*d^2 + \\
& 219744*a^5*b^5*c^5*d^2 - 215040*a^6*b^3*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c - \\
& b^2)^9)^{(1/2)} - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - 2077*a^4*b^9*c^2 \\
& *e^2 + 10656*a^5*b^7*c^3*e^2 - 30240*a^6*b^5*c^4*e^2 + 44800*a^7*b^3*c^5*e^ \\
& 2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9 \\
&)^{(1/2)} - 288*a^6*b^7*c^2*f^2 + 1504*a^7*b^5*c^3*f^2 - 3840*a^8*b^3*c^4*f^2 \\
& + 96*a^8*b^5*c^2*g^2 - 512*a^9*b^3*c^3*g^2 + 615*a*b^13*c*d^2 - 10*a^2*b^1 \\
& 3*d*f - 35840*a^8*c^7*d*e - 10*a^3*b^12*d*g + 6*a^3*b^12*e*f + 6*a^4*b^11*e \\
& *g + 7168*a^9*c^6*d*g + 15360*a^9*c^6*e*f - 2*a^5*b^10*f*g - 3072*a^10*c^5* \\
& f*g - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 724*a^2*b^12*c*d*e + 258*a^3* \\
& b^11*c*d*f - 43520*a^8*b*c^6*d*f + 168*a^4*b^10*c*d*g - 152*a^4*b^10*c*e*f \\
& - 98*a^5*b^9*c*e*g + 1536*a^9*b*c^5*e*g + 2*a^5*b*f*g*(-(4*a*c - b^2)^9)^{(1 \\
& /2)} - 10*a^5*c*e*g*(-(4*a*c - b^2)^9)^{(1/2)} + 36*a^6*b^8*c*f*g + 246*a^2*b^ \\
& 2*c^2*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^{(1/ \\
& 2)} + 7278*a^3*b^10*c^2*d*e - 39132*a^4*b^8*c^3*d*e + 119616*a^5*b^6*c^4*d*e \\
& - 201600*a^6*b^4*c^5*d*e + 161280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*f*(-(4*a* \\
& c - b^2)^9)^{(1/2)} - 2706*a^4*b^9*c^2*d*f + 14784*a^5*b^7*c^3*d*f - 44352*a^ \\
& 6*b^5*c^4*d*f + 69120*a^7*b^3*c^5*d*f + 10*a^3*b^3*d*g*(-(4*a*c - b^2)^9)^{(\\
& 1/2)} - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^{(1/2)} + 42*a^4*c^2*d*f*(-(4*a*c - b \\
& ^2)^9)^{(1/2)} - 1044*a^5*b^8*c^2*d*g + 1548*a^5*b^8*c^2*e*f + 2688*a^6*b^6*c \\
& ^3*d*g - 8064*a^6*b^6*c^3*e*f - 1152*a^7*b^4*c^4*d*g + 22400*a^7*b^4*c^4*e* \\
& f - 6144*a^8*b^2*c^5*d*g - 30720*a^8*b^2*c^5*e*f - 6*a^4*b^2*e*g*(-(4*a*c - \\
& b^2)^9)^{(1/2)} + 576*a^6*b^7*c^2*e*g - 1344*a^7*b^5*c^3*e*g + 512*a^8*b^3*c^4 \\
& ^4*e*g - 192*a^7*b^6*c^2*f*g + 128*a^8*b^4*c^3*f*g + 1536*a^9*b^2*c^4*f*g - \\
& 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 12*a^4*b*c*d*g*(-(4*a*c - b^2) \\
& ^9)^{(1/2)} + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^{(1/2)} + 184*a^2*b^3*c*d*e*(-(\\
& 4*a*c - b^2)^9)^{(1/2)} - 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 78*a^3 \\
& *b^2*c*d*f*(-(4*a*c - b^2)^9)^{(1/2)}/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8 \\
& *b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^ \\
& 12*b^2*c^5)))^{(1/2)}*(1048576*a^21*b*c^8 + 256*a^15*b^13*c^2 - 6144*a^16*b^1 \\
& 1*c^3 + 61440*a^17*b^9*c^4 - 327680*a^18*b^7*c^5 + 983040*a^19*b^5*c^6 - 15 \\
& 72864*a^20*b^3*c^7) + 320*a^12*b^14*c^2*d - 7936*a^13*b^12*c^3*d + 82816*a^ \\
& 14*b^10*c^4*d - 468480*a^15*b^8*c^5*d + 1536000*a^16*b^6*c^6*d - 2867200*a^ \\
& 17*b^4*c^7*d + 2719744*a^18*b^2*c^8*d - 192*a^13*b^13*c^2*e + 4672*a^14*b^1 \\
& 1*c^3*e - 47360*a^15*b^9*c^4*e + 256000*a^16*b^7*c^5*e - 778240*a^17*b^5*c^ \\
& 6*e + 1261568*a^18*b^3*c^7*e + 64*a^14*b^12*c^2*f - 1664*a^15*b^10*c^3*f + \\
& 17920*a^16*b^8*c^4*f - 102400*a^17*b^6*c^5*f + 327680*a^18*b^4*c^6*f - 5570 \\
& 56*a^19*b^2*c^7*f + 64*a^15*b^11*c^2*g - 1280*a^16*b^9*c^3*g + 10240*a^17*b \\
& ^7*c^4*g - 40960*a^18*b^5*c^5*g + 81920*a^19*b^3*c^6*g - 851968*a^19*b*c^8* \\
& e - 65536*a^20*b*c^7*g) + x*(204800*a^17*c^9*e^2 - 401408*a^16*c^10*d^2 - 7 \\
& 3728*a^18*c^8*f^2 + 8192*a^19*c^7*g^2 + 400*a^9*b^14*c^3*d^2 - 9440*a^10*b^ \\
& 12*c^4*d^2 + 92816*a^11*b^10*c^5*d^2 - 488096*a^12*b^8*c^6*d^2 + 1458688*a^ \\
& 13*b^6*c^7*d^2 - 2401280*a^14*b^4*c^8*d^2 + 1871872*a^15*b^2*c^9*d^2 + 144* \\
& a^11*b^12*c^3*e^2 - 3264*a^12*b^10*c^4*e^2 + 30112*a^13*b^8*c^5*e^2 - 14336 \\
& 0*a^14*b^6*c^6*e^2 + 365568*a^15*b^4*c^7*e^2 - 458752*a^16*b^2*c^8*e^2 + 16
\end{aligned}$$

$$\begin{aligned}
& *a^{13}b^{10}c^3f^2 - 416a^{14}b^8c^4f^2 + 4608a^{15}b^6c^5f^2 - 25600a^{16}b^4c^6f^2 + 69632a^{17}b^2c^7f^2 + 160a^{15}b^8c^3g^2 - 2048a^{16} \\
& *b^6c^4g^2 + 9216a^{17}b^4c^5g^2 - 16384a^{18}b^2c^6g^2 + 344064a^{17} \\
& *c^9d*f - 81920a^{18}c^8e*g - 1236992a^{16}b*c^9d*e + 40960a^{17}b*c^8d \\
& *g + 237568a^{17}b*c^8e*f + 40960a^{18}b*c^7f*g - 480a^{10}b^{13}c^3d*e + \\
& 11104a^{11}b^{11}c^4d*e - 105824a^{12}b^9c^5d*e + 530432a^{13}b^7c^6d* \\
& e - 1469440a^{14}b^5c^7d*e + 2121728a^{15}b^3c^8d*e + 160a^{11}b^{12}c^3 \\
& *d*f - 3968a^{12}b^{10}c^4d*f + 39488a^{13}b^8c^5d*f - 200704a^{14}b^6c^6 \\
& *d*f + 542720a^{15}b^4c^7d*f - 720896a^{16}b^2c^8d*f + 160a^{12}b^{11}c^3 \\
& *d*g - 96a^{12}b^{11}c^3e*f - 2528a^{13}b^9c^4d*g + 2336a^{13}b^9c^4e \\
& *f + 14336a^{14}b^7c^5d*g - 22528a^{14}b^7c^5e*f - 31744a^{15}b^5c^6d \\
& *g + 107520a^{15}b^5c^6e*f + 8192a^{16}b^3c^7d*g - 253952a^{16}b^3c^7e \\
& *f - 96a^{13}b^{10}c^3e*g + 1472a^{14}b^8c^4e*g - 7168a^{15}b^6c^5e*g \\
& + 6144a^{16}b^4c^6e*g + 40960a^{17}b^2c^7e*g + 32a^{14}b^9c^3f*g - 10 \\
& 24a^{15}b^7c^4f*g + 9216a^{16}b^5c^5f*g - 32768a^{17}b^3c^6f*g) * ((25 \\
& *b^6d^2*(-(4*a*c - b^2)^9)^(1/2) - 9*a^2*b^13e^2 - 25*b^15d^2 - a^4*b^11 \\
& *f^2 - a^6*b^9g^2 + a^6*g^2*(-(4*a*c - b^2)^9)^(1/2) + 80640a^7*b*c^7d^2 \\
& + 213*a^3*b^11*c*e^2 - 26880a^8*b*c^6e^2 + 27*a^5*b^9*c*f^2 + 3840a^9*b \\
& *c^5f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768a^{10}b*c^4g^2 + 30a \\
& *b^{14}d*e - 6366a^2*b^{11}c^2d^2 + 35767a^3*b^9c^3d^2 - 116928a^4*b^7c^4 \\
& d^2 + 219744a^5*b^5c^5d^2 - 215040a^6*b^3c^6d^2 + 9*a^2*b^4e^2*(- \\
& -(4*a*c - b^2)^9)^(1/2) - 49*a^3c^3d^2*(-(4*a*c - b^2)^9)^(1/2) - 2077a^4 \\
& *b^9c^2e^2 + 10656a^5*b^7c^3e^2 - 30240a^6*b^5c^4e^2 + 44800a^7*b^3 \\
& c^5e^2 + a^4*b^2f^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^4c^2e^2*(-(4*a*c \\
& - b^2)^9)^(1/2) - 288a^6*b^7c^2f^2 + 1504a^7*b^5c^3f^2 - 3840a^8*b^3 \\
& c^4f^2 + 96a^8*b^5c^2g^2 - 512a^9*b^3c^3g^2 + 615a*b^{13}c*d^2 - 1 \\
& 0*a^2*b^{13}d*f - 35840a^8*c^7d*e - 10*a^3*b^{12}d*g + 6*a^3*b^{12}e*f + 6*a \\
& ^4*b^{11}e*g + 7168a^9*c^6d*g + 15360a^9*c^6e*f - 2*a^5*b^{10}f*g - 3072* \\
& a^{10}c^5f*g - 30*a*b^5d*e*(-(4*a*c - b^2)^9)^(1/2) - 724a^2*b^{12}c*d*e + \\
& 258a^3*b^{11}c*d*f - 43520a^8*b*c^6d*f + 168a^4*b^{10}c*d*g - 152a^4*b^ \\
& 10*c*e*f - 98a^5*b^9c*e*g + 1536a^9*b*c^5e*g + 2*a^5*b*f*g*(-(4*a*c - b \\
& ^2)^9)^(1/2) - 10*a^5*c*e*g*(-(4*a*c - b^2)^9)^(1/2) + 36a^6*b^8*c*f*g + 2 \\
& 46a^2*b^2c^2d^2*(-(4*a*c - b^2)^9)^(1/2) - 165a*b^4c*d^2*(-(4*a*c - b^ \\
& 2)^9)^(1/2) + 7278a^3*b^{10}c^2d*e - 39132a^4*b^8c^3d*e + 119616a^5*b^6 \\
& c^4d*e - 201600a^6*b^4c^5d*e + 161280a^7*b^2c^6d*e + 10*a^2*b^4d* \\
& f*(-(4*a*c - b^2)^9)^(1/2) - 2706a^4*b^9c^2d*f + 14784a^5*b^7c^3d*f - \\
& 44352a^6*b^5c^4d*f + 69120a^7*b^3c^5d*f + 10a^3*b^3d*g*(-(4*a*c - \\
& b^2)^9)^(1/2) - 6a^3*b^3e*f*(-(4*a*c - b^2)^9)^(1/2) + 42a^4c^2d*f*(-(\\
& 4*a*c - b^2)^9)^(1/2) - 1044a^5*b^8c^2d*g + 1548a^5*b^8c^2e*f + 2688* \\
& a^6*b^6c^3d*g - 8064a^6*b^6c^3e*f - 1152a^7*b^4c^4d*g + 22400a^7*b^4 \\
& c^4e*f - 6144a^8*b^2c^5d*g - 30720a^8*b^2c^5e*f - 6a^4*b^2e*g*(- \\
& -(4*a*c - b^2)^9)^(1/2) + 576a^6*b^7c^2e*g - 1344a^7*b^5c^3e*g + 512* \\
& a^8*b^3c^4e*g - 192a^7*b^6c^2f*g + 128a^8*b^4c^3f*g + 1536a^9*b^2c^4 \\
& f*g - 51a^3*b^2c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 12a^4*b*c*d*g*(-(4*a \\
& *c - b^2)^9)^(1/2) + 44a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 184a^2*b^3 \\
& c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 186a^3*b*c^2d*e*(-(4*a*c - b^2)^9)^(1/2) \\
& - 78a^3*b^2c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^{12} + 4096a^{13}c^6 \\
& - 24a^8*b^{10}c + 240a^9*b^8c^2 - 1280a^{10}b^6c^3 + 3840a^{11}b^4c^4 \\
& - 6144a^{12}b^2c^5)))^(1/2) - (((25*b^6d^2*(-(4*a*c - b^2)^9)^(1/2) - 9a \\
& ^2*b^13e^2 - 25*b^15d^2 - a^4*b^11f^2 - a^6*b^9g^2 + a^6*g^2*(-(4*a*c - \\
& b^2)^9)^(1/2) + 80640a^7*b*c^7d^2 + 213a^3*b^11*c*e^2 - 26880a^8*b*c^6 \\
& *e^2 + 27*a^5*b^9*c*f^2 + 3840a^9*b*c^5f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^ \\
& 9)^(1/2) + 768a^{10}b*c^4g^2 + 30*a*b^{14}d*e - 6366a^2*b^{11}c^2d^2 + 357 \\
& 67a^3*b^9c^3d^2 - 116928a^4*b^7c^4d^2 + 219744a^5*b^5c^5d^2 - 2150 \\
& 40a^6*b^3c^6d^2 + 9*a^2*b^4e^2*(-(4*a*c - b^2)^9)^(1/2) - 49a^3c^3d^ \\
& 2*(-(4*a*c - b^2)^9)^(1/2) - 2077a^4*b^9c^2e^2 + 10656a^5*b^7c^3e^2 - \\
& 30240a^6*b^5c^4e^2 + 44800a^7*b^3c^5e^2 + a^4*b^2f^2*(-(4*a*c - b^2 \\
&)^9)^(1/2) + 25a^4c^2e^2*(-(4*a*c - b^2)^9)^(1/2) - 288a^6*b^7c^2f^2 \\
& + 1504a^7*b^5c^3f^2 - 3840a^8*b^3c^4f^2 + 96a^8*b^5c^2g^2 - 512a^
\end{aligned}$$

$$\begin{aligned}
& 9*b^3*c^3*g^2 + 615*a*b^{13}*c*d^2 - 10*a^2*b^{13}*d*f - 35840*a^8*c^7*d*e - 10 \\
& *a^3*b^{12}*d*g + 6*a^3*b^{12}*e*f + 6*a^4*b^{11}*e*g + 7168*a^9*c^6*d*g + 15360* \\
& a^9*c^6*e*f - 2*a^5*b^{10}*f*g - 3072*a^{10}*c^5*f*g - 30*a*b^5*d*e*(-(4*a*c - \\
& b^2)^9)^{(1/2)} - 724*a^2*b^{12}*c*d*e + 258*a^3*b^{11}*c*d*f - 43520*a^8*b*c^6*d \\
& *f + 168*a^4*b^{10}*c*d*g - 152*a^4*b^{10}*c*e*f - 98*a^5*b^9*c*e*g + 1536*a^9* \\
& b*c^5*e*g + 2*a^5*b*f*g*(-(4*a*c - b^2)^9)^{(1/2)} - 10*a^5*c*e*g*(-(4*a*c - \\
& b^2)^9)^{(1/2)} + 36*a^6*b^8*c*f*g + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^{(1/2)} \\
& - 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + 7278*a^3*b^{10}*c^2*d*e - 3 \\
& 9132*a^4*b^8*c^3*d*e + 119616*a^5*b^6*c^4*d*e - 201600*a^6*b^4*c^5*d*e + 16 \\
& 1280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^{(1/2)} - 2706*a^4*b \\
& ^9*c^2*d*f + 14784*a^5*b^7*c^3*d*f - 44352*a^6*b^5*c^4*d*f + 69120*a^7*b^3* \\
& c^5*d*f + 10*a^3*b^3*d*g*(-(4*a*c - b^2)^9)^{(1/2)} - 6*a^3*b^3*e*f*(-(4*a*c \\
& - b^2)^9)^{(1/2)} + 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^{(1/2)} - 1044*a^5*b^8*c^ \\
& ^2*d*g + 1548*a^5*b^8*c^2*e*f + 2688*a^6*b^6*c^3*d*g - 8064*a^6*b^6*c^3*e*f \\
& - 1152*a^7*b^4*c^4*d*g + 22400*a^7*b^4*c^4*e*f - 6144*a^8*b^2*c^5*d*g - 307 \\
& 20*a^8*b^2*c^5*e*f - 6*a^4*b^2*e*g*(-(4*a*c - b^2)^9)^{(1/2)} + 576*a^6*b^7*c \\
& ^2*e*g - 1344*a^7*b^5*c^3*e*g + 512*a^8*b^3*c^4*e*g - 192*a^7*b^6*c^2*f*g + \\
& 128*a^8*b^4*c^3*f*g + 1536*a^9*b^2*c^4*f*g - 51*a^3*b^2*c*e^2*(-(4*a*c - b \\
& ^2)^9)^{(1/2)} + 12*a^4*b*c*d*g*(-(4*a*c - b^2)^9)^{(1/2)} + 44*a^4*b*c*e*f*(-(\\
& 4*a*c - b^2)^9)^{(1/2)} + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 186*a^ \\
& ^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^{ \\
& (1/2)}/(32*(a^7*b^{12} + 4096*a^{13}*c^6 - 24*a^8*b^{10}*c + 240*a^9*b^8*c^2 - 12 \\
& 80*a^{10}*b^6*c^3 + 3840*a^{11}*b^4*c^4 - 6144*a^{12}*b^2*c^5)))^{(1/2)}*(917504*a^ \\
& ^{19}*c^9*d - 393216*a^{20}*c^8*f + x*((25*b^6*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - 9* \\
& a^2*b^{13}*e^2 - 25*b^{15}*d^2 - a^4*b^{11}*f^2 - a^6*b^9*g^2 + a^6*g^2*(-(4*a*c \\
& - b^2)^9)^{(1/2)} + 80640*a^7*b*c^7*d^2 + 213*a^3*b^{11}*c*e^2 - 26880*a^8*b*c^ \\
& ^6*e^2 + 27*a^5*b^9*c*f^2 + 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2) \\
& ^9)^{(1/2)} + 768*a^{10}*b*c^4*g^2 + 30*a*b^{14}*d*e - 6366*a^2*b^{11}*c^2*d^2 + 35 \\
& 767*a^3*b^9*c^3*d^2 - 116928*a^4*b^7*c^4*d^2 + 219744*a^5*b^5*c^5*d^2 - 215 \\
& 040*a^6*b^3*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 49*a^3*c^3*d \\
& ^2*(-(4*a*c - b^2)^9)^{(1/2)} - 2077*a^4*b^9*c^2*e^2 + 10656*a^5*b^7*c^3*e^2 \\
& - 30240*a^6*b^5*c^4*e^2 + 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*c - b^ \\
& ^2)^9)^{(1/2)} + 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 288*a^6*b^7*c^2*f^2 \\
& + 1504*a^7*b^5*c^3*f^2 - 3840*a^8*b^3*c^4*f^2 + 96*a^8*b^5*c^2*g^2 - 512*a \\
& ^9*b^3*c^3*g^2 + 615*a*b^{13}*c*d^2 - 10*a^2*b^{13}*d*f - 35840*a^8*c^7*d*e - 1 \\
& 0*a^3*b^{12}*d*g + 6*a^3*b^{12}*e*f + 6*a^4*b^{11}*e*g + 7168*a^9*c^6*d*g + 15360 \\
& *a^9*c^6*e*f - 2*a^5*b^{10}*f*g - 3072*a^{10}*c^5*f*g - 30*a*b^5*d*e*(-(4*a*c - \\
& b^2)^9)^{(1/2)} - 724*a^2*b^{12}*c*d*e + 258*a^3*b^{11}*c*d*f - 43520*a^8*b*c^6* \\
& d*f + 168*a^4*b^{10}*c*d*g - 152*a^4*b^{10}*c*e*f - 98*a^5*b^9*c*e*g + 1536*a^9* \\
& b*c^5*e*g + 2*a^5*b*f*g*(-(4*a*c - b^2)^9)^{(1/2)} - 10*a^5*c*e*g*(-(4*a*c - \\
& b^2)^9)^{(1/2)} + 36*a^6*b^8*c*f*g + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^{ \\
& (1/2)} - 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + 7278*a^3*b^{10}*c^2*d*e - \\
& 39132*a^4*b^8*c^3*d*e + 119616*a^5*b^6*c^4*d*e - 201600*a^6*b^4*c^5*d*e + 1 \\
& 61280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^{(1/2)} - 2706*a^4*b \\
& ^9*c^2*d*f + 14784*a^5*b^7*c^3*d*f - 44352*a^6*b^5*c^4*d*f + 69120*a^7*b^3* \\
& c^5*d*f + 10*a^3*b^3*d*g*(-(4*a*c - b^2)^9)^{(1/2)} - 6*a^3*b^3*e*f*(-(4*a*c \\
& - b^2)^9)^{(1/2)} + 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^{(1/2)} - 1044*a^5*b^8*c \\
& ^2*d*g + 1548*a^5*b^8*c^2*e*f + 2688*a^6*b^6*c^3*d*g - 8064*a^6*b^6*c^3*e*f \\
& - 1152*a^7*b^4*c^4*d*g + 22400*a^7*b^4*c^4*e*f - 6144*a^8*b^2*c^5*d*g - 30 \\
& 720*a^8*b^2*c^5*e*f - 6*a^4*b^2*e*g*(-(4*a*c - b^2)^9)^{(1/2)} + 576*a^6*b^7*c \\
& ^2*e*g - 1344*a^7*b^5*c^3*e*g + 512*a^8*b^3*c^4*e*g - 192*a^7*b^6*c^2*f*g \\
& + 128*a^8*b^4*c^3*f*g + 1536*a^9*b^2*c^4*f*g - 51*a^3*b^2*c*e^2*(-(4*a*c - \\
& b^2)^9)^{(1/2)} + 12*a^4*b*c*d*g*(-(4*a*c - b^2)^9)^{(1/2)} + 44*a^4*b*c*e*f*(-(\\
& 4*a*c - b^2)^9)^{(1/2)} + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 186*a^ \\
& ^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9) \\
& ^{(1/2)}/(32*(a^7*b^{12} + 4096*a^{13}*c^6 - 24*a^8*b^{10}*c + 240*a^9*b^8*c^2 - 1 \\
& 280*a^{10}*b^6*c^3 + 3840*a^{11}*b^4*c^4 - 6144*a^{12}*b^2*c^5)))^{(1/2)}*(1048576* \\
& a^{21}*b*c^8 + 256*a^{15}*b^{13}*c^2 - 6144*a^{16}*b^{11}*c^3 + 61440*a^{17}*b^9*c^4 - \\
& 327680*a^{18}*b^7*c^5 + 983040*a^{19}*b^5*c^6 - 1572864*a^{20}*b^3*c^7) - 320*a^{11}
\end{aligned}$$

$$\begin{aligned}
& 2*b^{14}*c^2*d + 7936*a^{13}*b^{12}*c^3*d - 82816*a^{14}*b^{10}*c^4*d + 468480*a^{15}*b^{8}*c^5*d - 1536000*a^{16}*b^6*c^6*d + 2867200*a^{17}*b^4*c^7*d - 2719744*a^{18}*b^2*c^8*d + 192*a^{13}*b^{13}*c^2*e - 4672*a^{14}*b^{11}*c^3*e + 47360*a^{15}*b^9*c^4*e - 256000*a^{16}*b^7*c^5*e + 778240*a^{17}*b^5*c^6*e - 1261568*a^{18}*b^3*c^7*e - 64*a^{14}*b^{12}*c^2*f + 1664*a^{15}*b^{10}*c^3*f - 17920*a^{16}*b^8*c^4*f + 102400*a^{17}*b^6*c^5*f - 327680*a^{18}*b^4*c^6*f + 557056*a^{19}*b^2*c^7*f - 64*a^{15}*b^{11}*c^2*g + 1280*a^{16}*b^9*c^3*g - 10240*a^{17}*b^7*c^4*g + 40960*a^{18}*b^5*c^5*g - 81920*a^{19}*b^3*c^6*g + 851968*a^{19}*b*c^8*e + 65536*a^{20}*b*c^7*g) + x*(204800*a^{17}*c^9*e^2 - 401408*a^{16}*c^10*d^2 - 73728*a^{18}*c^8*f^2 + 8192*a^{19}*c^7*g^2 + 400*a^9*b^{14}*c^3*d^2 - 9440*a^{10}*b^{12}*c^4*d^2 + 92816*a^{11}*b^{10}*c^5*d^2 - 488096*a^{12}*b^8*c^6*d^2 + 1458688*a^{13}*b^6*c^7*d^2 - 2401280*a^{14}*b^4*c^8*d^2 + 1871872*a^{15}*b^2*c^9*d^2 + 144*a^{11}*b^{12}*c^3*e^2 - 3264*a^{12}*b^{10}*c^4*e^2 + 30112*a^{13}*b^8*c^5*e^2 - 143360*a^{14}*b^6*c^6*e^2 + 365568*a^{15}*b^4*c^7*e^2 - 458752*a^{16}*b^2*c^8*e^2 + 16*a^{13}*b^{10}*c^3*f^2 - 416*a^{14}*b^8*c^4*f^2 + 4608*a^{15}*b^6*c^5*f^2 - 25600*a^{16}*b^4*c^6*f^2 + 69632*a^{17}*b^2*c^7*f^2 + 160*a^{15}*b^8*c^3*g^2 - 2048*a^{16}*b^6*c^4*g^2 + 9216*a^{17}*b^4*c^5*g^2 - 16384*a^{18}*b^2*c^6*g^2 + 344064*a^{17}*c^9*d*f - 81920*a^{18}*c^8*e*g - 1236992*a^{16}*b*c^9*d*e + 40960*a^{17}*b*c^8*d*g + 237568*a^{17}*b*c^8*e*f + 40960*a^{18}*b*c^7*f*g - 480*a^{10}*b^{13}*c^3*d*e + 11104*a^{11}*b^{11}*c^4*d*e - 105824*a^{12}*b^9*c^5*d*e + 530432*a^{13}*b^7*c^6*d*e - 1469440*a^{14}*b^5*c^7*d*e + 2121728*a^{15}*b^3*c^8*d*e + 160*a^{11}*b^{12}*c^3*d*f - 3968*a^{12}*b^{10}*c^4*d*f + 39488*a^{13}*b^8*c^5*d*f - 200704*a^{14}*b^6*c^6*d*f + 542720*a^{15}*b^4*c^7*d*f - 720896*a^{16}*b^2*c^8*d*f + 160*a^{12}*b^{11}*c^3*d*g - 96*a^{12}*b^{11}*c^3*e*f - 2528*a^{13}*b^9*c^4*d*g + 2336*a^{13}*b^9*c^4*e*f + 14336*a^{14}*b^7*c^5*d*g - 22528*a^{14}*b^7*c^5*e*f - 31744*a^{15}*b^5*c^6*d*g + 107520*a^{15}*b^5*c^6*e*f + 8192*a^{16}*b^3*c^7*d*g - 253952*a^{16}*b^3*c^7*e*f - 96*a^{13}*b^{10}*c^3*e*g + 1472*a^{14}*b^8*c^4*e*g - 7168*a^{15}*b^6*c^5*e*g + 6144*a^{16}*b^4*c^6*e*g + 40960*a^{17}*b^2*c^7*e*g + 32*a^{14}*b^9*c^3*f*g - 1024*a^{15}*b^7*c^4*f*g + 9216*a^{16}*b^5*c^5*f*g - 32768*a^{17}*b^3*c^6*f*g))*((25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) - 9*a^2*b^{13}*e^2 - 25*b^{15}*d^2 - a^4*b^{11}*f^2 - a^6*b^9*g^2 + a^6*g^2*(-(4*a*c - b^2)^9)^(1/2) + 80640*a^7*b*c^7*d^2 + 213*a^3*b^{11}*c*e^2 - 26880*a^8*b*c^6*e^2 + 27*a^5*b^9*c*f^2 + 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^{10}*b*c^4*g^2 + 30*a*b^{14}*d*e - 6366*a^2*b^{11}*c^2*d^2 + 35767*a^3*b^9*c^3*d^2 - 116928*a^4*b^7*c^4*d^2 + 219744*a^5*b^5*c^5*d^2 - 215040*a^6*b^3*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^2 + 10656*a^5*b^7*c^3*e^2 - 30240*a^6*b^5*c^4*e^2 + 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^6*b^7*c^2*f^2 + 1504*a^7*b^5*c^3*f^2 - 3840*a^8*b^3*c^4*f^2 + 96*a^8*b^5*c^2*g^2 - 512*a^9*b^3*c^3*g^2 + 615*a*b^{13}*c*d^2 - 10*a^2*b^{13}*d*f - 35840*a^8*c^7*d*e - 10*a^3*b^{12}*d*g + 6*a^3*b^{12}*e*f + 6*a^4*b^{11}*e*g + 7168*a^9*c^6*d*g + 15360*a^9*c^6*e*f - 2*a^5*b^{10}*f*g - 3072*a^{10}*c^5*f*g - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) - 724*a^2*b^{12}*c*d*e + 258*a^3*b^{11}*c*d*f - 43520*a^8*b*c^6*d*f + 168*a^4*b^{10}*c*d*g - 152*a^4*b^{10}*c*e*f - 98*a^5*b^9*c*e*g + 1536*a^9*b*c^5*e*g + 2*a^5*b*f*g*(-(4*a*c - b^2)^9)^(1/2) - 10*a^5*c*e*g*(-(4*a*c - b^2)^9)^(1/2) + 36*a^6*b^8*c*f*g + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 7278*a^3*b^{10}*c^2*d*e - 39132*a^4*b^8*c^3*d*e + 119616*a^5*b^6*c^4*d*e - 201600*a^6*b^4*c^5*d*e + 161280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*d*f*(-(4*a*c - b^2)^9)^(1/2) - 2706*a^4*b^9*c^2*d*f + 14784*a^5*b^7*c^3*d*f - 44352*a^6*b^5*c^4*d*f + 69120*a^7*b^3*c^5*d*f + 10*a^3*b^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 1044*a^5*b^8*c^2*d*g + 1548*a^5*b^8*c^2*e*f + 2688*a^6*b^6*c^3*d*g - 8064*a^6*b^6*c^3*e*f - 1152*a^7*b^4*c^4*d*g + 22400*a^7*b^4*c^4*e*f - 6144*a^8*b^2*c^5*d*g - 30720*a^8*b^2*c^5*e*f - 6*a^4*b^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 576*a^6*b^7*c^2*e*g - 1344*a^7*b^5*c^3*e*g + 512*a^8*b^3*c^4*e*g - 192*a^7*b^6*c^2*f*g + 128*a^8*b^4*c^3*f*g + 1536*a^9*b^2*c^4*f*g - 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a^4*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2)
\end{aligned}$$

$$\begin{aligned}
&) - 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 78*a^3*b^2*c*d*f*(-(4*a*c \\
& - b^2)^9)^{(1/2)}/(32*(a^7*b^{12} + 4096*a^{13}*c^6 - 24*a^8*b^{10}*c + 240*a^9*b^8*c^2 \\
& - 1280*a^{10}*b^6*c^3 + 3840*a^{11}*b^4*c^4 - 6144*a^{12}*b^2*c^5)))^{(1/2)} \\
& - 128000*a^{15}*c^9*e^3 + 1024*a^{18}*c^6*g^3 + 476672*a^{13}*b*c^{10}*d^3 - 4608*a \\
& ^{16}*b*c^7*f^3 - 250880*a^{14}*c^{10}*d^2*e + 50176*a^{15}*c^9*d^2*g - 46080*a^{16}* \\
& c^8*e*f^2 + 76800*a^{16}*c^8*e^2*g - 15360*a^{17}*c^7*e*g^2 + 9216*a^{17}*c^7*f^2 \\
& *g + 1800*a^9*b^9*c^6*d^3 - 29080*a^{10}*b^7*c^7*d^3 + 176032*a^{11}*b^5*c^8*d^3 \\
& - 473216*a^{12}*b^3*c^9*d^3 - 504*a^{11}*b^8*c^5*e^3 + 8112*a^{12}*b^6*c^6*e^3 \\
& - 48704*a^{13}*b^4*c^7*e^3 + 129280*a^{14}*b^2*c^8*e^3 + 40*a^{13}*b^7*c^4*f^3 - \\
& 608*a^{14}*b^5*c^5*f^3 + 2944*a^{15}*b^3*c^6*f^3 + 48*a^{15}*b^6*c^3*g^3 - 320*a^ \\
& ^{16}*b^4*c^4*g^3 + 256*a^{17}*b^2*c^5*g^3 + 215040*a^{15}*c^9*d*e*f - 43008*a^{16}* \\
& c^8*d*f*g + 442880*a^{14}*b*c^9*d*e^2 - 433664*a^{14}*b*c^9*d^2*f + 109056*a^{15} \\
& *b*c^8*d*f^2 + 84480*a^{15}*b*c^8*e^2*f + 43520*a^{16}*b*c^7*d*g^2 - 7680*a^{17}* \\
& b*c^6*f*g^2 - 1400*a^9*b^{10}*c^5*d^2*e + 21680*a^{10}*b^8*c^6*d^2*e + 1680*a^{10} \\
& *b^9*c^5*d*e^2 - 121648*a^{11}*b^6*c^7*d^2*e - 27176*a^{11}*b^7*c^6*d*e^2 + 27 \\
& 5264*a^{12}*b^4*c^8*d^2*e + 164448*a^{12}*b^5*c^7*d*e^2 - 121088*a^{13}*b^2*c^9*d \\
& ^2*e - 441216*a^{13}*b^3*c^8*d*e^2 + 1000*a^9*b^{11}*c^4*d^2*f - 17800*a^{10}*b^9 \\
& *c^5*d^2*f + 124280*a^{11}*b^7*c^6*d^2*f + 400*a^{11}*b^9*c^4*d*f^2 - 422944*a^ \\
& ^{12}*b^5*c^7*d^2*f - 6600*a^{12}*b^7*c^5*d*f^2 + 694912*a^{13}*b^3*c^8*d^2*f + 40 \\
& 416*a^{13}*b^5*c^6*d*f^2 - 108928*a^{14}*b^3*c^7*d*f^2 - 600*a^9*b^{12}*c^3*d^2*g \\
& + 10960*a^{10}*b^{10}*c^4*d^2*g - 78904*a^{11}*b^8*c^5*d^2*g + 360*a^{11}*b^9*c^4* \\
& e^2*f + 278096*a^{12}*b^6*c^6*d^2*g - 5736*a^{12}*b^7*c^5*e^2*f - 240*a^{12}*b^8* \\
& c^4*e*f^2 + 120*a^{12}*b^9*c^3*d*g^2 - 472000*a^{13}*b^4*c^7*d^2*g + 33888*a^{13} \\
& *b^5*c^6*e^2*f + 3792*a^{13}*b^6*c^5*e*f^2 - 2216*a^{13}*b^7*c^4*d*g^2 + 284416 \\
& *a^{14}*b^2*c^8*d^2*g - 87936*a^{14}*b^3*c^7*e^2*f - 21696*a^{14}*b^4*c^6*e*f^2 + \\
& 14688*a^{14}*b^5*c^5*d*g^2 + 52992*a^{15}*b^2*c^7*e*f^2 - 41856*a^{15}*b^3*c^6*d \\
& *g^2 - 216*a^{11}*b^{10}*c^3*e^2*g + 3744*a^{12}*b^8*c^4*e^2*g - 25200*a^{13}*b^6*c \\
& ^5*e^2*g - 72*a^{13}*b^8*c^3*e*g^2 + 81984*a^{14}*b^4*c^6*e^2*g + 1296*a^{14}*b^6 \\
& *c^4*e*g^2 - 128256*a^{15}*b^2*c^7*e^2*g - 7872*a^{15}*b^4*c^5*e*g^2 + 19200*a^ \\
& ^{16}*b^2*c^6*e*g^2 - 24*a^{13}*b^8*c^3*f^2*g + 336*a^{14}*b^6*c^4*f^2*g + 24*a^{14} \\
& *b^7*c^3*f*g^2 - 960*a^{15}*b^4*c^5*f^2*g - 672*a^{15}*b^5*c^4*f*f*g^2 - 2304*a^ \\
& ^{16}*b^2*c^6*f^2*g + 4224*a^{16}*b^3*c^5*f*f*g^2 - 306176*a^{15}*b*c^8*d*e*g + 21504 \\
& *a^{16}*b*c^7*e*f*f*g - 1200*a^{10}*b^{10}*c^4*d*e*f + 20240*a^{11}*b^8*c^5*d*e*f - 1 \\
& 30656*a^{12}*b^6*c^6*d*e*f + 394368*a^{13}*b^4*c^7*d*e*f - 528896*a^{14}*b^2*c^8* \\
& d*e*f + 720*a^{10}*b^{11}*c^3*d*e*g - 12816*a^{11}*b^9*c^4*d*e*g + 89264*a^{12}*b^7 \\
& *c^5*d*e*g - 302400*a^{13}*b^5*c^6*d*e*g + 493824*a^{14}*b^3*c^7*d*e*g - 240*a^ \\
& ^{11}*b^{10}*c^3*d*f*g + 3872*a^{12}*b^8*c^4*d*f*g - 22368*a^{13}*b^6*c^5*d*f*f*g + 51 \\
& 840*a^{14}*b^4*c^6*d*f*f*g - 25088*a^{15}*b^2*c^7*d*f*f*g + 144*a^{12}*b^9*c^3*e*f*f*g \\
& - 2256*a^{13}*b^7*c^4*e*f*f*g + 12480*a^{14}*b^5*c^5*e*f*f*g - 28416*a^{15}*b^3*c^6*e \\
& *f*f*g))*((25*b^6*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - 9*a^2*b^{13}*e^2 - 25*b^{15}*d^2 \\
& - a^4*b^{11}*f^2 - a^6*b^9*g^2 + a^6*g^2*(-(4*a*c - b^2)^9)^{(1/2)} + 80640*a^ \\
& ^7*b*c^7*d^2 + 213*a^3*b^{11}*c*e^2 - 26880*a^8*b*c^6*e^2 + 27*a^5*b^9*c*f^2 + \\
& 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + 768*a^{10}*b*c^4 \\
& *g^2 + 30*a*b^{14}*d*e - 6366*a^2*b^{11}*c^2*d^2 + 35767*a^3*b^9*c^3*d^2 - 1169 \\
& 28*a^4*b^7*c^4*d^2 + 219744*a^5*b^5*c^5*d^2 - 215040*a^6*b^3*c^6*d^2 + 9*a^ \\
& ^2*b^4*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^{(1/2)} \\
&) - 2077*a^4*b^9*c^2*e^2 + 10656*a^5*b^7*c^3*e^2 - 30240*a^6*b^5*c^4*e^2 + \\
& 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^{(1/2)} + 25*a^4*c^2*e \\
& ^2*(-(4*a*c - b^2)^9)^{(1/2)} - 288*a^6*b^7*c^2*f^2 + 1504*a^7*b^5*c^3*f^2 - \\
& 3840*a^8*b^3*c^4*f^2 + 96*a^8*b^5*c^2*g^2 - 512*a^9*b^3*c^3*g^2 + 615*a*b^{11} \\
& *c*d^2 - 10*a^2*b^{13}*d*f - 35840*a^8*c^7*d*e - 10*a^3*b^{12}*d*g + 6*a^3*b^{11} \\
& *2*e*f + 6*a^4*b^{11}*e*g + 7168*a^9*c^6*d*g + 15360*a^9*c^6*e*f - 2*a^5*b^{10}* \\
& f*g - 3072*a^{10}*c^5*f*f*g - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^{(1/2)} - 724*a^2*b \\
& ^{12}*c*d*e + 258*a^3*b^{11}*c*d*f - 43520*a^8*b*c^6*d*f + 168*a^4*b^{10}*c*d*g - \\
& 152*a^4*b^{10}*c*e*f - 98*a^5*b^9*c*e*g + 1536*a^9*b*c^5*e*g + 2*a^5*b*f*f*g*(\\
& -(4*a*c - b^2)^9)^{(1/2)} - 10*a^5*c*e*g*(-(4*a*c - b^2)^9)^{(1/2)} + 36*a^6*b^8 \\
& *c*f*f*g + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - 165*a*b^4*c*d^2*(- \\
& (4*a*c - b^2)^9)^{(1/2)} + 7278*a^3*b^{10}*c^2*d*e - 39132*a^4*b^8*c^3*d*e + 11 \\
& 9616*a^5*b^6*c^4*d*e - 201600*a^6*b^4*c^5*d*e + 161280*a^7*b^2*c^6*d*e + 10
\end{aligned}$$

```

*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 2706*a^4*b^9*c^2*d*f + 14784*a^5*b^
7*c^3*d*f - 44352*a^6*b^5*c^4*d*f + 69120*a^7*b^3*c^5*d*f + 10*a^3*b^3*d*g*
(-(4*a*c - b^2)^9)^(1/2) - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 42*a^4*
c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 1044*a^5*b^8*c^2*d*g + 1548*a^5*b^8*c^2*
e*f + 2688*a^6*b^6*c^3*d*g - 8064*a^6*b^6*c^3*e*f - 1152*a^7*b^4*c^4*d*g +
22400*a^7*b^4*c^4*e*f - 6144*a^8*b^2*c^5*d*g - 30720*a^8*b^2*c^5*e*f - 6*a^
4*b^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 576*a^6*b^7*c^2*e*g - 1344*a^7*b^5*c^3
*e*g + 512*a^8*b^3*c^4*e*g - 192*a^7*b^6*c^2*f*g + 128*a^8*b^4*c^3*f*g + 15
36*a^9*b^2*c^4*f*g - 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a^4*b*c
*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 1
84*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 186*a^3*b*c^2*d*e*(-(4*a*c - b^
2)^9)^(1/2) - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 40
96*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^
11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*2i

```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**6+f*x**4+e*x**2+d)/x**4/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.129

$$\int x^2 (a + bx^2 + cx^4)^p (3a + b(5 + 2p)x^2 + c(7 + 4p)x^4) dx$$

Optimal. Leaf size=20

$$x^3 (a + bx^2 + cx^4)^{p+1}$$

Rubi [A] time = 0.04, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.024$, Rules used = {1588}

$$x^3 (a + bx^2 + cx^4)^{p+1}$$

Antiderivative was successfully verified.

[In] Int[x^2*(a + b*x^2 + c*x^4)^p*(3*a + b*(5 + 2*p)*x^2 + c*(7 + 4*p)*x^4),x]

[Out] x^3*(a + b*x^2 + c*x^4)^(1 + p)

Rule 1588

Int[(Pp_)*(Qq_)^(m_.), x_Symbol] := With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]

Rubi steps

$$\int x^2 (a + bx^2 + cx^4)^p (3a + b(5 + 2p)x^2 + c(7 + 4p)x^4) dx = x^3 (a + bx^2 + cx^4)^{1+p}$$

Mathematica [A] time = 0.14, size = 20, normalized size = 1.00

$$x^3 (a + bx^2 + cx^4)^{p+1}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(a + b*x^2 + c*x^4)^p*(3*a + b*(5 + 2*p)*x^2 + c*(7 + 4*p)*x^4),x]

[Out] x^3*(a + b*x^2 + c*x^4)^(1 + p)

IntegrateAlgebraic [F] time = 0.36, size = 0, normalized size = 0.00

$$\int x^2 (a + bx^2 + cx^4)^p (3a + b(5 + 2p)x^2 + c(7 + 4p)x^4) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2*(a + b*x^2 + c*x^4)^p*(3*a + b*(5 + 2*p)*x^2 + c*(7 + 4*p)*x^4),x]

[Out] Defer[IntegrateAlgebraic][x^2*(a + b*x^2 + c*x^4)^p*(3*a + b*(5 + 2*p)*x^2 + c*(7 + 4*p)*x^4), x]

fricas [A] time = 1.06, size = 31, normalized size = 1.55

$$(cx^7 + bx^5 + ax^3)(cx^4 + bx^2 + a)^p$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(c*x^4+b*x^2+a)^p*(3*a+b*(5+2*p)*x^2+c*(7+4*p)*x^4),x, algorithm="fricas")

[Out] (c*x^7 + b*x^5 + a*x^3)*(c*x^4 + b*x^2 + a)^p

giac [B] time = 0.61, size = 58, normalized size = 2.90

$$(cx^4 + bx^2 + a)^p cx^7 + (cx^4 + bx^2 + a)^p bx^5 + (cx^4 + bx^2 + a)^p ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(c*x^4+b*x^2+a)^p*(3*a+b*(5+2*p)*x^2+c*(7+4*p)*x^4),x, algorithm="giac")

[Out] (c*x^4 + b*x^2 + a)^p*c*x^7 + (c*x^4 + b*x^2 + a)^p*b*x^5 + (c*x^4 + b*x^2 + a)^p*a*x^3

maple [A] time = 0.01, size = 21, normalized size = 1.05

$$x^3 (cx^4 + bx^2 + a)^{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(c*x^4+b*x^2+a)^p*(3*a+b*(5+2*p)*x^2+c*(7+4*p)*x^4),x)

[Out] x^3*(c*x^4+b*x^2+a)^(p+1)

maxima [A] time = 1.03, size = 31, normalized size = 1.55

$$(cx^7 + bx^5 + ax^3)(cx^4 + bx^2 + a)^p$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(c*x^4+b*x^2+a)^p*(3*a+b*(5+2*p)*x^2+c*(7+4*p)*x^4),x, algorithm="maxima")

[Out] (c*x^7 + b*x^5 + a*x^3)*(c*x^4 + b*x^2 + a)^p

mupad [B] time = 1.10, size = 31, normalized size = 1.55

$$(cx^7 + bx^5 + ax^3)(cx^4 + bx^2 + a)^p$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(3*a + b*x^2*(2*p + 5) + c*x^4*(4*p + 7))*(a + b*x^2 + c*x^4)^p,x)

[Out] (a*x^3 + b*x^5 + c*x^7)*(a + b*x^2 + c*x^4)^p

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(c*x**4+b*x**2+a)**p*(3*a+b*(5+2*p)*x**2+c*(7+4*p)*x**4),x)

[Out] Timed out

$$3.130 \quad \int \frac{x^5(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}} dx$$

Optimal. Leaf size=210

$$\frac{(d-ex)^{5/2}(d+ex)^{5/2}(ae^4+3bd^2e^2+6cd^4)}{5e^{10}} + \frac{d^2(d-ex)^{3/2}(d+ex)^{3/2}(2ae^4+3bd^2e^2+4cd^4)}{3e^{10}} - \frac{d^4\sqrt{d-ex}\sqrt{d+ex}}{3e^{10}}$$

Rubi [A] time = 0.31, antiderivative size = 278, normalized size of antiderivative = 1.32, number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {520, 1251, 897, 1153}

$$\frac{(d^2-e^2x^2)^3(ae^4+3bd^2e^2+6cd^4)}{5e^{10}\sqrt{d-ex}\sqrt{d+ex}} + \frac{d^2(d^2-e^2x^2)^2(2ae^4+3bd^2e^2+4cd^4)}{3e^{10}\sqrt{d-ex}\sqrt{d+ex}} - \frac{d^4(d^2-e^2x^2)(ae^4+bd^2e^2+cd^4)}{e^{10}\sqrt{d-ex}\sqrt{d+ex}} + \frac{(d^2-e^2x^2)^4(be^2+4cd^2)}{7e^{10}\sqrt{d-ex}\sqrt{d+ex}} - \frac{c(d^2-e^2x^2)^5}{9e^{10}\sqrt{d-ex}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

[In] Int[(x^5*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]

[Out] -((d^4*(c*d^4 + b*d^2*e^2 + a*e^4)*(d^2 - e^2*x^2))/(e^10*Sqrt[d - e*x]*Sqrt[d + e*x])) + (d^2*(4*c*d^4 + 3*b*d^2*e^2 + 2*a*e^4)*(d^2 - e^2*x^2)^2)/(3*e^10*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((6*c*d^4 + 3*b*d^2*e^2 + a*e^4)*(d^2 - e^2*x^2)^3)/(5*e^10*Sqrt[d - e*x]*Sqrt[d + e*x]) + ((4*c*d^2 + b*e^2)*(d^2 - e^2*x^2)^4)/(7*e^10*Sqrt[d - e*x]*Sqrt[d + e*x]) - (c*(d^2 - e^2*x^2)^5)/(9*e^10*Sqrt[d - e*x]*Sqrt[d + e*x])

Rule 520

Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n))^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]

Rule 897

Int[((d_.) + (e_.)*(x_)^(m_.))*((f_.) + (g_.)*(x_)^(n_.))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^(2))^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e + a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2]^p, x], x, (d + e*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && FractionQ[m]

Rule 1153

Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]

Rule 1251

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]

Rubi steps

$$\int \frac{x^5 (a + bx^2 + cx^4)}{\sqrt{d - ex} \sqrt{d + ex}} dx = \frac{\sqrt{d^2 - e^2 x^2} \int \frac{x^5 (a + bx^2 + cx^4)}{\sqrt{d^2 - e^2 x^2}} dx}{\sqrt{d - ex} \sqrt{d + ex}}$$

$$= \frac{\sqrt{d^2 - e^2 x^2} \operatorname{Subst} \left(\int \frac{x^2 (a + bx + cx^2)}{\sqrt{d^2 - e^2 x}} dx, x, x^2 \right)}{2\sqrt{d - ex} \sqrt{d + ex}}$$

$$= -\frac{\sqrt{d^2 - e^2 x^2} \operatorname{Subst} \left(\int \left(\frac{d^2}{e^2} - \frac{x^2}{e^2} \right)^2 \left(\frac{cd^4 + bd^2 e^2 + ae^4}{e^4} - \frac{(2cd^2 + be^2)x^2}{e^4} + \frac{cx^4}{e^4} \right) dx, x, \sqrt{d^2 - e^2 x^2} \right)}{e^2 \sqrt{d - ex} \sqrt{d + ex}}$$

$$= -\frac{\sqrt{d^2 - e^2 x^2} \operatorname{Subst} \left(\int \left(\frac{cd^8 + bd^6 e^2 + ad^4 e^4}{e^8} - \frac{d^2 (4cd^4 + 3bd^2 e^2 + 2ae^4)x^2}{e^8} + \frac{(6cd^4 + 3bd^2 e^2 + ae^4)x^4}{e^8} - (4cd^4 + 3bd^2 e^2 + ae^4)x^6 \right) dx, x, \sqrt{d^2 - e^2 x^2} \right)}{e^2 \sqrt{d - ex} \sqrt{d + ex}}$$

$$= -\frac{d^4 (cd^4 + bd^2 e^2 + ae^4) (d^2 - e^2 x^2)}{e^{10} \sqrt{d - ex} \sqrt{d + ex}} + \frac{d^2 (4cd^4 + 3bd^2 e^2 + 2ae^4) (d^2 - e^2 x^2)^2}{3e^{10} \sqrt{d - ex} \sqrt{d + ex}} - \frac{(6cd^4 + 3bd^2 e^2 + ae^4) (d^2 - e^2 x^2)^3}{3e^{10} \sqrt{d - ex} \sqrt{d + ex}}$$

Mathematica [C] time = 1.38, size = 265, normalized size = 1.26

$$\frac{630d^{9/2} \sqrt{d+ex} \sin^{-1}\left(\frac{\sqrt{d-ex}}{\sqrt{d+ex}}\right) (ae^4 + bd^2 e^2 + cd^4) - 630d^5 \tan^{-1}\left(\frac{\sqrt{d-ex}}{\sqrt{d+ex}}\right) (ae^4 + bd^2 e^2 + cd^4) + \sqrt{d-ex} \sqrt{d+ex} (21ae^4 (8d^4 + 4d^2 e^2 x^2 + 3e^4 x^4) + 9b (16d^6 e^2 + 8d^4 e^4 x^2 + 6d^2 e^6 x^4 + 5e^8 x^6) + c (128d^8 + 64d^6 e^2 x^2 + 48d^4 e^4 x^4 + 40d^2 e^6 x^6 + 35e^8 x^8))}{315e^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
[Out] -1/315*(Sqrt[d - e*x]*Sqrt[d + e*x]*(21*a*e^4*(8*d^4 + 4*d^2*e^2*x^2 + 3*e^4*x^4) + 9*b*(16*d^6*e^2 + 8*d^4*e^4*x^2 + 6*d^2*e^6*x^4 + 5*e^8*x^6) + c*(128*d^8 + 64*d^6*e^2*x^2 + 48*d^4*e^4*x^4 + 40*d^2*e^6*x^6 + 35*e^8*x^8)) + (630*d^(9/2)*(c*d^4 + b*d^2*e^2 + a*e^4)*Sqrt[d + e*x]*ArcSin[Sqrt[d - e*x]/(Sqrt[2]*Sqrt[d])])/Sqrt[1 + (e*x)/d] - 630*d^5*(c*d^4 + b*d^2*e^2 + a*e^4)*ArcTan[Sqrt[d - e*x]/Sqrt[d + e*x]])/e^10
```

IntegrateAlgebraic [B] time = 0.37, size = 615, normalized size = 2.93

$$\frac{2\sqrt{d+ex} \left(\frac{315cd^9}{(d-e)x} + \frac{315bd^7e^2}{(d-e)x} + \frac{315ad^5e^4}{(d-e)x} + (840cd^9*(d+e*x))/(d-e*x) + (1260bd^7e^2*(d+e*x))/(d-e*x) + (1680ad^5e^4*(d+e*x))/(d-e*x) + (4788cd^9*(d+e*x)^2)/(d-e*x)^2 + (4284bd^7e^2*(d+e*x)^2)/(d-e*x)^2 + (4788ad^5e^4*(d+e*x)^2)/(d-e*x)^2 + (5112cd^9*(d+e*x)^3)/(d-e*x)^3 + (7956bd^7e^2*(d+e*x)^3)/(d-e*x)^3 + (9072ad^5e^4*(d+e*x)^3)/(d-e*x)^3 + (10658cd^9*(d+e*x)^4)/(d-e*x)^4 + (9234bd^7e^2*(d+e*x)^4)/(d-e*x)^4 + (11298ad^5e^4*(d+e*x)^4)/(d-e*x)^4 + (5112cd^9*(d+e*x)^5)/(d-e*x)^5 + (7956bd^7e^2*(d+e*x)^5)/(d-e*x)^5 + (9072ad^5e^4*(d+e*x)^5)/(d-e*x)^5 + (4788cd^9*(d+e*x)^6)/(d-e*x)^6 + (4284bd^7e^2*(d+e*x)^6)/(d-e*x)^6 + (4788ad^5e^4*(d+e*x)^6)/(d-e*x)^6 + (840cd^9*(d+e*x)^7)/(d-e*x)^7 + (1260bd^7e^2*(d+e*x)^7)/(d-e*x)^7 + (1680ad^5e^4*(d+e*x)^7)/(d-e*x)^7 + (315cd^9*(d+e*x)^8)/(d-e*x)^8 + (315bd^7e^2*(d+e*x)^8)/(d-e*x)^8 + (315ad^5e^4*(d+e*x)^8)/(d-e*x)^8 \right) / (315e^{10} \sqrt{d-ex} (1 + (d+e*x)/(d-e*x))^9)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^5*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
[Out] (-2*Sqrt[d + e*x]*(315*c*d^9 + 315*b*d^7*e^2 + 315*a*d^5*e^4 + (840*c*d^9*(d + e*x))/(d - e*x) + (1260*b*d^7*e^2*(d + e*x))/(d - e*x) + (1680*a*d^5*e^4*(d + e*x))/(d - e*x) + (4788*c*d^9*(d + e*x)^2)/(d - e*x)^2 + (4284*b*d^7*e^2*(d + e*x)^2)/(d - e*x)^2 + (4788*a*d^5*e^4*(d + e*x)^2)/(d - e*x)^2 + (5112*c*d^9*(d + e*x)^3)/(d - e*x)^3 + (7956*b*d^7*e^2*(d + e*x)^3)/(d - e*x)^3 + (9072*a*d^5*e^4*(d + e*x)^3)/(d - e*x)^3 + (10658*c*d^9*(d + e*x)^4)/(d - e*x)^4 + (9234*b*d^7*e^2*(d + e*x)^4)/(d - e*x)^4 + (11298*a*d^5*e^4*(d + e*x)^4)/(d - e*x)^4 + (5112*c*d^9*(d + e*x)^5)/(d - e*x)^5 + (7956*b*d^7*e^2*(d + e*x)^5)/(d - e*x)^5 + (9072*a*d^5*e^4*(d + e*x)^5)/(d - e*x)^5 + (4788*c*d^9*(d + e*x)^6)/(d - e*x)^6 + (4284*b*d^7*e^2*(d + e*x)^6)/(d - e*x)^6 + (4788*a*d^5*e^4*(d + e*x)^6)/(d - e*x)^6 + (840*c*d^9*(d + e*x)^7)/(d - e*x)^7 + (1260*b*d^7*e^2*(d + e*x)^7)/(d - e*x)^7 + (1680*a*d^5*e^4*(d + e*x)^7)/(d - e*x)^7 + (315*c*d^9*(d + e*x)^8)/(d - e*x)^8 + (315*b*d^7*e^2*(d + e*x)^8)/(d - e*x)^8 + (315*a*d^5*e^4*(d + e*x)^8)/(d - e*x)^8))/(315*e^10*Sqrt[d - e*x]*(1 + (d + e*x)/(d - e*x))^9)
```

fricas [A] time = 1.36, size = 138, normalized size = 0.66

$$\frac{(35ce^8x^8 + 128cd^8 + 144bd^6e^2 + 168ad^4e^4 + 5(8cd^2e^6 + 9be^8)x^6 + 3(16cd^4e^4 + 18bd^2e^6 + 21ae^8)x^4 + 4(16cd^6e^2 + 18bd^4e^4 + 21ad^2e^6)x^2)\sqrt{ex+d}\sqrt{-ex+d}}{315e^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fricas")

[Out] $-1/315*(35*c*e^8*x^8 + 128*c*d^8 + 144*b*d^6*e^2 + 168*a*d^4*e^4 + 5*(8*c*d^2*e^6 + 9*b*e^8)*x^6 + 3*(16*c*d^4*e^4 + 18*b*d^2*e^6 + 21*a*e^8)*x^4 + 4*(16*c*d^6*e^2 + 18*b*d^4*e^4 + 21*a*d^2*e^6)*x^2)*\text{sqrt}(e*x + d)*\text{sqrt}(-e*x + d)/e^{10}$

giac [A] time = 0.89, size = 269, normalized size = 1.28

$$\frac{\frac{1}{315}(((6(7((ax+dx^{10}-8cd^{10}))^{10}+d)+3(68cd^{10}+3bd^{10}))^{10}+d)-2(220cd^{10}+27bd^{10}))^{10}+d)+(3098cd^{10}+729bd^{10}+63a^{10})^{10}+d)-36(82cd^{10}+31bd^{10}+7ad^{10}))^{10}+d)+21(92cd^{10}+51bd^{10}+22ad^{10}))^{10}+d)-210(4cd^{10}+3bd^{10}+2ad^{10}))^{10}+d)+315(cd^{10}+bd^{10}+ad^{10}))^{10}+d)\sqrt{ex+d}\sqrt{-ex+d}^{-1}}{315e^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="giac")

[Out] $-1/315*(((5*((7*((x*e+d)*c*e^{-9})-8*c*d*e^{-9})*(x*e+d)+3*(68*c*d^2*e^{81}+3*b*e^{83})*e^{-90})*(x*e+d)-2*(220*c*d^3*e^{81}+27*b*d*e^{83})*e^{-90})*(x*e+d)+(3098*c*d^4*e^{81}+729*b*d^2*e^{83}+63*a*e^{85})*e^{-90})*(x*e+d)-36*(82*c*d^5*e^{81}+31*b*d^3*e^{83}+7*a*d*e^{85})*e^{-90})*(x*e+d)+21*(92*c*d^6*e^{81}+51*b*d^4*e^{83}+22*a*d^2*e^{85})*e^{-90})*(x*e+d)-210*(4*c*d^7*e^{81}+3*b*d^5*e^{83}+2*a*d^3*e^{85})*e^{-90})*(x*e+d)+315*(c*d^8*e^{81}+b*d^6*e^{83}+a*d^4*e^{85})*e^{-90})*\text{sqrt}(x*e+d)*\text{sqrt}(-x*e+d)*e^{-1}$

maple [A] time = 0.01, size = 145, normalized size = 0.69

$$\frac{\sqrt{ex+d}\sqrt{-ex+d}(35cx^8e^8+45be^8x^6+40cd^2e^6x^6+63ae^8x^4+54bd^2e^6x^4+48cd^4e^4x^4+84ad^2e^6x^2+72bd^4e^4x^2+64cd^6e^2x^2+168ad^4e^4+144bd^6e^2+128cd^8)}{315e^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)

[Out] $-1/315*(e*x+d)^{(1/2)}*(-e*x+d)^{(1/2)}*(35*c*e^8*x^8+45*b*e^8*x^6+40*c*d^2*e^6*x^6+63*a*e^8*x^4+54*b*d^2*e^6*x^4+48*c*d^4*e^4*x^4+84*a*d^2*e^6*x^2+72*b*d^4*e^4*x^2+64*c*d^6*e^2*x^2+168*a*d^4*e^4+144*b*d^6*e^2+128*c*d^8)/e^{10}$

maxima [A] time = 1.03, size = 295, normalized size = 1.40

$$\frac{\sqrt{-e^2x^2+d^2}cx^8}{9e^2} - \frac{8\sqrt{-e^2x^2+d^2}cd^2x^6}{63e^4} - \frac{\sqrt{-e^2x^2+d^2}bd^4x^4}{7e^2} - \frac{16\sqrt{-e^2x^2+d^2}ad^4x^2}{105e^6} - \frac{6\sqrt{-e^2x^2+d^2}bd^2x^4}{35e^4} - \frac{\sqrt{-e^2x^2+d^2}ax^4}{5e^2} - \frac{64\sqrt{-e^2x^2+d^2}cd^6x^2}{315e^8} - \frac{8\sqrt{-e^2x^2+d^2}bd^4x^2}{35e^6} - \frac{4\sqrt{-e^2x^2+d^2}ad^2x^2}{15e^4} - \frac{128\sqrt{-e^2x^2+d^2}d^6}{315e^{10}} - \frac{16\sqrt{-e^2x^2+d^2}bd^6}{35e^8} - \frac{8\sqrt{-e^2x^2+d^2}ad^4}{15e^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="maxima")

[Out] $-1/9*\text{sqrt}(-e^2*x^2 + d^2)*c*x^8/e^2 - 8/63*\text{sqrt}(-e^2*x^2 + d^2)*c*d^2*x^6/e^4 - 1/7*\text{sqrt}(-e^2*x^2 + d^2)*b*x^4/e^2 - 16/105*\text{sqrt}(-e^2*x^2 + d^2)*c*d^4*x^4/e^6 - 6/35*\text{sqrt}(-e^2*x^2 + d^2)*b*d^2*x^4/e^4 - 1/5*\text{sqrt}(-e^2*x^2 + d^2)*a*x^4/e^2 - 64/315*\text{sqrt}(-e^2*x^2 + d^2)*c*d^6*x^2/e^8 - 8/35*\text{sqrt}(-e^2*x^2 + d^2)*b*d^4*x^2/e^6 - 4/15*\text{sqrt}(-e^2*x^2 + d^2)*a*d^2*x^2/e^4 - 128/315*\text{sqrt}(-e^2*x^2 + d^2)*c*d^8/e^{10} - 16/35*\text{sqrt}(-e^2*x^2 + d^2)*b*d^6/e^8 - 8/15*\text{sqrt}(-e^2*x^2 + d^2)*a*d^4/e^6$

mupad [B] time = 1.65, size = 287, normalized size = 1.37

$$\frac{\sqrt{d-ex}\left(\frac{128cd^8+144bd^6e^2+168ad^4e^4}{315e^{10}} + \frac{x^2(40c^2e^2+45be^2)}{315e^{10}} + \frac{x^2(64cd^2e^2+72bd^2e^2+84ad^2e^2)}{315e^{10}} + \frac{c^2}{9e} + \frac{x^2(48cd^4e^2+54bd^2e^2+63ae^2)}{315e^{10}} + \frac{x(128cd^6e^2+144bd^4e^2+168ad^4e^2)}{315e^{10}} + \frac{x^2(40c^2e^2+45bd^2e^2)}{315e^{10}} + \frac{x^2(48cd^4e^2+54bd^2e^2+63ad^2e^2)}{315e^{10}} + \frac{cdx^8}{9e^2}\right)}{\sqrt{d+ex}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*x^2 + c*x^4))/((d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

```
[Out] -((d - e*x)^(1/2)*((128*c*d^9 + 168*a*d^5*e^4 + 144*b*d^7*e^2)/(315*e^10) +
(x^7*(45*b*e^9 + 40*c*d^2*e^7))/(315*e^10) + (x^2*(84*a*d^3*e^6 + 72*b*d^5
*e^4 + 64*c*d^7*e^2))/(315*e^10) + (x^3*(84*a*d^2*e^7 + 72*b*d^4*e^5 + 64*c
*d^6*e^3))/(315*e^10) + (c*x^9)/(9*e) + (x^5*(63*a*e^9 + 54*b*d^2*e^7 + 48*
c*d^4*e^5))/(315*e^10) + (x*(168*a*d^4*e^5 + 144*b*d^6*e^3 + 128*c*d^8*e))/
(315*e^10) + (x^6*(40*c*d^3*e^6 + 45*b*d*e^8))/(315*e^10) + (x^4*(54*b*d^3*
e^6 + 48*c*d^5*e^4 + 63*a*d*e^8))/(315*e^10) + (c*d*x^8)/(9*e^2)))/(d + e*x
)^(1/2)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(c*x**4+b*x**2+a)/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```


$$3.131 \quad \int \frac{x^3(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}} dx$$

Optimal. Leaf size=159

$$\frac{(d-ex)^{3/2}(d+ex)^{3/2}(ae^4+2bd^2e^2+3cd^4)}{3e^8} - \frac{d^2\sqrt{d-ex}\sqrt{d+ex}(ae^4+bd^2e^2+cd^4)}{e^8} - \frac{(d-ex)^{5/2}(d+ex)^{5/2}(bd^2e^2+3cd^4)}{5e^8}$$

Rubi [A] time = 0.19, antiderivative size = 213, normalized size of antiderivative = 1.34, number of steps used = 4, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {520, 1251, 771}

$$\frac{(d^2-e^2x^2)^2(ae^4+2bd^2e^2+3cd^4)}{3e^8\sqrt{d-ex}\sqrt{d+ex}} - \frac{d^2(d^2-e^2x^2)(ae^4+bd^2e^2+cd^4)}{e^8\sqrt{d-ex}\sqrt{d+ex}} - \frac{(d^2-e^2x^2)^3(be^2+3cd^2)}{5e^8\sqrt{d-ex}\sqrt{d+ex}} + \frac{c(d^2-e^2x^2)^4}{7e^8\sqrt{d-ex}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

[In] Int[(x^3*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]

[Out] -((d^2*(c*d^4 + b*d^2*e^2 + a*e^4)*(d^2 - e^2*x^2))/(e^8*Sqrt[d - e*x]*Sqrt[d + e*x])) + ((3*c*d^4 + 2*b*d^2*e^2 + a*e^4)*(d^2 - e^2*x^2)^2)/(3*e^8*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((3*c*d^2 + b*e^2)*(d^2 - e^2*x^2)^3)/(5*e^8*Sqrt[d - e*x]*Sqrt[d + e*x]) + (c*(d^2 - e^2*x^2)^4)/(7*e^8*Sqrt[d - e*x]*Sqrt[d + e*x])

Rule 520

Int[(u_)*((c_) + (d_)*(x_)^(n_)) + (e_)*(x_)^(n2_)]^(q_)*((a1_) + (b1_)*(x_)^(non2_))^(p_)*((a2_) + (b2_)*(x_)^(non2_))^(p_), x_Symbol] :> Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n))^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]

Rule 771

Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))

Rule 1251

Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :> Dist[1/2, Subst[Int[x^((m-1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m-1)/2]

Rubi steps

$$\int \frac{x^3 (a + bx^2 + cx^4)}{\sqrt{d - ex} \sqrt{d + ex}} dx = \frac{\sqrt{d^2 - e^2 x^2} \int \frac{x^3 (a + bx^2 + cx^4)}{\sqrt{d^2 - e^2 x^2}} dx}{\sqrt{d - ex} \sqrt{d + ex}}$$

$$= \frac{\sqrt{d^2 - e^2 x^2} \operatorname{Subst} \left(\int \frac{x (a + bx + cx^2)}{\sqrt{d^2 - e^2 x}} dx, x, x^2 \right)}{2\sqrt{d - ex} \sqrt{d + ex}}$$

$$= \frac{\sqrt{d^2 - e^2 x^2} \operatorname{Subst} \left(\int \left(\frac{cd^6 + bd^4 e^2 + ad^2 e^4}{e^6 \sqrt{d^2 - e^2 x}} + \frac{(-3cd^4 - 2bd^2 e^2 - ae^4) \sqrt{d^2 - e^2 x}}{e^6} + \frac{(3cd^2 + be^2)(d^2 - e^2 x)^{3/2}}{e^6} \right) dx, x, x^2 \right)}{2\sqrt{d - ex} \sqrt{d + ex}}$$

$$= -\frac{d^2 (cd^4 + bd^2 e^2 + ae^4) (d^2 - e^2 x^2)}{e^8 \sqrt{d - ex} \sqrt{d + ex}} + \frac{(3cd^4 + 2bd^2 e^2 + ae^4) (d^2 - e^2 x^2)^2}{3e^8 \sqrt{d - ex} \sqrt{d + ex}} - \frac{(3cd^2 + be^2) (d^2 - e^2 x)^{3/2}}{5e^8 \sqrt{d - ex} \sqrt{d + ex}}$$

Mathematica [C] time = 1.07, size = 232, normalized size = 1.46

$$\frac{210d^{5/2} \sqrt{d+ex} \sin^{-1} \left(\frac{\sqrt{d-ex}}{\sqrt{2}\sqrt{d}} \right) (ae^4 + bd^2 e^2 + cd^4)}{\sqrt{\frac{ex}{d} + 1}} + \sqrt{d - ex} \sqrt{d + ex} (35ae^4 (2d^2 + e^2 x^2) + 7b (8d^4 e^2 + 4d^2 e^4 x^2 + 3e^6 x^4) + 3c (16d^6 + 8d^4 e^2 x^2 + 6d^2 e^4 x^4 + 5e^6 x^6)) - 210d^3 \tan^{-1} \left(\frac{\sqrt{d-ex}}{\sqrt{d+ex}} \right) (ae^4 + bd^2 e^2 + cd^4)}{105e^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -1/105*(Sqrt[d - e*x]*Sqrt[d + e*x]*(35*a*e^4*(2*d^2 + e^2*x^2) + 7*b*(8*d^4*e^2 + 4*d^2*e^4*x^2 + 3*e^6*x^4) + 3*c*(16*d^6 + 8*d^4*e^2*x^2 + 6*d^2*e^4*x^4 + 5*e^6*x^6)) + (210*d^(5/2)*(c*d^4 + b*d^2*e^2 + a*e^4)*Sqrt[d + e*x]*ArcSin[Sqrt[d - e*x]/(Sqrt[2]*Sqrt[d])])/Sqrt[1 + (e*x)/d] - 210*d^3*(c*d^4 + b*d^2*e^2 + a*e^4)*ArcTan[Sqrt[d - e*x]/Sqrt[d + e*x]])/e^8
```

IntegrateAlgebraic [B] time = 0.29, size = 477, normalized size = 3.00

$$\frac{2\sqrt{d+ex} \left(\frac{105a^2e^4(d+e^2x)^2}{(d-e^2x)^2} + \frac{210a^2e^4(d+e^2x)}{(d-e^2x)} + \frac{105a^2e^4(d+e^2x)^2}{(d-e^2x)^2} + \frac{120a^2e^4(d+e^2x)}{(d-e^2x)} + \frac{105a^2e^4(d+e^2x)^2}{(d-e^2x)^2} + \frac{850a^2e^4(d+e^2x)}{d-e^2x} + 105a^2e^4 + \frac{105a^2e^4(d+e^2x)^2}{(d-e^2x)^2} + \frac{350a^2e^4(d+e^2x)}{(d-e^2x)} + \frac{791a^2e^4(d+e^2x)}{(d-e^2x)} + \frac{1092a^2e^4(d+e^2x)^2}{(d-e^2x)^2} + \frac{791a^2e^4(d+e^2x)}{(d-e^2x)} + \frac{350a^2e^4(d+e^2x)}{d-e^2x} + 105a^2e^4 + \frac{105a^2e^4(d+e^2x)^2}{(d-e^2x)^2} + \frac{210a^2e^4(d+e^2x)}{(d-e^2x)} + \frac{903a^2e^4(d+e^2x)}{(d-e^2x)} + \frac{903a^2e^4(d+e^2x)}{(d-e^2x)} + \frac{903a^2e^4(d+e^2x)}{(d-e^2x)} + \frac{210a^2e^4(d+e^2x)}{d-e^2x} + 105a^2e^4 \right)}{105e^8 \sqrt{d-ex} \left(\frac{ex}{d} + 1 \right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(105*c*d^7 + 105*b*d^5*e^2 + 105*a*d^3*e^4 + (210*c*d^7*(d + e*x))/(d - e*x)))/(d - e*x) + (350*b*d^5*e^2*(d + e*x))/(d - e*x) + (490*a*d^3*e^4*(d + e*x))/(d - e*x) + (903*c*d^7*(d + e*x)^2)/(d - e*x)^2 + (791*b*d^5*e^2*(d + e*x)^2)/(d - e*x)^2 + (1015*a*d^3*e^4*(d + e*x)^2)/(d - e*x)^2 + (636*c*d^7*(d + e*x)^3)/(d - e*x)^3 + (1092*b*d^5*e^2*(d + e*x)^3)/(d - e*x)^3 + (1260*a*d^3*e^4*(d + e*x)^3)/(d - e*x)^3 + (903*c*d^7*(d + e*x)^4)/(d - e*x)^4 + (791*b*d^5*e^2*(d + e*x)^4)/(d - e*x)^4 + (1015*a*d^3*e^4*(d + e*x)^4)/(d - e*x)^4 + (210*c*d^7*(d + e*x)^5)/(d - e*x)^5 + (350*b*d^5*e^2*(d + e*x)^5)/(d - e*x)^5 + (490*a*d^3*e^4*(d + e*x)^5)/(d - e*x)^5 + (105*c*d^7*(d + e*x)^6)/(d - e*x)^6 + (105*b*d^5*e^2*(d + e*x)^6)/(d - e*x)^6 + (105*a*d^3*e^4*(d + e*x)^6)/(d - e*x)^6)/(105*e^8*Sqrt[d - e*x]*(1 + (d + e*x)/(d - e*x))^7)
```

fricas [A] time = 1.25, size = 104, normalized size = 0.65

$$\frac{(15ce^6x^6 + 48cd^6 + 56bd^4e^2 + 70ad^2e^4 + 3(6cd^4e^4 + 7be^6)x^4 + (24cd^4e^2 + 28bd^2e^4 + 35ae^6)x^2)\sqrt{ex + d}\sqrt{-ex + d}}{105e^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fricas")

[Out]
$$-1/105*(15*c*e^6*x^6 + 48*c*d^6 + 56*b*d^4*e^2 + 70*a*d^2*e^4 + 3*(6*c*d^2*e^4 + 7*b*e^6)*x^4 + (24*c*d^4*e^2 + 28*b*d^2*e^4 + 35*a*e^6)*x^2)*\sqrt{e*x + d}*\sqrt{-e*x + d}/e^8$$

giac [A] time = 0.75, size = 194, normalized size = 1.22

$$-\frac{1}{105}(((3((5((x+d)ce^{7-7})-6cd^{6-7})(x+d) + (81cd^2e^{49} + 7b^2e^{51})e^{-56})(x+d) - 4(31cd^2e^{49} + 7bd^2e^{51})e^{-56})(x+d) + 7(51cd^2e^{49} + 22bd^2e^{51} + 5ae^{53})e^{-56})(x+d) - 70(3cd^2e^{49} + 2bd^2e^{51} + ade^{53})e^{-56})(x+d) + 105(cd^6e^{49} + bd^4e^{51} + ad^2e^{53})e^{-56})\sqrt{x+d}\sqrt{-x+d}e^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="giac")

[Out]
$$-1/105*(((3*((5*((x*e + d)*c*e^{-7} - 6*c*d*e^{-7})*(x*e + d) + (81*c*d^2*e^{49} + 7*b*e^{51})*e^{-56})*(x*e + d) - 4*(31*c*d^3*e^{49} + 7*b*d*e^{51})*e^{-56})*(x*e + d) + 7*(51*c*d^4*e^{49} + 22*b*d^2*e^{51} + 5*a*e^{53})*e^{-56})*(x*e + d) - 70*(3*c*d^5*e^{49} + 2*b*d^3*e^{51} + a*d*e^{53})*e^{-56})*(x*e + d) + 105*(c*d^6*e^{49} + b*d^4*e^{51} + a*d^2*e^{53})*e^{-56})*\sqrt{x*e + d}*\sqrt{-x*e + d}*e^{-1}$$

maple [A] time = 0.01, size = 109, normalized size = 0.69

$$\frac{\sqrt{ex+d}\sqrt{-ex+d}(15cx^6e^6 + 21be^6x^4 + 18cd^2e^4x^4 + 35ae^6x^2 + 28bd^2e^4x^2 + 24cd^4e^2x^2 + 70ad^2e^4 + 56bd^4e^2 + 48cd^6)}{105e^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)

[Out]
$$-1/105*(e*x+d)^{(1/2)}*(-e*x+d)^{(1/2)}*(15*c*e^6*x^6+21*b*e^6*x^4+18*c*d^2*e^4*x^4+35*a*e^6*x^2+28*b*d^2*e^4*x^2+24*c*d^4*e^2*x^2+70*a*d^2*e^4+56*b*d^4*e^2+48*c*d^6)/e^8$$

maxima [A] time = 1.06, size = 217, normalized size = 1.36

$$\frac{\sqrt{-e^2x^2+d^2}cx^6}{7e^2} - \frac{6\sqrt{-e^2x^2+d^2}cd^2x^4}{35e^4} - \frac{\sqrt{-e^2x^2+d^2}bx^4}{5e^2} - \frac{8\sqrt{-e^2x^2+d^2}cd^4x^2}{35e^6} - \frac{4\sqrt{-e^2x^2+d^2}bd^2x^2}{15e^4} - \frac{\sqrt{-e^2x^2+d^2}ax^2}{3e^2} - \frac{16\sqrt{-e^2x^2+d^2}cd^6}{35e^8} - \frac{8\sqrt{-e^2x^2+d^2}bd^4}{15e^6} - \frac{2\sqrt{-e^2x^2+d^2}ad^2}{3e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="maxima")

[Out]
$$-1/7*\sqrt{-e^2*x^2 + d^2}*c*x^6/e^2 - 6/35*\sqrt{-e^2*x^2 + d^2}*c*d^2*x^4/e^4 - 1/5*\sqrt{-e^2*x^2 + d^2}*b*x^4/e^2 - 8/35*\sqrt{-e^2*x^2 + d^2}*c*d^4*x^2/e^6 - 4/15*\sqrt{-e^2*x^2 + d^2}*b*d^2*x^2/e^4 - 1/3*\sqrt{-e^2*x^2 + d^2}*a*x^2/e^2 - 16/35*\sqrt{-e^2*x^2 + d^2}*c*d^6/e^8 - 8/15*\sqrt{-e^2*x^2 + d^2}*b*d^4/e^6 - 2/3*\sqrt{-e^2*x^2 + d^2}*a*d^2/e^4$$

mupad [B] time = 1.49, size = 215, normalized size = 1.35

$$\frac{\sqrt{d-ex}\left(\frac{48cd^7+56bd^5e^2+70ad^3e^4}{105e^8} + \frac{x^3(18cd^2e^5+21be^7)}{105e^8} + \frac{cx^7}{7e} + \frac{x^3(24cd^4e^3+28bd^2e^5+35ae^7)}{105e^8} + \frac{x(48cd^6e+56bd^4e^3+70ad^2e^5)}{105e^8} + \frac{x^4(18cd^3e^4+21bd^5e^6)}{105e^8} + \frac{x^2(24cd^5e^2+28bd^3e^4+35ad^7e^6)}{105e^8} + \frac{cdx^6}{7e^2}\right)}{\sqrt{d+ex}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3*(a + b*x^2 + c*x^4))/((d + e*x)^(1/2)*(d - e*x)^(1/2)),x)

[Out]
$$-((d - e*x)^{(1/2)}*((48*c*d^7 + 70*a*d^3*e^4 + 56*b*d^5*e^2)/(105*e^8) + (x^5*(21*b*e^7 + 18*c*d^2*e^5))/(105*e^8) + (c*x^7)/(7*e) + (x^3*(35*a*e^7 + 8*b*d^2*e^5 + 24*c*d^4*e^3))/(105*e^8) + (x*(70*a*d^2*e^5 + 56*b*d^4*e^3 + 48*c*d^6*e))/(105*e^8) + (x^4*(18*c*d^3*e^4 + 21*b*d^5*e^6))/(105*e^8) + (x^2$$

$$\frac{(28*b*d^3*e^4 + 24*c*d^5*e^2 + 35*a*d*e^6)/(105*e^8) + (c*d*x^6)/(7*e^2)}{(d + e*x)^{1/2}}$$

sympy [C] time = 135.14, size = 367, normalized size = 2.31

$$\frac{{}_2F_2\left(\frac{5}{4}, \frac{3}{4}; -1, -\frac{1}{2}; \frac{x}{4e^2}\right)}{4e^{3/2}} - \frac{{}_2F_2\left(-2, -\frac{3}{2}; \frac{5}{4}, -1, 1; -\frac{x}{4e^2}\right)}{4e^{3/2}} - \frac{{}_2F_2\left(\frac{9}{4}, \frac{7}{4}; -2, -\frac{3}{2}; \frac{x}{4e^2}\right)}{4e^{3/2}} - \frac{{}_2F_2\left(-3, -\frac{5}{2}; \frac{3}{4}, -2, 1; -\frac{x}{4e^2}\right)}{4e^{3/2}} - \frac{{}_2F_2\left(\frac{13}{4}, \frac{11}{4}; -3, -\frac{5}{2}; \frac{x}{4e^2}\right)}{4e^{3/2}} - \frac{{}_2F_2\left(-4, \frac{15}{4}, \frac{7}{4}, \frac{13}{4}, -3, 1; -\frac{x}{4e^2}\right)}{4e^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**4+b*x**2+a)/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] -I*a*d**3*meijerg((( -5/4, -3/4), (-1, -1, -1/2, 1)), ((-3/2, -5/4, -1, -3/4, -1/2, 0), ()), d**2/(e**2*x**2))/(4*pi**(3/2)*e**4) - a*d**3*meijerg((( -2, -7/4, -3/2, -5/4, -1, 1), ()), ((-7/4, -5/4), (-2, -3/2, -3/2, 0)), d**2*exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*e**4) - I*b*d**5*meijerg((( -9/4, -7/4), (-2, -2, -3/2, 1)), ((-5/2, -9/4, -2, -7/4, -3/2, 0), ()), d**2/(e**2*x**2))/(4*pi**(3/2)*e**6) - b*d**5*meijerg((( -3, -11/4, -5/2, -9/4, -2, 1), ()), ((-11/4, -9/4), (-3, -5/2, -5/2, 0)), d**2*exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*e**6) - I*c*d**7*meijerg((( -13/4, -11/4), (-3, -3, -5/2, 1)), ((-7/2, -13/4, -3, -11/4, -5/2, 0), ()), d**2/(e**2*x**2))/(4*pi**(3/2)*e**8) - c*d**7*meijerg((( -4, -15/4, -7/2, -13/4, -3, 1), ()), ((-15/4, -13/4), (-4, -7/2, -7/2, 0)), d**2*exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*e**8)
```

$$3.132 \quad \int \frac{x(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}} dx$$

Optimal. Leaf size=109

$$\frac{\sqrt{d-ex}\sqrt{d+ex}(ae^4+bd^2e^2+cd^4)}{e^6} + \frac{(d-ex)^{3/2}(d+ex)^{3/2}(be^2+2cd^2)}{3e^6} - \frac{c(d-ex)^{5/2}(d+ex)^{5/2}}{5e^6}$$

Rubi [A] time = 0.12, antiderivative size = 149, normalized size of antiderivative = 1.37, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {520, 1247, 698}

$$-\frac{(d^2-e^2x^2)(ae^4+bd^2e^2+cd^4)}{e^6\sqrt{d-ex}\sqrt{d+ex}} + \frac{(d^2-e^2x^2)^2(be^2+2cd^2)}{3e^6\sqrt{d-ex}\sqrt{d+ex}} - \frac{c(d^2-e^2x^2)^3}{5e^6\sqrt{d-ex}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

[In] Int[(x*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]

[Out] -(((c*d^4 + b*d^2*e^2 + a*e^4)*(d^2 - e^2*x^2))/(e^6*Sqrt[d - e*x]*Sqrt[d + e*x])) + ((2*c*d^2 + b*e^2)*(d^2 - e^2*x^2)^2)/(3*e^6*Sqrt[d - e*x]*Sqrt[d + e*x]) - (c*(d^2 - e^2*x^2)^3)/(5*e^6*Sqrt[d - e*x]*Sqrt[d + e*x])

Rule 520

Int[(u_.)*((c_.) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_.) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_.) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n))^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]

Rule 698

Int[((d_.) + (e_.)*(x_)^(m_.))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))

Rule 1247

Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]

Rubi steps

$$\begin{aligned}
\int \frac{x(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}} dx &= \frac{\sqrt{d^2-e^2x^2} \int \frac{x(a+bx^2+cx^4)}{\sqrt{d^2-e^2x^2}} dx}{\sqrt{d-ex}\sqrt{d+ex}} \\
&= \frac{\sqrt{d^2-e^2x^2} \operatorname{Subst}\left(\int \frac{a+bx+cx^2}{\sqrt{d^2-e^2x}} dx, x, x^2\right)}{2\sqrt{d-ex}\sqrt{d+ex}} \\
&= \frac{\sqrt{d^2-e^2x^2} \operatorname{Subst}\left(\int \left(\frac{cd^4+bd^2e^2+ae^4}{e^4\sqrt{d^2-e^2x}} + \frac{(-2cd^2-be^2)\sqrt{d^2-e^2x}}{e^4} + \frac{c(d^2-e^2x)^{3/2}}{e^4}\right) dx, x, x^2\right)}{2\sqrt{d-ex}\sqrt{d+ex}} \\
&= -\frac{(cd^4+bd^2e^2+ae^4)(d^2-e^2x^2)}{e^6\sqrt{d-ex}\sqrt{d+ex}} + \frac{(2cd^2+be^2)(d^2-e^2x^2)^2}{3e^6\sqrt{d-ex}\sqrt{d+ex}} - \frac{c(d^2-e^2x^2)^3}{5e^6\sqrt{d-ex}\sqrt{d+ex}}
\end{aligned}$$

Mathematica [C] time = 0.69, size = 194, normalized size = 1.78

$$\frac{\sqrt{d-ex}\sqrt{d+ex} \left(5(3ae^4+2bd^2e^2+be^4x^2)+c(8d^4+4d^2e^2x^2+3e^4x^4)\right) + \frac{30\sqrt{d}\sqrt{d+ex} \sin^{-1}\left(\frac{\sqrt{d-ex}}{\sqrt{2}\sqrt{d}}\right) (ae^4+bd^2e^2+cd^4)}{\sqrt{\frac{ex}{d}+1}} - 30d \tan^{-1}\left(\frac{\sqrt{d-ex}}{\sqrt{d+ex}}\right) (ae^4+bd^2e^2+cd^4)}{15e^6}$$

Antiderivative was successfully verified.

[In] Integrate[(x*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]), x]

[Out] -1/15*(Sqrt[d - e*x]*Sqrt[d + e*x]*(5*(2*b*d^2*e^2 + 3*a*e^4 + b*e^4*x^2) + c*(8*d^4 + 4*d^2*e^2*x^2 + 3*e^4*x^4)) + (30*Sqrt[d]*(c*d^4 + b*d^2*e^2 + a*e^4)*Sqrt[d + e*x]*ArcSin[Sqrt[d - e*x]/(Sqrt[2]*Sqrt[d])])/Sqrt[1 + (e*x)/d] - 30*d*(c*d^4 + b*d^2*e^2 + a*e^4)*ArcTan[Sqrt[d - e*x]/Sqrt[d + e*x]])/e^6

IntegrateAlgebraic [B] time = 0.21, size = 329, normalized size = 3.02

$$\frac{2\sqrt{d+ex} \left(\frac{15ade^4(d+ex)^4}{(d-ex)^4} + \frac{60ade^4(d+ex)^3}{(d-ex)^3} + \frac{90ade^4(d+ex)^2}{(d-ex)^2} + \frac{60ade^4(d+ex)}{d-ex} + 15ade^4 + \frac{15bd^2e^2(d+ex)^4}{(d-ex)^4} + \frac{40bd^2e^2(d+ex)^3}{(d-ex)^3} + \frac{50bd^2e^2(d+ex)^2}{(d-ex)^2} + \frac{40bd^2e^2(d+ex)}{d-ex} + 15bd^2e^2 + \frac{15cf^5(d+ex)^4}{(d-ex)^4} + \frac{20cf^5(d+ex)^3}{(d-ex)^3} + \frac{58cf^5(d+ex)^2}{(d-ex)^2} + \frac{20cf^5(d+ex)}{d-ex} + 15cd^5\right)}{15e^6\sqrt{d-ex} \left(\frac{d+ex}{d-ex} + 1\right)^5}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]), x]

[Out] (-2*Sqrt[d + e*x]*(15*c*d^5 + 15*b*d^3*e^2 + 15*a*d*e^4 + (20*c*d^5*(d + e*x))/(d - e*x) + (40*b*d^3*e^2*(d + e*x))/(d - e*x) + (60*a*d*e^4*(d + e*x))/(d - e*x) + (58*c*d^5*(d + e*x)^2)/(d - e*x)^2 + (50*b*d^3*e^2*(d + e*x)^2)/(d - e*x)^2 + (90*a*d*e^4*(d + e*x)^2)/(d - e*x)^2 + (20*c*d^5*(d + e*x)^3)/(d - e*x)^3 + (40*b*d^3*e^2*(d + e*x)^3)/(d - e*x)^3 + (60*a*d*e^4*(d + e*x)^3)/(d - e*x)^3 + (15*c*d^5*(d + e*x)^4)/(d - e*x)^4 + (15*b*d^3*e^2*(d + e*x)^4)/(d - e*x)^4 + (15*a*d*e^4*(d + e*x)^4)/(d - e*x)^4)/(15*e^6*Sqrt[d - e*x]*(1 + (d + e*x)/(d - e*x))^5)

fricas [A] time = 1.39, size = 71, normalized size = 0.65

$$\frac{(3ce^4x^4 + 8cd^4 + 10bd^2e^2 + 15ae^4 + (4cd^2e^2 + 5be^4)x^2)\sqrt{ex+d}\sqrt{-ex+d}}{15e^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2), x, algorithm="fricas")

[Out] -1/15*(3*c*e^4*x^4 + 8*c*d^4 + 10*b*d^2*e^2 + 15*a*e^4 + (4*c*d^2*e^2 + 5*b*e^4)*x^2)*sqrt(e*x + d)*sqrt(-e*x + d)/e^6

giac [A] time = 0.63, size = 121, normalized size = 1.11

$$-\frac{1}{15} \left((3((xe+d)ce^{(-5)} - 4cde^{(-5)})(xe+d) + (22cd^2e^{25} + 5be^{27})e^{(-30)})(xe+d) - 10(2cd^3e^{25} + bde^{27})e^{(-30)}(xe+d) + 15(cd^4e^{25} + bd^2e^{27} + ae^{29})e^{(-30)} \right) \sqrt{xe+d} \sqrt{-xe+d} e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="giac")

[Out] -1/15*((3*((x*e + d)*c*e^(-5) - 4*c*d*e^(-5))*(x*e + d) + (22*c*d^2*e^25 + 5*b*e^27)*e^(-30))*(x*e + d) - 10*(2*c*d^3*e^25 + b*d*e^27)*e^(-30))*(x*e + d) + 15*(c*d^4*e^25 + b*d^2*e^27 + a*e^29)*e^(-30))*sqrt(x*e + d)*sqrt(-x*e + d)*e^(-1)

maple [A] time = 0.01, size = 73, normalized size = 0.67

$$\frac{\sqrt{ex+d} \sqrt{-ex+d} (3cx^4e^4 + 5be^4x^2 + 4cd^2e^2x^2 + 15ae^4 + 10bd^2e^2 + 8cd^4)}{15e^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)

[Out] -1/15*(e*x+d)^(1/2)*(-e*x+d)^(1/2)*(3*c*e^4*x^4+5*b*e^4*x^2+4*c*d^2*e^2*x^2+15*a*e^4+10*b*d^2*e^2+8*c*d^4)/e^6

maxima [A] time = 1.03, size = 139, normalized size = 1.28

$$\frac{\sqrt{-e^2x^2+d^2}cx^4}{5e^2} - \frac{4\sqrt{-e^2x^2+d^2}cd^2x^2}{15e^4} - \frac{\sqrt{-e^2x^2+d^2}bx^2}{3e^2} - \frac{8\sqrt{-e^2x^2+d^2}cd^4}{15e^6} - \frac{2\sqrt{-e^2x^2+d^2}bd^2}{3e^4} - \frac{\sqrt{-e^2x^2+d^2}a}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="maxima")

[Out] -1/5*sqrt(-e^2*x^2 + d^2)*c*x^4/e^2 - 4/15*sqrt(-e^2*x^2 + d^2)*c*d^2*x^2/e^4 - 1/3*sqrt(-e^2*x^2 + d^2)*b*x^2/e^2 - 8/15*sqrt(-e^2*x^2 + d^2)*c*d^4/e^6 - 2/3*sqrt(-e^2*x^2 + d^2)*b*d^2/e^4 - sqrt(-e^2*x^2 + d^2)*a/e^2

mupad [B] time = 1.38, size = 143, normalized size = 1.31

$$\frac{\sqrt{d-ex} \left(\frac{8cd^5+10bd^3e^2+15ade^4}{15e^6} + \frac{x^3(4cd^2e^3+5be^5)}{15e^6} + \frac{cx^5}{5e} + \frac{x^2(4cd^3e^2+5bde^4)}{15e^6} + \frac{x(8cd^4e+10bd^2e^3+15ae^5)}{15e^6} + \frac{cdx^4}{5e^2} \right)}{\sqrt{d+ex}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*(a + b*x^2 + c*x^4))/((d + e*x)^(1/2)*(d - e*x)^(1/2)),x)

[Out] -((d - e*x)^(1/2))*((8*c*d^5 + 10*b*d^3*e^2 + 15*a*d*e^4)/(15*e^6) + (x^3*(5*b*e^5 + 4*c*d^2*e^3))/(15*e^6) + (c*x^5)/(5*e) + (x^2*(4*c*d^3*e^2 + 5*b*d*e^4))/(15*e^6) + (x*(15*a*e^5 + 10*b*d^2*e^3 + 8*c*d^4*e))/(15*e^6) + (c*d*x^4)/(5*e^2))/((d + e*x)^(1/2))

sympy [C] time = 90.67, size = 350, normalized size = 3.21

$$\frac{\int \frac{a + bx^2 + cx^4}{\sqrt{d+ex}\sqrt{d-ex}} dx}{4n^{\frac{1}{2}}e^{\frac{1}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x**4+b*x**2+a)/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)

```
[Out] -I*a*d*meijerg((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4, 1/2, 0),
(), d**2/(e**2*x**2))/(4*pi**(3/2)*e**2) - a*d*meijerg((-1, -3/4, -1/2,
-1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), d**2*exp_polar(-2*I
*pi)/(e**2*x**2))/(4*pi**(3/2)*e**2) - I*b*d**3*meijerg((-5/4, -3/4), (-1,
-1, -1/2, 1)), ((-3/2, -5/4, -1, -3/4, -1/2, 0), ()), d**2/(e**2*x**2))/(4
*pi**(3/2)*e**4) - b*d**3*meijerg((-2, -7/4, -3/2, -5/4, -1, 1), ()), ((-7
/4, -5/4), (-2, -3/2, -3/2, 0)), d**2*exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi
**(3/2)*e**4) - I*c*d**5*meijerg((-9/4, -7/4), (-2, -2, -3/2, 1)), ((-5/2,
-9/4, -2, -7/4, -3/2, 0), ()), d**2/(e**2*x**2))/(4*pi**(3/2)*e**6) - c*d*
*5*meijerg((-3, -11/4, -5/2, -9/4, -2, 1), ()), ((-11/4, -9/4), (-3, -5/2,
-5/2, 0)), d**2*exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*e**6)
```


$$3.133 \quad \int \frac{a+bx^2+cx^4}{x\sqrt{d-ex}\sqrt{d+ex}} dx$$

Optimal. Leaf size=93

$$-\frac{a \tanh^{-1}\left(\frac{\sqrt{d-ex}\sqrt{d+ex}}{d}\right)}{d} - \frac{\sqrt{d-ex}\sqrt{d+ex}(be^2+cd^2)}{e^4} + \frac{c(d-ex)^{3/2}(d+ex)^{3/2}}{3e^4}$$

Rubi [A] time = 0.16, antiderivative size = 151, normalized size of antiderivative = 1.62, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {520, 1251, 897, 1153, 208}

$$-\frac{a\sqrt{d^2-e^2x^2} \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d\sqrt{d-ex}\sqrt{d+ex}} - \frac{(d^2-e^2x^2)(be^2+cd^2)}{e^4\sqrt{d-ex}\sqrt{d+ex}} + \frac{c(d^2-e^2x^2)^2}{3e^4\sqrt{d-ex}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2 + c*x^4)/(x*Sqrt[d - e*x]*Sqrt[d + e*x]),x]

[Out] -(((c*d^2 + b*e^2)*(d^2 - e^2*x^2))/(e^4*Sqrt[d - e*x]*Sqrt[d + e*x])) + (c*(d^2 - e^2*x^2)^2)/(3*e^4*Sqrt[d - e*x]*Sqrt[d + e*x]) - (a*Sqrt[d^2 - e^2*x^2]*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(d*Sqrt[d - e*x]*Sqrt[d + e*x])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 520

Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^ (q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^ (p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^ (p_.), x_Symbol] :> Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n))^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]

Rule 897

Int[((d_.) + (e_.)*(x_)^(m_.))*((f_.) + (g_.)*(x_)^(n_.))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^ (p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e + a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2]^p, x], x, (d + e*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && FractionQ[m]

Rule 1153

Int[((d_) + (e_.)*(x_)^2)^ (q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^ (p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]

Rule 1251

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^ (q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^ (p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte

gerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned}
\int \frac{a + bx^2 + cx^4}{x\sqrt{d - ex}\sqrt{d + ex}} dx &= \frac{\sqrt{d^2 - e^2x^2} \int \frac{a + bx^2 + cx^4}{x\sqrt{d^2 - e^2x^2}} dx}{\sqrt{d - ex}\sqrt{d + ex}} \\
&= \frac{\sqrt{d^2 - e^2x^2} \operatorname{Subst}\left(\int \frac{a + bx^2 + cx^4}{x\sqrt{d^2 - e^2x}} dx, x, x^2\right)}{2\sqrt{d - ex}\sqrt{d + ex}} \\
&= -\frac{\sqrt{d^2 - e^2x^2} \operatorname{Subst}\left(\int \frac{\frac{cd^4 + bd^2e^2 + ae^4}{e^4} - \frac{(2cd^2 + be^2)x^2}{e^4} + \frac{cx^4}{e^4}}{\frac{d^2 - x^2}{e^2}} dx, x, \sqrt{d^2 - e^2x^2}\right)}{e^2\sqrt{d - ex}\sqrt{d + ex}} \\
&= -\frac{\sqrt{d^2 - e^2x^2} \operatorname{Subst}\left(\int \left(b + \frac{cd^2}{e^2} - \frac{cx^2}{e^2} + \frac{a}{\frac{d^2 - x^2}{e^2}}\right) dx, x, \sqrt{d^2 - e^2x^2}\right)}{e^2\sqrt{d - ex}\sqrt{d + ex}} \\
&= -\frac{(cd^2 + be^2)(d^2 - e^2x^2)}{e^4\sqrt{d - ex}\sqrt{d + ex}} + \frac{c(d^2 - e^2x^2)^2}{3e^4\sqrt{d - ex}\sqrt{d + ex}} - \frac{(a\sqrt{d^2 - e^2x^2}) \operatorname{Subst}\left(\int \frac{1}{\frac{d^2 - x^2}{e^2}} dx, x, \sqrt{d^2 - e^2x^2}\right)}{e^2\sqrt{d - ex}\sqrt{d + ex}} \\
&= -\frac{(cd^2 + be^2)(d^2 - e^2x^2)}{e^4\sqrt{d - ex}\sqrt{d + ex}} + \frac{c(d^2 - e^2x^2)^2}{3e^4\sqrt{d - ex}\sqrt{d + ex}} - \frac{a\sqrt{d^2 - e^2x^2} \tanh^{-1}\left(\frac{\sqrt{d^2 - e^2x^2}}{d}\right)}{d\sqrt{d - ex}\sqrt{d + ex}}
\end{aligned}$$

Mathematica [B] time = 0.88, size = 217, normalized size = 2.33

$$\frac{\frac{3a\sqrt{d^2 - e^2x^2} \tanh^{-1}\left(\frac{\sqrt{d^2 - e^2x^2}}{d}\right)}{d\sqrt{d - ex}} + \frac{(e^2x^2 - d^2)(3be^2 + 2cd^2 + ce^2x^2)}{e^4\sqrt{d - ex}} + \frac{6d\sqrt{d + ex}(be^2 + cd^2) \tan^{-1}\left(\frac{\sqrt{d - ex}}{\sqrt{d + ex}}\right)}{e^4} - \frac{6d^{3/2}\sqrt{\frac{ex}{d} + 1}(be^2 + cd^2) \sin^{-1}\left(\frac{\sqrt{d - ex}}{\sqrt{2}\sqrt{d}}\right)}{e^4}}{3\sqrt{d + ex}}$$

Antiderivative was successfully verified.

`[In] Integrate[(a + b*x^2 + c*x^4)/(x*Sqrt[d - e*x]*Sqrt[d + e*x]), x]`

```
[Out] (((-d^2 + e^2*x^2)*(2*c*d^2 + 3*b*e^2 + c*e^2*x^2))/(e^4*Sqrt[d - e*x]) - (6*d^(3/2)*(c*d^2 + b*e^2)*Sqrt[1 + (e*x)/d]*ArcSin[Sqrt[d - e*x]/(Sqrt[2]*Sqrt[d])])/e^4 + (6*d*(c*d^2 + b*e^2)*Sqrt[d + e*x]*ArcTan[Sqrt[d - e*x]/Sqrt[d + e*x]])/e^4 - (3*a*Sqrt[d^2 - e^2*x^2]*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(d*Sqrt[d - e*x]))/(3*Sqrt[d + e*x])
```

IntegrateAlgebraic [B] time = 0.20, size = 199, normalized size = 2.14

$$\frac{a \log\left(\frac{\sqrt{d+ex}}{\sqrt{d-ex}} - 1\right)}{d} - \frac{a \log\left(\frac{d\sqrt{d+ex}}{\sqrt{d-ex}} + d\right)}{d} - \frac{2\sqrt{d+ex} \left(\frac{3bde^2(d+ex)^2}{(d-ex)^2} + \frac{6bde^2(d+ex)}{d-ex} + 3bde^2 + \frac{3cd^3(d+ex)^2}{(d-ex)^2} + \frac{2cd^3(d+ex)}{d-ex} + 3cd^3\right)}{3e^4\sqrt{d-ex} \left(\frac{d+ex}{d-ex} + 1\right)^3}$$

Antiderivative was successfully verified.

`[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)/(x*Sqrt[d - e*x]*Sqrt[d + e*x]), x]`

```
[Out] (-2*Sqrt[d + e*x]*(3*c*d^3 + 3*b*d*e^2 + (2*c*d^3*(d + e*x))/(d - e*x)))/(d - e*x) + (6*b*d*e^2*(d + e*x))/(d - e*x) + (3*c*d^3*(d + e*x)^2)/(d - e*x)^2 + (3*b*d*e^2*(d + e*x)^2)/(d - e*x)^2)/(3*e^4*Sqrt[d - e*x]*(1 + (d + e*x)/(d - e*x)))
```

$)^3) + (a \cdot \text{Log}[-1 + \text{Sqrt}[d + e \cdot x]/\text{Sqrt}[d - e \cdot x]])/d - (a \cdot \text{Log}[d + (d \cdot \text{Sqrt}[d + e \cdot x])/\text{Sqrt}[d - e \cdot x]])/d$

fricas [A] time = 1.34, size = 80, normalized size = 0.86

$$\frac{3 a e^4 \log\left(\frac{\sqrt{e x+d} \sqrt{-e x+d}-d}{x}\right)-\left(c d e^2 x^2+2 c d^3+3 b d e^2\right) \sqrt{e x+d} \sqrt{-e x+d}}{3 d e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/x/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fricas")

[Out] 1/3*(3*a*e^4*log((sqrt(e*x + d)*sqrt(-e*x + d) - d)/x) - (c*d*e^2*x^2 + 2*c*d^3 + 3*b*d*e^2)*sqrt(e*x + d)*sqrt(-e*x + d))/(d*e^4)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/x/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: schur row 1 1.55494e-10Francis algorithm not precise enough for[1.0,-220.8624746 43,10162.5484803,-174574.213802,1032773.91614]schur row 1 3.66198e-10Francis algorithm not precise enough for[1.0,-467.909596927,45612.3731035,-165996 9.6644,20804885.8013]Bad conditioned root j= 2 value 38.9905751966 ratio 0 .000133135092941 mindist 0.00241522618125-a*ln(abs(2*sqrt(d+x*exp(1)))/(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))-1/2*(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))/sqrt(d+x*exp(1)+2))/d+a*ln(abs(2*sqrt(d+x*exp(1)))/(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))-1/2*(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))/sqrt(d+x*exp(1)-2))/d+2*((-24*c*exp(1)^12/144/exp(1)^16*sqrt(d+x*exp(1))*sqrt(d+x*exp(1))+48*c*exp(1)^12*d/144/exp(1)^16)*sqrt(d+x*exp(1))*sqrt(d+x*exp(1))+(-72*c*exp(1)^12*d^2-72*exp(1)^14*b)/144/exp(1)^16)*sqrt(d+x*exp(1))*sqrt(-d-x*exp(1)+2*d)

maple [C] time = 0.04, size = 143, normalized size = 1.54

$$\frac{\sqrt{-e^2 x^2+d} \sqrt{e x+d}\left(\sqrt{-e^2 x^2+d^2} c d e^2 x^2 \operatorname{csgn}(d)+3 a e^4 \ln\left(\frac{2\left(d+\sqrt{-e^2 x^2+d^2} \operatorname{csgn}(d)\right) d}{x}\right)+3 \sqrt{-e^2 x^2+d^2} b d e^2 \operatorname{csgn}(d)+2 \sqrt{-e^2 x^2+d^2} c d^3 \operatorname{csgn}(d)\right) \operatorname{csgn}(d)}{3 \sqrt{-e^2 x^2+d^2} d e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^2+a)/x/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)

[Out] -1/3*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/d*(csgn(d)*x^2*c*d*e^2*(-e^2*x^2+d^2)^(1/2)+3*csgn(d)*(-e^2*x^2+d^2)^(1/2)*b*d*e^2+2*csgn(d)*(-e^2*x^2+d^2)^(1/2)*c*d^3+3*ln(2*d*((-e^2*x^2+d^2)^(1/2)*csgn(d)+d)/x)*a*e^4)*csgn(d)/(-e^2*x^2+d^2)^(1/2)/e^4

maxima [A] time = 1.00, size = 105, normalized size = 1.13

$$-\frac{\sqrt{-e^2 x^2+d^2} c x^2}{3 e^2}-\frac{a \log\left(\frac{2 d^2}{|x|}+\frac{2 \sqrt{-e^2 x^2+d^2} d}{|x|}\right)}{d}-\frac{2 \sqrt{-e^2 x^2+d^2} c d^2}{3 e^4}-\frac{\sqrt{-e^2 x^2+d^2} b}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/x/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="maxima")

[Out] -1/3*sqrt(-e^2*x^2 + d^2)*c*x^2/e^2 - a*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d - 2/3*sqrt(-e^2*x^2 + d^2)*c*d^2/e^4 - sqrt(-e^2*x^2 + d^2)*b/e^2

mupad [B] time = 2.95, size = 161, normalized size = 1.73

$$a \frac{\left(\ln \left(\frac{(\sqrt{d+ex}-\sqrt{d})^2}{(\sqrt{d-ex}-\sqrt{d})^2} - 1 \right) - \ln \left(\frac{\sqrt{d+ex}-\sqrt{d}}{\sqrt{d-ex}-\sqrt{d}} \right) \right)}{d} - \frac{\sqrt{d-ex} \left(\frac{2cd^3}{3e^4} + \frac{cx^3}{3e} + \frac{cdx^2}{3e^2} + \frac{2cd^2x}{3e^3} \right)}{\sqrt{d+ex}} - \frac{\left(\frac{bd}{e^2} + \frac{bx}{e} \right) \sqrt{d-ex}}{\sqrt{d+ex}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^2 + c*x^4)/(x*(d + e*x)^(1/2)*(d - e*x)^(1/2)),x)

[Out] (a*(log(((d + e*x)^(1/2) - d^(1/2))^2/((d - e*x)^(1/2) - d^(1/2))^2 - 1) - log(((d + e*x)^(1/2) - d^(1/2))/((d - e*x)^(1/2) - d^(1/2))))/d - ((d - e*x)^(1/2)*((2*c*d^3)/(3*e^4) + (c*x^3)/(3*e) + (c*d*x^2)/(3*e^2) + (2*c*d^2*x)/(3*e^3)))/(d + e*x)^(1/2) - (((b*d)/e^2 + (b*x)/e)*(d - e*x)^(1/2))/(d + e*x)^(1/2)

sympy [C] time = 91.28, size = 304, normalized size = 3.27

$$\frac{{}_2F_1\left(\frac{3}{4}, \frac{5}{4}, 1, \frac{x}{d}\right)}{4\pi^{3/2}d} - \frac{{}_2F_1\left(\frac{1}{4}, \frac{3}{4}, 1, \frac{x}{d}\right)}{4\pi^{3/2}d} - \frac{{}_2F_1\left(\frac{1}{2}, \frac{3}{2}, 1, \frac{x}{d}\right)}{4\pi^{3/2}d} - \frac{{}_2F_1\left(\frac{3}{4}, \frac{5}{4}, 1, \frac{x}{d}\right)}{4\pi^{3/2}d} - \frac{{}_2F_1\left(\frac{1}{4}, \frac{3}{4}, 1, \frac{x}{d}\right)}{4\pi^{3/2}d} - \frac{{}_2F_1\left(\frac{1}{2}, \frac{3}{2}, 1, \frac{x}{d}\right)}{4\pi^{3/2}d} - \frac{{}_2F_1\left(\frac{3}{4}, \frac{5}{4}, 1, \frac{x}{d}\right)}{4\pi^{3/2}d} - \frac{{}_2F_1\left(\frac{1}{4}, \frac{3}{4}, 1, \frac{x}{d}\right)}{4\pi^{3/2}d} - \frac{{}_2F_1\left(\frac{1}{2}, \frac{3}{2}, 1, \frac{x}{d}\right)}{4\pi^{3/2}d} - \frac{{}_2F_1\left(\frac{3}{4}, \frac{5}{4}, 1, \frac{x}{d}\right)}{4\pi^{3/2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**4+b*x**2+a)/x/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)

[Out] I*a*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0,)), d**2/(e**2*x**2))/(4*pi**(3/2)*d) - a*meijerg(((0, 1/4, 1/2, 3/4, 1, 1), ()), ((1/4, 3/4), (0, 1/2, 1/2, 0)), d**2*exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*d) - I*b*d*meijerg((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4, 1/2, 0), ()), d**2/(e**2*x**2))/(4*pi**(3/2)*e**2) - b*d*meijerg((-1, -3/4, -1/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), d**2*exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*e**2) - I*c*d**3*meijerg((-5/4, -3/4), (-1, -1, -1/2, 1)), ((-3/2, -5/4, -1, -3/4, -1/2, 0), ()), d**2/(e**2*x**2))/(4*pi**(3/2)*e**4) - c*d**3*meijerg((-2, -7/4, -3/2, -5/4, -1, 1), ()), ((-7/4, -5/4), (-2, -3/2, -3/2, 0)), d**2*exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*e**4)

$$3.134 \quad \int \frac{a+bx^2+cx^4}{x^3 \sqrt{d-ex} \sqrt{d+ex}} dx$$

Optimal. Leaf size=99

$$-\frac{(ae^2 + 2bd^2) \tanh^{-1}\left(\frac{\sqrt{d-ex} \sqrt{d+ex}}{d}\right)}{2d^3} - \frac{a\sqrt{d-ex} \sqrt{d+ex}}{2d^2x^2} - \frac{c\sqrt{d-ex} \sqrt{d+ex}}{e^2}$$

Rubi [A] time = 0.25, antiderivative size = 155, normalized size of antiderivative = 1.57, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {520, 1251, 897, 1157, 388, 208}

$$-\frac{\sqrt{d^2 - e^2x^2} (ae^2 + 2bd^2) \tanh^{-1}\left(\frac{\sqrt{d^2 - e^2x^2}}{d}\right)}{2d^3 \sqrt{d-ex} \sqrt{d+ex}} - \frac{a(d^2 - e^2x^2)}{2d^2x^2 \sqrt{d-ex} \sqrt{d+ex}} - \frac{c(d^2 - e^2x^2)}{e^2 \sqrt{d-ex} \sqrt{d+ex}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2 + c*x^4)/(x^3*sqrt[d - e*x]*sqrt[d + e*x]),x]

[Out] -((c*(d^2 - e^2*x^2))/(e^2*sqrt[d - e*x]*sqrt[d + e*x])) - (a*(d^2 - e^2*x^2))/(2*d^2*x^2*sqrt[d - e*x]*sqrt[d + e*x]) - ((2*b*d^2 + a*e^2)*sqrt[d^2 - e^2*x^2]*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^3*sqrt[d - e*x]*sqrt[d + e*x])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 388

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*x*(a + b*x^n)^(p+1))/(b*(n*(p+1)+1)), x] - Dist[(a*d - b*c*(n*(p+1)+1))/(b*(n*(p+1)+1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p+1)+1, 0]

Rule 520

Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n))^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]

Rule 897

Int[((d_.) + (e_.)*(x_)^(m_.))*((f_.) + (g_.)*(x_)^(n_.))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*(m+1)-1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e + a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2]^p, x], x, (d + e*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && FractionQ[m]

Rule 1157

Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2

```
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{a + bx^2 + cx^4}{x^3 \sqrt{d - ex} \sqrt{d + ex}} dx = \frac{\sqrt{d^2 - e^2 x^2} \int \frac{a + bx^2 + cx^4}{x^3 \sqrt{d^2 - e^2 x^2}} dx}{\sqrt{d - ex} \sqrt{d + ex}}$$

$$= \frac{\sqrt{d^2 - e^2 x^2} \text{Subst}\left(\int \frac{a + bx + cx^2}{x^2 \sqrt{d^2 - e^2 x}} dx, x, x^2\right)}{2\sqrt{d - ex} \sqrt{d + ex}}$$

$$= -\frac{\sqrt{d^2 - e^2 x^2} \text{Subst}\left(\int \frac{\frac{cd^4 + bd^2 e^2 + ae^4}{e^4} - \frac{(2cd^2 + be^2)x^2}{e^4} + \frac{cx^4}{e^4}}{\left(\frac{d^2}{e^2} - \frac{x^2}{e^2}\right)^2} dx, x, \sqrt{d^2 - e^2 x^2}\right)}{e^2 \sqrt{d - ex} \sqrt{d + ex}}$$

$$= -\frac{a(d^2 - e^2 x^2)}{2d^2 x^2 \sqrt{d - ex} \sqrt{d + ex}} + \frac{\sqrt{d^2 - e^2 x^2} \text{Subst}\left(\int \frac{-a - \frac{2(cd^4 + bd^2 e^2)}{e^4} + \frac{2cd^2 x^2}{e^4}}{\frac{d^2}{e^2} - \frac{x^2}{e^2}} dx, x, \sqrt{d^2 - e^2 x^2}\right)}{2d^2 \sqrt{d - ex} \sqrt{d + ex}}$$

$$= -\frac{c(d^2 - e^2 x^2)}{e^2 \sqrt{d - ex} \sqrt{d + ex}} - \frac{a(d^2 - e^2 x^2)}{2d^2 x^2 \sqrt{d - ex} \sqrt{d + ex}} + \frac{\left(e^2 \left(\frac{2cd^4}{e^6} + \frac{-a - \frac{2(cd^4 + bd^2 e^2)}{e^4}}{e^2}\right)\right) \sqrt{d^2 - e^2 x^2}}{2d^2 \sqrt{d - ex} \sqrt{d + ex}}$$

$$= -\frac{c(d^2 - e^2 x^2)}{e^2 \sqrt{d - ex} \sqrt{d + ex}} - \frac{a(d^2 - e^2 x^2)}{2d^2 x^2 \sqrt{d - ex} \sqrt{d + ex}} - \frac{(2bd^2 + ae^2) \sqrt{d^2 - e^2 x^2} \tanh^{-1}\left(\frac{\sqrt{d - ex}}{\sqrt{d + ex}}\right)}{2d^3 \sqrt{d - ex} \sqrt{d + ex}}$$

Mathematica [B] time = 0.21, size = 233, normalized size = 2.35

$$\frac{-e^2 x^2 \sqrt{d^2 - e^2 x^2} (ae^2 + 2bd^2) \tanh^{-1}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right) - ad^3 e^2 + ade^4 x^2 - 4cd^2 x^2 \sqrt{d - ex} \sqrt{\frac{ex}{d} + 1} \sin^{-1}\left(\frac{\sqrt{d - ex}}{\sqrt{2} \sqrt{d}}\right) - 2cd^5 x^2 + 4cd^4 x^2 \sqrt{d - ex} \sqrt{d + ex} \tan^{-1}\left(\frac{\sqrt{d - ex}}{\sqrt{d + ex}}\right) + 2cd^3 e^2 x^4}{2d^3 e^2 x^2 \sqrt{d - ex} \sqrt{d + ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^3*Sqrt[d - e*x]*Sqrt[d + e*x]), x]
```

```
[Out] (-(a*d^3*e^2) - 2*c*d^5*x^2 + a*d*e^4*x^2 + 2*c*d^3*e^2*x^4 - 4*c*d^(9/2)*x^2*Sqrt[d - e*x]*Sqrt[1 + (e*x)/d]*ArcSin[Sqrt[d - e*x]/(Sqrt[2]*Sqrt[d])]) + 4*c*d^4*x^2*Sqrt[d - e*x]*Sqrt[d + e*x]*ArcTan[Sqrt[d - e*x]/Sqrt[d + e*x]] - e^2*(2*b*d^2 + a*e^2)*x^2*Sqrt[d^2 - e^2*x^2]*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^3*e^2*x^2*Sqrt[d - e*x]*Sqrt[d + e*x])
```

IntegrateAlgebraic [B] time = 0.26, size = 223, normalized size = 2.25

$$\frac{(-ae^2 - 2bd^2) \tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d-ex}}\right) \sqrt{d+ex} \left(\frac{ae^4(d+ex)^2}{(d-ex)^2} + \frac{2ae^4(d+ex)}{d-ex} + ae^4 + \frac{2cd^4(d+ex)^2}{(d-ex)^2} - \frac{4cd^4(d+ex)}{d-ex} + 2cd^4\right)}{d^3} - \frac{d^3 e^2 \sqrt{d-ex} \left(\frac{\sqrt{d+ex}}{\sqrt{d-ex}} - 1\right)^2 \left(\frac{\sqrt{d+ex}}{\sqrt{d-ex}} + 1\right)^2 \left(\frac{d+ex}{d-ex} + 1\right)}{d^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)/(x^3*sqrt[d - e*x]*sqrt[d + e*x]),x]

[Out] -((sqrt[d + e*x]*(2*c*d^4 + a*e^4 - (4*c*d^4*(d + e*x))/(d - e*x) + (2*a*e^4*(d + e*x))/(d - e*x) + (2*c*d^4*(d + e*x)^2)/(d - e*x)^2 + (a*e^4*(d + e*x)^2)/(d - e*x)^2))/(d^3*e^2*sqrt[d - e*x]*(-1 + sqrt[d + e*x]/sqrt[d - e*x])^2*(1 + sqrt[d + e*x]/sqrt[d - e*x])^2*(1 + (d + e*x)/(d - e*x)))) + ((-2*b*d^2 - a*e^2)*ArcTanh[sqrt[d + e*x]/sqrt[d - e*x]])/d^3

fricas [A] time = 1.41, size = 98, normalized size = 0.99

$$\frac{2cd^4x^2 - (2bd^2e^2 + ae^4)x^2 \log\left(\frac{\sqrt{ex+d}\sqrt{-ex+d}-d}{x}\right) + (2cd^3x^2 + ade^2)\sqrt{ex+d}\sqrt{-ex+d}}{2d^3e^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/x^3/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fricas")

[Out] -1/2*(2*c*d^4*x^2 - (2*b*d^2*e^2 + a*e^4)*x^2*log((sqrt(e*x + d)*sqrt(-e*x + d) - d)/x) + (2*c*d^3*x^2 + a*d*e^2)*sqrt(e*x + d)*sqrt(-e*x + d))/(d^3*e^2*x^2)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/x^3/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: schur row 1 1.55494e-10Francis algorithm not precise enough for[1.0,-220.8624746 43,10162.5484803,-174574.213802,1032773.91614]schur row 1 3.66198e-10Francis algorithm not precise enough for[1.0,-467.909596927,45612.3731035,-165996 9.6644,20804885.8013]Bad conditioned root j= 2 value 38.9905751966 ratio 0 .000133135092941 mindist 0.002415226181251/exp(1)*(-(2*a*(2*sqrt(d+x*exp(1)))/(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))-1/2*(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))/sqrt(d+x*exp(1)))^3*exp(1)^3+8*a*(2*sqrt(d+x*exp(1)))/(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))-1/2*(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))/sqrt(d+x*exp(1)))^2*exp(1)^3)/d^3/((2*sqrt(d+x*exp(1)))/(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))-1/2*(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))/sqrt(d+x*exp(1)))^2-4)^2-1/2*(a*exp(1)^3+2*b*d^2*exp(1))*ln(abs(2*sqrt(d+x*exp(1)))/(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))-1/2*(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))/sqrt(d+x*exp(1))+2))/d^3+1/2*(a*exp(1)^3+2*b*d^2*exp(1))*ln(abs(2*sqrt(d+x*exp(1)))/(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))-1/2*(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))/sqrt(d+x*exp(1))-2))/d^3-2*c*exp(1)/2/exp(1)^2*sqrt(d+x*exp(1))*sqrt(-d-x*exp(1)+2*d))

maple [C] time = 0.02, size = 163, normalized size = 1.65

$$\frac{\sqrt{-ex+d}\sqrt{ex+d} \left(a e^4 x^2 \ln\left(\frac{2(d+\sqrt{-e^2x^2+d^2} \operatorname{csign}(d))d}{x}\right) + 2b d^2 e^2 x^2 \ln\left(\frac{2(d+\sqrt{-e^2x^2+d^2} \operatorname{csign}(d))d}{x}\right) + 2\sqrt{-e^2x^2+d^2} c d^3 x^2 \operatorname{csign}(d) + \sqrt{-e^2x^2+d^2} a d e^2 \operatorname{csign}(d) \right) \operatorname{csign}(d)}{2\sqrt{-e^2x^2+d^2} d^3 e^2 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^2+a)/x^3/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)

[Out] $-1/2*(-e*x+d)^{(1/2)}*(e*x+d)^{(1/2)}/d^3*(2*c*sgn(d)*x^2*c*d^3*(-e^2*x^2+d^2)^{(1/2)} + \ln(2*(d+(-e^2*x^2+d^2)^{(1/2)})*c*sgn(d))*d/x)*x^2*a*e^4 + 2*\ln(2*(d+(-e^2*x^2+d^2)^{(1/2)})*c*sgn(d))*d/x)*x^2*b*d^2*e^2 + c*sgn(d)*a*d*e^2*(-e^2*x^2+d^2)^{(1/2)})*c*sgn(d)/(-e^2*x^2+d^2)^{(1/2)}/e^2/x^2$

maxima [A] time = 1.02, size = 123, normalized size = 1.24

$$\frac{b \log\left(\frac{2d^2}{|x|} + \frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{d} - \frac{ae^2 \log\left(\frac{2d^2}{|x|} + \frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{2d^3} - \frac{\sqrt{-e^2x^2+d^2}c}{e^2} - \frac{\sqrt{-e^2x^2+d^2}a}{2d^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/x^3/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="maxima")

[Out] $-b*\log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d - 1/2*a*e^2*\log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^3 - sqrt(-e^2*x^2 + d^2)*c/e^2 - 1/2*sqrt(-e^2*x^2 + d^2)*a/(d^2*x^2)$

mupad [B] time = 5.15, size = 422, normalized size = 4.26

$$\frac{b \left(\ln\left(\frac{\sqrt{d+ex}-\sqrt{d}}{\sqrt{d-ex}-\sqrt{d}}\right) - 1 \right) - \ln\left(\frac{\sqrt{d+ex}-\sqrt{d}}{\sqrt{d-ex}-\sqrt{d}}\right)}{d} - \frac{\left(\frac{cd}{2} + \frac{cx}{e}\right) \sqrt{d-ex}}{\sqrt{d+ex}} - \frac{ae^2(\sqrt{d+ex}-\sqrt{d})^2}{(\sqrt{d-ex}-\sqrt{d})^2} - \frac{ae^2}{2} + \frac{15ae^2(\sqrt{d+ex}-\sqrt{d})^4}{2(\sqrt{d-ex}-\sqrt{d})^4} - \frac{ae^2 \ln\left(\frac{\sqrt{d+ex}-\sqrt{d}}{\sqrt{d-ex}-\sqrt{d}}\right)}{2d^3} + \frac{ae^2 \ln\left(\frac{\sqrt{d+ex}-\sqrt{d}}{\sqrt{d-ex}-\sqrt{d}}\right)^2 - 1}{2d^3} + \frac{ae^2(\sqrt{d+ex}-\sqrt{d})^2}{32d^3(\sqrt{d-ex}-\sqrt{d})^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^2 + c*x^4)/(x^3*(d + e*x)^(1/2)*(d - e*x)^(1/2)),x)

[Out] $(b*(\log(((d + e*x)^{(1/2)} - d^{(1/2)})^2/((d - e*x)^{(1/2)} - d^{(1/2)})^2 - 1) - \log(((d + e*x)^{(1/2)} - d^{(1/2)})/((d - e*x)^{(1/2)} - d^{(1/2)}))))/d - ((c*d)/e^2 + (c*x)/e)*(d - e*x)^{(1/2)}/(d + e*x)^{(1/2)} - ((a*e^2*((d + e*x)^{(1/2)} - d^{(1/2)})^2)/((d - e*x)^{(1/2)} - d^{(1/2)})^2 - (a*e^2)/2 + (15*a*e^2*((d + e*x)^{(1/2)} - d^{(1/2)})^4)/((d - e*x)^{(1/2)} - d^{(1/2)})^4)/((16*d^3*((d + e*x)^{(1/2)} - d^{(1/2)})^2)/((d - e*x)^{(1/2)} - d^{(1/2)})^2 - (32*d^3*((d + e*x)^{(1/2)} - d^{(1/2)})^4)/((d - e*x)^{(1/2)} - d^{(1/2)})^4 + (16*d^3*((d + e*x)^{(1/2)} - d^{(1/2)})^6)/((d - e*x)^{(1/2)} - d^{(1/2)})^6) - (a*e^2*\log(((d + e*x)^{(1/2)} - d^{(1/2)})/((d - e*x)^{(1/2)} - d^{(1/2)})))/(2*d^3) + (a*e^2*\log(((d + e*x)^{(1/2)} - d^{(1/2)})^2/((d - e*x)^{(1/2)} - d^{(1/2)})^2 - 1))/(2*d^3) + (a*e^2*((d + e*x)^{(1/2)} - d^{(1/2)})^2)/(32*d^3*((d - e*x)^{(1/2)} - d^{(1/2)})^2)$

sympy [C] time = 133.79, size = 270, normalized size = 2.73

$$\frac{iacG_{66}^{53} \left(\frac{7}{4}, \frac{9}{4}, 1, 2, 2, \frac{5}{2} \right) \left(\frac{5}{4}, \frac{3}{4}, 2, 1, \frac{3}{2}, 2, 1 \right) \left(\frac{5}{4}, \frac{3}{4}, 1, 1, 1, \frac{3}{2} \right) \left(\frac{5}{4}, \frac{3}{4}, 1, 1, 1, \frac{3}{2} \right)}{4\pi^{\frac{3}{2}}d^3} + \frac{ae^2G_{66}^{26} \left(\frac{5}{4}, \frac{3}{4}, 2, 1, \frac{3}{2}, 2, 1 \right) \left(\frac{5}{4}, \frac{3}{4}, 1, 1, 1, \frac{3}{2} \right) \left(\frac{5}{4}, \frac{3}{4}, 1, 1, 1, \frac{3}{2} \right)}{4\pi^{\frac{3}{2}}d^3} + \frac{ibG_{66}^{53} \left(\frac{3}{4}, \frac{5}{4}, 1, 1, 1, \frac{3}{2} \right) \left(\frac{3}{4}, \frac{5}{4}, 1, 1, 1, \frac{3}{2} \right) \left(\frac{3}{4}, \frac{5}{4}, 1, 1, 1, \frac{3}{2} \right)}{4\pi^{\frac{3}{2}}d} + \frac{bG_{66}^{26} \left(\frac{0}{4}, \frac{1}{4}, \frac{3}{4}, 1, 1, 1, \frac{3}{2} \right) \left(\frac{0}{4}, \frac{1}{4}, \frac{3}{4}, 1, 1, 1, \frac{3}{2} \right) \left(\frac{0}{4}, \frac{1}{4}, \frac{3}{4}, 1, 1, 1, \frac{3}{2} \right)}{4\pi^{\frac{3}{2}}d} + \frac{icdG_{66}^{52} \left(\frac{-1}{4}, \frac{1}{4}, 0, 0, \frac{1}{2}, 1 \right) \left(\frac{-1}{4}, \frac{1}{4}, 0, 0, \frac{1}{2}, 1 \right) \left(\frac{-1}{4}, \frac{1}{4}, 0, 0, \frac{1}{2}, 1 \right)}{4\pi^{\frac{3}{2}}e^2} + \frac{cdG_{66}^{26} \left(-1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, 1 \right) \left(-1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, 1 \right) \left(-1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, 1 \right)}{4\pi^{\frac{3}{2}}e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**4+b*x**2+a)/x**3/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)

[Out] $I*a*e**2*meijerg(((7/4, 9/4, 1), (2, 2, 5/2)), ((3/2, 7/4, 2, 9/4, 5/2), (0,)), d**2/(e**2*x**2))/(4*pi**(3/2)*d**3) - a*e**2*meijerg(((1, 5/4, 3/2, 7/4, 2, 1), ()), ((5/4, 7/4), (1, 3/2, 3/2, 0)), d**2*exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*d**3) + I*b*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0,)), d**2/(e**2*x**2))/(4*pi**(3/2)*d) - b*meijerg(((0, 1/4, 1/2, 3/4, 1, 1), ()), ((1/4, 3/4), (0, 1/2, 1/2, 0)), d**2*exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*d) - I*c*d*meijerg((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4, 1/2, 0), ()), d**2/(e**2*x**2))/(4*pi**(3/2)*e**2) - c*d*meijerg((-1, -3/4, -1/2, -1/4, 0, 1), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), d**2*exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*e**2)$

$$3.135 \quad \int \frac{a+bx^2+cx^4}{x^5 \sqrt{d-ex} \sqrt{d+ex}} dx$$

Optimal. Leaf size=126

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d-ex}\sqrt{d+ex}}{d}\right)(3ae^4 + 4bd^2e^2 + 8cd^4)}{8d^5} - \frac{\sqrt{d-ex}\sqrt{d+ex}(3ae^2 + 4bd^2)}{8d^4x^2} - \frac{a\sqrt{d-ex}\sqrt{d+ex}}{4d^2x^4}$$

Rubi [A] time = 0.28, antiderivative size = 182, normalized size of antiderivative = 1.44, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {520, 1251, 897, 1157, 385, 208}

$$\frac{\sqrt{d^2 - e^2x^2} \tanh^{-1}\left(\frac{\sqrt{d^2 - e^2x^2}}{d}\right)(3ae^4 + 4bd^2e^2 + 8cd^4)}{8d^5\sqrt{d-ex}\sqrt{d+ex}} - \frac{(d^2 - e^2x^2)(3ae^2 + 4bd^2)}{8d^4x^2\sqrt{d-ex}\sqrt{d+ex}} - \frac{a(d^2 - e^2x^2)}{4d^2x^4\sqrt{d-ex}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2 + c*x^4)/(x^5*sqrt[d - e*x]*sqrt[d + e*x]),x]

[Out] -(a*(d^2 - e^2*x^2))/(4*d^2*x^4*sqrt[d - e*x]*sqrt[d + e*x]) - ((4*b*d^2 + 3*a*e^2)*(d^2 - e^2*x^2))/(8*d^4*x^2*sqrt[d - e*x]*sqrt[d + e*x]) - ((8*c*d^4 + 4*b*d^2*e^2 + 3*a*e^4)*sqrt[d^2 - e^2*x^2]*ArcTanh[sqrt[d^2 - e^2*x^2]/d])/(8*d^5*sqrt[d - e*x]*sqrt[d + e*x])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 520

Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n))^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]

Rule 897

Int[((d_.) + (e_.)*(x_)^(m_))*((f_.) + (g_.)*(x_)^(n_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e + a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2]^p, x], x, (d + e*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && FractionQ[m]

Rule 1157

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2

```
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1251

```
Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{a + bx^2 + cx^4}{x^5 \sqrt{d - ex} \sqrt{d + ex}} dx = \frac{\sqrt{d^2 - e^2 x^2} \int \frac{a + bx^2 + cx^4}{x^5 \sqrt{d^2 - e^2 x^2}} dx}{\sqrt{d - ex} \sqrt{d + ex}}$$

$$= \frac{\sqrt{d^2 - e^2 x^2} \text{Subst}\left(\int \frac{a + bx + cx^2}{x^3 \sqrt{d^2 - e^2 x}} dx, x, x^2\right)}{2\sqrt{d - ex} \sqrt{d + ex}}$$

$$= -\frac{\sqrt{d^2 - e^2 x^2} \text{Subst}\left(\int \frac{cd^4 + bd^2e^2 + ae^4 - (2cd^2 + be^2)x^2 + cx^4}{\left(\frac{d^2}{e^2} - \frac{x^2}{e^2}\right)^3} dx, x, \sqrt{d^2 - e^2 x^2}\right)}{e^2 \sqrt{d - ex} \sqrt{d + ex}}$$

$$= -\frac{a(d^2 - e^2 x^2)}{4d^2 x^4 \sqrt{d - ex} \sqrt{d + ex}} + \frac{\sqrt{d^2 - e^2 x^2} \text{Subst}\left(\int \frac{-3a - \frac{4(cd^4 + bd^2e^2)}{e^4} + \frac{4cd^2 x^2}{e^4}}{\left(\frac{d^2}{e^2} - \frac{x^2}{e^2}\right)^2} dx, x, \sqrt{d^2 - e^2 x^2}\right)}{4d^2 \sqrt{d - ex} \sqrt{d + ex}}$$

$$= -\frac{a(d^2 - e^2 x^2)}{4d^2 x^4 \sqrt{d - ex} \sqrt{d + ex}} - \frac{(4bd^2 + 3ae^2)(d^2 - e^2 x^2)}{8d^4 x^2 \sqrt{d - ex} \sqrt{d + ex}} - \frac{\left(4b + \frac{8cd^2}{e^2} + \frac{3ae^2}{d^2}\right) \sqrt{d^2 - e^2 x^2}}{8d^2 \sqrt{d - ex} \sqrt{d + ex}}$$

$$= -\frac{a(d^2 - e^2 x^2)}{4d^2 x^4 \sqrt{d - ex} \sqrt{d + ex}} - \frac{(4bd^2 + 3ae^2)(d^2 - e^2 x^2)}{8d^4 x^2 \sqrt{d - ex} \sqrt{d + ex}} - \frac{(8cd^4 + 4bd^2e^2 + 3ae^4) \sqrt{d^2 - e^2 x^2}}{8d^5 \sqrt{d - ex} \sqrt{d + ex}}$$

Mathematica [A] time = 0.16, size = 134, normalized size = 1.06

$$\frac{-\left(x^4 \sqrt{d^2 - e^2 x^2} \tanh^{-1}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right) (3ae^4 + 4bd^2e^2 + 8cd^4)\right) - d(d^2 - e^2 x^2)(2ad^2 + 3ae^2 x^2 + 4bd^2 x^2)}{8d^5 x^4 \sqrt{d - ex} \sqrt{d + ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^5*Sqrt[d - e*x]*Sqrt[d + e*x]), x]
```

```
[Out] (-(d*(d^2 - e^2*x^2)*(2*a*d^2 + 4*b*d^2*x^2 + 3*a*e^2*x^2)) - (8*c*d^4 + 4*b*d^2*e^2 + 3*a*e^4)*x^4*Sqrt[d^2 - e^2*x^2]*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(8*d^5*x^4*Sqrt[d - e*x]*Sqrt[d + e*x])
```

IntegrateAlgebraic [A] time = 0.24, size = 207, normalized size = 1.64

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d-ex}}\right) (-3ae^4 - 4bd^2e^2 - 8cd^4)}{4d^5} - \frac{e^2 \sqrt{d + ex} \left(\frac{d+ex}{d-ex} + 1\right) \left(\frac{5ae^2(d+ex)^2}{(d-ex)^2} - \frac{2ae^2(d+ex)}{d-ex} + 5ae^2 + \frac{4bd^2(d+ex)^2}{(d-ex)^2} - \frac{8bd^2(d+ex)}{d-ex} + 4bd^2\right)}{4d^5 \sqrt{d - ex} \left(\frac{d+ex}{d-ex} - 1\right)^4}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)/(x^5*sqrt[d - e*x]*sqrt[d + e*x]),x]

[Out]
$$-1/4*(e^2*\sqrt{d + e*x}*(1 + (d + e*x)/(d - e*x))*(4*b*d^2 + 5*a*e^2 - (8*b*d^2*(d + e*x))/(d - e*x) - (2*a*e^2*(d + e*x))/(d - e*x) + (4*b*d^2*(d + e*x)^2)/(d - e*x)^2 + (5*a*e^2*(d + e*x)^2)/(d - e*x)^2))/(d^5*\sqrt{d - e*x}*(-1 + (d + e*x)/(d - e*x))^4) + ((-8*c*d^4 - 4*b*d^2*e^2 - 3*a*e^4)*\text{ArcTan}(\sqrt{d + e*x}/\sqrt{d - e*x}))/d^5$$

fricas [A] time = 1.23, size = 102, normalized size = 0.81

$$\frac{(8cd^4 + 4bd^2e^2 + 3ae^4)x^4 \log\left(\frac{\sqrt{ex+d}\sqrt{-ex+d}-d}{x}\right) - (2ad^3 + (4bd^3 + 3ade^2)x^2)\sqrt{ex+d}\sqrt{-ex+d}}{8d^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/x^5/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fricas")

[Out]
$$1/8*((8*c*d^4 + 4*b*d^2*e^2 + 3*a*e^4)*x^4*\log((\sqrt{e*x + d})*\sqrt{-e*x + d} - d)/x) - (2*a*d^3 + (4*b*d^3 + 3*a*d*e^2)*x^2)*\sqrt{e*x + d}*\sqrt{-e*x + d})/d^5*x^4$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/x^5/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: schur row 1 1.55494e-10Francis algorithm not precise enough for[1.0,-220.8624746 43,10162.5484803,-174574.213802,1032773.91614]schur row 1 3.66198e-10Francis algorithm not precise enough for[1.0,-467.909596927,45612.3731035,-165996 9.6644,20804885.8013]Bad conditioned root j= 2 value 38.9905751966 ratio 0 .000133135092941 mindist 0.002415226181251/exp(1)*(1/2*(-5*a*(2*sqrt(d+x*exp(1)))/(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))-1/2*(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))/sqrt(d+x*exp(1)))^7*exp(1)^5-4*b*d^2*(2*sqrt(d+x*exp(1)))/(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))-1/2*(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))/sqrt(d+x*exp(1)))^7*exp(1)^3-12*a*(2*sqrt(d+x*exp(1)))/(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))-1/2*(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))/sqrt(d+x*exp(1)))^5*exp(1)^5+16*b*d^2*(2*sqrt(d+x*exp(1)))/(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))-1/2*(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))/sqrt(d+x*exp(1)))^5*exp(1)^3-48*a*(2*sqrt(d+x*exp(1)))/(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))-1/2*(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))/sqrt(d+x*exp(1)))^3*exp(1)^5+64*b*d^2*(2*sqrt(d+x*exp(1)))/(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))-1/2*(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))/sqrt(d+x*exp(1)))^3*exp(1)^3-320*a*(2*sqrt(d+x*exp(1)))/(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))-1/2*(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))/sqrt(d+x*exp(1)))^5-256*b*d^2*(2*sqrt(d+x*exp(1)))/(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))-1/2*(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))/sqrt(d+x*exp(1)))^3/d^5/((2*sqrt(d+x*exp(1)))/(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))-1/2*(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))/sqrt(d+x*exp(1)))^2-4)^4-1/8*(3*a*exp(1)^5+4*b*d^2*exp(1)^3+8*c*d^4*exp(1))*ln(abs(2*sqrt(d+x*exp(1)))/(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))-1/2*(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))/sqrt(d+x*exp(1))+2))/d^5+1/8*(3*a*exp(1)^5+4*b*d^2*exp(1)^3+8*c*d^4*exp(1))*ln(abs(2*sqrt(d+x*exp(1)))/(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))-1/2*(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))/sqrt(d+x*exp(1))-2))/d^5

maple [C] time = 0.03, size = 222, normalized size = 1.76

$$\frac{\sqrt{-cx+d}\sqrt{cx+d}\left(3ae^4x^4\ln\left(\frac{2(d+\sqrt{-e^2x^2+d^2})\operatorname{csgn}(d)}{x}\right)+4bd^2e^2x^4\ln\left(\frac{2(d+\sqrt{-e^2x^2+d^2})\operatorname{csgn}(d)}{x}\right)+8cd^4x^4\ln\left(\frac{2(d+\sqrt{-e^2x^2+d^2})\operatorname{csgn}(d)}{x}\right)+3\sqrt{-e^2x^2+d^2}ad^2x^2\operatorname{csgn}(d)+4\sqrt{-e^2x^2+d^2}bd^3x^2\operatorname{csgn}(d)+2\sqrt{-e^2x^2+d^2}ad^3\operatorname{csgn}(d)\right)\operatorname{csgn}(d)}{8\sqrt{-e^2x^2+d^2}d^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^2+a)/x^5/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)

[Out] $-1/8*(-e*x+d)^{(1/2)}*(e*x+d)^{(1/2)}/d^5*(3*\ln(2*(d+(-e^2*x^2+d^2)^{(1/2)})*\operatorname{csgn}(d))*d/x)*x^4*a*e^4+4*\ln(2*(d+(-e^2*x^2+d^2)^{(1/2)})*\operatorname{csgn}(d))*d/x)*x^4*b*d^2*e^2+8*\ln(2*(d+(-e^2*x^2+d^2)^{(1/2)})*\operatorname{csgn}(d))*d/x)*x^4*c*d^4+3*\operatorname{csgn}(d)*x^2*a*d*e^2*(-e^2*x^2+d^2)^{(1/2)}+4*\operatorname{csgn}(d)*x^2*b*d^3*(-e^2*x^2+d^2)^{(1/2)}+2*\operatorname{csgn}(d)*a*d^3*(-e^2*x^2+d^2)^{(1/2)})*\operatorname{csgn}(d)/(-e^2*x^2+d^2)^{(1/2)}/x^4$

maxima [A] time = 1.03, size = 193, normalized size = 1.53

$$\frac{c \log\left(\frac{2d^2}{|x|} + \frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{d} - \frac{be^2 \log\left(\frac{2d^2}{|x|} + \frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{2d^3} - \frac{3ae^4 \log\left(\frac{2d^2}{|x|} + \frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{8d^5} - \frac{\sqrt{-e^2x^2+d^2}b}{2d^2x^2} - \frac{3\sqrt{-e^2x^2+d^2}ae^2}{8d^4x^2} - \frac{\sqrt{-e^2x^2+d^2}a}{4d^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/x^5/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="maxima")

[Out] $-c*\log(2*d^2/\operatorname{abs}(x) + 2*\sqrt{-e^2*x^2 + d^2}*d/\operatorname{abs}(x))/d - 1/2*b*e^2*\log(2*d^2/\operatorname{abs}(x) + 2*\sqrt{-e^2*x^2 + d^2}*d/\operatorname{abs}(x))/d^3 - 3/8*a*e^4*\log(2*d^2/\operatorname{abs}(x) + 2*\sqrt{-e^2*x^2 + d^2}*d/\operatorname{abs}(x))/d^5 - 1/2*\sqrt{-e^2*x^2 + d^2}*b/(d^2*x^2) - 3/8*\sqrt{-e^2*x^2 + d^2}*a*e^2/(d^4*x^2) - 1/4*\sqrt{-e^2*x^2 + d^2}*a/(d^2*x^4)$

mupad [B] time = 10.82, size = 932, normalized size = 7.40

$$\frac{c \log\left(\frac{2d^2}{|x|} + \frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{d} - \frac{be^2 \log\left(\frac{2d^2}{|x|} + \frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{2d^3} - \frac{3ae^4 \log\left(\frac{2d^2}{|x|} + \frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{8d^5} - \frac{\sqrt{-e^2x^2+d^2}b}{2d^2x^2} - \frac{3\sqrt{-e^2x^2+d^2}ae^2}{8d^4x^2} - \frac{\sqrt{-e^2x^2+d^2}a}{4d^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^2 + c*x^4)/(x^5*(d + e*x)^(1/2)*(d - e*x)^(1/2)),x)

[Out] $((a*e^4)/4 + (6*a*e^4*((d + e*x)^{(1/2)} - d^{(1/2)})^2)/((d - e*x)^{(1/2)} - d^{(1/2)})^2 - (53*a*e^4*((d + e*x)^{(1/2)} - d^{(1/2)})^4)/(2*((d - e*x)^{(1/2)} - d^{(1/2)})^4) - (87*a*e^4*((d + e*x)^{(1/2)} - d^{(1/2)})^6)/((d - e*x)^{(1/2)} - d^{(1/2)})^6 + (657*a*e^4*((d + e*x)^{(1/2)} - d^{(1/2)})^8)/(4*((d - e*x)^{(1/2)} - d^{(1/2)})^8) - (121*a*e^4*((d + e*x)^{(1/2)} - d^{(1/2)})^{10})/((d - e*x)^{(1/2)} - d^{(1/2)})^{10})/((256*d^5*((d + e*x)^{(1/2)} - d^{(1/2)})^4)/((d - e*x)^{(1/2)} - d^{(1/2)})^4 - (1024*d^5*((d + e*x)^{(1/2)} - d^{(1/2)})^6)/((d - e*x)^{(1/2)} - d^{(1/2)})^6 + (1536*d^5*((d + e*x)^{(1/2)} - d^{(1/2)})^8)/((d - e*x)^{(1/2)} - d^{(1/2)})^8 - (1024*d^5*((d + e*x)^{(1/2)} - d^{(1/2)})^{10})/((d - e*x)^{(1/2)} - d^{(1/2)})^{10} + (256*d^5*((d + e*x)^{(1/2)} - d^{(1/2)})^{12})/((d - e*x)^{(1/2)} - d^{(1/2)})^{12}) - ((b*e^2*((d + e*x)^{(1/2)} - d^{(1/2)})^2)/((d - e*x)^{(1/2)} - d^{(1/2)})^2 - (b*e^2)/2 + (15*b*e^2*((d + e*x)^{(1/2)} - d^{(1/2)})^4)/(2*((d - e*x)^{(1/2)} - d^{(1/2)})^4))/((16*d^3*((d + e*x)^{(1/2)} - d^{(1/2)})^2)/((d - e*x)^{(1/2)} - d^{(1/2)})^2 - (32*d^3*((d + e*x)^{(1/2)} - d^{(1/2)})^4)/((d - e*x)^{(1/2)} - d^{(1/2)})^4 + (16*d^3*((d + e*x)^{(1/2)} - d^{(1/2)})^6)/((d - e*x)^{(1/2)} - d^{(1/2)})^6) + (c*(\log(((d + e*x)^{(1/2)} - d^{(1/2)})^2/((d - e*x)^{(1/2)} - d^{(1/2)})^2 - 1) - \log(((d + e*x)^{(1/2)} - d^{(1/2)})/((d - e*x)^{(1/2)} - d^{(1/2))}))/d - (3*a*e^4*\log(((d + e*x)^{(1/2)} - d^{(1/2)})/((d - e*x)^{(1/2)} - d^{(1/2))}))/((8*d^5) - (b*e^2*\log(((d + e*x)^{(1/2)} - d^{(1/2)})/((d - e*x)^{(1/2)} - d^{(1/2))}))/((2*d^3) + (3*a*e^4*\log(((d + e*x)^{(1/2)} - d^{(1/2)})^2/((d - e*x)^{(1/2)} - d^{(1/2)})^2 - 1))/((8*d^5) + (b*e^2*\log(((d + e*x)^{(1/2)} - d^{(1/2)})^2/((d - e*x)^{(1/2)} - d^{(1/2)})^2 - 1))/((2*d^3) + (7*a*e^4*((d + e*x)^{(1/2)} - d^{(1/2)})^2)/(256*d^5*((d - e*x)^{(1/2)} - d^{(1/2)})^2) + (a*e^4*((d + e*x)^{(1/2)} - d^{(1/2)})$

$$\frac{\sqrt{d^4/(1024*d^5*((d - e*x)^{1/2} - d^{1/2}))^4 + (b*e^2*((d + e*x)^{1/2} - d^{1/2}))^2}}{32*d^3*((d - e*x)^{1/2} - d^{1/2})^2}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**4+b*x**2+a)/x**5/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)

[Out] Timed out

$$3.136 \quad \int \frac{a+bx^2+cx^4}{x^7 \sqrt{d-ex} \sqrt{d+ex}} dx$$

Optimal. Leaf size=212

$$\frac{e^2 \sqrt{d^2 - e^2 x^2} \tanh^{-1} \left(\frac{\sqrt{d^2 - e^2 x^2}}{d} \right) (5ae^4 + 6bd^2 e^2 + 8cd^4)}{16d^7 \sqrt{d - ex} \sqrt{d + ex}} - \frac{\sqrt{d - ex} \sqrt{d + ex} (5ae^4 + 6bd^2 e^2 + 8cd^4)}{16d^6 x^2} - \frac{\sqrt{d - ex} \sqrt{d + ex}}{16d^6 x^2}$$

Rubi [A] time = 0.37, antiderivative size = 248, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {520, 1251, 897, 1157, 385, 199, 208}

$$\frac{(d^2 - e^2 x^2)(5ae^4 + 6bd^2 e^2 + 8cd^4)}{16d^6 x^2 \sqrt{d - ex} \sqrt{d + ex}} - \frac{e^2 \sqrt{d^2 - e^2 x^2} \tanh^{-1} \left(\frac{\sqrt{d^2 - e^2 x^2}}{d} \right) (5ae^4 + 6bd^2 e^2 + 8cd^4)}{16d^7 \sqrt{d - ex} \sqrt{d + ex}} - \frac{(d^2 - e^2 x^2)(5ae^2 + 6bd^2)}{24d^4 x^4 \sqrt{d - ex} \sqrt{d + ex}} - \frac{a(d^2 - e^2 x^2)}{6d^2 x^6 \sqrt{d - ex} \sqrt{d + ex}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2 + c*x^4)/(x^7*Sqrt[d - e*x]*Sqrt[d + e*x]),x]

[Out] -(a*(d^2 - e^2*x^2))/(6*d^2*x^6*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((6*b*d^2 + 5*a*e^2)*(d^2 - e^2*x^2))/(24*d^4*x^4*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((8*c*d^4 + 6*b*d^2*e^2 + 5*a*e^4)*(d^2 - e^2*x^2))/(16*d^6*x^2*Sqrt[d - e*x]*Sqrt[d + e*x]) - (e^2*(8*c*d^4 + 6*b*d^2*e^2 + 5*a*e^4)*Sqrt[d^2 - e^2*x^2]*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(16*d^7*Sqrt[d - e*x]*Sqrt[d + e*x])

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 520

Int[(u_.)*((c_) + (d_.)*(x_)^(n_.)) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] := Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n))^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]

Rule 897

Int[((d_.) + (e_.)*(x_)^(m_))*((f_.) + (g_.)*(x_)^(n_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := With[{q = Denominator[m]}, Dist[q/e, S

`ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e + a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2]^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && FractionQ[m]`

Rule 1157

`Int[((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]`

Rule 1251

`Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]`

Rubi steps

$$\begin{aligned}
 \int \frac{a + bx^2 + cx^4}{x^7 \sqrt{d - ex} \sqrt{d + ex}} dx &= \frac{\sqrt{d^2 - e^2 x^2} \int \frac{a + bx^2 + cx^4}{x^7 \sqrt{d^2 - e^2 x^2}} dx}{\sqrt{d - ex} \sqrt{d + ex}} \\
 &= \frac{\sqrt{d^2 - e^2 x^2} \operatorname{Subst}\left(\int \frac{a + bx + cx^2}{x^4 \sqrt{d^2 - e^2 x}} dx, x, x^2\right)}{2\sqrt{d - ex} \sqrt{d + ex}} \\
 &= -\frac{\sqrt{d^2 - e^2 x^2} \operatorname{Subst}\left(\int \frac{\frac{cd^4 + bd^2 e^2 + ae^4}{e^4} - \frac{(2cd^2 + be^2)x^2}{e^4} + \frac{cx^4}{e^4}}{\left(\frac{d^2}{e^2} - \frac{x^2}{e^2}\right)^4} dx, x, \sqrt{d^2 - e^2 x^2}\right)}{e^2 \sqrt{d - ex} \sqrt{d + ex}} \\
 &= -\frac{a(d^2 - e^2 x^2)}{6d^2 x^6 \sqrt{d - ex} \sqrt{d + ex}} + \frac{\sqrt{d^2 - e^2 x^2} \operatorname{Subst}\left(\int \frac{-5a - \frac{6(cd^4 + bd^2 e^2)}{e^4} + \frac{6cd^2 x^2}{e^4}}{\left(\frac{d^2}{e^2} - \frac{x^2}{e^2}\right)^3} dx, x, \sqrt{d^2 - e^2 x^2}\right)}{6d^2 \sqrt{d - ex} \sqrt{d + ex}} \\
 &= -\frac{a(d^2 - e^2 x^2)}{6d^2 x^6 \sqrt{d - ex} \sqrt{d + ex}} - \frac{(6bd^2 + 5ae^2)(d^2 - e^2 x^2)}{24d^4 x^4 \sqrt{d - ex} \sqrt{d + ex}} - \frac{\left(\left(6b + \frac{8cd^2}{e^2} + \frac{5ae^2}{d^2}\right) \sqrt{d^2 - e^2 x^2}\right)}{16d^6 x^2 \sqrt{d - ex} \sqrt{d + ex}} \\
 &= -\frac{a(d^2 - e^2 x^2)}{6d^2 x^6 \sqrt{d - ex} \sqrt{d + ex}} - \frac{(6bd^2 + 5ae^2)(d^2 - e^2 x^2)}{24d^4 x^4 \sqrt{d - ex} \sqrt{d + ex}} - \frac{(8cd^4 + 6bd^2 e^2 + 5ae^4)(d^2 - e^2 x^2)}{16d^6 x^2 \sqrt{d - ex} \sqrt{d + ex}} \\
 &= -\frac{a(d^2 - e^2 x^2)}{6d^2 x^6 \sqrt{d - ex} \sqrt{d + ex}} - \frac{(6bd^2 + 5ae^2)(d^2 - e^2 x^2)}{24d^4 x^4 \sqrt{d - ex} \sqrt{d + ex}} - \frac{(8cd^4 + 6bd^2 e^2 + 5ae^4)(d^2 - e^2 x^2)}{16d^6 x^2 \sqrt{d - ex} \sqrt{d + ex}}
 \end{aligned}$$

Mathematica [A] time = 0.19, size = 173, normalized size = 0.82

$$\frac{-3e^2x^6\sqrt{d^2 - e^2x^2} \tanh^{-1}\left(\frac{\sqrt{d^2 - e^2x^2}}{d}\right) (5ae^4 + 6bd^2e^2 + 8cd^4) - d(d^2 - e^2x^2) (a(8d^4 + 10d^2e^2x^2 + 15e^4x^4) + 6(2bd^4x^2 + 3bd^2e^2x^4 + 4cd^4x^4))}{48d^7x^6\sqrt{d - ex}\sqrt{d + ex}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2 + c*x^4)/(x^7*Sqrt[d - e*x]*Sqrt[d + e*x]), x]

[Out] $(-(d*(d^2 - e^2*x^2)*(6*(2*b*d^4*x^2 + 4*c*d^4*x^4 + 3*b*d^2*e^2*x^4) + a*(8*d^4 + 10*d^2*e^2*x^2 + 15*e^4*x^4))) - 3*e^2*(8*c*d^4 + 6*b*d^2*e^2 + 5*a*e^4)*x^6*\text{Sqrt}[d^2 - e^2*x^2]*\text{ArcTanh}[\text{Sqrt}[d^2 - e^2*x^2]/d])/(48*d^7*x^6*\text{Sqrt}[d - e*x]*\text{Sqrt}[d + e*x])$

IntegrateAlgebraic [A] time = 0.44, size = 397, normalized size = 1.87

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d-ex}}\right) (-5ad^6 - 6bd^4e^4 - 8cd^4e^2) - e^2\sqrt{d+ex}\left(\frac{d+ex}{d-ex} + 1\right) \left(\frac{33ae^4(d+ex)^4}{(d-ex)^4} - \frac{28ae^4(d+ex)^3}{(d-ex)^3} + \frac{118ae^4(d+ex)^2}{(d-ex)^2} - \frac{28ae^4(d+ex)}{d-ex} + 33ae^4 + \frac{30bd^2e^2(d+ex)^4}{(d-ex)^4} - \frac{72bd^2e^2(d+ex)^3}{(d-ex)^3} + \frac{84bd^2e^2(d+ex)^2}{(d-ex)^2} - \frac{72bd^2e^2(d+ex)}{d-ex} + 30bd^2e^2 + \frac{24cd^4e^4}{(d-ex)^4} - \frac{96cd^4e^4}{(d-ex)^3} + \frac{144cd^4e^4}{(d-ex)^2} - \frac{96cd^4e^4}{d-ex} + 24cd^4\right)}{24d^7\sqrt{d-ex}\left(\frac{d+ex}{d-ex} - 1\right)^6}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)/(x^7*Sqrt[d - e*x]*Sqrt[d + e*x]), x]

[Out] $-1/24*(e^2*\text{Sqrt}[d + e*x]*(1 + (d + e*x)/(d - e*x))*(24*c*d^4 + 30*b*d^2*e^2 + 33*a*e^4 - (96*c*d^4*(d + e*x))/(d - e*x) - (72*b*d^2*e^2*(d + e*x))/(d - e*x) - (28*a*e^4*(d + e*x))/(d - e*x) + (144*c*d^4*(d + e*x)^2)/(d - e*x)^2 + (84*b*d^2*e^2*(d + e*x)^2)/(d - e*x)^2 + (118*a*e^4*(d + e*x)^2)/(d - e*x)^2 - (96*c*d^4*(d + e*x)^3)/(d - e*x)^3 - (72*b*d^2*e^2*(d + e*x)^3)/(d - e*x)^3 - (28*a*e^4*(d + e*x)^3)/(d - e*x)^3 + (24*c*d^4*(d + e*x)^4)/(d - e*x)^4 + (30*b*d^2*e^2*(d + e*x)^4)/(d - e*x)^4 + (33*a*e^4*(d + e*x)^4)/(d - e*x)^4))/((d^7*\text{Sqrt}[d - e*x]*(-1 + (d + e*x)/(d - e*x))^6) + ((-8*c*d^4*e^2 - 6*b*d^2*e^4 - 5*a*e^6)*\text{ArcTanh}[\text{Sqrt}[d + e*x]/\text{Sqrt}[d - e*x]])/(8*d^7))$

fricas [A] time = 1.30, size = 137, normalized size = 0.65

$$\frac{3(8cd^4e^2 + 6bd^2e^4 + 5ae^6)x^6 \log\left(\frac{\sqrt{ex+d}\sqrt{-ex+d-d}}{x}\right) - (8ad^5 + 3(8cd^5 + 6bd^3e^2 + 5ade^4)x^4 + 2(6bd^5 + 5ad^3e^2)x^2)\sqrt{ex+d}\sqrt{-ex+d}}{48d^7x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/x^7/(-e*x+d)^(1/2)/(e*x+d)^(1/2), x, algorithm="fricas")

[Out] $1/48*(3*(8*c*d^4*e^2 + 6*b*d^2*e^4 + 5*a*e^6)*x^6*\log((\text{sqrt}(e*x + d)*\text{sqrt}(-e*x + d) - d)/x) - (8*a*d^5 + 3*(8*c*d^5 + 6*b*d^3*e^2 + 5*a*d*e^4)*x^4 + 2*(6*b*d^5 + 5*a*d^3*e^2)*x^2)*\text{sqrt}(e*x + d)*\text{sqrt}(-e*x + d))/(d^7*x^6)$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/x^7/(-e*x+d)^(1/2)/(e*x+d)^(1/2), x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: schur row 1 1.55494e-10Francis algorithm not precise enough for[1.0,-220.8624746 43,10162.5484803,-174574.213802,1032773.91614]schur row 1 3.66198e-10Francis algorithm not precise enough for[1.0,-467.909596927,45612.3731035,-165996 9.6644,20804885.8013]Bad conditioned root j= 2 value 38.9905751966 ratio 0 .000133135092941 mindist 0.002415226181251/exp(1)*(-1/12*(33*a*(2*sqrt(d+x*exp(1)))/(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))-1/2*(2*sqrt(2)*sqrt(d)-

$$\begin{aligned}
& 2*\sqrt{-d-x*\exp(1)+2*d})/\sqrt{d+x*\exp(1)})^{11}*\exp(1)^{7+30*b*d^2}*(2*\sqrt{d+x* \\
& * \exp(1))/(2*\sqrt{2}*\sqrt{d}-2*\sqrt{-d-x*\exp(1)+2*d})-1/2*(2*\sqrt{2}*\sqrt{d} \\
& -2*\sqrt{-d-x*\exp(1)+2*d})/\sqrt{d+x*\exp(1)})^{11}*\exp(1)^{5+24*c*d^4}*(2*\sqrt{d+x* \\
& * \exp(1))/(2*\sqrt{2}*\sqrt{d}-2*\sqrt{-d-x*\exp(1)+2*d})-1/2*(2*\sqrt{2}*\sqrt{d} \\
&)-2*\sqrt{-d-x*\exp(1)+2*d})/\sqrt{d+x*\exp(1)})^{11}*\exp(1)^{3+20*a}*(2*\sqrt{d+x*e \\
& xp(1))/(2*\sqrt{2}*\sqrt{d}-2*\sqrt{-d-x*\exp(1)+2*d})-1/2*(2*\sqrt{2}*\sqrt{d}-2 \\
& * \sqrt{-d-x*\exp(1)+2*d})/\sqrt{d+x*\exp(1)})^9*\exp(1)^{7-168*b*d^2}*(2*\sqrt{d+x* \\
& exp(1))/(2*\sqrt{2}*\sqrt{d}-2*\sqrt{-d-x*\exp(1)+2*d})-1/2*(2*\sqrt{2}*\sqrt{d}- \\
& 2*\sqrt{-d-x*\exp(1)+2*d})/\sqrt{d+x*\exp(1)})^9*\exp(1)^{5-288*c*d^4}*(2*\sqrt{d+x* \\
& * \exp(1))/(2*\sqrt{2}*\sqrt{d}-2*\sqrt{-d-x*\exp(1)+2*d})-1/2*(2*\sqrt{2}*\sqrt{d} \\
& -2*\sqrt{-d-x*\exp(1)+2*d})/\sqrt{d+x*\exp(1)})^9*\exp(1)^{3+1440*a}*(2*\sqrt{d+x*e \\
& xp(1))/(2*\sqrt{2}*\sqrt{d}-2*\sqrt{-d-x*\exp(1)+2*d})-1/2*(2*\sqrt{2}*\sqrt{d}-2 \\
& * \sqrt{-d-x*\exp(1)+2*d})/\sqrt{d+x*\exp(1)})^7*\exp(1)^{7+192*b*d^2}*(2*\sqrt{d+x* \\
& exp(1))/(2*\sqrt{2}*\sqrt{d}-2*\sqrt{-d-x*\exp(1)+2*d})-1/2*(2*\sqrt{2}*\sqrt{d}- \\
& 2*\sqrt{-d-x*\exp(1)+2*d})/\sqrt{d+x*\exp(1)})^7*\exp(1)^{5+768*c*d^4}*(2*\sqrt{d+x* \\
& * \exp(1))/(2*\sqrt{2}*\sqrt{d}-2*\sqrt{-d-x*\exp(1)+2*d})-1/2*(2*\sqrt{2}*\sqrt{d}-2 \\
& * \sqrt{-d-x*\exp(1)+2*d})/\sqrt{d+x*\exp(1)})^7*\exp(1)^{3+5760*a}*(2*\sqrt{d+x*e \\
& xp(1))/(2*\sqrt{2}*\sqrt{d}-2*\sqrt{-d-x*\exp(1)+2*d})-1/2*(2*\sqrt{2}*\sqrt{d}-2 \\
& * \sqrt{-d-x*\exp(1)+2*d})/\sqrt{d+x*\exp(1)})^5*\exp(1)^{7+768*b*d^2}*(2*\sqrt{d+x* \\
& exp(1))/(2*\sqrt{2}*\sqrt{d}-2*\sqrt{-d-x*\exp(1)+2*d})-1/2*(2*\sqrt{2}*\sqrt{d}- \\
& 2*\sqrt{-d-x*\exp(1)+2*d})/\sqrt{d+x*\exp(1)})^5*\exp(1)^{5+3072*c*d^4}*(2*\sqrt{d+ \\
& x*\exp(1))/(2*\sqrt{2}*\sqrt{d}-2*\sqrt{-d-x*\exp(1)+2*d})-1/2*(2*\sqrt{2}*\sqrt{d} \\
&)-2*\sqrt{-d-x*\exp(1)+2*d})/\sqrt{d+x*\exp(1)})^5*\exp(1)^{3+1280*a}*(2*\sqrt{d+x* \\
& exp(1))/(2*\sqrt{2}*\sqrt{d}-2*\sqrt{-d-x*\exp(1)+2*d})-1/2*(2*\sqrt{2}*\sqrt{d}- \\
& 2*\sqrt{-d-x*\exp(1)+2*d})/\sqrt{d+x*\exp(1)})^3*\exp(1)^{7-10752*b*d^2}*(2*\sqrt{d \\
& +x*\exp(1))/(2*\sqrt{2}*\sqrt{d}-2*\sqrt{-d-x*\exp(1)+2*d})-1/2*(2*\sqrt{2}*\sqrt{d} \\
&)-2*\sqrt{-d-x*\exp(1)+2*d})/\sqrt{d+x*\exp(1)})^3*\exp(1)^{5-18432*c*d^4}*(2*\sqrt{d \\
& +x*\exp(1))/(2*\sqrt{2}*\sqrt{d}-2*\sqrt{-d-x*\exp(1)+2*d})-1/2*(2*\sqrt{2}*\sqrt{d} \\
&)-2*\sqrt{-d-x*\exp(1)+2*d})/\sqrt{d+x*\exp(1)})^3*\exp(1)^{3+33792*a}*(2*\sqrt{d+x* \\
& exp(1))/(2*\sqrt{2}*\sqrt{d}-2*\sqrt{-d-x*\exp(1)+2*d})-1/2*(2*\sqrt{2}*\sqrt{d} \\
&)-2*\sqrt{-d-x*\exp(1)+2*d})/\sqrt{d+x*\exp(1)})*\exp(1)^{7+30720*b*d^2}*(2*\sqrt{d+x* \\
& exp(1))/(2*\sqrt{2}*\sqrt{d}-2*\sqrt{-d-x*\exp(1)+2*d})-1/2*(2*\sqrt{2}*\sqrt{d} \\
&)-2*\sqrt{-d-x*\exp(1)+2*d})/\sqrt{d+x*\exp(1)})*\exp(1)^{5+24576*c*d^4}*(2*\sqrt{d+x* \\
& exp(1))/(2*\sqrt{2}*\sqrt{d}-2*\sqrt{-d-x*\exp(1)+2*d})-1/2*(2*\sqrt{2}*\sqrt{d} \\
&)-2*\sqrt{-d-x*\exp(1)+2*d})/\sqrt{d+x*\exp(1)})*\exp(1)^3/d^7/((2*\sqrt{d+x* \\
& exp(1))/(2*\sqrt{2}*\sqrt{d}-2*\sqrt{-d-x*\exp(1)+2*d})-1/2*(2*\sqrt{2}*\sqrt{d} \\
&)-2*\sqrt{-d-x*\exp(1)+2*d})/\sqrt{d+x*\exp(1)})^2-4)^6-1/16*(5*a*\exp(1)^7+6*b* \\
& d^2*\exp(1)^5+8*c*d^4*\exp(1)^3)*\ln(\text{abs}(2*\sqrt{d+x*\exp(1)})/(2*\sqrt{2}*\sqrt{d} \\
& -2*\sqrt{-d-x*\exp(1)+2*d})-1/2*(2*\sqrt{2}*\sqrt{d}-2*\sqrt{-d-x*\exp(1)+2*d}))/\sqrt{ \\
& d+x*\exp(1)+2})/d^7+1/16*(5*a*\exp(1)^7+6*b*d^2*\exp(1)^5+8*c*d^4*\exp(1)^ \\
& 3)*\ln(\text{abs}(2*\sqrt{d+x*\exp(1)})/(2*\sqrt{2}*\sqrt{d}-2*\sqrt{-d-x*\exp(1)+2*d})-1/ \\
& 2*(2*\sqrt{2}*\sqrt{d}-2*\sqrt{-d-x*\exp(1)+2*d})/\sqrt{d+x*\exp(1)-2}))/d^7)
\end{aligned}$$

maple [C] time = 0.04, size = 306, normalized size = 1.44

$$\frac{\sqrt{-cx+d}\sqrt{cx+d}\left(15a^2d^2\ln\left(\frac{2(d+\sqrt{-2d^2-d^2}\text{csign}(d))}{d}\right)+18b^2d^2\ln\left(\frac{2(d+\sqrt{-2d^2-d^2}\text{csign}(d))}{d}\right)+24c^2d^2\ln\left(\frac{2(d+\sqrt{-2d^2-d^2}\text{csign}(d))}{d}\right)+15\sqrt{-d^2+d^2}ad^2\text{csign}(d)+18\sqrt{-d^2+d^2}bd^2\text{csign}(d)+24\sqrt{-d^2+d^2}cd^2\text{csign}(d)+10\sqrt{-d^2+d^2}ad^2\text{csign}(d)+12\sqrt{-d^2+d^2}bd^2\text{csign}(d)+8\sqrt{-d^2+d^2}cd^2\text{csign}(d)\right)\text{csign}(d)}{48\sqrt{-d^2+d^2}d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^2+a)/x^7/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)

[Out] $-1/48*(-e*x+d)^{(1/2)}*(e*x+d)^{(1/2)}/d^7*(15*\ln(2*(d+(-e^2*x^2+d^2)^{(1/2)})*\text{csign}(d))*d/x)*x^6*a*e^6+18*\ln(2*(d+(-e^2*x^2+d^2)^{(1/2)})*\text{csign}(d))*d/x)*x^6*b*d^2*e^4+24*\ln(2*(d+(-e^2*x^2+d^2)^{(1/2)})*\text{csign}(d))*d/x)*x^6*c*d^4*e^2+15*(-e^2*x^2+d^2)^{(1/2)}*\text{csign}(d)*d*x^4*a*e^4+18*(-e^2*x^2+d^2)^{(1/2)}*\text{csign}(d)*d^3*x^4*b*e^2+24*(-e^2*x^2+d^2)^{(1/2)}*\text{csign}(d)*d^5*x^4*c+10*\text{csign}(d)*x^2*a*d^3*e^2*(-e^2*x^2+d^2)^{(1/2)}+12*\text{csign}(d)*x^2*b*d^5*(-e^2*x^2+d^2)^{(1/2)}+8*\text{csign}(d)*a*d^5*(-e^2*x^2+d^2)^{(1/2)})*\text{csign}(d)/(-e^2*x^2+d^2)^{(1/2)}/x^6$

maxima [A] time = 1.03, size = 271, normalized size = 1.28

$$\frac{ce^2 \log\left(\frac{2d^2}{|x|} + \frac{2\sqrt{-e^2x^2 + d^2}d}{|x|}\right)}{2d^3} - \frac{3be^4 \log\left(\frac{2d^2}{|x|} + \frac{2\sqrt{-e^2x^2 + d^2}d}{|x|}\right)}{8d^5} - \frac{5ae^6 \log\left(\frac{2d^2}{|x|} + \frac{2\sqrt{-e^2x^2 + d^2}d}{|x|}\right)}{16d^7} - \frac{\sqrt{-e^2x^2 + d^2}c}{2d^2x^2} - \frac{3\sqrt{-e^2x^2 + d^2}be^2}{8d^4x^2} - \frac{5\sqrt{-e^2x^2 + d^2}ae^4}{16d^6x^2} - \frac{\sqrt{-e^2x^2 + d^2}b}{4d^2x^4} - \frac{5\sqrt{-e^2x^2 + d^2}ae^2}{24d^4x^4} - \frac{\sqrt{-e^2x^2 + d^2}a}{6d^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^7/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*c*e^2*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^3 - 3/8*b*e^4*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^5 - 5/16*a*e^6*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^7 - 1/2*sqrt(-e^2*x^2 + d^2)*c/(d^2*x^2) - 3/8*sqrt(-e^2*x^2 + d^2)*b*e^2/(d^4*x^2) - 5/16*sqrt(-e^2*x^2 + d^2)*a*e^4/(d^6*x^2) - 1/4*sqrt(-e^2*x^2 + d^2)*b/(d^2*x^4) - 5/24*sqrt(-e^2*x^2 + d^2)*a*e^2/(d^4*x^4) - 1/6*sqrt(-e^2*x^2 + d^2)*a/(d^2*x^6)
```

mupad [B] time = 20.05, size = 1621, normalized size = 7.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(x^7*(d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

```
[Out] ((b*e^4)/4 + (6*b*e^4*((d + e*x)^(1/2) - d^(1/2))^2)/((d - e*x)^(1/2) - d^(1/2))^2 - (53*b*e^4*((d + e*x)^(1/2) - d^(1/2))^4)/(2*((d - e*x)^(1/2) - d^(1/2))^4) - (87*b*e^4*((d + e*x)^(1/2) - d^(1/2))^6)/((d - e*x)^(1/2) - d^(1/2))^6 + (657*b*e^4*((d + e*x)^(1/2) - d^(1/2))^8)/(4*((d - e*x)^(1/2) - d^(1/2))^8) - (121*b*e^4*((d + e*x)^(1/2) - d^(1/2))^10)/((d - e*x)^(1/2) - d^(1/2))^10)/((256*d^5*((d + e*x)^(1/2) - d^(1/2))^4)/((d - e*x)^(1/2) - d^(1/2))^4 - (1024*d^5*((d + e*x)^(1/2) - d^(1/2))^6)/((d - e*x)^(1/2) - d^(1/2))^6 + (1536*d^5*((d + e*x)^(1/2) - d^(1/2))^8)/((d - e*x)^(1/2) - d^(1/2))^8 - (1024*d^5*((d + e*x)^(1/2) - d^(1/2))^10)/((d - e*x)^(1/2) - d^(1/2))^10 + (256*d^5*((d + e*x)^(1/2) - d^(1/2))^12)/((d - e*x)^(1/2) - d^(1/2))^12) - ((c*e^2*((d + e*x)^(1/2) - d^(1/2))^2)/((d - e*x)^(1/2) - d^(1/2))^2 - (c*e^2)/2 + (15*c*e^2*((d + e*x)^(1/2) - d^(1/2))^4)/(2*((d - e*x)^(1/2) - d^(1/2))^4))/((16*d^3*((d + e*x)^(1/2) - d^(1/2))^2)/((d - e*x)^(1/2) - d^(1/2))^2 - (32*d^3*((d + e*x)^(1/2) - d^(1/2))^4)/((d - e*x)^(1/2) - d^(1/2))^4 + (16*d^3*((d + e*x)^(1/2) - d^(1/2))^6)/((d - e*x)^(1/2) - d^(1/2))^6) + ((a*e^6)/6 + (4*a*e^6*((d + e*x)^(1/2) - d^(1/2))^2)/((d - e*x)^(1/2) - d^(1/2))^2 + (71*a*e^6*((d + e*x)^(1/2) - d^(1/2))^4)/((d - e*x)^(1/2) - d^(1/2))^4 - (1558*a*e^6*((d + e*x)^(1/2) - d^(1/2))^6)/(3*((d - e*x)^(1/2) - d^(1/2))^6) - (540*a*e^6*((d + e*x)^(1/2) - d^(1/2))^8)/((d - e*x)^(1/2) - d^(1/2))^8 + (4248*a*e^6*((d + e*x)^(1/2) - d^(1/2))^10)/((d - e*x)^(1/2) - d^(1/2))^10 - (7683*a*e^6*((d + e*x)^(1/2) - d^(1/2))^12)/((d - e*x)^(1/2) - d^(1/2))^12 + (5558*a*e^6*((d + e*x)^(1/2) - d^(1/2))^14)/((d - e*x)^(1/2) - d^(1/2))^14 - (3643*a*e^6*((d + e*x)^(1/2) - d^(1/2))^16)/(2*((d - e*x)^(1/2) - d^(1/2))^16))/((4096*d^7*((d + e*x)^(1/2) - d^(1/2))^6)/((d - e*x)^(1/2) - d^(1/2))^6 - (24576*d^7*((d + e*x)^(1/2) - d^(1/2))^8)/((d - e*x)^(1/2) - d^(1/2))^8 + (61440*d^7*((d + e*x)^(1/2) - d^(1/2))^10)/((d - e*x)^(1/2) - d^(1/2))^10 - (81920*d^7*((d + e*x)^(1/2) - d^(1/2))^12)/((d - e*x)^(1/2) - d^(1/2))^12 + (61440*d^7*((d + e*x)^(1/2) - d^(1/2))^14)/((d - e*x)^(1/2) - d^(1/2))^14 - (24576*d^7*((d + e*x)^(1/2) - d^(1/2))^16)/((d - e*x)^(1/2) - d^(1/2))^16 + (4096*d^7*((d + e*x)^(1/2) - d^(1/2))^18)/((d - e*x)^(1/2) - d^(1/2))^18) - (5*a*e^6*log(((d + e*x)^(1/2) - d^(1/2))/((d - e*x)^(1/2) - d^(1/2))))/(16*d^7) - (3*b*e^4*log(((d + e*x)^(1/2) - d^(1/2))/((d - e*x)^(1/2) - d^(1/2))))/(8*d^5) - (c*e^2*log(((d + e*x)^(1/2) - d^(1/2))/((d - e*x)^(1/2) - d^(1/2))))/(2*d^3) + (5*a*e^6*log(((d + e*x)^(1/2) - d^(1/2))^2/((d - e*x)^(1/2) - d^(1/2))^2 - 1))/((16*d^7) + (3*b*e^4*log(((d + e*x)^(1/2) - d^(1/2))^2/((d - e*x)^(1/2) - d^(1/2))^2 - 1))/(8*d^5) + (c*e^2*log(((d + e*x)^(1/2) - d^(1/2))^2/((d - e*x)^(1/2) - d^(1/2))^2 - 1))
```

$$\begin{aligned} & 1)) / (2*d^3) + (197*a*e^6*((d + e*x)^{(1/2)} - d^{(1/2)})^2) / (8192*d^7*((d - e*x)^{(1/2)} - d^{(1/2)})^2) + (5*a*e^6*((d + e*x)^{(1/2)} - d^{(1/2)})^4) / (4096*d^7*((d - e*x)^{(1/2)} - d^{(1/2)})^4) + (a*e^6*((d + e*x)^{(1/2)} - d^{(1/2)})^6) / (24576*d^7*((d - e*x)^{(1/2)} - d^{(1/2)})^6) + (7*b*e^4*((d + e*x)^{(1/2)} - d^{(1/2)})^2) / (256*d^5*((d - e*x)^{(1/2)} - d^{(1/2)})^2) + (b*e^4*((d + e*x)^{(1/2)} - d^{(1/2)})^4) / (1024*d^5*((d - e*x)^{(1/2)} - d^{(1/2)})^4) + (c*e^2*((d + e*x)^{(1/2)} - d^{(1/2)})^2) / (32*d^3*((d - e*x)^{(1/2)} - d^{(1/2)})^2) \end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**4+b*x**2+a)/x**7/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)

[Out] Timed out

3.137 $\int \frac{x^2(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}} dx$

Optimal. Leaf size=216

$$\frac{d^2\sqrt{d^2 - e^2x^2} \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right) (8ae^4 + 6bd^2e^2 + 5cd^4) x\sqrt{d - ex}\sqrt{d + ex} (8ae^4 + 6bd^2e^2 + 5cd^4) x^3\sqrt{d - ex}\sqrt{d + ex}}{16e^7\sqrt{d - ex}\sqrt{d + ex} 16e^6}$$

Rubi [A] time = 0.21, antiderivative size = 245, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {520, 1267, 459, 321, 217, 203}

$$\frac{x(d^2 - e^2x^2)(8ae^4 + 6bd^2e^2 + 5cd^4)}{16e^6\sqrt{d - ex}\sqrt{d + ex}} + \frac{d^2\sqrt{d^2 - e^2x^2} \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right) (8ae^4 + 6bd^2e^2 + 5cd^4)}{16e^7\sqrt{d - ex}\sqrt{d + ex}} - \frac{x^3(d^2 - e^2x^2)(6be^2 + 5cd^2)}{24e^4\sqrt{d - ex}\sqrt{d + ex}} - \frac{cx^5(d^2 - e^2x^2)}{6e^2\sqrt{d - ex}\sqrt{d + ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -((5*c*d^4 + 6*b*d^2*e^2 + 8*a*e^4)*x*(d^2 - e^2*x^2))/(16*e^6*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((5*c*d^2 + 6*b*e^2)*x^3*(d^2 - e^2*x^2))/(24*e^4*Sqrt[d - e*x]*Sqrt[d + e*x]) - (c*x^5*(d^2 - e^2*x^2))/(6*e^2*Sqrt[d - e*x]*Sqrt[d + e*x]) + (d^2*(5*c*d^4 + 6*b*d^2*e^2 + 8*a*e^4)*Sqrt[d^2 - e^2*x^2]*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(16*e^7*Sqrt[d - e*x]*Sqrt[d + e*x])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] := Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n))^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/2]
```

2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]

Rule 1267

Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[(c^p*(f*x)^(m + 4*p - 1)*(d + e*x^2)^(q + 1))/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1)), x] + Dist[1/(e*(m + 4*p + 2*q + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]

Rubi steps

$$\int \frac{x^2(a + bx^2 + cx^4)}{\sqrt{d - ex}\sqrt{d + ex}} dx = \frac{\sqrt{d^2 - e^2x^2} \int \frac{x^2(a + bx^2 + cx^4)}{\sqrt{d^2 - e^2x^2}} dx}{\sqrt{d - ex}\sqrt{d + ex}}$$

$$= -\frac{cx^5(d^2 - e^2x^2)}{6e^2\sqrt{d - ex}\sqrt{d + ex}} - \frac{\sqrt{d^2 - e^2x^2} \int \frac{x^2(-6ae^2 - (5cd^2 + 6be^2)x^2)}{\sqrt{d^2 - e^2x^2}} dx}{6e^2\sqrt{d - ex}\sqrt{d + ex}}$$

$$= -\frac{(5cd^2 + 6be^2)x^3(d^2 - e^2x^2)}{24e^4\sqrt{d - ex}\sqrt{d + ex}} - \frac{cx^5(d^2 - e^2x^2)}{6e^2\sqrt{d - ex}\sqrt{d + ex}} + \frac{((5cd^4 + 6bd^2e^2 + 8ae^4)\sqrt{d^2 - e^2x^2})}{8e^4\sqrt{d - ex}\sqrt{d + ex}}$$

$$= -\frac{(5cd^4 + 6bd^2e^2 + 8ae^4)x(d^2 - e^2x^2)}{16e^6\sqrt{d - ex}\sqrt{d + ex}} - \frac{(5cd^2 + 6be^2)x^3(d^2 - e^2x^2)}{24e^4\sqrt{d - ex}\sqrt{d + ex}} - \frac{cx^5(d^2 - e^2x^2)}{6e^2\sqrt{d - ex}\sqrt{d + ex}}$$

$$= -\frac{(5cd^4 + 6bd^2e^2 + 8ae^4)x(d^2 - e^2x^2)}{16e^6\sqrt{d - ex}\sqrt{d + ex}} - \frac{(5cd^2 + 6be^2)x^3(d^2 - e^2x^2)}{24e^4\sqrt{d - ex}\sqrt{d + ex}} - \frac{cx^5(d^2 - e^2x^2)}{6e^2\sqrt{d - ex}\sqrt{d + ex}}$$

$$= -\frac{(5cd^4 + 6bd^2e^2 + 8ae^4)x(d^2 - e^2x^2)}{16e^6\sqrt{d - ex}\sqrt{d + ex}} - \frac{(5cd^2 + 6be^2)x^3(d^2 - e^2x^2)}{24e^4\sqrt{d - ex}\sqrt{d + ex}} - \frac{cx^5(d^2 - e^2x^2)}{6e^2\sqrt{d - ex}\sqrt{d + ex}}$$

Mathematica [A] time = 0.79, size = 202, normalized size = 0.94

$$\frac{ex\sqrt{d - ex}\sqrt{d + ex} (6(4ae^4 + 3bd^2e^2 + 2be^4x^2) + c(15d^4 + 10d^2e^2x^2 + 8e^4x^4)) + 96d^2 \tan^{-1}\left(\frac{\sqrt{d - ex}}{\sqrt{d + ex}}\right)(ae^4 + bd^2e^2 + cd^4) - \frac{6d^{3/2}\sqrt{d + ex} \sin^{-1}\left(\frac{\sqrt{d - ex}}{\sqrt{2}\sqrt{d}}\right)(8ae^4 + 10bd^2e^2 + 11cd^4)}{\sqrt{\frac{ex}{d} + 1}}}{48e^7}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]), x]
 [Out] -1/48*(e*x*Sqrt[d - e*x]*Sqrt[d + e*x]*(6*(3*b*d^2*e^2 + 4*a*e^4 + 2*b*e^4*x^2) + c*(15*d^4 + 10*d^2*e^2*x^2 + 8*e^4*x^4)) - (6*d^(3/2)*(11*c*d^4 + 10*b*d^2*e^2 + 8*a*e^4)*Sqrt[d + e*x]*ArcSin[Sqrt[d - e*x]/(Sqrt[2]*Sqrt[d])])/Sqrt[1 + (e*x)/d] + 96*d^2*(c*d^4 + b*d^2*e^2 + a*e^4)*ArcTan[Sqrt[d - e*x]/Sqrt[d + e*x]])/e^7

IntegrateAlgebraic [A] time = 0.29, size = 401, normalized size = 1.86

$$\frac{\tan^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d-ex}}\right)(8ad^2e^4 + 6bd^4e^2 + 5c^2d^4) - \sqrt{d+ex}\left(\frac{d^2(d+ex)}{d-ex} - d\right)\left(\frac{24ae^4(d+ex)^4}{(d-ex)^3} + \frac{96ae^6(d+ex)^3}{(d-ex)^2} + \frac{144ae^8(d+ex)^2}{(d-ex)} + \frac{96ae^6(d+ex)}{d-ex} + 24ae^4\right) + \frac{30bd^2e^2(d+ex)^4}{(d-ex)^3} + \frac{72bd^4e^4(d+ex)^3}{(d-ex)^2} + \frac{84bd^6e^6(d+ex)^2}{(d-ex)} + \frac{72bd^8e^8(d+ex)}{d-ex} + 30bd^6e^6}{24e^7\sqrt{d-ex}\left(\frac{d+ex}{d-ex} + 1\right)} + \frac{33c^3e^6(d+ex)^4}{(d-ex)^3} + \frac{28c^5e^8(d+ex)^3}{(d-ex)^2} + \frac{118c^7e^{10}(d+ex)^2}{(d-ex)} + \frac{28c^9e^{12}(d+ex)}{d-ex} + 33c^7d^4}{48e^7}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^2*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]), x]

```
[Out] -1/24*(Sqrt[d + e*x]*(-d^2 + (d^2*(d + e*x))/(d - e*x))*(33*c*d^4 + 30*b*d^2*e^2 + 24*a*e^4 + (28*c*d^4*(d + e*x))/(d - e*x) + (72*b*d^2*e^2*(d + e*x))/(d - e*x) + (96*a*e^4*(d + e*x))/(d - e*x) + (118*c*d^4*(d + e*x)^2)/(d - e*x)^2 + (84*b*d^2*e^2*(d + e*x)^2)/(d - e*x)^2 + (144*a*e^4*(d + e*x)^2)/(d - e*x)^2 + (28*c*d^4*(d + e*x)^3)/(d - e*x)^3 + (72*b*d^2*e^2*(d + e*x)^3)/(d - e*x)^3 + (96*a*e^4*(d + e*x)^3)/(d - e*x)^3 + (33*c*d^4*(d + e*x)^4)/(d - e*x)^4 + (30*b*d^2*e^2*(d + e*x)^4)/(d - e*x)^4 + (24*a*e^4*(d + e*x)^4)/(d - e*x)^4))/(e^7*Sqrt[d - e*x]*(1 + (d + e*x)/(d - e*x))^6) + ((5*c*d^6 + 6*b*d^4*e^2 + 8*a*d^2*e^4)*ArcTan[Sqrt[d + e*x]/Sqrt[d - e*x]])/(8*e^7)
```

fricas [A] time = 1.28, size = 134, normalized size = 0.62

$$\frac{(8ce^5x^5 + 2(5cd^2e^3 + 6be^5)x^3 + 3(5cd^4e + 6bd^2e^3 + 8ae^5)x)\sqrt{ex+d}\sqrt{-ex+d} + 6(5cd^6 + 6bd^4e^2 + 8ad^2e^4)\arctan\left(\frac{\sqrt{ex+d}\sqrt{-ex+d}-d}{ex}\right)}{48e^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/48*((8*c*e^5*x^5 + 2*(5*c*d^2*e^3 + 6*b*e^5)*x^3 + 3*(5*c*d^4*e + 6*b*d^2*e^3 + 8*a*e^5)*x)*sqrt(e*x + d)*sqrt(-e*x + d) + 6*(5*c*d^6 + 6*b*d^4*e^2 + 8*a*d^2*e^4)*arctan((sqrt(e*x + d)*sqrt(-e*x + d) - d)/(e*x)))/e^7
```

giac [A] time = 0.75, size = 208, normalized size = 0.96

$$\frac{1}{48} \left(6(5cd^6 + 6bd^4e^2 + 8ad^2e^4)\arcsin\left(\frac{\sqrt{2}\sqrt{ex+d}}{2\sqrt{d}}\right)e^{-6} - \left((2((4((xe+d)cd^{40} - 5cd^{40})(xe+d) + 3(15cd^{20} + 2bc^{20})d^{40})(xe+d) - (55cd^6e^{20} + 18bd^{20})d^{40})(xe+d) + (85cd^6e^{20} + 54bd^6e^{20} + 24ad^{40})d^{40})(xe+d) - 3(11cd^6e^{20} + 10bd^6e^{20} + 8ad^{40})d^{40})\sqrt{ex+d}\sqrt{-xe+d} \right) e^{(-1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(6*(5*c*d^6 + 6*b*d^4*e^2 + 8*a*d^2*e^4)*arcsin(1/2*sqrt(2)*sqrt(x*e + d)/sqrt(d))*e^(-6) - ((2*((4*((x*e + d)*c*e^(-6) - 5*c*d*e^(-6))*(x*e + d) + 3*(15*c*d^2*e^36 + 2*b*e^38)*e^(-42))*(x*e + d) - (55*c*d^3*e^36 + 18*b*d*e^38)*e^(-42))*(x*e + d) + (85*c*d^4*e^36 + 54*b*d^2*e^38 + 24*a*e^40)*e^(-42))*(x*e + d) - 3*(11*c*d^5*e^36 + 10*b*d^3*e^38 + 8*a*d*e^40)*e^(-42))*sqrt(x*e + d)*sqrt(-x*e + d))*e^(-1)
```

maple [C] time = 0.04, size = 273, normalized size = 1.26

$$\frac{\sqrt{-ex+d}\sqrt{ex+d}\left(8\sqrt{-2x^2+d^2}c^5\operatorname{csign}(e)+12\sqrt{-2x^2+d^2}b^5e^5\operatorname{csign}(e)+10\sqrt{-2x^2+d^2}cd^5\operatorname{csign}(e)-24ad^5\operatorname{arctan}\left(\frac{c\operatorname{csign}(e)}{\sqrt{-2x^2+d^2}}\right)+24\sqrt{-2x^2+d^2}a^5e^5\operatorname{csign}(e)-18bd^5\operatorname{arctan}\left(\frac{c\operatorname{csign}(e)}{\sqrt{-2x^2+d^2}}\right)+18\sqrt{-2x^2+d^2}bd^5e^5\operatorname{csign}(e)-15cd^5\operatorname{arctan}\left(\frac{c\operatorname{csign}(e)}{\sqrt{-2x^2+d^2}}\right)+15\sqrt{-2x^2+d^2}cd^5e^5\operatorname{csign}(e)\right)\operatorname{csign}(e)}{48\sqrt{-2x^2+d^2}e^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] -1/48*(-e*x+d)^(1/2)*(e*x+d)^(1/2)*(8*csign(e)*x^5*c*e^5*(-e^2*x^2+d^2)^(1/2)+12*csign(e)*x^3*b*e^5*(-e^2*x^2+d^2)^(1/2)+10*csign(e)*x^3*c*d^2*e^3*(-e^2*x^2+d^2)^(1/2)+24*(-e^2*x^2+d^2)^(1/2)*csign(e)*e^5*x*a+18*(-e^2*x^2+d^2)^(1/2)*csign(e)*e^3*x*b*d^2+15*(-e^2*x^2+d^2)^(1/2)*csign(e)*e*x*c*d^4-24*arctan(csign(e)*e*x/(-e^2*x^2+d^2)^(1/2))*a*d^2*e^4-18*arctan(csign(e)*e*x/(-e^2*x^2+d^2)^(1/2))*b*d^4*e^2-15*arctan(csign(e)*e*x/(-e^2*x^2+d^2)^(1/2))*c*d^6)*csign(e)/e^7/(-e^2*x^2+d^2)^(1/2)
```

maxima [A] time = 1.11, size = 190, normalized size = 0.88

$$\frac{\sqrt{-e^2x^2+d^2}cx^5}{6e^2} - \frac{5\sqrt{-e^2x^2+d^2}cd^2x^3}{24e^4} - \frac{\sqrt{-e^2x^2+d^2}bx^3}{4e^2} + \frac{5cd^6\arcsin\left(\frac{cx}{d}\right)}{16e^7} + \frac{3bd^4\arcsin\left(\frac{cx}{d}\right)}{8e^5} + \frac{ad^2\arcsin\left(\frac{cx}{d}\right)}{2e^3} - \frac{5\sqrt{-e^2x^2+d^2}cd^4x}{16e^6} - \frac{3\sqrt{-e^2x^2+d^2}bd^2x}{8e^4} - \frac{\sqrt{-e^2x^2+d^2}ax}{2e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="maxima")

[Out]
$$-1/6\sqrt{-e^2x^2 + d^2}cx^5/e^2 - 5/24\sqrt{-e^2x^2 + d^2}cd^2x^3/e^4 - 1/4\sqrt{-e^2x^2 + d^2}bx^3/e^2 + 5/16cd^6\arcsin(e*x/d)/e^7 + 3/8bd^4\arcsin(e*x/d)/e^5 + 1/2ad^2\arcsin(e*x/d)/e^3 - 5/16\sqrt{-e^2x^2 + d^2}cd^4x/e^6 - 3/8\sqrt{-e^2x^2 + d^2}bd^2x/e^4 - 1/2\sqrt{-e^2x^2 + d^2}ax/e^2$$

mupad [B] time = 23.12, size = 1132, normalized size = 5.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*(a + b*x^2 + c*x^4))/((d + e*x)^(1/2)*(d - e*x)^(1/2)),x)

[Out]
$$\begin{aligned} & ((14ad^2((d + ex)^{1/2} - d^{1/2})^3)/((d - ex)^{1/2} - d^{1/2})^3 - (14ad^2((d + ex)^{1/2} - d^{1/2})^5)/((d - ex)^{1/2} - d^{1/2})^5 + (2ad^2((d + ex)^{1/2} - d^{1/2})^7)/((d - ex)^{1/2} - d^{1/2})^7 - (2ad^2((d + ex)^{1/2} - d^{1/2}))/((d - ex)^{1/2} - d^{1/2}))/((e^3((d + ex)^{1/2} - d^{1/2})^2/((d - ex)^{1/2} - d^{1/2})^2 + 1)^4) - ((175cd^6((d + ex)^{1/2} - d^{1/2})^3)/((12((d - ex)^{1/2} - d^{1/2})^3) + (311cd^6((d + ex)^{1/2} - d^{1/2})^5)/(4((d - ex)^{1/2} - d^{1/2})^5) - (8361cd^6((d + ex)^{1/2} - d^{1/2})^7)/(4((d - ex)^{1/2} - d^{1/2})^7) + (42259cd^6((d + ex)^{1/2} - d^{1/2})^9)/(6((d - ex)^{1/2} - d^{1/2})^9) - (25295cd^6((d + ex)^{1/2} - d^{1/2})^{11})/(2((d - ex)^{1/2} - d^{1/2})^{11}) + (25295cd^6((d + ex)^{1/2} - d^{1/2})^{13})/(2((d - ex)^{1/2} - d^{1/2})^{13}) - (42259cd^6((d + ex)^{1/2} - d^{1/2})^{15})/(6((d - ex)^{1/2} - d^{1/2})^{15}) + (8361cd^6((d + ex)^{1/2} - d^{1/2})^{17})/(4((d - ex)^{1/2} - d^{1/2})^{17}) - (311cd^6((d + ex)^{1/2} - d^{1/2})^{19})/(4((d - ex)^{1/2} - d^{1/2})^{19}) - (175cd^6((d + ex)^{1/2} - d^{1/2})^{21})/(12((d - ex)^{1/2} - d^{1/2})^{21}) - (5cd^6((d + ex)^{1/2} - d^{1/2})^{23})/(4((d - ex)^{1/2} - d^{1/2})^{23}) + (5cd^6((d + ex)^{1/2} - d^{1/2}))/((4((d - ex)^{1/2} - d^{1/2}))/((e^7((d + ex)^{1/2} - d^{1/2})^2/((d - ex)^{1/2} - d^{1/2})^2 + 1)^{12}) - ((23bd^4((d + ex)^{1/2} - d^{1/2})^3)/(2((d - ex)^{1/2} - d^{1/2})^3) - (333bd^4((d + ex)^{1/2} - d^{1/2})^5)/(2((d - ex)^{1/2} - d^{1/2})^5) + (671bd^4((d + ex)^{1/2} - d^{1/2})^7)/(2((d - ex)^{1/2} - d^{1/2})^7) - (671bd^4((d + ex)^{1/2} - d^{1/2})^9)/(2((d - ex)^{1/2} - d^{1/2})^9) + (333bd^4((d + ex)^{1/2} - d^{1/2})^{11})/(2((d - ex)^{1/2} - d^{1/2})^{11}) - (23bd^4((d + ex)^{1/2} - d^{1/2})^{13})/(2((d - ex)^{1/2} - d^{1/2})^{13}) - (3bd^4((d + ex)^{1/2} - d^{1/2})^{15})/(2((d - ex)^{1/2} - d^{1/2})^{15}) + (3bd^4((d + ex)^{1/2} - d^{1/2}))/((2((d - ex)^{1/2} - d^{1/2}))/((e^5((d + ex)^{1/2} - d^{1/2})^2/((d - ex)^{1/2} - d^{1/2})^2 + 1)^8) + (2ad^2\operatorname{atan}(((d + ex)^{1/2} - d^{1/2})/((d - ex)^{1/2} - d^{1/2}))) / e^3 + (3bd^4\operatorname{atan}(((d + ex)^{1/2} - d^{1/2})/((d - ex)^{1/2} - d^{1/2}))) / (2e^5) + (5cd^6\operatorname{atan}(((d + ex)^{1/2} - d^{1/2})/((d - ex)^{1/2} - d^{1/2}))) / (4e^7) \end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(c*x**4+b*x**2+a)/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)

[Out] Timed out

$$3.138 \quad \int \frac{a+bx^2+cx^4}{\sqrt{d-ex}\sqrt{d+ex}} dx$$

Optimal. Leaf size=128

$$\frac{\tan^{-1}\left(\frac{\sqrt{d-ex}}{\sqrt{d+ex}}\right)(8ae^4 + 4bd^2e^2 + 3cd^4)}{4e^5} - \frac{x\sqrt{d-ex}\sqrt{d+ex}(4be^2 + 3cd^2)}{8e^4} + \frac{cx^3(ex-d)\sqrt{d+ex}}{4e^2\sqrt{d-ex}}$$

Rubi [A] time = 0.09, antiderivative size = 179, normalized size of antiderivative = 1.40, number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {520, 1159, 388, 217, 203}

$$\frac{\sqrt{d^2 - e^2x^2} \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)(8ae^4 + 4bd^2e^2 + 3cd^4)}{8e^5\sqrt{d-ex}\sqrt{d+ex}} - \frac{x(d^2 - e^2x^2)(4be^2 + 3cd^2)}{8e^4\sqrt{d-ex}\sqrt{d+ex}} - \frac{cx^3(d^2 - e^2x^2)}{4e^2\sqrt{d-ex}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2 + c*x^4)/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]

[Out] -((3*c*d^2 + 4*b*e^2)*x*(d^2 - e^2*x^2))/(8*e^4*Sqrt[d - e*x]*Sqrt[d + e*x]) - (c*x^3*(d^2 - e^2*x^2))/(4*e^2*Sqrt[d - e*x]*Sqrt[d + e*x]) + ((3*c*d^4 + 4*b*d^2*e^2 + 8*a*e^4)*Sqrt[d^2 - e^2*x^2]*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(8*e^5*Sqrt[d - e*x]*Sqrt[d + e*x])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 388

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(d*x*(a + b*x^n)^(p+1))/(b*(n*(p+1)+1)), x] - Dist[(a*d - b*c*(n*(p+1)+1))/(b*(n*(p+1)+1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p+1)+1, 0]

Rule 520

Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] := Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n))^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]

Rule 1159

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Simp[(c^p*x^(4*p-1)*(d + e*x^2)^(q+1))/(e*(4*p+2*q+1)), x] + Dist[1/(e*(4*p+2*q+1)), Int[(d + e*x^2)^q*ExpandToSum[e*(4*p+2*q+1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p-1)*x^(4*p-2) - e*c^p*(4*p+2*q+1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && !LtQ[q, -1]

Rubi steps

$$\begin{aligned}
\int \frac{a + bx^2 + cx^4}{\sqrt{d - ex} \sqrt{d + ex}} dx &= \frac{\sqrt{d^2 - e^2 x^2} \int \frac{a + bx^2 + cx^4}{\sqrt{d^2 - e^2 x^2}} dx}{\sqrt{d - ex} \sqrt{d + ex}} \\
&= \frac{cx^3 (d^2 - e^2 x^2)}{4e^2 \sqrt{d - ex} \sqrt{d + ex}} - \frac{\sqrt{d^2 - e^2 x^2} \int \frac{-4ae^2 - (3cd^2 + 4be^2)x^2}{\sqrt{d^2 - e^2 x^2}} dx}{4e^2 \sqrt{d - ex} \sqrt{d + ex}} \\
&= \frac{(3cd^2 + 4be^2)x (d^2 - e^2 x^2)}{8e^4 \sqrt{d - ex} \sqrt{d + ex}} - \frac{cx^3 (d^2 - e^2 x^2)}{4e^2 \sqrt{d - ex} \sqrt{d + ex}} - \frac{\left((-8ae^4 + d^2(-3cd^2 - 4be^2))\right)}{8e^4 \sqrt{d - ex} \sqrt{d + ex}} \\
&= \frac{(3cd^2 + 4be^2)x (d^2 - e^2 x^2)}{8e^4 \sqrt{d - ex} \sqrt{d + ex}} - \frac{cx^3 (d^2 - e^2 x^2)}{4e^2 \sqrt{d - ex} \sqrt{d + ex}} - \frac{\left((-8ae^4 + d^2(-3cd^2 - 4be^2))\right)}{8e^4 \sqrt{d - ex} \sqrt{d + ex}} \\
&= \frac{(3cd^2 + 4be^2)x (d^2 - e^2 x^2)}{8e^4 \sqrt{d - ex} \sqrt{d + ex}} - \frac{cx^3 (d^2 - e^2 x^2)}{4e^2 \sqrt{d - ex} \sqrt{d + ex}} + \frac{(3cd^4 + 4bd^2e^2 + 8ae^4) \sqrt{d^2 - e^2 x^2}}{8e^5 \sqrt{d - ex} \sqrt{d + ex}}
\end{aligned}$$

Mathematica [A] time = 0.56, size = 157, normalized size = 1.23

$$\frac{16 \tan^{-1}\left(\frac{\sqrt{d-ex}}{\sqrt{d+ex}}\right) (ae^4 + bd^2e^2 + cd^4) + ex\sqrt{d-ex}\sqrt{d+ex} (4be^2 + 3cd^2 + 2ce^2x^2) - \frac{2d^{5/2} \sqrt{\frac{ex}{d} + 1} (4be^2 + 5cd^2) \sin^{-1}\left(\frac{\sqrt{d-ex}}{\sqrt{2}\sqrt{d}}\right)}{\sqrt{d+ex}}}{8e^5}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2 + c*x^4)/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]

[Out] -1/8*(e*x*Sqrt[d - e*x]*Sqrt[d + e*x]*(3*c*d^2 + 4*b*e^2 + 2*c*e^2*x^2) - (2*d^(5/2)*(5*c*d^2 + 4*b*e^2)*Sqrt[1 + (e*x)/d]*ArcSin[Sqrt[d - e*x]/(Sqrt[2]*Sqrt[d])])/Sqrt[d + e*x] + 16*(c*d^4 + b*d^2*e^2 + a*e^4)*ArcTan[Sqrt[d - e*x]/Sqrt[d + e*x]])/e^5

IntegrateAlgebraic [A] time = 0.19, size = 207, normalized size = 1.62

$$\frac{\tan^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d-ex}}\right) (8ae^4 + 4bd^2e^2 + 3cd^4)}{4e^5} - \frac{d^2\sqrt{d+ex} \left(\frac{d+ex}{d-ex} - 1\right) \left(\frac{4be^2(d+ex)^2}{(d-ex)^2} + \frac{8be^2(d+ex)}{d-ex} + 4be^2 + \frac{5cd^2(d+ex)^2}{(d-ex)^2} + \frac{2cd^2(d+ex)}{d-ex} + 5cd^2\right)}{4e^5\sqrt{d-ex} \left(\frac{d+ex}{d-ex} + 1\right)^4}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]

[Out] -1/4*(d^2*Sqrt[d + e*x]*(-1 + (d + e*x)/(d - e*x))*(5*c*d^2 + 4*b*e^2 + (2*c*d^2*(d + e*x))/(d - e*x) + (8*b*e^2*(d + e*x))/(d - e*x) + (5*c*d^2*(d + e*x)^2)/(d - e*x)^2 + (4*b*e^2*(d + e*x)^2)/(d - e*x)^2))/(e^5*Sqrt[d - e*x]*(1 + (d + e*x)/(d - e*x))^4) + ((3*c*d^4 + 4*b*d^2*e^2 + 8*a*e^4)*ArcTan[Sqrt[d + e*x]/Sqrt[d - e*x]])/(4*e^5)

fricas [A] time = 1.25, size = 100, normalized size = 0.78

$$\frac{(2ce^3x^3 + (3cd^2e + 4be^3)x)\sqrt{ex+d}\sqrt{-ex+d} + 2(3cd^4 + 4bd^2e^2 + 8ae^4) \arctan\left(\frac{\sqrt{ex+d}\sqrt{-ex+d}-d}{ex}\right)}{8e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fricas")

[Out] $-1/8*((2*c*e^3*x^3 + (3*c*d^2*e + 4*b*e^3)*x)*\text{sqrt}(e*x + d)*\text{sqrt}(-e*x + d) + 2*(3*c*d^4 + 4*b*d^2*e^2 + 8*a*e^4)*\text{arctan}((\text{sqrt}(e*x + d)*\text{sqrt}(-e*x + d) - d)/(e*x)))/e^5$

giac [A] time = 0.59, size = 135, normalized size = 1.05

$$\frac{1}{8} \left(2(3cd^4 + 4bd^2e^2 + 8ae^4) \arcsin\left(\frac{\sqrt{2}\sqrt{xe+d}}{2\sqrt{d}}\right) e^{(-4)} - \left((2((xe+d)ce^{(-4)} - 3cde^{(-4)})(xe+d) + (9cd^2e^{16} + 4be^{18})e^{(-20)})(xe+d) - (5cd^3e^{16} + 4bde^{18})e^{(-20)}\sqrt{xe+d}\sqrt{-xe+d} \right) e^{(-1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="giac")`

[Out] $1/8*(2*(3*c*d^4 + 4*b*d^2*e^2 + 8*a*e^4)*\text{arcsin}(1/2*\text{sqrt}(2)*\text{sqrt}(x*e + d)/\text{sqrt}(d))*e^{(-4)} - ((2*((x*e + d)*c*e^{(-4)} - 3*c*d*e^{(-4)})*(x*e + d) + (9*c*d^2*e^{16} + 4*b*e^{18})*e^{(-20)})*(x*e + d) - (5*c*d^3*e^{16} + 4*b*d*e^{18})*e^{(-20)})*\text{sqrt}(x*e + d)*\text{sqrt}(-x*e + d))*e^{(-1)}$

maple [C] time = 0.02, size = 191, normalized size = 1.49

$$\frac{\sqrt{-ex+d}\sqrt{ex+d}\left(2\sqrt{-e^2x^2+d^2}ce^3x^3\text{csgn}(e) - 8ae^4\arctan\left(\frac{ex\text{csgn}(e)}{\sqrt{-e^2x^2+d^2}}\right) - 4bd^2e^2\arctan\left(\frac{ex\text{csgn}(e)}{\sqrt{-e^2x^2+d^2}}\right) + 4\sqrt{-e^2x^2+d^2}be^3x\text{csgn}(e) - 3cd^4\arctan\left(\frac{ex\text{csgn}(e)}{\sqrt{-e^2x^2+d^2}}\right) + 3\sqrt{-e^2x^2+d^2}cd^2ex\text{csgn}(e)\right)\text{csgn}(e)}{8\sqrt{-e^2x^2+d^2}e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)`

[Out] $-1/8*(-e*x+d)^{(1/2)}*(e*x+d)^{(1/2)}*(2*c*\text{sgn}(e)*x^3*c*e^3*(-e^2*x^2+d^2)^{(1/2)} + 4*(-e^2*x^2+d^2)^{(1/2)}*c*\text{sgn}(e)*e^3*x*b + 3*(-e^2*x^2+d^2)^{(1/2)}*c*\text{sgn}(e)*e*x*c*d^2 - 8*\text{arctan}(1/(-e^2*x^2+d^2)^{(1/2)}*e*x*c*\text{sgn}(e))*a*e^4 - 4*\text{arctan}(1/(-e^2*x^2+d^2)^{(1/2)}*e*x*c*\text{sgn}(e))*b*d^2*e^2 - 3*\text{arctan}(1/(-e^2*x^2+d^2)^{(1/2)}*e*x*c*\text{sgn}(e))*c*d^4)*c*\text{sgn}(e)/e^5/(-e^2*x^2+d^2)^{(1/2)}$

maxima [A] time = 1.02, size = 113, normalized size = 0.88

$$-\frac{\sqrt{-e^2x^2+d^2}cx^3}{4e^2} + \frac{3cd^4\arcsin\left(\frac{ex}{d}\right)}{8e^5} + \frac{bd^2\arcsin\left(\frac{ex}{d}\right)}{2e^3} + \frac{a\arcsin\left(\frac{ex}{d}\right)}{e} - \frac{3\sqrt{-e^2x^2+d^2}cd^2x}{8e^4} - \frac{\sqrt{-e^2x^2+d^2}bx}{2e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="maxima")`

[Out] $-1/4*\text{sqrt}(-e^2*x^2 + d^2)*c*x^3/e^2 + 3/8*c*d^4*\text{arcsin}(e*x/d)/e^5 + 1/2*b*d^2*\text{arcsin}(e*x/d)/e^3 + a*\text{arcsin}(e*x/d)/e - 3/8*\text{sqrt}(-e^2*x^2 + d^2)*c*d^2*x/e^4 - 1/2*\text{sqrt}(-e^2*x^2 + d^2)*b*x/e^2$

mupad [B] time = 12.86, size = 651, normalized size = 5.09

$$\frac{\frac{14bd^2(\sqrt{d^2-x^2})^2}{(\sqrt{d^2-x^2})^2} \cdot \frac{14bd^2(\sqrt{d^2-x^2})^2}{(\sqrt{d^2-x^2})^2} \cdot \frac{2bd^2(\sqrt{d^2-x^2})^2}{(\sqrt{d^2-x^2})^2} \cdot \frac{2bd^2(\sqrt{d^2-x^2})^2}{(\sqrt{d^2-x^2})^2} \cdot \frac{4a\text{atan}\left(\frac{e(\sqrt{d^2-x^2})}{\sqrt{d^2-x^2}}\right)}{\sqrt{d^2-x^2}} \cdot \frac{23cd^4(\sqrt{d^2-x^2})^2}{2(\sqrt{d^2-x^2})^2} \cdot \frac{333bd^4(\sqrt{d^2-x^2})^2}{2(\sqrt{d^2-x^2})^2} \cdot \frac{671cd^4(\sqrt{d^2-x^2})^2}{2(\sqrt{d^2-x^2})^2} \cdot \frac{671cd^4(\sqrt{d^2-x^2})^2}{2(\sqrt{d^2-x^2})^2} \cdot \frac{333bd^4(\sqrt{d^2-x^2})^2}{2(\sqrt{d^2-x^2})^2} \cdot \frac{23cd^4(\sqrt{d^2-x^2})^2}{2(\sqrt{d^2-x^2})^2} \cdot \frac{23cd^4(\sqrt{d^2-x^2})^2}{2(\sqrt{d^2-x^2})^2} \cdot \frac{3cd^4(\sqrt{d^2-x^2})^2}{2(\sqrt{d^2-x^2})^2} \cdot \frac{3cd^4(\sqrt{d^2-x^2})^2}{2(\sqrt{d^2-x^2})^2} \cdot \frac{2bd^2\text{atan}\left(\frac{\sqrt{d^2-x^2}}{\sqrt{d^2-x^2}}\right)}{e^2} \cdot \frac{3cd^4\text{atan}\left(\frac{\sqrt{d^2-x^2}}{\sqrt{d^2-x^2}}\right)}{2e^2}}{e^5\left(\frac{\sqrt{d^2-x^2}}{\sqrt{d^2-x^2}}+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*x^2 + c*x^4)/((d + e*x)^(1/2)*(d - e*x)^(1/2)),x)`

[Out] $((14*b*d^2*((d + e*x)^{(1/2)} - d^{(1/2)})^3)/((d - e*x)^{(1/2)} - d^{(1/2)})^3 - (14*b*d^2*((d + e*x)^{(1/2)} - d^{(1/2)})^5)/((d - e*x)^{(1/2)} - d^{(1/2)})^5 + (2*b*d^2*((d + e*x)^{(1/2)} - d^{(1/2)})^7)/((d - e*x)^{(1/2)} - d^{(1/2)})^7 - (2*b*d^2*((d + e*x)^{(1/2)} - d^{(1/2)}))/((d - e*x)^{(1/2)} - d^{(1/2)})/(e^3*((d + e*x)^{(1/2)} - d^{(1/2)})^2/((d - e*x)^{(1/2)} - d^{(1/2)})^2 + 1)^4 - (4*a*\text{atan}((e*((d - e*x)^{(1/2)} - d^{(1/2)}))/((e^2)^{(1/2))*((d + e*x)^{(1/2)} - d^{(1/2)}))))/(e^2)^{(1/2)} - ((23*c*d^4*((d + e*x)^{(1/2)} - d^{(1/2)})^3)/(2*((d - e*x)^{(1/2)} - d^{(1/2)})^3) - (333*c*d^4*((d + e*x)^{(1/2)} - d^{(1/2)})^5)/(2*((d - e*x)^{(1/2)} - d^{(1/2)})^5) + (671*c*d^4*((d + e*x)^{(1/2)} - d^{(1/2)})^7)/(2*((d - e*x)^{(1/2)} - d^{(1/2)})^7) + (671*c*d^4*((d + e*x)^{(1/2)} - d^{(1/2)})^7)/(2*((d - e*x)^{(1/2)} - d^{(1/2)})^7) + (333*c*d^4*((d + e*x)^{(1/2)} - d^{(1/2)})^5)/(2*((d - e*x)^{(1/2)} - d^{(1/2)})^5) - (23*c*d^4*((d + e*x)^{(1/2)} - d^{(1/2)})^3)/(2*((d - e*x)^{(1/2)} - d^{(1/2)})^3))$

$$\begin{aligned} & \frac{1}{2} - d^{(1/2)})^7) - (671*c*d^4*((d + e*x)^{(1/2)} - d^{(1/2)})^9)/(2*((d - e*x) \\ &)^{(1/2)} - d^{(1/2)})^9) + (333*c*d^4*((d + e*x)^{(1/2)} - d^{(1/2)})^{11})/(2*((d - \\ & e*x)^{(1/2)} - d^{(1/2)})^{11}) - (23*c*d^4*((d + e*x)^{(1/2)} - d^{(1/2)})^{13})/(2*(\\ & (d - e*x)^{(1/2)} - d^{(1/2)})^{13}) - (3*c*d^4*((d + e*x)^{(1/2)} - d^{(1/2)})^{15})/(\\ & 2*((d - e*x)^{(1/2)} - d^{(1/2)})^{15}) + (3*c*d^4*((d + e*x)^{(1/2)} - d^{(1/2)}))/ \\ & (2*((d - e*x)^{(1/2)} - d^{(1/2)})))/(e^5*(((d + e*x)^{(1/2)} - d^{(1/2)})^2/((d - e \\ & *x)^{(1/2)} - d^{(1/2)})^2 + 1)^8) + (2*b*d^2*atan(((d + e*x)^{(1/2)} - d^{(1/2)})/ \\ & ((d - e*x)^{(1/2)} - d^{(1/2)})))/e^3 + (3*c*d^4*atan(((d + e*x)^{(1/2)} - d^{(1/2) \\ & })/((d - e*x)^{(1/2)} - d^{(1/2)})))/(2*e^5) \end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**4+b*x**2+a)/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)

[Out] Timed out

$$3.139 \quad \int \frac{a+bx^2+cx^4}{x^2\sqrt{d-ex}\sqrt{d+ex}} dx$$

Optimal. Leaf size=102

$$-\frac{a\sqrt{d-ex}\sqrt{d+ex}}{d^2x} - \frac{(2be^2 + cd^2)\tan^{-1}\left(\frac{\sqrt{d-ex}}{\sqrt{d+ex}}\right)}{e^3} + \frac{cx(ex-d)\sqrt{d+ex}}{2e^2\sqrt{d-ex}}$$

Rubi [A] time = 0.12, antiderivative size = 155, normalized size of antiderivative = 1.52, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {520, 1265, 388, 217, 203}

$$-\frac{a(d^2 - e^2x^2)}{d^2x\sqrt{d-ex}\sqrt{d+ex}} + \frac{\sqrt{d^2 - e^2x^2}(2be^2 + cd^2)\tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{2e^3\sqrt{d-ex}\sqrt{d+ex}} - \frac{cx(d^2 - e^2x^2)}{2e^2\sqrt{d-ex}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2 + c*x^4)/(x^2*Sqrt[d - e*x]*Sqrt[d + e*x]),x]

[Out] -((a*(d^2 - e^2*x^2))/(d^2*x*Sqrt[d - e*x]*Sqrt[d + e*x])) - (c*x*(d^2 - e^2*x^2))/(2*e^2*Sqrt[d - e*x]*Sqrt[d + e*x]) + ((c*d^2 + 2*b*e^2)*Sqrt[d^2 - e^2*x^2]*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^3*Sqrt[d - e*x]*Sqrt[d + e*x])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 388

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]

Rule 520

Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] := Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p]]/(a1*a2 + b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n))^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]

Rule 1265

Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]}, Simp[(R*(f*x)^(m + 1)*(d + e*x^2)^(q + 1))/(d*f*(m + 1)), x] + Dist[1/(d*f^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[(d*f*(m + 1)*Qx)/x - e*R*(m + 2*q + 3), x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ

[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]

Rubi steps

$$\begin{aligned}
 \int \frac{a + bx^2 + cx^4}{x^2 \sqrt{d - ex} \sqrt{d + ex}} dx &= \frac{\sqrt{d^2 - e^2 x^2} \int \frac{a + bx^2 + cx^4}{x^2 \sqrt{d^2 - e^2 x^2}} dx}{\sqrt{d - ex} \sqrt{d + ex}} \\
 &= -\frac{a(d^2 - e^2 x^2)}{d^2 x \sqrt{d - ex} \sqrt{d + ex}} - \frac{\sqrt{d^2 - e^2 x^2} \int \frac{-bd^2 - cd^2 x^2}{\sqrt{d^2 - e^2 x^2}} dx}{d^2 \sqrt{d - ex} \sqrt{d + ex}} \\
 &= -\frac{a(d^2 - e^2 x^2)}{d^2 x \sqrt{d - ex} \sqrt{d + ex}} - \frac{cx(d^2 - e^2 x^2)}{2e^2 \sqrt{d - ex} \sqrt{d + ex}} + \frac{\left(2b + \frac{cd^2}{e^2}\right) \sqrt{d^2 - e^2 x^2} \int \frac{1}{\sqrt{d^2 - e^2 x^2}} dx}{2 \sqrt{d - ex} \sqrt{d + ex}} \\
 &= -\frac{a(d^2 - e^2 x^2)}{d^2 x \sqrt{d - ex} \sqrt{d + ex}} - \frac{cx(d^2 - e^2 x^2)}{2e^2 \sqrt{d - ex} \sqrt{d + ex}} + \frac{\left(2b + \frac{cd^2}{e^2}\right) \sqrt{d^2 - e^2 x^2} \operatorname{Subst}\left(\frac{1}{\sqrt{d^2 - e^2 x^2}}, d - ex\right)}{2 \sqrt{d - ex} \sqrt{d + ex}} \\
 &= -\frac{a(d^2 - e^2 x^2)}{d^2 x \sqrt{d - ex} \sqrt{d + ex}} - \frac{cx(d^2 - e^2 x^2)}{2e^2 \sqrt{d - ex} \sqrt{d + ex}} + \frac{(cd^2 + 2be^2) \sqrt{d^2 - e^2 x^2} \tan^{-1}\left(\frac{\sqrt{d - ex}}{\sqrt{d + ex}}\right)}{2e^3 \sqrt{d - ex} \sqrt{d + ex}}
 \end{aligned}$$

Mathematica [A] time = 0.56, size = 135, normalized size = 1.32

$$\frac{\frac{e \sqrt{d - ex} \sqrt{d + ex} (2ae^2 + cd^2 x^2)}{d^2 x} + 4 (be^2 + cd^2) \tan^{-1}\left(\frac{\sqrt{d - ex}}{\sqrt{d + ex}}\right) - \frac{2cd^{5/2} \sqrt{\frac{ex}{d} + 1} \sin^{-1}\left(\frac{\sqrt{d - ex}}{\sqrt{2} \sqrt{d}}\right)}{\sqrt{d + ex}}}{2e^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2 + c*x^4)/(x^2*Sqrt[d - e*x]*Sqrt[d + e*x]),x]

[Out] -1/2*((e*Sqrt[d - e*x]*Sqrt[d + e*x]*(2*a*e^2 + c*d^2*x^2))/(d^2*x) - (2*c*d^(5/2)*Sqrt[1 + (e*x)/d]*ArcSin[Sqrt[d - e*x]/(Sqrt[2]*Sqrt[d])])/Sqrt[d + e*x] + 4*(c*d^2 + b*e^2)*ArcTan[Sqrt[d - e*x]/Sqrt[d + e*x]])/e^3

IntegrateAlgebraic [A] time = 0.16, size = 192, normalized size = 1.88

$$\frac{(2be^2 + cd^2) \tan^{-1}\left(\frac{\sqrt{d + ex}}{\sqrt{d - ex}}\right) - \sqrt{d + ex} \left(\frac{2ae^4(d + ex)^2}{(d - ex)^2} + \frac{4ae^4(d + ex)}{d - ex} + 2ae^4 + \frac{cd^4(d + ex)^2}{(d - ex)^2} - \frac{2cd^4(d + ex)}{d - ex} + cd^4\right)}{e^3 d^2 e^3 \sqrt{d - ex} \left(\frac{d + ex}{d - ex} - 1\right) \left(\frac{d + ex}{d - ex} + 1\right)^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)/(x^2*Sqrt[d - e*x]*Sqrt[d + e*x]),x]

[Out] -((Sqrt[d + e*x]*(c*d^4 + 2*a*e^4 - (2*c*d^4*(d + e*x))/(d - e*x) + (4*a*e^4*(d + e*x))/(d - e*x) + (c*d^4*(d + e*x)^2)/(d - e*x)^2 + (2*a*e^4*(d + e*x)^2)/(d - e*x)^2))/(d^2*e^3*Sqrt[d - e*x]*(-1 + (d + e*x)/(d - e*x))*(1 + (d + e*x)/(d - e*x))^2) + ((c*d^2 + 2*b*e^2)*ArcTan[Sqrt[d + e*x]/Sqrt[d - e*x]])/e^3

fricas [A] time = 1.18, size = 90, normalized size = 0.88

$$\frac{2(cd^4 + 2bd^2e^2)x \arctan\left(\frac{\sqrt{ex+d}\sqrt{-ex+d}-d}{ex}\right) + (cd^2ex^2 + 2ae^3)\sqrt{ex+d}\sqrt{-ex+d}}{2d^2e^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/x^2/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fricas")

[Out] -1/2*(2*(c*d^4 + 2*b*d^2*e^2)*x*arctan((sqrt(e*x + d)*sqrt(-e*x + d) - d)/(e*x)) + (c*d^2*e*x^2 + 2*a*e^3)*sqrt(e*x + d)*sqrt(-e*x + d))/(d^2*e^3*x)

giac [B] time = 1.05, size = 257, normalized size = 2.52

$$\frac{1}{2} \left(\left(\pi + 2 \arctan \left(\frac{\sqrt{xe+d} \left(\frac{(\sqrt{2}\sqrt{d}-\sqrt{-xe+d})^2}{xe+d} - 1 \right)}{2(\sqrt{2}\sqrt{d}-\sqrt{-xe+d})} \right) \right) \left(cd^2 + 2be^2 \right) e^{(-2)} - ((xe+d)ce^{(-2)} - cde^{(-2)})\sqrt{xe+d}\sqrt{-xe+d} - \frac{8a \left(\frac{\sqrt{2}\sqrt{d}-\sqrt{-xe+d}}{\sqrt{xe+d}} - \frac{\sqrt{xe+d}}{\sqrt{2}\sqrt{d}-\sqrt{-xe+d}} \right) e^2}{\left(\left(\frac{\sqrt{2}\sqrt{d}-\sqrt{-xe+d}}{\sqrt{xe+d}} - \frac{\sqrt{xe+d}}{\sqrt{2}\sqrt{d}-\sqrt{-xe+d}} \right)^2 - 4 \right) d^2} \right) e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/x^2/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="giac")

[Out] 1/2*((pi + 2*arctan(1/2*sqrt(x*e + d)*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))^2/(x*e + d) - 1)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d))))*(c*d^2 + 2*b*e^2)*e^(-2) - ((x*e + d)*c*e^(-2) - c*d*e^(-2))*sqrt(x*e + d)*sqrt(-x*e + d) - 8*a*(sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d))*e^2/(((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^2 - 4)*d^2))*e^(-1)

maple [C] time = 0.02, size = 148, normalized size = 1.45

$$\frac{\sqrt{-ex+d}\sqrt{ex+d}\left(-2bd^2e^2x\arctan\left(\frac{ex\operatorname{csgn}(e)}{\sqrt{-e^2x^2+d^2}}\right)-cd^4x\arctan\left(\frac{ex\operatorname{csgn}(e)}{\sqrt{-e^2x^2+d^2}}\right)+\sqrt{-e^2x^2+d^2}cd^2ex^2\operatorname{csgn}(e)+2\sqrt{-e^2x^2+d^2}ae^3\operatorname{csgn}(e)\right)\operatorname{csgn}(e)}{2\sqrt{-e^2x^2+d^2}d^2e^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^2+a)/x^2/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)

[Out] -1/2*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/d^2*(c*sgn(e)*x^2*c*d^2*e*(-e^2*x^2+d^2)^(1/2)-2*arctan(1/(-e^2*x^2+d^2)^(1/2)*e*x*csgn(e))*x*b*d^2*e^2-arctan(1/(-e^2*x^2+d^2)^(1/2)*e*x*csgn(e))*x*c*d^4+2*(-e^2*x^2+d^2)^(1/2)*csgn(e)*e^3*a)*csgn(e)/e^3/(-e^2*x^2+d^2)^(1/2)/x

maxima [A] time = 1.05, size = 73, normalized size = 0.72

$$\frac{cd^2 \arcsin\left(\frac{ex}{d}\right)}{2e^3} + \frac{b \arcsin\left(\frac{ex}{d}\right)}{e} - \frac{\sqrt{-e^2x^2+d^2}cx}{2e^2} - \frac{\sqrt{-e^2x^2+d^2}a}{d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/x^2/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="maxima")

[Out] 1/2*c*d^2*arcsin(e*x/d)/e^3 + b*arcsin(e*x/d)/e - 1/2*sqrt(-e^2*x^2 + d^2)*c*x/e^2 - sqrt(-e^2*x^2 + d^2)*a/(d^2*x)

mupad [B] time = 7.00, size = 306, normalized size = 3.00

$$\frac{\frac{14cd^2(\sqrt{d+ex}-\sqrt{d})^3}{(\sqrt{d-ex}-\sqrt{d})^3} - \frac{14cd^2(\sqrt{d+ex}-\sqrt{d})^5}{(\sqrt{d-ex}-\sqrt{d})^5} + \frac{2cd^2(\sqrt{d+ex}-\sqrt{d})^7}{(\sqrt{d-ex}-\sqrt{d})^7} - \frac{2cd^2(\sqrt{d+ex}-\sqrt{d})}{\sqrt{d-ex}-\sqrt{d}}}{e^3 \left(\frac{(\sqrt{d+ex}-\sqrt{d})^2}{(\sqrt{d-ex}-\sqrt{d})^2} + 1 \right)^4} - \frac{4b \operatorname{atan}\left(\frac{e(\sqrt{d-ex}-\sqrt{d})}{\sqrt{2}(\sqrt{d+ex}-\sqrt{d})}\right)}{\sqrt{e^2}} + \frac{2cd^2 \operatorname{atan}\left(\frac{\sqrt{d+ex}-\sqrt{d}}{\sqrt{d-ex}-\sqrt{d}}\right)}{e^3} - \frac{\left(\frac{a}{d} + \frac{aex}{d^2}\right)\sqrt{d-ex}}{x\sqrt{d+ex}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^2 + c*x^4)/(x^2*(d + e*x)^(1/2)*(d - e*x)^(1/2)),x)

[Out] ((14*c*d^2*((d + e*x)^(1/2) - d^(1/2))^3)/((d - e*x)^(1/2) - d^(1/2))^3 - (14*c*d^2*((d + e*x)^(1/2) - d^(1/2))^5)/((d - e*x)^(1/2) - d^(1/2))^5 + (2*

$$c*d^2*((d + e*x)^{(1/2)} - d^{(1/2)})^7/((d - e*x)^{(1/2)} - d^{(1/2)})^7 - (2*c*d^2*((d + e*x)^{(1/2)} - d^{(1/2)})/((d - e*x)^{(1/2)} - d^{(1/2)})/(e^3*((d + e*x)^{(1/2)} - d^{(1/2)})^2/((d - e*x)^{(1/2)} - d^{(1/2)})^2 + 1)^4 - (4*b*atan((e*((d - e*x)^{(1/2)} - d^{(1/2)})/(e^2)^{(1/2)*((d + e*x)^{(1/2)} - d^{(1/2)})}))/((e^2)^{(1/2)} + (2*c*d^2*atan(((d + e*x)^{(1/2)} - d^{(1/2)})/((d - e*x)^{(1/2)} - d^{(1/2)})))/e^3 - ((a/d + (a*e*x)/d^2)*(d - e*x)^{(1/2)})/(x*(d + e*x)^{(1/2)})$$

sympy [C] time = 104.02, size = 287, normalized size = 2.81

$$\frac{i a e C_{66}^{33} \left(\begin{matrix} \frac{5}{4}, \frac{7}{4}, 1 \\ \frac{3}{4}, \frac{3}{2}, 2 \end{matrix} \middle| \frac{\rho}{2e^2} \right)}{4\pi^{\frac{3}{2}} d^2} + \frac{a e C_{66}^{28} \left(\begin{matrix} \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2}, 1 \\ \frac{3}{4}, \frac{3}{4} \end{matrix} \middle| \frac{\rho e^{-2a}}{2e^2} \right)}{4\pi^{\frac{3}{2}} d^2} - \frac{i b C_{66}^{42} \left(\begin{matrix} \frac{1}{4}, \frac{3}{4} \\ 0, \frac{1}{2}, \frac{3}{2}, 1, 0 \end{matrix} \middle| \frac{\rho}{2e^2} \right)}{4\pi^{\frac{3}{2}} e} + \frac{b C_{66}^{28} \left(\begin{matrix} -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 1 \\ -\frac{1}{2}, \frac{1}{4} \end{matrix} \middle| \frac{\rho e^{-2a}}{2e^2} \right)}{4\pi^{\frac{3}{2}} e} - \frac{i c d^2 C_{66}^{42} \left(\begin{matrix} \frac{3}{4}, -\frac{1}{4} \\ -1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, 0 \end{matrix} \middle| \frac{\rho}{2e^2} \right)}{4\pi^{\frac{3}{2}} e^3} + \frac{c d^2 C_{66}^{28} \left(\begin{matrix} -\frac{3}{2}, -\frac{5}{4}, -1, -\frac{3}{4}, -\frac{1}{2}, 1 \\ -\frac{5}{4}, -\frac{3}{4} \end{matrix} \middle| \frac{\rho e^{-2a}}{2e^2} \right)}{4\pi^{\frac{3}{2}} e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**4+b*x**2+a)/x**2/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)

[Out] I*a*e*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7/4, 2), (0,)), d**2/(e**2*x**2))/(4*pi**(3/2)*d**2) + a*e*meijerg(((1/2, 3/4, 1, 5/4, 3/2, 1), ()), ((3/4, 5/4), (1/2, 1, 1, 0)), d**2*exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*d**2) - I*b*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()), d**2/(e**2*x**2))/(4*pi**(3/2)*e) + b*meijerg(((-1/2, -1/4, 0, 1/4, 1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), d**2*exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*e) - I*c*d**2*meijerg(((-3/4, -1/4), (-1/2, -1/2, 0, 1)), ((-1, -3/4, -1/2, -1/4, 0, 0), ()), d**2/(e**2*x**2))/(4*pi**(3/2)*e**3) + c*d**2*meijerg(((-3/2, -5/4, -1, -3/4, -1/2, 1), ()), ((-5/4, -3/4), (-3/2, -1, -1, 0)), d**2*exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*e**3)

$$3.140 \quad \int \frac{a+bx^2+cx^4}{x^4\sqrt{d-ex}\sqrt{d+ex}} dx$$

Optimal. Leaf size=157

$$-\frac{(d^2 - e^2x^2)(2ae^2 + 3bd^2)}{3d^4x\sqrt{d-ex}\sqrt{d+ex}} - \frac{a(d^2 - e^2x^2)}{3d^2x^3\sqrt{d-ex}\sqrt{d+ex}} + \frac{c\sqrt{d^2 - e^2x^2} \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{e\sqrt{d-ex}\sqrt{d+ex}}$$

Rubi [A] time = 0.12, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {520, 1265, 451, 217, 203}

$$-\frac{(d^2 - e^2x^2)(2ae^2 + 3bd^2)}{3d^4x\sqrt{d-ex}\sqrt{d+ex}} - \frac{a(d^2 - e^2x^2)}{3d^2x^3\sqrt{d-ex}\sqrt{d+ex}} + \frac{c\sqrt{d^2 - e^2x^2} \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{e\sqrt{d-ex}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2 + c*x^4)/(x^4*Sqrt[d - e*x]*Sqrt[d + e*x]),x]

[Out] -(a*(d^2 - e^2*x^2))/(3*d^2*x^3*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((3*b*d^2 + 2*a*e^2)*(d^2 - e^2*x^2))/(3*d^4*x*Sqrt[d - e*x]*Sqrt[d + e*x]) + (c*Sqrt[d^2 - e^2*x^2]*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(e*Sqrt[d - e*x]*Sqrt[d + e*x])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 451

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)), x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1]))

Rule 520

Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] := Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n))^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]

Rule 1265

Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]}, Simp[(R*(f*x)^(m + 1)*(d + e*x^2)^(q + 1))/(d*f*(m + 1)), x] + Dist[1/(d*f

$\wedge 2*(m + 1))$, Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[(d*f*(m + 1)*Qx)/x - e*R*(m + 2*q + 3), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]

Rubi steps

$$\begin{aligned} \int \frac{a + bx^2 + cx^4}{x^4 \sqrt{d - ex} \sqrt{d + ex}} dx &= \frac{\sqrt{d^2 - e^2 x^2} \int \frac{a + bx^2 + cx^4}{x^4 \sqrt{d^2 - e^2 x^2}} dx}{\sqrt{d - ex} \sqrt{d + ex}} \\ &= -\frac{a(d^2 - e^2 x^2)}{3d^2 x^3 \sqrt{d - ex} \sqrt{d + ex}} - \frac{\sqrt{d^2 - e^2 x^2} \int \frac{-3bd^2 - 2ae^2 - 3cd^2 x^2}{x^2 \sqrt{d^2 - e^2 x^2}} dx}{3d^2 \sqrt{d - ex} \sqrt{d + ex}} \\ &= -\frac{a(d^2 - e^2 x^2)}{3d^2 x^3 \sqrt{d - ex} \sqrt{d + ex}} - \frac{(3bd^2 + 2ae^2)(d^2 - e^2 x^2)}{3d^4 x \sqrt{d - ex} \sqrt{d + ex}} + \frac{(c\sqrt{d^2 - e^2 x^2}) \int \frac{1}{\sqrt{d^2 - e^2 x^2}}}{\sqrt{d - ex} \sqrt{d + ex}} \\ &= -\frac{a(d^2 - e^2 x^2)}{3d^2 x^3 \sqrt{d - ex} \sqrt{d + ex}} - \frac{(3bd^2 + 2ae^2)(d^2 - e^2 x^2)}{3d^4 x \sqrt{d - ex} \sqrt{d + ex}} + \frac{(c\sqrt{d^2 - e^2 x^2}) \text{Subst}\left(\int \frac{1}{\sqrt{d^2 - e^2 x^2}}\right)}{\sqrt{d - ex} \sqrt{d + ex}} \\ &= -\frac{a(d^2 - e^2 x^2)}{3d^2 x^3 \sqrt{d - ex} \sqrt{d + ex}} - \frac{(3bd^2 + 2ae^2)(d^2 - e^2 x^2)}{3d^4 x \sqrt{d - ex} \sqrt{d + ex}} + \frac{c\sqrt{d^2 - e^2 x^2} \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right)}{e\sqrt{d - ex} \sqrt{d + ex}} \end{aligned}$$

Mathematica [A] time = 0.12, size = 81, normalized size = 0.52

$$-\frac{\sqrt{d - ex} \sqrt{d + ex} (a(d^2 + 2e^2 x^2) + 3bd^2 x^2)}{3d^4 x^3} - \frac{2c \tan^{-1}\left(\frac{\sqrt{d - ex}}{\sqrt{d + ex}}\right)}{e}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2 + c*x^4)/(x^4*Sqrt[d - e*x]*Sqrt[d + e*x]), x]

[Out] -1/3*(Sqrt[d - e*x]*Sqrt[d + e*x]*(3*b*d^2*x^2 + a*(d^2 + 2*e^2*x^2)))/(d^4*x^3) - (2*c*ArcTan[Sqrt[d - e*x]/Sqrt[d + e*x]])/e

IntegrateAlgebraic [A] time = 0.15, size = 168, normalized size = 1.07

$$\frac{2c \tan^{-1}\left(\frac{\sqrt{d + ex}}{\sqrt{d - ex}}\right)}{e} - \frac{2\sqrt{d + ex} \left(\frac{3ae^3(d + ex)^2}{(d - ex)^2} - \frac{2ae^3(d + ex)}{d - ex} + 3ae^3 + \frac{3bd^2e(d + ex)^2}{(d - ex)^2} - \frac{6bd^2e(d + ex)}{d - ex} + 3bd^2e \right)}{3d^4 \sqrt{d - ex} \left(\frac{d + ex}{d - ex} - 1 \right)^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)/(x^4*Sqrt[d - e*x]*Sqrt[d + e*x]), x]

[Out] (-2*Sqrt[d + e*x]*(3*b*d^2*e + 3*a*e^3 - (6*b*d^2*e*(d + e*x)))/(d - e*x) - (2*a*e^3*(d + e*x))/(d - e*x) + (3*b*d^2*e*(d + e*x)^2)/(d - e*x)^2 + (3*a*e^3*(d + e*x)^2)/(d - e*x)^2)/(3*d^4*Sqrt[d - e*x]*(-1 + (d + e*x)/(d - e*x))^3) + (2*c*ArcTan[Sqrt[d + e*x]/Sqrt[d - e*x]])/e

fricas [A] time = 1.22, size = 90, normalized size = 0.57

$$\frac{6cd^4x^3 \arctan\left(\frac{\sqrt{ex+d}\sqrt{-ex+d-d}}{ex}\right) + (ad^2e + (3bd^2e + 2ae^3)x^2)\sqrt{ex+d}\sqrt{-ex+d}}{3d^4ex^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/x^4/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fricas")

[Out] -1/3*(6*c*d^4*x^3*arctan((sqrt(e*x + d))*sqrt(-e*x + d) - d)/(e*x)) + (a*d^2*e + (3*b*d^2*e + 2*a*e^3)*x^2)*sqrt(e*x + d)*sqrt(-e*x + d)/(d^4*e*x^3)

giac [B] time = 1.47, size = 555, normalized size = 3.54

$$\frac{1}{3} \left(\pi + 2 \arctan \left(\frac{\sqrt{ax+d} \left(\frac{\sqrt{2}\sqrt{-x+d}}{2\sqrt{d}} - 1 \right)}{\sqrt{2}\sqrt{d} - \sqrt{-x+d}} \right) \right) \left(\frac{4 \left(3 \operatorname{Re} \left(\frac{\sqrt{2}\sqrt{-x+d}}{\sqrt{d}} - \frac{\sqrt{-x+d}}{\sqrt{2}\sqrt{-x+d}} \right)^2 + 3 \operatorname{Im} \left(\frac{\sqrt{2}\sqrt{-x+d}}{\sqrt{d}} - \frac{\sqrt{-x+d}}{\sqrt{2}\sqrt{-x+d}} \right)^2 \right) e^4 - 24 \operatorname{Re} \left(\frac{\sqrt{2}\sqrt{-x+d}}{\sqrt{d}} - \frac{\sqrt{-x+d}}{\sqrt{2}\sqrt{-x+d}} \right)^2 - 8 \operatorname{Im} \left(\frac{\sqrt{2}\sqrt{-x+d}}{\sqrt{d}} - \frac{\sqrt{-x+d}}{\sqrt{2}\sqrt{-x+d}} \right)^2 \right) e^4 + 48 \operatorname{Re} \left(\frac{\sqrt{2}\sqrt{-x+d}}{\sqrt{d}} - \frac{\sqrt{-x+d}}{\sqrt{2}\sqrt{-x+d}} \right)^2 + 48 \operatorname{Im} \left(\frac{\sqrt{2}\sqrt{-x+d}}{\sqrt{d}} - \frac{\sqrt{-x+d}}{\sqrt{2}\sqrt{-x+d}} \right)^2 \right) e^4}{\left(\frac{\sqrt{2}\sqrt{-x+d}}{\sqrt{d}} - \frac{\sqrt{-x+d}}{\sqrt{2}\sqrt{-x+d}} \right)^2 - 4} e^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/x^4/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="giac")

[Out] 1/3*(3*(pi + 2*arctan(1/2*sqrt(x*e + d)*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))^2/(x*e + d) - 1)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d))))*c - 4*(3*b*d^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^5*e^2 + 3*a*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^5*e^4 - 24*b*d^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^3*e^2 - 8*a*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^3*e^4 + 48*b*d^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))e^2 + 48*a*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))e^4)/(((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^2 - 4)^3*d^4))e^(-1)

maple [C] time = 0.03, size = 146, normalized size = 0.93

$$\frac{\sqrt{-ex+d} \sqrt{ex+d} \left(-3c d^4 x^3 \arctan \left(\frac{ex \operatorname{csgn}(e)}{\sqrt{-e^2 x^2 + d^2}} \right) + 2\sqrt{-e^2 x^2 + d^2} a e^3 x^2 \operatorname{csgn}(e) + 3\sqrt{-e^2 x^2 + d^2} b d^2 e x^2 \operatorname{csgn}(e) + \sqrt{-e^2 x^2 + d^2} a d^2 e \operatorname{csgn}(e) \right) \operatorname{csgn}(e)}{3\sqrt{-e^2 x^2 + d^2} d^4 e x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^2+a)/x^4/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)

[Out] -1/3*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/d^4*(-3*arctan(1/(-e^2*x^2+d^2)^(1/2)*e*x*csgn(e))*x^3*c*d^4+2*(-e^2*x^2+d^2)^(1/2)*csgn(e)*e^3*x^2*a+3*(-e^2*x^2+d^2)^(1/2)*csgn(e)*e*x^2*b*d^2+a*(-e^2*x^2+d^2)^(1/2)*d^2*csgn(e)*e)*csgn(e)/(-e^2*x^2+d^2)^(1/2)/x^3/e

maxima [A] time = 1.00, size = 85, normalized size = 0.54

$$\frac{c \arcsin \left(\frac{ex}{d} \right)}{e} - \frac{\sqrt{-e^2 x^2 + d^2} b}{d^2 x} - \frac{2 \sqrt{-e^2 x^2 + d^2} a e^2}{3 d^4 x} - \frac{\sqrt{-e^2 x^2 + d^2} a}{3 d^2 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/x^4/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="maxima")

[Out] c*arcsin(e*x/d)/e - sqrt(-e^2*x^2 + d^2)*b/(d^2*x) - 2/3*sqrt(-e^2*x^2 + d^2)*a*e^2/(d^4*x) - 1/3*sqrt(-e^2*x^2 + d^2)*a/(d^2*x^3)

mupad [B] time = 2.27, size = 138, normalized size = 0.88

$$\frac{4c \operatorname{atan} \left(\frac{e(\sqrt{d-ex}-\sqrt{d})}{\sqrt{e^2}(\sqrt{d+ex}-\sqrt{d})} \right)}{\sqrt{e^2}} - \frac{\left(\frac{b}{d} + \frac{bex}{d^2} \right) \sqrt{d-ex}}{x \sqrt{d+ex}} - \frac{\sqrt{d-ex} \left(\frac{a}{3d} + \frac{2ae^2x^2}{3d^3} + \frac{2ae^3x^3}{3d^4} + \frac{aex}{3d^2} \right)}{x^3 \sqrt{d+ex}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*x^2 + c*x^4)/(x^4*(d + e*x)^(1/2)*(d - e*x)^(1/2)),x)`

[Out] $-(4*c*\operatorname{atan}((e*(d - e*x)^{(1/2)} - d^{(1/2)}))/((e^2)^{(1/2)}*(d + e*x)^{(1/2)} - d^{(1/2)})))/((e^2)^{(1/2)} - ((b/d + (b*e*x)/d^2)*(d - e*x)^{(1/2)})/(x*(d + e*x)^{(1/2)})) - ((d - e*x)^{(1/2)}*(a/(3*d) + (2*a*e^2*x^2)/(3*d^3) + (2*a*e^3*x^3)/(3*d^4) + (a*e*x)/(3*d^2)))/(x^3*(d + e*x)^{(1/2)})$

sympy [C] time = 116.43, size = 257, normalized size = 1.64

$$\frac{i a e^3 C_{6,6}^{5,3} \left(\begin{matrix} \frac{9}{4}, \frac{11}{4}, 1 \\ 2, \frac{9}{4}, \frac{5}{2}, \frac{11}{4}, 3 \\ 0 \end{matrix} \middle| \frac{d^{\rho}}{d^2, d^2} \right) + a e^3 C_{6,6}^{2,6} \left(\begin{matrix} \frac{3}{2}, \frac{7}{4}, \frac{9}{4}, \frac{5}{2}, 1 \\ \frac{7}{4}, \frac{9}{4} \\ \frac{3}{2}, 2, 2, 0 \end{matrix} \middle| \frac{d^{\rho}}{d^2, d^2} \right) + i b e C_{6,6}^{5,3} \left(\begin{matrix} \frac{5}{4}, \frac{7}{4}, 1 \\ 1, \frac{5}{4}, \frac{3}{2}, \frac{7}{4}, 2 \\ 0 \end{matrix} \middle| \frac{d^{\rho}}{d^2, d^2} \right) + b e C_{6,6}^{2,6} \left(\begin{matrix} \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, 1 \\ \frac{3}{4}, \frac{5}{4} \\ \frac{1}{2}, 1, 1, 0 \end{matrix} \middle| \frac{d^{\rho}}{d^2, d^2} \right) - i c C_{6,6}^{2,6} \left(\begin{matrix} \frac{1}{4}, \frac{3}{4}, \frac{1}{2}, 1, 1 \\ 0, \frac{1}{4}, \frac{3}{4}, 1, 0 \\ \frac{1}{2}, 1, 1, 0 \end{matrix} \middle| \frac{d^{\rho}}{d^2, d^2} \right) + c C_{6,6}^{2,6} \left(\begin{matrix} -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, 1 \\ -\frac{1}{4}, \frac{1}{4} \\ -\frac{1}{2}, 0, 0, 0 \end{matrix} \middle| \frac{d^{\rho}}{d^2, d^2} \right)}{4\pi^{\frac{3}{2}} d^4} + \frac{3}{4\pi^{\frac{3}{2}} d^4} + \frac{3}{4\pi^{\frac{3}{2}} d^2} + \frac{3}{4\pi^{\frac{3}{2}} d^2} - \frac{3}{4\pi^{\frac{3}{2}} e} + \frac{3}{4\pi^{\frac{3}{2}} e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**4+b*x**2+a)/x**4/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)`

[Out] $I*a*e**3*\operatorname{meijerg}((9/4, 11/4, 1), (5/2, 5/2, 3)), ((2, 9/4, 5/2, 11/4, 3), (0,)), d**2/(e**2*x**2))/(4*pi**(3/2)*d**4) + a*e**3*\operatorname{meijerg}((3/2, 7/4, 2, 9/4, 5/2, 1), ()), ((7/4, 9/4), (3/2, 2, 2, 0)), d**2*\exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*d**4) + I*b*e*\operatorname{meijerg}((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7/4, 2), (0,)), d**2/(e**2*x**2))/(4*pi**(3/2)*d**2) + b*e*\operatorname{meijerg}((1/2, 3/4, 1, 5/4, 3/2, 1), ()), ((3/4, 5/4), (1/2, 1, 1, 0)), d**2*\exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*d**2) - I*c*\operatorname{meijerg}((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()), d**2/(e**2*x**2))/(4*pi**(3/2)*e) + c*\operatorname{meijerg}((-1/2, -1/4, 0, 1/4, 1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), d**2*\exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*e)$

$$3.141 \quad \int \frac{a+bx^2+cx^4}{x^6\sqrt{d-ex}\sqrt{d+ex}} dx$$

Optimal. Leaf size=160

$$-\frac{(d^2 - e^2x^2)(8ae^4 + 10bd^2e^2 + 15cd^4)}{15d^6x\sqrt{d-ex}\sqrt{d+ex}} - \frac{(d^2 - e^2x^2)(4ae^2 + 5bd^2)}{15d^4x^3\sqrt{d-ex}\sqrt{d+ex}} - \frac{a(d^2 - e^2x^2)}{5d^2x^5\sqrt{d-ex}\sqrt{d+ex}}$$

Rubi [A] time = 0.15, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {520, 1265, 453, 264}

$$-\frac{(d^2 - e^2x^2)(8ae^4 + 10bd^2e^2 + 15cd^4)}{15d^6x\sqrt{d-ex}\sqrt{d+ex}} - \frac{(d^2 - e^2x^2)(4ae^2 + 5bd^2)}{15d^4x^3\sqrt{d-ex}\sqrt{d+ex}} - \frac{a(d^2 - e^2x^2)}{5d^2x^5\sqrt{d-ex}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2 + c*x^4)/(x^6*sqrt[d - e*x]*sqrt[d + e*x]),x]

[Out] -(a*(d^2 - e^2*x^2))/(5*d^2*x^5*sqrt[d - e*x]*sqrt[d + e*x]) - ((5*b*d^2 + 4*a*e^2)*(d^2 - e^2*x^2))/(15*d^4*x^3*sqrt[d - e*x]*sqrt[d + e*x]) - ((15*c*d^4 + 10*b*d^2*e^2 + 8*a*e^4)*(d^2 - e^2*x^2))/(15*d^6*x*sqrt[d - e*x]*sqrt[d + e*x])

Rule 264

Int[((c_.)*(x_.))^(m_.)*((a_.) + (b_.)*(x_.)^(n_.))^(p_.), x_Symbol] :> Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]

Rule 453

Int[((e_.)*(x_.))^(m_.)*((a_.) + (b_.)*(x_.)^(n_.))^(p_.)*((c_.) + (d_.)*(x_.)^(n_.)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)), x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]

Rule 520

Int[(u_.)*((c_.) + (d_.)*(x_.)^(n_.) + (e_.)*(x_.)^(n2_.))^(q_.)*((a1_.) + (b1_.)*(x_.)^(non2_.))^(p_.)*((a2_.) + (b2_.)*(x_.)^(non2_.))^(p_.), x_Symbol] :> Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n))^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]

Rule 1265

Int[((f_.)*(x_.))^(m_.)*((d_.) + (e_.)*(x_.)^2)^(q_.)*((a_.) + (b_.)*(x_.)^2 + (c_.)*(x_.)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]}, Simp[(R*(f*x)^(m + 1)*(d + e*x^2)^(q + 1))/(d*f*(m + 1)), x] + Dist[1/(d*f^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[(d*f*(m + 1)*Qx)/x - e*R*(m + 2*q + 3), x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]

Rubi steps

$$\begin{aligned}
\int \frac{a + bx^2 + cx^4}{x^6 \sqrt{d - ex} \sqrt{d + ex}} dx &= \frac{\sqrt{d^2 - e^2 x^2} \int \frac{a + bx^2 + cx^4}{x^6 \sqrt{d^2 - e^2 x^2}} dx}{\sqrt{d - ex} \sqrt{d + ex}} \\
&= \frac{a(d^2 - e^2 x^2)}{5d^2 x^5 \sqrt{d - ex} \sqrt{d + ex}} - \frac{\sqrt{d^2 - e^2 x^2} \int \frac{-5bd^2 - 4ae^2 - 5cd^2 x^2}{x^4 \sqrt{d^2 - e^2 x^2}} dx}{5d^2 \sqrt{d - ex} \sqrt{d + ex}} \\
&= \frac{a(d^2 - e^2 x^2)}{5d^2 x^5 \sqrt{d - ex} \sqrt{d + ex}} - \frac{(5bd^2 + 4ae^2)(d^2 - e^2 x^2)}{15d^4 x^3 \sqrt{d - ex} \sqrt{d + ex}} + \frac{\left((15cd^4 - 2e^2(-5bd^2 - 4ae^2) - 5cd^2 x^2) \right)}{15d^4 \sqrt{d - ex} \sqrt{d + ex}} \\
&= \frac{a(d^2 - e^2 x^2)}{5d^2 x^5 \sqrt{d - ex} \sqrt{d + ex}} - \frac{(5bd^2 + 4ae^2)(d^2 - e^2 x^2)}{15d^4 x^3 \sqrt{d - ex} \sqrt{d + ex}} - \frac{(15cd^4 + 10bd^2 e^2 + 8ae^4)}{15d^6 x \sqrt{d - ex} \sqrt{d + ex}}
\end{aligned}$$

Mathematica [A] time = 0.12, size = 87, normalized size = 0.54

$$\frac{\sqrt{d - ex} \sqrt{d + ex} \left(a(3d^4 + 4d^2 e^2 x^2 + 8e^4 x^4) + 5bd^2 x^2 (d^2 + 2e^2 x^2) + 15cd^4 x^4 \right)}{15d^6 x^5}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2 + c*x^4)/(x^6*sqrt[d - e*x]*sqrt[d + e*x]),x]

[Out] -1/15*(sqrt[d - e*x]*sqrt[d + e*x]*(15*c*d^4*x^4 + 5*b*d^2*x^2*(d^2 + 2*e^2*x^2) + a*(3*d^4 + 4*d^2*e^2*x^2 + 8*e^4*x^4)))/(d^6*x^5)

IntegrateAlgebraic [B] time = 0.17, size = 329, normalized size = 2.06

$$\frac{2\sqrt{d + ex} \left(\frac{15ae^2(d+ex)^4}{(d-ex)^4} - \frac{20ae^2(d+ex)^3}{(d-ex)^3} + \frac{58ae^2(d+ex)^2}{(d-ex)^2} - \frac{20ae^2(d+ex)}{d-ex} + 15ae^5 + \frac{15bd^2e^2(d+ex)^4}{(d-ex)^4} - \frac{40bd^2e^2(d+ex)^3}{(d-ex)^3} + \frac{50bd^2e^2(d+ex)^2}{(d-ex)^2} - \frac{40bd^2e^2(d+ex)}{d-ex} + 15bd^2e^3 + \frac{15cd^4e(d+ex)^4}{(d-ex)^4} - \frac{60cd^4e(d+ex)^3}{(d-ex)^3} + \frac{90cd^4e(d+ex)^2}{(d-ex)^2} - \frac{60cd^4e(d+ex)}{d-ex} + 15cd^4e \right)}{15d^6 \sqrt{d - ex} \left(\frac{d+ex}{d-ex} - 1 \right)^5}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)/(x^6*sqrt[d - e*x]*sqrt[d + e*x]),x]

[Out] (-2*sqrt[d + e*x]*(15*c*d^4*e + 15*b*d^2*e^3 + 15*a*e^5 - (60*c*d^4*e*(d + e*x))/(d - e*x) - (40*b*d^2*e^3*(d + e*x))/(d - e*x) - (20*a*e^5*(d + e*x))/(d - e*x) + (90*c*d^4*e*(d + e*x)^2)/(d - e*x)^2 + (50*b*d^2*e^3*(d + e*x)^2)/(d - e*x)^2 + (58*a*e^5*(d + e*x)^2)/(d - e*x)^2 - (60*c*d^4*e*(d + e*x)^3)/(d - e*x)^3 - (40*b*d^2*e^3*(d + e*x)^3)/(d - e*x)^3 - (20*a*e^5*(d + e*x)^3)/(d - e*x)^3 + (15*c*d^4*e*(d + e*x)^4)/(d - e*x)^4 + (15*b*d^2*e^3*(d + e*x)^4)/(d - e*x)^4 + (15*a*e^5*(d + e*x)^4)/(d - e*x)^4)/(15*d^6*sqrt[d - e*x]*(-1 + (d + e*x)/(d - e*x))^5)

fricas [A] time = 1.14, size = 76, normalized size = 0.48

$$\frac{\left(3ad^4 + (15cd^4 + 10bd^2e^2 + 8ae^4)x^4 + (5bd^4 + 4ad^2e^2)x^2 \right) \sqrt{ex + d} \sqrt{-ex + d}}{15d^6 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/x^6/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fricas")

[Out] -1/15*(3*a*d^4 + (15*c*d^4 + 10*b*d^2*e^2 + 8*a*e^4)*x^4 + (5*b*d^4 + 4*a*d^2*e^2)*x^2)*sqrt(e*x + d)*sqrt(-e*x + d)/(d^6*x^5)

giac [B] time = 2.55, size = 1103, normalized size = 6.89

giac

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^6/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] -4/15*(15*c*d^4*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^9*e^2 + 15*b*d^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^9*e^4 - 240*c*d^4*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^7*e^2 + 15*a*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^9*e^6 - 160*b*d^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^7*e^4 + 1440*c*d^4*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^5*e^2 - 80*a*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^7*e^6 + 800*b*d^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^5*e^4 - 3840*c*d^4*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^3*e^2 + 928*a*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^5*e^6 - 2560*b*d^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^3*e^4 + 3840*c*d^4*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^e^2 - 1280*a*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^3*e^6 + 3840*b*d^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^e^4 + 3840*a*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^e^6)*e^(-1)/((((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^2 - 4)^5*d^6)
```

maple [A] time = 0.00, size = 82, normalized size = 0.51

$$\frac{\sqrt{ex+d} \sqrt{-ex+d} (8ae^4x^4 + 10bd^2e^2x^4 + 15cd^4x^4 + 4ad^2e^2x^2 + 5bd^4x^2 + 3ad^4)}{15d^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^6/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] -1/15*(e*x+d)^(1/2)*(-e*x+d)^(1/2)*(8*a*e^4*x^4+10*b*d^2*e^2*x^4+15*c*d^4*x^4+4*a*d^2*e^2*x^2+5*b*d^4*x^2+3*a*d^4)/x^5/d^6
```

maxima [A] time = 1.02, size = 148, normalized size = 0.92

$$\frac{\sqrt{-e^2x^2+d^2}c}{d^2x} - \frac{2\sqrt{-e^2x^2+d^2}be^2}{3d^4x} - \frac{8\sqrt{-e^2x^2+d^2}ae^4}{15d^6x} - \frac{\sqrt{-e^2x^2+d^2}b}{3d^2x^3} - \frac{4\sqrt{-e^2x^2+d^2}ae^2}{15d^4x^3} - \frac{\sqrt{-e^2x^2+d^2}a}{5d^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^6/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-e^2*x^2 + d^2)*c/(d^2*x) - 2/3*sqrt(-e^2*x^2 + d^2)*b*e^2/(d^4*x) - 8/15*sqrt(-e^2*x^2 + d^2)*a*e^4/(d^6*x) - 1/3*sqrt(-e^2*x^2 + d^2)*b/(d^2*x^3) - 4/15*sqrt(-e^2*x^2 + d^2)*a*e^2/(d^4*x^3) - 1/5*sqrt(-e^2*x^2 + d^2)*a/(d^2*x^5)
```

mupad [B] time = 1.73, size = 146, normalized size = 0.91

$$\frac{\sqrt{d-ex} \left(\frac{a}{5d} + \frac{x^4(15cd^5+10bd^3e^2+8ade^4)}{15d^6} + \frac{x^5(15cd^4e+10bd^2e^3+8ae^5)}{15d^6} + \frac{x^2(5bd^5+4ad^3e^2)}{15d^6} + \frac{x^3(5bd^4e+4ad^2e^3)}{15d^6} + \frac{aex}{5d^2} \right)}{x^5 \sqrt{d+ex}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(x^6*(d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

```
[Out] -((d - e*x)^(1/2)*(a/(5*d) + (x^4*(15*c*d^5 + 10*b*d^3*e^2 + 8*a*d*e^4))/(15*d^6) + (x^5*(8*a*e^5 + 10*b*d^2*e^3 + 15*c*d^4*e))/(15*d^6) + (x^2*(5*b*d^5 + 4*a*d^3*e^2))/(15*d^6) + (x^3*(4*a*d^2*e^3 + 5*b*d^4*e))/(15*d^6) + (a*e*x)/(5*d^2)))/(x^5*(d + e*x)^(1/2))
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**6/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

$$3.142 \quad \int \frac{a+bx^2+cx^4}{x^8 \sqrt{d-ex} \sqrt{d+ex}} dx$$

Optimal. Leaf size=226

$$\frac{2e^2(d^2 - e^2x^2)(24ae^4 + 28bd^2e^2 + 35cd^4)}{105d^8x\sqrt{d-ex}\sqrt{d+ex}} - \frac{(d^2 - e^2x^2)(24ae^4 + 28bd^2e^2 + 35cd^4)}{105d^6x^3\sqrt{d-ex}\sqrt{d+ex}} - \frac{(d^2 - e^2x^2)(6ae^2 + 7bd^2)}{35d^4x^5\sqrt{d-ex}\sqrt{d+ex}}$$

Rubi [A] time = 0.18, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {520, 1265, 453, 271, 264}

$$\frac{2e^2(d^2 - e^2x^2)(24ae^4 + 28bd^2e^2 + 35cd^4)}{105d^8x\sqrt{d-ex}\sqrt{d+ex}} - \frac{(d^2 - e^2x^2)(24ae^4 + 28bd^2e^2 + 35cd^4)}{105d^6x^3\sqrt{d-ex}\sqrt{d+ex}} - \frac{(d^2 - e^2x^2)(6ae^2 + 7bd^2)}{35d^4x^5\sqrt{d-ex}\sqrt{d+ex}} - \frac{a(d^2 - e^2x^2)}{7d^2x^7\sqrt{d-ex}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2 + c*x^4)/(x^8*Sqrt[d - e*x]*Sqrt[d + e*x]),x]

[Out] -(a*(d^2 - e^2*x^2))/(7*d^2*x^7*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((7*b*d^2 + 6*a*e^2)*(d^2 - e^2*x^2))/(35*d^4*x^5*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((35*c*d^4 + 28*b*d^2*e^2 + 24*a*e^4)*(d^2 - e^2*x^2))/(105*d^6*x^3*Sqrt[d - e*x]*Sqrt[d + e*x]) - (2*e^2*(35*c*d^4 + 28*b*d^2*e^2 + 24*a*e^4)*(d^2 - e^2*x^2))/(105*d^8*x*Sqrt[d - e*x]*Sqrt[d + e*x])

Rule 264

Int[((c_.)*(x_.))^(m_.)*((a_.) + (b_.)*(x_.)^(n_.))^(p_.), x_Symbol] :> Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]

Rule 271

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Simp[(x^(m + 1)*(a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m + 1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]

Rule 453

Int[((e_.)*(x_.))^(m_.)*((a_.) + (b_.)*(x_.)^(n_.))^(p_.)*((c_.) + (d_.)*(x_.)^(n_.)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)), x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]

Rule 520

Int[(u_.)*((c_.) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_.) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_.) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n))^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]

Rule 1265

Int[((f_.)*(x_.))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_.)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},

Simp[(R*(f*x)^(m + 1)*(d + e*x^2)^(q + 1))/(d*f*(m + 1)), x] + Dist[1/(d*f^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[(d*f*(m + 1)*Qx)/x - e*R*(m + 2*q + 3), x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]

Rubi steps

$$\int \frac{a + bx^2 + cx^4}{x^8 \sqrt{d - ex} \sqrt{d + ex}} dx = \frac{\sqrt{d^2 - e^2 x^2} \int \frac{a + bx^2 + cx^4}{x^8 \sqrt{d^2 - e^2 x^2}} dx}{\sqrt{d - ex} \sqrt{d + ex}}$$

$$= \frac{a(d^2 - e^2 x^2)}{7d^2 x^7 \sqrt{d - ex} \sqrt{d + ex}} - \frac{\sqrt{d^2 - e^2 x^2} \int \frac{-7bd^2 - 6ae^2 - 7cd^2 x^2}{x^6 \sqrt{d^2 - e^2 x^2}} dx}{7d^2 \sqrt{d - ex} \sqrt{d + ex}}$$

$$= \frac{a(d^2 - e^2 x^2)}{7d^2 x^7 \sqrt{d - ex} \sqrt{d + ex}} - \frac{(7bd^2 + 6ae^2)(d^2 - e^2 x^2)}{35d^4 x^5 \sqrt{d - ex} \sqrt{d + ex}} + \frac{\left((35cd^4 - 4e^2(-7bd^2 - 6ae^2) - 7cd^2 x^2) \int \frac{1}{x^5 \sqrt{d^2 - e^2 x^2}} dx \right)}{35d^4 \sqrt{d - ex} \sqrt{d + ex}}$$

$$= \frac{a(d^2 - e^2 x^2)}{7d^2 x^7 \sqrt{d - ex} \sqrt{d + ex}} - \frac{(7bd^2 + 6ae^2)(d^2 - e^2 x^2)}{35d^4 x^5 \sqrt{d - ex} \sqrt{d + ex}} - \frac{(35cd^4 + 28bd^2 e^2 + 24ae^4)}{105d^6 x^3 \sqrt{d - ex} \sqrt{d + ex}}$$

$$= \frac{a(d^2 - e^2 x^2)}{7d^2 x^7 \sqrt{d - ex} \sqrt{d + ex}} - \frac{(7bd^2 + 6ae^2)(d^2 - e^2 x^2)}{35d^4 x^5 \sqrt{d - ex} \sqrt{d + ex}} - \frac{(35cd^4 + 28bd^2 e^2 + 24ae^4)}{105d^6 x^3 \sqrt{d - ex} \sqrt{d + ex}}$$

Mathematica [A] time = 0.14, size = 124, normalized size = 0.55

$$\frac{\sqrt{d - ex} \sqrt{d + ex} (3a(5d^6 + 6d^4 e^2 x^2 + 8d^2 e^4 x^4 + 16e^6 x^6) + 7b(3d^6 x^2 + 4d^4 e^2 x^4 + 8d^2 e^4 x^6) + 35cd^4 x^4 (d^2 + 2e^2 x^2))}{105d^8 x^7}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2 + c*x^4)/(x^8*sqrt[d - e*x]*sqrt[d + e*x]), x]

[Out] -1/105*(sqrt[d - e*x]*sqrt[d + e*x]*(35*c*d^4*x^4*(d^2 + 2*e^2*x^2) + 7*b*(3*d^6*x^2 + 4*d^4*e^2*x^4 + 8*d^2*e^4*x^6) + 3*a*(5*d^6 + 6*d^4*e^2*x^2 + 8*d^2*e^4*x^4 + 16*e^6*x^6)))/(d^8*x^7)

IntegrateAlgebraic [B] time = 0.22, size = 477, normalized size = 2.11

$$\frac{2\sqrt{d+ex} \left(\frac{105b^2 d^4 e^3}{16e^2 x^7} - \frac{210a^2 d^4 e^3}{16e^2 x^7} + \frac{903a^2 d^4 e^3}{16e^2 x^7} - \frac{630a^2 d^4 e^3}{16e^2 x^7} + \frac{903a^2 d^4 e^3}{16e^2 x^7} - \frac{210a^2 d^4 e^3}{16e^2 x^7} + 105b^2 e^3 + \frac{105b^2 d^2 e^5}{16e^2 x^5} - \frac{350b^2 d^2 e^5}{16e^2 x^5} + \frac{791b^2 d^2 e^5}{16e^2 x^5} - \frac{1092b^2 d^2 e^5}{16e^2 x^5} + \frac{791b^2 d^2 e^5}{16e^2 x^5} - \frac{350b^2 d^2 e^5}{16e^2 x^5} + 105b^2 e^3 + \frac{105a^2 d^4 e^3}{16e^2 x^7} - \frac{490a^2 d^4 e^3}{16e^2 x^7} + \frac{1015a^2 d^4 e^3}{16e^2 x^7} - \frac{1260a^2 d^4 e^3}{16e^2 x^7} + \frac{1015a^2 d^4 e^3}{16e^2 x^7} - \frac{490a^2 d^4 e^3}{16e^2 x^7} + 105c^2 d^4 \right)}{105d^8 \sqrt{d - ex} (d - e^2 x^2)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)/(x^8*sqrt[d - e*x]*sqrt[d + e*x]), x]

[Out] (-2*sqrt[d + e*x]*(105*c*d^4*e^3 + 105*b*d^2*e^5 + 105*a*e^7 - (490*c*d^4*e^3*(d + e*x))/(d - e*x) - (350*b*d^2*e^5*(d + e*x))/(d - e*x) - (210*a*e^7*(d + e*x))/(d - e*x) + (1015*c*d^4*e^3*(d + e*x)^2)/(d - e*x)^2 + (791*b*d^2*e^5*(d + e*x)^2)/(d - e*x)^2 + (903*a*e^7*(d + e*x)^2)/(d - e*x)^2 - (1260*c*d^4*e^3*(d + e*x)^3)/(d - e*x)^3 - (1092*b*d^2*e^5*(d + e*x)^3)/(d - e*x)^3 - (636*a*e^7*(d + e*x)^3)/(d - e*x)^3 + (1015*c*d^4*e^3*(d + e*x)^4)/(d - e*x)^4 + (791*b*d^2*e^5*(d + e*x)^4)/(d - e*x)^4 + (903*a*e^7*(d + e*x)^4)/(d - e*x)^4 - (490*c*d^4*e^3*(d + e*x)^5)/(d - e*x)^5 - (350*b*d^2*e^5*(d + e*x)^5)/(d - e*x)^5 - (210*a*e^7*(d + e*x)^5)/(d - e*x)^5 + (105*c*d^4*e^3*(d + e*x)^6)/(d - e*x)^6 + (105*b*d^2*e^5*(d + e*x)^6)/(d - e*x)^6 + (105*a*e^7*(d + e*x)^6)/(d - e*x)^6)/(105*d^8*sqrt[d - e*x]*(-1 + (d + e*x)/(d - e*x))^7)

fricas [A] time = 1.52, size = 110, normalized size = 0.49

$$\frac{(15ad^6 + 2(35cd^4e^2 + 28bd^2e^4 + 24ae^6)x^6 + (35cd^6 + 28bd^4e^2 + 24ad^2e^4)x^4 + 3(7bd^6 + 6ad^4e^2)x^2)\sqrt{ex+d}\sqrt{-ex+d}}{105d^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/x^8/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fricas")

[Out] -1/105*(15*a*d^6 + 2*(35*c*d^4*e^2 + 28*b*d^2*e^4 + 24*a*e^6)*x^6 + (35*c*d^6 + 28*b*d^4*e^2 + 24*a*d^2*e^4)*x^4 + 3*(7*b*d^6 + 6*a*d^4*e^2)*x^2)*sqrt(e*x + d)*sqrt(-e*x + d)/(d^8*x^7)

giac [B] time = 4.73, size = 1517, normalized size = 6.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/x^8/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="giac")

[Out] -4/105*(105*c*d^4*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^13*e^4 + 105*b*d^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^13*e^6 - 1960*c*d^4*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^11*e^4 + 105*a*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^13*e^8 - 1400*b*d^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^11*e^6 + 16240*c*d^4*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^9*e^4 - 840*a*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^11*e^8 + 12656*b*d^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^9*e^6 - 80640*c*d^4*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^7*e^4 + 14448*a*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^9*e^8 - 69888*b*d^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^7*e^6 + 259840*c*d^4*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^5*e^4 - 40704*a*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^7*e^8 + 202496*b*d^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^5*e^6 - 501760*c*d^4*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^3*e^4 + 231168*a*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^5*e^8 - 358400*b*d^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^3*e^6 + 430080*c*d^4*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))*e^4 - 215040*a*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^3*e^8 + 430080*b*d^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))*e^6 + 430080*a*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))*e^8)*e^(-1)/(((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^2 - 4)^7*d^8)

maple [A] time = 0.01, size = 118, normalized size = 0.52

$$\frac{\sqrt{ex+d}\sqrt{-ex+d}(48ae^6x^6 + 56bd^2e^4x^6 + 70cd^4e^2x^6 + 24ad^2e^4x^4 + 28bd^4e^2x^4 + 35cd^6x^4 + 18ad^4e^2x^2 + 21bd^6x^2 + 15ad^6)}{105d^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((c*x^4+b*x^2+a)/x^8/(-e*x+d)^{(1/2)}/(e*x+d)^{(1/2)},x)$

[Out] $-1/105*(e*x+d)^{(1/2)}*(-e*x+d)^{(1/2)}*(48*a*e^6*x^6+56*b*d^2*e^4*x^6+70*c*d^4*e^2*x^6+24*a*d^2*e^4*x^4+28*b*d^4*e^2*x^4+35*c*d^6*x^4+18*a*d^4*e^2*x^2+21*b*d^6*x^2+15*a*d^6)/x^7/d^8$

maxima [A] time = 1.01, size = 226, normalized size = 1.00

$$\frac{2\sqrt{-e^2x^2+d^2}ce^2}{3d^4x} - \frac{8\sqrt{-e^2x^2+d^2}be^4}{15d^6x} - \frac{16\sqrt{-e^2x^2+d^2}ae^6}{35d^8x} - \frac{\sqrt{-e^2x^2+d^2}c}{3d^2x^3} - \frac{4\sqrt{-e^2x^2+d^2}be^2}{15d^4x^3} - \frac{8\sqrt{-e^2x^2+d^2}ae^4}{35d^6x^3} - \frac{\sqrt{-e^2x^2+d^2}b}{5d^2x^5} - \frac{6\sqrt{-e^2x^2+d^2}ae^2}{35d^4x^5} - \frac{\sqrt{-e^2x^2+d^2}a}{7d^2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((c*x^4+b*x^2+a)/x^8/(-e*x+d)^{(1/2)}/(e*x+d)^{(1/2)},x, \text{algorithm}=\text{"maxima"})$

[Out] $-2/3*\text{sqrt}(-e^2*x^2 + d^2)*c*e^2/(d^4*x) - 8/15*\text{sqrt}(-e^2*x^2 + d^2)*b*e^4/(d^6*x) - 16/35*\text{sqrt}(-e^2*x^2 + d^2)*a*e^6/(d^8*x) - 1/3*\text{sqrt}(-e^2*x^2 + d^2)*c/(d^2*x^3) - 4/15*\text{sqrt}(-e^2*x^2 + d^2)*b*e^2/(d^4*x^3) - 8/35*\text{sqrt}(-e^2*x^2 + d^2)*a*e^4/(d^6*x^3) - 1/5*\text{sqrt}(-e^2*x^2 + d^2)*b/(d^2*x^5) - 6/35*\text{sqrt}(-e^2*x^2 + d^2)*a*e^2/(d^4*x^5) - 1/7*\text{sqrt}(-e^2*x^2 + d^2)*a/(d^2*x^7)$

mupad [B] time = 1.82, size = 218, normalized size = 0.96

$$\frac{\sqrt{d-ex} \left(\frac{a}{7d} + \frac{x^2(21bd^7+18ad^5e^2)}{105d^8} + \frac{x^4(35cd^7+28bd^5e^2+24ad^3e^4)}{105d^8} + \frac{x^7(70cd^4e^3+56bd^2e^5+48ae^7)}{105d^8} + \frac{x^3(21bd^6e+18ad^4e^3)}{105d^8} + \frac{x^5(35cd^6e+28bd^4e^3+24ad^2e^5)}{105d^8} + \frac{x^6(70cd^5e^2+56bd^3e^4+48ad^2e^6)}{105d^8} + \frac{aex}{7d^2} \right)}{x^7\sqrt{d+ex}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((a + b*x^2 + c*x^4)/(x^8*(d + e*x)^{(1/2)}*(d - e*x)^{(1/2)}),x)$

[Out] $-((d - e*x)^{(1/2)}*(a/(7*d) + (x^2*(21*b*d^7 + 18*a*d^5*e^2))/(105*d^8) + (x^4*(35*c*d^7 + 24*a*d^3*e^4 + 28*b*d^5*e^2))/(105*d^8) + (x^7*(48*a*e^7 + 56*b*d^2*e^5 + 70*c*d^4*e^3))/(105*d^8) + (x^3*(18*a*d^4*e^3 + 21*b*d^6*e))/(105*d^8) + (x^5*(24*a*d^2*e^5 + 28*b*d^4*e^3 + 35*c*d^6*e))/(105*d^8) + (x^6*(56*b*d^3*e^4 + 70*c*d^5*e^2 + 48*a*d*e^6))/(105*d^8) + (a*e*x)/(7*d^2)))/(x^7*(d + e*x)^{(1/2)})$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((c*x**4+b*x**2+a)/x**8/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)$

[Out] Timed out

$$3.143 \quad \int \frac{a+bx^2+cx^4}{x^{10}\sqrt{d-ex}\sqrt{d+ex}} dx$$

Optimal. Leaf size=292

$$\frac{8e^4(d^2 - e^2x^2)(16ae^4 + 18bd^2e^2 + 21cd^4)}{315d^{10}x\sqrt{d-ex}\sqrt{d+ex}} - \frac{4e^2(d^2 - e^2x^2)(16ae^4 + 18bd^2e^2 + 21cd^4)}{315d^8x^3\sqrt{d-ex}\sqrt{d+ex}} - \frac{(d^2 - e^2x^2)(16ae^4 + 18bd^2e^2 + 21cd^4)}{105d^6x^5\sqrt{d-ex}\sqrt{d+ex}}$$

Rubi [A] time = 0.24, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {520, 1265, 453, 271, 264}

$$\frac{8e^4(d^2 - e^2x^2)(16ae^4 + 18bd^2e^2 + 21cd^4)}{315d^{10}x\sqrt{d-ex}\sqrt{d+ex}} - \frac{4e^2(d^2 - e^2x^2)(16ae^4 + 18bd^2e^2 + 21cd^4)}{315d^8x^3\sqrt{d-ex}\sqrt{d+ex}} - \frac{(d^2 - e^2x^2)(16ae^4 + 18bd^2e^2 + 21cd^4)}{105d^6x^5\sqrt{d-ex}\sqrt{d+ex}} - \frac{(d^2 - e^2x^2)(8ae^2 + 9bd^2)}{63d^4x^7\sqrt{d-ex}\sqrt{d+ex}} - \frac{a(d^2 - e^2x^2)}{9d^2x^9\sqrt{d-ex}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2 + c*x^4)/(x^10*sqrt[d - e*x]*sqrt[d + e*x]), x]

[Out] -(a*(d^2 - e^2*x^2))/(9*d^2*x^9*sqrt[d - e*x]*sqrt[d + e*x]) - ((9*b*d^2 + 8*a*e^2)*(d^2 - e^2*x^2))/(63*d^4*x^7*sqrt[d - e*x]*sqrt[d + e*x]) - ((21*c*d^4 + 18*b*d^2*e^2 + 16*a*e^4)*(d^2 - e^2*x^2))/(105*d^6*x^5*sqrt[d - e*x]*sqrt[d + e*x]) - (4*e^2*(21*c*d^4 + 18*b*d^2*e^2 + 16*a*e^4)*(d^2 - e^2*x^2))/(315*d^8*x^3*sqrt[d - e*x]*sqrt[d + e*x]) - (8*e^4*(21*c*d^4 + 18*b*d^2*e^2 + 16*a*e^4)*(d^2 - e^2*x^2))/(315*d^10*x*sqrt[d - e*x]*sqrt[d + e*x])

Rule 264

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]

Rule 271

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x^(m + 1)*(a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m + 1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]

Rule 453

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_)), x_Symbol] := Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)), x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]

Rule 520

Int[(u_)*((c_) + (d_)*(x_)^(n_)) + (e_)*(x_)^(n2_))^(q_)*((a1_) + (b1_)*(x_)^(non2_))^(p_)*((a2_) + (b2_)*(x_)^(non2_))^(p_), x_Symbol] := Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n))^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]

Rule 1265

Int[((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := With[{Qx = PolynomialQuotient[(a + b*x^2 +

```
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x],
Simp[(R*(f*x)^(m + 1)*(d + e*x^2)^(q + 1))/(d*f*(m + 1)), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[(d*f*(m + 1)*Qx)/x
- e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{a + bx^2 + cx^4}{x^{10}\sqrt{d-ex}\sqrt{d+ex}} dx &= \frac{\sqrt{d^2 - e^2x^2} \int \frac{a+bx^2+cx^4}{x^{10}\sqrt{d^2-e^2x^2}} dx}{\sqrt{d-ex}\sqrt{d+ex}} \\ &= -\frac{a(d^2 - e^2x^2)}{9d^2x^9\sqrt{d-ex}\sqrt{d+ex}} - \frac{\sqrt{d^2 - e^2x^2} \int \frac{-9bd^2-8ae^2-9cd^2x^2}{x^8\sqrt{d^2-e^2x^2}} dx}{9d^2\sqrt{d-ex}\sqrt{d+ex}} \\ &= -\frac{a(d^2 - e^2x^2)}{9d^2x^9\sqrt{d-ex}\sqrt{d+ex}} - \frac{(9bd^2 + 8ae^2)(d^2 - e^2x^2)}{63d^4x^7\sqrt{d-ex}\sqrt{d+ex}} + \frac{\left((63cd^4 - 6e^2(-9bd^2 - \dots)\right)}{63d^4} \\ &= -\frac{a(d^2 - e^2x^2)}{9d^2x^9\sqrt{d-ex}\sqrt{d+ex}} - \frac{(9bd^2 + 8ae^2)(d^2 - e^2x^2)}{63d^4x^7\sqrt{d-ex}\sqrt{d+ex}} - \frac{(21cd^4 + 18bd^2e^2 + 16ae^4)}{105d^6x^5\sqrt{d-ex}\sqrt{d+ex}} \\ &= -\frac{a(d^2 - e^2x^2)}{9d^2x^9\sqrt{d-ex}\sqrt{d+ex}} - \frac{(9bd^2 + 8ae^2)(d^2 - e^2x^2)}{63d^4x^7\sqrt{d-ex}\sqrt{d+ex}} - \frac{(21cd^4 + 18bd^2e^2 + 16ae^4)}{105d^6x^5\sqrt{d-ex}\sqrt{d+ex}} \\ &= -\frac{a(d^2 - e^2x^2)}{9d^2x^9\sqrt{d-ex}\sqrt{d+ex}} - \frac{(9bd^2 + 8ae^2)(d^2 - e^2x^2)}{63d^4x^7\sqrt{d-ex}\sqrt{d+ex}} - \frac{(21cd^4 + 18bd^2e^2 + 16ae^4)}{105d^6x^5\sqrt{d-ex}\sqrt{d+ex}} \end{aligned}$$

Mathematica [A] time = 0.18, size = 158, normalized size = 0.54

$$\frac{\sqrt{d-ex}\sqrt{d+ex} \left(a(35d^8 + 40d^6e^2x^2 + 48d^4e^4x^4 + 64d^2e^6x^6 + 128e^8x^8) + 9b(5d^8x^2 + 6d^6e^2x^4 + 8d^4e^4x^6 + 16d^2e^6x^8) + 21cd^4x^4(3d^4 + 4d^2e^2x^2 + 8e^4x^4) \right)}{315d^{10}x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^10*Sqrt[d - e*x]*Sqrt[d + e*x]), x]
```

```
[Out] -1/315*(Sqrt[d - e*x]*Sqrt[d + e*x]*(21*c*d^4*x^4*(3*d^4 + 4*d^2*e^2*x^2 +
8*e^4*x^4) + 9*b*(5*d^8*x^2 + 6*d^6*e^2*x^4 + 8*d^4*e^4*x^6 + 16*d^2*e^6*x^
8) + a*(35*d^8 + 40*d^6*e^2*x^2 + 48*d^4*e^4*x^4 + 64*d^2*e^6*x^6 + 128*e^8
*x^8)))/(d^10*x^9)
```

IntegrateAlgebraic [B] time = 0.28, size = 615, normalized size = 2.11

$$\frac{-2\sqrt{d+ex} \left(315cd^4e^5 + 315bd^2e^7 + 315ae^9 - (1680cd^4e^5(d+ex)) - (1260bd^2e^7(d+ex)) - (840ae^9(d+ex)) + (4788cd^4e^5(d+ex)^2) + (4284bd^2e^7(d+ex)^2) + (4788ae^9(d+ex)^2) - (9072cd^4e^5(d+ex)^3) - (7956bd^2e^7(d+ex)^3) - (5112ae^9(d+ex)^3) + (11298cd^4e^5(d+ex)^4) + (9234bd^2e^7(d+ex)^4) + (10658ae^9(d+ex)^4) - (9072cd^4e^5(d+ex)^5) - (7956bd^2e^7(d+ex)^5) - (5112ae^9(d+ex)^5) \right)}{315d^{10}x^9}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)/(x^10*Sqrt[d - e*x]*Sqrt[d + e*x]), x]
```

```
[Out] (-2*Sqrt[d + e*x]*(315*c*d^4*e^5 + 315*b*d^2*e^7 + 315*a*e^9 - (1680*c*d^4*
e^5*(d + e*x))/(d - e*x) - (1260*b*d^2*e^7*(d + e*x))/(d - e*x) - (840*a*e^
9*(d + e*x))/(d - e*x) + (4788*c*d^4*e^5*(d + e*x)^2)/(d - e*x)^2 + (4284*b
*d^2*e^7*(d + e*x)^2)/(d - e*x)^2 + (4788*a*e^9*(d + e*x)^2)/(d - e*x)^2 -
(9072*c*d^4*e^5*(d + e*x)^3)/(d - e*x)^3 - (7956*b*d^2*e^7*(d + e*x)^3)/(d
- e*x)^3 - (5112*a*e^9*(d + e*x)^3)/(d - e*x)^3 + (11298*c*d^4*e^5*(d + e*x
)^4)/(d - e*x)^4 + (9234*b*d^2*e^7*(d + e*x)^4)/(d - e*x)^4 + (10658*a*e^9*
(d + e*x)^4)/(d - e*x)^4 - (9072*c*d^4*e^5*(d + e*x)^5)/(d - e*x)^5 - (7956
```

$$\begin{aligned} & *b*d^2*e^7*(d + e*x)^5)/(d - e*x)^5 - (5112*a*e^9*(d + e*x)^5)/(d - e*x)^5 \\ & + (4788*c*d^4*e^5*(d + e*x)^6)/(d - e*x)^6 + (4284*b*d^2*e^7*(d + e*x)^6)/(d - e*x)^6 \\ & + (4788*a*e^9*(d + e*x)^6)/(d - e*x)^6 - (1680*c*d^4*e^5*(d + e*x)^7)/(d - e*x)^7 \\ & - (1260*b*d^2*e^7*(d + e*x)^7)/(d - e*x)^7 - (840*a*e^9*(d + e*x)^7)/(d - e*x)^7 \\ & + (315*c*d^4*e^5*(d + e*x)^8)/(d - e*x)^8 + (315*b*d^2*e^7*(d + e*x)^8)/(d - e*x)^8 \\ & + (315*a*e^9*(d + e*x)^8)/(d - e*x)^8)/(315*d^10*sqrt[d - e*x]*(-1 + (d + e*x)/(d - e*x))^9) \end{aligned}$$

fricas [A] time = 2.02, size = 144, normalized size = 0.49

$$\frac{(35ad^8 + 8(21cd^4e^4 + 18bd^2e^6 + 16ae^8)x^8 + 4(21cd^6e^2 + 18bd^4e^4 + 16ad^2e^6)x^6 + 3(21cd^8 + 18bd^6e^2 + 16ad^4e^4)x^4 + 5(9bd^8 + 8ad^6e^2)x^2)\sqrt{ex+d}\sqrt{-ex+d}}{315d^{10}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/x^10/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fricas")

[Out] -1/315*(35*a*d^8 + 8*(21*c*d^4*e^4 + 18*b*d^2*e^6 + 16*a*e^8)*x^8 + 4*(21*c*d^6*e^2 + 18*b*d^4*e^4 + 16*a*d^2*e^6)*x^6 + 3*(21*c*d^8 + 18*b*d^6*e^2 + 16*a*d^4*e^4)*x^4 + 5*(9*b*d^8 + 8*a*d^6*e^2)*x^2)*sqrt(e*x + d)*sqrt(-e*x + d)/(d^10*x^9)

giac [B] time = 7.35, size = 1931, normalized size = 6.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/x^10/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="giac")

[Out] -4/315*(315*c*d^4*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^17*e^6 + 315*b*d^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^17*e^8 - 6720*c*d^4*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^15*e^6 + 315*a*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^17*e^10 - 5040*b*d^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^15*e^8 + 76608*c*d^4*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^13*e^6 - 3360*a*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^15*e^10 + 68544*b*d^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^13*e^8 - 580608*c*d^4*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^11*e^6 + 76608*a*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^13*e^10 - 509184*b*d^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^11*e^8 + 2892288*c*d^4*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^9*e^6 - 327168*a*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^11*e^10 + 2363904*b*d^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^9*e^8 - 9289728*c*d^4*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^7*e^6 + 2728448*a*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^9*e^10 - 8146944*b*d^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^7*e^8 + 19611648*c*d^4*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^5*e^6 - 5234688*a*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^7*e^10

0 + 17547264*b*d^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^5*e^8 - 27525120*c*d^4*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^3*e^6 + 19611648*a*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^5*e^10 - 20643840*b*d^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^3*e^8 + 20643840*c*d^4*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^e^6 - 13762560*a*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^3*e^10 + 20643840*b*d^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^e^8 + 20643840*a*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^e^10)*e^(-1)/((((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^2 - 4)^9*d^10)

maple [A] time = 0.01, size = 154, normalized size = 0.53

$$\frac{\sqrt{ex+d}\sqrt{-ex+d}(128ae^8x^8+144bd^2e^6x^8+168c^4e^4x^8+64ad^2e^6x^6+72bd^4e^4x^6+84cd^6e^2x^6+48ad^4e^4x^4+54bd^6e^2x^4+63cd^8x^4+40ad^6e^2x^2+45bd^8x^2+35a^8d^8)}{315d^{10}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^2+a)/x^10/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)

[Out] -1/315*(e*x+d)^(1/2)*(-e*x+d)^(1/2)*(128*a*e^8*x^8+144*b*d^2*e^6*x^8+168*c*d^4*e^4*x^8+64*a*d^2*e^6*x^6+72*b*d^4*e^4*x^6+84*c*d^6*e^2*x^6+48*a*d^4*e^4*x^4+54*b*d^6*e^2*x^4+63*c*d^8*x^4+40*a*d^6*e^2*x^2+45*b*d^8*x^2+35*a*d^8)/x^9/d^10

maxima [A] time = 1.03, size = 304, normalized size = 1.04

$$\frac{8\sqrt{-e^2x^2+d^2}ce^4}{15d^8x} - \frac{16\sqrt{-e^2x^2+d^2}be^6}{35d^8x} - \frac{128\sqrt{-e^2x^2+d^2}ae^8}{315d^{10}x} - \frac{4\sqrt{-e^2x^2+d^2}ce^2}{15d^4x^3} - \frac{8\sqrt{-e^2x^2+d^2}be^4}{35d^6x^3} - \frac{64\sqrt{-e^2x^2+d^2}ae^6}{315d^8x^3} - \frac{\sqrt{-e^2x^2+d^2}c}{5d^2x^5} - \frac{6\sqrt{-e^2x^2+d^2}be^2}{35d^4x^5} - \frac{16\sqrt{-e^2x^2+d^2}ae^4}{105d^6x^5} - \frac{\sqrt{-e^2x^2+d^2}b}{7d^2x^7} - \frac{\sqrt{-e^2x^2+d^2}a}{63d^4x^7} - \frac{\sqrt{-e^2x^2+d^2}}{9d^2x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/x^10/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="maxima")

[Out] -8/15*sqrt(-e^2*x^2 + d^2)*c*e^4/(d^6*x) - 16/35*sqrt(-e^2*x^2 + d^2)*b*e^6/(d^8*x) - 128/315*sqrt(-e^2*x^2 + d^2)*a*e^8/(d^10*x) - 4/15*sqrt(-e^2*x^2 + d^2)*c*e^2/(d^4*x^3) - 8/35*sqrt(-e^2*x^2 + d^2)*b*e^4/(d^6*x^3) - 64/315*sqrt(-e^2*x^2 + d^2)*a*e^6/(d^8*x^3) - 1/5*sqrt(-e^2*x^2 + d^2)*c/(d^2*x^5) - 6/35*sqrt(-e^2*x^2 + d^2)*b*e^2/(d^4*x^5) - 16/105*sqrt(-e^2*x^2 + d^2)*a*e^4/(d^6*x^5) - 1/7*sqrt(-e^2*x^2 + d^2)*b/(d^2*x^7) - 8/63*sqrt(-e^2*x^2 + d^2)*a/(d^2*x^7) - 1/9*sqrt(-e^2*x^2 + d^2)*a/(d^2*x^9)

mupad [B] time = 1.87, size = 290, normalized size = 0.99

$$\frac{\sqrt{d-ex}\left(\frac{a}{9d} + \frac{x^2(45bd^4+40ad^2e^2)}{315d^{10}} + \frac{x^4(84cd^2e^2+72bd^4e^4+64a^2e^6)}{315d^{10}} + \frac{x^6(84cd^4e^2+72bd^6e^4+64a^2e^6)}{315d^{10}} + \frac{x^8(63cd^6e^2+54bd^8e^4+48a^2e^6)}{315d^{10}} + \frac{x^{10}(168cd^8e^2+144bd^{10}e^4+128a^2e^6)}{315d^{10}} + \frac{x^{12}(45bd^8e^2+40ad^6e^4)}{315d^{10}} + \frac{x^{14}(63cd^8e^4+54bd^{10}e^6+48a^2e^8)}{315d^{10}} + \frac{x^{16}(168cd^8e^6+144bd^{10}e^8+128ad^8e^2)}{315d^{10}} + \frac{ax}{9d^2}\right)}{x^9\sqrt{d+ex}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^2 + c*x^4)/(x^10*(d + e*x)^(1/2)*(d - e*x)^(1/2)),x)

[Out] -((d - e*x)^(1/2)*(a/(9*d) + (x^2*(45*b*d^9 + 40*a*d^7*e^2))/(315*d^10) + (x^6*(64*a*d^3*e^6 + 72*b*d^5*e^4 + 84*c*d^7*e^2))/(315*d^10) + (x^7*(64*a*d^2*e^7 + 72*b*d^4*e^5 + 84*c*d^6*e^3))/(315*d^10) + (x^4*(63*c*d^9 + 48*a*d^5*e^4 + 54*b*d^7*e^2))/(315*d^10) + (x^9*(128*a*e^9 + 144*b*d^2*e^7 + 168*c*d^4*e^5))/(315*d^10) + (x^3*(40*a*d^6*e^3 + 45*b*d^8*e))/(315*d^10) + (x^5*(48*a*d^4*e^5 + 54*b*d^6*e^3 + 63*c*d^8*e))/(315*d^10) + (x^8*(144*b*d^3*e^6 + 168*c*d^5*e^4 + 128*a*d^7*e^2))/(315*d^10) + (a*e*x)/(9*d^2)))/(x^9*(d + e*x)^(1/2))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**4+b*x**2+a)/x**10/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)

[Out] Timed out

Chapter 4

Appendix

Local contents

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4.1 Download section

The following zip files contain the raw integrals used in this test.

This is a subset of Rubi test suite thanks to Albert Rich, which includes only the algebraic integrals with elementray optimal antiderivatives. It also includes a subset of a test file provided thanks to Sam Blake.

Mathematica format Mathematica_syntax_CAS_integration_elementary_version.zip

Maple and Mupad format Maple_syntax_CAS_integration_elementary_version.zip

Sympy format SYMPY_syntax_CAS_integration_elementary_version.zip

Sage math format SAGE_syntax_CAS_integration_elementary_version.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*      GradeAntiderivative[result,optimal] returns*)
(* "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(* "C" if result involves higher level functions than necessary*)
(* "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(* "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
        "B"],
      "C"],
    If[FreeQ[result,Integrate] && FreeQ[result,Int],
      "C",
      "F"]]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
```

```

(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)

```

```

ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
    If[ListQ[expn],
      Max[Map[ExpnType, expn]],
      If[Head[expn]===Power,
        If[IntegerQ[expn[[2]]],
          ExpnType[expn[[1]]],
          If[Head[expn[[2]]]===Rational,
            If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
              1,
              Max[ExpnType[expn[[1]], 2]],
            Max[ExpnType[expn[[1]], ExpnType[expn[[2]], 3]]],
          If[Head[expn]===Plus || Head[expn]===Times,
            Max[ExpnType[First[expn]], ExpnType[Rest[expn]]],
            If[ElementaryFunctionQ[Head[expn]],
              Max[3, ExpnType[expn[[1]]]],
              If[SpecialFunctionQ[Head[expn]],
                Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 4]],
                If[HypergeometricFunctionQ[Head[expn]],
                  Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 5]],
                  If[AppellFunctionQ[Head[expn]],
                    Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 6]],
                    If[Head[expn]===RootSum,
                      Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 7]],
                      If[Head[expn]===Integrate || Head[expn]===Int,
                        Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 8]],
                        9]]]]]]]]]]

```

```

ElementaryFunctionQ[func_] :=
  MemberQ[{
    Exp, Log,
    Sin, Cos, Tan, Cot, Sec, Csc,
    ArcSin, ArcCos, ArcTan, ArcCot, ArcSec, ArcCsc,
    Sinh, Cosh, Tanh, Coth, Sech, Csch,
    ArcSinh, ArcCosh, ArcTanh, ArcCoth, ArcSech, ArcCsch
  }, func]

```

```

SpecialFunctionQ[func_] :=
  MemberQ[{
    Erf, Erfc, Erfi,
    FresnelS, FresnelC,
    ExpIntegralE, ExpIntegralEi, LogIntegral,
    SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
    Gamma, LogGamma, PolyGamma,
    Zeta, PolyLog, ProductLog,
    EllipticF, EllipticE, EllipticPi
  }, func]

```

```

HypergeometricFunctionQ[func_] :=

```

```
MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]

AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017 Use Maple leaf count instead since buildin
#Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017 corrected the check for complex result
#Nasser 10/27/2017 check for leafsize and do not call ExpnType()
#                    if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
#                    see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

    leaf_count_result:=leafcount(result);
    #do NOT call ExpnType() if leaf size is too large. Recursion problem
    if leaf_count_result > 500000 then
        return "B";
    fi;

    leaf_count_optimal:=leafcount(optimal);

    ExpnType_result:=ExpnType(result);
    ExpnType_optimal:=ExpnType(optimal);

    if debug then
        print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
ExpnType_optimal);
    fi;

# If result and optimal are mathematical expressions,
# GradeAntiderivative[result,optimal] returns
# "F" if the result fails to integrate an expression that
#     is integrable
# "C" if result involves higher level functions than necessary
# "B" if result is more than twice the size of the optimal
#     antiderivative
# "A" if result can be considered optimal

#This check below actually is not needed, since I only
#call this grading only for passed integrals. i.e. I check
#for "F" before calling this. But no harm of keeping it here.
#just in case.

if not type(result,freeof('int')) then
    return "F";
end if;

if ExpnType_result<=ExpnType_optimal then
    if debug then
        print("ExpnType_result<=ExpnType_optimal");
    fi;
end if;
```

```

if is_contains_complex(result) then
  if is_contains_complex(optimal) then
    if debug then
      print("both result and optimal complex");
    fi;
    #both result and optimal complex
    if leaf_count_result<=2*leaf_count_optimal then
      return "A";
    else
      return "B";
    end if
  else #result contains complex but optimal is not
    if debug then
      print("result contains complex but optimal is not");
    fi;
    return "C";
  end if
else # result do not contain complex
  # this assumes optimal do not as well
  if debug then
    print("result do not contain complex, this assumes optimal do
not as well");
  fi;
  if leaf_count_result<=2*leaf_count_optimal then
    if debug then
      print("leaf_count_result<=2*leaf_count_optimal");
    fi;
    return "A";
  else
    if debug then
      print("leaf_count_result>2*leaf_count_optimal");
    fi;
    return "B";
  end if
end if
else #ExpnType(result) > ExpnType(optimal)
  if debug then
    print("ExpnType(result) > ExpnType(optimal)");
  fi;
  return "C";
end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function

```

```

# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'^^') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
      max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'+' or type(expn,'*') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
    9
  end if
end proc:

ElementaryFunctionQ := proc(func)
  member(func,[
    exp,log,ln,
    sin,cos,tan,cot,sec,csc,
    arcsin,arccos,arctan,arccot,arcsec,arccsc,
    sinh,cosh,tanh,coth,sech,csch,
    arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
    erf,erfc,erfi,
    FresnelS,FresnelC,
    Ei,Ei,Li,Si,Ci,Shi,Chi,
    GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
    EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

```

```

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product. rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u)
  else
    apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
  MmaTranslator[Mma][LeafCount](u);
end proc:

```

4.2.3 Sympy grading function

```

#Dec 24, 2019. Nasser M. Abbasi:
#           Port of original Maple grading function by
#           Albert Rich to use with Sympy/Python
#Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
#           added 'exp_polar'
from sympy import *

def leaf_count(expr):
  #sympy do not have leaf count function. This is approximation
  return round(1.7*count_ops(expr))

def is_sqrt(expr):
  if isinstance(expr,Pow):
    if expr.args[1] == Rational(1,2):
      return True
    else:
      return False
  else:
    return False

def is_elementary_function(func):
  return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
                 asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
                 asinh,acosh,atanh,acoth,asech,acsch
                 ]

def is_special_function(func):
  return func in [ erf,erfc,erfi,
                 fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
                 gamma,loggamma,digamma,zeta,polylog,LambertW,
                 elliptic_f,elliptic_e,elliptic_pi,exp_polar
                 ]

def is_hypergeometric_function(func):
  return func in [hyper]

def is_appell_function(func):
  return func in [appellf1]

def is_atom(expn):

```

```

try:
    if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
    else:
        return False

except AttributeError as error:
    return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow): #type(expn,'^^')
        if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
            return expnType(expn.args[0]) #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'+`') or
    type(expn,'*`)
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
    expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
    ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function

```



```

def grade_antiderivative(result,optimal):

    leaf_count_result = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"

```

4.2.4 SageMath grading function

```

#Dec 24, 2019. Nasser: Ported original Maple grading function by
#           Albert Rich to use with Sagemath. This is used to
#           grade Fracas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#           'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow: # isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False

```

```

else:
    return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
        'sin','cos','tan','cot','sec','csc',
        'arcsin','arccos','arctan','arccot','arcsec','arccsc',
        'sinh','cosh','tanh','coth','sech','csch',
        'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
    ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")

    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
sinh_integral'
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e',
        'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func , " is special_function")
        else:
            print ("func ", func , " is NOT special_function")

    return m

def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M',
        hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in
    sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
    sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):

```

```

        return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
:
            return expn in expn.parent().base_ring() or expn in expn.parent().
gens()
        return False

    except AttributeError as error:
        return False

def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list: #isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
    elif expn.operator() == operator.pow: #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer: #isinstance(expn.args[1],Integer
)
            return expnType(expn.operands()[0]) #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational: #isinstance(expn.args[1],
Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0])) #max(2,expnType(
expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn)))
    elif is_elementary_function(expn.operator()): #is_elementary_function(expn
.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
        return max(4,m1) #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
        return max(5,m1) #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))

```

```

    return max(6,m1)    #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
    #is checked before calling the grading function that is passed.
    #but kept it here.
    m1 = max(map(expnType, expn.operands()))    #max(map(expnType, list(
expn.args)))
    return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"

```